

Performance Evaluation for Adaptive Modulation Wireless System over Rayleigh  
Fading Channel Using Finite State Markov Chain (FSMC) Technique

by

Khamis Elnawaa  
B.Sc., University of Benghazi, 2009

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### ABSTRACT

In this project, adaptive modulation for end to end wireless link system using MQAM modulation scheme over Rayleigh fading channel is addressed. Selecting the modulation schemes, which maintain appropriate frame error rate and maximum throughput, according to channel states is discussed. First order finite state Markov chain (FSMC) model is utilized to model the channel states in terms of SNR. In order to get the best choice modulation scheme selection for the adaptive modulation system, a new method for channel partitioning the is proposed. In this method, we select the SNR levels of the channel based on a desired (target) FER of the system with respect to the average time of channel state duration. We present another method for channel petitioning, which is called equilibrium steady state method. for both methods, performance measures of the FSMC model for Rayleigh channel are derived, plotted and analyzed. The performance analysis of the system and numerical results of both methods are compared and discussed.

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*In The Name of Allah, The Beneficent, The Merciful. “Read In the Name of your Lord Who created.” “Created man, out of a clot.” “Read, and your Lord is the Most Generous.” “Who taught by the Pen.” “Taught man that which he knew not.”*

Quran Book, Surah ‘Alaq, Chapter 96

## DEDICATION

To my mother, **Khouloud Gatan** and my father, **Hamid Elnawaa** and my wife, **khadeejah Hamzah**, they were the secret of my diligence. It would not be possible without their support.

To my supervisor, **Dr. Fayez Gebali**, he is always supportive and helpful. I appreciate his invaluable time he spent with me for his supervision.

# Chapter 1

## Introduction

The use of wireless communication networks in multimedia makes the need for high data rates grow rapidly. Providing a high data rate in networks is challenging due to several issues. For instance, Bottleneck phenomenon that occurs in wireless links, is one of the challenged problems. The cause of the Bottleneck phenomenon is not only because of wireless resources such as bandwidth availability and power expensive, but also because of Doppler frequency, multipath fading, and wireless propagation that degrades the overall system performance. One of the ways to enhance performance of the wireless link is called Adaptive transmission. This technique targets the enhancement of the spectral efficiency of the network. In this technique, a target error rate over wireless channel has to be chosen to ensure high performance of the network. Adaptive transmission has been widely used to match transmission parameters to time-varying channel conditions [1] [6] [11] [12] [18]. Due to the high performance of the adaptive transmission technique, it has been connected to the physical layer of several standards, e.g., IEEE 802.11a, IEEE 802.15.3, IEEE 802.16, and 3GPP [16] [8] [9] [15].

In order to model wireless fading channels and to evaluate the performance of certain signal transmission techniques over wireless channels, finite state Markov chain (FSMC) is widely used to describe the wireless fading channels behavior and to evaluate the performance of a particular system over different types of wireless fading channels. In radio mobile communication, FSMC model was used to evaluate the performance of the fading channels [24], where each set of Signal-to-Noise Ratio (SNRs) was represented by a state in the FSMC. In [5], the order of fading channel memory is explained in auto regressive modeling (AR) of time varying flat fading channel, and that models help to describe the time variation of the fading channel gain accu-

rately. In [20] the authority and the accurateness of the FSMC as a model for the Rayleigh fading channels is presented through the state balance equations. In [2], received SNRs that have Lognormal, K and Chi-square distribution is represented using FSMC model. The performance measurement and parameters such as steady state probability, level crossing rate, and state transition probability are derived. In [19], the first order FSMC model, which can be acquired for fading channels, is described. The paper also discusses its applications.

In [14], the author presents a technique to calculate the parameters of FSMC that matches a Nakagami-m distribution - slow fading channel. The author in [10] presents a finite state Markov model of Nakagami fading to evaluate the performance of adaptive coding transmission technique. The throughput analytical evaluation of the adaptive coding is obtained. In [23], the author presents an analytical method utilizing FSMC to implement an error model for performance estimation of adaptive modulation system (AMS) combined with automatic repeat request (ARQ) schemes in slow fading channels.

## 1.1 Contributions

In adaptive modulation systems, selecting the modulation schemes, which maintain appropriate frame error rate and maximum throughput according to the channel state, is interesting problem. In order to select the modulation schemes of adaptive modulation technique, and to investigate the complicity of mapping the received SNR into FSMC channel's thresholds to make the right decision of modulation switching as in [23], a new method for FSMC channel partitioning is proposed. This work includes the following contributions:

1- The received SNR threshold of each mode of the adaptive modulation and the average time duration  $\tau_i$  are taken into account for FSMC channel partitioning calculation

2- The relationship between the average time duration and the SNR thresholds is theoretically plotted and discussed.

## 1.2 Project Organization

This section gives a brief remainder of this project . For each of the chapters below, there is a short summary of what the project focus is.

Chapter 2 discusses statistical fading channel models and adaptive modulation technique.

Chapter 3, present Finite State Markov Chain FSMC model, and performance measures of the model such as state time duration, state transition probability, steady state probability, and crossing rate are analyzed. These parameters are used to evaluate End-to-End system with adaptive modulation technique over Rayleigh fading channel.

Chapter 4 presents the portioning methods of the channels and describes the relationship between the average time channel state duration and the SNR thresholds.

Chapter 5 presents the system model, and the assumptions that related to the channel environment are pointed out. Also ,FSMC model parameters are plotted, and the performance evaluation results such as the average throughput and the average FER of the system are discussed and plotted.

Chapter 6, presents the project conclusion.

## Chapter 2

# Overview of Channel Models and Adaptive Modulation Technique

## 2.1 Channel Models

### 2.1.1 Statistical Fading Channel Models

The electromagnetic wave propagation affects the transmitted signal on wireless channels. The multiple propagation paths between the sender and the receiver appear when radio waves propagate through several mechanisms such as scattering, reflection, diffraction, and LOS. Modeling of wireless channels is challenging since the nature of the propagation is unpredictable, and the propagation environment is complicated. Usually to characterize the wireless channels, there are three major effects which have to be considered: path loss, shadowing, and fading. These effects will be discussed briefly in the following sections.

### 2.1.2 Pathloss and Shadowing

Free space propagation (Pathloss) happens since the wave spreads over distance between the transmitter and the receiver; thus, power loss through the channel. The Path loss has a large scale propagation effect because the variation in the signal occurs over a large distance compared to the wavelength. Linear path loss is known as the ratio between the power of the transmitted signal  $P_t$  over the power of the received signal  $P_r$ , i.e.  $P_l = P_t/P_r$ . For high level system analysis, the log-distance model is the most suitable model for this kind of analysis [21]. With regards to the log-normal

model, the path loss at distance  $d$  can be predicted by the following formula [21]:

$$P_l(d)_{db} = P_l(d_0)_{db} + 10 \gamma \log\left(\frac{d}{d_0}\right) \quad (2.1)$$

where  $d_0$  is the reference distance,  $P_l(d_0)_{db}$  is the path loss at  $d_0$ , and  $\gamma$  is the path loss exponent.

Shadowing is a phenomenon that appears due to a large objects (building) presence between the transmitter and the receiver and it has a large scale propagation effect. These objects could attenuate the magnitude of the transmitted signal due to its dielectric properties. Using the log-normal shadowing model, we could find the PDF distribution of the received power by [21]:

$$p(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right] \quad (2.2)$$

where  $x = P_t/P_r$ ,  $\sigma$  is the standard deviation of  $x$ , and  $\mu$  is the mean of  $x$ .

### 2.1.3 Multipath Fading

Multipath fading describes the impact of random overlap among arrived copies of the original transmitted signal at the receiver. In other words, it characterizes the effects of the received signal's copies from different propagation paths on the desired received signal. These signal's copies generated from the fact that the transmitted signal could be scattered, or reflected, depends on the channel environment. Fading occurs in short distance compared to the signal wavelength, and is classified as a small-scale phenomenon.

To write the formulation of the received signal over multipath fading, let us assume  $s(t)$  is a signal transmitted over a wireless channel as follows [21]:

$$s(t) = \text{Re}[u(t)e^{j2\pi f_c t}] \quad (2.3)$$

where  $u(t)$  is complex baseband envelope. The received signal over the multipath channel will be as follows [21]:

$$r(t) = \text{Re} \left\{ \sum_{n=0}^{N(t)} \alpha_n(t) u(t - \tau_n(t)) e^{j2\pi f_c(t - \tau_n(t)) + \theta_{Dn}(t)} \right\} \quad (2.4)$$

where  $N(t)$  is the number of paths,  $\alpha_n(t)$  is the amplitude,  $\tau_n(t)$  is the delay, and

$\emptyset_{Dn}(t)$  is the phase shift, which is equal to  $\int 2\pi f_{Dn}(t) .dt$  , where  $f_{Dn}(t)$  is Doppler frequency.

The multipath fading channel can be classified as frequency selective or flat fading. Also it can be classified as fast or slow fading. These classifications are based on the relative severity of the time-domain variation and power delay spread that cause the transmitted signal over the wireless channel. Multipath channel introduces power spread, and to quantify it along the delay axis, Root Mean Square (RMS) delay spread ( $\sigma_T$ ) can be used. The RMS delay spread can be calculated as follows [21]:

$$\sigma_T = \sqrt{\frac{\sum_0^N \alpha_n^2 (\tau_n - \mu_T)^2}{\sum_{n=0}^N \alpha_n^2}} \quad (2.5)$$

where  $\mu_T$  is the average delay spread, and it given as follows:

$$\mu_T = \frac{\sum_{n=0}^N \alpha_n^2 \tau_n}{\sum_0^N \alpha_n^2} \quad (2.6)$$

If the symbol period of the transmitted signal is small compared to the delay spread  $\sigma_T$  , then intersymbol interference (ISI) will occur, and the time-domain delay spread will translate to selective frequency in frequency domain. If  $\sigma_T$  is very small compared to the symbol time  $T_s$ , the wireless channel will be considered flat fading. Otherwise the channel will be considered selective fading. We can convert frequency selective channel into multiple parallel frequency-flat fading channels with the well-known multicarrier transmission over OFDM technique. (Reference 2 in [21]).

#### 2.1.4 Frequency-Flat Fading

The scenario where the delay spread is smaller than the transmit signal symbol period i.e.  $\sigma_T \ll T_s$  reflects the flat fading phenomenon. The multipath signals can be considered to reach the receiver side at the same time, and the complex baseband input/output relationship of the channel is as in [21]:

$$r(t) = z(t) u(t) + n(t) \quad (2.7)$$

where  $u(t)$  is the transmitted complex envelop,  $n(t)$  is the additive Gaussian noise, and  $z(t)$  is equal to  $z(t) = z_i(t) + jz_q(t)$ . Note,  $z(t)$  can be modeled as a Gaussian random process with the application of central limit theorem (CLT) [21].

In the case where there is no LOS component, the random process  $z(t)$  has zero

mean, thus, the channel amplitude  $|z(t)| = \sqrt{z_i(t)^2 + z_q(t)^2}$  is Rayleigh distributed with distribution function as follows [21]:

$$P_{|z|}(x) = \frac{x}{\sigma^2} \exp\left[-\frac{x^2}{2\sigma^2}\right] \quad (2.8)$$

where  $\sigma^2$  is the variance of  $z(t)$ .

## 2.2 Performance Analysis over Fading Channels

The complex baseband channel model for flat fading is mentioned in equation (2.7). The instantaneous power of the received signal can be shown as  $P_r = |z(t)|^2 E_s / T_s$  and will be random variable with  $|z(t)|^2$  values. The instantaneous SNR is also shown as  $\gamma_s = |z(t)|^2 E_s / N_o$ .

The overall system performance can not be reflected by the instantaneous system performance. Therefore, the average performance measures should be taken to reflect the overall system performance. The average error rate can be calculated by averaging the instantaneous error rate over the distribution of SNR. It is important to know that at any time instant, the fading channel can be viewed as an AWGN channel with SNR,  $\gamma_s = |z(t)|^2 E_s / N_o$ , so over flat fading channel, the average error rate for a certain modulation scheme can be calculated by averaging the instantaneous error rate over the distribution function of  $\gamma_s$ . Theoretically, as in [21], the average error rate is giving by:

$$\bar{P}_E = \int_0^\infty P_E(\gamma) p_\gamma(\gamma) d\gamma \quad (2.9)$$

where  $P_E(\gamma)$  is the error rate over the AWGN channel with SNR and the distribution is function of  $\gamma$ .

## 2.3 Adaptive Modulation

In a wireless link with fading channels, adaptive transmission can be utilized to achieve high spectral and power efficiency with low error rate [21]. Basically, adaptive transmission technique varies the transmission parameters and/or the transmission schemes such as modulation mode, coding rate, or transmission power, depend on in current fading channel state. The system chooses the best channel condition to send the data with high rate and low power level, and responds to channel degradation to reduce data rate or increase power level. As a result, a certain desired error rate will be reached, and thus overall system throughput will be maximized. As such, this technique has recently seen growing interest in academia to meet the demands of high transmission efficiency over fading channels. Now, adaptive transmission schemes are incorporate in GSM/CDMA cellular systems and wireless LAN systems [21].

The fundamental requirement of adaptive transmission techniques is the availability of certain channel state information (CSI) at the transmitter. With perfect CSI at the transmitter, Shannon capacity can be reached over fading channels using optimal adaptive transmission scheme involving continuous rate and power adaptation. However, the condition of perfect channel state at the transmitter is a challenging task in reality even for recent advanced wireless systems. Also, continuous rate adaptation will be highly complex. As a result, most current wireless standards assume adaptive transmission schemes employing discrete adaptation, which requires only limited channel state information at the transmitter, achieved through feedback signaling. The constant -power variable-rate adaptive M-QAM scheme is employed in this work.

### 2.3.1 Constant-Power Variable-Rate Adaptive M-QAM

In the constant-power variable-rate adaptive M-QAM scheme, the system uses a fixed power level for transmission. Assuming an adaptive M-QAM system uses a certain power level, the system selects one of  $N$  dissimilar modulation schemes based on channel conditions. Each modulation scheme has a different constellation size. The constellation size for rectangular or squared M-QAM schemes is denoted by  $M$ , and it is chosen to be  $M = 2^n$ ,  $n = 1, 2, 3, \dots$ , where  $n$  in bps/Hz. The modulation schemes are chosen to reach the highest spectral value. The value range of the channel quality is indicated by dividing the received SNR in to  $(N + 1)$  regions with threshold values  $\gamma_{t_0} < \gamma_{t_1} < \gamma_{t_2} < \dots < \gamma_{t_3} < \gamma_{\infty}$ .

When the received SNR falls into the  $n^{th}$  region, i.e.  $t_n \leq t_{n+1}$ , the constellation size ( $M$ ) will be selected for transmission. In reality, the channel estimator estimates the received SNR at the receiver, and the modulation mode selection chooses the mode depending on the received SNR. The receiver feeds back the selected mode to the transmitter over the control channels [21].

The thresholds are chosen under the condition that the instantaneous bit error rate of the chosen modulation mode is below a certain target value, denoted by  $BER_0$ . For instance, as in reference [4], instantaneous bit error rate of square  $2^n$ - QAM with two-dimensional Grey coding over AWGN channel with SNR can be calculated by:

$$BER_n(\gamma) = \frac{2}{\sqrt{M} \log_2 \sqrt{M}} \times \sum_{k=1}^{\log_2 \sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} (-1)^{\lfloor \frac{i^{k-1}}{\sqrt{M}} \rfloor} \left( 2^{k-1} - \left\lfloor \frac{i 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \times Q \left( (2i+1) \sqrt{\frac{6 \log_2 M}{2(M-1)} \gamma} \right) \quad (2.10)$$

As in reference [23], equation (2.10) can be approximated using a simple formula as follows:

$$BER_n(\gamma) = \begin{cases} 1 & 0 < \gamma < \gamma_{pn} \\ a_M \exp(-g_M \gamma) & \gamma \geq \gamma_{pn} \end{cases} \quad (2.11)$$

where  $n$  is the mode index, and,  $a_M$ ,  $g_M$ , and  $\gamma_{pn}$  are state dependent parameters which are obtained by fitting the curve of equation (2.11), to the exact curve  $BER_n(\gamma)$  of equation (2.10) by using least-mean-square method [17]. In the upcoming sections the parameters  $a_M$ ,  $g_M$ , and  $\gamma_{pn}$  are calculated to fit frames error rate curves. The fitting graphs are shown too. For a target bit error rate ( $BER_0$ ), the threshold values can be calculated as follows:

$$\gamma_{Tn} = -\frac{1}{g_M} \ln \left( \frac{BER_0}{a_M} \right), \quad n = 0, 1, 2, \dots, N. \quad (2.12)$$

### 2.3.2 Packet and Frame Structures

Adaptive transmissions systems deal with frames in the physical layer. The signal will be sent in term of frames. Each frame contains a fixed number of symbols  $N_F$ . Assuming the symbol rate is fixed, the frame duration time will be constant.

Having constant frame duration can be used to calculate the parameters of Markov model for the channel model, as it shown in next sections. Each frame contains a number of packets from the data layer, and each packet contains number of bits  $N_b$ , which include packet header, cyclic redundancy check, and payload. After modulation and coding with rate  $R_m = (\text{bits/symbol})$ , each packet is mapped to a symbol block containing  $N_b / R_m$  symbols. These blocks are used to build the frame, so the data can be transmitted in the physical layer. The number of symbols per frame can be calculated as follows [17]:

$$N_F = N_c + N_b N_p / R_m \quad (2.13)$$

where  $N_c$  contains the pilot symbols and control part, and  $N_p$  is the number of packets per frame. The value of the number of packets per frame depends on the rate  $R_m$  of the modulation and coding schemes. Also we can calculate the frame time duration as follows [17]:

$$T_f = (N_c + N_p) / R \quad (2.14)$$

where  $R$  is the rate in bits per second of the system. The packet and the frame structures are shown in Figure 2.1.

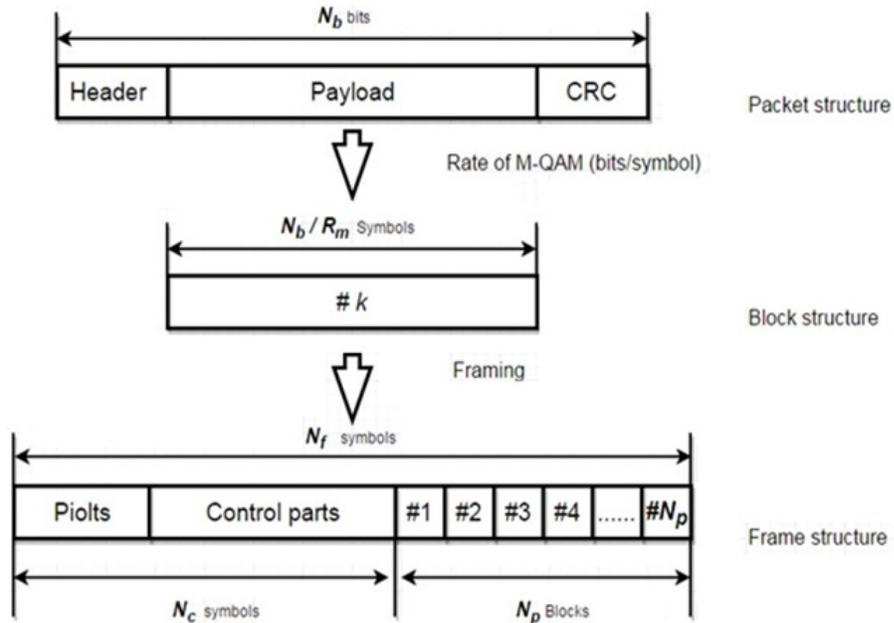


Figure 2.1: The packet and the frame structures [17].

## 2.4 Performance Analysis of Adaptive Modulation System Model

### 2.4.1 Frame Error Rate

Having the instantaneous bit error rate  $BER_{M(\gamma)}$  of M-QAM modulation format (2.10), we can calculate the exact-packet-error rate  $PER_{M(\gamma)}$  as follows [23]:

$$PER_{M(\gamma)} = 1 - (1 - BER_{M(\gamma)})^{N_b} \quad (2.15)$$

where  $N_b$  is number of bits per packets. Having the exact packet error rate  $PER_{M(\gamma)}$ , we also can calculate the frame error rate  $FER_{M(\gamma)}$  as follows [23]:

$$FER_{M(\gamma)} = 1 - (1 - BER_{2(\gamma)})^{N_c} (1 - PER_{M(\gamma)})^{N_p(M)} \quad (2.16)$$

where  $N_c$  is the total number of symbols in the header and in the control part. As in [17] for the  $PER_{M(\gamma)}$  approximation, we could also find  $FER_{M(\gamma)}$  approximation as follows [23]:

$$FER_{M(\gamma)} = \begin{cases} 1 & 0 < \gamma < \gamma_{pn} \\ a_M \exp(-g_M \gamma) & \gamma \geq \gamma_{pn} \end{cases} \quad (2.17)$$

where  $M(\gamma)$  is the state index, and the state dependent parameters,  $a_M$ ,  $g_M$ , and  $\gamma_{pn}$  are obtained by fitting the curve of equation (2.17), to the exact curve  $FER_{M(\gamma)}$  of equation (2.16) by using least-mean-square method [23]. The state dependent parameters for different M-QAM modulation modes are calculated and they are shown in Table 2.1.

In order to calculate adaptive modulation thresholds in terms of packet error rate or in term of frame error rate, we can rewrite equation (2.12) to the follows:

$$\gamma_{Tn} = -\frac{1}{g_M} \ln \left( \frac{FER_0}{a_M} \right) \quad , \quad n = 0, 1, 2, \dots, N \quad (2.18)$$

where  $FER_0$  is the desired or target frame error rate of a system . The curves in Figure 2.2 display the FER fitting curve per mode calculated by equation (2.17), and the exact FER per mode calculated by equation (2.16). these results are regenerated and are matched the results in reference [23].

Table 2.1: State Dependent Parameters of  $FER_{M(\gamma)}$  Equation.

Mode ( $M$ )	Rate bit/symbol	$a_M$	$g_M$	$\gamma_{pn}$ (db)
4-QAM	2	70.21	0.9929	7.5
8-QAM	3	87.98	0.4971	10.73
16-QAM	4	99.19	0.3948	11.76
32-QAM	5	106	0.1896	14.99
64-QAM	6	118.4	0.1417	16.399

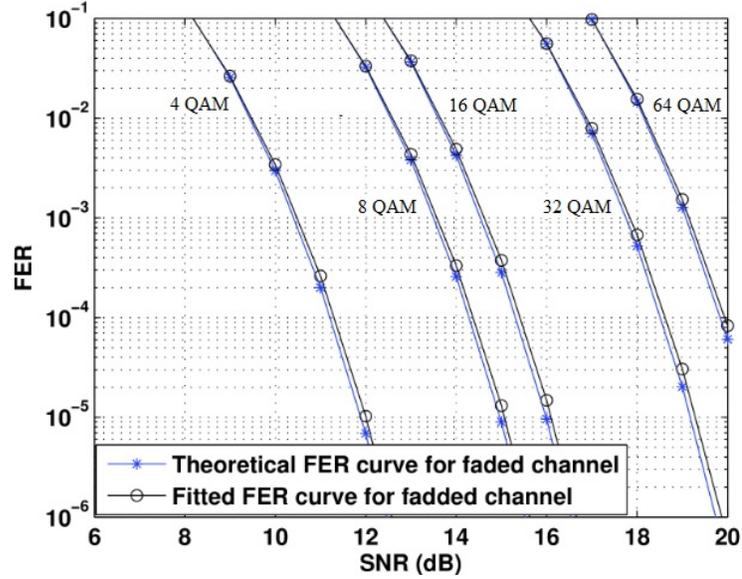


Figure 2.2: Theoretical FER curve and FER fitted curve of M-QAM adaptive modulation modes over Rayleigh fading channel.

In order to calculate average frame error rate for each mode of adaptive modulation model over Rayleigh, The average frame error rate per mode can be calculated as follows:

$$\overline{FER}_M = \frac{1}{\pi_M} \int_{\gamma_t}^{\gamma_{t+1}} FER_{M(\gamma)} p_\gamma(\gamma) d\gamma \quad (2.19)$$

where  $\pi_M$  is the probability of being in the current mode, and it will be shown in next chapter.

## Chapter 3

# Overview of Finite State Markov Chain (FSMC)

### 3.1 Finite State Markov Chain (FSMC) for radio channel

Finite State Markov Chain protrudes from early the works of GIBERT and ELLIOTT. Modeling the Radio channel as two states was not enough in order to form channel variation; the solution to forming channel variation is to form the channel with more than two states. Let us assume vector  $s = \{s_0, s_1, \dots, s_{k-1}\}$  denote a finite set of states in the channel and  $s_n$  be a constant Markov process.  $s_n$  is a constant which has the property of stationary transition, so the transition probability between the states is independent of the time index  $n$  and it can be written as in reference [20] as follows:

$$p_{j,k} = Pr (s_{n+1} = s_k / s_n = s_j) \quad (3.1)$$

where  $n = \{0,1,2, \dots\}$ ,  $j$  and  $k$  are current and next states respectively  $(j,k) \in (0, 1,2, \dots, K-1)$ , and  $K$  is number of states. With these definitions, we can calculate the state transitions probability matrix  $\mathbf{P}$  with elements  $p_{j,k}$  as in (3.1). The probability of staying in state  $k$  at any possible time index  $n$  is called stationary transition property and it can be defined as follows:

$$\pi_k = Pr( s_n = s_k), k \in \{0, 1, 2, 3, \dots, K-1\}. \quad (3.2)$$

For a state  $k$ , the outcome and income flows must be equal. This assumption is called

equilibrium condition, and is shown as follows:

$$\sum_{j=0}^{k-1} \pi_j P_{j,k} = \sum_{i=0}^{k-1} \pi_k P_{k,i} \quad (3.3)$$

We can write (3.3) simply as  $\boldsymbol{\pi}^t \mathbf{P} = \boldsymbol{\pi}^t$ . Where,  $\boldsymbol{\pi}^t$  is matrix [20]. Also the sum of all  $\boldsymbol{\pi}$  elements have to equal to one.

### 3.1.1 Finite State Markov Chain (FSMC) Model for Rayleigh Fading Channel

Rayleigh fading is a model for a received signal envelop through typical wireless channel with multipath propagation and non-line-of sight (NLOS) frequency-nonselctive (flat) fading. Assuming a certain modulation and coding schemes are given; the channel fading characteristics can be mapped to the packet level (cross-layer). Using this approach for the performance analysis of the upper layer protocols is quite complex. Alternatively, the Rayleigh fading channel can be represented by a FSMC[17].

FSMC model can be built by partitioning the received instantaneous SNR into levels. Let  $s_i$  denotes the  $i^{th}$  state at level  $i$  and  $\Gamma_i$  denote SNR at level  $i$ , and  $K$  denote the number of levels. As we mentioned in the previous section,  $s$  vector includes all  $s_i$  states,  $s = (s_1, s_2, \dots, s_{k-1})$ , and the radio channel evolves as  $K - 1$  states of Markov chain. We assume all packets and all frames have the same size so the channel keeps staying in one state during the transmission time of each frame. If the received SNR is located between  $\Gamma_i$  and  $\Gamma_{i+1}$  thresholds then the channel will be considered in state  $s_i$ . The instantaneous SNR ( $\gamma$ ) for a Rayleigh fading channel with additive white Gaussian noise is exponentially distributed as follows [23]:

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \quad (3.4)$$

where  $\bar{\gamma}$  is the average of the received signal to noise ratio. The steady state probability, which is the probability of staying at state  $s_i$  can be calculated as follows [23]:

$$\pi_i = \int_{\Gamma_i}^{\Gamma_{i+1}} p_\gamma(\gamma) d\gamma \quad (3.5)$$

As in reference [23], the crossing rate  $N(\Gamma_i)$  at a specific threshold level is defined as

the number of times per second that the fading amplitude envelop crosses the level  $\Gamma_i$  in the downward direction and is given by:

$$N(\Gamma_i) = \sqrt{\frac{2 \pi \Gamma_i}{\bar{\gamma}}} f_m \exp\left(-\frac{\Gamma_i}{\bar{\gamma}}\right) \quad (3.6)$$

where  $f_m$  is the Doppler frequency which can be calculated as  $f_m = v f_c / c$ . Where,  $v$  is the velocity of motion,  $c$  is light speed, and  $f_c$  is the carrier frequency. Assume the modulation scheme and a forward error correcting (FEC) are given, the instantaneous SNR can be mapped to packet error rate PER then to frame error rate FER. The average error  $e_i$  of the state  $s_i$  is given as follows:

$$e_i = \frac{1}{\pi_i} \int_{\Gamma_i}^{\Gamma_{i+1}} p(e/\gamma) p_\gamma(\gamma) .d\gamma = \overline{FER}_i \quad (3.7)$$

where  $p(e/\gamma)$  is the FER given the signal to noise ratio is equal to the instantaneous  $SNR(\gamma)$ , and  $\overline{FER}_i$  is the average frame error rate for state  $i$ . In our system model we used M-QAM modulation to send the packets, so the  $p(e/\gamma)$  is equal to (2.17). We assume  $p_{j,k}$  is the state transition probability from state  $s_j$  to  $s_k$  and  $T_F$  is the time duration of a frame. For simplicity, we assume the current state  $j = i$  and the adjacent states  $k = i + 1$ , or  $k = i - 1$ . We also assume that there is no state transition within a frame time, and the transition between the states occurs between the adjacent states as in Figure 3.1.

Assume that  $N(\Gamma_i)T_F$  and  $N(\Gamma_{i+1})T_F$  are less than  $\pi_i$ , which indicates the slow fading channel, the state transition probabilities can be approximated as follows:

$$p_{i,i+1} = \frac{N(\Gamma_{i+1})T_F}{\pi_i} \quad \text{if } i = 0, 1, 2, \dots, K - 1 \quad (3.8)$$

$$p_{i,i-1} = \frac{N(\Gamma_i)T_F}{\pi_i} \quad \text{if } i = 1, 2, \dots, K \quad (3.9)$$

$$p_{i,i} = \begin{cases} 1 - p_{-(i, i+1)} - p_{-(i, i-1)} & \text{if } 0 < i < K \\ 1 - p_{0,1} & \text{if } i = 0 \\ 1 - p_{K,K-1} & \text{if } i = K \end{cases} \quad (3.10)$$

Figure 3.1 shows FSMC model with transition state probability between adjacent states, and it implements the slow fading channel model.

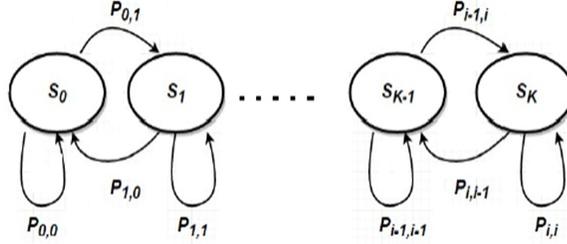


Figure 3.1: FSMC model illustrated the state transitions probabilities for  $k$  number of states

As in reference [3], the average time duration of state  $i$  is defined as follows:

$$\tau_i = \frac{\pi_i}{N(\Gamma_i) + N(\Gamma_{i+1})} = c_k T_F \quad (3.11)$$

### 3.1.2 Average Throughput Analysis Using FSMC's Parameters

In [17], the throughput has been calculated in terms of average packet error rate, and in [22], the throughput is calculated in terms of average frame error rate and in terms of Markov parameter which is the transition matrix between the states. As in [22], the expected average throughput of mode  $i$  in state  $j$ , is called  $T_{ij}$ , and can be calculated as follows:

$$T_{ij} = \sum_{k=0}^{m-1} N_i P_{jk} (1 - FER_{ik}) \quad (3.12)$$

where  $N_i$  is the number of bits in a frame using the  $i^{th}$  mode,  $m$  is number of states,  $P_{jk}$  is the transition matrix probability from state  $j$  to state  $k$ , and  $(1 - FER_{ik})$  is the probability of correct transmission if the  $i^{th}$  Mode is selected when the Markov chain is in state  $k$ .

## Chapter 4

# Channel Partitioning Methods of FSMC

### 4.1 Signal to noise ratio partitioning

In this section, two different methods about SNR partition are presented. The first one is called Steady State Equilibrium method, which is presented in [24][23], and the second method is called Error Rate Curve Partitioning method. In this method, the SNR is desecrated into levels (thresholds) regards to the curve of frame error rate (FER) verses signal to noise ratio (SNR). The time interval of the frames is taken into account.

#### 4.1.1 Steady State Equilibrium Method

This method is also called Equal-Probability method and it is used to calculate the channel thresholds of FSMC model. In this method, the SNR thresholds of the channel are determined by  $[\pi_1 = \pi_2 = \pi_3, \dots \pi_i = 1/K]$ . Where,  $\pi_i$  can be writing as follows [24]:

$$\pi_i = \exp\left(-\frac{\Gamma_i}{\bar{\gamma}}\right) - \exp\left(-\frac{\Gamma_{i+1}}{\bar{\gamma}}\right) = 1/K. \quad (4.1)$$

In the first case study of this project, we use this method to estimate the thresholds (SNR) of the states to build the Markov model [24].

### 4.1.2 Error Rate Curve Partitioning Method

From the fact that the relationship between SNR and FER is a non linear curve as shown in Figure 4.1, which shows the FER versus SNR curves for adaptive transmission system, uses three different M-QAM modulation schemes.

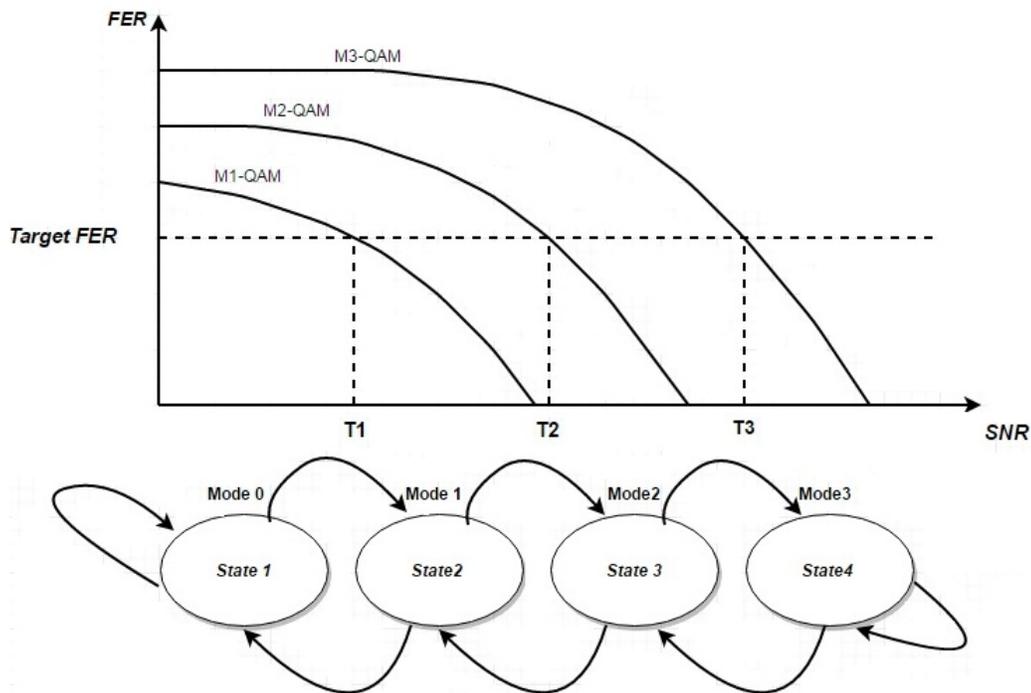


Figure 4.1: The FER versus SNR curves for adaptive transmission system, uses three different M-QAM modulation schemes.

If we consider that an adaptive transmission wireless system works perfectly at a certain FER, which is called target FER, we can calculate target SNR for each curve by using equation (2.10). From Figure 4.1, we can determine three different target SNR's for each curve. These SNR's can be considered as thresholds that the adaptive transmission system uses to switch from current mode to other. The channel performance of adaptive transmission systems can be evaluated using the Finite State Markov Chain (FSMC) model. The target SNRs can be utilized as thresholds which are used to calculate the parameter of FSMC model. Each mode can be considered as a state in the FSMC model. In order to calculate the threshold of each state of FSMC model, signal to noise ratio should be large enough for each state to cover the SNR variation during a time frame  $T_F$ . However, the signal to noise ratio ranges cannot be too large so the states have different range of Frame error rates [24]. Based on these

considerations, there is a parameter used to calculate the time duration for a state in order to estimate SNR partitioning. This parameter is called average time duration  $\tau_i$ , which is the average time interval of the received instantaneous SNR between two thresholds  $(\Gamma_i - \Gamma_{i+1})$ . Average time duration  $\tau_i$  is shown as follows [24]:

$$\tau_i = \frac{\pi_i}{N(\Gamma_i) + N(\Gamma_{i+1})} = c_k T_F \quad (4.2)$$

where  $c_k$  is a constant and it must be greater than 1. The constant  $c_k$  can be calculated by:

$$c_k = \frac{1}{T_F} \frac{\pi_i}{N(\Gamma_i) + N(\Gamma_{i+1})} \quad (4.3)$$

The SNR thresholds can be calculated using equations (2.12),(2.18), and it is called main thresholds (adaptive modulation thresholds). The constants  $c_k$  are calculated from main thresholds, with regard to target FER and consultation size  $M$ . The constant  $c_k$  is large when the number of states is small and vice versa [5]. From Figure 4.1, we have three main thresholds or four states which are not enough states to get appropriate values of  $\tau_i$ ; that's because the range between  $(\Gamma_i - \Gamma_{i+1})$  is large and not uniform, which makes the parameter  $c_k$  large too. We proved this phenomenon in Figure 4.3, and it will be discussed later in the relationship between  $c_k$  and SNR thresholds section. For these reasons, we calculate new thresholds in order to get reasonable values of  $c_k$  and  $\tau_i$ . These new thresholds are calculated for each mode or state between  $(\Gamma_i - \Gamma_{i+1})$  to consider the average time duration  $\tau_i$  in the calculation. These new thresholds introduce sub states in each mode or in each main state, and they are chosen in order to ensure that the time duration of each sub state is not too large or too small compared with the frame time duration  $T_F$ . Now each mode has sub states. The overall sub states for all modes are the new states for the system and they will be used to calculate the parameters of the FSMC model for the adaptive transition channel. The new thresholds can be calculated as in the following steps:

Step 1: calculate the main thresholds using formula (2.18).

Step 2: calculate the constant  $c_k$  for each state from formula (4.3); if  $c_k < 1$ , end the calculation, else if  $c_k > 1$ , go to step 3.

Step 3: calculate the sub thresholds for each mode as follows:

- Choose number of sub thresholds  $N$  of each mode.
- Set the vector  $\{k_n\} = \{1, 2, 3, \dots, N\}$ , where  $\{k_n\}$  is number of sub thresholds vector.

- Calculate the parameter Delta ( $\Delta n$ ) for each mode as follows:

$$\Delta n = ((\Gamma_{i+1} - \Gamma_i)/N), \text{ where } i = [1, 2, 3, 4, \dots, N] \quad (4.4)$$

- Calculate the sub thresholds for each mode as follows:

$$\gamma_j = (\Gamma_i + (\Delta n \times (kn(n) - 1))), \text{ where } n = [1, 2, \dots, N], j = [1, 2, \dots, K] \quad (4.5)$$

where  $\gamma_j$  is the new sub thresholds of the FSMC channel model,  $\Delta_n$  is the step size among sub thresholds ( $\gamma_j - \gamma_{j+1}$ ). The main thresholds ( $\Gamma_{i+1} - \Gamma_i$ ) are assumed to equal to adaptive transmission thresholds as we assumed previously in this method. The parameters of the FSMC model can be calculated using the new thresholds (all sub thresholds of all modes in one vector) where taking into account the time duration of each state.

With this method we could eliminate the value of constant  $c_k$ , and thus control the average time duration  $\tau_i$ , so it is reasonable value to calculate the thresholds of the FSMC channel regards to the target FER of the system.

The advantages of this method are the flexibility of choosing the thresholds of the channel directly from the target FER of the system, with low number of states  $K$  and less average time duration  $\tau_i$ . It can give a good evaluation for higher consultation size of modulation schemes without increasing the number of states as in equilibrium steady states method that we discussed previously. Average time duration  $\tau_i$  can be controllable using this method by changing the parameter ( $\Delta n$ ). We can add value of the parameter  $\Delta n$  to SNR thresholds of adaptive modulation system to ensure the best time to switch from one mode to another safely with respect the average time duration among modes.

### 4.1.3 Relationship between Average Time Duration of a State and SNR Thresholds of the Same State

Figure 4.2 and Figure 4.3 show the relationship between  $c_k$  and the step size from one state to another in FSMC model, which implement ( $\Gamma_i - \Gamma_{i+1}$ ) range for different values of SNR thresholds. In Figure 4.2, the step size for all states are small and

equal. Figure 4.3 shows the relation between  $c_k$  and the step size of each state, with non-uniform step size.

To plot these figures, we used equation (4.3) for different values of the thresholds range ( $\Gamma_i - \Gamma_{i+1}$ ). We set the number of thresholds  $K$  equal to nine, which gives eight values of constant  $c_k$ , and we plot the results in two cases. In the first case, we assume the step size is equal for all states and is equal to delta ( $\Delta n$ ). In the second case we assume a random or non-equally step size. The values of  $\Gamma_i$ ,  $\Delta n$  and  $c_k$  are shown in Table 4.1, for the first case, and in Table 4.2 for the second case.

Table 4.1: Constant  $c_k$  and  $\Gamma_i$  thresholds values calculation based on equal step size  $\Delta n$  for all states.

$\Gamma_i$	0	3	6	9	12	15	18	21	24
$\Gamma_{i+1}$	3	6	9	12	15	18	21	24	-
$\Delta n$	3	3	3	3	3	3	3	3	-
$c_k$	9.6	3.82	2.92	2.45	2.16	1.95	1.79	1.67	-

Table 4.2: Constant  $c_k$  and  $\Gamma_i$  thresholds values calculation based on non-equal step size  $\Delta n$  for all states.

$\Gamma_i$	0	3	6	10	13	16	19	22	25
$\Gamma_{i+1}$	3	6	10	13	16	19	22	25	-
$\Delta n$	3	3	4	3	3	3	3	3	-
$c_k$	9.6	3.82	<b>3.79</b>	2.34	2.08	1.89	1.75	1.63	-

Figure 4.2 and Figure 4.3 show the relationship between constant  $c_k$  and SNR thresholds curve with different state's step size range ( $\Gamma_i - \Gamma_{i+1}$ ). Figure 4.2 shows the curve with equal step size for all states  $\Delta n$ , and Figure 4.3 shows the curve with non-equal step size for all states.

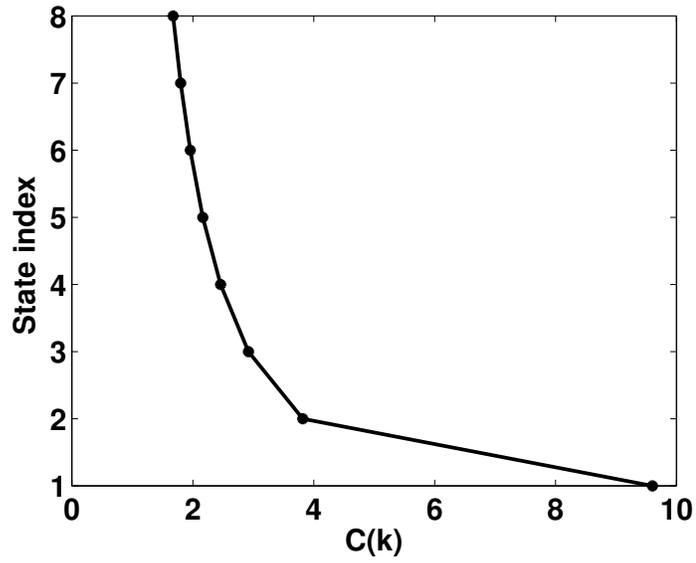


Figure 4.2: The relationship between constant  $c_k$  and thresholds  $\Gamma_i$  with equal step size ( $\Delta n$ ) for all states.

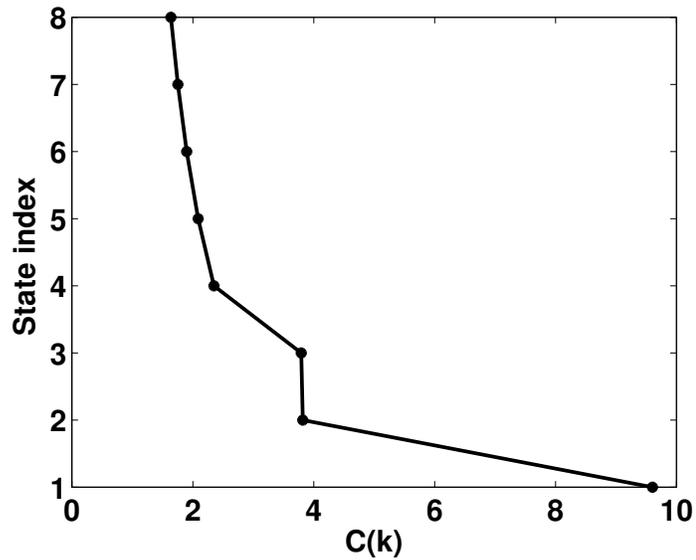


Figure 4.3: The relationship between constant  $c_k$  and thresholds  $\Gamma_i$  with non-equal step size ( $\Delta n$ ) for all states.

# Chapter 5

## System Setup and Results Discussion

### 5.1 Model of End to End Adaptive Modulation System

Figure 5.1 shows end to end system connection from sender to receiver with a wireless link working with a single-transmit antenna and a single-receive antenna. Even though we focus on Downlink systems here, the results are valid to Uplink systems as well. A buffer with a first-in-first-out (FIFO) basis is used at the transmitter. The buffer feeds the adaptive modulation (AM) controller, and the AM selector is fixed at the receiver. We assume that the transmission has multiple modes to transmit the data. Each mode represents a modulation format, and a forward error correction code as in IEEE 802.11a. The AM selector determines the modulation mode based on the channel state information (CSI) that is available at the destination, and sends the decision back through a feedback channel to the AM controller to reselect the transmission mode. A maximum likelihood decoding, and coherent demodulation are used at the receiver. The decoded bits are mapped to packets so it can be pushed up to layers above the physical layer.

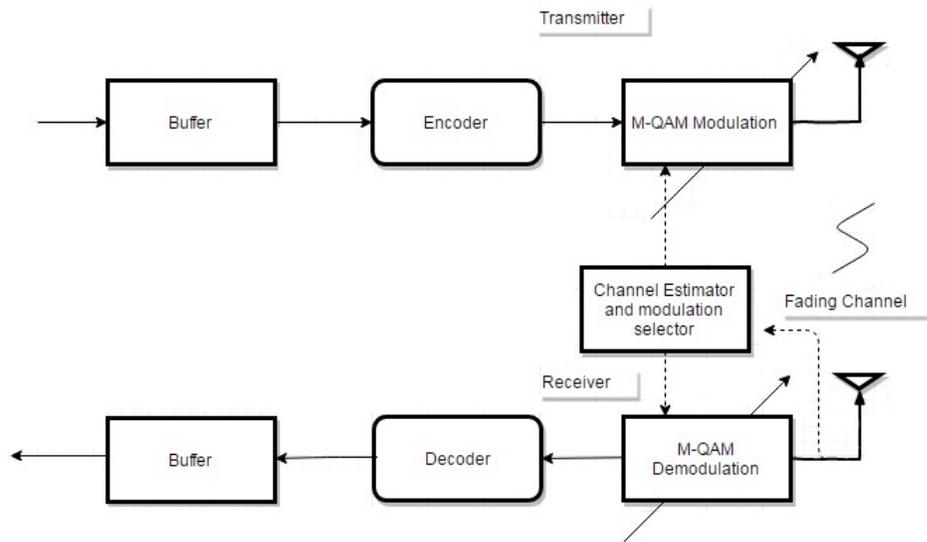


Figure 5.1: End to End wireless link system model.

### 5.1.1 System Model Assumptions

1. The channel is frequency flat-slow fading. It keeps invariant per frame, but it varies from one frame to another. The frame is a group of packets that contains the bits stream. This assumption can be implemented by a block fading model that is suitable for slow variation channel behavior. the AM mode is adjusted to change from mode to another based on frame-by-frame basis.
2. We assume a perfect channel state (CSI) at the receiver, and the mode selection is fed back to the AM controller without any latency or errors.
3. We assume the packets transmitted through a first in-first out queue. If the queue become full, the new incoming packets will be dropped and will not be recovered or retransmitted by end-to-end (sender to receiver) link. This assumption can be made available by using User Datagram Protocol (UDP).
4. We assume perfect Cyclic Redundancy Check (CRC) will detect the error. The CRC parity bits per packet is not incorporated in the throughput calculation.
5. The packet is dropped if it is not received correctly after error detection.

The aim of this project lies in finite-state-Markov modeling of received SNRs that are assumed to follow Rayleigh fading distribution. Performance evaluation done by finite state modeling and performance measures such as state time duration, state transition probability, steady state probability, and level crossing rates are plotted and presented.

## 5.2 Results of Steady State Equilibrium Partitioning Method

In this section, numerical results for the Markov channel model and performance evaluation of the adaptive modulation system are presented. We consider the length of frame  $N_F = 424$  symbols, which includes  $N_C = 40$  Symbols, and  $N_P = 384$  symbols. Adaptive modulation modes with (4- 16 -32) QAM schemes are used to send the signal through slow Rayleigh distribution fading channel. Note a bit error rate Matlab function that support both square and rectangular M-QAM modulations is employed in this project.

For the fading process based on Clarke's and Jakes' models that was generated in [7] [13], 5m/s vehicular speed is used, and 1 M bit/s is set for the transmission rate at carrier frequency equal to 1.9GHz . The allowed data rates of the adaptive modulation modes  $N_i$  are (848, 1696, and 2544) bits per frame respectively. The fading rate is set to equal to  $F_m T_f = 0.0134$ , and Steady State Equilibrium Partitioning Method for thresholds calculations is employed. Number of states  $K= 10$  is set, and number of SNR thresholds is 9. the first state, which is called state 0 is neglected, so the performance calculation will include from state 1 to state 9.

In order to calculate the SNR thresholds of the adaptive modulation with (4- 16 -32) QAM schemes, equation (2.18) is employed. The target frame error rate FER is set to equal  $10^{-3}$  . The state dependent parameters values of each mode in Table 2.1 and the target FER are substituted in equation (2.18) to calculate the SNR thresholds for each mode. The SNR thresholds values are presented in Table 5.1.

The channel SNR thresholds  $\Gamma_i$  can be calculated using equation (4.1). Note that the first threshold is neglected (the system is off at low SNR ranges). The channel's SNR thresholds values and the states boundaries are presented in Table 5.1.

Figure 5.2 shows the steady state probabilities of Rayleigh fading distribution obtained analytically. In the figure, the steady state probabilities are equal for all states. This confirms the partition assumption of the Steady State Equilibrium Partitioning Method. In this method the effect of fading on the signal is assumed to be constant and not varied. Based on the results, it can be concluded that this method is limited to constant fading effects.

Figure 5.3 shows the level crossing rate verses the total number of states for Rayleigh distribution. Based on the vehicular speed and the carrier frequency, the Doppler frequency is set to 31.66Hz. It is obvious that when the number of state

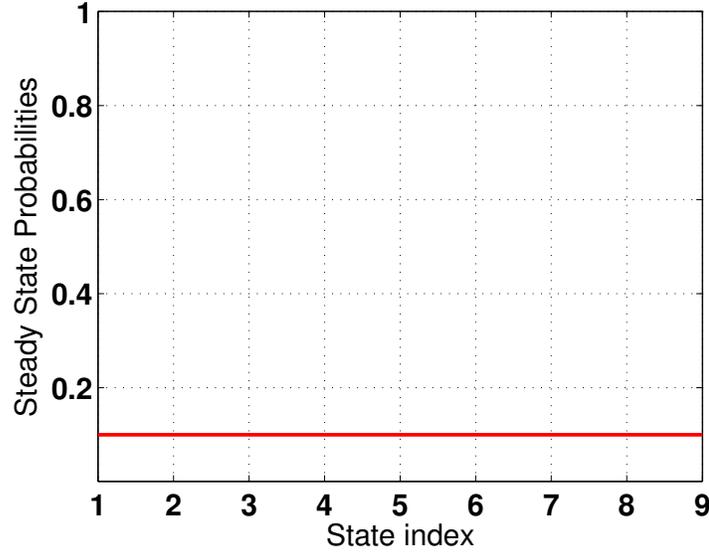


Figure 5.2: Steady state probabilities verses state index for Rayleigh fading channel using steady state equilibrium partitioning method.

index increases, the SNRs increases, and the effects of the fading will be lower. From the figure, the crossing rate increases from state 1 to state 3, and decreases from state 4 to 7. If the fading effect is low the channel will be considered good quality. The crossing rate here depends on the value of the average SNR value and the thresholds values, so the curve in the figure increases and decreases from the first state to the last state regularly. this result is regenerated and is matched the results in [24].

Figure 5.4a, Figure 5.4b , and Figure 5.4c show the transition probabilities  $p_{i,i+1}$  ,  $p_{i,i-1}$ , and  $p_{i,i}$  verses the states index. For the transition  $p_{i,i+1}$  the figure shows the transitions from state 1 to 3 is increased and form state 4 to 7 decreased. For the transition  $p_{i,i-1}$  , the figure shows the transitions from state 2 to 4 is increased and form state 5 to 8 decreased. Note here the  $p_{i,i+1} = p_{i,i-1}$  ; this is because the steady state probabilities for each state are equal. The probability of not making any transition  $p_{i,i}$  record high values, which represents slow fading, and it increases when the number of states increases. these result are regenerated and are matched the results in [24].

In Figure 5.5, the average frame error rates per states are presented. From Table 2.1, that includes the values of the state dependent parameters,  $a_M$  ,  $g_M$ , and  $\gamma_{pn}$  for all M-QAM modes, and from equations (2.17) , (2.19) and (3.7), We calculate the actual average FER per state. Since the state index increases, the SNR increases and

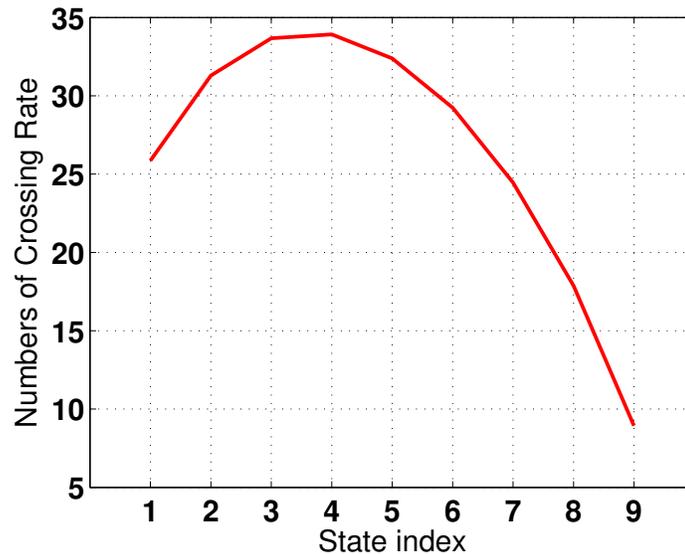


Figure 5.3: Number of crossing rate verses state index for Rayleigh fading channel using steady state equilibrium partitioning method.

the average FER decreases. For the first mode which uses 4-QAM modulation scheme, The channel introduces poor quality in the first two states, which is considered a noisy channel that has significant effect on the received signal. States 3 to 9 introduce a superior quality, which reflects the decreasing FER for these states. For the second mode, which uses 16-QAM modulation scheme, the channel introduces poor quality in the first four states, which is considered a noisy channel that has high effect on the received signal. States 5 to 9 introduce improved quality. For the third mode, which uses 32-QAM modulation scheme, the channel introduces poor quality from state 1 to state 7, which is considered as noisy channel that effects on the received signal. States 8 to 9 introduce a good quality.

Figure 5.6 shows the average throughput per state of each M-QAM mode of the system model individually. First, we run 4-QAM modulation scheme of the adaptive modulation system over all channel states and we calculate the average throughput per each state, then we repeat this step with the 16-QAM and 32-QAM modulation schemes modes respectively over all channel states. For all modes, we can see the average throughput at the first states are low, and they increase dramatically toward last states. For each state, we can compare the average throughput of all modes; so we can decide at which SNR range the system should switch among modes. In Table 5.1, we record the best mode switches decisions to get the highest performance

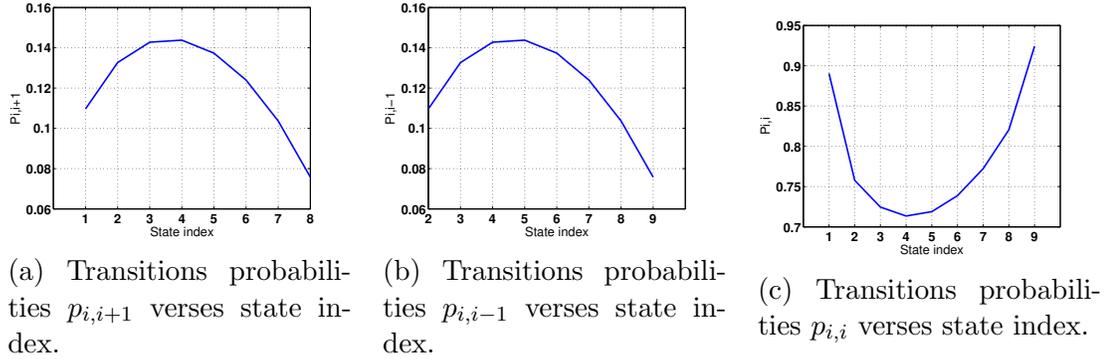


Figure 5.4: The transition probabilities for Rayleigh fading channel using steady state equilibrium partitioning method.

of the system. As in the table, we can see the best switching choice from 4-QAM to 16-QAM is at state 4. In other words, the system should switch whenever the received SNR is located in the SNR channel's thresholds range of state 4. The best time to switch from mode 16-QAM to 32-QAM is at state 7 and then it keeps going to with 32-QAM mode.

In Table 5.1, Interesting result came up when we sign in the boundaries of the adaptive modulation thresholds into the boundaries of the channel thresholds, especially at the state 4. The thresholds boundaries of the adaptive modulation to switch from the first mode to the second is different compared with channel thresholds Boundaries, and that is because we set the target FER a little bit high ( $10^{-3}$ ). So at that mode threshold value, the average throughput is less than the average throughput of the first mode, thus no switching.

The constant  $c_k$  in all states recorded high values, which range between 3 and 9. The constant  $c_k$  in this method is not controllable, so we cannot ensure the average time duration of state  $k$  is a reasonable value.

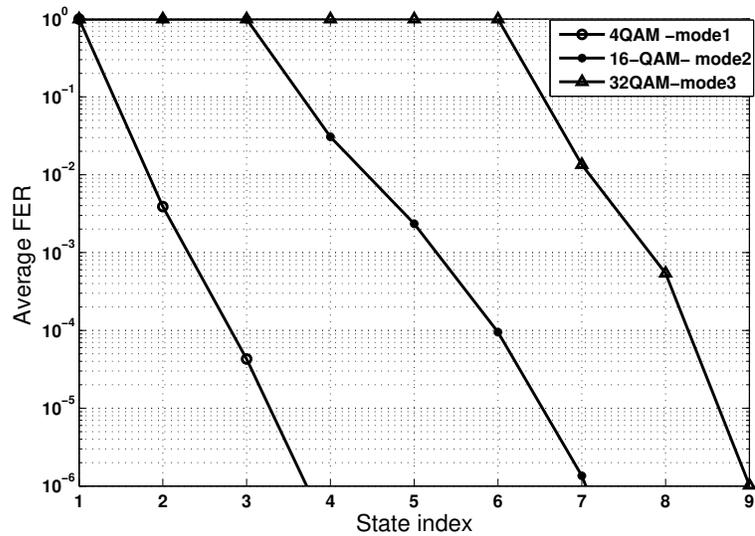


Figure 5.5: Average frame error rate per mode verses state index for the steady state equilibrium partitioning method.

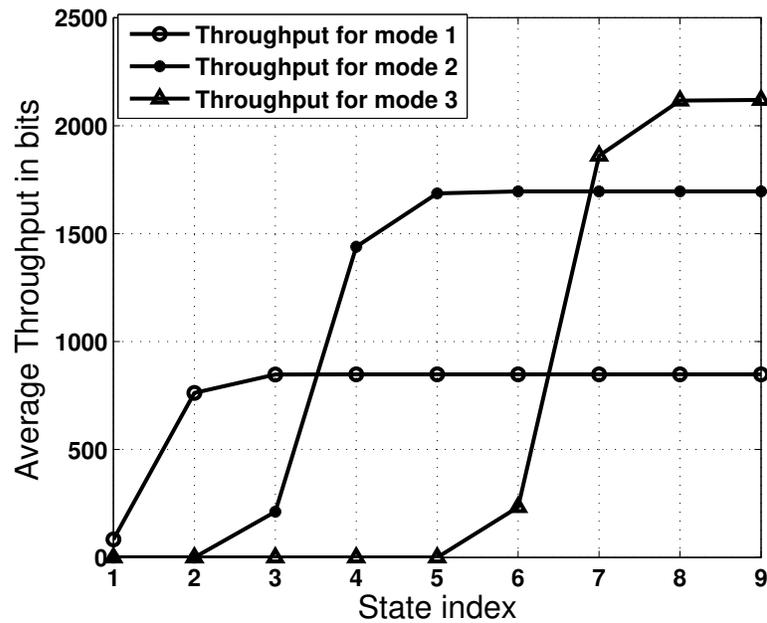


Figure 5.6: Average throughput per state in bits of each mode in case target FER =  $10^{-3}$  for the steady state equilibrium partitioning method.

Table 5.1: Values of channel thresholds and the adaptive modulation thresholds at target FER =  $10^{-3}$  calculated using steady state equilibrium partitioning method.

Channel thresholds ( $\Gamma_i$ ) <sub>db</sub>	Selected mode	Mode's thresholds ( $\gamma_M$ ) <sub>db</sub>	Constant ( $c_k$ )
6.4763	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	4.5855
9.1941	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	4.03415
11.0412	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	3.8774
12.5107	16-QAM	$10.3021 < \gamma_{t2} < 13.5125$	3.95230
13.7905	16-QAM	$10.3021 < \gamma_{t2} < 13.5125$	4.2521
14.9854	16-QAM	$10.3021 < \gamma_{t2} < 13.5125$	4.8801
16.1815	32-QAM	$13.5125 < \gamma_{t3} < \text{INF}$	6.1901
17.4980	32-QAM	$13.5125 < \gamma_{t3} < \text{INF}$	9.7727
19.2715	32-QAM	$13.5125 < \gamma_{t3} < \text{INF}$	-

### 5.3 Results of Error Rate Curve Partitioning Method

In this section, numerical results of the system performance evaluation and the Markov channel (FSMC) model are presented. We consider the same system model and the same scenario as the previous method except the channel partitioning method is changed to become Error Rate Curve Partitioning method. We assume mode 0 in Figure 4.1 is neglected, so the SNR thresholds  $\gamma_m$ , which in this case study assumed as  $\Gamma_i$ , start from mode 1 to mode 3. In addition, we assume there are two cases based on target FER. The first case assumed the target FER =  $10^{-3}$ , and second case assumed the target FER =  $10^{-6}$ . These two cases applied to all modes individually. The effect of fading on the signal is assumed to be varied.

We first set the Target Frame error rate equal to  $10^{-3}$ . We calculated the parameters of the FSMC channel model and the performance evaluation of the system based on this setting, then we did the same procedure using Target FER =  $10^{-6}$  to compare the average throughput of the first case with the second case. Note that when we change the value of the target FER, the thresholds of the channel will be changed, so the parameters of the channel will also be changed. In this work, we only present the channel parameter from the first case. The steady state probability, which is the probability of being in state  $k$ , depending on the available number of FSMC states, which is presented for the first case. Figure 5.7 shows the steady state probabilities

of 9 states available for FSMC. The probabilities of states 1, 2, and 3 are pretty close. This means there is no switching among modulation modes in these states. States 4, 7, and 9 record the highest values among the states, which means at these states the modulation mode should be switched.

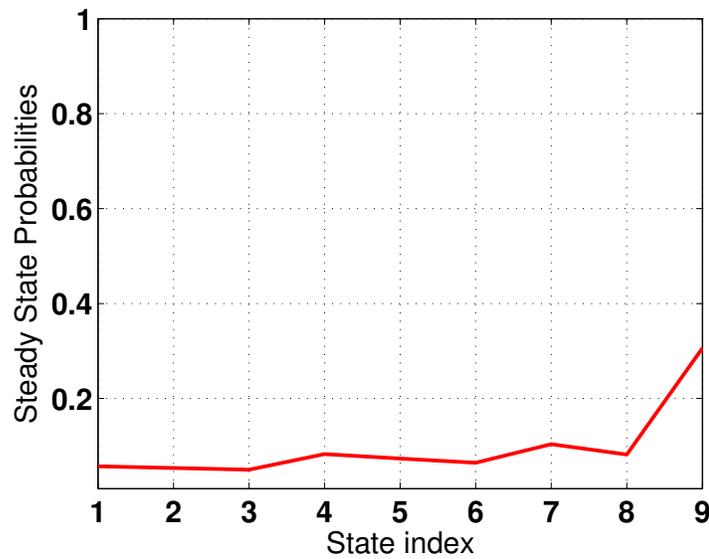


Figure 5.7: Steady state probabilities verses states in case target FER=  $10^{-3}$  using error rate curve partitioning method.

Figure 5.8 shows the level crossing rate verses the total number of states for Rayleigh distribution. The Doppler frequency is set to equal 31.66Hz. From the figure, the crossing rate increases from state 1 to state 3, and decreases from state 5 to 9. If the fading effect is low, the channel will be considered as good quality.

Figure 5.9a , Figure 5.9b, and Figure 5.9c show the transition probabilities  $p_{i,i+1}$  ,  $p_{i,i-1}$ , and  $p_{i,i}$  verses the total number of states. Figure 5.9a, shows the transitions of  $p_{i,i+1}$ . The figure shows the transitions from state 1 to state 3 randomly increases, and from states 4 to 7 randomly decreases compared with state 3. Figure 5.9b shows the transitions of  $p_{i,i-1}$ , the figure shows the transitions from states 2 to 3 increases, and from states 4 to 9 randomly decreases compared with state 3. The probability of not making any transition  $p_{i,i}$  is shown in Figure 5.9c. It records high values which represents slow fading, and it increases when the number of states increases.

In Figure 5.10, the average frame error rate per states is presented. As in the previous section, we calculate the actual average FER per state by substituting the parameter in Table 2.1 in equations (2.17) and (3.7) . Since the state index increases,

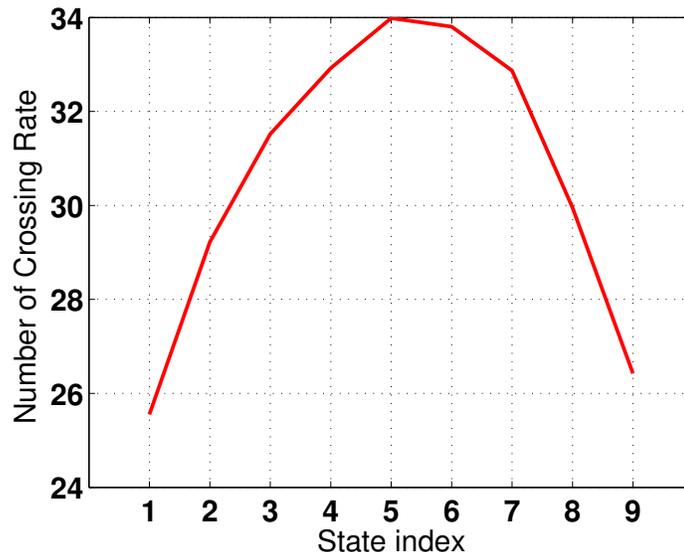


Figure 5.8: Number of crossing rate verses states in case target FER=  $10^{-3}$  using error rate curve partitioning method.

the SNR increases and the average FER decreases. For the first mode, which uses 4-QAM modulation scheme, the channel introduces low quality in the first state, which is considered as a noisy channel that has significant effects on the received signal. State 3 to 9 introduce an excellent quality, which reflects the decreasing of the FER for these states.

For the second mode which uses 16-QAM modulation scheme, the channel introduces bad quality in the first three states, which is considered as a noisy channel that has increased effects on the received signal. States 5 to 9 introduce a good quality. For the third mode which uses 32-QAM modulation scheme, the channel introduces bad quality from state 1 to state 6, which is considered as a noisy channel that has considerable effects on the received signal. States 7 to 9 introduce an acceptable quality.

Figure 5.12 and Figure 5.13 show the average throughput per state in cases the target FER=  $10^{-3}$  and  $10^{-6}$ . We calculate the throughput of each mode for each case as in the previous scenario.

For all modes and in both cases, the average throughput at the first states are low, and they increase dramatically toward last states. For each channel state, we can compare the value of average throughput of all states, so we can decide at which channel SNR range the system should switches among modes. In Table 5.2, we record

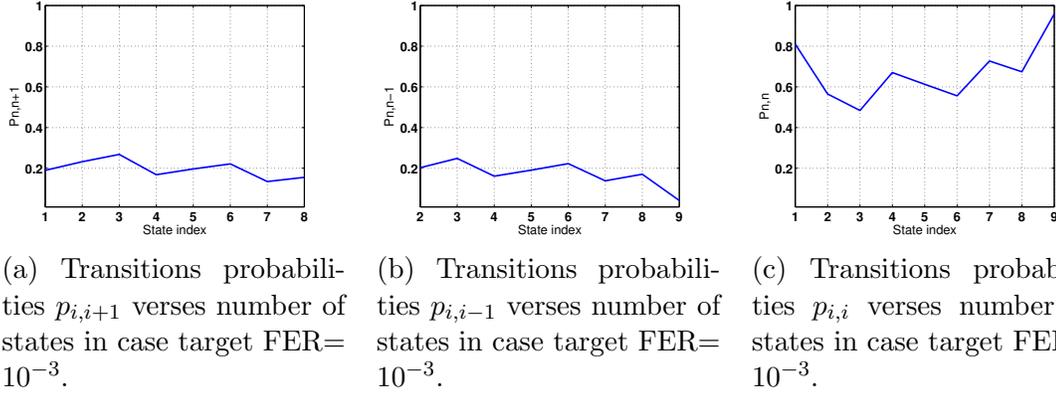


Figure 5.9: The transition probabilities for Rayleigh fading channel in case FER =  $10^{-3}$  using error rate curve partitioning method.

the results for the first case which the target FER=  $10^{-3}$ . In the table, we record the channel's SNR thresholds, the mode's received SNR boundaries, the constant  $c_k$ . Also we record the best switch's mode decisions to get the highest performance of the system. As in the table and from the Figure 5.12, the best mode switching from 4-QAM to 16-QAM is at state 6. In other words, the system should switch the modes whenever the received SNR is located in the SNR channel's threshold range of state 6. The best time to switch from mode 16-QAM to 32-QAM is at state 9 and then it keeps going with 32-QAM mode.

An interesting results came up in this case, which is the SNR channel threshold at state 5 is out of the first mode's SNR boundaries, but the system keeps the first mode on. This is because of the average throughput of the 16-QAM mode is lower than the average throughput of the 4-QAM mode at the same state. The same scenario is noticed in state 8 in the case of mode switching from 16-QAM to 32-QAM. The constant  $c_k$  in all states recorded good values, which are in the range between 2 and 4. In the case where the target FER =  $10^{-6}$ , we plot the results as in Figure 5.13, and we record the results in Table 5.3. In this case, the best mode to switch from 4-QAM to 16-QAM is at state 4. The best choice to switch from mode 16-QAM to 32-QAM is at state 7, and then it keeps going with 32-QAM mode. The constant  $c_k$  in all states recorded good values, which are the between 3 and 5. We can control the average time step by increasing the number of sub states and decreasing delta ( $\Delta n$ ) as we mentioned in Chapter 4.

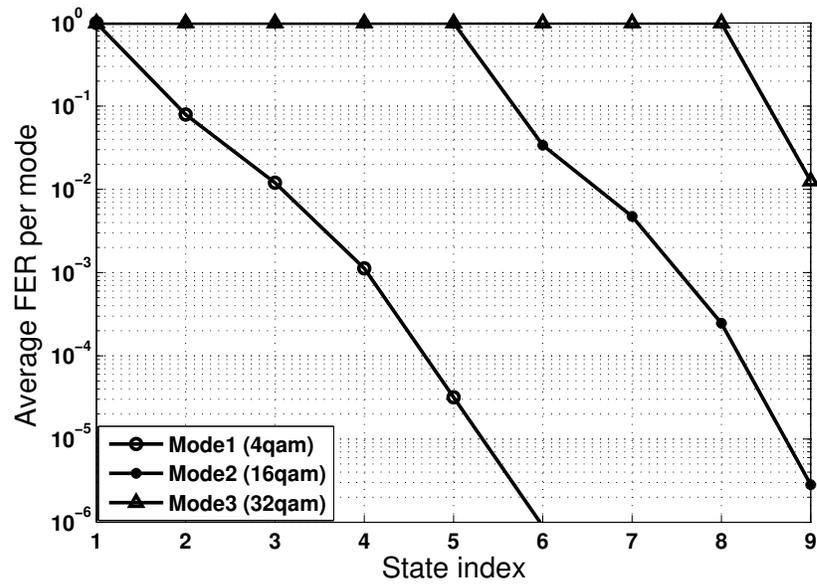


Figure 5.10: Average frame error rate per mode verses state index in case target FER=  $10^{-3}$  using error rate curve partitioning method

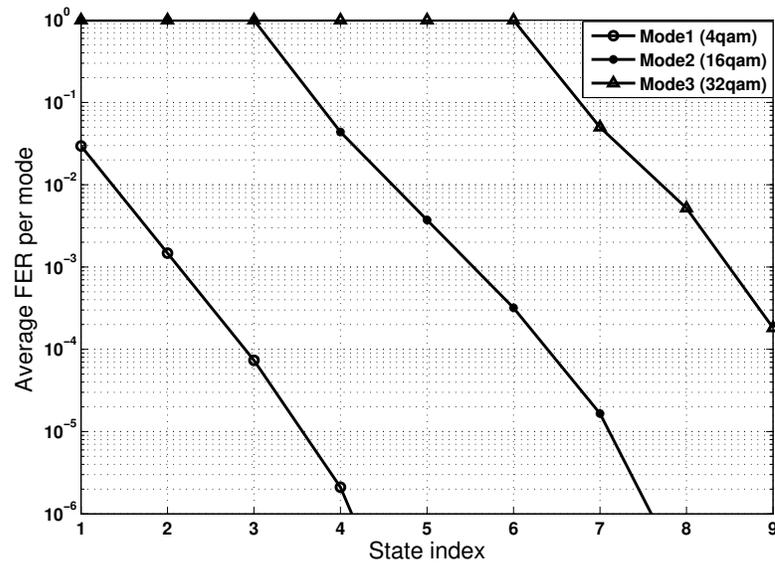


Figure 5.11: Average frame error rate per mode verses state index in case target FER=  $10^{-6}$  using error rate curve partitioning method

Table 5.2: Values of channel thresholds and the adaptive modulation thresholds at target FER =  $10^{-3}$  calculated using error rate curve partitioning method.

Channel thresholds ( $\gamma_i$ ) <sub>db</sub>	Selected mode	Mode's thresholds ( $\gamma_M$ ) <sub>db</sub>	Constant ( $c_k$ )
6.5349	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	2.3446
8.1792	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	1.9942
9.3691	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	1.7655
10.3021	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	2.9225
11.6527	4-QAM	$6.5349 < \gamma_{t1} < 10.3021$	2.5487
12.6814	16-QAM	$10.3021 < \gamma_{t2} < 13.5125$	2.2898
13.5125	16-QAM	$10.3021 < \gamma_{t2} < 13.5125$	3.89
14.7618	16-QAM	$10.3021 < \gamma_{t2} < 13.5125$	3.42
15.7309	32-QAM	$13.5125 < \gamma_{t3} < \text{INF}$	-

Table 5.3: Values of channel thresholds and the adaptive modulation thresholds at target FER =  $10^{-6}$  calculated using error rate curve partitioning method.

Channel thresholds ( $\gamma_i$ ) <sub>db</sub>	Selected mode	Mode's thresholds ( $\gamma_M$ ) <sub>db</sub>	Constant ( $c_k$ )
8.6274	4-QAM	$8.6274 < \gamma_{t1} < 12.3444$	2.9327
10.2445	4-QAM	$8.6274 < \gamma_{t1} < 12.3444$	2.4990
11.4202	4-QAM	$8.6274 < \gamma_{t1} < 12.3444$	2.2147
12.3444	16-QAM	$12.3444 < \gamma_{t2} < 15.5454$	3.6862
13.6902	16-QAM	$12.3444 < \gamma_{t2} < 15.5454$	3.2140
14.7162	16-QAM	$12.3444 < \gamma_{t2} < 15.5454$	3.21
15.5454	32-QAM	$15.5454 < \gamma_{t2} < \text{INF}$	4.9057
16.7948	32-QAM	$15.5454 < \gamma_{t2} < \text{INF}$	4.3089
17.7639	32-QAM	$15.5454 < \gamma_{t2} < \text{INF}$	-

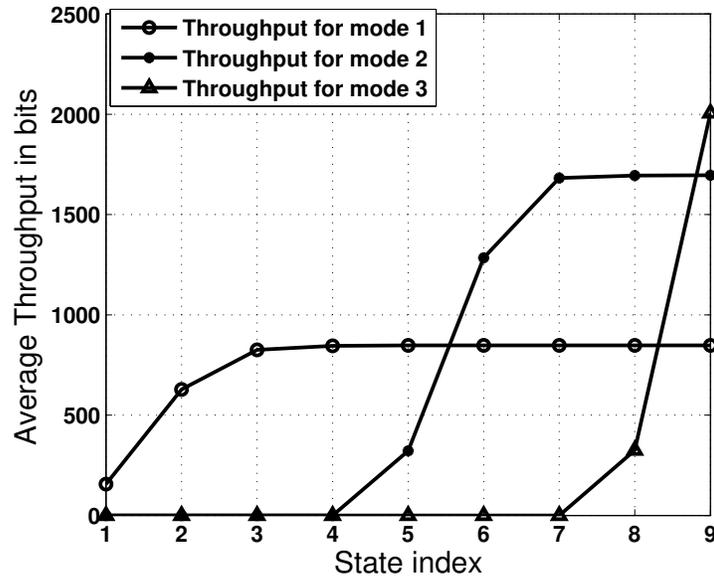


Figure 5.12: Average Throughput per state in bits of each mode in case target FER= $10^{-3}$  using error rate curve partitioning method

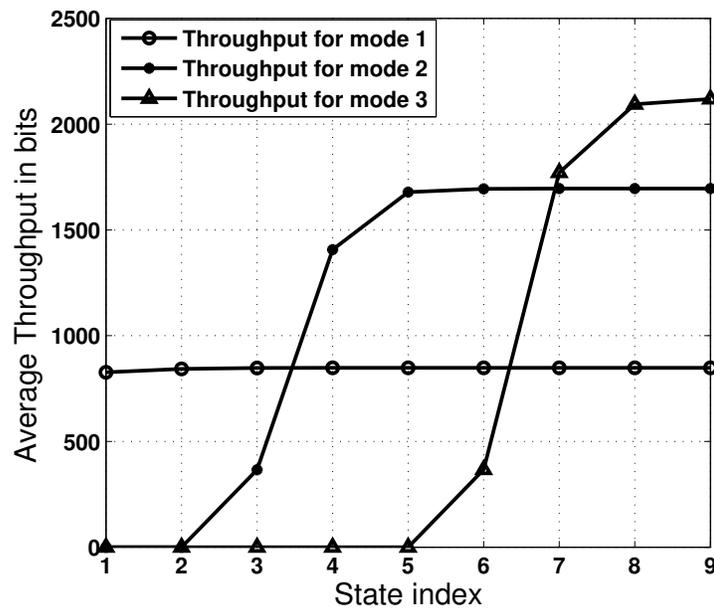


Figure 5.13: Average throughput per state in bits of each mode in case target FER= $10^{-6}$  using error rate curve partitioning method.

# Chapter 6

## Conclusion

In this project, we presented adaptive modulation for end to end wireless link system, which uses M-QAM modulation scheme over Rayleigh fading channel. We discussed the mechanism of selecting the modulation schemes of adaptive modulation system, which maintain appropriate frame error rate, according to the channel state.

Finite State Markov Chain (FSMC) model is used to model the channel states in terms of SNR. We presented two different FSMC channel partitioning methods. The first method is called Equilibrium Steady State method. We discussed the FSMC parameters of this method, and we used them to evaluate the system performance. In this method, the fading effects on the received signal is assumed to be constant, which means this method is specific to constant fading effects. Also, the average time duration of the channel state is not included in the SNR thresholds calculation. The second method is a new method for partitioning the SNRs of the channel states. It is used in order to get best choice of modulation scheme selection for the adaptive modulation system. In this method, we select the SNR levels of the channel based on a desired FER of the system with respect to the average time of channel state duration. We also introduce the relationship between the average time of channel state duration and the SNR levels.

The advantages of this method are the flexibility of choosing the thresholds of the channel directly from the desired (target) FER of the system, with lower number of states ( $K$ ) and less average time duration  $\tau_i$ . It can give a good evaluation for higher consultation size of modulation schemes without increasing the number of states  $K$  as in Equilibrium Steady States method that we motioned previously. Average time

duration  $\tau_i$  can be controllable using this method by changing the parameter Delta ( $\Delta n$ ).

For both methods, we discuss channel parameters and formulas that show the behaviors of channels. Then we use these parameters to evaluate the performance of the adaptive modulation system model. Performance measures of the FSMC model such as state time duration, state transition probability, steady state probability, and crossing rate for Rayleigh distribution are derived, plotted and analyzed. The average FER for each channel state and the average throughput per channel state for each modulation scheme (mode) are plotted. The numerical results of both methods are compared and discussed.

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