Designing an Online Mathematics Resource

by

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We acknowledge with respect the Lekwungen peoples on whose traditional territory the university stands and the Songhees, Esquimalt and WSÁNEĆ peoples whose historical relationships with the land continue to this day.

Supervisory Committee

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Abstract

Students are much more mobile than mathematics curricula allow. A motivated student, when changing schools or even while attending a British Columbia (BC) school, could use a well-planned online mathematics resource to prepare themselves for a course placement evaluation that suits their university plans. An effective online resource for BC would include references to not only the specific curriculum skills, but also the BC curriculum big ideas, the core competencies, and the curricular competencies (BC's Course Curriculum, n.d.). It would incorporate Indigenous ways of knowing and be focused on inquiry-based learning and authentic real-life applications of knowledge. A constructivist approach including some persuasive pedagogy would be suitable for a static resource. Videos, quizzing apps and tasks, and real data and/or articles using math are readily available. Although there are many online resources accessible, high school students need guidance on what concepts and skills they need to learn while ensuring their privacy is not compromised as it could be by many of the online resources hosted in areas of the world, which do not have the strict laws BC has. An asynchronous self-directed resource requires automated feedback, motivational elements such as gamification, and the opportunity for self-regulation and exploration of self-chosen topics of interest. Self-assessment and the development of a portfolio of work to show evidence of learning will be used instead of assigning a numerical percentage or level to the work completed.

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Dedication

This work is dedicated to all the wonderful teachers I have had the opportunity to work with, especially those who are trying to find ways to connect mathematics to age-appropriate real-life problems.

Chapter One: Introduction

My classroom teaching career started as a substitute teacher in northern Saskatchewan schools. Before moving to La Ronge, I was in the Music Education program at the University of Saskatchewan and, at the time, the practicum was split into parts so prospective teachers could experience different situations. My last student teaching practicum was in an elementary school with a high Indigenous population in Prince Albert. For my secondary English minor, I was in Meadow Lake teaching middle and high school English, mathematics and social studies, again in a school with a high Indigenous population. Being raised in southern Saskatchewan, I was exposed to stereotypes that were uncomplimentary about Indigenous peoples. My practicum and substitute teaching experiences revealed to me that kids are kids and each one wants to be listened to, met where they are, cared about, and part of the conversation about how and where they need to go next.

Once I had full-time employment, part of my teaching load always included a mathematics class. A regular problem was that students joining our programs had not been taught or had not learned mathematical concepts that were required for success in our mathematics course. Sometimes, this was due to different curriculum being used while other students just needed more time to understand and be capable of applying the concepts, including more personalized explanations, or possibly reasons to be engaged with the material. If a mathematics situation was based in a real-life situation, such as actually building a shed in a carpentry class and needing to understand geometry and trigonometry, students with less confidence and interest in mathematics exhibited more willingness to engage in learning mathematics skills.

In my most recent teaching position, although students were willing to put in time, they lacked foundational knowledge of concepts. A great deal of teacher time is required to track back through a student's knowledge base to see where they have misconceptions or lack of information. Once the deficiency is identified, students can be directed to online mathematics resources or further print resources to build the skills, but shortcomings are usually diagnosed near the end of the unit or once it is too late for students to be proactive about their learning.

The Problem

It has become clear to me recently that although there are many online resources for learning mathematics, it is difficult for students to independently find suitable learning tools and practice exercises (Greene et al., 2015). They may lack the vocabulary for searches and they definitely lack the pedagogical knowledge necessary to know where they need to start. If they know the correct mathematical terminology for the skill they are looking for, they can Google a resource such as a Khan Academy mathematics instructional video and do some practice exercises, or they can read through an explanation on a mathematics instructional resource such as mathisfun.com or purplemath.com and do some exercises. Many resources available online are just a lecture-style explanation (video or print), with limited interactivity, limited motivational factors, no skill building in recognizing patterns or looking at habitual mathematical errors, and no relevance to using mathematics in real-life situations (Bates, 2019). There are some wonderful online mathematical resources, but many sites require membership or a subscription to access questions or track progress, therefore harnessing motivational and self-regulation elements. The majority of the sites require a subscription fee and/or do not adhere to British Columbia's privacy laws (OIPCBC, 2012).

Professional Context

British Columbia (BC) has many international students coming to our schools whose curricula do not follow the same order as ours. In many BC mathematics classes, students either repeat a significant amount of material or they flounder without sufficient background in some areas. Some of these students are very motivated, but their English ability may not be sufficient to use the videos from Khan Academy or the print explanations impactfully. Others may come from the American system that does not organize its courses with mixed mathematics including algebra, statistics and geometry as Canadian courses do and, therefore, the students may have limited background in some topics. Some students are very unfamiliar with the inquiry-based instruction, which is being encouraged by the BC Ministry of Education and which is required in the International Baccalaureate Middle Years Programme. Students in BC are allowed to challenge a math course based on school and BC Education requirements. This means they can, with documented evidence of prior learning, complete a challenge assessment at their BC school and earn credit for a course. The challenge assessment covers all course learning standards and can include hands-on demonstrations, oral performances, interviews, written examinations, or presentations of a collection of work. It is possible for students to study between school years to conquer the areas in mathematics they may be missing in order to have a smooth transition to BC mathematics courses, including earning required course credits towards graduation. If students had a resource where they could diagnose which required mathematics areas they have not studied, check their depth of understanding of areas they have studied, and further their knowledge in the topics in which they lack mastery, they could choose their BC mathematics course with more accuracy and expectation of potential success without either wasting their time or being forced to put in many extra hours of study during the regular school year. Khan Academy is a useful resource, as it has explanatory videos and basic knowledge level questions. It meets the National Council of Teachers of Mathematics (NCTM) 2000 standards of providing equity for students, providing effective mathematics teaching, and enhancing learning through the use of technology. Khan Academy curriculum is organized according to the American system which has independent courses for algebra, geometry, trigonometry, and statistics, whereas BC courses are integrated with all branches studied every year and therefore Khan Academy has discrepancies as to grade level for many mathematics skills.

Research Questions

The focus of this project is to research instructional design and the elements required to create an asynchronous online resource that would be useful to international students coming to BC. Ideally, the resource created could be useful for mathematics students to use for upgrading as well as for those students who desire the opportunity to work ahead. For teachers, this resource could provide ideas for summative assessments, as none of the summative assessments will have one correct answer. The summative tasks will be based on real-life situations, which will have multiple possibilities for answers, depending upon the limitations the student chooses and how they justify their choices. Students would be able to self-assess their summative tasks by comparing their answer to a sample answer which includes comments on possibly differences and considerations. Though current online resources will be utilized, the hope is that this resource will have some longevity for its focus on the needs of BC students and the opportunity to interact with up-to-date real mathematical problems.

The goal of this project is to design an asynchronous online mathematics resource for the BC Foundations of Mathematics and Pre-Calculus 10 course that:

- provides a diagnostic tool to direct a student to areas requiring further study
- assumes a certain entry point but also provides support to develop and practice the missing skills and the skills required
- has real-life applications to keep a student returning to continue their learning
- supports the BC curriculum big ideas
- incorporates Indigenous ways of knowing and topics
- supports the International Baccalaureate Middle Years Programme focus on inquiry-based learning and authentic real-life applications of knowledge
- makes use of available resources while ensuring student privacy
- enables the student to self-assess

For this project, real-life refers to situations based in reality that could actually take place. This does not include the type of 'real-life' questions which include someone buying seventeen cantaloupes and trying to divide them between four people, as though this is potentially possible in real-life, it is unlikely. Although a student may not actually be able to attend Hogwarts, this is a potential real-life opportunity they would like to have, and thus authentic. Authentic, for this project, will refer to situations that might be of interest to a Grade 10 student, as well as situations in which the student may, in fact, experience.

Search Methodology

Both technology in general and the use of technology in the classroom has changed significantly in the past decade and even more in the last year due to the COVID 19 pandemic. The availability of open digital resources, which require no payment or login details, and the opportunity to access those resources from smart phones, which a large amount of the population now have in comparison to 10 years ago, has changed how students choose to interact with their world. The pivot to remote online learning has changed how teachers are able to interact with their students. Many articles from even two years ago are not referring to the type of student who is as comfortable using technology as our current or future students. Therefore, the initial research focus was on articles that are recent, released within the past two years, including those presented at conferences and conventions and available in online journals, but not necessarily peer reviewed. Authors were reputable and not employed with the purpose to promote a particular resource or idea. One of the first useful resources found was a journal publication from 2018, which included a University of Victoria (UVic) mathematics professor's contribution. This journal led to another journal in the same series that was published even more recently, though sourced from papers in the 2017 International Conference on Technology in Mathematics Teaching (ICTMT). In the UVic Library search engine, search parameters were developed where articles from both these journals would be prominent, but other relevant articles were included (the star indicates a wildcard): 'Technolog* Teach* Math*' Jan 2019 to March 2020. Further readings came from searching ‘Indigenous mathematics’ from the last five years and from interesting quotations in the original 33 useful articles. As searches narrowed to more specific mathematic and mathematic educational technology topics, the search parameters were widened to include peer-reviewed articles from the last ten years. To further explore the design of one of the most successful online mathematics teaching tools, I also searched ‘Khan Academy design’ and ‘asynchronous learn* design.’

Project Description

This project aims to develop diagnostic tools for elements of BC curriculum mathematics skills required by Foundations of Mathematics and Pre-Calculus 10, where students can find out what elements they understand and can apply, and whether they are ready for Pre-Calculus 11. In order to develop their skills, the resource will include a curated list of videos, interactive exercises, and real-life applications that are not behind sites that require payment or which have questionable privacy settings. Ideally, most would have the opportunity for translation into other languages. If needed, some resources would be created. All activities would be linked to the mathematics content areas from the diagnostic tools, to a list of mathematics content, to a list of BC mathematics competencies, to a list of BC required big ideas, and to a list of BC curricular competencies. The resource would include motivational elements such as the opportunity to track their improvement and use of all resources, application to authentic real-life problems and real-life solutions to these problems offered by professionals, puzzle/inquiry challenges to improve pattern recognition, and links to what they already know well. Indigenous cultural content and respect for Indigenous ways of knowing will be honoured in the resource (First Nations

Education Steering Committee, 2000). Formative and summative assessments will include the types of questions found on our BC Education Numeracy Assessment which is now required of students graduating in BC. The cultural basis of some of the questions excludes international students from experiencing success. Although the graduation assessment should not require preparation, those new to BC or Canadian culture might find that some of the situational questions posed are inaccessible.

When a student is working without the guidance of a teacher, it is difficult to determine how deep to explore in any topic area. Online resources such as Khan Academy and IXL, a drill-based site which provides excellent explanations of possible errors when one gets the answer incorrect, allow users to explore according to their individual curriculum outline, which may not match BC's. They only meet the base requirements of remembering and understanding according to Bloom's Taxonomy. They do not supply the opportunity to apply mathematics or analyze and evaluate the application of mathematics. In the classroom, a teacher may choose to use these types of resources as supplements as they work with their students towards a real-life application of mathematics which may include creating, analyzing and evaluating. Students working without a teacher need to be able to:

- diagnose the areas they need to work on,
- find useful interactive resources for remembering and understanding without having to sign in,
- be able to track and collate their work simply for evidence,

- experience enjoyable math activities that spark their interest and require them to apply, analyze and evaluate the mathematics they utilize, and
- engage with an online community of math users that inspire them to feel confident in using and developing their math skills.

Chapter Two: Literature Review

In order to create a digital resource, a theoretical framework needs to be decided upon and the elements required for a successful resource need to be explored. These would include video, self-directed modules, the virtual reality environment, gamification, and whether inquiry-based resources work in an online educational environment. Decisions on what will motivate an online learner to be self-regulated need consideration and research. Options for successful digital self-assessment also need to be analyzed. A summary of literature is presented in this chapter.

A difficulty with implementing any digital resource for the teaching of math is the historical practice of mathematics teaching as procedural knowledge instead of conceptual knowledge. As Monaghan and Trouche (2019) affirm, this practice was formed over the course of many years before digital technologies arrived. Denham (2019) agrees that mathematics teaching often involves teaching shortcuts as opposed to teaching the concepts. For example, dividing a fraction requires multiplying using the inverse and how and why it works is not taught as a concept, but is taught by the algorithm of ‘copy, dot (for multiplication, not a decimal), flip’. Digital resources can guide students through an inquiry process and help develop conceptual knowledge before students memorize the procedure. A goal of this project is to focus on the understanding of the concept rather than just a memorization and application of procedural knowledge. We would not consider teaching our students their native language through only spelling, grammar and amassing vocabulary, rather than through these elements plus reading, speaking and writing, so we should include the real-life use of mathematics and the exploration of mathematical patterns in our teaching of mathematics instead of focusing on speed during timed tests.

According to Devlin (2011), although teachers feel everything matters, college teachers want to have students with good command of the four mathematical operations including using the operations with fractions, understand data analysis including rates, ratios and proportions, and how to graph and use linear equations. Students think mathematics means arithmetic and that 'do the math' means calculate. Higher-order thinking is what professional mathematicians and statisticians do (Copes, 2020). By encouraging students to use their skills in real-life situations and compare their solutions to mathematicians and statisticians, students will see the usefulness of mathematics study and develop the understanding that mathematics is not a set of procedures to memorize but a necessary set of skills.

Song et al. (2019) broke down the advantages and disadvantages of digital resources in their study:

> Online pedagogic tools remove many limits of conventional classrooms, such as the need for synchronization (time) and co-location (space), limitations on class size, and the one-size-fits-all problem, but these tools also have disadvantages, such as less direct student-instructor interaction and difficulty in the design of effective course assessment. (p. 39)

Therefore, care is needed when designing online resources to minimize disadvantages. Students need to have elements that buoy motivation and inspire self-motivation when they do not have an overseer. Students also need systems that will address comprehension issues to minimize frustration. If students are not encouraged to use the resource for long enough to address their learning requirements, there is no use in developing the resource.

According to the National Research Council (2001), mathematical proficiency has five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Much mathematics instruction has focused on procedural fluency, the ability to carry out correct procedures accurately and efficiently, as opposed to conceptual understanding, the ability to understand the relationship between the operations and the reason for carrying out those operations. Word problems are used traditionally to develop strategic competence, which is the ability to represent and solve mathematical problems set in real-life situations. Adaptive reasoning refers to the ability to think logically about the relationship among concepts and situations and reason how to use and extend these relationships to new situations. Adaptive reasoning requires reflection on whether current strategies are appropriate. With the lack of focus on conceptual knowledge, this area is often missing in current mathematics instruction. Thus, productive disposition, which refers to the tendency to see mathematics as accomplishable with steady effort and see it as useful and worthwhile, is also missing from our culture of mathematics. As many students are perfectly willing to say they are ‘bad at mathematics’, it is obvious this area is missing from much current mathematics instruction.

In 2000, the National Council of Teachers of Mathematics put out a set of Principles and Standards to guide mathematics education. The six principles listed outline what is fundamental to a high-quality mathematics education:

1. Equity – all students have access to appropriately challenging content and high expectations with strong support for success
2. Curriculum – the curriculum is focused on preparing students for solving problems encountered in everyday life (strategic competence) as well as future mathematics study. The curriculum links to previous knowledge and encourages students to apply their mathematics to increasingly complex situations.
3. Teaching – teachers of mathematics must understand their subject deeply in order to be flexible in their teaching and their tasks and to scaffold the learning for their students so students can employ adaptive reasoning.
4. Learning – factual knowledge and procedural proficiency is aligned with conceptual knowledge so students are capable of reflecting on their thinking and learning from their mistakes. New knowledge needs to be built on experience and prior knowledge.
5. Assessment – formative and summative assessment are important for helping students learn and set their own goals. Assessment should provide useful information for a student and help build a productive disposition.
6. Technology – the use of technology enhances student learning

Almost 20 years later, little has changed in many secondary classrooms (NCTM, 2018): assessment is still primarily summative and does not contribute towards a productive disposition; technology use is limited to drill and practice; and real-life problems are rarely encountered in secondary courses. All the five processes presented by the NCTM in 2000 are included in many current secondary classrooms: problem solving, reasoning and proof, communication, connections, and representation, but not to the depth suggested. Most problem solving and use of reasoning and proofs at the secondary level is completely mathematical in context and does not include any real-life situation. Communication is usually limited to showing working through notation, diagrams, tables, charts, graphs, models, and short sentences, and rarely includes lengthy explanations of analysis or evaluation of mathematical thinking and strategies. Connections are made between mathematical ideas yet are rarely applied to contexts outside of mathematics study. Using representations to model and interpret physical, social and mathematical phenomena occurs infrequently. According to Goodman et al. (2015), when analyzing data from the Programme for the International Assessment of Adult Competencies (PIAAC) of for young adults aged 16 to 34, Americans lack basic skills for success in the workplace and postsecondary education; their numeracy and problem-solving skills in current technology-rich environments are inadequate for making wise decisions in their personal lives.

In 2018, the NCTM put forth its recommendations for transforming secondary mathematics education. Essential concepts were listed for the content domains of number, algebra and functions, statistics and probability, and geometry and measurement (NCTM, 2018). This resource will focus on conceptual knowledge, linking it to real-life application, and, through seeing the usefulness of mathematical skills, encouraging students to continue with their effort to learn. Skill practice will be included so the student can build sufficient factual knowledge and procedural proficiency to be successful on the real-life summative task.

Theoretical Framework

Constructivism

This resource will be providing task-oriented self-directed learning and, therefore, is suited to the constructivist educational approach (Von Glasersfeld, 1998, Conole et al., 2004). Attempting the real-life summative task will be the reason to interact with the hands-on and self-directed activities provided, with a focus on discovery through inquiry-based experiences, a process in line with constructivist principles. Students will need to learn the mathematics skills in order to complete the real-life authentic task. By working through the tasks, learners build their own mental structures, in agreement with the pedagogical mapping suggested by Conole et al. (2004). Their suggested framework requires the individual to be the focus of the learning, that learning is demonstrated through interaction with others and experienced through practical application of real information, and that reflection on the individual's experience is required. Edwards (2017) agrees that by designing challenges which represent real-world actions, the student is motivated to engage and then creates a meaningful and memorable experience.

Minarni and Napitupulu's (2020) study employed various types of constructivism with Indonesian grade 8 students, who are regularly taught mathematics using a lecture and skill practice method. The study explored problem-based learning, cooperative learning, contextual learning using Geogebra software, real-life authentic problems using the Autograph software, open-ended approach to questions, and discovery learning. Both Geogebra and Autograph software includes a variety of ways to visualise mathematics and see how the manipulation of the equations or expressions affects the visual representation. Minarni and Napitupulu's study concluded that constructivism-based teaching and learning approaches significantly improved higher order thinking skills in mathematics.

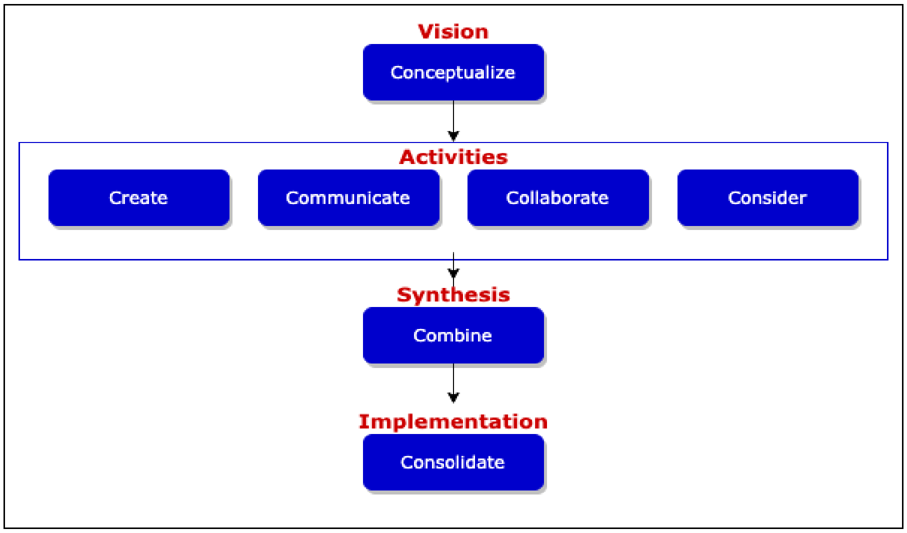
Since the constructivist approach builds on students' current mental structures, there needs to be elements of persuasive pedagogy included in case the current understandings are flawed (Hennessey et al., 2011). Students need to understand why they chose a particular path to solve a problem and understand why it may or may not be a valid method. Through using persuasive text, including hints, students can be encouraged to adapt or alter what they originally thought about a mathematics concept or method of solving a question. Persuasive pedagogy promotes problem solving, reasoning, links to prior knowledge, but more directly corrects misconceptions as opposed to having students take time drilling to resolve their underlying problems. Both constructivism and persuasive pedagogy acknowledge that there is no one correct view in complex problems, but persuasive pedagogy will help guide students through activities to deepen their understanding by suggesting next steps based on what students accomplished on previous tasks. Hints will help steer students towards successful completion of the summative tasks. If students do not meet the requirements of the answer key, suggestions will provide further examples to try, other instructional videos to watch, or a different document to read. As Baliya and Aman (2013) found, by examining evidence, discussing alternate views, and providing justification, students can come to a more complete and correct understanding of mathematics. If there is no teacher to do this, as in an online resource, these views and justifications need to be part of hints and keys for the tasks. Mathematics principles have been discovered over many years by experienced mathematicians; we cannot expect high school aged students to discover these principles without some guidance.

Online Learning Design

In articles from 2018, Conole alone as well as with Brown revisit Conole's 2015 7C's of learning design. The first 5 C's deal with the vision and activities for the course and the last 2 C's are about reflecting and evaluating the created design. The vision of the course needs to be 'Conceptualised' as the designer thinks about the learner anticipated to take their course. In this project, the anticipated learner is fairly self-motivated and looking to self-assess for areas where they need study in order to be successful in mathematics courses in BC. For the 'Create' C, the designer needs to use or repurpose open education resources and add some activities where the learners can create their own content. The designer needs to have plans for methods to facilitate 'Communication' between the learners and the teacher and 'Collaboration' between learners. This planned static resource will be asynchronous and will have limited communication and collaboration. The 'Consider' stage requires the designer to consider different ways for students to demonstrate their learning. These can be through assessments that have a rubric for evaluation of understandings considered, opportunities to check against the actual real-life situation, as well as through formative and summative assessments that have a more traditional key of correct answers.

Figure 1

The 7 Cs of Learning Design



Note: The 7 Cs of Learning Design. Adapted from "Learning design and open education" by G. Conole, 2018, *International Journal of Open Educational Resources*. Retrieved from https://www.ijoer.org/learning-design-and-open-education_doi-10-18278-ijoer-1-1-6/ Adapted with permission.

A sensitivity to the learners' reasons to be interacting with the resources also needs to be taken into consideration. The learner may be a student aiming to learn new material, it may be a struggling student aiming to upgrade, or it may be an average student looking for materials to use for review for testing. The use of language, the support for vocabulary, and choices of types of data and subjects for situational problem solving are affected when considering potential students (Veletsianos & Shaw, 2018). Visual interpretations will need to be prevalent for international students to learn vocabulary for math they already understand as well as for struggling students to have an image to which to link their learning. Using the correct vocabulary is important but the definitions should be available in an easily accessible manner, possibly hyperlinked to a dictionary on the resource. The introduction of cultural information which will enable student success in the BC Numeracy Assessment also needs to be addressed in a manner that does not require prior knowledge or cultural understanding for international students but also display sensitivity to the strong understanding of our Indigenous students.

Create - Asynchronous Design of Learning

Using technology in an educational manner appropriate to today's very tech-aware student is complex. As Symons and Pierce (2019) so ably summarized, we live in a time where web applications (apps) allow even young children to create, share, collaborate and communicate. What will be available to our students in the near future is probably not yet imaginable, as even the National Academy of Engineering in 2004 anticipated technology availability for 2020 might be computers which would fit in a pocket, which we have had for many years already! Current use of technology in the mathematics classroom is primarily engaging children in fluency practice with immediate feedback. Some teachers make use of online graphing tools, such as Desmos, in the classroom. Desmos includes options to use it as an online graphing tool without any log in or saving available, to sign in and save graphs and exercises created, and to sign in as a teacher with a class and use pre-existing assignments. Khan Academy videos are frequently used as a review or reteach option provided by teachers. Khan Academy videos have clear lecture-style explanations, complete with transcript and many have some practice questions with hints available. Kahoot!, a user-created multiple-choice quiz game, Quizlet, an online flashcard tool with a matching game, and quizzes on Google Forms are used by many teachers to test simple knowledge-based understanding in class or as homework. All of these methods are enjoyable in the face-to-face environment as they provide variety from the traditional classroom. Using technology to entice a learner in an online course should not be limited to duplicating a traditional face-to-face experience in an online environment through just providing a lecture presentation and a set of practice questions, sometimes with an answer key.

According to Calder et al. (2018), instead of focusing on apps where students follow a task to a set level, such as what is provided on Wolfram Alpha and Khan Academy, there should be a focus on apps that enable students to create screencasts of their mathematical thinking or enable students to create visual, dynamic representations of mathematical situations. The challenge of relying on apps to motivate students is that access is inconsistent. I suggested the use of one phone app for my Grade 9 and 10 classes and by the next year, the app was no longer compatible with newer phones and the following year, the app was no longer available. Focusing on any app in any resource developed will limit the longevity of the resource. By allowing students to choose their method of demonstrating their learning while considering real-life problems, the resource will fulfill Calder et al.'s (2018) suggestion of enabling student creation of visual and dynamic representations of mathematical situations.

Use of a Virtual Reality Environment (VRE) or Augmented Reality (AR) may be beneficial as it provides immediate visual feedback. Between immediate feedback and the opportunity to engage with technology in a novel manner, a VRE/AR can be highly motivating for both the tech-aware student and for those that are not very tech savvy. Khlaisang and Songkram (2019) encourage the use of a learning environment which combines the benefits of a

Learning Management System (LMS) and 3D virtual world (e.g., OpenSimulator). Cai et al. (2019) found that the introduction of AR-based learning games made students with higher levels of motivation and self-regulation pay more attention to the higher-level conceptions in learning mathematics, but there was no significant influence on lower level conceptions in any of the students. Although use of VRE/AR technology has the potential for extending classroom resources, insufficient research has been done to address the learning curve necessary to include the new technology or the need for realistic rendering of interactions (Bujak et al., 2013). There are many obstacles to using VRE/AR as most require specialized equipment which students will not own. For example, few students will own virtual reality goggles. Even simpler versions of VREs have limited effectiveness on mobile devices and may be completely inaccessible on older devices of any sort. Focusing on specialized equipment in any resource developed will limit the accessibility of the resource as well as limit the resource's longevity since this type of technology becomes obsolete rapidly. Requiring a LMS for the resource involves privacy management and a level of overseeing that this static resource is unable to satisfy.

Vast amounts of mathematical data are available, which can be used as an Open Educational Resource (OER). Using data to frame a student's own question using the mathematical skills they have learned and apps available at the time is a way to insert real-life experiences for mathematics students. Webb and Ifenthaler (2018) recommend the use of open data to create OER. If students can research a real-life question (with guidance from the resource on how to frame the question), find information from which to deduce their answer, and then check their result against the result of professionals who have answered the question, they will be acting as a real mathematician, thereby figuring out ‘why do we have to learn this?’ and hopefully be excited about their accomplishments.

Having access to technologically supported mathematics learning can help overcome socio-cultural diversity and achievement diversity in students according to Huda's (2019) research and findings. Even if students do not use the online resource for long, Radović et al. (2019) found that within eight weeks of using a process driven technological system rather than a product driven system, pupils build a more positive attitude towards work and teaching activities in mathematics. In a technology supported environment, students can get the immediate feedback they need to know if they are completing the assignments correctly, especially if the feedback includes hints on what to consider and/or on possible errors. Students find properly placed and immediate feedback engaging and supportive in overcoming barriers in their learning. The technology encourages and enables students to focus on patterns and connections which helps self-diagnose difficulties, even though the learning process may be procedurally heavy.

Encouraging engagement can take many forms, but according to Jo Boaler (2011) the 5 Cs of mathematics engagement combines curiosity, connection making, challenge, creativity, and collaboration. In this online resource, collaboration will not be available, but the design of the lessons and tasks can be capable of stimulating curiosity through using inquiry. Through working towards a final summative real-life task while modelling a similar task, curiosity as well as connection making can be encouraged. The level of challenge and creativity will be individualized by the student’s choice of summative assessment task and the information they choose to use in their task.

Video. Students do need some explanation and direct teaching, so although video is duplicating the traditional lecture style of lesson, it is still useful in an asynchronous resource. Whether the video is a recorded lecture, a presentation of slides, animations, or a combination of the three, a visual combined with audio improves student progress (Eyyam & Yarata, 2014). The combination of visual, audio, and text is not critical if the task is not very complex (Bhowmick et al., 2007). For complex procedures and tasks requiring problem-solving skills, a combination of audio, video and synchronized text yielded the best results in student final performance as well as in student efficiency in learning the material. According to Berk (2009), using video is appropriate for university introductory courses, for introducing any complex topic, for any lower-achieving students, and for visual/spatial learners.

Battle et al. (2020) recorded video lessons for their students using prepared slideshows on which the instructor wrote while recording audio narration and explanation. When they found some students needed even more detail, they added additional examples and easily edited the details into the original single video file. Video is fairly easy to edit, even when using older digital files, as most formats will convert to current technology when imported into an editing platform. Recording clear audio is easier every year as microphones included in the most recent hardware (computers, tablets, and phones) are increasingly sophisticated. Camera quality and video capability has expanded with improved clarity and editing options within the hardware without extra software. Free versions of video and audio editing software are progressively more sophisticated as developers encourage people to try out their software (e.g., Explain Everything, iMovie, Loom) and providers encourage people to commit to their platforms (e.g., Apple:

iMovie, Microsoft: Microsoft Whiteboard). The OER movement has also provided many free open source software programs, some even cross-platform (e.g., Audacity, OBS).

Videos can also be accessed easily on almost any type of device. Difficulties occur when file sizes are large and students need to download the videos or slideshows to view. Using video hosting platforms such as YouTube or Vimeo is common. Students in some countries have difficulty accessing certain platforms that host videos. For example, YouTube is blocked in China, Iran, Syria, and Turkmenistan and Vimeo is blocked in China and Indonesia. Students can access these sites through a Virtual Private Network (VPN), which creates a secure connection between a user and the internet so the user can access whatever they want without being traced, but this is technically illegal in these countries. Videos included in a resource should be downloadable from a website in the most recent common form for usage on devices, currently MP4 (Elmokadem, 2018). The videos should also be made available on a cloud-based platform for streaming from the web.

Videos provide the option to replay any length of section, re-watch multiple times, and often have automatic subtitles in the original language or the opportunity for automatic translation, depending on the platform on which they have been stored. There are few studies on what makes an effective video when teaching mathematics at the secondary school level. Andra et al.'s (2020a, 2020b) two articles studying the use of videos in four different classes at the secondary level found that it is advisable to introduce struggling mathematics students to online resources (videos) in a peripheral way. Students need guidance on how to use instructional videos to direct and analyze their learning. Suggestions need to be made when designing the order of items in the resource about when to watch the video, how to watch the video (including stopping and trying the question) and when to re-watch portions of the video (for example, when practice exercises following the video are not being completed successfully).

Self-Directed Modules. Higgins et al. (2017) performed a meta-analysis of 56 articles about using technology to intervene in mathematical instruction for grades kindergarten to 8. They found short tasks with frequent feedback were the most effective to ensure each student was progressing correctly and staying motivated. Longer tasks that use already learned skills or longer durations spent on simpler tasks were also helpful. Longer exercises should use multiple approaches, such as watching a video and completing exercises in another application simultaneously, as they found students' achievement, motivation and attitudes were more positively influenced when multiple technologies were employed. It can be extrapolated that these guides on length and type of technologies used would be suitable in self-directed modules which are not connected to a classroom intervention situation.

Jedtke and Greefrath (2019) developed their approximately six-hour computer-based learning environment including a welcome unit, which listed how to use the website, prerequisite skills and the learning objectives, a revision unit, and then eight 'chapters.' The self-directed modules allowed students to decide their own sequence, time frame spent and number of exercises to complete. Interactive exercises could be completed multiple times. Their course also included a paper booklet for extra exercises, formulas, and space for planning and self-assessment activities. Although the self-directed modules Jedtke and Greefrath created were for research on types of electronic feedback, their design choices were made carefully to ensure they could collect accurate data on grade 8-9 students studying quadratics. Each unit contained multiple elements, each using different technologies, aiming for short tasks with frequent feedback. In their reflections, students appreciated the paper booklet as a study and review tool for future testing.

Scaffolding skill acquisition may encourage self-regulation and motivation according to Greene et al. (2015). Allowing every student to learn at their own pace and in their own time, with the opportunity to self-test for mastery and create their own real-life assignments, will improve student accomplishment, according to Sal Khan as reported by Brown (2019). Dorko (2020) referenced that students 'like' when questions offer immediate feedback, which may include correct solutions and suggestions on errors that may have occurred, and when self-tests have multiple attempts; these elements of self-directed modules would encourage self-regulation and motivation.

The University of Toronto Scarborough had particular goals for their preparedness for first-year mathematics course. One design goal was to provide students with a set of resources that would increase their interest in mathematics while enabling identification of their weaknesses and monitoring of their progress. Another was to develop their mathematical thinking and understanding by providing challenging questions which required deep thinking rather than just memorizing the rules and procedures (Shahbazi, 2020). Each module included an introduction, between two and five video lessons, a diagnostic test with solutions, practice questions with solutions, a summative assessment test with solutions, and a handbook/notebook. Again, elements were short or had the ability to be chunked into smaller sections. This was a self-directed course with no teacher planning activities for the students to follow in a linear fashion, so students had the opportunity to learn and explore where they felt they needed or had interest. Research based on the 2012 edition of the Programme for International Student Assessment (PISA) by Tourón et al. (2019) found the students who state that their teachers never or hardly ever plan their activities are those who achieve the highest scores in mathematics. In comparison with students of teachers that plan everything, the students who have more options learn more. Although this finding appears odd, perhaps having choice is a positive motivator. Over-planned lessons may be inflexible and/or unable to react to evidence of student learning and therefore spend too much time reviewing areas students know. Another possibility in interpreting this data is that maybe it only appears to students that their teachers do not plan, but the teacher planned activities are ones where students need to inquire and persevere through challenges without a great deal of teacher support. The students may view the requirement to research the answer themselves while the teacher only poses more questions in answer to student questions as a lack of planning on the teacher’s part instead of a way for teachers to encourage student engagement and problem-solving abilities.

Gamification. Denham (2019) found that digital games may address students' inability to fluently translate mathematics from theory to practical symbolic format by allowing students to interact with mathematics in an environment that encourages exploration through critical thinking and provides metaphoric representation of concepts. Games can be used to reward, through feedback, the testing of ideas including failure of those tests, while fostering high levels of engagement and motivation (Devlin, 2011). Cai et al.’s (2019) research confirmed Garfield and Ben-Zvi (2007) findings that the purposeful design of activities through the use of games can improve students' reasoning and understanding at the elementary and secondary levels. The research reveals that gamification may be an approach to helping some learners, and may be more enjoyable and impactful if trying to change attitudes towards learning mathematics.

The resource could be set up as a serious game. Serious games are those designed to train users in a specific skill set, such as the types of games the military use for training soldiers (Cyr et al., 2019). Cyr et al. set up a serious game about fractions for their study. They considered Prensky's 2007 model for serious games: representation, rules, goals and objectives, results and feedback, conflict-challenges, and interactions. Representation refers to the story, or situation the player is in. Rules refer to the limitations on their actions. Goals and objectives are the steps along the way that need to be completed in order to progress and 'win' the game. Results and feedback are what the player earns by completing or being unsuccessful completing the goals. Conflict-challenges are the random limitations on completing a goal, which can include limited time. Interactions refers to how the player can communicate or be affected by other players or non-players within the game. Cyr et al. (2019) chose to follow the elements defined by Annetta in 2010 as all of Prensky's elements were included except for representation: identity, immersion, interactivity, increasing complexity, informed teaching, and instructional. Identity is the idea that each player has a unique character when playing the game. Immersion refers to the idea that the individual feels like they are part of the game as their unique character. Interactivity is how the game allows players to interact with each other or non-player characters. Informed teaching is the ability to gather information about how a player is playing and progressing and being able to provide individual feedback. Instructional includes the elements needed for the player to learn on their own while meeting the various challenges. The final two elements of Annetta's system, informed teaching and instructional, were important for the educational components. Cyr et al. (2019) did find the game they created, which included 120 levels on fraction concepts with increasing difficulty every 10 levels, produced significant learning on the conceptual level, even though the game did not include the element of interactivity.

The Play Curricular-activity Reflection and Discussion (PCaRD) model Denham (2019) discusses attempts to address the lack of pedagogical models available for mathematics teachers who are interested in digital game-based learning but PCaRD is missing the needed exploratory research on its implementation within formal mathematics classrooms. There is value in considering all sets of elements, Parensky, Annetta and PCaRD, but developing a serious game for learning is well beyond the time commitment available by a regular classroom teacher or even a Masters student. Dalby and Swan (2019) found even simple games can be helpful, as they provide a rich formative assessment process which can enhance student learning. The interactive features of digital curriculum resources are useful as formative assessment practices, because they can inform the next learning steps (Pepin et al., 2017). Many of the developments Pepin et al. reviewed displayed a weakening of traditional demarcations between pedagogy and assessment and between summative and formative techniques of assessment.

Lewis (2014) clarified elements of motivational design that could be useful in the gamification of a resource. Lewis's gameful patterns focus on quick feedback loops and include specialization badges, which indicate the reaching of a certain goal and is a means for users to recognize their progress, and score, where points are awarded in response to actions. Score is more focused on comparative positioning and could be equated to grading. Interface patterns could be useful, as they mirror what happens in a classroom. These include: praise; where users earn rewards for performing actions, predictable results; where users realize certain actions will have certain results which lessens nervousness, state preservation; where users feel confident to drop in and out at will and not fear they will lose progress, and the ability to undo (or redo). Information motivation patterns include search, where users can search for specific content, and task queue, which tells users what can be completed next. Lewis's book included many more patterns, some of which were aimed at creating addicting games and immersive games beyond the needs of a free educational game.

Inquiry-Based. Inquiry-based learning includes many teaching techniques which focus on students developing ideas on their own as opposed to an instructor presenting the knowledge (Cushman et al., 2018). In a study of 120 first-year high school students, Laudano et al. (2018) found that inquiry-based learning increased student interest in mathematics in general, as well as in abstract mathematics specifically. With inquiry learning's discovery approach and focus on creative and critical thinking, as opposed to the more traditional teaching's focus on memorization and practice, students found the study of mathematics more enjoyable. A variety of Flash-based interactive mathematics discovery tools have been created, and Yan et al.'s (2018) study found that well-designed topic-specific technical tools are valuable to mathematics teachers. Unfortunately, Flash is no longer supported on the internet, so many great tools are not available.

Using authentic real-life situations offers the opportunity to incorporate a variety of content. For much of the learning, inquiry, based on investigating mathematical patterns to discover the general mathematic rule, would be the introduction rather than having the introduction presented by a traditional lecture video on the rule and the algorithmic procedures which the student then memorizes through practice. The inquiry would be followed by a video explanation, or the option to read an explanation, and the practice exercises to solidify the student's inquiry. As realized by Pepin et al. (2017), flipping the classroom and developing instructional videos does not present much difference from regular sage-on-the-stage instruction. Lecture videos already exist in diverse online versions; Khan Academy, WooTube, and ProfRobBob are just some of the many YouTube channels providing online lectures on mathematics topics. According to Trenholm and Peschke (2020), many fully online courses focus on this method of instruction – requiring students to watch videos of instructors. Much of the other technological resources used rely upon factual implementation, which typically involve students playing procedurally-focused games that encourage students to practice routine procedures (Symons & Pierce, 2019). These games usually provide limited multiple-choice questions that allow students to succeed as obviously inaccurate choices are options or students can use trial and error without requiring development of understanding. Koh's (2019) research found that mathematics teachers face challenges when designing technology-integrated lessons to support mathematical inquiry with authentic problems. Mathematics teachers' lesson plans show more evidence of technology use to support recall and application rather than the analysis, comparison or evaluation of mathematical concepts (Dawson et al., 2013).

By providing students with a real-life problem, including a set of resources to help students develop their mathematical thinking and understanding and increase their productivity, interest, and self-regulation for successful completion of the real-life summative task, students could successfully be prepared for university-level mathematics (Shahbazi, 2020). In each module of the resource created by the University of Toronto Scarborough, the inquiry process guided students in a model of asking and answering questions. For example, in Module Four Lesson One, as students are reviewing functions by watching an instructional video, students are led through explaining why a particular relation, $y^2 = x$, is not a function. Later, they are reminded to ask themselves if the function they are graphing actually fits the definition of a function; does the function pass the vertical test which means that any vertical line intersects the curve at exactly one point. The purpose is to check their work as the question stated that the graph would be a function. This modelling can help the student in their future learning, as it teaches them how to learn. Mainali (2020) also found that providing students with the opportunity to explore, experiment and discover mathematical ideas made mathematics courses more effective.

Conceptualize - The Anticipated Learner

The anticipated learner for this resource may be a student that wants to prepare themselves for a BC mathematics course if they are coming from a different part of the country or world, or they may be a BC student that wants to upgrade their skills. The expectation is that any student looking for this resource will be motivated and have some self-regulation skills, but there is also the potential that the motivation to explore this resource will be parent or teacher instigated. There is also the potential that this will be used by a teacher as a resource for their own teaching. Providing summative assessments that do not have a single correct answer will also afford teachers with the opportunity to use the assessments in their classroom.

Cultural Background. The learner may or may not have strong English skills, so visuals beyond simply text will be important. Indigenous students have also been documented to react well to visuals (Kitchenham, 2016). Many mathematics curricula are still very procedural-based and our provincial curriculum is focused on inquiry and authentic, real-life applications (BC's Course Curriculum, n. d.). Students will potentially be more successful in BC mathematics courses if they gain some experience in the different methods of showing their learning. Students with more experience in inquiry-based learning show a stronger preference for inquiry (Huang et al., 2020), with the implication that more experience in inquiry-based learning will enable a student to be more successful with that format.

Growth Mindset. The fixed mindset that "I am not good at math" is a cultural element of North Americans and should be replaced by a growth mindset (Boaler, 2015). A growth mindset is the idea that dedication and hard work will improve intelligence and ability whereas a fixed mindset is the idea that intelligence and ability are fixed traits and cannot be developed (Dweck, 2009). Students provided with the opportunity to Licensed London, England taxi drivers, who must learn the complex and irregular layout of London's streets as well as the locations of thousands of places of interest, take between 3 and 4 years to complete a set of examinations to earn their license. A study of their brains revealed that the hippocampus, the area specializing in acquiring and using spatial information, grew significantly over those 3 to 4 years (Woollett &

Maguire, 2011). If adult brains can change with the desire to learn, school age students should have just as much ability. Even short amounts of daily practice can initiate neural processes that continue for hours afterwards (Karni et al., 1998). Engaging in mathematical activities daily will have lasting long-term effects. Developing a growth mindset will enable students to achieve accuracy after mistakes more quickly than students with a fixed mindset as the neural mechanisms associated with awareness of errors in growth mindset individuals enables them to rebound from mistakes more efficiently and rapidly (Moser et al., 2011).

Motivation. Although students may be motivated to do their best and prepare for their courses, this can fade quickly when they are not recognizing that they are learning and experiencing success. According to Radović et al. (2019), a well-designed game to practice or learn skills can encourage a sustained task commitment and positive, dynamic attitude. Sun-Lin and Chiou (2019) found game elements used in learning content attracted students' attention and motivated them to complete learning tasks. Even if the game was only used for drill-and-practice, it has the ability to increase student motivation (Beserra et al., 2019). Sampayo-Vargas et al. (2013) realized that even if the game was non-adaptive (did not provide harder questions when correct answers were presented or easier questions when incorrect answers were presented), motivation was higher when games were provided. However, Sun-Lin and Chiou (2019) discovered that simple rules for games were important for motivation.

Huda's (2019) research presented many articles which confirmed that technology use has a positive influence on students as it is able to improve student achievement, motivation and attitude toward mathematics learning. As a tool, it did not matter whether technology use was limited to drill practice; use still would boost student engagement and keep students motivated to stay engaged long enough to work out whatever they need to be successful in completing their high school mathematics courses (Findley et al., 2019). Gaming can be helpful to encourage students from lower socio-economic backgrounds to stay on-task and involved in continuing to learn (Beserra et al., 2019).

Being involved in tasks which students find inherently interesting or enjoyable enhances motivation (Kobylanski, 2019). This is important when choosing both the formative and summative task real-life problems. Higgins et al. (2017) found using technology made a significant impact on student motivation when in the content areas of reasoning with data, thinking with models, and numerical reasoning, but not in the areas of abstract reasoning or spatial reasoning. Therefore, linking abstract reasoning to thinking with models, and linking thinking with models or numerical reasoning to spatial reasoning, is important when developing motivational real-life problem situations. A proper design of support for the tasks is also important for motivation and engagement, according to Kim et al. (2015). There is a fine balance between enjoyability, difficulty level which causes anxiety, and boredom. Providing optional scaffolding to guide student efforts is important, even in tasks which include a great deal of choice.

Two professors, Deci and Ryan, based at the University of Rochester, discovered the self-determination theory in 1985 which focuses on three areas: autonomy (the ability to make choices), competence (the task is challenging but achievable) and the opportunity to receive meaningful feedback. Providing the opportunity for the summative assessment to have some autonomy allows students to decide on their competence level and purpose and hopefully will encourage motivation to persevere through the preparatory exercises. Feedback is provided on exercises, and researching to find online solutions will also provide meaningful feedback. If the final tasks have personal relevance, where the learning is related to real-world actions, and help create a meaningful and memorable experience, the asynchronous e-learning experience will be more motivating (Edwards, 2017).

Self-Regulation. Bandura (1991) clarified his three stages of self-regulation, self-observation, self-judgement and self-reaction, as have multiple sub-routines. Self-monitoring, a sub-routine of self-observation, includes knowing when to move on to the next topic or when to continue to practice a skill, choices normally overseen by a teacher. Students will need to identify when they need to persevere with their own discoveries or when to connect to other resources, a peer, or a teacher to help sort out problems they may not be able to isolate without some intervention. Since Beserra et al. (2019) suggests that the maximum duration of any activity should be around 20 minutes, it will be useful to identify general time frames for each activity so students know when they are spending too much or too little time and can gauge their need to look for more assistance or review work. Students could also use this information to compare themselves to an imagined ‘norm’ and decide whether they are regularly quicker or slower than the ‘norm’ to improve their self-diagnostic function, another sub-routine of self-observation.

Providing answer keys and rubrics to self-assess also affects the self-judgement stage of self-regulation. The inclusion of optional activities and activities which can be completed multiple times should convey the idea that success on the initial attempt is unexpected and that perseverance is required. Students have the opportunity to revise when their original short task was unsuccessful. By dividing difficult sets of questions into shorter chunks, with immediate feedback, the energy needed to persist in learning is lower and the student can maintain a positive attitude towards self-judgement.

Naranjo et al. (2019) discovered that including a dashboard to disclose data of an overview of the work performed by the student fosters self-regulation since students become aware not only of the activities completed but also of those still pending, encouraging time management and planning. This goal setting is part of Bandura's (1991) self-motivating subfunction. The gamification suggested by Lewis (2014) would support Naranjo et al.'s dashboard and a less technical form: a downloadable tracking document.

Positive reinforcement can be heightened through ticking off successfully completed activities in a tracking document, but also through understanding why a particular activity was not successful. Both increase a student's self-satisfaction, a sub-routine of self-reaction. Clear explanations of what may have been incorrect solution strategies or other controllable processes needs to be included (Zimmerman, 2002).

Indigenous Ways of Knowing. For many Indigenous peoples, community is at the centre of interactions (Donald, 2009; FNESC, 2020). Focusing on the community and building up each other rather than competition for the 'highest grade' or getting done first becomes more possible in the digital classroom where asynchronous work takes place and where student ability to monitor others' progress is limited. Students are put into a situation to learn and seek 'wisdom' where they can, such as on the internet. Tessaro et al. (2018) discuss the 5Rs of Indigenous education: respect (the need to recognize and respect First Nations cultural norms and values), relevance (the need to go beyond books according to First Nations culture and ways of knowing, based typically in oral communication), reciprocity (the need to recognize the goals and concerns of the student), responsibility, and relationships. In an asynchronous resource, relationships are difficult to create, responsibility will be to provide what a student requires to be successful, relevance can be assisted by adding audio to resources, reciprocity is the resource itself by providing something the student needs, and respect needs to be conveyed through some of the topics in the real-life questions (FNESC, 2020).

The University of Toronto's open online course on Aboriginal Education (n.d.) identified five characteristics of Indigenous knowledge: it is personal, it is orally transmitted, it is experiential, it is holistic, and it is often conveyed through narrative. These ideas echo some of the same values expressed by Tessaro et al. and the First Peoples Principles of Learning (FNESC, 2020), strengthening the requirement for real-life topics of interest to Indigenous students which will also be educational and interesting for other students. McNamara and Naepi (2018) confirmed that narrative stories used by Indigenous elders provide the tools to find answers just as books can provide answers. Using a narrative story as a basis for learning also conveys cultural norms that are helpful for any student integrating into a different culture. BC's Grade 10 Numeracy Assessment also has this focus on narrative stories in its questions, so using this method to frame units and real-life problems will benefit all students planning to earn a BC Dogwood diploma.

Frequent mention of Indigenous students' connection to the land were made in the University of Alberta's open online course on Indigenous Canada (n.d.) and Sterenberg's (2013) study of teaching methods. Learning from place recognizes the intimate relationship that Indigenous peoples have with the land. Learning from place continues to be a valid and meaningful method of interpreting and understanding the world, including mathematics (FNESC, 2020). Young people use Google Maps and Google Earth more often than they ever used maps, so connecting this personal experience to cultural experience and mathematics could have a strong motivating factor for students. Care is needed when developing any task in order to avoid incorporating stereotypes or romanticized identities based in Western notions of the traditional 'Indian' (Stavros & Miller, 2017, FNESC, 2020).

When considering Indigenous learning preferences, Kitchenham (2016) recommended using interactive teaching in the classroom. This allows for observational learning, flexibility to account for individual strengths and needs of students, contextualized instruction and an emphasis on students' experiences and knowledge in a collaborative approach. Visual aids to support visual-spatial learning and engaging students using activities like arts, crafts, drumming, singing, cooking and other traditional Indigenous methods were encouraged. The biggest emphasis was that teachers should use multi-method and multi-modality instruction to incorporate experiential learning techniques and focus on process rather than product. Supporting Indigenous ways of knowing will be of benefit to Indigenous students but will also benefit all learners as they will need to complete the BC Numeracy Assessment which is written in a manner that is responsive to Indigenous ways of knowing. The First Nations Education Steering

Committee (FNESC) and the First Nations Schools Association (FNSA) created a *Math First Peoples: Teacher resource guide* (2020) which has many ideas that can be developed as part of this online resource.

Communicate – Teacher Role

The resource planned is a static resource that motivated students can access to help place them in a correct course or to upgrade their knowledge. Communication with a teacher and the building of a relationship between student and teacher is an important motivator for students, particularly at the beginning of the course (Ni Shé et al., 2019; Baran et al., 2011). There may be an opportunity for teacher/student contact to be made with a student's current mathematics teacher.

Consider - Online Learning Design of Assessments

The design of assessments needs to take into consideration both formative and summative types of assessment for the student to track and provide evidence of their learning. As there is no interaction between teacher and student, automated feedback systems will need to be included as well as directions on how the students can self-assess their work. In order to provide for varying levels of students, an optional as well as a suggested amount of formative assessments will need to be in place to lead towards the summative real-life problem tasks.

Formative Assessment. At the time of the first ICTMT in 1993, Monaghan and Trouche (2019) ascertained that assessment in mathematics classrooms was usually summative reports of students' mathematics skills. Much can be learned about a student's understanding through formative assessment. A study by Findley et al. (2019) considered the use of simulations for learning mathematics in terms of student and teacher roles. Some of the student roles included lower level uses where the students just recorded or described what happened, but there were also levels in which students used the simulation to construct scenarios to meet specific requirements and moved on to explore experimental situations. Aiming for the higher levels of these roles is preferable when preparing for summative assessments. Huda (2019) stated that when redesigning tasks from paper-and-pencil tasks to online tasks, the technology should allow for self-reflection, promote learning during the assessment, and guide the students to focus on important details of the tasks. Sun-Lin and Chiou (2019) found that after doing an example and then a similar problem, providing a message to the student to recognize their performance and encouraging them to solve a challenging problem for practice helped students gain a sense of achievement which could motivate them to attempt to practice more related problems, enhancing their transfer ability. In general, students tend to spend more time on online homework when there is immediate feedback, especially when hints are included or similar examples are available (Dorko, 2020).

Many apps provide an answer but do not provide a reasoning as to why the incorrect answer is wrong. When using games like *Kahoot* in the face-to-face classroom, teachers have the time to explain incorrect answers before moving forwards to the next question. Ruchniewicz and Barzel's (2019) study of apps including *Math 42, Photomath, Geogebra, Mathway, Cymath* and *Socratic* found the apps focus on procedure without any connected aspects to develop a concept or an overall idea. Many apps include no opportunity to connect to why the procedure is meaningful or to connect the procedure to previous knowledge. It is important in math to reinforce any new ideas learned, and the best way to do this is by using it in different ways (Boaler, 2015), but feedback in the form that would be present in classroom use of these apps would be helpful so students can connect their thinking to the different ways the new idea can be used.

Automated Feedback Systems. Setting any type of goal, whether high or low, and receiving feedback significantly increases motivation (Bandura, 1991) and self-regulation (Jedke & Greefrath, 2019). El-Demerdash et al.'s (2019) study into the design and evaluation of digital resources for creative mathematical thinking found two types of useful mathematical feedback: educational feedback and technical feedback. Educational feedback includes the hints and comments that encouraged creative mathematical thinking. Technical feedback provides hints on how to use the technology. Jedke and Greefrath's (2019) research supported the idea that feedback messages need to be timely so students can self-correct their mistakes and not perpetuate learning misconceptions.

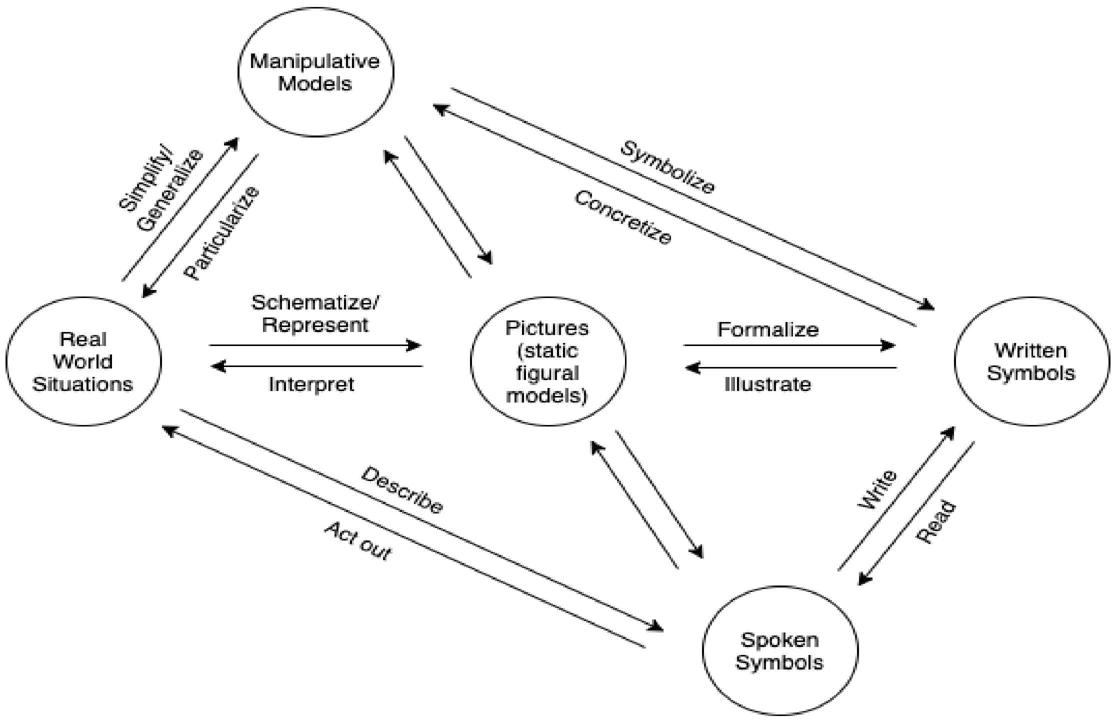
When students have the opportunity to receive feedback for any part of a summative assessment before it is completed, Dorko (2020) found they usually use the feedback as formative assessment and incorporate the suggestions into their work. Providing an extended formative assessment, built similarly to the summative assessment, provides students feedback on the elements required to be successful on the summative assessment. When students are looking for their own real-life task, based on parameters provided, they should be guided as to how to search, as Dorko (2020) also found that many students lack the understanding of how to conduct internet word searches. By providing rubrics with successful strategies, students understand how to evaluate their own work. Ruchniewicz and Barzel (2019) used a rubric version for their study which they called a 'Check'; students were provided with six statements in this section which included six statements of important aspects of the question alongside common mistakes that could arise.

Real-life Authentic Problems. Why does mathematics need to include real-life situations? Devlin (2011) confirmed that our current culture of mathematics separates real life from math use. Students may find they can do the mathematics in a real-life situation, yet are unable to complete similar problems when presented on paper. Is this because of our focus on converting problems into symbolic form and then back into real-life? Devlin suggests that if the problem is based in real-life first and students explore through context, the learner is more motivated to carry out the tasks that involve mathematics. If the task is creative, then students are much more persistent and willing to take on academic risk because of the authenticity of the task (FitzPatrick and Dominguez, 2017). Even Sal Khan of Khan Academy (Brown, 2019) recognized that a focus on creativity in online education can have a positive effect on self-regulation and motivation. This activates higher-order thinking, something that is often missing in online resources which focus on the remember and understand elements of Bloom's taxonomy because it is simpler to do.

Lesh's (1981) translation processes model articulates how the more ways of knowing a student experiences while exploring and interacting with mathematics concepts, the more successfully they will internalize the concepts. When students can explore and make connections through multiple forms of a mathematics concept, the process encourages deep understanding and enhances the ability to apply, analyze, and evaluate the use of that concept.

Figure 2

Translation Processes



Note: Translation Processes. Adapted from "Applied mathematical problem solving" by R. Lesh, 1981, *Educational Studies in Mathematics*, *12*(2). Retrieved from https://www.jstor.org/stable/3482367. Adapted with permission.

Being able to use an idea or concept involves something more than simply remembering it (Lesh, 1985). Being able to use the ideas requires students using them in familiar or authentic situations. Although problems set in cooking, carpentry, or business may be real-life, students may be totally unfamiliar with the techniques or work-specific procedures required (Lesh, 1985). In order to make them authentic, the real-life problems must be something the student might have the opportunity to do and they must be provided with the real-life procedural or techniques required for successful completion of the problem. Lesh's (1985) camp problem involved placing 12 students at a track-and-field camp into 3 groups. Speed in the 50- and 100-yard run, height in high jump, distance in long jump and swim levels were provided, as well as a comment on effort and attitude. The problem was presented at varying times of the year to groups of students, and as the year progressed, students were more successful supplying a justified and viable solution as they experienced more of these types of problems in their mathematics course. By using carefully sequenced mathematical problems, the students learned to use processes they use in other problem-solving areas such as social problem solving and decision making (Lesh, 1981).

The importance of real-life, authentic problems is faced in all mathematics courses. Recent reforms in mathematics education emphasize the need for students to understand and experience the authentic applications of mathematics to their lives (Koh, 2019). Koh's research into the kinds of knowledge teachers use to design lessons using technological tools confirmed that mathematics educators aspire to use technology to support mathematical inquiry and the understanding of real-world mathematical applications through authentic real-world problems. They aim to engage students in higher-order knowledge construction such as analysis, creation and evaluation techniques. Through the study of the kinds of pathways teachers use to create technological pedagogical mathematical knowledge, Koh also revealed that student-centered learning encourages learning through doing and manipulating information with technology tools, is collaborative, and supports student control over their learning goals and processes. In Thailand, Khlaisang and Songkram (2019) recognized that a traditional classroom setting involving lectures on procedures and then practicing those procedures does not meet the needs of higher education learners and the use of student-centered teaching methods and virtual learning tools for inquiry, as well as for skill practice, is increasing. Presenting authentic activities is a key component of designing an online course. Activities can include, but are not limited to, using graphics, audio, video, animation, hypertext, providing links to databases or knowledge repositories that provide real-time data or developing a direct manipulation interface for students to use for exploratory-type activities (Dabbagh, 2005).

Using real, up-to-date data available for free on the internet, known as open data, is a great tool for encouraging critical thinking and research skills, developing the mathematical skills of statistics and data information management and curation, and improving data visualization skills (Atenas et al., 2015). Another benefit of using open data is that it provides students with the opportunity to run their own tests on the data and then compare their results to that of professionals, a self-assessment opportunity.

Conclusion

The resource created will provide a collection of interactive tools and already created activities that are currently available to use for free without signing into another website. Units will be framed through a narrative story for a real-life application of the knowledge. Inquiry will be an important learning tool, so students develop mathematical reasoning skills. The tools and activities will also include instructional videos with visuals, text, and translation ability. Links to already developed apps with skill practice will be provided. Keys and opportunities for multiple attempts will be available. If not already included, hints will be created so students can learn what questions to be asking themselves. As this resource is meant to be asynchronous without teacher evaluation, screencasts of mathematical thinking created by students, which are aimed at teacher assessment, will not be required. If a student chooses to present their work through a screencast for their self-assessment, that is a choice they are free to make.

This online resource will be flexible as students will have the ability to choose their path to the summative assessment, it will be based in real-life contexts and relate to student experiences in order to make the contexts authentic, it will use many visual aids as there is no lecturing teacher, and it will be a multi-method resource. It will combine all of the aspects suggested by Kitchenham (2016): textual, audio, visual and recordings. Use of VRE which requires specialized equipment will not be included, as the focus will be from a larger conceptual approach rather than a procedural approach. This, according to Kitchenham's study, will support Indigenous learners as they tend to learn better when they see the larger view of a concept rather than the finite details.

In a resource that is not aiming to be a grading system but a system for diagnosis and learning what is required to be successful in BC mathematics courses, formative assessment is essential for the student to understand their next learning step. Unfortunately, although developing an app specific to this resource would be beneficial, it requires more time and effort than is available. Some gamification patterns as explained by Devlin (2011) will be used.

Providing a downloadable tracking document would satisfy the state preservation pattern, allowing for multiple attempts would satisfy the undo and predictable results option, and adding feedback to unsuccessful and successful completion of elements would satisfy the praise option. The task queue would be part of the downloadable tracking document, and clear search by topic and possibly the activity's numbered title according to the tracking document would need to be included.

For summative assessment, to enable concept and context connection as well as reasoning and reflection, there will be a rubric prepared so students can self-assess their response to their own questions which they have framed according to the narrative story. How to use open data will be explained. A downloadable and printable notebook will be provided so students can track their learning through a clear outline of the order of learning suggested. The book will include formulas, a space for notes, space for planning time to complete activities as well as how long the activities may take, question levels (understand, apply, analyze), optional extra practice and space to plan the summative assessments.

Chapter Three: BC Math 10 Resources Website

The website with updated content can be accessed at: http://bcmath10resources.opened.ca/. A captured PDF of the website from April 2021 can be found uploaded to UVic Space under the file name Tradewell_Cheryl_MEdProject_2021_WebsiteCapture_002. All components of this project are licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. Readers and website viewers are free to copy and redistribute the material in any medium or format, or remix, transform, and build upon the material, as long as they present appropriate credit, provide a link to the license and indicate if changes were made. Attribution may be made in any reasonable manner, but not in any way that suggests my work endorses you or your use. The materials in this work or the website may not be used for commercial purposes. If you remix, transform or build upon the material, you must distribute your contributions under the same license as the original. You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. Should you reuse my work, you can provide attribution as follows: "This work [or give title] by Cheryl Tradewell is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License."

Further resources considered for creation of the website can be found on a padlet accessed at: https://padlet.com/cherylltradewell/f9guda7y5zcf, which is also included in pdf form uploaded to UVic Space under the file name: Tradewell_Cheryl_MEdProject_2021_Resources_003.

Chapter Four: Reflections

My reflection can be found in video format uploaded to UVic Space under file name: Tradewell_Cheryl_MEdProject_2021_Reflection_004.mp4 and on YouTube at https://youtu.be/fbWcVKduKis

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