

***THE DECISION TO CONSERVE OR HARVEST
OLD-GROWTH FOREST***

by

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DMS-616-IR

August 1992

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* Research supported by NSERC Grant No. 0GP7252, and by Forest Economics and Policy Analysis (FEPA) Research Unit.

ABSTRACT

The decision as to whether to harvest or conserve old-growth forest is formulated as a stochastic decision problem in continuous time. Uncertainty in future amenity values for standing forest and in future timber revenues for harvested forest are included in the model, along with the risk of catastrophic destruction by fire, pest infestation, etc. It is shown how the decision problem can be expressed as an optimal stopping problem, which can be solved analytically. The optimal decision rule is shown to depend on how the ratio of current timber value to the current expected present value of amenity benefits foregone through harvesting compares with some critical level. The effects of changes in uncertainty and other parameters on the optimal rule are discussed. Also it is shown how the cost-benefit analysis and certainty-equivalence procedures lead to premature harvesting, and the expected loss in survival time for these sub-optimal procedures is calculated.

1. Introduction

As old-growth forest becomes increasingly scarce the conflict between user groups intensifies. To the logging industry old-growth forest is a source of cheap high-quality timber, while to environmentalists and recreational users it represents an irreplaceable natural treasure which should be preserved indefinitely. Often the arguments put forward in favour of logging by the industry and its supporters are expressed in economic terms (*e.g.* the wealth generated and the number of jobs preserved through logging, *etc.*). Environmentalists on the other hand often tend to reject economic arguments, basing their claims on philosophical and moral grounds.

However, if economics can be defined as the study of "how men and society end up choosing, with or without the use of money, to employ scarce productive resources, which could have alternative uses . . . " (Samuelson and Scott, 1975), then it would appear that the question facing society as to whether to harvest or conserve old-growth forest is preeminently an economic one. Once one recognizes that standing forest has a value¹ *per se*, other than as a source of timber, then the problem falls squarely into the framework of economic analysis.

The standard economic methodology for addressing such public policy issues is that of *cost-benefit analysis* (see *e.g.* Porter, 1982) which in its naive form would prescribe a harvest provided the expected present value of benefits from harvesting exceeded the expected present value of the costs (including the amenity benefits foregone through harvesting). A more sophisticated application of the method would time the harvest to maximize the expected present value of benefits net of costs. In spite of the fact that there are many serious shortcomings with cost-benefit analysis when applied to resource and environmental problems (including

distributional and equity issues; difficulties in measuring non-marketed costs and benefits; the choice of an appropriate discount rate *etc.*) it can nonetheless provide a useful starting point for the decision-making problems.

By and large forestry harvesting models have ignored the amenity services provided by old-growth forest although they can easily be included in forest-level harvest scheduling procedures such as FORPLAN (Johnson, Jones and Kent, 1980). The first analytic treatment to include amenity service values appears to be the stand-level model of Hartman (1976), who shows that if amenity values are increasing with age then the optimal rotation age for the stand will exceed the Faustmann rotation age, thereby delaying the harvest and indeed that it may be optimal (in the sense of maximizing net present value) in some circumstances to deter harvesting indefinitely. Porter (1982) considers the cost-benefit analysis of projects involving wilderness destruction and shows how such projects may not be viable if the discount rate is too high or too low.

There are two major shortcomings with the Hartman and Porter models. The first is that they deal with a single stand or project in isolation, and the second is that they are deterministic, regarding future timber or development benefits and amenity values as known. Attempts to address the first of these shortcomings lead back to the forest-level harvesting models of the FORPLAN type, for which analytic solutions appear impossible (see Bowes and Krutilla (1985) for an economic forest-level harvesting model which includes amenity values).

The problem of future uncertainty in timber and amenity values has not been addressed explicitly in the framework of the Hartman model (although for a stochastic generalization of Porter's (1982) model see McDonald & Siegel (1986)). However, there is now a considerable body of literature which deals in general with the problem of irreversible decisions in the face

of uncertainty. The pioneering paper is that of Arrow and Fisher (1974) who considered the problem of developing wilderness when the future benefits from conservation and development are uncertain. They show that since development is irreversible, while conservation is not, there is a value, which they called the *quasi-option value*, associated with the reversible decision to conserve. Simply put, with uncertainty present, there is some value associated with keeping one's options open.

This literature for the most part has dealt with highly stylized two-period models with linear benefit functions, leading therefore to total development or total conservation in period 1 or period 2. A more general model which allows for partial development at any time over an infinite time horizon is that of Clarke and Reed (1990). This model assumes uncertain future amenity values for wilderness which depend on the amount of wilderness surviving, but it assumes that the costs (or immediate revenues) from developing are known. The model might thus be applicable to the problem of harvesting old-growth forest if one could reasonably assume that future timber prices were fixed and known, and that the forest exhibited no growth nor was subject to catastrophic destruction.

It is the purpose of this paper to address the harvesting decision problem, when both future amenity values and future timber values are uncertain. Growth in the volume of timber is not explicitly included, it being assumed that biologically the old-growth forest has reached something close to a steady-state. However, the possibility of unpredictable catastrophic destruction *via* fire, tempest or pest infestation is included. Also, the model is formulated for a single stand of old-growth. The possibility of the amenity value of the stand depending on the amount of old-growth forest remaining globally, or regionally, has not been considered. This is undoubtedly a shortcoming of the model, which can be defended only on the grounds that to

include this aspect would lead to a model for which analytic results would not be forthcoming.

In Section 2 the model is developed. The objective considered is the maximization of the expected present value of the flow of amenity services plus timber revenues up until the time of destruction of the stand, whether through harvesting or through catastrophe. If the latter occurs, of course, there will be no timber revenues. The decision problem faced by the managers of the resource at each instant in time is whether to conserve (no harvest) or to harvest. It is assumed that current timber and amenity values are known but that future values are not known with certainty. Both are assumed to follow (possibly correlated) geometric Brownian motion processes. Thus both timber values and amenity values are assumed to grow (or decay) at a proportional rate, which at all times is made up of a fixed component and a random component. The problem of whether to harvest or not is formulated and solved as an optimal stopping problem in Section 3. Conditions which guarantee conservation at all times are given. Formally the optimization model is similar to that of McDonald and Siegel (1986) developed in the context of investment timing, although it differs from that model in the inclusion of the risk of catastrophic destruction.

In Section 4 the sub-optimality of harvest rules based on the naive application of cost-benefit analysis, and on certainty equivalence, is discussed.

2. A Model for the Harvest-Conservation Decision Problem.

Consider an area of old-growth forest which can be either conserved *in toto*² or clear-cut harvested. Clearly the decision to conserve is reversible, in that the forest can be harvested at a later date, while the decision to harvest is irreversible. Suppose that if the forest is harvested at time t a net revenue $V(t)$ is realized. This includes the revenue from the sale of timber net

of harvest costs,³ plus the value of the bare land on which future rotations may possibly be grown. We shall assume that $V(t)$ is an observable stochastic process following *geometric Brownian motion*, governed by the stochastic differential equation

$$\frac{dV}{V} = b dt + \sigma_1 dw_1 \quad (1)$$

where b is a drift parameter, $\{w_1(t)\}$ is a standard Wiener process, and σ_1^2 is a variance parameter. Thus we assume that the proportional change in net value over time dt is a constant $b dt$ plus or minus a random component $\sigma_1 dw_1(t)$. Large values of the parameter σ_1 will correspond to a large amount of uncertainty in future timber values. If harvest and replanting costs are neglected, $V(t)$ should be proportional to the current price of timber, since both the revenues from the sale of timber and the land value will then be proportional to timber price (see Clarke and Reed, 1989). The assumption that a commodity price follows geometric Brownian motion is very reasonable. A similar assumption for asset prices lies at the heart of much of the recent literature in financial economics.

The realization of the immediate benefit $V(t)$ resulting from harvesting at time t is offset by the foregoing of the option to harvest at a later date when timber prices might conceivably be higher, as well as by the foregoing of the flow of amenity services from the old-growth forest at all times beyond t .

Suppose that the flow of amenity services at time t has social valuation $A(t)$. It should be emphasized that $A(t)$ is a flow and has units such as dollars per month. It represents the rent that society is willing to pay to preserve the old-growth forest.⁴ To reflect the fact that future valuations of amenity services are uncertain, we shall assume that $A(t)$ is a stochastic

process. Specifically we shall assume it follows geometric Brownian motion

$$\frac{dA}{A} = a dt + \sigma_2 dw_2 \quad (2)$$

As before, a and σ_2^2 are drift (mean growth rate) and variance (uncertainty) parameters while $\{w_2(t)\}$ is another standard Wiener process. To reflect the fact that future changes in amenity value and timber value may be correlated, we shall assume that the white-noise processes $\{dw_1(t)\}$ and $\{dw_2(t)\}$ have correlation ρ . Positive values of ρ will reflect a positive association between fluctuations in timber and amenity values, and negative values of ρ a negative association.

Note that the assumption that the stochastic process $\{A(t)\}$ is observable is something of a simplification since societal valuations of amenity services can be determined only through sampling or other imprecise methods and thus will contain an element of error. Similarly, even though current timber prices may be known exactly, the net revenue $V(t)$ to be realized through an immediate clear-cut harvest of the old growth cannot be known exactly prior to the harvest. For example, harvesting costs (and certainly environmental costs) can only be estimated and the volume of usable timber is usually known only via sampling methods. However, uncertainty in future values of $V(t)$ and $A(t)$ is much greater than uncertainty concerning current values. Abstracting from the former source of uncertainty captures the essence of the problem of when, if ever, a clear-cut harvest should take place.

The fact that a harvest does not take place does not guarantee the survival of the forest forever. There is always the possibility that it may be destroyed by some natural catastrophe such as fire, tempest or pest infestation. In assessing the total value of amenity services obtained

from the forest before a scheduled clear-cut harvest, this fact must be taken into account.

If there were to be no harvest, amenity services would accrue up until the time at which catastrophic destruction occurred, if indeed it ever did. Thus the expected present value of amenity services, using an *instantaneous discount rate*, δ , would be⁵

$$\bar{A}_f(0) = E \left\{ \int_0^{\tau} e^{-\delta t} A(t) dt \right\} \quad (3)$$

where τ is a random variable denoting the time of destruction (τ would be infinity if destruction never occurs). The expectation in (3) is taken with respect to both τ and the stochastic process $\{A(t)\}$. The integral can be re-expressed as

$$\bar{A}_f(0) = E \left\{ \int_0^{\infty} e^{-\delta t} A(t) S(t|0) dt \right\} \quad (4)$$

where $S(t|0)$ is the *survivor function* denoting the probability that the stand survives catastrophic destruction until time t , given that it is 'alive' at time 0.

The survivor function is determined by the *hazard-rate*. Although many functional forms could be used here (see *e.g.* Thompson, 1988), for simplicity, and in absence of any direct indication to the contrary, we shall assume a constant hazard rate λ , *i.e.* assume that the hazard of catastrophic destruction is time-independent. In this case the survivor function can be written as

$$S(t|0) = e^{-\lambda t}$$

and the expectation in (4) can be written as

$$\bar{A}_f(0) = E \left\{ \int_0^{\infty} e^{-(\delta + \lambda)t} A(t) dt \right\} \quad (5)$$

which can be evaluated (see Appendix 1) to give the expected present value, at time zero, of amenity services as

$$\bar{A}_f(0) = \frac{A(0)}{\delta + \lambda - a}. \quad (6)$$

More generally we define $\bar{A}_f(t)$ as the expected present value (at time t) of future amenity services conditional on the stand not being destroyed by time t , and on the value $A(t)$ of the amenity flow at that time. Specifically

$$\begin{aligned} \bar{A}_f(t) &= E \left\{ \int_t^{\infty} e^{-\delta(z-t)} A(z) S(z|t) dz | A(t) \right\} \\ &= \frac{A(t)}{\delta + \lambda - a}. \end{aligned} \quad (7)$$

Note that the presence of a constant risk of destruction affects the evaluation of future amenity services in the same way as the addition of a premium λ to the discount rate δ .

3. The Optimal Harvest Rule

To determine the optimal harvest rule, consider the *expected present value* (E.P.V.) of benefits if a harvest is scheduled to take place at time T . They include the E.P.V. of timber benefits

$$e^{-\delta T} S(T|0) V(T) = e^{-(\delta+\lambda)T} V(T) \quad (8)$$

and the E.P.V. of amenity benefits

$$\begin{aligned} E \left\{ \int_0^T e^{-\delta t} A(t) S(t|0) dt \right\} &= E \left\{ \int_0^\infty e^{-(\delta+\lambda)t} A(t) dt \right\} - E \left\{ \int_T^\infty e^{-(\delta+\lambda)t} A(t) dt \right\} \\ &= \bar{A}_f(0) - e^{-(\lambda+\delta)T} \bar{A}_f(T). \end{aligned} \quad (9)$$

Thus the net E.P.V. of a harvest scheduled at time T is

$$e^{-(\lambda+\delta)T} V(T) - e^{-(\lambda+\delta)T} \bar{A}_f(T) + \bar{A}_f(0). \quad (10)$$

Note that to ensure convergence of the integrals in (9) we require the condition

$$\delta + \lambda - a > 0. \quad (11)$$

We shall assume that this condition, and in addition the condition

$$\delta + \lambda - b > 0, \quad (12)$$

hold. If either of these conditions is not met it will be optimal to never harvest the forest.⁶ Thus sufficient conditions for *conservation in perpetuity* to be optimal are that $\delta + \lambda < a$ or $\delta + \lambda < b$. If the former is met, amenity services are growing faster in expectation than the risk-adjusted discount rate and it therefore pays to conserve forever. If the latter condition is met, timber values are growing faster in expectation than the risk-adjusted discount rate, and so at all times it will pay to postpone a clear-cut harvest.⁷

The problem of determining an optimal harvest rule can be expressed as that of finding a *stopping time* T to maximize the total expected present value:

$$e^{-(\delta+\lambda)T} [V(T) - \bar{A}_f(T)] + \bar{A}_f(0). \quad (13)$$

This can be formulated as a problem of *optimal stopping* (see *e.g.* Brock, Rothschild and Stiglitz, 1988) with *intrinsic value* function

$$R(V,A) = V(T) - \bar{A}_f(T) = V(T) - \frac{A(T)}{\Delta - a} \quad (14)$$

and the objective of maximizing the expected present value

$$E\{e^{-\Delta T} R(V(T), A(T))\} \quad (15)$$

where $\Delta = \delta + \lambda$ is the *risk-adjusted discount rate*. (Note that (14) omits the last term on the right-hand side of (13) which is a constant independent of T , $A(T)$ and $R(T)$.) Note also that in general a stopping time T may depend on past and current values of the bivariate stochastic process $\{V(t), A(t)\}$, and that the expectation in (15) depends on the initial values (V_0, A_0) of this process.

It is shown in Appendix 2 that the *optimal stopping rule* depends only on the current state and involves a harvest at time t if and only if

$$V(t) \geq \left(\frac{1+\theta}{\theta} \right) \bar{A}_f(t) \quad (16)$$

where θ is the positive root of the characteristic equation

$$\frac{1}{2} \sigma^2 \theta^2 + (b - a - \frac{1}{2} \sigma^2) \theta + (b - \Delta) = 0 \quad (17)$$

and σ^2 is the variance of the $\left\{ \ln \left(\frac{V(t)}{A(t)} \right) \right\}$ stochastic process, *i.e.*

$$\sigma^2 = \sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2. \quad (18)$$

Since θ is positive it is clear that *optimally one harvests if and only if the net timber revenues, $V(t)$ ("benefits") exceed the E.P.V. of amenity benefits foregone ("costs") by a factor*

$$q^* = \left(1 + \frac{1}{\theta} \right) \quad (19)$$

which is greater than one. Thus the optimal harvest rule is more conservative than that arising from the naive application of cost-benefit analysis.

Another way of expressing the optimal harvest rule is in terms of the ratio of current timber value to current amenity service value $V(t)/A(t)$. Optimally the stand is harvested if and only if this ratio exceeds

$$C^* = \frac{1}{\Delta - a} \left(\frac{1 + \theta}{\theta} \right) \quad (20)$$

It is fairly easy to show (see Appendix 3) that the critical ratio q^* , of benefits to expected cost (benefits foregone), *increases* with increases in

- (a) the expected growth rate of timber values b ,
- (b) the uncertainty in the growth of timber values σ_1 ,
- (c) the uncertainty in the growth of amenity values σ_2 .

Conversely q^* decreases with increases in

- (d) the expected growth rate of amenity service values, a ,
- (e) the discount rate δ ,
- (f) the hazard rate λ ,
- (g) the correlation ρ between amenity service values and timber values.

Table 1 displays the critical q^* for various values of the parameters a , b , δ and the total variance parameter σ^2 . In all cases the hazard rate is set at $\lambda = .005$ per year corresponding approximately to a 1 in 200 chance of destruction in any given year. The blank entries (where a or b exceed $\Delta = \delta + \lambda$) correspond to cases where it is never optimal to harvest. Note how the critical ratio q^* is large when the difference $b - a$ in expected growth rates for timber values and amenity value is large, and how q^* increases with the variance σ^2 . For example with $a = 0$, $b = 0.05$, $\delta = .05$ and $\sigma^2 = .02$ the benefits (the current timber value) would have to exceed the costs (expected present value of amenity services foregone) by a factor of more than 13 before, optimally, a harvest would be undertaken.

With σ^2 set at 0.02 one could be fairly confident ($\approx 95\%$ confidence) that within one year the ratio of timber value to amenity flow $V(t)/A(t)$ would lie within between 0.75 and 1.33 times its initial value;⁸ with $\sigma^2 = 0.01$ the corresponding range would be 0.82 and 1.22 times the initial value. Such fluctuation seems well within the range of possibility.

The critical ratio, C^* , for timber values to amenity values increases with increases in (a), (b), (c) and (d) but decreases with increases in (e), (f) and (g). Thus qualitatively it behaves like q^* except with respect to the parameter a , the expected growth rate of amenity service values.

4. Other Harvest Rules

In this section the optimal harvest rule developed in Section 3 is compared with other policies which might be employed in making a decision as to whether to harvest or not.

It has already been noted that naive application of cost-benefit analysis would prescribe a harvest whenever the expected benefits exceeded the expected costs *i.e.* whenever

$$V(t) \geq \bar{A}_f(t) \quad (21)$$

(which would occur whenever the ratio of timber values to amenity values $(V(t)/A(t))$ exceeded $(\Delta - a)^{-1}$), and that this procedure would prescribe a premature harvest. To measure the degree to which the harvest would be premature, we can look at the additional survival time of the stand using the optimal rule rather than the sub-optimal cost-benefit procedure. Of course this is a random quantity and we must use some characteristic of its distribution, such as the mean, to characterize the degree to which cost-benefit analysis prescribes a premature harvest. It can be shown (see Appendix 4) that *the expected additional survival time* is

$$E(\tau_{ex}) = \frac{1}{\lambda} [1 - (q^*)^{-\phi}] \quad (22)$$

where ϕ is the *positive* root of

$$\frac{1}{2} \sigma^2 \phi^2 + \left(b - a + \frac{1}{2} \sigma^2 \right) - \lambda = 0. \quad (23)$$

Table 2 presents values of this expected additional survival time when the hazard rate λ is set at 0.005. In the absence of any harvesting the expected survival time of the old-growth forest would be $1/\lambda = 200$ years. It can be seen that in all cases (when $a < \Delta$, $b < \Delta$),

harvesting under the cost-benefit rule is considerably premature with the expected difference ranging from a low of 17 years to a high of 115 years. In cases with no variability in both $A(t)$ and $V(t)$ ($\sigma^2 = 0$ -- deterministic model) the optimal rule collapses to the cost-benefit rule when $a \geq b$ (hence the zero loss in survival time), but not when $a < b$, (see later).

A more sophisticated application of cost-benefit analysis might recognize that the net present value (timber revenues net of expected amenity services foregone) could vary over time and seek a harvest rule to maximize the expected value of this difference. The harvesting rule which accomplishes this is, of course, the optimal rule of Section 3. In practice however when cost-benefit analysis is applied it is common to replace uncertain future quantities by their expected values. In the jargon of stochastic optimization this is known as a *certainty-equivalence* procedure. In the current context it can be shown (see Appendix 5) that the certainty equivalence harvest rule is to harvest whenever timber benefits exceed expected costs of amenity services foregone by a factor q_1 i.e. whenever

$$V(t) \geq q_1 \bar{A}_f(t) \quad (24)$$

where

$$q_1 = \begin{cases} 1 & \text{if } b \leq a \\ \frac{\Delta - a}{\Delta - b} & \text{if } b > a \end{cases} \quad (25)$$

Thus when amenity flow values are growing in expectation faster than timber values ($b \leq a$) the certainty-equivalence rule is identical to that obtained from the naive application of cost-benefit analysis.

In the other case ($b > a$) it can easily be verified that the certainty-equivalence rule is

the same as the stochastic version of the *Wicksell rule* (Wicksell, 1934) for the disposal of a capital asset *i.e.* the rule resulting from harvesting the stand when the intrinsic net revenue $R(V,A)$ (in (14)) is growing *in expectation* at the discount rate *i.e.* when $E(dR) = \Delta R dt$.⁹ Table 3 displays the critical ratio q_1 for various values of the parameters a , b and σ^2 . It can easily be verified that in general (when $\sigma^2 > 0$) that $q^* > q$, *i.e.* that the certainty-equivalence procedure will prescribe a premature harvest.

As with the naive cost-benefit rule the degree to which the certainty-equivalence rule prescribes premature harvesting can be characterized by the expected additional survival time, which is given by the formula

$$\frac{1}{\lambda} [1 - (q_1/q^*)^\phi] \quad (26)$$

(see Appendix 4). This is tabulated in Table 4, from which it can be seen that the expected loss in lifetime of the stand is not large in the cases when $b > a$. In such cases the certainty-equivalence procedure provides a reasonable approximation to the optimal rule (see also Table 3). However such cases ($b > a$) are for (the fairly unlikely) situations in which timber benefits grow faster (in expectation) than amenity service benefits. As old-growth forest becomes more scarce one could expect that the social valuation of the amenity services provided by the remaining stands of old-growth would increase. Thus it seems likely that the mean growth rate parameter, a , for amenity services would exceed that for timber benefits, b . In this case the certainty-equivalence procedure is identical to the naive cost-benefit procedure, which as has already been demonstrated, can lead to the considerably premature destruction of the stand. The sub-optimality of certainty-equivalence procedures for irreversible decisions

taken in the face of uncertainty has been well-documented (Arrow and Fisher; 1974, Henry, 1974; Conrad, 1980), using simple two-period models. The results established here concur with the earlier findings and enlarge upon them by quantifying the sub-optimality in terms of expected survival time.¹⁰

5. Concluding Remarks

In this paper an "optimal" harvest rule for old-growth forest has been developed and discussed. It is "optimal" in the narrow sense of maximizing the expected net present value of timber benefits and amenity benefits. By making certain plausible assumptions about the way in which timber values and amenity service values might grow in the future, it has been possible to determine analytically the optimal harvest rule, which turns out to be of a particularly simple form *viz.* harvest only when the immediate timber benefit exceeds the expected present value of amenity services foregone, by a factor, larger than one (or alternatively when the ratio of current timber value to amenity service value exceeds a threshold value).

Of course any model is a simplification of reality, with the model of this paper being no exception. For example we have conveniently ignored external environmental costs (or subsumed them in the net timber benefit variable $V(t)$, along with harvest costs; or alternatively included the environmental services provided by old-growth forest in the amenity service value variable $A(t)$). The assumptions that net timber benefits and amenity service benefits grow at rates made up of fixed and serially uncorrelated random components is also undoubtedly a great simplification, although it is a considerable generalisation over related deterministic models, (*e.g.* Porter, 1982) and does capture the essence of the future uncertainty in these values. Again the assumption that both current amenity values and current net timber values can be measured

without error, is also a simplification. Another is the assumption that the amenity service value of the stand can be dealt with as an exogenous variable, when in fact it may very well depend on decisions made with respect to other stands of old-growth *i.e.* it could be an endogenous variable dependent on public policy with respect to old-growth forest.

Notwithstanding all these limitations of the model we claim that it satisfies the most important requirement of any mathematical model, namely that it is *useful*. Its primary usefulness is in demonstrating how uncertainty in future timber and amenity values and the risk of catastrophic destruction interact in determining the policy which maximizes expected present value and furthermore how they affect the adequacy of other procedures such as cost-benefit analysis.

For example it has been demonstrated that the naive application of cost-benefit analysis (the "new" cost-benefit analysis of Porter, 1982) or the application of certainty-equivalence procedures can lead to the premature harvesting of the forest; and furthermore how such policies exhibit greater deviations from the optimal policy, the greater the degree of future uncertainty. Of course this phenomenon has been known for a long time (Arrow and Fisher, 1974), but until now the quantitative aspects have not been closely examined. In the current article the degree of sub-optimality has been quantitatively assessed by examining the expected shortening of the lifetime of the stand through using such sub-optimal policies. Also the effect of the risk of catastrophic loss on the optimal (and suboptimal) policies has been considered. The presence of such risk has often been used by the proponents of logging to justify rapid harvesting (the "use it or lose it to Nature" argument). As one might expect the presence of the inclusion of a time-independent hazard rate has the same effect as the addition of a premium to the effective discount rate. This in turn has the effect of lowering the expected present value of amenity

benefits foregone through harvesting and of lowering the ratio of timber value to amenity service value at which harvesting becomes optimal. Indeed there are situations in which the presence of the risk of catastrophic loss could change the optimal policy from one in which it would never be optimal to harvest to one in which one would harvest if the ratio of timber value to amenity value were suitably high.¹¹

As it stands the model is not intended to be an operational one for determining whether a particular stand of old-growth should be harvested or not. There are many complexities to such decisions that have been ignored here. For example the question of the distribution of benefits and costs associated with various decisions has not been addressed, but is undoubtedly an important one. In this respect the analysis is similar to cost-benefit analysis in that it deals with aggregated benefits and costs ignoring the issue of who reaps the benefits and who suffers the costs. Very often the benefits of logging go to a relatively small group (the forest industry, its workers and dependents), while the benefits of conservation (and the environmental costs of logging) are more diffuse, often spread throughout the whole of society as public goods (or bads). Distributional issues such as these do not readily lend themselves to quantification and modelling and are probably best dealt with in the political arena. However, even though the model is not intended to provide hard and fast decisions, it can still be of use in the decision making process. It can do this by providing a starting point, and a yardstick against which other policies can be measured, *i.e.* the cost (in terms of loss in expected present value to society) of "sub-optimal" policies can be calculated. In order to do this however one needs estimates of the model parameters and data on current timber value and current amenity service value. While timber values are relatively easy to obtain, timber being a marketed good, amenity services, generally being non-marketed, are much more difficult to evaluate. However much recent work

has been done in this area and there are now a number of methods of assessing amenity values, such as the travel-cost method, contingent valuation *etc.*, (see *e.g.* Pearce & Turner, 1990, Chapt. 9). While estimates of the timber price parameters could be obtained from historical time series of timber prices, there is no guarantee that these values would hold in the future. The same caveat holds for the amenity service parameters, only in this case, with much less historical data available, even the estimation of the parameters might not be possible. Probably the best approach is to regard these parameters as unknowns and perform the analysis for various different plausible values of them, and see how the conclusions concur or differ. For example it is customary to perform cost-benefit analysis under a number of different interest rate scenarios to assess the sensitivity of the conclusions to the choice of discount rate. The same procedure can be followed for the other unknown (or unknowable) parameters, when using the model and analytic methods developed in this paper.

Appendix 1

Evaluation of the Expected Present Value of Amenity Services

The expected present value (at time 0) of amenity services over an infinite time horizon is (see (5))

$$\bar{A}_f(0) = E \left\{ \int_0^{\infty} e^{-(\delta + \lambda)t} A(t) dt \right\} = \int_0^{\infty} e^{-(\delta + \lambda)t} E(e^{y(t)}) dt \quad (\text{A1.1})$$

where $y(t) = \ln A(t)$; the interchange of the order of the expectation and integration operators is justified by the convergence of the right-hand side (see below).

Now $\{y(t)\}$ is Brownian motion with drift $a - \frac{1}{2}\sigma_2^2$ and thus for $t > 0$, $y(t) = y(0) + (a - \frac{1}{2}\sigma_2^2)t + \sigma_2\sqrt{t}Z$, where Z is a standard normal random variable. From well-known results on the log-normal distribution it follows that

$$\begin{aligned} E(e^{y(t)}) &= \exp\left\{y(0) + (a - \frac{1}{2}\sigma_2^2)t + \frac{1}{2}\sigma_2^2 t\right\} \\ &= A(0) \exp\{at\}. \end{aligned} \quad (\text{A1.2})$$

Substituting this in the right-hand side of (A1.1) and performing the integration gives

$$\bar{A}_f(0) = \frac{A(0)}{\delta + \lambda - a}. \quad (\text{A1.3})$$

Note that for convergence of the integral we require $\delta + \lambda - a > 0$, as assumed in Section 2 (see (11)).

The derivation of the expression (7) for $\bar{A}_f(t)$ is essentially similar.

Appendix 2

Solution of the Optimal Stopping Problem

Since the stochastic processes (1) and (2) are stationary Markov processes we can confine our attention to *stationary* decision or *stopping rules*, which in this case are simply partitions of the $V - A$ space into two regions -- a *stopping region* and a *continuation region*. The process is stopped (forest clear-cut harvested) when the $\{V(t), A(t)\}$ process leaves the continuation region for the first time.

For purposes of analysis it is convenient to consider a transformation of variables V and A . Let

$$x(t) = \ln V(t) \quad \text{and} \quad y(t) = \ln A(t). \quad (\text{A2.1})$$

Then from (1) and (2) using Itô's lemma

$$\begin{aligned} dx &= (b - \frac{1}{2}\sigma_1^2)dt + \sigma_1 dw_1 \\ dy &= (a - \frac{1}{2}\sigma_2^2)dt + \sigma_2 dw_2. \end{aligned} \quad (\text{A2.2})$$

Consider now any stopping rule S which partitions the $V - A$ space (and hence the $x - y$ space) into stopping and continuation regions. For such a stopping rule we define a *value function*

$$W^S(x, y) = E\left\{e^{-\Delta\tau_s} R(\exp(\tau_s), \exp(y(\tau_s))) | x(0) = x, y(0) = y\right\} \quad (\text{A2.3})$$

where τ_s is a random variable denoting the time at which the process first leaves the continuation region, and the expectation is taken with respect to τ_s , $x(\tau_s)$ and $y(\tau_s)$. The function R is as specified in (14). The function W^S simply represents the expected present

value of timber revenues net of foregone amenity services, given that initially the timber and amenity service values are $A(0) = e^x$ and $V(0) = e^y$, and using the stopping rule S . Adding $A(0)/(\Delta - a)$ to W^S gives the expected present value of all benefits from the forest (see (13)).

It is easy to show that for (x,y) belonging to the continuation region, the function $W^S(x,y)$ satisfies the following *Hamilton-Jacobi-Bellman* equation

$$\Delta W^S = (b - \frac{1}{2} \sigma_1^2) W_x^S + (a - \frac{1}{2} \sigma_2^2) W_y^S + \frac{1}{2} \sigma_1^2 W_{xx}^S + \rho \sigma_1 \sigma_2 W_{xy}^S + \frac{1}{2} \sigma_2^2 W_{yy}^S \quad (\text{A2.3})$$

where subscripts denote partial derivatives. Also, on the boundary of the stopping region,

$$W^S(x,y) = R(e^x, e^y). \quad (\text{A2.4})$$

In addition, if the stopping rule S is optimal (*i.e.* if $W^S(x,y) \geq W^U(x,y)$ for all stopping rules U), Then W^S satisfies the so-called "*smooth-pasting*" conditions on the boundary of the stopping region

$$W_x^S(x,y) = \frac{\partial}{\partial x} R(e^x, e^y) = e^x \quad (\text{A2.5})$$

$$W_y^S(x,y) = \frac{\partial}{\partial y} R(e^x, e^y) = -e^y/(\Delta - a) \quad (\text{A2.6})$$

It can be shown (see *e.g.* Brock, Rothschild and Stiglitz (1988)) that the conditions (11), (A2.4-A2.6), along with the condition that $W^S(x,y) > R(e^x, e^y)$ on the continuation region, determine an optimal stopping rule provided certain regularity conditions are satisfied. Thus the optimal stopping problem can be solved as a *free-boundary problem* given by the partial differential equation (A2.3) with free-boundary conditions (A2.4-A2.6).

In general, solution of free-boundary problems in two or more dimensions is very

difficult. However, the problem above turns out to have a particularly simple solution, viz. a stopping rule which prescribes stopping whenever $x - y$ exceeds some critical level. To see this consider a stopping rule which partitions $x - y$ space into the continuation region $\{x - y < k\}$ and the stopping region $\{x - y \geq k\}$ where k is some, as yet unspecified, constant. This stopping rule prescribes stopping as soon as the ratio $V(t)/A(t)$ reaches, from below, a threshold level, $C = e^k$. Consider also a value function $W(x, y)$ of the form

$$W(x, y) = B \exp\{(1 + \theta)x - \theta y - \theta k\} \quad (\text{A2.7})$$

on the continuation region $\{x - y < k\}$ (where B and θ are as yet unspecified constants), and of the form

$$W(x, y) = R(e^x, e^y) \quad (\text{A2.8})$$

on the stopping region $\{x - y \geq k\}$, where R is as specified in (14). We shall show that there are particular values of the constants k , B and θ for which W satisfies the conditions above sufficient for an optimum.

Before proceeding with the analysis we discuss briefly the reasoning behind specifying a value function of the form (A2.7). It can be shown that for a stopping rule of the above type which is a simple barrier rule on $z(t) = x(t) - y(t)$, the associated value function must be of this form. This follows from writing the expectation in (14) as

$$E_{\tau}\{e^{-\delta\tau} E_{x(\tau)|x(\tau)-y(\tau)=k} [R(e^{x(\tau)}, e^{y(\tau)})]\}, \quad (\text{A2.9})$$

and evaluating the inner conditional expectation using well-known results on Brownian motion and the log-normal distribution. The outside expectation can then be evaluated using, again well-known, results on the moment generating function of the first-passage time of Brownian motion with drift. Full details are given for a similar problem in a paper of Clarke and Reed

(1988). Rather than repeat all this detailed analysis (which is in fact unnecessary), we shall start with a value function of the form (A2.7) and find values of the unspecified constants which allow it to satisfy the conditions for an optimum.

Firstly, to satisfy the Hamilton-Jacobi-Bellman equation (A2.3), θ must satisfy the quadratic equation

$$\frac{1}{2} \sigma^2 \theta^2 + (b - a + \frac{1}{2} \sigma^2) \theta + (b - (\delta + \lambda)) = 0 \quad (\text{A2.10})$$

where

$$\sigma^2 = \sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2$$

is the variance parameter for the (Brownian motion with drift) process $\{z(t) = x(t) - y(t)\}$.

Next, to satisfy the continuity condition (A2.4) at the stopping boundary, the condition

$$B \exp\{x + \theta x - y - k\} = e^x - \frac{e^y}{\Delta - a} \quad (\text{A2.12})$$

must be satisfied on the stopping boundary $x - y = k$.

This reduces to the condition

$$B = 1 - \frac{e^{-k}}{\Delta - a}. \quad (\text{A2.13})$$

To satisfy the smooth-pasting conditions (A2.5-A2.6) the conditions

$$B(1 + \theta) \exp\{x - \theta(x - y - k)\} = e^x \quad (\text{A2.14})$$

and

$$-B\theta \exp\{x + \theta(x - y - k)\} = \frac{-e^y}{\Delta - a} \quad (\text{A2.15})$$

must be satisfied on $x - y = k$. These reduce to

$$(1 + \theta)B = 1 \quad (\text{A2.16})$$

$$\theta B = \frac{e^{-k}}{\Delta - a}. \quad (\text{A2.17})$$

Note that (A2.16) is the sum of (A2.13) and (A2.17), so we have any two independent conditions on the two unknowns B and k (θ is determined by (A2.10)). Dividing (A2.16) by (A2.17) gives

$$e^k = \frac{1}{\Delta - a} \left(\frac{1 + \theta}{\theta} \right) \quad (\text{A2.18})$$

and

$$B = \left[1 - \frac{1}{C(\Delta - a)} \right] = \frac{1}{1 + \theta}. \quad (\text{A2.19})$$

To complete the proof of optimality we need only show that the condition $W(x, y) \geq R(e^x, e^y)$ on the continuation region $\{x - y < k\}$ is met. This can be confirmed provided we choose θ to be the *positive* root to the quadratic (A2.10). (The fact the quadratic has real roots follows from the assumptions that $\delta + \lambda > a$ and $\delta + \lambda > b$; that they are of different signs follows similarly.)

Using the above expressions for B the value function $\{x - y < k\}$ can be expressed as

$$W(x, y) = \left[1 - \frac{e^{-k}}{\Delta - a} \right] e^x e^{\theta(x-y-k)}. \quad (\text{A2.20})$$

This will be strictly greater than

$$R(e^x, e^y) = e^x - \frac{e^y}{\Delta - a} \quad (\text{A2.21})$$

if
$$e^{\theta(x-y-k)} - 1 > \frac{e^{-k}}{\Delta - a} [e^{\theta(x-y-k)} - e^{y-x+k}]$$

which on using (A2.17), and some simplification leads to the condition

$$\frac{\theta}{1 - e^{-\theta(y-x+k)}} > \frac{1}{e^{(y-x+k)} - 1}. \quad (\text{A2.22})$$

Now since on $x - y \leq k$ the right hand side is strictly bounded above by $1/(y-x+k)$, and the left hand side bounded below by the same quantity (this follows from showing that for $\theta > 0$ the left hand side is increasing in θ , and by L'Hôpital's rule, that in the limit as $\theta \rightarrow 0$, it assumes the value $1/(y-x+k)$), it follows that the condition will always be met for $\theta > 0$.

Since θ is the positive root of (A2.10) it follows that the condition $W(x, y) > R(e^x, e^y)$ is met on the continuation region $\{x - y < k\}$ and hence that stopping rule derived above is optimal.

In terms of the processes $\{V(t), A(t)\}$ the optimal stopping rule is stop whenever

$$V(t)/A(t) \geq C^* = e^k = \frac{1}{\Delta - a} \left[\frac{1+\theta}{\theta} \right], \quad (\text{A2.22})$$

and in terms of benefits $V(t)$ and "costs", $\bar{A}_f(t)$, it is stop whenever

$$\frac{V(t)}{\bar{A}_f(t)} \geq q^* = \left(\frac{1+\theta}{\theta} \right). \quad (\text{A2.23})$$

Appendix 3

Effects of Parameter Changes on the Optimal Critical Levels q^*

The optimal barrier q^* for $V(t)/\bar{A}_f(t)$ is

$$q^* = 1 + \frac{1}{\theta} \quad (\text{A3.1})$$

where θ is the *positive* root to the quadratic equation (17).

We shall consider the partial derivative of θ with respect to various parameters. First we note that

$$D = b - a + \frac{1}{2}\sigma^2 + \sigma^2\theta > 0. \quad (\text{A3.2})$$

This follows from the fact that (from (17))

$$D = b - a + \frac{1}{2}\sigma^2 + \sigma^2\theta = \frac{1}{\theta}(\Delta - b + \frac{1}{2}\sigma\theta^2) > 0. \quad (\text{A3.3})$$

Differentiating (17) implicitly with respect to a , b , σ^2 and Δ gives:

$$\begin{aligned} \frac{\partial \theta}{\partial a} &= \frac{\theta}{D} > 0 \\ \frac{\partial \theta}{\partial b} &= \frac{-(1+\theta)}{D} < 0 \\ \frac{\partial \theta}{\partial \sigma^2} &= \frac{-\theta(\theta+1)}{2D} < 0 \\ \frac{\partial \theta}{\partial \Delta} &= \frac{1}{D} > 0. \end{aligned} \quad (\text{A3.4})$$

Since $\sigma^2 = \sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$ and $\Delta = \delta + \lambda$, it follows that q^* decreases with small increases in a , λ , δ and ρ and decreases with small increases in b , σ_1^2 , σ_2^2 .

Appendix 4

Expected Additional Survival Time

In this appendix we consider the expected survival time of the old-growth forest under various harvest rules.

Let τ be a random variable denoting the time at which the forest stand is destroyed either by fire or other catastrophe or by a clear-cut harvest. We shall consider harvest rules of the type discussed in Sections 3 and 4, viz. harvest as soon as the ratio $V(t)/A(t)$ reaches some critical level C . If T_c is the first passage time to this boundary with distribution function $F(t)$ and the probability density function $f(t)$, then the probability that the stand is still surviving t periods into the future is

$$\begin{aligned} P(\tau > t) &= S(t|0)[1 - F(t)] \\ &= e^{-\lambda t}[1 - F(t)]. \end{aligned} \tag{A4.1}$$

The *expected survival time* is (using integration by parts)

$$\begin{aligned} E(\tau) &= \int_0^{\infty} P(\tau > t) dt = \frac{1}{\lambda} \left[1 - \int_0^{\infty} e^{-\lambda t} f(t) dt \right] \\ &= \frac{1}{\lambda} [1 - E(e^{-\lambda T_c})]. \end{aligned} \tag{A4.2}$$

The right-hand side of (A4.2) involves the Laplace transform of the first-passage time of Brownian motion with drift to a fixed barrier. Using well-known results for this (*e.g.* Karlin and Taylor (1975), p. 362) gives:

$$E(\tau) = \frac{1}{\lambda} [1 - \exp\{-(k - z_0)\phi\}] \quad (\text{A4.3})$$

where ϕ is the *positive* root to

$$\frac{1}{2} \sigma_z^2 \phi^2 + (b - a + \frac{1}{2} \sigma_z^2) \phi - \lambda = 0, \quad (\text{A4.4})$$

$k = \ell \ln C$ and $z_0 = x_0 - y_0 = \ell \ln(V_0/A_0)$ which depends on the current (initial) ratio of timber to amenity values.

An alternative form of (38) is

$$E(\tau) = \frac{1}{\lambda} \left[1 - \left(\frac{V_0}{C A_0} \right)^\phi \right]. \quad (\text{A4.5})$$

It is assumed here that $V_0/A_0 < C$; if not, a harvest would take place immediate and τ would be zero.

To assess the effects on survival time of using a sub-optimal policy we can consider the process starting on the boundary of the suboptimal stopping region, and consider its expected future survival time if the optimal stopping rule is used.

For the naive cost-benefit rule then we consider an initial state $V_0 = \bar{A}_f(0)$ or $V_0/A_0 = (\Delta - a)^{-1}$. Substituting this and (20) in (A4.5) above gives

$$E(\tau_{ex}) = \frac{1}{\lambda} [1 - (q^*)^{-\phi}] \quad (\text{A4.6})$$

which is (22).

Similarly for the certainty-equivalence rule we consider an initial state $V_0 = q_1 \bar{A}_f(0)$

which gives the expected additional survival time as

$$\frac{1}{\lambda} \left[1 - \left(\frac{q_1}{q^*} \right)^\phi \right] \quad (\text{A4.7})$$

which is (26).

Appendix 5

The Certainty Equivalence Harvesting Rule

The solution to the stochastic differential equation (1) is

$$V(t) = V(0) \exp \left\{ (b - \frac{1}{2} \sigma_1^2) t + w_1(t) \right\} \quad (\text{A5.1})$$

which has expected value

$$E(V(t)) = V_0 e^{bt} \quad (\text{A5.2})$$

Similarly

$$E(A(t)) = A_0 e^{at}$$

The certainty equivalence harvest rule can be found by solving the *deterministic* problem of finding a harvest time T_1 to maximize the present value of timber benefits plus accumulated amenity benefit up until harvest time, assuming that $V(t)$ and $A(t)$ are deterministically given by the r.h.s. of (A5.1) and (A5.2) *i.e.* to maximize

$$e^{-\Delta T_1} V_0 e^{bT_1} + \int_0^{T_1} e^{-\Delta t} A_0 e^{at} dt. \quad (\text{A5.3})$$

Upon evaluating the integral and using elementary calculus it can be verified that the maximum is obtained for $a \geq b$ by harvesting immediately if $V_0 > A_0/\Delta - a$ but not at all otherwise. If $b > a$ the maximum is at the time T which solves $(\Delta - b) V(t) - A(t) = 0$. Expressed in terms of $\bar{A}_f(t)$ this is exactly the rule given by (24).

The same rule arises by taking the limit (as $\sigma^2 \rightarrow 0$) of the optimal stopping rule. The asymmetry arises because of the fact that the root θ of the quadratic converges to different values for $a \geq b$ and $a < b$. In the former case $\theta \rightarrow \infty$, while in the latter case $\theta \rightarrow (\Delta - b)/(\Delta - a)$.

FOOTNOTES

1. The value of standing old-growth forest comprises many components. Old-growth forest can provide positive amenity services as one or more the following: (a) a locus for recreational and touristic activities, (b) a habitat for wildlife, (c) a generator of oxygen, (d) an environmental sink for carbon, (e) a regulator of water flow, (f) a repository of genetic diversity, (g) a regulator of local and even possibly global climate. In addition many people are coming to recognize that it has an intrinsic *existence* value (apart from the 'use' values listed above), simply because it is a part of a vanishing pristine Nature. Like diamonds or any other economic good it has value simply because it is simultaneously wanted and scarce.

2. We do not consider the possibility of harvesting part of the old-growth area. For a treatment of that problem with known future timber revenues but uncertain future amenity values see Clarke and Reed, 1989.

3. We are assuming no 'external' environmental costs here. The inclusion of such would clearly reduce the net value V , thereby making both the optimal policy and the cost-benefit policy more conservative.

4. Some of the amenity services provided by old-growth forest are listed in Footnote 1. If there are positive environmental costs associated with harvesting, they could either be

subtracted from the value V , or alternatively in an amortized form, be added to the amenity service value, A . In other words one of the services provided by old-growth forest could be the *prevention* of environmental damage. The services of regulating water flow and/or climate can be thought of as being of this type. It has typically been the case that it is only after the forest has been logged and consequent flooding, erosion and silting experienced that the value, in regulating water flow, of the forest has been recognized.

5. The bar is used to indicate an expectation, and the argument 0 to indicate the present value at time *zero* of the flow of amenity benefits from time *zero* on. The subscript f is used to signify that \bar{A}_f represents the present value of *future* amenity flows. Also it suggests the expected present values of amenity services *foregone* if a harvest takes place at time zero.
6. In the technical parlance of optimal stopping the problem would not be 'stable' (see Ross, 1983, p. 53).
7. In this case the stopping problem resembles the famous *St. Petersburg Paradox* (see *e.g.* Smith, 1988, p. 23). While at each point in time it pays, in expectation, to defer the harvest, the eventual return in timber benefits will be zero since if the hazard rate λ is positive, then with probability one, the stand will eventually be destroyed.

8. These calculations are based on a two standard deviation band for $\ln(V(t)/A(t))$ after one year, which would exhibit a normal distribution with variance σ^2 .
9. This form of stopping rule is also known as the myopic look ahead (M.L.A.) rule, and can be shown to be optimal for monotone processes.
10. Another way of quantifying the sub-optimality is to look at the reduction in expected present value from the optimal value through using the sub-optimal procedure. An upper bound on this reduction for the naive cost-benefit procedure is given in Reed (to appear). For the range of parameter values considered in this paper the reduction can be as high as 43%.
11. This would occur if $\delta < \max(a,b)$ but $\delta + \lambda > \max(a,b)$ i.e. if $\lambda > \max(a,b) - \delta > 0$.

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