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The extractable power from a channel linking a bay to the open ocean

J Blanchfield^{1*}, C Garrett², P Wild¹, and A Rowe¹

¹Department of Mechanical Engineering, Institute for Integrated Energy Systems, University of Victoria, Victoria, British Columbia, Canada

²Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, Canada

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Abstract: Interest in the power potential of tidal streams is growing worldwide. While the latest assessment for Canadian coastlines estimates a resource of approximately 42 GW, these results are based on the average kinetic energy flux through the channel. It has been shown, however, that this method cannot be used to obtain the maximum extractable power for electricity generation. This work presents an updated theory for the extractable power from a tidal stream in a channel linking a bay to the open ocean. The maximum average extractable power from a channel linking a bay to the open ocean may be estimated, within approximately 15 per cent, as $0.22\rho gaQ_0$, where a is the amplitude of the dominant tidal constituent in the open ocean and Q_0 is the maximum volumetric flowrate in the undisturbed state.

Keywords: tidal power, tidal stream, renewable energy, resource assessment

1 INTRODUCTION

While the development of tidal stream power is still in its infancy, interest continues to grow among utility companies and governments of maritime countries in assessing its true potential [1–4]. This interest is driven by the desire for energy supply security and the environmental impacts associated with fossil fuel combustion. As a result, there has been continued development of theories for an accurate assessment of this resource [5–9].

Although the latest assessment for Canadian coastlines estimates a resource potential of approximately 42 GW, these results are based on the kinetic energy flux in the undisturbed state [2]. Here, the undisturbed state represents the natural state prior to installing turbines. Bryden and Couch [7] and Bryden and Melville [8] have shown a relationship between the extractable power and the undisturbed energy flux in a one-dimensional model for a simple uniform channel

when flow acceleration and variations in the channel bathymetry are neglected. This method, however, has been shown by Garrett and Cummins [5, 6] to have no simple relationship to the maximum extractable power in a more complicated flow regime.

The present work uses the model of a channel linking two large basins, developed in reference [6], to extend the model of a channel linking a bay to the open ocean, presented in reference [5], by including local flow acceleration, bottom drag, and exit flow separation effects within the dynamical balance. This updated one-dimensional model describes the flow through a channel, of varying cross-sectional area, linking a bay to the open ocean.

2 TIDAL STREAM MODEL

A tidal stream may be found in a channel linking a bay to the open ocean. A mathematical model is developed to solve for the flowrate, Q , and the water surface elevation within the bay, ζ_{Bay} , given a bay surface area, A , and channel cross-sectional area, $E(x)$, which may vary along the length of the channel, L . The water surface elevation just outside the channel in the open ocean, ζ_0 , is assumed to be unaffected by the flow through

*Corresponding author: Institute for Integrated Energy Systems, Department of Mechanical Engineering, University of Victoria, PO Box 3055, STN CSC, Victoria, British Columbia, Canada V8W 3P6. email: jblanchf@uvic.ca

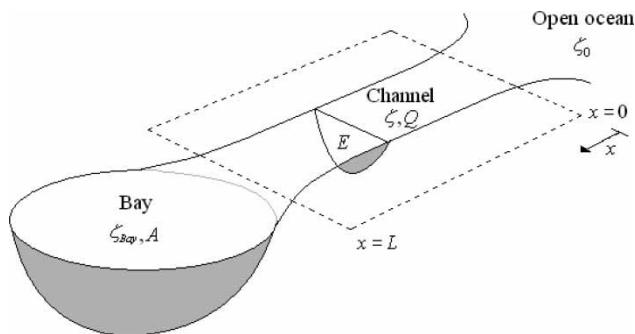


Fig. 1 Schematic drawing of a channel linking a bay to the open ocean

the channel. Here, water surface elevation refers to the height above or below the average tide height. A schematic drawing of this model is presented in Fig. 1. The momentum balance, continuity, and power equations are derived and presented in their general forms. This is convenient as it will allow for multiple solutions based on alternative assumptions. These solutions will be discussed in section 3.

2.1 Momentum balance

For a one-dimensional flow along the x -axis, the Navier–Stokes equation may be written as

$$-\frac{\partial p(x, t)}{\partial x} - \rho F(x, t) = \rho \left(\frac{\partial u(x, t)}{\partial t} + u(x, t) \frac{\partial u(x, t)}{\partial x} \right) \quad (1)$$

The resistance forces per unit mass of the sea water, $F(x, t)$, may include bottom drag and turbine drag associated with power generation. The flow velocity is represented as $u(x, t)$, ρ is the density of the sea water, and $p(x, t)$ is the pressure. Assuming the pressure to be hydrostatic, the pressure gradient, $\partial p / \partial x$, may be taken as $g \partial \zeta / \partial x$, where g is the acceleration due to gravity and $\zeta(x, t)$ is the local water surface elevation along the channel.

Turbines are assumed to be deployed such that all the flow passes through the turbines [6]. While installing uniform ‘fences’ of turbines may be impractical in engineering terms, the assumption significantly simplifies the analysis and provides insight into the maximum extractable power from the tidal stream. The reduction in maximum power associated with using partial fences that do not occupy the whole cross-section has been analysed in reference [10].

The one-dimensional flow assumption implies that the resistance forces and flow velocity are independent of the cross-channel position and vary only along the length of the channel.

The momentum equation may now be written as

$$-F = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} \quad (2)$$

Integrating equation (2) over the length of the channel, the momentum equation becomes

$$-\int_0^L F dx = \int_0^L \frac{\partial u}{\partial t} dx + \int_0^L u \frac{\partial u}{\partial x} dx + \int_0^L g \frac{\partial \zeta}{\partial x} dx \quad (3)$$

Assuming that the tidal wavelength is much longer than the channel length and that the surface area of the channel is much less than that of the basin, volume conservation implies that the volume flux is independent of position and may be expressed only as a function of time as

$$Q(t) = Eu \quad (4)$$

The cross-sectional area of the channel, $E(x)$, is assumed not to be significantly affected by the tidal changes in elevation.

The local acceleration term may then be written as

$$\int_0^L \frac{\partial u}{\partial t} dx = c \frac{\partial Q}{\partial t} \quad (5)$$

where the channel geometry term, c , is now introduced and defined as

$$c = \int_0^L E^{-1} dx \quad (6)$$

If the cross-sectional area of the channel is assumed to be constant along its entire length, then the channel geometry term is

$$c = \frac{L}{E} \quad (7)$$

A more general model, however, allows the cross-sectional area to vary along the channel.

The flow is assumed to be drawn in smoothly at the channel entrance from a region with a large surface area, weak currents, and a prescribed tidal elevation [6]. The integral of the convective acceleration term, therefore, describes the flow separation at the channel exit and may be written as a function of the volumetric flowrate as

$$\int_0^L u \frac{\partial u}{\partial x} dx = \frac{1}{2} u_e |u_e| = \frac{1}{2E_e^2} Q |Q| \quad (8)$$

where u_e and E_e represent the flow velocity and the local cross-sectional area at the exit of the channel, respectively. In the momentum balance, the channel exit switches from one end of the channel to the other during the flood and ebb tides. For simplicity, the same

cross-sectional area is assumed at the separation for flood and ebb tides.

The forcing term, representing the change in water surface elevation along the channel, is

$$-\int_0^L \frac{\partial \zeta}{\partial x} dx = \zeta_0 - \zeta_{\text{Bay}} \quad (9)$$

where ζ_{Bay} is the water surface elevation within the bay and ζ_0 represents the water surface elevation just outside the channel in the open ocean. The water surface elevation in the bay is assumed to rise and fall uniformly.

The momentum equation for the flow through the channel may now be written as

$$c \frac{dQ}{dt} = g(\zeta_0 - \zeta_{\text{Bay}}) - \int_0^L F dx - \frac{1}{2E_e^2} Q|Q| \quad (10)$$

The resistance force, F , representing the turbine drag and bottom drag within the channel, is defined as

$$\int_0^L F dx = \int_0^L F_{\text{turb}} dx + \int_0^L \frac{C_d}{h} u|u|^{n_2-1} dx \quad (11)$$

where $\int_0^L F_{\text{turb}} dx$ represents the turbine drag and $\int_0^L (C_d/h) u|u|^{n_2-1} dx$ represents the bottom drag, in which C_d is the bottom drag coefficient and $h(x)$ is the average water depth [6, 9] at position x .

The bottom drag is expressed as a function of the flowrate as

$$\int_0^L \frac{C_d}{h} u|u|^{n_2-1} dx = \lambda_2 Q|Q|^{n_2-1} \quad (12)$$

where the bottom drag parameter is defined as

$$\lambda_2 = \int_0^L C_d(hE_e^{n_2})^{-1} dx \quad (13)$$

The turbine drag is expressed as a function of the flowrate as

$$\int_0^L F_{\text{turb}} dx = \lambda_1 Q|Q|^{n_1-1} \quad (14)$$

where λ_1 is related to the number of turbines and their location along the channel, and will be referred to as the turbine drag parameter.

The relationship between drag and the flowrate, defined by n_1 and n_2 , is left arbitrary at this point. Substituting equations (14) and (12) into equation (11), the

total drag within the channel is

$$\int_0^L F dx = \lambda_1 Q|Q|^{n_1-1} + \lambda_2 Q|Q|^{n_2-1} \quad (15)$$

Substituting equation (15) into equation (10), the momentum equation is

$$c \frac{dQ}{dt} = g(\zeta_0 - \zeta_{\text{Bay}}) - \lambda_1 Q|Q|^{n_1-1} - \lambda_2 Q|Q|^{n_2-1} - \frac{1}{2E_e^2} Q|Q| \quad (16)$$

Dimensional analysis eliminates the need for site-specific parameters and reduces the number of variables. The momentum equation is non-dimensionalized with the following non-dimensional variables: $\zeta^* = \zeta/a$, $t^* = \omega t$, $Q^* = (c\omega/ga)Q$, $\lambda_1^* = \lambda_1(ga)^{n_1-1}/(c\omega)^{n_1}$, and $\lambda_2^* = \lambda_2(ga)^{n_2-1}/(c\omega)^{n_2}$ to become

$$\frac{dQ^*}{dt^*} = \zeta_0^* - \zeta_{\text{Bay}}^* - \lambda_1^* Q^*|Q^*|^{n_1-1} - \lambda_2^* Q^*|Q^*|^{n_2-1} - \frac{ga}{2(c\omega E_e)^2} Q^*|Q^*| \quad (17)$$

Continuity is now applied to determine the relationship between the water surface elevation within the bay and the flowrate through the entrance.

2.2 Continuity

The water surface elevation in the bay may be related to the flowrate through the channel and the surface area of the bay as

$$\frac{d\zeta_{\text{Bay}}}{dt} = \frac{Q}{A} \quad (18)$$

As the surface area of the bay increases toward infinity, the dependence of the tidal regime inside the bay on the flowrate through the channel approaches zero. For this reason, Garrett and Cummins [6] neglected this dependence when modelling the extractable power from a channel connecting two large basins. The surface area of the bay, A , is assumed to be unaffected by the rise and fall of the tides.

In the present analysis, the general case of a channel connecting a bay to the open ocean will be examined. This general case effectively describes both scenarios presented in references [5] and [6], since the surface area may equal any positive value greater than zero. The model will solve for a channel connecting a bay to the open ocean for any value of surface area greater than zero.

The continuity equation is non-dimensionalized using the previously defined non-dimensional variables, giving

$$\frac{d\zeta_{\text{Bay}}^*}{dt^*} = \beta Q^* \quad (19)$$

where

$$\beta = \frac{g}{cA\omega^2} \quad (20)$$

The bay geometry term, β , is the ratio of the square of the bay's Helmholtz frequency to the square of the forcing frequency, ω . The Helmholtz resonance, therefore, corresponds to $\beta = 1$. As an example, imagine two bays with the same surface area, A , and uniform channels of equal cross-sectional area, E . If the first bay corresponds to $\beta = 1.5$, the second bay would correspond to $\beta = 0.75$ if its channel were twice the length of the first.

The governing equations (17) and (19) represent a coupled, first-order, non-linear, non-homogeneous ordinary differential equation system and may be solved to determine the flowrate and bay height for varying levels of turbine drag, bottom drag, exit separation effects, and open ocean tidal amplitude as a function of time. Once the system of equations is solved, the extractable power from the channel may be calculated as a function of the flowrate.

2.3 Extractable power

The total extracted power, P_{Total} , over the entire channel includes power extracted from the channel due to bottom drag and the power extracted for electricity generation using tidal turbines. At any given time during the tidal cycle, the total power [6] may be defined as

$$P_{\text{Total}} = \rho Q \int_0^L F \, dx \quad (21)$$

The average extractable power for electricity generation, P_{avg} , over a tidal cycle, as indicated by the overbar, is then

$$\overline{P_{\text{avg}}} = \rho Q \overline{\int_0^L F_{\text{turb}} \, dx} \quad (22)$$

Substituting equation (14) into equation (22)

$$\overline{P_{\text{avg}}} = \rho \lambda_1 \overline{Q^2 |Q|^{n_1-1}} \quad (23)$$

Using the previously defined non-dimensional variables, the non-dimensional average extractable

power is

$$\overline{P_{\text{avg}}}^* = \lambda_1^* \overline{Q^{*2} |Q^*|^{n_1-1}} \quad (24)$$

where the dimensional average power is

$$P_{\text{avg}} = \frac{\rho(ga)^2}{c\omega} \overline{P_{\text{avg}}}^* \quad (25)$$

It was shown in reference [6] that the maximum extractable power is conveniently expressed as

$$(P_{\text{avg}})_{\text{max}} = \gamma \rho g a Q_0 \quad (26)$$

where Q_0 is the maximum volume flowrate in the undisturbed state. It was shown in reference [6] that when drag is quadratic, the multiplier, γ , only varies between 0.24 and 0.20 for a channel connecting two large basins ($\beta = 0$), depending on the mix of acceleration and drag in the undisturbed state.

In terms of dimensionless variables, the relative power is

$$P_{\text{rel}}^* = \frac{\overline{P_{\text{avg}}}^*}{Q_0^*} \quad (27)$$

where $(P_{\text{rel}}^*)_{\text{max}} = \gamma$.

3 RESULTS

Three scenarios are explored and compared below. The first two scenarios assume that bottom drag and exit flow separation within the channel are negligible. Two drag laws are explored. The first scenario assumes that the turbine drag is linearly proportional to the flowrate, whereas the second scenario assumes that the turbine drag is quadratic in the flowrate. The third scenario includes bottom drag and exit flow separation in the momentum balance and assumes that the bottom drag and turbine drag are quadratic in the flowrate. The average extractable power for electricity generation for all scenarios is calculated and comparisons are made.

The water surface elevation just outside the channel in the open ocean is approximated by a single sinusoid as

$$\zeta_0 = a \cos \omega t \quad (28)$$

where a and ω are the amplitude and frequency of the dominant tidal constituent, respectively.

3.1 Negligible bottom drag and exit flow separation

3.1.1 Linear drag – analytic solution

Linear drag may be unrealistic, but allows for an analytic solution that gives insight into the underlying

physics of the problem and provides validation for the numerical solver used for the case of quadratic drag.

The momentum balance is now

$$\frac{dQ^*}{dt^*} = \cos t^* - \zeta_{\text{Bay}}^* - \lambda_1^* Q^* \quad (29)$$

Equations (29) and (19) are solved simultaneously, giving

$$Q^* = \frac{\lambda_1^* \cos t^* - (\beta - 1) \sin t^*}{(\beta - 1)^2 + \lambda_1^{*2}} \quad (30)$$

and

$$\zeta_{\text{Bay}}^* = \frac{\beta(\beta - 1) \cos t^* + \beta \lambda_1^* \sin t^*}{(\beta - 1)^2 + \lambda_1^{*2}} \quad (31)$$

The complex modulus and phase of the non-dimensional water surface elevation within the bay are

$$|\zeta_{\text{Bay}}^*| = \frac{\beta}{[(\beta - 1)^2 + \lambda_1^{*2}]^{1/2}} \quad (32)$$

and

$$\theta = \tan^{-1}\left(\frac{\lambda_1}{\beta - 1}\right) \quad (33)$$

The value of θ is assumed to be between 0° and 180° . The phase lag is plotted in Fig. 2 as a function of the bay geometry term and the turbine drag parameter.

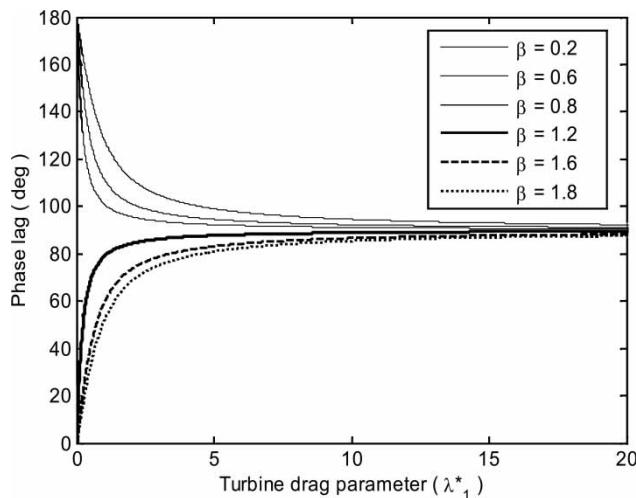


Fig. 2 Phase lag as a function of the turbine drag parameter for varying bay geometries when drag is assumed to be linear with the flowrate, and bottom drag and exit flow separation are assumed negligible

The non-dimensional average extractable power is

$$P_{\text{avg}}^* = \frac{1}{2} \frac{\lambda_1^*}{(\beta - 1)^2 + \lambda_1^{*2}} \quad (34)$$

and is plotted in Fig. 3 as a function of the turbine drag parameter, λ_1^* , for varying bay geometries. Since bottom drag is assumed negligible, all of the power from the channel is assumed to be extracted for electricity generation. The maximum non-dimensional average extractable power increases as the bay geometry approaches the Helmholtz frequency corresponding to $\beta = 1$.

The relative power, from equation (27), is

$$P_{\text{rel}}^* = \frac{1}{2} \frac{\lambda_1^*(\beta - 1)}{(\beta - 1)^2 + \lambda_1^{*2}} \quad (35)$$

since the maximum flowrate in the undisturbed state, Q_0^* , is equal to $(\beta - 1)^{-1}$. The relative power is plotted in Fig. 4, as a function of the turbine drag parameter for varying bay geometries.

The relative power is maximized when $\lambda_1^* = \beta - 1$ resulting in a multiplier, γ , equal to 0.25. This is also shown in Fig. 4. Thus

$$(P_{\text{avg}})^*_{\text{max}} = 0.25 \rho g a Q_0 \quad (36)$$

is independent of the geometry parameter β , and $Q_0 = \omega A |\zeta_{\text{Bay}}|$ when $\beta > 0$.

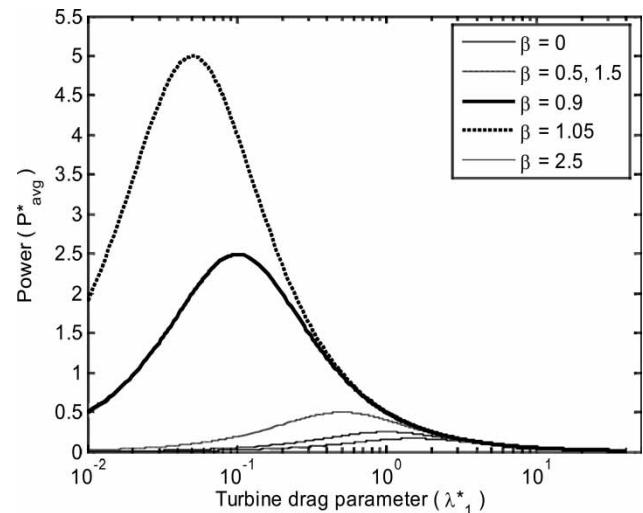


Fig. 3 Non-dimensional average extractable power as a function of the turbine drag parameter for varying bay geometries when friction is assumed linearly proportional to the flowrate, and bottom drag and exit separation effects are negligible

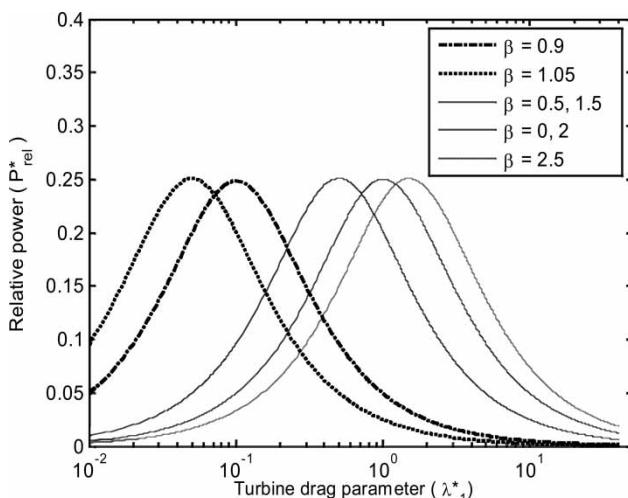


Fig. 4 Relative power as a function of the turbine drag parameter for varying bay geometries when friction is assumed linearly proportional to the flowrate, and bottom drag and exit separation effects are negligible

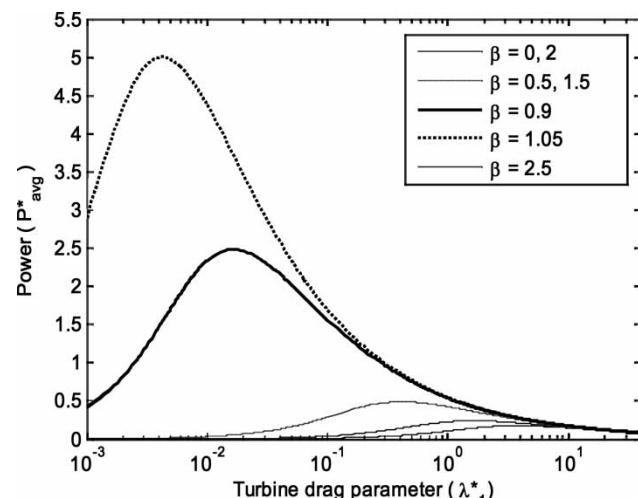


Fig. 5 Non-dimensional average extractable power as a function of the turbine drag parameter for varying bay geometries when drag is assumed to be quadratic in the flowrate, and bottom drag and exit separation effects are negligible

3.1.2 Quadratic drag – numerical solution

A numerical solver was developed to calculate the average extractable power when the turbine drag is assumed to be quadratic in the volume flowrate. The program calculates the average power curves using equations (24) and (27), and solves both the linear and quadratic cases by allowing the exponent n_1 to vary arbitrarily. The numerical solver is validated by setting n_1 equal to unity and comparing the numerical and analytical solutions.

With $n_1 = 2$ substituted into equation (24)

$$P^*_{avg} = \lambda_1^* Q^{*2} |Q^*| \quad (37)$$

This is shown in Fig. 5 as a function of the turbine drag parameter, λ_1^* . The relative power curves are plotted in Fig. 6. The multiplier, γ , for maximum power is only slightly different than in the linear case, varying between 0.25 and 0.24 depending on the value of β .

3.2 Including bottom drag and exit separation effects – quadratic drag

Bottom drag and exit separation effects are now included in the momentum balance and both bottom drag and turbine drag are assumed to be quadratic in the flowrate [11]. The bottom drag and separation terms may be grouped into the non-dimensional loss parameter, λ_0^* , where

$$\lambda_0^* = \lambda_2^* + \frac{ga}{2(c\omega E_e)^2} \quad (38)$$

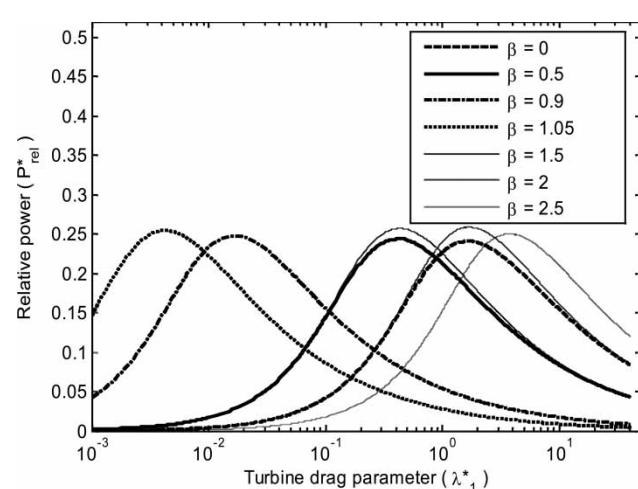


Fig. 6 Relative power as a function of the turbine drag parameter for varying bay geometries when drag is assumed to be quadratic in the flowrate, and bottom drag and exit separation effects are negligible

The momentum balance may then be written as

$$\frac{dQ^*}{dt^*} = \cos t^* - \zeta_{Bay}^* - (\lambda_1^* + \lambda_0^*)|Q^*|Q^* \quad (39)$$

The multiplier, γ , plotted in Fig. 7 varies between approximately 0.26 and 0.19 depending on the geometry parameter, β , but converges to approximately 0.21 for $\lambda_0^* \geq 10$. This is a particularly interesting result since the maximum average extractable power for electricity generation may now be estimated, within

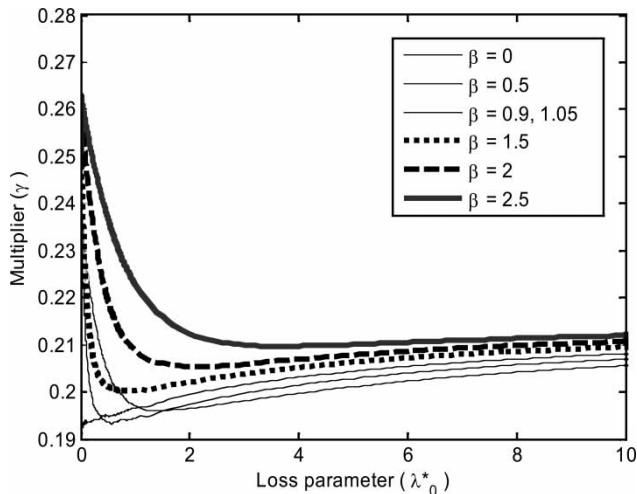


Fig. 7 Multiplier as a function of the loss parameter for varying bay geometries when drag is assumed to be quadratic with the flowrate

approximately 15 per cent, as $0.22 \rho g a Q_0$ for any bay geometry without a need to understand the basic dynamical balance.

3.3 Determining bay geometry term and loss parameter

The bay geometry term was defined in equation (20) to be equal to $g(cA\omega^2)^{-1}$. Since g and ω are known, it is possible to measure both the surface area of the bay, A , and the channel geometry term, c , to calculate the value of the geometry parameter, β . This method, however, is time-consuming and requires significant knowledge of the channel bathymetry to calculate the channel geometry term. A less time-consuming method is developed and presented below.

Both β and λ_0^* for a bay are determined from the observable amplitude ratio and phase lag of the tidal regime within the bay in the undisturbed state. The amplitude ratio is defined as $(\zeta_{\text{Bay}})_{\text{max}}/a$ and represents the maximum non-dimensional water surface elevation in the undisturbed state. The phase lag was defined in equation (33) as the lag of the maximum water surface elevation within the bay behind the maximum water surface elevation in the open ocean, just outside the channel.

A contour plot of the amplitude ratio and phase lag is plotted in Fig. 8. The bay geometry term and loss parameter may now be determined using Fig. 8 if the magnitude and phase of the dominant tidal constituent at each end of the channel are known. For example, a bay with an amplitude ratio of 0.6 and a phase lag of 80° may be modelled based on $\beta = 2$ and $\lambda_0^* = 12.5$. It is also apparent from Fig. 8 that the model begins to break down as the loss parameter

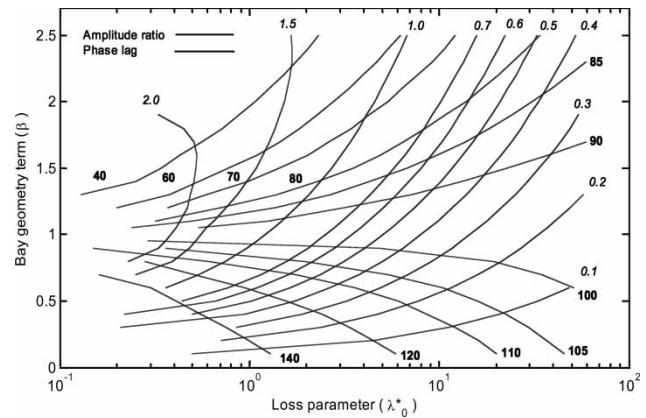


Fig. 8 Solid line represents various values of the amplitude ratio (in italics). Dashed line represents various values of phase lag in degrees (in bold). Contours are a function of the loss parameter and the bay geometry term

approaches zero and the bay approaches resonance at $\beta = 1$.

3.4 Bottom drag coefficient

The bottom drag coefficient, C_d , may be calculated once the loss parameter is determined. Since the loss parameter may be expressed as

$$\lambda_0^* = \frac{ag}{(c\omega)^2} \left[\int_0^L C_d(hE^2)^{-1} dx + (2E_e^2)^{-1} \right] \quad (40)$$

the drag coefficient is

$$C_d = \left[\lambda_0^* \frac{(c\omega)^2}{ag} - (2E_e^2)^{-1} \right] \left(\int_0^L \frac{1}{hE^2} dx \right)^{-1} \quad (41)$$

and is insensitive to the exit cross-section if $E_e^2 \gg ag/2\lambda_0^*(c\omega)^2$.

4 DISCUSSION

The model assumes that the tidal regime just outside the channel exit in the open ocean may be approximated by a single sinusoid. It is apparent that the majority of tidal regimes are composed of many tidal constituents, resulting in a difference in magnitudes between successive high and low tides.

Garrett and Cummins [6] concluded that multiple constituents can be included in the analysis when $\beta = 0$, where $\zeta_0 = a \cos \omega t + a_1 \cos \omega_1 t + a_2 \cos \omega_2 t + \dots$, but the results depends on the basic dynamical balance. Denoting $r_1 = a_1/a$, $r_2 = a_2/a \dots$, the extractable power calculated when $\zeta_0 = a \cos \omega t$ is multiplied by a factor of $1 + (9/16)(r_1^2 + r_2^2 + \dots)$ if the basic state is frictional, and $1 + (r_1^2 + r_2^2 + \dots)$ if the

basic state is frictionless [6]. A similar analysis would be beneficial for $\beta > 0$.

The model presented in Garrett and Cummins [6] has been shown to predict, within approximately 2 per cent, the results predicted by a two-dimensional numerical model [12]. A two-dimensional model for a channel linking a bay to the open ocean is required to provide further validation of the current model for $\beta > 0$.

5 CONCLUSIONS

A one-dimensional mathematical model, describing the average extractable power in a channel linking a bay to the open ocean, has been developed. The dynamic balance includes flow acceleration, exit separation effects, and frictional losses associated with turbine drag and bottom drag. A numerical solver was developed to determine the flowrate through the channel and the water surface elevation within the bay for various bay geometries.

The maximum average extractable power from a channel linking a bay to the open ocean may be estimated, within approximately 15 per cent, as $0.22\rho gaQ_0$, for any bay geometry, where a is the magnitude of the dominant tidal constituent in the open ocean just outside the channel and Q_0 is the maximum volumetric flowrate in the undisturbed state.

A contour plot has been developed to determine the bay geometry term and loss parameter based solely on the observed water surface elevation at each end of the channel and the phase lag of the maximum water surface elevation within the bay behind the open ocean. These two parameters are required to calculate the extractable power from the tidal stream. This method is valuable since it eliminates the time and expense involved in obtaining the geometric values required to calculate the bay geometry term. The contour plot is only applicable to a channel linking a bay to the open ocean. A detailed analysis for a bay linking two large basins is presented in reference [6].

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APPENDIX

Notation

a	amplitude of the dominant tidal constituent outside the channel in the open ocean (m)
A	surface area of the bay (m^2)
c	channel geometry term ($1/m$)
C_d	bottom drag coefficient
E	cross-sectional area of the channel linking a bay to the open ocean (m^2)
E_e	exit cross-sectional area of the channel linking a bay to the open ocean (m^2)
F	resistance force (m/s^2)
F_{turb}	turbine drag force (m/s^2)
g	acceleration due to gravity (m/s^2)
h	depth of the water (m)
L	length of the entrance channel (m)
n_1	dependence of turbine drag on the flowrate
n_2	dependence of bottom drag on the flowrate

p	pressure (N/m^2)	ζ_{Bay}	water surface elevation of the bay (m)
P_{total}	total power extracted from the channel (W)	ζ_0	water surface elevation just outside the bay in the open ocean (m)
P_{avg}	average extractable power for electricity generation (W)	λ_0	loss parameter ($1/\text{m}^4$)
Q	volume flowrate through the channel (m^3/s)	λ_1	turbine drag parameter
Q_0	maximum flowrate in the undisturbed state (m^3/s)	λ_2	bottom drag parameter
t	time (s)	ρ	density of the sea water (kg/m^3)
u	flow velocity (m/s)	ω	frequency of the dominant tidal constituent outside the channel in the open ocean (1/s)
u_e	exit flow velocity (m/s)	<i>Subscript</i>	
x	Cartesian coordinate	max	maximum
β	bay geometry term	<i>Superscript</i>	
ζ	water surface elevation (m)	*	non-dimensional value