AIP Journal of Applied Physics

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Citation: J. Appl. Phys. **93**, 1672 (2003); doi: 10.1063/1.1536016
View online: http://dx.doi.org/10.1063/1.1536016
View Table of Contents: http://jap.aip.org/resource/1/JAPIAU/v93/i3
Published by the American Institute of Physics.

JOURNAL OF APPLIED PHYSICS VOLUME 93, NUMBER 3 1 FEBRUARY 2003

Ideal magnetocaloric effect for active magnetic regenerators

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(Received 15 July 2002; accepted 14 November 2002)

The active magnetic regenerator (AMR) uses a magnetic solid as a thermal storage medium and as a working material in a refrigeration cycle. Thermodynamically coupled to a heat transfer fluid, the regenerator produces a cooling effect and generates a temperature gradient across the AMR. The coupling between the heat transfer fluid and the magnetic refrigerant is a key aspect governing the operating characteristics of an AMR. To increase our understanding of AMR thermodynamics, we examine the entropy balance in an idealized active magnetic regenerator. A relation for the entropy generation in an AMR with varying fluid capacity ratios is derived. Subsequently, an expression describing the ideal magnetocaloric effect (MCE) as a function of temperature is developed for the case of zero entropy generation. Finally, the link between ideal MCE and refrigerant symmetry is discussed showing that an ideal reverse Brayton-type magnetic cycle cannot be achieved using materials undergoing a second-order magnetic phase transition. *© 2003 American Institute of Physics.* [DOI: 10.1063/1.1536016]

I. INTRODUCTION

The active magnetic regenerative refrigerator (AMRR) is a device that could be used to produce efficient and compact cooling over a broad range of temperatures. The heart of such a device is the active magnetic regenerator (AMR) which exploits the magnetocaloric effect (MCE) of some materials. The MCE, as it will be used in this article, is the property exhibited by a material whereby there is a characteristic reversible temperature change induced when the material is adiabatically exposed to a magnetic field change. By using such a material in a regenerator as the heat storage medium and as the means of work input, one creates an AMR. Unlike traditional working materials in many heat engines (such as gases), there is no single cycle describing the thermodynamic process (i.e., Stirling, Brayton, Carnot, etc.) undergone by all of the working material. Each section of the AMR experiences a unique cycle which may be similar to some traditional gas processes. The solution of the coupled energy equations for the fluid and magnetic material is a necessary step in predicting AMR performance. However, as with other cycles, it is useful to examine the process in the limit of reversible operation to gain a basic level of understanding.

The MCE is a key parameter determining the magnitude of cooling and the maximum temperature span that can be established. For all magnetic refrigerants near their ordering temperature both the MCE and heat capacity are strong functions of temperature and magnetic field. The highly nonlinear nature of these properties and the fact that they are material dependent complicates any analysis of an AMR. How the MCE should vary as a function of temperature so as to maximize cooling capacity is a fundamental question that has been studied since the idea of the AMR was developed. Implicit in determining the ideal MCE is the desire to have minimum entropy generation. This requirement arises from the fact that the refrigeration cycle must satisfy the second law and should be as efficient as possible. An early analysis of an AMR by Cross *et al.*[1] based upon entropy balance determined that the ideal MCE should vary linearly with temperature throughout the bed according to

$$\Delta T_{ad}^{ideal}(T) = \frac{\Delta T_{ad}(T_{ref})}{T_{ref}} T, \tag{1}$$

where ΔT_{ad} is the MCE at a temperature T, and the subscript ref is a reference point which could be the Curie temperature. Equation (1) is derived assuming the net entropy flows entering and leaving the AMR are equal and determined by the adiabatic temperature change of the material at the ends of the regenerator. It was assumed that this relation holds throughout the regenerator.

Further analysis of the problem was performed by Hall *et al.*[2] who determined that the ideal MCE need only satisfy Eq. (1) at the ends of the AMR and not throughout the bed. They also suggested that no unique ideal MCE exists for an AMR, however the material should satisfy the constraint:

$$\frac{d\Delta T_{ad}}{dT} \geqslant -1. \tag{2}$$

This constraint was further supported by modeling results of Smaili and Chahine.[3]

A recent study reports that the ideal MCE(T) profile is a function of AMR operating conditions and is given by

$$\Delta T_{ad}^{ideal}(T) = f(B) T^{m_{fC}/m_{fH}} - T, \tag{3}$$

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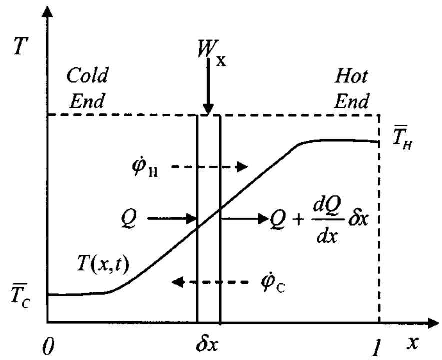

FIG. 1. Schematic representation of an AMR showing the net work, W, and heat flux, Q, at a differential section of the bed.

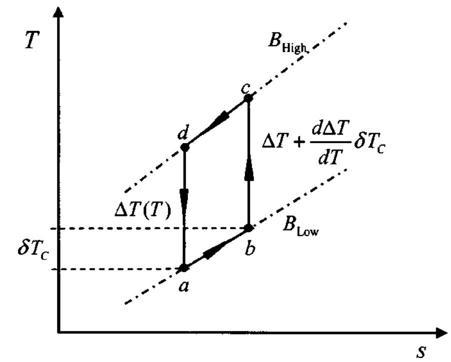

FIG. 2. Hypothetical cycle for the magnetic refrigerant at some cross section of the AMR.

where $f(B)$ is a function of magnetic field strength B, m_f is the fluid mass flow rate for the hot, H, and cold, C, blows, and T is the temperature of the bed at the cold end.[4] The details of this derivation are not published.

The purpose of this article is to investigate idealized AMR behavior via a simplified model of the coupled solid–fluid system. An expression for entropy generation in the AMR is derived and is then used to determine an analytic expression for the ideal MCE as a function of temperature. The ideal MCE function is then related to refrigerant properties in the case of a four-step isofield–isentropic process.

II. ANALYSIS

The system under consideration is graphically depicted in Fig. 1. The envelope of an AMR bed is shown with a dashed line while a section of differential thickness is highlighted. Hot and cold heat exchangers are not shown, but are assumed to be present on each end of the AMR. The bed is made up of a porous solid material that is the magnetic refrigerant and a fluid within the pores acts as the heat transfer medium. The fluid transfers heat between the cold heat exchanger, the refrigerant, and the hot heat exchanger. It is assumed that the heat transfer coefficient is sufficiently large so that the fluid and solid temperatures are essentially equal at all times. Furthermore, the thermal mass of the interstitial fluid is assumed to be negligible relative to the solid matrix. The fluid capacity rate $(\dot{m}c_p)$ for the cold and hot blows are shown as $\dot{\varphi}$. The *hot* blow is defined as the period when the AMR is in a high magnetic field and the fluid flux is from the cold end to the hot end of the regenerator bed. Likewise, the *cold* blow is when the fluid flux is in reverse and the bed is in a low magnetic field. Over a complete cycle, heat is absorbed by the fluid in the cold heat exchanger, rejected in the hot heat exchanger and the total work input is the volume integration of the work performed by each section of the bed. The AMR should be recognized as the combined solid–fluid system.

Many AMR devices built and tested to date have mimicked a reverse magnetic Brayton cycle in each section of the regenerator bed by using four distinct steps in a cycle:

(1) while the AMR is in a low magnetic field the fluid is blown from the hot side to the cold side of the bed, thereby warming the refrigerant;

(2) the AMR is exposed to a high magnetic field in a nearly adiabatic process, thereby causing a temperature rise at each section of the bed equal to the MCE at the local temperature;

(3) heat transfer fluid is blown through the bed from the cold side to the hot side causing a small constant-field temperature change in each section, and

(4) the bed is adiabatically removed from the magnetic field thus reducing the temperature of each section by the local MCE.

In the analysis that follows, the adiabatic steps are assumed to occur instantaneously and isentropically while the hot and cold blows occur over some equal time, τ_B.

Figure 2 shows the assumed refrigerant cycle occurring in a differential section of width δx at some location in the AMR. The cycle as described above is equivalent to the process starting at point "a" and proceeding alphabetically to return to the starting point. The refrigerant temperature change in the low isofield process, δT_C, is due to regeneration occurring during the cold blow, $\dot{\varphi}_C$. It is assumed that the fluid capacity rate is small relative to the thermal mass of the bed (a so-called, *short blow*) so that the isofield temperature changes are small and the magnitude of the MCE for the process b-c can be described by a first order Taylor series approximation in reference to point a. The resulting area within the T-s diagram is equivalent to the magnetic work input per unit mass for the material at location x.

A. Entropy generation

Focusing on the heat transfer fluid, an expression for the entropy generation per unit length in the AMR will be derived. In the following derivation, thermal diffusion is assumed negligible and the mass flow rates for each blow phase are assumed constant. The general entropy balance equation per unit length in the differential section δx is,

$$\frac{\partial S'}{\partial t} + \nabla \cdot \dot{S} = \dot{S}'_S + \dot{\sigma}', \tag{4}$$

where S' is the entropy per unit length, $\dot{S}$ is the rate of entropy flux through the section, $\dot{S}'_S$ is an entropy source per unit length due to heat transfer with the solid refrigerant, and $\dot{\sigma}'$ is the entropy generation rate per unit length due to irreversibilities in the fluid.

If a complete cycle for the AMR is considered when periodic steady-state operation has been achieved, the net entropy generation is found by integrating over a cycle:

$$\sigma' = \oint \frac{\partial S'}{\partial t} dt + \oint \nabla \cdot \dot{S} dt - \oint \dot{S}'_S dt. \tag{5}$$

Using Fig. 2 as a guide, an equation describing the local cyclic entropy generation for the fluid can be derived. For periodic steady state and assuming the refrigerant undergoes a reversible cycle as shown, the entropy change of the refrigerant during hot and cold blows are equal and, therefore, the last term in Eq. (5) is zero. Furthermore, for periodic steady-state conditions the material starts and ends at point a, the first term on the right-hand side is zero and the entropy generation relation becomes

$$\sigma' = \oint \nabla \cdot \dot{S} dt. \tag{6}$$

Assuming the mass flow rate is position independent, Eq. (6) can be written using the mass flow rate and mass specific entropy explicitly in one dimension:

$$\sigma' = \oint \dot{m} \frac{ds}{dx} dt. \tag{7}$$

For an ideal gas with negligible pressure drop, the mass specific entropy is related to heat capacity by

$$c_p dT = T ds. \tag{8}$$

Thus, the local entropy generation becomes

$$\sigma' = \oint \frac{\dot{m} c_p}{T} \frac{dT}{dx} dt. \tag{9}$$

If the thermal capacity of fluid is small, we may assume that the local temperature gradient remains constant over the duration of a blow and the temperature change of the material is small. The cycle integral can then be easily evaluated for the hypothetical process consisting of two isentropic steps and two isofield blows by noting that the mass flux is zero in the two adiabatic steps. A piecewise integration then gives

$$\sigma' = \frac{(\dot{m} c_p \tau_B)_H}{T + \Delta T} \frac{dT}{dx_H} - \frac{(\dot{m} c_p \tau_B)_C}{T} \frac{dT}{dx}. \tag{10}$$

Finally, using the relations

$$T_H = T + \Delta T(T)$$
$$\frac{dT}{dx_H} = \frac{dT}{dx} + \frac{d\Delta T(T)}{dx}, \tag{11}$$
$$\frac{dT}{dx_H} = \left(1 + \frac{d\Delta T}{dT}\right) \frac{dT}{dx}$$

the entropy generation per unit length is

$$\sigma' = \left[\frac{(\dot{m} c_p \tau_B)_H}{T + \Delta T} \left(1 + \frac{d\Delta T}{dT}\right) - \frac{(\dot{m} c_p \tau_B)_C}{T} \right] \frac{dT}{dx}. \tag{12}$$

Equation (12) can be written using the following definitions

$$\varphi_C \equiv (\dot{m} c_p \tau_B)_C,$$
$$\varphi_H \equiv (\dot{m} c_p \tau_B)_H, \tag{13}$$

resulting in

$$\sigma' = \left[\frac{\varphi_H}{T + \Delta T} \left(1 + \frac{d\Delta T}{dT}\right) - \frac{\varphi_C}{T} \right] \frac{dT}{dx}. \tag{14}$$

The parameters φ are seen to be the total fluid thermal capacity over the cold and hot blows. If these fluxes are equal, the AMR is said to be operating in a balanced condition; however, in general this need not be true. A balance parameter can be defined as

$$\beta \equiv \frac{\varphi_C}{\varphi_H}. \tag{15}$$

We know from the second law that entropy generation is a positive quantity; therefore, the following must be satisfied for real conditions (entropy generation >0):

$$\left[\frac{\varphi_H}{T + \Delta T} \left(1 + \frac{d\Delta T}{dT}\right) - \frac{\varphi_C}{T} \right] \frac{dT}{dx} > 0. \tag{16}$$

When the temperature increases monotonically, i.e.,

$$\frac{dT}{dx} > 0, \tag{17}$$

at all locations in the AMR, then the following inequality must be true:

$$\left(1 + \frac{d\Delta T}{dT}\right) > \frac{\varphi_C}{\varphi_H} \left(1 + \frac{\Delta T}{T}\right). \tag{18}$$

B. Ideal MCE

The purpose of this section is to derive an analytic expression for the ideal MCE as a function of temperature. If the ‘‘ideal’’ AMR is defined as one with zero entropy generation, then, using Eq. (14), the following differential equation is true:

$$\frac{\varphi_H}{T + \Delta T} \left(1 + \frac{d\Delta T}{dT}\right) - \frac{\varphi_C}{T} = 0. \tag{19}$$

Equation (19) can be rewritten as

$$\frac{d\Delta T^{\text{ideal}}}{dT} - \frac{\varphi_C}{\varphi_H} \frac{\Delta T^{\text{ideal}}}{T} = \frac{\varphi_C}{\varphi_H} - 1. \tag{20}$$

Equation (20) is an ordinary first-order differential equation for the ideal MCE as a function of temperature and can be solved using the boundary condition $\Delta T(T_{\text{ref}}) = \Delta T_{\text{ref}}$:

$$\Delta T^{\text{ideal}}(T) = (\Delta T_{\text{ref}} + T_{\text{ref}}) \left(\frac{T}{T_{\text{ref}}} \right)^{\beta} - T, \tag{21}$$

where the balance parameter has been used. As can be seen, Eq. (21) is similar in form to Eq. (3), and, if the AMR is balanced ($\beta = 1$), the resulting expression is the same as Eq. (1). Thus, Eq. (1) is a particular case of the more general expression, Eq. (21). Figure 3 shows some ideal MCE curves for various conditions of balance. The reference conditions are for Gd with a field change of 0–2 T shown as the dashed curve.

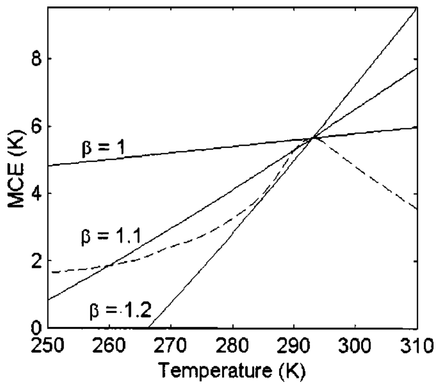

FIG. 3. Ideal MCE curves for various conditions of balance. The reference condition is for Gd with a field change of 0–2 T (dashed line).

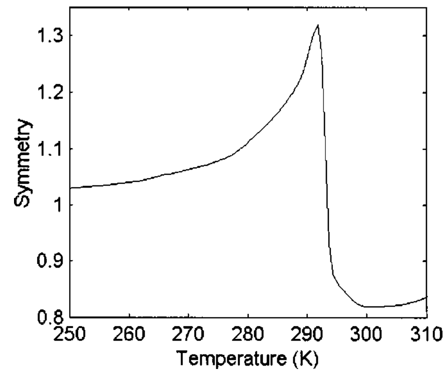

FIG. 4. Symmetry of Gd for a 0–2 T field change.

C. Refrigerant cycle

In the preceding analysis the entropy generation is derived by an entropy balance focusing on the heat transfer fluid. The assumed cycle for the solid, shown in Fig. 2, is reversible; thus, if there is entropy generation in the AMR it is assumed to be external to the refrigerant (in the fluid). Using the short blow assumption, a simple entropy balance on the refrigerant is easily derived. The temperature change of the refrigerant during the cold blow is δT_C. The temperature change during the hot blow can be found using the Taylor series expansion and subtracting the temperature at c from d:

$$\delta T_H = \left(T + \delta T_C + \Delta T + \frac{d\Delta T}{dT}\delta T_C \right) - (T + \Delta T). \tag{22}$$

Further manipulation gives the following:

$$\frac{\delta T_H}{\delta T_C} = 1 + \frac{d\Delta T}{dT}. \tag{23}$$

Now, for the assumed cycle, the refrigerant entropy change during the hot blow equals the entropy change during the cold blow. The entropy change can be approximated by:

$$ds = \frac{c_B \delta T}{T}, \tag{24}$$

so, equating hot to cold gives

$$\frac{\delta T_H}{\delta T_C} = \frac{T_H}{T}\frac{c_{BC}}{c_{BH}} = \frac{T + \Delta T}{T}\frac{c_{BC}}{c_{BH}}. \tag{25}$$

The isentropic ratio of the low-field heat capacity to the high-field heat capacity is defined as the refrigerant symmetry, ς:

$$\varsigma \equiv \frac{c_{BC}}{c_{BH}} = \frac{c_B(T, B_L)}{c_B(T + \Delta T, B_H)}, \tag{26}$$

where B_L is the low-field strength, and B_H is the strength of the high field.

Using the definition of symmetry, the equivalence of Eqs. (23) and (25) results in the following differential equation:

$$\frac{d\Delta T}{dT} - \varsigma \frac{\Delta T}{T} - (\varsigma - 1) = 0. \tag{27}$$

D. "Ideal" material properties

There are some interesting implications of the basic thermodynamic analysis. Two key differential equations were derived, one for zero entropy generation for the fluid and the other for the refrigerant:

$$\text{Fluid ideal MCE:} \quad \frac{d\Delta T}{dT} - \beta \frac{\Delta T}{T} = \beta - 1, \tag{28}$$

$$\text{Solid ideal MCE:} \quad \frac{d\Delta T}{dT} - \varsigma \frac{\Delta T}{T} = \varsigma - 1. \tag{29}$$

If both equations are to be satisfied and entropy generation is to remain zero, then the solid and fluid temperatures must be equal at all locations and the following must be true

$$\varsigma = \beta. \tag{30}$$

Thus, for an ideal AMR, the condition of balance must match the refrigerant symmetry. Because the ideal MCE as a function of temperature is determined by the balance, intuitively, there should be a relationship between balance and symmetry since heat capacity and MCE are derived from the entropy curves. In practice, balance is generally constant throughout the AMR (i.e., the fluid capacity rate at all locations is the same during a blow); therefore, the refrigerant symmetry must be independent of position. Thus, for a position independent field change in the AMR, the symmetry must be independent of temperature to satisfy the constraint of Eq. (30). Carpetis[5] qualitatively discussed this inherent cycle irreversibility in an AMR due to nondeal entropy curves of the refrigerant. Here, we have linked the material symmetry to AMR balance.

Figure 4 shows the symmetry of Gd for a 0–2 T field change using the data of Dan'kov *et al.*[6] Near the phase transition temperature, the refrigerant symmetry is a strong nonlinear function of temperature. Thus, for an AMR with constant β the constraint of Eq. (30) will not be satisfied over any significant temperature span using such a material. Moreover, even if the AMR has zero longitudinal conduction, the heat transfer coefficient is infinite, and the fluid has zero viscosity the simple isofield–adiabatic cycle does not satisfy a local solid–fluid entropy balance. This suggests that the ideal isofield–adiabatic cycle shown in Fig. 2 cannot satisfy a zero entropy generation constraint and, therefore, the real AMR cycle must be different when using refrigerants undergoing a second order magnetic phase transition.

The properties of an ideal refrigerant can be further specified. The functional relationship between the low and high field entropy curves for an ideal material is defined using Eqs. (21) and (30) and the definition of heat capacity

$$c_B \equiv T\left(\frac{\partial s}{\partial T}\right)_B . \tag{31}$$

An expression of the following form results giving the relationship between the slopes of the low field and high field entropy curves:

$$\frac{\left(\frac{\partial s}{\partial T}\right)_L}{\left(\frac{\partial s}{\partial T}\right)_H} = \beta\left(\frac{\Delta T_{\mathrm{ref}} + T_{\mathrm{ref}}}{T}\right)\left(\frac{T}{T_{\mathrm{ref}}}\right)^{\beta}, \tag{32}$$

where H and L signify the derivatives at the high and low fields respectively on an isentrope. Thus, as previously reported for the specific case of a balanced AMR,[1] the entropy curves for an ideal refrigerant are diverging for all conditions of balance greater than one.

IV. CONCLUSIONS

An AMR operating with a four-step isofield–adiabatic cycle and where the fluid thermal flux is small relative to the bed thermal mass is studied. An expression for local entropy generation is derived and used to produce a relation describing the ''ideal'' MCE as a function of balance and temperature. An entropy balance on the refrigerant produces a relation for the ideal MCE similar in form to the fluid-derived expression, but in terms of refrigerant symmetry and temperature. It is shown that a zero-entropy generation condition can only be satisfied for this cycle when the symmetry and fluid capacity balance are equal. Using the symmetry relationship of a prototypical second-order magnetic refrigerant (Gd), it is shown that for this type of magnetic refrigerant such a cycle cannot be satisfied without entropy generation. Finally, the relationship between the slopes of the low and high field entropy curves of an ideal AMR refrigerant is derived in terms of MCE and balance.

ACKNOWLEDGMENT

The support of Natural Resources Canada is greatly appreciated.

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