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ORIGINAL RESEARCH PAPER

Stability analysis of 2-D discrete and continuous state-space systems

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Abstract

This paper presents new stability conditions for two-dimensional (2-D) systems in state-space description. Both discrete and continuous systems are studied. These results are based on the criteria first presented by Huang, De Carlo, Strintzis, Murray, Delsarte, et al. and on the discrete Lyapunov equation with complex elements for 2-D systems. The stability properties of the Mansour matrix are also used for stability testing in state-space. Criteria for the VSHP property of 2-D polynomials are further presented using the continuous Lyapunov equation with complex elements and the stability properties of the Schwarz matrix form. The stability properties of the Schwarz matrix are also used for testing the VSHP property of 2-D polynomials in state-space. The proposed new criteria are non-conservative for the stability analysis of 2-D discrete and continuous systems and achieve the aim of reducing the original 2-D problem as much as possible to a set of 1-D stability tests. Numerical examples are given to illustrate the utility of the proposed conditions.

1 | INTRODUCTION

The study of two and higher dimensional systems has attracted significant attention in recent years due to a variety of applications such as broadband beamforming [1], multipass processes [2], video and lightfield processing [3], digital filtering, image processing, gas filtration, thermal processes, geophysics, medical electronics, sensor networks, two-dimensional (2-D) discrete control systems, and so forth [4–8]. The common feature of these applications is that the signals to be processed are functions of two or more variables, which can be various combinations of space and/or time and so forth.

2-D linear systems are dynamical systems whose signals propagate in two independent dimensions but reciprocally influence each other. 2-D systems can be discrete (in both dimensions), continuous (in both dimensions), or mixed (continuous in one dimension and discrete in the other dimension). An important property of the system designed for such applications is stability. The stability conditions developed for 2-D discrete systems require to test whether the 2-D characteristic polynomial is devoid of zeros in the unit bidisc. On the other hand, for the 2-D continuous systems it is required to test whether a 2-D con-

tinuous polynomial is a very strict Hurwitz polynomial (VSHP), which is equivalent to say that the polynomial has no zeros in the left half bi-plane including the points at infinity.

The first studies of the stability analysis of 2-D discrete systems were carried out in the frequency domain leading to many well known tests [9], [10–29]. The introduction of state-space models for 2-D systems allowed the investigation of stability using the state-space approach and several stability criteria have been formulated in state-space using either the Roesser model [30] or the Fornasini-Marchesini model [31–62, 73]. In particular in [37] the state-space versions for several internally stability criteria of 2-D discrete systems have been presented.

It is a heavy computational task to test whether the 2-D characteristic polynomial is devoid of zeros in the unit bidisc. Therefore efforts have been directed to reducing the original 2-D problem as much possible to a set of one-dimensional stability tests [9]. The purpose of this paper is to establish new stability criteria for 2-D discrete/continuous systems described in state space. The proposed new criteria achieve the aim of reducing the original 2-D problem as much possible to a set of one-dimensional stability tests.

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The paper is organized as follows. In Section 2, some preliminary results on 2-D stability analysis and the Mansour matrix are presented. In Section 3, the new criteria for the stability of 2-D discrete systems are presented. These new stability criteria are the state space versions of the stability conditions in [11, 12, 63, 64]. Also they are formulated using the Lyapunov equation with complex elements. Further, the stability properties of the Mansour matrix in [65] are used for stability testing in discrete state-space. In Section 4, the very strict Hurwitz character of 2-D polynomials VSHP is studied. New criteria for the VSHP property are presented. The formulation of these criteria is based on the stability condition by Delsarte et al. in [64] on the Lyapunov equation formulated with complex elements and the stability properties of the Schwarz matrix in [65]. Further, the stability properties of the Schwarz matrix are used for testing the VSHP property of 2-D polynomials in state-space. Numerical examples are given to illustrate the utility of the proposed conditions in Sections 3 and 4.

Notation: U^2 denotes the open unit bidisc

$$U^2 = \{(\zeta_1, \zeta_2) \mid |\zeta_1| < 1, |\zeta_2| < 1\}$$

\overline{U}^2 denotes the closed unit bidisc

$$\overline{U}^2 = \{(\zeta_1, \zeta_2) \mid |\zeta_1| \leq 1, |\zeta_2| \leq 1\}$$

and T^2 denotes the distinguished boundary of the unit bidisc

$$T^2 = \{(\zeta_1, \zeta_2) \mid |\zeta_1| = 1, |\zeta_2| = 1\},$$

Similar definitions will hold for the one-variable regions U , \overline{U} and T . Notations such as $P(T^2) \neq 0$ will be used to mean that the polynomial $P(\zeta_1, \zeta_2)$ has no zeros on T^2 . I_n is the $n \times n$ unity matrix, the transpose (conjugate transpose) of any matrix A is denoted by A^t (A^*), the determinant is denoted by $\det[A]$ and the entries of the matrix are denoted by a_{kl} . The set of real numbers and complex numbers are denoted by \mathbf{R} and \mathbf{C} , respectively. The stability region of a stable polynomial is defined as a region in which the polynomial is allowed to have its roots.

2 | PRELIMINARIES

A single input-single output 2-D linear, shift invariant, discrete system can be represented by the local state-space model (LSSM) due to Roesser [30] given by

$$\begin{bmatrix} x^b(k+1, l) \\ x^v(k, l+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x^b(k, l) \\ x^v(k, l) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(k, l) \quad (1)$$

$$y(k, l) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x^b(k, l) \\ x^v(k, l) \end{bmatrix} + Du(k, l) \quad (2)$$

where $x^b(k, l) \in \mathbf{R}^n$ and $x^v(k, l) \in \mathbf{R}^m$ are the horizontal state vector and the vertical state vector, respectively, $u(k, l) \in \mathbf{R}$ is the input and $y(k, l) \in \mathbf{R}$ is the output.

The system matrix A is given by

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (3)$$

where the matrices $A_{i,j}$, $i, j = 1, 2$ have appropriate dimensions.

If the 2-D discrete system (1), (2) has no non-essential singularities of the second kind on the unit bidisc T^2 [66], then the internal stability of the 2-D discrete system represented with a model of the above type depends on the zeros of the characteristic polynomial $b(\zeta_1, \zeta_2)$ given by:

$$b(\zeta_1, \zeta_2) = \det \begin{bmatrix} I_n - \zeta_1 A_{11} & -\zeta_1 A_{12} \\ -\zeta_2 A_{21} & I_m - \zeta_2 A_{22} \end{bmatrix} \quad (4)$$

$$b(\zeta_1, \zeta_2) = \sum_{p=0}^n \sum_{r=0}^m \alpha_{pr} \zeta_1^p \zeta_2^r \quad (5)$$

The polynomial $b(\zeta_1, \zeta_2)$ given in (5) can be rewritten as

$$\begin{aligned} b(\zeta_1, \zeta_2) &= \det[I_n - \zeta_1 A_{11}] \cdot \\ &\det[I_m - \zeta_2(A_{22} + A_{21}(\zeta_1^{-1} I_n - A_{11})^{-1} A_{12})] \\ &= \det[I_m - \zeta_2 A_{22}] \cdot \\ &\det[I_n - \zeta_1(A_{11} + A_{12}(\zeta_2^{-1} I_m - A_{22})^{-1} A_{21})] \end{aligned} \quad (6)$$

if the involved inverses exist.

The condition for internal stability of the 2-D discrete system (1), (2) is given by [9]

$$b(\zeta_1, \zeta_2) \neq 0 \quad (7)$$

in the closed unit bidisc \overline{U}^2 . Important stability criteria for 2-D discrete systems in the frequency domain have been presented in the literature. These criteria are presented in the following statements.

C_1 [63, 64]: The 2-D discrete system (1), (2) is internally stable if and only if

$$b(\zeta, \zeta e^{j\omega}) \neq 0, |\zeta| \leq 1, \omega \in [0, 2\pi] \quad (8)$$

C_2 [11, 12, 64]: The 2-D discrete system (1), (2) is internally stable if and only if

- (i) $b(1, \zeta) \neq 0$ in \overline{U}
- (ii) $b(\zeta, 1) \neq 0$ in \overline{U}
- (iii) $b(T^2) \neq 0$

C_3 [11, 12, 64]: The 2-D discrete system (1), (2) is internally stable if and only if

- (i) $b(z_1, 0) \neq 0$ in \overline{U}
- (ii) $b(1, z_2) \neq 0$ in \overline{U}
- (iii) $b(T^2) \neq 0$

The state space versions of the above criteria will be presented in the next section. Also a 2-D stability testing method based on the Mansour matrix will be presented.

Consider now the basic statements of the proposed method. Let $f(z) = z^n + \sum_{i=1}^n \alpha_i z^{n-i}$ be n th degree polynomial with complex coefficients $\alpha_i \in \mathbf{C}$. We define a sequence of polynomials $f(z) = F_n(z), F_{n-1}(z), F_{n-2}(z), \dots$ of degree $n, n-1, n-2, \dots$ via the table in [65, 74]. Let

$$F_q(z) = \sum_{i=0}^q \alpha_{iq} z^{q-i}, \alpha_{0q} = 1 \quad (9)$$

with the table

$$\left. \begin{array}{cccccc} 1 & \alpha_{1n} = \alpha_1 & \alpha_{2n} = \alpha_2 & \dots & \dots & \alpha_{nn} = \alpha_n \\ \alpha_{nn}^* & \alpha_{n-1,n}^* & \alpha_{n-2,n}^* & \dots & \dots & 1 \\ 1 & \alpha_{1,n-1} & \alpha_{2,n-1} & \dots & \alpha_{n-1,n-1} & \\ \alpha_{n-1,n-1}^* & \alpha_{n-2,n-1}^* & \alpha_{n-3,n-1}^* & \dots & 1 & \\ \vdots & & & & & \end{array} \right\} \quad (10)$$

where the superscript star denotes a complex conjugation of coefficients. The formula giving entries of any odd numbered row in terms of entries of the previous two rows is

$$\alpha_{i,q-1} = \frac{\alpha_{i,q} - \alpha_{qq} \alpha_{q-1,q}^*}{1 - |\alpha_{qq}|^2} \quad (11)$$

Define

$$\Delta_q = \alpha_{qq}, q = 1, 2, \dots, n$$

The matrix in companion form associated with $f(z)$ is given by

$$E = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -\alpha_n & -\alpha_{n-1} & \dots & -\alpha_2 & -\alpha_1 \end{bmatrix} \quad (12)$$

Theorem 1. [65] Let E be the companion matrix associated with $f(z)$ as defined in (12); with the Δ_q as defined above, let the Mansour matrix associated with this system is given by (14) and with the α_{iq} as defined

above, let

$$T = \begin{bmatrix} 1 & \alpha_{1,n-1}^* & \alpha_{2,n-1}^* & \dots & \alpha_{n-1,n-1}^* \\ 0 & 1 & \alpha_{1,n-2}^* & \dots & \alpha_{n-2,n-2}^* \\ 0 & 0 & 1 & \dots & \alpha_{n-3,n-3}^* \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & 1 & \alpha_{11}^* \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad (13)$$

Then $TET^{-1} = \Sigma$

$$\Sigma = \begin{bmatrix} -\Delta_n \Delta_{n-1}^* & 1 - |\Delta_{n-1}|^2 & 0 & \dots & 0 \\ -\Delta_n \Delta_{n-2}^* & -\Delta_{n-1} \Delta_{n-2}^* & 1 - |\Delta_{n-2}|^2 & 0 & 0 \\ \dots & \dots & \dots & \dots & 0 \\ -\Delta_n \Delta_1^* & -\Delta_{n-1} \Delta_1^* & \dots & -\Delta_2 \Delta_1^* & 1 - |\Delta_1|^2 \\ -\Delta_n & -\Delta_{n-1} & \dots & -\Delta_2 & -\Delta_1 \end{bmatrix} \quad (14)$$

The necessary and sufficient conditions for the eigenvalues of the matrix E to be inside the unit disc (stability condition) is

$$|\Delta_q| < 1, q = 1, 2, \dots, n \quad (15)$$

where Δ_q are the appropriate entries of the Jury stability table. For continuous systems, analogous results as in Theorem 1 have been presented in [65], but are omitted here for brevity.

3 | STABILITY FOR 2-D DISCRETE SYSTEMS IN STATE-SPACE

In this section, the new stability criteria for 2-D discrete systems will be presented.

Theorem 2. The following statements are equivalent:

- 2.1: The 2-D discrete system (1), (2) is internally stable
- 2.2: The matrix

$$P(e^{j\omega}) = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ e^{j\omega} \mathcal{A}_{21} & e^{j\omega} \mathcal{A}_{22} \end{bmatrix}, \omega \in [0, 2\pi] \quad (16)$$

is stable

- 2.3: The matrix Lyapunov equation with complex elements

$$G(e^{j\omega}) - P(e^{j\omega})^t G(e^{j\omega}) P(e^{j\omega}) = W(e^{j\omega}) \quad (17)$$

has a positive definite and Hermitian solution $G(e^{j\omega})$ for any given $(n+m) \times (n+m)$ positive definite and Hermitian matrix $W(e^{j\omega})$ for $\omega \in [0, 2\pi]$ and $P(e^{j\omega})$ is given by (16)

2.4:

(i) The matrix $A_{22} + A_{21}(\bar{z}_1^{-1}I_n - A_{11})^{-1}A_{12}$ with $|\bar{z}_1| = 1$ has no eigenvalues on T ,

(ii) The matrix $A_{22} + A_{21}(I_n - A_{11})^{-1}A_{12}$ is stable

(iii) The matrix A_{11} is stable and $\det[I_m - [A_{22} + A_{21}(\bar{z}_1^{-1}I_n - A_{11})^{-1}A_{12}]] \neq 0$ in \bar{U}

2.5:

(i) The matrix $A_{11} + A_{12}(\bar{z}_2^{-1}I_m - A_{22})^{-1}A_{21}$ with $|\bar{z}_2| = 1$ has no eigenvalues on T

(ii) The matrix $A_{11} + A_{12}(I_m - A_{22})^{-1}A_{21}$ is stable

(iii) The matrix A_{22} is stable and $\det[I_n - [A_{11} + A_{12}(\bar{z}_2^{-1}I_m - A_{22})^{-1}A_{21}]] \neq 0$ in \bar{U}

2.6:

(i) The matrix A_{11} is stable

(ii) The matrix $A_{22} + A_{21}(\bar{z}_1^{-1}I_n - A_{11})^{-1}A_{12}$ with $|\bar{z}_1| = 1$ has no eigenvalues on T

(iii) The matrix $A_{22} + A_{21}(I_n - A_{11})^{-1}A_{12}$ is stable

2.7:

(i) The matrix A_{22} is stable

(ii) The matrix $A_{11} + A_{12}(\bar{z}_2^{-1}I_m - A_{22})^{-1}A_{21}$ with $|\bar{z}_2| = 1$ has no eigenvalues on T

(iii) The matrix $A_{11} + A_{12}(I_m - A_{22})^{-1}A_{21}$ is stable

Proof. It follows from (4), (5), (8) that

$$b(\bar{z}, \bar{z}e^{j\omega}) = \det \begin{bmatrix} I_n - \bar{z}A_{11} & -\bar{z}A_{12} \\ -\bar{z}e^{j\omega}A_{21} & I_m - \bar{z}e^{j\omega}A_{22} \end{bmatrix} = \det [I_{n+m} - \bar{z}P(e^{j\omega})] \text{ with } P(e^{j\omega}) = \begin{bmatrix} A_{11} & A_{12} \\ e^{j\omega}A_{21} & e^{j\omega}A_{22} \end{bmatrix}$$

Therefore the statement 2.2 gives the state space version for the statement C_1 or equivalently for the statement 2.1. The statement 2.3 gives an equivalent state space version for the statement C_1 using the Lyapunov equation formulated with complex elements.

It follows from (6) that the statements 2.4 and 2.5 give the state space version for the statement C_2 .

Also it follows from (6) that the statements 2.6 and 2.7 give the state space version for the statement C_3 . \square

Remark 1. Statement C_2 implies also the following conditions for 2.4:

(iv) The matrix A_{11} has no eigenvalues on T

(v) $\det[I_n - A_{11}] \neq 0$

These conditions are satisfied because of the requirement of stability for A_{11} . Similar comments apply to statements 2.5, 2.6, 2.7.

In statement 2.2, the 2-D stability problem has been reduced in a 1-D stability problem with a matrix $P(e^{j\omega})$ with entries which are functions of $e^{j\omega}$ for $\omega \in [0, 2\pi]$.

In statement 2.3, the 2-D stability problem has been reduced in the solution of a 1-D Lyapunov equation formulated with matrices with entries which are functions of $e^{j\omega}$ for $\omega \in [0, 2\pi]$.

In statements 2.4–2.7, the 2-D stability problem has been reduced in 1-D stability problems and checking for existence of eigenvalues on the unit disc of matrices with entries which are functions of $e^{j\omega}$ for $\omega \in [0, 2\pi]$ and matrices with entries which are real numbers.

It should be mentioned that using the arguments in [63, 64] numerous other stability tests in state space can be derived from the statement C_1 by replacing one or both variables by a finite Blaschke product since this is precisely the class of functions analytic in U , continuous on U and having unit modulus on T . An introduction to finite Blaschke products is presented in [72].

Also the statement 2.2 is interesting because it is shown below that it leads in a natural way to a stability test in the open unit disc. Hence the following statement can be stated.

C_4 [63, 64]: The 2-D characteristic polynomial $b(\bar{z}_1, \bar{z}_2)$ has no zeros in U^2 if and only if

$$b(\bar{z}, \bar{z}e^{j\omega}) \neq 0, |\bar{z}| < 1, \omega \in [0, 2\pi] \quad (18)$$

The following theorem gives the state space version for the statement C_4 .

Theorem 3. The 2-D characteristic polynomial $b(\bar{z}_1, \bar{z}_2)$ has no zeros in U^2 if and only if the matrix

$$P(e^{j\omega}) = \begin{bmatrix} A_{11} & A_{12} \\ e^{j\omega}A_{21} & e^{j\omega}A_{22} \end{bmatrix}, \omega \in [0, 2\pi] \quad (19)$$

has all its eigenvalues in U .

The 2-D stability problem in statement C_4 has been reduced in a 1-D stability problem with a matrix $P(e^{j\omega})$ with entries which are functions of $e^{j\omega}$ for $\omega \in [0, 2\pi]$.

Corollary 1. A necessary condition for the system in (1), (2) to be internally stable is the matrix

$$P(1) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \text{ is stable.}$$

For a given positive definite and Hermitian matrix $W(e^{j\omega})$, one may solve the Lyapunov matrix equation (17) to obtain a matrix solution $G(e^{j\omega})$ which is a Hermitian matrix. Denoting the λ th-order principal minor of $G(e^{j\omega})$ by $g_\lambda(\omega)$, we note that the principal minors $g_\lambda(\omega)$, $1 \leq \lambda \leq (m+n)$ are real rational functions of one real variable ω over the closed interval $[0, 2\pi]$. For checking the positive definiteness for the matrix $G(e^{j\omega})$, one has to check for positivity of all the principal minors. According to a result by Siljak in [67], one has to check for positivity of all the principal minors at one point, say $\omega = 0$ and the determinant of the matrix for all $\omega \in [0, 2\pi]$. We thus have the following corollary:

Corollary 2. The system in (1), (2) is internally stable if and only if

(i) $g_\lambda(1) > 0, \lambda = 1, 2, \dots, (n+m) - 1$

(ii) $g_{n+m}(\omega) > 0, \omega \in [0, 2\pi]$

At this point, an equivalent formulation for theorem 2 is presented. This formulation gives a new stability test for 2-D discrete systems described in state space.

Theorem 4. The following statements are equivalent:

- 4.1: The 2-D discrete system (1), (2) is internally stable
 4.2: The matrix

$$P(e^{j\omega}) = \begin{bmatrix} A_{11} & A_{12} \\ e^{j\omega}A_{21} & e^{j\omega}A_{22} \end{bmatrix}, \omega \in [0, 2\pi] \quad (20)$$

is stable

- 4.3: The matrix Lyapunov equation with complex elements

$$G(e^{j\omega}) - P(e^{j\omega})^t G(e^{j\omega}) P(e^{j\omega}) = W(e^{j\omega}) \quad (21)$$

has a positive definite and Hermitian solution $G(e^{j\omega})$ for any given $(n+m) \times (n+m)$ positive definite and Hermitian matrix $W(e^{j\omega})$ for $\omega \in [0, 2\pi]$ and $P(e^{j\omega})$ is given by (20).

- 4.4: The quantities $\Delta(e^{j\omega})$ of the Mansour matrix associated with the matrix $P(e^{j\omega})$ satisfy the following conditions:

$$(i) |\Delta_\lambda(1)| < 1, \lambda = 1, 2, \dots, n+m \quad (22)$$

$$(ii) \prod_{\lambda=1}^{n+m} (1 - |\Delta_\lambda(e^{j\omega})|^2) > 0, \forall \omega \in [0, 2\pi] \quad (23)$$

Proof. To prove the statement 4.4 we use the arguments presented in [65]. The matrix $P(e^{j\omega})$ can be transformed in the Mansour form with complex elements, see (14). The sufficient and necessary conditions for stability are given by

$$|\Delta_\lambda(e^{j\omega})| < 1, \lambda = 1, 2, \dots, n+m \quad (24)$$

where $\Delta_\lambda(e^{j\omega}), \lambda = 1, 2, \dots, n+m$ are the quantities of the Mansour matrix associated with the matrix $P(e^{j\omega}), \omega \in [0, 2\pi]$.

Consider the matrix Lyapunov equation with complex elements given in (17). Using the arguments presented in [65] we choose

$$W(e^{j\omega}) = \text{diag}\{(1 - |\Delta_{n+m}(e^{j\omega})|^2)^2, 0, \dots, 0\} \quad (25)$$

then the solution of the Lyapunov equation with complex elements (17) is given by

$$G(e^{j\omega}) = \text{diag}\{1 - |\Delta_{n+m}(e^{j\omega})|^2, \dots, \prod_{\lambda=1}^{n+m} (1 - |\Delta_\lambda(e^{j\omega})|^2)\} \quad (26)$$

The positive definiteness of the matrix $G(e^{j\omega})$ implies the stability of the 2-D discrete system. Based on the results by Siljak in [67], the positive definiteness of the matrix $G(e^{j\omega})$ is equivalent to conditions (22) and (23).

Theorems 2 and 3 give new criteria for the stability of 2-D discrete systems in state-space description.

Theorem 4 gives a new method for checking the stability of 2-D discrete systems in state-space description. It should be mentioned that there are advantages of the checking the conditions (22) and (23) instead of checking whether the matrix $P(e^{j\omega})$ in (20) is stable $\forall \omega \in [0, 2\pi]$.

The proposed new criteria are non-conservative for the analysis of the 2-D discrete systems and achieve the aim of reducing the original 2-D problem as much possible to a set of one-dimensional stability tests. The conditions are applicable to 2-D linear systems described by any of state-space and transfer function models.

The following example illustrates the utility of the proposed conditions. \square

Example 1. Consider the 2-D discrete system with order (2,1) with the following system matrix

$$A = \begin{bmatrix} 0.4 & 0.5 & -0.5 \\ 0 & 0.25 & -0.25 \\ -0.4 & 0 & 0.25 \end{bmatrix}$$

The matrix $P(e^{j\omega})$ is given by

$$P(e^{j\omega}) = \begin{bmatrix} 0.4 & 0.5 & -0.5 \\ 0 & 0.25 & -0.25 \\ -0.4e^{j\omega} & 0 & 0.25e^{j\omega} \end{bmatrix}$$

Let us check the stability of the system using the arguments in statement 4.4.

The matrix $P_c(e^{j\omega})$ in companion form associated with the matrix $P(e^{j\omega})$ is given by

$$P_c(e^{j\omega}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.025e^{j\omega} & 0.0375e^{j\omega} - 0.1 & 0.25e^{j\omega} + 0.65 \end{bmatrix}$$

The quantities $\Delta_\lambda(e^{j\omega}), \lambda = 1, 2, 3$ are

$$\Delta_3(e^{j\omega}) = -0.025(e^{j\omega})$$

$$\Delta_2(e^{j\omega}) = \frac{0.099063 - 0.035e^{-j\omega}}{0.999375}$$

$$\Delta_1(e^{j\omega}) = \frac{-0.594709668 - 0.2245625e^{j\omega} + 0.024517968e^{-j\omega}}{0.987712011 - 3.4671875(10)^{-3}(e^{j\omega} + e^{-j\omega})},$$

$$\omega \in [0, 2\pi]$$

or equivalently

$$\Delta_1(e^{j\omega}) = \frac{-0.594709668 - 0.2245625e^{j\omega} + 0.024517968e^{-j\omega}}{0.987712011 - 6.934375(10)^{-3}x},$$

$$x = \frac{e^{j\omega} + e^{-j\omega}}{2}, x \in [-1, 1]$$

The quantities must satisfy the condition $|\Delta_\lambda(e^{j\omega})| < 1, \lambda = 1, 2, 3$ for all $\omega \in [0, 2\pi]$.

According to a result by Siljak in [67] and Jury in [9] one has to check for positivity the all $\Delta_\lambda(e^{j\omega}), \lambda = 1, 2, 3$ at one point, say $\omega = 0$ and the $\Delta_1(e^{j\omega})$ for all $\omega \in [0, 2\pi]$.

$$|\Delta_3(1)| = 0.025 < 1$$

$$|\Delta_2(1)| = 0.0641025 < 1$$

$$\text{Let us check the condition } |\Delta_1(e^{j\omega})| < 1.$$

For the numerator of $\Delta_1(e^{j\omega})$ it follows

$$|-0.594709668 - 0.2245625e^{j\omega} + 0.024517968e^{-j\omega}| <$$

$$|-0.594709668| + |-0.2245625e^{j\omega}| +$$

$$|0.024517968e^{-j\omega}| = 0.84379014$$

$$\text{For } x \in [-1, 1] \text{ it follows that } 0.98077764 < 0.987712011 - 6.934375(10)^{-3}x < 0.9946464$$

The value of the numerator of the fraction is less than 0.84379014 and the value of denominator is greater than 0.9807776. Therefore the fraction has value less than 1 for all $\omega \in [0, 2\pi]$. The condition is satisfied and the system is stable.

Let us check the stability of the system using the arguments in statement 4.3.

Choosing

$$W(e^{j\omega}) = \text{diag}\{(1 - |\Delta_3(e^{j\omega})|^2)^2, 0, \dots, 0\}$$

the solution of the Lyapunov equation with complex elements in (17) is given by

$$G(e^{j\omega}) = \text{diag}\{1 - |\Delta_3(e^{j\omega})|^2, \dots, \prod_{\lambda=1}^3 (1 - |\Delta_\lambda(e^{j\omega})|^2)\}$$

where $\Delta_\lambda(e^{j\omega}), \lambda = 1, 2, 3$ are the quantities of the Mansour matrix associated with the matrix $P(e^{j\omega}), \omega \in [0, 2\pi]$. The quantities satisfy the following conditions

$$|\Delta_\lambda(e^{j\omega})| < 1, \lambda = 1, 2, 3$$

Thus the matrix $G(e^{j\omega})$ is positive definite. The matrix $G(e^{j\omega})$ is the solution of the matrix Lyapunov equation in statement 4.3, hence the system is stable.

The proposed new stability test for 2-D discrete systems requires testing the positivity of one or more functions of ω for all $\omega \in [0, 2\pi]$ as the other stability tests presented in the literature. It allows the extension of frequency domain 2-D stability analysis methods presented in the literature to 2-D discrete systems described in state-space using Roesser's model.

4 | THE 2-D VERY STRICT HURWITZ CHARACTER IN STATE-SPACE

In this section, the Hurwitz character of 2-D polynomials will be considered. The open right-half plane is defined as $D = \{s : \text{Re}(s) > 0\}$ and the closed right-half plane as $\bar{D} = \{s : \text{Re}(s) \geq 0\}$. The extended region including the point at infinity is given by $\bar{\bar{D}} = \bar{D} \cup \{\infty\}$ where ∞ stands for $1/0$ and \cup indicates union. The extension of the definition above to 2-D case renders

$$\bar{\bar{D}}^2 = [\bar{D} \cup \{\infty\}]^2 = \bar{\bar{D}} \times \bar{\bar{D}} \quad (27)$$

where

$$\bar{\bar{D}}^2 = \{(s_1, s_2), \text{Re}(s_1) \geq 0, \text{Re}(s_2) \geq 0\} = \bar{\bar{D}} \times \bar{\bar{D}} \quad (28)$$

Let the polynomial $c(s_1, s_2)$ of a 2-D continuous system [9, 71]:

$$c(s_1, s_2) = \sum_{k=0}^n \sum_{l=0}^m c_{kl} s_1^k s_2^l \quad (29)$$

The polynomial $c(s_1, s_2)$ can be rewritten as the characteristic polynomial of the matrix A, that is

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (30)$$

$$c(s_1, s_2) = \det \begin{bmatrix} s_1 I_n - A_{11} & -A_{12} \\ -A_{21} & s_2 I_m - A_{22} \end{bmatrix} \quad (31)$$

The polynomial $c(s_1, s_2)$ is a very strict Hurwitz polynomial (VSHP) [9] if $1/c(s_1, s_2)$ does not possess any singularities in the region

$$\{(s_1, s_2) | \text{Re}(s_1) \geq 0, \text{Re}(s_2) \geq 0, |s_1| \leq \infty, |s_2| \leq \infty\} \quad (32)$$

The above corresponds to $\bar{\bar{D}}^2$ given in (27). The VSHP polynomial does not have any zeros in the closed right half of the biplane including infinite distant points. The properties of VSHP are studied in [70].

The VSHP character can be tested in the frequency domain using any of the existing tests [9, 68, 69]. Delsarte et al. in [64] proposed a criterion to test the stability of 2-D continuous polynomials. This condition was later extended in [9] to deal with the problems discussed in [69] for the points at infinity in the (s_1, s_2) biplane. The criterion for the 2-D VSHP in state-space will be presented in the following statement.

C_5 [9, 64]: The polynomial $c(s_1, s_2)$ in (29), (31) has no zeros in $\bar{\bar{D}} \times \bar{\bar{D}}$ if and only if

$$(i) c(s, s + j\beta) \neq 0, \text{ in } \bar{\bar{D}}, \forall \beta \in \mathbf{R} \cup \{\infty\} \quad (33)$$

$$(ii) c(\infty, j\omega) \neq 0, \forall \omega \in \mathbf{R} \quad (34)$$

The state space versions for the conditions (33), (34) are presented in the following theorem:

Theorem 5. *The following statements are equivalent:*

5.1: *The polynomial $c(s_1, s_2)$ in (29), (31) is a VSHP*

5.2: *(i) $c(\infty, j\omega) \neq 0, \forall \omega \in \mathbf{R}$*

(ii) *The matrix $P(j\beta) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} - j\beta I_m \end{bmatrix}$ is stable for all $\beta \in \mathbf{R} \cup \{\infty\}$*

5.3: *(i) $c(\infty, j\omega) \neq 0, \forall \omega \in \mathbf{R}$*

(ii) *The matrix Lyapunov equation with complex elements*

$$P^*(j\beta)G(j\beta) + G(j\beta)P(j\beta) = -W(j\beta) \quad (35)$$

has a positive definite and Hermitian solution for any given $(n+m) \times (n+m)$ positive definite and Hermitian matrix $W(j\beta)$ for all $\beta \in \mathbf{R} \cup \{\infty\}$.

5.4: *(i) $c(\infty, j\omega) \neq 0, \forall \omega \in \mathbf{R}$*

(ii) *The quantities $\delta_\lambda(\beta)$ of the Schwarz matrix $H(j\beta)$ associated with the matrix $P(j\beta)$ are positive at one point, say, $\beta = 0$ for $\lambda = 1, 2, \dots, (n+m)$ and the quantity $\delta_{n+m}(\beta)$ is positive for all $\beta \in \mathbf{R} \cup \{\infty\}$.*

Proof. It follows from (31) that

$$c(s, s + j\beta) = \det \begin{bmatrix} sI_n - A_{11} & -A_{12} \\ -A_{21} & (s + j\beta)I_m - A_{22} \end{bmatrix} = \det [sI_{n+m} - \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} - j\beta I_m \end{bmatrix}] = \det [sI_{n+m} - P(j\beta)]$$

Then the VSHP condition can be formulated as follows:

$$c(s, s + j\beta) = \det [sI_{n+m} - P(j\beta)] \neq 0 \quad (36)$$

in the region $\text{Re}(s) \geq 0, \beta \in \mathbf{R} \cup \{\infty\}$. This leads to the state space conditions given in statement 5.2.

Another formulation using the Lyapunov equation with complex elements is given in statement 5.3.

To prove the statement 5.4 we use the arguments presented in [65]. It is shown there that the matrix $P(j\beta)$ can be transformed in the Schwarz form with complex elements, see (37). The sufficient and necessary conditions for stability are given by

$$\delta_\lambda(\beta) > 0, \beta \in \mathbf{R} \cup \{\infty\}, \lambda = 1, 2, \dots, (n+m) \quad (38)$$

Consider the matrix Lyapunov equation with complex elements given in (35). We choose

$$W(j\beta) = \text{diag}\{2\delta_1^2(\beta), 0, 0, \dots, 0\} \quad (39)$$

then the solution of the Lyapunov equation with complex elements (35) is given by

$$G(j\beta) = \text{diag}\{\delta_1(\beta), \delta_1(\beta)/\delta_2(\beta), \delta_1(\beta)/\delta_2(\beta)\delta_3(\beta), \dots\} \quad (40)$$

where $\delta_\lambda(\beta), \lambda = 1, 2, \dots, n+m$ are the quantities of the Schwarz matrix associated with the matrix $P(j\beta), \beta \in \mathbf{R} \cup \{\infty\}$. Thus the matrix $G(j\beta)$ is positive definite provided that (38) holds.

The positive definiteness of the matrix $G(j\beta)$ implies the VSHP character of the 2-D polynomial.

Based on the results by Siljak in [67], the positivity of these quantities $\delta_\lambda(\beta)$ is equivalent to the following conditions:

$$\delta_\lambda(0) > 0, \lambda = 1, 2, \dots, (n+m-1) \quad (41)$$

$$\delta_{n+m}(\beta) > 0, \beta \in \mathbf{R} \cup \{\infty\} \quad (42)$$

This constitutes a simplification by not requiring all the quantities $\delta_\lambda(\beta)$ to be positive for all $\beta \in \mathbf{R} \cup \{\infty\}$.

In statement 5.2 the 2-D VSHP problem has been reduced in a 1-D stability problem with a matrix $P(j\beta)$ with entries which are functions of $j\beta$ for all $\beta \in \mathbf{R} \cup \{\infty\}$.

In statement 5.3 the 2-D VSHP problem has been reduced in the solution of a 1-D Lyapunov equation formulated with

$$H(j\beta) = \begin{bmatrix} -\delta_1(\beta) + j\gamma_1(\beta) & 1 & 0 & \dots & \dots & 0 \\ -\delta_2(\beta) & j\gamma_2(\beta) & 1 & 0 & \dots & 0 \\ 0 & -\delta_3(\beta) & j\gamma_3(\beta) & 1 & \dots & 0 \\ \dots & \dots & -\delta_4(\beta) & j\gamma_4(\beta) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & 1 \\ 0 & 0 & 0 & \dots & -\delta_{n+m}(\beta) & j\gamma_{n+m}(\beta) \end{bmatrix} \quad (37)$$

□

matrices with entries which are functions of $j\beta$ for all $\beta \in \mathbf{R} \cup \{\infty\}$.

In statement 5.4 the 2-D VSHP problem has been reduced in a 1-D stability problem with a matrix $H(j\beta)$ with entries which are functions of $j\beta$ for all $\beta \in \mathbf{R} \cup \{\infty\}$. The matrix $H(j\beta)$ is in Schwarz form and the stability properties of this matrix are applied for the testing of the 1-D stability problem.

Theorem 5 gives new criteria and tests for the 2-D very strict Hurwitz character in state-space description. It should be mentioned that there are advantages of the checking the conditions (41) and (42) instead of checking whether the matrix $P(j\beta)$ in the statement 5.2 is stable $\forall \beta \in \mathbf{R} \cup \{\infty\}$.

Also the same theorem gives an alternative condition to the ones proposed in [9, 36 68] for checking the very strict Hurwitz property of a 2-D polynomial.

The proposed new criteria are non-conservative for the analysis of the 2-D VSHP property and achieve the aim of reducing the original 2-D problem as much possible to a set of one-dimensional stability tests. The conditions are applicable to 2-D polynomials described by any of state-space models.

The following example illustrates the utility of the proposed conditions.

Example 2. Consider the 2-D continuous system with order (2; 1) with the following system matrix A

$$A = \begin{bmatrix} -3 & -2 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The characteristic polynomial is given by

$$C(s_1, s_2) = s_1^2 s_2 + s_1^2 + 3s_1 s_2 + 3s_1 + 2s_2 + 2$$

The matrix $P_c(j\beta)$ in companion form is given by

$$P_c(j\beta) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 - j2\beta & -5 - j3\beta & -4 - j\beta \end{bmatrix}$$

We will check the VSHP character of the system using the arguments in statement 5.4.

$$C(\infty, j\omega) \neq 0, \omega \in \mathbf{R}$$

The quantities $\delta_\lambda(\beta)$ satisfy the conditions

$$\delta_1 = 4 > 0,$$

$$\delta_2(\beta) = \frac{9}{2} + \frac{3}{16}\beta^2 > 0, \beta \in \mathbf{R} \cup \{\infty\} \text{ and}$$

$$\delta_3(\beta) = \frac{1}{2} + \frac{1.59375\beta^2(4.5 + 4.703\beta^2)}{(4.5 + 0.1875\beta^2)^2} > 0, \beta \in \mathbf{R} \cup \{\infty\}$$

Hence the characteristic polynomial $C(s_1, s_2)$ is very strict Hurwitz polynomial.

Let us check the VSHP character of the polynomial using the arguments in statement 5.3.

Consider the Hermitian matrices $G(j\beta)$ and $W(j\beta)$ given by

$$G(j\beta) = \begin{bmatrix} \delta_1 & 0 & 0 \\ 0 & \frac{\delta_1}{\delta_2(\beta)} & 0 \\ 0 & 0 & \frac{\delta_1}{\delta_2(\beta)\delta_3(\beta)} \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & \frac{4}{\frac{9}{2} + \frac{3}{16}\beta^2} & 0 \\ 0 & 0 & g_{33}(\beta) \end{bmatrix}$$

with

$$g_{33}(\beta) = \frac{4}{\left(\frac{9}{2} + \frac{3}{16}\beta^2\right) \left(\frac{1}{2} + \frac{1.59375\beta^2(4.5 + 4.703\beta^2)}{\left(\frac{9}{2} + \frac{3}{16}\beta^2\right)^2}\right)}$$

$$W(j\beta) = \begin{bmatrix} 2\delta_1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 32 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the matrix $G(j\beta)$ is positive definite for all $\beta \in \mathbf{R} \cup \{\infty\}$. The matrix $G(j\beta)$ is the solution of the matrix Lyapunov equation in statement 5.3, therefore the characteristic polynomial $C(s_1, s_2)$ is very strict Hurwitz polynomial.

The proposed new test for the VSHP property of 2-D continuous polynomials requires testing the positivity of one or more functions of β for all $\beta \in \mathbf{R} \cup \{\infty\}$ as the other tests presented in the literature. It allows the extension of frequency domain analysis methods for the VSHP property of 2-D continuous polynomials presented in the literature to 2-D continuous systems described in state-space.

5 | CONCLUSION

This paper presents the stability analysis of 2-D discrete and continuous state-space systems. The study was carried out using the Roesser state-space model. The state-space versions for several stability criteria of 2-D discrete systems have been presented. Several sufficient and necessary stability conditions have been developed for the 2-D discrete systems in state-space form based on the 1-D Lyapunov theory with a complex parameter and the properties of the Mansour matrix. Criteria for the VSHP property of 2-D polynomials were further presented using the continuous Lyapunov equation with complex elements and the stability properties of the Schwarz matrix form. The obtained conditions are applicable to 2-D linear systems

described by any of state-space and transfer function models. The proposed conditions have been demonstrated by numerical examples.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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