Bivariate Extreme Value Analysis of Commodity Prices

by

Matthew Joyce
BSc. Economics, University of Victoria, 2011

A Thesis Submitted in Partial Fulfillment
of the Requirements for the Degree of

Masters of Arts

in the Department of Economics

© Matthew Joyce, 2017
University of Victoria

All rights reserved. This thesis may not be reproduced in whole or in part, by photocopy or other means, without the permission of the author.
Supervisory Committee

Bivariate Extreme Value Analysis of Commodity Prices

by

Matthew Joyce
BSc. Economics, University of Victoria, 2011

Supervisory Committee

Dr. David E.A. Giles, (Department of Economics)
Supervisor

Dr. Judith A. Clarke, (Department of Economics)
Departmental Member
Abstract

Supervisory Committee
Dr. David E.A. Giles, (Department of Economics)
Supervisor

Dr. Judith A. Clarke, (Department of Economics)
Departmental Member

The crude oil, natural gas, and electricity markets are among the most widely traded and talked about commodity markets across the world. Over the past two decades each commodity has seen price volatility due to political, economic, social, and technological reasons. With that comes a significant amount of risk that both corporations and governments must account for to ensure expected cash flows and to minimize losses. This thesis analyzes the portfolio risk of the major US commodity hubs for crude oil, natural gas and electricity by applying Extreme Value Theory to historical daily price returns between 2003 and 2013. The risk measures used to analyze risk are Value-at-Risk and Expected Shortfall, with these estimated by fitting the Generalized Pareto Distribution to the data using the peak-over-threshold method. We consider both the univariate and bivariate cases in order to determine the effects that price shocks within and across commodities will have in a mixed portfolio. The results show that electricity is the most volatile, and therefore most risky, commodity of the three markets considered for both positive and negative returns. In addition, we find that the univariate and bivariate results are statistically indistinguishable, leading to the conclusion that for the three markets analyzed during this period, price shocks in one commodity does not directly impact the volatility of another commodity’s price.
# Table of Contents

Supervisory Committee .................................................................................................................. ii
Abstract ........................................................................................................................................ iii
Table of Contents ............................................................................................................................ iv
List of Tables .................................................................................................................................... v
List of Figures ............................................................................................................................... vi
Acknowledgments ......................................................................................................................... vii
Chapter 1 – Introduction ................................................................................................................ 1
Chapter 2 – Extreme Value Theory and Statistical Approaches .................................................. 4
  2.1 Generalized Pareto Distribution ............................................................................................. 4
    2.1.1 Univariate Case ......................................................................................................... 4
    2.1.2 Multivariate Case .................................................................................................... 5
  2.2 Peak over Threshold Model ................................................................................................... 6
  2.3 Parameter Estimation ............................................................................................................. 7
  2.4 Bias-Correction of Maximum-Likelihood Estimators of Parameters ................................. 8
Chapter 3 – Measures of Extreme Risk ....................................................................................... 10
  3.1 Calculation of VaR and ES .................................................................................................... 11
Chapter 4 – Modelling the Tails of the Distribution using EVT ................................................ 13
  4.2 Preliminary & Exploratory Analysis ..................................................................................... 15
  4.3 Determination of Thresholds ............................................................................................... 17
Chapter 5 - Computation of Extreme Risk Measures ................................................................. 20
  5.1 Point Estimates of Univariate VaR and ES ......................................................................... 20
    5.1.1 Interpretation of Univariate Results ......................................................................... 21
  5.2 Point Estimates of Bivariate VaR and ES ......................................................................... 22
    5.2.1 Interpretation of Bivariate Results ......................................................................... 22
Chapter 6 - Conclusions ............................................................................................................... 25
Bibliography .................................................................................................................................. 27
Appendix ........................................................................................................................................ 29
  Appendix A – QQ Plots & Histograms ...................................................................................... 29
  Appendix B - ME and TC Plots ............................................................................................... 39
List of Tables

Table 1 - Series threshold selections and number of observations ........................................ 19
Table 2 - Point Estimates and Standard Errors of Univariate VaR and ES - Positive Returns .......................................................... 20
Table 3 - Point Estimates and Standard Errors of Univariate VaR and ES - Negative Returns .................................................................................................. 20
Table 4 - Point Estimates and Standard Errors of Bivariate VaR and ES - Positive Returns .................................................................................................. 24
Table 5 - Point Estimates and Standard Errors of Bivariate VaR and ES - Negative Returns .................................................................................................. 24
List of Figures

Figure 1 - Daily Commodity Prices (2003-2013) ................................................................. 13
Figure 2 - PJM Daily Returns ................................................................................................. 14
Figure 3 - WTI Daily Returns ................................................................................................. 15
Figure 4 - Henry Hub Daily Returns .................................................................................... 15
Figure 5 – Histogram & Descriptive Statistics of PJM Daily Returns .............................. 16
Figure 6 - QQ plots of PJM daily returns against the Student's t distribution (left) and normal distribution (right) .................................................................................. 17
Figure 7 - Mean Excess Plot, WTI ...................................................................................... 18
Figure 8 - Threshold Choice Plot, WTI ............................................................................ 18
Acknowledgments

I would first like to thank my thesis supervisor, Dr. David E. A. Giles, professor of the Department of Economics, University of Victoria. Your extensive knowledge, overwhelming support and patience contributed significantly to the success and completion of this thesis.

I would also like to thank Dr. Judith Clarke, Department of Economics, University of Victoria and Dr. Farouk Nathoo, Department of Mathematics and Statistics, University of Victoria, as the second reader and external member, respectively, of this thesis. Your comments on this thesis were invaluable, and I thank you for your contribution.

Finally, I must express my utmost gratitude to my parents, Jim and Kathleen Joyce, and my wife, Cayley Joyce, for the continuous encouragement and support throughout my years of study, especially while writing this thesis. This accomplishment would not have been possible without you. Thank you.
Chapter 1 – Introduction

Over the past two decades, both utilities and oil & gas markets have seen times of extreme volatility, so much so that it is now becoming the norm. In the crude market, we have seen two major swings over the past 10 years, the first coming in 2008 during the financial crisis, where in July, the WTI market peaked at 147.27 US Dollars per barrel (USD/bbl). Almost six months later, in January 2009, the same market hit bottom at 32.07 USD/bbl. Within that time, there had been one-day declines in value of up to 22%. The most recent crisis in the crude market occurred from 2015-2016, where the price of crude peaked at 107 USD/bbl in June 2014, and steeply declined for over 18 months, where crude hit its lowest price since May 1999 at 26.05 USD/bbl. This decline has put dozens of companies out of business, and resulted in several mergers to keep companies afloat.

The gas market has seen a drastic decline in value over the past decade, with its price dropping over 85% since 2005. The Henry Hub gas market (the largest hub in the world) has also had two large swings since 2005. The first took place in October 2005, when natural gas rose to an all-time peak at 13.42 US Dollars per one million British Thermal Units (USD/MMBtu), only to drop to 4.90 USD/MMBtu 11 months later, resulting in an over 60% decline in value. The second volatile period took place in 2008, as gas prices followed oil prices from its rise and fall. The value of gas more than doubled in less than a year, rising from 6.08 USD/MMBtu in September 2007 to 12.69 USD/MMBtu in June 2008. The rise was short lived, as prices then dropped to under 3 USD/MMBtu in September 2009, and has not recovered since that time, with prices steadily hovering between 2-4 USD/MMBtu, and tend to have large percentage changes day-over-day due to its low value.

The electricity market continues to be one of the most volatile markets, globally. The uncertainty in demand, supply, asset maintenance, and cash flow inherently contribute to the volatility and risk involved in the market. It is quite common to see deregulated markets selling electricity at sub-zero prices in order to avoid shutting down and starting up operations, due to the inability to store the commodity. PJM Interconnection is the largest wholesale competitive electricity market in North America, and delivers a large portion of electricity across the United States. Similarly to oil and gas markets, PJM wholesale prices changed drastically between 2008 and 2009, where we saw prices as high as 152.59 US Dollars per Megawatt hour (USD/MWh)
(July 2008), and as low as 27.10 USD/MWh (September 2009), seeing as high as 63% day-over-day changes.

Extreme fluctuations in prices across these three commodities motivates this thesis. The goal is to understand how price shocks behave, and how to mitigate risk within a portfolio that may contain one or several commodities. Furthermore, with the three examples noted above, it has also become increasingly important to research how price variability in one commodity can affect price risk of another. For example, it would seem that the natural gas price is highly determined by the price of oil, especially when daily variability of oil is higher than normal levels, such as the 2008 financial crisis. The question then becomes: Does the variability in gas have a significant impact on oil price volatility?

To answer this, we must first define an approach to model risk, and determine a measurement to understand the maximum losses a portfolio can take, given extreme conditions. Extreme value theory (EVT) models the tail of the distribution, while assessing the probability distribution of extreme values. Potential risk can be estimated based on the assumption that previous risks can provide distributional information for future extremes (e.g., see McNeil, 1999). EVT, derived from probability theory, provides a parametric form for the tail of the distribution, which makes it an attractive technique when measuring risks. As EVT allows us to model the tails of the distribution, we can define an appropriate technique to measure the risk of a portfolio. Two common measures to calculate risk are Value-at-Risk (VaR) and Expected Shortfall (ES). The statistic VaR is defined as the maximum loss or gain of a portfolio in a given time horizon for a specific level of confidence. The Expected Shortfall is the average of losses that equal or exceed VaR. VaR has become the standard risk measure across many industries including oil & gas, utilities, financial services, and insurance, among others. In this thesis, we use EVT models to calculate both univariate and bivariate VaR and ES estimates across the major oil, gas, and electricity hubs in the United States.

There is a significant amount of literature supporting the use of EVT when dealing with highly volatile markets, not only in commodities, but across several financial markets as well. Within the financial and insurance markets, Embrechts et al. (1997), Reiss and Thomas (1997), Danielsson and de Vires (2000), McNeil and Frey (2000), and Gilli and Këllezi (2006) have conducted research on the topic. Research focused on commodities markets has increased
steadily over the past decade, including that of Ren and Giles (2010), who look at estimating VaR and ES using EVT in the Edmonton crude oil market. In addition, Krehbiel and Adkins (2005) examine risk in the US gas market, while Chen and Giles (2017) analyze risk in the precious metals markets. Unfortunately, there is little research focused on multivariate analysis across commodities. The most notable study on multivariate modelling of extreme values is Coles (2001), who focuses on modelling multivariate extremes (limited to the bivariate case). We also analyze the bivariate case using the methodology described by Coles (2001). Although this methodology has been used in a variety of applications, this is the first time it has been used in the context of commodity markets.

Specifically, we analyze daily one-month-forward oil (WTI), gas (Henry Hub), and electricity (PJM) prices between 2003 and 2013. We look at both univariate and bivariate risk measurements using extreme value theory, utilizing the peak-over-threshold (POT) method. The parameters are calculated by bootstrapping the MLE’s of the GPD distribution, then bias-corrected due to the small amount of observations used to calculate our risk measures. We undertake the work using the POT (Ribatet, 2006) package in the R environment.

The remainder of this thesis is as follows: Section 2, which presents an overview of extreme value theory and its components, first describes the generalized Pareto distribution (GPD), and the statistical approach to implementing GPD through the peak-over-threshold (POT) method. We also detail how the parameter estimates are calculated by maximizing the log-likelihood function with the GPD. Section 3 elaborates on our risk measurements, VaR and ES, and their calculation. In Section 4 we detail the underlying data and threshold selections. The VaR and ES point estimates for both univariate and bivariate measures are provided in Section 5. Closing remarks are given in Section 6.
Chapter 2 – Extreme Value Theory and Statistical Approaches

Extreme value theory (EVT), which has become increasingly popular within the insurance and finance industries over the past 20 years, focuses on modelling stochastic behavior of rare events. Specifically, EVT provides an estimate of the probability of rare events, which are then used to determine tail risk measures. There are two methods to model extremes: threshold models, and block maxima models. Block maxima models focus on the distribution of block maxima using a generalized extreme value distribution. Threshold modelling considers extreme values over a specified threshold using a generalized Pareto distribution. Block maxima models have a drawback due to the fact that it only uses partial information within a chosen block size (e.g., see Ren and Giles, 2010). Threshold models, on the other hand, use all information above the specified threshold, which allows for more efficient parameter estimators. With the availability of daily data, it seems sensible to use the threshold method for our analysis. The foundation behind threshold models is now discussed in this section.

2.1 Generalized Pareto Distribution

2.1.1 Univariate Case

The generalized Pareto distribution function is defined as follows:

\[
G_{\xi,u,\beta}(x) = \begin{cases} 
1 - \left(1 + \xi \left(\frac{x-u}{\beta}\right)\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\
1 - e^{-\left(\frac{x-u}{\beta}\right)} & \text{if } \xi = 0
\end{cases}
\]  

(1)

where (i) \(x - u \geq 0\) when \(\xi \geq 0\), and \(0 \leq x - u \leq -\beta/\xi\) when \(\xi < 0\); and (ii) \(\beta > 0\).

The excess distribution above a certain threshold \(u\) for a random variable \(X\) is referred to as the excess distribution function \(F_u\), and is defined as

\[
F_u(x) = P(X - u \leq x | X > u)
\]

for \(0 \leq x < x_F - u\) where \(x_F \leq \infty\) is the right endpoint of \(F\). The variable \(x\) represents the exceedances over \(u\). The excess distribution function can also be written as

\[
F_u(x) = \frac{F(x + u) - F(u)}{1 - F(u)}
\]
Pickands-Balkema-de Haan Theorem (Pickands, 1975; Balkema and De Haan, 1974):

It is possible to find a positive measurable function $\beta$, where $\beta$ is a function of $u$, such that

$$\lim_{u \to x_p} \sup_{0 \leq x \leq x_p - u} |F_u(x) - G_{\xi,\beta}(x)| = 0 \text{ iff } F \in MDA(H_\xi(x))^3.$$

To expand, as the threshold $u$ becomes large, the excess distribution function of the extremes that are greater than $u$ is approximately a generalized Pareto distribution (GPD). With this, conditional on a high threshold and positive $\beta$, we are able to use the following within our risk measures:

$$F_u(x - u) \approx G_{\xi,\beta}(x) \quad u \to \infty$$

where the exceedances $(x - u) > 0$ and $G_{\xi,\beta}(x - u) = G_{\xi,u,\beta}(x)$.

The generalized Pareto distribution describes the limiting distribution of exceedances beyond a chosen high threshold. The Pickands-Balkema-de Haan Theorem is important as it explains that if block maxima have a GEV distribution, then the values above the specified threshold will follow a GPD distribution. Given a GEV can be represented as

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} \exp \left( - \left( 1 + \frac{x - \mu}{\xi} \frac{\sigma}{\sigma} \right)^{-\frac{1}{\xi}} \right) & \text{if } \xi \neq 0 \\ \exp \left( - e^{-\frac{x - \mu}{\sigma}} \right) & \text{if } \xi = 0 \end{cases}$$

the relationship between the GEV and GPD is represented by $\beta = \sigma + \xi (u - \mu)$. The shape parameter $\xi$, identical between the GEV and GPD distributions, is the key factor describing the characteristics of the tail of the GPD distribution (Coles, 2001). When $\xi > 0$, the GPD can be described as having fat tails, the tail of the distribution will continue to grow as $\xi$ increases from 0, and the distribution is unbounded. On the other hand, when $\xi < 0$, the excess distribution is bounded and can be described as being Pareto type II. For this reason, it is beneficial for our study to have $\xi > 0$.

2.1.2 Multivariate Case

When we have multivariate extremes it is standard to transform the data to some particular distribution. The most common choice is the Fréchet distribution, for which $\Pr[X < x] = \ldots$
exp\((-1/x)\) (Smith, 1994; Smith et al., 1997; Bortot and Coles, 2000). If \(Y\) denotes the original data, then the margins are transformed as:

\[
Z_j = -1/\log[F_j(Y_j)] ,
\]

where \(F_j\) is the distribution function for the \(j\)th margin. In the case of the peak over threshold analysis, these marginal distributions will be generalized Pareto, with different shape and scale parameters.

Then, a multivariate extreme value distribution in dimension \(d\) has the following representation:

\[
G(y_1, y_2, \ldots, y_d) = \exp[-V(z_1, z_2, \ldots, z_d)] ,
\]

where the function, \(V\), is called the “exponential measure” (Klüppelberg and May, 2006), and is an homogeneous function of order -1.

In the multivariate case (and in contrast to the univariate case), there is an infinity of possible \(V\) functions. Some parametric families for \(V\) that are commonly used, and which are considered in our own analysis for the bivariate modelling in this thesis, include the logistic model, the asymmetric logistic model, the negative logistic model, the asymmetric negative logistic model, the “mixed” model, and the asymmetric “mixed” model.

By way of example, in the bivariate case the logistic model for \(V\) takes the form:

\[
V(z_1, z_2) = [z_1^{-1/\alpha} + z_2^{-1/\alpha}]^\alpha ; \quad 0 \leq \alpha \leq 1 .
\]

The parameters of the marginal generalized Pareto distributions are estimated, as usual by Maximum Likelihood.

### 2.2 Peak over Threshold Model

The Pickands-Balkema-de Haan Theorem allows us to use the peak over threshold (POT) model for our study, which focuses on the distribution of exceedances above a chosen high threshold. With the POT model, we are able to use all available information in order to obtain the most accurate results in estimating the properties of the tail distribution.

Coles (2001) shows that the excess distribution function can be re-written as
\[ F_u(x - u) = \frac{F(x) - F(u)}{1 - F(u)} \], \text{when } x - u \geq 0 \quad (3) \]

where \( x \) represents the exceedances over the threshold \( u \). Rearranging, we have

\[ F(x) = (1 - F(u))F_u(x - u) + F(u). \]

First, we must choose an appropriate threshold to allow us to fit the GPD distribution function. Assuming that the threshold is chosen correctly (i.e., it is sufficiently high such that the distribution follows GPD) we are then able to estimate \( F(u) = 1 - N_u/n \), where \( n \) is the total number of observations, and \( N_u \) represents the number of observations above the threshold \( u \), using historical sample data. The Pickands-Balkema-de Haan Theorem allows us to apply maximum likelihood estimation to estimate \( F_u(x - u) \) by a GPD approximation. The tail estimator can then be obtained as

\[ \hat{F}(x) = \frac{N_u}{n} \left( 1 - \left( 1 + \frac{x - u}{\hat{\beta}} \right)^{-\frac{1}{\hat{\xi}}} \right) + \left( 1 - \frac{N_u}{n} \right), \]

which can be simplified to

\[ \hat{F}(x) = 1 - \frac{N_u}{n} \left( 1 + \frac{x - u}{\hat{\beta}} \right)^{-\frac{1}{\hat{\xi}}} \]

(4)

where \( \hat{\xi} \) and \( \hat{\beta} \) are the shape and scale parameter estimators, respectively. Methods of threshold determination are discussed later in Section 4.3.

2.3 Parameter Estimation

The scale parameter \( \beta \) and shape parameter \( \xi \) are estimated by maximizing the log-likelihood function for the GPD. The resulting estimators will be weakly consistent and asymptotically efficient. To illustrate, consider the probability density function of the GPD. As long as the sample data \( \{x_1 - u, \ldots, x_n - u\} \) follow a GPD distribution, where \( x_i - u \geq 0 \) for \( \xi > 0 \) and \( 0 \leq x_i - u \leq -\frac{\beta}{\xi} \) for \( \xi < 0 \), the individual density function in logarithmic form (Azzalini, 1996) is

\[ \log f(x_i) = -\log(\beta) - \frac{1 + \xi}{\xi} \log \left( 1 + \xi \frac{x_i - u}{\beta} \right). \]
When $\xi = 0$, the individual density is
\[
\log f(x_i) = -\log(\beta) - \frac{1}{\beta} (x_i - u).
\]

Assuming independence\(^1\) of the sample data, the logarithm of the joint density function, which is equivalent to the log-likelihood function $L(\xi, \beta|x_i - u)$ for the GPD, is:

\[
L(\xi, \beta|x_i - u) = \begin{cases} 
-n\log(\beta) - \frac{1 + \xi}{\xi} \sum_{i=1}^{n} \log \left(1 + \xi \left(\frac{x_i - u}{\beta}\right)\right) & \text{if } \xi \neq 0 \\
-n\log(\beta) - \frac{1}{\beta} \sum_{i=1}^{n} (x_i - u) & \text{if } \xi = 0
\end{cases}
\]

Obtaining the parameter estimates by maximizing the log-likelihood function, (5), for a chosen threshold, requires the use of a numerical algorithm to solve the resulting non-linear first-order conditions.

2.4 Bias-Correction of Maximum-Likelihood Estimators of Parameters

When examining extreme values, it is typical to estimate parameters of the GPD from a small sample of observations, particularly when using the peak-over-threshold methodology. In this case, applying bias-correction techniques to the MLEs becomes important (Giles et al., 2016). As Giles et al. (2016) explain, bias-correcting the MLEs of the parameters of the GPD effectively reduces absolute relative bias, and can also result in a lower relative mean squared error.

Following Giles et al. (2016), the approximate biases of the MLEs of the scale and shape parameter are, respectively:

\[
\text{Bias}(\hat{\xi}) = -(1 + \xi)(3 + \xi)/[n(1 + 3\xi)] + O(n^{-2})
\]

\[
\text{Bias}(\hat{\beta}) = \frac{\beta(3 + 5\xi + 4\xi^2)}{[n(1 + 3\xi)]} + O(n^{-2}).
\]

\(^1\) The independence requirement is met in our study given that we are analyzing the daily percentage change in price, not the price levels themselves.
Using estimators of these biases (replacing the unknown parameters with their MLEs) we subtract these from the original MLEs of the scale and shape parameters. The bias-corrected parameter estimates are then used to calculate our risk measures, to be explained in Section 3.
Chapter 3 – Measures of Extreme Risk

Since the early 1990’s, Value-at-Risk (VaR) has become the standard in measuring market risk within the financial and energy industry. The Basel II Accord\(^2\), originally implemented in 2004, mandated that VaR was the preferred risk measurement for market risk. While allowing flexibility for each firm to create their own models, strict guidelines and procedures were required by law. VaR measures the maximum potential losses of a given portfolio with a specific confidence interval, typically over a one-day period. Effectively, VaR evaluates the extreme tail of daily returns using historical price data. With \( X \) being a random variable with a continuous cumulative distribution function \( F \), VaR can be defined as

\[
VaR_p = F^{-1}(1 - p)
\]

where \( p \) is some probability of exceeding VaR, and \( F^{-1} \), the inverse of the distribution function, is the associated quantile function.

Although VaR is the preferred measure of risk from the industry perspective, it does not come without its shortcomings, as it often does not give the full picture of the aspects of risk that a portfolio faces. For example, a 1% one-day VaR of $1 million denotes that there is a 1% probability that losses will exceed $1 million on a given day, which does not estimate the loss if the loss exceeds the VaR. The area of concern is based on the unknown of whether a loss greater than the VaR of $1 million will be $2 million, or $500 million, and so on. Typically, the Expected Shortfall (ES) is considered to measure this expected loss, conditional that the loss exceeds the VaR. Artzner \textit{et al.} (1997, 1999) conclude that the ES is a coherent risk measure\(^3\), and can be expressed as:

\[
ES_p = E((X|X > VaR_p))
\]

for risk \( X \) at a given probability \( p \) (Artzner, 1997).


\(^3\) A coherent risk measure is described by Artzner \textit{et al.} as a risk measure that satisfies 4 properties: monotonicity, subadditivity, homogeneity, and translational invariance.
3.1 Calculation of VaR and ES

The VaR and ES statistics can be computed in two ways – by using either parametric or non-parametric methods. The parametric approach relies on historical data and assumes that these data follow a normal distribution. As commodity returns tend to demonstrate heavy tailed and skewed distributions, this normality assumption can lead to inaccurate VaR and ES results. On the other hand, non-parametric approaches, such as Monte-Carlo simulation and so-called historical simulation calculate VaR by first estimating future values, and incorporating those into the VaR calculation. These methods are considered more sophisticated than parametric models by dropping the assumption of normality, however they do make the assumption that the distribution of past returns provides a good guide to the distribution of future returns; e.g., see Li et al. (2012). As an alternative, we turn to extreme value theory using a Generalized Pareto Distribution (GPD). This method allows us to analyze just the tail section of the data, perhaps resulting in more accurate estimators of the VaR and ES measures.

Mathematically, for a given probability \( p \), the tail of the distribution can be calculated as the inverse of the tail estimator formula provided in Section 2

\[
\hat{x}_p = u + \hat{\beta} \left( \frac{n}{N_u} \right)^{-\hat{\xi}} - 1
\]

where \( \hat{\xi} \) and \( \hat{\beta} \) are the maximum likelihood estimators of the GPD parameters. Since VaR is defined as an extreme quantile (e.g. 95th percentile), it can be estimated using the estimator:

\[
\hat{VaR}_p = u + \hat{\beta} \left( \frac{n}{N_u} \right)^{-\hat{\xi}} - 1
\] (8)

We can then estimate the ES using that:

\[
\hat{ES}_p = \hat{VaR}_p + E(X - \hat{VaR}_p | X > \hat{VaR}_p)
\] (9)

where the second term represents the mean of the excess distribution \( F_{\hat{VaR}_p}(x) \) above the threshold \( \hat{VaR}_p \). By applying the Pickands-Balkema-de Haan Theorem, the excess distribution above the threshold \( \hat{VaR}_p \) is a GPD as long as the threshold for \( 1 - p > F(u) \) is sufficiently high. This implies the relationship (McNeil, 1999),

\[
F_{\hat{VaR}_p}(x) = G_{\xi,\beta+\xi(\hat{VaR}_p-u)}(x).
\]
With this, the mean of the excess distribution can be calculated as

\[
(\beta + \xi (V aR_p - u))/(1 - \xi)
\]

where \( \xi < 1 \). Substituting this into equation (9), the estimator of the ES can be written as

\[
\hat{ES}_p = \frac{\hat{VaR}_p}{1 - \hat{\xi}} + \frac{\hat{\beta} - \hat{\xi} u}{1 - \hat{\xi}}
\]

or,

\[
\hat{ES}_p = \frac{1}{1 - \hat{\xi}} \left( u + \frac{\hat{\beta}}{\hat{\xi}} \left( \left( \frac{n}{\xi N_p} \right)^{-\hat{\xi}} - 1 \right) \right) + \frac{\hat{\beta} - \hat{\xi}}{1 - \hat{\xi}}.
\]

The standard errors of these estimators are calculated using the delta method, which is derived based on the central limit theorems and a truncated Taylor series expansion, described by Greene (2003, p.914). As Ren and Giles (2010) state, the shape and scale parameter estimators are asymptotically normally distributed since both parameters are maximum likelihood estimators. This allows us to calculate the standard errors by taking the square root of the associated variance, calculated as:

\[
\text{var}(G(\hat{\mu})) = \frac{\partial G(\hat{\mu})}{\partial \hat{\mu}} \text{cov}(\hat{\mu}) \frac{\partial G(\hat{\mu})}{\partial \hat{\mu}^T}
\]

where \( \hat{\mu} \) is the vector of the scale and shape parameter estimators, \( \hat{\beta} \) and \( \hat{\xi} \), \( G(\hat{\mu}) \) is a function of \( \hat{\mu} \), and \( \text{cov}(\hat{\mu}) \) is the variance-covariance matrix of \( \hat{\beta} \) and \( \hat{\xi} \).
Chapter 4 – Modelling the Tails of the Distribution using EVT

The markets of focus in this thesis within the U.S. oil, gas, and electricity markets are the Western Texas Intermediate (WTI), Henry Hub (HH), and Pennsylvania-New Jersey-Maryland Interconnection (PJM), respectively between 2003 and 2013. The WTI market is used as the benchmark for oil pricing, and is the underlying commodity of the Chicago Mercantile Exchange oil futures contracts. The Henry Hub market is the distribution hub location for natural gas in the United States, and is used as the benchmark price traded for the New York Mercantile Exchange (NYMEX) and Intercontinental Exchange (ICE) futures contracts. The PJM market is the largest deregulated electricity market globally, serving a large portion of the Northeast United States. All prices used coincide with the one-month forward contract for each series. After the exclusion of daily returns that are equal to zero, which arise with non-trading days/holidays, we analyze the daily returns of each price set.

Figure 1 - Daily Commodity Prices (2003-2013)

From Figure 1, which shows the daily price for each market from 2003 to 2013, it is evident that there is substantial price variability within the PJM and WTI markets throughout the eleven year period. From 2003 to 2009, the gas market prices also showed high variance at times, but volatility has moderated over the past several years. The trend between the power and crude market was similar from 2003 to 2009, with substantial upward trends from 2003-2007, and a common downfall from 2008-2009 during the financial crisis, when the price of oil dropped from $150 to under $40 in under 12 months. The relationship between the power and gas prices
also follows a similar trend throughout, with both having significant rises in mid-2005 and early 2008.

Figures 2-4 show the daily returns for each series, defined as (natural) logarithmic differences, and expressed in percentage terms. Each price series contains varying numbers of observations for gains and losses day-over-day. The PJM series contains 1122 positive and 1170 negative daily return values; the WTI data contains 1317 positive and 1193 negative returns; while the Henry Hub series contains 1212 positive and 1284 negative returns. It is evident that power returns are extremely volatile, with several gains and losses reaching between 40-60% in a single day. The WTI series is less volatile, however high volatility clusters are evident for a period in 2006, as well as in 2008, relating to the price drop during the financial crisis. The Henry Hub returns, on the other hand, show very little volatility over our data sample, and this continues to be a low risk, low reward investment.

![Figure 2 - PJM Daily Returns](image-url)
Preliminary & Exploratory Analysis

Although we do not expect the daily returns to be normally distributed, we perform the Jarque-Bera (JB) test, and assess the QQ plots for each series. The Jarque-Bera test (Jarque and Bera, 1980) tests the null hypothesis that the data are normally distributed against the alternative that they are non-normally distributed. The test is constructed using sample skewness and kurtosis sample statistics. The JB test statistic is:

\[ JB = \frac{n}{6} (S^2 + \frac{1}{4} (K - 3)^2), \]

where \( n \) is the number of observations, \( S \) is the sample skewness coefficient, and \( K \) is the sample kurtosis coefficient. Skewness and excess kurtosis are both zero under the null
hypothesis. For the three series, the JB statistics are very large with a p-values that are essentially zero, leading us to the conclusion that returns are not approximated well by a normal distribution. The histogram and sample statistics for the PJM returns are shown in Figure 5 below, while those for the WTI and HH series can be found in the Appendix. The high values of the sample kurtosis coefficients for each series (PJM =13.03, WTI=13.17, HH=21.62) suggests fat tails within the distributions. In addition, each sample distribution is skewed to the right, leading us to model the left and right tails separately in order to analyze their distinct qualities.

![Histogram of PJM Returns](image)

<table>
<thead>
<tr>
<th>Observations</th>
<th>2292</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.00007</td>
</tr>
<tr>
<td>Median</td>
<td>-0.00118</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.57278</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.63687</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.07412</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.34053</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>13.03144</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>9654.446894</td>
</tr>
<tr>
<td>Probability</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Figure 5 – Histogram & Descriptive Statistics of PJM Daily Returns

The quantile-quantile plots allow us to assess whether the data is consistent with a pre-defined distribution. In this case, we assess goodness of fit to a normal and Student’s t distribution (with three degrees of freedom). The quantile function Q is the generalized inverse function of the cumulative distribution function F:

\[ Q(p) = F^{-1}(p) \text{ where } 0 < p < 1 \]

A QQ plot suggests a match between the empirical and theoretical distributions if the points within the plot are roughly on the 45-degree line. A plot that shows deviations from this line, finishing in an “S” shape, suggests heavy tails relative to the hypothesized distribution. Figure 6 shows the QQ plots of the PJM series against a normal and Student-t distribution. Although a Student’s t distribution (with three degrees of freedom) provides a better fit to the empirical quantiles than does the normal distribution, it is evident that there is still a clear deviation at the tails, suggesting that the returns data are not compatible with a normal distribution or a Student-t
distribution. The WTI and HH series show similar results (see the Appendix). For this reason, we do not proceed with estimating Value-at-risk and Expected Shortfall under the assumption of a normal or Student’s t distribution, as is sometimes done in the literature.

Figure 6 - QQ plots of PJM daily returns against the Student’s t distribution (left) and normal distribution (right)

4.3 Determination of Thresholds

The POT method of calculating VaR and ES requires us to choose a specific threshold for the returns data to analyze and model the tails of the distribution. Threshold selection is a subjective process, so we are best to use multiple approaches to confirm the selection results. In turn, our threshold determinations were assisted by two plots: a Threshold Choice (TC) plot and a Mean Excess (ME) plot; e.g., see McNeil (1997), Danielsson and de Vires (1997). The ME plot involves plotting the mean excess function (the mean of the exceedances over a certain threshold) for a range of threshold values, \( u \), which is linear and positively sloped for an underlying generalized Pareto distribution. Meanwhile, the TC plot (also known as the parameter stability plot), plots the scale and shape parameter estimates from the GPD for a range of thresholds. The shape and scale parameter estimates under GPD should be stable above a certain threshold, therefore the threshold is chosen where the two parameters begin to vary (e.g., see Wang, 2016). The goal of choosing an approximate threshold is to have the optimal data points in order to minimize variance and bias associated with our parameter estimators, while also
making sure that we are analyzing only the tail of the distribution. That is, we want to select a threshold that is high enough such that we know the data follows a generalized Pareto distribution, while also low enough to reduce the estimator variance such that arises when there are very few observations in the tail of the empirical distribution for our data.

![Mean Residual Life Plot](image)

**Figure 7 - Mean Excess Plot, WTI**

![Threshold Choice Plot](image)

**Figure 8 - Threshold Choice Plot, WTI**

For both plots, the threshold is chosen based on where instability in the plots begins. As seen in Figures 7 and 8, instability in the plots for the WTI gains series occurs around 0.038. The same procedure is used to choose thresholds for both gains and losses of the returns data for the
other commodities. The thresholds and number of observations used to calculate the MLE’s are shown in Table 1. The TC and MRL plots for the other series can be found in the Appendix.

<table>
<thead>
<tr>
<th>Threshold Selection</th>
<th>Positive Returns</th>
<th>Negative Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PJM</td>
<td>WTI</td>
</tr>
<tr>
<td>Threshold</td>
<td>0.060</td>
<td>0.038</td>
</tr>
<tr>
<td>Observations</td>
<td>263</td>
<td>119</td>
</tr>
</tbody>
</table>

Table 1 - Series threshold selections and number of observations
Chapter 5 - Computation of Extreme Risk Measures

5.1 Point Estimates of Univariate VaR and ES

Now that we have a defined distribution for the tail modelling of the commodity returns data, we can estimate separate models for each tail of the returns distributions for each of the commodities. These results are then used as a baseline when we analyze the differences between bivariate risk measures. The calculated estimates of VaR and ES for each commodity for both positive and negative returns are given in Tables 2 and 3 below. These estimates are calculated using equations (8) and (11) in Section 3.1.

Value-at-Risk & Expected Shortfall – Positive Returns

<table>
<thead>
<tr>
<th>Market</th>
<th>VaR 95%</th>
<th>VaR 99%</th>
<th>ES 95%</th>
<th>ES 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>PJM (Threshold = 0.06)</td>
<td>0.1609 (0.0074)</td>
<td>0.2941 (0.0218)</td>
<td>0.2310 (0.0117)</td>
<td>0.3743 (0.0241)</td>
</tr>
<tr>
<td>WTI (Threshold = 0.038)</td>
<td>0.0461 (0.0011)</td>
<td>0.0752 (0.0123)</td>
<td>0.0847 (0.0071)</td>
<td>0.1450 (0.0403)</td>
</tr>
<tr>
<td>Henry Hub (Threshold = 0.05)</td>
<td>0.0737 (0.0025)</td>
<td>0.1344 (0.0100)</td>
<td>0.1092 (0.0073)</td>
<td>0.1892 (0.0196)</td>
</tr>
</tbody>
</table>

Table 2 - Point Estimates and Standard Errors of Univariate VaR and ES - Positive Returns

Value-at-Risk & Expected Shortfall – Negative Returns

<table>
<thead>
<tr>
<th>Market</th>
<th>VaR 95%</th>
<th>VaR 99%</th>
<th>ES 95%</th>
<th>ES 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>PJM (Threshold = 0.06)</td>
<td>0.1490 (0.0063)</td>
<td>0.2693 (0.0189)</td>
<td>0.2134 (0.0102)</td>
<td>0.3584 (0.0210)</td>
</tr>
<tr>
<td>WTI (Threshold = 0.038)</td>
<td>0.0506 (0.0016)</td>
<td>0.0930 (0.0075)</td>
<td>0.0769 (0.0070)</td>
<td>0.1374 (0.0245)</td>
</tr>
<tr>
<td>Henry Hub (Threshold = 0.06)</td>
<td>0.0636 (0.0006)</td>
<td>0.0983 (0.0052)</td>
<td>0.0870 (0.0041)</td>
<td>0.1302 (0.0125)</td>
</tr>
</tbody>
</table>

Table 3 - Point Estimates and Standard Errors of Univariate VaR and ES - Negative Returns
5.1.1 Interpretation of Univariate Results

In this study, we look at two measures; Value-at-Risk and Expected Shortfall. Value-at-Risk (VaR) measures the one-day worst case (negative returns) and best case (positive returns) performance of a portfolio given a certain probability. The three industries of focus typically look at VaR from a negative perspective, determining how much of their portfolio they should hedge, based on maximizing profits, while maintaining a certain risk tolerance (i.e. minimizing expected losses).

Focusing on the negative returns first, we see that the electricity market (PJM) has significant day-over-day variability in comparison to the crude oil (WTI) and gas (Henry Hub) markets. This feature is expected given the fact that electricity demand can vary based on a number of factors in a given day, in addition to the fact that electricity is extremely difficult, if not impossible, to store. The VaR(0.95) for PJM was calculated at 0.149. That can be interpreted as, with 95% probability, the drop in value of PJM electricity in a given day will not exceed 14.9%. In other words, an investment of $1 million would expect to lose no more than $149,000 in a given day, with 95% probability. In comparison, VaR(0.95)’s for the WTI and Henry Hub markets are estimated to be 0.051 and 0.064, respectively. By comparison, Ren and Giles (2010) did a similar study using EVT, applying VaR to the Edmonton crude market between 1998-2006. They calculated a VaR(0.95) of 0.034. It is not unexpected that WTI’s VaR(0.95) is slightly higher, due to the higher trade volume in US markets (resulting in higher volatility), as well as our historical data incorporating data from the 2008-2009 financial crisis, which saw extremely high daily volatility during that time. Although it might seem unusual that the natural gas market has slightly higher risk associated with it than does the crude oil market, given the industry decline over the past decade, the risk difference likely comes from a low value by comparison. For example, a shift of $1 in crude oil has much less impact than a $1 shift in natural gas, therefore we will often see higher volatility in the gas market.

Comparing negative returns to positive returns, there is a consistent trend that the right tail VaR and ES estimates are often larger than their left tail counterparts by one to two percentage points, with the exception of the estimates for the WTI market, which are slightly lower in the right tail. With this result, we can assume that the behavior difference between the two tails is
small, which can be interpreted as players on both sides of the market (i.e. bear and bull) face similar risks.

Tables 2 and 3 also provide estimates for Expected Shortfall, which represents the best and worst case scenarios if price shocks were to exceed the estimated VaR. For example, the left-tail VaR\(_{(0.95)}\) estimate for WTI is 0.051, whereas the ES\(_{(0.95)}\) estimate is 0.077. This means that if the one-day value drops more than 5.1%, the worst case expected scenario with 95% probability is a 7.69% drop in value. That the ES estimate is higher than the VaR estimate is expected, given that the ES is a coherent risk measure that estimates the excepted loss (or gain) that exceeds a given VaR. We see that the Expected Shortfall results often suggest a 30-50% higher return/loss if the estimated VaR were to be breached.

There are three takeaway messages from our point estimates of VaR and ES: (1) the point estimates at both confidence levels are in line with previous research; (2) the risk within the electricity market far outweighs that of the crude and natural gas markets; (3) the difference in VaR and ES estimates between the tails is small, therefore both sides of the markets face similar risks;

5.2 Point Estimates of Bivariate VaR and ES

Now that we have defined our univariate baseline, we now estimate the effects of one-day price shocks across the different commodity markets. Using the same thresholds as in our univariate analysis, we apply the same methods to our bivariate models to calculate VaR and ES estimates for each commodity by including another commodity as a variable within the Pareto distribution.

5.2.1 Interpretation of Bivariate Results

Tables 4 and 5 below show the bivariate results of the right and left tail VaR and ES at the 95\(^{th}\) and 99\(^{th}\) percentile for each pair of commodity data. Focusing on Table 5 (left tail), we see that for each commodity, the results are very similar across the bivariate analyses using different pairs of commodity data. For example, the PJM market has an estimated VaR\(_{(0.95)}\) of 0.1614 and 0.1619 under the PJM/HH bivariate case. This relationship is consistent across all commodities, and for both tails, in that both the
VaR and ES estimates do not differ by more than 1 percentage point between combinations within a commodity.

The differences between the left and right tail estimates of VaR and ES also behave similarly in the bivariate case as compared to the univariate case. Specifically, there is a consistent trend that the positive tail results in larger VaR and ES estimates by one to two percentage points across the board, with the exception of VaR and ES estimates for the WTI market, which again has a slightly lower statistic in the right tail.

Finally, we analyze the differences between the bivariate and baseline results. There is very little change between the two approaches, with most results only varying within less than 1% at the two provided significance levels. For example, the WTI market’s left tail estimated VaR$(0.95)$’s are 0.0508 and 0.0510 from the bivariate analysis, while the univariate estimate is 0.0506 with 95% probability. The Henry Hub market’s left tail estimates of the VaR are 0.0875 and 0.0865 from the bivariate analysis, compared with 0.0870 for the univariate baseline case. This feature of minor differences between the univariate and bivariate VaR and ES estimates is consistent across the three markets with the data that has been analyzed. This suggests that price shocks in one commodity do not directly impact variability in other commodity prices. This could be possible for a variety of reasons, most likely due to the fact that there may be a multi-day lag between price shocks, especially between the gas and crude oil markets. For instance, if there is a one-day shock in the price of WTI crude oil, the shock may not be felt in the Henry Hub gas market for several days or weeks. Various possible lag structures can be put in place to assess the effects of the bivariate case, and would be valuable to investigate in future studies in order to incorporate better decision making in the industry.

The characteristics of the estimates from the bivariate analysis can be summarized as follows: (1) the VaR and ES estimates are numerically very similar for every combination of each commodity; (2) the difference between the left and right tail estimates of ES and VaR are small, numerically; (3) electricity remains the most volatile commodity regardless of approach; (4) price shocks in commodities do not seem to effect volatility in other commodity prices, given that the univariate and bivariate outcomes are similar.
### Value-at-Risk & Expected Shortfall – Positive Returns

<table>
<thead>
<tr>
<th>Price Combination</th>
<th>PJM</th>
<th>WTI</th>
<th>Henry Hub</th>
<th>Henry Hub</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VaR 95%</td>
<td>ES 99%</td>
<td>VaR 95%</td>
<td>ES 99%</td>
</tr>
<tr>
<td>PJM/WTI</td>
<td>0.1614 (0.0074)</td>
<td>0.2951 (0.0219)</td>
<td>0.3758 (0.0242)</td>
<td>0.0465 (0.0012)</td>
</tr>
<tr>
<td>PJM/HH</td>
<td>0.1619 (0.0075)</td>
<td>0.2961 (0.029)</td>
<td>0.3766 (0.0242)</td>
<td>-</td>
</tr>
<tr>
<td>WTI/HH</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0461 (0.0012)</td>
</tr>
</tbody>
</table>

Table 4 - Point Estimates and Standard Errors of Bivariate VaR and ES - Positive Returns

### Value-at-Risk & Expected Shortfall – Negative Returns

<table>
<thead>
<tr>
<th>Price Combination</th>
<th>PJM</th>
<th>WTI</th>
<th>Henry Hub</th>
<th>Henry Hub</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VaR 95%</td>
<td>ES 99%</td>
<td>VaR 95%</td>
<td>ES 99%</td>
</tr>
<tr>
<td>PJM/WTI</td>
<td>0.1494 (0.0064)</td>
<td>0.2732 (0.0198)</td>
<td>0.3543 (0.0226)</td>
<td>0.0508 (0.0017)</td>
</tr>
<tr>
<td>PJM/HH</td>
<td>0.1493 (0.0064)</td>
<td>0.2707 (0.0192)</td>
<td>0.3482 (0.0215)</td>
<td>-</td>
</tr>
<tr>
<td>WTI/HH</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0510 (0.0017)</td>
</tr>
</tbody>
</table>

Table 5 - Point Estimates and Standard Errors of Bivariate VaR and ES - Negative Returns
Chapter 6 - Conclusions

The oil and gas and utilities markets have seen significant volatility over the past decade. So much so, that both organizations and governments across North America have implemented regulations to ensure structured and reliable risk management measures. In turn, Value-at-Risk and Expected Shortfall have become the most common measures of risk to better understand expected cash-flow and maximum losses in cases of extreme events. Although there are many ways to implement these measures, we explore using extreme value theory to estimate the risk associated with extreme events in these highly volatile markets. We estimated VaR and ES from univariate and bivariate models in three major hubs using daily prices of the one-month futures markets. Parameter estimates and extreme value thresholds were determined using the peak-over-threshold method in order to apply EVT. To incorporate bias-reduction due to the low amount of data above the threshold used to determine the distribution of the tails, we have bias-corrected the maximum likelihood estimates of the GPD parameters.

Our results suggest that electricity is undoubtedly the most volatile commodity market of the three we examined, exhibiting VaR and ES estimates that are almost double those of the oil and gas markets, at least for the two significance levels we considered. In addition, we find that when modelling the right and left tails separately, expected positive returns are slightly higher than the expected negative losses but the differences are minor. Finally, we find that when estimating VaR and ES from a bivariate model, two observations are worth stating; first, with the prices analyzed, price shocks in the three commodities measured do not seem to affect risk measures for another market, due to the VaR and ES estimates being very similar between our univariate and bivariate analysis across the three commodities. Second, this is the first application of the bivariate modelling in markets of this type. Even though we found that our bivariate results were numerically very similar to the univariate case, we should not conclude that the bivariate analysis is invaluable, and suggest using this methodology in other markets in the future. Organizations that incorporate a mixed-commodity portfolio can incorporate both the univariate and bivariate methodology towards their hedging and trading strategies to maximize expected cash-flow and minimize potential losses.

As our research used one-month forward prices as a proxy for the markets spot prices, it would be valuable to calculate risk measures using extreme value theory using daily spot prices.
in future research. Futures prices can incorporate political, social, and economical uncertainty, which creates higher variability than spot prices typically will present over time. Therefore, spot prices would create a more accurate risk measure, and would be valuable to investigate in the future. Furthermore, further research is recommended on incorporating various lag structures when undertaking bivariate analysis to determine other effects of price shocks across commodity markets.
Bibliography


Appendix

Appendix A – QQ Plots & Histograms

a) Henry Hub All Returns

![Quantile-Quantile (QQ) plot of Henry Hub All Returns against Normal distribution and Student's t distribution.](image)

- Quantiles of HHA vs. Quantiles of Normal distribution.
- Quantiles of HHA vs. Quantiles of Student's t distribution.
b) PJM All Returns

Quantiles of PJMA

Quantiles of Normal

Quantiles of Student's t
c) WTI All Returns

Quantiles of WTIA

Quantiles of Normal

Quantiles of Student's t
d) Henry Hub Positive Returns

Quantiles of Normal

Quantiles of Student's t
e) Henry Hub Negative Returns

Quantiles of Normal vs. Quantiles of RHN

Quantiles of Student's t vs. Quantiles of RHN
f) PJM Positive Returns

![Graphs showing quantiles of Normal, RPP, and Student's t distributions.](image-url)
g) PJM Negative Returns

Quantiles of RPN vs Quantiles of Normal

Quantiles of RPN vs Quantiles of Student's t
h) WTI Positive Returns

![Graph showing quantiles of Normal and Student's t distributions compared to quantiles of RWP.](image)

- Quantiles of Normal
- Quantiles of Student's t
- Quantiles of RWP
i) WTI Negative Returns
Histgrams of returns

Histgram of HH

Histgram of PJM

Histgram of WTI
Appendix B - ME and TC Plots

Positive Returns
PJM

Mean Residual Life Plot

Modified Scale

Shape
Negative Returns
PJM

Mean Residual Life Plot

Threshold
Mean Excess

0.00 0.02 0.04 0.06 0.08 0.10

0.00 0.02 0.04 0.06 0.08 0.10

Modified Scale

0.00 0.02 0.04 0.06 0.08 0.10

Threshold

Shape

0.00 0.02 0.04 0.06 0.08 0.10
WTI

Mean Residual Life Plot

Threshold

Mean Excess

Modified Scale

Shape
Henry Hub

Mean Residual Life Plot

Modified Scale

Shape

Threshold

Threshold