Quantifying Peak Freshwater Ice across the Northern Hemisphere using a Regionally Defined Degree-day Ice-growth Model

by

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B. Sc., University of Victoria, 2010

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Freshwater ice (river and lake ice), a key component of the cryosphere, plays a dominant role in the hydrology of northern climates. Although freshwater ice has been modelled at small geographic scales, it remains the only major unquantified component of the cryosphere. Therefore, the goal of this thesis is to quantify peak freshwater ice across the Northern Hemisphere using a regionally defined degree-day ice-growth model. To address this the ecological and climatic importance of freshwater ice are reviewed, as well as the physical processes that govern freshwater-ice growth, the existing approaches to modelling freshwater ice, and the major climate classification methods. Using a degree-day ice-growth model, ice-growth coefficients are defined by hydro-climatic region, and validated using maximum observed seasonal ice thickness values from across the Northern Hemisphere. The maximum seasonal extent of freshwater ice is then estimated over a 44-year temporal period and the areal extent and volume of freshwater ice quantified.
# TABLE OF CONTENTS

SUPERVISORY COMMITTEE ..................................................................................... ii  
ABSTRACT .................................................................................................................. iii  
TABLE OF CONTENTS ......................................................................................... iv  
LIST OF TABLES .................................................................................................... vii  
LIST OF FIGURES ................................................................................................... viii  
ACKNOWLEDGMENTS .............................................................................................. ix  

CHAPTER 1: INTRODUCTION ............................................................................. 1  
1.1 Purpose and Objectives .................................................................................. 2  
1.1.1 Study area ................................................................................................. 3  
1.1.2 Purpose of study ..................................................................................... 3  
1.1.3 Objectives ................................................................................................. 3  
1.2 Thesis Structure ............................................................................................. 4  
References ............................................................................................................... 5  

CHAPTER 2: LITERATURE REVIEW .................................................................. 7  
2.1 Importance of Freshwater Ice ....................................................................... 7  
2.1.1 Freshwater ecosystems ........................................................................... 8  
2.1.2 Climate change and freshwater ice ......................................................... 11  
2.2 Freshwater-ice Growth ............................................................................... 13  
2.2.1 Ice freeze-up and growth processes ....................................................... 13  
2.2.2 Climatic and non-climatic index relationships ...................................... 16  
2.3 Freshwater-ice Modelling Approaches ....................................................... 21  
2.3.1 History .................................................................................................... 22
| 2.3.2 Physically-based models | 23 |
| 2.3.3 Regression models | 31 |
| 2.3.4 Degree-day models | 32 |
| 2.4 Climate Classifications | 36 |
| 2.4.1 Existing climate classifications | 36 |
| 2.4.2 Weighted linear combination | 38 |
| 2.4.3 Weights-of-evidence | 40 |
| 2.4.4 Spatial clustering | 41 |
| References | 44 |

CHAPTER 3: DEFINING FRESHWATER-ICE GROWTH COEFFICIENTS BY HYDRO-CLIMATIC REGION

Abstract | 52 |
3.1 Introduction | 54 |
3.2 Methodology and Data | 57 |
| 3.2.1 Model background | 57 |
| 3.2.2 Classification of hydro-climatic regions | 60 |
| 3.2.3 Calibration of model coefficients by hydro-climatic region | 62 |
| 3.2.4 Validation | 69 |
3.3 Results and Discussion | 69 |
| 3.3.1 Classification of hydro-climatic regions | 69 |
| 3.3.2 Calibration of model coefficients by hydro-climatic region | 73 |
| 3.3.3 Validation | 76 |
3.4 Conclusion and Future Research | 78 |
LIST OF TABLES

Table 3.1: Typical $\alpha$ values, derived for the Stefan equation by Michel (1971, p. 79). ... 60

Table 3.2: Freshwater-ice thickness datasets compiled for model calibration and validation across the Northern Hemisphere, split by water-body type. ... 66

Table 3.3: Total number of observation sites used in model calibration and validation, split by dataset and water-body type, for datasets which contained both river and lake-ice data. ... 67

Table 3.4: Total number of observation sites used in model calibration and validation, split by dataset, for datasets which contained only river sites. ... 67

Table 3.5: Cluster means and standard deviations (in parentheses) for January precipitation and mean January temperature for the 14-cluster hydro-climatic region definition, as well as sample size and area (in parentheses) per cluster. ... 73

Table 3.6: Calibration results by hydro-climatic region definition, stratified by water-body type. ... 75

Table 3.7: Ice-growth coefficients defined during calibration for 14 hydro-climatic regions, stratified by water-body type. Mean January precipitation and temperature are provided for comparison of clusters. ... 75

Table 3.8: Model validation results by water body type and hydro-climatic definition, with and without infilling using a single optimal coefficient for regions lacking observational data. ... 77

Table 4.1: Area and volume of freshwater ice. Values represent peak freshwater ice, averaged between 1957 and 2002. ... 90
LIST OF FIGURES

Figure 3.1: All freshwater-ice thickness observation sites used in model calibration and validation, split by water-body type.......................................................... 68

Figure 3.2: Fourteen hydro-climatic regions, defined using two-step clustering method, and latitude, elevation, and mean January temperature and precipitation, north of the January 0°C isotherm. Note: the numbers assigned to each hydro-climatic region are arbitrary................................................................................................................................................ 71

Figure 3.3: Fourteen hydro-climatic regions, overlaid with all freshwater ice thickness observation sites.......................................................................................................................... 72

Figure 3.4: Maximum observed seasonal ice thickness measurements compared to modelled ice thicknesses during model calibration, using the 14 hydro-climatic region definition and optimal coefficients defined by region, stratified by water-body type..... 76

Figure 3.5: Maximum observed seasonal ice thickness measurements compared to modelled ice thicknesses during model validation, using the 14 hydro-climatic region definition and optimal coefficients defined by region, stratified by water-body type..... 78

Figure 4.1: Accumulated freezing degree-days averaged between 1957 and 2002, north of the January 0°C isotherm. ................................................................................................................................. 91

Figure 4.2: Hydro-climatic regions north of the January 0°C isotherm defined in Chapter 3................................................................................................................................................ 92

Figure 4.3: Freshwater-ice distribution north of the January 0°C isotherm.................. 93
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CHAPTER 1: INTRODUCTION

To set the thesis in context, this introductory chapter identifies research gaps, outlines the purpose and objectives of this thesis and presents the thesis structure.

Freshwater makes up a small portion of the overall water volume on the earth, and consists of ice caps, glaciers, groundwater and soil, lakes, swamps, rivers and atmospheric moisture (Ashton, 1986). The components that make up freshwater ice include ice caps, glaciers, river and lake ice, however for the purpose of this thesis, freshwater ice is defined as floating river and lake ice, excluding icings, anchor ice and hanging dams. A high density of freshwater lake and river ecosystems exist within the Northern Hemisphere (Downing et al., 2006), many of which are located at higher latitudes where the air temperature drops below 0°C in the winter months. Such northern climates are characterized by the formation of an ice cover on many of the water bodies (Prowse & Beltaos, 2002). Within these northern climates, the seasonal duration and extent of freshwater ice has both ecological and socio-economic implications discussed further in section 2.1.1.

Freshwater ice is one of the main components of the cryosphere, along with seasonal snow, sea ice, mountain glaciers, ice sheets, ice caps, permafrost and seasonally frozen ground (Fitzharris, 1996). The cryosphere and its components integrate climate variations over a wide temporal scale through their direct connection to the surface energy budget, the water cycle and the surface gas exchange. These detectible variations in climate provide a visible expression of our changing climate (Lemke et al., 2007); implying variations in freshwater ice can be used to monitor variations in the overall climate.
The current state of freshwater ice, in terms of quantity and distribution, must be established in order to estimate temporal change and variation of freshwater ice. To date, only one estimate of the areal extent of freshwater ice has been cited in the literature (Ashton, 1986) and it reports the area of seasonal snow and freshwater ice to be \(45 \times 10^6\) km\(^2\) (credited to Untersteiner, 1975). Upon inspection, however, this estimate was found to be incorrect, because Untersteiner (1975) provided only an estimate for seasonal snow (\(45 \times 10^6\) km\(^2\) in January). Although considerable research has focused on quantifying the area and volume of other cryospheric components (IPCC, 2007), there remains no global, hemispheric or even broad regional estimates for freshwater ice quantities.

### 1.1 Purpose and Objectives

Observations of freshwater-ice thickness are sparse (Lemke et al., 2007), and to date freshwater-ice cover research has focused on ice phenology trends and temporal variations, but future research recommendations such as those given by Beltauos and Prowse (2009) state additional work should focus on ice-cover thickness trends and duration. Both the Intergovernmental Panel on Climate Change (IPCC; 2007) and the Canadian Council of Ministers of the Environment (CCME, 2003) have designated river and lake ice as a top indicator of climate change, suggesting changes to freshwater ice will require further investigation by the scientific community. There are gaps in scientific knowledge of freshwater-ice distribution and quantity, and gaps in understanding of freshwater-ice processes in relation to a changing climate. This thesis will address this gap in knowledge.
1.1.1 Study area

General Circulation Model’s (GCM’s) have suggested that climate change will be amplified in the high-latitude regions (Kattsov and Källén, 2005) and interest has been focused on these areas where cryospheric components (e.g. freshwater ice) are most sensitive (Prowse et al., 2002). The Northern Hemisphere is where the largest temperature increase has occurred in recent decades (Serreze et al., 2000), and where the main cryospheric components, including freshwater ice, are largely located. For these reasons, the Northern Hemisphere, covering the area from the equator to the North Pole, will be the focus of this research.

1.1.2 Purpose of study

Although the distribution of freshwater ice has been examined at a small scale (e.g. Assel et al., 2003), a comprehensive quantification of the area and volume of freshwater ice across the Northern Hemisphere has not been conducted. This thesis addresses this shortcoming through the use of Geographic Information Science (GIS), which allows the first assessment of large-scale spatial and temporal variability in freshwater-ice thickness.

1.1.3 Objectives

The primary goal of this research is to quantify the aerial extent and volume of freshwater ice across the Northern Hemisphere during its maximum seasonal extent. The specific objectives are to:

(i) develop a degree-day ice-growth model that addresses regional hydro-climatic variations across the Northern Hemisphere;
(ii) calibrate the degree-day ice-growth model by define optimal ice-growth coefficients by hydro-climatic region, using historical peak-ice thickness data from locations across the Northern Hemisphere;

(iii) validate the degree-day ice-growth model using historical peak-ice thickness data from locations across the Northern Hemisphere and coefficients defined in (ii);

(iv) model peak freshwater-ice thickness across the Northern Hemisphere using a suitable spatial dataset of rivers and lakes and the degree-day ice-growth model from (iii); and,

(v) quantify the areal extent and volume of peak freshwater ice for the Northern Hemisphere using the modelled peak-ice thickness in (iv).

1.2 Thesis Structure

This thesis is comprised of five chapters. Chapter 1 introduces the thesis topic, identifies gaps in the literature, and describes the purpose and objectives of the thesis in detail. Chapter 2 presents a literature review to put the thesis in context, through the assessment of the current state of knowledge in the field of study. Objectives i, ii and iii outlined above are addressed in Chapter 3, which is written as a scientific journal-style manuscript. Objectives iv and v outlined above are addressed in Chapter 4, again written in journal style. Chapter 5 concludes this thesis with a summary of the research findings presented in Chapters 3 and 4, and identifies future research avenues in freshwater-ice thickness modelling. Because Chapters 3 and 4 are written as scientific journal-style manuscripts, some material presented in Chapters 1 and 2 will be repeated in Chapters 3 and 4 when necessary.
References


CHAPTER 2: LITERATURE REVIEW

Modelling of freshwater-ice growth is difficult due to complex hydro-climatic processes at work during ice cover development. These complexities will be addressed in this chapter through the review of the ecological and climatic importance of freshwater ice, and the hydrologic, hydraulic, meteorological and thermodynamic processes that govern freshwater-ice growth. Following this, different freshwater-ice modelling approaches used in the literature are explored for their strengths, weaknesses and relevance to modelling freshwater ice on a hemispheric scale. Finally, existing climate classifications and regional classification approaches are reviewed for their applicability to the goal of defining hydro-climatic regions. These four sections are to provide background about the methodology selected in this thesis to address the objectives presented in 1.1.3. This literature review does not cover the topics in detail, but instead provides the rational why the final methodological approaches described in Chapters 3 and 4 were selected.

2.1 Importance of Freshwater Ice

To provide background on the broad importance of freshwater ice, its biological, chemical, hydrological and geomorphological influences on freshwater ecology, as well as its relevance in climate change and variability, are discussed below for lotic and lentic system. The discussion follows a seasonal timeframe because the influences of freshwater ice vary by season (Prowse, 2001a, b; Walsh, 2005; Prowse et al., 2007). The effects of freshwater-ice freeze-up, growth and breakup are often similar between rivers and lakes, however, there are certain processes that are specific to the water-body type, and these are described when required. The three ice-related periods are considered freeze-up, main
winter and breakup (Prowse, 2001a). For a physical review of the freeze-up and ice-growth period on rivers and lakes, see section 2.2.1.

2.1.1 Freshwater ecosystems

The processes of freeze-up, ice growth, and breakup play important roles in lake and river ecosystems. As the water temperature begins to cool in the fall, many aquatic species respond to these thermal changes through a lowering of their metabolism and reduced activities (Prowse, 2001b). As ice begins to form along the banks of the water body, it provides a low flow refuge for fish, protecting them from predation (Maciolek and Needham, 1952). Water quantity and productivity are also affected. As the ice grows, water is abstracted from the water column into the ice cover, resulting in reduced water volume, and reduction or elimination of downstream flow in rivers systems (Gray and Prowse, 1993). On rivers, this abstraction can occasionally trigger low flows severe enough to affect downstream aquatic habitat due to ecological and water quality issues (Beltaos, 2000; Prowse, 2001a). Conversely, elevated water levels upstream of the ice aid in the replenishing of water in riparian areas due to flooding over the rivers banks (Prowse, 2001a, b).

Once an ice cover has formed, many ecological changes occur. Hydrologic processes important in the summer months, such as evaporation and condensation, slow or stop, as the ice cuts the water body off from direct heat and moisture exchanges with the atmosphere (Adams, 1981). The formation of an ice cover can reduce or eliminate flows to and from outlets and inlets, as the ice cover acts as a dam, restricting the movement of water. This restrictive movement can lead to a reduction in productivity and oxygen supply, causing eutrophication in some lakes (Adams, 1981). As an ice cover
halts direct gaseous exchange between the water body and the atmosphere, dissolved-
oxygen (DO) levels can decrease. Lower DO levels have been shown to reduce biological
productivity, diversity, and robustness of aquatic communities (e.g. Barton and Taylor,
1996). As well, an ice cover alters the radiation regime of a water body through the
reflection of incoming solar radiation, thereby reducing the radiation reaching the water
column and reducing the biological activity under the ice cover (Prowse, 2001a). An ice
cover also reduces the ability of a river to transport dissolved and suspended sediments
downstream (Prowse, 2001a). As the ice grows vertically into the water column, it can
reach the bed in shallow locations. Invertebrate may become trapped in the ice, where
they may overwinter without detrimental effects or become subjected to the mechanical
effects of freeze-up, and not survive the winter (Prowse, 2001b).

The ice composition and presence or absence of snow also affects winter lake
productivity. The presence of a snow cover on the ice sheet will alter the type and amount
of solar radiation penetrating the ice, with snow-free black ice allowing near-maximum
penetration of short-wave radiation through the ice cover, and white ice and snow
effectively reflecting a large portion of it, and thereby reducing the amount reaching the
underlying water column (Prowse, 2001a). This also serves to reduce radiative heating of
the lake water, although it can also be heated from below through heat released from
bottom sediments (Gu and Stefan, 1990). Heating of the water column allows many
organisms to over-winter in lakes, which would otherwise be too cold for survival, with
heating from below often having a larger role than that from above (Adams, 1981). The
presence of snow and snow ice on an ice cover also limits the penetration of ultraviolet
radiation into the water column, which affects both aquatic biota and the ecosystem
through photochemical alterations of molecules and oxidative damage to the cellular structure of organisms (Wrona et al., 2006).

Frozen lakes and rivers are also important economically as they provide major transportation corridors to access and deliver supplies to remote northern communities. Frozen lakes and rivers make ideal winter roads, as they are flat and smooth and since the onset of oil and gas exploration in the 1960’s, these roads have also been used for exploring the Arctic regions (Adam, 1981). These frozen water bodies are also integral to the indigenous peoples who inhabit the area as lakes provide camps during ice fishing, and rivers provide travel routes for snow mobile or more traditional modes of transportation (Nuttall et al., 2005).

As the temperatures warm, and the ice thins, the breakup period begins. Breakup can be characterized as thermal or dynamic, and often associated with lake- or river-ice breakup respectively. During a thermal breakup, spring discharge remains low as the ice sheet detaches from the banks, thins and dissipates, while dynamic breakup occurs as a large spring flood from melting snow and ice upstream comes in contact with an intact ice sheet, breaking it up and driving it downstream (Gray and Prowse, 1993). The melting ice cover and often-accompanying snow cover provide a freshwater influx to the water body, the chemistry of which influences the nutrient budget of a lake in the spring, and can cause such things as phosphorous loading in some lakes (Adams, 1981). This sudden influx of nutrients can also have positive ecological effects in more remote areas where nutrient supply is low (Barica and Armstrong, 1971).

Dynamic breakup can have major effects on communities and habitat in close proximity, as ice jams associated with dynamic breakup can cause flooding upstream
followed by large surges downstream when the ice jam releases (Beltoos, 1997). Large chunks of ice can scour the riverbed and banks, eroding sediment and transporting it downstream. This can lead to bank destabilization and slope failure (Prowse, 2001a). Such dynamic break-ups will also remove riverine vegetation along the banks and are considered a reset mechanism for the physical characteristics of the river channel, allowing certain biological activity to thrive after a breakup event (Prowse, 2001b).

The ecological and socio-economic effects of freshwater-ice processes are far reaching and climate-influenced changes to freshwater ice can have major consequences for both the natural environment and human activities (Lemke et al., 2007). This implies a need for more research on the topic of freshwater ice to understand how our changing climate will influence these processes. The following section will address this through the review of the link between climate and freshwater ice.

2.1.2 Climate change and freshwater ice

In the context of freshwater ice, Beltoos and Burrell (2003) define climate as the weather experienced over time at any one location whilst climate change is defined as changes in the entire climate, not just a single component of the weather. Furthermore, they define global climate change as changes in all interconnected weather components.

Temperatures have shifted considerably throughout the history of the earth, but over the past 100 years this variability has been more noticeable and accelerated with the world’s average temperature increasing by 0.6°C over the twentieth century (CCME, 2003; IPCC, 2007). The difference in global temperatures between now and the peak of the last ice age is a mere 5°C (CCME, 2003). Accompanying this change in temperatures are changes in the large-scale hydrological cycle including changes in precipitation
patterns, reduced snow cover, and widespread melting of ice, particularly in the Northern Hemisphere (Bates et al., 2008). This widespread melting of ice is resulting in a shrinking cryosphere and an influx if freshwater to the ocean, contributing to sea level rise (Lemke et al., 2007). With global climate change affecting the hydrologic cycle, there is a necessity to understand how this will affect freshwater ice as well as other cryospheric components.

Historical freeze-up, breakup and ice cover data have been used as quantitative indicators of climate change by, for example, Palecki and Barry (1986) and Robertson et al. (1992). As freshwater ice serves as a climate indicator (Walsh et al., 2005), and therefore a proxy for climate change, changes in freshwater ice reflect changes in our climate. This link has been explored with a focus on ice phenology and how the timing of freeze-up and breakup has changed with a warming climate. Studies such as Duguay et al. (2006) examined the trends in lake freeze-up and breakup across Canada (1951-2000) and found variable trends in freeze-up dates and contrasting trends in breakup dates between the eastern and western portions of Canada. Lakes east of Hudson Bay are experiencing earlier breakup while those to the west are experiencing later breakup. River freeze-up and breakup have shown more consistent trends across Canada (1951-1998), with a significant trend towards earlier breakup and a variable trend towards later freeze-up (Lacroix et al., 2005). On a larger scale, Magnuson et al. (2000) examined trends in lake- and river-ice cover across the Northern Hemisphere for the period 1846-1995 and found similar trends of later freeze-up and earlier breakup over the 150 year period of study. These results indicate the robustness of river and lake ice as a proxy indicator of climate variability and change. With this strong link between changes in the cryosphere
and changes in the climate, freshwater ice, and changes in its quantity and distribution, is important to our understanding of the global climate system.

Issues of climate change and climate variability are driving current freshwater-ice research (Beltaos and Burrell, 2003), but little research has examined the impacts climate change has had on river- and lake-ice thickness over the past decade (Beltaos and Prowse, 2009). As indicated in the most recent report by the IPCC, there is inadequate information regarding the water-related impacts of climate change (Bates et al., 2008), and large-scale ice thickness datasets required to explore these areas of research are sparse (Prowse et al., 2007). There are gaps in scientific knowledge of freshwater-ice distribution and quantity, and gaps in understanding of freshwater-ice processes in relation to a changing climate.

2.2 Freshwater-ice Growth

To better understand these freshwater-ice processes, and to place river and lake ice in a more detailed hydro-climatic context, a brief hydrological review is presented in four parts, including: 1) freeze-up and ice-growth processes on lakes and rivers, 2) energy exchanges controlling freshwater-ice processes, 3) climatic controls of freshwater-ice processes, 4) related non-climatic controls. As the focus of this thesis is on quantifying peak-ice thickness occurring before the ice begins to decay, the later end of season breakup process is not reviewed, but details about this process can be found in, for example, Gray and Prowse (1993).

2.2.1 Ice freeze-up and growth processes

The following section provides a physical description of the freeze-up and ice-growth processes that occur on both lakes and rivers during the winter months. These
processes are similar for both water-body types, while there are also some clear differences between them, described below when required. The timing of freeze-up is dependent of the heat storage of the water body and the cooling in the fall, as discussed in section 2.3.2. Ice will form at different times on water bodies experiencing the same meteorological conditions if they have different surface area to volume ratios, meaning wide shallow water bodies will freeze sooner than deep narrow water bodies under the same meteorological conditions (Gray and Prowse, 1993). The review of freeze-up and growth follows the seasons as they move from fall to winter, and as stated earlier, does not cover breakup.

Late in the summer and into the fall, as the mean daily air temperature begins to move towards 0°C, the surface water of a lake will also begin to cool as the heat exchange between the lake surface and the surrounding air takes place and the heat from the lake is drawn out into the cooler air. This transition from open-water conditions to freeze-up involves two distinct phases. The first is the isothermal phase, during which the water cools to reach a uniform temperature of maximum density at 4°C. This process is driven naturally by density instability, as the surface water cools and becomes denser than that below it, or by wind, which cools the surface of the water and in turn creates density instability (Gerard, 1990). A lake will then experience a full turnover when half of the water body reaches its maximum density and sinks, to be replaced by warmer less dense water on top (Gray and Prowse, 1993). After this thermal stratification and overturning, the second phase begins, where the less dense surface water cools to the freezing point and an ice cover begins to develop on top (Michel, 1971). Rivers follow a similar process, experiencing the first isothermal phase of cooling, however, due to the
turbulent nature of rivers a mixing of the warmer and cooler water precludes the thermal stratification observed on lakes, and thus the entire water column cools at approximately the same rate (Gray and Prowse, 1993).

The subsequent process of ice formation requires nucleation, which naturally occurs in water containing foreign particles that act as freezing nuclei (Prowse, 1995). Once nucleation has occurred, lake ice will begin to form along the shores first, in calmer weather conditions, before progressing towards the center of the water body (Michel, 1971). On lakes, as the ice cover begins to form, a thin layer of ice known as skim ice will typically develop over night, and break during the day due to wind and warming. This step will repeat itself over several days before a permanent ice cover develops (Gray and Prowse, 1993). Lake currents will then control the progression of the ice sheet on a lake (Michel, 1971), moving the ice around the lake until it drifts into shore ice or other solid chunks and forms larger sheets, eventually forming a permanent ice cover.

The process of ice formation on a river is different, again due to the turbulent nature of rivers. After nucleation, frazil ice begins to form in the water column as it mixes and cools (Prowse, 1995). Frazil ice is defined as the small disks of ice forming in turbulent, slightly supercooled waters (Michel, 1971). After formation these frazil ice particles float to the surface and begin to cluster together to form large “flocs” and eventually large sheets of ice (Prowse, 1995). Under freeze-up conditions, these sheets of ice will continue to grow and agglomerate (Gray and Prowse, 1993). Eventually they will stretch across and attach to the border ice on the opposite side of the riverbank, creating an obstruction or “bridge” to objects down the river channel (Prowse, 1995). Ice is swept downriver where it builds up against this bridge, often producing a 15m/day upstream
advance of a freeze-up front (Martin, 1981). Frazil ice also adds to the ice thickness on a river over the ice-growth period. As frazil ice is generated, it becomes deposited beneath the ice cover, and with further ice-growth progression vertically into the water column, the frazil is encapsulated and becomes part of the solid ice cover (Gray and Prowse, 1993). As ice sheets completely cover a river, frazil ice stops being produced (Prowse, 1995).

As the ice sheet develops over the winter, the presence or absence of snow will dictate whether white ice or black ice will form on both rivers and lakes. White ice forms under the presence of snow at the ice/snow interface, while black ice forms under the absence of snow. As snow falls on a water body it insulates the ice cover and slows the ice-growth process through restriction of heat loss to the atmosphere. If enough snow falls onto an ice cover to cause a positive hydrostatic water level, the ice cover will develop cracks under the weight of the snow, and water will seep through. This water will mix with the snow cover, and, once frozen, form white ice (Gray and Prowse, 1993). Ice will then continue to grow vertically into the water column throughout the winter, as air temperatures remain below freezing (Prowse, 1995), reaching maximum or peak-ice thickness just before the onset of spring warming and breakup.

2.2.2 Climatic and non-climatic index relationships

Processes that govern ice freeze-up and growth on both rivers and lakes described above have been linked to several climatic indexes including seasonal air temperature, geographic location of the 0°C isotherm, presence and quantity of snow accumulation, and wind speed, in studies that focus on regions across the Northern Hemisphere (Brown and Duguay, 2010). This section reviews the relationship between climatic and non-
climatic indexes and the basic physical processes of freeze-up and ice growth described in section 2.2.1.

Of all climatic indexes, freeze-up correlates best with air temperature. Bilello (1964) was one of the first to use the relationship between air temperature and the date of ice formation, previously defined by Rodhe (1952) for the Baltic Sea, to forecast the first appearance of ice in the fall and subsequent ice formation along the Mackenzie River. Because the Baltic Sea has low salinity, it was assumed to freeze at around 0°C, similar to freshwater bodies. This assumption allowed the model to easily be extended to lake systems and even rivers. First-ice and freeze-over forecast curves were developed for Fort Good Hope, Northwest Territories using previously observed data and the relationship between air temperature and ice formation, but Bilello (1964) warned this method was not suitable for water bodies susceptible to wind-induced breakup, as only thermal influences on breakup were considered in the development of the curves.

This relationship between seasonal air temperature and freshwater-ice processes has been further explored for both rivers and lakes across a wide range of geographic locations. Williams (1965) also found air temperature to be a climatic index of freshwater-ice freeze-up and breakup on lakes in the Ottawa region. Air temperatures were correlated with surface water temperatures during the first stage of freeze-up, and accumulated freezing degree-days (AFDD; daily air temperature below 0°C) were correlated with the time taken for a solid ice cover to form during the second stage of freeze-up. It was found that predicting freeze-up was almost entirely dependent on weather conditions, while breakup was dependent on weather conditions, ice thickness, and snow cover. This implies that the energy budget controlling freeze-up is simpler than
that controlling breakup. The link between air temperature and ice on rivers was further explored by Rannie (1983), who found air temperature to be the dominant factor in both freeze-up and breakup events along the Red River, Winnipeg. Robertson et al. (1992) then used this ice phenology/air-temperature relationship and worked backwards to estimate historical air-temperature changes using historical freeze-up and breakup dates for the Lake Mendota, Wisconsin area. Lake-ice phenology was used as a proxy for local and regional surface air temperatures. This approach was also explored for breakup dates on three alpine lakes in the Swiss Alps (Livingston, 1997). Based on an uninterrupted ice breakup record from 1832, results showed the timing of breakup strongly related to local and regional surface air temperatures.

To further establish this relationship between freshwater-ice processes and air temperature, it was explored at a larger geographic scale than site or region specific water bodies. Magnuson et al. (2000) explored the relationship between ice phenology and air temperature on a much larger geographic scale, analysing the trends in breakup and freeze-up dates for both rivers and lakes throughout the Northern Hemisphere over a 150 year period (1864-1995). They found that trends towards earlier breakup and later freeze-up corresponded to an increase in air temperature over the same time period (~1.2°C/100 years), providing further evidence to the strong link between air temperature and ice phenology. Weyhenmeyer et al. (2010) also used this link between air temperature and ice phenology to model ice-on and ice-off dates for a global dataset of 1213 lakes and 236 rivers. The timing and duration of ice cover was modelled using air temperature, altitude, and latitude, a proxy for solar radiation. Results showed that ice-off dates were
strongly related to latitude, but ice-on dates were not, and both were weakly related to altitude.

More specifically, freeze-up and breakup on a water body has been related to the advance and retreat of the 0°C isotherm (Bonsal and Prowse, 2003), which allows for the estimation of these significant changes in freshwater ice based on the geographic location. Prowse et al. (2002) linked the 0°C isotherm to indicators of river-ice breakup across northern Canada, while Duguay et al. (2006) showed similar spatial and temporal trends between the 0°C isotherm dates and freeze-up/breakup for lake ice across Canada, with trends towards earlier lake-ice breakup dates over the majority of western Canada. Bonsal and Prowse (2003) also examined the spatial and temporal variability of the 0°C isotherm across Canada for the years 1900-1998 and 1961-1990. They found that the spring and autumn 0°C isotherms tend to be earlier along the west coast of British Columbia and southern Ontario due to Pacific and Atlantic oceanic influences for the years 1961-1990. Trends in the 0°C isotherm are also linked to large-scale oscillations over the Pacific and Atlantic for the years 1950-1998. More recently, the 0°C isotherm has been used to quantify the extent of ice-covered rivers across the Northern Hemisphere (Bennett and Prowse, 2010). Air temperature for the Northern Hemisphere was used to define three isotherms: January isotherm, mean winter isotherm, and annual isotherm. All rivers north of the defined isotherms were considered to have river ice present on them and, using GIS, the surface area of ice-covered rivers was quantified (Bennett and Prowse, 2010).

Snow accumulation also plays a role during the freeze-up and growth of an ice cover as described in section 2.1.1. In a study by Duguay et al. (2003), snow cover was
shown to govern the mean absolute error in ice-off dates during model simulation of lake-ice phenology, which ranged from 1 to 8 days depending on the snow cover scenario input to the model for the Churchill, Manitoba study site. Varying the modelled snow depth at the study sites also influenced maximum ice thickness. This indicates that snow cover influences lake ice-off dates as well as maximum lake-ice thickness, as it acts as an insulator, preventing initial ice melt when air temperatures increase. Duguay et al. (2006) also found that locations where trends towards later freeze-up were observed corresponded to locations with increasing fall snow cover, again indicating snow cover has a direct influence on lake-ice phenology.

Lastly, wind speed can play a significant role during the freeze-up process on both lakes and rivers (e.g. Duguay et al., 2006), as well as during breakup (e.g. Williams, 1965). Wind will dictate where snow will accumulate on a water body, and create a more dynamic breakup on a lake through the movement and collision of the ice cover into the shores and other ice chunks. Using available break-up data for across Canada, Williams (1965) found that breakup on lakes, rivers and salt-water harbours driven by heat gain from the atmosphere occurred at a rate of ~ 2.5 cm/day, where breakup driven by wind and currents occurred at a rate more than 10 times faster, at ~38 cm/day. These results illustrate the difference between atmosphere-driven breakup and wind-driven breakup, indicating winds and currents often influence breakup more than heat gain. An energy balance model by Liston and Hall (1995) also concludes that wind plays a key role in the growth and decay of lake ice, through its ability to affect lake surface temperature and lake-ice surface snow accumulation and location.
Non-climatic controls governing ice freeze-up and growth include lake and river morphology, latitude and elevation, inflow from streams, and land runoff (Brown and Duguay, 2010). Model simulations by Duguay et al. (2003) show that timing of breakup is related to lake depth, with deeper lakes remaining ice-covered later in the spring, and Korhonen (2006) found that deeper lakes can freeze up to one month later in the fall when compared to shallower lakes in the same region. Mean lake depth also has a stronger influence than surface area on ice-in, ice-out, and ice thickness measurements (Williams and Stefan, 2006), and morphology has also been poorly correlated with freeze-up and breakup on Alaskan water bodies (Jeffries and Morris, 2007). These findings indicate the important role of lake depth on ice phenology, attributed to the heat storage of deeper lakes, which influences the heat budget of the lake, and therefore may delay freeze-up.

The process of freeze-up and growth, as well as the climatic and non-climatic indexes influencing freshwater-ice growth, can be described by a series of heat fluxes and modelled using both physical and degree-day based models. These will be reviewed below.

2.3 Freshwater-ice Modelling Approaches

Much research over the last few decades has focused on modelling freshwater-ice processes described in section 2.2. The origins of these approaches can be traced back to the link between air temperature and ice growth, first established in the literature by Stefan (1891), as it applied to sea-ice growth. This basic relationship between air temperature and sea-ice growth has since been utilized in a broad range of ice-growth models from simplistic degree-day equation, to physically-based ice-growth estimations,
to more complex energy-balance approaches, to multidimensional thermodynamic and multivariate linear regression models. This evolution of ice-growth models is reviewed below.

2.3.1 History

Stefan (1891) has been credited in the literature for establishing the connection between air temperature and sea-ice thickness mathematically, where the squared ice thickness is proportional to the difference of the freezing point of water (saline or fresh) and the air temperature at the time of ice thickness measurement. This theory is best known as the Stefan Law or Stefan equation (Lepparanta, 1983). Rodhe (1952) developed a method for calculating the weighted mean temperature from an air-temperature series, where temperatures read further in the past have less weight than those taken closer to the present, using a coefficient, further advancing the relationship between the date of sea-ice formation and the weighted mean temperatures on the Baltic Sea presented by Stefan (1891). Bilello (1961) also used air temperatures to predict formation, growth, and decay of sea-ice around the Queen Elizabeth Islands in the Canadian Arctic Archipelago, based on the relationship established by Rodhe (1952). Sea-ice formation was predicted using a "Z" function, where constants used in the function were determined by trial and error, and the weighted mean daily air temperatures were calculated from a base of -1.8°C. The "Z" function predicted freeze-up dates within three days of observed dates, except for two cases out of the 25 station-year record. An equation for predicting sea-ice growth by increments, based on air temperature and snow depth, was also developed for the study site. The decay of sea-ice thickness was plotted against accumulated degree-days above -1.8°C to establish a linear relationship between the two. A correlation coefficient of 0.93
with a standard deviation of 16.4 cm was obtained. The relationship between decreasing ice thickness and accumulated freezing degree-days using other temperature bases (e.g. -5.0°C) was also explored, with results showing a base of -1.8°C achieving the highest correlation coefficient at 0.93.

Bilello (1964) then applied previous work by Rodhe (1952) and Bilello (1961) to freshwater ice at Fort Good Hope, Canada. Although the relationship between air temperature and ice was established for the Baltic Sea, it was assumed to freeze at 0°C due to its brackishness; therefore, the relationship was also assumed to hold for freshwater bodies. As conducted by Bilello (1961), the numerical constants used for prediction were established through trial and error, relating temperature to previously observed first-ice and freeze-over dates. Results showed that predicted dates were within three days of observed first-ice and freeze-up dates at three test sites. These established constants were then plotted with observed temperatures to establish freeze-up forecast curves at the test sites.

2.3.2 Physically-based models

The timing of freeze-up is dependent of the heat storage on the water body and the cooling in the fall, forced by a combination of heat fluxes. The following section describes the energy exchanges that take place during freeze-up and ice growth on rivers and lakes.

As a river cools in the fall, the water temperature is controlled by several major heat fluxes:

\[ Q^* = Q_{sn} + Q_{ln} + Q_h + Q_e + Q_p + Q_{gw} + Q_b + Q_f \]  

(2.1)
where $Q^*$ is the net heat flux to the water column, $Q_{sn}$ and $Q_{ln}$ are the net short- and long-wave radiation, $Q_e$ and $Q_h$ are convective heat flux terms for latent heat flux from water vapor and sensible heat flux from air, $Q_p$ is the precipitation heat flux, $Q_{gw}$ is the groundwater heat flux, $Q_b$ is the combined geothermal and sediment heat flux, and $Q_f$ is the heat from fluid friction, all in Wm$^{-2}$. The same equation can be modified for lakes by removing $Q_f$, adding a term for heat conduction or heat storage from the underlying water ($Q_s$) to reflect heat storage in deeper lakes, and combining $Q_{gw}$ and $Q_b$ for shallow lakes (Gray and Prowse, 1993). As an ice cover forms, it reduces the effects of the atmospheric exchange, and $Q_{gw}$ and $Q_b$ becomes comparatively more important heat sources (Prowse, 1996).

Determining these heat fluxes requires micrometeorological scale data, not often readily available at a large geographic scale, therefore a more common empirical approach is to estimate surface heat loss using a difference in temperatures between the air/water interface and a heat transfer coefficient:

$$
Q^* = C_o(T_w - T_a)
$$

(2.2)

where $C_o$ is a heat transfer coefficient (Wm$^{-2}$°C$^{-1}$) and $T_w$ and $T_a$ are water and air temperatures (°C), respectively (Prowse, 1996).

Once the surface water has cooled to the freezing point, due to net energy loss to the atmosphere, and an ice cover is established, the rate of ice growth at the base of the ice sheet is determined by the difference between the heat exchanges at the ice undersurface and the heat supplied to the base of the ice cover by water:

$$
\frac{dh_i}{dt} \rho \lambda = \left[ \frac{T_m - T_a}{\frac{1}{k_i} + \frac{1}{c_i}} \right] - C_w(T_w - T_m)
$$

(2.3)
where $\rho_l$ is density of ice (kg m$^{-3}$), $\lambda$ is latent heat of fusion of ice (J kg$^{-1}$), $h_i$ and $k_i$ are the thickness (m) and thermal conductivity (Wm$^{-1}$°C$^{-1}$) of ice, $C_a$ and $C_w$ are the heat transfer coefficients (Wm$^{-2}$°C$^{-1}$) from ice to air and water to ice respectively, and $T_m$ is the basal ice temperature (°C) (commonly assumed to be 0°C) (Gray and Prowse, 1993). This ice cover will continue to grow vertically into the water column, as heat is lost throughout the winter. To model these energy exchanges, many ice-growth models have been developed. Although not exhaustive, several relevant physically-based models will be reviewed here.

Using the theory of the Stefan Law, Lepparanta (1983) developed a physically-based model for black ice, snow ice, and snow thickness for sea ice in the subarctic basins of Finland. The model takes as input temperature at the upper surface of the ice sheet, snowfall, and the water to ice heat flux. The model outputs estimated thickness of black ice, snow ice, and snow, as well as the snow density and thermal conductivity. Ice was modelled for observed dates at Virpiniemi for the 1976/1977 winter season, and results showed that by varying the packing rate of snow, the model fit also varied for all outputs. Maximum annual ice thickness was also modelled using long-term averages of model inputs and thickness observations, with results again showing the sensitivity of the model to snow as well as snow packing. These results highlight the particular importance snow has on sea-ice growth and development, and the author strongly recommended future fieldwork be undertaken to further examine the effects of snow on sea-ice.

The ability to predict air temperatures from lake-ice cover records has also been established in the literature. Robertson et al. (1992) explored three approaches to determining the relationship between lake-ice cover records and air temperature on Lake
Mendota, as often reliable historical meteorological data do not exist for these sites, but ice phenology data do. By linking ice cover and air temperature, future predictions can be made as to how the ice cover will react to a changing climate. The authors explore fixed period regression analyses, variable length air-temperature integration, and dynamic freeze and breakup models to link ice cover characteristics between 1855 to 1992 with air temperature. The sensible heat transfer model used in the dynamic modelling approach gave best results out of the three approaches tested. This model was then used to predict how the ice cover may change with predicted air-temperature changes.

More complex physically-based ice-growth modelling approaches were also developed, such as MINLAKE (Minnesota Lake Model), developed by Riley and Stefan (1988). Gu and Stefan (1990) advanced the existing MINLAKE model, to include the ice cover period. MINLAKE is a dynamic one-dimensional, unsteady model, which uses heat flux properties of lakes to estimate hydrologic conditions. To incorporate winter conditions into the model, two sub-models were developed to simulate the growth and decay of ice and snow, as well as calculate the heat flux of lake sediments. Results from the model indicate that lake conditions fit within "typical" ranges of observed data from the region. Fang and Stefan (1996) then advanced this model further by including new vertical thermal diffusion coefficients, an added sediment heat flux, and a modified computation scheme. Results show improvement with a standard error between 0.48°C and 0.60°C for observed and measured water temperatures, as well as a standard error between 0.06m and 0.11m for observed and measured ice and snow thicknesses for the two study sites.
Liston and Hall (1995) used a one-dimensional energy-balance model, driven by observed daily atmospheric forcing of precipitation, wind speed and air temperature, to explore and better understand ice-growth mechanisms and other energy-related processes. Focusing on high latitude and high elevation lakes, the model was composed of four sub-models which include: a surface-energy balance sub-model to determine lake surface temperature and energy availability for the freezing/melting process; a lake mixing energy-transport sub-model to determine the evolution of lake-water temperature and stratification; a snow sub-model to determine snow depth, density, accumulation, metamorphoses and melt; and a lake-ice growth sub-model to determine white-ice and black-ice thickness. Two study sites were used to validate the model using identical atmospheric forcing, except the first site (Lower Two Medicine Lake) was assumed to have a lower wind speed and higher snow accumulation than the second site (St. Mary Lake). St. Mary Lake simulated the average of the observations well, while Lower Two Medicine Lake simulated the observations well except for freeze-up, which lagged behind observed dates by approximately one week. The snow-ice depth varied between study sites, with enhanced snow-ice formation on the Lower Two Medicine Lake due to the light winds. These findings illustrate the important influence wind has on the ice-growth process.

Shen et al. (1995) used a thermal river-ice growth and decay model developed by Shen and Lal (1986) to model black ice, white ice, snow, and frazil-ice slush within an ice cover. This growth and decay model is a key component of the larger refined one-dimensional river-ice process model called RICEN, which also simulates water-
temperature and ice discharge distribution, ice-cover evolution, undercover deposition and erosion, and skim-ice and border-ice formation.

Fang *et al.* (1996) looked more specifically at the date of ice formation on a lake using a physically based algorithm requiring wind speed and water temperature as inputs. The model was tested against nine Minnesota lakes during the 1989-90 winter and results show simulated and observed permanent-ice formation dates to be within 6 days for all nine lakes.

Another physically based model, LIMNOS (lake ice model-numerical operational simulation), was designed by Vavrus *et al.* (1996) after a sea-ice model by Maykut and Untersteiner (1971). This numerical thermodynamic process model requires maximum and minimum air temperature, wind speed, snowfall rate, humidity, solar radiation, and cloud fraction as atmospheric input variables to the model. LIMNOS was applied to three lakes in the Wisconsin area, and accurately predicted ice-on and ice-off dates within two days of their long-term means and maximum ice depth within 7.6 cm.

Walsh *et al.* (1998) followed research conducted by Vavrus *et al.* (1996) using the LIMNOS model, and examined the large-scale patterns of lake-ice phenology by applying the LIMNOS model to the entire globe on a 0.5° by 0.5° grid. Ice phenology is simulated using 30-year (1931-1960) average climatic data and hypothetical 5 m and 20 m lake depths for each grid cell to quantify the role of lake morphology on ice phenology, as lake depth will influence heat storage and thus lake phenology. Thirty lakes with associated lake depth and long-term (10-year) ice phenology information were chosen from the Lake Ice Analysis Group (LIAG) data set for model validation across the Northern Hemisphere. LIMNOS most accurately predicted ice duration ($r^2 = 0.86$)
followed by ice-on dates ($r^2 = 0.83$) and ice-off dates ($r^2 = 0.79$). These results indicate the level of accuracy that can be achieved when modelling lake-ice phenology on a global scale using only simple climate and morphometric parameters to capture the major heat fluxes controlling the process.

Stefan and Fang (1997) developed another one-dimensional water-temperature model for lake ice and snow cover. The lake characteristics required for input were surface area, maximum depth, and Secchi depth and the model was driven by daily weather data. The model simulated ice-in and ice-out dates, ice cover, maximum ice thickness, average snow depth, and a continuous snow-cover ratio. The model was applied to 27 lakes in the Minnesota region, and resulted in a standard error between measured and simulated values within 6 days of ice formation dates, 0.12 m for ice thickness and 0.07 m for snow cover.

CLIMo (Canadian Lake Ice Model), described by Duguay et al. (2003), is another one-dimensional thermodynamic ice model like LIMNOS, except CLIMo differs in its parameterization of snow conductivity and surface albedo. This study by Duguay et al. (2003) also differs from other thermodynamic modelling studies in that it used both in situ and remote-sensing data to validate the model. The requisite CLIMo model input data include: air temperature, humidity, cloud cover, wind speed, snowfall and snow density. Model outputs include daily snow depth, daily ice thickness (black and snow ice), end-of-season clear ice, snow ice, total ice thickness, and freeze-up/breakup dates. Results from a Churchill, Manitoba validation site show ice-on dates can be modelled within 2 days of observed dates, and mean absolute error for ice-off dates can vary from 1 to 8 days depending on snow cover input, thereby illustrating the strong influence snow
cover has on lake-ice phenology. The CLIMo model was also used to simulate maximum lake-ice thickness and results from a Poker Flats, Alaska site showed that snow-ice thickness was underestimated by 7 cm. This again indicates the strong influence snow can have on ice thickness (Duguay et al., 2003).

More recently, lake-ice phenology, thickness and composition was modelled using MyLake (Multi-year simulation model for Lake thermo- and phytoplankton dynamics; Saloranta and Andersen, 2007), a one-dimensional process-based lake model, for hypothetical lake depths of 5, 20, and 40m between 40 and 75°N across North America (Dibike et al., 2012). Using daily gridded data of atmospheric variables MyLake was run for current (1979-2006) and future scenarios (2041-2070) to identify the effects of climate change on lake-ice phenology, thickness, and cover composition. In the future scenario, maximum lake-ice thickness is expected to decrease 10-30cm, freeze-up is delayed by ~10 days, breakup is advance by ~10-20 days, and black-ice will be reduced while white-ice increased due to increased snowfall in most locations except the coast and extreme south. Although MyLake was run across North America, the intension was not to model specific lake systems, but rather examine general patterns of change.

Although these complex physically-based models perform well for site specific studies, their need for large amounts of micro-meteorological data makes them inappropriate for use at a larger geographic scale, as desired in this research, with the exception of MyLake. However, MyLake was designed to explore effects of climate change on lake ice phenology, not to quantify freshwater ice thickness. Therefore, less data-intensive models will be explored for their relevance to this research in the following section.
2.3.3 Regression models

More statistical approaches to lake-ice modelling have also been explored, which use regression to establish relationships between dependent and independent variables to predict freshwater-ice processes. Williams et al. (2004) used single-variable linear regression to establish correlations between air temperature, morphology, latitude and topography for 143 lakes across North America. Multivariate regression was then used to create predictive equations for ice-in and ice-out dates, ice cover duration and maximum ice thickness, based on the established correlations. Although the variables explored are known to influence lake-ice characteristics, the relationships were established statistically, and not empirically. Results suggest that factors influencing ice cover dynamics are still missing from the equations.

Subsequently, Williams and Stefan (2006) used a multivariable linear regression model, a log-transform model and a hybrid model that combined the multivariable linear regression and log-transform models to model ice characteristics for over 128 lakes in Canada and the United States. Requisite input data for all three modelling approaches included mean air temperature, latitude, average lake depth, elevation, and surface area. Results showed that the log-transform model best estimated ice-in dates, while the linear regression model proved superior for ice-out dates and the hybrid model for maximum ice thickness. The input variables that had the dominant influence on ice phenology were air temperature and latitude, while air temperature had the most influence on maximum ice thickness. The results suggest air temperature, latitude and elevation have a dominant influence on lake-ice characteristics, while bathymetric variables play a smaller role. However, as discussed for physically-based model, the need for large amounts of micro-
meteorological data to run regression models at large geographic scale makes them inappropriate for this research, therefore more simplistic degree-day models which require only air-temperature data are often used (Ashton, 1986; Gray and Prowse, 1993).

2.3.4 Degree-day models

If a steady state is assumed, with the ice temperature in contact with the air equal to the air temperature, and the heat transfer between the ice/water interface is negligible, the heat flux through the ice can be determined using an energy balance at the ice/water interface:

\[ Q_i = k_i(T_m - T_a)/h_i \]  \hspace{1cm} (2.4)

The ice-growth rate at the ice/water interface is then:

\[ Q_i = \frac{dh_i}{dt} \rho_i \lambda \] \hspace{1cm} (2.5)

Integrating equation 2.4 and 2.5, assuming \( t \) and \( h = 0 \) results in:

\[ h_i = \left(\frac{2k_i}{\rho_i \lambda}\right)^{1/2} [(T_m - T_a)t]^{1/2} \] \hspace{1cm} (2.6)

This approach is commonly known as the Stefan equation, where subsurface heat flow is ignored, and ice growth is based on a degree-day function:

\[ h_i = \alpha (D_f)^{1/2} \] \hspace{1cm} (2.7)

where \( h_i \) is total ice thickness (mm), \( D_f \) is the sum of Accumulated Freezing Degree Day (AFDD) value in degrees Celsius (°C), and \( \alpha \) is a numerical ice-growth coefficient (mm°C⁻¹/² day⁻¹/²) which has a theoretical maximum of \( (2k_i/\rho_i \lambda)^{1/2} \), but is varied to account for conditions of exposure, surface insulation and subsurface heat flux (Michel, 1971; Ashton, 1986; Prowse, 1996). It is important to note that for thin ice (i.e. <10 cm), \( h_i \) increases in direct proportion to \( D_f \) (Ashton, 1986).
The degree-day approach is considered to be a statistical method, as degree-days are first correlated with the phenomena of study, and then used to predict how the phenomena will change as the degree-days change (Greene, 1981). As such, they can be considered a simplified form of the regression models discussed in section 2.3.3. The most common degree-day definition is that of the American Meteorological Society which considers a degree-day "a measure of departure of the mean daily temperature from a given standard" (Huschke, 1959). From this definition, freezing degree-days (FDD) and thawing degree-days (TDD) are defined as departures below freezing and above freezing temperatures, respectively. Accumulated FDD (AFDD) are a summation of degree-days over a specified period, typically the winter period and can be calculated one of two ways. The first sums only those FDD that are below freezing, and ignores days of above freezing temperatures, generating gross AFDD. The second approach takes sums both above and below freezing temperatures, where below freezing temperatures are given a positive value, and above freezing temperatures are given a negative value, generating net AFDD (Schmidlin and Dethier, 1985). The gross AFDD approach is taken in this thesis, as it is the common approach employed in the freshwater-ice thickness literature when applying the degree-day ice-growth equation (equation 2.7) (e.g. Bilello, 1961; Shen and Yapa, 1985; Walsh et al., 1998; Prowse and Conly, 1998; Prowse et al., 2002; Prowse and Carter, 2002).

In the context of freshwater ice, the degree-day approach reflects the major heat fluxes significant in ice formation and growth described in section 2.3.2, and has long been employed in the cryospheric science. The degree-day method has been used extensively to explore ice cover characteristics on the Great lakes using FDD and TDD.
Richards (1964) related FDD and TDD to ice cover, with FDD providing a measure of winter severity, and TDD providing a measure of antecedent heat or an index of the heat available for storage in a lake. From the strong relationship between FDD, TDD and ice cover on all the Great Lakes, multiple regression equations were derived for each lake for use in lake cover forecasting. Assel (1976) extended this work on the Great Lakes, calculating regression equations to predict ice thickness. Assel (1980) classified winter severity in the Great Lakes using maximum FDD accumulations. More recently, Assel et al. (2003) related annual maximum ice concentrations with AFDD and lake depth for the Great Lakes.

The use of FDD and TDD has also been applied in other areas of research including the estimating snow cover density (e.g. Bruce and Clark, 1966), snowmelt runoff (e.g. Rango and Martinec, 1995), and static mass-balance sensitivity of Arctic glaciers and ice caps (e.g. de Woul and Hock, 2005). The degree-day approach has also been employed in other areas of research including estimating crop productivity using growing degree-days (e.g. Russell et al., 1984) and analysing building energy demands in a warming climate, using predicted heating and cooling degree-days (e.g. Christenson et al., 2006).

The most well known and most often used degree-day equation is equation 2.7, the Stefan equation. This equation has been used, for example, to predict ice thickness along the St. Lawrence River (Shen and Yapa, 1985), to estimate ice thickness within the Peace-Athabasca Delta (Prowse and Conly, 1998), to calculate total ice growth along the Mackenzie River (Prowse and Carter, 2002), and to explore trends in river-ice breakup across Northern Canada (Prowse et al., 2002). Although this equation is inaccurate during
initial freeze-up and ice growth, its accuracy improves once a stable ice cover has formed (Prowse and Conly, 1998), and is therefore more appropriately used in peak-ice thickness estimation rather than initial ice-cover thickness estimation.

As described in section 2.2.2, freshwater-ice formation is governed by a number of climatic indexes including air temperature, precipitation, solar and long-wave radiation, cloudiness, humidity, and wind speed (Ashton, 1986). Although combinations of these variables can be used to model the growth of ice, the more common method is to employ the simplified degree-day index described above. It has been suggested that by applying such a simplistic model, we mask our understanding of underlying meteorological and hydrological processes (e.g. Greene, 1981). Arguments, however, have been made for the necessity of a simplistic model, as more complex theoretical models require large multivariate in situ datasets of micrometeorological variables, which are difficult to acquire (USACE, 2002).

Given that air temperature is a dominant factor in freshwater-ice growth and the degree-day ice-growth equation provides a simplified and accurate method for predicting ice thickness over a large geographic region, the degree-day ice-growth equation (equation 2.7), also known as the Stephen equation, is adopted in this study as the basis for modelling freshwater-ice thickness, and the specific methodology used to defining the model is detailed in section 3.2. Furthermore, the high confidence in future estimates of air temperature makes the degree-day approach a logical one for exploring future changes to freshwater-ice thickness. Given the dominant role precipitation, air temperature, latitude and elevation play in ice thickness, these variables are incorporated into the
degree-day ice-growth model using an ice-growth specific regional climate classification described in section 2.4 and Chapter 3.

2.4 Climate Classifications

To improve the degree-day ice-growth model described above, a regional climate classification that captures hydro-climate influences on freshwater-ice thickness can be employed. This section explores the dominant techniques used for defining geographic regions, beginning with a review of the original founding climate classifications by Köppen and Thornthwaite, which classify climate zones based on temperature and precipitation indexes, then moving towards statistical approaches of classification including weighted linear combination, weights-of-evidence and spatial clustering.

2.4.1 Existing climate classifications

Vladimir Köppen produced one of the first global climate classifications in 1900, which, despite its simplistic approach, is one of the most widely used today (Peel et al., 2007). This first classification was based on vegetation zones, largely influenced by a global vegetation map published by Grisebach in 1866 (Wilcock, 1968). Köppen did not use formulae, but rather classified climate zones using empirical methods based on mean annual temperature and precipitation (Lockwood, 1995). Rudolf Geiger updated the Köppen classification in 1961 (Kottek et al., 2006), and this updated Köppen-Geiger climate classification map has been used, for example, to validate GCMs (Lohmann et al., 1993; Kalvova et al., 2003), to detect Arctic climate change (Wang and Overland, 2004), to examine the change in areal extent of alpine tundra (Diaz and Eischeid, 2007), and to monitor changes in the arid climates of northern China (Kim et al., 2008).
Thornthwaite (1948) followed Köppen's approach, but used a moisture index and a thermal index, rather than precipitation and temperature directly. This water-balance approach is considered an improvement on the Köppen classification, as it distinguishes between moist and dry climates more clearly, is more closely tied to plant and energy usage, provides a more systematic approach to classification, and has boundaries that are not as closely tied to vegetation boundaries (Feddema, 2005). The Thornthwaite climate model has also been used in similar ways as the Köppen model, including the monitoring of changing climates across the United States (Grundstein, 2008), as well as relating historic climatic sequences derived from pollen analysis to current climates (Beenhouwer, 1953).

Recent approaches to climate classification have employed observed station data or gridded data sets. Peel et al. (2007) used 12396 precipitation stations (1909-1991) and 4844 temperature stations (1923-1993) to update the Köppen-Geiger climate map. Feddema (2005) revised Thornthwaite's classification using 0.5° gridded monthly climatologies, interpolated from land-based gauge measurements and shipboard measurements across the globe. Mean monthly climatologies with a 0.5° resolution from the Climate Research Unit (CRU) were also used to recreate the Köppen-Geiger climate for the period 1901-1921 and 1961-1990 to validate global GCMs (Kalvova et al., 2003). Gridded temperature data from the CRU at a 0.5° resolution was also used along with a 0.5° resolution precipitation data from the Global Precipitation Climatology Center to update the Köppen-Geiger classification for the period 1951-2000 (Kottek et al., 2006).

Although these climate classifications have been used extensively, one of the objective of this thesis is to develop a degree-day ice-growth model that incorporates
regional climatic variations shown to influence freshwater-ice growth, a purpose for which the Köppen-Geiger and Thornthwaite climate definitions were not intended. As existing global climate classifications do not fit with the objectives of this research, a global climate classification that reflects freshwater-ice growth processes, henceforth know as hydro-climatic regions, will be defined. In the following three sections, more statistical and spatial approaches are explored for their relevance to defining these.

Hydro-climatic regions are considered in this research to be spatial regions in which both hydrologic and climatic parameters influence the ice-growth environment of freshwater bodies (rivers and/or lakes). Identifying these regions can be considered a type of suitability analysis, which in its most general definition aims to "identify spatial patterns of requirements, preferences, or predictors of some activity" (Hopkins, 1977, p. 386). In the context of this thesis, the aim is to identify the spatial patterns of freshwater-ice growth processes. To define hydro-climatic regions, three suitability analysis approaches are reviewed: weighted linear combination, weights-of-evidence and spatial clustering.

2.4.2 Weighted linear combination

An intuitive and easy-to-understand approach to identifying regions is that of weighted linear combination (WLC), which has been applied to land use/habitat suitability analysis, site selection, and resource evaluation (Malczewski, 2000). In its most basic form, WLC takes maps of each variable or criterion deemed important in identifying regions, weights them relative to their importance and combines all weighted variables mathematically to arrive at a final suitability map (Eastman et al., 1995).

Based on steps outlined in Malczewski (2000), employing WLC to define hydro-climatic regions, would follow five general steps:
1. identify maps of each of the variables that influence freshwater-ice growth processes;

2. standardize the range of values of all maps to ensure they are grouped using the same criteria;

3. define the weights of each variable according to how important it is to ice growth;

4. combine the variable layers together according to their weights;

5. classify the resulting image into hydro-climatic regions using the range of values output, and evaluate the results.


Although this approach is feasible, it has limitations. First, it is assumed that WLC can be performed using Boolean or presence/absence options, as well as continuous criteria, and that results will be the same, however in practice this is not the case. Second, the need to standardize the variables before inputting them into the WLC equation may skew or results in a loss of information (Drobne and Liseč, 2009). Third, WLC assumes interdependence among criterion, where the suitability of one variable is not dependent on the suitability of another (Hopkins, 1977). In the context defining hydro-climatic regions, it assumes, for example, that a high value in one dataset is independent of a high value in another dataset, an assumption often violated during implementation. Finally, deciding on weights of each variable requires expert knowledge is biased to the decision maker (O'Sullivan and Unwin, 2003). In an attempt to overcome these limitations, a second approach, weights-of-evidence, is explored below.
2.4.3 Weights-of-evidence

A second approach, which is a variation of WLC, is the weights-of-evidence (WoE) model, used extensively in mineral exploration to produce maps of potential mineral deposits (Bonham-Carter et al., 1989), as well as for predicting other natural phenomena such as landslide occurrences (Dahal et al., 2007). The WoE model is based on Bayesian probability, where available data are used to estimate the probability of occurrence of an event, and predict its spatial pattern (O'Sullivan and Unwin, 2003), thus removing the need for expert knowledge to decide weights of each variable such as employed in WLC.

Following the methodology outlined by Romero-Calcerrada and Luque (2006), defining hydro-climatic regions would follow five main steps:

1. calculate weights for each predictive map, where each map represents a variable found to influence ice growth;

2. generalized or standardize the predictive maps, similar to step 2 in the WLC approach;

3. test for conditional independence between the evidence maps to identify which maps will be accepted (conditionally independent) or rejected (not conditionally independent);

4. create a posterior probability theme, or final predictive map, by combining the weighted evidence maps not rejected in step 3;

5. classify the final predictive map and evaluate the results.

See Bonham-Carter et al. (1989) and Bonham-Carter (1994) for a more detailed description of the WoE approach.
The primary disadvantage to this approach is the assumption of conditional independence between the data, similar to WLC, however various statistical tests do exist to evaluate the degree to which this assumption is violated (Franca-Rocha et al., 2003), an improvement on WLC. A second disadvantage in employing the WoE method is the poor predictability in regions of sparse data (Romero-Calcerrada and Luque, 2006), and in the context of this research, regions of sparse ice thickness and influencing variables. As the spatial distribution of ice thickness data employed in this research are sparse in many locations across the Northern Hemisphere, this approach is not a valid one, and a third approach to defining hydro-climatic regions, spatial clustering, is evaluated below.

2.4.4 Spatial clustering

A third approach to defining hydro-climatic regions is that of spatial clustering, which identifies homogeneous regions explicitly (Hopkins, 1977), removing the issue of interdependence which exists for WLC and WoE. Clustering techniques can be applied to defining hydro-climatic regions, and are categorized by their approach as either hierarchical or non-hierarchical (O'Sullivan and Unwin, 2003). Hierarchical clustering techniques build a nested hierarchy of clusters, where each observation is initially its own cluster, then similar clusters are grouped together to form a new cluster, and these new clusters are again grouped together, and so on until the desired number of clusters is reach, forming a hierarchy (Hopkins, 1977). In contrast, non-hierarchical clustering techniques establish the number of desired clusters at the outset, and observations are randomly assigned to these initial clusters. Observations are then reassigned to other clusters based on a set of criteria, until the desired number of clusters is reached (O'Sullivan and Unwin, 2003).
One particular hierarchical clustering method, the two-step clustering method, has proved efficient in handling very large datasets (SPSS, 2001), such as those employed in defining hydro-climatic regions across the Northern Hemisphere. The two-step clustering method first runs a pre-clustering algorithm, which clusters the data into a maximum of 585 sub-clusters by default. These pre-clusters are then grouped into the desired number of clusters for output, using an agglomerative hierarchical clustering technique, where similar clusters are grouped together to form a new cluster. The number of final clusters can be given explicitly or auto-clustered based on Bayesian information criterion and distance change between clusters (SPSS, 2001). To determine the optimal number of clusters, or hydro-climatic regions, the difference between the means of the variables within each cluster can be assessed using a one-way ANOVA and post-hoc test. ANOVA identifies if the clusters variable means are statistically different from the other clusters variable means, and the post-hoc test identifies which clusters variable means differ and by how much. A two-step clustering method has been adopted in this study and the specific methodology used to defining hydro-climatic regions is detailed further in section 3.2.2.

Given that air-temperature is a dominant factor in freshwater-ice growth and the degree-day ice-growth equation provides a simplified and accurate method for predicting ice thickness over a large geographic region, the degree-day ice-growth equation (equation 2.7), is adopted in this study as the basis for modelling freshwater-ice thickness. Furthermore, the dominant role precipitation, air temperature, latitude and elevation play in ice thickness is incorporated into the degree-day ice-growth model using hydro-climatic regions, defined by a two-step clustering method. In effect, such an
approach implicitly integrates the role of climatic variables into the degree-day ice-growth model. Details of the degree-day ice-growth model are reviewed in section 3.2.1.
References


CHAPTER 3: DEFINING FRESHWATER-ICE GROWTH COEFFICIENTS BY HYDRO-CLIMATIC REGION

Abstract

Freshwater ice is a major component of the cryosphere, and a key component of the North, as well as important to numerous physical, ecological, and socio-economic processes. As found in analyses of temporal trends, freshwater ice is highly variable due to climatic influences such as air temperature. Given the importance, concern has been raised over the future changes to freshwater ice given the predicted changes in climate. Small-scale studies have explored freshwater-ice thickness and phenology using detailed models that require large input datasets, however, a large-scale quantification has not been performed, as the input data required are not yet readily available at the hemispheric scale. A more common approach is to use a simplified degree-day index that requires only air temperatures as input, making it applicable at large geographic scales. The most common degree-day index is the Stefan equation, which requires air temperature and ice-growth coefficients. These have been defined by Michel (1971) to account for surface insulation and exposure, varied by water-body type, however, these ice-growth coefficients have not been advanced. This paper advances the work of Michel (1971) through the definition of ice-growth coefficients, spatially stratified by hydro-climatic region and water-body type, and empirically defined using a degree-day ice-growth model. This study employs 46 large lake and reservoir observation sites, 130 small to medium lake and reservoir observation sites and 256 river observation sites to calibrate the ice-growth coefficients by hydro-climatic region and subsequently validate the model. Using a 14 hydro-climatic region definition, model validation achieved an $r^2$ of 0.45 for
large lakes and reservoirs, 0.73 for small to medium lakes and 0.44 for rivers ($p$-value $< 0.001$ for all).
3.1 Introduction

A high density of freshwater ecosystems exists within the Northern Hemisphere (Downing et al., 2006), many of which are located at higher latitudes where the air temperature drops below 0°C in the winter months. Freezing temperatures result in seasonal to annual ice cover on these water bodies, establishing freshwater ice, defined in this thesis as floating river and lake ice, as a key characteristic of cold northern regions (Bennett and Prowse, 2010). Freshwater ice makes up one of the main components of the cryosphere (Lemke et al., 2007), which integrates climate variations over a wide temporal scale through a direct connection to the surface energy budget, the water cycle, and the surface gas exchange. These detectible variations in climate provide a visible expression of our changing climate (Walsh et al., 2005); implying variations in freshwater ice can be used to monitor variations in the overall climate.

The seasonal duration and extent of freshwater ice has important physical, ecological, and socio-economic implications. Reduced stream flow during freeze-up can eliminate downstream aquatic habitat in extreme cases, and the formation of an ice cover reduces oxygen exchange at the air-water interface, lowering the dissolve-oxygen (DO) levels, impacting aquatic species (Prowse, 2001a). Many aquatic species in cold regions have adapted their life cycles to benefit from the freshwater-ice processes that occur throughout the season, however river flooding, for example, is responsible for both the replenishment of many nearby lakes and ponds, as well as severe fish mortality and destruction of spawning grounds (Beltaos, 2000). Furthermore, extreme events including ice jams and flooding on rivers are responsible for many economic losses caused by
damage to property and infrastructure, constraints to hydropower generation, and interference with navigation (Gerard and Davar, 1995).

As found in analyses of temporal trends, freshwater ice is highly variable due to climatic influences. Much freshwater-ice research has focused on ice phenology and ice season duration and their link to temperature and the 0°C isotherm, which illustrates, for example, that an increase in air temperature results in later freeze-up and earlier breakup, leading to a longer open water season on many water bodies (e.g. Magnuson et al., 2000; Bonsal and Prowse, 2003; Lacroix et al., 2005; Duguay et al., 2006). Furthermore, historical freeze-up, breakup, and ice cover data have been used as quantitative indicators of climate change (Palecki and Barry, 1986; Robertson et al., 1992). With this strong link between changes in the cryosphere and changes in the climate, freshwater ice, and changes in its quantity and distribution, is important to our understanding of the global climate system. However, trends in seasonal ice thickness and extent have been largely ignored, due to more complex interactions with climate and sparse observational datasets (Lemke et al., 2007). There are gaps in scientific knowledge of freshwater-ice distribution and quantity, emphasising a need to predict its current state.

Given the importance of freshwater ice, concern has been raised by the Intergovernmental Panel on Climate Change (Lemke et al., 2007), the Arctic Climate Impacts Assessment (Walsh et al., 2005), and most recently the Arctic Monitoring and Assessment Programme (Prowse et al., 2011) over future changes to freshwater ice given the predicted changes in climate. Despite this, a baseline assessment of the current state of freshwater ice has yet to be conducted. Modelling freshwater-ice formation and growth on both rivers and lakes is difficult due to complex hydro-climatic processes, governed
by a number of climatic indexes (Ashton, 1986). Although several complex physically-based models have been developed to model this process (e.g. Lepparanta, 1983; Gu and Stefan, 1990; Duguay et al., 2003), the need for large amounts of micrometeorological input data makes these models inappropriate for use at a larger geographic scale. Freshwater-ice growth can be modelled using a simplified degree-day approach that relies on air-temperature data as its climatic index, making it more appropriate for large geographic scales (Ashton, 1986; Gray and Prowse, 1993). Degree-days have been used extensively to capture the energy budget of freshwater-ice growth, as well as for estimating snow cover density, snowmelt and runoff, static mass-balance sensitivity of Arctic glaciers and ice caps, crop productivity and energy consumption (e.g. Bruce and Clark, 1966; Russelle et al., 1984; Rango and Martinec, 1995; de Woul and Hock, 2005; Christenson et al., 2006).

The most common degree-day ice-growth model is one described by Michel (1971), which requires a numerical ice-growth coefficient as input. Typical coefficient values have been defined by Michel (1971) for regional climatic controls, by water-body type, and this simplistic model has been used extensively (e.g. Shen and Yapa, 1985; Prowse and Conly, 1998; Prowse and Carter, 2002; Prowse et al., 2002; Beltaos et al., 2006). However, the ice-growth coefficients defined by Michel (1971) have never been advanced to account for spatial variations of climatic drivers or heat storage of large lakes. Prowse and Beltaos (2002) argue that a simple heat index such as air temperature does not capture all the climatic controls that influence ice growth, as other climate variables play a role in ice development. By varying the numerical coefficient by hydro-
climatic region, this paper aims to capture the influence of other climatic variables on the ice-growth process, and advance ice-growth coefficients.

Given that freshwater ice is a key characteristic of cold regions, that changes in freshwater ice have physical, biological and socioeconomic implications, that to date the distribution of freshwater ice has not been examined on a large geographic scale, and that the most common approach to estimating freshwater-ice thickness has not been advanced since its development, the objective of this research is to define freshwater-ice growth coefficients, stratified by hydro-climatic region, for input to the degree-day ice-growth model.

3.2 Methodology and Data

To address the goal of advancing freshwater-ice growth coefficients for input to the degree-day ice-growth model, freshwater-ice growth coefficients are derived empirically by hydro-climatic region using the degree-day ice-growth model and observed maximum seasonal ice thickness data for study sites across the Northern Hemisphere. By calibrating the model by hydro-climatic region, using observed maximum seasonal ice thickness data, optimal ice-growth coefficients are defined. Validation is carried out using a second set of observational ice thickness data to assess the regionally stratified ice-growth coefficients.

3.2.1 Model background

The timing of freeze-up and ice growth on a water body is dependent on the heat storage of the water body and the rate of cooling in the fall, controlled by a combination of heat fluxes (Prowse, 1996). These heat fluxes between the water, ice, and air interfaces are important to describe the phase changes between liquid and solid during ice formation
and growth, based on the transfer of heat to and from the interfaces (Gray and Prowse, 1993). Determining these heat fluxes requires micrometeorological scale data, not often readily available at a large geographic scale; therefore, a more common approach is to determine the relationship between freshwater ice and climatic variables. The major climatic variables that govern the heat exchange process between the air/water interface, and thus ice formation and growth, include air temperature and precipitation, wind velocity, barometric pressure, and sun and cloud conditions (Ashton, 1986). Heat exchange at the base of the ice is the controlling factor in ice growth; therefore, an empirical approach to estimate surface heat loss using a difference in temperatures between the air/water interface and a heat transfer coefficient is often applied to estimating ice thickness using a degree-day function (Gray and Prowse, 1993).

The most well known and most often used degree-day ice-growth function is the Stefan equation,

\[ h_t = \alpha (D_f)^{1/2} \]  

(3.1)

where \( h_t \) is total ice thickness (mm), \( D_f \) is the sum of Accumulated Freezing Degree Day (AFDD) value in degrees Celsius (°C), and \( \alpha \) is a numerical ice-growth coefficient (mm°C\(^{-1/2}\)day\(^{-1/2}\)) defined to account for conditions of exposure, surface insulation and subsurface heat flux (Michel, 1971).

Although the Stefan equation has been shown to poorly predict ice thickness during initial freeze-up and growth, its accuracy improves once a stable ice cover has formed (Ashton, 1986), and is therefore more appropriately used in maximum seasonal ice thickness estimation rather than initial ice-cover thickness estimation. Arguments have
been made for the necessity of a simplistic model, as more complex theoretical models require large multivariate *in situ* datasets that are difficult to acquire (USACE, 2002).

Typical values of $\alpha$ have been established by Michel (1971), based on the authors' experience, and are shown in Table 3.1. Ice-growth coefficients range from a theoretical maximum of 34 to a low of 7, therefore using a high coefficient to estimate ice thickness will result in thicker ice, while using a low coefficient will result in thinner ice under similar hydro-climatic conditions. Furthermore, for the same water-body type, snow-covered lakes have a lower coefficient compared to lakes without snow in the same region, due to the insulating effects of snow on early season ice development, which results in slower ice growth. Lakes also have a higher ice-growth coefficient compared to rivers, indicating a river and lake in the same region will develop different ice thicknesses during the winter under similar hydro-climatic conditions. This is due to the turbulent nature of rivers, which forces the entire water column to reach a freezing point before ice will begin to form on the surface, resulting in a later freeze-up on rivers compared to lakes, and thus a thinner ice sheet compared to that of a lake in the same region. From this, we can expect that a water body in a hydro-climatic region with high precipitation (snow cover) and low (cold) temperatures would produce a thick ice cover and thus require a high ice-growth coefficient when modelling ice thickness. Conversely, a water body in a hydro-climatic region with low precipitation (no snow cover) and high (warm) temperatures would produce a thin ice cover and thus require a very low ice-growth coefficient when modelling ice thickness. Furthermore, a hydro-climatic region with low precipitation (no snow cover) and low (cold) temperatures would produce thick ice and
require a high coefficient, while a region with high precipitation (snow cover) and high (warm) temperatures would produce thin ice and require a low coefficient.

**Table 3.1: Typical α values, derived for the Stefan equation by Michel (1971, p. 79).**

<table>
<thead>
<tr>
<th></th>
<th>α (mm°C⁻¹/₂ day⁻¹/₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theoretical maximum</strong></td>
<td>34</td>
</tr>
<tr>
<td>Windy lakes with no snow</td>
<td>27</td>
</tr>
<tr>
<td><strong>Average lake with snow</strong></td>
<td>17-24</td>
</tr>
<tr>
<td>Average river with snow</td>
<td>14-17</td>
</tr>
<tr>
<td>Sheltered small river with rapid flow</td>
<td>7-14</td>
</tr>
</tbody>
</table>

The typical range of ice-growth coefficient values given in Table 3.1 does not capture the influence of other physical and climatic variables on the ice-growth process. Along with the climatic variables described, lake size plays a role in ice formation and growth, as large lakes will typically form ice later in the season due to their heat storage capacity (Dereki, 1976). To incorporate these physical and climatic variables into the degree-day ice-growth model, new ice-growth coefficients, stratified by hydro-climatic region, water-body type and lake size are defined.

### 3.2.2 Classification of hydro-climatic regions

Global climate classifications such as the Köppen classification (Wilcock, 1968) and the Thornthwaite classification (Thornthwaite, 1948) classify climate zones based on temperature and precipitation indexes. Although these regional classifications have been used extensively (e.g. Kottke et al., 2006; Diaz & Eischeid, 2007; Kim et al., 2008; Grundstein, 2008), they do not represent the hydro-climatic variables specific to freshwater-ice growth. To address this need hydro-climatic regions are defined spatially using a two-step clustering method. Hydro-climatic regions are considered in this
research to be spatial regions in which both hydrologic and climatic parameters influence the ice-growth environment of freshwater bodies.

Given that elevation, latitude, air temperature and snow have a dominant influence on maximum seasonal ice thickness (Huntington et al., 2003; Williams et al., 2004; William and Stefan, 2006) these variables are used to define hydro-climatic regions. Mean January air temperature and mean January precipitation capture climatic conditions during the coldest month of the year, and are considered within this research to be indicative of the winter ice-growth conditions. For simplicity, it is assumed that precipitation falls in the form of snow when the air temperature drops below freezing. Therefore, elevation, latitude, mean January air temperature and mean January precipitation are employed here to define hydro-climatic regions.

A two-step clustering method is used to define hydro-climatic regions, as this method is effective in handling very large datasets using the Statistical Package for the Social Sciences (SPSS, 2001). The two-step clustering algorithm is run to identify areas with similar hydro-climatic variable values. As the goal of this paper is to define ice-growth coefficients by hydro-climatic region, empirically derived using the degree-day ice-growth model, several criteria are taken into account when selecting the optimal number of hydro-climatic regions. Using a one-way ANOVA and Games-Howell post-hoc test, as Leven's test for homogeneity of variances indicates unequal variances between regions and sample sizes differ between clusters (Field, 2009), the difference between the means of the variables within each region are assessed. Furthermore, a sensitivity analysis is performed, where the model is run using the optimal cluster definition indicated by the ANOVA test, as well as for several cluster definitions above
this, ranging from 2 to 15 clusters. The number of hydro-climatic regions considered optimal within this research will give the best model performance ($r^2$) and maximize the between-cluster variance, while minimizing the within-cluster variance. The optimal number of hydro-climatic regions is also evaluated qualitatively to ensure the spatial coverage captures meso-scale climatic influences. To explore the properties of the hydro-climatic regions and their relationship to the ice-growth coefficients, means for both precipitation and temperature are calculated by hydro-climatic region.

High-resolution reanalysis data, Climate Research Unit Climatology (CRU CL 2.0), is employed as the source of global land surface climate information. CRU CL 2.0 is a gridded climatology dataset containing mean monthly global land surface climate information (excluding Antarctica), interpolated from global land station means between 1961 and 1990, and is considered an improvement on earlier gridded climatology products (New et al., 2002). Its key advantages for this study included high spatial resolution (10' latitude/longitude), 30-year temporal coverage, and global spatial coverage. The area analyzed is north of the January 0°C isotherm, as it represents all ice-affected areas during the coldest month of the year, and the most southern extent of freshwater-ice cover (Bennett and Prowse, 2010).

3.2.3 Calibration of model coefficients by hydro-climatic region

With the establishment of hydro-climatic regions and a degree-day ice-growth model, ice-growth coefficients are defined empirically by hydro-climatic region. The model is run using AFDD and maximum observed seasonal ice thickness values to predict optimal ice-growth coefficients (coefficients that model observed data most accurately) by hydro-climatic region. Optimal ice-growth coefficients are defined by
observation site, then averaged by hydro-climatic region, separately by water-body type. For this research, three water-body types are considered: rivers, large lakes and reservoirs (surface area > 500 km$^2$; Herdendorf, 1982), and small to medium lakes and reservoirs. A single ice-growth coefficient is also calculated for each water-body type over the entire study area to use in hydro-climatic regions without observational data.

Optimal ice-growth coefficients are evaluated using the Nash-Sutcliffe efficiency $E$ (Nash and Sutcliffe, 1970), where $E$ ranges from a value of 1.0 (perfect model fit) to $-\infty$. What makes $E$ more suitable to this analysis is the division of the mean squared error over the observed variance when calculating the statistic, as opposed to the division of the explained variance over the total variance calculated by $r^2$. Adjusting the ice-growth coefficient within the model to determine optimal coefficients simply changes the variance of the modelled ice thickness values, and using $r^2$ as an evaluation statistic ignores this variance when the division is applied. $E$ considers this variance, and is therefore more suitable for this research when evaluating optimal ice-growth coefficients. Once optimal ice-growth coefficients are defined by hydro-climatic region, the model is evaluated using $r^2$.

To calibrate ice-growth coefficients by hydro-climatic region, all maximum observed ice thickness measurements from the calibration dataset are combined into one dataset for each water-body type, and the model run with coefficients ranging from the theoretical minimum to maximum (7.0 to 34.0; Michel, 1971), incrementing by 0.1 every iteration. Optimal ice-growth coefficients are defined for each calibration site, then averaged by hydro-climatic region to produce a single optimal ice-growth coefficient by hydro-climatic region. The model is then run using these new hydro-climatic coefficients,
where every observation site that falls within a particular hydro-climatic region is modelled using that assigned hydro-climatic coefficient. A single optimal coefficient is also defined for all calibration sites across the Northern Hemisphere by averaging optimal coefficients defined for each observation site, and employed in hydro-climatic regions where coefficients are undefined due to lack of observational data.

Given the large geographic scope of this study, daily temperature data on a hemispheric scale are required for calculating AFDD values as input to the degree-day ice-growth model. The European Centre for Medium-Range Weather Forecasts 40-year re-analysis gridded dataset (ERA-40) is employed as the source of daily air temperatures for AFDD calculations. ERA-40 is a gridded reanalysis dataset containing daily air temperatures for the globe, based on meteorological observations between September 1957 and August 2002 (Uppala et al., 2005). Its key advantages for this study are the daily temporal resolution, the 2.5° spatial resolution, and the 45-year record. Furthermore, ERA-40 has been shown to capture interannual variability well and provides an excellent spatial temperature field for use in climate change and variability modelling (Frauenfeld et al., 2005).

AFDDs are calculated as described by Walsh et al. (1998) by summing the daily average air temperatures on days below 0°C throughout the hydrologic year (hydro-year), defined here as August 1st to July 31st to capture the ice-growth season. Because of the inherent variability in daily temperature, the 0°C threshold is crossed several times in the spring and fall. To smooth this variability, a 31-day running mean is used to filter the temperature data, as described by Bonsal and Prowse (2003). AFDDs are calculated from the smoothed temperature dataset, summing the smoothed temperatures from the first day
below freezing in the fall.

The search for ice thickness datasets is constrained to large geographical areas, such as by country, whenever possible. Maximum observed seasonal ice thickness datasets are compiled for Canada (Canadian Ice Database [CID]; Lenormand et al., 2002), Sweden (Swedish Meteorological and Hydrological Institute [SMHI]), Finland (Finnish Environmental Institute [SYKE]), Russia (Vuglinsky, 2000), and Alaska (Pacific River Forecast Center [PRFC]). Smaller datasets for the Yukon (Water Survey of Canada [WSC], unpublished data), British Columbia (WSC, unpublished data), and the Great Lakes (Great Lakes Environmental Research Laboratory [GLERL] ice thickness database; Sleator, 1995), are employed to supplement the Canadian dataset. A total of 46 large lake and reservoir observation sites with 548 seasonal maximum ice thickness measurements, 130 small to medium lake and reservoir observation sites with 2468 seasonal maximum ice thickness measurements, and 256 river observation sites with 5219 seasonal maximum ice thickness measurements are compiled from across the Northern Hemisphere (Table 3.2).
Table 3.2: Freshwater-ice thickness datasets compiled for model calibration and validation across the Northern Hemisphere, split by water-body type.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Large lakes and reservoirs</th>
<th>Small to medium lakes and reservoirs</th>
<th>Rivers</th>
<th>Start year</th>
<th>End year</th>
<th>Record count</th>
<th>Measurement type*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada (CID)</td>
<td>5</td>
<td>48</td>
<td>94</td>
<td>1958</td>
<td>2002</td>
<td>42</td>
<td>Total ice thickness</td>
</tr>
<tr>
<td>Canada (GLERL)</td>
<td>39</td>
<td>0</td>
<td>0</td>
<td>1966</td>
<td>1979</td>
<td>14</td>
<td>Lake-ice Thickness</td>
</tr>
<tr>
<td>Finland (SYKE)</td>
<td>2</td>
<td>70</td>
<td>16</td>
<td>1961</td>
<td>2002</td>
<td>42</td>
<td>Total ice thickness</td>
</tr>
<tr>
<td>Russia (NSIDC)</td>
<td>0</td>
<td>0</td>
<td>46</td>
<td>1917</td>
<td>1992</td>
<td>35</td>
<td>Ice thickness</td>
</tr>
<tr>
<td>Sweden (SMHI)</td>
<td>0</td>
<td>0</td>
<td>22</td>
<td>1939</td>
<td>2010</td>
<td>43</td>
<td>Total ice thickness</td>
</tr>
<tr>
<td>Alaska (PRFC)</td>
<td>0</td>
<td>13</td>
<td>29</td>
<td>1960</td>
<td>2002</td>
<td>37</td>
<td>Ice thickness</td>
</tr>
<tr>
<td>Yukon (WSC)</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>1930</td>
<td>2004</td>
<td>43</td>
<td>Average actual ice thickness</td>
</tr>
<tr>
<td>BC (WSC)</td>
<td>0</td>
<td>0</td>
<td>26</td>
<td>1916</td>
<td>2005</td>
<td>44</td>
<td>Ice thickness</td>
</tr>
</tbody>
</table>

* Unless stated otherwise, ice thickness measurements are assumed to be total ice thickness, including white ice and black ice.

All observations taken less than 30 days after the first day of freezing are removed, as the Stefan equation is known to predict poorly during initial ice cover formation (Ashton, 1986). Furthermore, little research has focused on using the Stefan equation for predicting ice thickness once thinning has begun; therefore, it is not recommended it be used for predicting ice thickness after peak AFDD (White, 2004). To ensure this does not occur, all observations taken more than 15 days after the last day of freezing are also removed, as ice will begin to decay as temperatures rise above freezing. Additionally, ice thickness observations taken outside the temporal timeframe of this research (1958/59 to 2001/02) are not used. If more than one observed ice thickness measurement is recorded throughout the hydro-year at a particular observation site, the maximum ice thickness observation is selected to represent the peak-ice thickness measured at that station, for that given hydro-year. Finally, only those observation sites...
with greater than three years of observations are considered in this analysis, following Bilello (1980). The datasets are split into two data subsets, one for model calibration, and the other for model validation, based on temporal and spatial coverage, taking care to balance the spatial and temporal representation within both the calibration and validation data subsets. A summary of the datasets is presented in Table 3.3 and Table 3.4, and a map of the observation sites is presented in Figure 3.1.

**Table 3.3:** Total number of observation sites used in model calibration and validation, split by dataset and water-body type, for datasets which contained both river and lake-ice data.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Large lakes and reservoirs</th>
<th>Small to medium lakes and reservoirs</th>
<th>Rivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada (GLERL)</td>
<td>Calibration: 19</td>
<td>Validation: 0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Validation: 20</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Canada (CID)</td>
<td>Calibration: 3</td>
<td>Validation: 23</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>Validation: 2</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Finland (SYKE)</td>
<td>Calibration: 1</td>
<td>Validation: 1</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Validation: 1</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>Alaska (PRFC)</td>
<td>Calibration: 0</td>
<td>Validation: 7</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Validation: 0</td>
<td></td>
<td>16</td>
</tr>
</tbody>
</table>

**Table 3.4:** Total number of observation sites used in model calibration and validation, split by dataset, for datasets which contained only river sites.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Rivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Russia (NSIDC)</td>
<td>Calibration: 24 Validation: 22</td>
</tr>
<tr>
<td>Sweden (SMHI)</td>
<td>Calibration: 11 Validation: 11</td>
</tr>
<tr>
<td>Yukon (WSC)</td>
<td>Calibration: 12 Validation: 12</td>
</tr>
<tr>
<td>BC (WSC)</td>
<td>Calibration: 13 Validation: 13</td>
</tr>
</tbody>
</table>
Figure 3.1: All freshwater-ice thickness observation sites used in model calibration and validation, split by water-body type.
3.2.4 Validation

To validate the degree-day ice-growth coefficients defined in the calibration step, the model is run for each maximum observed ice thickness measurement from the validation datasets using the numerical ice-growth coefficients defined by hydro-climatic region during model calibration. Every validation site that falls within a particular hydro-climatic region is modelled using that assigned hydro-climatic coefficient, and validation results evaluated for the Northern Hemisphere. Validation sites that fall within a hydro-climatic region for which a coefficient has not been defined due to lack of observation sites in that hydro-climatic region are modelled using the single optimal coefficient defined during model calibration.

3.3 Results and Discussion

3.3.1 Classification of hydro-climatic regions

Results of the two-step cluster analysis indicate 6 clusters as the statistically optimal number of cluster that should be used to define hydro-climatic regions, as it maximizes the number of hydro-climatic regions, while maintaining a statistically significant difference between region for each variable. Results from the ANOVA tests indicate, with greater than 95% confidence, that there is a statistically significant difference between at least one of the variable means in each region and all other variable means, and the differences between the variable means are not likely due to chance ($p=0.00$). Furthermore, results from the Games-Howell post-hoc tests indicate that the means of all variables between regions are statistically significantly different ($p < 0.05$).

The sensitivity analysis indicates that the 14 hydro-climatic region definition (Figure 3.2) maximized $r^2$ for small to medium lakes and reservoirs, and rivers. Large
lake and reservoir observation sites do not have a large enough spatial distribution to run a sensitivity analysis of this type. Results from the ANOVA tests indicate with greater than 95% confidence that there is a statistically significant difference between at least one of the variable means in each region and all other variable means. Results from the Games-Howell post-hoc tests indicate that the means of all other variables between regions are statistically significantly different ($p < 0.05$) for all except a few cluster variables. Variable means and standard deviations ($\sigma$) for each hydro-climatic region are shown in Table 3.5 for the 14 hydro-climatic regions. Mean January precipitation varies from 4 ($\sigma = 5$) mm/month to 145 ($\sigma = 56$) mm/month. Mean January temperature varies from -5 ($\sigma = 4$) to -30 ($\sigma = 3$) °C/month.

The spatial patterns of the 6 hydro-climatic regions definition broadly reflects macro-scale climatic processes, while the 14 hydro-climatic regions definitions captures more meso-scale processes (Figure 3.2), making it the more suitable hydro-climatic region definition to apply on this research. Region 14 reflects the Maritime influence, while region 9 and 10 represents the Arctic and Subarctic climates respectively. Within North America, the Grasslands and Temperate climates are visible in region 4 and 2, while region 5 represents the Boreal or Taiga climates. In Europe, the Humid Continental climate is represented by region 6, and there is a visible divide between Eastern and Western Siberia. Although the 14-region definition captures meso-scale processes more successfully than the 6-region definition, optimal coefficients will be defined and evaluated for both, as the most suitable hydro-climatic region definition should capture both the meso-scale processes as well as achieve the best $r^2$ results during calibration.
Figure 3.2: Fourteen hydro-climatic regions, defined using two-step clustering method, and latitude, elevation, and mean January temperature and precipitation, north of the January 0°C isotherm. Note: the numbers assigned to each hydro-climatic region are arbitrary.
Figure 3.3: Fourteen hydro-climatic regions, overlaid with all freshwater ice thickness observation sites.
Table 3.5: Cluster means and standard deviations (in parentheses) for January precipitation and mean January temperature for the 14-cluster hydro-climatic region definition, as well as sample size and area (in parentheses) per cluster.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Number of pixels (Area km²)</th>
<th>Mean January Precipitation Cluster Mean (mm/month)</th>
<th>Mean January Temperature Cluster Mean (°C/month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,276 (4,812,331)</td>
<td>5 (9)</td>
<td>-14.6 (4.7)</td>
</tr>
<tr>
<td>2</td>
<td>10,572 (4,102,033)</td>
<td>61 (18)</td>
<td>-4.8 (3.9)</td>
</tr>
<tr>
<td>3</td>
<td>10,919 (3,338,675)</td>
<td>4 (5)</td>
<td>-21.7 (4.3)</td>
</tr>
<tr>
<td>4</td>
<td>15,598 (6,061,980)</td>
<td>9 (10)</td>
<td>-7.5 (3.8)</td>
</tr>
<tr>
<td>5</td>
<td>14,021 (4,984,179)</td>
<td>13 (7)</td>
<td>-7.2 (3.8)</td>
</tr>
<tr>
<td>6</td>
<td>25,213 (7,049,014)</td>
<td>17 (8)</td>
<td>-17.7 (3.6)</td>
</tr>
<tr>
<td>7</td>
<td>38,088 (7,177,782)</td>
<td>27 (7)</td>
<td>-21.6 (3.1)</td>
</tr>
<tr>
<td>8</td>
<td>14,581 (3,106,401)</td>
<td>15 (10)</td>
<td>-29.8 (4.8)</td>
</tr>
<tr>
<td>9</td>
<td>36,684 (4,333,432)</td>
<td>11 (6)</td>
<td>-37.0 (4.7)</td>
</tr>
<tr>
<td>10</td>
<td>38,929 (6,094,191)</td>
<td>16 (8)</td>
<td>-30.0 (3.3)</td>
</tr>
<tr>
<td>11</td>
<td>16,197 (4,873,123)</td>
<td>41 (9)</td>
<td>-4.9 (2.9)</td>
</tr>
<tr>
<td>12</td>
<td>20,411 (4,039,703)</td>
<td>37 (6)</td>
<td>-12.3 (2.9)</td>
</tr>
<tr>
<td>13</td>
<td>12,461 (3,333,039)</td>
<td>66 (14)</td>
<td>-15.1 (5.1)</td>
</tr>
<tr>
<td>14</td>
<td>5,929 (1,526,157)</td>
<td>145 (56)</td>
<td>-6.1 (3.8)</td>
</tr>
</tbody>
</table>

3.3.2 Calibration of model coefficients by hydro-climatic region

Results for model calibration are presented in Table 3.6 and Table 3.7, as well as Figure 3.4. The single optimal ice-growth coefficient for large lakes and reservoirs is 19.4, 21.2 for small to medium lakes and reservoirs, and 19.9 for rivers, with an adjusted r² of 0.64 for large lakes and reservoirs, 0.71 for small to medium lakes and reservoirs and 0.50 (p-value < 0.001) for rivers when modelled ice thickness values are compared to observed. When ice-growth coefficients are defined for the 6 hydro-climatic region definition, infilling regions with missing coefficients using the single optimal ice-growth coefficient, values range from 21.7 to 23.6 for large lakes and reservoirs, 20.6 to 21.8 for small to medium lakes and reservoirs and 20.5 to 22.5 for rivers, with an adjusted r² of 0.63, 0.72 and 0.50, respectively (p-value < 0.001). The 14 region classification gives
best results when compared to the 6-region and single coefficient definitions, with coefficient values ranging from 18.0 to 26.4 for large lakes and reservoirs, 17.7 to 24.6 for small to medium lakes and reservoirs and 14.0 to 27.5 for rivers, with an adjusted $r^2$ of 0.65, 0.75 and 0.52, respectively ($p$-value < 0.001). The spatial distribution of the calibration and validation datasets overlaid on the 14-region definition can also be seen in Figure 3.3, and indicates hydro-climatic regions where more observational datasets are needed in future research.

In general, when precipitation and temperature means are compared to ice-growth coefficients by hydro-climatic region, trends that fit the theory of ice-growth coefficient values described in section 3.2.1 are identified (Table 3.7). Regions with high precipitation and cold temperatures have higher ice-growth coefficients, as seen in region 12, which has a high mean January precipitation of 37mm/month, and cold mean January temperature of -12.3°C/month, and high ice-growth coefficients of 20.7, 21.7 and 20.5 for large lakes and reservoirs, small to medium lake and reservoirs, and rivers. Conversely, regions with low precipitation and warmer temperatures have lower ice-growth coefficients for both rivers and lakes, such as in region 5, with ice-growth coefficients of 18, 21.2 and 19.9 for large lakes and reservoirs, small to medium lake and reservoirs, and rivers. Regions with low precipitation and cold temperatures have higher ice-growth coefficients. For example, region 10 has a low mean January precipitation of 16mm/month and a cold mean January temperature of -30°C/month, resulting in ice-growth coefficients of 21.7, 23.7 and 20.7 for large lakes and reservoirs, small to medium lake and reservoirs, and rivers. Finally, regions with high precipitation and warm temperatures have lower ice-growth coefficients, as seen in region 11, which has a high
mean January precipitation of 41mm/month, and warm mean January temperature of -4.9°C/month, and low ice-growth coefficients of 18.2, 20.1 and 18.8 for large lakes and reservoirs, small to medium lake and reservoirs, and rivers.

Table 3.6: Calibration results by hydro-climatic region definition, stratified by water-body type.

<table>
<thead>
<tr>
<th>Water-Body Type</th>
<th>Single Coefficient $R^2$</th>
<th>Optimal Coefficient $R^2$</th>
<th>Hydro-climatic region 6 $R^2$</th>
<th>Hydro-climatic region 14 $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large lakes and reservoirs</td>
<td>0.64</td>
<td>0.86</td>
<td>0.63</td>
<td>0.65</td>
</tr>
<tr>
<td>Small to medium lakes and reservoirs</td>
<td>0.71</td>
<td>0.87</td>
<td>0.72</td>
<td>0.75</td>
</tr>
<tr>
<td>Rivers</td>
<td>0.50</td>
<td>0.82</td>
<td>0.50</td>
<td>0.52</td>
</tr>
</tbody>
</table>

* p-value < 0.001 for all regression results

Table 3.7: Ice-growth coefficients defined during calibration for 14 hydro-climatic regions, stratified by water-body type. Mean January precipitation and temperature are provided for comparison of clusters.

Cluster | Mean January Precipitation Cluster Mean (mm/month) | Mean January Temperature Cluster Mean (°C/month) | Large lake and reservoir coefficients | Small to medium lake and reservoir coefficients | River coefficients |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>-14.6</td>
<td>19.4*</td>
<td>21.2*</td>
<td>19.9*</td>
</tr>
<tr>
<td>2</td>
<td>61</td>
<td>-4.8</td>
<td>19.4*</td>
<td>21.2*</td>
<td>16.7</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>-21.7</td>
<td>19.4*</td>
<td>21.2*</td>
<td>19.9*</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>-7.5</td>
<td>19.4*</td>
<td>21.2*</td>
<td>19.9*</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>-7.2</td>
<td>9.2</td>
<td>21.2*</td>
<td>19.9*</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>-17.7</td>
<td>23.2</td>
<td>19.6</td>
<td>20.7</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
<td>-21.6</td>
<td>19.4*</td>
<td>19.7</td>
<td>22.0</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>-29.8</td>
<td>19.4*</td>
<td>17.7</td>
<td>14.0</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>-37</td>
<td>19.4*</td>
<td>24.6</td>
<td>18.2</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>-30</td>
<td>21.7</td>
<td>23.7</td>
<td>20.7</td>
</tr>
<tr>
<td>11</td>
<td>41</td>
<td>-4.9</td>
<td>17.8</td>
<td>20.1</td>
<td>18.8</td>
</tr>
<tr>
<td>12</td>
<td>37</td>
<td>-12.3</td>
<td>20.7</td>
<td>21.7</td>
<td>20.5</td>
</tr>
<tr>
<td>13</td>
<td>66</td>
<td>-15.1</td>
<td>20.7</td>
<td>18.2</td>
<td>21.7</td>
</tr>
<tr>
<td>14</td>
<td>145</td>
<td>-6.1</td>
<td>19.4*</td>
<td>21.2*</td>
<td>27.5</td>
</tr>
</tbody>
</table>

* Denotes hydro-climatic regions lacking observational data, therefore employing the single optimal coefficient defined during calibration by water-body type.
Figure 3.4: Maximum observed seasonal ice thickness measurements compared to modelled ice thicknesses during model calibration, using the 14 hydro-climatic region definition and optimal coefficients defined by region, stratified by water-body type.

3.3.3 Validation

Ice thickness is modelled for the validation dataset employing ice-growth coefficients, defined in the calibration step, for the 14-region definition given it captured the meso-scale processes and gives best results during calibration. Validation results give an $R^2$ of 0.45 for large lakes and reservoirs, 0.73 for small to medium lakes and reservoirs and 0.44 ($p$-value < 0.001 for all) for rivers (Table 3.8, Figure 3.5). When model
validation results from this research are compared against more complex ice thickness models from the literature, results are encouraging. Modelling maximum lake-ice thickness using multivariate linear regression, incorporating mean air temperature, latitude, elevation, mean lake depth, and snow depth into the regression, resulted in an $r^2$ of 0.58 with a standard error of 12.9 cm (Williams et al., 2004). In comparison, modelling maximum large lake- and reservoir-ice thickness by hydro-climatic region results in an $r^2$ of 0.45 and a standard error of 15.5 cm, an $r^2$ of 0.73 and a standard error of 14.6 cm for small to medium lake- and reservoir-ice thickness, and an $r^2$ of 0.44 and a standard error of 24.2 cm for river-ice thickness in this research. These results indicate that modelling small to medium lake- and reservoir-ice thickness by hydro-climatic region using a simple degree-day ice-growth model produces comparable results to other more complex lake-ice models, while modelling river-ice thickness may require a more comprehensive set of variables to capture the complexities of river-ice formation.

Table 3.8: Model validation results by water body type and hydro-climatic definition, with and without infilling using a single optimal coefficient for regions lacking observational data.

<table>
<thead>
<tr>
<th>Hydro-climatic region 14</th>
<th>Large lakes and reservoirs</th>
<th>Small to medium lakes and reservoirs</th>
<th>Rivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.45</td>
<td>0.73</td>
<td>0.44</td>
</tr>
</tbody>
</table>

*p-value < 0.001 for all regression results.
Figure 3.5: Maximum observed seasonal ice thickness measurements compared to modelled ice thicknesses during model validation, using the 14 hydro-climatic region definition and optimal coefficients defined by region, stratified by water-body type.

3.4 Conclusion and Future Research

Regional ice-growth coefficients have been defined across the Northern Hemisphere, using a simple degree-day ice-growth model and maximum observed seasonal ice thickness measurements. The model was calibrated and validated using 46 large lake and reservoir observation sites with 548 seasonal maximum ice thickness measurements, 130 small to medium lake and reservoir observation sites with 2468 seasonal maximum ice thickness measurements, and 256 river observation sites with
5219 seasonal maximum ice thickness measurements from across the Northern Hemisphere. Ice-growth coefficients were defined by hydro-climatic region, and validation results produce an $r^2$ of 0.45 for large lakes and reservoirs, 0.73 for small to medium lakes and reservoirs and an $r^2$ of 0.44 ($p$-value < 0.00 for all) for rivers using a 14 hydro-climatic region definition. This new set of ice-growth coefficients expands on the work of Michel (1971), who defined coefficients for rivers and lakes based on surface insulation and exposure, to define coefficients by hydro-climate, based on mean January precipitation and temperature, latitude and elevation. These ice-growth coefficients can be used in many applications including modelling freshwater-ice thickness across the Northern Hemisphere using a degree-day approach, or quantifying freshwater ice on a hemispheric scale, as done for other cryospheric components (Lemke et al., 2007). Modelling freshwater ice thickness on an annual basis may also be used to explore variability and change over time and space. Chapter 4 will apply this degree-day ice-growth model to rivers and lakes across the Northern Hemisphere to quantify its areal extend and volume over a 44-year temporal time frame.
References


CHAPTER 4: QUANTIFYING PEAK FRESHWATER ICE ACROSS THE NORTHERN HEMISPHERE

Abstract

The average areal extent and volume of peak freshwater (river and lake) ice is quantified across the Northern Hemisphere for the period 1958-2002. Quantification is conducted using a degree-day ice-growth model and ice-growth coefficients defined for fourteen (ice-specific) hydro-climatic regions. The model is driven by ERA-40 gridded daily air-temperature data, and the Global Lakes and Wetlands Database (GLWD) is employed to spatially define rivers and lakes. Results indicate that the total area covered by freshwater ice, during peak seasonal thickness, north of the January 0°C isotherm (excluding the Greenland ice sheet) is $1.7 \times 10^6$ km$^2$ and the total freshwater-ice volume is $1.6 \times 10^3$ km$^3$. Such values complete a hemispheric quantification of major cryospheric components (the others, for example, snow, glaciers or sea ice, have been previously evaluated) and provides a reference dataset for assessing future climate-related changes.
4.1 Introduction

Through their influence on surface energy budgets, gas exchanges and the hydrologic cycle, components of the cryosphere are influenced by, and feedback to, climatic conditions across a wide range of temporal scales (Fitzharris, 1996). Moreover, detecting and modelling climate variations in such components provides a visible analysis of the changing climate (Lemke et al., 2007), particularly in higher latitudes affected by polar amplification of climate change (e.g. Serreze et al., 2000; Kattsov and Källén, 2005). To date, efforts have been made to quantify all components of the cryosphere across the Northern Hemisphere, except that of freshwater ice (Lemke et al., 2007). This is especially surprising considering the increasing recognition of the broad environmental significance of freshwater ice to bio-geo-chemical and socio-economic systems (e.g. Prowse, 2001a,b; Walsh et al., 2005; Wrona et al., 2006; Prowse et al., 2011). Given the above, the goal of this paper is to provide the first quantification of the areal extent and volume of freshwater ice across the Northern Hemisphere.

4.2 Methodology and Data

Freshwater-ice quantification was conducted using a degree-day ice-growth model (Michel, 1971) based on the Stefan equation,

\[ h_i = \alpha (D_f)^{1/2} \]  

(4.1)

where \( h_i \) is total ice thickness (mm), \( D_f \) is the sum of accumulated freezing degree day value in degrees Celsius (°C), and \( \alpha \) is a numerical ice-growth coefficient (mm °C^{-1/2} day^{-1/2}). Although more complex physically based models exist (e.g. Riley and Stefan, 1988; Vavrus et al., 1996; Duguay et al., 2003), the multivariate datasets required to drive such models are difficult to acquire for large-scale applications as addressed in this study (e.g. ...
USACE, 2002). Moreover, as noted by, for example, Shen and Yapa (1985) and Prowse et al. (2002), such degree-day models have been shown to produce reliable estimates of ice thickness except in the early stages of ice growth (Ashton, 1986) and are therefore more appropriately used in peak ice thickness estimation. Until recently, the only values for $\alpha$ were the very generalized ones first defined by Michel (1971) to try and account for variations in controlling factors such as water-body type, exposure, surface insulation and subsurface heat flux. These, however, have been updated to consider variations in ice-growth conditions experienced across a range of fourteen different ice-specific hydro-climatic regions (section 3.3.1), and hence are employed in this chapter.

The gridded daily air-temperature data employed to calculate $D_f$ and drive the model were obtained from the ERA-40, a gridded climatology dataset covering the period September 1957 and August 2002, produced by the European Centre for Medium-Range Weather Forecasts (Uppala et al., 2005). It was selected for this study because of its daily temporal resolution, 2.5° spatial resolution, and lengthy 44 hydrologic-year record. Moreover, ERA-40 has been shown to capture interannual variability well and provides an excellent spatial temperature field for use in climate change and variability modelling (Frauenfeld et al., 2005).

Spatial data used to represent the areal extent of lakes and rivers for the freshwater-ice quantification was acquired from the GLWD (Lehner & Doll, 2004). The GLWD was selected because it is the only global dataset that represents both lakes and rivers as polygons, thereby allowing for the calculation of surface area. It contains three categories of global data encompassing: a) shoreline polygons representing lakes with a surface area $\geq 50$ km$^2$ and reservoirs with a storage capacity $\geq 0.5$ km$^3$; b) lake, reservoir,
and river polygons with a surface area ≥ 0.1 km², excluding those water bodies in category a), and; c) all water bodies in categories a and b in raster format. For this analysis, ice thickness on large lakes is modelled separately due to their large heat storage capacity (Dereki, 1976); therefore, lakes and reservoirs in categories a) and b) are separated by surface area, where those with a surface area > 500 km² are considered "large" (Herdendorf, 1982), to create two lake and reservoir datasets split by surface area. Lake depth also plays an important role in the heat storage of large lakes; however, these data are not available on a hemispheric scale.

The spatial extent of these datasets was restricted to north of the January 0°C isotherm, and excludes areas of large permanent ice coverage including Greenland. This northern region has previously been noted to represent all ice-affected areas during the coldest month of the year (Chapter 3) and the most southern extent of freshwater-ice cover (Bennett and Prowse, 2010). Within this area, the GLWD was found to contain 245,880 lake and 713 reservoir polygons with a surface area ≥ 0.1 km² but ≤ 500 km² (classified here as representing "small" to "medium" lakes and reservoirs), 255 lake and 109 reservoir polygons with a surface area > 500 km² (considered in this paper to represent "large" lakes and reservoirs), and 1,656 river polygons within this region.

The degree-day ice-growth model was run for small to medium and large lakes and reservoirs, as well as rivers, using a single 44-year average $D_f$ grid at the spatial resolution of 2.5° (Figure 4.1), and hydro-climatically derived ice-growth coefficients defined in section 3.2 (Figure 4.2). Annual $D_f$ grids were calculated for each hydrologic year (August 1st to July 31st) contained in the ERA-40 dataset (1958/59-2001/02) by summing the air temperatures on days below 0°C (Walsh et al., 1998). These grids were
then averaged to produce a single 44-year average $D_f$ grid. Because of the inherent variability in daily air temperature in the spring and autumn, the 0°C threshold is crossed several times at the beginning and end of the ice season, therefore the data were filtered using a 31-day running mean to identify the crossing of the 0°C threshold (Bonsal and Prowse; 2003). $D_f$ grids are calculated from the running mean daily air-temperature dataset between the first and last date of below 0°C temperatures in autumn and spring.

Ice-growth coefficients were empirically defined by water-body type using a degree-day ice-growth model, spatially stratified using a fourteen hydro-climatic region definition (section 3.3.1). These regions capture the spatial variability inherent in freshwater-ice growth, and were defined using a two-step clustering approach and latitude, elevation, and mean January precipitation and temperature. Resulting ice thickness grids were restrained to the boundaries of the three water-body categories from the GLWD and total ice thickness area and volume were calculated for each (Figure 4.3).

### 4.3 Results and Discussion

The total area and volume of freshwater ice north of the January 0°C isotherm, excluding the Greenland ice sheet, is 1.71x10^6 km^2 and 1.56x10^3 km^3, respectively (Table 4.1). When total freshwater-ice quantities are compared to other cryospheric components, the areal extent is equal to that of the Greenland ice sheet (1.7x10^6 km^2; Bamber and Layberry, 2001), similar, for example, to the ice shelves of Antarctica (1.5x10^6 km^2; Lythe et al., 2001) and similar to mean August estimates (1966-2004) of snow on land across the Northern Hemisphere (1.9x10^6 km^2; Lemke et al., 2007). To put this in context, if all freshwater-ice were represented as a single water body, its area of ice cover would be similar in size to the drainage area of the Mackenzie River basin (1.8x10^6 km^2;
Culp et al., 2005). The volume of freshwater ice is within the range of mean estimates for snow on land across the Northern Hemisphere at $0.5 \times 10^3 \text{ km}^3$ to $5 \times 10^3 \text{ km}^3$ (Lemke et al., 2007). It is important to note that these estimates of freshwater-ice area and volume represent peak seasonal ice thickness, while estimates of the areal extent of the Greenland ice sheet and ice shelves of Antarctica represent perennial ice thicknesses in general.

Although the GLWD represents lakes accurately, with an estimated global area coverage of $3.2 \times 10^6 \text{ km}^2$ (Lehner and Doll, 2004), more recent estimates of the global abundance of natural lakes and ponds suggest small water bodies have been underestimated and put the global area covered by natural lakes and ponds at $4.2 \times 10^6 \text{ km}^2$ (Downing et al., 2006). This suggests that this initial quantification of freshwater ice area and volume using the GLWD are conservative estimates.

### Table 4.1: Area and volume of freshwater ice. Values represent peak freshwater ice, averaged between 1957 and 2002.

<table>
<thead>
<tr>
<th></th>
<th>Area ($10^6 \text{ km}^2$)</th>
<th>Ice Volume ($10^3 \text{ km}^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rivers (NH)</strong></td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Large lakes and reservoirs (NH)</strong></td>
<td>0.79</td>
<td>0.56</td>
</tr>
<tr>
<td><strong>Small to medium lakes and reservoirs (NH)</strong></td>
<td>0.80</td>
<td>0.86</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1.71</td>
<td>1.56</td>
</tr>
</tbody>
</table>
Figure 4.1: Accumulated freezing degree-days averaged between 1957 and 2002, north of the January 0°C isotherm.
Figure 4.2: Hydro-climatic regions north of the January 0°C isotherm defined in Chapter 3.
Figure 4.3: Freshwater-ice distribution north of the January 0°C isotherm.
4.4 Conclusion and Future Research

Using a degree-day ice-growth model and ice-growth coefficients defined by hydro-climatic region, the areal extent and volume of freshwater ice has been predicted north of the January 0°C isotherm, excluding the Greenland ice sheet. Results of this research complete the first global quantification of all major cryospheric components and provide a reference point for assessing future climate-related changes to freshwater ice. Quantities are comparable to those of other cryospheric components quantified, including snow on land and the Greenland ice sheet (Lemke et al., 2007). The success of this model indicates that it is possible to use hydro-climatically pre-defined ice-growth coefficients and a simplistic degree-day ice-growth model over broad temporal and spatial extents to effectively model freshwater-ice thickness.

This model could also be applied to predicting future changes in freshwater ice thickness through the employment of future climate scenarios. Although the employment of such a model for future predictions assumes a stationary relationship between degree-days and the heat fluxes governing freshwater ice growth, one that may not hold in future climate (Beltaos and Burrell, 2003), confidence in estimates of future climate variables at larger geographic scales is high, especially air temperature and to a lesser extent precipitation (Randall et al., 2007). This makes the use of the degree-day ice-growth model developed within this research a logical approach to predicting future changes in the quantity of freshwater ice given the reliability of future climate scenarios. Modelling this process is integral to our understanding of the hydrologic cycle and its impacts on the cryosphere. Results presented here expand hydrological knowledge and provide a robust
and viable method for analyzing climate variations over a wide spatial and temporal range.
References


CHAPTER 5: CONCLUSION

This thesis presents the first quantification of peak freshwater-ice thickness across the Northern Hemisphere, using a degree-day ice-growth model, stratified by hydro-climatic region. Chapter 1 introduces the purpose of this thesis, which was to quantify the aerial extent and volume of freshwater ice across the Northern Hemisphere during its maximum seasonal extent. Specifically, the objectives were to:

(i) develop a degree-day ice-growth model that addresses regional hydro-climatic variations across the Northern Hemisphere;

(ii) calibrate the degree-day ice-growth model by define optimal ice-growth coefficients by hydro-climatic region, using historical peak-ice thickness data from locations across the Northern Hemisphere;

(iii) validate the degree-day ice-growth model using historical peak-ice thickness data from locations across the Northern Hemisphere and coefficients defined in (ii);

(iv) model peak freshwater-ice thickness across the Northern Hemisphere using a suitable spatial dataset of rivers and lakes and the degree-day ice-growth model from (iii); and,

(v) quantify the areal extent and volume of peak freshwater ice for the Northern Hemisphere using the modelled peak-ice thickness in (iv).

Chapter 2 reviews the bio-geo-chemical and socio-economic importance of freshwater ice, the physical processes that govern freshwater-ice growth, the different freshwater-ice modelling approaches, and the existing climate classifications and regional
classification approaches applicable to defining hydro-climatic regions, to provide the necessary background to the methods applied in Chapters 3 and 4.

The first journal style manuscript presented in Chapter 3 advances the work of Michel (1971), who defined basic ice-growth coefficients for rivers and lakes based on surface insulation and exposure. Addressing objectives i, ii and iii, this work produced more advanced coefficients that are spatially stratified by hydro-climatic region and water-body type, and empirically defined using a degree-day ice-growth model across the Northern Hemisphere. Such regions were defined using a two-step clustering method, employing latitude, elevation, and mean January air temperature and precipitation as driver variables. A 14-region definition proved optimal for this research. Freshwater-ice growth coefficients were then defined by hydro-climatic region and water-body type during the model calibration phase, using maximum observed seasonal ice thickness data from across the Northern Hemisphere and the degree-day ice-growth model. Validation results produce an $r^2$ of 0.45 for large lakes and reservoirs, 0.73 for small to medium lakes and reservoirs and an $r^2$ of 0.44 for rivers ($p$-value < 0.001), results comparable to those achieved by others when applying more complex freshwater-ice growth models.

The second manuscript presented in Chapter 4 addresses objectives iv and v through the modelling and quantification of freshwater ice using the degree-day ice-growth model and related ice-growth coefficients developed in Chapter 3. Results present the first quantification of freshwater ice north of the January 0°C isotherm and provides the first estimate of the areal extent and volume of freshwater ice, equal to $1.71 \times 10^6$ km$^2$ and $1.56 \times 10^3$ km$^3$, respectively, an area roughly equivalent to the entire area of the Mackenzie River basin. Quantities are also comparable to those of other cryospheric...
components, including snow on land and the Greenland ice sheet. These results complete the quantification of all major cryospheric components across the Northern Hemisphere, and the success of this model indicates that it is possible to use hydro-climatically pre-defined ice-growth coefficients and climate data over broad temporal and spatial extents to effectively model freshwater-ice growth.

The results of the work presented in this thesis serve to expand hydrological knowledge and provide a robust and viable method for analyzing climate variations over a wide spatial and temporal range. The ice-growth coefficients developed in Chapter 3 can be used in a range of bio-geo-physical applications requiring information about ice-cover thickness, or for projecting future estimates of freshwater-ice quantities and how these may be altered under expected climate change. Such projections, however, should be viewed circumspectly, as this degree-day approach assumes that the current-climate relationship between an air-temperature index and ice growth will remain the same under future climate. As new higher resolution datasets become available, particularly those including information about lake bathymetry, it is recommended that additional quantifications be preformed to further validate and advance these results.
APPENDIX A

Modelling by Köppen region

Optimal coefficients were also defined by Köppen-Geiger region using a recently updated Köppen-Geiger climate map (Peel et al., 2007) and the approach described in Chapter 3 for model calibration by hydro-climatic region. As the Köppen-Geiger classification is an existing climate classification, the model was calibrated using this regional classification for comparison to calibration results using the hydro-climatic region definition presented in Chapter 3. Results are presented below (Figure A.1), which indicates defining the ice-growth coefficients by hydro-climatic region as opposed to by Köppen-Geiger region produces better results.

![Figure A.1 Model calibration results by Köppen-Geiger classification for rivers, small to medium lakes and reservoirs, and large lakes and reservoirs.](image)
APPENDIX B

Modelling by hydro-climatic region using an alternative definition

Hydro-climatic regions were initially defined using three variables, mean January air temperature, latitude, and elevation. Methodology for model calibration is presented in Chapter 3. Results are presented below (Figure B.1) for the optimal hydro-climatic region definition using three variables.

Figure B.1 Model calibration results by hydro-climatic regions using three variables, for rivers and small to medium lakes and reservoirs.
APPENDIX C

Approaches to estimating ice-growth coefficient

To define ice-growth coefficients within hydro-climatic regions lacking observational data, multiple linear regression and stepwise regression were applied to establish a relationship between regionally defined ice-growth coefficients and the variables used to define them, namely mean January temperature and precipitation, latitude and elevation. A statistically significant relationship between these variables and the ice-growth coefficients was unable to be established, and this approach was therefore deemed unsuitable. Spearman's rank correlation was run which established there was no statistically significant correlation, at the 99th percentile, between the ice-growth coefficients and any of the variables used to define hydro-climatic regions. Both multiple linear and stepwise regressions were run and results for both regression attempts, for both river and lake coefficients, were not statistically significant. Furthermore, the variables used during the regressions did not cover the full range of values within the dataset; therefore, the regression equation was unable to accurately predict coefficients within the regions defined by variables outside the range. Regression results are presented below for rivers (Table C.1) and lakes (Table C.2), as well as coefficients estimated from the regression equations.
Table C.1 River-ice growth coefficients defined using multiple linear regression and stepwise regression, not statistically significant.

<table>
<thead>
<tr>
<th>Cluster #</th>
<th>Mean January Precipitation Cluster Mean (mm/month)</th>
<th>Mean January Temperature Cluster Mean (°C/month)</th>
<th>Elevation Cluster Mean (km)</th>
<th>Latitude Cluster Mean (DD)</th>
<th>River Coefficient Calculated by Regression Equation</th>
<th>Coefficient Calculated by Stepwise Regression Equation</th>
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</thead>
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<td>53.23</td>
<td>41.5</td>
<td>40.3</td>
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Table C.2 Lake-ice growth coefficients defined using multiple linear regression and stepwise regression, not statistically significant.

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<th>Cluster #</th>
<th>Mean January Precipitation Cluster Mean (mm/month)</th>
<th>Mean January Temperature Cluster Mean (°C/month)</th>
<th>Elevation Cluster Mean (km)</th>
<th>Latitude Cluster Mean (DD)</th>
<th>Lake Coefficient Calculated by Regression Equation</th>
<th>Coefficient Calculated by Stepwise Regression Equation</th>
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APPENDIX D

Freshwater-ice dataset details

Each freshwater-ice thickness dataset used in this research had its own unique structure and relevant data, with some containing only ice thickness measurements and others containing ice thickness, ice phenology, and water-body characteristics information. These datasets are described in Chapter 3 briefly, and in more detail below.

The Canadian Ice Database (CID), developed by Lenormand et al. (2002), is the first database to combine data from the 3 main departments of Environment Canada, resulting in 63546 indexed records from 757 lake and river sites across Canada (Lenormand et al., 2002). The CID has been used in lake-ice phenology (Latifovic & Pouliot, 2007; Duguay et al., 2006) as well as river-ice trends research (Lacroix et al., 2005). The CID contained five large lake and forty-eight small to medium lake sites and ninety-four river sites with ice thickness measurements, ranging from 1958 to 2002 and containing up to forty-two years of observations.

The Great Lakes Environmental Research Laboratory (GLERL) ice thickness database (Sleator, 1995), available through the National Snow and Ice Data Center (NSIDC), contains snow ice, lake ice and total ice thickness (in centimeters) observations for 121 sites across the Great Lakes and surrounding areas, between 1966 and 1979. Only those observation sites from the Great Lakes themselves are used in this analysis, resulting in thirty-nine lake sites, with up to 14 years of data.

The Finnish Environmental Institute (SYKE) has publically available freshwater-ice dataset, which can be downloadable from their website at http://wwwp2.ymparisto.fi. Two large lake and sixty-eight small to medium lake and reservoir sites, as well as
sixteen river sites, each containing up to forty-two years of ice thickness measurements from 1961 to 2002, taken on the 15th and last day of each month throughout the winter, are used from this dataset.

Russian river-ice data is publically available from the NSIDC, an organization dedicated to the management and distribution of cryospheric data (Vuglinsky, 2000). Forty-six sites covering Northern Russia, from 1917 to 1992, are used from this dataset. Each site contains up to thirty-five years of observations. Two outlier sites are removed.

The Swedish Meteorological and Hydrological Institute (SMHI) licenses Swedish river-ice data for the period 1939 to 2010, available upon request for a minimal fee. Total ice thickness measurements taken weekly are available for twenty-two small to medium lake sites across Sweden, with up to forty-three years of observations per site.

The Pacific River Forecast Center, part of the National Oceanic and Atmospheric Administration (NOAA) Alaska National Weather Service, provides publicly available river- and lake-ice thickness data via their website at aprfc.arh.noaa.gov. Thirteen small to medium lake and twenty-nine river sites containing up to thirty-seven years of data from 1960 to 2002 are used in this research.

Unpublished river-ice thickness data for the Yukon Territory, obtained from Water Survey of Canada (WSC), are used to supplement the CID (WSC, unpublished data). This smaller dataset contains both 'water surface to bottom ice' and 'actual ice thickness' measurements in meters. Before the 1989-1990 ice season, WSC personnel measured 'water surface to bottom ice', whereas after this point 'actual ice thickness' measurements were reported. As stated by Beltaos et al. (2006), when snow cover is absent, the 'water surface to bottom of ice' represents 92% of the 'actual ice thickness'. To
account for this difference and standardize all measurements to 'actual ice thickness', all 'water surface to bottom' measurements are multiplied by 1.08. Measurements are taken on a semi-regular basis, anywhere from one to seven times per winter between 1930-2004, at twenty-four sites across the Yukon.

Unpublished river-ice data for BC are also used to supplement the CID (WSC, unpublished data), and contained 'end of season' measurements in feet and meters, taken at the end of the season just before breakup, between 1916 and 2005. This dataset contains a total of twenty-six sites, with up to forty-four years of 'end of season' measurements.

**Table D.1 Summary of dataset references and web addresses to access data if available**

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Reference; Web addresses (if applicable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada (CID)</td>
<td>Lenormand <em>et al.</em>, 2002</td>
</tr>
<tr>
<td>Canada (GLERL)</td>
<td>Sleator, 1995; nsidc.org/data/g00803.html</td>
</tr>
<tr>
<td>Finland (SYKE)</td>
<td>wwwp2.ymparisto.fi</td>
</tr>
<tr>
<td>Russia (NSIDC)</td>
<td>Vuglinsky, 2000; nsidc.org/data/docs/noaa/g01187_russian_river_ice_thickness/index.html</td>
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<tr>
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<td>Alaska (PRFC)</td>
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<tr>
<td>Yukon (WSC)</td>
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<td>BC (WSC)</td>
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</tbody>
</table>
APPENDIX E

AFDD calculations

The ERA-40 re-analysis dataset contains meteorological observations, including daily surface temperature, from September 1957 to August 2002 (Uppala et al., 2005), and was used to calculate AFDDs. The procedure for generating annual and 44-year average AFDD grids is outlined here in detail. The ERA-40 gridded surface temperature dataset for the Northern Hemisphere contains 44 individual files, one for each hydrological year (August 1st to July 31st) covered by the dataset. Each file contain 5328 rows and 365 columns, each row of data corresponding to a unique 2.5° x 2.5° grid cell and its 365 days of temperature data for that hydrological year (Table E.1).

<table>
<thead>
<tr>
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<th>Longitude</th>
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<th>3</th>
<th>4</th>
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</table>

AFDDs were calculated as described by Walsh et al. (1998) by summing the daily average air temperatures on days below zero degrees Celsius throughout the hydrologic year. The hydrological year captures the total period of river-ice coverage for the entire Northern Hemisphere, except in rare cases of multi-year river ice. Summing starts on the first day below freezing in the fall, and continues to the last day below freezing in the spring as described by Bonsal and Prowse (2003). Because of the inherent variability in daily temperature, the 0°C threshold is crossed several times in the spring and fall (Dugay
et al., 2006). To smooth this variability, a 31-day running mean or moving average smoothes the temperature data, as described by Dugay et al. (2006) and Bonsal and Prowse (2003).

The 31-day running mean works by averaging the temperature over a 31-day window. This research employs a specific type of running mean known as a central moving average, where the mean is taken from an equal number of temperature values on either side of the central value. This avoids introducing a lag in the moving average for any particular point in the data series. Because the 15 temperature values on either side are used to calculate the running mean, the initial 15 dates and last 15 dates cannot be smoothed as there is no data to use. Some running means will pad either side of a data series with zeros to facilitate the calculation of a running mean value for all real data entries, using false values to generate real smoothed data, and is therefore not appropriate here.

AFDD’s were first calculated for the 44 hydrological years individually, resulting in 44 AFDD files, each grid cell having its own AFDD value. A script, written in the programming language R®, was used to transform all 44-year temperature data files into a single 44-year average AFDD file for output from the program. Initially, the temperature file for each hydrologic year is read in, and the latitude, longitude and temperature data extracted. A 31-day running mean is applied to the temperature data, and the Julian dates of the first and last day when temperature remains below 0°C are identified. Temperatures are summed between the first and last dates identified from the running mean for all grid squares, and output along with the latitude and longitude of each. The output file consists of the latitude, longitude, first and last Julian dates of smoothed
negative temperatures, and their sum or AFDD. This was repeated for all 44 years of temperature data available.

The values corresponding to each grid cell for each of the 44 years were then averaged to create a single AFDD value for each grid cell corresponding to a 44-year AFDD average (Table E.2). This single file consisted of a latitude and longitude position for each grid cell and a corresponding 44-year average AFDD value. The MATLAB code used to generate these grids can be seen in Appendix G.

**Table E.2 Example of 44-year average AFDD dataset format**

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Longitude</th>
<th>AFDD total</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0°</td>
<td>6002.47</td>
</tr>
<tr>
<td>90°</td>
<td>2.5°</td>
<td>6002.47</td>
</tr>
<tr>
<td>90°</td>
<td>5°</td>
<td>6002.47</td>
</tr>
<tr>
<td>90°</td>
<td>7.5°</td>
<td>6002.47</td>
</tr>
<tr>
<td>90°</td>
<td>10°</td>
<td>6002.47</td>
</tr>
<tr>
<td>90°</td>
<td>12.5°</td>
<td>6002.47</td>
</tr>
</tbody>
</table>
APPENDIX F

Determining appropriate statistic for evaluating the degree-day model

Evaluating the degree-day ice-growth model as well as determining the optimal ice-growth coefficient of each observation site could not be accomplished using the same statistic. Using a coefficient of determination ($r^2$) is often the most common statistic when it comes to evaluating models, but when determining the optimal ice-growth coefficient for each observation site, regression does not capture the variance change, which occurred when the coefficient is varied. Optimal ice-growth coefficients are selected using the Nash-Sutcliffe efficiency $E$ (Nash and Sutcliffe, 1970), where $E$ ranges from a value of 1.0 (perfect model fit) and $-\infty$. $E$ is calculated as

$$E = 1 - \frac{\sum_{i=1}^{n} (O_i - P_i)^2}{\sum_{i=1}^{n} (O_i - \bar{O})^2}$$

(2)

where $O$ is observed and $P$ is predicted values. What makes $E$ more suitable to determining the optimal ice-growth coefficient is the division of the mean squared error over the observed variance, rather than the division of the explained variance over the total variance calculated by $r^2$. Adjusting the ice-growth coefficient within the model simply changes the variance of the modelled ice thickness values, and using $r^2$ as an evaluation statistic ignores this variance when the division is applied. $E$ considers this variance, and is therefore more suitable for this research. To illustrate this, two graphs are presented in Figure F.1 and F.2, using two very different ice-growth coefficients, and respective $R^2$ and $E$ statistics.
Figure F.1 Model calibration using a low coefficient of 7.0, with corresponding $R^2$ and $E$ statistics.

Figure F.2 Model calibration using a high coefficient of 34.0, with corresponding $R^2$ and $E$ statistics.
APPENDIX G

Scripts used in the research

This section contains both R and MATLAB code used in this thesis. The Module.bas contains all the VBA macros written in excel, used to format the ice thickness data, resulting in a final worksheet containing only annual peak-ice thickness measurements. AFDD values are extracted for each observation and optimal coefficients calculated. Within Module.bas, there are several sub functions which each contain a brief description. As mentioned, each ice thickness dataset had a different format, therefore not all datasets required the same formatting. As such, presented here are the most common VBA functions used on all datasets. The HydroYearToAFDD.m is a MATLAB file used to calculate AFDD values for each grid point, for each hydrologic year of temperature data (1958/59 - 2001/02). AverageAFDD.R is an R file used to calculate the average AFDD values at each grid point using the 44 AFDD grids.

Module.bas

Sub remove_zeros()
' This VBA function removes all zero values from the ice thickness measurements, row by row
  i = 1
  Range("H2").Offset(i, 0).Select
  ' loop for each record of station
  Do While Not IsEmpty(ActiveCell.Offset(1, -1))
    Range("H2").Offset(i, 0).Select
    ' if ice thickness is zero remove it
    If ActiveCell.Value = 0 Or IsEmpty(ActiveCell.Value) Or IsNull(ActiveCell.Value) Then
      Selection.EntireRow.Select
      Selection.Delete Shift:=xlUp
      Range("H2").Offset(i, 0).Select
    Else
      i = i + 1
    End If
  Loop
' Next j
End Sub

Sub remove_outlier_years()
' This VBA function removes all years of data not included in the study, namely before 1958 or after 2002
Dim ws As Worksheet
Dim CRLF As String          ' Carriage return/line feed.
Dim TheDate As Date         ' The serial date.
  i = 1
  Range("G2").Offset(i, 0).Select
  ' loop for each record of station
  Do While Not IsEmpty(ActiveCell)
    ' get julian date & year
    CRLF = Chr$(13)
'NormalDate variable.
TheDate = Range("G2").Offset(i, 0).Value
'Assign TheMonth the month number
TheMonth = Format(TheDate, "mm")
TheMonth = CInt(TheMonth)
'Assign DateYear the year number
DateYear = Format(TheDate, "yyyy")
DateYear = CInt(DateYear)
'if = 2002 but month is in 2002/2003 hydro year (august of later) of if greater than 2002 or less than 1958
If (DateYear = 2002 And TheMonth > 7) Or DateYear > 2002 Or (DateYear = 1958 And TheMonth < 7) Or DateYear < 1958
Then
  Range("G2").Offset(i, 0).EntireRow.Select
  Selection.EntireRow.Select
  Selection.Delete Shift:=xlUp
Else
  i = i + 1
End If
Loop
'Next j
End Sub

Sub JulianDate()
' This VBA function converts dates to Julian dates
'Must 1st use text-to-column to split date up
  Dim i
  Dim CRLF As String   'Carriage return/line feed.
  Dim NormalDate As Date       'The serial date.
  Dim DateYear As String     'The year of the serial date.
  Dim JulianDay As String  'The converted Julian date value
  Range("Q2").Select
  ActiveCell.Value = "JULIAN_DATE"
  i = 0
  Do While Not IsEmpty(ActiveCell.Offset(1, -10))
    'get julian date & year
    CRLF = Chr$(13)
    'Prompt for input and assign the value entered to the
    'NormalDate variable.
    NormalDate = ActiveCell.Offset(1, -10)
    'Assign TheMonth the month number
    TheMonth = Format(NormalDate, "mm")
    'Assign DateYear the year number
    DateYear = Format(NormalDate, "yy")
    'Find the day number for NormalDate
    If (TheMonth = "08" Or TheMonth = "09" Or TheMonth = "10" Or TheMonth = "11" Or TheMonth = "12") Then
      If (DateYear = "00") Then
        JulianDay = Format(Str(NormalDate - DateValue("1/8/" & Str(99)) + 1), "000")
      Else
        JulianDay = Format(Str(NormalDate - DateValue("1/8/" & Str(DateYear - 1)) + 1), "000")
      End If
    Else
      If (DateYear = "00") Then
        JulianDay = Format(Str(NormalDate - DateValue("1/8/" & Str(99)) + 1), "000")
      Else
        JulianDay = Format(Str(NormalDate - DateValue("1/8/" & Str(DateYear - 1)) + 1), "000")
      End If
    End If
  End If
  Range("Q3").Offset(i, 0).Select
  ActiveCell.Value = JulianDay
  i = i + 1
Loop
End Sub

Sub CreateInputFileNames()
' This VBA function creates the input file name required to extract AFDD values
' Run 1st, then CreateOutputFileNames()
  Range("P2").Select
  ActiveCell.Value = "INPUT_FILE"
Temp = "Hydro_
Range("P3").Select
' loop through the sheet
' This loop runs as long as there is something in the next Row
j = 0
Do While Not IsEmpty(Range("P3").Offset(j, 1))
' NormalDate variable.
NormalDate = Range("P3").Offset(j, -9)
' Assign TheMonth the month number
TheMonth = Format(NormalDate, "mm")
' Assign DateYear the year number
TheYear = Format(NormalDate, "yy")
If (TheMonth = "08" Or TheMonth = "09" Or TheMonth = "10" Or TheMonth = "11" Or TheMonth = "12") Then
    If (TheYear = "99") Then
        Temp3 = "00"
        Range("P3").Offset(j, 0) = Temp + TheYear + ".txt"
    ElseIf (TheYear = "00") Then
        Temp3 = "01"
        Range("P3").Offset(j, 0) = Temp + TheYear + ".txt"
    Else '98/99 etc.
        Temp3 = TheYear + 1
        Temp3 = Format(Temp3, "00")
        Temp3 = CStr(Temp3)
        Range("P3").Offset(j, 0) = Temp + TheYear + ".txt"
    End If
Else
    If (TheYear = "00") Then
        Temp3 = "99"
        Temp3 = CStr(Temp3)
        Range("P3").Offset(j, 0) = Temp + TheYear + ".txt"
    ElseIf (TheYear = "01") Then
        Temp3 = "00"
        Range("P3").Offset(j, 0) = Temp + TheYear + ".txt"
    Else
        Temp3 = TheYear - 1
        Temp3 = Format(Temp3, "00")
        Temp3 = CStr(Temp3)
        Range("P3").Offset(j, 0) = Temp + TheYear + ".txt"
    End If
End If
j = j + 1
Loop
End Sub

Sub CreateOutputFileNames()
' This VBA function creates output names for the AFDD files created for each observation
Dim TheDay As String
i = 1
j = 2
Range("R2").Select
ActiveCell.Value = "OUTPUT_FILE"
Temp = ActiveSheet.Name
' loop through the sheet
' This loop runs as long as there is something in the next Row
Do While Not IsEmpty(ActiveCell.Offset(1, -1))
    Range("R2").Offset(i, 0).Select
    Temp2 = ActiveCell.Offset(0, -2).Value
    TheDay = ActiveCell.Offset(0, -1).Value
    ActiveCell.Value = Temp + "." + TheDay + ".txt"
    ActiveCell.Replace What:="\", Replacement:="\", MatchCase:=False
    i = i + 1
Loop
i = 1
End Sub

Sub Extract_LN_Date()
' This VBA function extracts the Julian date for the last day of freezing temperatures
' number of records for station

Dim a As Integer
'latitude used to locate AFDD
Dim lat As Double
'longitude used to locate AFDD
Dim lon As Double
'name of file to open to look for AFDD
Dim File
Dim File2
'AFDD value
Dim Julian As Double
'current sheet to work on
Dim cur
i = 1
Range("Q2").Activate
'loop for each record of station
Do While Not IsEmpty(ActiveCell.Offset(1, -1))
  Windows("CID_Lake_Calibration_15days++.xlsm").Activate
  lat = Range("N3").Value
  lon = Range("O3").Value
  Range("P2").Activate
  File2 = (Left$(ActiveCell.Offset(i, 0).Value, 11)) & "_AFDD_Fix.txt"
  File = "C:\Users\rbrooks\Desktop\rbrooks\rbrooksBackUp\AFDD\HydroYearAFDDFixed" & File2
  If Not (Left$(ActiveCell.Offset(i, 0).Value, 11)) = (Left$(ActiveCell.Offset(i - 1, 0).Value, 11)) Then
    'open file
    Workbooks.OpenText Filename:=File
    .Origin:=xlMSDOS, StartRow:=1, DataType:=xlDelimited, TextQualifier:=
    xlDoubleQuote, ConsecutiveDelimiter:=False, Tab:=True, Semicolon:=False, 
    Comma:=False, Space:=False, Other:=False, FieldInfo:=Array(Array(1, 1), 
    Array(2, 1), Array(3, 1), Array(4, 1), Array(5, 1), Array(6, 1)), TrailingMinusNumbers 
    :=True
    Range("B1").Select
    'find AFDD
    For j = 0 To 5298
      'compare lat
      If ActiveCell.Offset(j, 0).Value = lat Then
        j = 2
      'compare lon
      For k = j To 5298
        If ActiveCell.Offset(k, 1).Value = lon Then
          Julian = ActiveCell.Offset(k, 3).Value
        j = 5298
        Exit For
      End If
    Next k
  End If
'add AFDD to results sheet
Windows("CID_Lake_Calibration_15days++.xlsm").Activate
'Sheets(s).Select
Range("S2").Activate
ActiveCell.Offset(i, 0).Select
ActiveCell.Value = Julian + 15
'close workbook without saving changes
Windows(File2).Activate
ActiveWorkbook.Close False
Windows("CID_Lake_Calibration_15days++.xlsm").Activate
i = i + 1
Else
  Range("S2").Activate
  ActiveCell.Offset(i, 0).Select
  ActiveCell.Value = Julian + 15
  i = i + 1
End If
Loop
Range("Q2").Activate
'Next x
End Sub
Sub Extract_FN_Date()
  'This VBA function extracts the Julian date for the first day of freezing temperatures
  'number of records for station
  Dim a As Integer
  'latitude used to locate AFDD
  Dim lat As Double
  'longitude used to locate AFDD
  Dim lon As Double
  'name of file to open to look for AFDD
  Dim File
  Dim File2
  'AFDD value
  Dim Julian As Double
  'current sheet to work on
  Dim cur

  i = 1
  'Sheets(x).Select
  Range("U2").Value = "FN Julian"
  Range("Q2").Activate
  'loop for each record of station
  Do While Not IsEmpty(ActiveCell.Offset(1, -3))
    Windows("CID_Lake_Calibration_15days+.xlsm").Activate
    'Sheets(x).Select
    lat = Range("N3").Value
    lon = Range("O3").Value
    Range("P2").Activate
    File2 = (Left$(ActiveCell.Offset(i, 0).Value, 11)) & ":AFDD_Fix.txt"
    File = "C:\Users\rbrooks\Desktop\rbrooksBackUp\AFDD\HydroYearAFDDFixed" & File2
    If Not (Left$(ActiveCell.Offset(i, 0).Value, 11)) = (Left$(ActiveCell.Offset(i - 1, 0).Value, 11)) Then
      'open file
      Workbooks.OpenText Filename:= File
      Range("B1").Select
      'find AFDD
      For j = 0 To 5298
        'compare lat
        If ActiveCell.Offset(j, 0).Value = lat Then
          j = 2
        End If
        'compare lon
        For k = j To 5298
          If ActiveCell.Offset(k, 1).Value = lon Then
            Julian = ActiveCell.Offset(k, 2).Value
            j = 5298
            Exit For
          End If
        Next k
        Exit For
      Next j
    'add AFDD to results sheet
    Windows("CID_Lake_Calibration_15days+.xlsm").Activate
    'Sheets(x).Select
    Range("U2").Activate
    ActiveCell.Offset(i, 0).Select
    ActiveCell.Value = Julian + 15
    'close workbook without saving changes
    Windows(File2).Close False
    Windows("CID_Lake_Calibration_15days+.xlsm").Activate
    i = i + 1
  Else
    Range("U2").Activate
    ActiveCell.Offset(i, 0).Select
    ActiveCell.Value = Julian + 15
    i = i + 1
  End If
End Sub
Sub Delete_Late_Season_Measurements()
' This VBA function deletes measurements taken more than 15 days after the last day of freezing temperatures
' first compare Julian dates manually
' if Julian dates more than 15 after last negative temp, remove it
' also manually look for early season dates to remove
i = 0
Loop
Range("T3").Offset(i, 0).Select
' loop for each record of station
Do While Not IsEmpty(ActiveCell.Offset(1, 0))
Range("T3").Offset(i, 0).Select
If ActiveCell.Value = 1 Then
Selection.EntireRow.Select
Selection.Delete Shift:=xlUp
Else
i = i + 1
End If
Loop
End Sub

Sub Delete_Early_Season_Measurements()
' This VBA function deletes measurements taken less than 15 days after the first day of freezing temperatures
For x = 6 To 28
i = 0
Range("V3").Offset(i, 0).Select
' loop for each record of station
Do While Not IsEmpty(ActiveCell.Offset(1, 0))
Range("V3").Offset(i, 0).Select
If ActiveCell.Value = 1 Then
Selection.EntireRow.Select
Selection.Delete Shift:=xlUp
Else
i = i + 1
End If
Loop
End Sub

Sub select_max_thickness()
' This VBA function deletes all but max ice thickness measurement for each year of observations
i = 1
NormalDate = Range("G2").Offset(i, 0)
TheMonth1 = Format(NormalDate, "mm")
TheYear1 = Format(NormalDate, "yy")
Thickness1 = Range("H2").Offset(i, 0)
NormalDate = Range("G2").Offset(i + 1, 0)
TheMonth2 = Format(NormalDate, "mm")
TheYear2 = Format(NormalDate, "yy")
Thickness2 = Range("H2").Offset(i + 1, 0)
Do While Not IsNull(NormalDate)
' if year1 = year2
If (TheYear1 = TheYear2) Then
' if month1 & month2 > 8 (aug.) or month1 & month2 < 8 (aug.)
If (TheMonth1 >= 8 And TheMonth2 >= 8) Or (TheMonth1 < 8 And TheMonth2 < 8) Then
' if thickness1 > thickness2
If Thickness1 > Thickness2 Then
'delete thickness2
Range("H2").Offset(i + 1, 0).Select
Selection.EntireRow.Select
Selection.Delete Shift:=xlUp
' reset values
NormalDate = Range("G2").Offset(i + 1, 0)
TheMonth2 = Format(NormalDate, "mm")
TheYear2 = Format(NormalDate, "yy")
Thickness2 = Range("H2").Offset(i + 1, 0)
' if thickness1 < thickness2
ElseIf Thickness1 < Thickness2 Then
    'delete thickness1
    Range("H2").Offset(i, 0).Select
    Selection.EntireRow.Select
    Selection.Delete Shift:=xlUp
    'reset values
    TheYear1 = TheYear2
    TheMonth1 = TheMonth2
    Thickness1 = Thickness2
    NormalDate = Range("G2").Offset(i + 1, 0)
    TheMonth2 = Format(NormalDate, "mm")
    TheYear2 = Format(NormalDate, "yy")
    Thickness2 = Range("H2").Offset(i + 1, 0)
    'if thickness1 = thickness2
    ElseIf Thickness1 = Thickness2 Then
        'delete first entry (thickness2)
        Range("H2").Offset(i, 0).Select
        Selection.EntireRow.Select
        Selection.Delete Shift:=xlUp
        'reset values
        TheYear1 = TheYear2
        TheMonth1 = TheMonth2
        Thickness1 = Thickness2
        NormalDate = Range("G2").Offset(i + 1, 0)
        TheMonth2 = Format(NormalDate, "mm")
        TheYear2 = Format(NormalDate, "yy")
        Thickness2 = Range("H2").Offset(i + 1, 0)
    End If
    'if month1 < 8 (aug.) and month2 > 8 (aug.)
    ElseIf (TheMonth1 < 8 And TheMonth2 >= 8) Then
        'move to month2, leave month1 alone
        i = i + 1
        NormalDate = Range("G2").Offset(i, 0)
        TheMonth1 = Format(NormalDate, "mm")
        TheYear1 = Format(NormalDate, "yy")
        Thickness1 = Range("H2").Offset(i, 0)
        NormalDate = Range("G2").Offset(i + 1, 0)
        TheMonth2 = Format(NormalDate, "mm")
        TheYear2 = Format(NormalDate, "yy")
        Thickness2 = Range("H2").Offset(i + 1, 0)
        'else condition I haven't thought of
    End If
    'if year1 != year2
    ElseIf Not (TheYear1 = TheYear2) Then
        'if month1 > 8 (aug.) and month2 < 8 (aug.)
        If (TheMonth1 >= 8 And TheMonth2 < 8) Then
            'if thickness1 > thickness2
            If Thickness1 > Thickness2 Then
                'delete thickness2
                Range("H2").Offset(i + 1, 0).Select
                Selection.EntireRow.Select
                Selection.Delete Shift:=xlUp
                'reset values
                TheYear1 = TheYear2
                TheMonth1 = TheMonth2
                Thickness1 = Thickness2
                NormalDate = Range("G2").Offset(i + 1, 0)
                TheMonth2 = Format(NormalDate, "mm")
                TheYear2 = Format(NormalDate, "yy")
                Thickness2 = Range("H2").Offset(i + 1, 0)
                'if thickness1 < thickness2
                ElseIf Thickness1 < Thickness2 Then
                    'delete thickness1
                    Range("H2").Offset(i, 0).Select
                    Selection.EntireRow.Select
                    Selection.Delete Shift:=xlUp
                    'reset values
                    TheYear1 = TheYear2
                    TheMonth1 = TheMonth2
                    Thickness1 = Thickness2
        End If
    End If
NormalDate = Range("G2").Offset(i + 1, 0)
TheMonth2 = Format(NormalDate, "mm")
TheYear2 = Format(NormalDate, "yy")
 Thickness2 = Range("H2").Offset(i + 1, 0)
 'if thickness1 = thickness2
 ElseIf Thickness1 = Thickness2 Then
 'delete second entry (thickness2)
 Range("H2").Offset(i + 1, 0).Select
 Selection.EntireRow.Select
 Selection.Delete Shift:=xlUp
 'reset values
 TheYear1 = TheYear2
 TheMonth1 = TheMonth2
 Thickness1 = Thickness2
 NormalDate = Range("G2").Offset(i + 1, 0)
TheMonth2 = Format(NormalDate, "mm")
TheYear2 = Format(NormalDate, "yy")
 Thickness2 = Range("H2").Offset(i + 1, 0)
End If
'else data already sorted up to this point
Else
 'move to next row
 i = i + 1
 NormalDate = Range("G2").Offset(i, 0)
TheMonth1 = Format(NormalDate, "mm")
TheYear1 = Format(NormalDate, "yy")
 Thickness1 = Range("H2").Offset(i, 0)
NormalDate = Range("G2").Offset(i + 1, 0)
TheMonth2 = Format(NormalDate, "mm")
TheYear2 = Format(NormalDate, "yy")
 Thickness2 = Range("H2").Offset(i + 1, 0)
End If
End If
Loop
'x = x + 1
i = 1
'Next x
End Sub

Sub Extract_AFDD_Data()
'This VBA function extracts AFDD values for each maximum ice thickness observation, so that ice thickness can be modelled for each observation and results compared to evaluate model
'number of records for station
Dim a As Integer
'latitude used to locate AFDD
Dim lat As Double
'longitude used to locate AFDD
Dim lon As Double
'name of file to open to look for AFDD
Dim File
Dim File2
'AFDD value
Dim AFDD As Double
'current sheet to work on
Dim cur
Range("L3").Activate
i = 0
'loop for each record of station
Do While Not IsEmpty(ActiveCell.Offset(1, -5))
 Windows("CID_Lake_Calibration_15days+.xlsm").Activate
 lat = Range("N3").Value
 lon = Range("O3").Value
 Range("R3").Activate
 File2 = ActiveCell.Offset(i, 0).Value
 File = "C:\Users\brooks\Desktop\brooks\Masters_Work\Ice Thickness Data\Canada - CID\CIDModelApr2011\Calibration - Large Lakes Removed\04Results\MATLAB_Output_AllSites_Lake_15days" & File2
 On Error Resume Next
 'open file
Workbooks.OpenText Filename:= _
File _
  , Origin:=xlMSDOS, StartRow:=1, DataType:=xlDelimited, TextQualifier:= _
  xlDoubleQuote, ConsecutiveDelimiter:=False, Tab:=True, Semicolon:=False, _
Comma:=True, Space:=False, Other:=False, FieldInfo:=Array(Array(1, 1), _
  Array(2, 1), Array(3, 1), Array(4, 1), Array(5, 1), Array(6, 1)), TrailingMinusNumbers _
:=True
If (Err.Number = 1004) Then
  Range("L3").Offset(i, 0).Select
  i = i + 1
Else
  Range("B1").Select
  'find AFDD
  For j = 0 To 5298
    'compare lat
    If ActiveCell.Offsets(j, 0).Value = lat Then
      'compare lon
      For k = j To 5298
        If ActiveCell.Offsets(k, 1).Value = lon Then
          AFDD = ActiveCell.Offsets(k, 4).Value
          j = 5298
          Exit For
        End If
      Next k
    End If
  Next j
  'add AFDD to results sheet
  Windows("CID_Lake_Calibration_15days+.xlsm").Activate
  Range("L3").Offset(0, 0).Select
  ActiveCell.Value = AFDD
  Windows(File2).Activate
  ActiveWorkbook.Close False
  Windows("CID_Lake_Calibration_15days+.xlsm").Activate
  i = i + 1
End If
Loop
End Sub
Sub Opt_Coef()
  'This VBA macro calculates the optimal ice-growth coefficient for each measurement
  'number of reacords to run through
  Dim a As Integer
  'lowest coefficient to start with
  Dim coef As Double
  'coefficient + 0.1
  Dim higher As Double
  'coefficient - 0.1
  Dim lower As Double
  'CHANGE
  a = 428
  coef = 7#
  Range("N3").Activate
  'loop for each observed ice thickness record
  For i = 1 To a
    'at each cell, begin by calculating ice thickness using the lowest coefficient (7)
    ActiveCell.Value = (Math.Sqr(ActiveCell.Offsets(0, -2).Value) * coef) / 10
    For j = 1 To 200
      'compare the modelled ice thickness to observed
      If ActiveCell.Value = ActiveCell.Offsets(0, -6).Value Then
        Exit For
      Else
        If ActiveCell.Value < ActiveCell.Offsets(0, -6).Value Then
          're-calculate ice thickness using slightly larger coef
          coef = coef + 0.1
          ActiveCell.Value = (Math.Sqr(ActiveCell.Offsets(0, -2).Value) * coef) / 10
        Else
          're-calculate ice thickness using slightly smaller coef
          coef = coef - 0.1
          ActiveCell.Value = (Math.Sqr(ActiveCell.Offsets(0, -2).Value) * coef) / 10
        End If
      End If
    Next j
End If
Next i
End Sub
function[] = HydroYearToAFDD(fileName)

Temp_File = importdata('C:\Users\rbrooks\Desktop\rbrooks\rbrooksBackUp\AFDD\HydroYearsValidation\Hydro_61_62.txt');
fileName = strcat('C:\Users\rbrooks\Desktop\rbrooks\rbrooksBackUp\AFDD\HydroYearsCalVal\', fileName);
Temp_File = importdata(fileName);

ID_Lat_Long = Temp_File(:,1:3);

Daily_Temp = Temp_File(:,4:368);

span = 31;
window = ones(span,1)/span;

Daily_Temp_T = Daily_Temp';
RM = conv2(Daily_Temp_T, window, 'valid');
RM = RM';

for i=1:size(RM,1);
    FN(i,:) = find((RM(i,:)< 0),1,'first');
    LN(i,:) = find((RM(i,:)< 0),1,'last');
end

tot= zeros(size(RM,1),1);

for i=1:size(RM,1); %row counter
    for j=FN(i,:):LN(i,:); %Column counter
        if(RM(i,j) < 0);
            FN(i, :) = find((RM(i,:)< 0),1,'first');
            LN(i, :) = find((RM(i,:)< 0),1,'last');
        else
            FN(i,:) = FN(i,:);
            LN(i,:) = LN(i,:);
        end
        tot(i) = tot(i) + RM(i,j);
    end
end

end

\( \text{tot}(i) = \text{tot}(i) + \text{RM}(i,j); \) \%Used in Duguay et al., 2006 & Bonsal and Prowse 2003 paper
end
end
end

\%Round to one decimal place
for i=1:size(RM,1); \%row counter
\text{tot}(i) = \text{roundn(tot}(i),-1);
end

\%Covert to positive value
for i=1:size(RM,1); \%row counter
\text{tot}(i) = \text{abs(tot}(i));
end

\%Output ID, Lat, Long, FN, LN, and sum(AFDD)
\text{AFDD} = [\text{ID}_i, \text{Lat}, \text{Long}, \text{FN}, \text{LN}, \text{sum(AFDD)}];

\%write content to txt file
\%create empty file with write permission
[pathstr, name, ext] = fileparts(fileName);
newName = [’C:\Users\rbrooks\Desktop\rbrooks\rbrooksBackUp\AFDD\HydroYearAFDDFixed\MATLAB\name ‘_\text{AFDD}_Fix’ ext];
csvwrite(newName, \text{AFDD});

#AverageAFDD.R
#Read in 44 text files 58_59 to 01_02
\text{Y}_58\_59 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_58\_59\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_59\_60 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_59\_60\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_60\_61 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_60\_61\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_61\_62 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_61\_62\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_62\_63 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_62\_63\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_63\_64 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_63\_64\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_64\_65 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_64\_65\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_65\_66 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_65\_66\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_66\_67 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_66\_67\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_67\_68 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_67\_68\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_68\_69 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_68\_69\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_69\_70 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_69\_70\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_70\_71 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_70\_71\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_71\_72 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_71\_72\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
\text{Y}_72\_73 \leftarrow 
\text{read.table(“C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_72\_73\_AFDD\_Fix.txt”, header=TRUE, sep=”,”)}
Y_73_74 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_73_74_AFDD_Fix.txt", header=TRUE, sep=";")
Y_74_75 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_74_75_AFDD_Fix.txt", header=TRUE, sep=";")
Y_75_76 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_75_76_AFDD_Fix.txt", header=TRUE, sep=";")
Y_76_77 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_76_77_AFDD_Fix.txt", header=TRUE, sep=";")
Y_77_78 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_77_78_AFDD_Fix.txt", header=TRUE, sep=";")
Y_81_82 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_81_82_AFDD_Fix.txt", header=TRUE, sep=";")
Y_83_84 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_83_84_AFDD_Fix.txt", header=TRUE, sep=";")
Y_84_85 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_84_85_AFDD_Fix.txt", header=TRUE, sep=";")
Y_86_87 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_86_87_AFDD_Fix.txt", header=TRUE, sep=";")
Y_89_90 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_89_90_AFDD_Fix.txt", header=TRUE, sep=";")
Y_90_91 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_90_91_AFDD_Fix.txt", header=TRUE, sep=";")
Y_95_96 <- read.table("C://Users//rbrooks//Desktop//rbrooks//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_95_96_AFDD_Fix.txt", header=TRUE, sep=";")
Y_96_97 <- read.table("C://Users//rbrooks//Desktop//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_96_97_AFDD _Fix.txt", header=TRUE, sep=" ")
Y_99_00 <- read.table("C://Users//rbrooks//Desktop//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_99_00_AFDD _Fix.txt", header=TRUE, sep=" ")
Y_00_01 <- read.table("C://Users//rbrooks//Desktop//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_00_01_AFDD _Fix.txt", header=TRUE, sep=" ")
Y_01_02 <- read.table("C://Users//rbrooks//Desktop//rbrooksBackUp//AFDD//HydroYearAFDDFixed//MATLAB//Hydro_01_02_AFDD _Fix.txt", header=TRUE, sep=" ")

#combine AFDD for each year with lat/long
AFDD <- cbind(Y_58_59[,6],Y_59_60[,6],Y_60_61[,6],Y_61_62[,6],Y_62_63[,6],Y_63_64[,6],Y_64_65[,6],Y_65_66[,6],Y_66_67[,6],Y_67_68[,6],Y_68_69[,6],Y_69_70[,6],Y_70_71[,6],Y_71_72[,6],Y_72_73[,6],Y_73_74[,6],Y_74_75[,6],Y_75_76[,6],Y_76_77[,6],Y_77_78[,6],Y_78_79[,6],Y_79_80[,6],Y_80_81[,6],Y_81_82[,6],Y_82_83[,6],Y_83_84[,6],Y_84_85[,6],Y_85_86[,6],Y_86_87[,6],Y_87_88[,6],Y_88_89[,6],Y_89_90[,6],Y_90_91[,6],Y_91_92[,6],Y_92_93[,6],Y_93_94[,6],Y_94_95[,6],Y_95_96[,6],Y_96_97[,6],Y_97_98[,6],Y_98_99[,6],Y_99_00[,6],Y_00_01[,6],Y_01_02[,6])

#sum AFDD for each year to get a 44 yr average AFDD
AFDDtotal <- rowSums(AFDD)
AFDDtotal <- abs(AFDDtotal)
AFDDtotal <- AFDDtotal/44

AFDDtotal <- cbind(Y_58_59[,2],Y_58_59[,3],AFDDtotal)
APPENDIX H

Model calibration and validation results without coefficient restraints

The model was also calibrated and validated without restraints on minimum or maximum ice-growth coefficient values, resulting in the use of coefficient values above 34 and below 7. Results for model calibration and validation are shown below in Table H.1 and H.2.

Table H.1 Model calibration results when run without ice-growth coefficient minimum of maximum restraints.

<table>
<thead>
<tr>
<th>Hydro-climatic region</th>
<th>Single Coefficient</th>
<th>Optimal Coefficient</th>
<th>Köppen Coefficient</th>
<th>Hydro-climatic region 6</th>
<th>Hydro-climatic region 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large lakes and reservoirs</td>
<td>0.64</td>
<td>0.86</td>
<td>0.60</td>
<td>0.67</td>
<td>0.61</td>
</tr>
<tr>
<td>Small to medium lakes and reservoirs</td>
<td>0.71</td>
<td>0.87</td>
<td>0.71</td>
<td>0.71</td>
<td>0.74</td>
</tr>
<tr>
<td>Rivers</td>
<td>0.50</td>
<td>0.82</td>
<td>0.49</td>
<td>0.50</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table H.2 Model validation results when run without ice-growth coefficient minimum of maximum restraints.

<table>
<thead>
<tr>
<th>Hydro-climatic region 14</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large lakes and reservoirs</td>
<td>0.46</td>
</tr>
<tr>
<td>Small to medium lakes and reservoirs</td>
<td>0.69</td>
</tr>
<tr>
<td>Rivers</td>
<td>0.44</td>
</tr>
</tbody>
</table>