Numerical Simulation of Landslide-Generated Tsunamis with Application to the 1975 Failure in Kitimat Arm, British Columbia, Canada

by

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Abstract

Numerical modeling was carried out for both the underwater landslide and the associated tsunami which occurred near Kitimat, British Columbia, Canada on April 27, 1975. The subaqueous slope failure was modeled as a Bingham visco-plastic fluid (debris flow) based on previous geotechnical investigations at the site. A Bingham fluid is determined by two rheologic parameters (yield stress and viscosity) and provides more realistic debris flow representation than a Newtonian fluid. A long wave approximation was utilized for the hydrodynamic equations of the landslide and the resultant tsunami waves. The landslide-generated tsunami wave and debris flow equations were solved numerically using a finite-volume Godunov-type scheme. This method resolves abrupt wave and landslide front interactions and remains oscillation-free.

The computed motion of the debris flow was generally consistent with observations from earlier surveys; simulations indicated that the failure continued approximately 4.5 km down the axis of the fjord. Survey results showed that it extended about 5 km; the difference can be, in part, attributable to hydroplaning, as documented by survey results, that was not included in the modeled landslide behaviour. Computed amplitudes for the tsunami wave crest at the coast of Kitimat Arm were between 6 and 11 m; these values are somewhat higher than previous simplistic solitary wave theory estimates of 6.3 m and 8.2 m based on observations of high water marks along coastline of Kitimat Arm.
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To my son Alexey ...
1 Introduction

1.1 General overview

Tsunamis are ocean waves triggered by impulsive geologic events such as sea floor deformation (faulting), submarine landslides, slumps, subsidence, volcanic eruptions and bolide impacts. They can inflict significant damage and casualties both nearfield and after evolving over long propagation distances, impacting distant coastlines. Understanding tsunami generation and evolution is of paramount importance for protecting coastal populations, coastal structures and the natural environment. A major tsunami in the open ocean is several hundred of kilometers in wavelength and up to a meter high. It is long and low amplitude compared to the ocean depth of several kilometers. Approaching the shore, the wave height increases as the water depth decreases.

Tsunamis generated by tectonic displacement faults often cause widespread damage. The enormously destructive tsunami at the end of 2004, produced by a strong earthquake in the Indian Ocean, caused many casualties and significant devastation over wide areas. Protection from natural disasters is now one of the priorities all over the world. Due to the steady growth in coastal development over the last fifty years, future tsunamis could be even more catastrophic than historic ones. Rapid progress in tsunami science and engineering is needed for mitigating this deadly hazard.

Tsunamis can also be produced by masses entering the sea from elevations above sea level or by submarine landslides. Although landslide-generated tsunamis are more localized than seismically generated tsunamis, they still produce catastrophic waves and cause significant coastal run-up, especially where the wave energy is trapped by the confines of inlets or fjords. Recently it has been recognized that this generation mechanism was overlooked by tsunami research in the past; tsunami studies were biased toward those induced by large earthquakes in offshore subduction zones. But today, especially after several catastrophic tsunamis caused by a huge submarine landslides, the general consensus has been reached on the need to improve our understanding of tsunamis induced by mass movements underwater (Tinti, 2002).
Landslides are often the secondary effects of strong earthquakes. For example, the Alaska earthquake of July 10, 1958 triggered a rockslide at the head of Lituya Bay, Southeast Alaska, causing a giant tsunami that impacted sides of the inlet to a height of 517m (Miller, 1960; Lander, 1996).

Theoretical investigations of waves generated by moving underwater bodies (Harbitz & Pedersen, 1992; Pelinovsky & Poplavsky, 1996; Tinti & Bortolucci, 2000) help to understand the basic processes of energy transfer from the body to the wave, but the interaction is certainly very complex and demands more complex models to simulate the slide, slump or rockfall dynamics. This is a field where important progress is needed.

1.2 Project intent

Earthquakes, landslides, volcanic eruptions and resultant tsunamis are usually quite complex and need very sophisticated modeling to simulate the physical evolution of the processes and their impact on land and communities. Significant effort is needed to develop tools that are simple and practical, but, most importantly, to clarify the assumptions that are the basis of the model as well as the reliability of the results. The latter is crucial, especially for infrequent events, such as catastrophic tsunamigenic processes, since it is always difficult and often even impossible to develop statistical approaches and to use concepts such as probability of occurrence (Tinti, 2002).

The intent of this work is to demonstrate a capability of modeling for tsunami risk assessment and to improve upon previous studies of tsunami risk, based on reasonable submarine landslide scenarios. In this work we derive and solve numerically a 3d mathematical model of the landslide-generated tsunami, using various assumptions related to different types and volumes of landslides. It provides a good mathematical framework to analyze, estimate and predict tsunami wave heights and times of propagation for areas where a landslide-generated tsunami has occurred or may occur.

Numerical modeling of tsunamis caused by submarine slides and slumps is a much more complicated problem than simulation of seismically generated tsunamis. Durations of the slide deformation and propagation are sufficiently long that they
affect the characteristics of the surface waves. As a consequence, the interaction between a landslide body and surface waves must be considered (Fine et al., 2002).

Using this model I present a geotechnically reasonable scenario of a submarine slide which occurred on April 27, 1975 in Kitimat Arm on the west coast of British Columbia, generating tsunami waves which swept along the shores of the inlet. Information about sediments in the Kitimat area, the seabed morphology and a range of likely values of physical properties are utilized.

The thesis is organized in the following way. In the second chapter a variety of slope failures, landslide-generated tsunamis and the “state-of-the-art” of tsunami and landslide modelling is reviewed. In the third chapter differential equations for the mathematical model are derived which I use for the simulation of landslide-generated tsunamis. In the fourth chapter I present a numerical approach to the solution of the problem and also provide some numerical test results. The last chapter is devoted to the modeling of the underwater failure in the Kitimat Arm, British Columbia in 1975. I provide a comparison of computed and observed water levels at the coastline during this tsunami.

Landslide-generated tsunami modeling is a major challenge because of its difficulty and importance. Can scientists predict the consequences of an underwater landslide or slump for a given continental margin morphology, based on minimal sedimentary and geologic characterization? This is a valid question to estimate correctly the possible risk due to this natural phenomenon.
2 Literature review

2.1 Tsunamis

2.1.1 Occurrence of landslide-generated tsunamis

In the world ocean tsunamis occur due to underwater fault movement, submarine slides, volcanic eruptions, human activity, such as explosions, and other factors. The main interest in the scope of this present study is to address landslide-generated tsunamis as a result of a failure of sediments along steep fjords, banks, on continental slopes and within submarine canyon systems. These tsunamis are produced because of underwater or subaerial mass movements of soil or rock driven by gravity. Landslide tsunamis remain one of the least studied generation mechanisms, in part because their occurrence is concealed and many of the events remain undetected (Miller, 1960; Hamilton & Wigen, 1987; Watts, 2001).

The identification of slide-generated tsunamis is sometimes possible from the historical catalogs of tsunamis. Despite the fact that parametric tsunami catalogs contain very limited information on a particular event, the preliminary identification of landslide-generated events in the catalogs is possible on the basis of several criteria such as the width of the area with maximum run-up values. The world-wide catalog of tsunamis and tsunami-like events covers the period from 1628 B.C. to 2000 A.D. and contains nearly 2250 historical events that occurred in almost all parts of the world ocean, in many marginal seas as well as in lakes and inland reservoirs. The catalog gives many examples of historical events where involvement of subaerial and submarine landslides in the tsunami generation was clearly observed and well documented (Gusiakov, 2002).

Most susceptible to landslide-generated tsunamis are the regions of the Pacific and Atlantic coasts of Canada, Alaska, Norwegian fjords (Bjerrum, 1971; Karlsrud & Edgers, 1980), the coast of New Zealand, The Netherlands (Silvis & de Groot, 1995; Stoutjesdijk & de Groot, 1997), the coast of The Levant, Israel (Miloh & Striem, 1978) in Yanahuin Lake, Peru, Japan, Mediterranean Sea coasts (Heinrich, 1992), and the coasts of Kamchatka and Indonesia (Gusiakov, 2002).
2.1 Tsunamis

One of the largest prehistoric submarine landslides with an estimated volume of 1700 km³ occurred 7000 B.C. in the North Sea at the edge of the continental shelf in Norway (the Storrega slide). The resultant tsunami hit a large part of the Scottish coast with heights up to 6-8 meters (Harbitz, 1992).

Among the best known examples of extreme events in recent history is a well documented 517 meter run-up in Lituya Bay (Alaska) caused by a massive landslide occurring after the magnitude 7.8 earthquake of July 10, 1958 in south-eastern Alaska. Less well-known cases of extreme run-up heights in the same bay were the 1936 and 1853 events with maximum run-up heights of 150 and 120 meters, respectively (Gusiakov, 2002).

Local submarine landslides occurring during the 1992 Flores Island earthquake (Indonesia) of M = 7.5 generated destructive tsunami waves with heights up to 26.2 m, with catastrophic consequences, including 1713 casualties (Imamura & Gica, 1996).

One of the most fatal cases occurred on October 9, 1963 in Italy, when a massive rock slide fell into a water reservoir in the Vaiont Valley, resulting in a wave that destroyed a town and killed 3000 people (Wiegel et al., 1970).

A very common trigger of landslides and subsequent tsunamis is an earthquake. For example the Grand Banks earthquake on Nov. 18, 1929 in the Atlantic Ocean, southwest of Newfoundland triggered turbidity currents, which caused numerous cable breaks in the Atlantic Ocean. Resultant tsunami amplitudes were estimated to have been of at least 12.2m in Burin Inlet on the south coast of Newfoundland; 26 people were killed (Murty, 2000; Rabinovich et al., 2001).

In the inlets and narrow straits of the Pacific coast of North America (e.g. Lituya Bay, Yakutat, Russel Fjord, Skagway Harbor, Kitimat Arm, Tacoma) landslide-generated tsunamis occur frequently and are accompanied by significant run-up (Soloviev & Go, 1975; Lander, 1996; Palmer, 1999; Evans, 2001).

In many cases catastrophic submarine landslides occurred because of local processes in the absence of seismic events. These are often related to construction activities, which can also be coincident with other factors (Prior et al., 1983). Low tides, as well as certain hydrometeorological factors such as rainfall, strong winds, atmospheric pressure change can help trigger coastal landslides (Bornhold et al., 1994; Ren et al., 1996).
A well-known example is the event of October 16, 1979, when part of the Nice International Airport on the French Riviera slumped into the Mediterranean Sea during landfilling operations for the airport expansion. The tsunami generated by the submarine landslide was observed near Nice and Antibes and resulted in the deaths of several people (Assier-Rzadkiewicz et al., 1997).

A catastrophic underwater landslide occurred in Skagway Harbor, southeast Alaska. On November 3, 1994 a 250 m section of the Pacific and Arctic Railway
and Navigation Company (PARN) under construction on the eastern side of the harbor slid rapidly into the water. The event occurred about 25 min after an extreme low tide. The landslide and accompanying tsunami claimed the life of one worker and caused an estimated $21 million damage (Kulikov et al., 1996; Cornforth & Lowell, 1996; Lander, 1996).

In August 1905, a large landslide took place on the right bank of the Thompson River at Spences Bridge in southwestern British Columbia. The landslide generated a displacement wave in the river that ran up the opposite valley wall to a height of 22.5 m. It destroyed many buildings in the settlement and killed 15 people (Evans, 2001).

On April 27th, 1975, a major submarine landslide occurred in the Kitimat Inlet in the Douglas Channel system of the northern part of the coast of British Columbia (Prior et al., 1983). Water waves with ranges up to 8.2 m were generated (Murty, 1979).

Other locations in British Columbia where landslides have been reported include Howe Sound (Terzaghi, 1956) and the Fraser River delta region (Hamilton & Wigen, 1987; McKenna & Luternauer, 1987, 1992) (Figure 2.1).

Construction sites, buildings, and submarine cables in the areas of potential landslide-tsunamis are at significant risk to the direct damage from subaerial or submarine landslides. Tsunamis generated by the failure events in these areas probably pose an even greater threat in terms of damage and loss of life than tsunamis generated directly by faulting associated with earthquakes. In this respect, it is important to define areas of high landslide risk (especially in new construction zones) and to provide appropriate computations of possible landslide motions and associated tsunamis (Rabinovich et al., 2001).

2.1.2 Landslide-generated tsunami mitigation

A basic activity in the tsunami mitigation effort is hazard assessment. It is necessary for coastal communities to identify populations and assets at risk and to quantify the level of that risk. This requires knowledge of probable tsunami sources, their likelihood of occurrence, and specific characteristics of the resulting tsunamis. For a few coastal regions there are data from historical tsunamis on
2.1 Tsunamis

which to base an assessment of the hazard. For most places, however, only very limited or no past data exist. In those cases, numerical modelling studies must be carried out to produce synthetic data for study. The precise estimation of possible tsunami wave heights along the coast is of prime importance. Tsunami wave overestimation greatly increases construction costs, while underestimation significantly increases the risk of destruction including death. Whatever the method, accurate hazard assessment is essential for motivating and designing other aspects of mitigation such as warning systems, land use, public education and engineering works (Charles, 1998).

There are two different approaches for estimating tsunami risk. One is based on historical precedents; i.e., on analysis of tsunami run-up observed at a specific site in the past and application of methods of extreme statistics (Go et al., 1985; Rabinovich & Shevchenko, 1990). The other method is based on numerical modelling of historical and design earthquakes or submarine landslides with associated tsunamis (Hebenstreit & Murty, 1989; Dunbar et al., 1991; Mofjeld et al., 1999).

The best way is to combine these two methods; i.e., to use observational run-up data to verify a numerical model, and to use a numerical simulation to extend observational results. Unfortunately, there are usually very few or no data for important areas. For these, numerical models (verified with existing data where possible) are the only means by which it is possible to obtain estimates of tsunami risk (Bernard, 1998). Tsunami risk predictions based on numerical simulation of potential tsunamis for existing coastal regions have become an important branch of modern coastal engineering (Mofjeld et al., 1999).

The risk reduction or mitigation of landslide-generated tsunamis has a number of specific features that are different from seismogenic tsunamis:

- Numerical simulations of earthquake-generated tsunamis are normally based on historical seismic parameters (source characteristics) or on parameters of hypothetical earthquakes. For constructing a model of landslide-generated tsunamis it is possible to use actual parameters of the unstable sediment body estimated by geotechnical or geophysical methods.
2.1 Tsunamis

- Tsunami-warning, which is so important for open ocean tsunamis, has little application to landslide-generated tsunamis because normally the time interval between the event (landslide, slump, or rock fall) and tsunami waves affecting coastal areas is negligible.

- Based on present capabilities, it is not possible to release the accumulated energy of a pending earthquake in order to prevent any associated catastrophic tsunamis. It may be possible in specific cases to incrementally trigger subaerial or submarine sediment slides (in the same manner as for avalanches) to prevent sediment from accumulating in dangerous amounts and generating significant tsunamis. Using numerical modelling, it is straightforward to consider various scenarios and define the corresponding “triggering” strategy (Rabinovich et al., 2001).

- Seismically generated tsunamis are natural phenomena which occur independently of human activity. In contrast, landslide-generated tsunamis are often the direct result of construction activity in coastal areas (Bjerrum, 1971). Seismically generated tsunamis are induced by the impulsive displacements of the sea floor during undersea earthquakes. Because the duration of the earthquakes is very short (a few seconds), the interaction between the tectonically induced motions and the surface waves is unimportant. In such cases, the classical Cauchy-Poisson initial value problem is valid for earthquake-induced tsunamis. However, for landslide-generated tsunamis, the duration of the slide deformation and propagation is sufficiently long that it affects the characteristics of the surface waves. As a consequence, the Cauchy-Poisson model is not valid, and we must take into account the effect of the slide body on surface waves. The landslide itself evolves significantly during its movement. The dynamic changes of the slide body are interesting themselves, and they also significantly influence the generated surface waves (Jiang & LeBlond, 1992, 1994; Rubino et al., 1994; Assier-Rzadkiewicz et al., 1997).
2.2 Landslide Mechanics

2.2.1 Classification of landslides

A landslide is a general term used to describe the down-slope movement of soil, rock and organic materials under the influence of gravity. Many definitions have been applied to the term and they all vary depending on the objective of the author. For example, Cruden (1991) defined a landslide as, “a movement of a mass of rock, earth or debris down a slope”. However, Hutchinson (1988) gave a more detailed interpretation. He suggested that slope movements could be categorized into individual groups based on the mechanism of failure. Hutchinson classified them as: fall, topple, rotational and translational slides, lateral spreading, flow and complex. The common types of landslide according to the above criteria are seen as slope failures, mudflows, rock falls and rock slides (Hutchinson, 1988).

Landslides vary in size from an individual boulder to large rock masses that form mountainsides, and involve materials ranging from muds to massive rocks. Landslides can be extremely rapid or very slow and occur in a wide variety of environments, including underwater. They display movement modes ranging from sliding of relatively intact masses of rock or soil to the flow of completely disaggregated materials (Evans, 2001). The horizontal scales of coastal and submarine landslides typically range from a few hundred to a few thousand meters. Some large continental slope slides, such as the Storrega Slides, which occurred on the Norwegian continental slope, have scales of 20-30 km; however they remain much smaller than scales of typical seismic sources (Harbitz, 1992).

Observations show there is much in common between underwater failures and subaerial landslides (Figure 2.2). This classification is proposed for mass movements by the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE TC-11). The schema is modified for underwater failures to include turbidity currents, which are solely a submarine phenomenon.
2.2 Landslide Mechanics

All the types indicated here (Figure 2.2) are mutually exclusive; for instance, a slide cannot be a fall. But in nature complex landslides do occur, when two or more of these mass movements are seen together. Basic movement types have different characteristics:

- Slides are recognizable by the step-like morphology indicative of little disruption of the failed mass. In a slide, displaced material moves on a relatively thin zone of intense strain. They can be subdivided into rotational and translational types. A rotational slide is a land motion in which the slip surface of failure closely follows the arc of a circle. In translational slide the mass moves down along a relatively planar surface and has little to no rotational movement.

- Topples involve rock or soil that tilts and/or rotates forward on a pivot point. There is not necessarily much displacement, however it may lead to falls or slides of the displaced material.

- Spreads are sudden horizontal movements on very gentle terrain. They are often initiated by earthquakes that liquefy the layer below the moving material.

- Falls occur when a rock and/or soil detaches from the slope and moves rapidly to its new resting place. They are often associated with undercut cliffs and riverbanks. In a fall, the displaced mass falls, bounces, and rolls.
2.2 Landslide Mechanics

- Flows vary in type of material and speed. Soil and rock can move by less than 1.5 cm per year (classed as a very slow flow) to over 5 m/s (classed as a rapid flow). Flows commonly occur in steeper terrain where there are already landslides or where intensive farming has stripped the land of vegetation. Heavy rainfall loosens soil and rock, creating a flow of sediment. In a typical flow the source area and main pathway fan out as soil and rock collect at the bottom of the slide. At the extreme case for flows in the marine environment, the slide area will be emptied and the failed mass may be deposited hundreds of kilometers away from the source (Schwab et al., 1996).

- Turbidity currents occur only in the subaqueous environment. These flows are highly turbid and carry large quantities of clay, silt, and sand in suspension, flowing down a slope beneath less dense sea water.

Certainly, one type of mass movement can lead to another; e.g. a slide can transform into a flow. One could introduce subdivisions, but the terms presented (Figure 2.2) cover most of the observed phenomena (Prior, 1984; Norem et al., 1990; Mulder & Cochinat, 1996).

2.2.2 Underwater failures

Submarine slides have created much geotechnical interest because they damage dock facilities and often break buried conduits and cables at considerable distances from the original failure site. They are also a cause of secondary effects such as tsunamis, which can be destructive, particularly in confined fjords and bays. Submarine landslides are commonly observed on continental slopes where the steeper part of the margin increases the effect of gravity on the downslope forces acting on a certain volume of sediment.

Submarine slides present serious problems for coastal engineering. Often a submarine failure starts from the initially solid state. If this initial slide involves large quantities of saturated, loosely deposited fine sand and silt, it will evolve into a debris flow (Bjerrum, 1971). The underwater mass movement is a continuous
phenomenon as illustrated by the diagram proposed by Meunier (1993) (Figure 2.3).

The diagram classifies landslides according to the concentration of solid particles and water. It has two axes, granular and cohesive, and takes into account the relative proportion of solids and water. Therefore, depending on the type of mixture, the behavior of the failure will be best analyzed by soil-rock mechanics principles, fluid mechanics or torrential hydraulics. If a slide has a very high density and does not disaggregate during its motion, mechanics of solids is most applicable. But if, for example, a slide develops into a mudflow, the rate of movement is fast enough and there is no time for excess pore-water dissipation, the mechanics of the movement cannot be adequately explained by soil mechanics but rather by fluid mechanics principles (Locat & Lee, 2002).

The driving mechanisms of submarine mass movements will vary according to the causes, but also according to the environment in which the mass movements occur. For example, the Grand Banks slide was triggered by an earthquake,
2.2 Landslide Mechanics

but the open ocean margin provided ideal conditions for the development of a large turbidity current. In the case of debris flows on the Mississippi Fan, the large travel distances of the sediments can only be explained by the presence of a well-developed channel system (Locat et al., 1996; Schwab et al., 1996). Therefore, considering the various stages of mass movement it is an important step to bring together the various driving mechanisms. It has been observed that at the failure stage, soil or rock mechanics principles are needed to explain or predict the stability. However, for the post-failure stage, very often the approach must rely on fluid mechanics principles (Locat & Lee, 2002).

2.2.3 Classification of debris flows

Debris flows occur very often in a subaqueous setting. They slide when masses of poorly sorted sediment, agitated and saturated with water, surge down slopes in response to gravitational forces. Both solid and fluid forces vitally influence the motion, distinguishing debris flows from related phenomena such as rock avalanches and sediment-laden water floods. Whereas solid grain forces dominate the physics of avalanches, and fluid forces dominate the physics of floods, solid and fluid forces must act in concert to produce a debris flow. Other criteria for defining debris flows emphasize sediment concentrations, grain size distributions, flow front speeds, shear strengths, and shear rates (Beverage & Culberson, 1964; Varnes, 1978; Pierson & Costa, 1987).

Description of the physics of debris flows remains an active research topic. The most important aspect of the theory is to correctly understand the dynamics of the motion; i.e. rheology of the debris flow. By definition, rheology is the study of the deformation and flow of matter. The rheology of a flow depends on the material, which forms the landslide, and changes in the nature of the slide as it moves. There are essentially two ways to investigate this:

- Consider water and coarse materials separately. These are often referred to as grain-flow models. These complicated models rely on a detailed knowledge of particles and their size distribution.
- Consider the entire mass (fluid and solids) as one fluid with particular properties.
2.2 Landslide Mechanics

Let us first discuss the grain-flow approach as it is more accurate and shows the internal nature of debris flows. The following classification schema (Figure 2.4) for debris flows in terms of the internal properties of the flow material, such as shear stresses, concentration, size of particles, density and thickness of the flow was proposed by Takahashi (2001).

The vertical axis represents the mean coarse particle concentration $C_{\text{0}}$ in the flow. If coarse particles are not present (at the lowest point on the vertical axis) (i.e., the concentration of solids is very low) a flow is a fluid or a slurry. In water flow, almost all shear stresses are shared by the turbulent Reynolds stress and the higher the viscosity, the higher the viscous stress becomes. Therefore, the flow regime changes along the lowest horizontal axis.

The criteria for the existence of various sediment motions, in which the fluid is water or a slurry are shown in the ternary diagram. The two end members at the apexes are the ratios $\tau_t/\tau$ and $\tau_c/\tau$. The ratio $\tau_t/\tau$ is a function of $H/d_p$, so that as the relative depth of the flow becomes larger, the larger the relative effect of turbulence becomes, compared to the effect of collision. The ratio $\tau_c/\tau_\mu$ ($\tau_\mu$ is approximated as $\tau_\mu$) represents Bagnold’s number (Bagnold, 1954), which classifies the flow as inertial or viscous, and the ratio $\tau_\mu/\tau_t$ is the inverse of the Reynolds number. Therefore, the three axes of the ternary diagrams represent relative depth, Bagnold’s number, and Reynolds number, respectively.

As the concentration increases, but is still less than about $C_4$, a flow starts to exhibit bedload or suspended load depending on the turbulence and viscosity. A flow becomes a debris flow if the mean coarse particle concentration becomes higher than $C_4$ but less than $C_3$. In this case possible dominant stresses include a particle collision stress, a turbulent mixing stress and a viscous stress. The region where the Bagnold’s number is high and the relative depth is small is for stony debris flows. If Bagnold’s and Reynolds numbers are small, a viscous debris flow occurs. The region where the relative depth and the Reynolds number are large corresponds to turbulent muddy debris flows. Thus, the areas close to the three corners are occupied by stony, viscous and turbulent flows, respectively. The rest of the area in the triangle is shared by immature and hybrid debris flows. The boundary lines shown in the ternary diagrams are arbitrary. The shape of the
2.2 Landslide Mechanics

Figure 2.4: Criteria for various sediment motions (Takahashi, 2001)

\( \tau \) - total shear stress
\( \tau_t \) - shear stress due to turbulent mixing and migration of coarse particles
\( \tau_c \) - shear stress due to particle collisions
\( \tau_v \) - shear stress due to viscoplasticity of the material
\( \tau_p \) - viscous stress due to deformation of fluid
\( \tau_s \) - static stress
\( C \) - volume concentration of particles
\( C_1 \approx 0.02 \quad C_3 \approx 0.5 \quad C_2 \approx 0.62 \quad C_* \approx 0.65 \)
\( d_p \) - particle diameter
\( H \) - depth of the flow
\( \rho_t \) - density
2.2 Landslide Mechanics

Boundary lines and the area shared by the respective type of flows should change with concentration $C$ (Takahashi, 2001).

If the mean concentration of particles is high and exceeds $C_3$, collision, turbulent and viscous stresses become small, the quasi-static Coulomb stress becomes dominant, and the flow exhibits quasi-static motion. For concentrations higher than $C_2$ a dislocation of the particles cannot take place and the material becomes rigid (Takahashi, 2001).

The modelling of landslides is a complicated problem which is why the second approach, where a mixture of water and solids is considered as one fluid, is usually utilized. It also has been found that yield-strength fluid models work well for flows with considerable fines (Johnson, 1970). In the next section I discuss this type of rheology model, where a slide is studied as a single fluid.

2.2.4 Rheological models

An emphasis is placed on muddy debris flows, in the present analysis, because they very often occur in underwater settings and this type of failure is a generator of destructive tsunamis. These dilute flows are less common in the subaerial setting, but are still known to occur. Field and laboratory data suggest that most muddy debris flows can be approximately modeled as a linear or nonlinear viscous or viscoplastic material (Johnson, 1970; O'Brien & Julien, 1988; Locat, 1997).

A rheology model for a fluid is defined through a dependence of shear stress on a shear rate for the given laminar flow (Figure 2.5). When fluid flow over a surface is laminar, fluid layers near the boundary move at a slower rate than those further from the surface. The shear rate $\gamma$ is the velocity of a layer relative to the layers next to it or precisely a velocity gradient measured perpendicular to the fluid flow. A shear stress $\tau$ is the resistance to flow developed between fluid layers, due to the shear rate. The value of $\tau_0$ is the yield stress - a stress threshold to be overcome to make one layer of the fluid move over another. Rheological models which describe viscous or viscoplastic behavior are presented in Figure 2.5. The viscous cases are given by the Newtonian and Power law while the viscoplastic are Bingham and Herschel - Bulkley rheological models.
Newtonian is the simplest type of rheology. It is characterized by a constant ratio of the shear stress to the shear rate, which defines a dynamic viscosity of the fluid. Other fluids are called non-Newtonian as their viscosity is dependent on a shear rate. Rather than describing the stress-rate relationship with one constant, as for a Newtonian fluid, two or more constants are required for non-Newtonian fluids. The Herschel-Bulkley rheology is the general case for the rheologies in Figure 2.5. The equation for the Herschel-Bulkley rheology is obtained from the relation (2.1).

\[ \gamma = \begin{cases} 
0 & \tau \leq \tau_0 \\
\frac{\gamma_r}{\tau_0^n} (\tau - \tau_0)^{1/n} & \tau > \tau_0 
\end{cases} \quad (2.1) \]

Here \( \gamma_r \) denotes a reference strain rate, \( n \) is a power parameter. For the case \( \tau > \tau_0 \), we define viscosity \( \mu = \frac{\tau_0}{\gamma_r} \) and the rheology reduces to the simple form (2.2). Equations for other curves in Figure 2.5 are easily obtained.
2.2 Landslide Mechanics

1. Herschel-Bulkley
\[ \tau = \tau_0 + \mu \gamma^n \]  
(2.2)

2. Power-Law (\( \tau_0 = 0 \))
\[ \tau = \mu \gamma^n \]  
(2.3)

3. Bingham (n=1)
\[ \tau = \tau_0 + \mu \gamma \]  
(2.4)

4. Newtonian (\( \tau_0 = 0, \ n=1 \))
\[ \tau = \mu \gamma \]  
(2.5)

There is a major difference between viscous and visco-plastic fluids. Visco-plastic fluids have 2 zones: a shear zone and a distinct plug zone in which there is no deformation. Viscous fluids have only a shear zone, so deformation occurs in every part of the fluid and all layers in the laminar flow move with different speeds relative to each other (Imran et al., 2001).

Besides the rheologies presented above there exist modified/mixed models. For example, Locat (1997) introduced a new rheological model for mud and debris flows based on the original work of Patton (1966) on rock failure. His bilinear model does not distinguish between the plug flow layer and the viscous shear layer. It uses an apparent yield strength, and allows a smooth transition from Newtonian to Bingham behavior as the shear stress increases. The bilinear model assumes that the initial phase of the flow is Newtonian and evolves, after reaching a threshold shear rate value, into a Bingham type flow (Figure 2.6).

The equation proposed by Locat (1997) for the bi-linear flow is expressed as follows:

\[ \tau = \tau_0 + \mu_{dh} \gamma + \frac{\tau_0 + \gamma_0}{\gamma + \gamma_0} \]  
(2.6)

Here \( \mu_{dh} \) - the viscosity in the region (2) (Figure 2.6) and \( \gamma_0 \) - the shear rate at the transition from Newtonian to Bingham behavior. The equation (2.6) can be rearranged into the following convenient form

\[ \frac{\tau}{\tau_0} = 1 + \frac{\gamma}{\gamma_r} + \frac{1}{1 + r \frac{\gamma}{\gamma_r}}, \quad r = \gamma_r / \gamma_0 \]  
(2.7)
In the limit, as $\gamma/\gamma_r$ becomes large (i.e. high shear stresses) (2.7) reduces to the Bingham relation (2.4). If $r(\gamma/\gamma_r) \ll 1$ it corresponds to very low shear stresses and thus the equation (2.7) reduces to the Newtonian relation (2.5) (Locat, 1997). The bilinear model provides a more realistic representation of a stress-strain relationship, especially at low shear rates, and therefore should be considered as a viable alternative to the more popular Bingham model. The experimental data in the work of O'Brien & Julien (1988) shows a good match with the bilinear model (Imran et al., 2001).

The field of landslide modelling is extensive and I have only reviewed what is necessary to understand my modelling approach. Various researchers have developed and applied models of mud and debris flow rheology. These models can be classified as: viscous models (Johnson, 1970; Trunk et al., 1989; Hunt, 1994), linear and nonlinear viscoplastic models (Johnson, 1970; O'Brien & Julien, 1988; Liu & Mei, 1989; Jiang & LeBlond, 1993; Huang & Garca, 1994), dilatant fluid models (Bagnold, 1954; Takahashi, 1978; Mainali & Rajaratnam, 1994) dispersive or turbulent stress models (Arai & Takahashi, 1986; O'Brien et al., 1993; Hunt, 1994) and frictional models (Iverson, 1997).
2.3 Landslide-generated tsunami modelling approaches

2.3.1 General overview

Tsunami modelling involves large scale simulations in the sense that the computational domain covers many characteristics such as velocities, heights, wavelengths and topographic details. Hence there is a strong need to economize, with respect both to resolution and choice of mathematical model. The mathematical models for wave propagation can be divided into three categories:

1. Shallow water equations based on the hydrostatic approximation for pressure;

2. Weakly dispersive theory, such as Boussinesq equations;

3. Fully dispersive theory, where long wave assumptions are invoked. This corresponds to either the Navier-Stokes equations or full potential theory.

The computational cost is markedly increased, by a factor 10 or more, when advancing from hydrostatic equations to, for instance, the Boussinesq equations. Even considering the power of present computers, fully dispersive theory is still too computationally demanding to be applied to anything but idealized or local studies. Most of the current tsunami computations are based on the shallow water theory (Pedersen & Langtangen, 1998).

The problem of modelling tsunamis generated by coastal and submarine landslides is initially related to the problem of describing landslide motions. All known models for landslide-generated tsunami modelling may be classified into several main groups, depending on physical properties of the landslide.

- Solid body models (Heinrich, 1992), in which a landslide is represented by a rigid body that slides as a whole along the slope with allowance made
2.3 Landslide-generated tsunami modelling approaches

for friction between a bed surface and the slide. The slide consists of a well-consolidated substance and retains its shape during movement. Such models better describe tsunami waves generated by underwater rock masses and slumps, or subaerial failures which fall or slide into the ocean.

- Viscous or Newtonian models (Johnson, 1970; Jiang & LeBlond, 1992) better describe landslide processes in the case of fine-grained water-saturated deformable sediments, in particular, oozy deposits. In this process, waves at the sea surface are generated as a consequence of the formation of a submarine debris flow.

- Visco-plastic Bingham models (Liu & Mei, 1989; Jiang & LeBlond, 1993) unite the features of both of the above theories and incorporate a transition from solid body to viscous type models. They are often used to describe landslides consisting of different substances such volcanic lava or marine sediments. The visco-plasticity of a landslide allows one not only to more adequately model the process of tsunami generation by landslide masses moving at the seafloor, but also to consider the problem of the stability of sediments over an inclined seafloor and the probability of a self-generated detachment of a landslide body (Kulikov et al., 1998).

- Rheology models such as Herschel-Bulkley (Coussot, 1994), Bilinear (Locat & Lee, 2002), “Diffusion model” (Heinrich et al., 1998) and other sophisticated approaches, depending on the complexity of the modeled problem and landslide behavior.

2.3.2 Mixture of fluids or “Diffusion model”

An accurate simulation of a slide tsunami involves modelling of the landslide and the generated water waves, considering the interaction between the slide body and the water. To date, few numerical studies have taken into account this interaction. Recently certain authors have modeled such interactions, assuming that a submarine “flowslide” may be modelled as the flow of a dense viscous fluid. For instance, Heinrich et al. (1998) developed a 3D model solving the
full Navier-Stokes equations. In order to simulate the interaction between the debris avalanche and the water column, the full 3D model was based on the Euler equations where water and debris avalanche were considered as a mixture of two fluids.

The mixture was composed of sea water taken as a fluid of density $\rho_1$ and of the debris material treated as a homogeneous fluid of density $\rho_2$. The density of the mixture $\rho$ is defined by the relation $\rho = (1 - c)\rho_1 + c\rho_2$, where $c$ is the volume fraction of sediments. $c = 1$ corresponds to total sediments and $c = 0$ indicates completely water.

The 3D model was based on the 3D hydrodynamics code of Torrey et al. (1987) developed for a single fluid. It solves the Euler equations with a free-surface for a mixture of two incompressible fluids using an Eulerian finite-difference method. The debris avalanche was assumed to be a non-viscous fluid flowing down a frictionless slope and is considered to be non-porous while sliding into water. Friction between the two media was also neglected. The nonlinear governing equations of the mixture were formulated as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \tag{2.8}
\]

\[
\frac{\partial U}{\partial t} + U \cdot \nabla U = g - \frac{\nabla P}{\rho} \tag{2.9}
\]

\[
\frac{\partial F}{\partial t} + \nabla \cdot (FU) = 0 \tag{2.10}
\]

\[
\nabla \cdot U = \frac{\rho_2 - \rho_1}{\rho_1 \rho_2} \nabla \cdot j \tag{2.11}
\]

These are continuity, momentum, transport and diffusion equations, respectively. $U$ is the 3D fluid velocity vector of the mixture; $g$ is the gravity acceleration; $p$ is the pressure; $j$ is the diffusion flux of the fluids mixture; $F$ is the fractional volume of the cell occupied by the mixture and is used to calculate the free-surface evolution. The 3D mixture model was used to calculate water waves generated for a debris avalanche entering the sea. Water waves propagated at further distances were computed by a shallow water model.
2.3 Landslide-generated tsunami modelling approaches

The model of Heinrich is particularly appropriate to masses flowing down steep slopes, where vertical acceleration of the landslide and water cannot be neglected compared to the gravitational acceleration. For gentle slopes and assuming large horizontal dimensions of the landslide compared to the thickness, the shallow water approximation may be used for both water waves and the landslide (Heinrich et al., 1998).

2.3.3 Solid body models

The dynamics of a submarine slide is very important for determining the characteristics of the landslide-generated tsunami. The magnitude of these tsunamis depends on many factors such as seafloor geometry, volume of debris avalanche, acceleration, velocity and extent of the displaced mass. Most tsunami models assume a rigid block slide (Grilli & Watts, 1999; Watts, 2000). Use of the rigid-body model of submarine slides has a long history. Some simple estimates for this model may be obtained analytically (Harbitz & Pedersen, 1992; Pelinovsky & Poplavsky, 1996; Watts, 2000).

Rigid-body models assume that the shape and dimensions of the initial slide remain invariant during the slide motion. All points of the rigid body move with the same velocity \( U = U(t) \) and the position of the slide changes with time through the relation:

\[
D(x, y; t) = D_0(x - X(t), y - Y(t)) \\
X = \int_0^t U dt; \quad Y = \int_0^t V dt
\]  

(2.12)

where \( D_0 \) is the initial slide distribution.

In solving the equations of motion Fine et al. (2002) assume that:

- Bottom friction on the slide is proportional to the integrated normal pressure, \( P \)
- There are no hydraulic forces ("drag") on the slide
- The bottom slope is small, \( \|\nabla h\| << 1 \)
2.3 Landslide-generated tsunami modelling approaches

Under these assumptions, the momentum equation of the slide is formulated as

\[ \rho_2 \frac{dU}{dt} \int_S D ds = P \nabla h - k \frac{U}{|U|} P \tag{2.13} \]

\[ P = g \int_S (\rho_1 \eta + (\rho_2 - \rho_1) D) ds \tag{2.14} \]

where \( k \) is the non-dimensional coefficient of kinetic friction (Coulomb friction coefficient), \( S \) is the surface area of the slide. \( \rho_2 \) and \( \rho_1 \) are the densities of the slide and seawater respectively (Fine et al., 2002).

Next we consider a source of tsunami generation as the simplest 2D \((x, z)\) bottom displacement due to a horizontally moving slide with a constant speed \( U \). In this case we have a uni-dimensional problem and the solution can be found explicitly. The slide (Figure 2.7) is assumed to maintain its initial shape and move in the positive \( x \)-direction on a horizontal seabed. It is at rest for the moment of time \( t < 0 \), moves with constant velocity \( U \) for \( 0 < t < T \), and stops instantaneously at \( t = T \). The length of the slide is \( L \), the height is \( D(x - Ut) \) and the distance from the horizontal sea bed to the undisturbed free surface is denoted by \( H \), \( g \) is the acceleration due to gravity (Harbitz & Pedersen, 1992).

![Figure 2.7: Solid body, simple case.](image)

Tsunamis are classified as long waves. In other words most of the energy that is transferred from the slide to water motion is distributed on waves with typical wavelength much larger that the characteristic water depth. Furthermore, the characteristic amplitude of the waves is much less than the characteristic water depth. Hence, the surface elevation \( \eta \) and the averaged horizontal velocity \( u \)
at time $t$ can be determined by linear, non-dispersive shallow water equations for conservation of mass and momentum for a two-dimensional version with the constant $H \gg D$ (Wu, 1981; Pedersen, 1989),

\[
\begin{bmatrix}
\frac{\partial \eta}{\partial t} = -H \frac{\partial u}{\partial x} + \frac{\partial D}{\partial t} \\
\frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x}
\end{bmatrix}
\]

(2.15)

Under the assumptions listed above, the set of the equations is easily solved by integration along characteristics $x = x_0 \pm ct$ and $x = x_0 - (U \pm c)t$ in the $x, t$ space. The final expression for the surface displacement has the form (Harbitz & Pedersen, 1992).

\[
\eta(x, t) = \frac{U^2}{U^2 - c^2} D(x - Ut) - \frac{U}{2(U + c)} D(x + ct) - \frac{U}{2(U - c)} D(x - ct)
\]

(2.16)

The analytical solution is a superposition of 3 waves, where wave heights are a function of $U$ and $c$. For a rigid body, which moves as an entity with the same speed $U$, the Froude number is defined as

\[
F_r = \frac{U}{c}
\]

(2.17)

where $c = \sqrt{gH}$ is the long-wave speed.

Let us study the nature of the first component in (2.16). In the case of steady slide motion with constant speed we can assume the slide to be static and water running over it; i.e., associate our frame of reference to the slide. The physics of the problem transforms to the idealized case of the laminar flow over a topographic feature or a hump. As the horizontal flux $Q = h(x)u(x)$ is constant in the domain, a simple solution can be obtained. The energy balance equation can be expressed as follows:

\[
\frac{Q^2}{2h(x)^2 g} + h(x) + D(x) = H + \frac{U^2}{2g}
\]

(2.18)

taking derivative $d/dx$ and using $H = D(x) + h(x) - \eta(x)$ produces an expression for the horizontal gradient of the flow (wave) displacement:
2.3 Landslide-generated tsunami modelling approaches

\[
\frac{d\eta}{dx} = \frac{\frac{dD}{dx}}{1 - \frac{gh(x)}{u(x)^2}} \tag{2.19}
\]

linearizing assuming

\[
Fr = \frac{u(x)}{\sqrt{gh(x)}} \approx \frac{U}{\sqrt{gH}} \tag{2.20}
\]

The expression (2.19) transforms to:

\[
\frac{d\eta}{dx} = \frac{U}{U^2 - c^2} \frac{dD}{dx} \tag{2.21}
\]

The equation (2.21) is essentially the same as the first term in equation (2.16). Therefore the nature of the phenomenon is similar and the Froude number considerations for the flow over a bump are applicable in the same manner to the simple landslide-tsunami problem.

The second and third components in (2.16) are free wave solutions of the well-known wave equation. They represent propagation of the initial disturbance in positive and negative directions with velocity \(c\).

The plot (Figure 2.8) represents a dependence of the normalized amplitude versus Froude number for the 3 components in equation (2.16): \(h_1\) is the first, \(h_2\) is the second, and \(h_3\) is the last component (wave). The lower plot is an enlarged version of the upper one and gives a view for \(Fr < 0.5\).

The Froude number for submarine landslides plays the same role as the Mach number for high-speed aircraft. As seen in Figure 2.8 for different values of Froude number the wave components exhibit different behaviors; three regimes are possible:

- \(Fr < 1\) is a subcritical motion (slow) of the slide. \(h_1\) is the trough wave, bound to the landslide and propagating with the velocity \(U\). \(h_2\) is the trough wave, propagating with velocity \(c\) in the opposite direction relative to the slide. \(h_3\) is the crest wave, propagating with velocity \(c\) in the direction of the slide.
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Figure 2.8: Wave amplitude analysis for the linear 2D simple case

- $Fr \approx 1$ Resonant case. The solution (2.16) is not applicable; amplitudes of the trough wave $h_1$ and the crest wave $h_3$ run to infinity. In practice it means that the height of the positive forced wave is significantly amplified and the frontal side of the wave becomes very steep forming a bore. Behind the slide an intensified trough forms.

- $Fr > 1$ is a supercritical motion (fast) of the slide. $h_1$ is the leading forced crest wave propagating with the slide velocity $U$. Its amplitude is higher than the slide thickness and decreases with increasing velocity. $h_2$ is the small trough wave, propagating with velocity $c$ in the direction opposite to that of the slide. $h_3$ is the trough wave, propagating with velocity $c$ in the positive direction behind a slide.

The most efficient generation occurs near resonance when $Fr = 1$. For purely submarine slides, Froude numbers are usually less than unity and resonance cou-
pling of slides and surface waves is physically impossible. For subaerial slides there is always a resonant point where \( Fr = 1 \) for which there is a significant transfer of energy from a slide into surface waves (Fine et al., 2002).

Because of the simplicity of the rigid body slide formulation it makes sense that use of this type of model has been so widespread and that the majority of publications examining slide-generated tsunamis are based on this approximation.

### 2.3.4 Viscous slide models

Another type of model describes a flow as a Newtonian fluid. Jiang & LeBlond (1992) presented a numerical model to study water waves generated by underwater landslides on a gentle uniform slope in shallow water. A formulation of the dynamics of the problem was presented where the landslide was treated as a laminar flow of an incompressible viscous fluid and the water motion was assumed irrotational. For simplicity in theoretical analysis, it was assumed that a finite solid mass suddenly liquefies and reaches a well-mixed phase without appreciable movement down a slope. The landslide material was treated as an incompressible viscous fluid; the long-wave approximation was employed for both the water waves and the mudslide.

Jiang and LeBlond presented numerical results and found that the density of sliding material and the depth of water at the mudslide site are important parameters and dominate the interaction between the slide and the waves it produces. They examined the behavior of the mud flow in the presence of one-way coupling (bottom deformations affect the free surface) and with a full coupling (surface pressure gradients react on the mud flow). They noted that two-way interactions were significant for the cases of lower mud density and shallower water. The possibility of resonance between the slide and the waves was examined and numerical results indicated that it was not expected in most real cases. It was also confirmed that three main waves were generated by a landslide, starting from rest on a gentle uniform slope, in the same manner as with solid models (Jiang & LeBlond, 1992).

In 1993 Jiang & LeBlond (1993) extended their 2D shallow water model to 3 dimensions. Comparisons of 3D calculations with 2D results indicated small
differences for large length/width slide ratios and within a brief internal initiation of the slide. However the water surface profiles differed significantly from the 2D results. Hence an adequate 3D representation of the slide is required for numerical simulations. A finite-difference numerical method was used for the computational solution of the resulting differential equations (Jiang & LeBlond, 1994).

The study of the PARN Dock failure and associated slide in Skagway Harbor on November 3, 1994. was based on the 3D viscous landslide model of Jiang & LeBlond (1994). Several corrections were made to the 3D equations by (Fine et al., 1998). The slight differences between models arose from minor errors in the constant coefficients in the advective terms of the momentum equations of (Jiang & LeBlond, 1992) and (Jiang & LeBlond, 1994). The model was generalized to include the actual bottom topography (Fine et al., 1998; Thomson et al., 2001).

The main assumptions for the viscous model are:

- The slide is an incompressible, isotropic viscous fluid, and seawater is an incompressible inviscid fluid.
- The density difference between a flowslide and seawater is large, viz. \((\rho_2 - \rho_1) \geq 0.2 g cm^{-3}\).
- The slide is characterized by laminar, quasi-steady viscous flow. For a finite mass of sediment released on a slope, there will be two distinct flow regimes; inertial and viscous (Simpson, 1987). It is assumed that the viscous regime is rapidly reached after any failure.
- Mixing at the water-mud interface is negligible, whereby the slide material is not significantly diluted while flowing downslope.

The physical background of these assumptions was thoroughly discussed by Jiang and Leblond (Jiang & LeBlond, 1992).

A conceptual model for the slide and associated waves is presented in Figure 2.9 (Rabinovich et al., 2001). The upper layer consists of seawater with density \(\rho_1\), surface elevation \(\eta(x, y; t)\), and horizontal velocity \(u\). The lower layer consists of sediments of density \(\rho_2\), kinematic viscosity \(\nu\), and horizontal velocity \(U\). The slope is gentle, so that the motion is essentially horizontal. The slide is bounded
2.3 Landslide-generated tsunami modelling approaches

by an upper surface \( z = -h(x,y,t) \), the seabed is designated by \( z = -h_s(x,y) \) and the thickness of the slide is \( D(x,y,t) = h_s(x,y) - h(x,y,t) \). At the seabed, the tangential velocity of the slide is set to zero, while at the upper surface of the slide the normal gradient in tangential velocity is set to zero. At steady state, horizontal velocities in the slide have a parabolic profile:

\[
U_1(x,y,z,t) = U(x,y,t)(2\xi - \xi^2),
\]

where

\[
\xi = (z + h_s)/D
\]

is a normalized depth.

Conservation of mass and momentum for a viscous slide have the form:

\[
\frac{\partial D}{\partial t} + \frac{2}{3} (\nabla \cdot D U) = 0
\]

\[
\frac{2}{3} \frac{\partial U}{\partial t} - \frac{2}{15} \frac{U \partial D}{\partial t} + \frac{8}{15} (U \cdot \nabla) U = -\frac{g}{\rho_2} [ (\rho_2 - \rho_1) \nabla (D - h_s) + \rho_1 \nabla \eta] - \frac{2\nu U}{D^2}
\]
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Here (2.25) is subjected to the condition of no slide transport through the coastal boundary, and the assumption that the slide does not cross the outer (open) boundary (Fine et al., 1998).

The upper layer of the model is governed by the nonlinear shallow water equations:

\[
\frac{\partial (h + \eta)}{\partial t} + \nabla \cdot [(h + \eta) \mathbf{u}] = 0
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -g \nabla \eta
\]  

In effect, the slide generates water waves through the continuity equation (2.26) only. The waves then propagate within the restrictions imposed by the boundary conditions and the nonlinear momentum equation (2.27).

At the open boundary, Fine et al. (1998) used the one-dimensional radiation condition for outgoing waves:

\[
u_n = \eta \sqrt{\frac{g}{h}}
\]  

where \(u_n\) is the velocity component normal to the boundary. At the shore, a vertical wall is assumed as

\[
u_n = 0
\]

The viscous slide model was verified against observations for the Skagway tsunami, recorded by the NOAA tide gauge in the harbor. The results of numerical simulations were in good agreement with the tide gauge record. More specifically, the simulated wave heights for the tide gauge site are well correlated with the tide records (Kulikov et al., 1996; Lander, 1996).

The viscous fluid slide model for the tsunami modelling, first rigorously formulated by Jiang and Leblond (Jiang & LeBlond, 1993, 1994) is now widely used to simulate catastrophic tsunamis arising from submarine landslides. Examples include Nice, France (1979) (Assier-Rzadkiewicz et al., 2000), Skagway (1994) (Fine et al., 1998; Rabinovich et al., 1999; Thomson et al., 2001), and PNG (1999) (Heinrich et al., 2000; Imamura et al., 2001; Titov & Gonzalez, 2001).
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In all of the above events, the viscous model gave reasonable agreement with the existing empirical data (Fine et al., 1998).

2.3.5 Bingham visco-plastic models

Bingham visco-plasticity is most applicable to the dynamics of cohesive muds. A cohesive mud is a mixture of water and very fine particles composed largely of clay minerals and sometimes organic materials. Mud in different locales can have different rheological behavior, partially as a consequence of the varied chemical composition. Krone (1963) reported viscosometric tests of mud samples from seven different coastal cites along the east and west coasts of the United States and found that for the most common concentrations, mud behaves like a Bingham visco-plastic fluid (Mei & Liu, 1987).

The dynamics of both debris flows and mudflows can be considered essentially the same, within the present limits of measurement and the ability to physically represent these phenomena. During debris flows, the source sediment is remolded and reconstituted; the degree to which this occurs determines the rheological properties and flow type. The soil mass usually travels as a visco-plastic material, with distinct stress-strain characteristics (Niedoroda et al., 2003).

A landslide typically exhibits transformation during a failure. It is usually a solid mass in the beginning stages, but then is diluted by water entrainment, flowing often more as a viscous fluid than moving as a solid body. While some models consider landslides as viscous, very few studies on modelling landslide-generated tsunamis consider a landslide as a Bingham fluid, a better rheological model. Bingham visco-plastic models incorporate the transition from a solid body to viscous materials, hence have a potential advantage.

Bingham-fluids are characterized by a yield stress. In order to make the Bingham-fluid flow, the driving shear stress has to be larger than the yield stress. Below this yield stress the moving fluid will behave almost like a solid body and above this threshold as a liquid.

The Bingham rheology is defined by the following equation.
2.3 Landslide-generated tsunami modelling approaches

\[
\begin{cases}
    \frac{\partial u}{\partial x} = 0 & \tau < \tau_0 \\
    \frac{\partial u}{\partial x} = \frac{1}{\mu} (\tau - \tau_0) & \tau > \tau_0
\end{cases}
\]  

(2.30)

where \( \tau \) is the shear stress, \( \tau_0 \) is the yield stress, and \( \mu \) is the coefficient of dynamic viscosity.

The nonlinear equation (2.30) can only be used if the flow is laminar. To understand the Bingham flow velocity structure let us consider a simple case - a developed flow in a constant cross-section horizontal channel (Figure 2.10).

![Figure 2.10: Bingham channel flow](image)

It is assumed that the flow is constant along the channel length (conservation of mass) and, in the stationary case, the forces acting on an infinitely small fluid element are as in Figure 2.11.

![Figure 2.11: Forces on a fluid element](image)

From the necessity that the forces to be balanced, it follows from Figure 2.11:

\[ \frac{dP}{dx} = \frac{d\tau}{dz} \]  

(2.31)
for the shear zone from (2.30)

\[ \tau = \tau_0 + \mu \frac{du}{dz} \tag{2.32} \]

Introducing (2.32) into (2.31) leads to:

\[ \frac{dP}{dx} = \mu \frac{d^2u}{dz^2} \tag{2.33} \]

Assuming constant horizontal pressure gradient and integrating twice results in

\[ \frac{du}{dz} = \frac{1}{\mu} \frac{dP}{dx} z + C_1 \tag{2.34} \]

\[ u = \frac{1}{2\mu} \frac{dP}{dx} z^2 + C_1 z + C_2 \tag{2.35} \]

In order to determine the constants \( C_1 \) and \( C_2 \) boundary conditions are necessary:

- At \( z = 0 \) (bottom) \( u(z = 0) = 0 \) thus it follows from (2.35) \( C_2 = 0 \).

- The friction at the walls has to be balanced by the driving force. The friction force is a product of the wall shear stress by the area. The pressure force results from multiplying the cross-sectional area of the channel by the pressure.

\[ -2 \left( \tau_0 + \mu \frac{du}{dz} \bigg|_{z=0} \right) = \frac{dP}{dx} h \tag{2.36} \]

substituting it into (2.34) we get the constant \( C_1 \)

\[ C_1 = -\frac{1}{\mu} \left( \frac{h}{2} \frac{dP}{dx} + \tau_0 \right) \tag{2.37} \]

And finally the velocity profile in the shear part of the flow is expressed as:

\[ u(z) = \frac{1}{2\mu} \frac{dP}{dx} z^2 - \frac{1}{\mu} \left( \frac{h}{2} \frac{dP}{dx} + \tau_0 \right) z \quad 0 \leq z \leq z_0 \tag{2.38} \]
2.3 Landslide-generated tsunami modelling approaches

\[ u(z) = \frac{1}{2\mu} \frac{dP}{dx} (h - z)^2 - \frac{1}{\mu} \left( \frac{h}{2} \frac{dP}{dx} + \tau_0 \right) (h - z) \quad (h - z_0) \leq z \leq h \]  

(2.39)

The flow is uniform in the plug zone:

\[ u(z) = u(z_0) = u_m \quad z_0 < z < (h - z_0) \]

(2.40)

The magnitude of \( z_0 \) can also be obtained. Above \( z_0 \) the Bingham-fluid is like a solid body, resulting from (2.30):

\[ \frac{\partial u}{\partial z} \bigg|_{z=z_0} = 0 \]

(2.41)

Introducing this condition into (2.34) and using (2.37) obtains the equation for \( z_0 \) as:

\[ z_0 = \frac{h}{2} + \frac{\tau_0}{dP/dx} \]

(2.42)

In the shear zone, the shear stress exceeds the yield stress, and the velocity varies in \( z \). In the plug zone, the stress is less than the yield stress and the horizontal velocity is uniform in \( z \) (Assier-Rzedkiewicz et al., 1997). The velocity profile is similar to that of a Newtonian viscous fluid except an additional layer (plug zone) appears in the Bingham fluid. A velocity profile for the channel flow of the Newtonian fluid has a velocity gradient which decreases towards the center of the channel. Because the Bingham fluid becomes solid when the applied shear rate becomes lower than the yield stress, it becomes solid in the central part of the channel. This solid plug zone moves with the flow.

Jiang & LeBlond (1993) introduced a 2D model for the Bingham landslide and the water wave which it generates. For surface waves they utilized a similar approach to that in their work for viscous slides. They discussed the dynamics of a submarine Bingham mudslide flowing on a gentle uniform slope in shallow water. The long wave approximation was adopted for both waves and the mudslide (Jiang & LeBlond, 1992, 1993).

The numerical results for the Bingham plastic slides, with various yield stresses, indicate that the plasticity of the mud significantly reduces the mobility of the
slide and magnitude of generated surface waves. A Bingham plastic slide can travel only a finite distance in contrast to the flow pattern of a viscous flow, which can spread very far away from its origin. It will stop at its equilibrium thickness on the slope when the shear stress exerted on the bottom of the flow becomes smaller than the yield stress. The slide with a larger yield stress flows much more slowly than those with a smaller yield stress. The major parameters that dominated the interaction between a Bingham-plastic slide and the waves were Bingham yield stress, density of the mud, and depth of initiation of the landslide (Jiang \\& LeBlond, 1993).

To date the area of 3D modelling of landslide-generated tsunamis using a Bingham landslide model has not been developed. The author is aware only of a few applications of this type of model for practical full scale 3D simulations of landslide-tsunamis. The Bingham visco-plastic rheology was utilized in the works of Heinrich et al. (1998) and Assier-Rzadkiewicz et al. (1997). This type of rheological model is quite common, however, in the simulation of terrestrial landslides and avalanches. In this research I try to unite the advances in the areas of landslide mechanics with tsunami modelling and in the next sections I introduce the 3D mathematical Bingham model for the landslide with a numerical approach to the solution. Computational results for the real case study of a landslide-generated tsunami are also provided.
3 Landslide-Tsunami model

3.1 Introduction

The aim of the work is to present a 3D numerical model that simulates a Bingham plastic mudslide on a gentle slope with the surface water waves which it generates. I present a formulation of the dynamics of the problem wherein the landslide is treated as an incompressible Bingham plastic flow and the water motion is assumed to be irrotational. The long-wave approximation is adopted for both the water wave and mudslide \(^1\).

The dispersion of waves and turbulent mixing are not considered. Resulting differential equations are solved by a finite-volume Godunov-type method. The model is utilized to simulate a landslide-generated tsunami which occurred on April 27, 1975 in Kitimat Arm on the west coast of British Columbia. I present numerical results for successive profiles of the mud surface and the evolution of the water surface elevations.

3.2 Landslide model

3.2.1 Shallow water equations for the submarine landslide

The mathematical model, used for the submarine landslide and tsunami waves, is based on the 3D shallow water equations. The model is obtained by averaging Navier-Stokes equations along the vertical direction using several assumptions:

1. The slide satisfies a long-wave (hydrostatic) approximation, implying that that the width and length are much greater than its thickness. Velocity dominates in the x and y directions. The pressure within the mud layer is assumed to be hydrostatic.

2. The slide is an incompressible, isotropic, quasi-steady fluid.

\(^1\)To avoid misinterpretation the shallow water equations are referred as 3D throughout this thesis. In the literature full systems of SWE equations are called 2D and 3D sometimes, though it should be clear that shallow water equations resolve only 2 dimensions; all the computational arrays are 2D not 3D fields.
3. The effects of turbulence and surface tension are not taken into account.

According to assumption (1) vertical accelerations of the Bingham fluid are neglected, this allows the integration of part of the vertical momentum equation and leads to an expression for pressure which, in turn, can be eliminated. Let us consider a layer of the incompressible fluid. The Navier-Stokes equations for the incompressible fluid have the following form:

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) = -\frac{1}{\rho_2} \left( \frac{\partial P}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial y} \right) \quad (3.1)
\]

\[
\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) + \frac{\partial}{\partial z}(vw) = -\frac{1}{\rho_2} \left( \frac{\partial P}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \tau_{yx}}{\partial x} \right) \quad (3.2)
\]

\[
\frac{\partial w}{\partial t} + \frac{\partial}{\partial x}(uw) + \frac{\partial}{\partial y}(vw) + \frac{\partial}{\partial z}(w^2) = -g - \frac{1}{\rho_2} \left( \frac{\partial P}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} \right) \quad (3.3)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.4)
\]

Where \(x, y, z\) are Cartesian coordinates (\(z\) is directed upward), \(u, v, w\) are velocity components, \(\rho_2\) fluid density, \(P\) pressure, \(\tau_{pq}\) is a \(q = x, y, z\) component of the stress tangent to the square \(p = x, y, z\). In terms of the stress, a pressure \(P\) is a diagonal component of the stress tensor, i.e. \(\tau_{qq}\).

The \(z\) scale is assumed to be much less than the \(x\) and \(y\) scales (shallow water assumption). The component \(w\) is much less than the components \(u\) and \(v\); the derivative \(\frac{\partial \rho}{\partial z}\) is much larger than the derivatives \(\frac{\partial \rho}{\partial y}\) and \(\frac{\partial \rho}{\partial y}\).

So we obtain:

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) = -\frac{1}{\rho_2} \left( \frac{\partial P}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right) \quad (3.5)
\]

\[
\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) + \frac{\partial}{\partial z}(vw) = -\frac{1}{\rho_2} \left( \frac{\partial P}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right) \quad (3.6)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.7)
\]
3.2 Landslide model

\[ P = -\rho_2 \ddot{z} + \varphi(x, y, t) \]  \hspace{1cm} (3.8)

\[ \varphi(x, y, t) = (\rho_1 - \rho_2)gh(x, y, t) + \rho_1 \eta(x, y, t) = \rho_2 g(D(x, y, t) - h_s(x, y)) + \rho_1 gh_w \]  \hspace{1cm} (3.9)

The symbols \( h, \eta \) denote the upper slide boundary and the water surface variations respectively; \( D \) and \( h_w = h + \eta \) are the slide thickness and the water column thickness, \( \rho_1 \) is the water density, \( \rho_2 \) is the density of the slide. Equation (3.8) is just a hydrostatic statement, achieved by integrating (3.3) in \( z \) and neglecting vertical acceleration.

Figure 3.1: Landslide - Tsunami Model 3D sketch
(Jiang & LeBlond, 1994)

The 3D perspective sketch of the problem is in Figure 3.1, taken from Jiang & LeBlond (1994). At the bottom of the shallow water basin is a massive landslide...
body. It is assumed that it starts moving at the moment $t = 0$. During the motion the slide transmits its energy to the water surface and generates tsunami waves, as shown on the sketch. The body may have various rheologic properties, including those of a solid block. In this particular case shallow water Navier-Stokes equations are derived for the Bingham visco-plastic rheological model of the landslide.

The slide body occupies the area:

$$-h_s(x, y) < z < -h_s(x, y) + D(x, y, t) = -h(x, y, t) \quad x, y \in \Omega$$

Equations (3.5)-(3.7) are integrated on $z$ from $z = -h_s(x, y)$ to $z = -h(x, y, t)$:

$$
\int_{z=-h_s(x,y)}^{z=-h(x,y,t)} \left[ \frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) \right] \, dz = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial x} - \tau^u_{zz} + \tau^b_{zz} \right) \\
(3.10)
$$

$$
\int_{z=-h_s(x,y)}^{z=-h(x,y,t)} \left[ \frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) + \frac{\partial}{\partial z}(vw) \right] \, dz = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial y} - \tau^u_{yz} + \tau^b_{yz} \right) \\
(3.11)
$$

$$
\int_{z=-h_s(x,y)}^{z=-h(x,y,t)} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \, dz + w^u = 0 \\
(3.12)
$$

Boundary conditions are imposed at the upper surface and the bottom as:

$$w^u = w(x, y, -h, t) = -\frac{\partial h}{\partial t} - u^u \frac{\partial h}{\partial x} - v^u \frac{\partial h}{\partial y} \quad (3.13)$$

$$u^u = u(x, y, -h, t), \quad v^u = v(x, y, -h, t), \quad u^b = 0, \quad v^b = 0, \quad \tau^u = \tau(x, y, -h, t), \quad \tau^b = \tau(x, y, -h_s, t) \quad (3.14)$$

After integration we get:

$$
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \int_{z=-h_s}^{z=-h} u \, dz + \frac{\partial}{\partial y} \int_{z=-h_s}^{z=-h} u \, dz = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial x} - \tau^u_{zz} + \tau^b_{zz} \right) \\
(3.15)
$$
3.2 Landslide model

\[ \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \int_{z=-h_s}^{z=-h} u v dz + \frac{\partial}{\partial y} \int_{z=-h_s}^{z=-h} v v dz = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial y} - \tau^u_{yz} + \tau^b_{yz} \right) \]  
\[ \frac{\partial s}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \]  
\[ M = \int_{z=-h_s}^{z=-h} u dz, \quad N = \int_{z=-h_s}^{z=-h} v dz \]

### 3.2.2 Bingham rheology model

A Bingham plastic fluid (also called visco-plastic fluid) is one in which no deformation takes place until a specified shear stress, called the yield stress or Bingham stress, is applied to the fluid, after which the deformation is driven by the excess of the stress beyond the yield stress. Many materials such as snow, volcanic lava, marine and river sediments behave approximately as Bingham plastic fluid (Mei & Liu, 1987).

As was discussed in the previous chapter, the Bingham rheology is determined by the equation:

\[ \begin{cases} \frac{\partial u}{\partial x} = 0 & \tau < \tau_0 \\ \frac{\partial u}{\partial x} = \frac{1}{\mu} (\tau - \tau_0) & \tau > \tau_0 \end{cases} \]  
\[ \tau = \tau_{xz} (1 - \xi) \]
\[ \tau_{yz} = \tau_{yz}^b (1 - \xi) \]  
\[ \frac{z+h_s}{D} \]
3.2 Landslide model

Figure 3.2: Bingham velocity, stress profiles

$\xi[0; 1]$ denotes a vertical normalized non-dimensional coordinate, $\tau^b$ is the shear stress at the bottom.

For an open boundary flow it also leads to two distinct zones as for the tube case discussed earlier: 1) a shear zone near the bottom; and, 2) an upper plug zone (Figure 3.3).

Figure 3.3: Bingham flow profile

The vertical profile of the velocity components (Figure 3.3) contains a solid layer and a shear layer. The solid layer has a constant velocity and the shear layer has a parabolic velocity distribution (Figure 3.2). The equations for $x$ and $y$ velocity components are:

$$
\begin{align*}
\{u, v\} &= \{u^s, v^s\} & -h_s + D_1 < z < -h \\
\{u, v\} &= \{u^s, v^s\}(2\xi_1 - \xi_1^2) & -h_s < z < -h_s + D_1 \\
\xi_1 &= \frac{z + h_s}{D_1}
\end{align*}
$$

(3.21)
\[
D_1 \text{ is the thickness of the shear layer and } D_2 = D - D_1 \text{ is the thickness of the upper (solid) layer. Let } \tau = \sqrt{\tau_{xx}^2 + \tau_{yy}^2} \text{ be the magnitude of the stress vector. At the boundary of the shear layer, the stress gradient } \tau(D_1) = \tau_0 \text{ has to be the same in both layers:}
\]
\[
\frac{\tau_0}{D_2} = \frac{\tau^b - \tau_0}{D_1} = \frac{\tau^b}{D}
\]

At the bottom, (3.19) with (3.21) gives:
\[
\tau^b - \tau_0 = \mu \frac{2U}{D_1}
\]

Thus from (3.22) and (3.23) a square equation is derived to find the thickness of the shear layer:
\[
d_1^2 + 2d_1 \kappa - 2\kappa = 0
\]

Here \(d_1 = \frac{D_1}{D}\) is a relative thickness of the shear layer and \(\kappa = \frac{\mu U}{\tau_0 D}\) is a dimensionless parameter. The solution of (3.24) is
\[
d_1 = \sqrt{\kappa^2 + 2\kappa} - \kappa
\]

The bottom stress value can now be written in terms of the parameter \(\kappa\). From (3.22) we have: \(d_1 \tau^b = \tau^b - \tau_0\), and using (3.25) the following expression for the bottom stress \(\tau^b\) is obtained:
\[
\tau^b = \frac{\tau_0}{1 - d_1} = \frac{\tau_0}{1 + \kappa - \sqrt{\kappa^2 + 2\kappa}} = \tau_0 (1 + \kappa + \sqrt{\kappa^2 + 2\kappa})
\]

The direction of the bottom stress vector \(\tau^b\) is opposite to that of the velocity, so for bottom stress components we have:
\[
\begin{align*}
\tau^b_{xx} &= -\frac{u^b}{U} \tau_0 (1 + \kappa + \sqrt{\kappa^2 + 2\kappa}) \\
\tau^b_{yy} &= -\frac{v^b}{U} \tau_0 (1 + \kappa + \sqrt{\kappa^2 + 2\kappa})
\end{align*}
\]

Equation (3.26) is the main result of the analysis. If velocity \(U\) is small, the parameter \(\kappa \rightarrow 0\), and we get:
\[
d_1 \rightarrow \sqrt{2\kappa} \rightarrow 0
\]
In this case the bottom stress \( \tau^b \) is equal to the yield stress of the Bingham fluid. The plug zone is much thicker compared to the shear zone thus the dynamics of the slide becomes similar to that of the rigid body slide.

If the velocity \( U \) is high, than \( \kappa \to \infty \), and we get:

\[
d_1 = \sqrt{\kappa^2 + 2\kappa - \kappa} \to 1
\]

(3.29)

In this case the bottom stress \( \tau^b \) simplifies to that of a Newtonian fluid. The plug zone becomes thin and disappears so the slide becomes liquid in the whole volume.

Stress components for the equations (3.15 - 3.16) were derived. It is now necessary to consider the advective terms in these equations. For any velocity component \( q \) and the respective flux \( \Phi \) the integral value will be:

\[
\Phi = \int_{z=-h(x,y,t)}^{z=-h(x,y)+D_1} q \, dz = \int_{z=-h}^{z=-h+D_1} q \, dz + \int_{z=-h}^{z=-h+D} \phi \, dz = q^n \left( \frac{2}{3} D_1 + D_2 \right) = q^n D \left( 1 - \frac{1}{3} d_1 \right) = \alpha \frac{\Phi^2}{D}
\]

(3.30)

\[
\int_{z=-h(x,y,t)}^{z=-h(x,y)+D_1} q^2 \, dz = \int_{z=-h}^{z=-h+D_1} q^2 \, dz + \int_{z=-h}^{z=-h+D} q^2 \, dz = \left( \frac{8}{15} D_1 + D_2 \right) = \alpha \frac{\Phi^2}{D}
\]

(3.31)

where \( \alpha = \frac{1 - \frac{7}{2} d_1}{(1 - \frac{1}{3} d_1)^2} \).

The system (3.15 - 3.18) can be rewritten and the shallow water Bingham slide equations become:

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \frac{\alpha MM}{D} + \frac{\partial}{\partial y} \frac{\alpha MN}{D} = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial x} - \frac{u^n}{U} \tau_0 (1 + \kappa + \sqrt{\kappa^2 + 2\kappa}) \right)
\]

(3.32)

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \frac{\alpha MN}{D} + \frac{\partial}{\partial y} \frac{\alpha NN}{D} = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial y} - \frac{v^n}{U} \tau_0 (1 + \kappa + \sqrt{\kappa^2 + 2\kappa}) \right)
\]

(3.33)
3.2 Landslide model

\[
\frac{\partial D}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{3.34}
\]

Using the relation (3.25) and substituting for the fluxes

\[
\{M, N\} = \int_{z=--h}^{z=-h_s} \{u, v\} dz = \{u^*, v^*\} D \tag{3.35}
\]

we next rewrite the equations (3.32 - 3.34):

\[
\frac{\partial u D}{\partial t} + \frac{\partial \alpha u^2 D}{\partial x} + \frac{\partial \alpha uv D}{\partial y} = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial x} - \frac{u}{U_0} \tau_0 (1 + d_1) - \frac{2\mu u}{D(1 - \frac{d_4}{3})} \right) \tag{3.36}
\]

\[
\frac{\partial v D}{\partial t} + \frac{\partial \alpha uv D}{\partial x} + \frac{\partial \alpha v^2 D}{\partial y} = \frac{1}{\rho_2} \left( -D \frac{\partial \varphi}{\partial y} - \frac{v}{U_0} \tau_0 (1 + d_1) - \frac{2\mu v}{D(1 - \frac{d_4}{3})} \right) \tag{3.37}
\]

\[
\frac{\partial D}{\partial t} + \frac{\partial u D}{\partial x} + \frac{\partial v D}{\partial y} = 0 \tag{3.38}
\]

where \( \varphi = \rho_2 g (D - h_s) + \rho_1 g h_w \) is the forcing potential.

The water wave is assumed to have no effect on the landslide, i.e. no coupling; thus the expression for the forcing term simplifies to:

\[
h_w = h_s - D \quad \Rightarrow \quad \varphi = (\rho_2 - \rho_1) g (D - h_s)
\]

\[
g' = g \frac{\rho_2 - \rho_1}{\rho_2} \quad \Rightarrow \quad \frac{1}{\rho_2} \frac{\partial \varphi}{\partial x} = \frac{g'}{\rho_2} \left( \frac{\partial D}{\partial x} - \frac{\partial h_s}{\partial x} \right) \tag{3.39}
\]

The system of equations can be presented as a vector, which is a very convenient form for the further manipulation of the equations.

\[
\frac{\partial \bar{Q}}{\partial t} + \frac{\partial \bar{F}_i}{\partial x_i} = \bar{S} \tag{3.40}
\]

\[
\bar{Q} = \begin{pmatrix} D \\ uD \\ vD \end{pmatrix} \quad \bar{F}_x = \begin{pmatrix} udD \\ \alpha u^2 D + g' D^2/2 \\ \alpha uv D \end{pmatrix} \quad \bar{F}_y = \begin{pmatrix} vD \\ \alpha uv D \\ \alpha v^2 D + g' D^2/2 \end{pmatrix} \tag{3.41}
\]
\[ S = \begin{pmatrix} 0 \\ Dg \frac{\partial h_x}{\partial x} - \frac{1}{\rho_2} \frac{u}{U} \tau_0 (1 + d_1) - \frac{1}{\rho_2} \frac{2 \mu U}{D(1 - \frac{d_1}{3})} \\ Dg \frac{\partial h_y}{\partial y} - \frac{1}{\rho_2} \frac{v}{U} \tau_0 (1 + d_1) - \frac{1}{\rho_2} \frac{2 \mu V}{D(1 - \frac{d_1}{3})} \end{pmatrix} \quad (3.42) \]

where

\[ \alpha = \frac{1 - \frac{7}{15} d_1}{(1 - \frac{d_1}{3})^2} \quad \text{non-dimensional coefficient } 1 < \alpha < 1.2 \]

\[ d_1 = \sqrt{\kappa^2 + 2\kappa} - \kappa \quad \text{relative thickness of the shear area} \]

\[ \kappa = \frac{\mu |U|}{\tau_0 D (1 - \frac{d_1}{3})} \quad \text{visco-plastic dynamic ratio } 0 < k < \infty \]

\[ U = \sqrt{u^2 + v^2} \quad \text{full velocity} \]

In the system (3.40 - 3.42) \( Q \) is the vector of conserved variables, \( F_x(Q) \) and \( F_y(Q) \) are flux vectors, and \( S(Q) \) is the source term vector.

Simple analysis shows that for \( d_1 \to 0 \) we get \( \alpha \to 1 \). If this is the case, a plug layer in the Bingham flow becomes dominant and the fluid solidifies.

If \( d_1 \to 1 \), we get \( \alpha \to 1.2 \) and a shear layer dominates as the fluid becomes Newtonian. In the extreme case if \( d_1 = 1 \) the equations (3.40 - 3.42) will transform to the equations for the viscous slide.

The slide initiation requirement is that the force must exceed the yield stress \( \tau_0 \):

\[ D |\nabla \varphi| > \tau_0 \quad (3.43) \]

If the driving force is lower than the yield stress the Bingham fluid will eventually stop on the slope or will not start its motion. From a uniform mud layer on an underwater slope with no elevation at the water surface, the expression
(3.43) can be transformed to obtain the critical thickness at which deposits can stay stationary on a slope with an inclination of $\beta$.

$$D = \frac{\tau}{(\rho_2 - \rho_1)g \tan \beta}$$  

(3.44)

Thus a uniform mud layer remains stationary on a slope if its thickness $D$ is smaller than $D_c$. In contrast to Newtonian fluids it can achieve a state of equilibrium.

### 3.3 Tsunami model

#### 3.3.1 Shallow water equations for tsunami waves

The mathematical model for tsunami waves is based on the 3D shallow water equations. It is obtained by averaging the Navier-Stokes equations for the Newtonian fluid in the vertical direction under several assumptions:

1. The surface waves satisfy the long-wave (hydrostatic) approximation, implying that the wavelength of the water waves is much greater than the water depth, and that the width and length of the moving body are greater than its thickness.

2. Seawater is an incompressible inviscid fluid with a constant density.

3. The effects of turbulence, surface tension, and wind on waves are not taken into account.

4. The surface waves are generated only because of the displacement effect of the landslide body.

The assumption (1) is very important as it implies that the flow depth is small compared to the horizontal linear dimensions of the computational domain. It follows that vertical accelerations of the fluid can be neglected, allowing the integration of part of the vertical momentum equation and obtaining an expression for pressure which in turn can be eliminated. This is basically the same approach as for the landslide equations in the previous section.
3.3 Tsunami model

The derivation of the shallow water equations for an inviscid fluid can be found in the literature (Toro, 2001); here I present the final set of equations. The unsteady nonlinear 3D shallow water Navier-Stokes equations in differential conservation law form are:

\[
\begin{align*}
\frac{\partial h_w u}{\partial t} + \frac{\partial}{\partial x} \left( h_w u^2 + \frac{1}{2} g h_w^2 \right) + \frac{\partial}{\partial y} (h_w u v) &= g h_w \frac{\partial h}{\partial x} \\
\frac{\partial h_w v}{\partial t} + \frac{\partial}{\partial y} \left( h_w v^2 + \frac{1}{2} g h_w^2 \right) + \frac{\partial}{\partial x} (h_w u v) &= g h_w \frac{\partial h}{\partial y} \\
\frac{\partial h_w}{\partial t} + \frac{\partial (h_w u)}{\partial x} + \frac{\partial (h_w v)}{\partial y} &= 0
\end{align*}
\]

Equation (3.47) is the continuity equation and (3.45,3.46) are the momentum equations along the x and y Cartesian coordinates. The slide generates water waves through the continuity equation (3.47) only.

For the right hand side (RHS) of (3.45) and (3.46) a force term can be rewritten as:

\[
g h_w \frac{\partial \eta}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{1}{2} g (\eta^2 + 2 \eta h) \right) - g \eta \frac{\partial h}{\partial x_i}
\]

Substituting (3.48) into the system (3.45 - 3.47) and transforming it to vector form, we obtain:

\[
\frac{\partial \tilde{Q}}{\partial t} + \frac{\partial \tilde{F}_i}{\partial x_i} = \tilde{S}
\]

\[
\tilde{Q} = \begin{pmatrix} h_w \\ u h_w \\ v h_w \end{pmatrix} \quad \tilde{F}_x = \begin{pmatrix} u h_w \\ u^2 h_w + \frac{1}{2} g (\eta^2 + 2 \eta h) \\ u v h_w \end{pmatrix} \quad \tilde{F}_y = \begin{pmatrix} v h_w \\ v^2 h_w + \frac{1}{2} g (\eta^2 + 2 \eta h) \\ u v h_w \end{pmatrix}
\]

\[
\tilde{S} = \begin{pmatrix} 0 \\ \eta g \frac{\partial h}{\partial x} \\ \eta g \frac{\partial h}{\partial y} \end{pmatrix}
\]
3.3 Tsunami model

- water depth from zero level, \( \eta \) - water surface variation, \( h_w = \eta + h \) - total water depth, \( u \) and \( v \) are local wave horizontal velocities, \( g \) - acceleration due to gravity (3.1).

3.3.2 Energy estimates

In order to estimate efficiency of the wave generation a relative energy estimate of the generated tsunami waves can be obtained. Multiplying equation (3.47) by \( g\eta \), (3.45) by \( h_w u \), and (3.46) by \( h_w v \), and summing all three contributions, gives the time rate of change of total wave energy, \( \frac{\partial \varepsilon}{\partial t} \):

\[
\frac{\partial \varepsilon}{\partial t} = g\eta \frac{\partial D}{\partial t} - \frac{\partial}{\partial x}(h_w u \varepsilon_0) - \frac{\partial}{\partial y}(h_w v \varepsilon_0),
\]

(3.52)

where \( \varepsilon = \frac{1}{2}(h_w u^2 + h_w v^2 + g\eta^2) \) and \( \varepsilon_0 = \frac{1}{2}(u^2 + v^2 + 2g\eta) \). Integration of (3.52) over the area, \( \Omega \), occupied by the waves and slide yields,

\[
\frac{\partial E}{\partial t} = \iint_{\Omega} g\eta \frac{\partial D}{\partial t} \, ds - \int_{\Gamma} \varepsilon_0 h_w u_n \, dl,
\]

(3.53)

where \( \Gamma \) is the open boundary, \( u_n \) is the current velocity component normal to this boundary, and \( E = \iint_{\Omega} \varepsilon \, ds \) is the wave energy. Equation (3.53) has a simple interpretation: changes in wave energy in the tsunami domain are the sum of the wave generation within the domain and the energy flux through the open boundary. The first term in the right side of (3.53)

\[
W = \iint_{\Omega} g\eta \frac{\partial D}{\partial t} \, ds
\]

(3.54)

describes the rate-of-energy-generation (energy generated in the area, \( \Omega \), per unit time) (Fine et al., 2002).
3.4 Non-dimensional variables

We can employ dimensionless variables:

\[(x', y') = (x, y) / L_0; \]
\[(h', h'_w, \eta', D') = (h, h_w, \eta, D) / D_0; \quad \varepsilon = D_0 / L_0; \]
\[t' = t / t_0; \quad t_0 = (D_0 / g)^{1/2} \]
\[(u', v', U', V') = (u, v, U, V) / U_0; \quad U_0 = L_0 / t_0.\]

$L_0, D_0$ are characteristic length and thickness respectively.

The landslide equations (3.40 - 3.42) transform to:

\[
\frac{\partial \bar{Q}'}{\partial t'} + \frac{\partial \bar{F}_x'}{\partial x'} + \frac{\partial \bar{F}_y'}{\partial y'} = \bar{S}'
\]

\[
\bar{Q}' = \begin{pmatrix} D' \\ u'D' \\ v'D' \end{pmatrix} \quad \bar{F}_x' = \begin{pmatrix} \alpha u'^2 D' + \varepsilon^2 r D'^2/2 \\ \alpha u' u'^2 D' \end{pmatrix} \quad \bar{F}_y' = \begin{pmatrix} \varepsilon^2 r D' \partial h'_w / \partial x' - B \varepsilon r (1 + d_1) \frac{u'}{U'} - \frac{1}{\varepsilon Re} \frac{2u'}{D'(1 - d_1/3)} \\ \varepsilon^2 r D' \partial h'_w / \partial y' - B \varepsilon r (1 + d_1) \frac{v'}{U'} - \frac{1}{\varepsilon Re} \frac{2v'}{D'(1 - d_1/3)} \end{pmatrix}
\]

where

\[
\alpha = \frac{1 - \frac{7}{15} d_1}{(1 - \frac{2}{3} d_1)^2}; \quad \kappa = \frac{r U'}{\varepsilon Re B (1 - d_1/3) D'}; \quad U' = \sqrt{u'^2 + v'^2};
\]
\[
d_1 = \sqrt{r^2 + 2\kappa - \kappa}; \quad r = \frac{\rho_2 - \rho_1}{\rho_2};
\]
\[
B = \frac{\tau_0}{(\rho_2 - \rho_1) g D_0} \quad - \text{Critical angle}; \quad Re = \frac{\rho_2 D_0 U_0}{\mu} \quad - \text{Reynolds number}
\]

(3.59)
The tsunami waves equations (3.49 - 3.51) transform to:

\[
\frac{\partial \tilde{Q}'_w}{\partial t'} + \frac{\partial \tilde{F}'_{wx}}{\partial x'} + \frac{\partial \tilde{F}'_{wy}}{\partial y'} = \tilde{S}'_w
\]  

(3.60)

\[
\begin{align*}
\tilde{Q}'_w &= \begin{pmatrix} \eta' \\ u'h'_w \\ v'h'_w \end{pmatrix}, \\
\tilde{F}'_{wx} &= \begin{pmatrix} u'h'_w \\ u'^2 h'_w + \frac{1}{2} \varepsilon^2 (\eta'^2 + 2\eta' h'') \\ u'v'h'_w \end{pmatrix}, \\
\tilde{F}'_{wy} &= \begin{pmatrix} \eta h'_w \\ \eta \eta' + \frac{1}{2} \varepsilon^2 (\eta'^2 + 2\eta' h'') \\ \eta' \eta' h'_w + \frac{1}{2} \varepsilon^2 (\eta'^2 + 2\eta' h'') \end{pmatrix}, \\
\end{align*}
\]  

(3.61)

\[
\tilde{S}'_w = \begin{pmatrix} 0 \\ \varepsilon^2 \eta \frac{\partial h'}{\partial x'} \\ \varepsilon^2 \eta \frac{\partial h'}{\partial y'} \end{pmatrix}
\]  

(3.62)
4 Numerical solution

4.1 Introduction

The systems of differential equations (3.49 - 3.51) and (3.40 - 3.42) are of the strictly hyperbolic type and it is the hyperbolic character of the shallow water equations (SWE) that makes finding solutions to these equations difficult. Hyperbolic equations admit discontinuous and smooth solutions. Even for the case in which the initial conditions are smooth, the non-linear character combined with the hyperbolic nature of the equations can lead to discontinuous solutions in finite time. The non-linear character of the shallow water equations means that analytical solutions to these equations are limited to only very special cases. Numerical methods are generally used to obtain solutions to practical problems.

Several techniques are available to solve the two-dimensional shallow water equations for the simulation of free-surface flow transients. These include finite difference methods (FDM) (Garcia & Kahawitha, 1986; Fennema & Chaudhry, 1990; Bellos et al., 1991), finite element methods (FEM) (Akanbi & Katopodes, 1988) and the finite volume methods (FVM) (Bellos et al., 1991; Zhao et al., 1994).

Finite volume methods have several advantages over finite difference and finite element approaches for this type of problem. Finite volume methods combine the simplicity of finite difference methods with the geometric flexibility of finite element methods. FVM can be considered as finite difference methods applied to the differential conservative form of the conservation laws in arbitrary coordinates. Thus the methods can also be applied using an unstructured grid system as FEM; generally FVM need less computational effort then FEM. FVM are based on the integral form of the conservation equations; thus a scheme in conservation form can easily be constructed to capture shock waves. By discretization of the integral form of the conservation equations, mass and momentum remain conserved (Hirsch, 1990). The key problem in FVM is to estimate the normal flux through each cell interface. There are several algorithms to estimate this flux. Since the set of shallow water equations is hyperbolic, it has an inherent directional property with respect to signal propagation. Algorithms to estimate the flux are able to handle this property appropriately. To achieve this objective
upwind schemes are developed. There are two techniques known as flux vector splitting (FVS) and flux difference splitting (FDS). In particular, FDS schemes are derived from the Godunov method (Godunov, 1959) and are known as Godunov-type methods. In these schemes flow problems involving abrupt changes in flow variables at cell interfaces fall into the framework of Riemann problems (Valiani et al., 1999).

The numerical solution of the shallow water equations for gravity driven flows or wave propagation over real domains poses three specific problems. The first is the simulation of flow fronts or abrupt water waves that can be represented numerically as propagating discontinuities. The second problem derives from abrupt changes in bathymetry. As long as the bottom surface remains sufficiently smooth, most numerical techniques provide an accurate solution of the flow, but if the bottom surface is very rough the majority of methods fail. The third problem especially arises when these schemes are applied to study the front propagation over a dry bed (Toro, 1997).

In this work I utilize a finite volume schema in a rectangular computational domain. The numerical solution proposed is suitable to handle complex topography, to simulate subcritical and supercritical flow and to simulate steady or unsteady flow. It gives solutions which are both smooth and discontinuous.

In this chapter I devise a 1st order Godunov explicit finite volume method and discuss the approach to the solution of our sets of 3D SWEs (3.49 - 3.51), (3.40 - 3.42). First, I consider tsunami wave (3.49 - 3.51) equations, because they are easier to solve and next I extend the method to solve the system (3.40 - 3.42).

### 4.2 Numerical method

#### 4.2.1 Godunov's scheme

In finite volume methods the hyperbolic problems are known as Godunov's schemas. These methods consider local initial value problems in the computational domain. Local initial value problems which involve discontinuous neighbouring states are known as Riemann problems.
4.2 Numerical method

The Riemann problem takes the central role in Godunov's method for the hyperbolic differential equations.

\[ Q(x_j, t^n) = \begin{cases} Q_L(x, t^n) & x_j < 0 \\ Q_R(x, t^n) & x_j > 0 \end{cases} \]  

(4.1)

The Riemann problem is defined as the initial-value problem (4.1). In Figure 4.1, \( Q \) is any function of a coordinate which can be a height, velocity, flux, pressure, etc. The most important attribute is that there exist two constant states on the right and left side, separated by the discontinuity. The Riemann problem solution is the evolution of fluid initially composed of two states with different and constant values. An example of a classic Riemann problem in fluid dynamics is Sod's shock tube, where there are two states of gas with initial values of pressure, velocity and density separated by a diaphragm in a tube. The diaphragm is removed instantly (or breaks under pressure) at some moment and the process evolves, producing shock and rarefaction waves.

The idea behind Godunov's method (Godunov et al., 1979) consists of solving Riemann problems arising at each cell interface of a cell-wise constant finite-volume scheme. The solution obtained is then cell-averaged before the next time step. Figure (4.2) shows a finite volume discretization on the 2D grid of the Godunov scheme.
4.2 Numerical method

If $Q_i$ and $Q_{i-1}$ are cell averages in two neighbouring grid cells on a finite volume grid, then by solving the Riemann problem (4.1) with $Q_l = Q_{i-1}$ and $Q_r = Q_i$, we can obtain information that can be used to compute a numerical flux and update the cell averages over a time step. For hyperbolic problems the solution to the Riemann problem is a function of $x$ and $t$ which consists of a finite set of waves that propagates away from the origin with constant wave speeds.

Figure 4.3: Godunov's scheme

Finite volume discretization with the Riemann problems at the cell interfaces is depicted in Figure (4.3). The 1st (upper) figure shows the initial data for time $n$ at the cell boundaries $x_{j+1/2}$ and $x_{j-1/2}$. This representation is similar to the
so called dam-break problem. Assume this is a horizontal channel of uniform, rectangular cross-section and a free upper boundary. Suppose furthermore that the channel has several uniform water levels, both at rest separated by walls at the positions \( x_{j+1/2} \) and \( x_{j-1/2} \). If the walls separating the uniform levels collapse instantly two dominant features emerge from the process in the form of waves. A right-facing wave travels into the shallow portion of the fluid from the interface \( j + 1/2 \), raising the depth abruptly. The left-facing wave travels into the deep water from the interface \( j + 1/2 \) region and has the effect of reducing the free-surface height. The same is applicable to the interface \( j - 1/2 \) (Figure 4.3 bottom). The details of the physical processes occurring in the vicinity of the walls shortly after the collapse of the walls are indeed very complex and are not correctly modelled by the shallow water equations, though if the walls collapse in a sufficiently short time, the wave pattern emerging gives a left rarefaction and a right shock wave for the \( j + 1/2 \) interface and vice versa for the \( j - 1/2 \) interface; such wave systems may be approximated by the shallow water equations. The 2nd figure (middle) (Figure 4.3) shows the process as a function of space and time. In the linear case, a shock has a straight line characteristic curve, while a rarefaction has a fan like structure. The shock is a discontinuous wave and a rarefaction is a smooth wave. In the language of free-surface water flows or waves, a rarefaction is also called a depression and a shock is also called a bore. This physical approach to the mathematical problems in numerical methods was first developed by Godunov (1959).

Godunov's schemas for multidimensional problems do not appear to have matured enough to be used in the construction of multidimensional unsplit schemes. Even if such solvers were available, the resulting schemas are likely to be too complicated for common use (Billett & Toro, 1997). Therefore for the solution of the 3D systems (3.49 - 3.51), (3.40 - 3.42) I have applied a fractional step method (Yanenko, 1971). The method reduces the numerical solution of the 3D SWE into the consecutive solution of two 2D problems. The system (3.49 - 3.51) can be split into two 2D problems the following way:

\[
\frac{\partial \tilde{Q}}{\partial t} + \frac{\partial \tilde{F}_z}{\partial x} = \tilde{S}_x \tag{4.2}
\]
4.2 Numerical method

The system (4.2) includes derivatives only along $x$ and (4.3) along $y$ Cartesian coordinates, respectively. In the fractional step method I solve each of these systems sequentially, obtaining a solution for the new time step. The landslide equations (3.40 - 3.42) are handled the same way.

As a finite volume method, Godunov’s schema of 1st order for conservational 2D SWE can be formulated as follows:

$$
\bar{Q} = \begin{pmatrix} h_w \\ uh_w \\ vh_w \end{pmatrix}, \quad \bar{F}_x = \begin{pmatrix} uh_w \\ u^2 h_w + g \frac{h_w^2}{2} \\ uh_w \end{pmatrix}, \quad \bar{S} = \begin{pmatrix} 0 \\ h_w g \frac{\partial h}{\partial x} \\ 0 \end{pmatrix}
$$

$$
\frac{\partial \bar{Q}}{\partial t} + \frac{\partial \bar{F}_y}{\partial y} = \bar{S}_y
$$

(4.3)

$$
\bar{Q} = \begin{pmatrix} h_w \\ uh_w \\ vh_w \end{pmatrix}, \quad \bar{F}_y = \begin{pmatrix} vh_w \\ uh_w \\ \nu^2 h_w + g \frac{h_w^2}{2} \end{pmatrix}, \quad \bar{S} = \begin{pmatrix} 0 \\ 0 \\ h_w g \frac{\partial h}{\partial y} \end{pmatrix}
$$

The numerical flux function $F$ is the flux computed from the exact or approximate solution $Q^n_i$ of the Riemann problem, taken at the location of the cell-interfaces:

$$
\bar{Q}^{n+1}_i - \bar{Q}^n_i = \frac{\Delta t}{\Delta x} (\bar{F}_{i+1/2} - \bar{F}_{i-1/2}) + \Delta t \bar{S}^n_i
$$

(4.4)

The structure of the general solution for the x-split two-dimensional shallow water equations is depicted in Figure (4.4). The characteristic lines show directions of propagation of the solution in the $x, t$ domain. Because of the presence
of the $y$ momentum equation in the system (4.2), which is for derivatives along the $x$ coordinate, the middle wave $S_s$ appears in the general solution of the Riemann problem. There are several wave patterns possible, in general: left and right waves are shocks or rarefactions, while the middle wave is always a shear wave, across which the tangential velocity component $v$ changes discontinuously. The three waves separate four constant states denoted, from left to right, by $Q_l, Q_r, Q_s, Q_{sr}$. The region between the left and right waves is called the star region and subdivided into two subregions. The same is applicable to the $y$-split system (4.3) (Toro, 2001, 1997).

As for the purely 2D problem the system (4.2) would simplify to (4.7). For this type of problem there are only left and right waves in the solution.

\[
\frac{\partial \bar{Q}}{\partial t} + \frac{\partial \bar{F}_x}{\partial x} = \bar{S}_x.
\]

\[
\bar{Q} = \begin{pmatrix} h_w \\ uh_w \end{pmatrix}, \quad \bar{F}_x = \begin{pmatrix} uh_w \\ u^2 h_w + g \frac{h^2}{2} \end{pmatrix}, \quad \bar{S} = \begin{pmatrix} 0 \\ h_w g \frac{\partial h}{\partial x} \end{pmatrix}
\]

The usual steps in Godunov's scheme for solving a one-dimensional hyperbolic problem, using an exact solution of the Riemann problems involve:
4.2 Numerical method

- Discretization into cell-wise constant values.
- Solution of the Riemann problems at the cell-interfaces.
- Averaging over the cells.

In practical computations, an exact solution of the Riemann problem can become quite time-consuming due to the iterative approach to the solution of each Riemann problem. Moreover, the great detail of the cell-wise solution is partially lost during the averaging stage so that a simplified approach is often sufficient. Several efficient one-dimensional approximate Riemann solvers have been proposed such as Roe's (Roe & Pike, 1984), HLL ((Harten et al., 1983), HLLC (Toro et al., 1994), Osher-Solomon (Osher & Solomon, 1982), Vanleer (van Leer, 1977, 1985) and extended later to hydraulics (Glaister, 1987, 1988; Alcrudo & Garcia-Navarro, 1992, 1993; Nujic, 1995). In the framework of our model, I utilize Roe's and HLLC approximate Riemann solvers. In the next paragraphs I present the techniques involved in Roe's and HLLC solvers.

4.2.2 Roe’s solver

A successful approach to the solution of a hyperbolic problem was taken by Roe (1981). Roe’s idea is that an exact solution of the locally linearized problem (4.2) can be constructed yielding:

\[
\frac{\partial \bar{Q}}{\partial t} + A(\bar{Q}_{j+1}, \bar{Q}_j) \frac{\partial \bar{Q}}{\partial x} = \bar{S} \tag{4.8}
\]

\[
A = \frac{\partial \bar{F}}{\partial \bar{Q}} = \begin{pmatrix} 0 & 1 & 0 \\ c^2 - u^2 & 2u & 0 \\ -uv & v & u \end{pmatrix} \quad c = \sqrt{gh_w} \tag{4.9}
\]

\(A\) is a Jacobian matrix of constant coefficients. The interface flux \(F\) is expressed by incrementing across each wave either from the right or from the left state.

\[
\bar{F}_{i+1/2} = \frac{1}{2}(\bar{F}_{i+1} + \bar{F}_i - |\bar{A}_{i+1/2}|(\bar{Q}_{i+1} - \bar{Q}_i)) \tag{4.10}
\]

\[
\bar{A}_{i+1/2} = RAR^{-1} \tag{4.11}
\]
4.2 Numerical method

A—diagonal eigenvalue matrix, \( R \)- eigenvector matrix, \( R^{-1} \)—inverse to \( R \).

The approximated, diagonalized matrix \( \tilde{A}_{i+1/2} \) must satisfy the following properties:

1. \( \tilde{A}_{i+1/2}(\tilde{Q}_i, \tilde{Q}_i) = A(\tilde{Q}_i) = J(\tilde{Q}_i) \) - Jacobian matrix, which is diagonalizable
2. \( \tilde{A}_{j+1/2} = \tilde{A}_{j+1/2}(\tilde{Q}_{j+1}, \tilde{Q}_j) \)
3. \( \tilde{F}_{i+1} - \tilde{F}_i = \tilde{A}_{i+1/2}(\tilde{Q}_{i+1} - \tilde{Q}_i) \)

These conditions ensure that the resulting numerical scheme is conservative and consistent with the original hyperbolic problem. From the imposed properties 1-3, the Roe-Pike approach ((Roe & Pike, 1984) obtains the following averages for the primitive variables:

\[
\tilde{h}_{w,i+1/2} = \frac{h_{w,i+1} + h_{w,i}}{2} \tag{4.12}
\]

\[
\tilde{u}_{i+1/2} = \frac{u_{i+1} \sqrt{h_{w,i+1}} + u_i \sqrt{h_{w,i}}}{\sqrt{h_{w,i+1}} + \sqrt{h_{w,i}}} \quad \tilde{c}_{i+1/2} = \sqrt{gh_{w,i+1/2}} \tag{4.13}
\]

For the diagonalization procedure (4.11) eigenvalues and eigenvectors can be derived analytically.

Eigenvalues:

\[
\lambda_1 = u - c \quad \lambda_2 = u + c \quad \lambda_3 = u \tag{4.14}
\]

Eigenvector matrices:

\[
R = \begin{pmatrix}
1 & 1 & 0 \\
1 & u - c & u + c \\
v & v & 1
\end{pmatrix} \quad R^{-1} = \frac{1}{2c} \begin{pmatrix}
c + u & -1 & 0 \\
c - u & 1 & 0 \\
-2vc & 0 & 1
\end{pmatrix} \tag{4.15}
\]

After finding a diagonalized matrix, a numerical flux is updated using (4.10). The fluxes are then substituted into the conservative formula (4.4) to find a vector variable \( Q \) which in turn gives us values for the velocities \( u, v \) and the depth \( h_w \) to compute values for a new time step.
4.2 Numerical method

4.2.3 HLLC solver

The HLLC (Harten, Lax and van Leer Contact wave) approximate Riemann solver is a modification of the basic HLL scheme to account for the influence of intermediate waves. The HLL Riemann solver is correct for the purely 1d problem without extra species-like equations. For an augmented system allowing for shear waves the HLL approach is found to be inadequate, as it ignores the middle wave. This unsatisfactory situation motivated the introduction of the HLLC modification. The HLLC solver is an extension of the HLL solver to account for the aforementioned intermediate wave (Toro, 1997).

Both HLLC and HLL methods assume estimates $S_l$ and $S_r$ for the smallest and largest signal velocities of the Riemann problem with data $Q_l = Q_i$, $Q_r = Q_{i+1}$ and corresponding left and right fluxes $F_l = F(Q_l)$, $F_r = F(Q_r)$. For the interface wave speed estimate, there are several possible choices available. Toro suggests the estimate which leads to accurate and robust schemas:

$$S_l = u_l - c_l \quad S_r = u_r + c_r q_r$$

$$q_k = \begin{cases} \frac{1}{2} \left( \frac{(h_* + h_k) h_*}{h_k^2} \right) & h_* > h_k \\ \frac{1}{2} & h_* < h_k \end{cases}$$

(4.16)

$k = \{ l, r \}$

Here $h_*$ is an estimate for the exact solution for $h$ in the star region. The $h_*$ and also $u_*$ choice can be taken as follows:

$$h_* = \frac{1}{2} (h_l + h_r) - \frac{1}{4} (u_r - u_l)(h_l + h_r)/(c_l + c_r)$$

$$u_* = \frac{1}{2} (u_l + u_r) - (u_r - u_l)(c_l + c_r)/(h_l + h_r)$$

(4.17)

Figure 4.4 illustrates the assumed wave structure in the HLLC solver. There are 2 distinct fluxes for the star region. In the HLL solver we have no intermediate wave, thus no fluxes in the star region. In addition to the wave speed estimates $S_l$ and $S_r$ in the HLL solver, we now need an estimate $S_*$ for the speed of the middle wave. The following expression is used:

$$S_* = \frac{S_l h_r (u_r - S_r) - S_r h_l (u_l - S_l)}{h_r (u_r - S_r) - h_l (u_l - S_l)}$$

(4.18)
4.2 Numerical method

The HLLC numerical flux can be derived as

\[
F_{t+1/2}^{\text{HLLC}} = \begin{cases} 
F_l & 0 \leq S_l \\
F_{sl} & S_l \leq 0 \leq S_s \\
F_{er} & S_s \leq 0 \leq S_r \\
F_r & 0 \geq S_r 
\end{cases}
\] (4.19)

where

\[
F_{sl} = F_l + S_l (Q_{sl} - \tilde{Q}_l) \\
F_{er} = F_r + S_r (Q_{er} - \tilde{Q}_r)
\] (4.20)

The states \(Q_{sl}\) and \(Q_{er}\) are given by

\[
\tilde{Q}_{sk} = h_k \begin{bmatrix} S_k - u_k \\ S_k - S_s \\ u_k \end{bmatrix} \begin{bmatrix} 1 \\ S_s \\ u_k \end{bmatrix}
\] (4.21)

Computed fluxes (4.19) are now substituted into the finite volume expression (4.4) to find a solution for the new time step. From this point the procedure is analogous to that discussed for Roe's solver (Harten et al., 1983; Toro, 1997, 2001).

The advantage of the HLLC solver appears to be that it allows one to deal with dry bed states. A dry bed is where we have a zero value of our variables, which is the case for example with a landslide thickness \(D\). In our simulations the thickness of the slide \(D\) can be zero, where we have no landslide material at the bottom of our domain. A popular way of dealing with this kind of problem is by artificially wetting a dry bed, that is by setting the landslide depth \(D\) to some small positive value. This approach might seem to work, but it brings an error and it also does not work well with an arbitrary bathymetry, since at any moment during the evolution of the solution you may get a zero or close to zero value at some point and the schema becomes unstable. The dry bed problem is not easy to fix with approximate solvers, as discussed by Toro (2001). The HLLC solver handles these situations relatively well which is why I decided to use it.
4.2 Numerical method

instead of Roe's. For the surface waves this problem has not yet been much of an issue as I use the method for the simulation of underwater landslides only.

4.2.4 Ordinary Differential Equations

The systems (3.49 - 3.51), (3.40 - 3.42) are not homogeneous, as we have source terms on the right hand side. We can simply treat source terms for the conservation form of finite volume discretization (4.4), using finite differences, after we have solved partial differential equations (PDE) using Godunov's finite volume method. It also must be applied for every fractional step in the surface wave equations (4.2, 4.3). Here I give a discretization for the x-split equations. The procedure for the y-split is analogous.

For the surface waves, an update due to the source terms is given as:

$$u_i^{n+1} = u_i^{n+1/2} + g h_w^{n+1/2} \frac{\Delta t}{2\Delta x} (h_{i+1} - h_{i-1})$$

$$h_{w_i}^{n+1} = h_{w_i}^{n+1/2}$$

For the landslide equations (3.40 - 3.42), source terms along the x coordinate in the discretization are:

$$u_i^{n+1} = u_i^{n+1/2} +$$

$$\frac{\Delta t}{2\Delta x} \left( D_i^{n+1/2} g \frac{h_{w_{i+1}} - h_{w_{i-1}}}{2\Delta x} - \frac{1}{\rho_2} u_i^{n+1/2} - \frac{2\mu u_i^{n+1/2}}{\rho_2} \frac{D_i^{n+1/2}(1 - \frac{d_{14}^{n+1/2}}{3})}{D_i^{n+1/2}} \right)$$

$$D_i^{n+1} = D_i^{n+1/2}$$

where the superscript $^{n+1/2}$ denotes a half time step, i.e. right after the solution procedure of PDE by Godunov’s method.

4.2.5 Stability

Godunov's method is overall monotone, does not give a rise to oscillations in the presence of discontinuities, and is of first order accuracy in time and space.
4.2 Numerical method

The Courant-Frederic-Levy (CFL) number is given by (4.24).

\[ C = \frac{\lambda}{\Delta x / \Delta t} \]  

(4.24)

Note that \( C \) is the ratio of two speeds, namely the wave propagation speed \( \lambda \) in the differential equation and the grid speed \( \Delta x / \Delta t \). As the CFL number is effectively the fraction of the cell width \( \Delta x \) travelled by a signal of speed \( \lambda \) in one time step of size \( \Delta t \), the stability condition says that the scheme will allow time steps \( \Delta t \) such that the fastest signals/waves do not travel more than a single cell of width \( \Delta x \) in time \( \Delta t \). This also means that characteristic lines cannot intersect cell boundaries (Figure 4.3(2)). A generalization of this condition reads

\[ \Delta t = C \frac{\Delta x}{S_{\max}^n} \]  

(4.25)

where \( S_{\max}^n \) is the maximum propagation speed in the differential equations. For the linear case \( S_{\max}^n = \lambda \). The stability condition for the Godunov scheme is

\[ 0 < C \leq 1 \]  

(4.26)

For non-linear systems, such as the shallow water equations, one implements the linearized stability condition (4.26), where a reliable estimate for \( S_{\max}^n \) must be found. For 2D problems one may use the local solutions of Riemann problems to provide a reliable estimate of \( S_{\max}^n \). An unreliable estimate may cause the scheme to crash. A usual estimate for multidimensional problems is

\[ S_{\max}^n = max_i \{ |u_i^n| + c_i^n \} \]  

(4.27)

where \( u_i^n \) is the normal velocity component at time level \( n \) and \( c_i^n \) is the celerity. In practical computations one chooses a value of \( C \) close to the maximum allowed by the stability condition (4.26) (Toro, 2001).

4.2.6 Boundary Conditions

The system (3.40 - 3.42) is subject to the condition of no slide transport through the coastal boundary and the assumption that the slide does not cross the open boundary. Hence we do not have to impose special boundary conditions.
on the landslide equations. For the system of nonlinear shallow water equations (3.49 - 3.51) we need to set boundary conditions. At the open boundary we simulate transmissive boundaries, setting

\[
\begin{aligned}
    h^n_0 &= h^n_1, & u^n_0 &= u^n_1, \\
    h^n_{m+1} &= h^n_m, & u^n_{m+1} &= u^n_m.
\end{aligned}
\] (4.28)

At the shore we set solid reflective boundaries as

\[
\begin{aligned}
    h^n_0 &= h^n_1, & u^n_0 &= -u^n_1, \\
    h^n_{m+1} &= h^n_m, & u^n_{m+1} &= -u^n_m.
\end{aligned}
\] (4.29)

I have reviewed briefly the numerical method for the solution of hyperbolic type problems, which is utilized in this work. In the next section I demonstrate some computational results with simplified cases obtained applying the discussed Godunov scheme.

4.3 Computational Test Results

4.3.1 Introduction

Numerical tests have been provided to demonstrate that the method produces consistent results and also to examine the effects of various model parameters. For 2D simplistic cases I present a comparison of the results computed by Godunov’s method with results obtained by other methods. Tests for 3D shallow water waves and Bingham landslide are provided to examine the suitability for hydrostatic 3D simulations applied to real cases. The following computations are presented:

- Comparisons of 2D computations for water waves generated by a solid smooth body with an analytical solution for the simple case.
- Numerical tests for 3D water waves generated by a solid body.
- Comparisons of 2D computations for Bingham landslide profiles on the incline with those generated by other researchers’ programs.
- Numerical tests for the 3D Bingham body on an incline.
4.3.2 2D/3D Waves comparison with analytical solution for 2D

Two-dimensional waves have been generated by a solid slide moving with a constant speed over a horizontal bottom (Figure 4.5, left). An analytical solution of the linear system for the waves generated by this simple slide motion is given by equation (2.16).

![Figure 4.5: 2D and 3D slide profiles at the horizontal bottom](image)

A subcritical case of the slide body motion is considered. Parameters for the 2D run are as follows: $H = 100 \text{ m} - \text{water depth}$, $g = 9.8 \text{ m/s}^2$, $U = 5 \text{ m/s}$ - slide velocity. The initial slide has a smooth sinusoidal profile $h(x, t_0)$ with a maximum height $h_{\text{max}} = 20 \text{ m}$ and length $L = 1.5 \text{ km}$. Comparisons of wave profiles computed by Godunov's schema Roe's solver with the analytical solution (2.16) for different moments of time are in Figure 4.6.

The general solution (2.16) is a superposition of three waves. During the first seconds of tsunami generation crest and trough are produced because of the abrupt initiation of the slide at $t = 0$. These waves are free waves and propagate with celerity speeds $c = \pm \sqrt{gH}$. Crest and trough waves propagate away from the source of generation quickly particularly for greater depths and lower Froude numbers. There is also a smaller trough which moves with velocity $U$ associated with the slide. (Figure 4.6). The movement of this trough corresponds to the basic hydrodynamic problem of flow over a hump in a subcritical case if one associates the frame of reference with the moving slide body.

Equations (4.7) solved numerically contain advective terms, while the analytical solution (2.16) is given for the linear equations (2.15). This difference in
the formulated equations (2.15) and (4.7) can have an influence on a final result, depending on the chosen settings. In this particular case, I have been more interested in comparing numerical and analytical solutions than two different formulations. In order to diminish the effect of non-linear terms in the system of equations (4.7) I have adjusted physical parameters of the slide to minimize error. The following has been taken into account:

- To satisfy the shallow water condition, the wave length must be larger than the characteristic water depth; thus the length of the slide must be much
greater than the water depth, i.e. $L >> H$.

- For the linearized case the characteristic amplitude of the waves $\eta$ must be much less than the characteristic water depth, i.e. $\eta << H$.

To satisfy both conditions the slide body must be long compared to the water depth, and at the same time, its height and speed must be low to avoid generation of high amplitude waves.

Numerical parameters for the 2D run are as follows: grid step $\Delta x = 50$ m; time step $\Delta t = 1.5$ s; Courant number (4.24) $C = 0.95$. Comparisons between profiles in Figure 4.6 show satisfactory results; i.e., a good match in amplitudes between analytical and numerical solutions for the 1st order accuracy Godunov’s scheme. A slight lag for the left trough develops with time. This inaccuracy can be associated with the upwind character of the scheme. This means that, for Godunov’s fluxes, the spatial difference is computed between $F_{i+1}$ and $F_i$ cells. It is likely that central scheme would remove this artifact, utilizing flux difference between $F_{i+1}$ and $F_{i-1}$. But the fact is that the upwind methods are superior to the central methods in Godunov schemes, providing better accuracy (Toro, 2001). In this case, higher order methods diminish or eliminate the discrepancy.

In the 3D case, water wave generation was also tested for the solid slide moving over a horizontal bottom with a constant speed. As with the 2D case, settings were chosen as follows: $H = 100$ m - water depth; $U = 5$ m/s speed of the slide, $W = L = 150$ m - slide length and width, sinusoid profile $h(x, t_0)$ with maximum height $h_{max} = 15$ m. These parameters violate the shallow water condition, because the water depth is close to the slide length. As this was a test run to check the numerical schema, it is of less importance here; the result in Figure 4.7 indicates a type of wave profile one should expect from a tsunami wave simulation in the simplest 3D case under a hydrostatic assumption. The development of the wave pattern is similar to 2D, producing a crest and two troughs. One of the troughs is located above the slide and propagates with the slide velocity. The generated waves are cylindrical and wave energy spreads out in two dimensions. Amplitudes are thus smaller under the same conditions, compared to the 2D case where energy is confined along one axis.
4.3 Computational Test Results

4.3.3 2D/3D Bingham landslide, comparison with BING for 2D

Comparisons between Bingham 2D landslide profiles computed by my program (Godunov, HLLC) and those generated by BING software under similar settings are presented. BING is software designed to compute 2D flow characteristics for various rheology models and settings. It was developed by Imran et al. (1999) and utilizes a Lagrangian numerical method.

I have computed profiles of the slide with an initial parabolic shape and positioned on a surface with a slope of 5°. The slide height is \( h_{\text{max}} = 20 \text{ m} \) with length \( L = 600 \text{ m} \) at the initial position. Water and Bingham slide densities are taken as \( \rho_w = 1 \text{ g/cm}^3 \) and \( \rho_s = 1.8 \text{ g/cm}^3 \) respectively. The dynamic viscosity is \( \mu = 0.1 \text{ kPa} \cdot \text{s} \). Comparisons are given, varying the yield stress \( \tau_0 \) while keeping other parameters fixed (Figure 4.8).

Figure 4.7: 3D water waves
Bingham material stops if the driving shear force decreases below the yield stress. On an incline it means that the slide moves under gravity until it eventually stops. While the thickness of the plug zone remains constant the shear zone becomes thinner because the moving slide leaves a trace in behind. When the shear zone thickness is zero the slide stops on the incline (Figure 3.3). Bingham landslide profiles are computed for the moments of time when the motion of the slide passes its developed stage; it then slows down and stops on the incline. The initial state is plotted in the first row in Figure 4.8 (right BING).

The comparisons in Figure 4.8 indicate similar slide forms under same settings between two programs. The greatest difference in slide heights is up to 10% and can be seen in longer runs; i.e., for lower yield stress numbers. The Godunov scheme produces moderate smoothing. Due to the complex source terms in the landslide equations (3.40 - 3.42) to satisfy the stability condition, the Courant...
number is taken to be $C \leq 0.3$. This contributes to numerical diffusion in the scheme and creates smoothed profiles.

An interesting slide motion feature in this modelling is that the left part of the slide displaces in the uphill direction. This is back propagation as a result of the high horizontal gradients in the initial parabolic shape. As local angles are higher than the incline angle the total force acts in the uphill direction on the left side. The result is due to model formulation where a slide has fluid properties. In nature, it is difficult to imagine that a landslide moves upward, because it is mainly solid initially. In the experiment the assumed layer of fluid released on the low angle surface, flows upward a short distance.

In modelling the 3D Bingham fluid on an incline underwater, the slide has a cosine profile with $h_{\text{max}} = 15 \, \text{m}$, length and width $L = W = 180 \, \text{m}$, dynamic viscosity $\mu = 1 \, \text{kPa} \cdot \text{s}$, yield stress $\tau_0 = 5 \, \text{kPa}$, densities are $\rho_w = 1 \, \text{g/m}^3$ and $\rho_s = 1.8 \, \text{g/m}^3$ for water and the slide respectively.

Stability problems were encountered when trying to make a run over a completely "dry bed"; i.e., zero slide thickness in the computational domain areas which do not include a slide itself. To avoid associated instabilities, the bed was "wetted", setting the base ambient level to 2 m, although it was possible to set it as low as $\approx 0.3 \, \text{m}$. Changing this parameter the grid step was $\Delta x = 2 \, \text{m}$, time step $\Delta t = 0.05 \, \text{s}$, and Courant number $C < 0.3$.

Bingham fluid simulation snapshots are presented for different moments of time (Figure 4.9). The slide stopped when its thickness became $D \approx 3.0 \, \text{m}$. The cessation of motion coincides with a value of $D = 2.91 \, \text{m}$ which can be obtained from the formula for the critical slide thickness on a slope under a hydrostatic assumption (3.44). The maximum front velocity observed in this test was $10.4 \, \text{m/s}$. In contrast to Newtonian fluids, Bingham fluids stop on the incline and numerical results indicate that a run-out distance is sensitive to the yield stress. Large numbers can significantly reduce mobility of the flow. In the Bingham slide formulation it becomes possible to compare run-out distances in real case scenarios with known information from a site.

An assumption has been made that the flow remains laminar. A flow regime depends on the Reynolds number. Maximum Reynolds numbers for the Bingham slides will be smaller than those of viscous flows owing to the effect of Bingham
plasticity and lower viscosities (Mei & Liu, 1987). Wan (1982) has described the transition from laminar to turbulent states in terms of the following effective number:

\[
Re = \frac{4\rho UD}{\mu + \frac{10D}{2U}} < 2100 \tag{4.30}
\]

To estimate the approximate Re number I have taken the average velocity to be \( U = 7 \text{ m/s} \) and the average slide thickness to be \( D = 7 \text{ m} \). The effective Reynolds number is 103 in the 3D test and satisfies the laminar condition (4.30) of the flow, yet at a somewhat higher speeds and slide thicknesses turbulence may occur.

Figure 4.9: 3D Bingham landslide
Bingham visco-plasticity has a certain additional advantage over Newtonian fluids. Because Bingham material can stay stationary on a slope, it becomes possible to model processes such as the incorporation of sediments from the bottom during landslide motion. It is known that landslides starting from small failures can develop into massive slides because they incorporate more and more bottom sediments in the flow. In practical simulations it means that a Bingham material can be set on a slope with its thickness less than critical and it will stay stationary until a slide impacts it and incorporates the additional volume. For real case modelling it is of importance when working with accurate site bathymetry. In the next chapter a discussion on a landslide failure scenario with resultant tsunami waves near Kitimat is presented.
5 Kitimat, 1975 failure numerical simulation

5.1 Introduction

Kitimat Arm lies at the head of Douglas Channel, a major fjord system on the northern coast of British Columbia, Canada (53°59'N, 128°41'W) (Figure 2.1). It has a history of landslides with several occurring during the period 1952-1968 and in 1971. On October 17, 1974 following a submarine slide, a water wave of 2.8 m amplitude was generated. On April 27, 1975 following a major slide, water waves with amplitudes up to 8.2 m were generated, causing substantial damage to the coastline areas. The waves damaged a dock belonging to Northland Navigation Limited, destroyed dolphins at the Eurocan shipping dock, and damaged shore installations at the First Nations settlement of Kitimat Mission some 4 km across the inlet on the east shore (Golder Associates, 1975; Swan, 1978; Murty, 1979).

In this chapter a detailed description is given for the 1975 event and the area of the landslide-generated tsunami. This case presents a particular interest, since the only analysis of tsunami wave heights in this region was performed by Murty (1979) and was very simplistic. New tools available today such as computer simulations are able to improve on the older estimations. Numerical modelling was carried out for the landslide failure and resultant water waves for the event. Available bathymetric data are utilized to achieve realistic simulations. In later paragraphs results are discussed and compared with known information on the event. Significant characteristics such as water wave heights and timing are computed.

5.2 Event and site description

Douglas Channel, which is more than 70 km long, is divided into several basins by moraines and bedrock sills. The innermost basin floor has a general depth of 200-220 m and receives sediment from the Kitimat River, most of which are deposited in the prograding delta. The floor of the basin and the adjacent fjord walls are covered with gray marine muds, which were deposited after the retreat of Pleistocene glaciers (Bornhold, 1983).
At 10:05 a.m. on April 27, 1975, a major submarine slope failure occurred at the head of Kitimat Arm. The failure happened at approximately 53 min after an extreme low tide and was triggered by relatively minor construction operations on soil deposits that were in state of critical equilibrium (Murty, 1979).

Two major landslides were involved: 1) a landslide in sensitive marine clay on the west wall of the fjord; and, 2) a large-scale failure of deltaic sediments from the foreslope of the Kitimat River delta. The failed mass of delta and sidewall sediment (estimated volume $26 \times 10^6$ m$^3$) slid down into the fiord and apparently caused the mobilization, by undrained loading, of a further $28 \times 10^6$ m$^3$ of fiord-bottom sediments, creating a massive, submarine, kilometer-wide debris flow of about $55 \times 10^6$ m$^3$ that travelled over 5 km from the delta front over slopes of less than 0.4° to a water depth of 210 m. Individual blocks (outrunner blocks) of considerable size (up to $35 \times 10^3$ m$^3$) slid along the seafloor up to 1 km beyond the distal limit of the main flow. The landslides generated a series of displacement waves that caused substantial damage to nearby dock facilities and coastal settlements in the upper reaches of Kitimat Arm (Prior et al., 1983).

The slump was reported to have started at the breakwater, and within 2 minutes or so had worked its way around the shoreline along the road to the wharf. The log crib wharf structure was carried away by the slide. Slumping of the breakwater and shoreline was followed by propagation of the seawave, which fanned out from Moon Bay and struck Northland Navigation’s wharf, Eurocan’s dock, and swept across the inlet to the Kitimaat Haisla village (Figure 5.1). Backward and forward oscillations of the water across the inlet were observed for over 1 hour after the initial event; the height of the wave was reported to have been 8.2 m at Kitimaat village based on observations of water marks on pilings.

On Monday, April 28 two further slumps occurred in the general area of the wharf at the north end of the slump scarp, but these must have been of relatively minor proportions, as no waves were reported (Golder Associates, 1975).

5.3 Geotechnical characteristics; post failure analysis.

The Kitimat submarine slide occurred as a result of shearing failure in the soft marine clay that forms part of the seafloor sediments in Moon Bay. The change
5.3 Geotechnical characteristics; post failure analysis.

Figure 5.1: Kitimat area map
(Golder Associates, 1975)

in the distribution of loading as a result of construction of the breakwater was insignificant relative to the forces involved in the stability of such a large mass of slide material. Although construction of the timber crib dock, and the breakwater certainly contributed to the triggering of the slide, the slope was preconditioned by the following.

- The presence of soft, moderately sensitive marine clay near shore, and beneath the sloping seafloor.
- Excess pore water pressure within clay resulting from: 1) extreme low tide
5.3 Geotechnical characteristics; post failure analysis.

2) saturated soil conditions as a result of run-off from spring snow melt from higher elevations on the west shore of Kitimat Arm.

- An implied factor of safety (the ratio between the shear stress at which the ground failure can occur and the actual stress) is only very slightly greater than 1.0 along this segment of Kitimat Arm.

When shearing strains within the clay became large enough for the clay along the shearing surface to be remoulded, the soil suffered a significant loss in strength. Under these conditions, the slide mass could be expected to accelerate in the seaward direction (GolderAssociates, 1975).

Geotechnical characteristics of fjord sidewall sediments were determined through coring and laboratory analysis. Four distinct instability mechanisms were identified within the fjord, including failure of the fjord wall and delta-front sediments, initiation of movement of fjord-bottom clays, long-distance translational movement on the low-angle fjord floor, and block gliding at the downslope front of the landslide. Using post-failure geometries and geotechnical data, factors such as coastal construction, tidally induced drawdown, and undrained loading were evaluated as possible initiators of sediment instability (Johns et al., 1985).

A schematic sketch of the seafloor morphology (Figure 5.2) at the head of Kitimat Arm illustrates the geomorphology of failures on the Kitimat River delta and submarine debris flow (Prior, 1984). The entire feature assemblage is analogous to some types of terrestrial landslides such as fine-grained debris flows and mudflows described by Varnes (1978). The main elements are as follows:

- Shallow rotational movements, which have transported sediment to the base of the delta front.

- Lateral marginal zones of tensinal shearing along the edge of the displaced mass, which help to incorporate marine fjord-bottom sediments into the debris flow.

- A central area of differential sliding and flowage where longitudinal shearing is present.
5.3 Geotechnical characteristics; post failure analysis.

Figure 5.2: Kitimat seafloor morphology 
(Prior, 1984)

- A large area of accumulated debris in which compression has produced arcuate pressure ridges and fold systems toward the downslope limits of sediment movement.

- A distal blocky lobe with outrunner blocks that apparently have glided beyond the margin of the main debris flow.

The deposits resulting from slope instability and debris flow processes cover the bottom of the fjord from the base of the delta front to a water depth of 210 m over a maximum distance to 5.5 km. The depositional unit is about 1.8 km wide at the base of the delta front and tapers slightly downfjord to about 1.1 km in width at the distal margin. The average thickness of the deposit (from subbottom profiles) is approximately 8 m, with a maximum thickness upslope. The deposit therefore is extremely thin and elongated (Prior et al., 1984).

The steepest underwater slopes are found near the delta front, at the northern end of the inlet, and along the sidewall of the fjord. Slopes angle on the delta front achieve a maximum of 8°, whereas sidewall slopes near the delta have angles of up to 14°. Immediately seaward of the delta is an intermediate area where the
5.4 Kitimat Bathymetry

bottom slopes downfjord at 2.25°. For approximately 3 km farther down the fjord, the seafloor is covered with irregular sediment hummocks and blocks and has a surface slope of 0.56°. Beyond the slide area, the smooth, undisturbed fjord floor has a slope of 0.37° (Johns et al., 1985).

Extremely complex depositional geometries are found in front of the channels on the delta slope. Coalescing and overlapping masses of sediment, discharged from the gullies, form local lobes. These local depositional lobes probably represent several different delta-front failure events. The subbottom profile shows that about 4 m of the deposit is acoustically incoherent material, probably as a result of disturbance of the uppermost stratified Holocene sediments below the main body of the debris flow. There is also evidence of some marginal incorporation of fjord bottom sediments along the upslope edges of the debris flow (Bornhold, 1977).

Approximately half of the total volume of the delta debris flow deposits is of fjord bottom origin, whereas the remainder, about $27 \times 10^6$ m$^3$, was introduced from the sidewall and involved an area of about 1 km$^2$. The volume of the evacuation features is thus broadly equivalent to the volume of sediment required to make up the total debris flow unit (Prior et al., 1984).

5.4 Kitimat Bathymetry

The bathymetry of the Kitimat area was surveyed in 1952, before the occurrence of the 1975 slide events. Original sounding data were acquired and remapped using a computer mapping routine. The 1981 survey yielded high-density bathymetric information; comparison of these datasets allowed an evaluation of the seafloor geometry before and after the slope failures. Details of the seafloor morphology attributable to instability processes were mapped from the side-scan and sub-bottom data (Prior et al., 1983).

For the numerical simulation the bathymetric information obtained for 1952 and 1981 is utilized. An equidistant grid with 20 m steps was generated to represent a computational domain. Bottom profiles obtained for 1952 and 1981 are in Figure 5.3. The bathymetry of 1952 is smoother compared to 1981. The differences are mainly due to the series of landslides, which occurred in the period
between 1952 and 1981 and also a result of sediment transport from the Kitimat River. The actual 1975 landslide occupied the area at the top left corner on the bathymetry plots in Figure 5.3.

In order to use bathymetric data for numerical simulation, the slide material (mobile sediments) has to be separated from the bottom topography (static part). This is difficult as it is not possible to know exactly which part of the seafloor was displaced during the failure on April 27, 1975. As it was the largest underwater failure between 1952 and 1981 it is reasonable to assume that all changes in bathymetry at the left upper corner (Figure 5.3) are because of this event.

The grids of 1952 and 1981 bathymetries have been subtracted to get the change in bottom topography over that period of time. The assumed landslide location was cropped from the data; this volume is utilized as the initial slide in computations (Figure 5.3(3)).

The estimated volume of the landslide material after integration is $28 \times 10^6 \, m^3$, which coincides with the volume introduced by transport from the Kitimat sidewall and delta (Prior et al., 1984). An approximate average area occupied by the slide is $1 \times 1.5 \, km$ (Figure 5.4, 5.6(a)). The slide features a maximum thickness
of 62 m on the sidewall and has a rough, irregular form (Figure 5.4). These irregularities in the computational grid can introduce serious difficulties for numerical methods, being a significant source of instability. Therefore usage of surfaces with many discontinuities is a stress test for numerical methods.

Thinner parts of the slide have not enough thickness and angles to initiate failure under Bingham rheology model formulation. This potentially provides realistic slide simulation for arbitrary slide forms as in this case. It makes sense that thicker sediment masses are unstable and fail first, eventually incorporating other parts of the slide into the flow.

5.5 Landslide modelling

For adequate numerical modelling it is necessary to choose correct model parameters. The process of water wave generation is found to be controlled by the overall characteristics of the slide such as volume, location, and rheology. Several physical quantities affect run out distance, thickness, and velocity of the slide and include densities of the slide and water, viscosity, and a yield stress. Some researchers have provided sensitivity and experimental analysis of these factors
5.5 Landslide modelling

(Locat & Lee, 2002; Rabinovich et al., 2001). For example, a change in density by 20% results in an increase of simulated wave amplitudes of 20%, however for density variations $< 0.1 \ g \cdot cm^{-3}$, changes in tsunami wave heights are negligible (Rabinovich et al., 2001). From laboratory experiments it has been shown that the yield strength contributes about 1000 times more than the viscosity to the resistance to flow of the fluid (Locat & Lee, 2002). These experiments confirm that the yield stress is the most effective parameter influencing slide mobility, hence should be chosen with particular care.

In order to obtain Bingham slide settings for the present modelling Core 4 sediment samples obtained near Kitimat were used (Prior et al., 1984). The Core 4 samples were taken several kilometers downfjord within the main deposits, and show a mixture of silt (56%) and clay (43%) with only a very small amount of sand (< 1%); water contents are high and generally decline with depth. Laboratory values of water and slide densities are $\rho_w = 1 \ g/cm^3$ and $\rho_s = 1.65 \ g/cm^3$ respectively (Prior et al., 1984).

Values of yield stress and viscosity for the Bingham material are not straightforward to obtain. Geotechnical measurements of sediment static resistance for Kitimat samples indicate a shear strength of the order of 25-40 kPa, at least four times larger than the yield stress required for failure. Much lower yield stresses occurred during the event. This marked decrease in sediment resistance can be attributed to a sudden rearrangement of clay particles during the flow within the clay and extensive remolding and mobilizing in the initial stages of failure. An even more efficient mechanism for a progressive decrease of yield stress during flow is water incorporation into the sediment mass, a process promoted by high rates of dilution at the interface between the flowing sediment and water. Taking into account these factors, a reduction in the yield stress of 0.5-3 kPa can be achieved (DeBlasio et al., 2003).

It has been proposed that the yield stress and viscosity could be related to the liquidity index $I_L$. Experimental results by Locat & Lee (2002) and Locat (2001) indicated that they could be estimated from the empirical formulae:

$$\tau_0 = \left( \frac{5.81}{I_L} \right)^{4.55}$$ (5.1)
where \( w_n \) is the natural water content, and \( w_L \) and \( w_p \) are the liquid and plastic limits, respectively. For the computations, lower values of water content (those in deeper parts of the slide) (Prior et al., 1984), are chosen with respective plastic and liquid limit parameters of the material in the Core 4. Based on this estimation the following settings for the computations were used: kinematic viscosity \( \nu = 0.01 \, m^2/s \), yield stress \( \tau_0 = 2.5 \, kPa \).

The digitized numerical domain for the Kitimat region has grid dimensions of 300 \( \times \) 187 with spatial steps \( \Delta x = \Delta y = 20 \, m \). The time steps are \( \Delta t = 0.025 \, s \) and \( \Delta t = 0.2 \, s \) for a landslide and tsunami waves respectively.

Numerical simulations were intended to answer the following questions:

- What will be the wave heights and timings of the arrival of the landslide-generated tsunami waves for the Kitimat fjord?
- What is the maximum speed, propagation distance, and thickness of the debris flow?

Figures 5.5 and 5.6 present snapshots of the slide form for various moments of time. The landslide exhibits a complex motion, especially during initial phase of the collapse; i.e. first 30s. During this period the slide accelerates because of the steeper bottom slopes (up to 14\( ^\circ \)) and local slopes in the slide body itself. It exhibits complex mobilization and reorganization transforming itself into a debris flow. Half of the volume comes from the side wall in the perpendicular direction to the inlet. Two fronts collide and merge to produce a massive flow moving along the fjord to the outlet of Douglas Channel.

The maximum speed 22.5 \( m/s \) is reached at the 19\( ^{th} \) second of the modelled failure. After this point the gained momentum is gradually spent, and the slide slows down until it stops, approximately 4 min 30 s after the inception of failure. Most of the slide sediments stop closer to the delta, as they do not attain enough acceleration to run over the gentle slopes (\( \lesssim 0.5^\circ \)) of the seafloor. The thickest part of the slide found after the landslide stopped (18 m) is located in the Delta Front area. The slide front thickness is 4.3 m with run-out distance of about
Figure 5.5: Kitimat 1975, Landslide and bathymetry
5.5 Landslide modelling

Figure 5.6: Kitimat 1975, Landslide profile
4.5 km, in good agreement with findings of seafloor studies. The Reynolds number computed, using (4.30) for the velocity $U = 15 \text{ m/s}$ and average slide thickness $D = 15 \text{ m}$ is approximately $Re = 1172$ and is below the limit to produce turbulence.

5.6 Tsunami modelling

Next is presented modelling for the water waves created by the submarine slide. A simple solitary wave theory was used by Murty (1979) to estimate Kiti-mat wave heights, though the estimates were given without consideration of the area and essential slide characteristics, such as bathymetry, coastline, and rheology. It also neglected the complicated interaction of waves presented in 3D simulations. The main result of the analysis provided an estimate for the distance of 305 m from the slide origin and estimated a maximum 6.3 m height for the wave.

Coastal boundaries and associated model boundary conditions are an important part of the tsunami modelling. The east and west sides of the inlet are mainly steep and thus reflective boundary conditions are appropriate. At the outer part of the fjord a radiative boundary condition has been applied. Minette Bay (Figure 5.1) is an extremely shallow area (most of the time there is no water in it) and the landslide failure occurred during a very low tide. The bay bottom is very rough because of vegetation and logs which is why the wave could not have penetrated and travelled through Minette Bay (Murty, 1979). The upper Delta Front boundary has thus been modelled as a wall.

Snapshots of the wave evolution, generated by the landslide are in Figure 5.8. During the initial moments, a positive wave (crest) propagates in front of the submarine slide across the inlet; a negative wave (trough) significantly reduces the water level in the generation region. Following wave reflections from the coastline, tsunami waves form a complicated structure of motions in the inlet.

Coastal regions of the greatest interest, in terms of wave run-ups and destruction, are the Eurocan Terminal, positioned at the delta; Northland Navigations Wharf on the side wall, precisely at the source of the landslide; The Kitimaat Haisla First Nations settlement on the opposite side of the inlet (Figure 5.1).
5.6 Tsunami modelling

In Figure 5.8 these places are marked as T - terminal, W - wharf and I - First Nations Settlement on every subplot.

![Graphs of simulated Kitimat sea levels](image)

Figure 5.7: Simulated Kitimat sea levels

Simulated wave records (Figure 5.7) provide wave characteristics for these sites: sea levels and timings of arrival. During the first 40 s of the slide failure, water recession in the area of the Eurocan Terminal and Navigation Wharf is observed. The minimum trough amplitude is -13.3 m for the Navigation Wharf in 15 s and is quickly followed by a short 6.9 m crest wave after about 50 s. The highest wave hit the opposite side of the inlet where a crest arrived at the Haisla settlement in 72 s reaching a maximum height of 10.9 m at the coast. Although the Eurocan Terminal is located very close to the slide area, the destructive impact occurred there 2 min after the inception of the slide and the crest reached 7 m. This destructive wave had the longest period and should have substantially flooded the area; water retreat occurred only after 4-5 minutes. A secondary crest
5.6 Tsunami modelling

Figure 5.8: Kitimat 1975, Tsunami wave
hit the Navigation Wharf in 3 min 20 s and had practically the same amplitude as the first impact. This was the main tsunami crest, which had reflected from the opposite side of the bay (east coast).

To study the effectiveness of the tsunami generation and to estimate the duration of the most intensive energy transition to the water surface the following analysis is presented (Figure 5.9).

![Figure 5.9: Energy transfer rate](image)

The plot was computed using equation (3.54) and represents the rate of wave energy increase. Figure 5.9 indicates that the initial phase of landslide failure is the most important in terms of tsunami generation. Between 10 - 30 s, the highest energy transfer rate is reached with a peak of 26.5 MW; about 70% of the total energy is transfered to surface waves from the slide during this short period of time. This result correlates well with other characteristics of the slide such as acceleration, where it reaches its maximum speed at the 19th s. As the slide velocity is established and starts declining after 40 s, the energy transfer is significantly reduced. The total energy transferred to waves is 550 MJ for this modelling.

An important characteristic of the landslide - wave system is the Froude number. The Froude number is usually computed for rigid-body slides, which move as an entity. In contrast, different parts of a visco-plastic slide move with different
5.6 Tsunami modelling

speeds. For this case Froude number can be defined as the ratio of the slide front velocity to the local long-wave speed (Fine et al., 2002). The computed Froude number is presented in Figure 5.10. It rises significantly during the initial phase of the failure, due to the increasing velocity and shallower water depth. At this stage it correlates well with the above energy transfer plot. The maximum Froude number is 0.59 between 18 and 20 s. The relatively flat area of the plot between 70 and 180 s indicates stable values of Froude number around 0.4; at this time the slide moves with an average speed of 17-18 m/s over insignificantly changing depth where seafloor slopes are about 0.5°. Decreasing values occur when the slide slows down and stops in 285 s.

![Figure 5.10: Froude number](image)

The most important feature of the slide modelling is the ability to capture all the topographic details of the slide and bathymetry. Providing that the utilized sediment rheology model is adequate, and parameters are chosen correctly, it should provide competent landslide simulation and the basis for wave modelling. In fact, there are still certain factors, which may have occurred during the failure, but it is difficult to take them into account and are targets for future investigation. They are:
5.6 Tsunami modelling

- Incorporation of the debris from the fjord basin, during motion. As the seafloor of Kitimat has high concentrations of silt and clay, part of this sediment was apparently incorporated into the debris flow. This could introduce an additional volume and would influence the dynamics of the slide. This may not affect surface waves significantly however, because it would not sufficiently increase the slide elevation over the seafloor.

- Hydroplaning or block gliding. At a certain velocity, pressure is sufficient for a thin water layer to be intruded beneath the debris flow, producing a lubrication effect. When hydroplanning is established, the moving debris flow is decoupled from its bed, and runout and head velocity become independent of debris flow rheology. Hydroplanning is likely to occur on low slopes (Mohrig et al., 1999). In Kitimat there were outrunner blocks; i.e., sediments which travelled farther than the main volume. This can be explained by the hydroplanning phenomenon.

- Retrogressive slides. These are consecutive slides triggered by the main slide at different moments of time. The current modelling accounts only for the instant and simultaneous failure of all sediments; i.e., two main parts of the slide (sidewall and delta front) collapse simultaneously.

- Irregularity of the moving material; i.e., varying rheology and density. Landslide sediments have much more complicated structure and can deviate distinctly from the imposed rheology. Parameters of Bingham rheology such as a yield stress and viscosity can be locally highly variable.

Campbell & Skermer (1975) mentioned that the duration of the slide could have been about 2 min; Murty estimated it as 0.5 min. Current modelling suggest that the slide travelled for around 4.5 min, which also agrees well with run out distances and average speed. It should be mentioned that a wave solution is not very sensitive to the slide propagation after 0.5 min, which is indicated in the energy transfer plot (Figure 5.9).

The numerical modelling results show extreme wave crests of 6 to 11 m generated in Kitimat Arm and provide estimates of wave amplitudes in the region. Times for wave propagation across the bay (2 km) are about 1 min 30 s. The
5.6 Tsunami modelling

height estimate by Murty (1979), was 6.3 m, measured from trough to crest. I believe this is hardly an adequate approach as it is known that a crest is a flooding destructive wave, while a trough brings water retreat. The crest itself by Murty's calculations was 4.3 m (Murty, 1979), significantly lower than observations.

Unfortunately it is not possible to obtain measured run-ups for the area and provide accurate comparisons, as there was no tide-gauge at Kitimat, and the tidal curve for the Kitimat could only be deduced by the curve for Bella Bella, which is the closest tidal station to Kitimat and is several hundred kilometers away. Only a few observations based on water marks at the coast are available (Murty, 1979). The lack and reliability of these records make it difficult to provide an in-depth analysis. In the report by GolderAssociates (1975) it is given that waves of 8.2 m were generated at the coast near the Haisla Settlement. It is not mentioned, however, that these measurements by water marks on pilings were taken during a low tide. Variability in the tide level can add several meters. Current simulations indicate that water waves higher than 10 m might have been generated at the east side of the inlet and were of the highest amplitude for the area.
6 Discussion

6.1 Simulation accuracy

The goal of this project was to understand the complex physics of landslide-generated tsunamis and to develop mathematical and numerical models to simulate real events. An attempt has been made to reproduce the complicated picture of water surface disturbances associated with an underwater slope failure that occurred at Kitimat, BC in April 1975. The following are the most important characteristics which were introduced into the research:

- The problem of landslides and associated tsunamis under a hydrostatic assumption in a 3D case was considered.
- Bingham rheology was applied to the modelling of the landslide to account for its complex motion.
- The shock-capturing, finite volume Godunov scheme was used for the numerical solution.
- Computations for the landslide-generated tsunami at Kitimat that occurred in 1975 were performed.

The problem of landslide-generated tsunami modelling is challenging because many factors are involved in setting up the problem and its numerical solution. After a solution has been obtained the question of greatest interest is the validity of the results. Accuracy is the main concern in any kind of simulation and is very important for landslide-generated tsunamis because errors can be significant and also cannot be measured exactly. To address this issue I provide here an attempt to evaluate possible errors in such models.

There are several stages in the problems solution, each of which has a certain degree of inaccuracy. The cumulative error introduced by simulations can be divided into three main parts:

1. Simplification of the actual physics of the landslide-generated tsunamis. Assumptions regarding the physics of the process, suitable for more idealized cases, but nature is much more complicated.
6.1 Simulation accuracy

2. Input data problems. A lack and inaccuracy of information about the modelled region, landslide, bathymetry.


The starting point, and the most important, is the physics of the processes associated with landslides and tsunamis. Problems that currently exist can be classified as follows:

- The tsunami wave - landslide interaction. A mechanism of energy transition from the bottom to the surface.
- Tsunami wave modelling. Run-up and boundary problems.
- Landslide modelling. Rheology.

The mechanism of energy transition is poorly studied because of its complexity. Additional energy can be transferred to the surface due to the shearing at the slide - water interface. Also a certain amount of energy dissipation is introduced into the water and dependent on this process. A theoretical model which accounts for the effect of this shear stress at the interface between water and slide masses was described in Harbitz & Pedersen (1992).

Tsunami wave modelling is the most important part in the simulation of landslide-generated tsunamis. The majority of the models utilize the shallow water approximation. Conditions for the validity of this approximation were revised in this work. This approximation should be used carefully because in many practical cases it is incorrect. For example, it is not well suited for modelling in generation zones where the water depth changes rapidly and along the shore where run-up and wave-breaking occur (Heinrich, 1992). The long wave approximation also fails to work in deep parts of the ocean and neglects dispersion of waves, which can be of significance when timings of wave propagation are long.

The hydrostatic assumption is a compromise between computational time and accuracy. Even considering the power of existing computing technology, it is still very time consuming to solve a full 3D system of Navier-Stokes equations, using a conventional PC. Computer clusters and supercomputers can probably provide the necessary performance boost, but these solutions are expensive. The best
way is to combine methods to save computing time. For example, the Heinrich group uses the 3D Torrey code to simulate slides on steep slopes and to compute wave breaking and run-up in coastal areas, while they use shallow water assumptions to propagate waves to greater distances (Heinrich, 1992). Neglecting wave dispersion leads to an overestimation of wave heights at the coast.

In modelling the source of generation, the landslide itself is the central part of the problem. Changes in landslide motion are reflected in tsunami wave amplitudes. The simplest approach is to model the slide as a solid block. This approach may be suitable for subaerial slides because waves are generated primarily at the time when the slide enters the water and subsequent underwater deformation may not significantly influence surface waves. Submarine landslides behave differently and often disaggregate and liquefy rapidly during the initial stages of failure; this is important for wave generation and thus must be accounted for. Different rheologies can be chosen to approximate slide motion, according to the available information about sediments at the site. In this research several rheology models were reviewed and the Bingham fluid formulation was utilized for the simulation of the landslide. It is found to be more sophisticated compared to Newtonian formulations and enables one to adjust landslide behaviour more accurately. Landslide sediments can have changing rheological characteristics during a failure. These changes depend on the amount of water incorporated into the slide, which in turn depends on the velocity of the slide. The greater the slide velocity the greater the interaction at the slide-water interface. Thus rheological parameters (yield stress and viscosity) are highly dependent on time and location.

There are many observations of reported submarine slide velocities and run out distances that are much higher than can be explained simply based on a rheology model. The presence of a basal layer of water can severely reduce bed friction and initiate hydroplaning. For example, some Grand Bank and Storegga Slide evidence can be explained by hydroplaning (Harbitz, 1992). Hydroplaning occurs only at high velocities and is likely not present in the initial stages of tsunami wave generation because the slide is only beginning to accelerate. Therefore as a tsunami generator this can generally be considered of little concern, unless the speed of the slide reaches a supercritical state due to hydroplaning.
Another type of error introduced into simulations is input errors. The minimal requirements for real scenario modelling are bathymetric data, information about the morphology of the seabed, landslide position, and volume. Bathymetries can usually be obtained from soundings near port facilities and important coastal construction sites, though it is often inaccurate near a coastline and may not be present for distant areas. A correct position for the coastal boundaries is essential for the modelling in the closed fjords and confined areas such as Kitimat Arm. In simplified models, which do not implement run-up, coastal boundaries are usually modelled as reflective walls. Inaccurate location of them results in erroneous wave amplitudes after multiple wave reflections.

A difficult input parameter is determining accurate information about the initial slide position and volume. Ideally these can be obtained by comparing bathymetric data before and after the event. Unfortunately in many cases these two datasets are not available. In such cases other possible means are used to locate and estimate a volume and position of the slide. To estimate potential landslide failures a risk analysis can be undertaken and approximated volumes used to simulate landslide-generated tsunamis.

A numerical solution is the final stage and unites first two stages. It takes equations from (1), incorporates them into the numerical scheme framework, and adds input data (2). After that the output is only dependent on the ability of the numerical scheme to correctly generate a final solution. This is the point where a researcher meets many difficulties. A complicated formulation of the equations, non-linearities, coupled with realistic rough bathymetric input data often introduces stability problems. This can lead to simplifications in the physical model and to the processing of the original data to make numerical modelling easier.

In my research on numerical methods for the simulations of open boundary flows, such as landslides and tsunamis, I found Godunov type schemes to be very suitable. The application of this method to shallow water flows is relatively recent and the experience is encouraging. Godunov methods are attractive for their ability to resolve strong wave and flow interactions with steep gradients yet remaining free from unrealistic oscillations. Modern finite volume schemes of the Godunov-type achieve an accuracy that is higher than first-order accuracy. The extension of Godunov methods to second-order accuracy brought back the
relevance of Godunov’s theorem: “All schemes of accuracy greater than one will produce spurious oscillations in the vicinity of discontinuities”. The difficulty is that first-order methods are rather inaccurate and that high-order methods produce spurious oscillations; for a long time this was a stumbling block in the development of good numerical methods for hyperbolic conservation laws. Total Variation Diminishing (TVD) methods then came onto the scene. The fundamental works by Harten (1983); Roe (1983); Sweby (1984) and many others have led to a mature class of numerical methods that are oscillation-free near shock waves and other sharp flow features, and retain second-order accuracy in smooth parts of the flow.

The simple approach of TVD methods is to provide a better representation of a solution discretization. The basic idea of these methods is to use higher order interpolation at cell interfaces in a FV domain before carrying out a solution of the Riemann problem. The reconstruction of the initial data from the Riemann problem at the cell boundaries can be made with a number of second order algorithms also known as limiters; the best known are minmod, MC, Superbee, Vanleer (Toro et al., 2001).

As a simple example, a piecewise linear reconstruction is shown in Figure 6.1. By interpolation to the cell interface \( j+1/2 \), using values of the neighboring cells and slope limiter algorithms a better accuracy for the given function can be achieved, yet suppressing spurious oscillations.
A robust numerical method, such as Godunov's, becomes a useful tool when combined with adequate measurements of seafloor features. Although only a first order accuracy schema has been applied in this research it has an advantage over FD methods in aspects such as stability. Simple tests of second and third order accuracy schemas for the simple 1D/2D problems indicated higher accuracy, with more computing time needed for the interpolation.

The question of main concern is the validity of the results obtained for the Kitimat area. Since there are few observations there is no objective way to conduct an accuracy analysis. In the Kitimat simulation I rely on accurate modelling of the landslide, where parameters such as the volume, yield stress, viscosity and overall location are chosen carefully and according to the available information.

The Kitimat landslide-generated tsunami case is well suited for modelling using a hydrostatic assumption for the landslide because the bottom slopes are low and the slide became thin very quickly. The fact that vertical accelerations and associated pressure gradients were neglected could have affected the slide behaviour during the first few seconds of failure. The Bingham rheology model works well, knowing the type of sediments in the area. The shallow water assumption for the waves may not be satisfied, especially in distant areas, where depth reaches 250 m. These model assumptions likely overestimate wave heights no more than by 20%.

Input errors are believed to be essentially random and almost unpredictable. For the current modelling, accurate location of the coastline is of significant importance, but there were some missing portions of bathymetric data near the coasts. The upper boundary has been assumed to be a wall but is a very shallow area in reality, thus reflection from that boundary could have been partial and waves probably lost energy flooding into Minette Bay. I would assume this could produce an average error of \( \pm 25\% \) in wave height.

Numerical accuracy is the last factor which contributes to the total error. The first order Godunov's schema, keeping the solution oscillation free, produces a smoothing of wave fronts and thus decreases wave amplitudes. My early numerical tests revealed that for the Courant number 0.25 (average for the Kitimat waves) the numerical dissipation can damp maximum wave amplitudes by 15%.
6.2 Conclusions

The numerical error compensates for that of the simplified physics (neglecting dispersion) and the total variation for wave heights would probably not exceed ± 25-30%.

In order to substantially improve accuracy in landslide-generated tsunami simulations, progress in many areas of science is needed. Many theoretical investigations have been provided already but, related to numerical methods, it is important to clearly understand the assumptions and limitations of the model. As in other areas of computational fluid dynamics, one of the restraining factors in tsunami modelling is a lack of computational power; with rapidly advancing technology, however, it might be possible to use more complicated models, moving to dispersive 3D theory in several years. Numerical methods is a rapidly developing field too and tremendous progress has been made in shock-capturing methods in the framework of finite volumes and Galerkin discontinuous finite element methods. Correct physics and mathematics of the model are certainly important factors, but without a drastic improvement in the reliability of input data, they are useless. In oceanography and geoscience better and more reliable acquisition makes it now easier to obtain data and measure important characteristics at sites of interest. Better risk assessments of subaqueous landslides and associated tsunamis in hazardous coastal areas should be possible in the near future.

6.2 Conclusions

- A computational framework for hydrodynamical simulations, described by shallow water incompressible Navier-Stokes equations has been developed.

- The Bingham rheology of the landslide is a good approach and also allows modelling of incorporation of sediments from the bottom. It can be extended to more complex rheologies to achieve better flexibility and control of the landslide behaviour.

- The results of landslide-generated tsunami modelling for Kitimat 1975 are satisfactory, but there was no tide gauge records for comparisons.
6.2 Conclusions

- Accurate modelling of abrupt topographic features is possible using robust Godunov's scheme. Higher order methods achieve better accuracy and is the way to explore.

- A wave run-up at the coast and a subaerial landslide are the next steps to improve on this research. They are connected with a dry bed problem and may require the implementation of the exact Riemann solver in the Godunov scheme.

- The dispersion of waves is important, thus it is necessary to develop a more complex tsunami model.
Bibliography


CAMPBELL, D. & SKERMER, N. (1975). Investigation of sea wave at Kitimat, BC. *BC water resources service*, 9. 5.6


Bibliography


FINE, I., RABINOVICh, A., KULIKOV, E., THOMSON, R. & BORNHOLD, B. (1998). Numerical modelling of landslide-generated tsunamis with application to the Skagway Harbor tsunami of November 3, 1994. *Proc. Int. Conf. on Tsunamis, Paris*, 211–223. 2.3.4, 2.9, 2.3.4, 2.3.4, 2.3.4

FINE, I., RABINOVICh, A., THOMSON, R. & KULIKOV, E. (2002). Numerical modeling of tsunami generation by submarine and subaerial landslide. *Earth and Environmental Sciences*, **21**(4), 69–88. 1.2, 2.3.3, 2.3.3, 2.3.3, 3.3.2, 5.6


Bibliography


GOLDER ASSOCIATES (1975). Report to B.C. water resources service on investigation of seawave at Kitimat, B.C. Tech. Rep. 9/87, Consulting Geotechnical Engineers. 5.1, 5.2, 5.1, 5.3, 5.6


HARBITZ, C. & PEDERSEN, G. (1992). Model theory and large water waves due to landslides. Tech. Rep. 4, Preprint Series Dept of Mathematics, University of Oslo. 1.1, 2.3.3, 2.3.3, 2.3.3, 6.1


HEINRICH, P. (1992). Nonlinear water waves generated by submarine and aerial landslides. J. Waterway, Port, Coastal and Ocean Eng., ASCE, 118(3), 249-266. 2.1.1, 2.3.1, 6.1


HIRSCH, C. (1990). Numerical computation of internal and external flows. John Wiley and Sons, LTD, Baffins Lane Chichester West Sussex PO19 1UD England. 4.1


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KULIKOV, E., RABINOVICH, A., FINE, I., THOMSON, R. & BORNHOLD, B. (1998). Tsunami generation by landslides at the Pacific coast of North America and the role of tides. Oceanology, 38, 323-328. 2.3.1


LOCAT, J. (1997). Normalized rheological behavior of fine muds and their flow properties in a pseudo-plastic regime. In Proceedings of the First International Conference on Debris-Flow Hazards Mitigation, Mechanics, Prediction, and Assessment; ASCE, San Francisco, 260-269. 2.2.4, 2.2.4, 2.2.4, 2.6, 2.2.4


Bibliography


MILOH, T. & STRIEM, H. (1978). Tsunami effects at coast sites due to offshore faulting. Technophysics, 46, 247-356.  2.1.1


MOHRIG, D., ELVERHOI, A. & PARKER, G. (1999). Experiment on the relative mobility of muddy subaqueous and subaerial debris flows, and their capacity to remobilize antecedent deposits. Marine Geology.  5.6


Bibliography


NUJIC, M. (1995). Efficient implementation of non-oscillatory schemes for the computation of free-surface flows. J. Hydraulic Research, 33, 101-111. 4.2.1

O'BRIEN, J. & JULIEN, P. (1988). Laboratory analysis of mudflow properties. Journal of Hydraulics Engineering, ASCE, 114, 877-887. 2.2.4, 2.2.4


PALMER, S. (1999). Geotechnical considerations for the proposed southwest harbor CAD facility, Unpublished manuscript. DNR Geology Division, 42. 2.1.1

PATTON, F. (1966). Multiple modes of shear failure in rocks. 1st International Congress in Rock Mechanics, Lisbon, 1, 509-513. 2.2.4


Bibliography


Bibliography


Bibliography


TORO, E. (1997). Riemann Solvers and Numerical Methods for Fluid Dynamics. Springer-Verlag. 4.1, 4.2.1, 4.2.3, 4.2.3

TORO, E. (2001). Shock-capturing methods for free-surface flows. ISBN 0-471-98766-2, John Wiley and Sons, LTD, Baffins Lane Chichester West Sussex PO19 1UD England. 3.3.1, 4.2.1, 4.2.1, 4.2.3, 4.2.5, 4.3.2


WATTS, P. (2000). Tsunami features of solid block underwater landslides. J. Waterway, Port, Coastal and Ocean Eng., ASCE, 126(3), 144–152. 2.3.3


