Monetary Growth Models and the Neutrality of Money in the Long Run *

by

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I. Introduction

Some time ago there was an exchange involving Harry Johnson and James Tobin [1, 2] on the question of the neutrality of money. At the time it appeared that Tobin had carried the day with a demonstration that money was non-neutral in the sense that changes in the rate of growth of nominal money balances would entail changes in the capital-labour ratio in equilibrium. We wish in this note to argue that (uncharacteristic as it may seem) Harry Johnson capitulated too soon. To substantiate this argument, we sketch here the outlines of a simple one sector neo-classical model with outside money balances. This model is developed in more detail in our recent book [1], but without the interpretation we emphasize here.**

One conclusion is the existence of stabilization policies which seem paradoxical at first glance. Upon examination, however, they are revealed to be perfectly sensible for the model under discussion. What is paradoxical - as in all neo-classical monetary growth models to date - is

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** The relevant references are listed at the end of Chapter 6 in [1]. Of particular interest in Sidrauski's paper [1].
that the price of physical capital goods is set on markets involving the trading of assets for speculative purposes. The answer to the paradox is clearly that we require models in which paper assets are the instruments for speculation, and real asset values are linked only rather tenuously to the paper asset values through the mysterious processes of capital market valuation. We shall return to comment on this issue somewhat later.

In reviewing the evolution of the simple growth models, one observes a few major strands. Taking the neo-classical one-sector growth model without money as a starting point, we trace the development of a smooth, completely trouble-free, accumulating economy. We suppose that all saving is absorbed, and the process of accumulating capital goods in line with saving desires involves the deepening of capital in production to the point that in equilibrium, all saving is absorbed simply by the requirement that capital per man be maintained constant in the face of the growth of the labour force and physical depreciation. The well-known Solow model provides the economical statement of this process. Within its limits, the model is impeccable and its conclusions not subject to challenge. A major difficulty, however, has been the question whether investors would be prepared to absorb all the saving volunteered by the economy. If not, then the saving flow desired by the economy is not matched by an equal flow of resources directed to the accumulation of physical capital goods. One might view Harrod's development of the Keynesian problems of over-saving in a dynamic context as the principal statement of the argument that the presence of a floor to acceptable rates
of profit will prevent the capital deepening required in the Solow model to attain equilibrium. Under these circumstances, the problems of stagnation recur, and one is faced with a flow of desired savings exceeding the flow of resources directed to capital accumulation. That difficulty is, of course, the original Keynesian problem.

If we are not to accept the Solow path - that is, the process of continuing capital deepening to the point where the entire saving flow is absorbed simply in maintaining capital-labour ratios - as a route out of this difficulty, then we might turn to the development suggested by Tobin. He appeals to a second principle of Keynesian economics, that of a desire for liquid assets for speculative purposes. If there are other assets in the system to which the flow of saving may be directed, then it will not be necessary that the rate of accumulation of capital goods acceptable to managers of firms match the flow of saving desired by the community as a whole; the excess may be absorbed in accumulation of the alternative assets. Thus a second way out of the Keynesian difficulties of over-saving may be seen: by offering to the community a paper asset which provides an alternative form in which to hold wealth, a central government may in fact avoid the apparent difficulty of an excess of desired saving over attainable levels of investment expenditure. Of course it is necessary that real saving be reduced to a rate equal to an acceptable level of real investment; it is in this way that the burden of the debt emerges. But if savers in the economy are prepared to accumulate wealth in the form of paper assets, then equilibrium may be maintained at full employment indefinitely, with realization of all saving desires, even in the presence of a floor to the rate of profit (and therefore a ceiling
on the acceptable levels of investment in the accumulation of physical capital goods). It is such a model that we wish to analyze below.

II. The Model

II.a The Technology

We adopt for this model the standard one-sector technology with a single physical capital good. In this model a single output good which serves as the consumption good and the capital good is produced by use of the services of the stock of capital goods and the labour force. The production function is assumed to be homogeneous of degree one and to satisfy the following commonly-invoked regularity conditions on the per capital consumption function \( f(k) \), which gives the flow of output per capita as a function \( f(k) \) of the capital labour ratio \( k \):

\[
\begin{align*}
    f(k) &> 0, \ 0 < k < \infty; \quad f(0) = 0; \quad f(\infty) = \infty; \\
    f'(k) &> 0, \ 0 < k < \infty; \quad f'(0) = \infty; \\
    f'(\infty) &= 0; \quad f''(k) < 0, \ 0 < k < \infty.
\end{align*}
\]

The latter conditions are not indispensable, but they do avoid difficulties which might arise at boundaries and are therefore convenient for our present expositional purposes.

II.b Definitions

1. **Real wealth.** Letting \( K \) denote the level of physical capital stocks, \( M \) the level of nominal money balances, \( W \) the level of real wealth and \( p \) the price of the output (and capital) good in terms of money, we
define real wealth \( W \equiv K + \frac{M}{p} \).

2. **Real Net Disposable Income.** Since we wish real income and real consumption to satisfy the relationship \( Y = W + C \), we must define real net disposable income by the relationship

\[
Y \equiv F(K,L) - \delta K + \left( \frac{M}{p} \right) = C + \left( \frac{M}{p} \right)
\]  

(1)

3. **Rate of Return.** In line with the usual arguments as to the yield on assets over any particular holding period, we shall define the expected money yield on the capital good as \( r \), identical to the rate of real rental on the capital good plus the expected rate of capital gain. Thus we have

\[
r \equiv f'(k) - \delta + E(\frac{p}{p}.
\]  

(2)

II.c **Goods Market Equilibrium at Full Employment**

The first of the relationships which make up the model is that concerning the equilibrium on goods and factor markets, namely, the condition that desired and realized saving be equal to desired and realized investment, with factor markets clearing at full employment. In this model real wealth has been defined as \( W = K + \frac{M}{p} \), so that in equilibrium realized investment must equal the observed value for \( W \), that is, the observed value for \( K + \frac{d(M/p)}{dt} \). Since we assume that there are no active investment decisions to be taken into account, we may take realized investment to be equal to desired investment,
and also therefore to be equal to realized saving. Thus our first behavioral hypothesis is that determining desired saving. We take the simple hypothesis of a constant saving rate and assume that desired real saving is a constant fraction of real net disposable income. Hence we have \( w = sY \). Since \( Y \) is real net disposable income at full employment, the assumption of factor market clearing is built in, and the goods market equilibrium condition is that desired saving, \( sY \), should equal realized investment, \( K + d(M/p)dt \). This is the first of the two fundamental equilibrium conditions of this model.

II.d Money Market Equilibrium

The supply of nominal balances per capita is a given number at each instant. We shall assume the existence of a demand for money function in per capita terms which summarizes the results of portfolio decisions of asset holders in such a way as to show the desired nominal balances per capita for any combination of per capita money wealth, per capita value of output flow, and expected yield on the holding of the capital good. We summarize this money demand function as

\[
M_d = G(Y,W,r) .
\] (3)

Absence of money illusion would suggest that this function should be homogeneous of degree one in its first two arguments. Consequently we may write the demand for money function as the function

\[
M/p = G(Y/p,W/p, r)
\] (4)
or defining \( x \) as real cash balances per capita, we may write

\[
x = G[f(k), k + x, r].
\]  

(5)

Since the price of goods is determined at each instant, the actual level of real cash balances per capita is likewise determined at each instant. We shall impose the equilibrium condition that the desired level of real cash balances should always be the actual level of real cash balances per capita. This equilibrium must be brought about by alterations in the rate of change of prices which bring the yield on the capital good to whatever level is required to persuade people to hold the portfolio currently in existence. Setting out the equilibrium condition that the desired level of cash balances should equal the actual, we may solve the relationship to express the required yield \( r \) as a function

\[
r = \phi(k, x).
\]  

(6)

This relationship shows the yield required to maintain equilibrium in asset markets with the various portfolios of cash and real capital per capita.

The model is therefore very straightforward. We begin with given endowments of capital, nominal balances, and labour force. We assume that the labour force grows exponentially at an exogenously given rate \( g \), and that the money supply in nominal terms grows exponentially at an exogeneous rate \( \theta \). Finally, with a given initial price level, an initial level of real cash balances is determined. Thus the initial portfolio of real cash balances per capita and real capital per capita is given.
For that portfolio, a prescribed yield is necessary if equilibrium on the money markets is to be attained. Such a yield consists of a real rental rate determined completely by the level of the capital-labour ratio and an expected rate of price inflation. Only this expected rate of price inflation is available as an equilibrium variable to bring the yield into line with the portfolios and existing portfolio desires. We assume that the rate of price inflation is determined so as to bring about this equilibrium. Given this rate of price inflation, the rate of change of real cash balances is determined and real net disposable income therefore is also determined. From this, desired saving is calculated according to our proportional saving hypothesis. Since, however, all gains in real cash balances represent accumulation of wealth, and since desired saving is known, the residual saving must take the form of accumulation of physical capital. Thus the growth of the capital stock is determined in such a way as to bring total realized saving up to the level of saving warranted by the observed levels of net disposable income including increases in real cash balances. The model thus advances itself continuously in this fashion, evolving through the whole set of equilibrium portfolio configurations. The question is "To what end does this evolution lead?"

III. Analysis of the Model

With the initial capital stock, nominal money supply, labour force, and initial price level all given, the growth of nominal balances per
capita is an exogenous function of time. Two equilibrium conditions thus determine the path of the capital labour ratio required if all saving is to be absorbed (so that equilibrium on goods and factor markets prevails at all times) and the path of the price level required if stocks of money and capital are to be willingly held (so that there is equilibrium on asset markets at all times). Instead of dealing with the price level itself, however, it is more convenient to deal with the stock of capital per capita and the stock of real cash balances per capita as the two variables to be determined by the system. The model thus gives two differential equations to determine these variables from prescribed initial conditions. Substituting from the relations above, we obtain the following two differential equations as the final form of the system to be analyzed

\[ \dot{k} = \]  
\[ \dot{x} = \]  

The model thus gives two differential equations to determine these variables from prescribed initial conditions. The resulting trajectories then represent paths of equilibrium portfolios through which the economy must move.

Without going into details, we observe that under fairly general conditions on the demand for money function, it is possible to show that the system of two differential equations leads to trajectories as shown in the attached diagram. In this diagram, there is a saddlepoint equilibrium.
at the portfolio \((k^*, x^*)\), and there is a further equilibrium at the portfolio \((k^{**}, 0)\). We observe further that if the system begins with no money, it ends with no money and follows a trajectory along the \(x\) axis where the capital-labour ratio evolves exactly as dictated by the original Solow model to an equilibrium \(k^{**}\), where all saving is absorbed by the requirement to maintain the capital-labour ratio constant in the face of growing labour force and certain physical depreciation. On the other hand, if there are positive money balances in the system initially, various trajectories may be generated, depending on the nature of the initial configuration.

Though not exactly in this form, Tobin's original analysis [ ], led to the identification of the equilibrium portfolio \((k^*, x^*)\), at which the evolution of the system finds a sustainable equilibrium with constant real cash balances per capita and a constant capital-labour ratio. Tobin demonstrated that the location of this equilibrium portfolio \((k^*, x^*)\) depended crucially upon the exogenous rate of growth of the money stock, \(\theta\). In particular, the equilibrium capital-labour ratio, \(k^*\) depends upon the value assumed for \(\theta\), and thus it is possible to assert that money cannot be neutral in the long run: a change in the rate of growth of nominal money stocks alters the real equilibrium capital-labour ratio, and therefore real output per capita, and therefore also the real rate of interest in equilibrium. It was on this basis that Tobin asserted that Harry Johnson's claims [ ] for the neutrality of money were false. In his reply, Harry Johnson conceded on this point [ ].
However, as has been pointed out above, and earlier by Nagatani [ ], the equilibrium portfolio \((k^*, x^*)\), is in fact a saddlepoint equilibrium. Thus, although Tobin's comparative static results showing that \(k^*\) depends directly on the value of \(\theta\) are correct, their significance is called into question. For "almost any" initial condition, the system will diverge from the saddlepoint equilibrium \((k^*, x^*)\), and therefore in the long run, the value of \(k^*\) is "almost always" irrelevant. We must seek further elaboration of the trajectories of the model in order to determine the effects of changing rates of growth of money balances. Examination of Figure 1 suggests in fact that there is a stable equilibrium within the system, namely, at the portfolio \((k^{**}, 0)\) cited earlier as the well-known no-money "Solow" equilibrium. From the diagram it is seen that, with a given initial capital-labour ratio for all initial stocks of real cash balances lying below the stable arm marked aa, the system does not approach the saddlepoint equilibrium \((k^*, x^*)\), but rather it converges to this stable equilibrium \((k^{**}, 0)\). Thus if the initial money balances are sufficiently low, or if initial price levels are sufficiently high, the system converges ultimately to a no-money equilibrium and behaves like the one-sector Solow model. Changes in the rate of growth of the money supply which do not push the system from below the stable arm on to or above it, therefore, have no effect on the economy in the long run.

What about the trajectories beginning above the stable arm aa? Of these trajectories, little can be said in the context of this model, since all such trajectories ultimately violate a constraint that real consumption cannot exceed real output. On these trajectories, the system is led to
violate this constraint by virtue of the fact that capital gains are occurring at such a rate that real net disposable income is so high that desired saving can be accomplished even with consumption levels exceeding the flow of output goods in the economy. Along such trajectories, therefore, capital stocks would decumulate and real cash balances rise until the boundary at which consumption equals the flow of physical output is reached. Beyond such a boundary, this model can have no economic significance. Proper analysis of the model taking into account the constraint that consumption must be constrained to be not greater than the output flow must await elaboration of the process of goods market clearing.

III. Conclusions

The essence of this model (and the issue fundamental in the exchange between Johnson and Tobin) is that individuals count changes in real cash balances as part of their net real disposable income and wish to save some portion and consume some portion. In fact, however, all of the gains in real cash balances accrue as increments to wealth. To the extent that individuals wish to consume some part of their gains in real cash balances, they are forced to reduce their saving in the form of physical capital goods. (That is, being misled by the presence of paper gains into a feeling of high incomes, they take out a greater portion of the flow of real goods in consumption.) Thus, in the model with money and rising real cash balances, the level of capital per worker that yields equilibrium is reduced. This insight is fundamental to an understanding of how these models work, and the clarification of these points has been a major contribution of the exchange between Tobin and Johnson. But analysis of the differential
equation system implicit in their exchange shows that in fact the economy circumvents the effects of the introduction of paper money by the paradoxical process of reaching for money stocks too intensely. That is, when money stocks are introduced in such proportion as initially to put the economy below the stable arm aa in Figure 1, one may interpret the economy as being in a position of relatively scarce real cash balances. Favoring real money balances relatively over goods, the economy can be persuaded to hold the existing capital stock only by the promise of capital gains. Realization of these promised capital gains, however, has the effect of reducing real cash balances still further and increasing their relative scarcity. The process continues, forcing prices to rise faster than nominal balances per capita with the result that in the end the level of real balances per capita falls to zero, and the economy approaches the no-money "Solow point". That Solow point being independent of initial money stock or its rate of growth, one can assert the long run neutrality of money: unless the initial stock of real balances places the economy exactly (in a knife-edge position) on the stable arm leading to Tobin's saddlepoint equilibrium, changes in the money stock or its rate of growth do not affect real equilibrium capital-labour ratio, real output per capita, or the real rate of return in balanced growth.

It is interesting to note that a process of endogenous growth of the money supply may stabilize this system. In some simple models a rule by which the growth of the money supply is related directly to the yield on the capital goods is sufficient to ensure that the equilibrium portfolio (k*, x*)
with positive money balances becomes a saddlepoint toward which the 
economy may converge from arbitrary initial conditions. This stabilizing 
policy, however, seems somewhat perverse at first glance, as Hahn [ ] has 
pointed out: when the yield on assets is high, the rate of price inflation 
is also high, and it is precisely then that this rule calls for the greatest 
rate of increase in the nominal balances. This means that we are arguing 
for a stabilization policy based on adding nominal balances more and more 
quickly the higher is the rate of price increase. However, the perversity 
of this policy is only an illusion; review of the principles on which this 
model operates will illustrate that there is no sense in which the price 
increases of this model may be related to standard questions of price inflation. 
In this model, price increases are generated in an attempt to raise the 
expected yield on capital goods and thereby persuade people to hold existing 
stocks of physical assets. Yields are highest when people are most reluctant 
to hold these assets; only by promise of the reward of significant capital 
gains will they be prepared to move from holding money stocks to holding 
physical capital goods. High yields in this model, in other words, are a 
signal of relative scarcity of cash. The stabilizing rule calls for there 
to be cash supplied when the signals point to it being scarce. The results 
therefore of this stabilizing policy are not perverse in the context of the 
model we are analyzing. What is perverse is that we have the price of goods 
determined strictly on the basis of the capital gains required to persuade 
people to hold physical assets in their portfolio, for speculative reasons 
alone.
It is evident that monetary growth models demand the introduction of alternative paper assets which will be the instrument held by households accumulating wealth and holding portfolios for speculative reasons. The accumulation of physical capital goods will then be determined by managers of firms who hold the physical capital goods without anticipating the trade of these capital goods in realizations of capital gains. Under such circumstances, with an independent investment function, there must be some further consideration of the assumption of full employment. One approach to a consistent model embodying these features is a model with an explicit government stabilization policy determining expenditures; then the financing of the required stabilization expenditures would determine an endogenous money supply. Analysis of such a model can be expected to advance the state of monetary growth models to the point where some discussion of price inflation in the usual sense may indeed be possible.