Power and Channel Allocation for Broadband Wireless Networks

by

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B.Eng, Southeast University, Nanjing, China, 2009

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ABSTRACT

With the limited wireless spectrum and the ever-increasing demand for wireless services, two issues are pressing and difficult: efficient spectrum utilization and heterogeneous traffic management. Throughput and utility maximization problems are proposed to quantify these two issues. To exploit the wireless spatial multiplex gain, concurrent transmissions, if controlled appropriately, can lead to overall higher network throughput as well as utility. The optimal scheduling and power control for concurrent transmissions in rate-adaptive wireless networks is a very challenging NP-hard problem. In the thesis, we propose efficient power allocation and scheduling algorithms for concurrent transmissions which can improve network throughput and utility with fairness consideration. We first formulate the optimal power allocation and scheduling problem for network throughput and utility maximization individually, and convert the original non-convex problems into a series of convex problems using a two-phase approximation. Then, we propose power and channel allocation with fairness for network throughput maximization (PCAF-NTM) and for network
utility maximization (PCAF-NUM) algorithms to solve the converted problems. Extensive simulation results show the substantial improvement in terms of both network throughput and utility, comparing to the previous scheduling algorithms.
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DEDICATION

To my parents.
Chapter 1

Introduction

1.1 Research Motivation and Challenges

Wireless networks have become part of the modern way of life. Ubiquitous wireless networks and various of network services have penetrated into our lives. With the emergence of different network services and applications, such as Internet Protocol television (IPTV) [1], Video on Demand (VOD) [2], Peer-to-Peer (P2P) [3], Online Gaming [4] and etc., the demand of higher network throughput is never ended. Since currently, wired networks have bandwidth over-provisioned, the wireless access networks are usually considered as the bottleneck for achieving the full potential of high speed communication services. The task of increasing wireless network throughput is pressing.

With the limited wireless spectrum and the ever-increasing demand for wireless services [5], it is important to further exploit the wireless spatial capacity to improve the throughput. With the existing heterogenous applications using wireless networks, effective management of heterogeneous traffic [6] to achieve the maximized user satisfaction (utility) becomes an important issue. How to optimize the scheduling and
power control decisions to improve network throughput and utility in dense networks becomes a key issue, which is also very difficult [7].

1.1.1 Resource Allocation for Broadband Wireless Networks

Broadband wireless networks have become the new trend of network development [8]. The growth in broadband wireless networks can be attributed to the increasing demand for wireless multimedia services and the development of new wireless standards [9].

In a narrow-band wireless system, each flow can successfully transmit at a specified data rate with a fixed transmission power, if the received signal to interference and noise ratio (SINR) is greater than a threshold. There has been extensive research on the channel allocation algorithm for such systems [10, 11].

In broadband wireless systems, to better utilize the time-varying wireless channels, the physical (PHY) layer typically adjusts the transmission rate according to SINR, using adaptive modulation and coding (AMC) schemes [12]. With AMC, the transmitting power is also adjustable to optimize the SINR and transmission rate. Practically, power control is feasible to implement in a number of network environments. Therefore, power control provides another dimension to improve the network throughput and optimize network energy efficiency. We will discuss the related work in chapter 2 in detail.

1.1.2 Network Throughput Optimization

Concurrent transmission means letting multiple flows transmit simultaneously in a time slot, which may result in lower instantaneous data rate of each individual flow due to the interference from others; however, with more flows transmitting concurrently, each flow can be allocated with more time slots, so the overall throughput of each
Figure 1.1: Adaptive Modulation according to SINR
When applying concurrent transmissions to improve the spatial efficiency, the scheduling and power allocation problem for rate-adaptive wireless networks is inherently much more difficult than the classic scheduling or job assignment problems. This is because we do not know the achievable data rate of each flow before we determine which flows can transmit in the tagged time slot and at what power level. There is even no capacity bound for such networks, and the bounds derived for rate-nonadaptive networks [13] are no longer applicable.

For efficient spectrum utilization, we formulate the optimal scheduling and power allocation problem as the network throughput maximization (NTM) problem. The power and channel allocation for network utility maximization (PCAF-NTM) algorithm is proposed.
1.1.3 Network Utility Optimization

Broadband wireless networks are expected to support heterogeneous traffic such as audio and video streaming, web browsing and so on. To efficiently and fairly allocate resources such as bandwidth and power to heterogeneous traffic, utility can be an effective measure [14, 15]. Utility reflects the user’s relative satisfaction level to the assigned resource. Moreover, utility functions can easily incorporate price for differentiated services for heterogeneous applications [16, 17]. In this thesis, we consider user utility as a monotonic non-decreasing function of throughput. However, maximized network throughput does not always lead to the maximized network utility. Because the satisfaction level of users are not simply proportional to the throughput. Taking audio phone streaming as an example, a small increase of throughput to meet the minimum constant bit rate required for decoding may increase the users satisfaction level dramatically; on the other hand, further increase of throughput will not have significant effect for the users satisfaction level.

Considering the resource allocation problem for heterogeneous traffic, we formulate the optimal scheduling and power allocation problem as a network utility maximization (NUM) problem. The power and channel allocation for network utility maximization (PCAF-NUM) algorithm is proposed.

1.2 Research Contribution

The main contributions of this dissertation are listed as follows.

Considering the mutual interference and the adaptive link data rate, network throughput is an non-convex function [18] of the transmitting power vector. Considering heterogeneous traffic in the wireless network, network utility is the summation of a series of non-convex functions of flow throughputs, which is also a non-convex
function of the transmitting power vector. Both throughput and utility optimization problems are NP-hard. The majority of previous work assumed that throughput and utility are convex functions under specific network environment to simplify the problems. But the assumption does not hold.

Different from any optimizing scheme for specific network environment, we formulate the generic optimal power allocation and scheduling problems aimed to maximize the throughput for general wireless networks with rate adaptive techniques considering mutual interference. We also consider about the fairness among competing flows.

We first have formulated the network utility optimization problem for wireless networks with heterogeneous traffic including TCP-type traffic, audio and video streaming traffic with consideration of the long-term fairness.

Using a two-step approximation method, we have converted the original non-convex problems into a series of convex problems, and design the fast-ascent algorithms to solve the approximated network throughput and utility optimization problem.

Extensive simulations have been conducted. The results show a substantial improvement for throughput, while the fairness has been maintained. From the results, we have also provide an important guideline of how to adjust the system parameters to make a good tradeoff of network throughput and utility versus fairness.

1.3 Thesis Organization

The rest of the thesis is organized as follows. Chapter 2 reviews the background and related work for network throughput and utility optimization. In Chapter 3, the system model is presented and the optimal power allocation and scheduling problems
are formulated. In Chapter 4, we convert the primal problem and solve the converted problem by proposing the power and channel allocation with fairness algorithms for throughput maximization (PCAF-NTM) and for utility maximization (PCAF-NUM). Performance evaluations are presented in Chapter 5, followed by concluding remarks in Chapter 6.
Chapter 2

Background and Related Work

Power control and scheduling in wireless networks are the key issues in the management of wireless resources. There have been significant research effort research on improving the network throughput or utility using power control and channel allocation. In this chapter, we briefly review the background and related work of power control and channel allocation for wireless networks, from the aspect of power control with fixed and variable SINR and concurrent scheduling.

2.1 Power Control with Fixed SINR

For a narrow-band wireless system, the transmission data rate is fixed when SINR is larger than a threshold. A lot of studies on power control solved the basic problem formulation, where transmission power is the only adjustable parameter, constrained by a fixed SINR threshold, aimed to minimize the total power or maximize the network throughput. For example, the Distributed Power Control (DPC) algorithm was proposed by Foschini and Miljanic [19]. In [19], a iterative algorithm was proposed to solve power minimization problem successfully under fixed SINR threshold constrains. However, the extensions of the approach to variable SIR are much more challenging.
In [13], an significant analytical capacity bound was given. The analytical results show that the throughput capacity per node reduces significantly when the node density increases. However, the result does not hold when the data rate is adaptive to the received SINR and the power control mechanism was not incorporated. To capture the stochastic character of the wireless channel, the authors in [20] proposed an adaptive power control algorithm for random channels that predicts channel and other stochastic behaviors based on the past observations and then use these predictions to update the transmission power. In [21], the authors proposed heuristics algorithms for joint power control and channel assignment in order to ensure feasible power allocation in each channel. However, a joint distributed optimal power control and channel assignment algorithm remains an open problem.

2.2 Power Control with Variable SINR

For a broadband network with feasible power control according to different received SINR, the network can optimize the power allocation according to traffic requirements and channel conditions. A higher SINR channel can support higher data rates and possibly has better. There are also much study for the power control problem with variable SINR.

A binary power control algorithm was proposed in [22]; however, the algorithm tries to find the local optimal solution which works well with low network density. In addition, the fairness among different links has not been considered and thus the flows with poorer channel condition may get starved.

Many previous works use an assumption that the objective function, i.e. the network throughput or utility function is a convex function of power, which makes the problem much simplified. The work in [23] proposed to approximate data rates as
linear functions of SINRs, and proposed a distributed algorithm on a subset of the SIR feasibility set that is easier to decouple and leads to a heuristic solution. However, according to Shannon theory, the data rates cannot always be approximated linearly. In [24], the author solved the uplink power control problem in cellular system, however, the assumption of convex utility function does not hold for new type of applications such as scalable video streaming. In addition, heterogenous traffic was not considered. In the case of convex SINR feasibility set, the work in [25] presented a distributed algorithm that does not require coordination among cells which converges to the jointly optimal SINR and power. The main idea comes from a re-parametrization of the constraint set from power-interference representation to the so-called load-spillage representation, essentially a sophisticated change of coordinates, which then leads to an ascent search-direction for this optimization problem that is locally computable by each mobile user. The algorithm has also been recently adopted in the industry. In our work, we do not rely on the strong assumption that SINR must be a convex set.

2.3 Concurrent Scheduling Algorithm

The optimal scheduling problem has been extensively investigated [10, 26], where a conflict graph has been used to bound the mutual interference. However, these approaches are not suitable for rate-adaptive wireless networks where the transceivers can adjust the link data rate according to different levels of interference. In [27], the author developed an Deficit Max-Weight algorithm (DMW), using Max-Weight based on an alternative control process to make the scheduling decision. However, fairness issues were not considered in the work.

For rate adaptive networks, several heuristic concurrent scheduling algorithms
have been proposed in the literature. In [28, 29], the idea of exclusive region has been used to bound the mutual interference. By allowing concurrent transmissions without violating the exclusive region conditions, the network throughput can be higher than that with the one-by-one TDMA scheme. However, exclusive region based algorithm cannot fully exploit the spatial capacity of wireless networks and the fairness is not fully considered. In [30], concurrent scheduling algorithms using global search approach has been proposed. But the work did not consider the power control and the global search is time consuming.

In [31], an approximation was used to approximate the function of data rate considering mutual interference. The work ingeniously used a monotonically increasing lower bound to bound the approximated function. However, the result holds under the assumption that the approximation is relative accurate. For general wireless networks, this assumption does not always hold. In this thesis, we use a two-step approximation method to convert the challenging problem into a series of convex problems so that we can ensure the approximation is relatively accurate for a wide range of the SINR value.

2.4 Fairness Considerations

Simply maximizing network throughput or utility often makes the users with bad channel conditions starved. For network resource allocation, not only network throughput and utility, but also fairness among users should be considered as an important metric. In [32], different types of fairness was introduced, including max-min fairness and proportional fairness, for network congestion control. Adjustable fairness has been considered in the literature [33]. However, the fairness guarantee in [33] depends on the interference degree of the network. In [34], an optimal fairness achieving
strategy has been developed for networks with heterogeneous traffic. However, in [34], the computation complexity to achieve the optimal fairness is very high. In this thesis, we assign weights to flows dynamically to control the long-term fairness, the proposed algorithm is computationally feasible and found to achieve good performance for network throughput, utility as well as fairness.
Chapter 3

Power Allocation and Scheduling

In Chapter 2, we briefly introduced the background and related work of power allocation and scheduling for wireless networks. In this chapter, we will present our system model and formulate the optimal power allocation and scheduling problems.

3.1 System Model

Generally, wireless networks can be classified into different categories. Broadband wireless communication systems are emerging to support high data-rate services. Concurrent transmissions and rate adaptive technique such as adaptive modulation have been adopted to improve the spectrum efficiency.

In a small-scale mesh network, each node in the network is within the one-hop transmission range to other nodes; self organizing and healing make mesh topology a good choice for last-mile wireless access networks. Broadband wireless networks can support high data rate and heterogeneous applications such as video and audio streams and data transmission.
3.1.1 Network Structure

The resource allocation problem investigated is for general single-hop mesh-topology wireless networks. Consider a wireless network with $2N$ active nodes randomly deployed in a square area. $N$ of them are saturated senders belonging to set $S$, and the rest are receivers belonging to set $R$. The $i$-th source node $s_i \in S$ and the $i$-th destination node $r_i \in R$ form a transmission pair $(s_i, r_i)$ for flow $i$. The source-destination association does not change during the scheduling period.

The above network setting can be observed in many situations, such as in home networks and wireless access networks. In the following, we use the standard and the typical parameter settings in an IEEE 802.15.3a Ultra-wideband (UWB) based Wireless Personal Area Network (WPAN) as an example. Our approach is applicable to other broadband wireless networks.

According to the IEEE 802.15.3 standard, wireless devices can autonomously form
a piconet. One node is selected as the piconet coordinator (PNC). Resource allocation in the piconet is based on a superframe structure as shown in Fig. 3.2. Each superframe begins with a Beacon Period (BP) for network synchronization and control message broadcast. After the BP, devices use the carrier sensing multiple access/collision avoidance (CSMA/CA) MAC protocol to send requests to the PNC during the contention access period (CAP). The remaining channel time of the superframe is for the contention-free period which is called the channel time allocation period (CTAP) for data transmission.

In current 802.15.3 networks, during CTAP, each slot is allocated to one flow (with a fixed power) exclusively using the TDMA scheme. The one-by-one TDMA scheme is easy to implement; however, for broadband wireless systems such as UWB-based WPANs, it is inefficient in terms of spectrum utilization. Therefore, by exploiting the spatial multiplexing gain of broadband wireless communications, we can schedule flows to concurrently transmit at appropriate power levels to maximize the network throughput or utility under the fairness constraint.
3.1.2 Channel Model

Wireless transmissions generally suffer from path loss, fading and shadowing, and interference. Since it is difficult to know the instantaneous channel quality for all the flows, the power allocation and time scheduling decisions can be based on the estimated average link quality. Thus, we mainly consider path loss and large-scale fading, which determine the average link quality and thus link rate. Given the transmission power of $s_i$, $P_i$, the received signal power $P_{i,i}$ can be expressed as

$$P_{i,i} = P_i \kappa G_{i,i} d_{i,i}^{-\gamma},$$  \hspace{1cm} (3.1)$$

where the constant $G_{i,i}$ represents the fading gain for channel $i$, $\kappa$ is the constant scaling factor corresponding to the reference path loss, $d_{i,i}$ is the distance between $s_i$ and $r_i$, and $\gamma$ is the path-loss exponent. The received SINR can be written as

$$\text{SINR}_i = \frac{P_{i,i}}{N_0 + b \sum_{j \neq i} P_{r_j}} = \frac{P_i \kappa G_{i,i} d_{i,i}^{-\gamma}}{N_0 + b \sum_{j \neq i} P_j \kappa G_{j,i} d_{j,i}^{-\gamma}},$$  \hspace{1cm} (3.2)$$

where $N_0$ is the background noise power, and $b$ is the processing gain related to the cross-correlation of signals for different users.

For a network that allows concurrent transmissions, the transmission rate of each flow is constrained by the summation of mutual interference. We assume that the flow $i$ can achieve transmission rate $R_i$ according to the Shannon capacity estimation, which is

$$R_i = \eta W \cdot \log_2(1 + \text{SINR}_i),$$  \hspace{1cm} (3.3)$$

where $W$ is the transmission bandwidth and $\eta \in (0, 1)$ is a coefficient describing the efficiency of the transceiver design.
Using (3.2) and (3.3), the link data rate of flow $i$, $R_i$, can be written as

$$R_i = \eta W \cdot \log_2(1 + \frac{P_iG_{i,i}d_{i,i}^{-\gamma}}{N_0 + b\sum_{l\neq i} P_lG_{l,i}d_{l,i}^{-\gamma}}).$$ (3.4)

We assume that the scheduler can estimate the link data rate using the above equation.

### 3.1.3 Utility Function

Utility shows the satisfaction level of users corresponding to the amount of bandwidth allocated by the network. A series of utility functions have been proposed to describe the characteristics of different kinds of traffic. To deal with heterogeneous applications, we classify the traffic into three classes.

One class of traffic is for constant bit-rate (CBR) applications [35]. For these applications, such as audio streaming, the utility of a user remains very low when the expected bandwidth is not allocated; once the bandwidth exceeds a threshold, the utility will increase rapidly, and more results a minimal increase in utility. In this thesis, we use an exponential function to approximate such utility function.

Another type of traffic is for rate adaptive applications. For these applications, such as video streaming, the utility increases slowly when the allocated bandwidth is less than the minimum bandwidth requirement; it increase fast when the allocated bandwidth is greater than the minimum requirement; it increases slowly again when the allocated bandwidth is increased further. In this thesis, we use logistic functions to model the utility for rate adaptive applications.

For applications with no minimum bandwidth requirement such as many TCP controlled data applications, the utility increases almost linearly when the allocated bandwidth is larger than 0, and increases slowly after the allocated bandwidth is over
provisioned. By carefully tuning the parameter of the logistic function, the utility of the above three types of application can be modeled by the following function:

\[
U = \frac{1}{A + Be^{-(K(R - R_{cr})}}} - \frac{1}{A + Be^{KR_{cr}}},
\]  

(3.5)

where \( A, B, K \) and \( R_{cr} \) are the parameters describing the amplitude, slope and central point. These parameters are determined by the characteristics of the application.

In this thesis, we carefully choose the parameters to best reflect the utility of the types of applications. Examples of utility functions for different applications is shown in the Fig. 3.3.
3.2 Problem Formulation

With different optimization purpose, we formulate several non-convex optimization problems according to the system model described in the last section. We first formulate the network throughput and utility maximizing problems as (P1) and (P2). Then, we consider about the fairness control for both throughput and utility optimization problems and formulate (P3) and (P4).

3.2.1 Maximizing Network Throughput and Utility

To maximize the total network throughput, the problem can be formulated as the following constrained optimization problem.

**Problem 1. (P1)**

\[
\begin{align*}
\max & \sum_{i=1}^{N} R_i(P) = \sum_{i=1}^{N} & \eta W \cdot \log_2(1 + \frac{P_i G_{i,i} d_i^{-\gamma}}{N_0 + b \sum_{l \neq i} P_l G_{l,i} d_l^{-\gamma}}) \\
\text{s.t.} & 0 \leq P_i \leq P_{\text{max}}; i = 1, 2, 3, \ldots, N,
\end{align*}
\]

where \( P_{\text{max}} \) is the maximum transmission power. This is a constrained non-convex (non-concave) problem, and no algorithm exists to effectively solve it.

If a scheduling algorithm is used without power control, the algorithm controls which flows transmit in a time slot, and all transmissions are at the maximum power level \( P_{\text{max}} \). The problem without power control is also non-convex, so it is just as difficult to solve. In addition, the performance in terms of throughput may be degraded without power control.

For heterogeneous traffic, such as combined flows of audio and video transmission, the maximum network throughput does not necessarily lead to maximum network utility, due to the utility function for each type of application is different. Thus, we
formulate the NUM problem as follows

**Problem 2. (P2)**

\[
\max \sum_{i=1}^{N} U_i(R_i(P)) \\
\text{s.t. } 0 \leq P_i \leq P_{\text{max}}; i = 1, 2, 3, \ldots N,
\]

where the throughput of flow \( i \), \( R_i \), is a function of each transmission power vector \( P \) as show in (3.4).

The utility functions to model different application traffic are various. For each flow \( i \), the utility function can be written as follows

\[
U_i(R_i) = \frac{1}{(A_i + B_i e^{-K_i(R_i - R_{cri})})} - \frac{1}{(A_i + B_i e^{K_i R_{cri})}}.
\]

where the parameters \( A_i, B_i, K_i \) and \( R_{cri} \) describe the characteristics for the \( i \)-th flow.

### 3.2.2 Fairness Consideration

As different flows may have drastically different average link qualities, so do their achievable data rates. If we only consider maximizing the total throughput or utility, some flows may be starved. Therefore, we need to further consider how to maintain a level of long term fairness and avoid the starvation situation. We take the approach of weighted fair queueing (WFQ) [36]. The channel time is divided into equal-duration time slots. For each slot, we optimize the power level of all flows to maximize the sum of the weighted flow throughput and utility. To ensure fairness, the weight of each flow is determined as follows [36]

\[
\rho_{k,i} = \frac{w_i}{(\sum_{m=1}^{k-1} F_{m,i} + \epsilon)^{\alpha}},
\]
where $\epsilon$ is a small positive scalar in order to prevent a zero denominator, $w_i$ is a constant coefficient assigned to flow $i$ for providing differentiated services, and $\alpha$ is a parameter to make a tradeoff between fairness and network throughput. For the $k$-th slot, this dynamic flow weight, viewed as a control variable, is adjusted according to the total throughput or utility of flow $i$ in the previous $k - 1$ time slots, i.e., $R_1 + R_2 + \cdots + R_{k-1}$ or $U_1 + U_2 + \cdots + U_{k-1}$, $F_{m,i}$ represent throughput $R_i$ or utility $U_i$ in time slot $m$.

For time slot $k$, the power allocation and scheduling decision is represented by a control vector $P_k = (P_{k,1}, P_{k,2}, \cdots, P_{k,N})$, where $P_{k,i}$ is the transmission power for node $i$ in time slot $k$. Denote by $R_{k,i}$ the throughput of flow $i$ in time slot $k$. We have

$$R_{k,i} = \eta W \cdot \log_2(1 + \frac{P_{k,i} G_{i,i}^{-\gamma}}{N_0 + b \sum_{l \neq i} P_{k,l} G_{l,i}^{-\gamma}}), \quad (3.12)$$

Denote by $U_{k,i}$ the throughput of flow $i$ in time slot $k$. We have

$$U_{k,i}(R_{k,i}) = \frac{1}{(A_i + B_i e^{-K_i(R_{k,i} - R_{cri})})} - \frac{1}{(A_i + B_i e^{-K_i(R_{cri})})}, \quad (3.13)$$

Considering practical transmission power constraints, to maximize weighted network throughput considering long-term fairness, the problem of optimal power allocation and scheduling can be formulated as follows.

**Problem 3. (P3)**

$$\max \sum_{k=1}^{N} \sum_{i=1}^{N} \rho_{k,i} \cdot R_{k,i} \quad (3.14)$$

$$\text{s.t.} \quad 0 \leq P_{k,i} \leq P_{\text{max}}; \quad (3.15)$$

To maximize the weighted total network utility with long-term fairness, the prob-
Problem 4. \((P4)\)

\[
\max_{\{0,1\}} \sum_{k=1}^{N} \sum_{i=1}^{N} \rho_{k,i} \cdot U_{k,i}(R_{k,i}) \quad (3.16)
\]

\[
\text{s.t.} \quad 0 \leq P_{k,i} \leq P_{\text{max}}; \quad (3.17)
\]

We can consider the problem as maximizing the weighted sum of flow throughputs and utilities subject to the power constraints. The weight is to control the fairness for all flows. The initial values of \(\rho_{0,i}\) are all equal to \(1/\epsilon\), for \(i = 1, 2, ..., N\).

The optimization problems \((P3)\) and \((P4)\) are also non-convex (non-concave) problems. In the next chapter, we solve problem \((P1)\) and \((P2)\) first, and apply the technique to solve problems \((P3)\) and \((P4)\).
Chapter 4

Power and Channel Allocation with Fairness Algorithm Design

In Chapter 3, the system model was presented and the power allocation and scheduling problems was formulated as non-convex constrained optimization problems. In this chapter, the primal problem is converted to a simpler approximated model. An algorithm is designed to solve both the network throughput and utility optimization problems considering long-term fairness consideration.

4.1 Problem Conversion

4.1.1 Network Throughput optimization with Fairness

The network throughput maximization problem (P1) has been investigated extensively in the literature. Since no existing software can effectively solve non-convex (non-concave) optimization problems, a typical approach is to convert the non-convex problem into a convex problem or a series of convex problems. Approximation is needed for both kinds of conversion. Then the approximated convex problem or
provides can be solved efficiently with existing optimization tools.

Due to the impact of mutual interference, the expression for flow throughput in (3.6) is complicated. Converting the objective function in (P3) and (P4) to a convex function using a trivial approximation may introduce severe approximation errors, e.g., using a linear function to approximate the log function.

Different from the previous approaches, in this work, we used a two-step conversion so the non-convex primal problem is converted into a series of convex problems with reasonable accuracy. The tradeoff between accuracy and computational complexity can be adjusted by choosing a different order Taylor series approximation in each step.

In this work, we first calculate the first order Taylor series of the SINR for flow $j$ in time slot $k$,

\[
\text{SINR}_i(P_k + \delta_k) \approx \frac{\kappa G d^{-\gamma}_{i,i} P_{k,i}}{N_0 + b \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} P_{k,l}} + \frac{\kappa G d^{-\gamma}_{i,i} \delta_{k,i}}{N_0 + b \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} P_{k,l}}
\]

\[
- \frac{\kappa b G d^{-\gamma}_{i,i} P_{k,i}}{(N_0 + b \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} P_{k,l})^2} \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} \delta_{k,l},
\]

(4.1)

where $\delta_k$ is a vector with very small norm. For a given $P_k$

\[
T_{k,i} = 1 + \left( \frac{\kappa G d^{-\gamma}_{i,i} P_{k,i}}{N_0 + b \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} P_{k,l}} \right) ,
\]

(4.2)

and $T_{k,i}$ is a constant. Define

\[
\epsilon_{k,i} = \frac{\kappa G d^{-\gamma}_{i,i} \delta_{k,i}}{N_0 + b \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} P_{k,l}}
\]

\[
- \frac{\kappa b G d^{-\gamma}_{i,i} P_{k,i}}{(N_0 + b \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} P_{k,l})^2} \sum_{l \neq i}^N \kappa G d^{-\gamma}_{l,i} \delta_{k,l}.
\]

(4.3)
In the second step, we again use a Taylor series to approximate the log term in (3.6) with $T_{k,i}$ and $\epsilon_{k,i}$ for flow $i$. The network throughput for time $k$ is shown as follows

$$
\sum_{i=1}^{N} R_{k,i} \approx \eta W \sum_{i=1}^{N} \log T_{k,i} + \eta W \sum_{i=1}^{N} \frac{1}{T_{k,i}} \epsilon_{k,i}.
$$

(4.4)

With the above two-step approximation, the original objective function is converted into a constant term $\eta W \sum_{i=1}^{N} \log T_{k,i}$ and a variable term with $\epsilon_{k,i}$ as the optimization parameter. The variable term is determined by $\delta_{k,i}$, which must have a very small range to ensure the approximation is reasonably accurate.

To deal with the optimization problem (P3), we need to consider the dynamic weight $\rho_{k,i}$. Define a new constant

$$
I_{k,i} = \frac{\eta W \rho_{k,i} \kappa G}{(N_0 + b \sum_{i=1}^{N} \kappa G d_{l,i}^{-\gamma} \rho_{k,l}) T_{k,i}}.
$$

(4.5)

We have the following linear term

$$
E_k = \eta W \sum_{i=1}^{N} \rho_{k-1,i} \frac{1}{T_{k,i}} \epsilon_{k,i}
$$

$$
= \sum_{i=1}^{N} I_{k,i} (d_{i,i}^{-\gamma} \delta_{k,i} - \frac{bd_{i,i}^{-\gamma} P_{k,i}}{(N_0 + b \sum_{i=1}^{N} \kappa G d_{l,i}^{-\gamma} P_{k,l})} \sum_{l \neq i}^{N} \kappa G d_{l,i}^{-\gamma} \delta_{l,k})
$$

(4.6)

$$
= C \delta_k,
$$

where $I_{k,i} = I_{k,i}$ and $\delta_{k,i} = \epsilon_{k,i}$. Note that $\delta_k$ represents the transmission power to be optimized. At this point, the convex approximation about a fixed power vector $P_k$ has been converted to a local convex linear programming problem. The problem can be expressed in terms of $\delta_k$ as follows.
Problem 5. \((P5)\)

\[
\max_{\delta_{k,i} \in \{\delta_{\min}, \delta_{\max}\}} \quad C\tilde{\delta}_k + V_k, \tag{4.7}
\]

s.t. \[\tilde{\delta}_{\min} \leq \tilde{\delta}_{k,i} \leq \tilde{\delta}_{\max} \quad \forall i,\]

where \(V_k = \eta W \sum_{i=1}^{N} r_{k-1,i} \log T_{k,i}\) is the throughput in time slot \(k\) based on the allocated power \(P_k\), and \(C\) is a vector defined as

\[C = [C_1 C_2 \cdots C_j \cdots C_N], \tag{4.8}\]

where

\[C_j = (P_j^j d_{j,j}^{-\gamma} - \sum_{i \neq j}^{N} \frac{I_{k,i}d_{i,j}^{-\gamma} b P_{k,i} \kappa G d_{j,i}^{-\gamma}}{N_0 + b \sum_{i \neq j}^{N} \kappa G d_{i,j}^{-\gamma} P_{k,i}}).\]

The original non-convex problem around the point \(P_k\) has now been converted into a convex linear programming optimization problem \((P3)\) within a limited range \((\delta_{\min}, \delta_{\max})\). By solving this problem, we obtain a solution \(\tilde{\delta}_{\text{solution}}\) which maximizes the approximated objective function \(C\tilde{\delta} + V\). This leads to a new convex problem with starting point \(P_k + \tilde{\delta}_{\text{solution}}\). These iterative solutions lead to an approximation of the optimal solution of problem \((P3)\). If the approximations are accurate, these iterative solutions should approach the optimal solution of the original problem.

4.1.2 Network Utility optimization with Fairness

To deal with the problem \((P2)\). We first obtain the utility as a function of power. The rate as a function of power is given in \((3.3)\) and the utility as a function of throughput in \((3.13)\). Thus, we can derive the utility-power function as

\[U_i(P) = \frac{1}{A_i + B_i e^{(-K_i R_{cri})}} (1 + \frac{P_i \kappa G d_{i,i}^{-\gamma}}{N_0 + b \sum_{j \neq i} P_j \kappa G d_{i,j}^{-\gamma}}) \frac{K_i \rho W}{\eta d_i^2} - \frac{1}{A_i + B_i e^{K_i R_{cri}}}. \tag{4.9}\]
We applied the technique to get the approximation of the objective function in (P2), that is

\[
\sum_{i=1}^{N} U_{k,i}(P_k) \approx \sum_{i=1}^{N} \left( \frac{1}{(A_i + B_i e^{-K_i(\eta W(\log T_{k,i} + \frac{1}{R_{cr,i}}))})} - \frac{1}{(A_i + B_i e^{K_i R_{cr,i}})} \right)
\]

(4.10)

where \( T_{k,i} \) and \( \epsilon_{k,i} \) are the constant variables given by (4.2) and (4.3), respectively.

To deal with the optimization problem (P4), we use the same technique as for problem (3). Assume we have \( C \), and wish to calculate

\[
C' = C \cdot \nabla U(R)
\]

(4.11)

We can formulate a new problem (P6) as follows,

**Problem 6. (P6)**

\[
\begin{align*}
\max_{\delta_{k,i} \in \{\delta_{\min}, \delta_{\max}\}} & \quad C' \delta_{k,i} + W_k, \\
\text{s.t.} & \quad \delta_{\min} \leq \delta_{k,i} \leq \delta_{\max} \quad \forall i,
\end{align*}
\]

(4.12)

where the constant \( W_k = \eta W \sum_{i=1}^{N} \rho_{k-1,i} \left( \frac{1}{(A_i + B_i e^{-K_i(\eta W(\log T_{k,i} + \frac{1}{R_{cr,i}}))})} - \frac{1}{(A_i + B_i e^{K_i R_{cr,i}})} \right) \),

and

\[
C' = \left[ C'_1 C'_2 \cdots C'_j \cdots C'_N \right],
\]

(4.13)

where

\[
C'_j = \frac{B_j \cdot K_j \cdot (A_j + B_j e^{-K_j(R_{k,j} - R_{cr,j})}) \cdot (I_k d_{j,j}^{-\gamma} - \sum_{i \neq j}^{N} \frac{I_k d_{i,j}^{-\gamma} b P_{k,i} \kappa G d_{i,j}^{-\gamma}}{N_0 + b \sum_{l \neq j}^{N} \kappa G d_{i,j}^{-\gamma} P_{k,l}})}{(A_j + B_j e^{-K_j(R_{k,j} - R_{cr,j})})^2}.
\]
4.2 Optimization Algorithm Design

To solve (P5) based on the idea above, we design an algorithm named Power and Channel Allocation with Fairness (PCAF). For time slot $k$, we first initialize the transmission power of every flow with $P_{ini}$, and calculate the $C_0$ corresponding of the converted problem at this starting point. The algorithm stops when the difference between the current data rate and the previous data rate is less than $\epsilon$. With the interior point optimization algorithm [37], the flow transmission powers $P_{now}$ are adjusted simultaneously to maximize the weighted sum of the flow throughputs within the region $\bar{\delta}_{min}$ to $\bar{\delta}_{max}$. This region should be kept small enough to ensure the accuracy of approximation. The new starting point is set as $P_{now}$ and repeat the optimization algorithm until the difference between the new transmission power and the previous transmission power is less than a threshold $\epsilon$. The dynamic flow weight for fairness control is then calculated. When optimizing the transmission power for each flow in the following slots, the fairness coefficient is updated based on the sum of the flow throughputs. Pseudocode for the PCAF algorithm is given in Algorithm 1.

For network utility optimization with long-term fairness consideration, we follow an approach similar to that for network throughput optimization. Conversion of the primal problem results in the new problem (P6). The new vector $C'$ need to be calculated in order to start the PCAF-NUM algorithm. To reduce the computational complexity, we use $C$ to calculate $C'$ as shown in 4.14 and 4.11. The Pseudocode is given as Algorithm 2.

Note that the PCAF algorithm can only provide an approximation to the optimal solution. Thus, we cannot claim that the maximum network capacity is achieved even with continuous power control and without considering fairness. However, since the proposed solution is based on linear programming, it has a relatively low computational complexity and can be solved in polynomial time.
Algorithm 1 Network Throughput Maximization with PCAF(PCAF-NTM)

1: Initial $\rho_{0,i} = 1$ for $i = 1, 2, ..., N$
2: for $k = 1$ to $N$ (time slot) do
3: Initialize $P_{0,i} = P_{ini}$ for $i = 1, 2, ..., N$
4: Calculate the vector $C_0$ at point $P_0$, calculate $V_0$;
5: Set the stop condition $\epsilon$
6: repeat
7: Set $\delta_0 = \vec{0}$;
8: Calculate the gradient $C_{now}$ at point $P_{now}$;
9: Calculate constant $V_{now}$;
10: max $\delta_{k,i} \in \{(\delta_{min}, \delta_{max})\}$ \{\[C_{now}\delta_{now} + V_{now}\}$

with the interior point algorithm below:
Step 1: input $C_{now}$, initialize $\delta_0$, set loop=0; set tolerance $\epsilon$, evaluate $f(\delta_{loop} = C_{loop}\delta_{loop})$
Step 2: Calculate the moving direction $d_{loop}$ using the PAS method, calculate the step size $\alpha_{loop}$
Step 3: Set $\delta_{loop+1} = \delta_{loop} + \alpha_{loop}d_{loop}$
Step 4: If $\frac{f(\delta_{loop} - \delta_{loop+1})}{\max(1, \delta_{loop})} \leq \epsilon$ Output $\delta_{solution} = \delta_{loop+1}$
Otherwise: Repeat Step 2 and set loop=loop+1
10: Set $P_{now} = P_{now} + \delta_{solution}$
11: until norm ($\delta_{solution}$) $< \epsilon$
12: update the flow weights $\rho_{k+1,i}$;
13: end for
14: Sum the throughput of each flow $\sum_{i=1}^{N} R_i$
15: Update $\rho_{k+1}$
Algorithm 2 Network Utility Maximization with PCAF(PCAF-NUM)

1: Initialize \(\rho_{0,i} = 1\) for \(i = 1, 2, \ldots, N\)
2: for \(k=1\) to \(N\) (time slot) do
3: Initialize \(P_{0,i}=P_{ini}\) for \(i = 1, 2, \ldots, N\)
4: Calculate the vector \(C'_0\) at point \(P_0\), calculate the constant \(W_0\);
5: Set the stopping condition \(\epsilon\)
6: repeat
7: Set \(\delta_0 = \vec{0}\);
8: Calculate the gradient \(C'_{\text{now}}\) at point \(P_{\text{now}}\);
9: Calculate the constant \(W_{\text{now}}\);

\[
\max_{\delta'_{k,i} \in \{\delta'_\text{min}, \delta'_\text{max}\}} \{C'_{\text{now}}\delta_{\text{now}} + Q_{\text{now}}\}
\]

with the interior point algorithm below:

Step 1: input \(C_{\text{now}}\), initialize \(\tilde{\delta}'_0\), set loop= 0; set tolerance = \(\epsilon'\), evaluate \(f(\tilde{\delta}'_{\text{loop}} = C_{\text{loop}}\delta'_{\text{loop}})\)

Step 2: Calculate the moving direction \(d_{\text{loop}}\) using the PAS method, calculate the step size \(\alpha_{\text{loop}}\)

Step 3: Set \(\tilde{\delta}'_{\text{loop}+1} = \tilde{\delta}'_{\text{loop}} + \alpha_{\text{loop}}d_{\text{loop}}\)

Step 4: If \(\frac{f(\delta'_{\text{loop}} - \delta'_{\text{loop}+1})}{\max(1,\delta'_{\text{loop}})} \leq \epsilon'\), output \(\delta'_{\text{solution}} = \tilde{\delta}'_{\text{loop}+1}\)

Otherwise: Repeat Step 2 and set loop=loop+1

10: Set \(P_{\text{now}} = P_{\text{now}} + \delta'_{\text{solution}}\)
11: until \(\text{norm}(\delta'_{\text{solution}}) < \epsilon'\)
12: Update the flow weights \(\rho_{k+1,i}\)
13: end for
14: Sum the flow utilities \(\sum_{i=1}^N U_i\)
15: Update \(\rho_{k+1}\)
Chapter 5

Performance Evaluation

To evaluate the performance of the proposed PCAF algorithms, we have conducted simulations using Matlab. In this chapter, we present the simulation results which show a substantial improvement in throughput and utility, as well as fairness. The results also indicate an important guideline of how to adjust the system parameters to make obtain good tradeoff of network throughput and utility versus fairness.

5.1 Simulation Environment

In the simulations, $2N$ nodes are randomly located in a $10 \times 10 \ m^2$ square region, and they are randomly grouped into $N$ sender-receiver pairs. The simulation parameters follow the typical UWB network settings [28]. The fading gain for every flow in each time slot remains the same, and is set to a constant $G_{i,j} = 1$ in the simulation. The transmission power for each sender is adjustable from 0 to $0.0397 \ mW$. The background noise power spectral density is $3.9811 \times 10^{-9} \ mW/Hz$. We set the path-loss coefficient $\gamma$ to 4 and the processing gain $b$ to 0.01. The parameter settings are summarized in Table I.

From the aspect of throughput maximization, we evaluate our PCAF-NTM algo-
rithm with different node densities by varying the number of active nodes from 2 (1 flows) to 60 (30 flows). The scheduling duration is 15 slots. We repeat 1000 times using Monte Carlo simulations for each number of nodes, and calculate the average results with different random network topologies. We implement the heuristic algorithm based on the concept of exclusive region proposed in [28] for comparison. The exclusive region size is set to 2 m, which is the optimal value calculated according to the analysis in [28]. Another benchmark scheduling scheme is the one-by-one TDMA transmission. For both the exclusive region based scheme and the TDMA scheme, all senders transmit at the maximum transmission power. We use the same network configurations to fairly compare the performance of all scheduling algorithms.

From the aspect of utility maximization, we evaluate the PCAF-NUM algorithm in two ways. First, we evaluate the algorithm in a network with all nodes transmitting video streams using adaptive modulation. The utility function for adaptive video is applied. According to the standards for video transmissions, we choose the critical data rate for the utility function to be 5 Gbps, which is between the standard TV and HDTV video data rates. The utility function as shown in Fig. 5.1 is then

\[ U = \frac{1}{(A + Be^{(-K(R_i-R_{cr})})} - \frac{1}{(A + Be^{(K R_{cr})})}. \]  

(5.1)

where \( A = 1, B = 1, K = 0.5 \) and \( R_{cr} = 5 \).

Second, we evaluate the algorithm in a network with all nodes transmitting 3 types of heterogeneous flows as shown in Fig. 5.2: adaptive video flows, TCP controlled data of flows, and constant-bit-rate flows. The number of each flow type is the same. A separate utility function is used to describe each type of flow.

We evaluate the PCAF-NUM algorithm for adaptive video streams transmission with different node densities by varying the number of active nodes from 2 (1 flow) to 60 (30 flows). For heterogeneous traffic, the number of active nodes varies from 3 (1
Figure 5.1: Homogeneous network user utility vs user throughput.
Figure 5.2: Heterogeneous network user utility versus user throughput
flows) to 36 (12 flows). The scheduling duration is 10 slots. We repeat 1000 times with the same number of flows using Monte Carlo simulation for each number of nodes, and calculate the average results with different random network topologies. We also compare with the round-robin TDMA and the utility maximized TDMA, i.e., the Best Flow Always Transmit (BFAT) scheme. For TDMA and BFAT, all senders transmit at the maximum transmission power. We use the same network configurations to fairly compare the performance of all scheduling algorithms. The utility function is then

\[ U_i = \frac{1}{(A_i + B_i e^{-K_i (R_i - R_{cr_i})})} - \frac{1}{(A_i + B_i e^{K_i R_{cr_i}})}, \tag{5.2} \]

where \( A_1 = 0.75, B_1 = 2, K_1 = 0.4 \) and \( R_{cr_1} = 0 \), where \( A_2 = 1, B_2 = 2, K_2 = 10 \) and \( R_{cr_2} = 5 \), where \( A_3 = 1, B_3 = 2, K_3 = 1 \) and \( R_{cr_3} = 5 \),
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth ($W$)</td>
<td>1G(Hz)</td>
</tr>
<tr>
<td>Center frequency $f$</td>
<td>5.092G(Hz)</td>
</tr>
<tr>
<td>Noise power spectrum density $N_0$</td>
<td>$3.9811 \times 10^{-9}$ (mW)</td>
</tr>
<tr>
<td>Maximal transmission power $P_{max}$</td>
<td>0.0397 (mW)</td>
</tr>
<tr>
<td>Processing gain $b$</td>
<td>0.01</td>
</tr>
<tr>
<td>Path-loss exponent $\gamma$</td>
<td>4</td>
</tr>
</tbody>
</table>
5.2 Network Throughput and Fairness

5.2.1 Network Throughput

We investigate four performance metrics, the total network throughput, minimum flow throughput, total network utility, and minimum flow utility. In this thesis, network throughput is normalized to the average throughput when there is only one flow in the network.

In Fig. 5.3, we compare the network throughputs using TDMA, exclusive region based scheduling, pure scheduling exhaustive search, and the proposed PCAF algorithm with different values of the fairness parameter ($\alpha = 0, 0.5, 1, 2$). We see that the network throughput for TDMA scheduling does not change as the density increases, because there is always only one flow transmitting in each slot exclusively. For the concurrent transmission scheduling algorithms, the network throughput increases almost linearly when the number of nodes in the network is small. Different concurrent transmission scheduling algorithms achieve almost the same throughput when there are less than three flows in the network. As the network density is further increased, the PCAF algorithm can achieve a much higher throughput than that with the exclusive region based scheduling algorithm. In particular, when the flow number of flows exceeds 10, the exclusive region based scheduling does not increase the network throughput much. On the other hand, the PCAF algorithm can better utilize the spatial multiplexing gain, so the network throughput increases almost linearly with the node density. PCAF outperforms the TDMA and exclusive region based algorithm by 850% and 110%, with 15 flows. Compared with exhaustive search for pure scheduling that can be regarded as the performance upper bound of scheduling algorithm, our method has less than 10% margin, but the computational complexity is much less. Another observation is that a larger value of the fairness parameter $\alpha$
leads to a lower overall network throughput, as the flows with low throughput are scheduled more frequently.

### 5.2.2 Minimum Flow Throughput and Fairness Index

In Fig. 5.7, we evaluate the fairness of different scheduling algorithms in terms of the minimum flow throughput among all competing flows. When $\alpha = 0$, the minimum throughput declines rapidly to almost 0. This is because when the number of nodes is small, it is likely that all flows will be allowed to transmit simultaneously to achieve the maximum network throughput. As the number of nodes increases, flows with poor link quality are more likely to be skipped (minimum flow throughput is 0), in order to maximize network throughput. From the figure, by allowing concurrent transmissions, the exclusive region algorithm and PCAF with $\alpha = 0.5$ achieve a higher minimum flow throughput than that with TDMA. The minimum flow throughput is highest when $\alpha = 1$.

We further evaluate the fairness of different scheduling algorithms, in terms of the widely used Jain’s fairness index, defined as

$$f(R_1, R_2, \ldots, R_i) = \frac{\left(\sum_{i=1}^{N} R_i \right)^2}{N \sum_{i=1}^{N} R_i^2}.$$  

From Fig. 5.4, we can see that the PCAF algorithm generally achieves higher fairness index than the exclusive region based algorithm. When there is no fairness constraint, i.e., $\alpha=0$, the fairness index with PCAF is lower than that with TDMA scheme; when $\alpha \geq 0.5$, the fairness index with PCAF is higher than that with TDMA. We note that, when the fairness $\alpha$ is increasing, the overall network throughput is monotonically decreasing. The results are different for the fairness index, as shown in Fig. 5.4. However, unlike in classic job assignment problems, for concurrent transmission power allocation and scheduling. A larger $\alpha$ may not always result in a higher fairness index, e.g., the fairness index is smaller for $\alpha = 2$ compared with $\alpha = 1$. In the long term, the throughputs of different flows will not converge to 1, no matter
Figure 5.3: Network throughput with the TDMA, exclusive region, exhaustive search scheduling and PCAF-NTM algorithms.
Figure 5.4: Fairness index with the TDMA, exclusive region and PCAF-NTM algorithms.
how large $\alpha$ is, which is different from that in classic scheduling problems without concurrent transmissions. With a larger value of $\alpha$, flows assigned more time slots (but sharing with other flows) may not achieve a higher throughput. From our analysis of the throughput of each flow, the reason this occurs is when $\alpha = 2$ a poor quality channel will be scheduled more often, it will affect the whole network performance. On the other hand, our object function is the weighted sum of all flows' throughputs, so even the poor quality channel user has more chances to be scheduled. This user may share the slot with other users which leads to a low throughput for that particular user as shown in Fig. 5.5 and Fig. 5.6. This fact brings the overall fairness down as well as network throughput. Thus, a larger $\alpha$ does not necessary lead to a fairer throughput, but we can choose an optimal $\alpha$ to have both relative high throughput and fairness.
Figure 5.6: Network for Jain’s index with $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$, $\alpha = 2$
Figure 5.7: Minimum flow throughput with the TDMA, exclusive region and PCAF-NTM algorithms.
5.3 Network Utility and Fairness

5.3.1 Network Utility

From the network utility aspect, we evaluate the proposed algorithms with homogeneous traffic and heterogeneous traffic. We compare the network utility using the TDMA, BFAT, exclusive region, and the proposed power and channel allocation with fairness for network utility maximization (PCAF-NUM) algorithms with different values of the fairness parameter ($\alpha = 0, 0.5, 0.8, 1, 2$).

For networks with homogeneous traffic, as shown in Fig. 5.8, the network utility of TDMA scheduling does not change as the node density increases. Since with the TDMA, the network throughput does not change, the per flow throughput decreases inversely proportional to the number of flows, and the network utility remains flat. With the most greedy single-flow scheduling, the BFAT, the network utility is above that for TDMA, but always below that for the proposed PCAF algorithm. For the proposed PCAF algorithms, the network utility increases as the number of nodes in the network increases. The PCAF-NUM algorithm can achieve more than a 500% improvement when the number of flows in the network is 6, compare to the BFAT algorithm which can be considered as an upper bound for exclusive transmission algorithm. A larger value of the fairness parameter $\alpha$ also leads to a lower overall network utility, as the flows with low utility are scheduled more frequently.

5.3.2 Minimum User Utility

We evaluate the fairness of different scheduling algorithms in terms of minimum flow throughput among all competing flows. In Fig. 5.9, for the proposed PCAF-NUM algorithm, a large value of $\alpha$ leads to a higher minimum flow utility. When no fairness is considered, $\alpha = 0$, so the minimum flow utility drops quickly. When $\alpha = 0.5$, the
Figure 5.8: Network utility with the TDMA, BFAT, exclusive region and PCAF-NUM algorithms for a homogeneous traffic network.

The minimum flow utility is better than with TDMA. When $\alpha = 0.8$, the minimum flow utility of PCAF-NUM is better than the exclusive region based algorithm. Note that even when $\alpha = 0.8$, the network utility achieved with PCAF-NUM is much higher than with the exclusive region based and the TDMA algorithms.

For a network with heterogeneous traffic, we note that exclusive region based algorithm achieves a lower network utility than BFAT. The proposed PCAF-NUM algorithm always achieves a higher network utility. This indicates that PCAF-NUM is less sensitive to the node density and outperforms the exclusive region based algorithm when heterogeneous traffic exists in the network. Overall, PCAF-NUM achieves a higher minimum flow utility than the exclusive region based and TDMA algorithm.
Figure 5.9: Minimum flow utility with the TDMA, exclusive region and PCAF-NUM algorithms for a homogeneous traffic network.
Figure 5.10: Network utility with the TDMA, BFAT, exclusive region and PCAF-NUM algorithms for a heterogeneous traffic network.

when the number of flows is no larger than 6. When the number of flows in network increases further, the minimum flow utility of all algorithms approaches 0.
Figure 5.11: Minimum flow utility with the TDMA, exclusive region and PCAF-NUM algorithms for a heterogeneous traffic network.
Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, we have considered the scheduling and power control strategy for concurrent transmissions in broadband wireless networks. We formulated the power and channel allocation problem for both throughput and utility with fairness considerations. We converted the non-convex NP hard problem into a sequential of convex problems and proposed the PCAF-NTM for network throughput optimization and PCAF-NUM network utility optimization. Performance results were presented and shows the proposed algorithm can achieve high network throughput and utility as well as good long term fairness which is much better than TDMA and exclusive region based scheduling. The proposed algorithm can be applied to general broadband wireless networks to improve wireless spectrum utilization and traffic management.

6.2 Future Work

There are quite a lot of related research problems which are still open and worth further investigation.
From the theoretical point of view, to find the global optimal solution of power control and channel allocation for the non-convex throughput and utility maximization problem is still an open issue. Before we can get analytical global optimal solution, it is an optional solution for practical system to choose different starting points to get several local optimal solutions and pick the best one. More effort can be devoted to find a way to choose so called warm-start points, and eventually the analytical global optimization solution.

From the practical implementation point of view, the PCAF algorithms can deal with different network topologies and channel conditions if global position and channel information is known. Power and channel allocation follows a centralized pattern. A decentralized algorithm without requirement of the global topology and channel information will help to improve the practicality. The result for real discrete power control system can be different from the result of our algorithm, integer programming methods can be applied to modify our method to adapt the discrete power control system. There are several possible methods to reduce the computational complexity and the processing delay to adapt dynamic network environment. For iterative methods: optimal step size $\delta_{\text{min}}$ to $\delta_{\text{max}}$ for each converted problems to achieve reasonable accuracy while reduce the number of iterations remains a optimization problem. Non-iterative methods such as global relaxation and other optimization technique to reduce computational complexity is still challenging problem for future research.
Bibliography


