A MACROSCOPIC TRAFFIC FLOW MODEL FOR ADVERSE WEATHER CONDITIONS

by

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

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University of Victoria

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ABSTRACT

Adverse weather has a direct effect on traffic congestion and the time delay on roads. Weather conditions today are changing rapidly and are more likely to have a severe effect on traffic in the future. Although different measures have been taken to mitigate these conditions, it is important to study the impact of these events on road conditions and traffic flow. For example, the surface of a road is affected by snow, compacted snow and ice. The objective of this thesis is to characterize the effect of road surface conditions on traffic flow. To date, traffic flow under adverse weather conditions has not been characterized. A macroscopic traffic flow model based on the transition velocity distribution is proposed which characterizes traffic behavior during traffic alignment under adverse weather conditions. The model proposed realistically characterizes the traffic flow based on snow, compacted snow, and ice. Results are presented which show that this model provides a more accurate characterization of traffic flow behavior than the well known Payne-Whitham model.
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DEDICATION

To my father (late) Syed Wilayat Muhammad and my mother Naseem Akhtar for having a lifelong dream to see me achieve my graduate qualification at a world class foreign institution. In difficult times, this proved to be a key motivating factor and enabled me to maintain focus.

To my elder brother, Dr. Syed Iftikhar Ahmad for being a good mentor and for guiding me towards the right path in life. I will always be thankful to him.

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# List of Acronyms

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<td>PW model</td>
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<tr>
<td>$\rho$</td>
<td>Average traffic density</td>
</tr>
<tr>
<td>$v$</td>
<td>Average velocity</td>
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<tr>
<td>$\rho v$</td>
<td>Traffic flow</td>
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<td>$\rho_m$</td>
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<tr>
<td>$v_m$</td>
<td>Maximum velocity</td>
</tr>
<tr>
<td>$v(\rho)$</td>
<td>Equilibrium velocity distribution</td>
</tr>
<tr>
<td>$G$</td>
<td>Vector of data variables</td>
</tr>
<tr>
<td>$f(G)$</td>
<td>Vector of the functions of the data variables</td>
</tr>
<tr>
<td>$A(G)$</td>
<td>Jacobian matrix</td>
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<td>$S(G)$</td>
<td>Vector of source terms</td>
</tr>
<tr>
<td>$\delta t$</td>
<td>Time step</td>
</tr>
<tr>
<td>$\delta x$</td>
<td>Segment length</td>
</tr>
<tr>
<td>$\Delta G$</td>
<td>Change in data variables</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>Change in the functions of data variables</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of road segments</td>
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\( N \)  
Number of time steps

\( \lambda_k \)  
Eigenvalue \( k \)

\( \Lambda \)  
Diagonal matrix of eigenvalues

\( C_0 \)  
Velocity constant

\( \tau \)  
Relaxation time

\( e \)  
Eigenvector

\( \rho_0 \)  
Initial density at time zero

\( v_t(\beta) \)  
Transition velocity distribution

\( \beta \)  
Severity index

\( \beta_m \)  
Maximum severity index

\( H \)  
Distance headway

\( \alpha \)  
Time headway

\( t_N \)  
Total simulation time

\( x_M \)  
Length of road
Weather has a significant effect on traffic flow. During normal weather conditions, the traffic alignment is smooth as vehicles cover distances quickly to align. In adverse weather conditions, the traffic alignment depends on the surface conditions of the road. In the case of snow, ice and compacted snow, there is a slick pavement. The friction between tires and the road is reduced, so vehicles require longer distances and longer times to align than in normal weather. The alignment in adverse weather has an effect on traffic flow. Vehicles move slowly due to reduced friction and thus are more likely to form clusters [13]. The reduced friction and smaller distances between vehicles in the clusters is the leading cause of accidents in adverse weather conditions. Thus, weather has a significant impact on traffic mobility, planning and efficiency. Adverse weather causes a decline in traffic flow and increases the risk of accidents [5].

A realistic characterization is required to align traffic safely in such conditions to reduce accidents and to mitigate congestion on roads [9]. Adverse weather conditions include rain, fog, dust, snow and ice. These conditions affect traffic flow by reducing visibility, friction, and the maneuverability of vehicles. This causes delays, speed variability, and an increase in the accident rate. An 18\% time delay has been observed on roads in adverse weather [27]. In snow, when the temperature is above 0\(^\circ\) C, the injury ratio is increased by
6.6% as compared to normal weather and 15% when the temperature is below 0°C. In heavy rain, when the temperature is moderate, i.e. between 0°C and 20°C, the traffic volume increases by 12% while it is increased by 8.5% when the temperature is less than 0°C, and by 6.5% when the temperature is greater than 20°C.

In transportation, road safety is the major concern. Accidents cause serious injuries and loss of life as well as property damage. According to the World Health Organization, 1.24 million road fatalities were observed during 2013 worldwide [35]. The highest accident rates occur when the temperature is below freezing and in snowy weather. Traffic accidents disturb the flow and create congestion. A single collision can cause multiple vehicle collisions or chain reaction accidents. In adverse weather conditions, this is more prevalent. Light snow that falls for a period of time with no other precipitation can cause slippery pavement, and vehicle traction is reduced by roughly 10% in such conditions [17]. On the other hand, heavy snowfall and rainfall reduce visibility and thus increase the time and distance to align, and reduce speeds by 16% to 40%. The speed declines by 30% to 40% in snowfall and snow or slushy pavement [6]. Consequently, the traffic volume rises, which causes an increase in delay, congestion, and accident risk. Accident rates increase from 100% to 1,000% during snowfall [28]. There was a 20% increase in road accidents during the winter season in Ottawa, Canada, from 1990-1998 [35]. Snow impacts driving behavior more compared to other adverse weather conditions. The reason is that snow affects visibility and road traction. If the temperature is low on the day after a heavy snow fall or high precipitation, there is low traction due to reduced friction. The snow covers lane markings and road signs which are designed to aid drivers, which leads to a higher accident rate [36]. There is an increase in fuel consumption and travel delay in snow and ice conditions [21]. The traffic speed is reduced by 3% to 5% with light snow and 30% to 40% with heavy snow on a freeway in Canada [10]. A study of a busy freeway in Minnesota demonstrated that in snow there is a speed reduction of 40%, the maximum traffic flow is decreased by 11%, and the traffic alignment time increases from 2 to 3 s [20].
There are three main types of traffic flow models: macroscopic, microscopic and mesoscopic. Microscopic traffic flow models represent single vehicle position and velocity, while macroscopic traffic flow models represent system-level characteristics. A mesoscopic traffic flow model shares the properties of both microscopic and macroscopic models. Vehicles are modeled at an individual level, and the behavior of traffic flow is approximated aggregately. A macroscopic traffic flow model considers speed and density for determining the cumulative behavior of traffic flow. The density $\rho$ is the average number of vehicles on a road segment per unit length. The product of speed (velocity) $v$ and density $\rho$ determines the traffic flow per unit time. Lighthill, Whitham and Richards [19], [29] developed a macroscopic traffic flow model (known as the LWR model), which is based on the equilibrium flow of vehicles. The LWR model assumes that vehicles align in zero time but ignores the transition velocity during alignment [40]. Payne modified the LWR model and removed some of the deficiencies. Payne [25] and Whitham [39] independently developed a two equation model for traffic flow which is known as the Payne-Whitham (PW) model. The first equation of the PW model is the continuity equation for the conservation of vehicles on a road, and the second models the acceleration behavior of traffic based on driver presumption, relaxation and traffic acceleration. Driver presumption is based mainly on changes in the forward traffic density, while relaxation characterizes the alignment of traffic during transitions. The PW model is known to produce oscillatory behavior at traffic discontinuities. This is stop-and-go traffic flow due to changes in traffic density during transitions and is typical congestion. The traffic flow is assumed to be adjusted with a constant speed (velocity), which can produce unrealistic results such as velocities below zero, which is impossible. Due to the instability of the PW model, it is unable to predict the number of traffic clusters and their densities [12].

This thesis characterizes the traffic flow during adverse weather conditions based on road surface conditions such as snow, compacted snow, and ice. Tire friction on snow, compacted snow, and ice severely affects the traffic flow on the road surface. Traffic flow transitions occur due to a change in speed, and this is affected by weather conditions. The
transition velocity is derived for traffic during the alignment of flow at transitions. The distribution of this velocity is developed based on the surface conditions of the road. The road surface conditions are characterized by a severity index. The severity index is higher for a smaller friction and a larger density of snow, compacted snow, and ice. For a higher severity index, the velocity during transitions should be reduced in order to align safely. The effect of this transition velocity is included in the proposed macroscopic traffic flow model. This model is evaluated on a circular road to demonstrate the alignment of vehicles in adverse weather conditions. Results are presented which show that this model accurately characterizes the traffic flow during alignment. The flow has smaller variations than with the PW model, which indicates reduced acceleration and deceleration in adverse weather. This reduction is required to align vehicles safely during transitions as there is reduced friction between the vehicle tires and road surface.

The rest of the thesis is organized as follows. In Chapter 2 the transition velocity distribution based on adverse weather conditions is derived. Then, a macroscopic traffic flow model based on the transition velocity distribution is presented. Chapter 3 describes the decomposition of both the proposed and PW models according to the Roe scheme. The Jacobian matrix of both models is developed to derive the eigenvalues, eigenvectors and average velocities. Chapter 4 presents an evaluation of the proposed and PW models. The spatial and temporal evolution of traffic flow, velocity and density is presented for a circular road under different weather conditions. Finally, the contributions of the thesis and suggestions for future work are given in Chapter 5.
Chapter 2

Proposed Traffic Flow Model

Payne [25] and Whitham [39] independently developed a two equation model for the traffic flow which is known as the Payne-Whitham (PW) model. The first equation of the Payne-Whitham (PW) model is

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0. \quad (2.1)$$

This is a conservation relation which states that traffic flow remains conserved and there are no transitions on the road. The assumptions of the conservation relation are that the length of road is infinite and there is no acceleration or deceleration. The second equation of the PW model is

$$\frac{\partial (\rho v)}{\partial t} + v \frac{\partial v}{\partial x} = C_0^2 \frac{\partial \rho}{\partial x} + \left( \frac{v(\rho) - v}{\tau} \right). \quad (2.2)$$

This determines the acceleration and deceleration due to changes such as ingress and egress from the flow. $C_0$ is the velocity constant and $\tau$ is the relaxation time to align traffic velocities during transitions. $\frac{v(\rho) - v}{\tau}$ is called the relaxation term, which characterizes the adjustment of velocities during transition. $v(\rho)$ is the velocity distribution which is dependent on the density and is called the equilibrium velocity distribution. According to the relaxation term, traffic adjusts to this velocity distribution which is called alignment. Several models have been proposed for $v(\rho)$ [22]. A commonly employed model is the Greenshields model [7, 23].
which is given by

\[ v(\rho) = v_m \left( 1 - \frac{\rho}{\rho_m} \right). \] (2.3)

The anticipation term is

\[ \frac{C_0^2}{\rho} \frac{\partial \rho}{\partial x}, \] (2.4)

which accounts for spatial changes in the traffic density. It is a function of the spatial gradient of density \( \frac{\partial \rho}{\partial x} \). The anticipation term of the PW model cannot characterize the spatial change in density in adverse weather due to the fixed velocity constant. Thus, the PW model assumes drivers are alike and behave the same in all conditions.

Adverse weather conditions such as snow, compacted snow, and ice have a significant impact on the traffic flow. There is reduction in the speed (velocity) due to reduced friction between the tires and road surface. This friction is affected by the density of snow, compacted snow, and ice. The larger the density, the smaller the friction. The effect of snow, compacted snow, and ice can be characterized by a severity index \( \beta \) which is given by

\[ \beta = \frac{d_s}{f_s} + D_c \frac{d_c}{f_c} + D_i \frac{d_i}{f_i}, \] (2.5)

where \( d_s, d_i \) and \( d_c \) are the depth of snow, ice, and compacted snow on the road, respectively. \( f_s, f_c \) and \( f_i \) are the friction coefficients of the tires on snow, compacted snow, and ice, respectively, and \( D_c \) and \( D_i \) are the density ratios of compacted snow and ice to new snow, respectively. The distance covered by a vehicle to align its velocity during a transition is known as the distance headway. The time to cover this distance is known as the time headway. The minimum distance required for safe alignment between vehicles under normal weather conditions is the safe distance headway, \( H \). The time to cover the safe distance headway is the safe time headway, \( \alpha \). Traffic velocity during a transition is adjusted to align with traffic conditions ahead. In adverse weather, this velocity is reduced due to lower friction on the road and is zero \( (v_t = 0) \), for the minimum friction. It is also affected by the density of snow, compacted snow, and ice, and is smaller for a larger density and can be
Table 2.1: Transition Velocity Parameters

<table>
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<tr>
<th>Weather</th>
<th>Severity Index, $\beta$</th>
<th>Transition velocity, $v_t$</th>
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<tr>
<td>adverse</td>
<td>$\beta_m$</td>
<td>0</td>
</tr>
<tr>
<td>normal</td>
<td>0</td>
<td>$\frac{H}{\alpha}$</td>
</tr>
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characterized as

$$v_t(\beta) = \frac{H}{\alpha} \left(1 - \frac{\beta}{\beta_m}\right),$$

where $v_t(\beta)$ is the transition velocity based on the severity index $\beta$, and $\beta_m$ is the maximum severity index. When the severity index is maximum, that is $\beta = \beta_m$, the transition velocity is zero as there is minimum friction between the tires and road. When there is no snow, compacted snow, or ice on the road and the surface of the road is clear, $\beta = 0$ and the weather is considered normal. In normal weather, the transition velocity only depends on the safe distance and safe time headways, that is, $v_t = \frac{H}{\alpha}$.

During adverse weather conditions, the changes in traffic are based on the road conditions. A driver will align with a smaller velocity in adverse weather than in normal weather to avoid accidents. The transition velocity $v_t(\beta)$ is aligned during the relaxation time $\tau$, so

$$\frac{v_t(\beta)}{\tau},$$

is the rate at which the density changes. For a faster traffic flow, $\tau$ is smaller and the alignment is quicker. The relaxation time is the traffic alignment sensitivity, and traffic alignment is less sensitive for a larger relaxation time.

For the PW model, driver anticipation is characterized by the velocity constant $C_0$, so behavior is fixed for all transition conditions including adverse weather. The PW model does not account for traffic behavior based on the surface conditions of the road. Thus, to better characterize traffic behavior in adverse weather, a variable anticipation term based on the surface condition of the road is introduced. To achieve this, $C_0^2$ in (2.4) is replaced
with \( \frac{\partial}{\partial x} \left( \frac{v_t(\beta)}{\tau} \right) \) to obtain

\[
\frac{1}{\rho} \frac{\partial}{\partial x} \left( \frac{v_t(\beta)}{\tau} \right) \frac{\partial \rho}{\partial x}.
\]  \tag{2.7}

Substituting \( v_t(\beta) \) in (2.7) from (2.6), the anticipation term is obtained as

\[
\frac{\partial}{\rho \partial x} \left( \frac{H_\alpha \left(1 - \frac{\beta}{\beta_m}\right)}{\tau} \right) \frac{\partial \rho}{\partial x}.
\]  \tag{2.8}

This accounts for the spatial changes in density due to adverse weather conditions. It also characterizes the spatial changes because of vehicle egress and ingress to the traffic flow. For a smaller \( \tau \), a larger change in density occurs during transitions, so traffic alignment is faster and achieves a smooth flow in a shorter time. Substituting (2.7) in (2.2), the acceleration relation based on the transition velocity distribution for the proposed model takes the form

\[
\frac{\partial (\rho v)}{\partial t} + v \frac{\partial v}{\partial x} = \frac{\partial}{\rho \partial x} \left( \frac{H_\alpha \left(1 - \frac{\beta}{\beta_m}\right)}{\tau} \right) \frac{\partial \rho}{\partial x}.
\]  \tag{2.9}

The first equation of the proposed model is the same as in the PW model.

In the proposed model, driver anticipation and relaxation are characterized using (2.7), whereas in the PW model they are two different terms. These terms are based on the traffic density and does not consider the road surface conditions. Therefore, the relaxation term, \( \frac{v(\rho) - v}{\tau} \), in (2.2) is unrealistic in adverse weather. The alignment behavior in the proposed model is characterized by the transition velocity distribution \( v_t(\beta) \) and the relaxation time \( \tau \). Therefore, the relaxation and anticipation term of PW model is replaced with a single term (2.8) in the proposed model to better characterizes vehicle behavior in adverse weather.
Chapter 3

Traffic Model Decomposition

3.1 Roe Decomposition

The proposed and PW models are discretized using the Roe decomposition technique \[31\] to evaluate their performance. This technique can be used to approximate a nonlinear system of equations

\[ G_t + f(G)_x = S(G), \]  

(3.1)

where \( G \) denotes the vector of data variables, \( f(G) \) denotes the vector of functions of the data variables, and \( S(G) \) is the vector of source terms. The subscripts \( t \) and \( x \) denote the partial derivatives with respect to time and distance, respectively. Equation (3.1) can be expressed as

\[ \frac{\partial G}{\partial t} + \frac{\partial f}{\partial G} \frac{\partial G}{\partial x} = S(G), \]  

(3.2)

where \( \frac{\partial f}{\partial G} \) is the gradient of the function of data variables with respect to these variables. Let \( A(G) \) be the Jacobian matrix of the system. Then (3.2) can be written as

\[ \frac{\partial G}{\partial t} + A(G) \frac{\partial G}{\partial x} = S(G), \]  

(3.3)
setting the source term in (3.3) to zero gives the quasilinear form

$$\frac{\partial G}{\partial t} + A(G)\frac{\partial G}{\partial x} = 0. \quad (3.4)$$

The data variables in the PW and proposed models are density \(\rho\) and flow \(\rho v\). The Roe technique is used to linearize the Jacobian matrix \(A(G)\) by decomposing it into eigenvalues and eigenvectors. It is based on the concept that the data variables, eigenvalues and eigenvectors remain conserved for small changes in time and distance. This technique is widely employed because it is able to capture the effects of abrupt changes in the data variables.

Consider a road divided into \(M\) equidistant segments and \(N\) equal duration time steps. The total length is \(x_M\) so a segment has length \(\delta x = x_M/M\), and the total time duration is \(t_N\) so a time step is \(\delta t = t_N/N\). The Jacobian matrix is approximated for road segments \((x_i + \frac{\delta x}{2}, x_i - \frac{\delta x}{2})\). This matrix is obtained for all \(M\) segments in every time interval \((t_{n+1}, t_n)\), where \(t_{n+1} - t_n = \delta t\). Let \(\Delta G\) denote the change in the data variables \(G\), and \(\Delta f\) the corresponding change in the functions of these variables. Further, let \(G_i\) be the average values of the data variables in the \(i\)th segment. The change in flux at the boundary between the \(i\)th and \((i + 1)\)th segments is defined as

$$\Delta f_{i+\frac{1}{2}} = A(G_{i+\frac{1}{2}})\Delta G, \quad (3.5)$$

where \(A(G_{i+\frac{1}{2}})\) is the Jacobian matrix at the segment boundary, and \(G_{i+\frac{1}{2}}\) is the vector of data variables at the boundary obtained using the Roe technique. The flux approximates the change in traffic density and flow at the segment boundary, so then

$$\Delta f_{i+\frac{1}{2}} = A(G_{i+\frac{1}{2}})(G_{i+1} - G_i), \quad (3.6)$$

where the approximation \(\Delta G = (G_{i+1} - G_i)\) is used. The flux at the boundary between
segments $i$ and $i + 1$ at time $n$ can then be approximated by

$$f^m_{i+\frac{1}{2}}(G^n_i, G^n_{i+1}) = \frac{1}{2} \left( f(G^n_i) + f(G^n_{i+1}) \right) - \frac{1}{2} \Delta f_{i+\frac{1}{2}},$$  \hspace{1cm} (3.7)

where $f(G^n_i)$ and $f(G^n_{i+1})$ denote the values of the functions of the data variables in road segments $i$ and $i + 1$, respectively, at time $n$. Substituting (3.6) into (3.7) gives

$$f^m_{i+\frac{1}{2}}(G^n_i, G^n_{i+1}) = \frac{1}{2} \left( f(G^n_i) + f(G^n_{i+1}) \right) - \frac{1}{2} A(G^n_{i+\frac{1}{2}}) \left( G^n_{i+1} - G^n_i \right).$$  \hspace{1cm} (3.8)

This approximates the change in density and flow without considering the source. The updated data variables are obtained by including the source term which gives

$$G^{n+1}_i = G^n_i - \frac{\delta t}{\delta x} \left( f^m_{i+\frac{1}{2}} - f^m_{i-\frac{1}{2}} \right) + \delta t S(G^n_i).$$  \hspace{1cm} (3.9)

### 3.2 Jacobian Matrix

In this section, the Jacobian matrix $A(G)$ is derived. We first consider the PW model and convert it into conservation form giving

$$v\rho_t + v(\rho v)_x = 0,$$  \hspace{1cm} (3.10)

where the subscripts $t$ and $x$ denote the partial derivatives with respect to time and distance, respectively. Then

$$(\rho v)_t = \rho v_t + v \rho_t,$$  \hspace{1cm} (3.11)

and rearranging gives

$$v \rho_t = (\rho v)_t - \rho v_t.$$  \hspace{1cm} (3.12)
Substituting (3.12) into (3.10), we obtain

$$\rho v_t = v(\rho v)_x + (\rho v)_t.$$  \hfill (3.13)


Multiplying (2.2) by $\rho$ gives

$$\rho v_t + \rho v v_x + C_0^2 \rho_x = \rho \frac{v(\rho) - v}{\tau},$$  \hfill (3.14)

and substituting (3.13) in (3.14) results in

$$v(\rho v)_x + (\rho v)_t + \rho v v_x + C_0^2 \rho_x = \rho \frac{v(\rho) - v}{\tau}.$$  \hfill (3.15)

We have that

$$(\rho v v)_x = v(\rho v)_x + \rho v v_x,$$  \hfill (3.16)

and rearranging gives

$$v(\rho v)_x = (\rho v v)_x - \rho v v_x.$$  \hfill (3.17)

Substituting (3.17) in (3.15), we obtain

$$(\rho v v)_x + (\rho v)_t + C_0^2 \rho_x = \rho \frac{v(\rho) - v}{\tau}.$$  \hfill (3.18)

Now, using the fact that

$$(\rho v v)_x = \left( \frac{(\rho v)^2}{\rho} \right)_x,$$

(3.18) can be written as

$$(\rho v)_t + \left( \frac{(\rho v)^2}{\rho} + C_0^2 \rho \right)_x = \rho \left( \frac{v(\rho) - v}{\tau} \right),$$  \hfill (3.19)

which is in conservation form. The source term can be considered as traffic movement into and out of the traffic, which is ingress and egress to the flow. If the source term is assumed
to be zero and traffic mobility is conserved, then the RHS of (3.19) is zero which gives

$$\left(\rho v\right)_t + \left(\frac{(\rho v)^2}{\rho} + C_0^2 \rho \right)_x = 0.$$  \hfill (3.20)

The model in quasilinear form is then

$$G = \begin{pmatrix} \rho \\ \rho v \end{pmatrix}, f(G) = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \rho v \\ \frac{(\rho v)^2}{\rho} + C_0^2 \rho \end{pmatrix} \quad \text{and} \quad S(G) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \hfill (3.21)$$

The Jacobian matrix $A(G) = \frac{\partial f}{\partial G}$ from (3.21) is

$$A(G) = \begin{pmatrix} 0 & 1 \\ -\frac{(\rho v)^2}{\rho^2} + C_0^2 & 2v \end{pmatrix}, \hfill (3.22)$$

which gives

$$A(G) = \begin{pmatrix} 0 & 1 \\ -v^2 + C_0^2 & 2v \end{pmatrix}. \hfill (3.23)$$

The eigenvalues $\lambda_i$ of the Jacobian matrix are required to obtain the flux in (3.8), and are obtained from (3.23) as the solution of

$$\begin{vmatrix} A(G) - \lambda I \end{vmatrix} = \begin{vmatrix} -\lambda & 1 \\ -v^2 + C_0^2 & 2v - \lambda \end{vmatrix} = 0,$$ \hfill (3.24)

which gives

$$\lambda^2 - 2v\lambda + v^2 - C_0^2 = 0.$$ \hfill (3.25)

The eigenvalues are then

$$\lambda_{1,2} = \frac{2v \pm \sqrt{4v^2 - 4(v^2 - C_0^2)}}{2} = v \pm \sqrt{C_0^2}. \hfill (3.26)$$
For the PW model

\[ \lambda_{1,2} = v \pm C_0. \quad (3.27) \]

For the proposed model, we have

\[ C_0^2 = \int \frac{\partial}{\partial x} \left( \left( \frac{H(1 - \frac{\beta}{\beta_m})}{\alpha \tau} \right) \right) \, dx \]

and substituting this in (3.20) gives the eigenvalues

\[ \lambda_{1,2} = v \pm \sqrt{\left( \frac{H(1 - \frac{\beta}{\beta_m})}{\alpha \tau} \right)}. \quad (3.28) \]

The eigenvectors are obtained by solving

\[ |A(G) - \lambda I|x = 0, \quad (3.29) \]

where

\[ x = \begin{pmatrix} 1 \\ x_2 \end{pmatrix}. \quad (3.30) \]

For the PW model, using (3.23) and \( \lambda_1 = v + C_0 \) from (3.27), the eigenvectors obtained from (3.29) are

\[ e_1 = \begin{pmatrix} 1 \\ v + C_0 \end{pmatrix}, \quad (3.31) \]

and

\[ e_2 = \begin{pmatrix} 1 \\ v - C_0 \end{pmatrix}. \quad (3.32) \]

For the proposed model, using (3.23) and \( \lambda_1 = v + \sqrt{\left( \frac{H(1 - \frac{\beta}{\beta_m})}{\alpha \tau} \right)} \) from (3.28), (3.29) takes the form

\[
\begin{pmatrix}
-v - \sqrt{\left( \frac{H(1 - \frac{\beta}{\beta_m})}{\alpha \tau} \right)} & 1 \\
\left( \frac{H(1 - \frac{\beta}{\beta_m})}{\alpha \tau} \right) - v^2 & v - \sqrt{\left( \frac{H(1 - \frac{\beta}{\beta_m})}{\alpha \tau} \right)}
\end{pmatrix}
\begin{pmatrix} 1 \\ x_2 \end{pmatrix} = 0, \quad (3.33)
\]
so the eigenvectors are

\[ e_1 = \left( v + \sqrt{\frac{H(1 - \frac{\beta}{\alpha \tau})}{\alpha \tau}} \right), \quad (3.34) \]

and

\[ e_2 = \left( v - \sqrt{\frac{H(1 - \frac{\beta}{\alpha \tau})}{\alpha \tau}} \right). \quad (3.35) \]

To obtain the average speed for the improved model, using (3.5) and (3.23), \( \Delta f \) can be expressed as

\[
\begin{align*}
\Delta f &= \begin{pmatrix} \Delta f_1 \\ \Delta f_2 \end{pmatrix} = A(G)\Delta G = \begin{pmatrix} 0 & 1 \\ \frac{H(1 - \frac{\beta}{\alpha \tau})}{\alpha \tau} - v^2 & 2v \end{pmatrix} \begin{pmatrix} \Delta \rho \\ \Delta \rho v \end{pmatrix}. \\
\end{align*}
\quad (3.36)
\]

From (3.36), we have

\[
\Delta f_2 = \left( \left( \frac{H(1 - \frac{\beta}{\alpha \tau})}{\alpha \tau} - v^2 \right) \Delta \rho + 2v \Delta \rho v, 
\quad (3.37)
\]

and substituting \( C_0^2 = \int \frac{\partial}{\partial x} \left( \frac{H(1 - \frac{\beta}{\alpha \tau})}{\alpha \tau} \right) \) \( \text{d}x \) in (3.21) gives

\[
\begin{align*}
f(G) &= \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \rho v \\ \frac{(\rho v)^2}{\rho} + \rho \left( \frac{H(1 - \frac{\beta}{\alpha \tau})}{\alpha \tau} \right) \end{pmatrix}, \\
(3.38)
\end{align*}
\]

so then

\[
\begin{align*}
\Delta f(G) &= \begin{pmatrix} \Delta f_1 \\ \Delta f_2 \end{pmatrix} = \begin{pmatrix} \Delta (\rho v) \\ \Delta \left( \frac{(\rho v)^2}{\rho} + \rho \left( \frac{H(1 - \frac{\beta}{\alpha \tau})}{\alpha \tau} \right) \right) \end{pmatrix}. \\
(3.39)
\end{align*}
\]

Equating (3.37) with \( \Delta f_2 \) from (3.39), we obtain

\[
\bar{\nu}^2 \Delta \rho - 2\bar{\nu} \Delta \rho v + \Delta \rho v^2 = 0, \quad (3.40)
\]
and taking the positive root gives the average speed of the proposed model as

\[ \bar{v} = \frac{2\Delta \rho v + 2\sqrt{(\Delta \rho v)^2 - (\Delta \rho)(\Delta \rho v^2)}}{2\Delta \rho}. \]  

(3.41)

Substituting \( \Delta \rho v = \rho_{i+1}v_{i+1} - \rho_i v_i \), \( \Delta \rho v^2 = \rho_{i+1}v^2_{i+1} - \rho_i v^2_i \), and \( \Delta \rho = \rho_{i+1} - \rho_i \) in (3.41), the average speed at the boundary of segments \( i \) and \( i+1 \) is

\[ v_{i+\frac{1}{2}} = \frac{v_{i+1}\sqrt{\rho_{i+1}} + v_i\sqrt{\rho_i}}{\sqrt{\rho_{i+1}} + \sqrt{\rho_i}}. \]  

(3.42)

To obtain the average speed for the PW model, using (3.5) and (3.23), \( \Delta f \) can be expressed as

\[ \Delta f = \begin{pmatrix} \Delta f_1 \\ \Delta f_2 \end{pmatrix} = A(G)\Delta G = \begin{pmatrix} 0 & 1 \\ C_0^2 - v^2 & 2v \end{pmatrix} \begin{pmatrix} \Delta \rho \\ \Delta \rho v \end{pmatrix}. \]  

(3.43)

From (3.43), we obtain

\[ \Delta f_2 = (-v^2 + C_0^2)\Delta \rho + 2v\Delta \rho v, \]  

(3.44)

and using (3.21) gives

\[ f(G) = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \rho v \\ \frac{(\rho v)^2}{\rho} + C_0^2\rho \end{pmatrix}, \]  

so then

\[ \Delta f(G) = \begin{pmatrix} \Delta f_1 \\ \Delta f_2 \end{pmatrix} = \begin{pmatrix} \Delta (\rho v) \\ \Delta \left(\frac{(\rho v)^2}{\rho} + C_0^2\rho\right) \end{pmatrix}. \]  

(3.46)

Equating (3.44) with \( \Delta f_2 \) in (3.46) results in

\[ \bar{v}^2 \Delta \rho - 2\bar{v} \Delta \rho v + \Delta \rho v^2 = 0. \]  

(3.47)
The average density $ρ_{i+\frac{1}{2}}$ at the boundary of segments $i$ and $i+1$ is given by the geometric mean of the densities in these segments

$$ρ_{i+\frac{1}{2}} = \sqrt{ρ_{i+1}ρ_i}. \quad (3.48)$$

The eigenvalues of the Jacobian matrix $A(G_{i+\frac{1}{2}})$ of the proposed model are

$$λ_{1,2} = v_{i+\frac{1}{2}} \pm \sqrt{\left(\frac{H(1-\frac{β}{β_m})}{ατ}\right)}. \quad (3.49)$$

The eigenvalues of the Jacobian matrix for the PW model are

$$λ_{1,2} = v_{i+\frac{1}{2}} \pm C_0, \quad (3.50)$$

so the change in speed due to a transition is constant. The corresponding eigenvectors are

$$e_{1,2} = \begin{pmatrix} 1 \\ v_{i+\frac{1}{2}} \pm C_0 \end{pmatrix}. \quad (3.51)$$

### 3.3 Entropy Fix

Entropy fix is applied to Roe’s technique to smooth any discontinuities at the segment boundaries. The Jacobian matrix $A(G_{i+\frac{1}{2}})$ is decomposed into eigenvalues and eigenvectors to approximate the flux in the road segments (3.8). Thus, the Jacobian matrix for the road segments is replaced with the entropy fix solution given by

$$e|Λ|e^{-1},$$

where $|Λ| = [\hat{λ}_1, \hat{λ}_2, \ldots, \hat{λ}_k, \ldots, \hat{λ}_n]$ is a diagonal matrix which is a function of the eigenvalues $λ_k$ of the Jacobian matrix, and $e$ is the corresponding eigenvector matrix. The Harten and
Hayman entropy fix scheme \[8\] is employed here and is given by

\[
\hat{\lambda}_k = \begin{cases} 
\hat{\delta}_k & \text{if } |\lambda_k| < \hat{\delta}_k \\
|\lambda_k| & \text{if } |\lambda_k| \geq \hat{\delta}_k,
\end{cases}
\]  

(3.52)

with

\[
\hat{\delta}_k = \max \left( 0, \lambda_{i+1/2} - \lambda_i, \lambda_{i+1} - \lambda_i - \lambda_{i+1/2} \right).
\]  

(3.53)

This ensures that the \(\hat{\lambda}_k\) are not negative and are similar at the segment boundaries.

For the proposed model, we obtain

\[
e|\Lambda|e^{-1} = \begin{pmatrix}
1 & 1 \\
v_i + \frac{1}{2} + \sqrt{\frac{H(1-\beta/\beta_m)}{\alpha \tau}} & v_i + \frac{1}{2} - \sqrt{\frac{H(1-\beta/\beta_m)}{\alpha \tau}} \\
0 & 0 \\
v_i - \sqrt{\frac{H(1-\beta/\beta_m)}{\alpha \tau}} & -v_i - \sqrt{\frac{H(1-\beta/\beta_m)}{\alpha \tau}} \\
-1 & 1 \\
2 \sqrt{\frac{H(1-\beta/\beta_m)}{\alpha \tau}}
\end{pmatrix}
\times
\begin{pmatrix}
0 \\
0 \\
0 \\
-1 \\
1 \\
-1 \\
2C_0
\end{pmatrix},
\]

and for the PW model we have

\[
e|\Lambda|e^{-1} = \begin{pmatrix}
1 & 1 \\
v_i + \frac{1}{2} + C_o & v_i + \frac{1}{2} - C_o \\
0 & 0 \\
v_i + \frac{1}{2} + C_o & -v_i + \frac{1}{2} - C_o \\
0 & 0 \\
-1 & 1 \\
-1 & -1 \\
2C_0
\end{pmatrix}
\times
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
-1 \\
-1 \\
1 \\
2C_0
\end{pmatrix}.
\]

The corresponding flux is obtained from \[3.8\] using \(f(G_i)\) and \(f(G_{i+1})\) and substituting \(e|\Lambda|e^{-1}\) for \(A(G_{i+1/2})\). The updated data variables, \(\rho\) and \(\rho v\), are then obtained at time \(n\) using \(3.9\).
Chapter 4

Traffic Model Evaluation and Comparison

The performance of the proposed and PW models is evaluated in this chapter using the parameters given in Table 4.1. Periodic boundary conditions are used to evaluate the traffic evolution on a circular road. The total simulation time for the proposed model is 75 s in both Examples 1 and 2, whereas the PW model is evaluated for 300 s in example 1. The PW model is evaluated for a larger time to determine if the variation in traffic flow will reduce with time, as a large variation is observed at 20 s.

The proposed and PW models are evaluated with \( v_m = 20 \) m/s and 25 m/s in Examples 1 and 2, respectively. The initial velocity distribution at \( t = 0 \) is the Greenshields equilibrium velocity distribution given by (2.3) \[7\]. In Example 1, safe distance headway is chosen as \( H = 25 \) m, whereas in Example 2 the safe distance headway is \( H = 58 \) m \[30\]. A smaller safe distance headway in Example 1 is chosen to analyze a scenario with short distances between vehicles. For both examples, \( \alpha = 2 \) s.

Both models are evaluated using a relaxation time of \( \tau = 0.5 \) s which is suitable for transitions over small distances \[2\]. Further, a small value of \( \delta x \) is used to ensure accurate numerical results. Therefore, the road of length \( x_M = 3000 \) m is divided into \( M = 50 \)
equal segments with $\delta x = 60$ m for both models. To satisfy the Courant–Friedrichs–Lewy (CFL) condition \cite{18}, $\delta t$ is chosen as 1.25 s for the proposed model in Example 1, and 1.5 s in Example 2 for the proposed model. For the proposed model, $t_N = 75$ s is divided into $N = 60$ and $N = 50$ intervals in Examples 1 and 2, respectively. For the PW model, $t_N = 60$ s and is divided into $N = 60$ intervals in Example 1. The initial density $\rho_0$ at time $t = 0$ s for Example 1 has the following distribution

$$
\rho_0 = \begin{cases} 
0.01, & \text{for } x \leq 500; \\
0.3, & \text{for } 500 < x \leq 1300; \\
0.1, & \text{for } 1300 < x \leq 1750; \\
0.01, & \text{for } x > 1750, 
\end{cases}
$$

(4.1)

which spans the first 3000 m of the road. The maximum density is $\rho_m = 1$ which means that the road is 100% occupied. The initial density $\rho_0$ at time $t = 0$ s for example 2 has the following distribution

$$
\rho_0 = \begin{cases} 
0.05, & \text{for } x \leq 500; \\
0.3, & \text{for } 500 < x \leq 1300; \\
0.15, & \text{for } 1300 < x \leq 1750; \\
0.05, & \text{for } x > 1750, 
\end{cases}
$$

(4.2)

which spans the first 3000 m of the road. The velocity constant values used in the literature for the PW model vary between 2.4 m/s and 57 m/s to evaluate the performance with different traffic densities \cite{22, 37, 38}. Thus, the value of $C_0$ considered here is 15 m/s.

### 4.0.1 Example 1

In this example, the traffic behavior with the proposed model is evaluated for $\beta = 0.5$. Figure 4.1 presents the traffic density behavior with this model over a time span of 75 s for the 50
Table 4.1: Simulation Parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>equilibrium velocity distribution</td>
<td>$v(\rho)$</td>
<td>Greenshields distribution</td>
</tr>
<tr>
<td>length of road</td>
<td>$x_M$</td>
<td>3000 m</td>
</tr>
<tr>
<td>maximum normalized density</td>
<td>$\rho_m$</td>
<td>1</td>
</tr>
<tr>
<td>maximum severity index</td>
<td>$\beta_m$</td>
<td>1</td>
</tr>
<tr>
<td>relaxation time</td>
<td>$\tau$</td>
<td>0.5 s</td>
</tr>
<tr>
<td>safe time headway</td>
<td>$\alpha$</td>
<td>2 s</td>
</tr>
<tr>
<td>Example 1:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>total simulation time for the proposed model</td>
<td>$t_N$</td>
<td>75 s</td>
</tr>
<tr>
<td>total simulation time for the PW model</td>
<td>$t_N$</td>
<td>60 s</td>
</tr>
<tr>
<td>number of time steps for the proposed model</td>
<td>$N$</td>
<td>$\frac{75}{1.25} = 60$</td>
</tr>
<tr>
<td>safe distance headway for the proposed model</td>
<td>$H$</td>
<td>25 m</td>
</tr>
<tr>
<td>severity index</td>
<td>$\beta$</td>
<td>0.5</td>
</tr>
<tr>
<td>road step for the proposed and PW models</td>
<td>$\delta x$</td>
<td>60 m</td>
</tr>
<tr>
<td>time step for the PW model</td>
<td>$\delta t$</td>
<td>1 s</td>
</tr>
<tr>
<td>number of time steps for the PW model</td>
<td>$N$</td>
<td>$\frac{60}{1} = 60$</td>
</tr>
<tr>
<td>maximum velocity for the proposed and PW models</td>
<td>$v_m$</td>
<td>20 m/s</td>
</tr>
<tr>
<td>velocity constant</td>
<td>$C_o$</td>
<td>15 m/s</td>
</tr>
<tr>
<td>Example 2:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>maximum velocity for the proposed and PW models</td>
<td>$v_m$</td>
<td>25 m/s</td>
</tr>
<tr>
<td>total simulation time for the proposed model</td>
<td>$t_N$</td>
<td>75 s</td>
</tr>
<tr>
<td>time step for the proposed model</td>
<td>$\delta t$</td>
<td>1.5 s</td>
</tr>
<tr>
<td>road step for the proposed model</td>
<td>$\delta x$</td>
<td>60 m</td>
</tr>
<tr>
<td>number of time steps for the proposed model</td>
<td>$N$</td>
<td>$\frac{75}{1.5} = 50$</td>
</tr>
<tr>
<td>safe distance headway for the proposed model</td>
<td>$H$</td>
<td>58 m</td>
</tr>
<tr>
<td>severity index</td>
<td>$\beta$</td>
<td>0.0, 0.15, 0.95</td>
</tr>
</tbody>
</table>
road steps. At the 60th time step (after 75 s), the traffic density is highest between the 30th and 40th road steps and attains a maximum of 0.25, so the distance between vehicles is small. The density gradually reduces from 0.25 at the 37th road step to 0.1 at the 50th road step, so the distance between vehicles increases. The minimum traffic density of 0.01 occurs between the 15th and 30th road steps. The maximum change in density is 0.19 which occurs between the 30th and 40th road steps. This is a smaller variation in density than with the PW model as demonstrated in Figure 4.2.

The traffic density behavior on the circular road over a time span of 60 s with the PW model is given in Figure 4.2. The density at the 60th time step reaches a high of 0.28 at the 25th road step. For a density of 0.28, the distance between vehicles is small. The change in traffic density between the 20th and 50th road steps is large. At 60 s (60th time step), the density at the 20th road step is 0.01, increases to 0.28 at the 25th road step, and then reduces to 0.03 at the 50th road step. The density at the 50th time step is 0.01 from the 3rd road step to the 20th road step.

The velocity behavior on the circular road over a time span of 75 s with the proposed model is given in Figure 4.3. At the 60th time step (after 75 s), the velocity is 15 m/s at the 37th road step, and is the minimum velocity. At the 60th time step, the velocity increases gradually from 15 m/s at the 37th road step to 18 m/s at the 50th road step. The maximum velocity of approximately 20 m/s occurs at the 60th time step from the 15th to 30th road steps. The maximum variation of 5 m/s occurs at the 60th time step between the 30th and 40th road steps. This is smaller than with the PW model as shown in Figure 4.4.

The velocity behavior on the circular road over a time span of 60 s with the PW model is given in Figure 4.4. This shows that the velocity reaches 30 m/s at the 20th and 2nd time steps, even though 20 m/s is the maximum possible velocity. The minimum velocity is −8 m/s, which is below 0 m/s and is not realistic. The variation from −8 m/s to 30 m/s in velocity at the 2nd time step between the 20th and 30th road steps reduces as time evolves. The maximum velocity at the 20th time step is 22 m/s. With the proposed model, the
variations in velocity are much smaller and more realistic than with the PW model.

The traffic flow behavior with the proposed model over a time span of 75 s for the 50 road steps is presented in Figure 4.5. At the 60th time step (after 75 s), the traffic flow is high between the 30th and 40th road steps and attains a maximum of 3.75 veh/s. At the 60th time step, the traffic flow gradually reduces from 3.75 veh/s at the 37th road step to 2 veh/s at the 50th road step. The minimum traffic flow of approximately 0.2 veh/s at the 60th time step is between the 15th and 30th road steps. The maximum variation of 1.75 veh/s occurs at the 60th time step between the 30th and 40th road steps. This is smaller than with the PW model as shown in Figure 4.6.

The traffic flow behavior on the circular road over a time span of 60 s with the PW model is given in Figure 4.6. At the 2nd time step, the maximum flow of 5.5 veh/s occurs at the 20th road step. The flow goes below 0 veh/s at the 19th and 2nd time steps, which is unrealistic. The traffic flow at the 60th time step with the PW model reaches a high of 4 veh/s at the 30th road step. The change in traffic flow between the 20th and 50th road steps is large. At 60 s (60th time step), the flow at the 20th road step is 0.25 veh/s, increases to 4 veh/s at the 30th road step, and then changes to 0.7 veh/s at the 50th road step. The maximum variation of 3.75 veh/s occurs at the 60th time step between the 20th and 30th road steps.

These results show that the PW model does not accurately characterize the traffic flow during adverse weather. Further, the proposed model results in velocities between 0 m/s and 20 m/s, whereas the velocity with the PW model exceeds the maximum and minimum. The variations in traffic flow and velocity are small with the proposed model, which is expected in adverse weather.

4.0.2 Example 2

Figures 4.7 presents the traffic density behavior at 75 s with both the PW and proposed models on a circular road of length 3000 m. The proposed model traffic flow behavior is
given for $\beta = 0$, 0.15 and 0.95, corresponding to three different weather conditions. $\beta = 0$ represents normal weather, $\beta = 0.15$ characterizes low adverse weather, and $\beta = 0.95$ indicates severe adverse weather.

The traffic density with $\beta = 0.95$ increases from 0.05 at 1500 m to 0.32 at 2000 m and then reduces to 0.12 at 2700 m at 75 s. The traffic density varies from 0.12 at 2900 m to 0.05 at 500 m. The maximum density is 0.32 at 2000 m. This indicates that the distance between vehicles is small at 2000 m with $\beta = 0.95$. The density is approximately 0.05 between 500 m and 1500 m, which is the minimum density with $\beta = 0.95$. With $\beta = 0.15$, the density gradually reduces from 0.12 at 0 m to 0.05 at 1300 m, then gradually increases to 0.23 at 2400 m, and then reduces to 0.12 at 3000 m. The maximum density variation of 0.27 with $\beta = 0.95$ occurs between 1500 m to 2000 m, whereas with $\beta = 0.15$ the maximum density variation of 0.11 occurs between 1300 m and 2400 m. This indicates that a smaller variation in density occurs over a larger distance in low adverse weather as compared to severe adverse weather. In normal weather ($\beta = 0$), the traffic density is smoother than with $\beta = 0.15$. The maximum density is 0.21 with $\beta = 0$ which is less than the 0.23 with $\beta = 0.15$. The majority of vehicles with the PW model are between 1500 m and 700 m on the circular road. The density varies from 0.05 at 1500 to 0.27 at 1800 m and then reduces to 0.05 at 700 m. This large change in density (0.22) occurs over a small distance of 300 m and then the density reduces gradually to 0.05 over a distance of 1900 m. These results show that the proposed model can characterize traffic behavior in different weather conditions whereas the PW model exhibits the same behavior in all weather conditions.

Figure 4.8 presents the traffic density behavior with the proposed model on a circular road over a time span of 75 s with $\beta = 0.15$. In low adverse weather, the variation in density of 0.29 at $t = 0$ s gradually reduces to 0.11 at 75 s. The maximum density at 75 s is 0.23 at 2400 m. A variation in density of 0.11 occurs between 1300 m and 2400 m.

Figure 4.9 presents the proposed model traffic density behavior on a circular road with $\beta = 0.95$ over a time span of 75 s. This figure shows that the maximum change in
density of 0.27 occurs between 1500 m and 2000 m. The maximum density is 0.32 at 2000 m. Thus in severe adverse weather, the traffic is spread over a smaller distance with a larger variation in density than in low adverse weather.

The traffic velocity at 75 s is shown in Figure 4.10 for $\beta = 0.15$ and 0.95. For $\beta = 0.95$, the fastest traffic is between 500 m and 1000 m with a maximum velocity of 24 m/s. The slowest traffic is between 1800 m and 2300 m with a maximum velocity of 18 m/s. With $\beta = 0.15$, the velocity reaches 25 m/s between 500 m and 700 m. The traffic attains the maximum possible velocity as the surface friction is larger than with $\beta = 0.95$. The velocity with $\beta = 0.15$ ranges from 25 m/s at 500 m to 16 m/s at 1600 m. The velocity with $\beta = 0.95$ ranges from 24 m/s at 1000 m to 18 m/s at 2000 m. The variation in velocity is 6 m/s with $\beta = 0.95$, while with $\beta = 0.15$ it is 9 m/s. Smaller changes in velocity are expected in more adverse weather.

The traffic flow with the proposed model over a circular road of 3000 m with $\beta = 0.15$ and 0.95 at 75 s is presented in Figure 4.11. The traffic flow with $\beta = 0.95$ increases from 1.2 veh/s at 1500 m to 5.8 veh/s at 2000 m and then reduces to 2.2 veh/s at 2700 m. The traffic flow varies from 2.2 veh/s at 2700 m to 1.1 veh/s at 500 m. The maximum flow is 5.8 veh/s at 2000 m. The traffic flow is approximately 1.1 veh/s between 500 m and 1500 m, which is the minimum flow with $\beta = 0.95$. With $\beta = 0.15$, the traffic flow gradually reduces from 2.15 veh/s at 0 m to 1.1 veh/s at 1300 m and then gradually increases to 4 veh/s at 2400 m, which then reduces to 2.2 veh/s at 3000 m. With $\beta = 0.95$ the variation in flow is 4.7 veh/s between 1500 m to 2000 m, whereas with $\beta = 0.15$ the variation in flow is 2.9 veh/s between 1300 m and 2400 m.

The traffic flow behavior over a circular road of 3000 m with $\beta = 0.15$ and 0.95 over a time span of 75 s is presented in Figures 4.12 and 4.13 respectively. Figure 4.12 shows that with $\beta = 0.15$, the variation in flow after 75 s is 2.9 veh/s with a maximum flow of 4 veh/s at 2400 m. Figure 4.13 shows that with $\beta = 0.95$, the variation in flow is 4.7 veh/s with a maximum flow of 5.8 veh/s at 2000 m. This shows that in more severe weather, the majority
of vehicles are located over a smaller distance and have a smaller variation in velocity.

This example shows that the proposed model can characterize traffic under different weather conditions. The PW model has the same response for all weather conditions and so cannot characterize the traffic during adverse weather.
Figure 4.1: The proposed model density behavior on a circular road with $\beta = 0.5$, relaxation time $\tau = 0.5$ s, safe time headway $\alpha = 2$ s and safe distance headway $H = 25$ m.
Figure 4.2: The PW model density behavior with $C_o = 15$ m/s on a circular road.
Figure 4.3: The proposed model velocity behavior on a circular road with $\beta = 0.5$, relaxation time $\tau = 0.5$ s, safe time headway $\alpha = 2$ s and safe distance headway $H = 25$ m.
Figure 4.4: The PW model velocity behavior with $C_o = 15$ m/s on a circular road.
Figure 4.5: The proposed model flow behavior on a circular road with $\beta = 0.5$, relaxation time $\tau = 0.5$ s, safe time headway $\alpha = 2$ s and safe distance headway $H = 25$ m.
Figure 4.6: The PW model flow behavior with $C_o = 15$ m/s on a circular road.
Figure 4.7: The proposed model density behavior with, $\beta = 0.0$, 0.15 and 0.95, safe time headway $\alpha = 2$ s and safe distance headway $H = 58$ m at 75 s, on a circular road of length 3000 m. The PW model density behavior with $C_0 = 10$ m/s.
Figure 4.8: The proposed model density behavior with $\beta = 0.15$, safe time headway $\alpha = 2$ s and safe distance headway $H = 58$ m on a circular road of length 3000 m.
Figure 4.9: The proposed model density behavior with $\beta = 0.95$, safe time headway $\alpha = 2$ s and safe distance headway $H = 58$ m on a circular road of length 3000 m.
Figure 4.10: The proposed model velocity behavior with $\beta = 0.15$ and 0.95, safe time headway $\alpha = 2$ s and safe distance headway $H = 58$ m at 75 s on a circular road of length 3000 m.
Figure 4.11: The proposed model flow behavior with $\beta = 0.15$ and $0.95$, safe time headway $\alpha = 2$ s and safe distance headway $H = 58$ m at $75$ s on a circular road of length $3000$ m.
Figure 4.12: The proposed model flow behavior with $\beta = 0.15$, safe time headway $\alpha = 2 \text{ s}$ and safe distance headway $H = 58 \text{ m}$ on a circular road of length 3000 m.
Figure 4.13: The proposed model flow behavior with $\beta = 0.95$, safe time headway $\alpha = 2 \text{s}$ and safe distance headway $H = 58 \text{ m}$ on a circular road of length 3000 m.
Chapter 5

Contributions and Future Work

5.1 Contributions

Adverse weather has a significant impact on driving safety. Tire friction on a road is reduced with snow, ice, and compacted snow which leads to a higher accident rate. In this thesis, a model was developed to characterize traffic flow based on the surface conditions of the road. In adverse weather, traffic with the proposed model had less variations in velocity and density than with PW model. These results are more realistic than with the PW model. Traffic with the PW model had oscillatory behavior. The velocity behavior of vehicles with the PW model showed drastic changes, which is not expected in adverse weather conditions. In adverse weather, drivers are cautious so small changes in traffic density and velocity are expected. With the proposed model, traffic became smoother over time than with PW model, which is more realistic in adverse weather. The variations in velocity were not as large as with the PW model and stayed within the velocity limits. The small variations in traffic density and velocity with the proposed model during adverse weather are a better characterization of traffic flow. The PW model cannot characterize traffic flow in different weather conditions due to the use of the velocity constant $C_0$. Conversely, the proposed model employs a severity index based on road surface conditions to characterize the traffic
flow in normal and adverse weather.

5.2 Future Work

The future work proposed is as follows.

A macroscopic flow model can be developed to characterize traffic behavior based on visibility conditions. In low visibility, the probability of an accident increases. The effect of visibility on the traffic can be examined under different weather conditions.

The model proposed in this thesis can be compared with real data to examine the accuracy of the traffic behavior with the model during adverse weather conditions. This can lead to model improvements.

The distance headway distribution in adverse weather conditions can be examined. This distribution depends on driver behavior, visibility and road surface conditions and is an important factor in understanding traffic dynamics.

The effect of vulnerable road users such as pedestrians, cyclists and motor bikes on traffic flow in adverse weather conditions can be considered. The traffic behavior changes significantly under different conditions and therefore can create road safety issues for these users. Research is needed to study the effect of different weather conditions such as fog, hail, rain, wind, snow and ice on pedestrians, cyclists and motor bikes.

This thesis focused on the effect of snow, compacted snow and ice, but rain can also significantly affect traffic behavior. Thus, a study can be conducted to investigate the effect of rain on traffic.
Bibliography


