OPTIMAL HARVESTING MODELS IN FOREST MANAGEMENT -- A SURVEY

By

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Abstract

A survey of optimal timber harvesting models is presented. Even-aged and uneven-aged single-stand models, and forest-level harvest scheduling models are considered.

1. Introduction

The question of what should be the objective of forest management has been a subject of considerable discussion in the forestry literature. While, in part, this reflects the different interest of various user groups (e.g. timber supply, recreational use, environmental protection etc), it also reflects the distinct traditions of the different disciplines of the discussants. Foresters have been brought up with concepts such as culmination age, sustained yield, even-flow and normal forest forming the keystones of their thought. To economists, on the other hand, concepts such as time-discounting and the efficient allocation of resources, are equally central, and to them the forester's basic concepts remain irrelevant unless they can be justified within the framework of economic theory.

It is the purpose of this paper to provide a survey of the models of optimal forest harvesting that have been proposed from both disciplines. Broadly speaking the models can be divided into two categories: single-stand (or stand-level) models and whole-forest (or forest-level) models. In the former category, the management unit is a single-stand, and management decisions such as harvesting, thinning, silviculture, etc., are made only with respect to this stand; what is happennning in other parts of the forest or of the economy are considered to be exogeneous factors. Stand-level management models can be further subdivided into even-aged and uneven-aged stand models. In the even-aged stand management system a stand is envisioned as having a dated time of origin and a dated point of harvest and replacement. In contrast the uneven-aged system utilizes periodic entry and harvest with no identifiable time of stand initiation and replacement.

In forest-level models the management unit is a forest comprising many even-aged stands of different ages and possibly of different species, growing possibly on sites of different productivity etc. Management decisions are made globally for the forest as a whole.

In this paper we shall consider firstly, even-age stand-level models (Section 2) and then forest-level models (Section 3). Finally we shall look at the much sparser literature on uneven-aged stands (Section 4). Only models and methods that fall broadly into the category of "mathematical optimization" are discussed. Methods based on simulation are not included.

2. Even-aged Stand Models

The problem which has received the greatest amount of attention in single stand models, is the determination of the optimal cutting, or rotation age. We shall consider this problem first.

Starting with the simplest case, consider a forest site on which a stand of trees is growing. Suppose that the volume of usable timber in the stand is a function, $v(a)$ of stand age a and suppose further that once a stand is cut a new stand with identical growth characteristics is immediately regenerated. If one always cuts a stand whenever it reaches the age T, the long-run average volume yield from the site will be

$$\lim_{t\to\infty}\left[\frac{\text{volume cut in interval }[0,t]}{t}\right]$$

(1)

$$= \lim_{t\to\infty}\left[\frac{[t/T]v(T)}{t}\right],$$

where $[x]$ is the greatest integer less than x. On taking the limit it can be seen that the long-run average per annum volume yield is

$$v(T)/T,$$

which is known as the mean annual increment (MAI) in the forestry literature. The rotation age T_c which maximizes the MAI or long-run average volume yield is known as the culmination age and, from simple calculus, satisfies

$$(2) \qquad v'(T) = v(T)/T.$$

Thus at the culmination age the current rate of volume growth, or current annual increment, $v'(T)$, is equal to the average rate of volume growth, or MAI. The traditional objective of forestry management has been to obtain a normal forest, containing equal areas in each age class up to the culmination age, so that if each stand is harvested at the culmination age, an even-flow of timber volume, proportional to the maximum MAI, will be extracted from the forest.

Both the goal of obtaining an even-flow of timber from the forest, and the policy of cutting at the culmination age have been criticized on economic grounds. We shall defer discussion of the goal of even-flow until the next section and consider here only the appropriateness of the use of the culmination age to determine the rotation period.

Cutting at the culmination age maximizes the long-run physical output of timber from a site. Insofar as this maximization criterion ignores the costs of inputs, such as those for land and labour, cutting at the culmination age does not necessarily maximize economic productivity. Forest economists (e.g. Duerr, Fedkiw and Guttenburg, 1956) have used the term financial maturity to indicate the age at which a stand should optimally be cut from an economic point of view. There have been several different formulations and solutions to the problem of determining the rotation age corresponding to financial maturity (see Samuelson (1976) for a lively account), in spite of the fact that what is now accepted as the correct formulation of the problem was presented well over a century ago by Martin Faustmann (1849), albeit without providing explicit conditions for determining the optimal rotation. (For an English translation of Faustmann's paper from the original German, see Gane, 1968).

Suppose that the value of timber in a stand of age a, net of harvesting costs, is $V(a)$ dollars, and suppose that the case of reestablishing a stand of similar type on the site is C dollars. Suppose initially one has a newly established stand on the site. If one sets the rotation age at T years and reestablishes a new stand immediately after every harvest, then one earns a stream of net incomes each of size

$$V(T) - C$$

at times $T, 2T, 3T, \ldots$ years after the initial time. The net present value of this stream of incomes is

$$\text{(3)} \qquad PV = \sum_{j=1}^{\infty} e^{-\delta jT}\big(V(T)-C\big)$$

where δ is the instantaneous discount rate reflecting the opportunity cost of capital and is related to the per annum discount rate i, by $e^{\delta} = 1 + i$. Summing the geometric series in (3) gives

$$\text{(4)} \qquad PV = \frac{V(T) - C}{e^{\delta T} - 1}$$

and also

$$\text{(5)} \qquad PV = e^{-\delta T}\, V(T) + e^{-\delta T}(PV-C).$$

Setting the derivative of (4) equal to zero gives the following equation, known as the Faustmann equation, whose solution determines the optimal rotation age, T_F:

(6) $$V'(T) = \frac{\delta(V(T)-C)}{1-e^{-\delta T}} .$$

The value of PV with rotation at the optimum, T_F, minus the establishment costs, C, is known as the land-expectation value, and represents the market value of the bare site, assuming that the land has no value other than for growing trees, and that the market is perfect, in the sense that the selling price of the land is exactly equal to the maximum present value of revenues that can be earned from it. If we set (PV-C) equal to this land expectation value in the right side (r.h.s.) of (5), we see that the r.h.s. is equal to the sum of the revenue that can be earned through cutting in one rotation plus the selling price of the site after the cut, all discounted to a present value. Thus T_F maximizes the revenues earned over one rotation, through cutting the trees and immediately selling the site.

A marginal economic interpretation along the above lines can be given to the Faustmann equation by re-writing it as

(7) $$V'(T) = \delta V(T) + \delta\left[\frac{V(T) - C}{e^{\delta T} - 1} - C\right].$$

The left-hand side represents the marginal increase in value over the time period T to T + 1 if the stand is not cut. The first terms on the r.h.s. represents the interest that can be earned over the same period on the net revenue obtained from cutting at age T, while the second term represents the interest that can be earned on the revenue obtained from selling the site, after the cut. Thus at the optimum, the marginal growth in value through not cutting equals the marginal growth in value through cutting the stand and selling the site (Clark 1976, Pearse 1967).

The Faustmann formulation as it is commonly presented nowadays (see e.g. Pearse 1967, Gregory 1972) can include other costs, such as those for pre-commercial thinning and other silvicultural treatments etc. Also revenues from thinnings etc., prior to the terminal cut can be included.

Other formulations to the financial maturity problem have been proposed. They include the maximization of present net worth, and the maximization of the internal rate of return.

The present net worth (Duerr et al, 1956, Fisher, 1930) is defined as the present discounted value of the revenues minus that of the costs, over one rotation. Thus in the simplest case, with revenues and costs incurred only at the cutting time, the present net worth of the stand is equal to the first term on the r.h.s. of (5). It thus ignores the value of the site. As Samuelson (1976) points out the difference between the rotation lengths that maximize present net worth and maximize the net present value of the stream of all future revenues, will usually be small for realistic discount rates. However, the maximization of present net worth leads to a rotation period which is independent of costs of planting, which have to be considered as sunk costs (Gregory 1972, p. 287).

The maximization of internal rate of return (Boulding, 1955) seeks to maximize the effective rate of return on capital invested as costs (of planting, silviculture etc.) over one rotation. This formulation regards land rental costs, if it includes them at all, as being exogenously determined. However the only way to determine the correct market value for land rent is through the Faustmann formulation. The rotation age determined by the maximization of internal rate of return is independent of the interest rate.

Besides Samuelson (1976), several other authors have given accounts and comparisons of the various formulations. These include Gaffney (1960), Bentley & Teeguarden (1965), Gregory (1972) and Chang (1984). Numerical comparisons of the consequences of various procedures for determining rotation length are given in Riitters, Brodie and Kao (1982).

As Samuelson (1976) points out, in order to arrive at a simple solution to the Faustmann formulation of the rotation problem certain "heroic assumptions" must be made. These include: (i) fixed (or at least known) future lumber prices and costs; (ii) fixed (or known) future interest rates; (iii) identical (or known) biological growth characteristics for every stand established on the site; (iv) the assumption of a perfect market in forest land (or if a government owns public lands that it rents them at rates determined in a perfect market, and that it conducts its own forestry operations to earn the maximum rent using the market rate of interest). To these could be added: (v) that a more or less instantaneous harvest at the selected rotation age can be accomplished in spite of the location of the site relative to roads and established logging operations; (vi) that externalities such as the recreational value of forest land, its usefulness for wildlife habitat, flood prevention, oxygen production etc., can all be safely ignored; (vii) that one can be certain that catastrophies such as fire or pest infestation will not destroy a stand, and (viii) that a perfect capital market exists in which unrestricted amounts can be loaned or borrowed at the same interest rate. When one recognizes that rotation lengths of 60 to 100 years are not uncommon in forestry operations, one realizes that these assumptions are, indeed, truly heroic.

Many authors have investigated the consequences, for rotation length and land expectation value, of relaxing one or more of the above assumptions, Bare and Waggener (1980) present numerical results on the effect on land value, of prices and costs varying in time, but with a fixed rotation period. Hardie, Daberkow and McConnell (1984), and Lamberson and Barber (1985) consider a similar situation but with rotation periods allowed to be determined optimally.

Heaps (1981) investigates the effect on rotation age of allowing costs of logging to depend on the age of the stand, distance from mill centre etc. He identifies the minimal conditions that the harvesting net revenue function must satisfy, in order that the effects of changes in cost or revenue parameters on optimal rotation length, may be determined. He further generalizes the results to allow for variable replanting and silvicultural expenditures. Jackson (1980) provides a micro-economic analysis of the timber industry, in which the production function for a firm is allowed to depend on a variable level of establishment intensity.

Chang (1982) considers the impact of various forms of forest taxation on optimal rotation age and land expectation value. Nautiyal and Fowler (1980) consider the case of a downward sloping demand curve for lumber, although they admit, the applicability of such to a single-stand model is somewhat unrealistic.

Hartman (1976) and Nguyen (1979) consider the Faustmann rotation problem when there is a value associated with standing forest, corresponding to the recreational and environmental benefits it may provide. They show that the effect is to increase the rotation age, sometimes to as long as the culmination age or beyond. However, results by Calish, Fight and Teeguarden (1978), who carried out numerical calculations for the optimal rotation length of Douglas fir, when several non-timber values are included, contradict the above findings. They found that for some types of non-timber yield functions, the optimal rotation is shorter than the Faustmann rotation, while for other forms it is longer. In no case did they find optimal rotation lengths as long as the culmination age.

The effect on the optimal rotation age and on land expectation values, of the presence of the risk of destruction of a stand through fire or other catastrophe, has been considered by several authors. Routledge (1980), starting from the Faustmann formula in discrete-time derives a rule for determining the optimal rotation, while Martell (1980) presents a direct search numerical procedure for the same problem. Martell suggests that the benefits of fire protection can be evaluated in terms of the resulting decrease in land expectation value. Reed (1984) derives the appropriate form of the Faustmann formula when there is fire-risk present, and shows how the policy effect of the risk of fire is equivalent to adding a premium to the effective discount rate, thereby shortening the rotation period (see also Burt, 1965). Reed and Errico (1985a) present numerical results to show how even low rates of fire can cause considerable reductions in long-run average volume yield and in land-expectation value, and they point out that the benefits of reforestation and other silvicultural treatments need to be re-evaluated when there is a risk of fire present.

Walter (1980) has considered the land economics of the Faustmann problem when rotation length and the terminal date for the utilization of the site for timber growth are open to question. The problem of an imperfect capital market has been considered by Murphy, Fortson and Bethune (1977). They consider the objective of maximization of liquidation value, when there are debt limits imposed and when there is a difference between lending and borrowing rates.

Anderson (1976) has considered the rotation problem from an optimal control theory point of view, and concludes that the Faustmann rotation model is appropriate not only for private timber management decisions, but also for public policy, where the goal is the maximization of discounted social welfare. Heaps and Neher (1979) also use optimal control theory to address the problems of optimal harvesting when the rate of harvesting is constrained, and when the cost of harvesting varies with the rate. In the former case an instantaneous harvest of the whole stand is impossible and the even-aged property of the stand is quickly lost. Heaps and Neher show that in this case, with linear havesting costs, harvesting takes place over a number of disjoint intervals. The first harvest interval can be regarded as a transition period, in which the age distribution of the stand is modified to satisfy the harvesting constraint. Subsequently all trees are cut at the Faustmann age. The resulting age distribution of the stand is in general irregular and depends on the magnitude of the constraint. Heaps and Neher point out that the results can easily be applied to a stand with an initially uneven age distribution. In the case of non-linear harvesting costs, Heaps and Neher suggest that, asymptotically, the age-distribution of the stand converges to a uniform distribution (normal forest) with trees being cut at the Faustmann age, determined with harvest costs set at the minimum average cost level. Wan and Anderson (1983) and Anderson and Wan (1981) treat the problem of constrained harvesting and the problem of non-linear harvest costs in a novel way, by assuming that there is an order imposed (through regulation, economic considerations etc.) on the harvesting of trees in the stand. With this extra assumption the problem becomes mathematically tractable and the authors use the methods of optimal control theory to obtain a solution.

The problem of determining an optimal thinning pattern along with the rotation age has received considerable attention. The problem is sometimes referred to as finding the optimal level of growing stock. Chappelle and Nelson (1964) employed marginal analysis, equating the marginal percentage growth of the stand to the marginal cost of capital at each point in time. Amidon and Akin (1968) employed dynamic programming to obtain numerical solutions to the problem, as did Kilkki and Väisänen (1969). Naslund (1969) used the Pontryagin maximum principle of optimal control theory to derive necessary conditions for the optimal harvesting level at each point in time, but did not discuss the problems associated with solving the equations derived. Schreuder (1971) also discussed the use of the Pontryagin maximum principle, but recommended use of dynamic programming for obtaining numerical solutions, claiming that analytic solution of the differential equations resulting from application of the maximum principle, was not possible. Clark and de Pree (1979), however, formulated the problem as one of linear control, and in consequence, were able to use the maximum principle to obtain a qualitative description of the optimal solution, which involves thinning along a singular path during some intermediate interval between establishment and clear-cut harvesting. Clark and de Pree also gave numerical results. Cawrse, Betters and Kent (1984) also determined numerical solutions, and like Clark and de Pree, looked at the effects of varying the relative harvest costs (per unit volume), of thinning and clear-cut harvesting, and of varying the discount rate.

Riitters, Brodie and Kao (1982) use dynamic programming to numerically determine the optimal thinning regimes to maximize both volume and value for a Douglas-fir stand. Chen, Rose and Leary (1980) use the Bellman equation of dynamic programming, with a continuous state variable, to derive a simple algorithm for determining optimal stand densities over a single rotation. Lembersky and Johnson (1975) consider the problem of determining optimal management decisions (thinning, clearcut harvesting, planting etc.) when there is stochasticity in the price of lumber, and in the response to management decisions. They formulate the problem as a Markov decision process and obtain numerical solutions using dynamic programming. Kao (1982) uses dynamic programming to obtain optimal thinning and clear-cut harvest policies when there is stochastic growth, and in a further paper (Kao, 1984) considers the problem when there is uncertainty concerning the state transition probabilities. It is shown how adaptive optimization techniques, again based on dynamic programming, can be used.

As can be seen from the foregoing descriptions, the single even-aged stand model has been a very popular vehicle for the theoretical discussion of optimal forest harvesting and management. This is especially true on the economic side where it can be treated within the realm of capital theory. However in spite of the considerable attention is has received, the use of the single stand model as a practical tool in forestry management is quite limited. The main reason for this is that a single stand, by itself, seldom constitutes a management unit, as the single-stand models assume.

Typically a management unit will be a large area of forest comprising stands of many different ages, and possibly of different species etc. Harvesting decisions will be made globally with respect to the management unit, and thus for a given single stand, management decisions with respect to cutting, thinning etc., will depend not only on the evolution of the stand itself, as the single-stand models assume, but also on the evolution and management decisions made for all other stands in the management unit.

This is true both when the management agency is a government administering publicly owned lands, and when it is a private agency administering its own lands. In the former case, the objective is usually to provide a fairly even and large flow of timber from the management unit. Thus the stands in a management unit are linked, through the requirement of an even flow. In the latter case factors such as price elasticity, the efficiency of logging operations, the utilization of capital equipment, the development of roads etc., will militate against stands being managed independently and will require that management decisions be made globally.

In order to facilitate harvest scheduling decisions at the global level, 'whole-forest' or forest-level models have been developed, and will be discussed in the next section.

3. Forest-level Models

In order to bridge the gap between stand-level models and forest-level managment, traditional forestry theory has developed the ideal of a *normal forest*, from which an even flow of timber can in principle be extracted in perpetuity. Specifically suppose a forest management unit of area A hectares, is to sustain trees with similar growth characteristics throughout its extent and suppose a rotation age of T periods is chosen. A normal forest on this land would consist of equal areas each of extent A/T hectares in each of the T age-classes: 0-1, 1-2, ... $(T-1)$-T periods. If all trees were cut at the rotation age, a sustained yield of $Av(T)/T$ volume units would be extracted from the forest in each period, where $v(T)$ is the per hectare volume of usable timber of trees at age T. To maximize the sustainable yield the rotation age T would be chosen to maximize the mean annual increment $v(T)/T$, as in the single stand theory, and would in fact be the culmination age T_c.

The ideal of a normal forest is thus linked to that of sustained yield and it has, it seems, occupied a central place in forestry thinking, for as long as a scientific discipline called Forestry can be said to have existed. Osmaston (1968) traces the idea back to the late eighteenth century. Leaving aside for the moment economic considerations, there is a serious shortcoming in the prescription of a normal forest for optimal management in that the normal forest is essentially an equilibrium concept and ignores the dynamics of the forest; while a normal forest may indicate a desireable steady-state for the forest to end up in, it gives no indication of how one should optimally arrive at that steady-state. This is especially important for North American forests which tend to exhibit highly irregular age-class distributions, with much old growth timber yet to be harvested.

Traditional forestry theory seems to have considered the attainment of a normal forest as the primary goal of management, and sought ways to achieve this state as quickly as possible - usually within a conversion period no greater than one rotation. Various more or less ad-hoc methods of achieving this goal such as the Hanzlik Formula (Hanzlik 1922) and the Area-Volume Check (e.g. Chappell 1966) were employed (see Hennes, Irving and Navon, (1971) for a discussion of these and other methods).

Apparently the first authors to consider how an existing uneven age distribution could be optimally converted to a normal forest were Nautiyal and Pearse (1967). Rather than use their description of the method, we shall describe it in terms of the dynamics of the forest.

The dynamics of a forest comprising even-aged stands with similar growth characteristics can be conveniently represented (see Reed & Errico (1985(b), 1985(c)) by the equation

$$(7) \qquad \underset{\sim}{X}_{t+1} = R\,\underset{\sim}{X}_t - S\,\underset{\sim}{h}_t$$

where

$$(8) \qquad \underset{\sim}{X}_t = \begin{bmatrix} X_1^{\,t} \\ X_2^{\,t} \\ \vdots \\ X_k^{\,t} \end{bmatrix} \quad \text{and} \quad \underset{\sim}{h}_t = \begin{bmatrix} h_1^{\,t} \\ h_2^{\,t} \\ \vdots \\ h_k^{\,t} \end{bmatrix},$$

represent respectively, the areas in the forest in age-classes 1 through k at the start of period t, and the areas harvested in the various age-classes in period t, and R and S are transition matrices of the form

$$(9) \qquad R = \begin{bmatrix} 0, & 0, & \cdots & & 0 \\ 1, & & & & \\ & 1, & & & \\ & & \ddots & & \\ & & & 1, & 1 \end{bmatrix}, \quad S = \begin{bmatrix} -1, & -1, & \cdots & & -1 \\ 1, & & & & \\ & 1, & & & \\ & & \ddots & & \\ & & & 1, & 1 \end{bmatrix}$$

Note that we are assuming that age-class k comprises all stands of age $(k-1)$ periods or older at the start of a period.

It can easily be checked that the normal forest with an even flow of timber, cut at the rotation age T, represents an equilibrium for the dynamic system (7). Specifically

$$(10) \qquad \hat{\underset{\sim}{X}} = \begin{bmatrix} A/T \\ A/T \\ \vdots \\ A/T \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \text{and} \quad (11) \qquad \hat{\underset{\sim}{h}} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ A/T \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

in an equilibrium for (7).

In considering the problem of the optimal conversion of an existing forest to a normal forest, Nautiyal and Pearse (1967) specified a conversion period P, and a rotation age T. They sought the optimal harvest schedule to convert an initial inventory $\underset{\sim}{X}'_1 = (X_1^{\;1}, X_2^{\;1}, \ldots, X_k^{\;1})$ to a normal forest (10) in P time periods. The objective to be maximized was the net present value of the stream of harvests over the conversion period,

$$(12) \qquad J_{P,T} = \sum_{1}^{P} \alpha^t \, \underset{\sim}{V}' \underset{\sim}{h}_t$$

where α is the per period discount factor (related to the per-annum discount rate i by $\alpha = (1+i)^{-\ell}$, where ℓ is the length in years of a time period), and

$$(13) \qquad \underset{\sim}{V}' = (V_1, V_2, \ldots, V_k)$$

is a vector of net stumpage values per hectare, of stands in age classes 1 through k. The problem addressed by Nautiyal and Pearse was to maximize (12) for a given initial inventory $\underset{\sim}{X}_1$ and subject to the constraints (7) for $t = 1, \ldots, P-1$, the constraint $\underset{\sim}{X}_P = \hat{X}$ and the constraints $\underset{\sim}{X}_t \geq \underset{\sim}{h}_t \geq \underset{\sim}{0}$. They show how the problem could be solved by linear programming.

The formulation of Nautiyal and Pearse did not involve the intermediate state variables $\underset{\sim}{X}_2, \underset{\sim}{X}_3, \ldots, \underset{\sim}{X}_{P-1}$. It can be derived however from the above by removing these variables by applying (7) recursively to obtain

(14) $$\underset{\sim}{X}_t = R^{t-1}\underset{\sim}{X}_1 - (R^{t-2}S\underset{\sim}{h}_1 + R^{t-3}S\underset{\sim}{h}_2 + \ldots + S\underset{\sim}{h}_{t-1} + \underset{\sim}{h}_t)$$

While Nautiyal and Pearse showed how the problem could be solved for a fixed rotation period T and a fixed conversion period P, they went on to examine how the net present value of returns earned, (both over the conversion period and subsequently while in normal state),

(15) $$NPV_{P,T} = J^{max}_{P,T} + \frac{\alpha^{P+1}}{1 - \alpha} \cdot A \frac{v_T}{T}$$

depends on these two parameters. While they recognized that (15) would always have a maximum with respect to rotation age T, they also recognized that it would increase monotonically with respect to P; in other words that conversion to a normal forest in a finite number of periods would not be economically optimal. Indeed this is obvious when one recognizes that the optimal policy for the whole forest would be to manage each stand independently in an optimal manner, cutting it if and only if its age were to exceed the Faustmann rotation age T_F. The resulting equilibrium distribution, if such existed, would reflect the initial inventory distribution; only in exceptional special cases would the equilibrium state be a normal forest.

Nautiyal and Pearse thus recognized the economic sub-optimality of a policy of conversion to a normal forest. They pointed out how the opportunity cost of such a conversion in P_0 periods could be calculated as the difference in (15) with $P = \infty$ and $P = P_0$. (It should be noted, however, that the results of Heaps and Neher (op cit in previous section) suggest that, when the costs of harvesting depend, non-linearly, on the rate of harvesting, the optimal policy does involve the conversion to a normal forest, at least asymptotically.)

Although Nautiyal and Pearse did not indicate how they would go about maximizing (15) with $P = \infty$, it will be useful for us to look at the problem here for it represents a more correct formulation, from the economic point of view, of the harvest scheduling problem. With $P = \infty$ the net present value of the resource is

$$(16) \qquad J = \sum_{1}^{\infty} \alpha^t \, \underset{\sim}{V}' \underset{\sim}{h}_t$$

which is not dependent on a rotation age parameter T, because there is no conversion to a normal forest.

The problem of maximizing the net present value of harvests from the forest can be expressed as: - maximize (16) subject to the constraints (7) and the constraints $\underset{\sim}{X}_t \geq \underset{\sim}{h}_t \geq \underset{\sim}{0}$ for $t = 1, 2, \ldots$. Although the solution to this problem is obvious (one simply cuts stands if and only if their age exceeds the Faustmann rotation age), the problem itself is very close to one that has occupied a central place in the literature on forest-level harvest scheduling -- the so-called Model II formulation (see Johnson & Scheurman, 1977).

Model II in its simplest form is a linear program for maximizing the net present value of timber harvests from a forest over a finite time horizon, given terminal payoff values for standing inventory at the end of the problem. The activities, x_{ij}, represent areas planted or regenerated in period i and harvested in period j. There are also activities $w_{i,N}$ representing areas planted or regenerated in period i and left standing as ending inventory at the end of the planning horizon in period N. There are constraints reflecting the facts that the initial inventory must either be cut or left as ending inventory and that the total area regenerated in any period cannot exceed the area harvested. With only these constraints present the problem could be disaggregated into separate single-stand problems. However in applications there are usually other constraints which link the stands. These may include:- (a) harvest flow constraints which ensure that the period to period harvests either lie between certain limits, or do not fluctuate more than a specified absolute or proportional amount, (b) constraints on the areas of standing forest for recreational and wildlife purposes etc., (c) constraints on the ending inventory etc. In addition in its more sophisticated forms (e.g. FORPLAN (Johnson, Jones and Kent, 1980, Stuart and Johnson, 1985)) Model II can include many more options, such as multiple timber types and regeneration options, various silviculture treatments, commercial and non-commercial thinnings etc.

In spite of all its many complexities, Model II can be thought of as an essentially fairly straightforward linear control theory problem similar to that outlined earlier viz.

maximize

$$J = \sum_{1}^{N} \alpha^{t} \underset{\sim}{V}' \underset{\sim}{h}_{t} + \alpha^{N+1} \underset{\sim}{r}' \underset{\sim}{X}_{N+1} \tag{17}$$

subject to:-

$$\underset{\sim}{X}_{t+1} = R\underset{\sim}{X}_{t} - S\underset{\sim}{h}_{t} \qquad t = 1, \ldots, N \tag{18}$$

$$0 \le \underset{\sim}{h}_{t} \le \underset{\sim}{X}_{t} \qquad t = 1, \ldots, N \tag{19}$$

and possibly volume flow constraints of the form

$$(1-\gamma_1)\underset{\sim}{v}' \underset{\sim}{h}_{t} \le v' \underset{\sim}{h}_{t+1} \le (1+\gamma_2) v' \underset{\sim}{h}_{t}, \quad t = 1, 2, \ldots, N-1 \tag{20}$$

where γ_1 and γ_2 represent maximum allowable proportional increases and decreases in volumes cut from period to period, and $\underset{\sim}{v}' = (v_1, v_2, \ldots, v_k)$ is a vector of per hectare usable volumes for stands of age 1 through k. In addition there may be constraints on the standing areas $\underset{\sim}{X}_t$ etc. In (17) the vector $\underset{\sim}{r}'$ represents terminal payoff values for land with timber in the various age-classes, which in most cases would be set to the single-stand net present values, using the Faustmann formulation.

Reed and Errico (1985c) discuss this formulation and two methods of solving the problem using linear programming - the first (LP1) regarding both the $\underset{\sim}{h}_t$ and the $\underset{\sim}{X}_t$ $(t = 1,\ldots,N)$ as activities, and the second (LP2) with only the $\underset{\sim}{h}_t$ $(t = 1,\ldots,N)$ as activities, the $\underset{\sim}{X}_t$ having been removed by the use of (14). The linear program (LP2) arising from the second method is essentially the same as the Model II linear program, the only differences arising from the fact that Model II does not lump all timber above a certain age into a single age class. Reed and Errico argue in favour of the above control theory formulation of the problem with the LP1 method of solution on the grounds that it can handle many extensions in a straightforward way, and that in complex problems, it is computationally efficient both in execution time and programming time. Extensions considered include random losses through fire etc., with partial salvage (see later), the problem of a changing land base, multiple timber types and regeneration options etc. Furthermore the dual variable values at the optimum, corresponding to constraints (18), provide shadow values for land containing timber of various ages, i.e. they provide the net present value of forested land, at the margin, in the whole-forest context.

Model II can accommodate a (linear) downward sloping demand curve (Johnson and Scheurman, 1977), by specifying a quadratic objective which can be maximized by quadratic programming. Control theory formulations of the problem with downward sloping demand curves, and complex cost functions have been discussed by McDonough & Park (1975) and Lyon and Sedjo (1983). The latter authors use the

Discrete Maximum Principle (Halkin, 1966) to numerically determine long-run supply projections and stumpage prices.

Another linear programming approach to the harvest scheduling problem, termed Model I by Johnson and Scheurman (1977), has been used considerably and is included in large-scale packages such as FORPLAN (Johnson, Jones and Kent, 1980). Suggested by Kidd, Thomson and Hoepner (1966) and Ware and Clutter (1971) it was implemented by Navon (1971) as the Timber Resource Allocation Model (Timber RAM). In Model I a number of possible harvest sequences are specified, and the activities, y_{ij} represent the areas initially in timber class i assigned to harvest sequence j. An advantage claimed for Model I over Model II is that from the solution to Model I, a complete trajectory of future harvests, regenerations etc., for a given stand can be determined. Typically Model I will have more activities but fewer constraints than Model II. However since in such a case there will be a many-to-one relationship between the Model I activities, y_{ij}, and the Model II activities, x_{ij}, there are likely to be multiple optima to the Model I linear program. A full discussion of the Model I and Model II approaches, and their relationship to other methods is given in Johnson and Scheurman (1977).

Several authors have discussed methods of reducing the solution time for the linear programs of Model I and Model II. Nazareth (1978) uses Dantzig-Wolfe decomposition for a Model I-type problem, while Hoganson and Rose (1984) use a similar method involving a decomposition and solution of the dual problem. Dantzig-Wolfe decomposition has been applied to the Model II problem by Berck and Bible (1984). While these methods appear to be useful for the simplest problems, in which the only linkage between stands is through harvest-flow constraints, their applicability in more complex situations with other constraints etc., has yet to be established.

The problem of catastrophic loss through fire in the forest-level harvest problem has recently received attention. Reed and Errico (1985b) formulate the problem as one of stochastic control with the dynamics of the system represented by the stochastic difference equation

$$\underset{\sim}{X}_{t+1} = R_t \underset{\sim}{X}_t - S_t \underset{\sim}{h}_t \tag{21}$$

where $\{R_t\}$ and $\{S_t\}$ are sequences of random matrices, of the form

$$R_t = \begin{bmatrix} p_1^{\,t}, & p_2^{\,t}, & \cdots & & p_k^{\,t} \\ 1-p_1^{\,t}, & & & & \\ & 1-p_2^{\,t} & & & \\ & & \ddots & & \\ & & & 1-p_{k-1}^{\,t}, & 1-p_k^{\,t} \end{bmatrix}, \quad S_t = \begin{bmatrix} -1+p_1^{\,t}, & -1+p_2^{\,t}, & \cdots & & -1+p_k^{\,t} \\ 1-p_1^{\,t}, & & & & \\ & 1-p_2^{\,t} & & & \\ & & \ddots & & \\ & & & 1-p_{k-1}^{\,t}, & 1-p_k^{\,t} \end{bmatrix} \tag{22}$$

where $p_i^{\,t}$ is a random variable representing the proportion of the area in age class i destroyed by fire etc., in period t. Reed and Errico show how an approximately optimal feedback solution to the problem can be obtained by solving in each period a deterministic control problem like the one described above (maximize (17) subject to (18), (19) and (20)) only with the matrices R and S replaced by the expected value matrices $\bar{R}$ and $\bar{S}$ of R_t and S_t in (22). An adaption of the Model II linear programming formulation, which includes deterministic catastrophic losses has been discussed by Johnson and Stuart (1985) and is scheduled to be included in the next release of FORPLAN.

A different stochastic control approach to the problem of harvest scheduling was proposed by Dixon and Howitt (1980). They use LQG (Linear-Quadratic-Gaussian) control methods under the assumption of (a) linear dynamics with additive Gaussian noise, (b) additive Gaussian noise in observing the state (current inventory), and (c) a quadratic objective function. Casti (1983) discusses the use of feedback policies for harvest scheduling in the presence of randomness.

In most of the forest-level harvesting models discussed above, harvest-flow constraints are present. Indeed for models with a linear objective function, the forest-wide harvest flow constraints often play the major role in linking the separate stands of the forest together. Experience has shown that these flow constraints are usually more important than other factors, such as the discount rate, in determining the optimal harvest schedule. Constraints regulating the stability of volume flow from a forest are essentially non-economic and have often been criticized by economists and others. While it is straightforward to assess the opportunity costs of such flow constraints the benefits are harder to assess. The idea of even flow or regulated flow seems to be an extension of the forester's traditional idea of sustained yield, which is often justified in terms of achieving stability of forest-based communities and of long-run conservation of resources for the future benefit of society.

For privately-owned forest land it is doubtful whether the above are considered as primary objectives of management. Flow constraint, when they are used, probably are for the purpose of ensuring evenness of cash flow, taking into account price elasticity. For publicly owned land the even-flow idea seems to have reached its extreme in the U.S. National Forest Management Act of 1976 which specified that the harvest policy must "... limit the sale of timber from each national forest to a quantity equal to or less than a quantity which can be removed from such forest annually in perpetuity on a sustained-yield basis". In other words the annual harvest cannot <u>exceed</u> that which would be taken from a normal forest, cut at the culmination age. While such legislation undoubtedly meets the objective of long-run conservation for the future benefit of society it seems an extremely inefficient way of doing so. Future yields could be guaranteed just as easily by stipulating the maximum long-run sustained yield as a <u>floor</u> for annual harvests rather than as a ceiling (see Johnson & Beuter, 1977). The opportunity cost associated with the current legislation must be enormous since it prohibits any large-scale liquidation of old-growth forest. While it is true that old-growth forest has value for purposes other than providing timber, and while few would disagree that government has a responsibility in managing the forests for the benefits of all users, the current act again seems to be an extremely inefficient way of discharging this responsibility. If old-growth forest is required for non-timber uses this can be incorporated as a constraint in a harvest scheduling model, or alternatively if a utility value for growing forest can be specified, it can be incorporated into the objective function of such a model, as current versions of FORPLAN permit.

Understandably the 1976 Act, and the whole concept of sustained yield and even-flow have received harsh criticism from some economists. Dowdle (1976) regards the idea of an allowable cut determined within the framework of sustained yield thinking, as a historical mistake - an anachronism from the time when European forests were used in common, and a "bag limit" had to be set of the number of trees cut. Hyde (1976) and Waggener (1977) question whether the current Act can achieve the goal of "community stability" and point out that an even-flow of timber volume has the effect of amplifying short-run fluctuations in market price, since supply cannot respond to changes in price. Parry, Vaux and Dennis (1983) trace the history of the sustained yield idea for U.S. National Forests, and question the effectiveness of sustained yield policies in achieving their stated goals. They suggest that a policy which "explicitly recognizes growing stock considerations" could better meet these goals. It should be noted that the very concept of sustained yield is a fuzzy one, given that it may be possible to manipulate future yields by genetic improvement, silviculture etc.; and that what constitutes usable or economically recoverable timber will depend, in unforseeable ways on technological and market conditions.

If current models (such as Model II or the Reed-Errico (1985c) model) were used without harvest flow constraints, the prescribed optimal policy would involve a very rapid draw-down of old-growth forest, limited only by growing-stock constraints and demand. However such models recognize no upper limits on per period harvests as would in reality occur due to the limited capacity of logging operations, mills etc. Such capacity could presumably be increased, but it would involve capital investment. An economic optimization model which jointly determines optimal harvests and optimal investment in harvest capacity is currently being considered by Reed and Errico (1985d). This model will enable the determination of the optimal economic development of a forest industry.

A model for planning in a vertically integrated forest industry and using a linear programming formulation of the Model I type has been presented by Barros & Weintraub (1982). A model for short-rotation forestry has been presented by Giese and Jones (1984). They obtain optimal equilibrium harvesting strategies using the method of linear complementarity (Dantzig and Manne, 1974).

4. Uneven-Aged Stand Models

In even-aged stand models, both at the stand-level and at the forest level, the trees growing on a site are eventually clear-cut harvested and the site subsequently regenerated. This is not the case in uneven-aged stand models.

Probably the simplest uneven-aged stand model is the one discussed by Chang (1981) (see also Hall (1983)) who assumes that the state of the stand can be described by its growing volume, S. Harvests are assumed to occur cyclically at times T years apart, and it is assumed that there is a residual volume, G, of growing stock left standing after each harvest. It is further assumed that the stand volume, t years after a harvest, depends on t, and on the initial growing stock (i.e. residual growing stock from the last harvest), G, by a relation $S = Q(t,G)$. Under the assumptions of a constant price, Chang uses a Faustmann-type analysis to determine the optimal cutting cycle, T, and the optimal residual growing stock G.

Other uneven-age stand models explicitly include the size structure i.e. the distribution over diameter classes) of the stand. Usher (1969a and 1969b) proposed a linear model, in which in each period fixed proportions of trees in the various size classes, move to the next higher size-class, and in which in each period new trees are "born" the number of which depending on the total area cleared through the harvesting of old trees. Usher (1969a) showed that if equal proportions were to be removed from each size class there would be only one level of harvesting which would lead to equilibrium. He noted however (Usher 1969b) that higher sustainable yields could be realized by harvesting at different rates in different size classes. Rorres (1978) showed how the optimal equilibrium harvest schedule for the Usher model could be derived by linear programming.

Adams and Ek (1974) considered a non-linear size-structured model in which the ingrowth of new trees and the numbers of trees moving into higher size classes depend, in a non-linear fashion, on the current size distribution of the stand. The authors showed how the optimal equilibrium growing stock could be determined by non-linear programming methods. They also considered the conversion from an initial size distribution to the optimal equilibrium size distribution in a <u>pre-determined</u> number of periods. They showed how the problem of how to do this in such a way as to maximize present value, was analagous to the forest-level conversion problem considered by Nautiyal and Pearse (1967), (and discussed in Section 3 of this survey) and could be solved by a non-linear programming method analagous to the linear programming method of those authors. Adams (1976) considered the investment-efficient equilibrium size distribution and its relationship to stocking levels. Michie (1985) considers a conversion problem, in which in a predetermined number of periods, an initial size distribution is to be converted into an equilibrium size distribution. Both the equilibrium distribution and the harvest sequence required for the conversion are chosen so as to maximize net present value.

Haight, Brodie and Adams (1985) consider the dynamic optimization problem for a non-linear growth model with no specified ending size distribution. Using a gradient method of solution they obtain the optimal selection harvests and the resulting optimal sequence of size distributions to maximize the net present value of harvests from a northern hardwood stand. They report a pulse-like transition harvest regime, for the case when the largest size-classes have a higher per cubic metre value than the smaller size-classes. Haight (1985) discusses the dynamic optimization problem, in relation to the objective of maximizing equilibrium yield.

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