A Simulation Study of Walks in Large Social Graphs

by

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B.Sc., University of Dhaka, 1999
M.Sc., University of Dhaka, 2001

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ABSTRACT

Online Social Networks (OSNs) such as Facebook, Twitter, and YouTube are among the most popular sites on the Internet. Billions of users are connected through these sites, building strong and effective communities to share views and ideas, and make recommendations nowadays. Therefore, by choosing an appropriate user-base from billions of people is required to analyze the structure and key characteristics of the large social graphs to improve current systems and to design new applications. For this reason, node sampling technique plays an important role to study large-scale social networks. As a basic requirement, the sampled nodes and their links should possess similar statistical features of the original network, otherwise the conclusion drawn from the sampled network may not be appropriate for the entire population. Hence, good sampling strategies are key to many online social network applications.

For instance, before introducing a new product or adding new feature(s) of a product to the online social network community, that specific new product or the additional feature has to be exposed to only a small set of users, who are carefully chosen to represent the complete set of users. As such, different random walk-based sampling techniques have been introduced to produce samples of nodes that not only are internally well-connected but also capture the statistical features of the whole network. Traditionally, walk-based techniques do not have the restriction on the number of times that a node can be re-visited while sampling. This may lead to an inefficient sampling method, because the walk may be “stuck” at a small number of high-degree nodes without being able to reach out to the rest of the nodes. A random walk, even
after a large number of hops, may not be able to obtain a sampled network that captures the statistical features of the entire network.

In this thesis, we propose two walk-based sampling techniques to address the above problem, called K-Avoiding Random Walk (KARW) and Neighborhood-Avoiding Random Walk (NARW). With KARW, the number of times that a node can be re-visited is constrained within a given number $K$. With NARW, the random walk works in a “jump” fashion, since the walk starts outside of the $N$-hop neighborhood from the current node chosen randomly. By avoiding the current nodes neighboring area of level-$N$, NARW is expected to reach out the other nodes within the entire network quickly. We apply these techniques to construct multiple independent subgraphs from a social graph, consisting of 63K users with around a million connections between users collected from a Facebook dataset. By simulating our proposed strategies, we collect performance metrics and compare the results with the current state-of-the-art sampling techniques (Uniform Random Sampling, Random Walk, and Metropolis Hastings Random Walk). We also calculate some of the key statistical features (i.e., degree distribution, betweenness centrality, closeness centrality, modularity, and clustering coefficient) of the sampled graphs to get an idea about the network structures that essentially represent the original social graph.
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sincerely, Shahed Anwar
DEDICATION

Dedicated to my beloved wife, Nafisa Shahrin Bari and only son, Zawata Afnaan.
Chapter 1

Introduction

1.1 Social Networks

Online Social Networks (OSNs) have become immensely popular nowadays and the study of social graphs has attracted a large number of researchers all over the world. According to statistics [3] as of January 2015, it has been estimated that about 42% of active population around the world use the Internet and among the active internet users, more than 65% use OSNs actively. It has been projected that the user of OSNs would grow upto around 2.44 billion by the end of 2018 [4]. One of the biggest challenges of studying social graphs is its mammoth size and dynamic structure that changes over time. Due to the massive size and dynamic nature of the complex and scale-free graphs, it becomes almost impossible to analyze the structure of the entire social graph. Therefore, graph sampling techniques have been evolved over the years to obtain smaller subgraphs that can essentially inherit almost all the characteristics of the original graph.

OSNs have been popular all over the world noticeably in recent years because of the fast spreading of messages and information over OSNs. Social media such as Facebook, Twitter, YouTube, WikiLeaks, Wikipedia, QQ (Chinese Social Media), Tuenti (Spanish Social Media), and Naver (Korean Social Network) have been used extensively for social interactions, news, marketing products, rumors and political purposes. According to Facebook statistics [1], there are 936 million daily active users on average and in Twitter [2], it has been estimated that 302 million active users around the world generate 500 million “tweets” a day. To understand the information propagation over OSN [26] and the social interaction [44], and to characterize user-
behaviors [10], we need to investigate the topological features of social networks. For this purpose, sampling algorithms are designed to obtain a smaller social graph that exhibits same or similar topological features as the original graph.

1.1.1 Motivation

Billions of people around the world are using social networking sites and online social networking has become a part of their lives. With OSNs, it is the most effective way to interact with people of similar or different communities to share views and ideas, not only within communities but also across the boundary to reach out people around the world. In addition, the number of people that can be interacted with through social media has a huge benefit and significant impact for promoting business product(s) nowadays, because the cost involved using social media and social networking is considerably lower than that of the traditional marketing and advertising strategies. It has now become very much essential to select potential customers from online social networks to use new product(s) and expect unbiased feedback about features in order to decide whether to keep or enhance the quality before introducing to the entire community. Satisfying users among the consumer community can ultimately promote business by the referring the newly introduced product(s) to their friends and kins through online social networks. For this reason, companies around the world have adapted to use social media as their business promotion platform and follow this marketing strategy of introducing their product prototype to a small number of users before releasing it to the actual market. In this way, the companies can greatly reduce risk and save product features enhancement, modification cost before final release. Nevertheless, such practice is meaningful only when the selected small user base can effectively inherit all the statistical features to represent the large customer community. As such, good sampling method(s) is/are necessary for selecting a representative test-case user community.

Sampling technique plays a vital role for selecting potential users among millions in the social network. At times, selecting friends of certain users may end up getting similar feedback/criticism, which in turn has a good chance to become more biased as friends of users in social networks tend to have similar mentality and taste. Such biasness may be misleading when it comes to product development and marketing. For this reason, it is necessary to choose effective users who can potentially give unbiased feedback, help building a product with the highest standard over time.
From the sampling point of view, it may not be helpful if the sampled nodes are only concentrated in a small region even after so many times of sampling. In other words, an ideal and effective sampling technique would always sample users that are uniformly distributed over the network, resulting an ideal representation of the whole population.

1.2 Main Contributions

The main contributions of this thesis include two newly proposed sampling methods, called K-Avoiding Random Walk (KARW) and Neighborhood-Avoiding Random Walk (NARW). In our first method, KARW, we limit the number of re-visits to a node, at most $K - 1$ times and avoiding the node $K^{th}$ times during a walk. The intuition behind KARW is to avoid re-visiting nodes several times within a region and enforce the random walk to new regions. In the second method, we propose NARW, where $N$ stands for the level-$N$ of neighborhood nodes to be avoided during the walk. In this strategy, a list of neighborhood nodes up to level-$N$ is to be generated before the walk begins and avoids them intentionally to “push” sampling nodes from different regions in the graph. When $N$ is larger than 2, NARW actually works in a “jump” fashion, since the next sampled node is several hops away from the current node.

We compare our new methods with the three existing state-of-the-art algorithms, namely, Uniform Random Sampling (URS), Random Walk (RW), and Metropolis Hastings Random Walk (MHRW). After simulation, we compare results of KARW and NARW with the three most popular sampling techniques that are mentioned before, based on certain performance criteria, for instance, the actual length a walk is able to finish, number of total nodes sampled, number of nodes sampled uniquely while walking on the social graph. We also analyze statistical key features of all the subgraphs taken from the original graph, that include degree distribution, between-ness centrality, closeness centrality, modularity and clustering coefficient.

1.3 Thesis Outline

The rest of the thesis is organized as follows:

Chapter 2: This chapter introduces the existing approaches of sampling techniques
with their features and strategies.

**Chapter 3:** Two proposed algorithms, K-Avoiding Random Walk (KARW) and Neighborhood-Avoiding Random Walk (NARW) are described extensively in this chapter with their complexities.

**Chapter 4:** This chapter includes the simulation of the proposed two algorithms. Simulation environment, parameters and performance metrics have been defined in details.

**Chapter 5:** Simulation results of the two proposed algorithms based on performance metrics have been collected, analyzed and compared with the existing state-of-the-art algorithms. All of the sampled subgraphs with different variations of $K$ and $N$ in KARW and NARW respectively have been visualized based on key statistical features.

**Chapter 6:** The conclusion of this thesis has been drawn with final remarks, and future directions are discussed to wrap up the thesis.
Chapter 2

Background and Related Work

2.1 Background

There are some known techniques to measure and analyze interesting key features of large social graphs, such as the betweenness centrality, closeness centrality, degree distribution, number of shortest paths, modularity and clustering coefficient and so on. Due to the massive size and dynamicity of large graphs with billions of users, it is difficult and time consuming to measure these metrics overall. As a result, sampling large graph is essentially required to effectively and efficiently obtain data and analyze its features. In order to estimate these features with sampling, the questions that we need to answer include: (i) which sampling methods are appropriate? (ii) what is the proper size of samples? (iii) whether or not the sampling method can be easily performed in large graphs? As a background study, we are going to review several existing sampling methods in this chapter, including the basic idea and properties of each method.

2.2 Classification of Sampling Methods

Most of the real-world complex networks are dense graphs, each subgraph taken by sampling strategies represents a certain community and plays an important role in many application contexts. In order to collect information from online social networks, graph sampling is necessary and can be categorized into several different ways: (a) graph traversal techniques, (b) sampling by explorations, (c) random node selection sampling, and (d) random edge selection sampling.
2.3 Related Work on Graph Traversal-Based Sampling

In the graph traversal technique, each node in the connected component is visited exactly once until all nodes are visited. Graph crawling methods include Breadth-First Search (BFS), Snow-Ball Sampling (SBS), Respondent-Driven sampling, Forest Fire Sampling (FFS) and so on.

2.3.1 Breadth-First Search

Breadth-First Search (BFS) is a basic strategy that has been used comprehensively for sampling OSNs by many researches. Starting from an initial node, in each iteration, BFS samples the next neighbor of the initial node, which has not yet been visited. When all neighbors of the initial node are visited, BFS moves to the first neighbor of the initial node and repeats the same process from there. It has been shown that this method is biased towards nodes with high degrees [9][29][31]. In other words, after a limited number of steps, nodes with higher-degrees tend to be selected with a higher chance. This bias towards higher-degree of nodes has been confirmed in [24] with a measurement of Facebook [17] data, where their [24] BFS crawler found the average degree of nodes to be 324, while the real value was only 94 (i.e., about 3.5 times smaller).

2.3.2 Snowball Sampling

Snowball sampling was developed by Coleman (1958–1959) [15] and Goodman (1961) [19] to study the structure of social networks. It is a non-probabilistic sampling technique that existing subjects (e.g., persons or animals) recruit future subjects from their acquaintances. It is called snowball sampling because similar groups tend to grow larger as the sampling process moves ahead. The main advantage of snowball sampling is that it can help reach hard-to-reach population after several iterations. As such, snowball sampling has been used in many areas, including public health (e.g., drug users), public policy (e.g., illegal immigrants), and arts and culture (e.g., musicians) [22]. Clearly, due to the snowball nature, this method is also biased towards nodes with high degrees. For example, people who have many friends are more likely to be selected into the sample.
2.3.3 Respondent-Driven Sampling

Respondent-driven sampling is a variation of snowball sampling that was introduced by Coleman [15] in 1958, with the goal to avoid bias in snowball sampling. It works in the same way as snowball sampling, but unlike snowball sampling, respondent-driven sampling involves extra custom estimation procedure that identifies and corrects the homophily on attributes in the population [21]. In other words, with the extra custom estimation procedure, snowball sampling tends to select samples from friendship. This type of biasness has been corrected in respondent-driven sampling by avoiding homophily nature of samples.

2.3.4 Forest Fire Sampling

Strictly speaking, forest fire sampling (FFS) is a hybrid method and it combines snowball sampling and random walk sampling [5][6]. Starting from a randomly selected node, FFS “burns” a fraction of its outgoing links. The process is repeated to neighboring nodes just as the spread of forest fire. When no new node can be burned, FFS randomly selects from the graph a new seed node which has not been burned to start the burning process. In other words, the “forest fire” starts from another new region. This process continues until we get the desired sample size.

2.4 Related Work on Random Walk-Based Sampling

In random walk-based sampling, random walks allow each node in the graph to be visited randomly and potentially many times. Random walks have been used extensively for sampling the Web [25], P2P networks [38][18] and other large graphs [38]. The random walk-based methods include Uniform Random Sampling (URS), Metropolis Hastings Random Walk (MHRW), unweighted Random Walk (RW), Self-Avoiding Random Walk (SARW), and so on. Comparing with the graph crawling techniques, random walks have several advantages [34] where it can be used to capture the community structure as well as capturing the structure with overlapping communities that occur in real-world cases. Random walk-based methods are also very helpful in detecting malicious activities of fake users in social graphs [36].

Because of the advantages of using random walk-based method mentioned earlier,
we will mainly focus on random walk-based sampling techniques in this thesis, especially three of the most popular state-of-the-art strategies: Uniform Random Sampling (URS), Random Walk (RW) and Metropolis Hastings Random Walk (MHRW). Nevertheless, to provide a complete picture, we briefly introduce all other existing sampling methods later in this chapter.

2.4.1 Uniform Random Sampling (URS)

In this strategy, nodes of a graph are sampled uniformly [7]. With this sampling technique, subsets of equal size are selected with equal probability. Uniform sampling is considered to be the simplest and most useful sampling techniques. This sampling technique captures the intuitive concept of randomness and the sampled nodes are often considered representative to the whole population [17]. If no information about the entire population is available, uniform sampling is probably the only choice.

2.4.2 Random Walk (RW)

Random walk [7] starts at a random node and advances in each step to a neighbor of the current node at random. The fraction of times that the random walks visit node \( n \) after many hops is proportional to the degree of node \( n, d_n \). When the graph is unweighted, the next node the walk moves to, is chosen uniformly at random among the neighbors of the present node [7]. When the graph is weighted, it moves to a neighbor with the probability proportional to the weight of the corresponding edge.

2.4.3 Metropolis Hastings Random Walk (MHRW)

Metropolis Hastings Random Walk (MHRW) [7] avoids bias towards nodes of higher-degrees by modifying the transition probabilities. MHRW works in the two steps: proposal for the next move, and acceptance/rejection of the proposal. Assume that the current state of the walk is at node \( i \). In the proposal step, the algorithm chooses a node \( j \) uniformly at random from the neighbors of node \( i \) and proposes to move to \( j \). In the acceptance/rejection step [13], it generates a random number \( p \) uniformly distributed in \((0,1)\). If \( p \leq \min\{1, \frac{d_i}{d_j}\} \), where \( d_i \) and \( d_j \) represent the degree of node \( i \) and the degree of node \( j \), respectively, then the proposal is accepted, i.e., the next move of the walk is node \( j \). Otherwise, the walk stays at node \( i \). The
above proposal → acceptance/rejection process repeats again until a desired number of walks is reached.

MHRW suffers from their slow diffusion over the space, which can in turn lead to poor estimation accuracy. In particular, their fully random nature in selecting the next node, when making a transition, often causes the walk to go back to the previous node from where they just came. This behavior may produce many duplicate samples for a short to moderate time span. It is apparently desirable to avoid such backtracking transitions whenever possible, so as to steer them toward “unvisited” places (or to obtain new node samples), as long as such a modification does not affect the unbiased estimation.

2.4.4 Re-Weighted Random Walk (RWRW)

As discussed earlier that random walk is biased towards higher-degree nodes, this technique corrects that problem by assigning re-weighted values. It uses Hansen-Hurwitz estimator [20] to eliminate the problem of bias towards higher-degree nodes in random walks [16]. Hansen and Hurwitz (1943) introduced the notion of sampling unequal size clusters with Probabilities Proportionate to Size (PPS) to estimate $X$, the sum of the $x$-variates over a finite population of $M$ elements. Their procedure for sampling clusters with PPS and with replacement has been widely adopted in sample surveys.

2.4.5 Self-Avoiding Random Walk (SARW)

Self-avoiding random walk is defined as a walk along the path of a given graph or a network without any loops, i.e., the walk will avoid any node that has been visited already. This technique has been used extensively for modeling large-scale properties of long-flexible macromolecules in solution, study of polymers and to characterize complex crystal structures [23]. While being useful, SARW may end up with a dead-end path where the walk has nowhere to go.
2.5 Related Work on Random Node Selection Sampling

In this method, nodes in the graph have been sampled based on some criteria and they are classified in the following ways:

2.5.1 Random Node Sampling

A sampled graph with nodes is being created by selecting a set of nodes $N$, taken uniformly at random and then is induced by set of those $N$ nodes [30]. This algorithm does not retain the power-law degree distribution [37].

2.5.2 Random PageRank Node Sampling

In contrast to uniform sampling, authors in [30] explored a sampling strategy where they set the probability of a node being selected into the sample to be proportional to its PageRank weight. This sampling strategy is called Random PageRank Node (RPN) sampling.

2.5.3 Random Degree Node Sampling

In this sampling technique, nodes with higher-degree are being selected, therefore, Random Degree Node (RDN) sampling is non-uniform and has even more bias towards high-degree nodes [30]. Since too many nodes with high degree would be sampled, it may cause problems when matching the degree distribution and hence no exact information can be found that can represent the original graph completely.

2.6 Related Work on Random Edge Selection Sampling

In this strategy, edges in a graph rather than nodes have been chosen at random. It has been characterized by two different ways:
2.6.1 Random Edge Sampling

Similarly of selecting nodes at random, one can also select edges uniformly at random. This algorithm is referred to as Random Edge (RE) sampling. But RE suffers with drawbacks [30] as sampled graphs taken from the original graph would be connected sporadically and therefore will have large diameters, and will not fall under any community structure.

2.6.2 Random Node-Edge Sampling

A slight variation of random nodes is Random Node-Edge (RNE) sampling [30], where a node is picked uniformly randomly and choose an edge uniformly at random incident to the node. RNE sampling does not have a tendency to bias towards higher-degree nodes.

2.7 Summary

This chapter introduces several popular sampling techniques. The basic steps and the features of each sampling method are presented and explained. Motivated by the self-avoiding random walk, in the following chapters we will propose and evaluate two new sampling strategies, called K-Avoiding Random Walk (KARW) and Neighborhood-Avoiding Random Walk (NARW), to sample social graphs.
Chapter 3

New Sampling Methods

3.1 Motivation

Good sampling methods in large social networks should quickly obtain enough samples that capture the statistical features of the whole network. Sampling methods based on random walk have been shown to be effective but may need a large number of steps to collect enough samples. In some cases, random walks may end up with a dead end. Even in a large network, if the random walks move into a small region that has weak connection to other parts of the network, the random walks may be stuck within the region for a long time, without a chance to sample nodes in the rest of the network. In this case, the samples will lead to a biased view of the whole network. Motivated by the self-avoiding random walk (SARW), in this chapter we propose two new algorithms that help speed up the sampling process and reduce the chance of random walks being limited within small regions.

3.2 K-Avoiding Random Walk

In our first algorithm, we extend self-avoiding random walk to K-Avoiding Random Walk (KARW), where $K$ is a positive integer whose value will determine the maximum number of re-visiting nodes while sampling a graph. The constraint we pose on random walk is that a node should not be visited more than $K - 1$ times, avoiding a node $K$th times. Clearly, when $K = 1$, KARW is the same as the self-avoiding random walk. When $K \to \infty$, KARW is equivalent to the unweighted random walk. For the implementation detail, we show the pseudo-code of KARW here:
Algorithm 1 Node Sampling with K-Avoiding Random Walk

Require: \( G, \text{sample} \text{Size}, K \)

choose a seed number randomly
define list \( \text{ka\_rw\_sample} \) /* An array that allows an element to appear multiple times */
define list \( \text{currently\_Sampled\_Neighbors} \) /* A list that only records unique elements */

\( \text{starting\_Node} = \text{randomly\_choose\_a\_node\_from\_G} \)
\( \text{ka\_rw\_sample} = \text{starting\_Node} \)
\( \text{prev\_Node} = \text{starting\_Node} \)

while \( \text{len(ka\_rw\_sample)} < \text{sample\_Size} \) do

\( \text{currently\_Sampled\_Neighbors} = \text{neighbors}(\text{G.prev\_Node}) \) /* Record all the neighbors of the current node */
\( \text{temp\_Node} = \text{rand\_choice(\text{currently\_Sampled\_Neighbors})} \) /* \( \text{temp\_Node} \) is the next proposed move */

while \( \text{num\_occurrences(ka\_rw\_sampled, temp\_Node)} > K \) do

if \( \text{len(\text{currently\_Sampled\_Neighbors})} == 0 \) then

print 'Sorry; there has been a deadlock at \( \text{G.prev\_Node} \) because all of its neighbors have been visited \( k \) times'

return \( \text{ka\_rw\_sample} \)
exit;

else

remove \( \text{temp\_Node} \) from \( \text{currently\_Sampled\_Neighbors} \) /* The proposed move is invalid, update the valid neighbors */

\( \text{temp\_Node} = \text{rand\_choice(\text{currently\_Sampled\_Neighbors})} \) /* Propose another neighbor */

end if

end while

\( \text{ka\_rw\_sample}\_\text{append(temp\_Node)} \) /* The proposed move is valid and is added */

\( \text{prev\_Node} = \text{temp\_Node} \) /* Move to the proposed and valid node */

empty the list of \( \text{currently\_Sampled\_Neighbors} \)

end while

return \( \text{ka\_rw\_sample} \)
This algorithm takes the value of $K$ and sample size which essentially means the intended length of the walk (in hops) as input, returns the sampled nodes visited with the actual length of the walk (as actual hops). The algorithm terminates when it is not possible to move further or finished sampling all the neighbors from the current node. For instance, the walk may terminate prematurely if all the neighbors of the current node have been visited $K - 1$ times and there is essentially no node(s) to visit further.

Apart from the above algorithm, we also have developed three additional algorithms that are being used inside and are considered to be a part and parcel of the K-Avoiding Random Walk algorithm. These algorithms include:

- initialize_graph(filename, seed_Val) – This function is used to create a graph, $G$ from a given dataset. We need to generate this graph prior to applying our proposed algorithm.

- display_graph_info($G$) – This function is being used to determine the total number of nodes and the total number of edges. It also determines whether the graph is connected or not. If the graph is disconnected, the algorithm returns the number of connected components in the graph.

- num_occurrences(node_list, node) – This function will count the number of occurrence of nodes from a list and returns the exact numbers of presence of nodes in the list. This function is required to check and to make sure each node has been visited a maximum allowable $K - 1$ times.

### 3.3 Neighborhood-Avoiding Random Walk

Our second algorithm is called Neighborhood-Avoiding Random Walk (NARW). The walk starts from a random node in a graph. Depending upon the value of $N$ which determines the level of neighborhood that should be avoided in the walk, the next move is to a node that is outside the $N$-level neighborhood of the starting node. In other words, NARW moves in a jumping fashion. Note that, node $A$ is called the $N$-level neighbor of node $B$ if the shortest distance between nodes $A$ and $B$ is $N$ hops. The implementation detail of NARW is shown in the following pseudo-code:
Algorithm 2 Node Sampling with Neighborhood-Avoiding Random Walk

Require: $G, sample\_Size, starting\_Node, seed\_Val, N$

choose a seed number randomly

Define list na\_rw\_sample /* It is an array to hold all the sampled nodes */

neighbor\_List = determine\_neighbors($G, starting\_Node, N$) /* List all the neighbors from starting node to Level-$N$ */

skip\_Count = 0

max\_Nodes = $G$.number\_of\_nodes()

while len(na\_rw\_sample) < sample\_Size do

starting\_Node = rand.choice(neighbors($G, starting\_Node$)) /* Randomly choose a neighbor from starting node */

if starting\_Node not in neighbor\_List then

/* Check if the chosen node is in the neighborhood list */

na\_rw\_sample.append(starting\_Node) /* Add the node in the neighborhood list */

else

skip\_Count = skip\_Count + 1 /* Keeps track of how many nodes are skipped */

if skip\_Count > max\_Nodes then

print 'There are no nodes left to be sampled'

return na\_rw\_sample list

exit;

end if

end if

end while

return na\_rw\_sample list

The above algorithm takes the value of $N$ which represents the maximum level of neighborhood that needs to be avoided and sample size which determines the intended length of the walk (in hops) as input. Starting from the first node chosen randomly, the next move “jumps” to one of the current node’s $N$-level neighbors and samples nodes thereafter. The algorithm terminates when from the current node, there is no $N$-level neighbor remain unvisited.

For the graph initialization and the display of graph information, we have used two similar functions mentioned in the previous section: initialize\_graph( ) and dis-
play_graph_info(). Moreover, there is an additional function associated with NARW, which is briefly described as follows:

- determine_neighbors($G$, Node, $N$) – This algorithm is being used to determine neighbors of $N$-level from the starting node, chosen randomly.

### 3.4 Complexity of the Proposed Algorithms

Since at each step both algorithms only need to check the local information of the current node (direct neighbors for KARW or $N$-level neighbors for NARW), the complexities of both KARW and NARW are $O(n)$, where $n$ is the total number of samples.

### 3.5 Summary

In this chapter, we have proposed two random-walk based methods for sampling nodes in a large graph. The motivation of both algorithms is to avoid re-visiting a node multiple times or avoiding local regions and to “push” the walk quickly across the network. While behaviors of these sampling methods are straightforward, their performances are not being analyzed mathematically. To this end, we perform comprehensive simulation study to evaluate the performance of KARW and NARW by sampling a large-scale social network in the following chapter.
Chapter 4

Simulation and Performance Metrics

4.1 Introduction

In this chapter, we evaluate our proposed sampling algorithms named K-Avoiding Random Walk (KARW) and Neighborhood-Avoiding Random Walk (NARW). We compare these two algorithms with existing state-of-the-art algorithms: 1) Uniform Random Sampling (URS); 2) Random Walk (RW); and 3) Metropolis Hastings Random Walk (MHRW). We setup our simulation environment and implemented all the five algorithms using Python [35]. The algorithms were evaluated on a large publicly available real-life social network dataset (Mislove [41]) of Facebook user interactions. The simulation results from all algorithms are then statistically analyzed and visualized using Gephi [8]. The analysis results are shown in various tables while the visual comparisons are shown using graphs in the following chapter. The experimental setup, simulation environment, simulation parameters, the dataset, and performance metrics are explained in the following subsections:

4.1.1 Experiment Setup

The simulation was performed on a Linux (Ubuntu 14.04) machine with a quad-core Intel Xeon E5420 CPU and 8 GB of RAM. Different scenarios were created by changing the simulation parameters. The five algorithms were then compared for each scenario using the Facebook dataset. In many realistic scenarios, our proposed algorithms perform better than the other existing techniques.
4.1.2 Our Simulation Tool

We implemented a simulation tool in Python [35] to evaluate our algorithms. The simulation tool is independent and could be used for all evaluation scenarios by changing the performance parameters. The existing and the proposed algorithms are added as separate Python [35] modules to the tool. The advantage of implementing all the algorithms and simulation tool using single platform/programming language gives us the ease of its use.

4.1.3 The Dataset

We have studied and used a large publicly available social network dataset (Mislove et al [41]), collected from the most popular social network, Facebook. This dataset consists of user links and their wall posts from the New Orleans regional network with a total of 63,000 users and about 1 million user-to-user links. The dataset was collected for the duration of around 3 years. For further details on the dataset, and for its statistical features, we refer the interested readers to Mislove et al [41].

4.1.4 Simulation Parameters

The simulation parameters used in our simulation are described below:

- **Maximum number of re-visiting nodes** $K$: The parameter $K$ limits the re-visiting of a node. We test our algorithms for $K = 1$ to $K = 32$ incrementing it according to Geometric sequence. For $K = 1$, the algorithm will not allow any re-visit to a node i.e., a node can be visited only once at maximum. For $K = 32$, a node could be visited 32 times (31 re-visits) at maximum.

- **Level of neighborhood** $N$: The parameter $N$ represents how many levels of neighborhood nodes will be avoided during the walk when we apply NARW algorithm to sample original graph. This algorithm samples nodes by pushing outside of the $N^{th}$ level of the neighborhood, starting walk from a node, chosen randomly. The value of $N$ can be any positive number but for our simulation purpose we have used 1, 2, and 4. The algorithm terminates prematurely at $N = 4$ without completing the intended length of the walk when the number of neighborhood nodes becomes very high as the neighborhood size becomes 57K depicted in Table 5.2 in the next chapter, however there are 63K nodes in the
original graph and therefore there would be only 6K nodes left to be visited following this technique.

- **Sample size/initial length of the walk** $L_w$: The initial or intended length of the walk ($L_w$) is the size of the sample. We use $L_w = 5000$ to $1.28 \times 10^6$ incrementing each time by geometric series i.e., $5000, 10000, 20000, \ldots, 1.28 \times 10^6$. For smaller values of $K$ (and also larger values of $N$), the re-visits in the walks are restricted and both the algorithms terminate without reaching to the intended length of the walk, $L_w$.

### 4.1.5 Performance Metrics

The performance metrics to compare our proposed algorithms with the existing solutions are described as follows:

- **Length of the walk completed**: This metric explains the actual length of the walk completed for KARWs and NARWs for each entry of the intended length of the walk. For other current sampling algorithms, actual length and intended length were same, but in our two proposed algorithms, restriction in re-visiting nodes for small values of $K$ in KARWs and lack of nodes outside of the neighborhood in NARWs to visit for larger values of $N$ are not allowing walks to finish as they were intended. We are measuring the performance as how far (by hops) a walk can end up finishing by each algorithm as compared to what was targeted before starting the walk.

\[
\text{Length of the walk completed}\% = \left( \frac{p}{q} \right) \times 100 \quad (4.1)
\]

where $p = \text{Actual length completed by a walk}$ and $q = \text{Intended length of the walk}$

- **Number of nodes sampled over entire population**: The metric here explains the percentages of the number of nodes that are sampled from the entire population (63K nodes) during a walk with different lengths of intended walk using existing and proposed algorithms. This value has been taken as ratio with the entire population of the main graph and is being derived in the following
equation:

\[
\text{nodes\_sampled\_over\_entire\_population}(\%) = \left( \frac{y}{x} \right) \times 100 \quad (4.2)
\]

Where, \(x = \text{Total number of nodes in the main graph,}\)
\(y = \text{Total number of sampled nodes and }\)

- **Number of nodes sampled exactly once:** This performance metric is the ratio between the total number of nodes that are sampled exactly once and the total number of nodes sampled during a walk. For KARWs, the intention is to avoid re-visits by restricting values of \(K\). Due to the restriction in KARW, it may not be able to sample more nodes than expected, however, there is a likelihood of more nodes sampled uniquely than the existing state-of-the-art algorithms. For example, when \(K=1\) in KARW, there is no chance of re-visits, as a result, it will eventually sample every single nodes exactly once. For NARWs, this algorithm tends to push its locality and jumps to a different region, skipping level-\(N\) of neighborhood nodes and samples nodes thereafter. Due to less number of nodes at level-\(N\), there is a high probability of getting more uniquely sampled nodes when \(N=4\) in NARW than the other two variations of NARWs.

Based on the concept discussed here, we have developed an equation that is defined as follows:

\[
\text{nodes\_sampled\_exactly\_once}(\%) = \left( \frac{z}{y} \right) \times 100 \quad (4.3)
\]

\(y = \text{Total number of sampled nodes and }\)
\(z = \text{Total number of nodes sampled exactly once}\)

- **Degree Distribution:** The degree of a node in a network is the number of connections it has to other nodes and the degree distribution is the probability distribution of these degrees over the entire network.

Degree distribution plays a vital role in studying theoretical networks and real networks, such as the Internet and social networks. Networks like the Internet,
the world wide web, and some social networks are found to have degree distributions that approximately follow a Power-law [14]. Networks as such are called scale-free networks and have attracted specific attention for their structural and dynamic nature.

The degree distribution, $D(d)$, of a network is defined in [40] to be a fraction of nodes in the network with a degree $d$:

$$D(d) = \frac{n_d}{n}$$

(4.4)

Where, $n =$ Total number of nodes in a graph and $n_d =$ Number of nodes that have the degree, $d$

- **Betweenness Centrality:** Betweenness is considered as a measure of the centrality of a node in a network. It is calculated as the fraction of the shortest paths between pairs of nodes that pass through the node of interest. Betweenness is also a measure of the influence of a node in spreading information throughout the network. The measure is based on random walks, counting how often a node is traversed by a random walk between two other nodes [32]. Nodes with betweenness are used to connect different regions in the graph and provide seamless connection from end-to-end nodes even though they do not belong to a same region. The following is the standard measure of centrality:

$$C_B(x) = \sum_{a \neq x \neq b \in V} \frac{\sigma_{ab}(x)}{\sigma_{ab}}$$

(4.5)

In the above equation, $C_B(x)$ is the betweenness centrality of a vertex $x$, $\sigma_{ab}(x)$ denotes the number of shortest paths from node $a$ to node $b$ that some $x \in V$ lies on and $\sigma_{ab}$ is the total number of shortest paths between $a$ and $b$.

Betweenness centrality is an interesting metric, because it takes into account not only the local neighborhood for each node, but also the entire graph’s structure. Nodes with the highest betweenness centrality are like the junctions in network. The higher this measure is, the more likely it will cross those nodes
when traversing the network. Such nodes may not have the highest degree, but they are very influential, because they tie up different distinct clusters together.

- **Closeness Centrality**: Closeness can be regarded as a measure of how fast information spread from a node to all other nodes in a network [12]. This type of centrality has been defined as:

\[
C_C(n) = \frac{1}{\sum_{n \in V} d_G(n, b)}
\]

(4.6)

Where \(d_G(n, b)\) denotes the distance between vertices, which is the minimum length of any path connecting nodes \(n\) and \(b\) in a graph \(G\).

- **Modularity**: Modularity is a metric to define the structure of networks and graphs. It is also being used to measure the strength of groups, clusters, and communities within a network. Modularity also confirms the certain level of interactions among nodes in a network. Certain number of nodes that are closely connected form a strong social community which implies information propagation at a faster rate among members within the community. The algorithm for modularity is implemented in Gephi [8] based on [11], that checks step by step the densely connected nodes because nodes that are more densely connected to each other are considered to be belong to the same community. Communities within a network are sparsely connected to one another. Modularity plays a significant role to study many real world problems such as biological and social network phenomenon.

- **Clustering Coefficient**: Clustering coefficient is a measure of how nodes in social networks are tightly knitted to make a cluster. It has been categorized in two ways: the global and the local. Global clustering is a measure of the overall situation of clusters in a network. It is based on the ratio between number of close triplets and number of connected triplets of vertices in a network [43]. Triplets are three nodes that are connected by two or three undirected ties. Local clustering coefficient of a vertex in a graph measures how close the neighbors are to make a complete graph or clique [33].

As of the definition described above, the average of the local clustering coeffi-
coefficients of all the vertices \( n \), \( \bar{C} \) is:

\[
\bar{C} = \frac{1}{n} \sum_{i=1}^{n} C_i
\]

(4.7)

Where Clustering Coefficient,

\[
C_i = \frac{\text{no. of connections in the neighborhood of a node}}{\text{the number of connections if the neighborhood was fully connected}}
\]

(4.8)

and a fully connected group of \( n \) nodes has connections of

\[
\frac{n \cdot (n - 1)}{2}
\]

According to the definition above, the global clustering coefficient is:

\[
C_G = \frac{\text{no. of closed triplets}}{\text{number of connected triplets of vertices}}
\]

(4.9)

Triangles in clusters play an important role in complex network analysis. In particular, two prominent theories according to which triangles are generated in social networks are the homophily and the transitivity[42]. Based on homophily nature, people tend to choose friends that are similar to themselves, which is also known as “birds of a feather flock together” and due to transitivity nature, people who have common friends tend to become friends themselves [39].
Chapter 5

Simulation Results and Observations

5.1 Simulation Results

We ran our simulation for all possible and realistic scenarios of the simulation parameters. For KARW, the number of possible scenarios is $\text{sizeof}(K) \times \text{sizeof}(L_w)$ while for the NARW, it is $\text{sizeof}(N) \times \text{sizeof}(L_w)$. In our case, the Facebook dataset was exhausted for $\text{sizeof}(K) = 6$, $\text{sizeof}(N) = 3$, and $\text{sizeof}(L_w) = 9$. The performance values for all these scenarios are as follows:

5.1.1 Length of the walk completed

Our first result is based on the length of the walk completed by both algorithms; given a set of intended lengths of the walk ($L_w$) as input we need to observe the actual lengths of the walk completed by KARW and NARW. We ran our program five times and get the average actual lengths of the walk performed by three different variations of KARW and NARW algorithms. Initial input lengths are set from 5 Khops to a maximum of 1 Mhops, increasing the lengths each time by a factor of 2. All the outputs have been taken with different values of $K$ ($1 \leq K \leq 32$) and $N$ ($1 \leq N \leq 4$) in KARWs and NARWs respectively. We also measure the output for the existing state-of-the-art algorithms and as there is no restriction imposed of re-visiting the nodes while walking on the graph, all of them ended up completing walks as was intended. Lengths of the walk completed by our proposed algorithms are measured with different initial lengths ($L_w$) and are depicted in Table 5.1. We
have chosen to show the output of actual lengths for three different values of $K$ and $N$ in the following Table 5.1:

Table 5.1: Length of the walk completed by KARWs and NARWs

<table>
<thead>
<tr>
<th>Initial Length ($L_w$)</th>
<th>KARW ($K = 1$, $K = 8$, $K = 32$)</th>
<th>NARW ($N = 1$, $N = 2$, $N = 4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>34 5000 5000</td>
<td>5000 5000 5000</td>
</tr>
<tr>
<td>10000</td>
<td>36 10000 10000</td>
<td>10000 10000 335</td>
</tr>
<tr>
<td>20000</td>
<td>143 14057 20000</td>
<td>20000 20000 2557</td>
</tr>
<tr>
<td>40000</td>
<td>121 7013 12207</td>
<td>40000 40000 3162</td>
</tr>
<tr>
<td>80000</td>
<td>288 20784 55482</td>
<td>80000 80000 9792</td>
</tr>
<tr>
<td>160000</td>
<td>165 37832 87429</td>
<td>160000 160000 2430</td>
</tr>
<tr>
<td>320000</td>
<td>179 51639 89391</td>
<td>320000 305434 1657</td>
</tr>
<tr>
<td>640000</td>
<td>153 48392 85744</td>
<td>640000 453766 6297</td>
</tr>
<tr>
<td>1280000</td>
<td>220 509348 91369</td>
<td>819200 904194 12665</td>
</tr>
</tbody>
</table>

If we closely look at the previous table, we can observe that, for smaller values of $K$ in KARW, this algorithm is not able to complete the walk as intended. The reason behind is very obvious because when the value of $K$ is smaller, then the freedom of movement to re-visit nodes inside the graph is very much restricted. For instance, when $K = 1$, no nodes are allowed to be re-visited while walking on the graph. This constraint has resulted less hops completed by walks when we apply KARW algorithm to sample nodes from an original graph derived from the Facebook dataset. On the other hand, when the values of $K$ in KARW become higher, the more freedom this algorithm has to re-visit nodes in a graph, resulted finishing walks with more hops observed in Table 5.1 when the values of $K$ in KARW are, $K = 8$ and $K = 32$. More details of the actual walks with different values of $K$ and $N$ are depicted in Table A.1 with detail results for interested researchers.

In case of NARWs, there are no such restrictions of re-visiting nodes as opposed to KARWs. As a result, NARW performs better than KARW in finishing walks with more hops. However, there is a case where the actual lengths have been reduced significantly when the value of $N$ is increased to 4, observed in the Table 5.1. The reason behind finishing walks with less hops is, NARW algorithm pushes itself to a region outside of the neighborhood of level-4 and starts walking from there. If we
look at the Table 5.2, we can see that an average neighborhood size of each level of the original graph grows exponentially. As a result, the size of neighborhood of level-4 in NARW grows very quickly and has become 57K, leaving behind less nodes to be visited as we know the total number of nodes in the original graph is 63K. Actual lengths completed by the three variations of NARW are shown in Figure 5.1b. Moreover, details of the walks by NARWs have been summarized in Table A.2 for interested readers.

Table 5.2: Average neighborhood size of the graph in different levels of NARWs

<table>
<thead>
<tr>
<th>Level</th>
<th>Average Neighborhood Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level − 1 (N = 1)</td>
<td>29</td>
</tr>
<tr>
<td>Level − 2 (N = 2)</td>
<td>1000</td>
</tr>
<tr>
<td>Level − 4 (N = 4)</td>
<td>57043</td>
</tr>
</tbody>
</table>

Figure 5.1: Length completed with variations of KARWs and NARWs

Comparison of the actual lengths completed by three different values of $K$ and $N$ in KARW and NARW respectively have been shown by graphs in Figure 5.1. We can see that the performance of NARWs overall is much better than KARWs as NARW has more freedom of re-visiting nodes. Both algorithms suffer when $K = 1$ and $N = 4$ in KARW and NARW respectively, where KARW has no way to re-visit nodes in the first place and in the second place, there are not enough nodes outside of the neighborhood that NARW can complete as intended.
5.1.2 Number of nodes sampled over entire population

Our next simulation result is based on the number of nodes sampled by our proposed algorithms and then compare results with the existing state-of-the-art sampling techniques in different intended lengths. A comparison by percentages of nodes sampled over the entire population between the existing sampling techniques and our proposed algorithms with three different variations of each algorithm is depicted in the following Table 5.3. Interested researchers can consult more details of the results, summarized in Table A.3 and Table A.4.

Table 5.3: Nodes sampled over entire population(%)  

<table>
<thead>
<tr>
<th>$L_w$</th>
<th>URS</th>
<th>RW</th>
<th>MHRW</th>
<th>KARW ($K = 1$)</th>
<th>KARW ($K = 8$)</th>
<th>KARW ($K = 32$)</th>
<th>NARW ($N = 1$)</th>
<th>NARW ($N = 2$)</th>
<th>NARW ($N = 4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>8</td>
<td>5.73</td>
<td>3.1</td>
<td>0.054</td>
<td>6.7</td>
<td>8.26</td>
<td>7.9</td>
<td>7.65</td>
<td>6.87</td>
</tr>
<tr>
<td>10000</td>
<td>16</td>
<td>12</td>
<td>11.65</td>
<td>0.057</td>
<td>9.04</td>
<td>9.03</td>
<td>11.97</td>
<td>11.98</td>
<td>2.41</td>
</tr>
<tr>
<td>20000</td>
<td>27.43</td>
<td>20</td>
<td>11</td>
<td>0.7</td>
<td>2.1</td>
<td>16.37</td>
<td>20</td>
<td>19.89</td>
<td>2.85</td>
</tr>
<tr>
<td>40000</td>
<td>47.34</td>
<td>31.23</td>
<td>20</td>
<td>0.3</td>
<td>9.03</td>
<td>13.28</td>
<td>31.74</td>
<td>30.68</td>
<td>2.73</td>
</tr>
<tr>
<td>80000</td>
<td>72.65</td>
<td>44.17</td>
<td>34</td>
<td>0.3</td>
<td>5.28</td>
<td>21.46</td>
<td>43.54</td>
<td>43.57</td>
<td>8.77</td>
</tr>
<tr>
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<td>92</td>
<td>58.35</td>
<td>52</td>
<td>0.1</td>
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<td>58.3</td>
<td>56.39</td>
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<td>70.45</td>
<td>70</td>
<td>0.5</td>
<td>10.76</td>
<td>12.87</td>
<td>70.12</td>
<td>67.84</td>
<td>1.95</td>
</tr>
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<td>100</td>
<td>81.32</td>
<td>82</td>
<td>0.02</td>
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<td>5.3</td>
<td>80.43</td>
<td>72.59</td>
<td>4.78</td>
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<tr>
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<td>89.45</td>
<td>90</td>
<td>0.01</td>
<td>6.75</td>
<td>3.09</td>
<td>88.75</td>
<td>80.675</td>
<td>6.74</td>
</tr>
</tbody>
</table>

If we consult Table 5.3, we can see that in case of KARWs, there are less nodes sampled due to restriction of movements. As a result, the percentages of nodes sampled are the lowest when $K = 1$ in KARW. As the value of $K$ is increased, the number of sampled nodes is increased. The best result among these three variations can be obtained when $K = 8$ and is measured 22.58%. The rest of the values are lower than 22.58%, meaning KARW algorithm has less freedom of movements, resulted very small amount of nodes being sampled when values of $K$ are smaller. Among existing sampling techniques, URS performs best with sampling more nodes followed by RW and MHRW with a maximum of 90% nodes sampled for both cases. More details of the results of KARWs can be found in Table A.3. Comparison graphs between these three existing techniques with our proposed KARWs have been depicted in Figure 5.2.
In case of NARWs, simulation results based on the number of nodes being sampled from the entire population with different values of $N$ are illustrated in Table 5.3 and are much promising when values of $N$ are 1 and 2, and the maximum number of nodes sampled are 80.75% and 80.675% respectively of the entire population. However, lack of nodes outside of the neighborhood when $N = 4$, causes declining the rate of sampling nodes overall with a maximum of 8.77% of the nodes being sampled from the entire population. More details can be found in Table A.4 for interested readers.

Comparison graphs between NARWs with the existing sampling strategies have been depicted in Figure 5.3. It has been observed that the results of NARW with $N = 1$ and $N = 2$ are very close to RW and MHRW techniques whereas URS dominates all the way through over other algorithms especially when the initial lengths ($L_w$) are higher.
5.1.3 Number of nodes sampled exactly once

It is interesting to see how many nodes are sampled uniquely among the total number of sampled nodes during a walk. For example, think about a real situation where a company may want to introduce a new product online. Before it will be introduced to the entire online users, a user-base consisted of potential users has been chosen for their valuable feedback who essentially represent the entire online customers. If random sampling techniques choose similar set of people every time, it causes misleading feedback rather than constructive because of the flaw of sampling similar people time and time again. By introducing two proposed algorithms, we try to reduce the rate of choosing nodes multiple times as well as increasing the rate of uniquely sampled nodes over the other state-of-the-art techniques. The following Table 5.4 shows comparative results of existing strategies with our two proposed techniques with different values of $K$, $N$ and initial lengths ($L_w$). More details of
simulation results are illustrated in Table A.5 and Table A.6 for interested researchers.

Table 5.4: Nodes sampled exactly once (%)

<table>
<thead>
<tr>
<th>Lw</th>
<th>URS</th>
<th>RW</th>
<th>MHRW</th>
<th>KARW</th>
<th>NARW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>K = 1</td>
<td>K = 8</td>
</tr>
<tr>
<td>5000</td>
<td>96</td>
<td>85</td>
<td>57</td>
<td>100</td>
<td>85</td>
</tr>
<tr>
<td>10000</td>
<td>92</td>
<td>77</td>
<td>54</td>
<td>100</td>
<td>77</td>
</tr>
<tr>
<td>20000</td>
<td>85</td>
<td>66</td>
<td>52</td>
<td>100</td>
<td>72</td>
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<tr>
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<td>72</td>
<td>52</td>
<td>46</td>
<td>100</td>
<td>81</td>
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<tr>
<td>80000</td>
<td>50</td>
<td>40</td>
<td>36</td>
<td>100</td>
<td>93</td>
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<tr>
<td>160000</td>
<td>22</td>
<td>29</td>
<td>22</td>
<td>100</td>
<td>76</td>
</tr>
<tr>
<td>320000</td>
<td>3</td>
<td>21</td>
<td>9</td>
<td>100</td>
<td>77</td>
</tr>
<tr>
<td>640000</td>
<td>0</td>
<td>15</td>
<td>3</td>
<td>100</td>
<td>76</td>
</tr>
<tr>
<td>1280000</td>
<td>0</td>
<td>10</td>
<td>1</td>
<td>100</td>
<td>85</td>
</tr>
</tbody>
</table>

It has been seen from Table 5.4 that $K = 1$ in KARW performs best among all the other techniques with a 100% uniquely sampled nodes among total number of nodes being sampled. The other two variations of KARW have carried out sampling nodes uniquely within a range of 72% to 96%. The most compelling evidence of KARW is, even though increasing value of $K$ gives more freedom to re-visit nodes, the sampling tends to get evenly distributed. Among the state-of-the-art strategies, there are fewer nodes left to be sampled uniquely than the proposed KARW and NARW algorithms because when the lengths of the intended walk are increased, there is a likelihood of re-visiting nodes a number of times rather than visiting nodes exactly once. Comparison between the existing strategies and the proposed KARW technique is depicted in the following Figure 5.4 and found out that the KARW algorithm with variations have consistency of visiting more unique nodes than the existing state-of-the-art sampling algorithms.
For our second algorithm, NARW, if we consult Table 5.4, we observe that when $N = 1$ and $N = 2$ in NARW, the rate of uniquely sampled nodes are inversely proportional to the initial lengths of the walk ($L_w$) with a maximum rate of sampling 84% and 83% unique nodes respectively. When the size of the neighborhood grows to $N = 4$, the results are very consistent as compared to the other two variations of NARW, illustrated in Figure 5.5.
5.1.4 Degree Distribution

To obtain the degree distribution of KARWs and NARWs, we have used social network analysis tool called Gephi [8] and append all the subgraphs with three different variations of KARWs and NARWs and draw two separate graphs for each proposed algorithm. Afterwards, we calculate the degree distribution for each algorithm and illustrated in Figure 5.7 and Figure 5.8 and observe with the degree distribution of the main graph in Figure 5.6. It has been observed here that all the graphs obtained by different values of $K$ and $N$ in KARW and NARW respectively, have the property of scale-free networks which follow Power-law [14] distribution.

Figure 5.5: NARWs vs (URS, RW and MHRW) by nodes sampled exactly once (%)
Figure 5.6: Degree Distribution of the main graph (Log-Log plot).

(a) KARW with 5 Khops walk.  
(b) KARW with 80 Khops walk.  
(c) KARW with 1 Mhops walk.

Figure 5.7: Degree distribution of KARWs (Log-log plot).
Now we can show how many nodes are there for each range of degrees of nodes and calculate the scale down factor for both KARWs and NARWs with the original graph. Here is the summary of the observation that we find in Figure 5.9. It has been seen here that when the lengths of the walk ($L_w$) were over a million hops then the scaling factor of both KARW and NARW are higher than the other two versions of the lengths of the walk ($L_w$) which means the representation of the main graph is close when the lengths of the walk ($L_w$) are over a million hops.

Figure 5.8: Degree distribution of NARWs (Log-log plot).

Figure 5.9: Scale factor of two proposed algorithms with the original graph.
5.1.5 Betweenness Centrality

The betweenness centrality of the main graph is depicted in Figure 5.10 where we can see there are significant amount of higher betweenness nodes marked as deeper blue colors. In order to observe the betweenness centrality of the subgraphs consisting of sampled nodes, we have used a popular social network analysis tool, called Gephi [8] that accepts sampled data and graphically represents subgraphs with all the central nodes involved in the network, marked by blue-color shades. Higher values of $K$ mean more freedom of re-visiting nodes and we see betweenness nodes tend to be at the center of the sampled graphs. Higher betweenness nodes in the subgraphs have deeper shades of blue-color. Nodes with the highest betweenness have the deepest blue-color and shown in the following graphical representations. For the simplicity of representing graphs, we have taken three different variations of initial lengths (5 Khops, 80 Khops and 1 Mhops) for all possible combinations of $K$ and $N$ in KARW and NARW algorithms respectively, illustrated from Figure 5.11 – Figure 5.16. The other graphical representations for different initial lengths of the walk ($L_w$) have been depicted in the appendix section.

Figure 5.10: Betweenness centrality of the main graph.
Figure 5.11: Betweenness Centrality of sampled graphs for KARWs with 5 Khops walk

Figure 5.12: Betweenness Centrality of sampled graphs for NARWs with 5 Khops walk
Figure 5.13: Betweenness Centrality of sampled graphs for KARWs with 80 Khops walk

Figure 5.14: Betweenness Centrality of sampled graphs for NARWs with 80 Khops walk
Figure 5.15: Betweenness Centrality of sampled graphs for KARWs with 1 Mhops walk

Figure 5.16: Betweenness Centrality of sampled graphs for NARWs with 1 Mhops walk
5.1.6 Closeness Centrality

Closeness centrality measures how fast information spreads over a network from a specific node. Higher closeness nodes are determined and marked with deeper shades of blue-color whereas nodes with lighter shades mean closeness between nodes are lower. For the main graph, the number of nodes with least closeness centrality are seen in Figure 5.17. It has been observed that fewer nodes have higher closeness when the intended lengths of the walk are 5 Khops and 80 Khops. For KARW, when the value of $K$ is 1 which means re-visiting a node is prohibited, the number of closeness centrality nodes become least. As the value of $K$ in KARW is increased, the closeness between nodes increases which means nodes tend to get closer each other causing closeness between nodes gets higher as observed in the following graphical representations. For NARWs, even though the walk starts avoiding neighbors of a certain level, the closeness centrality of graphs tends to get higher when avoiding first and second level of neighborhood nodes.

Figure 5.17: Closeness centrality of the main graph.
Figure 5.18: Closeness Centrality of sampled graphs for KARWs with 5 Khops walk

Figure 5.19: Closeness Centrality of sampled graphs for NARWs with 5 Khops walk
Figure 5.20: Closeness Centrality of sampled graphs for KARWs with 80 Khops walk

(a) K=1  (b) K=2  (c) K=4  (d) K=8  (e) K=16  (f) K=32

(a) N=1  (b) N=2  (c) N=4

Figure 5.21: Closeness Centrality of sampled graphs for NARWs with 80 Khops walk
5.1.7 Modularity

It has been observed from the main graph in Figure 5.24 that a single community in the main graph has been dominated and the size of the other communities are
very small. For observing number of detected communities in sampled subgraphs, a specific algorithm [11] has been used in Gephi [8] for community detection and resolution; it also uses an algorithm mentioned in [27] to produce decomposition. We set resolution = 5 and choose a randomized option in this analysis tool, to produce a better decomposition and apply to all the sampled graphs and observe number of communities detected for both of our proposed algorithms depicted in Table 5.5.

Figure 5.24: Number of communities in the main graph.

Table 5.5: Average number of communities detected by KARWs and NARWs

<table>
<thead>
<tr>
<th>$L_w$ (in hops)</th>
<th>$KARWs$</th>
<th>$NARWs$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>10000</td>
<td>10</td>
<td>9</td>
</tr>
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<td>7</td>
<td>3</td>
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<tr>
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<tr>
<td>80000</td>
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<tr>
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<td>7</td>
</tr>
<tr>
<td>640000</td>
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<td>9</td>
</tr>
<tr>
<td>1280000</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>
Figure 5.25: Comparison of detected communities between KARWs and NARWs

It has been observed from Table 5.5 that for KARWs, the maximum number of communities detected using this algorithm is 12 when the initial length of the walk is 5 Khops. In general, the number of detected communities are fluctuating with a tendency of less number of communities being detected as the intended lengths of walk get higher, depicted in Figure 5.25a. The reason behind this is, when the intended lengths of the walk get higher, there are more nodes re-visited a number of times, means more connectivity among the nodes causing a reduction on the number of communities in the network. For NARW, the highest number of communities detected is 15 when the initial length of the walk is 80 Khops, illustrated in Table 5.5. Number of detected communities using NARW algorithm also fluctuates with the initial lengths being increased shown in Figure 5.25b. When the initial lengths of the walk reach upto a maximum of 1 Mhops, the number of communities detected using KARW and NARW are 5 and 6 respectively as shown in Figure 5.30 and Figure 5.31. In the following graphical representations, we can see that nodes from the same communities are identified with similar colors, therefore, different communities are identified in subgraphs with different colors. Bigger communities with increasing number of nodes are more visible than the rest. It has been estimated that 213 communities are being detected in the main graph.
Figure 5.26: Modularity of sampled graphs for KARWs with 5 Khops walk

Figure 5.27: Modularity of sampled graphs for NARWs with 5 Khops walk.
Figure 5.28: Modularity of sampled graphs for KARWs with 80 Khops walk.

Figure 5.29: Modularity of sampled graphs for NARWs with 80 Khops walk.
Figure 5.30: Modularity of sampled graphs for KARWs with 1 Mhops walk.

Figure 5.31: Modularity of sampled graphs for NARWs with 1 Mhops walk.
5.1.8 Clustering Coefficient

The mean value for the average clustering coefficient with the intended lengths of walk from 5 Khops to 1 Mhops are estimated to be 0.0156 for KARWs and 0.0312 for NARWs, calculated automatically by Gephi [8] using algorithm in [28]. Detail values of clustering coefficient are mentioned from Table A.7 to Table A.15. It is worthy to note here that the average clustering coefficient for $K = 1$ in KARW is zero since no cluster was formed due to restriction of re-visiting nodes. Another key feature is, average number of triangles encountered are estimated from Table A.7 to Table A.15 while calculating clustering coefficient and shown in Table 5.7. We are going to show the average number of triangles found using the two proposed algorithms and compare these to the total number of triangles in the main graph depicted in the following Figure 5.6:

Table 5.6: Average number of triangles of KARWs and NARWs with original graph.

<table>
<thead>
<tr>
<th>Original Graph</th>
<th>$L_w = 5Khops$</th>
<th>$L_w = 80Khops$</th>
<th>$L_w = 1280Khops$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KARWs</td>
<td>NARWs</td>
<td>KARWs</td>
</tr>
<tr>
<td>2228135</td>
<td>21</td>
<td>23</td>
<td>1595</td>
</tr>
</tbody>
</table>

Table 5.7: Average number of triangles in KARWs and NARWs.

<table>
<thead>
<tr>
<th>$L_w$ (in hops)</th>
<th>KARWs</th>
<th>NARWs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>21</td>
<td>23</td>
</tr>
<tr>
<td>10000</td>
<td>32</td>
<td>52</td>
</tr>
<tr>
<td>20000</td>
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<td>260</td>
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<tr>
<td>40000</td>
<td>436</td>
<td>937</td>
</tr>
<tr>
<td>80000</td>
<td>1595</td>
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</tr>
<tr>
<td>160000</td>
<td>4670</td>
<td>15674</td>
</tr>
<tr>
<td>320000</td>
<td>1604</td>
<td>59255</td>
</tr>
<tr>
<td>640000</td>
<td>7692</td>
<td>371380</td>
</tr>
<tr>
<td>1280000</td>
<td>10298</td>
<td>453822</td>
</tr>
</tbody>
</table>
In case of KARW, the number of triangles increase with the increase of initial lengths with an exception when initial length of the walk is 320 Khops. Further investigation is required for this specific case. On the other hand, in NARW algorithm, number of triangles are proportional to initial lengths of the walk depicted in Figure 5.32b. There were no triangles formed during $K = 1$ in KARW. It also shows from Table 5.7 that NARW has more triangles encountered than KARW because NARW has more freedom of movement between nodes while sampling graphs. Clustering coefficient of the main graph has been estimated to be 0.23 and 2,228,135 triangles have been encountered from the main graph.
Chapter 6

Conclusions and Future Directions

6.1 Final Remark

Online Social Networks (OSNs) play a very important role in the 21st century and it has now become significant in every aspect of our lives. It is a very powerful media where people get influenced by others very quickly and information, news, gossip of any kind spread at a very fast rate. Therefore, much of the attentions are required to OSNs for various interests such as business, politics, health and well-being, economics and so forth. Large organizations nowadays are choosing most influential people as their potential customers to promote their products using OSNs. New products are also being developed and introduced to customers with the help of OSNs platform. Therefore, effective and efficient sampling techniques are necessary in order to find potential customers online to promote and to improve business. Only the right way of gathering and overlaying data would help any organization to decide which potential customers would be the most valuable ones to approach. It is worth to remember that ubiquitous social networks like Facebook and Twitter are ten years and seven years old, respectively and we do not know what new technologies lie ahead. It will be a challenge for the business people to understand how strategies are evolving in existing social media and adapt newer techniques in the years to come.

With this research, we are able to find out how sampling can be done in most effective and efficient way on a large social graph by introducing the two newly proposed random walk-based techniques called K-Avoiding Random Walk (KARW) and Neighborhood-Avoiding Random Walk (NARW) in particular. Then we setup the parameters and apply these algorithms alongside with three existing state-of-the-art
algorithms to the Facebook dataset. Afterwards, we calculated simulation results of KARWs and NARWs and compared with the existing algorithms based on the performance metrics and then observed sampled subgraphs based on some of the key statistical features and tried to compare with the original graph. Based on the metrics, both of our algorithms outperformed existing state-of-the-art algorithms based on sampling unique nodes. Subgraphs constructed from sampling nodes from the original Facebook graph were analyzed with key statistical features such as degree distribution, centrality measurements, modularity and clustering coefficient and found out that the subgraphs inherited all the properties of the original graph as was expected at the beginning of our research. If we summarize our research work, we can say that KARW was able to minimize the number of re-visits by a controlling parameter, $K$, while walking on the graph and NARW was able to push the sampling technique to different region of the graph deliberately.

Besides being good sampling techniques, they both suffer in some of the areas that we observed throughout simulation and analysis phases. For KARWs, the intended lengths of the walk were not achieved even though the restrictions were more relaxed (i.e., when $K = 32$ in KARW). The walks also terminated prematurely when restrictions were at its best ($K = 1$ and $K = 2$) causing small number of nodes being sampled. With NARWs, the growth of the neighborhood size was exponential and the algorithm terminated prematurely, therefore, it was not possible to sample substantial amount of nodes when $N = 4$ in NARW.

### 6.2 Future Directions

We would also like to highlight some of the clues and ideas for researchers into their future endeavor. They are summarized as follows:

- A random walk is a stochastic process that starts at one node of a graph, and moves from the current node to an adjacent node, at each step, chosen randomly from the neighbors of the current node. Graph exploration problems such as hunting or tracking on a graph are particularly interesting where the environments are unknown and in that case multiple random walks can be used to traverse the graph and calculate the time to cover the graph, which is an important measure of the efficiency of random walks. This was not the intention of our research work but we can leave this strategy as the future endeavor.
• The main purpose of this research is to provide a sampling tool that can be easily adjusted with different parameter settings while selecting the value of $K$ and $N$ in KARW and NARW respectively depends on statistical features of the graph under investigation. The relationship between key statistical features of graphs and choosing the correct values of $K$ and $N$ is complicated and thus subject to the goal that we can to achieve with sampling. We leave this as an interesting future research.

• We can think about a hybrid approach in future where it may be possible to amalgamate the characteristics of both K-Avoiding and N-Avoiding random walks to develop a new technique for sampling nodes which would may perform better than many of the existing strategies.

• It would be more effective to see other random walk strategies besides what we have discussed in this research, that would help us to get some new ideas and directions near future.

• Another direction would be a random walk starting from the same source and observe the patterns of the walks rather than starting random walk choosing purely randomly.

• Lengths of the intended walk would also have to be extended over to few million hops and observe the nature of patterns of the walks.

• These proposed algorithms can also be applied to some other datasets (such as Twitter, YouTube, QQ and so forth) besides being used dataset only from Facebook and analyze, compare social behavior of these complex networks.
Appendix A

Additional Information

A.1 Length of the walk completed by KARWs and NARWs

Table A.1: Length of the walk completed by KARWs and NARWs

<table>
<thead>
<tr>
<th>$L_w$ (in hops)</th>
<th>KARW1</th>
<th>KARW2</th>
<th>KARW4</th>
<th>KARW8</th>
<th>KARW16</th>
<th>KARW32</th>
</tr>
</thead>
<tbody>
<tr>
<td>5K</td>
<td>34</td>
<td>619</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
</tr>
<tr>
<td>10K</td>
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<td>2126</td>
<td>7013</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
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<td>985</td>
<td>7027</td>
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<td>20000</td>
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<tr>
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<td>5187</td>
<td>20748</td>
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<td>55482</td>
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<td>1169</td>
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<td>37832</td>
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</tr>
</tbody>
</table>
A.2 Nodes sampled over entire population

Table A.3: KARW vs (URS, RW, MHRW) by nodes sampled nodes over entire population (%)

<table>
<thead>
<tr>
<th>$L_w$ (in hops)</th>
<th>URS</th>
<th>RW</th>
<th>MHRW</th>
<th>KARW1</th>
<th>KARW2</th>
<th>KARW4</th>
<th>KARW8</th>
<th>KARW16</th>
<th>KARW32</th>
</tr>
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<tr>
<td>5K</td>
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<td>.054</td>
<td>0.862</td>
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Table A.4: NARWs vs (URS, RW, MHRW) by sampled nodes over entire population (%)

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<th>NARW4</th>
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A.3 Nodes sampled exactly once

Table A.5: KARWS vs (URS, RW, MHRW) by nodes sampled exactly once (%)

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<th>MHRW</th>
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<th>KARW2</th>
<th>KARW4</th>
<th>KARW8</th>
<th>KARW16</th>
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Table A.6: NARWs vs (URS, RW, MHRW) by nodes sampled exactly once (%)

<table>
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<th>$L_w$ (in hops)</th>
<th>URS</th>
<th>RW</th>
<th>MHRW</th>
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<th>NARW2</th>
<th>NARW4</th>
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### A.4 Statistical data of sampled graphs for KARWs and NARWs

#### Table A.7: Statistical data of sampled graph when K=1 in KARW

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<th>APL</th>
<th>#Short.Path</th>
<th>Modularity</th>
<th>#Communities</th>
<th>ClustCoeff.</th>
<th>#Triangles</th>
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#### Table A.8: Statistical data of sampled graph when K=2 in KARW

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#### Table A.9: Statistical data of sampled graph when K=4 in KARW

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<th>Modularity</th>
<th>#Communities</th>
<th>ClustCoeff.</th>
<th>#Triangles</th>
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Table A.10: Statistical data of sampled graph when K=8 in KARW

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<th>#Communities</th>
<th>ClustCoeff.</th>
<th>#Triangles</th>
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Table A.11: Statistical data of sampled graph when K=16 in KARW

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<th>Modularity</th>
<th>#Communities</th>
<th>ClustCoeff.</th>
<th>#Triangles</th>
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Table A.12: Statistical data of sampled graph when K=32 in KARW

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<th>Modularity</th>
<th>#Communities</th>
<th>ClustCoeff.</th>
<th>#Triangles</th>
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### Table A.13: Statistical data of sampled graph when N=1 in NARW

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<th>ClustCoeff.</th>
<th>#Triangles</th>
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### Table A.14: Statistical data of sampled graph when N=2 in NARW

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<th>#Triangles</th>
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### Table A.15: Statistical data of sampled graph when N=4 in NARW

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A.5  Betweenness centrality of KARWs and NARWs with 10 Khops walk

Figure A.1: Betweenness Centrality of sampled graphs for KARWs with 10 Khops walk

Figure A.2: Betweenness Centrality of sampled graphs for NARWs with 10 Khops walk
A.6 Closeness centrality of KARWs and NARWs with 10 Khops walk

Figure A.3: Closeness Centrality of sampled graphs for KARWs with 10 Khops walk

Figure A.4: Closeness Centrality of sampled graphs for NARWs with 10 Khops walk
A.7 Modularity of KARWs and NARWs with 10Khops walk

Figure A.5: Modularity of sampled graphs for KARWs with 10 Khops walk

Figure A.6: Modularity of sampled graphs for NARWs with 10 Khops walk
A.8 Betweenness centrality for KARWs and NARWs with 20 Khops walk

Figure A.7: Betweenness Centrality of sampled graphs for KARWs with 20 Khops walk

Figure A.8: Betweenness Centrality of sampled graphs for NARWs with 20 Khops walk
A.9  Closeness centrality of KARWs and NARWs with 20 Khops walk

Figure A.9: Closeness Centrality of sampled graphs for KARWs with 20 Khops walk

Figure A.10: Closeness Centrality of sampled graphs for NARWs with 20 Khops walk
A.10 Modularity of KARWs and NARWs with 20 Khops walk

Figure A.11: Modularity of sampled graphs for KARWs with 20 Khops walk

Figure A.12: Modularity of sampled graphs for NARWs with 20 Khops walk
A.11 Betweenness centrality of KARWs and NARWs with 40 Khops walk

Figure A.13: Betweenness Centrality of sampled graphs for KARWs with 40 Khops walk

Figure A.14: Betweenness Centrality of sampled graphs for NARWs with 40 Khops walk
A.12 Closeness centrality of KARWs and NARWs with 40 Khops walk

Figure A.15: Closeness Centrality of sampled graphs for KARWs with 40 Khops walk

Figure A.16: Closeness Centrality of sampled graphs for NARWs with 40 Khops walk
A.13 Modularity of KARWs and NARWs with 40 Khops walk

Figure A.17: Modularity of sampled graphs for KARWs with 40 Khops walk

Figure A.18: Modularity of sampled graphs for NARWs with 40 Khops walk
A.14  Betweenness centrality of KARWs and NARWs with 160 Khops walk

Figure A.19: Betweenness Centrality of sampled graphs for KARWs with 160 Khops walk

Figure A.20: Betweenness Centrality of sampled graphs for NARWs with 160 Khops walk
A.15 Closeness centrality of KARWs and NARWs with 160 Khops walk

Figure A.21: Closeness Centrality of sampled graphs for KARWs with 160 Khops walk

Figure A.22: Closeness Centrality of sampled graphs for NARWs with 160 Khops walk
A.16 Modularity of KARWs and NARWs with 160 Khops walk

Figure A.23: Modularity of sampled graphs for KARWs with 160 Khops walk

Figure A.24: Modularity of sampled graphs for NARWs with 160 Khops walk
A.17 Betweenness centrality of KARWs and NARWs with 320 Khops walk

Figure A.25: Betweenness Centrality of sampled graphs for KARWs with 320 Khops walk

Figure A.26: Betweenness Centrality of sampled graphs for NARWs with 320 Khops walk
A.18 Closeness centrality of KARWs and NARWs with 320 Khops walk

Figure A.27: Closeness Centrality of sampled graphs for KARWs with 320 Khops walk

Figure A.28: Closeness Centrality of sampled graphs for NARWs with 320 Khops walk
A.19 Modularity of KARWs and NARWs with 320 Khops walk

Figure A.29: Modularity of sampled graphs for KARWs with 320 Khops walk

Figure A.30: Modularity of sampled graphs for NARWs with 320 Khops walk
A.20 Betweenness centrality of KARWs and NARWs with 640 Khops walk

Figure A.31: Betweenness Centrality of sampled graphs for KARWs with 640 Khops walk

Figure A.32: Betweenness Centrality of sampled graphs for NARWs with 640 Khops walk
A.21 Closeness centrality of KARWs and NARWs with 640 Khops walk

Figure A.33: Closeness Centrality of sampled graphs for KARWs with 640 Khops walk

Figure A.34: Closeness Centrality of sampled graphs for NARWs with 640 Khops walk
A.22 Modularity of KARWs and NARWs with 640 Khops walk

Figure A.35: Modularity of sampled graphs for KARWs with 640 Khops walk

Figure A.36: Modularity of sampled graphs for NARWs with 640 Khops walk
Bibliography


