Measurement of Sound-Speed Gradients in Deep-Ocean Sediments Using l_1 Deconvolution Techniques

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(Invited Paper)

Abstract—**A method is described for measuring the sound speed and the sound-speed gradient of surficial sea floor sediment from bottom-reflected signals recorded in marine seismic experiments. The technique makes use of the ocean-bottom impulse responses that are deconvolved from the data by means of a novel curve-fitting algorithm based on the l_1 norm (least absolute value) criterion. The algorithm constructs the impulse response by extracting spikes one at a time in a manner that causes the l_1 error to decrease by the maximum amount possible as each spike is chosen. The l_1 curve-fitting approach is a completely general strategy for deconvolution, and our algorithm can be used with data obtained from any type of marine seismic source. Since our experiments have been carried out with small explosive charges, we have also developed a method for estimating the bubble-pulse wavelet directly from the recorded bottom-reflected signal. In this paper, the l_1 algorithm is used to deconvolve impulse responses for data obtained in an experiment in the Alaskan Abyssal Plain. The sediment-sound-speed gradient determined from these results is typical of other values reported for turbidite abyssal plains where the surficial sediments are composed of unconsolidated silty deposits.**

I. INTRODUCTION

A REALISTIC MODEL of the acoustic interaction with ocean bottom is a necessary requirement for making predictions of the propagation loss of low-frequency sound in the deep ocean. The traditional approach in ocean acoustics has been to describe the bottom interaction by a single parameter—the bottom loss. This model is based on Rayleigh reflection at the sea floor, and is consequently not suitable for modeling the behavior at low frequencies (less than about 100 Hz) where the incident acoustic energy penetrates the seafloor and interacts with the subbottom structure [1]. In order to account for the interactions within the sediment column, it is necessary to develop a geophysical model of the ocean bottom. Relatively simple models have proven to be adequate for predicting the propagation loss in deep-water environments [2]. These models require measures of the sound speed, density, and attenuation profiles within 100-200 m of the seafloor. We shall consider here the measurement of sediment-sound-speed profiles.

Standard marine seismic techniques such as reflection and refraction profiling are not suitable for determining the sound speed in thin surficial sediment layers of thickness much less than 1/10-1/15 the water depth [3], [4]. In this paper we describe a method for estimating the subbottom sound speed and the sound-speed gradient near the seafloor from bottom-reflected signals measured in experiments using small explosive charges. The bottom reflections recorded at different ranges are analyzed to obtain ocean-bottom impulse responses which are then interpreted using simple geophysical models of the sound-speed profiles in the bottom. This technique provides a different approach from the ray-parameter method described by Bryan [4] for obtaining the sound speed in very thin subbottom layers from sonobuoy data.

Our method makes use of a curve-fitting technique based on the l_1 norm (least absolute value) criterion to deconvolve the sediment impulse response. This approach is attractive because the l_1 norm provides a more robust criterion than conventional (least squares) techniques for extraction of a sparse spike train, or impulse response [5]. The algorithm proceeds by extracting spikes one at a time in a manner that causes the l_1 norm to decrease by the maximum amount possible as each spike is located. In practice, relatively few spikes are required to achieve an adequate correspondence with the measured signal. An estimate of the source waveform must be provided to or generated by the algorithm, although the l_1 deconvolution strategy is completely general and can be used to analyze data obtained with any kind of seismic source. For our own experiments with small underwater explosives, we have developed a procedure for determining the bubble-pulse wavelet of the shot directly from the bottom-reflected signal. Consequently, the method presented here enhances our earlier approach which required a known wavelet synthesized from previous measurements to initiate the program [6].

In the remainder of the paper we shall describe the l_1 algorithm and the bubble-pulse-wavelet estimator. We will then illustrate the use of the algorithm in obtaining estimates of the sediment-sound-speed gradient by applying it to a data set recorded in an experiment carried out in the Alaskan Abyssal Plain. A geophysical model consisting of a single layer of constant sound-speed gradient is used to interpret the impulse responses deconvolved from the data. Finally, we present the sound-speed profile for the near-surface sediments determined from the analysis.

II. DECONVOLUTION WITH THE l_1 NORM

A. General

Our method for measuring sediment-sound-speed profiles proceeds in three stages: 1) we first estimate a bubble-pulse wavelet for each trace directly from the measured bottom-reflected signal; 2) each trace is then deconvolved with its own wavelet to obtain the impulse response for the particular grazing angle; and 3) the set of impulse responses for different ranges is then interpreted using a geophysical model to

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determine the sediment-sound-speed profile. The first two parts are described in this section, and the geophysical model is presented in the following section where the results of the technique are shown.

B. l_1 Deconvolution Algorithm

The bottom-reflected signal can be modeled as a convolution of a sparsely populated spike train (or ocean-bottom impulse response) s, of length m and a source wavelet w of length k, with an additive noise term r. Thus the measured signal t, of length $n = m + k - 1$, is defined by

$$t = w * s + r \tag{1}$$

where * denotes convolution. The analysis of bottom reflections from underwater explosive charges is complicated by the bubble pulses in the source waveform, which correspond to the successive expansions and contractions of the gas bubble following the initial shock pulse. The resulting train of pulses may mask, distort, or mimic the signals from the subbottom structure. Consequently, a general problem in processing the data is to deconvolve the impulse response from the measured signal and hence to deduce a layered structure for the ocean bottom. We shall assume that the bottom is composed of random layers such as in turbidite sediments. For this model the deconvolved impulse response will consist of a sparsely populated series of reflected arrivals from subbottom layers, and signals refracted by sound-speed gradients in the sediment [7]. The effect of water column multiples was not apparent in our experiments, which were designed to probe the surficial sediments with weak explosive charges.

Assuming first that the source wavelet is known, (1) can be written in matrix notation as

$$t - Ws = r \tag{2}$$

where the elements of the $n \times m$ matrix W are defined by $W_{ij} = w_{i-j+1}$, for $1 \leqslant i - j + 1 \leqslant k$, and $W_{ij} = 0$ otherwise; hence

$$W = \underset{\longleftarrow\; m \;\longrightarrow}{\begin{bmatrix} w_1 & & & & & & \\ w_2 & w_1 & & & & & \\ w_3 & w_2 & w_1 & & & & \\ \vdots & w_3 & w_2 & & & & \\ w_k & \vdots & w_3 & & & & \\ & w_k & \vdots & \ddots & w_1 & & \\ & & w_k & & w_2 & w_1 & \\ & & & & w_3 & w_2 & w_1 \\ & & & & \vdots & w_3 & w_2 \\ & & & & w_k & \vdots & w_3 \\ & & & & & w_k & \vdots \\ & & & & & & w_k \end{bmatrix}} \Bigg\updownarrow n. \tag{3}$$

Typical values for n exceed a few hundred while k is often less than 50, so the matrix W is usually large and slightly overdetermined. Although it is possible to solve the system of equations (2) completely, i.e., by considering every nonzero element of W in calculating the spike amplitudes [8], this approach is costly and inefficient because of the large computational effort required. Since a sparse solution s is sought in practice, it is more appropriate to develop algorithms which permit the system of equations to be solved efficiently and effectively by partially minimizing $\|t - Ws\|$, in order to obtain a spike train with relatively few nonzero elements.

The algorithm proceeds initially by setting $s = s^{(0)} = 0$ and $\|r^{(0)}\| = \|t - Ws^{(0)}\| = \|t\|$. In the first iteration all m possible spike positions are considered in determining the spike train $s = s^{(1)}$, with one nonzero spike, which minimizes $\|r^{(1)}\| < \|r^{(0)}\|$. In the second iteration the remaining $m - 1$ spike positions are considered in determining the spike train $s = s^{(2)}$, with two nonzero spikes, which minimizes $\|t - Ws\|$ subject to the constraint that the spike position chosen in $s^{(1)}$ (but not necessarily its amplitude) be preserved in $s^{(2)}$. Setting $\|r^{(2)}\| = \|t - Ws^{(2)}\|$, it follows that $\|r^{(2)}\| < \|r^{(1)}\|$. Continuing in this manner, $s^{(j+1)}$ is determined during the $(j + 1)$th iteration by minimizing $\|t - Ws\|$, where s is constrained to have $j + 1$ nonzero spikes, of which j have the same positions as in $s^{(j)}$ and the extra spike is positioned optimally from the remaining $m - j$ choices. Iterations continue until an upper limit for the number of nonzero spikes is reached, or until the quantity $\|r^{(j)}\|$ becomes sufficiently small compared to $\|r^{(0)}\|$, thereby yielding a spike train whose convolution with the wavelet closely approximates the trace.

This algorithm is *locally optimal*, in the sense that both the new spike position and all the amplitudes of $s^{(j+1)}$ are chosen to yield the maximum possible decrease in moving from $\|r^{(j)}\|$ to $\|r^{(j+1)}\|$.

We have carried out an extensive study using both synthetic data and measured bottom-reflected signals to determine the performance of the algorithm, and to compare it to the conventional l_2 deconvolution strategy. Our computational experience has indicated that the method based on one-at-a-time spike extraction with the l_1-norm performs better than the l_2 techniques in almost all cases [9]. We have also developed a more efficient l_1 algorithm in which the optimal spike amplitudes are found using a simplex pivot determined by the largest wavelet element. (The simplex method of linear programming proceeds from one iteration to the next by first choosing a pivotal element of the matrix and then transforming the matrix through the use of the pivot.) By using this easily determined pivot to calculate trial amplitudes for each possible spike location, the ultimate choice of new spike positions involves much less effort than is required by the locally optimal procedure. In practice the new algorithm is about ten times faster and requires much less storage space than the original, but the advantages over the l_2 algorithm are maintained.

C. Bubble-Pulse-Wavelet Estimation

It is possible to use a known wavelet in the deconvolution algorithm; however, we have developed a method to estimate a bubble-pulse wavelet directly from the bottom-reflected signal. This approach removes the need to record the shot waveform during the experiment or to simulate the wavelet from other information. There are several standard procedures for wavelet estimation, but these methods are based on the assumption that the waveform is of a minimum phase [10]. This assumption may be adequate for the pulses produced by the airgun arrays which are in widespread use in marine seismic exploration. However, we have found that our bubble-pulse wavelets are *not* of a minimum phase, and moreover, that minimum-phase wavelets derived from the measured signals do not provide satisfactory estimates for use in deconvolution [11]. Consequently, we have developed a method for estimating a three-peak bubble-pulse wavelet that does not depend on the minimum-phase assumption. The waveform consists of a shock pulse and only two bubble pulses; additional bubble pulses do not provide an improvement. This approach is empirical and differs from the work of Ziolkowski and co-workers who have developed a model for the signature of airguns [12] and airgun arrays [13] from the physics of the source.

Our algorithm proceeds in three stages, of which the first two use information derived directly from the individual bottom-reflected signals. These stages are: 1) measurement of the first and second bubble-pulse periods; 2) estimation of the relative amplitudes of the shock pulse and the two bubble pulses; and 3) filling in the negative phase portions by modeling these regions as exponentials or sums of exponentials. The details of this scheme have been published [11], and we shall include only a brief summary here.

The bubble-pulse periods are determined from the autocorrelation of the bottom-reflected signal. In practice it was found useful to confine the search for the maxima in the autocorrelation within time windows around the bubble-pulse periods expected for the shots used in the experiment. The relative amplitudes of the peaks were determined by identifying the largest peaks in the signal, and by using their positions to define amplitudes at the two appropriate bubble-pulse periods. From these values a least-squares estimate of the relative peak amplitudes was obtained. We have examined bottom-reflected signals from several sets of data recorded at 1500 samples/s, and have found that this procedure gave peak amplitudes relative to the shock pulse within the range of 0.7-1.15 for the first bubble pulse and of 0.15-0.40 for the second bubble pulse.

The negative phases between the pulses may be divided into two sections, an exponential decay from the peak to a negative offset value, and an exponential rise from the offset to the next peak. The first portion between the shock pulse and the first bubble pulse was modeled by fitting a sum of exponentials via Prony's method for each section, while the second negative phase and the portion following the second bubble pulse were each fitted by a single exponential term.

III. APPLICATION TO EXPERIMENTAL DATA

A. Experiment

The method described above has been used to analyze data recorded in several of our experiments in the northeast Pacific Ocean. In this section we shall demonstrate the technique by using a simple geophysical model to interpret the deconvolved impulse responses for one of the data sets. The experiment was carried out at a site in the Alaskan Abyssal Plain (51°N, 136°W) where the seafloor is uniformly flat with an average depth of 3625 m over the track of the short run. The ocean bottom in this region of the plain consists of turbidite layers, and the surficial sediments are composed of unconsolidated silty deposits [14]. The average shot depth was 188 m and the receiving hydrophone was suspended at 416 m. Bottom reflections for 0.82-kg charges deployed at intervals of 1 km were recorded out to a range of about 23 km. The sampling rate was 1500 samples/s, and the bandpass of the recording system was 5-630 Hz. An example of the data showing the first bottom-bounce signal measured for a grazing angle of 36° is presented in Fig. 1. The bubble-pulse wavelet which was derived from this signal using the method outlined in the previous section is shown in the inset. (The time scale in the inset is slightly smaller than for the measured trace.)

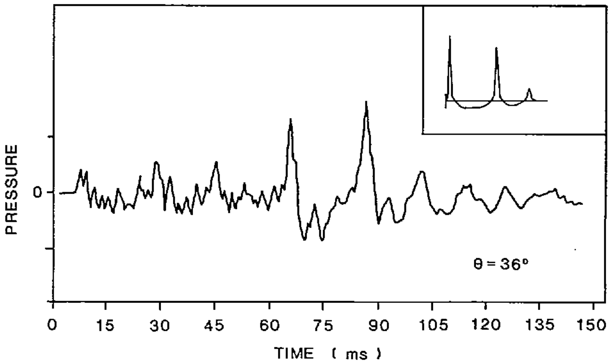

Fig. 1. First bottom bounce reflection signals measured at a grazing angle of 36°. Bubble pulses are clearly displayed in the sediment interacted arrival which lags the seafloor reflection by about 50-60 m•s. An example of the bubble-pulse wavelet is shown in the inset. The first bubble-pulse period is 23.0 m•s.

B. Geophysical Model

The model used in this analysis is similar to those proposed by Kaufman [15] and Dicus [16]. The unconsolidated sediments near the seafloor are modeled as a single layer of constant sound-speed gradient overlying a constant sound-speed half-space which represents deeper consolidated sediments. The impulse response of this model consists of a seafloor reflection, and a secondary pulse that will be reflected from the consolidated sediment layer for large grazing angles, or refracted entirely within the constant gradient layer for lower grazing angles [7].

In the top layer, the sound-speed profile of the unconsolidated sediments is given by

$$c(z) = c_0 \left(1 - \frac{2gz}{c_0}\right)^{-1/2}$$

where c_0 is the sediment sound speed at the seafloor, and g is the gradient. This profile has a singularity at $z_c = c_0/2g$ and so it is not suitable for very thick layers. Using values of c_0 and g reported by Hamilton [17] for turbidite abyssal plains, the critical depth is about 1 km. Therefore, the profile is satisfactory for our purposes since we are modeling the unconsolidated sediments within 100-200 m of the seafloor. At these depths the sound-speed variation is approximately linear, as can be seen by expanding $c(z)$ binomially for $2gz/c_0 \ll 1$.

Using this model, the travel time difference Δt between the arrival that is totally refracted within the constant gradient layer and the seafloor reflected arrival is given by

$$\Delta t = \frac{2}{3g}\left[1 - \left(\frac{c_0}{c_w}\right)^2 \cos^2\theta\right]^{3/2}$$

where θ is the grazing angle at the seafloor, and c_w is the sound speed at the bottom of the water column. Thus the ratio of the sound speeds at the ocean bottom and the sediment-sound-speed gradient can be determined from a linear least-squares fit of $(3/2\Delta t)^{2/3}$ v·s $\cos^2\theta$.

C. Interpretation of the Data

The broad-band bottom-reflected signals for grazing angles between 11-50° were deconvolved and the impulse responses were low-pass filtered at 200 Hz. These filtered impulse responses are plotted in Fig. 2 where the first peak at each grazing angle represents the seafloor reflection. The prominent secondary pulse which lags the seafloor reflected arrival by about 100 m·s at 50° is clearly observed at lower grazing angles, and at about 11° it becomes the dominant arrival. This pulse was interpreted as a sediment refracted arrival and was used to estimate the sound-speed gradient. For grazing angles less than 22.5°, the data were well described by this model, and the least-squares fit of $(3/2\Delta t)^{2/3}$ v·s $\cos^2\theta$ provided the parameter values

$$g = 1.12 s^{-1} + 0.05$$

and

$$c_0/c_w = 1.00 \pm 0.04$$

for the gradient and the sound-speed ratio at the seafloor, respectively. These values are within the range reported by Hamilton [17] for turbidite abyssal plains.

At the larger grazing angles the refraction model does not provide an adequate fit to the data. It is possible that for these angles the acoustic energy penetrates deep enough to interact with the unconsolidated sediments, or with a strong near-surface reflector, and thus is returned to the receiver by reflection. This interpretation was investigated using the thin layer model proposed by Bryan [4]. Assuming in this case that the upper layer is a constant sound-speed layer, the arrival time difference between the pulses is given by

$$\Delta t^2 = \left(\frac{2h}{c_s}\right)^2 - \left(\frac{2h}{c_w}\right)^2 \cos^2\theta$$

where h and c_s are the layer thickness and average sound speed, respectively. The parameter values obtained from a linear least-squares fit of Δt^2 v·s $\cos^2\theta$ were

$h = 0.11$ km ± 0.03, for the layer thickness

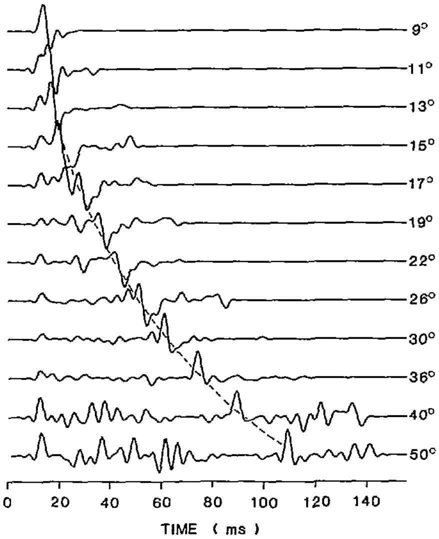

Fig. 2. Deconvolved ocean-bottom impulse responses, low-pass filtered at 200 Hz. The first peak at each grazing angle corresponds to the seafloor reflection. The secondary arrival used in the analysis is indicated by the broken curve through the plots.

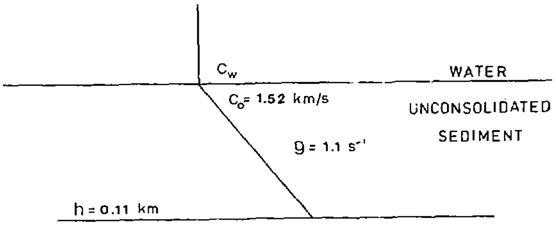

Fig. 3. The sound-speed profile for the sediments near the seafloor as determined from the deconvolved impulse responses.

and

$c_s = 1.61$ km/s ± 0.05, for the average sound speed.

The measured value of the bottom water sound speed c_w = 1.516 km/s, was used in the calculations. This average value for the sound speed within the first 100 m of the sediment column is consistent with the surficial sound speeds predicted using the gradient of 1.1 s^{-1} determined from the refraction model. Consequently, we have used the results of both models to obtain the sediment sound-speed profile shown in Fig. 3.

IV. SUMMARY

We have presented a method for measuring the sound-speed gradient in surficial sediment layers using ocean-bottom impulse responses deconvolved from bottom-reflected signals. The deconvolution algorithm is based on a curve-fitting technique using the l_1 norm criterion to construct a sparsely populated spike train. The l_1 strategy is completely general and can be used for data obtained with any type of marine seismic source. We have applied the method to our measurements made with small explosive charges, and have developed an empirical technique for estimating the bubble-pulse wavelets of the shots directly from the bottom-reflected signals. The method has been used to analyze data recorded in several experiments in the northeast Pacific Ocean. We have demonstrated the use of the technique by presenting the results of the analysis of data obtained in an experiment carried out in the Alaskan Abyssal Plain. A model of the surficial sediments consisting of a single layer of constant sound-speed gradient was used to interpret the data. The estimated value of 1.1 s^{-1} for the sediment-sound-speed gradient near the seafloor is typical of the unconsolidated silty turbidites which have been reported in this region of the plain.

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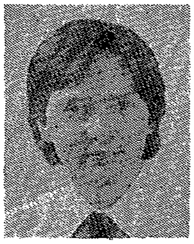

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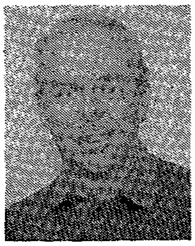

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He has been an editor of the SIAM Journal on Numerical Analysis and has served on advisory committees for the National Research Council and the Defense Research Board.

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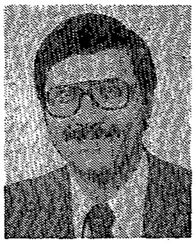

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During several years of post-doctoral research into mechanisms of transmembrane glucose transport at the Jewish General Hospital in Montreal, he became increasingly interested in computer applications, and in 1982 received a certificate in computer programming from McGill University, Montreal, Que., Canada. Shortly afterward, he joined the staff of Barrodale Computing Services in Victoria, where he is engaged in signal processing software development. His special interest is in geophysical applications of the l_1 norm.