A New Reddening Law for M4

by

Benjamin Hendricks

1. Staatsexamen, Philipps-Universität Marburg (Germany), 2008

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Supervisory Committee

Dr. Peter B. Stetson, Supervisor
(Herzberg Institute of Astrophysics)

Dr. Don A. VandenBerg, Supervisor
(Department of Physics & Astronomy)

Dr. Kim A. Venn, Departmental Member
(Department of Physics & Astronomy)

Dr. Jaymie M. Matthews, External Member
(Department of Physics & Astronomy (UBC))
ABSTRACT

We have used broad-band near infrared photometry in combination with optical Johnson-Cousins photometry to study the dust properties in the line of sight to the Galactic globular cluster M4. These data have been used to investigate the reddening effects in terms of absolute strength, distribution and variations across the cluster field, as well as the shape of the reddening law defined by the type of dust. All three aspects were poorly defined for this system and therefore there has been controversy about the absolute distance to the globular cluster which is closest to the sun.

Here, we introduce a new method to determine the ratio of absolute to selective extinction ($R_V$) in the line of sight toward resolved stellar populations, which is known to be a useful indicator for the type of dust and therefore characterizes the applicable reddening law. This method is independent of age assumptions and appears to be significantly more precise and accurate than existing approaches. In a first application, we determine $A_V/E(B-V) = 3.76 \pm 0.07$ (random error) for the dust in the line of sight to M4 for our set of filters. That corresponds to a dust-type
parameter $R_V = 3.62 \pm 0.07$ in the Cardelli, Clayton & Mathis (1989) reddening law. With this value, the distance to M4 is found to be $d = 1.80 \pm 0.05$ kpc, corresponding to a true distance modulus of $(m - M)_0 = 11.28 \pm 0.06$. These uncertainties do not include possible systematic errors in the theoretical isochrones.

A reddening map for M4 has been created which reveals a spatial differential reddening of $\delta E(B - V) \geq 0.2$ mag across the field within 10' around the cluster centre; this is about 50% of the total mean reddening, which has been determined to be $E(B - V) = 0.37 \pm 0.01$.

In order to provide accurate zero points for the extinction coefficients of our photometric filters, a computer code has been written to investigate the impact of stellar parameters such as temperature, surface gravity and metallicity on the extinction properties and the necessary corrections in different bandpasses. Using both synthetic ATLAS9 spectra and observed spectral energy distributions, we found similar sized effects for the range of temperature and surface gravity typical of globular cluster stars: both cause a change of about 3% in the necessary correction factor for each filter combination. Interestingly, variations in the metallicity cause effects of the same order when the assumed value is changed from the solar metallicity ([Fe/H] = 0.0) to [Fe/H]=−2.5. Our analysis showed that the systematic differences between the flux of a typical main-sequence turnoff star in a metal poor globular cluster and a Vega-like star are even stronger ($\sim 5\%$).

We compared the results from synthetic spectra to those obtained with observed spectral energy distributions and found significant differences in detail for temperatures lower than 5 000 K. We have attributed these discrepancies to the inadequate treatment of molecular bands in the $B$ filter within the ATLAS9 models. Accordingly, for those cooler temperatures we obtained corrections for temperature, gravity and metallicity primarily from the observed spectra. Fortunately, these differences do not affect our principal astrophysical conclusions in this study, which are based on stars hotter than 5 000 K.
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DEDICATION

Black and white and yet you make my life so colourful.
Chapter 1

Introduction

“Unfortunately, absolute age measurements are still affected by a large number of uncertainties, in particular due to significant uncertainties in globular cluster distances and reddening values.” (D’Antona et al. 2009).

Globular cluster systems rank among the oldest objects known in our Galaxy. Their ages set a lower limit for the age of the Universe. Unfortunately, a precise measurement of the absolute age of a globular cluster (GC) is strongly dependent on the knowledge of its precise distance and reddening (Gratton et al. 2003; Bolte & Hogan 1995).

M4 is a peculiar cluster in terms of interstellar reddening. It has a surprisingly large total amount of reddening for its relatively high galactic latitude ($b = 16^\circ$) and small distance ($\sim 2$ kpc), which is due to its location behind the Sco-Oph cloud complex. Moreover, the cluster suffers from a significant amount of spatially differential reddening (Cudworth & Rees 1990; Drake, Smith & Suntzeff 1994; Ivans et al. 1999) which is already taken into account in some CMD studies of M4 in recent publications (e.g. Marino et al. 2008, Mucciarelli et al. 2011). In the literature, approximations for peak-to-peak differences within a radius of the cluster centre larger than $10'$ range from $\delta E(B-V) = 0.05$ in Cudworth & Rees (1990) to $\delta E(B-V) = 0.25$ in Mucciarelli et al. (2011).

Furthermore, using a canonical reddening law with $R_V = 3.1$, we are not able to fit isochrones to the observed fiducial sequences in different filter combinations with a consistent assumption of $E(B-V)$, which leads us to the suspicion that the reddening law of M4 significantly differs from the standard value of $R_V \approx 3.1$ for the diffuse interstellar medium, as it is assumed for most galactic objects. However, problems
with the model $T_{eff}$ scale or the adopted colour-temperature transformations (CT-transformation) might also explain this difficulty, at least in part.

There are several hints in former studies of this cluster where the authors point out an abnormal dust type for M4, or at least remark on the discrepancies that arise when using $R_V = 3.1$ or similar values. For example, [Ivans et al. (1999)] compare their spectroscopic derived temperatures of bright RGB stars to the temperatures derived with photometric indices and find $R_V = 3.4 \pm 0.4$. [Dixon & Longmore (1993)] propose an $R_V$ “closer to 4 then to 3” by evaluating the relative location of RGB sequences from M4 compared to M3, M13 and M92 using data from [Frogel, Cohen & Persson (1983)]. Interestingly, the only study which does not rely on photometric magnitudes reaches a similar result: [Peterson, Rees and Cudworth (1995)] derived the distance modulus of M4 from proper motion and radial velocity measurements and find a value significantly smaller than the accepted value, a result which is only in agreement with canonical HB magnitude measurements by using a reddening law with $R_V \approx 4$. In addition to direct measurements of $R_V$ for M4, there are several indirect measurements for the Sco-Oph dust cloud complex which yield values of $R_V$ around 4 (see e.g., [Clayton & Cardelli 1988] or [Vrba, Coyne & Tapia 1993]). For a summary of derived $R_V$ values for M4 in the past, see Chapter 5.4.

Some publications on M4’s CMD already use a non-canonical higher value of $R_V$. [Richer et al. (1997)] (and later [Richer et al. 2004]) were the first to use a value of $R_V = 3.8$ for their distance estimate when determining the age of M4 from the white dwarf cooling sequence. However, this choice is solely based on the vague statements of [Peterson, Rees and Cudworth (1995)], and [Vrba, Coyne & Tapia (1993)] who only estimate the value to be “around 4”. Later, [Hansen et al. (2004)] and [Bedin et al. (2009)] follow the argumentation of Richer and furthermore use the results from [Clayton & Cardelli (1988)], who found a value of $R_V = 3.8$ for a star only one degree away from the line of sight of M4 to justify their choice. However, [Mathis (1990)] claims in his review about the properties of interstellar dust that “it is not possible to estimate $R_V$ quantitatively from the environment of a line of sight”. Moreover, this value for $R_V$ is based on the measurement of only one star using obsolete stellar models to estimate atmospheric parameters.

M4 is the closest GC to the sun. Its sparseness and its short distance make the stellar population accessible to especially deep photometric and spectroscopic analyses. For example, M4 hosts one of the largest populations of identified white dwarfs (WD), making it attractive for absolute age determinations and an excellent
laboratory for testing stellar evolutionary theory (e.g. Richer et al. 1997). It has a significant population of RR Lyrae stars and it shows a bimodal HB, with a well populated blue and red part, making it an interesting object to study the so called “Second Parameter Problem” (see, e.g., the review by Catelan 2009b or some recent work by Marino et al. 2011).

There is clearly a need for a detailed investigation of the specific, absolute reddening and the specific reddening law of M4, providing more precise information about its dust type, the effects of spatial differential reddening across the cluster face, and consequently its distance. By unraveling these uncertainties, which are the reason for the controversial and inaccurate cluster distance and reddening estimates in the past, M4 will increase its status as a most attractive object to investigate.

In this study, we introduce a new method to determine the dust type in the line of sight to a stellar population with high quantitative precision. With this method, we determine the properties of the dust in direction to M4 characterized by the appropriate value of $R_V$ and the assumed validity of the general reddening law given in Cardelli, Clayton & Mathis (1989). For that, we use near infrared (NIR) $J$ and $K_s$ photometry for the clusters M4 and NGC 6723 and combine those data with optical Johnson-Cousins $UBVRI$ photometry.

We want to determine the variations in the reddening across the face of the field of M4 that has been surveyed and then correct for those differential effects in order to reduce the observed scatter in the CMD and therefore increase the precision of parameters such as distance or age.

We investigate differential changes in the extinction properties of optical and NIR filters and filter combinations for stars with different intrinsic spectral-energy distributions as defined by their temperature, surface gravity and metallicity. We further want to test the consistency of the results obtained with synthetic and observed stellar fluxes by comparing synthetic spectra from the ATLAS9 library (Castelli & Kurucz 2003) to observational databases from Pickles (1998) and Sánchez-Blázquez et al. (2006). These results are supposed to be used to define an object-specific reddening law for M4 where the correction zero points are tailored for the atmospheric and chemical parameters of this cluster instead of using a Vega-like star as is the case for most literature values. This will be necessary to avoid the introduction of systematic errors to the determined value of $R_V$.

At last, we use the newest set of Victoria-Regina isochrones (VandenBerg et al. 2012) together with our optical and NIR photometry to confirm an abnormal dust
type for M4 and determine the value of $R_V$ with an observational precision several times higher than the estimates of previous studies. Implications for the absolute distance of M4 are discussed.

In Chapter 2 we summarize the data and data reduction process together with the criteria for our high-quality photometric sample. The procedure of generating a reddening map for M4 and the spatial differential reddening corrections is explained in Chapter 3. In Chapter 4 we investigate the effect of temperature, surface gravity, metallicity and extinction on the shape of the reddening law and calculate an object-specific law tailored for M4. Further, we discuss whether a constant law is sufficient to determine cluster parameters from photometry, or whether a star-by-star correction is required to avoid significant systematic errors. In Chapter 5 we introduce our new method to determine the dust type parameter $R_V$ for a stellar population, explaining the crucial parameters, and discuss random and systematic errors in detail. This method is then used to derive the dust type of M4 and its absolute distance. Finally, the summary of our work is given in Chapter 6 together with a discussion about the significance of the findings. The data reduction summary for NGC 6723, details about the spectral databases we used in Chapter 4 as well as extinction correction tables for different temperatures, metallicities and types of dust can be found in the Appendix A, B, and C respectively.
Chapter 2

Data and Data Reduction

2.1 Introduction

For this work, we use near infrared (NIR) $J$, $H$ and $K_s$ ground-based photometry for the globular clusters M4 (NGC 6121) and NGC 6723. The images for M4 were taken in 2002 with the large and the small field of SOFI, the infrared imaging camera on the New Technology Telescope of ESO, on Cerro La Silla in Chile as part of the program 69.D-0604(A) with principle investigator M. Zoccali. The images for NGC 6723 were taken in 2005 with the same telescope and instrument as part of the program 075.D-0372 with principle investigator F. R. Ferraro. The pre-processing of the CCD images for both clusters was done by our Italian collaborators (Massimo Dall’Ora), including flat-field corrections, bad pixel masking, bias and dark frame subtraction as well as sky subtraction. The latter is a major challenge and a quality-limiting factor in NIR photometry, since it is affected by the background thermal noise of our atmosphere to a much higher extent than, for example, the Johnson-Cousins $UBVRI$ filters in the optical wavelength range.

We use the data reduction packages DAOPHOT II (Stetson 1987; Stetson 1988) and ALLFRAME (Stetson 1994) together with programs therein to obtain the instrumental photometry for each cluster from the pre-processed CCD images and transform it to the 2MASS standard photometric system (Skrutskie et al. 2006).

Since the data reduction process for our two clusters is very similar, we will describe this work in detail for M4 only as an example. The most important corresponding numbers and figures for the reduction process of NGC 6723 are also recorded in the Appendix A.
Further, we make use of $K_s$ photometry of the globular cluster NGC 1851 which has been published by C. Brasseur (Brasseur et al. 2010) from observations with the Very Large Telescope of ESO on Cerro Paranal. For the data reduction process of NGC 1851 and the calibration to the standard system reference may be made to the aforementioned paper.

2.2 Instrumental Photometry

Figure 2.1 shows the distribution of stars from our data in the globular cluster M4. Observation fields for both $J$ and $K_s$ filters are over-plotted on the data in different colours to visualize the coverage of the cluster. In total, 50 frames for each filter were taken with a main focus on the central region of the cluster, which was observed 10 times, and one region outside the core, which was covered 20 times. The rest of the cluster is only covered by one exposure in each filter with slight overlap. The distribution of the frames over the cluster field has an important impact on the final photometric standard error of a star, since this value is a weighted mean of all individual detections. A large number of frames per observing field therefore decreases the photometric error of stars therein.

Blue dots indicate stars with no photometric information available in one or both filters. In most cases these are stars which were only been detected in one of the two NIR filters and we exclude them from our final photometric sample.

To extract the instrumental photometry of each individual frame, we define the best fitting point spread function (PSF) for the stars. The use of a PSF model is necessary for the best possible estimation of the stellar flux beyond the sky background noise and is particularly advantageous in photometry of crowded fields such as those containing GCs to accurately obtain photometry for blended stars. Since the PSF depends on such things as atmospheric conditions, which change on short timescales, we use for each exposure an individual PSF defined by a combination of an analytical Gaussian model and an interpolation table of empirical residuals. We further use a PSF which varies quadratically with the position in the frame, due to the fact that the SOFI large-field camera is not properly aligned, so that the images are radially elongated in the left part of the frame. This distortion affects a vertical strip of about 150 pixels and smoothly disappears toward the centre of the frame.

Figure 2.1: Distribution of detected stars in the globular cluster M4. The coloured boxes roughly indicate the different exposure frames used for the $J$ (upper panel) and $K_s$ filters (lower panel). There are 50 exposures taken in total, concentrating on two fields (the cluster centre and one field in a sparse outer region) where 10 and 20 integrations were taken respectively. The axes describe the distance for each star in arcsec to an arbitrary zero point at RA = 16h 23m 15.12s, Dec = 26°25’ 15.4”. Blue dots indicate stars with no photometric information in one or both bands. Large coloured dots mark the centre of each exposure.
study, we will show that we are able to correct the residual spatial inhomogeneities by applying a differential reddening correction (see Chapter 3).

The PSF is calculated using a sample of bright, isolated program stars, uniformly distributed over the individual frames. Although we have written a pipeline to process all frames through DAOPHOT and ALLFRAME automatically, we pick out these stars for each frame by eye to assure their quality and rule out blending. The same stars will later serve as local standards to calibrate our instrumental photometry to the 2MASS standard system.

So far, the positions of stars in each frame are only defined by their relative $x$ and $y$ coordinates within the exposure. We use DAOMATCH and DAOMASTER to find positional transformation equations between the frames, accounting for transversal offsets as well as for differences in scale and rotation. Those equations are then used to match up stars between different frames in order to get a complete starlist for our photometry.

Since every frame has different observational conditions, each frame may have different photon counts for the same star. To calibrate the frames among themselves, DAOGROW (Stetson 1990) is used to obtain the total integrated instrumental magnitudes (i.e., the total photon count) for our standard stars by multiple aperture photometry. Once we know the transformation equations between the frames and the calibrated photometry for each, we obtain cumulative instrumental magnitudes for all stars in M4.

2.3 Calibration to the Standard System

The instrumental photometry of M4 finally needs to be calibrated to the 2MASS standard system. For accurate comparisons of our observed data to stellar evolutionary models, it is especially important that systematic deviations between instrumental and standard magnitudes are small.

Here, the same hand-selected, bright and isolated program stars are now used to serve as local standards for our photometry. As an additional criterion, we only use stars below the 1% linearity limit of the detector ($\leq 13000$ ADU), and only those that could be cross-identified with a 2MASS standard star with a coordinate deviation less than 0.1′′. For M4, these are about 360 stars covering the whole observation field. Since they are furthermore distributed homogeneously over the different frames, photometric differences between different exposures as well as spatial inhomogeneities
within individual fields can be detected and compensated for (see Figure 2.2).

Before the local standards in our photometry can be matched up with 2MASS standard stars, the spherical coordinates of the 2MASS stars need to be transformed into the flat coordinate system of our CCDs. Following Smart (1965), the transformation equations between spherical right ascension ($\alpha$) and declination ($\delta$) to a flat system ($\xi, \eta$) are given by

$$\xi = \frac{\sin(\alpha - \alpha_0)}{\sin \delta_0 \tan \delta + \cos \delta_0 \cos(\alpha - \alpha_0)}$$

(2.1)

and

$$\eta = \frac{\cos \delta_0 \tan \delta - \sin \delta_0 \cos(\alpha - \alpha_0)}{\sin \delta_0 \tan \delta + \cos \alpha_0 \cos(\alpha - \alpha_0)}$$

(2.2)

where $\alpha_0$ and $\delta_0$ define the zero-point right ascension and declination used in the CCDs. An additional correction is applied to correct for distortions arising for stars with an angle close to 90° from the centre of the celestial sphere. The final $x$ and $y$ coordinates are therefore derived from $\xi$ and $\eta$ with the following equations:

$$x = \frac{\arctan \rho \xi}{\rho}$$

(2.3)

$$y = \frac{\arctan \rho \eta}{\rho}$$

(2.4)

where

$$\rho = \xi^2 + \eta^2.$$  

(2.5)

The equation that we use to transform the instrumental magnitudes ($m$) to the standard system ($M$) is of the general form

$$m_1 = M_1 + z_c + a_c \times X + c_c(m_1 - m_2)$$

(2.6)

and includes a zero-point correction ($z_c$), a term for atmospheric extinction ($a_c$) where $X$ is the airmass, and a colour-sensitive term ($c_c$) to account for differences in filter characteristics between the two systems.

DAOMASTER is used to match up 2MASS stars with corresponding local standards in our photometry and CCDSTD finds the best fitting solution for this equation.
by least-squares computation of the coefficients.

Once the transformation equations to the standard system are defined for each filter with the standard star sample, they are applied to all of the stars in the cluster.

### 2.3.1 Photometric Consistency

A comparison between our photometry and the 2MASS standard system is shown in Figures 2.3 and 2.4 where the deviation between the two systems is plotted as a function of magnitude and colour. There is neither a significant zero-point offset between the two systems nor a clear trend with colour which would arise from insufficient correction of the different filter characteristics. Only the $J$-band data might show a small trend in colour, indicating that our photometry is slightly too bright for warmer temperatures. However, this effect would only affect the very coolest and hottest stars and only by a systematic offset smaller than 0.05 mag.

### 2.3.2 Optical Photometry

We combine our NIR $J$ and $K_s$ photometry with Johnson-Cousins $UBVRI$ photometry provided by Peter Stetson. The optical data for M4 is a compilation of mainly archival data from 44 independent photometric nights distributed among 11 different observing runs. They have been obtained between 1994 and 2007 with several different ground-based telescopes ranging from 0.9 m to 3.6 m, and were analyzed and calibrated as described in Stetson (2000) and Stetson (2005a).

In total, the optical database covers a field of $25' \times 25'$ around the cluster centre, which is about four times the area that we cover with our NIR photometry. The database consists of $\sim 81,000$ stars and the photometry reaches down to $\sim 25$ mag, $\sim 23.5$ mag, and $\sim 21$ mag for the $B$, $V$, and $I$ bandpasses respectively.

We use DAOMASTER to match up stars between the two photometric sets, where only stars are considered for which the coordinate deviation between the optical and the NIR database is smaller than 0.1".

### 2.4 Selection of a High-Quality Sample

The initial output of the photometry packages DAOPHOT and ALLFRAME yields a total of $\sim 21,000$ stars with photometric information in both $J$ and $K_s$ and in at least
Figure 2.2: Large red dots indicate the location of bright, isolated program stars in the field of M4 which serve as local standards. They are used to calibrate our photometry to the 2MASS standard system.
Figure 2.3: Photometric differences between our photometry for M4 and 2MASS standard stars where $\Delta J = J - J_{\text{2MASS}}$ and $\Delta K = K - K_{\text{2MASS}}$. Only stars are shown that could be cross-identified with a 2MASS star to within $0.1''$. The solid black line indicate a theoretical zero offset. Stars used as local standards are viewed in orange, green dots indicate all other identified 2MASS stars from our final high-quality sample (see following paragraphs).

Figure 2.4: Photometric differences between our photometry for M4 and 2MASS standard stars as a function of colour to examine whether there are any trends with temperature.
one of the optical bands (which is usually $B$, $V$ and $I$ for the majority as well as $U$ and $R$ for some stars). This sample still includes stars with high photometric errors, blended stars, field stars in the line of sight to the actual GC system, variable stars, as well as poor cross-identifications between the optical and the NIR databases.

Since we are more interested in photometric precision than in photometric completeness, a high-quality subsample (hereafter: HQ-sample) was selected taking into consideration the aspects mentioned above.

### 2.4.1 Photometric Standard Error

Photometric uncertainty arises from a combination of inadequacies in the PSF fitting process and sky background subtraction during the image reduction as well as from Poisson photon statistics for a star observed on different frames. Since photometric uncertainty usually increases with larger magnitude due to a smaller signal-to-noise ratio for fainter stars, it is not appropriate to use the same upper uncertainty limit for all stars. Therefore a rejection function has been defined by choosing about 10 different upper limits depending on magnitude which then have been connected by linear interpolation. This has been done for each relevant filter ($B$, $V$, $I$, $J$, $K_s$) individually to account for the different characteristics of each bandpass and the different observing conditions for each dataset. The selected stars together with the particular rejection functions are plotted in Figures 2.5 and 2.6 for $BVI$ and $JK_s$ respectively.

It is important to note that we do not automatically choose a lower boundary for brighter stars. There are two main reasons for this: First, one focus of this work is on the horizontal branch (HB) and its morphological aspects. Consequently, we applied a less strict boundary for HB stars so as to obtain a larger, more complete sample in this evolutionary stage. It further helps to define the observed zero-age horizontal branch (ZAHB) luminosity which is important for the distance and age determination of the cluster. Second, the red giant branch (RGB) of M4 is generally relatively poorly populated. In addition, some pixels in the brighter stars occasionally overcome the saturation limit of the detector which increases their observed photometric standard error. To produce a clear RGB sequence extending as far as possible towards brighter magnitudes, we applied a less strict photometric rejection limit for the brightest stars on the RGB.
In this study we want to compare CMDs with different colours directly to each other. For that reason we do not create individual datasets by applying only those photometric rejections related to the current filter combination. Instead, we select one sample of stars satisfying the photometric error conditions for all filters at one time. The final selection now has the advantage that we can be confident that any relative effects we observe between CMDs of different filter combinations are of a physical nature and not due to different stellar samples.

This approach becomes necessary specifically because we expect significant differential reddening within the cluster (see Chapter 3): in combination with the unequal distribution of exposures over the cluster field of M4, the photometric error of a star is linked with its location in the cluster and therefore with a specific reddening. In the hypothetical case that we used different samples for each filter combination, our CMDs would be biased by the different reddening effects in each sample.

2.4.2 Field Stars

Field stars are stars in the line of sight to an observed GC, but with significantly different distances and chemical properties since they do not actually belong to the system. A significant fraction of field stars can be a problem in the analysis of GC CMDs since they are not located on the evolutionary sequence of the population. Therefore, they can smear the actual sequence and, in the worst case, lead to misinterpretation of observed features. The most secure way of identifying and rejecting field stars is by proper motion (PM) and radial velocity measurements. Richer et al. (2004) conducted PM measurements for main sequence (MS) stars in M4 with a time-shifted series of Hubble Space Telescope observations, and Anderson et al. (2006) provide similar measurements for the whole cluster sequence.

By comparing the total number of stars in our CMDs to those which clearly do not follow the common cluster sequence, it is clear that the fraction of field stars lying in the line of sight to the object is very small for the field of view of our observations. From this ratio, we assume the fraction of field stars to be $\leq 1\%$. Since the concentration of GC members decreases with radius, whereas it is nearly uniform for field stars, the relative occurrence of the latter should be higher for larger radii. We tested the impact of field stars by comparing the whole sample to one with an upper limit for the distance from the cluster centre, and find no significant decrease in the number of stars lying off as compared to the number lying on the sequence,
Figure 2.5: $B$-, $V$- and $I$-band photometric errors as a function of magnitude: Grey dots indicate the initial dataset and the applied rejection function is sketched as a red line with red dots indicating the interpolation points. Stars from the final HQ-sample are indicated as black dots. Note, that not every star below each rejection function makes it into the final HQ-sample because stars have to match rejection conditions for all bands. Note as well, that there are some stars in the final sample lying above the boundary: they are HB stars, which are treated separately and with less strict conditions to preserve a significantly large population to be used as luminosity standard candle.
Figure 2.6: Same as in Figure 8 except for $J$ and $K_s$: Note that the rejection level for RGB stars in $J$ is lower than for stars on the subgiant branch (SGB) having $J \geq 14$ to counteract the poorer photometric quality on the RGB due to saturation effects in our brightest stars.
so that we applied neither a rejection with cluster radius nor by cross-identification with PM stars.

### 2.4.3 RR Lyrae Stars

Since RR Lyrae stars show a time varying luminosity, where each star having its individual phase, they produce a significant magnitude scatter in a location on the HB called the instability strip, and hence are not immediately useful for studying GC isochrone sequences. Instead they add an additional uncertainty to the ZAHB luminosity and consequently to the distance and age determination.

RR Lyrae stars were been identified by their coordinates from the most recently updated database of C. Clement ([Clement et al. 2001](#)), updated in 2011. Here, similar coordinate transformations from spherical (celestial) to plane parallel (CCD) coordinates are necessary to match up the catalog RR Lyrae coordinates with our photometry, and we use the same transformation equations (2.1) and (2.2) which were used to do the match up to the 2MASS standards.

Furthermore, only cross-identifications with a coordinate deviation of less than 1" and only detections lying \( \sim \pm 1 \) mag around the ZAHB V-band magnitude have been considered to be true RR Lyrae identifications. By these criteria, 35 RR Lyrae stars were identified in our M4 sample. **Figure 2.8** shows the variable stars identified in M4 as red circles. Except for a few outliers, these stars fall very well in the predicted instability strip.

### 2.5 What does it look like: The Observed CMDs

With the incorporation of \( U, B, V, R, \) and \( I \) data from Peter Stetson to our own near infrared photometry (\( J \) and \( K_s \)), various possibilities for the combination of these bands in different CMDs become available. Especially interesting is the combination of optical with NIR bands to increase the horizontal (temperature) resolution of the sequence as compared to a pure optical or pure NIR combination. Since the \( J \) and \( K_s \) bands are much less sensitive to interstellar reddening than their optical siblings, a combination of both gives important insights on reddening effects. This is interesting for M4, being such a highly reddened system, but it can yield more generally important information for the treatment and understanding of other reddened systems when fiducial sequences or isochrones are used to determine system
parameters.

The photometry reaches to well below the main sequence turn-off (TO) to a magnitude of \( \sim 22.5 \) in \( V \) and \( \sim 18.5 \) in \( J \). Notably, it reaches below the MS “knee” for NIR photometry which might be an interesting key for the age determination (see [Bono et al. 2010]). On the luminous end, the photometry is limited by the linearity limit of the NIR data which lies at \( \sim 11.5 \) mag in \( V \) and \( \sim 9.0 \) mag for the \( J \) filter.

Both the HB and the RGB sequences are poorly defined (due to a large scatter in magnitude for the HB and a large scatter in colour for the RGB) compared to the well defined SGB and MS. More generally, parts of the sequence are especially affected if they are oriented in a specific direction within the CMD. Additionally, in CMDs with a combination of optical and NIR filters, the colour scatter of the MS increases with luminosity towards the TO. Taking into account only the photometric uncertainty one would expect the result to be the opposite. Furthermore, the scatter on the HB is much stronger in \( V \) than it is in the \( J \)-band, which suggests the inference that those effects are mainly caused by differential reddening instead of photometric error.
Figure 2.7: Location of stars from the HQ-sample in the field of M4: The final selection is not a homogeneous distribution over the whole cluster field, but prefers stars lying in areas with the most exposures (compare to Figure 2.1).
Figure 2.8: Different CMDs resulting from the HQ-sample of M4. Red circles indicate the identified RR Lyrae variable stars in the sample. In the bottom plot, the MS “knee", which is seen only in the NIR, is just visible at $J \sim 18$ mag.
Chapter 3

Differential Reddening

3.1 Introduction

Interstellar extinction is the diminution of radiation by intervening material between the observed object and the observer. The strength of the extinction is a function of wavelength and therefore affects both the magnitude and the colour index in an observed CMD. Furthermore the detailed functional form of the wavelength dependence of extinction depends upon the physical nature of the intervening material. For instance, coarse dust grains produce a different wavelength dependence than fine dust grains (see Figure 3 in Cardelli, Clayton & Mathis 1989). Significant systematic uncertainties will arise if photometric data are not properly corrected for extinction effects, since important GC properties such as distance, metallicity (from, e.g., the slope of the RGB) and age rely on either specific luminosity or colour fiducial points, or on a combination of both. The correction for interstellar reddening proceeds along a reddening vector whose direction depends upon the actual filters used in a specific CMD and the extinction relation between these filters, defined by the reddening law. The effect which is caused by different reddening laws in different filter combinations can be seen in Figure 3.1 where a green arrow indicates a Cardelli et al. law with \( R_V = 3.7 \) and a red arrow represents the same law with \( R_V = 3.1 \).[1]

M4 has the additional problem of suffering spatially differential reddening, which means that the total amount of extinction varies with the location of the stars in the cluster field. Generally, differential reddening induces the stars to scatter much more around the actual sequence than would be expected from their photometric values.

\(^{1}R_V = A_V/(A_B - A_V)\); we introduce this important parameter more detailed in Chapter 4.2.
Figure 3.1: Two isochrones with a different reddening do not show the same offset at every point of the sequence: the broadening due to differential reddening strongly depends on the angle between the sequence and the reddening vector. Furthermore, the direction of the vector and the sensitivity on the reddening law is different for each filter combination. For $V$ vs. $V-K$ for example, the reddening vector is basically independent of the reddening law (right panel) whereas significant differences can be observed in a $V$ vs. $B-V$ plane (left panel). Both panels show isochrones with a reddening difference of $\Delta E(B-V) = 0.05$ mag and reddening vectors that assume $R_V = 3.1$ (green) and $R_V = 3.7$ (red).

uncertainty. Higher uncertainties in the distance and age are the consequence. This additional scatter, however, is not uniformly evident throughout the GC fiducial sequence. In fact, it depends on the angle between the sequence and the reddening vector. Regions of the CMD where the sequence defines a wide angle with the reddening vector show a larger scatter than regions that are almost parallel. The apparent sequence broadening is illustrated in Figure 3.1 for two different filter combinations.

In the following, we correct the photometry of M4 for spatially differential reddening effects with the goal of reducing the additional scatter off the cluster sequence and consequently the observed uncertainty in the cluster locus, which finally improves the precision of our dust type determination.

3.2 Correction Procedure

The general idea for determining and correcting spatially differential reddening across the face of the cluster is to calculate the distance for a sample of reference stars from a fiducial sequence in a CMD along the reddening vector. In theory, this displacement
solely depends upon the individual reddening of the star and is therefore directly correlated with its spatial location in the cluster field. Now, the local reddening value defined by these reference stars can be applied to all program stars in their immediate neighbourhood. Such a correction for spatially differential effects in the photometry potentially includes a first-order correction for poor flat-fielding or spatially varying PSFs within different frames as well, in case they were not modelled appropriately during the data reduction process.

Researchers have tried to correct data for differential reddening effects for at least the last 15 years: Piotto et al. (1999) and later Sarajedini et al. (2007), for example, subdivide the cluster surface in small rectangular cells in order to correct each star in a cell according to the median reddening value derived from the reference stars therein. This approach has the disadvantage that its resolution is strictly limited by the cell size, which has to be chosen as a compromise between the needs to have sufficiently large cells to achieve a statistically significant sample of reference stars—even for the sparse outer regions—and small cells to increase the resolution in the dense interior. If the cells are constant in size, they are not flexible against changes in number density within the cluster. Furthermore, the use of rectangular cells does not allow a realistic representation of the smooth geometry of dust.

Our strategy for correcting differential reddening effects in M4 is a “method of closest neighbours,” where the reddening residual for each program star is determined as the median value of the closest 10 to 30 neighbours for which the residuals have been determined. With this approach, we are able to assign a reddening value on a star-by-star basis, allowing for any dust geometry. The resolution is automatically adjusted to the number density in a given area, determined by the neighbour with the largest distance used. The general method is adopted from Milone et al. (2011) and we will now describe the specific steps that are followed:

In order to obtain the maximal discrimination between the effects of differential reddening and the effects due to photometric error, we start with a $V$ vs. $V-K$ CMD. Compared to a $V$ vs. $V-I$ CMD, the effect of differential reddening is higher by about a factor of two (see Table 4.2). The choice of $V$ vs. $V-K$ provides another important advantage over other combinations: usually the direction of correction depends on the value of $R_V$ that describes the reddening law. This filter combination, however, is highly insensitive to the reddening law as can be seen in Figure 3.1. The reason can be understood after reading Chapter 4: a change in $R_V$ increases the ratio
between $E(V - K)$ and $E(B - V)$ in about the same way as it increases the ratio between $A_V$ and $E(B - V)$ and the two effects cancel out.

In the first step, we rotate the CMD counterclockwise so that the $x$-axis falls along the direction of the reddening vector, in which case the reddening residual of each reference star is given simply by the horizontal offset from the fiducial sequence:

\[
(V - K)_{\text{red}} = \cos \alpha (x - O_x) + \sin \alpha (y - O_y) \tag{3.1}
\]

\[
V_{\text{red}} = - \sin \alpha (x - O_x) + \cos \alpha (y - O_y) \tag{3.2}
\]

For the pivot we choose a point just above the MSTO at $O = (2.5, 16.5)$. The rotation angle $\alpha$ depends upon the choice of filters and, in the case of M4, on the value of $R_V$. It is determined by the extinction in the filters involved:

\[
\alpha = \arctan \frac{A_V}{A_V - A_K}. \tag{3.3}
\]

To define the rotation angle, we use $A_K/A_V = 0.124$ for a reddening law with $R_V = 3.70^2$

We choose reference stars from areas in the rotated CMD where the fiducial line defines a sufficiently wide angle with $(V - K)_{\text{red}}$ to ensure that the effect of differential reddening is not biased too much by photometric scatter. We further exclude the SGB region since here the shape of the fiducial line is more difficult to define compared to the MS or RGB. Since our data have low photometric uncertainty only for specific regions in the field of M4, we use almost all of the stars to be able to cover the majority of the surface area and only exclude those with particularly large error (see Table 3.1).

\begin{table}
\centering
\begin{tabular}{lccc}
\hline
Area & $V_{\text{red}}$ & $V$ & $\sigma(V - K)$ \\
\hline
MS & 16.4 - 17.3 & 16.7 - 18.3 & 0.04 \\
RGB & 14.0 - 15.8 & 13.6 - 15.7 & 0.03 \\
\hline
\end{tabular}
\caption{Selection criteria for reference stars to be used for the correction of spatial differential reddening.}
\end{table}

\footnote{Those numbers are the result of our object-specific extinction calculations in Chapter 4.}
Figure 3.2: The location of our reference stars in the field of M4. Only for areas in the field where we can define such stars will we be able to determine a local reddening value subsequently. The resolution of our reddening map depends upon the number density of the reference stars and is therefore higher close to the cluster centre.

Next, we define an empirical fiducial sequence along the MS and RGB by binning the data in magnitude intervals of 0.3 for the MS and 0.5 for the RGB and calculate the median value for each bin. The fiducial points are determined in the $V$ vs. $V - K$ plane and were then rotated to the new system. The fiducial sequence is defined by cubic spline interpolation between the fiducial points. Figure 3.3 shows the location of our reference stars in the rotated CMD together with the fiducial line so determined.

Now, the residual $\Delta(V - K)_{\text{red}}$ for each reference star along the reddening axis is determined. The result is shown in Figure 3.4. This distribution should not show a slope with $V_{\text{red}}$ if the fiducial line was correctly determined and, in fact, the vertical line corresponding to zero distance from the fiducial passes through the densest concentration. Moreover, there seem to be no significant systematic trends as a function of $V_{\text{red}}$. The small deviation for the very brightest stars on the RGB is caused by the low number density of stars here in combination with the relative large step size between two fiducial points. Since only a negligible small fraction of reference stars are affected, however, the general result will not be biased.

Finally, we use the median value of the closest 10 to 30 neighbours for which the
Figure 3.3: Rotated CMD of M4. The reddening vector is now aligned with the $(V - K)_{red}$ axis. Stars between the dashed lines (containing black points) are used as reference stars to determine the residual to the empirical fiducial sequence (red line) in a region along the MS and along the RGB.
differential reddening has been determined to derive an individual reddening value relative to the fiducial line for each star in the sample. On the one hand, a minimum number of neighbour reference stars is needed to obtain a confident estimate of the local reddening. On the other hand, the distance of those neighbours should be as small as possible to ensure that the reddening information they carry is actually valid for the desired location of each program star. To obtain the best resolution in the dense central regions and simultaneously be able to define differential reddening values for stars in sparse outer regions, we apply four different levels of significance: In a first step, the 30 closest neighbours to each star are selected. In case not all of them fall within a maximum radius of $R_{\text{max}} = 40''$, the process is started again, this time searching only for the closest 25 neighbours. If necessary, the iteration is repeated for 20 and finally 10 closest neighbours until all of them are located within $R_{\text{max}}$. With this approach, we assure the actual astrometric proximity of the closest
neighbours to each of the selected program stars by accepting a lower significance for values derived in sparse outer regions where fewer neighbours can be found in the local environment. All stars for which not all of the 10 closest neighbours are located within the critical radius are not assigned a differential reddening value.

After correcting the data for differential reddening, the new improved CMD is used to redefine the fiducial sequence and, in the next step, $\Delta(V - K)_{\text{red}}$ for the reference and program stars. We iterate this procedure three times until the slope of the fiducial line does not change significantly (see Figure 3.5).

Figure 3.5: After the first differential reddening correction, the location of the fiducial line is slightly different from the original definition. The left and right panels show the first three iteration steps for the MS and the RGB respectively. The fiducial sequence for both areas converges after no more than three iterations.
3.3 Results

3.3.1 Reddening Map

To visualize the spatially differential reddening across the cluster face of M4, we bin the surface in square cells of \(20'' \times 20''\) and calculate the median reddening residual for all stars that fall in this coordinate range and for which a reddening value has been assigned in the previous steps. Hereby cells with at least one “good” star are assigned a transparent colour where the colour intensity increases with increasing absolute value. Cells containing only undetermined stars inside are shaded dark grey. Cells saturate when the differential reddening value becomes larger than \(|\Delta E(B - V)| = 0.05\) (see Figure 3.6).

3.3.2 Corrected CMDs

Figures 3.7 and 3.8 show a comparison between our original CMDs and those corrected for differential reddening. With this correction, we are able to decrease the scatter in our data by about 50%; the reduction in the scatter along the RGB and the HB is especially dramatic. This allows us to determine crucial parameters like the HB luminosity or the mean value of \(E(B - V)\) with a much higher precision.

We are able to determine spatially differential effects for a total area of about \(10' \times 10'\) around the centre of M4. We estimate the size of the differentially reddening by evaluating the median values of \(40'' \times 40''\) cells and find a total range of \(\approx 0.2\) mag between the lowest and highest reddening values within the area analyzed. That is about half of the total mean reddening of M4 \((E(B - V) = 0.35\), according to the Harris catalog (Harris 1996).

Some of the spatially differential effects show strong evidence for systematic photometric errors within individual frames. In particular, the vertical feature at \(x \approx 0\) in Figure 3.6 falls along the boundaries of the two most frequently observed regions. These systematic photometric errors could be a consequence of the PSF variations within SOFI that were mentioned previously. Alternatively, they could be the result of inadequate flat-fielding of the individual images or possibly systematic errors in the 2MASS catalog itself.
Figure 3.6: Reddening map of the GC M4. Blue areas indicate a negative differential reddening compared to the empirical fiducial line and red areas indicate a positive differential effect. Some differential features are presumably caused by PSF variations within observation frames and are not due to differential reddening (see the text).
Figure 3.7: Direct comparison of our HQ-sample of M4 before applying an additional differential reddening correction (left panel) and after such a correction has been applied (right panel). Note, especially that the MSTO, the RGB and the HB are significantly sharper in the corrected version of the CMD. This will help us to define the distance and age of M4 with a higher precision.
Figure 3.8: Detailed view in the MSTO region before (left panel) and after (right panel) a differential reddening correction has been applied. We are able to decrease the observational scatter to about 50% of its original amount when we correct the photometry for spatial differential effects.
3.3.3 Fiducial Points

Some points in a CMD sequence of a globular cluster are especially important for estimating basic cluster parameters such as distance and age. After the correction for differential reddening, these points can now be determined with the highest possible precision. For M4, the most important fiducial points $V_{HB}$ (distance indicator), $\Delta V_{TO}^{HB}$ (age indicator) and $\Delta K_{TO}^{MSK}$ (age indicator, only in NIR) are estimated by eye and are listed below for different bands. The uncertainty is estimated from the observed scatter. The list is supplemented by the corresponding fiducial points for NGC 6723, which are estimated from its HQ-sample without the application of any additional reddening correction since the cluster reddening is very low ($E(B-V) = 0.05$ according to Harris).

<table>
<thead>
<tr>
<th>Fiducial Point</th>
<th>M4</th>
<th>NGC 6723</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{HB}$</td>
<td>13.46 ± 0.03</td>
<td>15.23 ± 0.03</td>
</tr>
<tr>
<td>$V_{TO}$</td>
<td>16.88 ± 0.10</td>
<td>19.05 ± 0.25</td>
</tr>
<tr>
<td>$V_{RC}$</td>
<td>13.57 ± 0.07</td>
<td></td>
</tr>
<tr>
<td>$V_{HB}^{TO}$</td>
<td>3.42 ± 0.11</td>
<td>3.82 ± 0.26</td>
</tr>
<tr>
<td>$K_{TO}$</td>
<td>14.27 ± 0.20</td>
<td>17.40 ± 0.30</td>
</tr>
<tr>
<td>$K_{RC}$</td>
<td>9.97 ± 0.08</td>
<td></td>
</tr>
<tr>
<td>$K_{MSK}$</td>
<td>16.10 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>$K_{TO}^{MSK}$</td>
<td>1.83 ± 0.28</td>
<td></td>
</tr>
<tr>
<td>$(J - K)_{MSK}$</td>
<td>0.89 ± 0.05</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Fiducial Points for M4 and NGC 6723.
Chapter 4

Reddening Law

4.1 Introduction

A reddening law describes the functional dependence of interstellar extinction with observed wavelength. With increases in the total amount of interstellar extinction that an observed object suffers from, minor discrepancies in the applied reddening law become more significant and can lead in the worst case to incorrect implications for distance and for basic atmospheric and chemical parameters.

When we apply the reddening correction factors given in McCall (2004) we are not able to match isochrones with the data in all filter combinations for a consistent assumption of $E(B - V)$ (see the upper panel of Figure 4.1). While the colour offset between the isochrone and the data in $B - V$ is $\approx 0.37$, a much higher value of $\approx 0.42$ becomes necessary to predict the observed colour offset in a $V - K$ CMD, for example. Similar discrepancies are observed when instead of the McCall values the equations in Cardelli, Clayton & Mathis (1989) are used to calculate correction factors for the effective wavelength given in McCall when their dust-type parameter $R_V$ is set to the standard value of 3.1.
Motivated by the inconsistency in the necessary assumption of $E(B - V)$ required to match model evolutionary sequences with observed data in different optical and near infrared colours for M4, we conduct a detailed investigation of the reddening law calculations with the following goals:

After the interstellar reddening has been directly measured from, for example, the colour excess of stars in the $B - V$ plane by comparison to isochrones, the corresponding (theoretical) colour-excess for other filter combinations can be derived with the extinction-relation of the involved filters defined by the reddening law. The final factor $F_{\lambda_1 - \lambda_2}$ is highly sensitive to the effective wavelength of each filter, where changes of only several ångstrom can produce visible shifts in the reddening corrected CMD, depending on the size of $E(B - V)$. One main goal for our reddening law corrections is therefore to calculate the precise total extinction ($A_\Lambda$) in each optical and infrared filter appropriate to the specific temperature ($T_{eff}$), surface gravity ($\log g$), and metallicity ([Fe/H]) for each star in an observed system like M4. The transformation factors so derived can now be tailored for the relatively cool, metal deficient GC stars and we are interested to know how big the difference is compared to extinction values provided in the literature (e.g. McCall 2004; Schlegel et al. 1998).
and Cardelli, Clayton & Mathis (1989) which are mostly derived from significantly hotter O-, B- or A-type stars.

Practically, most reddening corrections are done by assuming a constant value for $F_{\lambda_1} - F_{\lambda_2}$ for every star in the sample. However, this is only a first-order approximation to a more complex dependence on several factors—notably the stellar surface temperature, as pointed out in former studies (e.g. Bessell, Castelli & Plez 1998; McCall 2004; or Girardi et al. 2008). We want to determine the differential changes in the reddening value appropriate to stars of different intrinsic spectral-energy distributions, to determine whether the variations in those reddening corrections are significant with respect to the photometric uncertainty of the photometry, or whether a single reddening correction can be applied independently of the star’s intrinsic properties.

Moreover, the results of the investigation in this chapter will be used in Chapter 5 to determine the dust-type parameter $R_V$ in the reddening law of Cardelli, Clayton & Mathis (1989) and with this the properties of dust in the line of sight to M4. Our reddening law will allow us to keep $R_V$ as a free parameter to test which value best matches the observations. Moreover, the determination of the dust type depends on the zero points for the transformation factors and consequently on the stellar properties. Our reddening law calculations are necessary to provide appropriate zero points for M4 in order to minimize systematic errors in $R_V$.

4.2 A bit of Theory

4.2.1 The Source of Extinction

Interstellar dust is an important constituent of the Galaxy and obscures all but the relatively nearby regions in UV, optical and NIR wavelengths (Mathis 1990). Therefore it is an important factor to consider when analyzing and interpreting observational data. Light at UV, optical and NIR wavelengths is absorbed and reemitted in the far-infrared at wavelengths longer than $\lambda = 60 \mu m$. Here we use “interstellar dust” as an umbrella term for a variety of different materials and compositions; it therefore exhibits a variety of different properties.\footnote{Here, we only discuss the extinction properties of interstellar dust with its associated physical properties.}
The basic dust types can be associated with the state and appearance of interstellar hydrogen. The diffuse interstellar medium (ISM) contains the most common type of dust and populates basically the whole space in our Galaxy. At a gas temperature of $\sim 100$ K it mainly consists of neutral hydrogen. Molecular clouds are typically denser and colder than the ISM and contain mainly $H_2$-molecules. Here the dust grains are typically larger than in the ISM. Regions with significantly higher temperature ($T \geq 10000$ K), where hydrogen occurs in its ionized state $H^+$, are called $H\,\text{II}\,$ regions. Here, typically no dust is found because grains evaporate at those temperatures. However, the outer regions can be surrounded by dust where the temperature is significantly lower.

The continuous extinction for photons traveling through interstellar dust is caused by dust grains composed of mainly graphite and silicate, or a combination (Mathis).

Different extinction properties arise from variations in grain size and composition, as well as from the physical structure of those grains. The extinction caused by interstellar dust is generally a function of photon energy and decreases towards longer wavelengths. In particular, this extinction law ($A(\lambda)$) can be different for each line of sight, depending on the type of dust located between the observer and the object. In general, larger grains show a weaker dependence on wavelength. This means, for example, that the difference between NIR and optical extinction in a cool molecular cloud is smaller than it would be in the hot ISM since we expect the grains to be larger at lower temperatures. As a result, along diverse lines of sight a continuous graduation between the properties of all different basic types of dust has been observed (see Figure 4.2).

This makes it, in principal, necessary to determine an individual reddening law for each object that suffers significant interstellar extinction, before correcting for those effects appropriately without inducing systematic errors. In practice, however, for most objects a standard reddening law is applied, assuming dust characteristics of the diffuse ISM as the main origin of interstellar extinction. The validity of this assumption is part of our investigation.

Cardelli, Clayton & Mathis (1989) quantified the dust type according to its extinction properties through the ratio between absolute-to-selective extinction in the $V$-band ($R_V$) which we introduce in the next section.
4.2.2 Basic Equations

The extinction of a source is determined by how much the observed flux is suppressed relative to what it would have been if there were no dust along the line of sight. In general, the extinction can be expressed as

\[ A_{\lambda} = -2.5 \log\left(\frac{F_{\lambda}}{F_{\lambda}^0}\right) \]  \hspace{1cm} (4.1)

For a broadband filter such as Johnson-Cousins UBVRI and 2MASS JHKs, the total flux which passes through the filter and is measured by the detector depends upon

- The atmospheric transmission
- The mirror reflectivity of the telescope
- The filter transmission function
- The instrumental response (detector quantum efficiency)

The product of these functions considered together may be called *system response function* which we will designate \( T_{\lambda} \).

The second important component to know is the spectral energy distribution (SED) from the observed star. In a broadband filter such as B or V the flux from a hot star is predominantly on the short wavelength side of the bandpass while the flux from a cool star is predominantly on the long wavelength side of the filter. The flux from the hot star may therefore be more strongly attenuated by interstellar absorption than the flux from the cool star.

The effective wavelength of a bandpass is defined as the mean wavelength of the photons ultimately detected. If \( F_{\lambda} \) is the total flux in a given filter, the effective wavelength is given by

\[ \lambda_{\text{eff}} = \int_0^\infty \frac{\lambda F_{\lambda}}{F_{\lambda}} \, d\lambda. \]  \hspace{1cm} (4.2)

To a good approximation the total effect of interstellar extinction integrated across a photometric bandpass can be represented by the interstellar extinction evaluated at the effective wavelength. But all stellar parameters that can possibly have an effect on the SED, such as temperature, surface gravity, and metallicity can change the
effective wavelength of a given filter. Therefore, these parameters should be taken into consideration when calculating its extinction.

Knowing the system response and the stellar flux, the extinction for a given filter is given exactly by

$$A_\Lambda = -2.5 \log\left( \frac{F_{\lambda_{\text{eff}}}(\Lambda)}{F_{\lambda_{\text{eff}}}(\Lambda)} \right).$$  \hspace{1cm} (4.3)

The reddening or colour excess $E(B - V)$ is defined as the difference in extinction between the $B$ and the $V$ filter and can be observationally derived as the difference between the apparent colour $(B - V)$ and the intrinsic colour $(B - V)_0$:

$$E(B - V)_{th} = A_B - A_V$$  \hspace{1cm} (4.4)

$$E(B - V)_{obs} = (B - V) - (B - V)_0$$  \hspace{1cm} (4.5)

The relative strength of extinction in one filter compared to another does not solely depend on the effective wavelength where the observation is made. The physical properties of the dust itself along the line of sight to the observed object have a significant impact too.

A useful parameter to describe the dependence of the reddening on the type of dust is the ratio of total to selective extinction. For the $V$ filter, it has historically been defined as

$$R_V = \frac{A_V}{E(B - V)}$$  \hspace{1cm} (4.6)

where $E(B - V)$ indicates the change of extinction with wavelength for a particular target and $A_V$ is the total extinction in the $V$ bandpass for the same target. This ratio is a direct measure of the average properties of the dust along the line of sight. Cardelli, Clayton & Mathis (1989) found a “fairly tight linear relationship” between $A_\Lambda$ and $R_V$ which seems to be true for all photometric filters ranging from the UV to the NIR. Figure [4.2] taken from their paper, shows examples of the observed relationships between $R_V$ and the absolute extinction at several different wavelengths.
Figure 4.2: The linear relation between the normalized, absolute extinction $A_\lambda/A_V$ and $1/R_V$ is shown. Black dots indicate observed solar-neighbourhood stars with different $R_V$, other symbols are defined in the legend. The data were shifted vertically by a constant number in order to illustrate the different linear slopes for different wavelength. There is a continuous distribution of $R_V$ values, indicating a smooth graduation of properties between different dust types. $A_{12} = 1200\,\text{Å}$, $A_{22} = 2175\,\text{Å}$, $A_{28} = 2800\,\text{Å}$, $A_{70} = 7000\,\text{Å}$. Similar results are observed for effective wavelength in the NIR. The linear relation indicates the possibility to describe the different types of dust with only one parameter: $R_V$. The figure is taken from Cardelli, Clayton & Mathis (1989).

One important aspect of our study will be to compare the colour excess in different optical and NIR filters in order to derive a consistent reddening $E(B-V)$ from all filter combinations, or to explain the discrepancies. For this issue it is convenient to describe the reddening law in terms of the resulting transformation factors between the reddening in $(B-V)$ and any other colour $(\lambda_1 - \lambda_2)$. Those factors are defined by

$$F_{\lambda_1-\lambda_2} = \frac{E(\lambda_1 - \lambda_2)}{E(B-V)} \quad (4.7)$$

which can be calculated from the extinction values for each filter:

$$F_{\lambda_1-\lambda_2} = \frac{A_{\lambda_1} - A_{\lambda_2}}{A_B - A_V}. \quad (4.8)$$
4.2.3 Calculate Extinctions

We have written a computer code to assess the effects of the system response function, stellar SEDs, and the interstellar reddening law on stellar photometric observations. To calculate the actual extinction for each filter, we basically use Eq. (4.1) and express the fluxes with all necessary functions describing the system response and the stellar flux, where every distribution must be given as a function of $\lambda$:

$$A_\lambda = -2.5 \log \left( \frac{\int_0^\infty D_\lambda(\lambda)T_\lambda(\lambda)F_\lambda(\lambda)10^{-A_\lambda} \, d\lambda}{\int_0^\infty D_\lambda(\lambda)T_\lambda(\lambda)F_\lambda(\lambda) \, d\lambda} \right) \quad (4.9)$$

$T_\lambda$ is the response function of the system, including the filter and atmospheric transmission, the telescope mirror reflectivity and the quantum efficiency of the detector.

$F_\lambda$ is the spectral energy distribution from the observed star, which can be individually changed to the desired stellar parameters. The impact of temperature, surface gravity and metallicity on the reddening law can be examined by changing the spectrum used in that equation.

$A_\lambda$ describes the strength of the extinction as a function of $\lambda$ and represents what we call the “reddening law”. This function has $R_V$ as a free parameter so that the extinction and reddening values for different types of dust can be examined and compared.

Finally, $D_\lambda$ transforms the energy distribution of the spectrum from flux units to actual photon counts. If a photon counting detector, such as a CCD chip or a pulse counting photomultiplier, is used for the observations, $D_\lambda = \lambda$. In the case of a detector which measures energy directly, such as a bolometer, $D_\lambda$ becomes 1.0.

In the next section the different components of Eq. (4.9) are specified in detail.

4.3 Ingredients for the Reddening Law

The NIR data in $J$, $H$, and $K_s$ are calibrated to standard stars of the 2MASS system. For this reason we use the system response functions for the 2MASS filters provided in Cohen, Wheaton & Megeath (2003). Fortunately, these authors already combined very carefully all necessary contributions from the optics, detector, atmosphere and filters so that we can use their so-called “spectral response functions” directly as our
In their work, detailed, location-specific atmospheric predictions are implemented and the precise transmission through the optical system of the photometers is reproduced. The authors point out in their paper that those derived functions are well suited for synthetic photometry with the 2MASS photometric system.

For the optical filters and the Landolt standard stars, such a sophisticated implementation of detailed characteristics is not published and therefore we had to account for the basic contributions to the response function ourselves, as described in the next paragraphs.

4.3.1 Filter Transmission

The optical filter transmission curves for $U$, $B$, $V$, $R$, and $I$ standard bandpasses of our photometric system are defined in Landolt (1992). Since only the pure filter transmissions are given here, it is necessary to combine these functions with the atmospheric transmission, the telescope mirror reflectivity and the detector sensitivity to obtain the system response function ($T_{\lambda}$).

4.3.2 Atmospheric Transmission

The atmospheric transmission function has been adopted from Allen (1965). It provides a continuum transmission function with wavelength and does not include fine details of telluric absorptions. This function has been used only for the optical filters $U$, $B$, $V$, $R$ and $I$. For this study, we assume a typical airmass of 1.25 and take into account the elevation where observations are typically made. Major modern observatories typically are located at elevations of 2000 to 3000 meters which is above roughly one third of the sea level atmosphere, so that the final atmospheric transmission is defined as $A_T(\lambda) = (1.0 - 0.66(1.0 - A_T(\lambda_0)))^{1.25}$.

4.3.3 Telescope Reflectivity

The telescope reflectivity has to be taken into account since the telescope mirror does not show a constant reflectivity over all wavelengths. We used the reflectivity of aluminium, taken from Allen (1965), to quantify this effect. Since, for a Cassegrain instrument, the starlight is reflected twice on its way through the telescope, at the primary and the secondary mirrors, we used that function in its squared form to implement it in the system response for the optical bands.
4.3.4 Interstellar Extinction

One of the most crucial components of Eq. (4.9) is the dependence of the interstellar extinction on the wavelength. We are using the very well accepted empirically derived analytical expression from Cardelli, Clayton & Mathis (1989) and specifically their equations (1), (2a), (2b), (3a), and (3b) to describe the extinction function. Since equation (1) has $R_V$ as a free parameter, we are able to derive extinctions for different types of dust.

4.3.5 Stellar Spectra

With the use of different stellar spectra we are able to predict the reddening law for a specific combination of stellar parameters—like temperature, surface gravity and metallicity—and investigate the differential effects for a stellar sample found, for example, within a globular or an open cluster.

The basic problem in the selection of stellar spectra is the difficulty of finding a database covering all wavelengths from near UV ($U$ filter at $\sim 3600$ Å) to the $K_s$ filter at $\sim 21500$ Å in the NIR. In total we (must) use three different spectral databases to produce our results and to confidently confirm the predictions. The basic characteristics of the different databases and the sample of spectral fluxes used from each of them shall be described now.


This atlas contains observed stellar fluxes, compiled by combining data from several existing catalogues overlapping in wavelength coverage. In addition to the spectral type for each star, synthetic colours in $U$, $B$, $V$, $R$, $I$, $J$, $H$, and $K_s$ combinations are given together with the absolute magnitude $M(V)$ and the effective temperature $T_{\text{eff}}$. Synthetic colours and magnitudes are derived by multiplying the flux-calibrated spectra by filter transmission profiles and integrating over wavelength. The atlas provides 65 flux-calibrated spectra spanning a wavelength range of 1150 - 25000 Å and therefore covering all relevant optical and NIR filters. The spectra are sampled in intervals of 5 Å and thus have a resolution $\lambda/\Delta\lambda \sim 500$. Each spectrum is normalized to 1.0 in flux at a wavelength of 5556 Å.

Since we expect the reddening correction to depend on both dimensions (i.e., colour and magnitude) of a CMD (corresponding to effective temperature and surface gravity), we select a sample of spectra which follow a typical isochrone representation.
of a GC as closely as possible. Figure 4.10 illustrates the location of the selected spectra in a CMD together with the method we apply to combine different samples in order to create a smooth differential reddening law for the actual sequence of M4. The vast majority of the spectra have solar metallicity ([Fe/H] = 0.0) so that we are not able to make predictions for metal deficient stars from this atlas. The complete sample of spectra that was used for our study is listed in the Appendix B.

\section*{B) Castelli & Kurucz: ATLAS9 Model Atmospheres (Castelli & Kurucz 2003)}

In addition to the atmospheric models themselves, this atlas contains synthetic stellar fluxes for a fine grid in $T_{\text{eff}}$ and $\log g$. Furthermore, it provides synthetic fluxes for metal-poor stars and for stars with $[\alpha/\text{Fe}] = +0.4$, an important feature which is unfortunately missing in the Pickles atlas. The spectra cover a wavelength range from 300-800,000 Å and therefore include all filters of interest for our purpose. The wavelength sampling and the resolution changes continuously within the spectrum. In the crucial optical and NIR parts however, the resolution is somewhat below that of the observed spectra from Pickles. Temperatures from 3500 K to 50,000 K are provided with a sampling of 250 K in the range of typical GC temperatures. Values for $\log g$ range from 5.0 to 1.0 in steps of 0.5. This fine grid of $T_{\text{eff}}$ and $\log g$ is provided for $[\text{Fe/H}] = +0.5, +0.2, 0.0, -0.5, -1.0, -1.5, -2.0, -2.5, \text{ and } -4.0$ and also for $[\alpha/\text{Fe}] = 0.0 \text{ and } +0.4$.

\section*{C) MILES Library of Empirical Spectra (Sánchez-Blázquez et al. 2006)}

This catalog provides observed spectra for stars with a wide range of metallicities and atmospheric parameters. Although the atlas contains about 1000 individual spectra, it is unfortunately not possible to select a sufficiently large sample of stars with the same metallicity to occupy the whole evolutionary sequence of a GC. Furthermore, the wavelength coverage ranges from 3525 Å to 7500 Å and therefore fully covers only the $B$ and the $V$ filters. For this reason the MILES library will only be used to empirically confirm the results for low metallicities as derived from synthetic spectra (see Chapter 4.4.3). For this purpose we select a sample of spectra located close to the MSTO location in a CMD ($T_{\text{eff}} \approx 6000 \text{ K; } \log g \approx 4.5$) with different typical GC metallicities, which are listed in Table 4.1.
4.4 Results

With the reddening law defined by Eq. \((4.9)\) and the different components described in Chapter 4.3, we investigate the impact of different GC parameters on the reddening law which has to be applied to correct the data for interstellar reddening. In most cases we discuss the results using as an example the transformation factor \(F_{V-K}\) between the observed reddening in \(B-V\) and the derived colour excess in \(V-K\). That way, the impact on a CMD that combines optical and near infrared filters can be directly assessed, whereas a discussion on the basis of \(A_\lambda\) would keep the reader in the dark about the effects found when these extinctions are combined according to Eq. \((4.8)\). Throughout the whole section we assume a standard dust-type parameter of \(R_V = 3.1\). A higher value of \(R_V\) however, would only cause a constant shift in \(F_{\lambda_1-\lambda_2}\) but does not produce any differential effects.

In the following we will use the expression reddening-law zero point as the value of \(F_{\lambda_1-\lambda_2}\) or \(A_\lambda\) obtained for a star in the TO region of the CMD. It is therefore equated to the result of Eq. \((4.9)\) for a star with atmospheric parameters \(T_{\text{eff}} = 6000\) K and \(\log g = 4.5\). This definition has been adopted since, in most cases, this part of a GC evolutionary sequence is used to fit isochrone models to the data. In the case where the zero points are directly referred to M4, they further imply a metallicity of \([\text{Fe/H}] = -1.0\). In contrast to that, we discuss the differential effect in \(F_{\lambda_1-\lambda_2}\) towards a given zero point due to variations in stellar parameters.

4.4.1 Effects of Temperature

Figure 4.3 shows the effect of varying the temperature on the transformation factor \(F_{V-K}\). The results obtained with observed spectra are shown in direct comparison to those from synthetic spectra. For the comparison we choose a sample from the Pickles atlas which approximately follows the MS of a GC evolutionary sequence (see blue line in Figure 4.10). For the ATLAS9 spectra we choose a constant \(\log g = 4.5\) and a wide range of temperatures.

The most significant features of Figure 4.3 are:

1) The results obtained with the observed spectra show a relatively smooth increase for \(F_{V-K}\) with decreasing temperature; similar behaviour is observed for all other transformation factors. The dependence on temperature seems to be slightly stronger for the cool temperature regime below the TO corresponding
to $\log T_{eff} \leq 3.8$. Between the TO temperature and the coolest stars seen on the lower MS at $\log T_{eff} \sim 3.58$, the change in $F_{\lambda_1-\lambda_2}$ is about 3% for all filter combinations.

2) The synthetic spectra match the results from the observed sample very well for temperatures above 5 000 K, whereas there is a strong deviation for lower temperatures: for $T_{eff} \leq 5000$ K the synthetic spectra display a strong decrease in the transformation factor after reaching a maximum value, whereas the observed spectra show a constant increase for $F_{\lambda_1-\lambda_2}$ although a slight flattening can be observed as well. This discrepancy seem to be stronger for more metal rich stars since the synthetic spectra with a metallicity of $[Fe/H] = -2.0$ yield almost the same shape as the observed spectra. We further investigate the discrepancies between observed and synthetic spectra in the next section and discuss possible reasons for them.

3) A decrease in metallicity causes a significant decrease in the zero point of $F_{\lambda_1-\lambda_2}$ around $\log T_{eff} \sim 3.8$ (see also Figure 4.6) whereas the shape or the differential effect within the typical GC temperature range, from 3 000 to 5 000 K, is almost the same. For $T_{eff} \geq 9000$ K the same values of $F_{\lambda_1-\lambda_2}$ are found for all metallicities.

4) Principally, our zero points for the transformation factors which include $J$, $H$, or $K_s$ filters seem to be higher then the commonly used ones in the literature. For example, we find $F_{V-K} = 2.80 - 2.85$, dependent on the metallicity, whereas the canonical value is about 2.71 [McCall 2004]. This reference however gives zero points for a Vega-like star, much hotter than our zero-point reference at MSTO temperatures for GCs. A comparison for all transformation factors with the most important literature values is shown in Table 4.2.

5) There are no significant differences in shape between $F_{V-K}$ and any of the other $F_{\lambda_1-\lambda_2}$ that we calculated. Furthermore, quite similar comparisons between the real and synthetic spectra are found: generally good agreement above 5 000 K and increasing divergence below that temperature, with the synthetic spectra producing lower values of $F_{\lambda_1-\lambda_2}$. 
Figure 4.3: Dependence of the transformation factor $F_{V-K}$ on stellar temperature. Upper panel: Black dots indicate spectra from the observed atlas from Pickles (1998) and red lines show the results obtained with synthetic ATLAS9 spectra from Castelli & Kurucz (2003). Vertical, dashed lines indicate typical GC temperatures with the lower boundary at temperatures for the coolest MS stars and the upper boundary at roughly TO temperatures for old stellar populations. Lower Panel: Difference between the synthetic and observed spectra at solar metallicity. Good agreement is found for temperatures above $\sim 5000 \text{K.}$
4.4.2 Discrepancies between observed and synthetic Spectra

There are two main reasons why we need to know why there are differences between synthetic and observed spectra and how far we can trust either of them. First, the observational sample does not provide sufficiently metal poor spectra. For this reason we need the synthetic spectra to define at least the zero points of the transformation factors for metal-poor populations, if not the differential shape as well. Second, it is important to constrain and confirm the transformation factors with different spectral samples to be confident that the differential effects are significant with respect to the uncertainty coming from the spectral flux, and that the derived zero points are trustworthy.

According to Eq. (4.8), $F_{\lambda_1 - \lambda_2}$ includes the extinction for at least three different filters. To find the source of the discrepancy between the solid red and black loci in Figure 4.3 we looked into each component separately and compared the extinction in all filters for the observed and the synthetic spectra to find out whether there are general differences or whether they are only related to specific filters. The results are shown in Figure 4.4. We find that there is very good agreement for each filter except $U$ and $B$. However, only $B$ shows a significant difference in shape. This result explains very well the deviation between the Pickles and ATLAS9 spectra in Figure 4.3 since a higher value of $A_B$ results in a lower value of $F_{V-K}$. Because $A_B$ contributes to every transformation factor, it explains the observed deviations for all filter combinations.

To identify the difference explicitly in the spectral energy distribution, we over-plotted the observed spectrum with the synthetic spectrum for the $B$-filter in the wavelength range of the $B$ filter for $T_{\text{eff}} = 3500$ K, where we expect a strong deviation, and for $T_{\text{eff}} = 5000$ K where we expect almost no deviation. The results are shown in Figure 4.5. As expected, the difference in effective wavelength is much bigger for the comparison of spectra at 3500 K. For low temperatures, molecular lines and bands become more prominent in the optical part of the spectrum, including the wavelength range of the $B$ filter. In general, synthetic fluxes are not very well suited to model those very complex features (see e.g., Bessell, Castelli & Plez 1998). This fact, together with the increasing deviation towards lower temperatures where molecular bands become dominant features in the spectral flux, supports the assumption that the results obtained with the synthetic ATLAS9 spectra are not trustworthy at low temperatures due to inadequate treatment of molecular bands.

This comparison brings us to the conclusion that we can use the shape of the
transformation factors obtained with the observed spectra in the following work and that we cannot trust the results of a reddening law with ATLAS9 spectra for the coolest temperature regime.

4.4.3 Effects of Metallicity

Since the different spectral samples agree very well for temperatures above 5000 K and specifically in the MSTO region where isochrones are usually fitted to the data, we assume that the synthetic models predict the change of $F_{\lambda_1} - F_{\lambda_2}$ zero points with metallicity fairly well. To confirm this assumption, we use the spectral atlas from Sánchez-Blázquez et al. (2006) to compare the results of the ATLAS9 models with the observed spectra for a wide range of metallicity between $\text{[Fe/H]} = +0.2$ and $\text{[Fe/H]} = -2.5$ for a constant temperature of $T_{\text{eff}} \approx 6000$ K, which is roughly equivalent to the MSTO.

Unfortunately, we are not able to generate extinctions for each filter with the observed metal poor spectra, due to their limited wavelength coverage. For this reason we derive transformation factors where we use the observed metal poor spectra only for the $B$ and the $V$ filters, whereas we derive the extinction for the remaining bands with synthetic metal poor spectra. Then, we compare this hybrid model to a pure synthetic one where the extinctions for all filters are derived from synthetic spectra. Note that for $F_V - K$, for example, two of the three extinction coefficients involved ($A_V$ appears twice) can be calculated from the observed spectra, where only the effective wavelength for the $K_s$-filter is still calculated with synthetic fluxes. Since we expect that changes in the metallicity will mainly have impact on the optical part of the spectrum (and therefore $U$, $B$ and $V$), we are confident that the results obtained with the hybrid model reflect the true situation very well.

Table 4.1 summarizes the parameters of Sánchez-Blázquez and the ATLAS9 spectra which are used for the direct comparison. Figure 4.6 shows the derived values for $F_{V-K}$ with the synthetic and the hybrid model together with the differences between them. The consistency between synthetic metal poor spectra and observed ones is extremely good. Only the results from the most metal rich synthetic spectra for $\text{[Fe/H]} \geq 0.0$ deviate from the values obtained with observed fluxes. For the rest of the direct comparisons the difference in $F_{V-K}$ is considerably smaller than 0.01 without any systematic deviation. In practice, the observed deviations would clearly be below the photometric uncertainty of the data. Reviewing Figure 4.3 we want
Figure 4.4: Relative extinctions for all filters as derived from observed spectra from Pickles (1998) (blue) and synthetic ATLAS9 spectra (red) from Castelli & Kurucz (2003): Although the actual numbers are different, the y-axes show the same scale for each subplot ($A_X/A_V = 0.0075$ between tickmarks) so that relative changes with temperature and differences between synthetic and observed spectra can be compared directly. There is a good agreement for all photometric filters, including the absolute value for the $V$-band in the lower right corner. Only the extinction in the $B$-filter shows significant differences between the different types of spectra for temperatures below 5000 K. Typical GC temperatures are indicated as black dotted lines.
Figure 4.5: Top: Detailed plot of the different spectral energy distributions at $T_{\text{eff}} = 3500\,\text{K}$ to visualize what causes the extinction differences in the $B$-band. The dotted lines indicate the resultant effective wavelength for $B$ when the different spectra are used (red: synthetic ATLAS9; blue: observed from Pickles). The significantly shorter effective wavelength for ATLAS9 spectra results in the higher relative extinction in $B$ compared to observed spectra. Both spectra are normalized to the same wavelength so that intensity differences can be compared directly.

Bottom: As in the upper panel, except for $T_{\text{eff}} = 5000\,\text{K}$. The differences in the spectral flux are much smaller and the effective wavelength is basically the same for both types of spectra.
to point out that the solar metallicity synthetic spectra reproduce very well the zero point at TO temperatures and the shape of all $F_{\lambda_1-\lambda_2}$ derived from observed spectra above 5000 K. By comparing synthetic spectra with different metallicities, we find the shape to be very similar for all metallicities at higher temperatures. Altogether, a change in metallicity therefore mainly produces a shift in the zero-point value of the transformation factor between $E(B-V)$ and the other colour excesses. We can calculate these zero points from metal poor synthetic spectra in a temperature range where the results are confirmed by metal poor observed spectra. However, it is important to point out that the effect of metallicity on the reddening law is not constant over larger temperature scales and the use of a constant zero-point offset is an approximation which seems to be justified only for the small range in GC temperatures.

This comparison brings us to the conclusion that we can trust the zero point for the transformation factors around TO temperatures obtained with the synthetic metal poor spectra.

<table>
<thead>
<tr>
<th>Sanchez-B.</th>
<th>ATLAS9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[Fe/H]$</td>
<td>$\log T_{\text{eff}}$</td>
</tr>
<tr>
<td>+0.17</td>
<td>3.777</td>
</tr>
<tr>
<td>-0.01</td>
<td>3.772</td>
</tr>
<tr>
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<td>3.765</td>
</tr>
<tr>
<td>-2.05</td>
<td>3.772</td>
</tr>
<tr>
<td>-2.50</td>
<td>3.786</td>
</tr>
</tbody>
</table>

Table 4.1: Spectroscopic properties for observed and synthetic spectra in direct comparison.

### 4.4.4 Effects of Gravity

The absolute magnitude $M_V$ of a star in a simple stellar population such as a GC system is strongly correlated with its surface gravity and therefore has a possible impact on the stellar spectrum and the reddening law which has to be applied. Figure 4.7 shows the effect of different $\log g$ values on the transformation factors derived with synthetic spectra from ATLAS9. Ignoring the temperature range cooler than the peak in $F_{V-K}$ at 5000 K, and assuming that the peak itself still reproduces very
Figure 4.6: Top: Zero points for $F_{F-K}$ as a function of [Fe/H] for the hybrid combination (diamonds) and a pure synthetic sample (circles) are compared directly. The dashed line indicate the functional dependence of $F_{F-K}$ with [Fe/H] as derived from pure synthetic spectra with $T_{eff} = 6000$ K and log $g = 4.5$ which represents the atmospheric parameters at the MSTO. Note that these parameters are not used for all ATLAS9 spectra in the direct comparison sample (see Table 4.1) and therefore some circles does not fall exactly on the dashed line.

Bottom: Deviation in $F_{V-K}$ between pure synthetic and hybrid metal poor spectra. $\Delta F_{V-K} = F_{V-K}(syn) - F_{V-K}(hyb)$. 
well the real functional dependence, we observe a smooth increase for $F_{\lambda_1-\lambda_2}$ with decreasing surface gravity or decreasing $M_V$. The effect becomes smoothly more significant towards cooler temperatures where it is of the same order as the temperature effect itself. However, for temperatures between 5 600 K and 6 300 K (including TO temperatures), the differential effect with gravity practically disappears.

For the lowest GC temperatures, the change in $F_{V-K}$ between a dwarf and a giant is about +0.1 and about +3% for all transformation factors. This is especially important for stars on the RGB since the decrease in temperature and in surface gravity collaborate and enhance the differential effect of reddening along the RGB sequence.

Figure 4.8 shows the differences between the “dwarf” sample of the Pickles atlas and the “subgiants” and “giants”. These spectra are chosen in such a way that they follow a real GC evolutionary sequence as closely as possible. Therefore, the changes of both gravity and temperature are included as they are expected in a globular cluster. The general effect of a larger $F_{\lambda_1-\lambda_2}$ for smaller log g is confirmed. Interestingly, the decrease in the transformation factor for very low temperatures observed for the ATLAS9 spectra appears here as well for the empirical SEDs, but only for the giants.

It can be clearly seen that due to the effect of both temperature and surface gravity the differential effect along the RGB is significantly stronger than along the MS.

### 4.4.5 Effects of Extinction: The “Forbes Effect”

The total strength of interstellar extinction itself is a factor which affects the effective wavelength of the filters. This is because interstellar extinction is a function of wavelength and therefore changes the distribution of the incoming flux by damping the blue part of the spectrum more strongly than the red part. This effect has long been known for atmospheric extinction (Forbes 1842) and has been later applied to interstellar extinction, as well (Grebel & Roberts 1995).

Figure 4.9 shows the temperature dependent value of $F_{V-K}$ for different absolute extinctions $A_V$. Compared to the effects of temperature, the influence of $A_V$ on the transformation factor is small and although we use $E(B-V) = 0.36$ in our calculations to tailor the results for M4, they are basically valid for at least all $E(B-V) \leq 0.5$.

This assumption is supported by Girardi et al. (2008) who include the “Forbes Effect” in their calculations and found that it is not significant for $A_V \leq 4.0$ (see their
Figure 4.7: The effect of surface gravity for the transformation factor $F_{V-K}$ is shown for a wide range of temperatures. For this calculation we used synthetic spectra with $[\text{Fe/H}] = -1.0$ and $[\alpha/\text{Fe}] = +0.4$ to simulate the chemical composition of the stars in M4. As in previous plots, the vertical dotted lines indicate typical GC temperatures. In general, a lower surface gravity results in higher values for $F_{V-K}$, where this effect seem to increase with lower temperatures and is not observable for temperatures around the MSTO. Interestingly, the effect seem to be inverted for temperatures hotter than $T_{\text{eff}} \approx 6300 \, \text{K}$.
Figure 4.8: The actual combined effect of temperature and surface gravity as it is obtained from observed spectra that closely follow an actual GC sequence. The differential effect for giants on the RGB is about twice as big as the effect for stars along the MS.
Moreover, Schmidt-Kaler (1982) estimates

\[ R_V = R_V^{00} + 0.28(B - V) + 0.04E(B - V), \]  

where the effect of temperature in terms of \( B - V \) is seven times stronger than the effect of extinction itself in terms of \( E(B - V) \).

### 4.4.6 Differential Reddening Law

Despite the definition of appropriate zero-point values for relative extinctions \( A_A/A_V \) and transformation factors \( F_{\lambda_1 - \lambda_2} \) suited to the atmospheric and chemical conditions of GC stars, the question comes up whether changes of those parameters within a typical GC population cause any significant change in the appearance of a CMD compared to a constant correction. Such changes could affect the determination of [Fe/H], distance, age and other parameters as determined from photometric studies.

To determine the differential effect of reddening due to local changes in temperature and gravity we define a star-by-star reddening law in the following way:

- The zero points for the extinction in each band \( A_A \) and the resulting transformation factors to each colour are defined with ATLAS9 spectra with atmospheric parameters of a typical globular cluster MSTO (\( T_{\text{eff}} = 6000 \) K, \( \log g = 4.5 \)) because this region of a CMD has the greatest dependence on age (see e.g., Stetson, VandenBerg and Bolte 1996). For M4 specifically, we use a metallicity appropriate for the observed globular cluster, \([\text{Fe/H}] = -1.0\) and \( E(B - V) = 0.36 \) where the final input parameter \( A_V \) still depends on the choice of \( R_V \), which we set in this case to 3.70.

- Along the main sequence, the differential effects from temperature and surface gravity are defined by the change of \( F_{\lambda_1 - \lambda_2} \) with temperature calculated from the “dwarf” sample (blue line) with Pickles spectra. We use linear interpolation between different spectra since we could not observe a clear functional dependence. Stars are corrected with this function if \( M_V \geq 4.3 \).

- Along the SGB, the differential effects are calculated from the “subgiant” sample (green line) with Pickles spectra. We use linear interpolation between different spectra. Stars are corrected with this function if \( M_V < 4.3 \) and \((V - K)_0 \leq 1.98\).
Figure 4.9: The total extinction changes the shape of the incoming flux and therefore the reddening law. This is known as “Forbes Effect” and is shown here for several different amounts of total extinction between 0.31 and 3.1, which corresponds to $E(B-V) = 0.1, 0.3, 0.5$ and 1.0 assuming $R_V = 3.1$. The effect is small compared to other influences that have been discussed in the previous sections and has about the same impact for all temperatures.
• Along the RGB, the differential effects are calculated from the “giant” sample (red line) with Pickles spectra. We use linear interpolation between different spectra. Stars are corrected with this function if $M_V < 4.3$ and $(V - K)_0 > 1.98$.

See the right panel of Figure 4.10 to visualize these choices.

We apply the differential reddening law defined above on a $V$ vs. $V - K$ diagram of M4 in four steps:

1. We apply a constant reddening correction for each star and derive the distance modulus to the cluster by matching the observed ZAHB luminosity with that predicted by stellar models to get a first-order approximation of the intrinsic colour and magnitude for the stars.

2. We compile our own reddening law with spectra from three different luminosity regimes (“dwarf”, “subgiant”, “giant”), dependent on $M_V$ and $(V - K)_0$ for each star.

3. We use our reddening law to derive individual $F_{\lambda_1-\lambda_2}$ for each star using their atmospheric parameters. In addition, a constant value for the transformation factor is obtained on the assumption of a TO temperature and surface gravity.

4. We apply both the constant and the differential reddening law on the uncorrected data for M4 and calculate the difference in colour for both approaches.

The result is shown in Figure 4.11. Since the constant reddening law has been defined with TO parameters, the constant and the differential reddening laws come together here. To define the temperature dependence for the extinction values we used linear interpolation between spectra along the GC sequence. For this reason the correction is not smooth and reflects the small scatter from the observed fluxes in Figure 4.3.

In general, the differential effects for an intermediate-reddened cluster like M4 ($E(B-V) \approx 0.36$) are small and do not affect the shape of the evolutionary sequence significantly. For most stars in our sample the photometric uncertainty is larger than the differential correction effect due to variations in their temperature and surface gravity. That means that parameters like metallicity, distance and age which might be derived by isochrone fitting are not significantly biased by the effects of differential
Figure 4.10: How the different observed samples are applied to our photometry: On the left side, the location of the selection of observed spectra is shown in a CMD together with an isochrone for M4. The atmospheric conditions of the sample are close the cluster sequence and show slight differences due to the fact that all spectra have solar metallicity. Right side: We subdivide the cluster stars in three categories, each is treated by a different reddening law according to their atmospheric parameters: giants (red); subgiants (green); dwarfs (blue).
reddening. Specifically, the minimum luminosity of the HB, which is often used to
derive the distance modulus of GCs, is not affected by differential effects when the
zero-points are normalized to TO temperatures. At those temperatures, the change
in surface gravity does not affect the reddening zero points. However, the error caused
by the use of a constant reddening correction is of a systematic nature and a correction
for those effects should be considered, especially for RGB stars of highly reddened
Galactic bulge clusters.

4.4.7 Extinction Zero Points

We derive zero points for relative extinction values \((A_\lambda / A_V)\) and from them colour
transformation factors for a range of different metallicities using TO temperatures
and gravities with synthetic ATLAS9 spectra. Here, we assume \(A_V = 1.12\), but
the values are practically valid for all \(A_V \lesssim 3\). The transformation factors are sup-
posed to be applied to correct the colour excesses in filter combinations of optical
Johnson-Cousins and NIR 2MASS filters given a value of \(E(B-V)\). The values are
listed in Table 4.2 and compared to literature values from McCall (2004), Schlegel
et al. (1998) and to those derived with the equations given by Cardelli, Clayton &
Mathis (1989). For all values we assume a dust type with \(R_V = 3.1\). Note that we
use the effective wavelength from McCall (2004) for 2MASS \(J\), \(H\), and \(K_s\) to derive
Cardelli et al. transformation factors. Therefore all reference numbers are normalized
to a Vega-like star.

A simple calculation shows that, according to Eq. (4.8)

\[
F_{V*} + F_{BV} = \frac{A_V - A_* + A_B - A_V}{A_B - A_V} = \frac{A_B - A_*}{A_B - A_V} = F_{B*}
\]

and therefore

\[
F_{V\lambda_2} + 1.0 = F_{B\lambda_2}
\]

For this reason, all other possible colour combinations can be derived from the values
given in Table 4.2.
Figure 4.11: The differential effects of gravity and surface temperature on the reddening correction for stars in the globular cluster M4 compared to a constant reddening law: On the left side, the difference between a differential star-by-star correction (green) and a constant correction (red) is shown directly. The differential reddening law is applied for three different zones in the $V$ vs. $V - K$ plane and normalized to the constant law at the colour and magnitude of the MSTO. The differential effects are quite small in colour, though the effect on the RGB is about twice as big as on the MS, since here the change in temperature and surface gravity are additive. The middle panel shows the difference in $V - K$ between a constant and a differential reddening law. The effect for M4 is $\leq 0.05$ mag in $V - K$ and therefore not significantly larger than the photometric uncertainty in this colour, which is shown in the righthand panel.
### Table 4.2: Zero-point values for $F_{\lambda_1-\lambda_2}$ for spectra with $T_{eff} = 6000$ K, log $g = 4.5$, $R_V = 3.10$, and $E(B-V) = 0.36$ for different metallicities compared to literature values.

<table>
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<tr>
<th>[Fe/H]</th>
<th>$F_{VI}$</th>
<th>$F_{VJ}$</th>
<th>$F_{V-K}$</th>
<th>$F_{VH}$</th>
<th>$F_{VR}$</th>
<th>$F_{UK}$</th>
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<td>+0.50</td>
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<td>-</td>
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### Table 4.3: Effective Wavelength for Johnson-Cousins $UBVRI$ (Landolt 1992) and 2MASS $J, H,$ and $K_s$ filters for spectra with $T_{eff} = 6000$ K, log $g = 4.5$, $R_V = 3.10$, and $E(B-V) = 0.36$ for different metallicities.

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<th>[Fe/H]</th>
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<th>$V$</th>
<th>$R$</th>
<th>$I$</th>
<th>$J$</th>
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<td>-</td>
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4.4.8 Comparison to existing Work

Some studies have been made concerning the effects of temperature, surface gravity and other parameters on the reddening law. However, all of them use only synthetic spectra.

The recent work from Girardi et al. (2008) (hereafter G08) uses ATLAS9 spectra (the same generation than we adopt) to generate a differential reddening law with a grid of \( \log g \) and \( T_{\text{eff}} \) specifically applicable for the WFPC2 and ACS photometric systems from the Hubble Space Telescope. They found a differential effect of \( \sim 0.05 \) mag for a total hypothetical extinction of \( A_V \approx 3.0 \) mag which corresponds to a reddening of \( E(B-V) \approx 1.0 \), using a standard \( R_V \) value of 3.1. Comparing this result to our Figure 4.11 shows that we find a similar sized effect for about half the reddening using a \( V-K \) magnitude plane, which can be explained with the larger difference in the effective wavelength that we reach with this filter combination. All values from G08 are derived with solar metallicity spectra only; they claim that the variation with metallicity is negligible. That is not what we found in our study: even if the metallicity effect for the absolute extinction is very small in a single filter, it may increase significantly when combining four filters to derive the transformation factors for different colours; and those factors are actually used to define the colour shift to correct for extinction and therefore the visible effect in a CMD. Using the same thresholds for \([\text{Fe/H}]\) as G08, we find a difference of \( \delta F_{V-K} \approx 0.08 \) mag between \([\text{Fe/H}] = 0.0\) and \([\text{Fe/H}] = -2.5\), a change of 2.8% which is about ten times larger than G08 states. Overall, our work reveals a roughly equally sized effect for changes in \([\text{Fe/H}], T_{\text{eff}}, \) and \( \log g \) (for all \( \sim 3\)% encompassed by GC stars, whereas G08 claim that the effects of temperature variations dominate over those of surface gravity while those arising from metallicity variations are negligible.

Another interesting issue to discuss is the quality of ATLAS9 spectra, especially for low-temperature stars where molecular lines become important. Our work finds very good agreement between observed and synthetic spectra for all bands except \( B \) for \( T_{\text{eff}} \leq 5000 \) K (see Figure 4.4). Interestingly, Bessell, Castelli & Plez (1998) use an older version of ATLAS9 models (from 1993) and found a good fit only for temperatures above 4500 K due to their incomplete molecular opacity at lower temperatures. Although we are using a more recent version (2003) with an improved treatment of molecules, we seem to find a similar effect setting in at almost the same temperature. Carefully examining Figure 2 in G08 indicates that their study suffers
the same problem: the relative extinction $A_A/A_V$ ($A_{S_λ}/A_V$ in G08) for the F435W filter, which is closest to Johnson $B$, shows a dip at cooler temperatures similar to our results with ATLAS9, whereas the use of observed spectra reveals a constant decrease in relative extinction towards lower temperatures.

The work from McCall (2004) is focused on the description of a reddening law suitable for resolved extragalactic stellar populations like galaxies and dwarf spheroidals. The author discusses the effect of temperature on the reddening law but unfortunately only provides zero-point values normalized to a Vega-like star together with $R_V = 3.07$ which makes it difficult to transfer his results to globular cluster conditions and to compare them to our results.

4.5 Conclusions

With regard to a further use of the reddening law to determine the type of dust in the line of sight to M4 with our photometry, we found that, for a globular cluster with an intermediate reddening like M4, a star-by-star correction, which takes into account variations of $T_{eff}$ and $\log g$ within a stellar population, is not necessary to derive parameters from the photometry in an accurate way. However, the correction factors should be calibrated to the actual atmospheric and chemical conditions in the stellar population and not to a Vega-like star, if accurate results shall be achieved. Finally, synthetic spectra are not useful for predictions in the low temperature regime below 5000 K but can be used for all temperatures above where they have been approved by observed spectra for all metallicities.
Chapter 5

The Dust Type of M4

5.1 Introduction

The type of dust which causes the interstellar reddening can show variations for different lines of sight and is responsible for significant changes in the physics of absorption and scattering. Therefore the individual reddening law is mainly determined by the individual properties of the dust (Mathis 1990). The dependence on the dust type can be very well expressed by the ratio between total-to-selective extinction, \( R_V \), which seem to be strongly correlated with the grain properties of the dust and varies between \( R_V \approx 2.6 \) and a value up to 6.0 for dense molecular clouds or star forming regions. Usually a value of \( \sim 3.1 \) is assumed to describe the foreground reddening for GCs in the Milky Way as it is associated with the extinction properties of the diffuse interstellar medium. However, there is strong evidence in the literature that M4 follows a different, anomalous reddening law due to the fact that the cluster lies behind the Sco-Oph dark cloud complex which adds an additional uncertainty to the distance and age—if derived from the WD cooling sequence—of the cluster as well as to the assumed mean reddening expressed in \( E(B - V) \).

In this chapter, we want to determine the properties of the dust in the line of sight to M4, characterized by the appropriate value of \( R_V \) in the Cardelli reddening law (Cardelli, Clayton & Mathis 1989). From this, the actual ratio between \( A_V \) and \( E(B - V) \) for our photometry can be determined in order to derive the distance to the system accurately. To minimize the final uncertainty for both \( R_V \) and the true distance modulus, we make use of our differential reddening corrected HQ-sample (see Chapter 3) as well as of the results from Chapter 4 which allows us to use
extinction zero points precisely representing the actual atmospheric and chemical properties of M4 in order to minimize systematic uncertainties.

5.2 Method

In general, we want to find out which value of the dust type parameter $R_V$ produces a consistent match for isochrones with the observed CMDs of M4 in all filter combinations once the empirical offset is given for one colour. This is not the case for $R_V = 3.1$ (see Figure 4.1). In Chapter 4.4.7 we derived all zero-point values for extinction assuming a dust type with $R_V = 3.1$. Now, we make use of the possibility of treating $R_V$ as a free input parameter to calculate transformation factors $F_{\lambda_1 - \lambda_2}$ between different colours as a function of $R_V$. The dust-type parameter enters equation (4.9) twice: First, it defines the slope of the reddening law according to the Cardelli et al. equations. Second, it defines the actual extinction $A_V$ for a given reddening $E(B-V)$.

5.2.1 Empirical Color Offsets

To determine the empirical colour offset between the observed CMD and the theoretical isochrone for each combination of filters, we use an age-independent fiducial point on the MS, defined to be 5.5 mag below the red ZAHB V-band magnitude. For M4, this occurs at $m(V) = 18.96$. Our fiducial point is insensitive to age, since it lies well below the MSTO and it is further defined with respect to the ZAHB luminosity, a standard candle which is known to be nearly independent of age (e.g. Stetson, Vandenberg and Bolte 1996). This is important since ages are not precisely known for GCs and could cause a systematic error in the determination of $R_V$. We further choose the fiducial point intentionally below the MSTO to avoid known uncertainties within theoretical isochrones for evolutionary stages later than the TO due to the onset of various mixing and diffusion processes.

We find the colour of the fiducial point in the observed CMD by calculating fiducial sequences along the MS of M4 in each desired filter combination using data which have been corrected for spatial differential reddening. The fiducial sequences are obtained the same way as described in Chapter 3.2 by calculating the median colour in magnitude bins of 0.3 mag and by cubic spline interpolation. The fiducial lines so de-
derived match the actual slope of the observed cluster sequence very well, as illustrated in Figure 5.2 for several filter combinations. This approach has the advantage of being consistent for all inspected colours: since both the reference luminosity and the colour of the fiducial point are derived the same way for all filter combinations, any systematic uncertainty cancels out in the first order. In other words, even if the derived fiducial sequence might not represent the actual median reddening value for a given colour (e.g., due to a bias caused by binary stars), it still does represent the same reddening value for all colours.

Finally, each colour offset is given by the colour difference between the empirical fiducial point and its theoretical equivalent on the appropriate isochrone. The general fitting method is illustrated in Figure 5.1.

From the individual observed colour excesses we derive the necessary transformation factors $F_{\lambda_1-\lambda_2}$ for each filter combination and determine the corresponding $R_V$ values from our reddening law calculations. The zero points of the transformation factors are calculated for $[\text{Fe/H}] = -1.0$, $E(B-V) = 0.36$ and for the atmospheric parameters $T_{\text{eff}} = 5250$ K and $\log g = 4.5$ based on the location of the fiducial point which is used to determine the offset.

5.2.2 Error Estimation

In addition to the best fitting values of $R_V$, we estimate for each colour the observational standard error of the derived offset to the isochrone ($\sigma(\Delta\text{colour})$) and the uncertainty in $R_V$ ($\sigma(R_V)$) which defines the overall precision of our method. To do this, we have to take into account the photometric uncertainty of both the observed ZAHB luminosity ($\sigma(V_{HB})$) and the colour of the fiducial point at that luminosity ($\sigma(\text{colour})$):

$$\sigma^2(\Delta\text{colour}) = \sigma^2(V_{HB})(\frac{d_{\text{colour}}}{d_{\text{mag}}})^2 + \sigma^2(\text{colour}).$$  \hspace{1cm} (5.1)

To determine the first component of Eq. (5.1), we assume $\sigma(V_{HB}) = 0.03$ mag and calculate the change in colour of the fiducial point when its luminosity is shifted by that amount. For the latter part we calculate the median $\sigma(\text{colour})$ at the fiducial point luminosity. The quantity $\sigma(\Delta\text{colour})$ thus depends mainly on the photometric quality in each filter and to some extent on the slope of the MS at the fiducial luminosity. Consequently, each filter combination is assigned an individual value.
Figure 5.1: Schematic determination of the colour offset in a $V$ vs. $V - K$ plane: The ZAHB is used as an age-independent, metallicity-insensitive magnitude zero point in both the model and the data. The colour of the MS is determined which lies 5.5 mag in $V$ below that zero point. At this location, Victoria-Regina isochrones appear to describe the actual sequence very well, whereas they tend to deviate systematically to the blue for fainter magnitudes (e.g., VandenBerg, Casagrande and Stetson 2010). The empirical fiducial line is sketched into the observed sequence on the right panel with black dots marking the fiducial points.
Figure 5.2: Empirical fiducial sequences for different filter combinations along the MS of M4. These sequences are used to define the offset in colour between observed CMDs and model predictions. Only in the $I - J$ and $J - K$ plots is the cluster sequence not well described for $V \leq 19.5$, which, however, does not affect the determination of the colour offset in those filter combinations.
The uncertainty of $R_V$ for each colour is then determined by the propagation of those errors from $\sigma(\Delta \text{colour})$ to $\sigma(R_V)$. Here, the governing criterion is the sensitivity of each filter combination on $R_V$, which is discussed in the next section. Finally, each filter combination yields a best fitting value for $R_V$ together with an interval where the colour excess is reproduced to within its observational uncertainty limits. For the final individual colour offsets and their uncertainties, see the results listed in Chapter 5.4.

In addition to the observed random errors discussed here, the main sources of systematic errors are discussed in Chapter 5.5.

### 5.2.3 Independent Filter Combinations

Not all filter combinations do have the same uncertainty and not all of them yield independent results. We therefore have to decide which colours we want to use to minimize the final value of $\sigma(R_V)$.

The first decision which has to be made is the reference colour to which other combinations shall be compared, where the crucial criterion is to maximize the sensitivity of the ratio $E(\lambda_1 - \lambda_2)/E(\lambda_{\text{ref}1} - \lambda_{\text{ref}2})$ to $R_V$. This sensitivity is defined by the difference in the effect for the correction factor of a given filter combination compared to the correction factor of the reference colour when $R_V$ is changed. Figure 5.3 illustrates the effect of $R_V$ on different filters. It is based on Figure 3 in Cardelli, Clayton & Mathis (1989) and shows the relative extinctions for the effective wavelengths of our filters, assuming two different values of $R_V$. If we compare for example $E(I - J)$ to $E(V - K)$, we find that both colours are insensitive to $R_V$, but most importantly they are equally dependent on the dust type. Following this argumentation, $E(I - J)/E(V - K)$ would be insensitive to a change in $R_V$ and consequently $\sigma(R_V)$ would be large for this combination. For that reason, the most promising reference colour is $E(B - V)$, because its change with $R_V$ differs the most compared to all other combinations.

From this perspective, $U - V$ would be an even better choice. However, with an effective wavelength of $\sim 3560 \, \text{Å}$, the $U$ filter is located on the extreme short-wavelength end of common detector sensitivities and the atmospheric transmission and is therefore very sensitive to the exact observing conditions and the instrumental response. Moreover, the bandpass falls together with the region of the Balmer-jump in stellar spectra, associated with strong hydrogen lines, which makes the filter sensitive
for even small variations in $T_{\text{eff}}$ and $\log g$. Consequently, we do expect significant uncertainties in the synthetic $U$ magnitudes which would cause a systematic error in the determination of $R_V$.

The uncertainty of every individual comparison colour depends mainly on two factors: the photometric quality of the given filter combination and the sensitivity of the ratio $E(\lambda_1 - \lambda_2)/E(B - V)$ with $R_V$. Figure 5.4 shows the dependence of all individual $F_{\lambda_1 - \lambda_2}$ on $R_V$. The filter combination spanning the highest wavelength range and the biggest difference from $E(B - V)$ regarding $R_V$-sensitivity will have the steepest slope. An interesting fact to note is that the change in $F_{\lambda_1 - \lambda_2}$ is different only for some colours, whereas it is the same for colours where only $B$ and $V$ are exchanged (see Eq. (4.12) for explanation).

Our method uses as many different independent relations between $F_{\lambda_1 - \lambda_2}$ and $R_V$ as possible to find the value of $R_V$ for which isochrone fits converge to a consistent match for all of them.

Given $E(B - V)$ as the reference value, we can assign a maximum of four independent colours contributing to the determination of the dust type under the assumption that results which include the $U$ and $R$ filters are not trustworthy. To minimize systematic errors within the individual bandpasses, we choose the colours in a way such that each filter appears on alternating sides. Therefore we use $E(V - I)$, $E(I - J)$, $E(J - K)$, and $E(B - K)$ as independent colour measurements to $E(B - V)$. This choice might seem surprising at first, since those filter combinations all have relatively large individual $\sigma(R_V)$ compared to $E(V - J)$ or $E(V - K)$ for example (see Table 5.2). However, by using alternating combinations we are eliminating any systematic errors from faulty CT-relations used by the isochrones or bad zero-point calibrations of the observed data to the standard system; The resultant high accuracy overrules the high precision of some individual colours.

The method therefore necessarily relies on a combination of optical and NIR photometry, since otherwise only $E(V - I)$ could confidently be used to determine the dust type which would cause both high systematic and statistical errors in the result.
Figure 5.3: The Cardelli reddening law defines the extinction as a function of wavelength and the dust type parameter $R_V$. The effective wavelength of our filter set is indicated with red and green dots for $R_V = 3.1$ and $R_V = 4.1$ respectively. The change of the reddening law with the dust-type parameter is the crucial key to determine the properties of dust relevant to M4.
Figure 5.4: The dependence of different filter combinations on $R_V$: On the left side, all possible transformation factors are plotted as a function of $R_V$. Shown from top to bottom: $B - K$, $B - J$, $V - K$, $V - J$ (blue), $B - I$ (dashed green), $I - K$, $V - I$, $I - K$ and $J - K$. On the right side, the transformation factors are shifted to 1.0 at $R_V = 3.1$ for a direct comparison of the different slopes which are important for the precision of the determination of $R_V$ from individual colours. Whenever only $V$ is exchanged with $B$, the same slope is reproduced.
5.3 Isochrones

The method we describe here to determine the dust type toward a stellar population relies fundamentally on the assumption that theoretical isochrones fit observational data if either the correct reddening law is used, or, of course, when there is no reddening. The main source of uncertainty concerning the fit of isochrone models to observed CMDs comes from CT-relations which assign a synthetic colour from theoretical surface temperatures and gravities (from model atmospheres) as well as from the predicted $T_{\text{eff}}$-scales, while the theoretical luminosities are considered to be more robust.

We use the newest generation of Victoria-Regina isochrones to determine the colour offsets in different filter combinations. These models have been significantly updated since the latest description in VandenBerg, Bergbusch & Dowler (2006). The most important improvement is the incorporation of the treatment of the gravitational settling of helium (see Proffitt & Michaud [1991]). A detailed summary of all recent updates is provided in Brogaard et al. (2011). Besides the theoretical sequences reaching from the MS up to the RGB tip, we are provided with ZAHB models to determine $(m - M)_V$ which is necessary to match our fiducial point luminosity between observed CMD and isochrones. As soon as the reddening and the dust type are known, we can further use the HB as a standard candle to determine the distance to the cluster.

CT-relations and the overall fit of Victoria-Regina isochrones have been tested intensively for both optical and near infrared filters by several authors: For optical bands VandenBerg, Casagrande and Stetson (2010) found very good agreement between the isochrones and data for local Population II subdwarfs with Hipparcos-based distances. In a further test, the isochrones reproduced the observed fiducial sequences for several globular and open clusters with different metallicities very well. For their study, they tested three different CT-relations: the empirical transformations by Casagrande et al. (2010), those derived from MARCS model atmospheres and the semi-empirical relations by Vandenberg & Clem (2003) for which they found only minor differences. According to this paper, the models fail to match the observed sequences only for faint MS stars below a magnitude of $M_V \geq 6.5$.

Brasseur et al. (2010) tested Victoria-Regina isochrones using MARCS model atmosphere-based CT-relations for 2MASS $J$, $H$ and $K_s$ bands against the same sample of field subdwarfs and found good agreement between the models and the
data. Only a small offset was detected in $J - K$ and $V - K$ where the models seem to predict those colours too red by $\sim 0.03$ mag each. Comparing isochrone sequences to the observed CMDs of globular clusters generally yield good agreement for all except for the lowest metallicity systems ([Fe/H] $\leq -2.0$). Those discrepancies however are only limited to the lower RGB segments, where the isochrones seem to be significantly too red, whereas the location of the MS matches for all metallicities.

With the point on the MS 5.5 mag fainter than the ZAHB in M4, we satisfy both critical conditions for magnitude and metallicity very well, so that we are confident that the models should match the data if the correct reddening law is adopted.

### 5.3.1 The Guinea Pigs: NGC 6723 and NGC 1851

For both nearly unreddened globular clusters NGC 6723 ($E(B - V) \approx 0.05$) and NGC 1851 ($E(B - V) \approx 0.03$) we have optical and NIR photometry reaching well below the MSTO to test the morphological fit and the consistency of our isochrones in different observed filter combinations under optimal conditions. Specifically NGC 6723 is almost identical to M4 in chemistry and age, so it is therefore perfectly suited as an unreddened reference cluster. In Figure 5.5 we correct NGC 6723 to the distance modulus and reddening of M4 to obtain a direct overlay of the two systems. Both sequences show exactly the same slope along the RGB continuing to below the TO which implies almost identical age and chemical abundance pattern. On the lower MS, the photometric quality of NGC 6723 is too poor to allow a precise comparison. Remarkably, even the ZAHB sequences are superimposed precisely with a similar bimodal distribution of red and blue HB stars which is unusual, if one considers their relatively high metallicities.

For NGC 6723, a metallicity of [Fe/H] = $-1.0$ with an age of 11-12 Gyrs yields the best fit for our isochrones. With an adopted reddening of $E(B - V) = 0.05$ and a standard reddening law with $R_V = 3.1$ we find an excellent match between the observed CMDs and models for almost all filter combinations. Our best fitting parameters are in good agreement with the latest edition of the Harris catalog [Harris 1996] which gives [Fe/H] = $-1.1$ and $E(B - V) = 0.05$. We further adopt [$\alpha$/Fe] = +0.4, in good agreement with observational measurements from [Fullton & Carney 1996] who found a mean of +0.42 for the alpha elements in this cluster. Only in $J - K$ the isochrone colours are slightly too red and in apparent disagreement with $E(B - V) = 0.05$. 
Figure 5.5: Direct overlay of M4 (black) and NGC 6723 (green). The latter one has been shifted by $\Delta(V-K) = +1.07$ and $\Delta V = -2.06$ to match the data of M4. Although the MS for NGC 6723 is too poorly defined to compare the sequences here directly, both clusters show a remarkable consistency in their RGBs and HB sequences, implying that both systems are almost identical in metallicity and age.
Figure 5.6: Best fitting isochrone for the globular cluster NGC 6723.
Figure 5.7: Isochrone fits for different filter combinations of NGC 6723. Using a reddening of \( E(B - V) = 0.05 \) and a standard reddening law with \( R_V = 3.1 \), the isochrones show a consistently good fit for all combinations. The data in all diagrams are corrected for both reddening and distance.
Figure 5.8: Best fitting isochrone for the globular cluster NGC 1851.
Figure 5.9: The same comparison between the different filter combination as for NGC 6723 has been performed for NGC 1851. In the NIR, only $K_s$ has been available for this cluster. The data in all diagrams are corrected for both reddening and distance.
For NGC 1851, we find a slightly lower metallicity ([Fe/H] = −1.2) and a slightly younger age (∼10 Gyrs) than for NGC 6723, again in good agreement with the Harris catalog ([Fe/H] = −1.18; E(B − V) = 0.03). By adopting [α/Fe] = +0.4 \cite{Villanova, 2011}, E(B − V) = 0.03 and RV = 3.1, we find a consistent fit for all filters. Unfortunately, for this cluster we have NIR data only for the K_s filter so that we are not able to test the full range for colours that we are using for M4.

The results for both reference clusters are shown in Figures 5.6 to 5.9 and demonstrate that, at least in the intermediate metal poor regime, our isochrones fit data consistently well in all colours at absolute magnitudes of M(V) ≤ +8.0 in the case where there is no significant reddening. The small deviations for the tight NIR filter combination J − K in NGC 6723 is most probably due to slightly wrong CT-relations in J and K_s coupled with the extremely small colour range in the resulting CMD. Note that we eliminate any of such systematic effects in the final calculation of RV by using a set of colours where the individual filters appear on alternating sides of the filter combination.

5.3.2 Isochrone Parameters for M4

Faulty assumptions for the isochrone parameters [Fe/H], [α/Fe], helium mass fraction (Y) or age will inevitably lead to systematic errors in the determination of RV. For the determination of the dust type toward M4 we are using isochrones with the following parameters which will be justified in detail below:

\[
\begin{align*}
[\text{Fe/H}] &= -1.0 \\
[\alpha/\text{Fe}] &= +0.4 \\
\text{age} &= 12 \ \text{Gyrs} \\
Y &= 0.25
\end{align*}
\]

[Fe/H] & Age

Although the fiducial point which we are using to determine the colour excess is independent of age, it is important to know the metallicity of the cluster as accurately as possible since this parameter affects the shape and location of the isochrone, the absolute magnitude of the ZAHB, and also the transformation factor zero points from
the reddening law. There are many studies about the metallicity of M4, yielding values between $\text{[Fe/H]} = -1.4$ and $-1.0$ with recent results tending to favour the metal rich end of this interval. For example, Mucciarelli et al. (2011) found $\text{[Fe/H]} = -1.1 \pm 0.07$ for 87 MS and RGB stars in M4 and Marino et al. (2008) got $\text{[Fe/H]} = -1.07 \pm 0.01$ (and $\text{[\alpha/Fe]} = +0.39 \pm 0.05$) in their spectroscopic study of 105 RGB stars.

When we fit isochrones to our data for M4, we too find a best fit for a value of $\text{[Fe/H]}$ close to $-1.0$. **Figure 5.10** shows a comparison between isochrone fits with metallicities between $\text{[Fe/H]} = -1.4$ and $-1.0$ and shows that both the metal-sensitive slope of the RGB as well as the lower MS are matched best with $\text{[Fe/H]} = -1.0$ and an age of 12 Gyrs.

The recent work of Bedin et al. (2009) and Mucciarelli et al. (2011) supports this parameter choice: they both find a best fit for isochrones with $\text{[Fe/H]} = -1.01$ and age of 12 Gyrs derived from the WD cooling sequence. This number is in good agreement with the results from Hansen et al. (2004) who find a best fit for 12.1 Gyrs with the same method which gives us confidence that our choice of isochrone parameters does not add a significant systematic error to the derived value of $R_V$.

Since metallicity seems to be the parameter with the highest uncertainty and the biggest impact on $R_V$, the systematic error for $R_V$ caused by the uncertainty in $\text{[Fe/H]}$ is further discussed in Chapter 5.5.

**$\text{[\alpha/Fe]}$**

Variations in alpha-element abundances (Mg, Ca, Si, Ti) have basically the same effect on theoretical evolutionary sequences as variations in $\text{[Fe/H]}$: an increase in $\text{[\alpha/Fe]}$ leads to cooler and fainter isochrones (e.g., VandenBerg et al. 2000). Therefore, an underestimate of $\text{[\alpha/Fe]}$ would cause an overestimate of $\text{[Fe/H]}$ and, eventually, a systematically wrong $R_V$.

For M4, the most recent alpha-element measurements from Marino et al. (2008) yield $\text{[Mg/Fe]} = +0.50$, $\text{[Ca/Fe]} = +0.28$, $\text{[Si/Fe]} = +0.48$ and $\text{[Ti/Fe]} = +0.32$ and therefore confirms the generally enhanced alpha-elements in old, metal-poor GC stars.

**Helium**

Helium represents a significant amount of the total stellar mass ($\geq 24\%$) and isochrones react sensitively to the adopted abundance of this element.

In general, a higher helium abundance leads to an increase in the mean molecular weight and finally to higher envelope pressures and interior temperatures to maintain
Figure 5.10: The shape of the MS and the RGB is used to constrain the metallicity of M4 by isochrone fitting. Specifically the angle between those evolutionary sequences in a CMD is sensitive to the metallicity and yields a best fit for M4 when a value of $[\text{Fe/H}] = -1.0$ is used.
hydrostatic equilibrium. As a result, the star produces more energy in the burning regions. Practically, the HB becomes brighter, which affects the distance modulus to the cluster. For example an increase in helium from \( Y = 0.25 \) to \( Y = 0.33 \) would lead to an increase in the HB luminosity of \( M_V(HB) \approx -0.4 \text{ mag} \) (Dorman et al. 1989). An underestimate of \( Y \) therefore leads to an underestimation of the distance.

GC helium abundances are—still—assumed to be similar or equal to the primordial helium abundance arising from Big Bang nucleosynthesis. This would imply a helium abundance near a value of \( Y = 0.25 \) (Cyburt et al. 2008), at least for the vast majority of clusters. There is no evidence in the literature that M4 is an exception to that rule. A helium-enriched scenario would increase the luminosity of the blue HB more than the red, so that possible helium enhancements can theoretically be ruled out by HB observations (Catelan et al. 2009a).

**Figure 5.11:** Best fitting isochrone for the globular cluster M4: The CMD shows the HQ-sample of our photometry and is corrected for spatial differential effects. The derived cluster parameters are in very good agreement with recent results in the literature.
5.4 Results

Every colour we are using in direct comparison to $E(B-V)$ yields a best-fitting value together with an acceptable uncertainty interval for $R_V$ which produces the necessary observed colour offset for this filter combination. From those individual results we derive the final value of $R_V$ with an independent sample to gain the lowest possible random and systematic errors.

5.4.1 Individual Colors and the Independent Sample

In a first step, we want to find out whether, in general, it is possible to reach a consistent fit for all colours within their photometric uncertainty limits. For that purpose we superimpose the individual results by shading the acceptable $R_V$ intervals for each colour with the same intensity and in such a way that overlapping areas produce stronger shades. As more shaded areas overlap, the darker the area becomes. The top panel of Figure 5.12 shows such an overlay for the independent colours $V-I$, $I-J$, $J-K$ and $B-K$. Using this colour selection, we find best fitting values in the range of $R_V = 3.52 - 3.75$ with only a small interval where the model matches the data consistently for all colours at $3.62 \leq R_V \leq 3.65$. Note that each other filter combination can be deduced from this sample and will therefore have a best fitting value within the same range.

The rather symmetric distribution around this best fitting region by using colours where the filters alternate signs suggests that the deviations between different colours are mainly caused by systematic errors in individual filters. Similarly, it proves that those errors are small compared to the observational uncertainty, since a region of total overlap still exists.

Next, we find the best fitting dust type for M4 by evaluating the four selected colour combinations mentioned above assuming that each of them yields an independent measurement of $R_V$. We want to point out again that we eliminate any filter-specific systematic errors in $I$ and $J$ such as faulty CT-relations or bad standard-star calibrations by using filter combinations where each filter appears on alternating sides of the minus sign. We determine the best fitting value of $R_V$ and its uncertainty with their weighted mean and standard deviation of these for filter combinations:

$$R_V = 3.62 \pm 0.07.$$
Together with the systematic error of 0.06 due to uncertainties in [Fe/H], the total uncertainty including random and systematic effects can be estimated as

$$R_V = 3.62 \pm 0.07 \text{ (random)} \pm 0.06 \text{ (systematic)}.$$ 

This result is in good agreement with previous estimates in the literature, but with a total uncertainty several times smaller.

The individual best fitting $R_V$ values for the various colours as well as the measured colour offsets and the necessary values of $F_{\lambda_1-\lambda_2}$ are listed in Table 5.2 together with their uncertainties. Note that the value derived as the weighted mean nearly reproduces the region of total overlap. Furthermore, all other (deduced) colours yield best fitting values close to and symmetrically distributed around the mean. Note further that, for individual colours, the exact same result is obtained when $V$ is exchanged with $B$: the reddening law does not provide an independent solution for these combinations. This, in fact, shows further that the CT-relations for $B$ and $V$ are extremely well defined.

Interestingly, when using a reddening law with $R_V = 3.62$ for M4, we are actually able to reproduce the mismatch in $J-K$ which we observed in NGC 6723. Therefore, this discrepancy is not caused by an incorrect reddening law, but rather is provoked by problems which most probably arise from CT-relations. We want to point out that the mismatch of the model with the data in $J-K$ with an $R_V$ of 3.62 is therefore not an argument against, but much more a supporting argument for this number, since it reproduces the features of a cluster that is essentially unaffected by reddening.

In Table 5.3 we finally compare our result for $R_V$ and its uncertainty to other results obtained in the past. Here we only include papers where the value of $R_V$ is actually determined and not simply adopted or assumed from existing work. Note further that indirect (environmental) measurements can give a general hint concerning the dust type of M4, but cannot actually be used for a precise prediction.

It is necessary to mention that the determined dust type $R_V$ can generally be different from the actual total-to-selective extinction ratio $A_V/(A_B - A_V)$. This is due to the fact that the dust type parameter $R_V$ in the Cardelli equations is defined for Johnson’s $B$ and $V$ bands using O- and B-type stars. As soon as a different filter set is used, $R_V$ only describes the reddening law and not the extinction ratio. The same is true if the reddening law is used for stars with significantly different temperatures as in our study. In that case, the difference is caused by the change in
effective wavelengths for the $B$ and $V$ filters which is, in a sense, equivalent to using a different filter set.

For the reddening correction and the distance determination, however, we must use the actual ratio between $A_V$ and $A_B - A_V$. The extinction values we obtain with a reddening law using $R_V = 3.62$ as applied to a star with the atmospheric parameters of our fiducial point are listed in Table 5.1 and yield

$$\frac{A_V}{A_B - A_V} = 3.76 \pm 0.09$$

where the uncertainty represents the difference between the total-to-selective-extinction for a reddening law within its uncertainty $\sigma(R_V) = \pm 0.09$.

<table>
<thead>
<tr>
<th>$R_V$</th>
<th>$A_V$</th>
<th>$A_B$</th>
<th>$A_R$</th>
<th>$A_I$</th>
<th>$A_K$</th>
<th>$A_{J_{abs}}$</th>
<th>$A_{V_{abs}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.62</td>
<td>1.474</td>
<td>1.266</td>
<td>0.831</td>
<td>0.608</td>
<td>0.302</td>
<td>0.191</td>
<td>0.123</td>
</tr>
</tbody>
</table>

Table 5.1: Best fitting filter extinctions for M4, using a reddening law with $R_V = 3.62$.

5.4.2 A Quick Test

The two colours $V - J$ and $B - I$ provide a special case since the value of their transformation factors intersect roughly near the canonical dust type at $R_V = 3.2$ with $V - J$ being smaller for $R_V \leq 3.2$ and $B - I$ being smaller for $R_V > 3.2$ (see e.g. Figure 5.4). Therefore those colours provide a quick qualitative test for the dust type: if both $B - I$ and $V - J$ needs about the same colour shift to be matched with isochrones or unreddened reference data, a standard dust type can be assumed. If the colour shift for $V - J$ is significantly greater, $R_V > 3.2$; if the shift in $V - J$ is significantly less, $R_V < 3.2$. 
Figure 5.12: Determination of the dust type in the line of sight to M4: Coloured lines represent the dependence of the different photometric colours on $R_V$. Solid black horizontal lines indicate the best fitting value of $F_{\lambda_1-\lambda_2}$ for each colour and dotted black lines mark the uncertainty interval. Further, each filter combination produces a red shaded region where the boundaries are defined by the intersection of the dotted black lines with the corresponding coloured line. In the top panel, the results for the four independent combinations $V-I$, $I-J$, $J-K$ and $B-K$ are overplotted in a way that overlapping areas produce a more saturated red shade.
\[ \lambda_1 \lambda_2 \quad \text{Offset} \quad F_{\lambda_1-\lambda_2} \quad R_V \]

\begin{array}{cccc}
 B - V & 0.37 \pm 0.01 & 1.00 & - \\
 V - I & 0.53 \pm 0.01 & 1.45 \pm 0.03 & 3.52 \pm 0.13 \\
 V - J & 0.97 \pm 0.02 & 2.63 \pm 0.06 & 3.64 \pm 0.11 \\
 V - K & 1.21 \pm 0.04 & 3.28 \pm 0.10 & 3.60 \pm 0.11 \\
 B - I & 0.90 \pm 0.02 & 2.45 \pm 0.07 & 3.52 \pm 0.28 \\
 B - J & 1.34 \pm 0.03 & 3.64 \pm 0.09 & 3.65 \pm 0.15 \\
 B - K & 1.57 \pm 0.05 & 4.28 \pm 0.12 & 3.60 \pm 0.14 \\
 J - K & 0.24 \pm 0.03 & 0.65 \pm 0.07 & 3.52 \pm 0.28 \\
 I - J & 0.44 \pm 0.02 & 1.20 \pm 0.04 & 3.75 \pm 0.12 \\
 I - K & 0.68 \pm 0.03 & 1.84 \pm 0.08 & 3.66 \pm 0.13 \\
\end{array}

**Table 5.2:** Results for the individual colour measurements. Column (2): Empirical colour offsets together with observational uncertainties between our data and a theoretical isochrone with \([\text{Fe/H}] = -1.0\), \([\alpha/\text{Fe}] = +0.4\) and an age of 12 Gyrs. Column (3) and (4) are the resulting transformation factors and the ratio between total to selective extinction respectively, which determines the dust type in the line of sight toward M4. The uncertainties represent the propagation of the errorbars from the second column.

**Table 5.3:** Existing estimates of the dust type of M4 in terms of \(R_V\). Only publications are listed which actually derive \(R_V\) estimates in their work, instead of adopting or estimating \(R_V\) from previous studies.
Notes to Table 5.3

(1): This result was obtained by the measurement of 12 stars in the nearby Ophiuchi dark cloud. The stated value (which is used by Richer et al. (1997) and others) only represents the mean value of all measurements, where the individual results actually vary between $R_V = 3.15$ and $R_V = 5.25(!)$. This study is an excellent example of the fact that it is not possible to infer the value of $R_V$ for an object from its direct environment. Specifically, even if the lines of sight are similar, we cannot evaluate possible inhomogeneities transverse to the line of sight, or along the line of sight between the nearer and more distant objects.

(2): This result is obtained from the measurement of only one star.

(3) The value actually represents the observed ratio $A_V/(A_B - A_V)$ rather than the dust-type parameter for the Cardelli et al. reddening law.

5.5 Sources of Systematic Uncertainties

Since we are using observational data together with model assumptions, there are several different sources of uncertainty contributing to the total uncertainty of $R_V$. In the following, we discuss the different possible sources of systematic uncertainty and their significance.

5.5.1 [Fe/H]

It is important to know the metallicity of the cluster as accurately as possible since this parameter affects the position and shape of the isochrone, the magnitude of the ZAHB and also the zero points for the reddening law. We constrain the metallicity by isochrone fitting and find the best agreement for [Fe/H] = −1.0 and an age of 12 Gyrs. For the determination of the dust type we are therefore using isochrones with those parameters.

To estimate the change in $R_V$ arising from an uncertainty in [Fe/H], we perform the whole procedure to derive $R_V$ while increasing [Fe/H] by +0.2 dex. This includes the use of a different isochrone to determine the ZAHB luminosity and the colour offsets as well as a different set of zero points for the transformation factors in our reddening law.

The result for [Fe/H] = −1.2 yields a best-fitting value for $R_V = 3.56 \pm 0.07$ with an assumed reddening of $E(B - V) = 0.39$. 
By comparing this number to the result we got assuming \([Fe/H] = -1.0\), we conclude that an uncertainty of \(\sigma[Fe/H] = \pm 0.2\) dex results approximately in a change of \(R_V \approx \pm 0.06\). We conclude that an uncertainty in the metallicity does not strongly affect the determination of \(R_V\), making our method, in general, and our result for the dust type of M4, in particular, robust. Adding the systematic uncertainty coming from \(\sigma[Fe/H]\) quadratically with the photometric uncertainty for \(R_V\), we get

\[
R_V = 3.62 \pm 0.09
\]

### 5.5.2 Reddening Law

The value of \(R_V\) which best fits the data further depends upon the exact reddening law we are using, and explicitly on the zero points for the transformation factors \(F_{\lambda_1-\lambda_2}\). As can be seen in Table 4.2, the values differ between different literature sources and our own calculations and the same would be the case for the value of \(R_V\) needed to match isochrones with observations. In general, the larger the difference between the zero point and the required value, the larger the necessary change in \(R_V\) to reach it.

We can only estimate the uncertainty brought in by the definition of the reddening law zero points by assessing the change in \(R_V\) when we use different sets of values from the literature and our own. The effect on \(R_V\) is, in any case, smaller than 0.1 which might define an upper limit for the uncertainty due to the definition of the reddening law.

### 5.5.3 Isochrones and Colour-\(T_{eff}\)-Relations

When we determine observational colour excesses in different filter combinations relative to theoretical isochrones, we assume that the models actually predict the unreddened CMD colours correctly. If this is not the case, for example due to faulty CT-relations or \(T_{eff}\)-scales, the result of \(R_V\) is biased. We discussed this issue already in Chapter 5.3 where we found that the isochrones provide a consistent fit to the nearly unreddened clusters NGC 6723 and NGC 1851.
5.5.4 Standard System Calibration

The observed data are calibrated to the Landolt (1992) standard system for Johnson-Cousins UBVRI and to the 2MASS standard system for the NIR counterparts J and Ks. The accuracy of the match between our adopted instrumental bandpasses and those from the respective standard systems is determined by the level to which any systematic differences between both can be corrected. Inaccuracies in the calibration to the standard system have an impact similar to that of faulty CT-relations as they systematically bias the colour offset between model and data. Generally, the calibration for our data shows no systematic deviation from this standard system (see Figure 2.3 and 2.4). However, a final intrinsic uncertainty of as much as $\sigma(\text{colour}) \approx \pm 0.03$ (see discussion in Stetson 2005b) has to be considered, especially if the data were obtained in just one observing run, as is the case for our NIR photometry. The calibration of the optical photometry of M4 was derived independently on 20 photometric nights distributed among 9 observing runs, so the overall standard calibration should be significantly better than 0.01 mag in this case.

By using alternating colours such as $V - I$, $I - J$, $J - K$, and $B - K$ to determine the value of $R_V$, we eliminate any systematic error which is specific for the individual filters I and J. For this reason, the effects discussed in 5.5.3 and 5.5.4 have only a minor impact on the quality of our result. However, the uncertainty in our knowledge of [Fe/H] and the exact reddening law zero point is not filter-specific and therefore has to be taken into account when the total uncertainty of $R_V$ is estimated.

5.6 Implications for the Distance Modulus of M4

The absolute distance to a globular cluster, or its true distance modulus $DM_0$, strongly depends on the reddening law and the assumed dust type, if it is derived from standard candles of a stellar population such as variable stars or the apparent magnitude of the ZAHB. In this case

$$DM_0 = DM_{app} - A_\Lambda$$  \hspace{1cm} (5.2)

and

$$d[\text{kpc}] = \frac{10^\frac{DM_0}{5} + 1}{1000}$$  \hspace{1cm} (5.3)
where $DM_{app}$ is the observed or apparent distance modulus and $A_{\Lambda}$ the extinction correction for the filter $\Lambda$. However, the extinction is usually determined with the colour excess and expressed by $E(B - V)$. $A_{\Lambda}$ is then obtained with the ratio of absolute to relative extinction $R_{\Lambda}$:

$$A_{\Lambda} = R_{\Lambda} \cdot E(B - V)$$

(5.4)

Each $A_{\Lambda}$ is therefore unambiguously correlated with the dust type, as is $R_{\Lambda}$, and as a consequence $DM_0$ can only be determined as a function of that parameter.

We determine the apparent distance modulus by comparing the ZAHB $V$-band magnitude to models and find

$$(m - M)_V = 12.66 \pm 0.03$$

(see Figure 5.11) which is slightly smaller than most of the values we find in the literature, especially the value given in Harris (1996) of $(m - M)_V = 12.82$. This is partly because we are using the findings from the treatment of spatial differential reddening in this cluster, which defines the apparent magnitude of the ZAHB not with the faintest stars but with individually differential-reddening corrected stars which defines a slightly brighter zero age. The use of a slightly higher metallicity for the isochrones than Harris assumes additionally decreases the distance modulus.

Using the reddening law with $R_{V} = 3.62 \pm 0.09$ (including $\sigma([Fe/H])$) and a reddening of $E(B - V) = 0.37 \pm 0.01$ determined with isochrone fitting, we get a total-to-selective-extinction of $A_V/(A_B - A_V) = 3.76 \pm 0.09$ for our filters. The distance to M4 is finally calculated with Eq. (5.3):

$$d = 1.80 \pm 0.05 \text{kpc}$$

$$(m - M)_0 = 11.28 \pm 0.06$$

The stated uncertainty for the absolute distance of M4 includes the observational uncertainty for the dust-type parameter ($\sigma(R_V) = 0.09$; including a $\sigma([Fe/H])$ of $\pm 0.2$), the observational uncertainty for the apparent distance modulus ($\sigma(DM_{app}) = \pm 0.03$), and the uncertainty for the reddening ($\sigma(E(B - V) = 0.01)$.
Some publications already use a higher value of $R_V$, and found a distance to M4 of $d = 1.72 \pm 0.14 \text{kpc}$ (Hansen et al. 2004; Richer et al. 1997). Compared to these estimates, our distance is slightly larger but still within their observational uncertainty. It is unclear to us if those studies actually use $R_V$ as the actual total-to-selective extinction for their filters or as the dust-type parameter for the reddening law.

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<th>Distance</th>
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<th>$R_V$</th>
<th>Notes</th>
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<td>-</td>
<td>(3)</td>
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**Table 5.4:** Overview of recently derived absolute distances to M4 with different assumptions of $R_V$.

Notes to **Table 5.4**

(1): This distance is obtained when using the parameters for the apparent distance modulus and the reddening from this author, together with our value of $R_V$.

(2): This result is identical to Richer et al. (1997). The uncertainty does not include uncertainties in $R_V$.

(3): Derived from astrometric measurements; independent of photometric standard candles.
Figure 5.13: Direct comparison of two different reddening laws: The top panels show different CMDs corrected with a standard reddening law and $R_V = 3.1$. On the lower panels the same CMDs are shown if a reddening law with $R_V = 3.62$ is used instead. The isochrone parameters and distance modulus are the same as used in Figure 5.11.
Chapter 6

Summary

In this work we have investigated the dust properties in the direction to the globular cluster M4 using near infrared $J$ and $K_s$ observations obtained with SOFI on the New Technology Telescope in Chile which have been combined with optical Johnson-Cousins $UBVRI$ photometry. We determined the size and the distribution of spatial differential reddening across the cluster face, the mean absolute amount of reddening and finally the type of dust in the line of sight of M4. With this information we were able to determine the absolute distance to M4 with a higher precision than existing estimates.

We have introduced a new method to determine the dust type parameter represented by $R_V$ in the Cardelli reddening law (Cardelli, Clayton & Mathis [1989]) using a combination of NIR and optical photometry together with Victoria-Regina isochrones. This method is independent of age assumptions, insensitive to metallicity and appears to be significantly more precise and accurate than existing spectroscopic approaches.

In the following we list the detailed results:

- We investigated the reddening variations across the face of the field of M4 and found a total peak-to-peak difference of $\delta E(B - V) \approx 0.2$ mag from the north-east to the south-west part of the cluster within a radius of 10' around the cluster centre. Therefore the differential effects within the field of M4 are about half the size of its mean total reddening, which we found to be $E(B - V) = 0.37 \pm 0.01$. By correcting for the variations in reddening, we have been able to decrease the observational scatter in our photometry by about 50% and
consequently increase the precision of important parameters like the zero-age horizontal branch luminosity by the same factor.

- We have written a computer code to investigate the impact of stellar parameters such as temperature, surface gravity and metallicity on the extinction properties in different bandpasses and on the necessary reddening corrections. We found similar sized effects for temperature and surface gravity within typical globular cluster parameter limits; each causes a change of about 3% in the necessary correction factor for all filter combinations. However the impact of gravity seem to be significant only for the cool temperatures on the faint main sequence and the tip of the red giant branch. It is important to note that metallicity in terms of [Fe/H] causes the same changes in the correction factors as the atmospheric parameters when it is changed from solar metallicity ([Fe/H] = 0.0) to [Fe/H] = −2.5.

- We made use of three different spectral databases to compare the results obtained with synthetic ATALS9 spectra to observed spectra for all temperatures (using the atlas of [Pickles 1998]) and metallicity (using the atlas of Sánchez-Blázquez et al. 2006). We found an excellent agreement between the different types of spectra for all metallicities and for temperatures above 5000 K; however significant deviations are found for lower temperatures. We identify the discrepancies at low temperatures to be caused solely by variations in the extinction of the B-band filter and explain the effect by inadequate treatment of molecular bands within the ATLAS9 models.

- We have used our code to calculate object-specific reddening correction factors which are tailored for M4 and use the main-sequence turnoff temperature and gravity instead of a the parameters of a Vega-like star which is commonly used for published values in the literature. Moreover a metallicity of [Fe/H] = −1.0 has been implemented to account for the metal deficiency of M4.

- We have used a combination of optical and near infrared colours to determine the dust type of M4 as characterized by $R_V$ in the Cardelli et al. reddening law by fitting the newest generation of Victoria-Regina isochrones to our observed CMDs, requiring a consistent fit for all filter combinations. Here we have used the fact that the transformation factor $F_{\lambda_1-\lambda_2}$ changes differently with $R_V$ for
each colour. This method is independent of the assumed age of the system and insensitive to the assumed metallicity.

With our method, we have determined the dust-type parameter to be $R_V = 3.62 \pm 0.07 \pm 0.06$, where the first uncertainty is the observational precision and the second estimates possible systematic effects due primarily to uncertainties in [Fe/H]. Using this dust-type parameter, we find an actual ratio for the total-to-selective extinction in our filters of $A_V/(A_B - A_V) = 3.76$. A generally large number for $R_V$ has been proposed several times in the literature but never with such a strong quantitative justification.

- Beside metallicity, we have investigated different sources of systematic uncertainty such as incorrect assumptions for CT-relations and reddening-law zero points for the transformation factors and have discussed their impact on the determination of $R_V$.

- We have tested our isochrones for optical and NIR colours in the intermediate metal poor regime on the nearly unreddened globular clusters NGC 6723 and NGC 1851 and have found that they provide a good fit for magnitudes $M(V) \leq 8.0$ mag, consistent for all colours. It is worth noting that NGC 6723 is identical in age and metallicity to M4 within the photometric uncertainties, with a similar bimodal HB sequence and therefore is an essentially unreddened twin to the highly reddened M4.

- With the new dust type and the knowledge about the spatial differential reddening effects, we have reexamined the distance to M4 and find a significantly smaller ($\sim 10\%$) value for the absolute distance compared to the value provided in [Harris (1996)]. This makes M4 unambiguously the closest globular cluster to the sun with a distance of $1.80 \pm 0.05$ kpc. Our value is, however, somewhat larger than some recent estimates of $d = 1.72$ kpc, using $R_V = 3.8$ ([Hansen et al. 2004]).

The proper treatment of interstellar extinction is a crucial step to achieve accurate astrophysical results from photometric observations. As seen from this perspective, M4 is an excellent object to study the effect of interstellar extinction and its impact on the accuracy of the derived parameters due to the strong, differential and unusual extinction it suffers in combination with its extreme proximity which allows extremely
deep and precise photometry in all photometric filters. In a way, it serves as an in-situ laboratory from which we can learn about the application of extinction corrections for fainter objects such as stellar populations in the bulge of our Galaxy or resolved extragalactic populations.

Our study reveals that individual—object-specific—parameters of the target of investigations such as temperature, surface gravity and metallicity can all have a significant impact on the actual reddening law and consequently on the correction procedure. Even though variations of these parameters within the population do not seem to add a significant systematic error for systems with $E(B - V) \approx 0.4$, the systematic difference caused by the use of wrong zero points (e.g. defined for a Vega-like star) is larger and should be taken into account for all reddened systems. Therefore we recommend the use of an object-specific reddening law defined, for example, by the mean of the crucial target parameters for the correction procedure. However, for highly reddened systems with $E(B - V) = 0.5$ or more, use of a star-by-star correction should be considered to account for the differential effects within the population.

Moreover, our work shows that the shape of the reddening law defined by the type of dust in the line of sight can be as object-specific as the stellar parameters. Especially for highly reddened objects, the unconsidered use of the standard dust-type parameter for the interstellar medium can cause significant systematic errors in the interpretation of photometric data.

Finally, we find that model atmospheres and the resulting synthetic spectra still have problems adequately predicting the complex properties of molecular bands at cool temperatures. In terms of the derived reddening-correction factors, this effect is small. However, more work needs to be done to investigate the size of the effect on the actual CT-relations which are used to compare model predictions to observed results.
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Appendix A

Data Reduction of NGC 6723

Figure A.1: Large red dots indicate the location of bright, isolated program stars in the field of NGC 6723 serving as local standards. Our observations cover a field of about 5' x 5'. The zero point of the coordinate system is at RA = 18h 59m 17.15s and Dec = −36°30'58.9". In total, we pick 128 local standards homogeneously distributed over the individual observation frames. Seven frames are taken in $K_s$, two in $J$, and three in $H$ during one observational night.
Figure A.2: Comparison between stars in our sample to 2MASS photometric standard stars. Orange dots indicate standard stars, green dots indicate all other identified 2MASS stars from our HQ-sample.
Figure A.3: Comparison between stars in our sample to 2MASS photometric standards stars as a function of color. Orange dots indicate standard stars, green dots indicate all other identified 2MASS stars from our HQ-sample.
Figure A.4: Optical photometric uncertainty limits for the HQ-sample of NGC 6723.
Figure A.5: NIR photometric uncertainty limits for the HQ-sample of NGC 6723.
Figure A.6: Location of the stars from the HQ-sample of NGC 6723. For this cluster, we prefer to pick stars outside of a minimum distance to the cluster centre to avoid poor photometry from blended stars. In total, the HQ-sample consists of $\sim 3200$ stars.
Appendix B

Spectra Samples

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<th>$Fe/H$</th>
<th>$\log T_{\text{eff}}$</th>
<th>$\log g$</th>
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<td>3.777</td>
<td>4.02</td>
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Table B.1: Sample of spectral energy distributions from the Sanchez Atlas.
Table B.2: Our “dwarf” sample of spectral energy distributions from the Pickles Atlas.
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<th>Spectral Type</th>
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<th>$M_V$</th>
<th>[Fe/H]</th>
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**Table B.3:** Our “subgiant” and “giant” sample of spectral energy distributions from the Pickles Atlas.
## Appendix C

### Differential Extinction Tables

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Table C.1: Relative extinctions for Johnson-Cousins $UBVRI$ \cite{Landolt1992} and 2MASS $J$, $H$, and $K_s$ filters for spectra with $T_{\text{eff}} = 6000$ K, $\log g = 4.5$, [Fe/H] = 0.0 and $E(B - V) = 0.36$ for different $R_V$. 
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<th>log $T_{\text{eff}}$</th>
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<th>$A_G/A_V$</th>
<th>$A_I/A_V$</th>
<th>$A_J/A_V$</th>
<th>$A_H/A_V$</th>
<th>$A_Ks/A_V$</th>
<th>$A_V(\text{abs})$</th>
<th>reddening law</th>
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Table C.2: Relative extinctions for Johnson-Cousins $UBVRI$ [Landolt 1992] and 2MASS $J$, $H$, and $K_s$ filters for spectra with $R_V = 3.10$, log $g = 4.5$, $[\text{Fe/H}] = 0.0$ and $E(B - V) = 0.36$ for different temperatures.
<table>
<thead>
<tr>
<th>([Fe/H])</th>
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<th>(\frac{\Delta B}{A_V})</th>
<th>(\frac{\Delta R}{A_V})</th>
<th>(\frac{\Delta I}{A_V})</th>
<th>(\frac{\Delta J}{A_V})</th>
<th>(\frac{\Delta K_s}{A_V})</th>
<th>(A_V(\text{abs}))</th>
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<td>0.116</td>
<td>1.134</td>
</tr>
</tbody>
</table>

- 1.664  1.321  0.819  0.594  -  -  -  -  Schlegel (1998)

Table C.3: Relative extinctions for Johnson-Cousins \(UBVRI\) \(\text{[Landolt 1992]}\) and 2MASS \(J, H,\) and \(K_s\) filters for spectra with \(T_{\text{eff}} = 6000\) K, \(\log g = 4.5, R_V = 3.10\) and \(E(B-V) = 0.36\) for different metallicities.