MATERIALS FOR

"EDUCATION FINANCE AND OPTIMAL INVESTMENT 
WITh GESTATION LAGS"*

by

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These materials are preliminary, and circulated for discussion only. Please do not quote.

* The simulation work reported in this document was initiated at the Institute for Policy Analysis, University of Toronto, with the active support and encouragement of Prof. D.G. Hartle, now Deputy Secretary, Treasury Board Secretariat, Ottawa. Initial model design, programming, and subsequent development work has been carried out in collaboration with Morris A. Cohen.
1. The Macroeconomic Approach [3,4,5,7,8,11]

Model I

\[ \max J = \int_{0}^{t_f} U(c(t)) \exp(-\gamma t) dt \]

s.t. \( c(t) = (1-s(t)-e(t)) \frac{Q}{P(t)} = (1-s(t)-e(t)) \frac{w_f(k(t)/w(t))}{w_f} \)

\[ k = -\sigma k + sw_f(k/w) \]

\[ w = \frac{w_f}{d} - aw \]

\[ 0 \leq s(t) \leq 1 \]

\[ 0 \leq e(t) \leq 1 \]

\[ 0 \leq c(t) = (1-s-e) f(k) \]

\[ 0 \leq w \leq 1, \ 0 \leq k \]
Model II (more detail on population dynamics and skill structure)

Define the productivity-weighted measure of the effective labour force by:

\[ W^*(t) = \sum_{t-m}^{t} w(t,\tau)A(t,\tau) \text{d}\tau \]

where \( w(t,\tau) \) represents the fraction of the labour force entering employment at time \( \tau \) surviving to time \( t \), and \( A(t,\tau) \) represents the productivity factor attaching to them.

<table>
<thead>
<tr>
<th>criterion</th>
<th>( J = \int_{0}^{t_f} U(c(t))\exp[-\gamma t] \text{d}t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>dynamics</td>
<td>( w^<em>(t) = -aw^</em>(t) + b_1 u(t)A(t) - b_1 b_2 u(t-m)A(t-m) )</td>
</tr>
<tr>
<td></td>
<td>( k(t) = -ck(t) + s(t)w^<em>(t)f(k(t)/w^</em>(t)) )</td>
</tr>
<tr>
<td>control variables</td>
<td>( A(t), s(t), u(t) )</td>
</tr>
<tr>
<td>state variables</td>
<td>( k(t), w^*(t) )</td>
</tr>
<tr>
<td>constraints</td>
<td>( 1 \geq s \geq 0, 1 \geq u \geq 0, c = [(1-s)w^f(k/w^*) - b_1 u d(A)] \geq 0 )</td>
</tr>
</tbody>
</table>

Source: Budelis [3]
Model III (expand labour force block to take account of gestation lags)

\[
\begin{align*}
J_1 &= \int_0^T \frac{c(t)}{\theta} \exp[-\gamma t] dt \\
\dot{w}(t) &= -aw(t) + \exp[-bm_3]u(t-m_3) \\
\dot{n}(t) &= -bn(t) - \exp[-bm_3]u(t-m_3) + u(t) \\
k(t) &= -\sigma k(t) + s(t)w(t)f(k/w) \\
\end{align*}
\]

criterion

dynamics

control variables

state variables

constraints

Source: Budelis [3]
Simulation Model (Schematic Diagram)

Micro Problem: Given a policy θ, together with cost and income data, select δ so as to maximize $V_1(\theta)$.

Macro Problem: Given population data (cohort sizes) and a statistical approximation to the individual decision rules, select the policy θ yielding the "optimal" configuration of cohort monetary flows, CSS, fund flows, F, and distributional results, H.
BIBLIOGRAPHY ON CONTROL-THEORETIC APPROACHES TO INVESTMENT IN EDUCATION


