

**ANALYZING CATCH-EFFORT DATA
BY MEANS OF THE KALMAN FILTER**

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Analyzing Catch-Effort Data by Means of the Kalman Filter.

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Abstract

A method of analyzing catch-effort data which uses linear statistical theory, but which, at the same time, maintains a degree of biological realism, is described and applied to catch-effort data for two fisheries. The method is based on a linear state-space model for the evolution of an unobservable state variable, the natural logarithm of the stock biomass. An observable proxy for the state is the logarithm of catch per unit effort. Randomness can be present both in the stock dynamics and in the process of catching fish. In addition both uncertainty and the possibility of convexity or concavity (hyperdepletion or hyperstability) in the relationship between stock abundance and cpue are included. The stock dynamics are modelled by a power law (log-linear) relationship between escapement and returning stock, with multiplicative lognormal noise. Maximum likelihood is used to estimate model parameters, with the Kalman filter being employed to generate the likelihood function. The method can easily be extended to include other assessments (besides cpue) of stock abundance. When only catch-effort data are available it is recommended that the primary use of the methodology be for predictions of cpue and catch.

Keywords: Catch-effort data, Kalman filter, prediction, observation error.

1 Introduction.

The easiest data to obtain for many fisheries and sometimes the only data available for management, are the time series of annual aggregated catches and annual aggregated fishing effort (or some scaled estimate of “effective” effort). Such data provide a basis for stock assessment, and can provide an indication of how the fish stock will respond to exploitation.

Catch-effort data have been analyzed by a variety of different methods. On the one hand there are methods based on so-called *surplus production models* which attempt to model, in a biologically plausible way, the dynamics of the exploited population and sometimes as well the dynamics of the “population” of fishermen who are exploiting it. At the heart of all such methods lies a *nonlinear* model of the population dynamics. Early methods along these lines are those developed by Schaefer (1954), Pella & Tomlinson (1969) and Schnute (1977), all of which employed the deterministic continuous-time logistic growth model, or some simple variant of it, as a model of the stock dynamics. More sophisticated population dynamic models which included age structure and a flexible parametric class of stock-recruitment functions were employed by Deriso (1980) and Schnute (1985).

Reed (1986) considered a surplus production model which included a stochastic model of the fishing process which related catch to effort, arguing that much of the observed randomness in catches could be explained by the inherent stochasticity in the process of catching fish and that this source of

randomness could be more important than environmentally or biologically induced randomness. More recently Reed and Simons (1994) have extended this approach using a more sophisticated stochastic model of the fishing process, which includes a contagion effect *i.e.* in which it is assumed that the event of one fish or group of fish being caught influences the probability of other fish being caught. Ludwig, Walters and Cooke (1988) developed a model which included uncertainty in the catch-effort relationship as well as in the population dynamics. However in order to obtain parameter estimates the authors were forced to use approximate methods of unknown reliability. A somewhat different approach was adopted by Berck and Johns (1991), who considered the dynamics of effort in conjunction with those of the fish population. They assumed that effort would increase (or decrease) from year to year at a rate proportional to the profit (or loss) generated per unit of effort. They included stochasticity in the dynamics of both the fish stock and the fishing effort, and were able to estimate model parameters by maximum likelihood using the Kalman filter. They reported successful results in predicting catch-per-unit-effort, a well known proxy for stock abundance.

In contrast to the surplus production methods are a class of methods which eschew biological realism, for the sake of statistical simplicity. Such methods have been called **black box methods** (Reed ,1986), since the essential idea behind them is to develop a simple "black box" which processes the input series (efforts) to produce an output series which closely mimics the

observed output series (catches). At heart such models involve (for the sake of statistical simplicity) an assumption of a *linear* relationship between some combination of observed catches, efforts and cpues¹. The earliest methods along these lines involved the linear regression of cpue (or its logarithm) on some measure of average effort over the previous few years (Gulland 1961, Fox 1970). A comparison of the performance of the Gulland and Schaefer models was conducted by Huang and Redlack (1981). A more sophisticated black-box method was developed by Mendelsohn(1980) using Box-Jenkins ARIMA techniques. Roff (1983) used what he termed a “simple autoregressive” (SA) model, which involved the regression of catch in year t on the product of effort in year t and catch-per-unit-effort in year $t-1$. Roff demonstrated, using several data sets, that as a predictive tool, his SA model could outperform the surplus production methods of Deriso (1980) and Schnute (1977). Further he expressed considerable scepticism about methods which claim to be able to prescribe optimal exploitation on the basis of catch-effort data alone.

In this article we present a model, which has some degree of biological plausibility, which at the same time preserves some degree of statistical simplicity, and in this respect it bridges the gap between the surplus production and black-box methods. To reflect nonlinear population dynamics we employ a log-linear model, with multiplicative log-normal noise, to relate escapement

¹cpue = catch-per-unit-effort.

to next year's stock. Effort is assumed to be an exogenous variable, while cpue is considered to be an indicator of stock abundance (allowing however for a hyperdepletion or hyperstability phenomenon in the abundance-cpue relationship), which is of course not directly observable. It is assumed that effort is observed subject to error. This assumption can also encompass randomness in the relationship between effort and escapement, i.e randomness in the fishing process. By working in the logarithmic scale, linearity can be preserved resulting in a model in state-space form for which the Kalman filter can be used to generate the likelihood function. Numerical maximization then provides maximum likelihood estimates of the model parameters, with approximate standard errors and confidence intervals being obtained from the asymptotic properties of the likelihood function.

The method thus incorporates some of the more desirable features of both surplus production and black box types of models. Furthermore the model employed includes both system noise (randomness in the population dynamics *and* in the fishing process) along with observation error. However it turns out to be impossible statistically to separate some of the components of variance, and also to estimate the degree of hyperstability or hyperdepletion. In addition only *relative* estimates of stock abundance are possible.

In spite of these limitations the model turns out to be useful for predicting catches and cpues since the predictions do not depend on absolute stock size nor on the degree of hyperstability or hyperdepletion present. In principle

if the validity of the log-linear stock dynamic relation could be confirmed, the method could be used to prescribe an optimal level of effort. However since such an optimum is highly dependent on the parametric form specified for the dynamics, and since it is not possible to confirm or reject many such parametric forms, we do not recommend such a role for the method.

The method is applied to two distinct sets of catch-effort data for West Coast fisheries. Each data set was split into a set of “training data” and a set of “test data” (or validation data). Parameters were estimated from the training data and predictions of cpue and catch were made for the test data, thus providing a method of checking the statistical validity of the method.

Previous work in the fisheries literature employing state-space models and the use of the Kalman filter for maximum likelihood estimation include Berck and Johns (1991), Sullivan (1992), Speed (1993), Rein (1994) and Freeman & Kirkwood (1994). This last paper deals with catch and effort data. One way in which it differs from the current work is in the way the population dynamics are modelled. Freeman & Kirkwood assume that the population size at the start of year $t + 1$ is a fixed proportion of the escapement in year t plus a density-independent recruitment term, which is modelled as a random walk. Errors are assumed to follow a normal (Gaussian) distribution, thereby providing a linear model in state-space form. In contrast our model assumes (density-dependent) log-linear dynamics with log-normal errors.

The paper is organized in the following way. Section 2 describes the model

and how it can be represented in state-space form. It also includes a brief description of how the Kalman filter algorithm can be employed to evaluate the likelihood function for such a model. The reason for this inclusion is that this method shows great promise for application in fisheries science and although it has been used before in the fisheries context (see above), is not as yet widely known.

Section 3 describes the application of the proposed method of analysis to two sets of catch-effort data. Parameter estimates, standard errors and approximate confidence intervals are given, along with a brief description of how residuals can be determined. A method of predicting cpue and catch is described and the results of predictions for the two data sets presented.

The paper closes with a brief discussion section (Section 4).

2 A Statistical Model for Catch-Effort Data.

In this section a statistical model in **state space form** for the process by which the data are generated is formulated. The objective is to obtain a **likelihood function** which can be maximized to obtain **maximum likelihood (ML)** estimates of the model parameters. We are concerned with time series of observations of fishing effort E_t and catch $C_t(t = 1, 2, \dots, T)$. The effort series may be either (a) true aggregated effort over the whole fleet; or (b) an estimate of total effort obtained by some sampling method. *e.g.* a method of estimating effort commonly used for West Coast ground fisheries is via **ratio estimation** using the cpue for some sample of representative boats and then dividing total catch by this quantity.

The Model

Suppose that the population dynamics can be described by the **log-linear form**

$$X_{t+1} = e^a S_t^b e^{z_t}, \quad t = 1, \dots, T \quad (1)$$

where X_t denotes total fish biomass at the start of the fishing season in year t ; S_t denotes the escapement in that year and $\{z_t\}$ is a sequence of independent identically distributed normal random variables² *i.e*

²We use the notation $\sim NID(\mu, \sigma^2)$ to mean independent, identically distributed from a normal (Gaussian) distribution with mean μ and variance σ^2 .

$$z_t \sim NID(0, \tau^2). \quad (2)$$

Thus we are assuming multiplicative, log-normal randomness in the population dynamics (see *e.g.* Ludwig, Walters and Cooke, 1988). The parameters a , τ^2 and b are unknown and are to be estimated from the data; to make sense we require the constraints $0 < b < 1$, $a > 0$, $\tau^2 \geq 0$.

Suppose also that the escapement S_t is related to X_t and effort E_t through the equation

$$S_t = e^{-q(E_t + w_t)} X_t. \quad (3)$$

with

$$w_t \sim NID(0, \rho^2). \quad (4)$$

Thus we assume that aggregated fishing mortality is proportional to aggregated effort plus a random error term w_t . This error term can include sampling error in estimating total effort, if some sampling technique is used to estimate E_t (data of type (b) above), as well as intrinsic randomness in the relationships between escapement, fishing mortality and fishing effort. The constant q in (3) is the **catchability coefficient**. Note that **natural mortality** does not enter into (3). If the fishery is a seasonal one and if the fishing season is reasonably short, ignoring natural mortality should not cause serious problems.

Substituting the escapement equation (3) into the stock dynamic equation

(1) gives

$$X_{t+1} = e^{a-bq(E_t+w_t)} X_t^b e^{z_t}. \quad (5)$$

which can be written more simply as

$$X_{t+1} = e^{a-bqE_t+\varepsilon_t} X_t^b. \quad (6)$$

where

$$\varepsilon_t = z_t - bq w_t \quad (7)$$

If we make the plausible assumption that the errors, $\{z_t\}$, in the population dynamic equation are independent³ of those, $\{w_t\}$, in the escapement-effort equation then it will follow that

$$\varepsilon_t \sim NID(0, \omega^2) \quad (8)$$

where

$$\omega^2 = \tau^2 + (bq)^2 \rho^2 \quad (9)$$

Taking the natural logarithm of equation (6) and writing x_t for $\ln X_t$ gives

$$x_{t+1} = bx_t + a - bqE_t + \varepsilon_t. \quad (10)$$

This is a Gaussian linear model in **state space form (SSF)** (see *e.g.* Harvey, 1989 p. 100*ff.*). The state of the system is x_t , which is never directly observed, since stock biomass X_t is not directly observable. Equation (10) describes

³Lack of independence will cause the variance ω^2 to change by the inclusion of a covariance term. By itself this would introduce no extra complication into the statistical procedures that follow.

the evolution of the state, with exogenous input (or control) variables $\{E_t\}$. This is similar to the state equation considered by Berck and Johns (1991); however they regarded E_t as a state variable subject to its own state equation, rather than as an exogenous variable.

Although one cannot observe stock abundance, X_t , (and x_t) directly, often estimates of abundance are available (*e.g.* from survey trawls, catch per unit effort, *etc.*). If Y_t were an estimate of X_t with multiplicative log-normal errors, *i.e.* if

$$Y_t = e^k e^{\delta_t} X_t \quad (11)$$

where $e^k > 0$ is a constant of proportionality and

$$\delta_t \sim NID(0, \sigma^2), \quad (12)$$

then on taking logarithms we would have

$$y_t = \ln Y_t = x_t + k + \delta_t. \quad (13)$$

This is a **measurement equation** for a linear model in state space form (see *e.g.* Harvey, 1989 p. 100ff.). Iteratively via the **Kalman filter** one could obtain, for any given parameter values, the probability of observing the given data series $\{y_t\}$, given the exogenous variables E_t ; in other words one could determine the **likelihood** corresponding to any particular parameter values, and by maximizing the likelihood function could obtain **maximum likelihood** estimates of the parameters.

Often in practice there might be more than one (m , say) not necessarily independent estimates of stock abundance. The above method would still work in this case, however now $y_t = \ln(Y_t)$ would be a m -vector whose components would correspond to the individual estimates; $\{\delta_t\}$ would be multivariate normal random variables and k would be an m -vector and x_t would be preceded on the right-hand-side of equation (13) by an m -dimensional column vector of 1s.

In this paper we shall consider only catch-effort data, with the only proxy for stock abundance being based on the cpue. For a variety of reasons the relationship between cpue and stock biomass may be nonlinear. For example (see e.g Hilborn and Walters 1992, p175 ff) if handling time in catching fish is large compared to search time, and recorded effort includes both search and handling time, then one would expect the relationship between cpue and biomass to be concave (*i.e.* exhibit **hyperstability**); while if search time is large compared to handling time one would expect the relationship to be convex *i.e.* exhibit (**hyperdepletion**). Thus we shall assume a simple relationship between cpue and stock biomass which can be both concave and convex. To this end denote the cpue by $U_t = \frac{C_t}{E_t}$ and suppose that U_t is related to X_t through the equation

$$U_t = e^k e^{\delta_t} X_t^p \quad (14)$$

where $k > 0$ and $p > 0$ are parameters to be estimated and

$$\delta_t \sim NID(0, \sigma^2). \quad (15)$$

with $\sigma^2 \geq 0$ an unknown variance parameter. The parameter k is simply a scale factor, while p reflects the concavity or convexity of the cpue-stock biomass relationship. If $0 < p < 1$ concavity occurs and hyperstability is present, while if $p > 1$ convexity and hyperdepletion are present. The random term e^{δ_t} in (14) is included to reflect the fact that there is some random error in the relationship between cpue and stock biomass. The variance parameter σ^2 reflects the magnitude of the error. We make the plausible assumptions that the errors $\{e^{\delta_t}\}$ are log-normally distributed, serially uncorrelated, and uncorrelated with the log-normal errors $\{e^{\varepsilon_t}\}$ in the state dynamic equation (6).

On taking logarithms it follows from (14) that

$$y_t = \ln U_t = px_t + k + \delta_t. \quad (16)$$

Equation (16) in state space form is a **measurement equation** because the dependent variable y_t is observable depending linearly on the unobservable variable x_t .

Our model then in **state space form** comprises the state equation

$$x_{t+1} = bx_t + a - bqE_t + \varepsilon_t \quad t = 0, \dots, T-1 \quad (17)$$

with

$$\varepsilon_t \sim NID(0, \omega^2) \quad (18)$$

and the measurement equation

$$y_t = px_t + k + \delta_t, \quad t = 1, \dots, T \quad (19)$$

with

$$\delta_t \sim NID(0, \sigma^2), \quad (20)$$

The time index t runs from 0 (the season before the first catch-effort observations) to T , the last season for which such data are available.

It should be apparent that the two components τ^2 and ρ^2 of the variance ω^2 of the errors ε_t (equations (7) and (9)) are not estimable. However ω^2 is estimable, and can be regarded as a primary parameter⁴ independent of b and q .

In order to implement the Kalman filter, it is necessary to specify the mean (μ_0) and variance (P_0) of the (assumed normal) distribution of the initial state x_0 . To this end we shall assume that no significant fishing activity was taking place before the first catch-effort observation (*i.e.* in years 0 and earlier), and that the stock was in a stochastic equilibrium (*i.e.* a stationary state). Under these assumptions it follows that for $t < 0$ (before the start of fishing)

$$E(x_{t+1}) = E(x_t) = \mu_0 \quad (21)$$

$$\text{Var}(x_{t+1}) = \text{Var}(x_t) = P_0 \quad (22)$$

Using these results, and taking expected value and variance of both sides

⁴To see this simply reparameterize the model, replacing τ^2 and ρ^2 by ω^2 and say ρ^2 . Any likelihood based on (6) will involve the independent parameters a , b and ω^2 (which is estimable) but not ρ^2 (non-estimable).

of equation (18), gives

$$\mu_0 = b\mu_0 + a \quad (23)$$

so that

$$\mu_0 = \frac{a}{1-b} \quad (24)$$

and

$$P_0 = b^2 P_0 + \omega^2 \quad (25)$$

so that

$$P_0 = \frac{\omega^2}{1-b^2} \quad (26)$$

The Kalman Filter.

The model is now in SSF suitable for application of the the Kalman filter (KF) algorithm. The algorithm determines recursively, for given parameter values, the mean and variance of the (normally distributed) optimal predictor of the state (x_t) given the prior observations of the (logarithm of) cpue (y_s , $s = 1 \dots t$). From this the conditional distribution of the observation y_t given previous values, y_s , $s = 1 \dots t - 1$, can be obtained, whence through multiplication, the unconditional distribution of the T observations y_t , $t = 1 \dots T$ can be obtained. This provides the likelihood for the given parameter values. In more detail let

$$\mathbf{Y}_t = (y_1, y_2, \dots, y_t) \quad (27)$$

and let

$$\hat{x}_{t|s} = E[x_t|Y_s] \quad (28)$$

$$P_{t|s} = \text{Var}[x_t|Y_s] \quad (29)$$

It then follows from (18) that

$$\hat{x}_{t|t-1} = b\hat{x}_{t-1|t-1} + a - bqE_t \quad (30)$$

and

$$P_{t|t-1} = b^2P_{t-1|t-1} + \omega^2. \quad (31)$$

Once a new observation y_t becomes available at time t the mean $\hat{x}_{t|t-1}$ and variance $P_{t|t-1}$ of the prediction of x_t can be updated according to

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + pP_{t|t-1}f_t^{-1}(y_t - p\hat{x}_{t|t-1} - k) \quad (32)$$

and

$$P_{t|t} = P_{t|t-1} - \frac{P_{t|t-1}^2}{f_t}p^2 \quad (33)$$

where

$$f_t = p^2P_{t|t-1} + \sigma^2 \quad (34)$$

For details of the procedure see Harvey (1989, p.105-106).

Likelihood Formation

To determine the likelihood for particular values of the parameters, we first examine the conditional distribution of y_t given Y_{t-1} . Since the error

terms and the initial state variable in the state space model are normally distributed, the distribution of y_t conditional on Y_{t-1} , is also normal and its mean and variance are given directly by the Kalman filter (Harvey, 1989, p.125) The mean is

$$E[y_t|Y_{t-1}] = y_{t|t-1} = px_{t|t-1} + k \quad (35)$$

and

$$\text{Var}[y_t|Y_{t-1}] = f_t \quad (36)$$

Regarded as a function of the parameter vector Θ , the likelihood function expressed as a product of conditional probabilities can be written as

$$L(\Theta; y) = p(y_1, y_2, \dots, y_T) = \prod_{t=1}^T p(y_t|Y_{t-1}). \quad (37)$$

and the log-likelihood becomes

$$l = \log L(\Theta) = \text{constant} - \frac{1}{2} \sum_{t=1}^T \log f_t - \frac{1}{2} \sum_{t=1}^T \frac{v_t^2}{f_t} \quad (38)$$

where

$$v_t = y_t - y_{t|t-1}, \quad t = 1, \dots, T. \quad (39)$$

Note that the expression for the log-likelihood depends on f_t and $y_{t|t-1}$, which are found recursively via (36) and (35). Thus via the recursion the log-likelihood can be evaluated *numerically* for any particular parameter values. This permits the numerical maximization of the log-likelihood with respect to the unknown parameters which provides their ML estimates.

3 Applications.

The method was applied on two real data sets; (1) **Lingcod** (*Ophiodon elongatus*) off Vancouver Island, British Columbia, (Area 3D) from 1956 – 1989 (Richards and Hand 1991); Cass et al (1988) provide details about this fishery and compilation of catch statistics. The data here is made up of catch, effort and catch-per-unit effort. The catch-per-unit effort data is based on the 25% qualification method. (2) **Pacific Cod** (*Gadus macrocephalus*) from Georgia Strait and vicinity (Area 4B) from 1955-1989 (Foucher and Tyler 1991).

Each data set was split into two parts: all but the last five observations in the time series were used as **training data**, to which the model was fitted; the last five observations were kept apart as **test data**. Predictions using the fitted model on the training data, were made for the last five observations, and the predictions then compared with the actual test data. Scatter plots and time-series plots of the two data sets are displayed in Figs. 1-4.

It soon became apparent when attempting to fit the model to the training data by maximizing the log-likelihood function that one could not maximize simultaneously over all seven parameters (a , b , q , k , p , σ^2 and ω^2) Attempts to do this led to convergence to different maxima with the same value of the objective (log-likelihood) function using different starting values. The reason for this is that the parameters p , ω^2 and q are not jointly estimable; and similarly the parameters a and k . This can be seen by writing out

explicitly the first few iterations of the KF procedure, and observing, for example, that the log-likelihood does not change if as p changes, ω^2 changes to $p^2\omega^2$ and q changes to pq . Thus there is no unique maximum to the log-likelihood function; rather it attains its maximum over a one-dimensional manifold (in p - q - ω^2 space). The only way around this estimability problem is to specify (rather than estimate) one of these parameters. However the robustness of the results to the particular specification can be assessed by re-computing using a range of values for the specified parameter. We chose to specify the parameter p , using values $p = 0.5, 1.0$ and 2.0 , corresponding to hyperstability, proportionality and hyperdepletion, respectively, in the relationship between cpue and stock biomass. As will be seen many of the results do not depend on the value of p employed, which is of considerable importance since empirical determination of the degree of hyperstability-hyperinflation is a formidable problem.

A similar estimability problem occurs for the pair of parameters k and a . This is not surprising, since the parameter e^a is a **scale factor** relating the population size in non-dimensional units to its value in the particular biomass units employed. The parameter e^k on the other hand is a **proportionality constant** in the relationship between cpue and biomass in the particular units employed. Thus a change of biomass units would simultaneously change e^a and e^k by the same factor (i.e add the same constant to a and k); since we have no natural units for stock biomass, and no independent estimate

of it, the two parameters are inevitably confounded, and not simultaneously estimable. Because of this fact we have set the parameter a to zero, implying that the **median equilibrium stock size** before fishing was 1, and that estimates \hat{X}_t of stock size are expressed as fractions of this median equilibrium size. Thus the estimates \hat{X}_t are dimensionless quantities.

Parameter estimates.

Table 1 displays point estimates, approximate standard errors and approximate 95% confidence intervals for the parameters q , b , k , σ^2 and ω^2 for Lingcod and Pacific Cod fisheries using the value $p=1$ (i.e assuming proportionality between cpue and stock size). Tables 2 & 3 present the same information for the cases $p=0.5$ (hyperstability) and $p=2.0$ (hyperdepletion).

The standard errors were computed from the **observed information matrix** (obtained as the inverse of the matrix of second derivatives of the log-likelihood at its maximum). The confidence intervals were obtained by the **likelihood ratio method** i.e. they represent values of the individual parameters which would not be rejected using a likelihood ratio test at the 5% significance level. Note that the parameter estimates are correlated, so that the confidence intervals given are not independent. Both methods are known to be asymptotically correct. However since we are dealing with finite time-series, the results should be regarded only as approximations.

Model diagnostics based on residuals ($e_{t|T}$) showed no indication of model

parameter	Lingcod data	Pacific Cod data
\hat{q}	0.0152(0.0189) [0.01 – 0.16]	0.0059(0.0180) [0.0 – 0.35]
\hat{b}	0.7946(0.1178) [0.35 – 0.98]	0.7485(0.1825) [0.22 – 0.95]
\hat{k}	5.9548(0.2016) [5.6 – 6.36]	5.4612(0.1787) [5.10 – 5.92]
$\hat{\sigma}^2$	0.0341(0.0108) [0.0 – 0.06]	0.0561(0.0161) [0.0 – 0.095]
$\hat{\omega}^2$	0.0139(0.0096) [0.008 – 0.07]	0.0126(0.0120) [0.006 – 0.15]

Table 1: Maximum likelihood estimates, standard errors (in brackets) and approximate 95% confidence limits (in square brackets) for the model parameters for Lingcod and Pacific Cod data for $p = 1.0$.

parameter	Lingcod data	Pacific Cod data
\hat{q}	0.0303(0.0378) [0.0 – 0.32]	0.0118(0.0360) [0.0 – 0.8]
\hat{b}	0.7946(0.1178) [0.35 – 0.98]	0.7485(0.1825) [0.22 – 0.95]
\hat{k}	5.9548(0.2016) [5.6 – 6.36]	5.4612(0.1787) [5.10 – 5.92]
$\hat{\sigma}^2$	0.0341(0.0108) [0.0 – 0.06]	0.0561(0.0161) [0.0 – 0.095]
$\hat{\omega}^2$	0.05571(0.0382) [0.01 – 0.28]	0.0503(0.0479) [0.004 – 0.4]

Table 2: Maximum likelihood estimates, standard errors (in brackets) and approximate 95% confidence limits (in square brackets) for the model parameters for Lingcod and Pacific Cod data for $p = 0.5$.

parameter	Lingcod data	Pacific Cod data
\hat{q}	0.0076(0.0095) [0.0 – 0.08]	0.0029(0.0090) [0.0 – 0.2]
\hat{b}	0.7946(0.1178) [0.35 – 0.98]	0.7485(0.1825) [0.22 – 0.95]
\hat{k}	5.9548(0.2016) [5.6 – 6.38]	5.4612(0.1787) [5.10 – 5.92]
$\hat{\sigma}^2$	0.0341(0.0108) [0.0 – 0.06]	0.0561(0.0161) [0.0 – 0.95]
$\hat{\omega}^2$	0.0035(0.0024) [0.0 – 0.018]	0.0031(0.0030) [0.0 – 0.028]

Table 3: Maximum likelihood estimates, standard errors (in brackets) and approximate 95% confidence limits (in square brackets) for the model parameters for Lingcod and Pacific Cod data for $p = 2.0$.

misspecification. The residuals were calculated using the equation

$$e_{t|T} = y_t - p\tilde{x}_{t|T} - \hat{k} \quad (40)$$

where $\tilde{x}_{t|T}$ is the **fixed-interval smoother** of x_t given by

$$\tilde{x}_{t|T} = \hat{x}_{t|t} + P_t^*(\tilde{x}_{t+1|T} - \hat{x}_t) \quad (41)$$

and

$$P_t^* = \frac{\hat{b}P_{t|t}}{\hat{\omega}^2 + \hat{b}^2P_{t|t}} \quad t = T-1, \dots, 1 \quad (42)$$

(see Harvey, 1989 p.154).

Note in Tables 1, 2 & 3 that as p changes, the only estimates that change are those of q and ω^2 and that they change in proportion to p and p^2 respectively. Thus we can regard the estimates of the stock-dynamic parameter b , the proportionality constant k and the variance σ^2 (of stock dynamics plus error in effort observation) as being *robust* with respect to specification of the parameter p .

A glance at the standard errors and confidence intervals indicates the large degree of uncertainty in most of the parameter estimates. The confidence intervals for q and σ^2 often include the point 0. The coefficient of variation of \hat{q} is larger than 100% in all cases, while that of $\hat{\sigma}^2$ is of the order of 30%. The only parameter estimated with any degree of precision is the proportionality constant k in the relationship between cpue and non-dimensionalized stock biomass.

Figure 5 shows the **median** returning stock size ($median(X_{t+1})$) graphed against the previous season's escapement (S_t), (from the dynamic model (1)) for the two fisheries. The solid line uses the ML point estimate \hat{b} , while the dotted lines use the upper and lower limits of the 95% confidence interval for b . It can be seen that in both cases the maximum **median surplus production** could be anything from an almost negligible quantity to a fairly substantial amount. Likewise the escapement corresponding to the median surplus production, could lie in a very large range. Since the estimates of maximum surplus production and of the escapement which results in this maximum, depend very heavily on the parametric form specified for the stock-dynamic model, and since it is very difficult to determine the precise nature of this relationship, any attempt to estimate maximum surplus production and a corresponding optimal escapement level, based on catch-effort data alone, is likely to have very low reliability (see e.g. Roff, 1983 for a discussion of this issue). Going further, one could contemplate an optimal feedback control law, which would determine the optimal control (effort) for each prediction $\hat{x}_{t|t-1}$ of stock biomass, and in principle at least one could determine such a control law approximately using the **localized LQG (Linear-Quadratic-Gaussian)** method proposed by Horwood & Whittle (1986). However such a procedure is likely to be just as sensitive to the parametric specification of the stock-dynamic equation as the determination of the optimal equilibrium escapement, discussed above. In consequence we make no attempt to

prescribe either an optimal escapement or an optimal harvesting policy.

However as Roff (1983) points out catch-effort data can be used effectively to **predict** future levels of cpue and of relative stock size. The following section describes how predictions can be made and presents results on predictions and their closeness to the test data, for the two fisheries examined.

Predictions.

Once the ML estimates of the parameters based on data up to time $t = T$ have been obtained, one-step-ahead predictions for the unobservable relative stock size X_{T+1} and for the cpue in season $T + 1$ can be obtained. The one-step-ahead prediction of the log stock size is

$$\hat{x}_{T+1|T} = \hat{b}\hat{x}_{T|T} - \hat{b}\hat{q}E_{T+1} \quad (43)$$

(see (30)) with estimation error

$$P_{T+1|T} = \hat{b}^2 P_{T|T} + \hat{\omega}^2 \quad (44)$$

(see (31)). Also a one-step-ahead prediction of the observation $(\ln(\text{cpue}))$ is

$$\hat{y}_{T+1|T} = p\hat{x}_{T+1|T} + \hat{k} \quad (45)$$

which is unbiased (see (35)) with the variance of the **prediction error** given by

$$MSE(\hat{y}_{T+1|T}) = P_{T+1|T} + \hat{\sigma}^2 \quad (46)$$

Here $\hat{y}_{T+1|T}$ is the estimated conditional expectation of the observation ($\ln(\text{cpue})$) at time $T + 1$.

A $100(1 - \alpha)\%$ **prediction interval** for $\hat{y}_{T+1|T}$ is

$$\hat{y}_{T+1|T} \pm z_{\frac{\alpha}{2}} RMSE(\hat{y}_{T+1|T})$$

where $z_{\frac{\alpha}{2}}$ is the $\frac{\alpha}{2}$ percentage point of the standard normal distribution and $RMSE$ is the root mean square error (i.e the square root of MSE above). Exponentiating will lead to a similar prediction interval for cpue .

An important aspect of the one-step-ahead predictions is that they *do not depend on the value specified for the power parameter p in the cpue -biomass relationship*. This can be seen by observing that if the parameters q and ω^2 are adjusted respectively to pq and $p^2\omega^2$, then the quantities $p\hat{x}_{t|t-1}$, $p\hat{x}_{t|t}$, $p^2P_{t|t-1}$ and $p^2P_{t|t}$ are invariant with respect to changes in p .⁵ Thus as p is changed, since the MLEs of q and ω^2 change by the factors p and p^2 it follows that the predictions $p\hat{x}_{T+1|T}$ will be unchanged and thus via (45) that the predictions $\hat{y}_{T+1|T}$ will be unchanged. This is a very important point, since if it were not true predictions would depend on a parameter which would have to be guessed or estimated from other data. The fact that it is true makes the predictions much more robust.

Another point concerns the estimation of the prediction error variance (46) and the prediction intervals. The variance in (46) includes only **system**

⁵This can be verified by induction, or less formally by writing out the first couple of iterations of the KF algorithm.

error and not **sampling error**, i.e it includes only the uncertainty in the stock-dynamics (1) and in the relationship between cpue and stock biomass (14), but not the uncertainty in the parameter estimates. Since, as has been seen, there is a considerable amount of uncertainty in most of the parameter estimates, the true prediction error variance could be considerably bigger than that given in equation (46) and prediction intervals considerably wider than those obtained from the nominal interval above.

Predictions of the catch corresponding to a given level of effort can also be made. However the variance, ρ^2 , of the error in the observation and effect of effort and the variance, τ^2 , in the population dynamic relationship are confounded (equation (9)); it follows then that if effort is observed subject to error a catch prediction will be of that corresponding to an *observed* level of effort, E_T , say, rather than a true effort level. This distinction will probably be of little importance, if as is recommended, catch predictions are not used to determine some “optimal” level of effort.

Tables 4 & 5 show *one-step-ahead predictions* and 95% prediction intervals for the cpue and for the catch for the years in the test data for the two fisheries. The predictions for each year were made using the observed effort that year.

The point predictions and 95% prediction intervals for cpue and catch for Lingcod are plotted in Figs. 6 & 7, along with the observed catches. The point and prediction intervals are joined by line segments. The only year for

year	catch	effort	cpue	predicted cpue	predicted catch
1985	3623	7364	469	310.7 [194.9 – 495.2]	2288 [1435 – 3647]
1986	1075	3127	325	375.6 [233.0 – 605.3]	1174 [729 – 1893]
1987	726	3504	189	343.3 [214.1 – 550.4]	1203 [750 – 1929]
1988	674	2765	231	275.5 [165.7 – 456.9]	761 [458 – 1263]
1989	960	2989	283	263.8 [159.6 – 435.8]	788 [477 – 1303]

Table 4: One-step-ahead predictions of cpue and catch (with 95 percent prediction intervals in parentheses) for the test data (years 1985-1989) for the Lingcod fishery. Units of catch are metric tons, units of effort are hours and cpue is in kg. per hour.

year	catch	effort	cpue	predicted cpue	predicted catch
1984	652	3047	214	230.6 [132.2 – 402.0]	703 [403 – 1225]
1985	463	1508	307	226.1 [130.8 – 390.8]	341 [198 – 632]
1986	803	2974	270	242.2 [139.9 – 419.3]	720 [416 – 1247]
1987	1015	2934	346	245.0 [142.6 – 421.1]	719 [418 – 1236]
1988	1223	3012	406	264.8 [153.3 – 456.5]	798 [462 – 1375]
1989	602	2272	265	297.9 [171.0 – 519.0]	677 [388 – 1179]

Table 5: One-step-ahead predictions of cpue and catch (with 95 percent prediction intervals in parentheses) for the test data (years 1984-1989) for the Pacific cod fishery. Units of catch are metric tons, units of effort are hours and cpue is in kg. per hour.

which the observation lies outside the prediction interval is 1987. However it is only slightly outside, and given the fact that the prediction intervals do not include the sampling error of the parameter estimates, this prediction failure by itself does not invalidate the method. Including sampling error would almost certainly widen the prediction intervals enough to include the 1987 observations. The year 1987 is at the end of a fairly steep decline in cpue reflecting presumably a drop in stock abundance, and it is not surprising, given the simple nature of the stock-dynamic model, that the method has more difficulty in making predictions when stock abundance is changing relatively rapidly compared to when conditions are more stable.

Roff (1983) uses as a measure of prediction performance the **mean absolute percentage error (MAPE)** defined as

$$MAPE = \frac{100}{T_2 - T_1 + 1} \sum_{t=T_1}^{T_2} \frac{|O_t - P_t|}{O_t} \quad (47)$$

where O_t and P_t are the observed and predicted catches in year t , with T_1 and T_2 being the initial and final years over which predictions are made. The *MAPE* for the lingcod catch predictions above is 28.5%. In comparison Roff's SA method yielded a *MAPE* of 36.5%. Qualitatively the predictions from the two methods were similar. For example both under-estimated the catch in 1985, but over-estimated in 1986 and 1987, with, for both methods, the largest percentage error occurring in 1987.

The prediction results for Pacific cod are displayed in Figs. 8 & 9. The results here are very good, with all of the observations lying comfortably in-

side the 95% prediction intervals. The predictions of catch look particularly impressive with the highest and lowest catch predictions coinciding with the highest and lowest observed catches (but also with the highest and lowest levels of effort which is, of course, not surprising since effort is highly correlated with catch). The *MAPE* for catch predictions was 20.2%. Roff’s SA method also did well for this dataset. In fact the *MAPE* for the SA model was slightly superior at 17.5%. Again the predictions from the two methods were qualitatively similar.

The qualitative similarity of the predictions from the two methods, which are quite different in nature, suggests that both have captured more or less the “signal” in the data, and that what remains is essentially “noise” and that it may be difficult to improve upon these predictions, without incorporating more information in the form, say, of independent stock assessments. As discussed in Section 2, the method of this article can easily be adapted to incorporate such information.

4 Discussion

Existing methods of analyzing catch-effort data can broadly be classified as being either of the *surplus production* type or of the *black box* type, the former having a plausible nonlinear dynamic component to represent fishery dynamics and the latter eschewing biological detail for the sake of statistical tractability, via the use of linear statistical theory.

In this paper we have discussed a method of analyzing catch-effort data, which incorporates some of the better features of both kinds of models. In particular the method combines a plausible (albeit very simple) model of the fishery, with the statistical tractability of a linear state-space model. In addition the method recognizes both randomness in the population dynamics, and in the process of catching fish (system noise) along with uncertainty in the measurement of effort (observation error). In general one or more proxies for stock abundance can be used in the method, although in this article we have considered only the single proxy of catch-per-unit-effort. The method assumes that stock abundance is related to cpue according to a power law with multiplicative log-normal error. Thus both hyperstability and hyperdepletion (as well as proportionality) are possible in this relationship.

Not surprisingly not all of the parameters in the model are simultaneously estimable. In total there are three variance parameters, corresponding to noise in the stock dynamics, in the effort observations and/or catching process and in the relationship between cpue and stock abundance. It turns out that the first two variances are confounded, and one can at best get an estimate of a linear combination of the two. Similarly the proportionality constant in the relationship between cpue and stock abundance is not estimable. In consequence one can only estimate stock abundance up to a multiplicative constant i.e one can estimate relative abundance but not absolute abundance. Finally the power parameter reflecting the degree of

hyperstability-hyperdepletion is not estimable, and needs to be specified *a priori*.

However it turns out that, as far as predicting future catches is concerned, it is not necessary to have estimates of these parameters, i.e the point and interval predictions do not depend on the explicit values of these parameters. This is particularly fortunate since it implies a degree of robustness in the method.

In principle the model could be used to determine a level of effort to maximize mean or median surplus production, or in a dynamic framework to maximize expected discounted yield. However we do not recommend its use to this end, since, as with all surplus production models, the prescribed optimum depends critically on the parametric form employed for the stock dynamics. In our model if one changes this from a log-linear form, one loses the linearity of the state-space model. In other surplus production methods one can often use a variety of parametric forms and make prescriptions for approximately "optimal" exploitation based on results from all of them. This is not possible for our model without modification (see below). While the log-linear form captures some aspects of nonlinear dynamics, perhaps enough for prediction purposes, it is too limited to be regarded as a basis for optimal prescriptions⁶.

⁶Roff (1983) goes further and claims that "attempts to determine equilibrium yields from catch-and-effort data are as likely to be successful as finding the pot of gold at the end of the rainbow." He suggests that the very notion of an equilibrium is flawed because of the inherent variability in biological systems. This may be so, but if one broadens the

The method could be adapted with very little modification to incorporate estimates of population abundance other than one based on cpue. For example if there are one or more estimates of abundance (not necessarily independent) based on survey trawls, these can easily be incorporated into the measurement equation of the state space form. In this case the “observation” y_t would now be a vector of logarithms of the separate estimates. This causes no additional problems in the application of the Kalman filter to generate the likelihood function. Indeed it provides an appealing way of combining information from diverse stock assessments⁷.

The method could also be extended to include other models of the population dynamics. The main problem which arises with a nonlinear state-space model (which is what would result if the dynamics were not of the log-linear form) is that one loses the Gaussian property of the predictions and updated estimates when applying the Kalman filter. This means that one needs to keep track of more than simply means and variance-covariances (which characterize a Gaussian distribution). One approach which has been suggested is to locally linearize the dynamic equation, at each iteration, thereby preserving the Gaussian property, which one can regard as holding approximately.

notion of equilibrium to include a stochastic equilibrium or **stationary state**, one can seek a policy to maximize some central measure of yield in that stationary state. The second reason for Roff’s scepticism is his belief that catch-effort data may be simply too limited for use in determining optimal policies. He suggests that estimated equilibria from such data may be little more than statistical artifacts. Implicitly he thus rejects all surplus production methodologies.

⁷Relative credibilities for the assessments can be incorporated by specifying their variances as different fixed multiples of an unknown parameter.

This is the so-called **Extended Kalman Filter (EKF)**, and was used, for example, by Berck and Johns (1991) in an analysis of catch-effort data for the Pacific halibut fishery. It would present no major difficulty to incorporate the EKF in the present method. Doing so would permit use of a more flexible range of models for the population dynamics, which would certainly add to the robustness of the method, and make prescription of an approximately optimal exploitation more feasible. However since the validity the EKF is not well established the soundness of such prescriptions would necessarily be in doubt. We leave this question for exploration in a further paper.

Another more sophisticated approach to nonlinear state-space models is to numerically keep track of the complete distributions as the filter algorithm is iterated. This method has been termed the **Generalized Kalman Filter** and has recently been used in a fisheries context by Rein (1994). Of course it is much more numerically intensive than the simple KF algorithm employed here. Again application of this procedure to catch-effort data is a subject for further research.

Finally the method could be extended to include age-structure. In this case the unobserved state variable would be a vector of logarithms of abundances in various age-classes. The observation variable would be a vector including logarithms of abundance estimates in the various age-classes. However realistic dynamics would require an element of nonlinearity (in the log scale) since recruitment would be a function of a weighted sum of abundances,

i.e of exponentials of log-abundances⁸. An alternative approach, considered by Sullivan (1992), is to regard recruitment as density independent, which, if one assumes additive normal errors leads to a linear state-space model in the original variables. If used for prediction purpose only, such a model might be adequate. However if used for prescriptive purposes it would be unsuitable since it denies the possibility of “recruitment overfishing” to which ultimately almost all fisheries must be vulnerable (see e.g. Hutchins & Myers 1994, Myers & Cadigan, 1994 concerning overfishing of Atlantic northern cod). For prescriptive purposes a model must recognize the dependence of recruitment on spawning stock abundance. Furthermore, as we have emphasised in this paper, operational prescriptive models require a broader base of data than simply time series of catch and effort.

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⁸One possible exception to this would be if all sexually mature fish were to belong to one oldest “collector” age class, with immature fish in annual age-classes. This is one way of including a recruitment delay in a “lumped parameter” population dynamics model.

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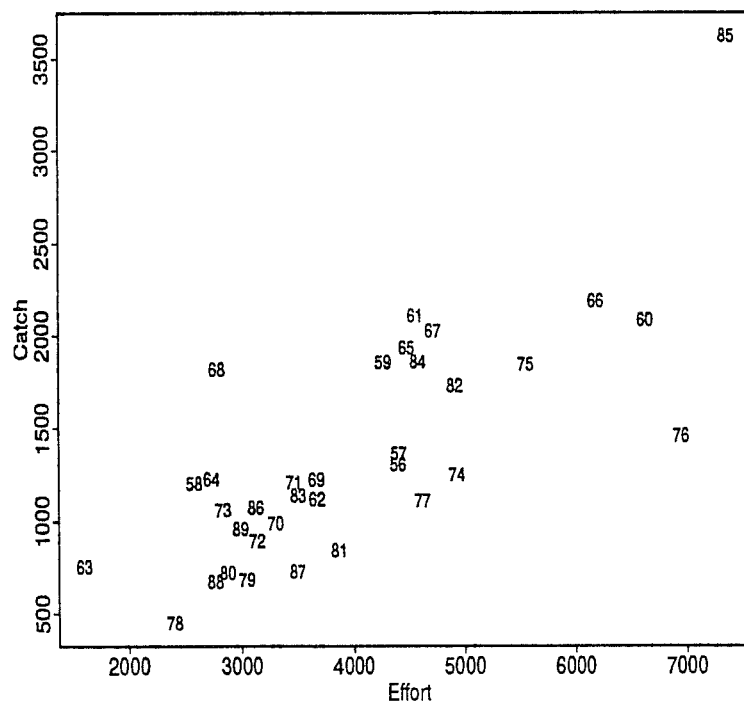


Figure 1: Total annual catch versus total annual effort for the Vancouver Is. Lingcod fishery (Area 3D) for the years 1956 – 1989. Units of effort are hours and units of catch are metric tons.

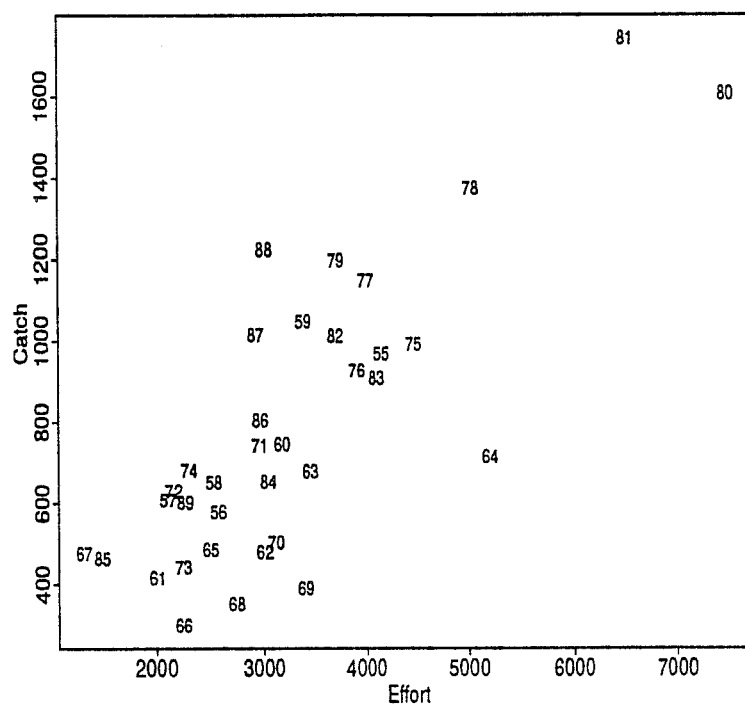


Figure 2: Total annual catch versus total annual effort for the Georgia Strait Pacific Cod fishery (Area 4B) for the years 1955 – 1989. Units of effort are hours and units of catch are metric tons.

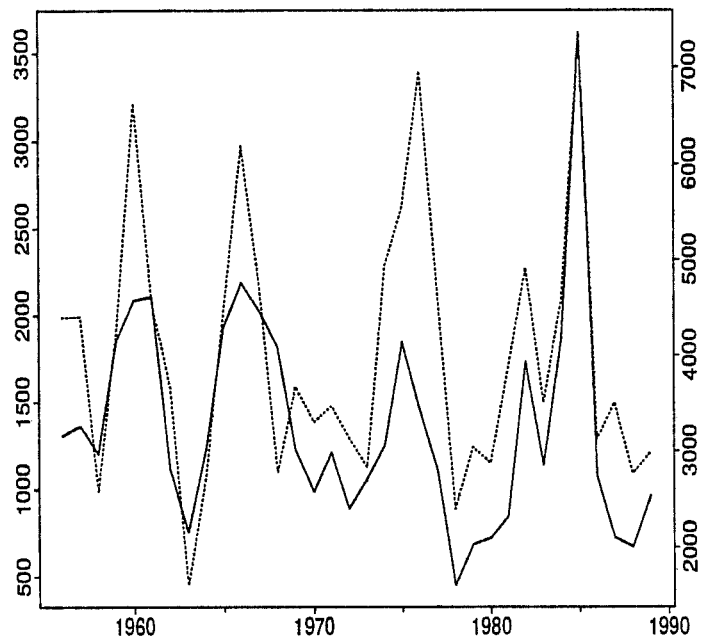


Figure 3: Time series plot of annual catch (left axis joined by solid line segments) and annual effort (right axis joined by dotted line segments) for the years 1956 – 1989 for the Vancouver Is. Lingcod fishery (Area 3D). Units of effort are hours and units of catch are metric tons.

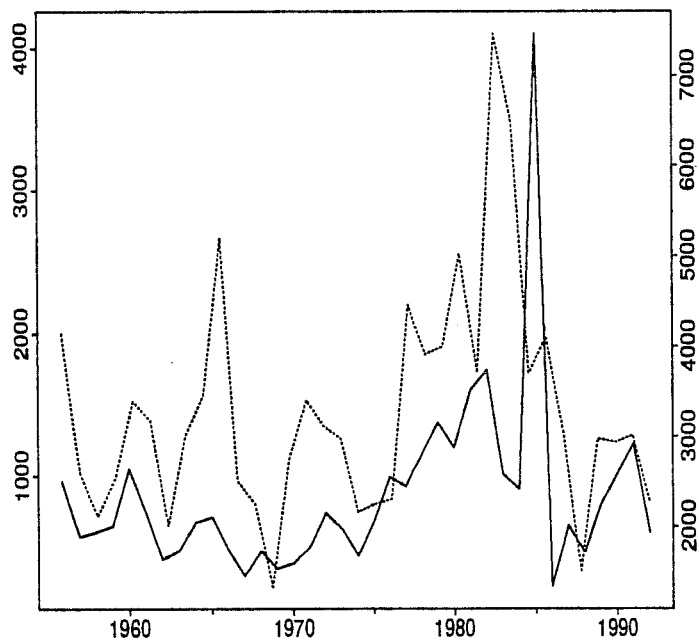


Figure 4: Time series plot of annual catch (left axis joined by solid line segments) and annual effort (right axis joined by dotted line segments) for the years 1956 – 1989 for the Georgia Strait Pacific Cod fishery (Area 4B). Units of effort are hours and units of catch are metric tons.

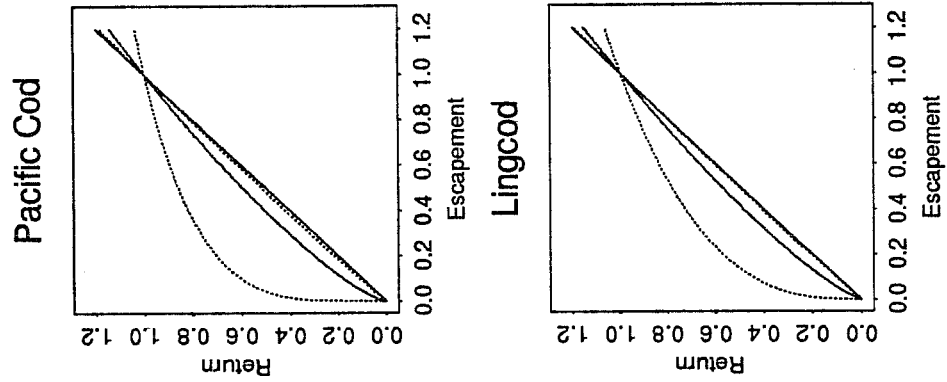


Figure 5: Estimated median returning stock as a function of escapement (solid line) and upper and lower 95 % confidence limits for this relationship (dotted lines). The units are non-dimensionalized i.e. expressed as fractions of the median unexploited equilibrium stock.

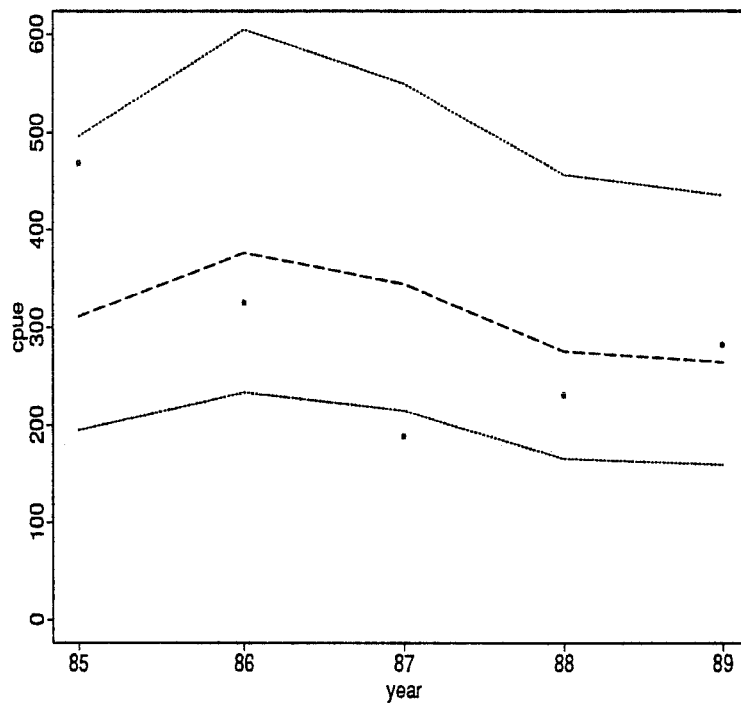


Figure 6: One-step-ahead predictions of cpue (in metric tons per thousand hours) for the Lingcod fishery (joined by pecked line segments) and boundaries of 95 % prediction intervals for cpue (joined by dotted line segments). Dots mark the observed cpue.

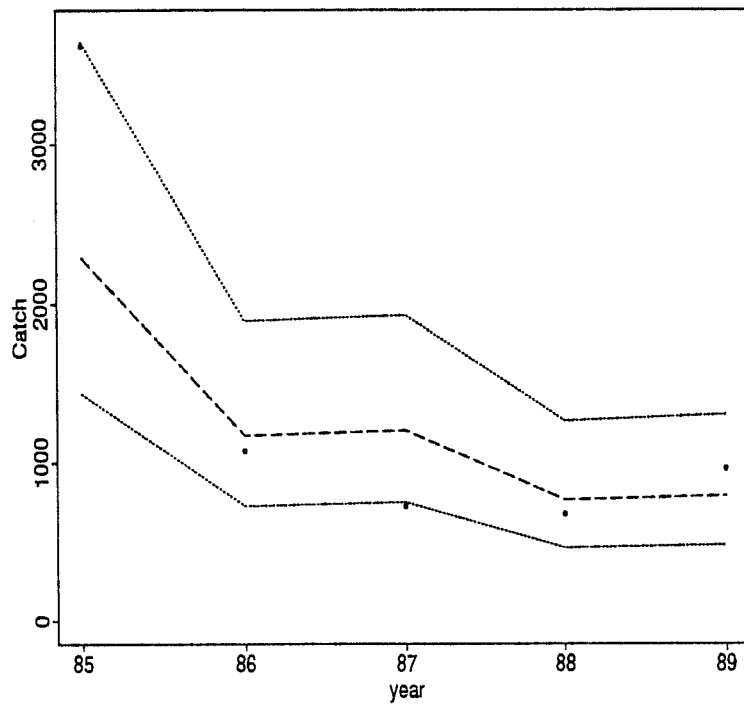


Figure 7: One-step-ahead predictions of catch (in metric tons), given the observed effort level, for the Lingcod fishery (joined by pecked line segments) and boundaries of 95 % prediction intervals for catch (joined by dotted line segments). Dots mark the observed catch.

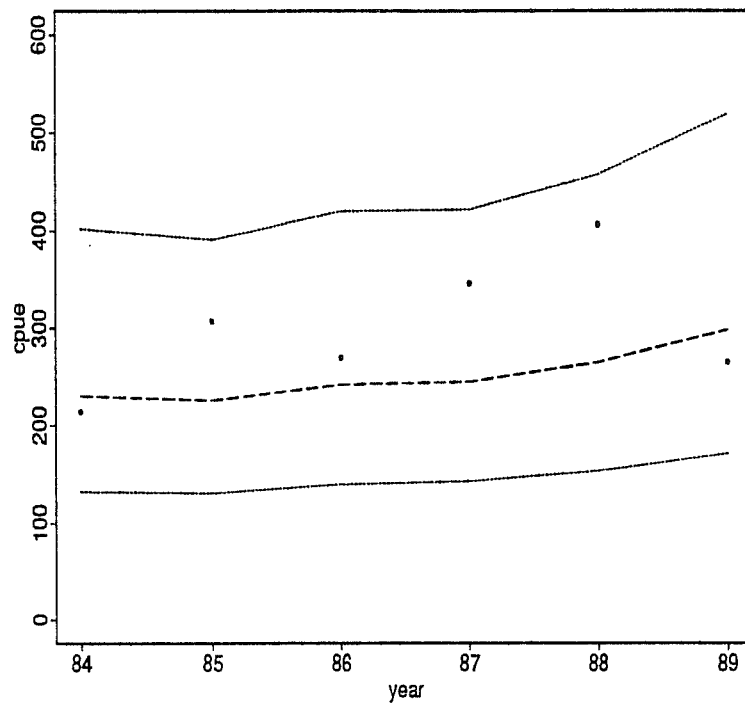


Figure 8: One-step-ahead predictions of cpue (in metric tons per thousand hours) for the Pacific Cod fishery (joined by pecked line segments) and boundaries of 95 % prediction intervals for cpue (joined by dotted line segments). Dots mark the observed cpue.

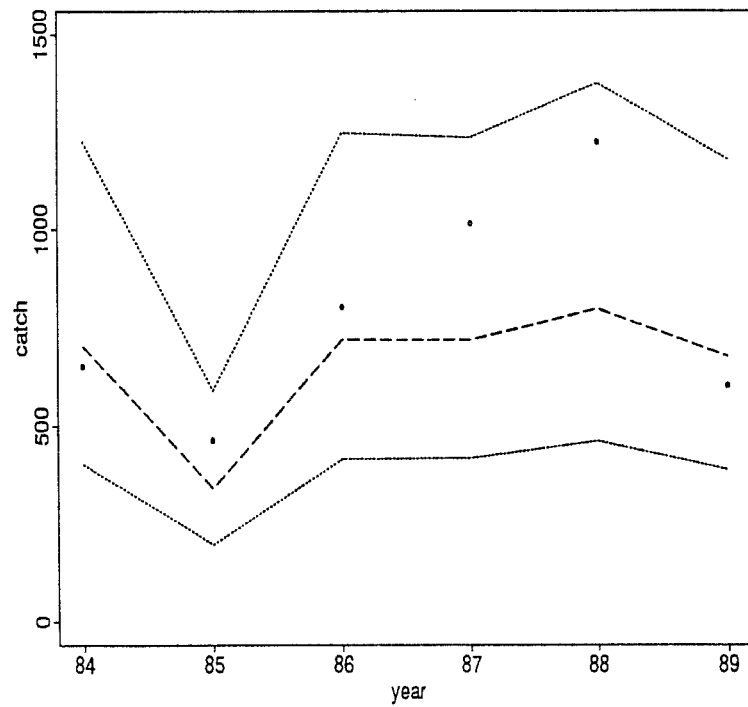


Figure 9: One-step-ahead predictions of catch (in metric tons), given the observed effort level, for the Pacific Cod fishery (joined by pecked line segments) and boundaries of 95 % prediction intervals for catch (joined by dotted line segments). Dots mark the observed catch.