Listening to Teach, Speaking to Learn: A Journey into Mathematical Learning through Conversation

by

Jeannie West DeBoice
BEd, University of Victoria, 1985

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Abstract

Supervisory Committee
Dr. Jennifer S. Thom, Department of Curriculum and Instruction
Supervisor
Dr. Ruthanne Tobin, Department of Curriculum and Instruction
Departmental Member

This project looks closely at the role of conversation in developing mathematical understandings in one third and fourth grade classroom. The data from this study comprises ten videotaped mathematics lessons that include 66 episodes of 2 to 10 minutes. These episodes feature conversations between the students and the teacher (Mrs. Howard), as a whole class and with me the researcher. This study focused on the examination of the different ways in which the students' conversations shaped their conceptual understanding. Five categories of conversations were identified and explored in this research: (a) student strategies, (b) playfulness of ideas in mathematics, (c) student misconceptions and/or misunderstandings, (d) student discoveries and (e) insights.

My goal with this project was to witness the effect of collaborative conversation between the teacher and students and as well, to allow the reader a glimpse into the workings of one elementary mathematics classroom.

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Acknowledgments

I would like to thank the Sooke School District for supporting my research through every phase. The idea for this project began 5 years ago when I first became the numeracy coordinator for Sooke district. Through this role, I had the opportunity to work with many teachers who shared the goal of making sense of mathematics with their students by creating rich learning experiences. I have learned more than I can say from all these students and teachers and would not have pursued this project had it not been for those experiences.

Thanks to Dr. Jennifer Thom for supervising this project. We had many hermeneutic conversations ourselves over these last 2 years, and I always came away richer. Thank you for taking this journey with me!

And thanks to Dr. Ruthanne Tobin for working with me as my committee member. Your thoughtful questions and engaging chats helped to deepen my understanding of conversations for learning.

Finally, there are two authors I want to acknowledge for inspiring me as I wrote. A.A. Milne, for providing me with some literary examples of collaborative conversations between two of his beloved characters, Christopher Robin and Winnie-the-Pooh. And Joseph Campbell, my all-time favourite author, with his writing of journeys – leaving the comfort of the familiar, journeying into the unknown, and returning the wiser for it. This is the journey on which I embarked, and I feel I *am* the wiser for it.

Dedication

This project is dedicated to my Mom and Dad, Val and Russ West, who taught me from an early age to love learning. They were the first in my life to model the art of listening and we spent many hours at the kitchen table over multiple cups of tea in deep, genuine conversation. They supported me in spirit, mind and soul all along the way, and continue to model a passion for learning in their lives today. Thanks, Mom and Dad.

I also want to dedicate this project to my husband, Randy, for his unfaltering support for this project, his energizing faith in me and undying patience these past three years. I couldn’t have done it without you!

Chapter 1: Departure

The Journey Begins

I have been an elementary teacher for over 20 years. In that time I taught in several different roles: as a music specialist, as a librarian, as a Reading Recovery teacher and as a generalist. Mathematics was never my forté, however after being asked to pilot a new mathematics program in my fourth grade classroom, I became very curious about how children learned mathematics and ways to improve my teaching of it. This quest led to my current position as the Numeracy coordinator for Sooke School District for the last five years. I am responsible for inservicing and supporting grades K – 8 teachers in their teaching of mathematics. I was hired in 2002 and began my steep learning curve, finding out all I could about the research behind such programs as *Math Makes Sense* (our new resource) and the new 2007 mathematics curriculum. This led me to begin my graduate work in 2005.

As part of this work, I earned a Certificate in School Management & Leadership through the CSML course at UVIC. I conducted a year-long action research project in a first and second grade classroom, focusing on how to help struggling learners gain confidence and understanding of number sense. This was where my interest in teacher-student conversation really began. As the classroom teacher and I worked to better understand how these students were developing number sense and why some students struggle to do so, we had to listen carefully and thoughtfully to what these young children had to say. We noticed that this kind of genuine listening often provoked more detailed, considered responses on the children's part–and, ultimately, what appeared to be deeper understanding for them as well. It also caused us to change *what* we listened for–the child grappling with ideas, striving to make sense of the concepts, rather than the right answer. The classroom teacher and I engaged in collaborative conversations as we too, worked to understand what we observed and how this impacted our next steps in planning activities for the classroom.

My reason for doing research is always with the aim to look for ways to improve the mathematics learning and teaching in classrooms. By improve, I mean helping children to make sense of mathematics and believe they can do so, at every level. I am in a unique position as a numeracy coordinator as I have access to many classrooms from Kindergarten through grade 8. I have the opportunity to observe children in their classrooms as well as see the results of the lessons that their teacher and I collaboratively plan and teach. I see firsthand the kinds of difficulties that children run into when learning mathematics and I am able to suggest ideas to their teachers and see the results. With this privilege comes great responsibility, I feel, to learn and do what I can to help the children in our district improve their understanding of mathematics.

Through five years of working closely with children and teachers in this capacity, I now realize that children do not only create mathematical knowledge for themselves but also with and for others in a social context. And it is in the social context that conversation plays an important role in students' development of deep mathematical understandings. I suspect that if children do not have frequent opportunities to make conjectures, justify ideas, collaborate and have mathematical discussions with others, their mathematical learning would most likely remain superficial and disconnected. Students need to be able to connect what they have learned to others' ideas time and again to mould and shape their deep understandings.

A Conflict

In education, it seems to be common knowledge that conversation is an intrinsic way we construct knowledge. If one of the ways in which we share and develop ideas is through explaining and justifying them to others then conversation plays a critical role in mathematical learning and understanding. Yet in many K-8 classrooms, conversation in mathematics lessons happens infrequently and often, superficially. For example, on the whole, teachers acknowledge the importance of allowing time for children to discuss mathematical ideas but then use short question and answer exchanges as the primary means with which to elicit group discussion. Even though teachers may agree that on-going conversation in small groups or one-on-one between student and teacher is a way to deepen learning, it appears to be the exception rather than the rule; typically, mathematics lessons remain quite clearly teacher-led in many K-8 classrooms with students for the most part acting the role of the listener. Brent Davis (1996) refers to this situation when he writes of the cultural norm of valuing the visual over the auditory: "In terms of mathematics teaching, a principal consequence of this loss of hearing is that learner – those we are to teach – have been reduced to silence; they are objects to be *seen and not heard"* (p. xxiii).

In talking with teachers about the use of conversation in their mathematics classes, some express frustration in the lack of conversation during their lessons. The reasons for it not occurring range from time constraints, inability on the children's part to sustain a mathematics-focused conversation and off-task behaviour among the other students while a teacher is conversing with one. Still other teachers question the value of conversations in the mathematics class and feel a need to teach the skills and procedures to the children through methods of teaching-by-telling and then set the children on to practice these skills and procedures.

A Quest

My experiences of working closely with other teachers helping students make sense of mathematics led me to my first question: Could changing the way we talk and listen to children in the mathematics classroom deepen their learning? More often than not, teachers seemed to spend a lot of the time in mathematics class telling and explaining ideas to listening students, yet in other subjects there seems to be more exchanging of ideas, more conjecturing, more imagining. What might happen if conversation was a regular part of mathematics lessons?

Conversation involves, among other aspects, listening and questioning on the part of everyone in the dialog. Certainly students and teachers talk during a mathematics classes, but often it is a query about a concept or a procedure from the student followed by instructions or explanations by the teacher. Or, if it is the teacher asking the question, it often centers on checking for basic understanding of a concept or procedure. I am talking about conversations that go deeper than this kind of question and answer, in which teachers use conversation with students as a way to support their learning, being aware of the *kind* of listening and questioning they are using. The same question can be asked in different ways: what counts is how we listen for the answer, and what we expect the answer to do for the child's learning. After I read Brent Davis' 1997 article, "Listening for Differences", I realized a teacher can ask, "What is a fraction?" and have three distinct purposes for listening to the answer. If the teacher has already taught a definition to the students and he/she is checking whether they have learned it, then he/she wants that specific answer; thus is listening to evaluate. Or the teacher might be interested in how students explain their reasoning for their definition of a fraction, in which case he/she is listening to interpret the thinking. A third way is to listen in order to *jointly* create a definition of what a fraction is to *both* the student and the teacher.

Hermeneutic listening, or listening to understand, unlike evaluative listening (listening for the right answer) or interpretive listening (listening to follow a line of reasoning) is collaborative in nature. In this case the teacher is a participant in the learning, a co-creator of the understanding, not a deliverer of information. Hermeneutic questioning is different from evaluative questioning in much the same way: the teacher who asks a hermeneutic question is not expecting one correct answer, but rather is interested in the thinking of the person questioned. “The hermeneutic question…is one for which the questioner does not know the answer and is sincere in his or her desire to learn it” (Davis, 1996, p. 250). This awareness of the kind of listening and questioning a teacher is using as well as a knowledge of the impact of each is important if he/she is to move towards hermeneutic conversations in the classroom.

It was when I read Davis that I began to wonder: could we as mathematics teachers consciously affect student learning by using more collaborative conversations with them? Is this a way to help students make sense of mathematics in a deep, personal way? Thus the central question of my research became: What role might genuine, hermeneutic conversation play in occasioning the growth of students’ conceptual understanding in the mathematics classroom?

It was from here that I set out on a journey to explore the different kinds of conversations that occurred in a grade 3/4 classroom and the kinds of mathematical understanding they engendered as the teacher employed hermeneutic listening and conversation while students engaged in and discussed their understanding of probing mathematical tasks. Additionally, I wanted to know how a teacher's perception and use of conversation might grow, change or develop over time.

The Way Forward

I decided to videotape and then examine mathematical conversations in the classroom; that is, one-on-one teacher-student conversations and interviews. In the beginning stages of this exploration, I used Gordon-Calvert's definition of conversation as "…open communication between teacher and students and among the students themselves" (2001, p. 47). I wanted to gather "thick" descriptions that would help the reader situate him/herself in the classroom experience as fully as possible. Unlike thin descriptions which focus only of the facts, a thick description "…gives the context of an experience, states the intentions and meanings that organized the experience, and reveal the experience as a process. Out of this process arises a text's claims for truth, or its verisimilitude" (Denzin, 1998, p. 324). To situate my story of this classroom in a greater context, I connected my findings to theories of learning and the role of conversation in learning mathematics, pushing the data against the ideas of Davis, Pirie & Kieren, Gordon-Calvert and Cobb.

I began my quest by obtaining consent to conduct the research from my district office. I then sent an advertisement for interested teachers through the district's Numeracy Networks. Mrs. Howard responded, (a teacher I know well from her attendance to many of my workshops) and we had a meeting to discuss the research and share some readings about the role of conversation in the mathematics classroom. After obtaining written consent from both the teacher and the parents of the students, I became part of the grade 3/4 classroom and followed this advice: “Place your best intellect into the thick of what is going on. The brain work ostensibly is observational, but, more basically, it is *reflective*” (Stake, 2003, p. 149 - 150). I wanted my project to be an account of my experience and reflections as a part of this learning community, not simply my views as a distant observer, so the reader could gain the full import of what happened for myself, the teacher and the learners over 10 weeks. To understand the impact of conversation on mathematical understanding, I had to be part of the conversation, or very close to it. By becoming part of the learning community, I was able to get to know the individual learners, and thus have a greater sense of when they were involved in a meaning-making conversation.

I hope to use these research findings in my work with teachers who strive to improve the mathematical understanding of their students but have not yet made use of conversation in their mathematics classes. With illustrative examples of collaborative conversations, the classroom contexts and the learning that occurs from them I wish to share my new understandings about mathematical conversations and how teachers might make them an integral and important part of their mathematics lessons.

Chapter 2: Initiation

Before I began my journey, I needed to gain an understanding of the key issues, questions and studies that related to my study. I examined professional/theoretical literature focusing on theories of learning mathematics, the role of conversation in learning and teaching mathematics and theories on the developmental stages of mathematical understanding.

Learning Mathematics

Before I explored the role of conversation in the mathematics classroom, I needed to examine the theories about how we learn mathematics. How we as teachers believe students learn directly impacts the choices we make in our instructional repertoire to help them do so. Thus my review of learning theories was the first path of my quest as I worked towards understanding how it was that my students were learning or not learning mathematics. My first exploration of this question seven years ago led me to read authors who ascribe to a constructivist perspective. This was a new idea for me and the teachers in my district. Some of these authors (e.g., von Glasersfeld, Fosnot, Hiebert) base their epistemology on Jean Piaget's writing of the developmental stages of cognition from the biological perspective. The basic idea is that knowledge is not something to be discovered but rather knowledge is constructed by the learner to make sense of the world. Questions about the role of community or social aspects of learning led me to examine the works concerning socioconstructivism. It seemed to me that my students learned as much mathematics through explorations and conversations with each other as they did listening to me. These conversations were more than just chatter or communicating information; rather these conversations engaged the students in meaning-making as they debated, conjectured, and explained their thinking to each other.

Constructivism

Over 60 years ago, Piaget set the stage for constructivism when he applied the idea of adaptation in the biological sense to genetics. Adaptation is how an organism survives through finding ways to "fit" the environment. He used the concept as a way to describe how a person's cognitive structures were closely connected to his/her experiences in the world. "He had realized early on that whatever knowledge was, it was not a copy of reality" (Glasersfeld, 2005, p. 4). Constructivism emphasizes the complexity of learning and the activity of the learner in contrast to behaviourism, which emphasizes learning as responses to stimuli and that learners are passive and require outside motivation:

> Rather than behaviors or skills as the goal of instruction *cognitive development* and *deep understanding* are the foci; rather than stages being the result of maturation, they are understood as *constructions of active learner reorganization*. Rather than viewing learning as a linear process, it is understood to be *complex* and fundamentally *nonlinear* in nature. (Fosnot, 2003, p. 10 – 11)

Fosnot (2003) cites the biological work of Piaget, the sociohistorical work of Lev Vygotsky and Jerome Bruner's work on the role of representation in learning as giving constructivism its start. She outlines general principles to keep in mind if teachers are to teach in a constructivist way. Learning does not result because of development; it <u>is</u> development. "It requires invention and self-organization on the part of the learner" (p. 33) and so teachers need to let the students take the lead in the learning. In addition, she points out, students need to learn from their errors, as this disequilibrium moves learning along. Students also need time to reflect and to dialogue with others to make connections, share strategies and defend ideas.

In 1975, during Ernst von Glasersfeld's work on knowing and learning mathematics, he presented a seminal interpretation of Piaget's work. In this work, Glasersfeld termed it "radical" or fundamental constructivism, to differentiate it from mainstream or "trivial" constructivism. Radical constructivism does not view concepts to be learned as knowledge that exists "out there" to be mirrored and reflected back. According to von Glasersfeld (1990) knowledge is actively built up and "…the function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability" (p. 23). So learners in mathematics are looking to construct knowledge that is the best fit for the moment: the learning will continually evolve as the learner repeatedly looks for viability of the concept. Glasersfeld (1990, 1995) also contends that the learner is not alone in creating his or her knowledge. So children construct their own knowledge of mathematics as a way to organize and understand their world and do so in relationship–through collaboration and communication–with others.

Socioconstructivism

Socioconstructivism is another discourse within constructivism which contends that knowledge is created within the learner. But there is a debate between those with constructivist and those with sociocultural/socioconstructivist perspectives. The argument seems to be where the mind is located: in the head of the individual (constructivism), or in the individual-in-social-action (sociocultural perspective). It is also about how and why one develops understanding: as a self-organizing entity or as an entity encultured by one's surroundings. Paul Cobb (1994) writes about the debate and expresses his opinion that one perspective compliments the other. The sociocultural perspective helps teachers understand what conditions are most conducive to learning mathematics (i.e., group settings where individuals can interact and share ideas) whereas the constructivist perspective helps us understand the process learners go through to construct knowledge about a subject: "It is as if one perspective constitutes the background against which the other comes to the fore" (p. 18). He takes the middle road, and does so for a specific reason: both can contribute to the betterment of mathematics education. He suggests we take the perspective which is most likely to improve a student's education, to coordinate perspective as we focus on what is best for children. Catherine Fosnot (2003) states it simply: "We do not act alone; humans are social beings" (p. 29).

As a result of the constructivist/radical constructivist and socioconstructivist theories, mathematics teaching and learning experienced a shift: away from teaching as telling with quiet, listening students and towards teaching as listening, too, with students talking and sharing their ideas in large and small group discussions. Glasersfeld (1995) writes of "perturbations", those instances where what the learner expected to happen does not. These perturbations are necessary, according to Glasersfeld, for any learning to take place, as the learner must either modify his/her thinking or his/her action to accommodate this dissonance. Glasersfeld adds that the most frequent way perturbations happen is in interaction with others. So the individual activity of meaning-making, of constructing one's own knowledge is situated in culture – in this case the classroom culture. And it is that negotiation of meaning through conversation that helps the learner clarify and solidify his/her understanding of mathematics.

Enactivism

These theories, constructivism and socioconstructivism, have a common thread: that knowledge is something to be had, found, uncovered or explored. The debate over what knowledge is has continued on for many years. Recently I have read about enactivism, which takes this idea even further. Knowledge, within an enactive perspective, is not something "out there" to be constructed in one's head, but rather is created in the interplay between one's nature and the environment. This work is carried forth by biologists such as Maturana & Varela and educators like Brent Davis. Davis (1996) writes of enactivism as the "middle way" with constructivism and socioconstructivism as steps toward the middle way. Davis is referring to the Buddhist idea of middle way as balance between the individual and the collective, of yin and yang. He sees enactivism as a way to challenge the divisions society draws between the self and others, between knowledge and action: "…enactivists (along with complexity theorists) conflate knowledge and action – both on collective and individual levels – and, in so doing, point to the co-emergence of individual knowing and collective knowledge and to the self-similarity of their underlying processes" (p. 190). This collective knowledge is neither found in the individual (as constructivists would see it) nor in the group of learners (the social cultural perspective). Rather, these understandings emerge in the interactions *between* individuals in the group.

From an enactive perspective, knowing is not some "thing" to be delivered or discovered by individuals. Rather knowledge is the result of the *interaction* between a person and the environment:

> In the interactions between the living being and the environment…the perturbations of the environment do not determine what happens to the living being; rather, it is the structure of the living being that determines what change

> occurs in it. This interaction is not instructive, for it does not determine what its effects are going to be… [The] changes that result from the interaction between the living being and its environment are brought about by the disturbing agent but *determined by the structure of the disturbed system.* (Maturana & Varela, 1987, p. 96)

The learner does not simply take in information from the environment but instead the environment, through perturbations, "…present an occasion for the person to act according to his or her structure" (Davis, 1996, p. 10). From an enactive perspective, we are always simultaneously living with and in our outer structure (physical-biological) and inner structure (lived-experiential-phenomenological). We are thus a product of both our biology and our interaction with the world. Additionally, as individuals we are affected by the environment, the environment is affected by the individual.

Teaching and learning cannot be seen as two opposite or even separate actions but instead entwined actions involving and affecting both the student and the teacher at the same time. For example, a teacher might use error-making in students' work as a place of learning – not to remediate, but to use probing questioning and hermeneutic, collaborative listening to "…excavate and interrogate such breeches" (Davis, 1996, p. 249). An enactive perspective emphasizes that the world that is constantly changing, and knowledge is not a set of predetermined truths, but that which is ever-evolving. "It is thus that, for the enactivist, the world is *not preformed*, but *performed.* We are constantly enacting our sense of the world – in the process, because we are part of it, altering it." (Davis, 1996, p. 13-14).

Davis, Sumara and Luce-Kapler (2008), in their book *Engaging Minds* acknowledge that the debate about what knowledge *is* has been going on for thousands of years. The authors instead attempt to "…explore what it *might look like*" (p. 65). They differentiate memory from knowledge and see them as complements of each other. "Briefly, whereas memory points to the internal dynamics of a complex unity, knowledge points to the dynamics of the unity-in-context" (p. 65). Memory refers to the dynamic in a system, and the system can be at the molecular level, the body level, or social, species or ecosphere levels. Knowledge, on the other hand, has to do with viability, with *fitness*. Davis, Sumara and Luce-Kapler (2008) explain this idea in the context of error-making. An error is not perceived as such until and unless the person recognizes that their thought does not fit with their understandings of the situation or idea at hand: "…interpretations are not errors until they are shown not to fit, because for something to be wrong it has to be manifest [sic] in a way that threatens the viability of the knower or the knower's knowledge" (p. 65) This idea of fit is important for educators of mathematics. Too often students are told their erroneous mathematical idea is incorrect, but this statement has little impact on learners if they do not see for themselves how the mistake does not fit into their mathematical understanding. For example, an error that young children often make when first learning to subtract double-digit numbers is to subtract *up*¸ rather than use the "regrouping" algorithm. A child might see 41 – 18 and, after lining them up one on top of the other and begin to subtract the ones (as they have been taught) they see 1 - 8 is not possible, so they simply reverse the digits and subtract 8 - 1. Even repeated demonstration on the teacher's part is not enough to convince some students, and it is only after working with the numbers themselves in a real life context - such as calculating how much money they have spent - coupled with a probing conversation that the learner sees the mis-match of their error and their understanding of subtraction.

Mathematical Conversations

Conversation in mathematics lessons has been the subject of much research exploring the uses and benefits. Steffe & Kieren (1994) explain it is in the conversations of teachers with students that we can begin to map out where we might guide the learner to next: “Observing and listening to the mathematical activities of students is a powerful source and guide for teaching, for curriculum, and for ways in which growth in student understanding could be evaluated” (p. 723). In Paul Cobb’s (1994) article, he talks about communication and refers to H. Bauersfeld’s discussion of “mathematizing”; thinking about mathematics at a deeper level than just skill building. Mathematizing, as Catherine Fosnot (2002) describes it, is students actively exploring mathematical ideas, explaining and justifying their thinking and noticing relationships between concepts. It is this deeper level that conversation addresses: the learning does not just happen as students in an interaction “explicitly negotiate” meanings or understandings of mathematical concepts, but also, and more importantly, it is when there are “implicit negotiations” (Cobb, 1994, p. 15) where meanings shift subtly even if participants are not consciously aware of this. For example, two students who try to solve a problem about fractions of a set may appear to be working to get to the one, right answer (i.e., 2/3 of 12 is 8) but also come to more deeply understand what fractions mean, how one compares to another, how fractions of a set are related to but different from fractions of a region.

Genuine Conversations

Many authors have written about the language of mathematics and the many facets of conversation in the classroom that support learning. Lynn Gordon Calvert (2001) emphasizes that, although teacher-led group discussions in mathematics class are valuable, it is the interaction between students or between a student and the teacher that she sees as “genuine conversations” (p. 46). She goes on to say that conversations are not random; instead they follow relevant concerns and ideas that come up in the moment. “Meanings are constantly being shaped and reshaped while the topic is molded and transformed within the course of the conversation” (p. 48). The idea that conversation continually and purposefully reshapes an idea is similar to one put forth by Gadamer (2004). He talks of participants in a conversation needing to allow themselves to be “…conducted by the subject matter” (p. 361) and to really attend to what the other is saying without focusing on winning the other over to a particular way of thinking.

Gadamer (2004) writes about the relationship forged through conversation and points out that the two engaged in conversation must be “on the same page”: “…that the partners do not talk at cross purposes” (p. 360). He defines questioning not as a way to argue the other person out of his/her opinion, but rather “…to lay open, to place in the open” (p. 361) the subject they share, the ideas that a person is exploring. He talks of the new understandings created between the two in conversation; that in fact the subject of the conversation necessarily exists collectively among and because of the conversants. So it is only through the act of conversing that the meaning of a conversation comes to exist: a conversations’ meaning does not reside in any one of the conversants or without them. Taking Gadamer's idea of meaning making in conversation and considering it within the learning context of the mathematics classroom, then teachers would enter into a conversation with a student, not to impose his or her understanding in order to teach, but rather to engage in a dialog, creating a common, shared language about the subject that is *between* them and thus greater than each of them as individual agents. This kind of conversation in a very real sense then transforms the conversants: “To reach an understanding in a dialogue is not merely a matter of putting oneself forward and successfully asserting one's own point of view, but being transformed into a communion in which we do not remain what we were" (Gadamer 2004, p. 371). Both the students and the teacher bring forth new understandings about the mathematics that they explore.

Given this participation between student and teacher in the conversation, there is the danger of misinterpreting the purpose of conversation as a letting go of the teacher's role as instructor. Teaching through conversation is neither tight control of all that our students learn and do nor is it letting the children learn (or not) on their own. Rather, a teacher's role focuses on finding a space between the two: "… curriculum designs and instructional strategies, if they are to be useful, need to lie in that space created by the dynamic interaction of the closed with the open (or in the interplay of the scientific with the storied and the spiritful)" (Doll, 2008, draft, p. 15). Doll calls this the "third space" and it is here where I believe conversation resides. It is this third space that is formed by the "…tensioned interaction between the open and the closed" (draft, p. 16) between the object and concept, between what is known and what is to be discovered.

Wang, (2004) in her book, "The Call from the Stranger on a Journey Home: Curriculum in a Third Space" describes the interaction between student and teacher as "… a process of simultaneously reaching inside and outside to meet the other…This pedagogical openness to student-as-stranger who has unrecognized potential with irreducible singularity reflects back to the teacher's necessity to confront her own otherness within" (p. 158). She goes on to caution teachers against thinking that by listening to and following our students' ideas and interests that we are give up our "…pedagogical responsibility and offering students only what they want" (p. 158). In fact, teaching with the idea of the third space requires more from us – and the students – emotionally and intellectually. It is through conversation that students and teachers discover new ways of connecting to and between ideas and thus continually bring forth ways of knowing. A subtle but critical point made by Wang (2004) is that the knowings which arise in conversation require not only what the conversants say but also how they are listening while engaged in the conversation. How one listens and speaks to the other in conversation shapes what is heard, what meanings emerge and thus, what new understandings are made possible. "It is the *tension* of the movement that issues new ways of connecting and constructing [knowledge]…Swinging in both directions simultaneously, one neither fully submits to the pull of any one pole, nor does one hold onto only one's own posture. One has to move with the swing but maintain balance. This is the teacher's position" (p. 178).

Real, genuine conversations occur when "…the participants in the conversation engage in a reciprocity of perspectives" (Aoki, 2005, p. 228) where one does not "win" over the other, but that each perspective informs and enriches the other. Aoki also writes of the "space in between" when he writes of conversation. He refers to conversation as a "bridge" in which the two parts – the two perspectives – are better understood in the context of the whole, and the whole is better understood in the context of the parts. "It is in this sense that I understand conversation as a bridging of two worlds by a bridge, which is not a bridge" (Aoki, 2005, p. 228).

I consider the work of Wells (2009) to be in keeping with Aoki's perspective when he writes of the importance of teacher engaging with individuals or groups of students to assist them through the use of "collaborative talk". Through this teacher-student conversation, the teacher may interact with the student and the new learning in a shared, supportive way. He believes this "collaborative knowledge building" is so critical that he "…cannot imagine achieving real understanding of new material without it" (p. 295). This knowledge building can take many forms, but most often it is through face-to-face conversation and can also include materials from the environment of the classroom – manipulatives, diagrams, charts and notes. "The aim is to create a common, or shared, understanding to which all participants contribute" (p. 294). Drawing from Vygotsky Wells asserts that our thoughts do not exist in a vacuum; rather we share ideas and perspectives "…to achieve a sharing of information and attitudes toward a common goal" (1984, p.190). Wells encourages teachers to consider collaborative talk in setting up their classrooms, forming student groups and providing students with opportunities to work independently to provide time for the teacher to dialog one-on-one with students.

Through all of the preceding viewpoints runs the idea that the concepts or understandings people develop in conversation exist in neither participant alone but rather *between* them both. The knowledge is created in the very act of genuine, collaborative conversation and both participants listening and speaking, connecting and creating ideas collaboratively. This then is a specific, considered type of conversation, very unlike the rapid-fire question and answer patter heard in some mathematics classrooms. A teacher, wanting to incorporate this type of dialogue with his/her students, must be both conscious of what collaborative conversation is and be alert for times in which to engage students in this kind of dialogue.

Growth of Mathematical Understanding through Talk

Susan Pirie (1997) explains that historically, mathematical language focused on "…symbolic representations, teacher talk and students' factual answers" (p. 229). She describes three facets of mathematical language that work together to help the learner create meaning: written symbols, early concepts embedded in everyday language, and mathematical language, or register, in which common words take on new and specialized mathematical meanings. Pirie's research focuses on understanding what actually happens in classroom conversations so that meaningful learning occurs. She and her colleague, Thomas Kieren (1994) developed a model and theory of the growth in mathematical understanding through the talk they observe in both student-student and student-teacher conversations (see fig. 1 below from p. 167).

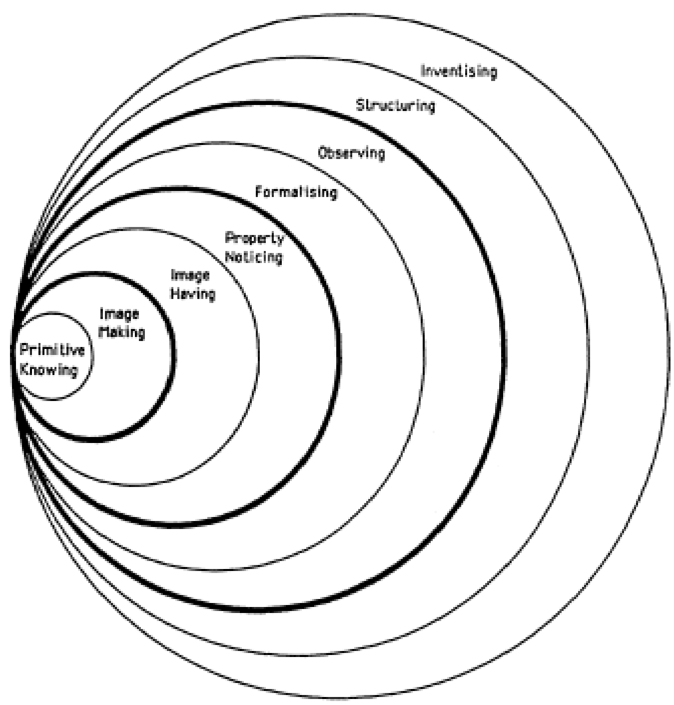

Fig. 1.

Pirie and Kieren (1994) state that their theory "… is a theory of the growth of mathematical understanding as a whole, dynamic, levelled but non-linear, transcendently recursive process" (p. 166). In other words, it is a departure from the idea that a child must have complete mastery of one idea (for example, addition) before they can be introduced to another (say, multiplication) that is prevalent in many mathematics textbooks written in the recent past. Mathematical understanding is not about mastering one skill or fact after another, but is rather a growth in understanding the connections and patterns *among and between* the concepts; returning to an idea, like addition, time and again, always making new connections and linking the idea to new, evolving understandings. The model has eight levels which are nonsequential or unidirectional through which learners move while their mathematical understandings grow and change. It is important for the reader to keep in mind that the ordered description that follows is only for the purpose of organization and therefore, does not imply that a learner's mathematical understanding proceeds from primitive knowing through inventising.

First is "primitive knowing" or the prior knowledge a learner brings to the task at hand. If a student is about to learn addition of fractions, the primitive knowing might be knowledge of fraction words and part-whole reasoning. Next is "image making" where the learner makes distinctions in his or her learning, followed by "image having" in which the learner can think about a topic without having to actually perform the activity that initiated it. To continue with the fraction example, image making would be working with fraction models such as fraction strips to create a way of adding say 1/2 and 1/4 for the first time. It is important to note that this is not a student simply making the image to represent what he or she already knows but that image making is actually making *new meaning* of a particular concept. Image having would be when a learner can do this addition without the need for the actual models. "Property noticing" follows, in which the learner draws on parts of images to identify specific qualities. For example, if a child is adding 1/2 and 3/4 and notices equivalent fractions, such as 1/2 and 2/4 in her models and uses this idea to add the fractions, the child is seen to be property noticing because the learner has noticed the area covered by both fractions is the same. "Formalising" is when the learner generalizes a common quality from earlier images: at this level, students are considered to be ready to generate algorithms as they make distinctions across many instances, not just one particular situation. This would be the case with a student who thinks about addition of unlike fractions by using number concepts and symbols of fractions and not having to rely on context specific models. "Observing" is when the learner expresses his or her understanding more formally as theorems, such as looking for patterns in a group of equations involving the addition of fractions to create a formula for predicting how many combinations add up to a given fraction. "Structuring" occurs when the learner tries to formulate a theory that is independent of models or even rules for adding fractions. A theory about the infinite number of factions between every whole number is an example of structuring because this understanding goes beyond what can be represented by models. And "inventising" is considered to be "…fully structured understanding" (p. 171) whereby the learner creates original questions and concepts such as seeing fractions as part of the set of rational numbers with the form a/b. "One might now *inventise* by asking: "What might numbers with the form of ordered quadruples a/b/c/d/ be like?" (p. 171).

Pirie and Kieren also highlight three features of their theory: "don't need boundaries", folding back and the complementarities of acting and expressing. Of these three, the one that they observed in student discussions is folding back. This is the idea that mathematical knowledge is not linear and subject to review (that is, a redundant act of repeating what has been done before) but rather is recursive in nature, and by which students revisit earlier learning to build thicker understandings. Said another way, folding back occurs when the learner returns to previous understanding(s) to revisit a concept in order to bring it forward and as a result extend and refine his/her thinking on the topic, to "…form a new way of looking" (Pirie & Kieren, 1994, p. 174) at the mathematical idea. Pirie and Kieren emphasize that it is only through externalization that a teacher might begin to understand what the child is constructing, and that one very effective way is through conversation. This they term "expressing" which, they caution, is not reflecting, which focuses on "…*how* previous understanding was constructed. Expressing, on the other hand, entails looking at and articulating *what* was involved in the actions" (1994, p. 175). They suggest one use of their model is for mapping how a child's mathematical understanding develops. Used in this way, the insights can provide a framework for planning and teaching mathematics lessons. For example, the authors describe a student who is learning about fractions. The teacher observes the learner to be at *image having* as he or she can add fractions by thinking about, but not manipulating, models. At this level, however, the teacher notices in their conversation that the student mistook some ideas about this addition. The teacher then provides her with a challenge: Given two or three fractional pieces, can the student find one that would fit over all of them? By intervening and providing the challenge, the teacher prompts the student to notice properties – leading to growth into the next level, *property noticing.*

Cobb, Boufi, McClain and Whitenack (1997) also examined the connection between classroom conversation and the development of mathematical understandings.

They focused specifically on "reflective discourse" which they define as being "…characterized by repeated shifts such that what the students and teacher do in action subsequently becomes an explicit object of discussion" (p. 258). Discourse, like conversations, involves the building of meaning between the conversants. Cobb et al. (1997) also coined the phrase "collective reflection" to refer to an action of the group becoming the object of discussion in the group. The authors focused for a year on a first-grade classroom, chosen because all of the 18 students had made significant gains in their mathematical understanding. They interviewed the students throughout the year, focusing on their "…evolving arithmetical conceptions and strategies" (p. 260) Cobb et al. (1997) assert that when children participate in group discussions it enables them to reflect on their recent mathematical explorations. "In other words, the children did not happen to spontaneously begin reflecting at the same moment. Instead, reflection was supported and enabled by participation in the discourse" (p. 264). Participation in the discourse (verbal discussion) can move mathematical learning along, they contend, but does not guarantee it. The individual child still must do his/her own reflecting and thinking about the mathematics while participating in the group conversations: "This implies that the discourse and the associated activity of collective reflection both support and are constituted by the constructive activities of individual children" (Cobb et al. 1997. p. 266).

The authors see the teacher's role, therefore, as a critical one: the teacher supports students' learning through guiding discussions after an activity to reflect on the mathematics just explored. Here, the questions that a teacher asks then become very important. The questions need to be open enough that the child reflects on his/her thinking about the concept in a deep way. Cobb et al. (1997) describe one conversation in which the teacher's well-chosen question ("Is there a way that we could be sure and know that we have gotten all the ways [that five monkeys could be in the two trees]?" helped to shift the conversation and lead to deeper reflection of the activity and the mathematics involved. "The role that the teacher's question played in this exchange was, in effect, that of an invitation, or an offer, to step back and reorganize what had been done thus far" (p. 269). This type of questioning takes skill on the teachers' part, to be able to read the group of students and observe when they reflect on the activity just completed: "The very real danger is, of course, that an intended occasion for reflective discourse will degenerate into a social guessing game in which students try to infer what the teacher wants them to say and do" (Cobb et al., 1997, p. 269).

Connected to this idea of learning mathematics through conversation is David Jardine's (2006) suggestion that there should also be a sense of "play" when teaching and learning mathematics, and this sense can be accessed through conversation. For too long, Jardine contends, routine school mathematics has been taught in a flat, lifeless way with an exaggerated focus on procedures and little time devoted to exploring, playing with ideas and concepts. He talks of play as the German word *spiel*, which "…is not a chaotic, unbounded space, but is full of character, full of characters. It is an open wisdom and open way in the world…It is precisely this sense of entering a world larger than ourselves that portends the freedom and ease experiences in playing" (p.59). The idea that we, through conversation, could follow the lead of the student in exploring an idea in mathematics opens it up to possibilities, treating mathematics as a living curriculum rather than a history-of-mathematics lesson. Jardine (2006) suggests we "…imagine the work of teaching as the work of exploring what it is that is so abundantly inviting regarding a particular curriculum topic and practicing the art of such invitation here, now, with these children" (p. 59). Through collaborative conversations with students, a teacher invites them to play with the mathematics that arises; encouraging the students to think like mathematicians - inventing, exploring, conjecturing, and experiencing.

Three Types of Teacher Questioning and Listening

Brent Davis (1997) researched teacher questioning and listening to examine a finding from an earlier (1990) study he conducted in which there seemed to be a relationship between the quality of student articulations and the teacher's mode of listening. He states that his focus on observing teachers listening to students resulted from his believing that past research has paid little attention to the importance of teacher listening when trying to teach students. Davis created three categories or types a teacher may use: evaluative listening, interpretive listening and hermeneutic listening. The first, evaluative listening, focuses on clear explanations, correct answers and is a listening that is "…a sort of telling" (Davis, 1997. p. 260). For example, when a teacher asks, "What is 28 and 36?" an evaluative listener expects to hear "64" and perhaps an explanation involving lining the numbers up vertically, adding the ones first to get 14, carrying the ten over, then adding the tens. Interpretive listening, or listening constructively, focuses more on checking for student interpretation of what has been taught. The same question (above) could be asked, but a teacher listening interpretively would want to hear other ways the students added 28 and 36 (for example, rounding 28 to 30, adding 30 and 36, then subtracting 2) and for an explanation of how they know their answer makes sense. Hermeneutic listening, or collaborative listening is when the teacher is a participant in the learning: "…concerned not merely with questions of knowing and doing, but with questions of personal and collective identity" (Davis, 1997, p. 262). Davis (1997) explains that "…hermeneutics is the art of interpretation. It is interested in meaning, in understanding, and in application. More particularly, hermeneutics is concerned with investigating the conditions that make certain understandings possible" (p. 18). Gadamer (2004) also uses this definition and adds that "…understanding must be conceived as a part of the event in which meaning occurs, the event in which the meaning of all statements – those of art and all other kinds of tradition – is formed and actualized" (p. 156). In other words, the conversation and its context are just as important as the words that are used.

Gallagher (1992) writes about hermeneutics in education and states that "[e]ducational experience is always hermeneutical experience. Put another way, learning always involves interpretation." (p. 39). A student is always interpreting what the teacher is saying and the student's interpretation may match what the teacher intended – or be a complete misunderstanding. Gallagher goes on to say that "[t]he interchange [between teacher and student] is an interchange of interpretations rather than an exchange of information" (p. 38). So teaching shifts from being a one-way, teaching-as-telling event to a two-way creation of meaning of the mathematics at hand. This is a significant shift, and one which brings hermeneutical conversation between teacher and student into the spotlight. Gadamer (2004) writes about the relationship that develops in conversation and how one should not be trying to convince the other or talk the other out of his or her position, but instead "…to place in the open" (p. 361) the subject at hand, exploring it together, to gain insight, both of the subject and of oneself.

Conversation and Conceptual Change

Teacher-student conversations that are of a hermeneutic nature offer a powerful way to enable learners to develop complex mathematical ideas. Merenluoto and Lehtinen (2004) are two researchers who explore the idea of conceptual change and its role in learning mathematics. "Conceptual change is used to characterize situations where learners' prior knowledge is incompatible with the new conceptualizations, and where learners are often disposed to make systematic errors or build misconception" (p. 519). Researchers and theorists (such as Dewey, Piaget, Vygotsky and von Glasersfeld) refer to the idea of "cognitive conflict" as a part of learning in their extensive writings. Researchers in cognitive change (Meremuoto & Lehtinen (2004) and Vosniadou (2003) make a distinction between qualities of learning processes. They talk of *continuous growth* in which the learning simply improves upon existing knowledge and *discontinuous change* which is more difficult and occurs when "…prior knowledge is incompatible with the new information and needs reorganization: where significant restructuring – not merely enrichment – of existing knowledge structures are needed" (Meremluoto & Lehtinen, 2004, p. 520). Vosniadou (2003) writes that research shows discontinuous change is more complex and old beliefs are only gradually replaced by new. She also explains that this change is resistant to teaching efforts that focus on explaining a new concept and requires that learners experience conflict with the concept and their current thinking around it. Learners who do not experience this conflict have an "illusion-of-understanding" that often develops a misconception. In this light, error-making takes on a new meaning - it transforms from a thing to be fixed into an insight into the child's mathematical growth. As Davis (1996) points out:

> Errors hint at false assumptions, over-generalizations, mistaken analogies, thus raising new questions and opening new possibilities. Far from being something to avoid at all costs, errors serve as important focal points of mathematical inquiry. They offer moments of interruptions, of bringing the unformulated (the enacted) to conscious awareness. Errors present for formulation things that we didn't know we knew – or for reformulation aspects of what we might have forgotten we knew. (pp. 248-9)

It is with this idea that I see a real possibility of merging Davis' hermeneutic listening on the teacher's part with the understanding of continuous and discontinuous growth in learning. I believe teachers can engage in conversation more often and more effectively with students to explore cognitive change and the growth of a child's conceptual mathematical understanding. Pirie and Kieren (1994) talk of teacher intervention, often in the form of conversing with students, which can aid in "property recording" – a solidifying of new understanding. If they do not record or articulate their findings in some way, they may not retain the new learning: "A lack of 'expressing' activity seems to inhibit the students from moving beyond their previous image" (p. 180).

Meremluoto's & Lehtinen's (2004) research focuses on the shift in learning mathematics from operations with natural numbers (1, 2, 3…) to ones with rational numbers (fractions and decimals). The big difference between these two types of numbers is that natural numbers are discrete (every number had a successor) while rational numbers are dense (between any two numbers are an infinity of numbers). Students come to school already confident in their understanding of natural numbers through their every day experience with counting. This is further solidified in the Primary years. Merenluoto & Lehtinen state: "Because of the early intuitive feeling for the discreteness of small cardinalities, and the abundant everyday experiences of the next object in the counting process, we claim that the change from the use [of] discrete natural numbers to the use of rational numbers requires a radical conceptual change for the learner" (p. 521).

Connections to Practice

During this project I worked in a third and fourth grade classroom, which happen to be the first two years (according to the provincial curriculum document) that students formally meet the ideas of rational numbers in the form of fractions and decimals. When teachers ask me for assistance in teaching mathematics, fractions and decimals and operations with fractions and decimals are among the most frequently cited topics that teachers of older students feel their students struggle to understand. Given this topic and my curiosity about the role of conversation in the mathematics classroom, I was curious how collaborative questioning and listening that involves first encounters with fractions and decimals might provide a way or insight into how young learners make sense of this shift from discrete to rational numbers. By listening and observing closely to the conversations that I have with the students, I wonder what conceptual changes might be revealed around their understanding of natural and rational numbers.

Chapter 3: Embarking on the Journey

The Setting

As I embarked on this journey to explore mathematical conversations, I sought out a teacher with whom I could not only visit in the classroom but with whom I could collaboratively teach as well. I envisioned this project as a partnership, rather than a clinical observation of the students and teacher. I work with many wonderful teachers in my district and opened up the invitation to work on this 10-week project with me. Mrs. Howard came forward and I knew right away this was going to be a memorable project. Mrs. Howard actively participated in several of the district numeracy workshops, bookclubs and meetings that I hosted aimed at helping teachers learn about the new (2007) mathematics curriculum and about teaching mathematics in a more sense-making manner. Recently she completed a workshop series I ran on multiplication and division, using the ideas from Fosnot's 2002 book, "Young Mathematicians at Work" and already used many ideas from it, sending me positive e-mails and talking excitedly about the insights and discoveries her class was making. Mrs. Howard was keen to talk about conversation in mathematics class and its role in deepening mathematics understanding, and was already employing some of the ideas around interpretive and collaborative conversations. I joined her grade 3/4 classroom here in the Sooke school district on January 26th, 2008, in their school of approximately 100 students and five teachers. Me included, now there were six!

Planning My Direction for Discovery

I was interested in exploring how mathematical conversations shaped a class of students' mathematical understandings occurring over several lessons. After reviewing different kinds of qualitative studies, I decided to do an instrumental case study. “In the social sciences and human services, the case has working parts; it is purposive; it often has a self. It is an integrated system” (Stake, 2003, p. 135). Rather than studying conversation apart from the workings of the classroom by, for example, pulling students aside to record their conversations, the case study is whole and takes into account the context of the conversations. I see a classroom as a community of learners and I view learning as an interaction between the learner and the environment. By studying one class deeply, I believed I would discover how that community creates mathematical meanings and understanding through their dialogs, both with each other and with the teacher.

A case study typically has four characteristics: it should be “…particularistic, descriptive, heuristic and inductive” (Merriam 1988 p. 11) As I planned out how I might join this classroom to experience close-up the conversations, discoveries and insights I hoped these students would share with me, I focused on conversation in a mathematics classroom, and I wrote the story of this class’s mathematics conversations and my interpretations of them. In this way, a descriptive narrative emerged that provides the reader an opportunity to experience my experiences of particular instances that happened in the flow of the 10 classes and hopefully, some of the richness of the conversations as they unfolded. It is rare that we, as teachers, have the luxury of revisiting a child’s thinking through conversation: so much happens in the thick of the moment for us. In this narrative mode of communicating my findings, I attempted to provide the reader with the opportunity to listen closely to the conversations as students make meaning out of the mathematics they are experiencing.

My hope was to provide insights into how conversation impacts students' learning of mathematics and how a teacher might occasion such conversations with young students. In this way I wanted the story of my quest to be heuristic in the same manner that a good story causes us to pause to reflect, and thereby compels us to think differently after reading it. Finally, this project was inductive because, although I began with an idea of what I might discover, I was also open to and welcomed the idea that this journey would involve moments of surprise for me and as well unexpected insights. This indeed was the case as I had not expected, for example, to see the power of play in the development of mathematical ideas when Anna suggested fractioning liquids (as the reader will hear) and took the class and me on an unexpected exploration of fractions in a unique and enriching manner.

How can one case study inform teachers' understandings of the role of conversation in mathematical learning? This is a question I asked myself as I imagined the contribution that my project might make in other teachers' learning. However, by writing a thick description of what I saw, heard and experienced in this classroom I was able to take the readers with me through this experience. Of course, as with all observation and description, these are my impressions, my interpretations of the experience and data I have collected, but done carefully "…[t]he reader comes to know some things told, as if he or she had experienced it. Enduring meanings come from encounter, and are modified and reinforced by repeated encounter" (Stake, 2003, p. 145). These stories from the field add vicariously to our collections of experiences, to our collective memory, and hopefully enrich understanding of the issue or idea under study.

To best capture the essence of what I experienced in this classroom, I chose to use direct observation (video-taped then partially transcribed), interviews and field notes as suggested by Tobin (2009) and Creswell (2007). From the beginning of my time with these students, I recognized that it was impossible for me to be a neutral observer. In one of the first sessions, the moment I took my camera to zoom in on the multiplication chart two boys, Jeff and Ron, were discussing, they brought me into the conversation by sharing their discoveries with me. Instead of being a detached observer I became part of this the classroom community by getting involved in the activities, asking questions and showing interest in what the students said and did, and as a result, was welcomed into many conversations with the students and the teacher. So, in fact, it was the students, their teacher and I who created mathematical meanings in our 10 weeks together. "…[R]esearchers, by their very presence, influence the research site in some ways, and to varying extents" (Tobin, 2009 p. 3). Often I returned home from the classroom and wrote in a journal reflecting on the experience and my impressions to preserve the context of the conversations and to record the impact the project was having on my own learning.

I relied heavily on the experiences of Susan Pirie who also researched discussions in mathematics classrooms using video recordings and writes of her research methods (Pirie 1996, 1997). In particular, Pirie's comment that videotaping, at first glance, may seem to be an efficient way to gather data for examination later, freeing us from scribbling masses of on-the-spot observational notes, but is in fact much more had great impact on me:

> Video-recording has been claimed as a way to capture everything that is taking place in the classroom, thus allowing us to postpone that moment of focusing, of decision taking. Yet this is misleading; who we are, where we place the cameras, even the type of microphone that we use governs which data we will gather and

> which we will lose. What video-tapes *can* do is give us the facility through which to re-visit the aspect of the classroom that we have recorded. (p.1, 1996)

In some very real sense, the video data *is* the data – not just a recording of it. How we use the camera (on a tripod or hand-held), what we choose to film and when (whole group or close up), as anonymous videographer or *camera vérité* (i.e., the camera as my eyes while I am in conversation with someone) even when we choose to turn it on and off – all these impact the outcome of the research and thus must be at the forefront of any analysis of videotaped data. Pirie also comments on the range of data videotape offers: unlike audiotape which give us only the words people say, or transcripts, which lose even more in terms of ways words are said, video gives us the nuances of actions, glances and body positions in conjuction with the inflection of the speaker's voice; all of which inform our analysis. However, with this enhanced data comes the difficulty of the sheer volume of things to analyze! Pirie (1996) suggests in some cases, transcripts may be easier to work with at times, "…but there will always be a loss of data and the researcher must *consciously* address the relevance of this loss" (p. 4). She adds that if the data is analyzed directly from video, the researcher must specify how that was done. Another benefit Pirie finds in using videotape is how a researcher can return to the same clip again and again, each time with a different purpose, resulting in thick description of a conversation. I was able to revisit clips several times, listening first to the types of questions Mrs. Howard asked, then to what the students said, then shifting to listening to how the concepts were developing among all the conversants, creating a rich impression of each conversation.

My Learning Along the Way

As I strove to make sense of my experiences in this classroom, my analysis in effect began as soon as I started videotaping the students. The very act of watching students through a lens made me focus more intently on their facial expressions, hand gestures, body position and tone of voice. I saw the students and the situations in which they worked through a kind of a tunnel vision, where only the immediate ideas and concepts of the conversation were in focus and other events in the classroom, other conversations and interactions were excluded. This focus of watching the students and the teacher through a lens also impacted what I videotaped next. For example, during one lesson I saw that more interactions between the teacher and students happened while the teacher squatted down by a child’s desk, so I quickly changed the way I filmed, moving from using a tripod to film the entire class to taking the hand-held and filming much more intimately to capture the nuances of each conversation – a child’s pencil sketches of a 6 by 6 array, a knowing nod as a misunderstanding is made clear, or the comment, “Now I get it!” were all made accessible by my moving in much closer with the camera.

> Data collection and analysis is a *simultaneous* activity in qualitative research. Analysis begins with the first interview, the first observation, the first document read. Emerging insights, hunches, and tentative hypotheses direct the next phase of data collection, which in turn leads to refinement or reformulation of one’s questions and so on. It is an interactive process throughout which the investigator is concerned with producing believable and trustworthy findings. (Merriam, 1988, pp. 123-4)

Just as Merriam points out, even after I gained insights into mathematical conversations during the filming, I discovered there was still a lot to do once I videotaped a lesson. Each day when I returned home, I watched that day’s filming and selected one or two episodes to transcribe. Transcribing individual episodes allowed me to take a closer look at the dialog, the dynamics of the partners or small group, the interactions between teacher and student as well as the understandings that I noticed emerging. In this way I was able to slow down real time, or more accurately, expand the temporal space in which the students' conversations took place. There were two kinds of contexts that I saw as relevant to my study of mathematical conversations and which I transcribed for further examination. Some episodes featured the interplay between the class as whole and Mrs. Howard in conversation while other episodes involved two students working together and Mrs. Howard or myself engaging them in a conversation about their strategies. When my visits were complete, I sat and watched the entire collection of videos sequentially from beginning to end and following Merriam's (1988) suggestion, I wrote notes, comments and questions as I watched.

As I studied the students' conversations with the teacher, I was also conscious of the fact that, although I framed my initial interpretations within Brent Davis' three categories of questions, I anticipated that other categories could, and most probably would, emerge. During the first viewing, I looked for examples of conversations that focused on "one right answer" (evaluative conversations), ones that focused on how a student got an answer (interpretive conversations) and ones in which the conversation centered on collaboration to understand a task at hand, in short, emergent ideas (hermeneutic conversations). I also examined the kinds of contexts in which the focused conversations occurred; such as, whether the conversations were initiated by a student, their teacher, by myself or arose spontaneously from some event that happened during the lesson.

In addition to these parts of my analysis, I also looked at the ways in which conversations appeared to lead to new or deeper mathematical understandings for the students and teacher involved. This required some speculation and theorizing, as one cannot be sure when and how learning has taken place. However, as my goal was to add to my understanding of how conversation might occasion the growth of mathematical conceptual understanding, I tried to speculate on when that was happening. Speculation is "…fraught with ambiguity…however, [it] is the key to developing theory in a qualitative study." (Merriam, 1988 p. 141). It was actually when I decided to begin the next phase by writing the first draft of a conversation that themes began to jump out at me. I started by writing about the first lesson I filmed–Mrs. Howard and the students discussing multiplication strategies–and I suddenly realized this itself was a theme I saw several times throughout the 10 weeks. Often conversations centered on students sharing different strategies for solving a number problem. Then, on further reflection, I realized not all the episodes fit this pattern. There were conversations about misconceptions, about discoveries, about playful mathematics ideas and about student insights into their own thinking.

So then, I wondered, how best to write about these themes so that I might share my discoveries with other teachers? Some who write about qualitative research advocate for the case "telling its own story" but a clinical reporting of the events of these 10 weeks would not reflect my interpretive analysis of these events. "We cannot be sure that a case, telling its own story, will tell all or tell well – but the ethos of *interpretive* study, seeking out emic meanings held by the people within the case, is strong" (Stake, 2003, p. 143-4). The subjective choices I made in presenting the discoveries made on my journey are in the same vein as the ones I made in choosing what and how to study.

In the following chapter, I explain the organization of my study as well as what I saw and heard during these conversations and the impact I observed the conversations to have on students' mathematical understandings. I also describe choices I made in how I presented and interpreted the conversations and evidenced student learning during them. It was also important that I connect my findings to theories of learning and the role of conversation in learning mathematics, so I pushed my observations against the ideas of Davis, Pirie & Kieren, Gordon-Calvert and Cobb, among others, looking for similarities, confirmations, insights and discrepancies that ultimately led to my interpretation of the role of conversation in *this* mathematics community.

Chapter 4: Returning Home

In the 10 weeks of Mondays that I was part of Mrs. Howard's third and fourth grade mathematics class, I came to recognize a definite rhythm, or flow, to the mathematics lessons that was orchestrated both by Mrs. Howard and the children. When I first arrived, the class was engaged in exploring the operations of multiplication and division. Later lessons moved on to fractions. Mrs. Howard treated the class as a whole, blurring the distinction between the two grades but still aware of each grade's curricular expectations and, more importantly, the needs of the individual children. Classes flowed from whole group exploration and discussion through group work, sometimes in their grade groups, at the meeting place (an area by the windows where they could sit in a circle, sharing ideas, while Mrs. Howard recorded findings on a chart or small blackboard). Students also worked in smaller groups of four and sometimes in pairs. In the few instances that Mrs. Howard asked the children to work alone, she suggested that students chat quietly with each other if they were confused or wanted to check their thinking. Collaboration was a foundational structure in this mathematics class, one that seemed very natural to all members.

I found this rhythm to exist in every lesson as I explored the video data as not unlike that of a fractal, such as a fern; scanning the whole plant, zooming in to view a cluster of fronds, an individual one, or even a part of a frond and then pulling back to see it in relation to the whole plant. In essence, my interpretation of these 10 lessons was a conversation with the data. This image of my analysis follows Aoki's (2005) depiction of genuine conversation in which the whole is best understood in the context of the parts, and the parts in context of the whole, and what is created in the conversation is qualitatively different than either the parts or the whole. When I first viewed the video clips, I watched them sequentially in one sitting while making notes on each episode in order to get a feeling for the whole story, the flow of the narrative. Next I focused in on a few conversations, re-reading the transcripts I made as I watched and listened, stopping and replaying the episode so that I could listen again and more carefully to shifts in students' meanings that revealed their conceptual understanding. Then when I looked at the episodes within the context of the entire 10 weeks, I began to see the emergence of particular themes.

Sharing the Adventure

During the first viewing of the video data, what struck me was how much Mrs. Howard asked more questions than she told facts, and how the classroom discussion was as much student voices as teacher voice. When I visit schools and walk down the halls and hear the talk of a mathematics lesson, I always listen for whose voice is predominant– the teacher's or the students'? And is the voice a questioning one or a telling one? And who is doing the questioning? The telling? In some classrooms, the style is very much one of teacher telling mathematical procedures and concepts and questioning to check that the ideas were heard. And the students' voices are confined to short answers, often repeating what they have just heard and questions back to the teacher are ones checking on procedures. This was not the impression I got in Mrs. Howard's classroom. The students' talk seemed to be routinely focused on understanding some idea, testing out ideas and creating meaning as a group. Students seemed quite comfortable with Mrs. Howard or me asking them questions like: "What's happening in your mind when you do this?" "Who did it a different way?" "Does that always work?"

"What patterns do you think you might find?" Also, students asked questions of the group themselves and Mrs. Howard followed the lead, whether exploring where 1/3 (a new fraction) was in comparison to the other fraction strips, or discussing a child's question about how, if we can talk about fractions of food, one could find the fraction of *liquid.*

Three Types of Questioning and Listening in Action

As I watched the videotape the first time through, I used Davis' three types of questioning and listening, noting whether a question and the listening by a teacher (Mrs. Howard or myself) was evaluative (asking for one correct answer), interpretive (asking for the thinking behind a strategy and listening constructively) or hermeneutic (an invitation to interact in exploring an idea; participatory). Davis (1997), in his article "Listening for Differences", collaborated with a new teacher over 2 years. He often observed her teaching, and noticed her questioning and listening evolving from mainly evaluative, through interpretive and then into hermeneutic as she learned more about the differences and the impact on student learning. As I watched the tapes from beginning to end, making notes on the type of questions and jotting down my first impressions on any impact I saw in the students' learning through their responses, I noticed that, rather than the questioning evolving from evaluative to hermeneutic over the 10 weeks, there was an ebb and flow between the three types, depending on several circumstances.

Mrs. Howard's questions were different if the lesson was taught to review and deepen a concept or to introduce it for the very first time; different again when she was talking to the whole group, a pair, or an individual student. Mrs. Howard also varied her questions based on whether the student(s) had a firm grasp on the mathematical idea or were struggling to make sense of it.

Overall, Mrs. Howard's type of questioning was more interpretive when the concept was explored for some time, as in multiplication and division. The children's responses entailed explanations of their various strategies for solving multiplication questions, adding on to each other's thinking, questioning and commenting on each other's strategies as well as creating new strategies on the spot. For example, Mrs. Howard asked the students to solve 8x6 using a strategy after solving 2x6, 4x6, 3x6, 6x6 as part of a series of "strings". One child stated: "I pretended the 6 was a 5 and I counted 8 fives to 40 then I added an 8 because those were the left-over ones."

The questions were more evaluative but also hermeneutic when a concept was new (as in the introduction of fractions). For example, Mrs. Howard asked, "In a fraction like 3/4, what do those numbers mean?" but also, in the next breath, she would ask, "What *is* a fraction? Discuss this with a partner." And then the whole group would collaboratively define, for themselves, what the working definition would be, adding to it as the days went on. She seemed to check both for surface knowledge of the symbolic meaning of a fraction but also digging deeper for conceptual knowledge of what a fraction means to the learners. The two episodes that contained the most hermeneutic questions were ones in which Mrs. Howard helped a small group make sense of the concept of odd and even and where students first explored fractions using paper strips. Mrs. Howard asked questions like: "John guessed I was thinking of thirds, but we haven't cut thirds yet…where would they be [on the number line]? Why? What does 1/3 mean? Who figured it out a different way?" And then she would listen intently, as would the rest of the children, in a manner that seemed part of the workings of this class. I believe these differences between Davis' research and mine occurred because of the experience of the teachers involved in the studies. Davis was working with a beginning teacher, who was learning how to explore and become aware of her listening and questioning styles. Mrs. Howard, on the other hand, is an experienced teacher who has taught for more than 10 years, actively working on her questioning and listening for the past 2-3 years.

The Runes of Wisdom

Going deeper into the use of questioning and listening by examining the video data, notes, observations and questions that I gathered on the clips, five themes emerged from the teacher-student collaborative conversations. The themes are: exploring student strategies, exploring "playfulness" with ideas in mathematics, exploring student misconceptions and/or misunderstandings, exploring student discoveries, and exploring student insights. These themes focused on the students' exploration of five aspects related to their conceptualization of mathematics that led to growth in student conceptual understanding. To me, I observed them as "pivot points" in the students' mathematical learning; that is, aspects that left unexplored, minimized or ignored by the teacher might deny the opportunities in which the mathematical understandings of the learner could be "thickened" (Pirie & Kieren, 1994) through collaborative conversation. In these cases, the conversations followed Gordon Calvert's (2001) definition of a genuine conversation in that they evolved out of the ideas and issues of the moment: the teacher involved in each of these conversations neither leads nor follows the student's thinking, but rather negotiates meaning with the child: "Meanings are constantly being shaped and reshaped while the topic is molded and transformed within the course of the conversation" (p. 48).

For the remainder of the chapter, I will expand on these themes, provide examples from my experiences on my journey and relate these to the research on learning and conversation.

Conversations Exploring Student Strategies

> 'I wonder if you've got such a thing as a balloon about you?'
> 'A Balloon?'
> 'Yes, I just said to myself coming along: "I wonder if Christopher Robin has such a thing as a balloon about him?" I just said it to myself, thinking of balloons, and wondering.'
> 'What do you want a balloon for?' you said.
> Winnie-the-pooh looked round to see that nobody was listening, put his paw to his mouth, and said in a deep whisper: *'Honey!'*
> 'But you don't get honey with balloons!'
> '*I* do,' said Pooh. (Milne, 1926, p.12)

The first theme I found in the video data was conversations that centered on students sharing or describing strategies they used to solve mathematical problems. When a teacher engages in conversations about students' strategies, several things might occur. Students may put words to their thinking, validating and strengthening their understanding of the concept discussed. In addition, errors in thinking may be exposed, opening up an opportunity for the teacher to probe more deeply and bring to light a misunderstanding, allowing the students to revise their ideas.

Multiplication strings and the community of learners.

My first few sessions with Mrs. Howard and her students involved the class' learning of multiplication and division concepts. Mrs. Howard began the first few lessons with "strings" – a series of connected multiplication or division questions constructed to provoke the use of strategies such as halving and doubling, "friendly numbers" such as 5 and 10, and the commutative principle (flipping the equation). Mrs. Howard followed the advice from "Young Mathematicians at Work" by Fosnot (2002) and had pre-cut arrays on hand so she could model a child's strategy by cutting or marking the array. The students listened as they took turns describing their strategies for solving 6x2, 6x4, 6x3 while Mrs. Howard folded the arrays to match what they described. For example, she hung a 6 by 4 array on the board. Then, as Ron explained he got 6x3 by just taking off a column of 6, Mrs. Howard cut that column off to show his thinking. The seeming ease with how the children shifted from listening, explaining and paraphrasing each other's strategies appeared effortless on Mrs. Howard's part, but this belies the work she did in the beginning of the year to create a community of learners who learned through conversation. When I commented on this during an interview with Mrs. Howard, she explained to me how she worked to create this in the classroom. She told me that at the beginning of the year, the students were not at all used to working on problems with a partner, talking about their thinking or having conversations about mathematics. She worked hard at teaching them *how* to share their thinking with someone else and how to really listen to what a person says, so you can explain someone else's thinking. Mrs. Howard stated: "I've done way more sharing, you know, having them share, than in previous years. I think – yeah, your right – it's the whole community thing. I really try to be encouraging and draw attention to learning from each other. It's more that a 'technique'."

This awareness of developing the community I believe is one part of Mrs. Howard's success with collaborative conversation. She works hard to ensure students understand and feel that they are part of a group of mathematicians who share ideas, listen to other's strategies and work together to create meaning. The fact that she said it is more than a technique told me she acknowledges this is a way of being, a way of teaching and learning, not a "quick fix" or entertaining game to get students' attention. Conversation and collaboration are part of the fabric of this class, and it is not just a device for keeping students "on task" but the way students share and create mathematical understandings.

Multiplicity and the outfit problem.

The day the students were working on a problem in which they had to figure out how many outfits could be made from three shirts and two pairs of pants, I wandered around the classroom observing and filming. I saw some children draw it out as they talked it through with their partner while others quickly noticed the multiplicity of the question and used the equation 2x3=6. I was curious to know how deeply these children understood why the equation worked, so I stopped and filmed a conversation between myself and Sara:

> Me: How does 3x2 tell you the answer? What made you think of that?"
> Sara: If we do 2 pairs of pants and there's 3 shirts and you match them with the other…with the pairs of pants.
> Me: How did you know that multiplication would give you the answer?
> Sara: I thought of groups.
> Me: How can you be sure you got the right answer? Is there a way to check?
> Sara: Because…if you have 3 shirts and 2 pairs of pants, you can make six outfits.
> Me: How can you prove it?
> Sara: You can do 1 shirt with 2 pairs of pants, another shirt with the 2 pairs of pants, and the other shirt with 2 pairs of pants.

When I pressed a little further, asking Sara to try a question with four shirts and three pairs of pants, her explanation became more refined and succinct: " 4x3=12, because if there's 4 shirts and 3 pants…every [one of the] 4 shirts would have 3 outfits." Some children are told, and subsequently memorize, that one uses multiplication to solve combination problems, and this shallow understanding is revealed when a teacher has a conversation such as this one with a child. Sara appeared to understand the connection between multiplication and combination problems in a deeper way. Pirie and Kieren see this kind of teacher-student conversation as important in exposing the levels of learning a child is moving through. In the context of Pirie and Kieren's (1994) theory of the growth of mathematical understanding, Sara appeared to have moved through "image having" in which she can talk about the clothing combinations without having to sketch them out (as some of her classmates were doing) and "property noticing" where she saw the multiplicity of other combination problems and was able to see the common qualities of the combination problems that led her to confidently use multiplication algorithms to solve them. Our conversation helped me to expose her understanding of multiplication as deep and confident.

Connecting division and multiplication.

Conversation between a teacher and a student can also help the child reflect more deeply on the activity they have just completed and the learning involved, as Cobb, Boufi, McClain and Whitenack (1997) explain. The questions teachers ask need to be carefully thought out so that real reflection is occurring and it does not dissolve into a "guessing game" where the child simply tries to say what they think the teacher wants to hear. In another episode, the children recorded the division sentences that go with a group of multiplication sentences and looked for patterns. I talked with Tom who explained that he found out how to make a division question from a multiplication one.

"You just write it backwards," he said.

But when he wrote "8x4=32" and "32 ÷ 4=8" he was unsure that this division sentence was really correct. I asked him if there was a way to prove it using an array. He drew an 8 by 4 array, and I got him to show me the parts of the multiplication sentence. I asked him if there is a connection between the array and his multiplication/division "flipping" or "writing it backwards" idea.

"Can you show me the division on that array?" I questioned.

I zoomed the camera in on his array as he pointed to the 32 squares, and found the "divided by 4" in the four rows.

"So what's 32 divided into 4 rows?"

"8!" he said confidently.

He then finished with, "I just found out today you *can* turn a multiplication question into a division question" and carried on with his work. In this case, our conversation took Tom from a superficial strategy – turning a multiplication equation into a division equation by reversing the order of the numbers – to a deeper understanding of the concept of multiplication and division and how closely they were connected at a concrete level. Tom moved from just a procedure for turning a multiplication equation into a division equation to understanding the meaning behind the equations. Wells (2009) would say Tom was "knowledge building" in the collaborative conversation between him and me. It is in this face-to-face discourse that the shared understanding emerges with both of us contributing to its creation.

Conversations Exploring Playfulness

> 'It's like this,' he said. 'When you go after honey with a balloon, the great thing is not to let the bees know you're coming. Now, if you have a green balloon, they might think you were only part of the tree, and not notice you, and if you have a blue balloon, they might think you were only part of the sky, and not notice you, and the question is: Which is most likely?'
> 'Wouldn't they notice *you* underneath the balloon?'
> 'They might or they might not,' said Winnie-the-Pooh. 'You can never tell with bees.' He thought for a moment and said: 'I shall try to look like a small black cloud. That will deceive them.'

> 'Then you had better have the blue balloon,' you said; and so it was decided. (Milne, 1926, p. 13)

Of all the elementary school subjects, mathematics has the unfortunate reputation for being difficult, boring and dry – for both students *and* teachers. Mathematics does not have to be taught in this way, and many teachers strive today to make it more alive, interesting and active. One way is to employ the idea of "playfulness" in mathematics lessons and this can be done through conversation. Jardine (2006) talks of play by referring to the German word *spiel* which does not mean unbridled, chaotic ramblings but rather explorative, curious exploring of a topic or idea. This playfulness also includes an openness and responsiveness on the part of the teacher to follow the curiosity of a student about a mathematical concept in an honouring way by accepting and regarding the student's inquiry as valid and worthy of consideration and exploration. Students are also engaged in trying to make sense of an idea in this playing of a concept and collaborate to do so.

Fractions of liquids.

The first instance of playfulness in conversation occurred during a discussion about fractions and the various ways fractions can be made – fractions of area, such as pieces of cake and fractions of a set, such as parts of a collection of candies. As the students gathered at the meeting place, Anna asked Mrs. Howard if you could have a fraction of liquid, and if so, how? So Mrs. Howard asked Anna to ask the class:

> Anna: I was thinking about, like, about the equal parts…if you were using food, what about drinks?
> *(The class immediately begins discussing this idea with each other)*
> Mrs. H: What do you think of Anna's question? Can you split liquid into equal parts like that?
> John: Yes you can – you would pour it into different cups – *equal* cups.
> Anna: What about…can you do it with *one* cup and inside the cup without using other cups…it's got to be *inside* the cup.

> Allan: Make it ice!
> Rachel: I was thinking…with a Sharpie, draw lines all around the cup.
> Mrs. H: Have you ever seen a measuring cup?
> Class: no…yeah…
> Rachel: …or you could put water half way in a measuring cup, then put plastic, then put more all the way up.
> John: You can't *slice* water, but you can pour it into equal cups…and even if you don't have the same [sized] cups, like a bigger cup and a smaller cup, just pour in the same amount - with a ruler.

Anna is a child who struggles with new concepts, taking longer than her peers to use new ideas independently, use procedures fluently and solve problems without help. But with the culture of collaboration and inquiry in this classroom she seemed comfortable asking questions and exploring ideas. Students are encouraged to ask questions, ponder possibilities and make conjectures and all are met with equal consideration from Mrs. Howard. Anna's question set out a discussion of measuring liquid, recording parts using a scale and the importance of equality when discussing fractions. The engagement and excitement among the children was evident. All were engaged in this play of ideas and the mathematics took on a living quality for them as well.

A new fraction strip.

This idea of questioning to explore mathematics in a playful way occurred again a week later, also in the context of fractions. Mrs. Howard had her class make "fraction strips" following a Marilyn Burn's unit. The students made halves, quarters, eighths and sixteenths and she was playing a game with the class called "Guess My Number" – the fraction version of a whole number game they played in the past, where they guessed Mrs. Howard's number between one and 100. Today, she explained, it was a fraction between zero and one. Students manipulated their strips to try and guess what her fraction was. She then told the class whether the guess was too low or too high and put the fraction on the number line. At one point, 1/4 was guessed – too low - and 1/2 was also too small. Ted guessed 1/3 – a fraction they had not made with their kit yet.

"We haven't cut thirds yet…where would that be [on the number line]? Talk to the person beside you…how would we figure it out?" Mrs. Howard said.

As they did, I moved in with my camera to focus on one group as they discussed it. Two boys sat beside each other. Using the quarter piece and the half piece, we talked about where 1/3 would be.

"What does 1/3 mean? Can you show me on the blue strip [the whole]?" I asked.

The boys mapped out thirds using their hands, and one boy states, "Between the two and the four! [1/2 and 1/4]".

Mrs. Howard called the class back to the front board and asked for ideas. Mark stated what the boys I filmed said, that 1/3 would fall between 1/4 and 1/2. When Mrs. Howard asked him to explain his thinking, he went up to the number line and traced a line with his finger from 1 back down towards zero.

"The numbers [denominators] get bigger as you get closer to zero, so 1/3 is here (pointing between 1/4 and 1/2)" he said.

"Did anyone do it differently?" Mrs. Howard asked.

"I took the 1/4 piece and put 1/16 beside it to make 1/3," Alec said.

When Mrs. Howard asked him to explain his thinking, he said, "It's like Mark says, it's between 1/4 and 1/2. (Interestingly, 1/4 +1/16 = 5/16, which in fact is very close to 1/3.)

Finally Ted, who initially guessed 1/3 told Mrs. Howard he *meant* to say 3/4. Mrs. Howard laughed, "But look at the interesting conversation we just had!"

Mrs. Howard recognized that, although the question led them to a concept she had not expected to teach yet (and it was actually a guess in error!) there was richness to be had going there. The act of exploring a fraction for which they had no strip could lead to a discussion that further deepened the students' beginning understanding of what a fraction is and how one fraction relates to another. In this way, I see the playfulness of exploring a fraction for which the students had no concrete representation was apparent in the *spiel* sense of the word – every child was engaged in trying to make sense of this new fraction, collaborating to bring meaning to these new numbers. The idea of taking a right turn in a mathematics lesson is disconcerting to some teachers, yet researchers such as Gordon Calvert (2001) would see this as a genuine conversation – not a random diversion, but rather following relevant concerns or ideas that come up at that moment. "Meanings are constantly being shaped and reshaped while the topic is molded and transformed within the course of the conversation" (p. 48). The richness of the conversation and the ease with which Mrs. Howard allowed the discussion to shift created a sense of mathematizing; of students involved in exploring and creating meaning in the mathematics.

Conversations Exploring Student Misconceptions and/or Misunderstandings

> 'Isn't that fine?' shouted Winnie-the-Pooh down to you. 'What do I look like?'
> 'You look like a Bear holding on to a balloon,' you said.
> 'Not,' said Pooh anxiously, '- not like a small black cloud in the sky?'
> 'Not very much'
> 'Ah, well, perhaps from up here it looks different. And, as I say, you never can tell with bees.' (Milne, 1926, p.14)

Brent Davis (2008) talks of knowing as having to do with viability, or the "fitness" of an idea in terms of how it fits in with a person's understanding. He explains this idea in the context of error-making. Davis states that a learner does not perceive they have made an error, nor have a misconception, until they see the mismatch between their understandings and the situation at hand. Simply telling a child their thinking is incorrect is not enough for cognitive change to take place. It is better addressed through collaborative conversations, in which probing questions help the child to see the mismatch for themselves.

Six times seven from six times six.

During the multiplication strategies lesson, Mrs. Howard spoke with Evan who was completing some multiplication questions using various strategies. She noticed he has written 6x7= 4**3** (not 4**2**) and asked him how he found the answer (not mentioning the answer was incorrect). Evan explained he thought of 6x6=36, then added another 7 on to get 43. At this point, Mrs. Howard could have simply stated he should have added on another **6** to 36, but instead she asked him to draw an array in his notebook (which was made up of grid paper) of 6x7 and find the 6x6 array within it. Evan began drawing the 6x7 array, but was confused about how to find the 6x6 in it. Sensing this was not helping, she instead suggested he draw a 6x6 array, which he did.

> Mrs. H: So there's a 6x6 and you know there's 36 in there. So how can you make 6x7 out of this one?
> Evan: Oh, I get it. (John draws another column of 6)
> Mrs. H: What happened?
> Evan: I drew another 6 onto it.
> Mrs. H: Another *six*…so how much is that?
> Evan: forty-*two*.

Mrs. Howard used a model the students were familiar with from her daily strings to talk about his strategy. And Evan was able to self-correct without Mrs. Howard actually *telling* him anything. In our first interview, Mrs. Howard explained how she had, for the first time, talked to her students about how feeling confused in mathematics is natural and part of the process of coming to understand an idea or concept:

> I never thought to tell my kids that before. So for a lot of them, they connect confusion with, 'I'm dumb, I don't get it' especially kids that tend to get things easily. So if they start feeling confused about something, their self-esteem starts to go and then they think they're embarrassed and they don't want to share that confusion so I've really tried to- to make that clear that, if you're feeling that confusion that's a good thing. That means your brain is trying to work something out, and that's exactly how you learn in math.

Listening and working with Mrs. Howard, I observed a genuine respect that she enacted in her conversations with students, especially when they found themselves perplexed about the mathematics at hand. She helped them to reflect on their understandings through conversations while preserving their dignity as learners, as she did with Evan; walking with him through the model and in doing so, occasioning the emergence of his new understanding.

What is an odd three-digit number?

During a group problem solving lesson, Mrs. Howard assigned small groups to work as teams to solve some word problems. The first one involved taking three digits and forming as many 3-digit odd numbers as possible. As Mrs. Howard traveled around the classroom checking that each group understood the task, she stopped at a group of three girls and began talking to them about "even and odd". I moved in with my camera as I sensed there was some confusion on Tina, Raven and Paula's part that was preventing them from progressing on the task. It seemed they had an idea of what a single-digit odd number was: "If there's two people (draws two circles) and we have five candies and we give one to Tina, me (draws sticks on circles, with one left over) 4 and one's left over." But when Mrs. Howard asked them to use that idea with a 3-digit number, she quickly realized they did not know what that was. So she proceeded with a 2-digit example:

> Mrs. H: Can you tell me a 2-digit number?
> Raven: 23.
> Mrs. H: Why is it a 2-digit number?
> Raven: Because it has a 2 and a 3.
> Mrs. H: So it's going to be 3 digits – have 3 different numbers. And it's going to be a number that can't be split between 2 people.

Then Mrs. Howard had the girls come up with a 3-digit number, and they wrote down "341". She asks, "Is it odd?" and the girls shrug.

At this point, a teacher might be tempted to simply tell Tina, Paula and Raven that it in fact *is* odd, because it ends in a one, and get on with the task. But these girls most likely *have* been told in the past exactly that, and still they have not made sense of it. If we do not face a new concept ourselves and test out its *viability* (to use a term from enactivism) then the best we can do is memorize this isolated bit of information. Without understanding how an idea fits in with other ideas, without a sense of it's connectedness to the rest of mathematics, it remains isolated and often unusable. Instead, Mrs. Howard proceeded with some probing questions to help them see for themselves what it is that makes a 3-digit number is odd or not– and she stayed with their initial idea: an odd number is one that can not be shared equally between two people.

> Mrs. H: Which number is really important when you're checking to see if it's odd or even? Which space? Which place value here?
> Paula: the four…[of 341]
> Mrs. H: The four's really important? Tell me why. What is it? In 341, how much is the four worth?
> Paula: 40.
> Mrs. H: Can you split 40 between two people?
> Paula: 40, um…wait – if there's four people, you could share 10, 10, 10, 10.
> Mrs. H: So this 40 is fine, but you can't really tell if the number is odd or even, because there's more than the 40. Three hundred, forty…

Paula:…one! You can't split the one.
Mrs. H: You can't split the 1. But can you split the 300?
Raven: Yes.
Mrs. H: Is the 300 odd?

The girls were not sure, so Mrs. Howard asked them to try 30, and Paula drew two circles and found they each get 15.

Mrs. H: Is there any way you can use the same idea to split 300? Is 300 odd or even?
Raven: I know half of 300, 150…
Mrs. H: OK, tell me what you're thinking.
Raven: Just like with the 30, she gets 100, I get 100, she gets 50, and I get 50.
Mrs. H: So you get a total of 150?
Paula: (drawing on her paper all this time) You'd get 150.
Mrs. H: So now you're telling me you can have an odd number in the hundreds place or the tens place and it would come out even as long as you have a zero in the ones place. So this number here (pointing to the 1 in 341) is the one that matters, because you can't split that one. (girls nod).
Mrs. H: Can you guys see if you can come up with some odd [3-digit] numbers now?

Mrs. Howard helped the girls develop understanding for knowing whether a number is even or odd by tapping into what fits for them in terms of what they already understand. Gallagher (1992) writes that "…learning always involves interpretation" (p. 39) and that this occurs in the interplay between teacher and student through conversation. But most importantly, this interchange "…is an interchange of interpretation rather than an exchange of information" (p.38). Mrs. Howard reflected this in her conversations with children; she aimed to interpret what they said while they built understanding more from the questions she asked than from the information she gave.

Using Pirie and Kieren's (1994) model Raven, Tina and Paula folded back on prior understanding of what "odd" meant to them in single digit numbers and brought it forward to grapple with this new idea of larger odd numbers, building a thicker understanding of all odd numbers. The students were "image having" when they made decisions about whether a number is odd or even without having to draw all the sticks, modeling the sharing technique that they start with. They moved into "property noticing" when they realized that the ones place is the most important place to look when trying to determine the odd-ness of a number. With collaborative questioning Mrs. Howard was able to create meaning about odd numbers between and among herself and Raven, Paula and Tina.

Conversations Exploring Student Discoveries

> 'Christopher – *ow!* – Robin,' called out the cloud.
> 'Yes?'
> 'I have just been thinking, and I have come to a very important decision. *These are the wrong sort of bees.*'
> 'Are they?'
> 'Quite the wrong sort. So I should think they would make the wrong sort of honey, shouldn't you?'
> 'Would they?'
> 'Yes. So I think I shall come down.' (Milne, 1926, p.18)

Another theme I found in the collaborative conversations in the classroom was talking about mathematical discoveries children were articulating as they came to understand a new concept. Unlike the first theme, where the teacher helped the student think more deeply about a strategy they are using, these conversations occurred at the moment students are meeting an idea for the first time, discovering for themselves the patterns of mathematics.

Multiplication 'flips'.

Early in the project, the class was finding patterns on a multiplication table with the goal of using these patterns as another multiplication strategy to add to their growing list. As Mrs. Howard assigned the task to partners, I noticed two boys, Jeff and Ron, deep in excited conversation about the patterns they were discovering. I recently decided to move away from the wide pan camera set up, as the conversations were too hard to make out and I could not see what the students were working on, and chose to carry the camera and interact with the children as if my camera were my face. The boys seemed unperturbed that I talked with them in this fashion and were quite open about letting me in on the conversation. After asking them what they discovered, they explained (as they pointed to their partially completed multiplication chart) they found a "shortcut". They found that for every equation, such as 6x3 (6 rows, 3 columns on the chart) there was a match – 3x6 – and you could "just copy the answer".

So then I asked the boys: "Is that a one-time happening, or do you think that happens more?" They felt it did happen more and found the 7x3/3x7 "flip". Then, as they begin to realize how vast this pattern is, excitement began to grow in their voices:

> Jeff: Oh! Yeah, right here! (indicates with his pencil) 7 [rows] times 3 [columns] is 21 and then…(traces the columns with his pencil) 7 [columns] times 3 [rows] is 21!
> Me: How much do you think that happens on this grid?
> Ron: Lots! Cause look, the numbers match from here (shows the rows with the side of his hand) you just flip it upside down (flipping his hand to the columns). It's long…it would be the exact same as that.

I asked them, "How could you check about how many times these flips happen?" and I left them to work it out…but not for long.

From across the room, I heard, "Oh-ho, oh no!" and laughter. As I returned, I was met with big smiles and Jeff holding the sheet up for the camera to see.

> Jeff: We found another one! "Cause there's 6 groups of 7…
> Ron: It happens every time!
> Jeff: …and, uh, uh, 7 groups of 6.
> Me: So for ev… it seems, you answer one, then you're able to copy it somewhere else.
> Ron: Yeah!
> Me: Wow! So are you going to be able to finish that grid quickly, then?
> Jeff: Probably.

> Ron: Say, um, if you doubled all of these like this (points at the column of numbers) this – all of this – is mostly…can be ‘flipparoo’ of all of this (pointing to the rows)– except this one (circles zero).

At this point, Ron created a word for what he discovered – “flipparoo” – which brings to mind the term “mathematizing” Fosnot (2002) uses to describe thinking about mathematics at a deeper level than just skill building. These students were acting more like mathematicians, doing and creating the mathematics rather than simply receiving the information in the form of rules. Jeff and Ron actively explored the concept of the commutative principle in multiplication with me as a collaborator, not as an information-giver, and creating their own terms to describe the property they discovered. They were on their way to formalizing (in Pirie and Kieren’s model) an understanding of this property and a deeper understanding of multiplication in general.

> Me: So, you’re thinking for every single question there’s a flip.
> Jeff & Ron: Yeah.
> Ron: *Mostly* every single question.
> Me: *Mostly*…so you say *mostly.* Something in your brain is telling you…what? Why did you say *mostly*?
> Ron: This one ‘cause this one (points to 81) the only ‘flipparoo’ for that one (the 9 row) is that one (the 9 column).
> Me: OK, hey I have an opposite question then. To double check our theory, what about the opposite. Is there any question here that does **not** have a flip? There’s a question that might help us understand about flips.
> Jeff: no…
> Ron: This one! (pointing to his 9x9=81)
> Me: Why? Why does 9x9 not have a flip?
> Jeff & Ron: same number – it’s the same numbers.
> Me: Oh my gosh! So maybe than rather…
> Ron: 8 and 8…all of them do!
> Me: What do you mean ‘all of them’?
> Ron: ‘Cause this one…(points with his pencil to the 1 row & 1 column, 2 row and 2 column, etc)
> Me: Oh, I think I see what you are saying. That on one side, if there are 9 rows and 9 columns there’s no flip. Could you do me a favour then? Could you circle all the ‘non-flips’ (the boys do, and begin speeding up as they see the pattern.)
> Me: Now the rest of those not circled. Do they all have flips, or not?
> Jeff: Yeah, mostly.

Ron: Yeah, probably.
Me: I'll leave you to it to find out.

What struck me about this conversation is how the boys naturally found the commutative property and squares (e.g., 8x8) and saw it as their *discovery* – not rules to be memorized. And they found them in the context of their usefulness and relevance; a method to completing the multiplication chart quickly. They were engaged in the mathematics and saw themselves as 'doers of mathematics'– as mathematicians. But also, as Steffe & Kieren (1994) and Pirie & Kieren (1994) point out, it was the conversation they had with me that allowed me to see where they needed to go next. Once I realized they were onto the commutative property, I asked them how often this happened. When they investigated that, they found the squares – those multiplication equations that had no "flips". By asking them to circle the "non-flips", we collaborated to bring these two important concepts to light.

This episode brings to mind what Doll (2008) says about finding the space between tight control of what students learn and allowing space for explorations, for discoveries, in what he terms "the third space": "…curriculum designs and instructional strategies, if they are to be useful, need to lie in that space created by the dynamic interaction of the closed with the open (or in the interplay of the scientific with the storied and the spiritful" (draft, p. 15). This is one of the powerful roles of collaborative conversation between students and a teacher, and was a memorable experience on my quest to explore collaborative conversations.

Geoboard fractions.

In an introductory lesson on fractions, Mrs. Howard had the students exploring fractions on geoboard with elastics as a way to begin talking about these special numbers.

It is the interpretive and collaborative questions she asked that moved the learning into that "third space" as the students began to move from thinking about natural numbers (1, 2, 3…) to rational numbers (fractions and decimals). Meremluoto & Lehtinen (2004) write about conceptual change and focused specifically on this shift young mathematics students need to make as they move from using discrete numbers (where every number has a successor) to dense numbers (where there is infinity between any two numbers). Mrs. Howard mentioned in one of our interviews how she was aware this year of taking more time to develop these ideas, knowing this is a big shift in understanding for her students. "I've been trying to teach the fractions with way more meaning instead of just symbols". In the conversations I videotaped of Mrs. Howard and individual students, there are numerous examples of collaboration between them as they grapple with ideas about fractions. As I filmed her moving from small group to small group, she asked individuals: "Can you think of a way that you could use more elastics and keep your whole square the same size but make your pieces smaller?" "That's interesting…you have smaller pieces, but the fraction number [denominator] got bigger…do you think that will keep happening?" In the background you can hear student voices saying, "That's equal. Those parts aren't equal!" as they refined their understanding through dialoguing. Conversation in which students explain their discoveries lets the mathematics be a living thing that grows in the classroom as the students add to the community's understanding in an evolving, dynamic way.

Conversations Exploring Student Insights

'Is that the end of the story?' asked Christopher Robin.
'That's the end of that one. There are others.'
'About Pooh and Me?'
'And Piglet and Rabbit and all of you. Don't you remember?'

'I do remember, and then when I try to remember, I forget.' (Milne, 1926, p.20)

Metacognition –knowing how we know what we know –is a focus of elementary education because students can better assess what they know and what they still have to learn if they can self-assess their learning. This self-awareness also helps students learn how they learn. Wood (1998) writes of learning taking place on two levels. Students learn about concepts through tasks, conversation and experiences, but at the same time "…he is also learning how to structure his own learning and reasoning" (p. 98) This is the fifth and final theme I found in the collection of videotaped conversations from Mrs. Howard's class.

"Division is hard."

I zoomed in on three boys working on a task in which they used multiplication facts to solve division questions.

Me: 36 divided by 4 – what does that mean?
Terry: 36 divided into 4 groups – how many in each group? - 9.
Dean: (muttering) If you ask me, I hate division. Division is hard.
Me: What makes it hard?
Dean: I don't know – it's just hard.
Me: What do you find easier?
Dean: Multiplication is easy for me.
Me: I wonder why?
Dean: I don't know, it just is.

I talked to Dean about how he might use multiplication to help himself with division, and he replied that he knows that is what they are to do; it is just not that easy. He then began to expand on his original statement to say that multiplication is easy because he has done it for a long time, and has lots of strategies. This idea of thinking multiplication to solve division felt a bit mysterious to him.

Me: And you have fewer division strategies.
Dean: We just started division, and we're already at 70 divided by 7, so…do you know what I mean?

> Me: Yes – suddenly the numbers have gotten big quick, whereas with multiplication you started with smaller numbers and developed your strategies slowly.
> Dean: Yeah.

Conversation continually shapes and reshapes an idea (Gadamer, 2004, Gordon Calvert, 2001). This is evident in the exchange between Dean and me: from his first somewhat defeatist comment, "Division is hard" through his description of why multiplication is easier for him, it became clearer to both of us that the idea, "division is hard" is already shifting as Dean described how he learned multiplication and how they have just started division. The process of learning a new concept and strategies is not without its setbacks and frustrations, but, as processes go, it can become easier with time. It would have been easy for me to tell him not to hate division, that it will get easier if he practices or some other encouraging statement. But would that really help Dean? Gadamer (2004) writes about the relationship that develops in conversation and how one should not be trying to convince the other or talk the other out of his or her position, but instead "…to place in the open" (p. 361) the subject at hand, exploring it together, to gain insight, both of the subject and of oneself. In this short conversation, Dean may now see more clearly how his understanding of division *will* develop through time.

"It all depends."

During a fraction review lesson, Carl had an insight into fractions of a set: a concept in fractions that can be a difficult one to master. To find 1/4 of 12 candies you have to flexibly move from thinking about the fraction – one out of four groups – then about the groups in the number – four, with three in each group – then about the fraction again – one of those four groups is three. Mrs. Howard created some interesting problems to do with fractions of a set for the students to do in pairs. One question had a few of the pairs stumped: “How can 2/3 of one set of candies be more than 2/3 of another set of candies? Use words, pictures and/or numbers to explain your thinking.” But Carl and Tammy were both drawing on their paper, heads together in discussion. I crouched beside them with the camera, filming over their shoulders, and asked them what they found. Carl talked me through the diagrams they made of 12 counters in 3 groups and 6 counters in 3 groups.

“12 is bigger than 6, so 2 groups in the 12 is 8, but 2 groups in the 6 is only 4,” Carl says. Just before I began filming, Carl said to me, “It all depends on the size you start with” and I asked him if he could repeat what he had said.

“It all depends on the size of the whole, or the groups,” Carl stated.

“Could you say it has to do with the size of the whole **and** the groups, as they sort of go together?” I replied.

“Yes,” he answers, and repeats the statement. So then I give him a different scenario – 2/3 of 30.

> Me: Would that be a small amount or a big amount?
> Carl: Big.
> Me: You answered that right away. How did you know?
> Carl: Because 30 is bigger than 12 and 6, so the thirds would be bigger. The thirds would be ten.
> Me: So how much would 2/3 of 30 be?
> Carl: 20…and 1/3 of 30 is close to that whole group [12] (pointing to the 12 counters drawn).

At the conclusion of the task, Mrs. Howard called the class back together to share their answers. Carl had his newly formulated idea down to a succinct statement which he shared with the class: “The fraction is useless if you don’t know what you’re starting with.” Again, Pirie and Kieren’s (1994) model of mathematical development shows that Carl was “property noticing” as he generalized his rule about how the size of the whole dictates how large the fraction actually is. He was able to move from working with drawings to handling another example (2/3 of 30) in his head. He refined his generalization about the properties of fractions during our conversation, creating a simple, but profound, statement in the end.

I believe this is also an example of what Pirie and Kieren (1994) call "acting and expressing" which occurs, they say, at each level beyond primitive knowing. This is necessary for a child to progress on to any level, and they write that one way to reveal acting and expressing is through conversation with the student. "Acting can encompass mental as well as physical activities and expressing is to do with making overt to others or to oneself the nature of those activities" (p. 175). And expressing is not reflection or explaining how they understand, but rather expressing "…entails looking at and articulating *what* was involved in the actions" (p. 175). Carl created and shaped his meaning as we conversed, and he came to build the properties of fractions of a set for us both.

Journeying into Collaboration

During an interview, Mrs. Howard said she is more aware now when a conversation with a student shifts into collaboration:

> I'm noticing when I'm asking…I'm actually not very often directing them, you know? Like I'm really trying to work with the 'mathematician'…When it does happen where we actually sort of seem to engage in it and it gets into this deeper conversation where the kid's not just responding to me as the teacher looking for the right answer but it's actually like a dialog that we're having where I can see that they're really thinking hard and they're really starting to piece things together and we're just both more engaged.

Mrs. Howard, through her efforts to incorporate collaborative conversations into her daily mathematics lessons, acknowledged that learning occurs when children interact with each other, the teacher and the environment in the classroom. In taking an enactive perspective which views learning as both personal and collective and highlights the self-similarity between personal knowledge and collective knowledge and the interrelatedness of the two, I got a real sense of this in action when observing the workings of Mrs. Howard's mathematics classroom.

Chapter 5: Lessons Learned

> "[I]n a successful conversation they [the conversers] both come under the influence of the truth of the object and are thus bound to one another in a new community. To reach an understanding in a dialogue is not merely a matter of putting oneself forward and successfully asserting one's own point of view, but being transformed into a communion in which we do not remain what we were" (Gadamer, 2004, p. 371).

This is the kind of conversation that was the focus of my project. And the students, Mrs. Howard and I were transformed into a communion, a partnership, of learning – and we do not "remain what we were". I was not only looking for conversations that led to new understandings for the students, but also conversations that led to new understandings for the teachers; about our practice of teaching as well as ourselves. When I think of Jeff and Ron and their discovery of "flips" on the multiplication table, I recall that their excitement was infectious and the connection they made to the patterns in mathematics powerful. Carl's discussion with me of how the whole is most important when thinking of fractions of a set highlighted his "property noticing", his shaping of meaning as we spoke. And Anna's playful problem about making fractions of liquids led the group into contemplating what it really meant to make a fraction of something – anything – through collaborative conversations.

For me, genuine conversations change teaching and learning from the teacher-learner dichotomy into a shared learning, a coming together and blending of roles: creating a teacher-as-learner as well as a learner-as-teacher. If our goal is to help teach students to be life-long learners rather than collectors of information, then collaborative conversations are essential. Collaborative conversations enable the student to think deeply about a concept, seeing it from different perspectives and exploring the veracity of their conceptions through the knowledgeable coaching questions from an engaged teacher. The conviction in Carl's voice as he explains how the whole is the most important referent when using fractions comes from the opportunity he had to formulate and try out his ideas through collaborative conversations with his partner, me and Mrs. Howard. Carl has this deep understanding now, unlike so many struggling grade 9 students I encounter who, after 5 years of the "teaching as telling" method are still confounded but the very same concept.

Taking the First Step

Talking to students about their thinking is not new in education; however, talking in this way *is* new – or not usual – for many. It is more than simply checking for understanding, or encouraging children through a discussion. It is much more profound than that. Conversation is a way into Pirie & Kieren's (1994) acting and expressing – "…articulating *what* was involved in the actions". It is a place for students to create and shape meaning as they speak, as the teacher listens. It is the bridge Aoki (2005) speaks of, the bridge "…which is not a bridge" (p. 228) but rather a creation of new understanding between two participants in a conversation that can not exist otherwise. For us teachers to begin to have hermeneutic conversations with our students, we need to understand how this kind of conversation is different. This is not the "one-word-answer" kind of question, or the right-and-wrong type, nor is it only probing into how a child got an answer. This is collaborative, where both we and the child are on the same page, on the same plane. Where meanings are made between the two, not delivered from teacher to child. When Mrs. Howard collaborated with Tina, Raven and Paula to create for themselves a place in their understanding where odd numbers could be created, the collaborative conversation demonstrated clearly how this was "an interchange of interpretation rather than an exchange of information" (Gallagher, 1992, p. 38). Often it is quicker to simply tell a child where their thinking has gone awry and how to fix it, but often that does not provoke changes in their understanding. For example, in the case of Evan and his use of arrays to solve 6x7, his answer was out by one. Mrs. Howard could have easily and quickly simply told him to either check his work or to fix his answer, adjusting it by one. But by exploring with him his thinking about the array, about where the seven he added on came from, they addressed the misconception and revised his understanding together: neither through Mrs. Howard's telling nor showing, but by asking and inquiring.

We can begin to journey into collaborative conversations as simply as asking a child, "What do you notice?" or "How did you figure that out?" and listen not only to the answer, but the meaning making behind it. Then we can ask, "Will that always work? Is there a pattern?" to engage the child in speculating, wondering and playing with the concepts – and let ourselves play with the ideas, too.

Listening to Teach, Speaking to Learn

How might we create the conditions where this kind of conversation is possible? On reflection, I believe that students need to feel they are part of a community of learners, where learning is collective and individual, with one being entwined with the other. They also need to experience the many ways in which learning happens in conversation with each other. By enabling a rich and varied learning setting where ideas are tried out, discussed, explored and enjoyed, conversations of many kinds can flourish. Of course, it is not a guarantee that hermeneutic conversations will arise, but doing so does offer invitations for this as well as collaborative, interpretive and other conversations to grow.

Conditions to Occasion Change

In an interview with Mrs. Howard, we discussed just how she felt she was moving from evaluative questions, through interpretive into collaborative, hermeneutic conversations with her class:

> Me: I notice as an outsider and having scanned through the clips …that a predominant feature of this classroom is that kids are…feel extremely comfortable talking about their thinking and explaining their thinking to each other, to you, that it's part of the fabric of this classroom.
> But how do you get there? So if we were trying to tell somebody about this, what would we say? Like how do you get from the one-word answer and the question for facts to that kind of collaboration between the teacher and the child?
> Mrs. Howard: I think it is that whole…unconsciously incompetent, then you're consciously incompetent – you start recognizing when you're do it, when you've got that kid sitting and guessing waiting for you to say, "Do it like this."
> Me: Right.
> Mrs. Howard: You start feeling that…those moments and you're going OK, this isn't what I want. I don't want them looking to me to tell them what to do. I want them…I want to talk to them in a way where they feel like they can work something out. So if the conversation we're having is too complicated and they're just waiting for me to have an answer, then I need to bring it down and do more asking, like…quit telling, and do more asking. So…and then…you know, at this point I would say I'm between 'consciously incompetent' and 'consciously competent', because I catch myself still doing that…
> Me: Sure.
> Mrs. Howard: …and then there's other times where I can…I see the opportunity and I take the opportunity to talk to them at a deeper level.

Teaching is such a busy profession that we sometimes do things automatically, without a lot of thought – we have to, or we would never get the job done! What I think Mrs. Howard is saying is, sometimes it is important to stop and take stock of what is happening in our classrooms, day to day. Talking and listening to students is what we do the most of in a day, and so I think sometimes we lose touch with what impact our speaking and listening to students has on their learning. But when Mrs. Howard began to think about what she asked of children and how she listened, she was not satisfied with the status quo. She decided to become more aware of her interactions with students, and to change to a more collaborative, careful and attentive way of listening first then deciding what to say next based on the conversation, not on her agenda for teaching a concept.

We cannot change that of which we are not conscious. When children are searching our face for the right answer, we are seeing the results of evaluative questions. Students can tell what we are listening for, and try to respond accordingly. When we shift into interpretive questioning, students begin to give us fuller explanations of their thinking, and begin to think more about how they came to understand a math idea or concept. With hermeneutic questioning, we and the student are beginning to create the understanding *between* ourselves, for ourselves and each other. It is a different way of thinking about how children come to understand and learn mathematics. We, as teachers, are often looking for ways to improve our teaching of children, ways that are not necessarily “harder” but definitely “smarter”. If we are going to truly help children mathematize, then collaborative conversations are worth exploring. It is within all of us to listen carefully and attentively, to ask more open (but definitely not directionless or loose questions) and collaborate to find the answers with each other. It is worth the effort, it is worth the time – and it is “smart” work.

Future Journeys

> *“The big question is whether you are going to be able to say a hearty yes to your adventure.” – Joseph Campbell*

This is the story of my journey with one class in which the students' teacher, Mrs. Howard, and I chose to change the way we ask questions and listen for the answers. This story, of course, continues as the students continue to develop mathematical understandings and Mrs. Howard and I continue to consider the role of conversation in creating the conditions for such learning. During our 10 weeks together, we had the opportunity to witness and be part of hermeneutic conversations about mathematics. The value of a case study is that "…[t]he reader comes to know some things told, as if he or she had experienced it. Enduring meanings come from encounter, and are modified and reinforced by repeated encounter" (Stake 2003, p. 145). From this work, I recognize that there is a definite need for more case studies of collaborative teacher-student conversation in mathematics classrooms to add to our collective experience of its affect on student learning. It is through taking such journeys, sharing our reflective stories and having conversations about our experiences that I believe teachers and others in education will begin to see the profound effect and importance that this kind of engagement between teacher and student invites. For a long time, teaching has been seen as a delivery of sorts – delivering the curriculum, delivering assessment of learning – and collaboration was something to be had between students or between teachers. Now we are stepping off that path and heading down a different path of teaching and learning with an enactive view, in which teachers and students collaborate to make meaning together. It is through studying many examples of collaborative classroom conversations that we can begin to collaborate with each other. This kind of learning is best done with others, with teachers who share the same interest in bringing mathematics alive and creating a culture of collaboration in their classrooms. When we know clearly what the goal of the journey is – in this case, collaborative conversations in the mathematics class – we have a better chance of reaching it.

I spoke at length with Mrs. Howard to hear her insights into what occurred in the classroom as she pushed the edges of her own learning to bring collaborative conversations to the forefront of her mathematics lessons. In future research, I would like to hear from the students themselves. How do students feel about these types of conversations? What can students teach us about the effects on their learning? It would be a whole new journey of discovery to compare the classrooms in which collaborative conversations are the norm with more traditional classrooms to see what differences and insights there are in how the children perceive their learning.

Bends in the Road

There are always challenges to address when conducting research of any kind. In this case study, one was the technology used to record the data. With only one camera, some rich conversations were never heard, some profound insights never recorded. Also, with a camera-mounted microphone, close-up filming was the only way to hear the conversations. What was gained with the immediacy of the close-up may have sacrificed spontaneity on the part of the children; it is hard to be yourself completely with a talking camera 30 centimetres from your face! As always, whatever technology we use and how we use it influences the data we collect. Future research in this area might consider lapel microphones and cameras with zoom lenses, multiple cameras with multiple videographers. It may have been disconcerting for me to travel around the classroom with the hand-held; on the other hand, students became quite used to me filming in this way, and I may not have captured the conversations I did using another technique.

Another challenge was that of time. In only 10 weeks, I was unable to capture the beginning of the year, how Mrs. Howard began transforming her class into one where collaborative conversations could take place with increasing frequency. Nor was I able to record how this played out – what kind of conversations were happening by the end of the year? This was only a glimpse into the workings of this classroom and a longer case study could potentially reveal much more.

Journey's End

In 10 weeks, I journeyed with a class of third and fourth-graders and their teacher as we explored conversations that arose between their teacher, me and the students in the mathematics classroom. I was most interested in how collaborative conversation affected a student's conceptual understanding of mathematics. I found that the conversations helped these students gain insight into their metacognition, articulate their discoveries of new ideas and concepts, share strategies, reveal and address misconceptions and find the playfulness in mathematics that is sometimes missing in more traditional textbooks and programs. Hermeneutic questions and listening brought a richness and abundance to mathematics through encouraging students to explore the diversity of patterns, the connectedness of ideas and the complex process of learning through creating meaning for themselves, between themselves and in collaboration with the teacher.

The students in this study appeared to be learning concepts in mathematics more conscious of what they were learning, how they were progressing and what they still needed to find out. As I listened to and watched the videotapes again, I was struck by how little these students cried out, "I don't get it!" or "Can you help me?" but rather create meaning in their dialogs with each other and the teacher in a constructive, sense-making fashion. Conversations in the mathematics classroom, if they are genuine and collaborative, can enrich the meaning-making for both the students and the teacher and help create a culture of caring, support, and joy in learning.

Through this project I was allowed the luxury and privilege of being party to and participating in many mathematical conversations between a teacher, Mrs. Howard, and her students as they created meaning together. This is rare in our profession as we are not usually offered the time to simply watch and listen to another classroom exploring mathematics. I learned how conversation can bring out nuances of meaning that ultimately lead to understanding. Through conversation students and teachers can take the time to explore these nuances fully, deepening understanding as they do. I was also able to engage in these hermeneutic conversations myself, in a classroom where I did not feel the pressure to finish a lesson, complete a unit or get through the curriculum. Instead I followed the rhythm of the teacher and her students as they explored ideas and conversed about mathematics in the moment: another rarity in our profession. This is what I will take with me into the classrooms I visit and into classrooms I may be responsible for in the future: to take time in conversation to investigate not only the mathematics at hand, but also the journey the children in my charge and I take to make mathematics meaningful for ourselves.

This project has not only changed what I think about learning, but it now shapes the way I am in the classroom. This experience has transformed my role as a teacher from an orchestrator of learning to what I would call a “mindful participant”. By this, I mean that I no longer see the purpose of my teaching to be a transmitter of knowledge nor a guide on the sidelines encouraging my students but rather a full participant in the creation of meaning through conversation. Certainly, I have different knowledge than my students, which allows me to help occasion their learning based on what I hear them saying about their thinking, mindfully aware of a framework (such as Pirie and Kieren's) to help me decide what question to ask or what task to suggest next. Before I began learning about constructivism, I was teaching as a transmitter of mathematics; getting through the curriculum in the recommended time, following the textbook in a linear way. As I learned more about constructivism, I moved from transmitting knowledge through a text and began working with the students as a guide, to help them create understanding for themselves. Now taking on an enactive perspective, I feel I see the mathematics, myself and the students in a much broader context. I am aware of the interrelatedness between us all, and sense how each one of us – the mathematics, the students and the teacher – relates to and impacts the other. Mathematics teaching to me is no longer necessarily a linear progression through a text or a means of guiding students through stimulating activities designed to build their knowledge. In a qualitatively different way, I am mindful that mathematics learning can be recursive, non-linear and occur in the space *between* myself and the learners participating fully in the mathematics in which we find ourselves. This requires more time for conversation in the classroom and awareness that the learning cannot be rushed, that it must first arise with the students and not the textbook. It requires me to be mindful of the curriculum, the mathematics and the students equally and fully. I am more present in the moments of learning, attentive and responding to students in an engaged manner. It is harder work, but infinitely more rewarding work.

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Appendix A: Letters of Consent

University of Victoria

Faculty of Graduate Studies
University of Victoria
PO Box 3025, STN CSC
Victoria BC V8W 3P2

Sooke School Board District Office
3143 Jacklin Road
Victoria, BC

Nov. 18th, 2008

Dear Mr. Fox and Sooke District Trustees:

I am seeking permission to conduct educational research in a classroom in our district from January through April for my Master's project, under the supervision of Dr. Jennifer S. Thom.

The purpose of this study is to understand the role that collaborative teacher-student conversation plays in intermediate student's learning of math concepts. In addition, this case study will also look at the teacher's role in both participating in and orchestrating the math dialogues.

I will begin by introducing the topic of my research to our Sooke District Numeracy Networks and ask the representatives to present the research topic through an information poster (see attached), to attract interested teachers to the project.

My study will involve one classroom. This research will entail in-class observations, samples of teacher and student work and videotaping of lessons every Monday in January through April for 12 weeks. Written consent will be sought from parents prior to the start of the study.

I am also seeking consent from the university human ethics review board and will submit a copy of the certificate once the study has been approved.

I thank you in advance for considering this request.

Sincerely,
Jeannie DeBoice

University of Victoria

PARTICIPANT CONSENT FORM

**Listening to Teach, Speaking to Learn:
The Role of Conversation in the Mathematics Classroom**

You are invited to participate in a study entitled "Listening to teach, speaking to Learn: The role of conversation in the math classroom" that is being conducted by myself, Jeannie DeBoice. I am a Graduate student in the department of Curriculum and Instruction at the University of Victoria. You may contact me if you have further questions by phoning 250-474-9851 or e-mailing jdeboice@sd62.bc.ca As a Graduate student, I am required to conduct research as part of the requirements for a degree in Masters of Education. It is being conducted under the supervision of Dr. Jennifer Thom. You may contact my supervisor at 250-721-7771 or e-mailing her at jethom@uvic.ca .

Purpose and Objectives

The purpose of this research project is to understand how teacher-student conversation helps students understand math concepts. In addition, this case study will also look at the teacher's role in both participating in and orchestrating the math dialogues. The central question is: What role might genuine, hermeneutic (collaborative) conversation play in occasioning the growth of students' conceptual understanding in the mathematics classroom?

Importance of this Research

Research of this type is important because we are always looking for ways to help students better understand mathematics and develop confidence in the subject.

Participants Selection

You are being asked to participate in this study because you expressed an interest after hearing about the project from your Numeracy Representative at your school.

What is Involved

If you agree to voluntarily participate in this research, your participation will include my observing, videotaping and conversing with students in your classroom for 10 -12 classes, every Monday from January through April. I will transcribe conversations I film and analyze this data for evidence of collaborative conversation aiding in the understanding of math concepts. I may ask you to keep a journal during the project, reflecting on your learning about the role of conversation. I will ask that we have 2-3 formal, videotaped conversations before or after school to collaboratively explore our thinking on the role of conversation.

Inconvenience
Participation in this study may cause some inconvenience to you, including some disruption to the flow of your day as I videotape portions of your math lessons and tutoring sessions between yourself and a student and talk to some of your students as they work on math in class. In anticipation of these possible effects, I am happy to come in to the classroom before the project begins so that the student will be familiar with me. During the videotaping, I will ensure that the camera(s) are positioned in the most unobtrusive places so as to not distract you or the students during the math lessons. I will use two to three 20-30 minute periods after school for one-on-one interviews with you.

Risks
There are no known or anticipated risks to you by participating in this research.

Benefits
The potential benefits of your participation in this research include exploring first hand the effectiveness of conversation in helping students construct ideas in mathematics. As I study the students' conversations with you, I will examine the data for emergent themes and patterns. As a result, it is my aim to find useful insights into how you and other teachers can make conversation an integral and important part of their mathematics lessons; ways that are not necessarily 'harder' but definitely 'smarter'. One of the ways in which we share and develop ideas is through explaining and justifying them to others; thus conversation plays a critical role in mathematical learning and understanding.

Voluntary Participation
Your participation in this research is completely voluntary. If at any time you do not wish to participate, you may withdraw without any consequences or any explanation. If you do withdraw from the study your data will be used only if you give explicit permission to do so. If you do not agree to do this, then all data collected will be destroyed.

Researcher's Relationship with Participants
Potential participants are colleagues of mine in the same school district. To help prevent this relationship from influencing your decision to participate, the following steps to prevent coercion have been taken: initial information about this project was disseminated to your school via your Numeracy Rep and/or through district workshops; then only when you expressed an interest and contacted me personally did I begin the process of recruiting you for the project.

Anonymity
Your anonymity will be protected by using a pseudonym for you, your students and your school when writing about this project. Videotape used in dissemination will reveal your identity, but your name will not be posted in the credits without your permission.

Confidentiality
Your confidentiality will be protected as the written data will be saved on a password protected flash drive and destroyed when the project has been written up. The data may be used for presentations and will continue to be saved on a password protected flashdrive.

Dissemination of Results
It is anticipated that the results of this study will be shared with others in the following ways: directly to you and your class at the end of the study, in a project for the University of Victoria and possibly at professional meetings in the future.

Disposal of Data
Data will be used for the study stated above and the other person that may see the video data will be my supervisor, Dr. Thom and perhaps, committee examiners; all data will be destroyed or shredded when the project is complete. Video clips may be retained for dissemination at professional meetings, with your permission (see below).

Contacts
Individuals that may be contacted regarding this study include me and my supervisor, Dr. Thom (see above for contact information).
In addition, you may verify the ethical approval of this study, or raise any concerns you might have, by contacting the Human Research Ethics Office at the University of Victoria (250-472-4545 or ethics@uvic.ca).
Your signature below indicates that you understand the above conditions of participation in this study and that you have had the opportunity to have your questions answered by the researchers.

Name of Participant (PRINT) : ____

Signature: ____________________________________

Consent for Visually Recorded Images/Data (please initial):

Videos may be taken of myself for analysis: ______ Dissemination: ________

Date: ____

University of Victoria

Faculty of Graduate Studies
University of Victoria
PO Box 3025, STN CSC
Victoria BC V8W 3P2

Jan. 1, 2009

Dear Parents:

Your child is being invited to participate in a study entitled "Listening to teach, speaking to learn: The role of conversation in the mathematics classroom" that is being conducted by myself, Jeannie DeBoice, Sooke district's Numeracy Curriculum advisor.

As a Graduate student at the University of Victoria, I am required to conduct research as part of the requirements for a degree in Curriculum and Instruction. It is being conducted under the supervision of Dr. Jennifer Thom. You may contact my supervisor at 250-721-7598.

The purpose of this study is to understand the role that collaborative teacher-student conversation plays in intermediate students' learning of math concepts. In addition, this case study will look at the teacher's role in both participating in and orchestrating the math dialogues.

Research of this type is important because we are always looking for ways to help students better understand mathematics and develop confidence in the subject. One of the ways in which we share and develop ideas is through explaining and justifying them to others; thus conversation plays a critical role in mathematical learning and understanding.

Your child is being asked to participate in this study because his/her teacher is interested in the role of conversation in learning mathematics and expressed an interest upon hearing about the project.

If you agree to your child's voluntary participation in this research, it will include my observing and conversing with students in the classroom for 12 of their regular classes, every Monday from January through April. In addition, I will be videotaping some of the lessons. If you agree to your child's participation, this means that you and your child are giving me permission to analyze samples of his/her classroom work as part of my research project.

There are no anticipated risks to your child by participating in this research, and the only effect to him/her would be my presence in the math classroom videotaping conversations for 12 weeks and asking questions about his/her learning.

Your child's participation in this research is completely voluntary. If you or s/he decides to participate, you or your child may withdraw at any time without any consequences or explanation. If you or your child withdraws from the study, his/her data will be used only if you give written permission. When your child's data is linked to group data (*e.g. group discussions*), it will be used in summarized form with no identifying information.

In terms of protecting you child's anonymity, I will not use his/her name but instead a pseudonym, in any subsequent articles or thesis. Data will be used for the study stated above and the other person that may see the video data will be my supervisor, Dr. Thom and perhaps, committee examiners; all data will be destroyed or shredded when the project is complete. Video collected may be used in the dissemeniation of results after the project is completed with your permission (please see below). You may contact me about this study at: 250-474-9851. In addition, you may verify the ethical approval of this study by contacting the Human Research Ethics Office at the University of Victoria, (250-474-4545 or ethics@uvic.ca)

Thank you!
Sincerely,

Jeannie DeBoice

(Please complete & return the bottom portion of this letter)

Your signature below indicates that you understand the above conditions of participation in this study.

______________________	______________________	__________
Name of Parent (PRINT)	*Signature*	*Date*

Consent for Visually Recorded Images/Data (please initial):

Videos may be taken of my child for analysis: _____ Dissemination*: ________

*Even if no names are used, your child may be recognizable if visual images are shown in the results.

Please talk about this with your child and if they consent, have then sign below:

Student Consent:
My parents/guardians and I have reviewed the information in the consent form together and I consent to participate in Mrs. DeBoice's Masters Project report.

______________________	______________________	__________
Name of Participating Student (PRINT)	*Student's Signature*	*Date*

Appendix B: Observation Form

Classroom Observation form Length of Activity: minutes Date: , 2009
What role might genuine, hermeneutic conversation play in occasioning the growth of students' conceptual understanding in the mathematics classroom?

Date: Lesson topic:	*Evidence and events that are brought forth by:* ☐ ***evaluative*** *questioning &listening* ☐ ***interpretive*** *(checking for understanding) questioning &listening* ☐ ***hermeneutic*** *(collaborative) questioning &listening*
Comments:	Transcript:
Date: Lesson topic:	*Evidence and events that are brought forth by:* ☐ ***evaluative*** *questioning &listening* ☐ ***interpretive*** *(checking for understanding) questioning &listening* ☐ ***hermeneutic*** *(collaborative) questioning &listening*
Comments:	Transcript:

Appendix C: Interview Questions for Teacher

Interview Protocol Project: The Role of Conversation in Deepening Understanding in Mathematics (Teacher Interviews)

Time of interview:
Date:
Place:
Interviewer:
Interviewee:

Questions/prompts:

1. The teacher & I will view parts of the video and pull out the questions he/she posed. We'll then review the different kinds of questions/listening that Davis talks about and together we'll discuss where these questions may fit in and why.

2. Davis states: "The same question, 'Where might we use positive and negative numbers?' might easily have become a hermeneutic question – an open one – the essential quality of which is that the answer not be settled. The questioner participates in the questionability of what is questioned; there is some indeterminacy. It is this that a hermeneutic question is not about reporting on truth, but about creating it…In effect, the essential difference between the teacherly question and the hermeneutic question is that the former has become a substitute for listening while the latter exists only in listening" (1996, p. 252).

It isn't the question but rather the frame of mind from which we ask it that makes it hermeneutic. It is asking question for which we don't have a complete answer already in our minds. We are digging deeper – looking beyond a 'right or wrong' response, participating in co-creation of a mathematical idea.

"Choose one of the questions that you posed in the video. What do you think of it in light of Davis' ideas? Explain your thinking.

How might the question be posed if it was not (hermeneutic, evaluative, interpretive) but one of the other two types?

What effects might each of these different prompts have on student's mathematical learning?

Is there a question that you did not ask and wished you had? What is it? Why do you wish you had asked this question?

3. What do you notice about your most recent conversations with students in math class compared with earlier ones?