

IMPROVING TRAFFIC CONTROL THROUGH  
SIMULATION AND OPERATIONS RESEARCH

by

ROBERT JOHN TAYLOR

Bachelor of Arts and Science, University of Lethbridge, 1971

A THESIS SUBMITTED IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in the Department

of

Mathematics

ACCEPTED  
FACULTY OF GRADUATE STUDIES

DATE

7 April 75

DEAN

We accept this thesis as conforming  
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UNIVERSITY OF VICTORIA

FEBRUARY 1975

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Supervisor: Dr. S. R. Clark

ABSTRACT

There are two major problems involved in traffic control in an urban area. The first is the adjustment of the individual signals to maximize the number of vehicles passing through the intersections in a given period of time. The second problem concerns the synchronization of the lights so that traffic can flow with minimal delay.

The problems of improving traffic control are dealt with in three stages. First it is shown that with only minor assumptions being made the problem of maximizing the traffic flow through the individual intersections can be treated as a linear programming problem. In the second stage, the problem of synchronizing the signals with each other to minimize delay is examined through the use of a computer model to simulate traffic flow along a roadway between two intersections. In the final stage several methods of synchronizing the signals are evaluated. The procedures developed are applied to data for the City of Victoria.

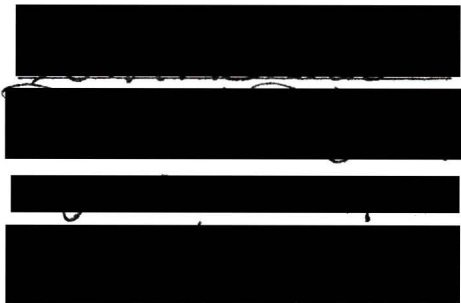


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## ACKNOWLEDGEMENTS

I would especially like to thank Mr. L. Roberts and Mr. D. Wild of the City of Victoria for their cooperation in providing the necessary data on traffic flow. In addition, they were generous enough to allow the use of the methods that developed from this research in adjusting the traffic signals for the off-peak hours. As a result of the success of this test, the City is currently in the process of modifying the signals for the peak periods and it is hoped that this will produce a noticeable improvement in the traffic flow.

In addition, I would like to express my gratitude to Dr. S. R. Clark, chairman of my advisory committee, for his guidance and encouragement throughout the stages of this study. Thanks are also due to the other members of my committee for their helpful suggestions and comments, the staff of the University of Victoria Computing Centre for their assistance, and Mrs. B. Hames for her careful typing of the manuscript.

CHAPTER 1  
INTRODUCTION

Recent growth of urban areas has placed tremendous strains on the transportation systems that serve them. There are two ways that the capacity of any such system can be increased: either the present system can be expanded; or, the efficiency can be improved. In the case of the network of roads in the core of a city expansion usually involves expensive and disruptive street widening. An alternative is to improve the efficiency by adjusting the signals that control the flow of traffic. We shall show that even minor adjustments to these signals can substantially effect the efficiency of the traffic system and so postpone the need for physical expansion.

There are essentially two major problems involved in traffic control in an urban area. The first is the adjustment of the individual signals to maximize the number of vehicles passing through the intersections in a given period of time. The second problem concerns the synchronization of the lights with each other so that traffic can flow smoothly and with minimal delay.

A signal controlling the flow of traffic at an intersection repeatedly goes through a set pattern of light changes. Green lights are displayed to some lanes of traffic allowing them to flow freely while other lanes are held back by red lights. Then the first lanes are halted and others allowed to pass through the intersection. This continues until all lanes have had a turn. Each separate combination of lights displayed to the traffic at an intersection is referred to as a phase of the signal. Thus, one phase may consist of green lights to both northbound and southbound traffic lanes while eastbound and

westbound vehicles are halted. This may be followed by a phase in which amber lights are displayed to northbound and southbound lanes so as to stem the flow of traffic from them. Hence, even the simplest of signals goes through a number of different phases. As the intersection becomes more complex (for example, by the inclusion of left turn only lanes and advanced or extended greens) the light pattern of the signal becomes much more complicated and the number of phases increase.

The complete set of phases of a signal constitutes a cycle of that signal. Thus, a given signal repeats a cycle of phases over and over. The time taken to pass through a set of phases is referred to as the cycle length of the signal.

A major problem facing traffic engineers is to determine the division of a cycle among the various phases to allow the maximum flow of traffic through the intersection. Traditionally this split was based upon the percentage of traffic utilizing the various lanes [13]. Thus, if northbound and southbound traffic constituted sixty per cent of the flow through an intersection they would be allotted a green phase for sixty per cent of the cycle length. This method is fairly crude and does not take into consideration either the number of lanes of traffic in each direction or the delays incurred if, say, numerous left turns are being made by the vehicles.

A second problem involving the traffic control signals is their synchronization by a master control so that traffic flows smoothly and with minimum delay through the entire street system. This master control regulates the time at which each signal begins its cycle. Thus, the signal at one intersection can start its cycle a number of seconds before the signal at another intersection. The difference

between the beginnings of the two cycles is referred to as the relative offset between the two signals. It is the objective of the traffic engineers to adjust these relative offsets so that traffic over the entire system is minimally delayed.

In the past, relative offsets have been set so that a hypothetical vehicle travelling at a constant rate of speed could progress through the signals without delay [14,11]. However, this method does not take into consideration the acceleration time required by the vehicle nor the influence of other traffic upon the time taken to traverse a block. Thus, while the signals may be adjusted so that a single vehicle travelling at a fixed rate of speed would have perfect progression through the signals, under more realistic traffic situations undue delays could result.

In this thesis we approach the problem of optimizing traffic signals in three stages. First, we show that with only a few assumptions being made, the problem of allotting a cycle among its phases can be treated as a linear programming problem. In the second stage, the problem of synchronizing the signals is examined through the use of a computer model to simulate traffic flow along a roadway between two intersections. In this way, large amounts of information about the delay that can be expected under various conditions can be gathered rapidly and conveniently without disturbing the actual system itself. Utilizing the model, we are able to describe the overall delay of the traffic in terms of the relative offsets of the signals involved. In the final stage several methods of synchronizing the signals are evaluated.

Chapter 2 is devoted to the explicit formulation of the problem of regulating the individual lights to maximize the traffic flow and the problem of synchronizing the lights to reduce delay. As the terminology of graph theory is particularly useful for formulating the problems, a brief introduction to the relevant terms is provided in the first section. The second section is devoted to the terminology of the signal system itself. The third and fourth sections deal with formal statements of the regulation of individual lights to maximize traffic flow (the cycle split problem) and the synchronization of the system to minimize the delay (the best offset problem). Historical background is provided in the final section.

The third chapter deals with reductions that are sometimes possible in the best offset problem. The first section shows how any signal in the system can be forced to have a fixed offset without changing the overall delay of the system. In the second section an algorithm is developed for totally solving the best offset problem under strict conditions. The third and fourth sections show how (under certain circumstances) the best offset problem can be simplified by decomposing it into a series of smaller problems. The final section of this chapter discusses some of the possible methods that could be used if no reductions to the problem are possible.

In order to investigate the nature of the delay in traversing a block between two signals, a computer model was developed. This model is described in the first section of Chapter 4 and supporting validation is given in the second section. The third section describes the typical pattern of delay that can be expected.

Chapter 5 applies the results of the previous chapters to the downtown signal system of The City of Victoria. The first section deals with the cycle split problem as it relates to Victoria while the best offset problem is examined in the second section.

Conclusions of this study and possible directions for further research are detailed in the final chapter. Program documentation appears in the appendices.

## CHAPTER 2

### FORMULATION OF THE PROBLEM

The terminology from graph theory provides an easy and concise way of formulating the problem of determining an optimal pattern of signals and so will be used extensively in this chapter. Most of the graph theoretic definitions are based on those of Frank Harary in his book Graph Theory [8]. The definitions used to describe the signal system itself are based on those used in a survey article by D.W. Ross [13].

#### 2.1 Introduction to the Terminology of Graph Theory

By a graph  $G$  we mean a non-empty finite set  $V(G)$  of elements called vertices together with a finite set  $E(G)$  of unordered pairs of distinct vertices referred to as edges. We will usually denote a vertex by  $v_i$  ( $i = 1, 2, 3, \dots, n$ ) and an edge by  $[v_i, v_j]$ . We say two vertices  $v_i$  and  $v_j$  are adjacent if  $[v_i, v_j]$  is an edge of  $G$ . Two edges are said to be incident if they have a vertex in common. An isolated vertex is one which is adjacent to no other vertex.

Traditionally a graph is represented by dots for the vertices and by joining two dots by a line if they are incident.

Hence, if  $G$  is a graph with:

$$V(G) = \{v_1, v_2, v_3, v_4\} \quad \text{and}$$

$$E(G) = \{[v_1, v_2], [v_2, v_3]\}$$

$G$  can be represented pictorially as in Figure 1.

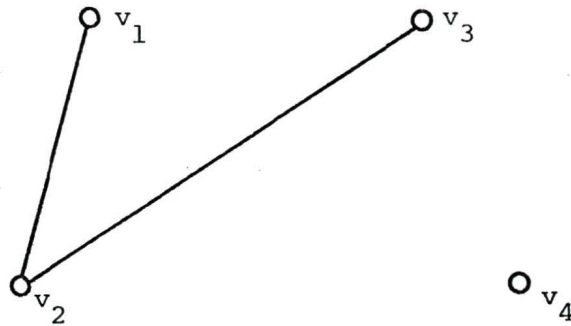


FIGURE 1. Pictorial representation of the graph  $G$ .

A subgraph  $H$  of a graph  $G$  is a graph with  $V(H) \subseteq V(G)$  and  $E(H) \subseteq E(G)$ .  $G$  is referred to as a supergraph of  $H$ . Hence the graph represented in Figure 2b is a subgraph of the graph represented in Figure 2a.

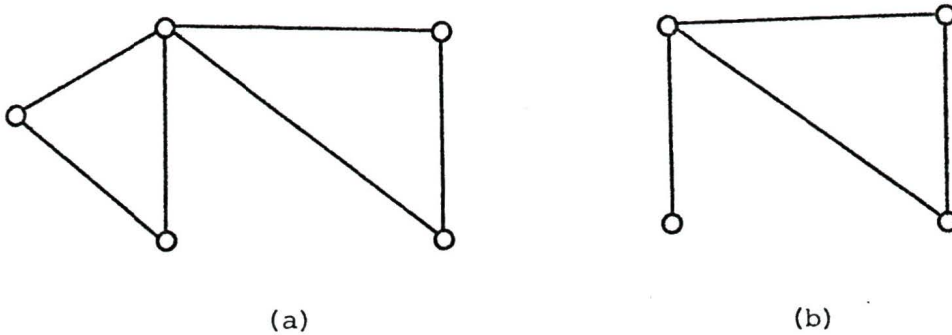


FIGURE 2. A graph and one of its subgraphs.

For a graph  $G$  we define the graph  $G' = G - \{v\}$  by:

$$V(G') = V(G) - \{v\} \quad \text{and}$$

$$[u_1, u_2] \in E(G') \text{ iff } [u_1, u_2] \in E(G), u_1 \neq v \text{ and } u_2 \neq v$$

For a graph  $G$  we define the graph  $G' = G - [v_i, v_j]$  by:

$$V(G') = V(G) \quad \text{and}$$

$$E(G') = E(G) - \{[v_i, v_j]\}$$

Hence if  $G$  is represented as in Figure 3a then  $G - \{v_1\}$  and  $G - [v_1, v_2]$  can be represented by Figures 3b and 3c respectively.

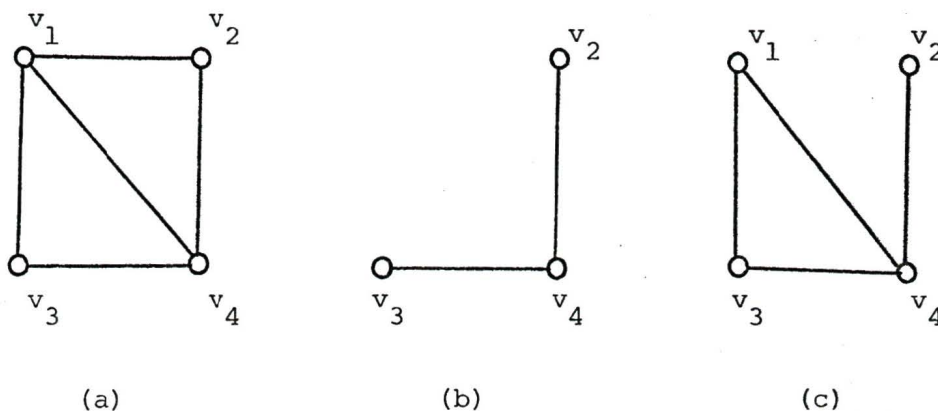


FIGURE 3. A graph  $G$ ,  $G - \{v_1\}$ , and  $G - [v_1, v_2]$

A path  $P$  in a graph  $G$  is a subgraph of  $G$  with  $V(P) = \{u_1, u_2, u_3, \dots, u_k\}$  and  $E(P) = \{[u_1, u_2], [u_2, u_3], \dots, [u_{k-1}, u_k]\}$  for some distinct  $u_i$ 's in  $V(G)$ . We say the path  $P$  joins  $u_1$  and  $u_k$ .

Thus if  $G$  can be represented as in Figure 4a the subgraphs represented in 4b and 4c are both paths while those in 4d and 4e are not.

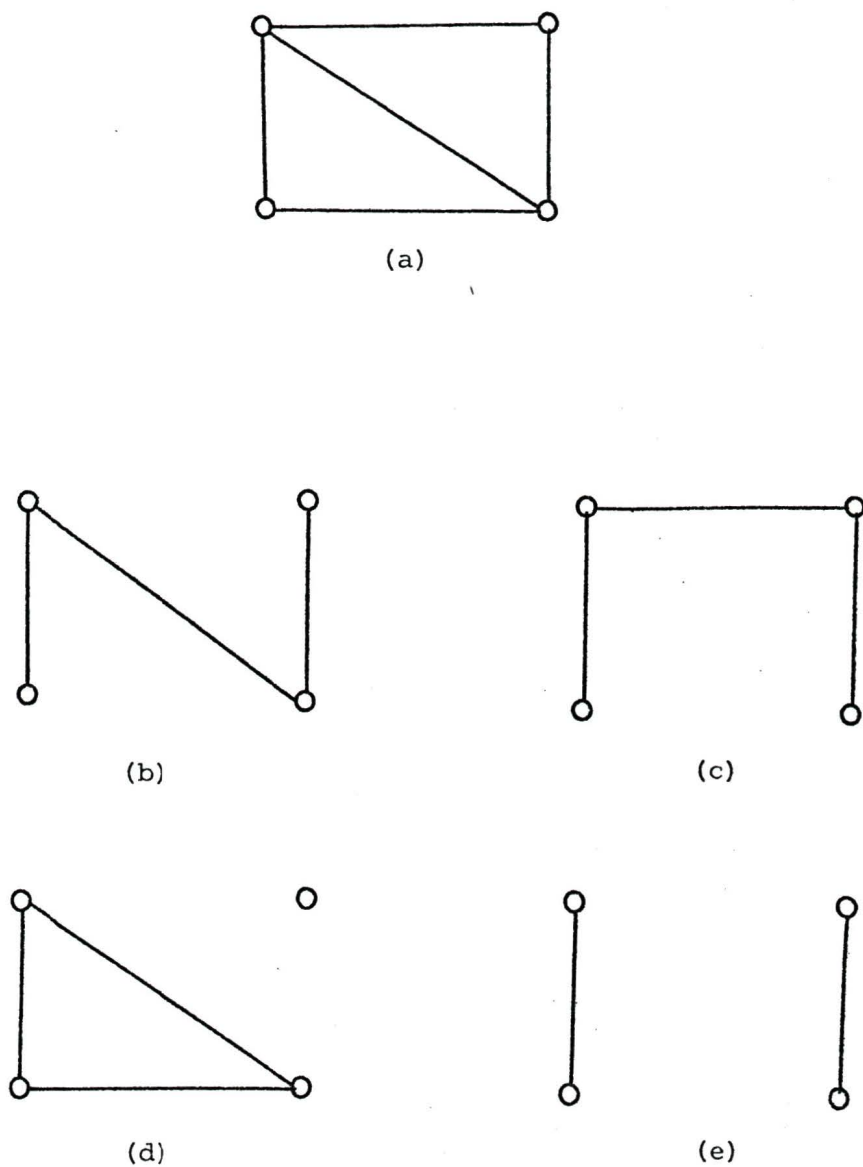


FIGURE 4. A graph  $G$ , two paths in  $G$ , and two subgraphs which are not paths.

A graph  $G$  is said to be connected if for any two distinct vertices  $v_i$  and  $v_j$  of  $G$  there exists at least one path joining  $v_i$  and  $v_j$ . Thus the graph represented in Figure 5a is connected while that in Figure 5b is not.

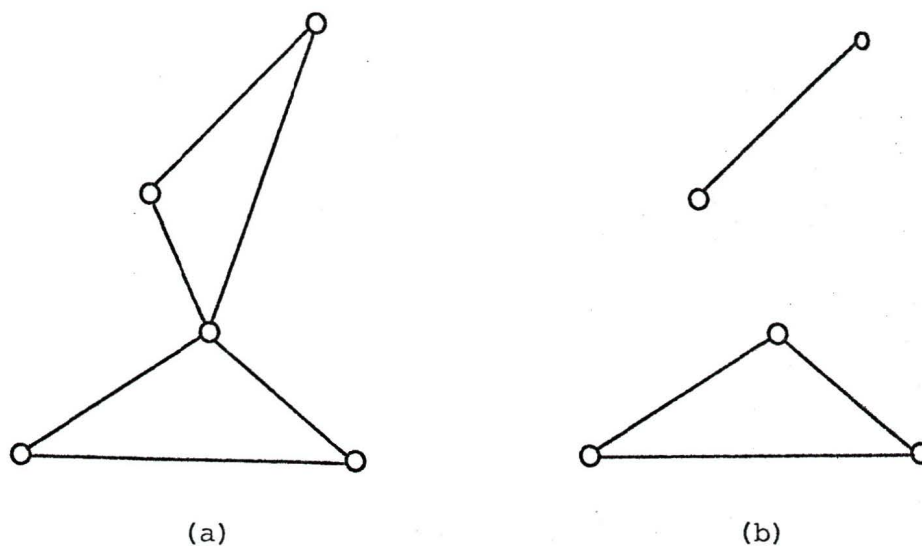


FIGURE 5. A connected graph and a graph which is not connected.

A graph  $G$  is said to be a tree if for any two vertices  $v_i$  and  $v_j$  ( $v_i$  and  $v_j$  distinct) there exists one and only one path joining  $v_i$  and  $v_j$ .

A component  $C$  of a graph  $G$  is a connected subgraph of  $G$  such that there exists no vertex in  $V(G)$  which is adjacent in the graph  $G$  to a vertex of  $C$  and yet is not in  $V(C)$ . That is,  $C$  is a maximal (with respect to the number of vertices) connected subgraph of  $G$ . The graph represented in Figure 6b is a component of the graph in Figure 6a.

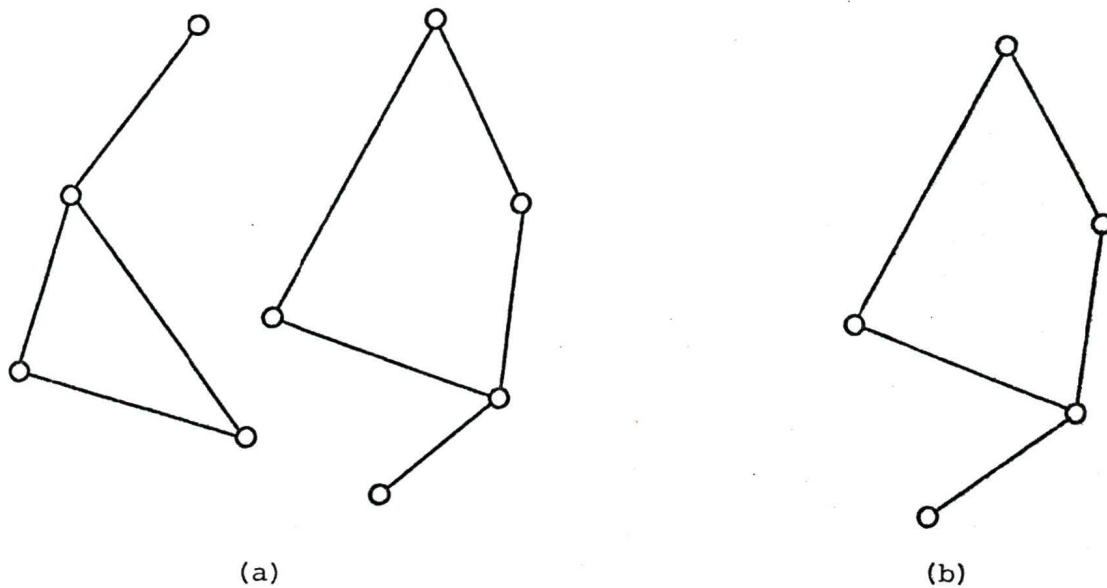


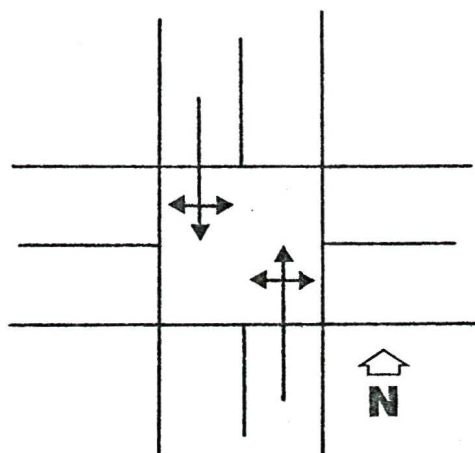
FIGURE 6. A graph and one of its components.

An edge  $[v_i, v_j]$  of a graph  $G$  is called a separating edge if  $G - [v_i, v_j]$  has one more component than  $G$ . If  $G$  contains a separating edge then we call  $G$  a separable graph.

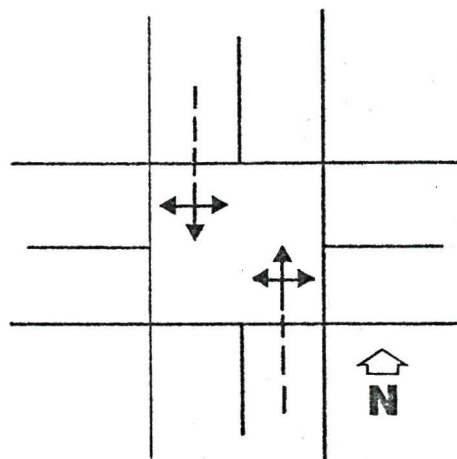
## 2.2 Introduction to the Terminology of the Signal System

In the type of street system with which we will be dealing each signal repeatedly goes through a sequence of phases during which certain traffic movements are allowed and others prohibited by displaying different coloured lights to the various lanes of vehicles.

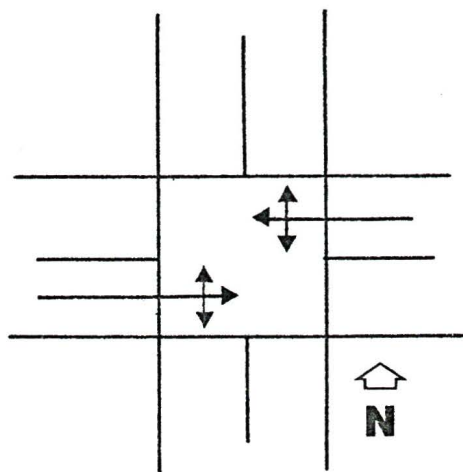
In Figure 7 a signal is represented which has four distinct phases ( $P_1, P_2, P_3,$  and  $P_4$ ). If we number the lanes as in Figure 8 the phases may be summarized as in Table I.



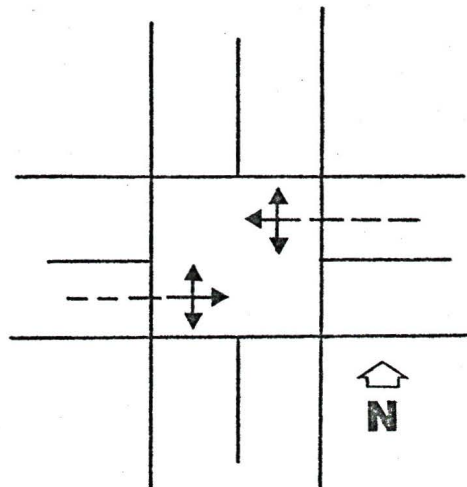
$P_1$ : Green for northbound and southbound traffic.



$P_2$ : Amber for northbound and southbound traffic.



$P_3$ : Green for eastbound and westbound traffic.



$P_4$ : Amber for eastbound and westbound traffic.

FIGURE 7. A simple intersection.

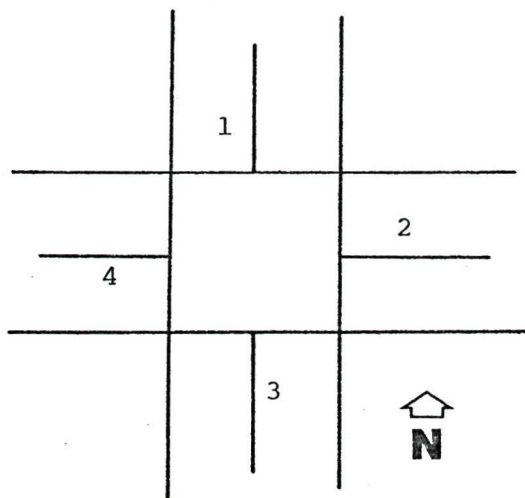


FIGURE 8. The numbering of the lanes of a simple intersection.

PHASE	LIGHT DISPLAYED TO LANE			
	1	2	3	4
$P_1$	G	R	G	R
$P_2$	A	R	A	R
$P_3$	R	G	R	G
$P_4$	R	A	R	A

G: Green

A: Amber

R: Red

TABLE I. Summary of the light pattern of a simple intersection.

Figure 9 represents a more complicated intersection involving left turn only lanes. Ten possible phases for this signal are summarized in Table II.

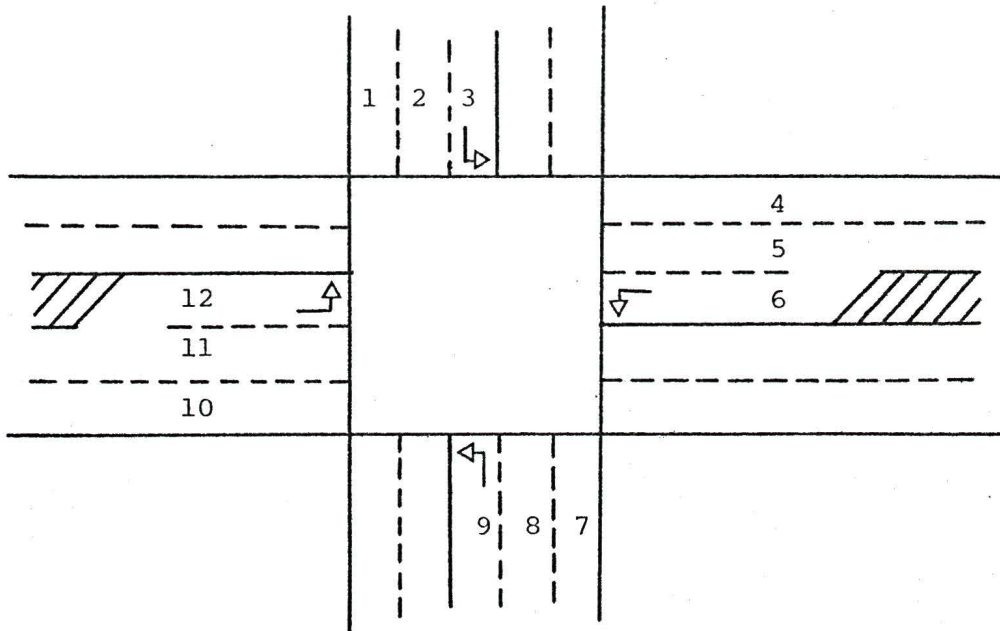


FIGURE 9. The numbering of the lanes of a more complex intersection.

PHASE	LIGHT DISPLAYED TO LANE											
	1	2	3	4	5	6	7	8	9	10	11	12
P <sub>1</sub>	R	R	G	R	R	R	R	R	G	R	R	R
P <sub>2</sub>	R	R	A	R	R	R	R	R	A	R	R	R
P <sub>3</sub>	G	G	R	R	R	R	G	G	R	R	R	R
P <sub>4</sub>	A	A	R	R	R	R	A	A	R	R	R	R
P <sub>5</sub>	R	R	R	R	R	R	R	R	R	G	G	G
P <sub>6</sub>	R	R	R	R	R	R	R	R	R	G	G	A
P <sub>7</sub>	R	R	R	G	G	R	R	R	R	G	G	R
P <sub>8</sub>	R	R	R	G	G	R	R	R	R	A	A	R
P <sub>9</sub>	R	R	R	G	G	G	R	R	R	R	R	R
P <sub>10</sub>	R	R	R	A	A	A	R	R	R	R	R	R

TABLE II. Summary of the light pattern of a more complex intersection.

We denote the length of time allocated to phase  $P_k$  by  $T_k$ . A repetition of a full sequence of phases is called a cycle while the time required to go through a cycle is referred to as the cycle length. The problem of determining how best to allocate the cycle length among the phases is the Cycle Split Problem. The allocation to the various phases can be easily represented as in Figure 10.

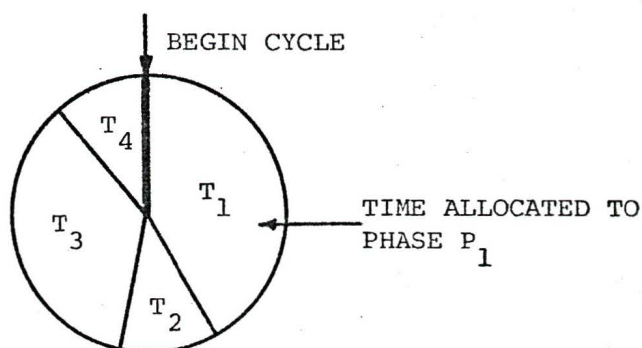


FIGURE 10. Allocation of the cycle to the phase.

In the system we will be dealing with, the cycle length of each signal is the same. Hence, we may talk about the cycle length  $C$  of a system of signals.

The signals of a light system are coordinated with respect to one another by reference to a master cycle. The offset of a signal is the difference in time between the beginning of the master cycle and the beginning of the signal cycle. Thus a signal with an offset of ten seconds begins its first phase ten seconds into the master cycle. This is illustrated in Figure 11.

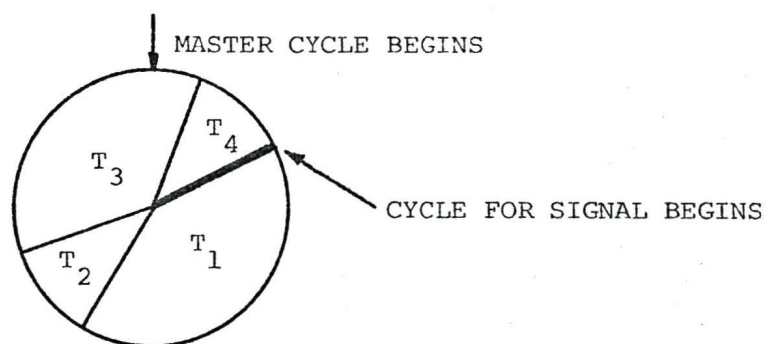


FIGURE 11. The offset of a signal.

A signal with offset equal to zero will be called a master signal. The problem of determining the set of offsets which will best serve to improve traffic flow is referred to as the Best Offset Problem (BOP).

We say two intersections are adjacent if there is a major roadway between the two. This roadway is sometimes referred to as a leg joining the two intersections.

We have now defined the terminology needed to formally state both the Cycle Split Problem and the Best Offset Problem. The Cycle Split Problem is formulated in Section 2.3 as a standard linear programming problem. The Best Offset Problem is formally stated in Section 2.4 and is dealt with in some detail in succeeding chapters.

### 2.3 Formal Statement of the Cycle Split Problem

The Cycle Split Problem may be formulated as a standard linear programming problem if certain assumptions about vehicle movements are made. We assume there is always a queue of vehicles waiting at an intersection (in effect assuming the intersection is working at capacity) and attempt to maximize the number of possible vehicle starts (S) through the intersection during a cycle.

Suppose the signal passes through phases  $P_k$  ( $k=1,2,3,\dots,m$ ). Let  $T_k$  denote the length of time allocated to phase  $P_k$ . If we assume for each phase  $P_k$ , for each leg  $j$  ( $j=1,2,3,\dots,n$ ) going into the intersection, and for each type of vehicle movement  $i$  ( $i=1,2,3,\dots,r$ ); there exists a constant  $c_{ijk}$  corresponding to the largest possible number of vehicle starts during a second of type  $i$  from leg  $j$  during phase  $P_k$  we may formulate the Cycle Split Problem as follows:

$$\text{Maximize } S = \sum_{k=1}^m \sum_{j=1}^n \sum_{i=1}^r c_{ijk} T_k$$

subject to the constraints:

- i) The sum of the times allocated to the phases must equal the cycle length, that is:

$$\sum_{k=1}^m T_k = C$$

- ii) Each phase must receive a minimum allotment of time (for example: sufficient amber to clear the intersection or sufficient green to allow pedestrians to cross), that is:

$$T_k \geq a_k \quad k=1,2,3,\dots,m$$

- iii) There must be a minimum amount of traffic flow of a given type passing through the intersection during a cycle from a given leg (for example: at least 25 vehicles per cycle must be able to turn left off the southbound leg), that is:

$$\sum_{k=1}^m c_{ijk} T_k \geq d_{ij} \quad i=1,2,\dots,r; \quad j=1,2,\dots,n$$

### 2.4 Formal Statement of the Best Offset Problem

For a given street system, we construct an associated graph  $G$  as follows:

Let  $V(G)$  be the set of all controlled intersections (labelled  $v_1, v_2, v_3, \dots, v_n$ ) and let  $[v_i, v_j]$  be in  $E(G)$  if and only if there is a uninterrupted leg between the two intersections. Figure 12 illustrates this construction.

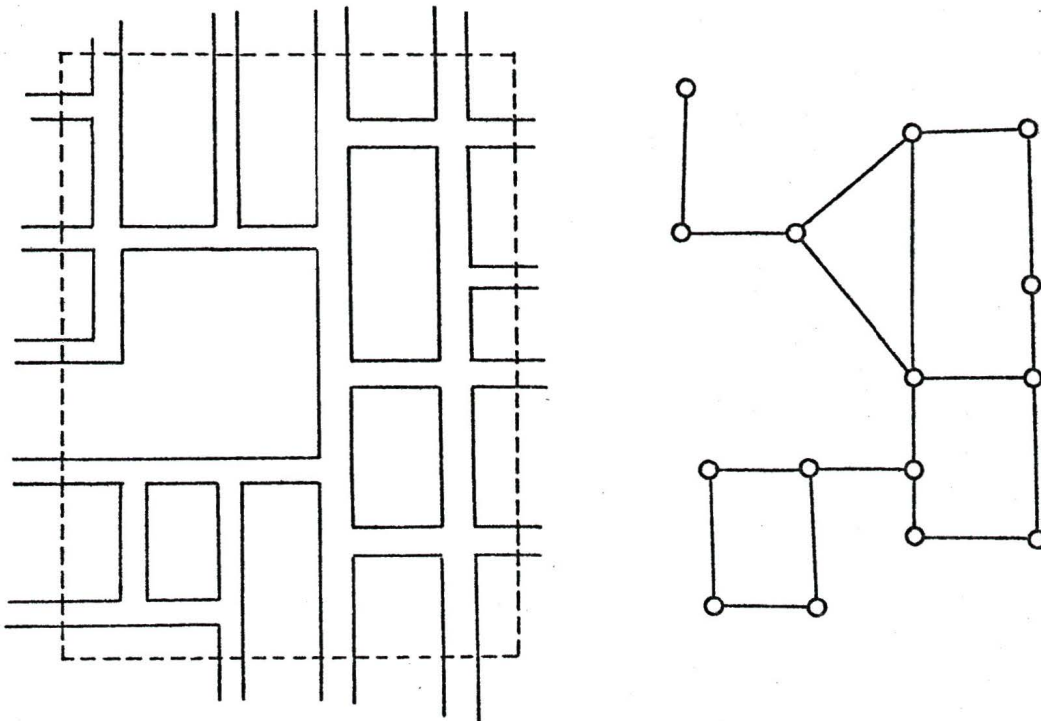


FIGURE 12. A street system and its associated graph.

As recent researchers in the field have done [13,1,9], we make the major assumption that with every edge  $[v_i, v_j]$  of the graph  $G$  we can associate a function  $f_{ij}$  ( $f_{ij}: \mathbb{R} \rightarrow \mathbb{R}^+ \cup 0$ ) which is a measure of the overall delay of vehicles using the leg as a function of the

difference between the offsets of the signals at either end of the leg.

We will also make certain assumptions about the nature of each of the functions  $f_{ij}$ . Because of the cyclic nature of the phases of each light, we assume  $f_{ij}$  is a periodic function with period equal to the cycle length of the system of lights. We will assume that the delay is always non-negative and that each  $f_{ij}$  obtains a minimum (denoted  $m_{ij}$ ).

For notational convenience let  $t_{ij} \in \mathbb{R}$  such that  $f_{ij}(t_{ij}) = m_{ij}$ . Let  $\theta_i$  denote the offset of signal  $v_i$ .

The Best Offset Problem may be formally stated as:

Given a street system with associated graph  $G$  ( $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ ) and delay functions  $f_{ij}$ , determine offsets  $(\theta_1, \theta_2, \theta_3, \dots, \theta_n)$  which will minimize

$$Z_G(\theta_1, \theta_2, \theta_3, \dots, \theta_n) = \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j)$$

As will be shown,  $Z_G$  is typically a very difficult function to minimize using standard methods. The next chapter is devoted to reducing the Best Offset Problem for a graph  $G$  to a series of smaller problems.

## 2.5 Historical Background

In the past, the allocation of time to the various phases that a signal repeatedly goes through has been in simple proportion to the traffic flow through the intersection [13]. In developments of late this has been augmented by simple simulation and "hill climbing" techniques designed to improve traffic flow through the intersections.[5]

The selection of offsets so as to synchronize the signals is somewhat more complicated. Hillier and Rothery [9] were among the first to consider the delay over a roadway between two signals as a function of the relative offsets of the signals.

A method developed under the auspices of the U.S. Federal Highway Administration [14] involves approximating the delay function by a series of parabolas. The optimization requires setting the derivatives of the overall delay function to zero and solving the resultant set of equations for a candidate minimum. This is repeated until one is "sufficiently certain that one has the global minimum or is at least close to it."

The first step in a method developed by the Road Research Laboratory [12] is to calculate a performance index based on the overall traffic delay and the number of vehicles stops on the leg for an initial set of signal offsets. These are altered by a fixed amount and the performance index is recalculated. This is done repeatedly as long as the performance index decreases. If it increases at any point, the search is made in another direction. This continues until a local minimum is obtained. The order in which the signals are to be adjusted must be stated. The City of Vancouver relies on a method very similar to that of the Road Research Laboratory written by J.C. Clapham [5].

The City of Victoria presently utilizes the more traditional method of selecting a number of roadways from the entire grid and setting the offsets of the signals so that a vehicle travelling at a fixed rate will have perfect progression through the lights. This method has the obvious advantage of not requiring extensive calculations, but the disadvantage that the overall delay may not be minimized.

This thesis approaches the problem of optimizing the signal system for the downtown core of the City of Victoria in three phases. In the first the problem of determining the optimal settings for individual lights is formulated as a linear programming problem. This approach was taken at the suggestion of J.C. Clapham [5].

In the second phase of the project a computer simulation of the traffic flow between two intersections is used to determine the characteristics of the delay functions. This model differs from those used previously in the field as it is somewhat more comprehensive. As suggested by Ross in his survey article, each delay function is then approximated by a sinusoidal function [13].

The third phase involves applying various function minimization techniques to the overall delay function to compare their effectiveness. Results show that the concept behind the traditional method used by the city (i.e. selecting a number of legs from the entire grid and optimizing over them) is satisfactory but the method of setting the offsets so that a hypothetical vehicle travelling at a fixed rate of speed would have perfect progression is shown to be unsatisfactory. By utilizing knowledge of the delay functions obtained through the computer simulation it is possible to adjust the offsets so that the delay is substantially reduced.

## REDUCTIONS TO THE BEST OFFSET PROBLEM

In this chapter we formalize some of the reductions known to traffic engineers and which can sometimes be applied to the Best Offset Problem. An algorithm for completely solving the problem when the graph involved consists of a set of components each of which is a tree or an isolated vertex is presented. A procedure for breaking the problem up into a series of smaller problems is also given.

Throughout this chapter we will assume (unless otherwise stated) that  $G$  is a graph with  $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ . We also assume a delay function  $f_{ij}$  ( $f_{ij}: \mathbb{R} \rightarrow \mathbb{R}^+ \cup 0$ ) is associated with every edge  $[v_i, v_j]$  of  $G$ .

### 3.1 The Selection of the Master Signal

The first proposition allows us to add a constant to the offset of every light without changing the overall delay. This enables us to make any one of the signals a master signal (i.e. have a zero offset).

#### Proposition 1:

If  $R$  is any constant then

$$Z_G(\theta_1+R, \theta_2+R, \dots, \theta_n+R) = Z_G(\theta_1, \theta_2, \dots, \theta_n).$$

PROOF:

$$\begin{aligned} Z_G(\theta_1+R, \theta_2+R, \dots, \theta_n+R) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}((\theta_i+R) - (\theta_j+R)) \\ &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j) \\ &= Z_G(\theta_1, \theta_2, \theta_3, \dots, \theta_n) \end{aligned}$$

### 3.2 The Solution of the BOP if G is a Tree

The next two propositions enable us to solve the BOP if G is a tree (or a tree combined with isolated vertices). Proposition 2 gives us a method of extending (under certain circumstances) a solution of a subgraph of G to a larger subgraph of G while increasing the delay minimally. Proposition 3 justifies assigning offsets arbitrarily to isolated vertices.

#### Proposition 2:

If  $G'$  is a supergraph of G such that:

$$V(G') = V(G) \cup \{v_{n+1}\} \text{ and}$$

$$E(G') = E(G) \cup \{[v_k, v_{n+1}]\} \text{ for some } v_k \text{ in } V(G)$$

then  $Z_{G'}(\theta_1, \theta_2, \theta_3, \dots, \theta_n, \theta_k - t_{k,n+1}) = Z_G(\theta_1, \theta_2, \dots, \theta_n) + m_{k,n+1}$   
 for any offsets  $(\theta_1, \theta_2, \dots, \theta_n)$ .

PROOF:

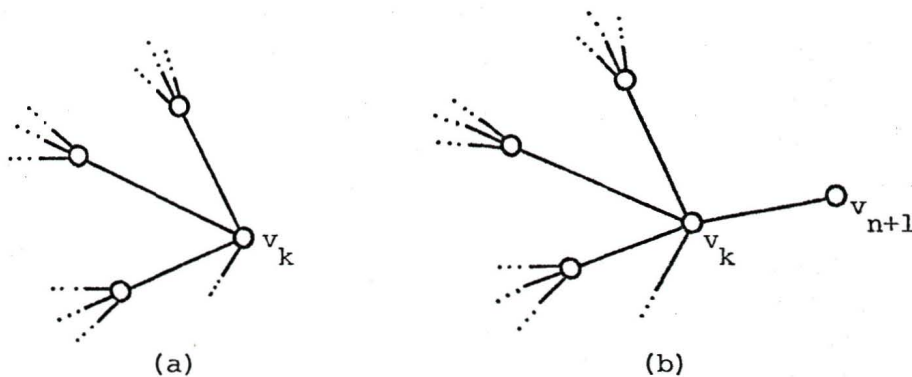


FIGURE 13. The graph G and the graph G'.

$$\text{Let } \theta_i^+ = \begin{cases} \theta_i & \text{for } i = 1, 2, 3, \dots, n \\ \theta_k - t_{k,n+1} & \text{for } i = n+1 \end{cases}$$

$$\begin{aligned}
\therefore Z_{G'}(\theta_1, \theta_2, \dots, \theta_n, \theta_k - t_{k,n+1}) &= Z_{G'}(\theta_1^+, \theta_2^+, \dots, \theta_n^+, \theta_{n+1}^+) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G')}} f_{ij}(\theta_i^+ - \theta_j^+) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G') - [v_k, v_{n+1}]}} f_{ij}(\theta_i^+ - \theta_j^+) + f_{k,n+1}(\theta_k^+ - \theta_{n+1}^+) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j) + f_{k,n+1}(\theta_k^+ - \theta_{n+1}^+) \\
&= Z_G(\theta_1, \theta_2, \dots, \theta_n) + f_{k,n+1}(t_{k,n+1}) \\
&= Z_G(\theta_1, \theta_2, \dots, \theta_n) + m_{k,n+1}
\end{aligned}$$

Proposition 3:

The offsets of isolated vertices may be chosen arbitrarily without affecting the overall delay.

PROOF:

Without loss of generality assume  $v_1, v_2, \dots, v_k$  are the isolated vertices. Let  $H = G - \{v_1, v_2, \dots, v_k\}$ . We will show for any choice of  $\theta_i$  ( $i=1, 2, \dots, n$ ) and  $\theta_j^i$  ( $j=1, 2, \dots, k$ ),  $Z_G(\theta_1, \theta_2, \dots, \theta_k, \theta_{k+1}, \dots, \theta_n) = Z_G(\theta_1^i, \theta_2^i, \dots, \theta_k^i, \theta_{k+1}, \dots, \theta_n)$  by showing both are equal to  $Z_H(\theta_{k+1}, \theta_{k+2}, \dots, \theta_n)$ .

Note that since  $v_1, v_2, \dots, v_k$  are isolated vertices  $E(H) = E(G)$ .

$$\begin{aligned}
\therefore Z_G(\theta_1, \theta_2, \dots, \theta_n) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(H)}} f_{ij}(\theta_i - \theta_j) \\
&= Z_H(\theta_k, \theta_{k+1}, \dots, \theta_n)
\end{aligned}$$

$$\text{Let } \theta_i^+ = \begin{cases} \theta_i' & \text{for } i = 1, 2, \dots, k \\ \theta_i & \text{for } i = k+1, k+2, \dots, n \end{cases}$$

$$\begin{aligned}
\therefore Z_G(\theta_1^+, \theta_2^+, \dots, \theta_n^+) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i^+ - \theta_j^+) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(H)}} f_{ij}(\theta_i^+ - \theta_j^+) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(H)}} f_{ij}(\theta_i - \theta_j) \\
&= Z_H(\theta_k, \theta_{k+1}, \dots, \theta_n)
\end{aligned}$$

In the case where  $G$  is a tree, or a tree combined with isolated vertices, Propositions 2 and 3 enable us to give an algorithm for determining  $(\theta_1, \theta_2, \dots, \theta_n)$  so that  $Z_G(\theta_1, \theta_2, \dots, \theta_n)$  is minimal, that is

$$Z_G(\theta_1, \theta_2, \dots, \theta_n) = \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} m_{ij}.$$

We will assume  $G$  has at least one edge, otherwise  $G$  consists of isolated vertices and from Proposition 3 we know that the offsets can be chosen arbitrarily.

The algorithm is as follows:

STEP 1. Choose any edge  $[v_r, v_s]$  in  $E(G)$  and let  $\theta_r = 0$  and  $\theta_s = -t_{rs}$ .

STEP 2. If all the edges have been chosen, set the offsets of the remaining (isolated) vertices arbitrarily to complete the algorithm, otherwise continue to STEP 3.

STEP 3. Choose any edge not already chosen but which is incident with a previously chosen one. Determine the offset of the vertex just added as in Proposition 2. Then go back to STEP 2.

Justification of the Algorithm:

STEP 1. If  $[v_r, v_s]$  is in  $E(G)$ ,  $\theta_r = 0$ , and  $\theta_s = -t_{rs}$  then  $f_{rs}(\theta_r - \theta_s) = f_{rs}(t_{rs}) = m_{rs}$  which is minimal.

STEP 2. If all the edges have been chosen, then the offsets of those vertices incident with at least one edge have been determined. The offsets of the remaining isolated vertices can be chosen arbitrarily by virtue of Proposition 3.

STEP 3. Any edge chosen from a tree so as to be incident with an already chosen edge must join a vertex which has not already been chosen to a vertex which has been chosen.

The requirements of Proposition 2 are met and so the delay incurred by adding the new edge is minimal. Since the delay over each edge is minimal the overall delay is minimal. Hence if  $(\theta_1, \theta_2, \dots, \theta_n)$  are the offsets

$$\text{determined in this manner } Z_G(\theta_1, \theta_2, \dots, \theta_n) = \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} m_{ij}$$

We illustrate the algorithm by application to the graph  $G$  illustrated in Figure 14a. As per step 1 we choose any edge (say  $[v_2, v_3]$ ) and set  $\theta_2 = 0$  and  $\theta_3 = -t_{23}$  yielding the graph in 14b. Since not all the edges have been chosen we apply step 3 choosing edge  $[v_1, v_2]$  and setting  $\theta_1 = +t_{12}$  thus yielding the graph in Figure 14c. Choosing edge  $[v_3, v_4]$  and setting  $\theta_4 = \theta_3 - t_{34} = -t_{23} - t_{34}$  we obtain the graph in Figure 14d. We apply step 3 once more setting  $\theta_5 = -t_{23} - t_{35}$ . Applying step 2 and setting  $\theta_6 = 0$  arbitrarily, we finish with the graph illustrated in Figure 14e.

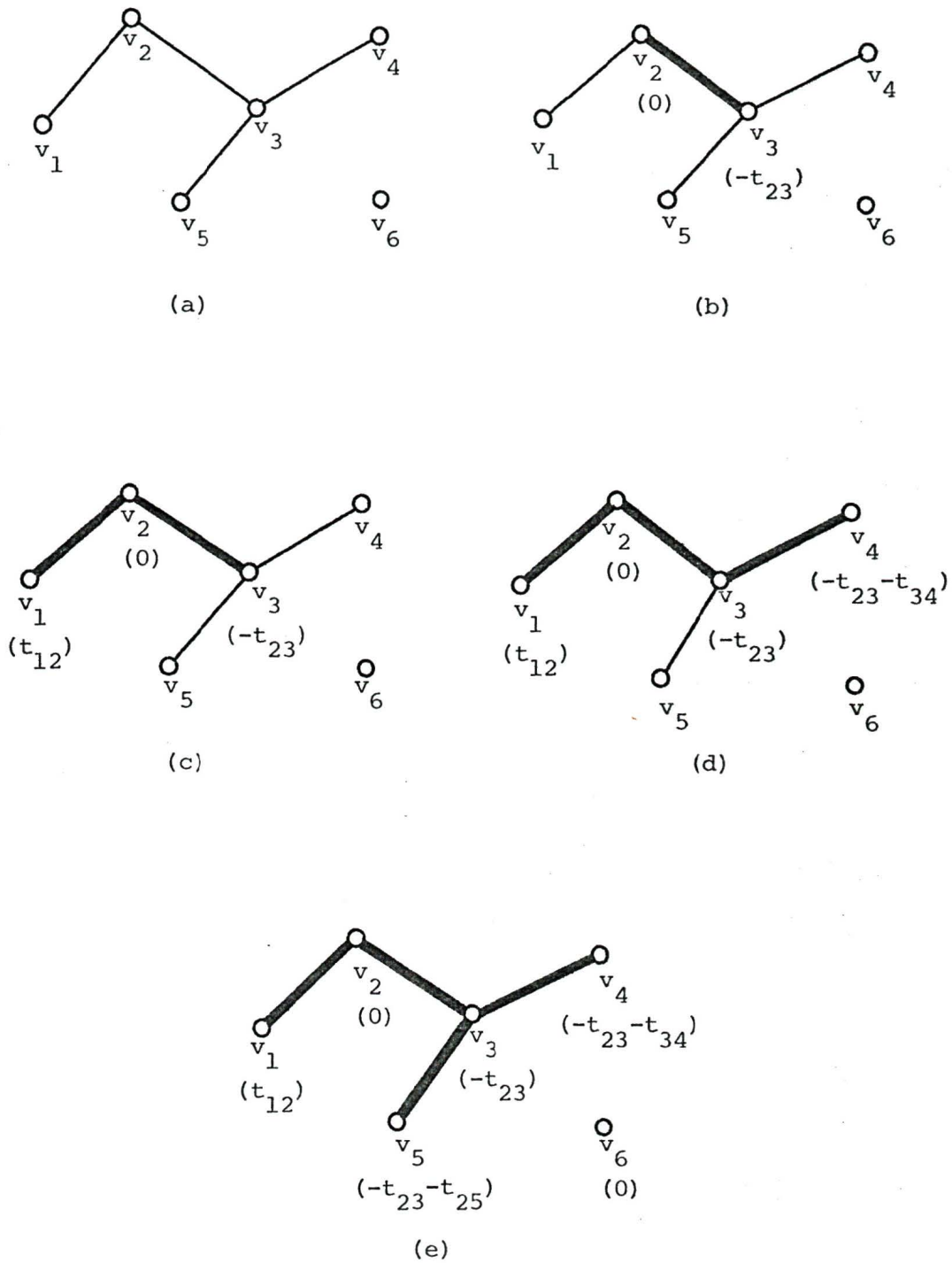


FIGURE 14. An example of the application of the algorithm for solving the BOP if the graph is a tree.

Hence,

$$\begin{aligned}
 z_G(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j) \\
 &= f_{12}(\theta_1 - \theta_2) + f_{23}(\theta_2 - \theta_3) + f_{34}(\theta_3 - \theta_4) + f_{35}(\theta_3 - \theta_5) \\
 &= f_{12}(t_{12}) + f_{23}(-(-t_{23})) + f_{34}(-t_{23} - (-t_{23} - t_{34})) \\
 &\quad + f_{35}(-t_{23} - (-t_{23} - t_{35})) \\
 &= f_{12}(t_{12}) + f_{23}(t_{23}) + f_{34}(t_{34}) + f_{35}(t_{35}) \\
 &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} m_{ij}
 \end{aligned}$$

### 3.3 The BOP On Graphs With More Than One Component

The following proposition allows us to solve the BOP on a graph with a number of components by solving it for each component separately. Hence if  $G$  consists of a number of components which are trees we may optimally solve the BOP on  $G$  by solving it for each of the trees separately as in the previous section.

Proposition 4:

If  $G$  is a graph with components  $C_1, C_2, \dots, C_m$  then the overall delay in  $G$  is the sum of the delays in each of the components.

PROOF:

Without loss of generality let  $V(C_k) = \{v_{i_{k-1}+1}, \dots, v_{i_k}\}$  for  $k = 1, 2, \dots, m$ .

$$\begin{aligned}
\therefore z_G(\theta_1, \theta_2, \dots, \theta_n) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in \bigcup_{k=1}^m E(C_k)}} f_{ij}(\theta_i - \theta_j) \\
&= \sum_{k=1}^m \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_k)}} f_{ij}(\theta_i - \theta_j) \\
&= \sum_{k=1}^m z_{C_k}(\theta_{i_{k-1}+1}, \dots, \theta_{i_k})
\end{aligned}$$

The next proposition is a variation of Proposition 1 which allows us to make one signal in each component a master signal.

Proposition 5:

If  $G$  is a graph with components  $C_1, C_2, C_3, \dots, C_m$  then for each of the components  $C_k$  we can add any constant  $R_k$  to the offsets of the vertices of  $C_k$  without changing the value of  $z_G(\theta_1, \theta_2, \dots, \theta_n)$ .

PROOF:

Let  $(\theta_1, \theta_2, \dots, \theta_n)$  be a set of offsets and let  $(\theta'_1, \theta'_2, \dots, \theta'_n)$  be the set of offsets obtained by adding  $R_k$  to  $\theta_i$  if  $v_i$  is in  $V(C_k)$  ( $k = 1, 2, \dots, m$ ).

$$\begin{aligned}
\therefore Z_G(\theta_1, \theta_2, \dots, \theta_n) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in \bigcup_{k=1}^m E(C_k)}} f_{ij}(\theta_i - \theta_j) \\
&= \sum_{k=1}^m \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_k)}} f_{ij}(\theta_i - \theta_j) \\
&= \sum_{k=1}^m \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_k)}} f_{ij}(\theta_i + R_k - (\theta_j + R_k)) \\
&= \sum_{k=1}^m \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_k)}} f_{ij}(\theta'_i - \theta'_j) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in \bigcup_{k=1}^m E(C_k)}} f_{ij}(\theta'_i - \theta'_j) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta'_i - \theta'_j) \\
&= Z_G(\theta'_1, \theta'_2, \dots, \theta'_n)
\end{aligned}$$

### 3.4 The BOP on Separable Graphs

Proposition 6 allows us to reduce the BOP on a separable graph  $G$  to two smaller problems and provides a method of combining the solutions of the two smaller problems together in a minimal manner thus forming a solution to the original problem.

Proposition 6:

If  $G$  is a separable connected graph with  $[v_r, v_s]$  being the separating edge and  $C_1, C_2$  the two components of  $G - [v_r, v_s]$  then there exists a constant  $R$  which can be added to the offsets of one of the components to make the delay over the edge  $[v_r, v_s]$  minimal.

PROOF:

Without loss of generality assume  $v_r \in C_1$  and  $v_s \in C_2$  and let  $R = -t_{rs} + \theta_r - \theta_s$  where  $(\theta_1, \theta_2, \dots, \theta_n)$  is any set of offsets. Let  $(\theta'_1, \theta'_2, \dots, \theta'_n)$  be the set of offsets obtained by adding  $R$  to the offsets of those vertices in  $C_2$ . We will show  $Z_G(\theta'_1, \theta'_2, \dots, \theta'_n) = Z_{G-[v_r, v_s]}(\theta_1, \theta_2, \dots, \theta_n) + m_{rs}$ .

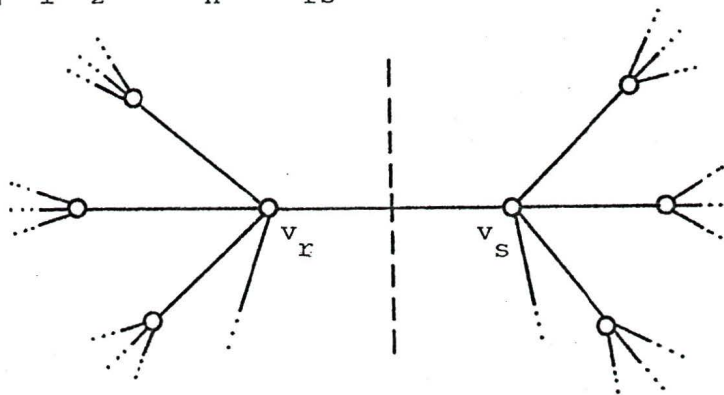


FIGURE 15. The separable connected graph  $G$ .

$$\begin{aligned}
z_G(\theta'_1, \theta'_2, \dots, \theta'_n) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta'_i - \theta'_j) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_1) \cup \{[v_r, v_s]\} \cup E(C_2)}} f_{ij}(\theta'_i - \theta'_j) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_1)}} f_{ij}(\theta'_i - \theta'_j) + \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_2)}} f_{ij}(\theta'_i - \theta'_j) \\
&\quad + f_{rs}(\theta'_r - \theta'_s) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_1)}} f_{ij}(\theta_i - \theta_j) + \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_2)}} f_{ij}((\theta_i + R) - (\theta_j + R)) \\
&\quad + f_{rs}(\theta_r - (\theta_s + R)) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(C_1) \cup E(C_2)}} f_{ij}(\theta_i - \theta_j) + f_{rs}(\theta_r - (\theta_s + R)) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G) - [v_r, v_s]}} f_{ij}(\theta_i - \theta_j) + f_{rs}(\theta_r - (\theta_s + R)) \\
&= z_{G-[v_r, v_s]}(\theta_1, \theta_2, \dots, \theta_n) + f_{rs}(\theta_r - (\theta_s + R)) \\
&= z_{G-[v_r, v_s]}(\theta_1, \theta_2, \dots, \theta_n) + f_{rs}(t_{rs}) \\
&= z_{G-[v_r, v_s]}(\theta_1, \theta_2, \dots, \theta_n) + m_{rs}
\end{aligned}$$

Proposition 6 allows us to state the following algorithm for solving the BOP on a graph  $G$  in terms of solving it for a series of subgraphs of  $G$ .

For a graph  $G$ , remove separating edges until the components are either: i) isolated vertices, ii) trees or iii) non-separable graphs. Solve the problem for each of the components, then reconstruct the original graph  $G$  by replacing the separating edges and making the delay over each of these minimal as in Proposition 6.

For example, given the graph  $G$  as in Figure 16 we may remove separating edges until we obtain the graph in Figure 17. The components of this graph can be solved separately and then the original graph  $G$  reconstructed as in Proposition 6.

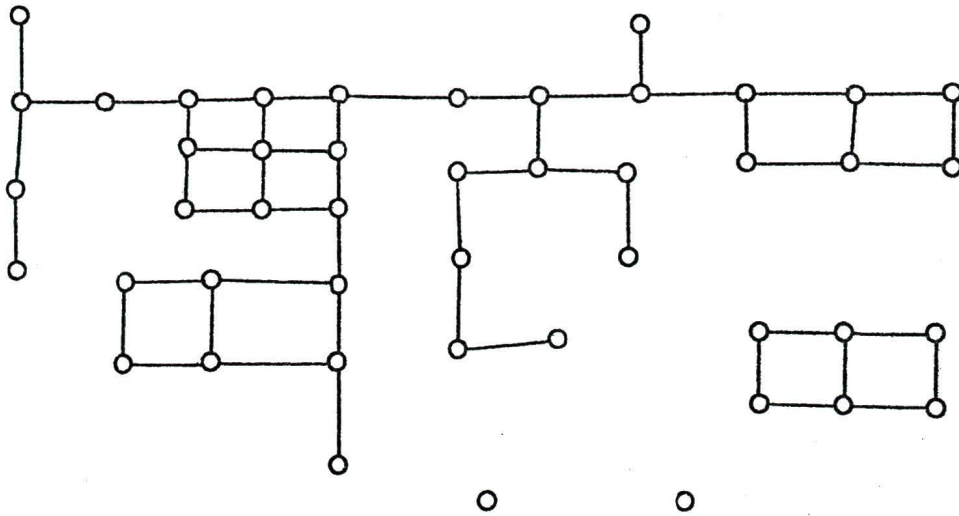


FIGURE 16. Graph of a street system.

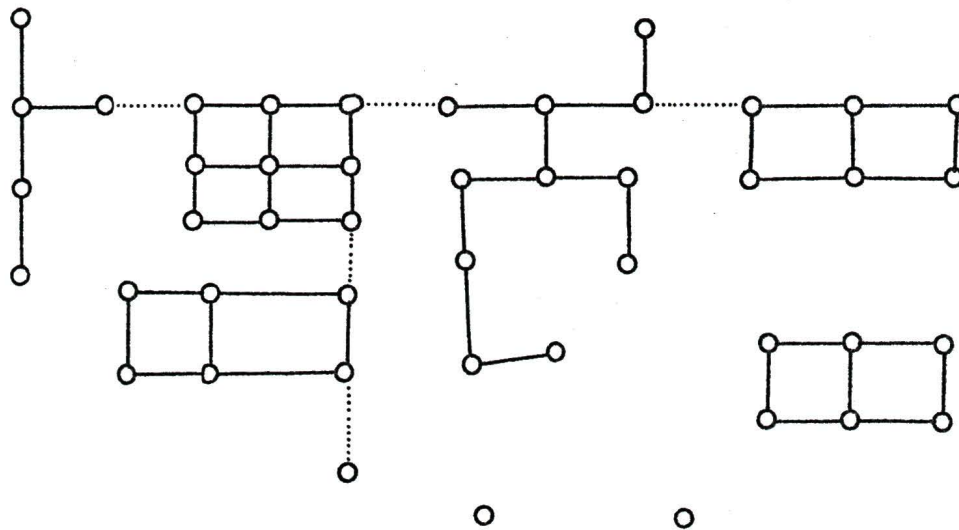


FIGURE 17. Graph of a street system with separating edges removed.

### 3.5 The BOP on Non-reducible Graphs

The problem remains of minimizing the overall delay function on graphs which are not separable, contain no isolated vertices and are not trees. The following is a brief discussion of possible ways of solving this problem.

#### a) The Davidon, Fletcher, Powell Method

This algorithm was originally proposed by W.C. Davidon [6] but did not become popular until 1963 when examined by R. Fletcher and M.J. Powell [7]. It is among the most efficient methods of function minimization currently available.

The central idea of the method is that by starting from a current point  $(x_i)$  a new point  $(x_{i+1})$  is found by the relation

$$x_{i+1} = x_i - \alpha_i H_i g_i$$

where  $g_i$  is the gradient vector evaluated at  $x_i$  and  $H_i$  is the  $i^{\text{th}}$  approximation to the matrix of second partial derivatives. By differentiating the second order Taylor expansion of the objective function about the minimum it is possible to show that  $-H_i g_i$  is a good direction in which to search while  $\alpha_i$  prescribes the step to the minimum in this direction. The algorithm does not guarantee location of a global minimum - only a local one.

#### b) The Complex Method

This method is based on the "complex" method of M.J. Box [3]. The algorithm proceeds as follows:

- 1) An original complex of  $K > N + 1$  points is formed consisting of a feasible starting point and  $K - 1$  additional randomly generated points.

- 2) If at any time the explicit constraints are violated the point is moved a small distance inside the violated limit. If an implicit constraint is violated, the point is moved one half of the distance to the centroid of the remaining points.
- 3) The objective function is evaluated at each point. The point having the greatest function value is replaced by a point located further away from the centroid of the remaining points and on a line joining the rejected point and the centroid.
- 4) If a point repeats in giving the greatest function value on consecutive trials it is moved one half the distance to the centroid of the remaining points.
- 5) The new point is checked against the constraints and is adjusted as before if the constraints are violated.
- 6) Convergence is assumed when the values of objective function at each of the points are within a given tolerance for a number of consecutive iterations.

c) Optimizing Over Selected Trees

Another possible solution to the BOP on non-reducible graphs involves the selection of a subgraph which is a tree and which has as its vertices all the vertices of the non-reducible graph. Since this subgraph is a tree we can apply the algorithm of Section 3.2. That is:

STEP 1. Choose any edge  $[v_r, v_s]$  in the tree and let

$\theta_r = 0, \theta_s = -t_{rs}$  (Recall that the delay function  $f_{rs}$  for this leg takes on its minimum at  $t_{rs}$ ). Hence,

$$\begin{aligned}
 f_{rs}(\theta_r - \theta_s) &= f_{rs}(0 - (-t_{rs})) \\
 &= f_{rs}(t_{rs}) \\
 &= m_{rs} .
 \end{aligned}$$

STEP 2. If all the edges of the tree have been chosen the algorithm is complete otherwise continue with STEP 3.

STEP 3. Choose any edge in the tree which has not already been chosen but which is incident with one which has. Determine the offset of the vertex just added as in Proposition 2 of Section 3.2. Go back to STEP 2.

The traditional method of approaching the BOP (used by the City of Victoria and numerous other cities) is a variation of this method. However, since no simulation is done, the traffic engineer must estimate the optimal relative offset ( $t_{rs}$ ). This is done by choosing a  $t_{rs}$  so that a vehicle travelling at a fixed rate of speed would progress through the intersections with no delay. This method has the advantage of specifying the legs over which the delay is to be minimal and requires little computation to determine the offsets. However, as will be shown, the overall delay is significantly greater than it need be.

Section 2 of Chapter 5 gives some computational results from these methods as they apply to the Best Offset Problem for the downtown core of the City of Victoria.

## CHAPTER 4

### THE TRAFFIC DELAY MODEL

To investigate the nature of the delay function  $f_{ij}$  associated with an edge  $[v_i, v_j]$  a model was developed to simulate the traffic flow along a leg between two intersections. This model is described in the first section of this chapter and supporting validation is given in the second section. The typical delay function  $f_{ij}$  is described in the last section.

#### 4.1 Description of the Model

The model is designed to simulate traffic flow along a leg of a traffic system given various parameters about the traffic flow, physical layout of the leg, and the light patterns of the intersections at each end. By timing vehicles traversing the leg, the model yields a value of the delay function  $f_{ij}$  for a particular difference in offsets of the two intersections  $(\theta_i - \theta_j)$ . Stepping one of the offsets through an offset of 0 to C (the cycle length of the system) while maintaining the other at a value of 0 yields information on the nature of the function  $f_{ij}$  over its entire domain and so allows us to approximate it by a function  $g_{ij}$ .

As the system to be simulated is continuous in nature, a "time-slice" model was utilized rather than one based on critical events. During each time-slice and for each vehicle in the system, the future velocity, acceleration and position of the vehicle are calculated based upon its present status, the road system, the light patterns and the positions of the other vehicles. In addition, vehicles are added to the leg and removed as the need arises. The time taken by each vehicle to traverse

the leg is noted and when a required number have finished, the simulation for that particular relative offset ceases. The offset for one intersection is then advanced and the simulation is repeated with the new relative offset. This continues until the offset for the intersection has been stepped through the entire length of the cycle.

There are essentially six factors that determine whether a driver would slow down, increase his speed, or maintain his present speed:

- i) The colour of the signal ahead. If it is red or amber and the vehicle is nearing the stop line, then the driver must slow down in order to stop at the light. If it is green then there is no need to slow down.
- ii) The movement of the vehicle required at the intersection. If the vehicle is to turn in either direction then it must slow up to make the corner. If the vehicle is going straight then no reduction in speed is necessary.
- iii) The distance to the vehicle directly ahead. If the distance to the vehicle ahead falls below a certain level the driver must reduce his speed so that if the vehicle ahead stops suddenly there will be no accident. If there is no vehicle ahead, or if the vehicle ahead is sufficiently distant, then there is no need for a reduction in speed.
- iv) The distance to the vehicle in the left turn lane. If the driver is going to enter the left turn lane then not only must he maintain a reasonable distance to the vehicle directly ahead but also between himself and the vehicle in the left turn lane. This is also true if the vehicle in the left turn lane is blocking the lane the driver is in presently.

- v) The distance to the vehicle in the right turn lane. As in the case of the left turn only lane the driver must maintain a reasonable distance to the car in the right turn lane if he is going to shift into that lane or if there is a vehicle in that lane blocking the present lane.
- vi) The speed limit. If the previous five factors indicate no need for a reduction in the speed of the vehicle then the driver is free to accelerate up to the speed limit.

The following is a step by step description of the calculation of the future status of a vehicle given the present status of the leg and the vehicles on it.

STEP 1. Compute the velocity required by the vehicle at the next time slice due to the colour of the relevant signal (VEL1). If no speed reduction is required set VEL1 equal to the speed limit.

STEP 2. Compute the velocity required by the vehicle at the next time slice in order that it be able to perform the particular movement required at the next corner (VEL2). For example, for a right turn, the vehicle is assumed to come to a stop to allow pedestrian traffic to pass.

STEP 3. Compute the velocity required by the vehicle at the next time slice in order to maintain a reasonable distance to the vehicle ahead (VEL3).

STEP 4. Compute the velocity required by the vehicle at the next time slice in order to maintain a reasonable distance to

the last vehicle in the left turn only lane (VEL4). (This is appropriate only if the vehicle is going to shift into that lane or if there is a vehicle in the left turn only lane which is blocking the present lane.)

STEP 5. Compute the velocity required by the vehicle at the next time slice in order to maintain a reasonable distance to the last vehicle in the right turn only lane (VEL5). (This is appropriate if and only if the vehicle is going to shift into that lane or if a vehicle in that lane blocks the present lane.)

STEP 6. Compute the minimum (VW) of the values of VEL1, VEL2, VEL3, VEL4 and VEL5. This is the velocity required by the vehicle at the next time slice.

STEP 7. Compute the acceleration required (AW) to obtain the velocity of VW by the next time slice.

STEP 8. Reset (if necessary) AW to within upper and lower bounds so as to maintain realistic accelerations and decelerations.

STEP 9. Compute the velocity of the vehicle at the next time slice (FV) based upon the present velocity and AW.

STEP 10. Calculate the distance traveled by the vehicle.

STEP 11. Calculate the new position of the vehicle.

STEP 12. If the vehicle has passed through the leg continue with STEP 13 otherwise go to STEP 14.

- STEP 13. Remove the vehicle from the system and record the time taken to traverse the leg. The iteration for this vehicle is complete.
- STEP 14. If the vehicle is in a position to shift into the left or right turn only lanes and if such a shift is desired continue with STEP 15 otherwise go to STEP 16.
- STEP 15. Relocate the vehicle in the left or right turn only lane.
- STEP 16. If the vehicle has just entered the leg continue with STEP 17 otherwise go to STEP 18.
- STEP 17. Record the time the vehicle entered the leg.
- STEP 18. Record the position, velocity, acceleration, etc. of the vehicle for the next time slice.
- STEP 19. End of iteration for one vehicle.

Vehicles may enter the simulation in two ways. At regular intervals (dependent upon the traffic volumes) vehicles are fed into the leg at the very ends of the lanes. This is in keeping with the assumption that a separate delay function exists for each leg. In addition, vehicles may enter the leg at one of the intersections thus simulating vehicles turning onto the leg from a side street. Whether a particular vehicle is to turn left or right, or proceed straight at the next intersection is randomly determined based on the proportion of vehicles making such maneuvers.

As described in Appendix B the model presently records the individual times required to traverse the leg. With slight modifications, the

simulation could also note the average velocity, amount of time vehicles spend not moving, the number of panic stops and similar measures of traffic flow. As the model is used to examine the delay functions  $f_{ij}$  it is sufficient for our purposes to note only the total simulated delay experienced by the vehicles traversing the leg.

As defined, the model simulates a leg with two way traffic and both left and right turn only lanes in both directions. By suitable choice of the parameters involved the model can also be used to simulate one way traffic as well as legs that do not have left or right turn only lanes.

In order to simulate one way traffic it is sufficient to specify that no vehicles enter lanes going in the other direction. To simulate a leg with no left turn only lane or no right turn only lane, the missing lane is given a length (say five feet) which always causes a vehicle in that lane to overhang into the adjacent lane thus simulating a single lane. This is illustrated in Figure 18.

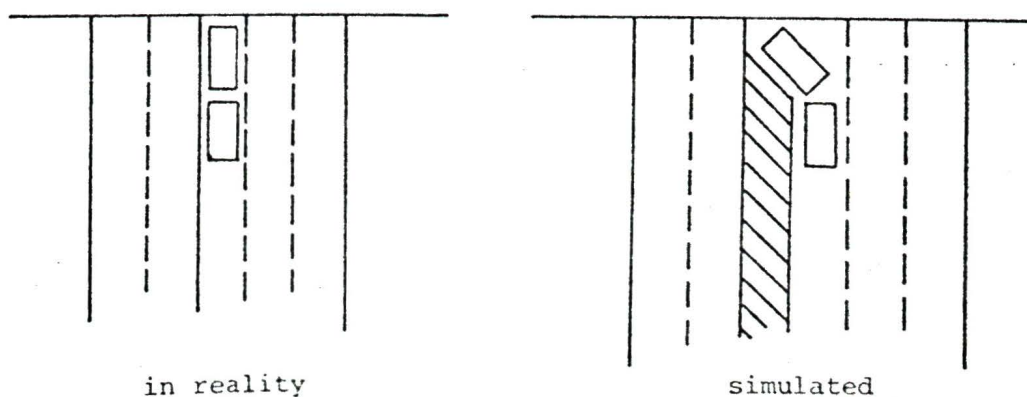


FIGURE 18. Simulation of a leg with no left turn only lane.

#### 4.2 Validation of the Model

The most important characteristic of any model is its ability to simulate real life situations. A model may be extremely elegant and efficient but if it does not realistically portray that which it is supposed to, it is virtually useless.

As a first step in validating the FORTRAN implementation of the model described in the previous section, a portion of Hillside Street in Victoria was marked off at 50 foot intervals to the East of the signal line at Quadra. An additional checkpoint was located 25 feet from the line. For each checkpoint fifty vehicles were timed from when the light turned green until they reached the checkpoint line. The average observed timings are plotted against those predicted by the model in Figure 19.

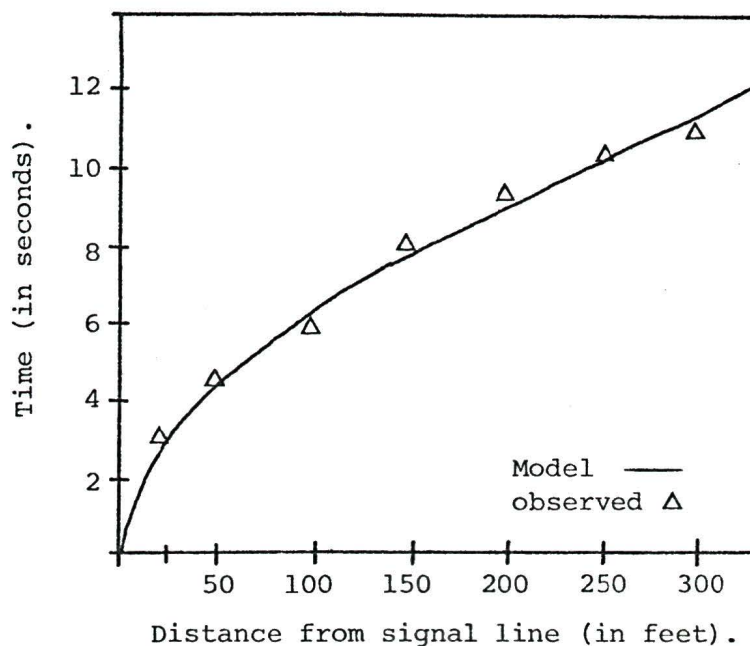


FIGURE 19. Acceleration away from a light.

In a similar fashion the times taken for vehicles to come to complete stops from given distances were also measured. The average observed timings are compared with the model in Figure 20. As can be seen, the model quite accurately simulates both acceleration from a light and braking to a stop.

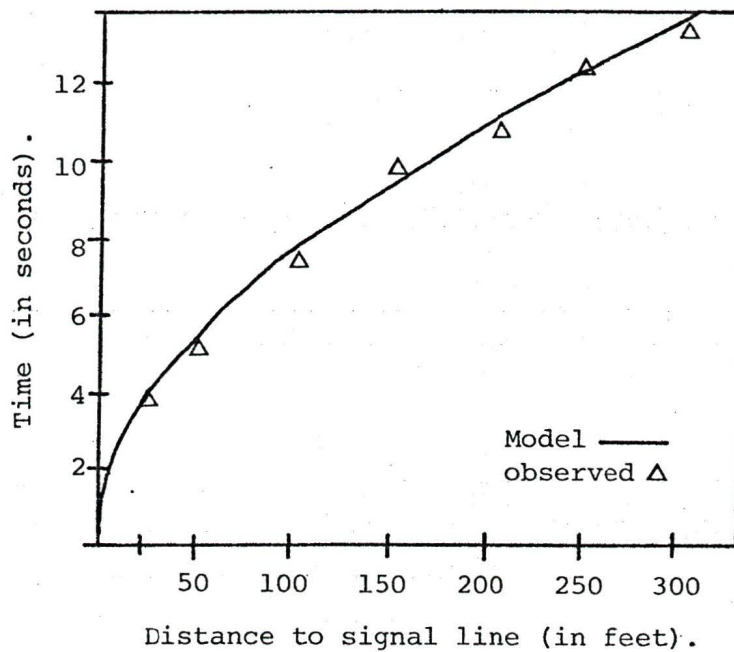


FIGURE 20. Deacceleration to a stop.

In order to validate the model under conditions including a number of vehicles and varying light patterns, fifty situations were observed and timed and were then simulated using the model. Figure 21 summarizes the observed and the model times.

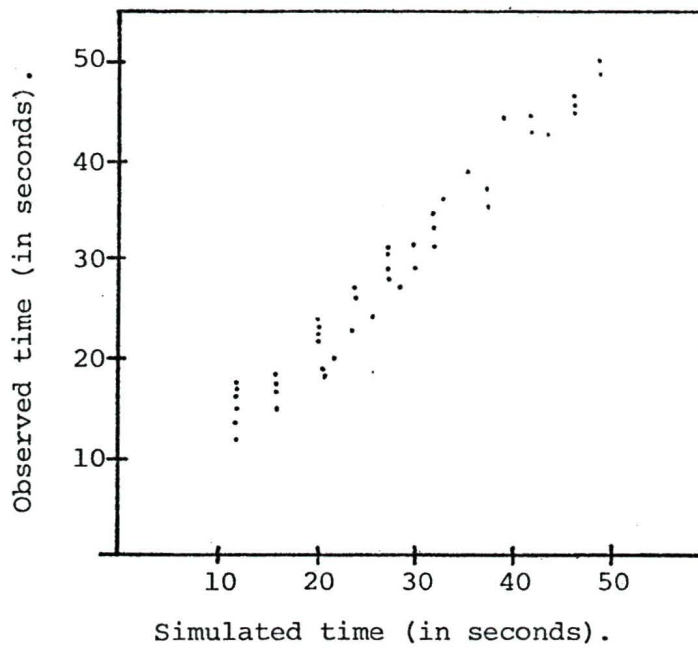


FIGURE 21. Observed vs. simulated times.

As a final check on the validity of the model a movie of the section of Douglas Street between Broughton and Fort was made. By replaying the film over and over it was possible to time all the vehicles traversing this block over a period of 10 minutes. The same block was simulated in the model with a similar flow of traffic. Figure 22 compares the two distributions of times taken to traverse the leg. A chi-squared test showed no significant difference between the two.

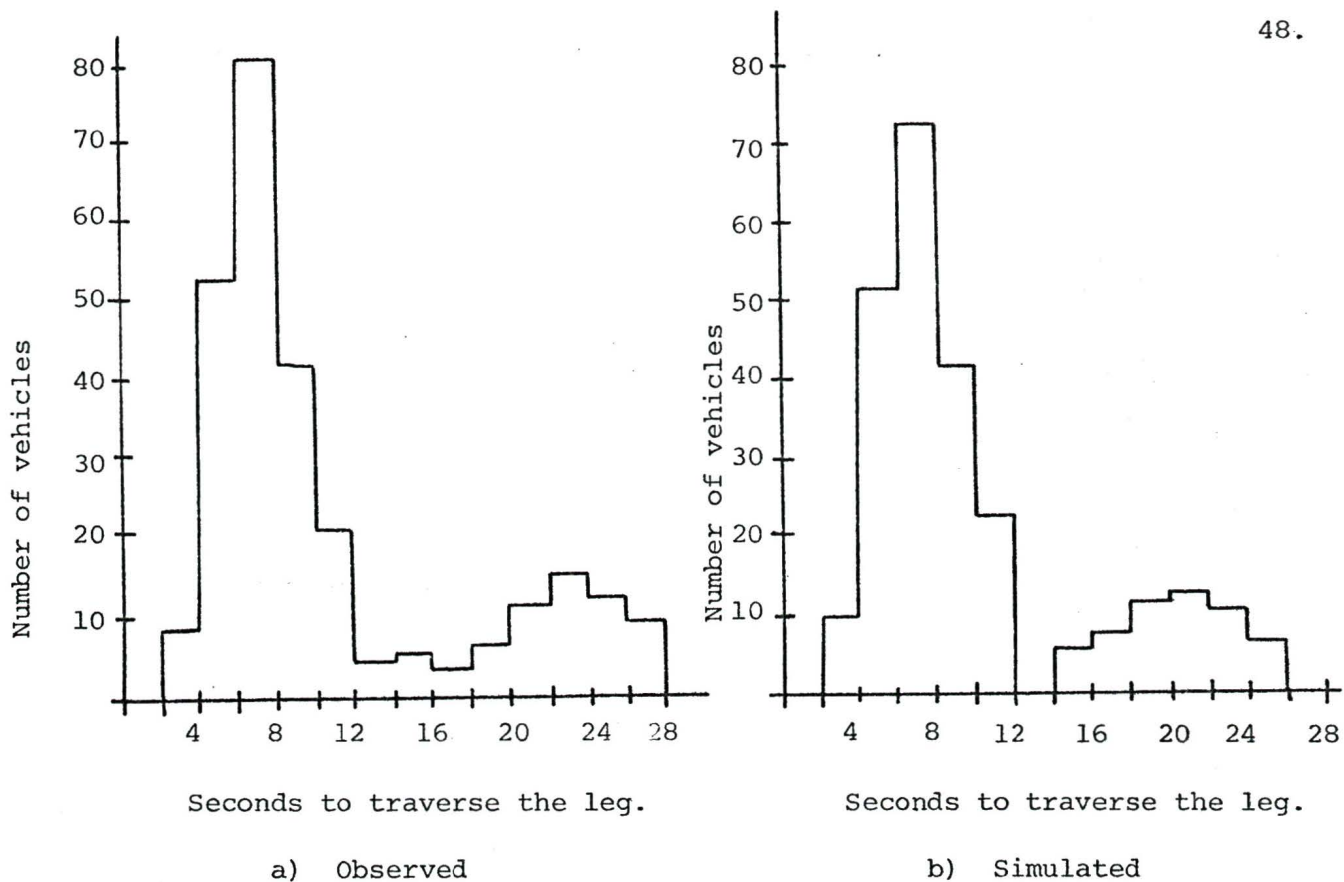


FIGURE 22. Distributions of times to traverse a leg.

#### 4.3 The Typical Delay Function

Utilizing the computer model described it is possible to examine the delay function for the various legs in some detail. By maintaining the offset of one light of the leg at a value of 0 while stepping the other through an offset from 0 to C (the cycle length of the light) and plotting the average time taken to traverse the leg, we obtain a function similar to the one in Figure 23. To obtain the total delay function for the leg, it is necessary to multiply the average delay by the number of vehicles utilizing the leg.

The form of the delay function shown in Figure 23 is fairly sinusoidal in nature, which agrees with the delay functions observed or predicted by other authors [13,9].

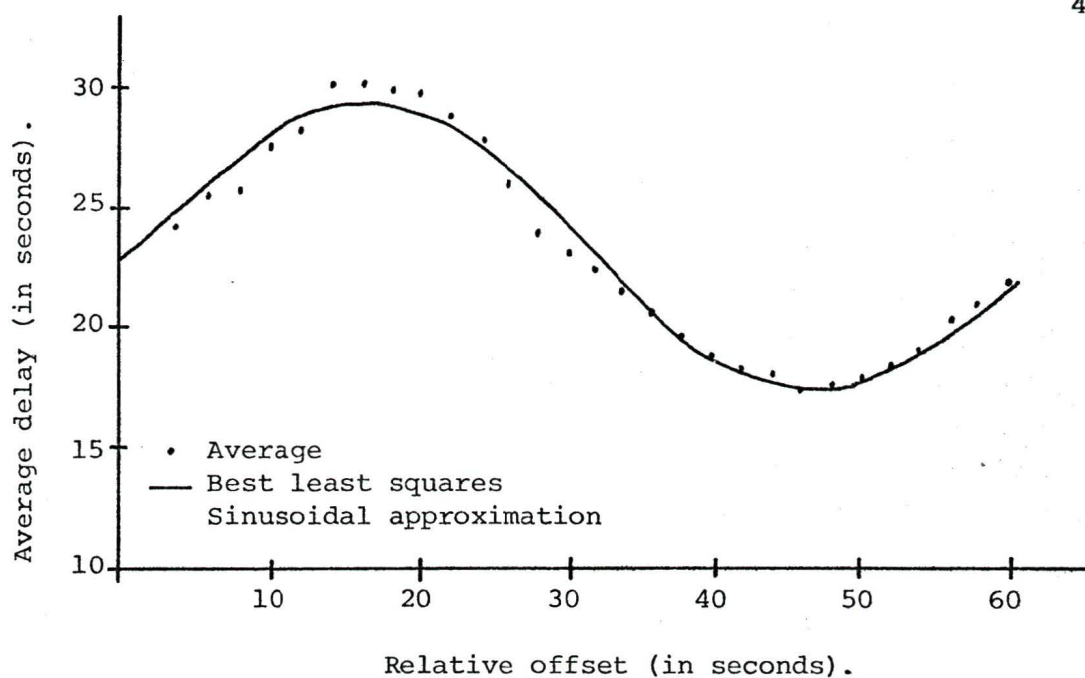


FIGURE 23. A typical average delay function.

Material in this section shows that not only is acceleration and deacceleration portrayed accurately by the model, but timings involving a number of vehicles in a variety of situations are comparable. Timings of vehicles traversing a leg in real life and the corresponding simulated timings produced distributions which were similar. On the basis of this evidence, it appears as if the model described in the first section of this chapter simulates real life situations accurately and so can be utilized in the investigation of the delay functions  $f_{ij}$ .

## CHAPTER 5

### APPLICATION TO THE CITY OF VICTORIA

Mr. D. Wilde and Mr. L. Roberts of the Traffic and Transportation Planning Department of the City of Victoria were very cooperative in providing traffic flow information for the downtown core of the City, thus enabling us to apply the theory described in the previous chapters to an actual urban situation. This was done in three separate stages. First, the linear programming technique described in Section 2.3 was applied to the data to optimize the allotment of time to the phases of the individual signals. Then, based on these phases times, the computer model outlined in Section 4.1 was utilized to determine the delay function for each leg of the test area. Finally, a number of different strategies for minimizing the overall traffic delay were compared.

The data provided covered most of downtown Victoria as shown in Figure 24 for the afternoon rush period. The associated graph, constructed in consultation with the Traffic and Transportation Planning Department, is illustrated in Figure 25.

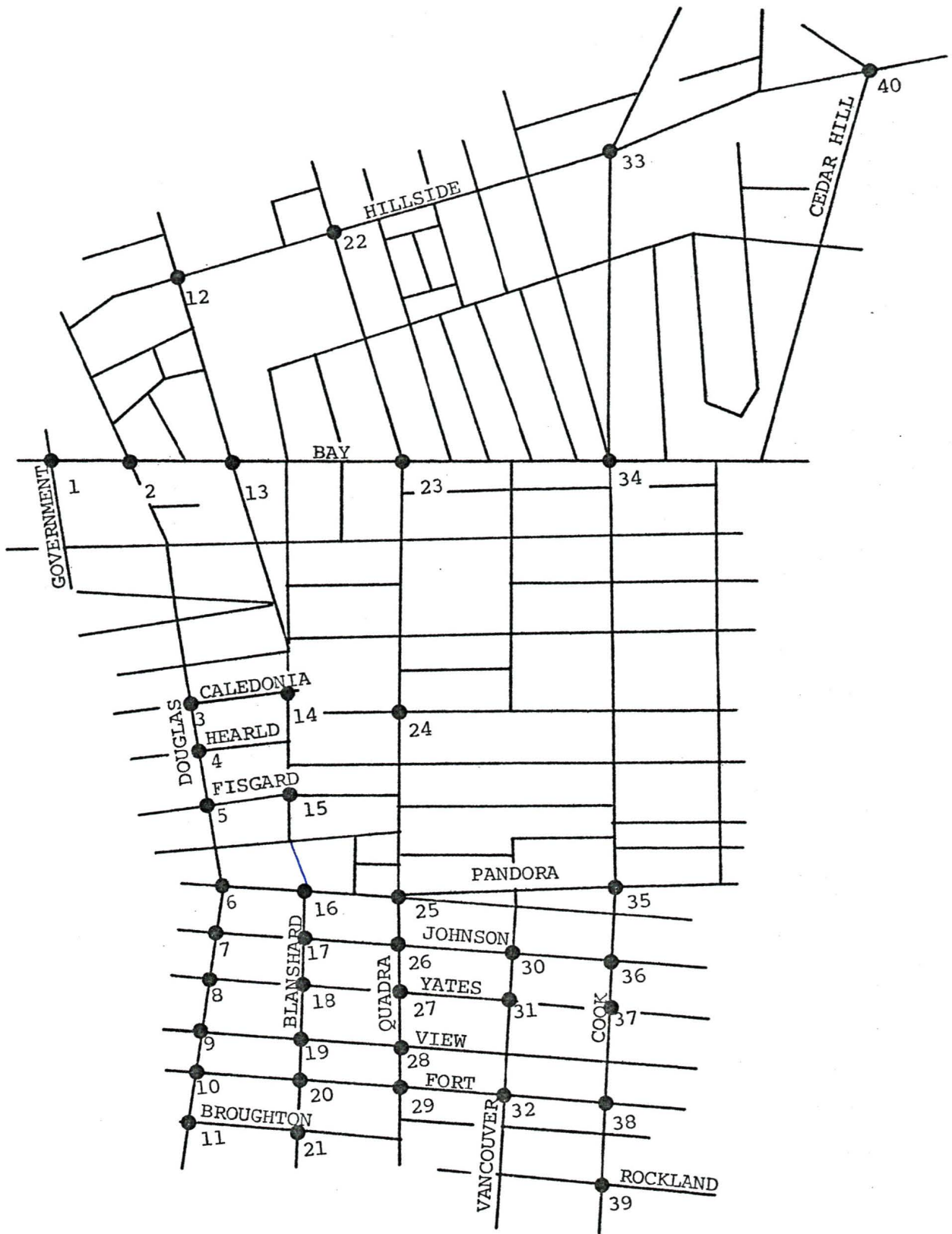


FIGURE 24. The test area.

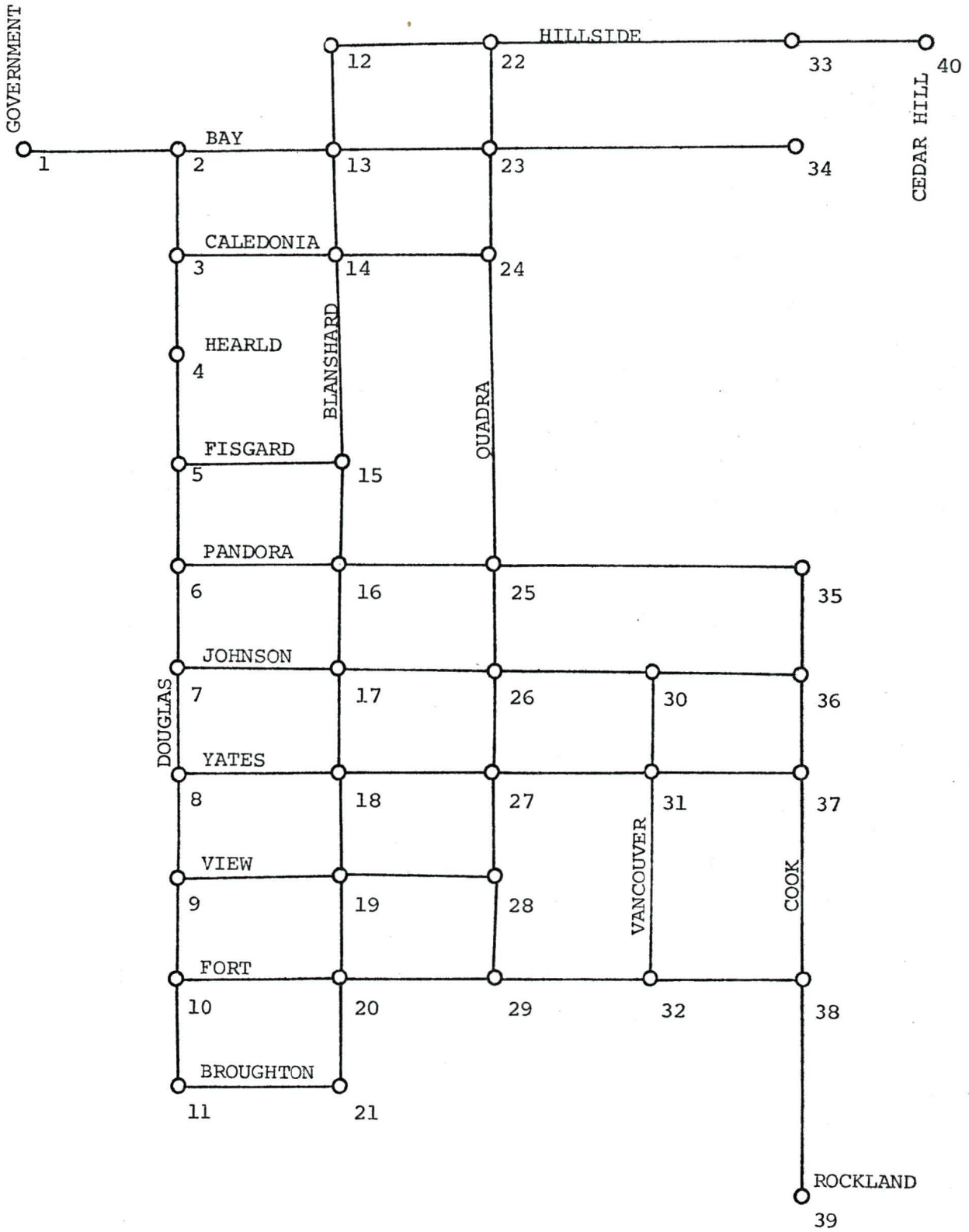


FIGURE 25. The graph associated with the test area.

### 5.1 The Cycle Split Problem as it Applies to the City of Victoria

In our application of the linear programming methods described in Section 2.3 to the Cycle Split Problem for the downtown area of Victoria, each intersection is assumed to have four incident legs numbered as in Figure 26.

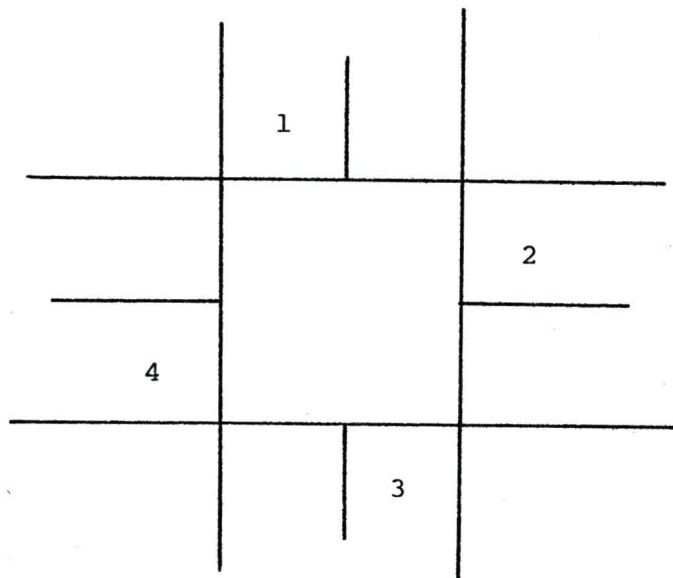


FIGURE 26. The numbering of the legs at an intersection.

The phases of all the signals in the test area fall into the 20 categories listed in Table III. Naturally no single signal goes through all 20 of these phases. For example, it would be illogical for an intersection to have an advanced green for northbound traffic followed immediately by an advance green for southbound traffic. Figure 27 illustrates the possible sequences of phases for a signal, while existing sequences are listed in Table IV.

PHASE DESCRIPTION	COLOR OF LIGHTS			
	SOUTH- BOUND	WEST- BOUND	NORTH- BOUND	EAST- BOUND
1 Advance green to southbound	G	R	R	R
2 Advance green to northbound	R	R	G	R
3 Green to northbound and southbound	G	R	G	R
4 Amber to southbound (before delayed green to northbound traffic)	A	R	G	R
5 Delayed green to northbound (Also one way with no southbound traffic)	R	R	G	R
6 Amber to northbound (after delayed green to northbound traffic)	R	R	A	R
7 Amber to northbound (before delayed green to southbound traffic)	G	R	A	R
8 Delayed green to southbound (Also one way with no northbound traffic)	G	R	R	R
9 Amber to southbound (after delayed green to southbound traffic)	A	R	R	R
10 Amber to northbound and southbound	A	R	A	R
11 Advance green to eastbound	R	R	R	G
12 Advance green to westbound	R	G	R	R
13 Green to eastbound and westbound	R	G	R	G
14 Amber to eastbound (before delayed green to westbound traffic)	R	G	R	A
15 Delayed green to westbound (Also one way with no eastbound traffic)	R	G	R	R
16 Amber to westbound (after delayed green to westbound traffic)	R	A	R	R
17 Amber to westbound (before delayed green to eastbound traffic)	R	A	R	G
18 Delayed green to eastbound (Also one way with no westbound traffic)	R	R	R	G
19 Amber to eastbound after delayed green to eastbound traffic)	R	R	R	A
20 Amber to eastbound and westbound	R	A	R	A

TABLE III: The 20 possible phases.

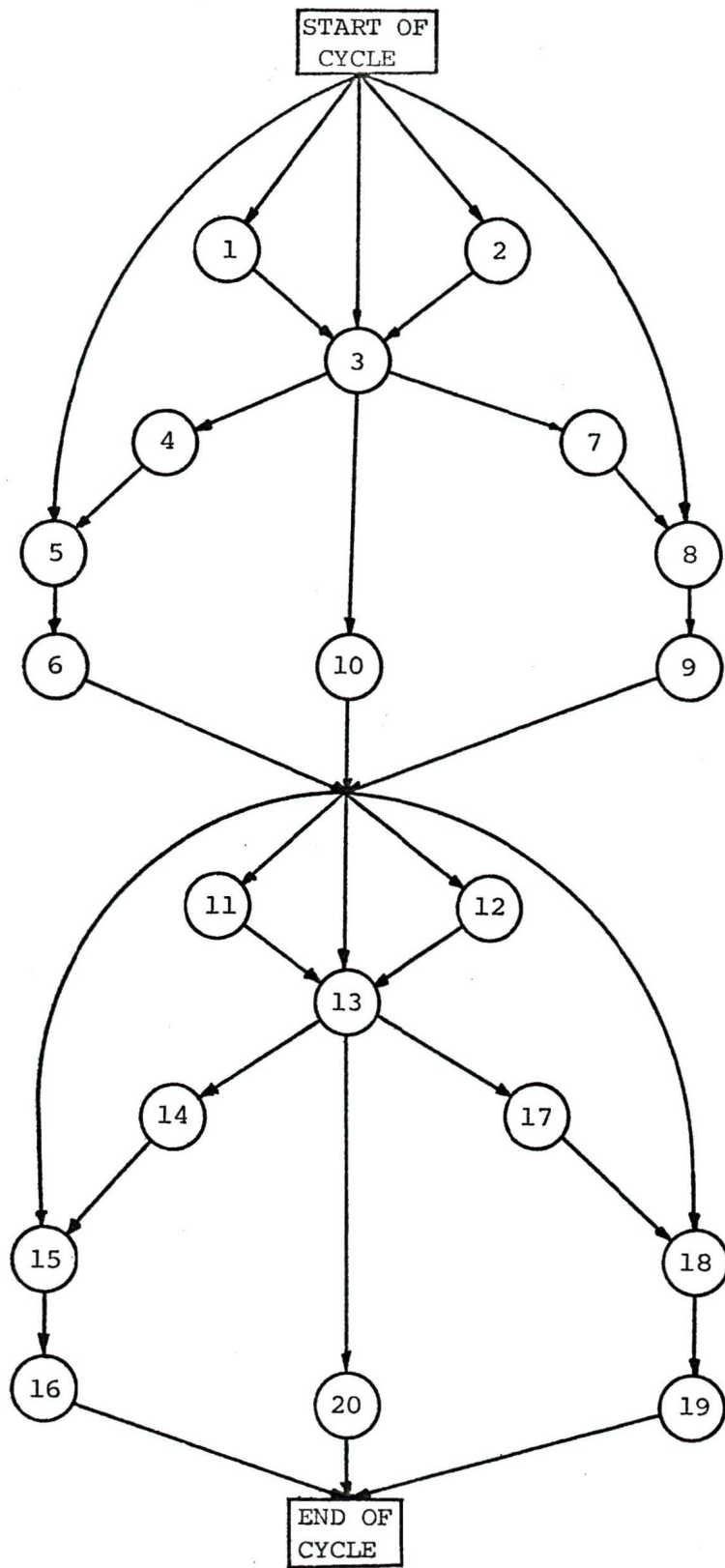


FIGURE 27. Possible sequences of phases.

INTERSECTION	PHASE SEQUENCE	INTERSECTION	PHASE SEQUENCE
1	2-3-10-13-20	21	3-10-13-20
2	3-10-13-20	22	3-10-13-20
3	3-10-13-20	23	3-10-13-20
4	3-7-8-9-13-20	24	3-10-13-20
5	3-10-13-20	25	3-10-15-16
6	3-10-15-16	26	3-10-18-19
7	3-10-18-19	27	3-10-13-20
8	3-10-15-16	28	3-10-13-20
9	3-10-13-20	29	3-10-13-20
10	1-3-10-13-20	30	3-10-18-19
11	3-10-13-20	31	3-10-13-20
12	1-3-10-12-13-20	32	3-10-13-20
13	3-10-13-20	33	3-10-11-13-20
14	3-10-13-20	34	3-10-13-20
15	2-3-10-23-20	35	3-10-15-16
16	3-10-15-16	36	3-10-18-19
17	3-7-8-9-18-19	37	3-10-13-20
18	3-10-15-16	38	3-10-13-20
19	3-10-13-20	39	3-10-13-20
20	3-7-8-9-18-19	40	3-10-13-20

Table IV: The existing sequences of phases.

Before applying the linear programming techniques discussed in Section 2.3 it is necessary to determine approximations to the constants  $c_{ijk}$ . That is, we must estimate the average number of vehicle starts from leg  $j$  ( $j = 1, 2, 3$ , or  $4$ ) of type  $i$  ( $i = 1$  for left turns,  $2$  for right turns and  $3$  for straight through) during phase  $k$  ( $k = 1, 2, 3, \dots$ , or  $20$ ).

To calculate these constants we make the assumptions about traffic flow summarized in Table V. These are based on figures used by the traffic departments of both the City of Victoria and the City of Vancouver. Most are self-explanatory but it should be noted that a vehicle start rate of  $1/20$  per second is assumed for vehicles turning left or going straight while facing a red or amber light. This was an arbitrary constant designed to allow for the vehicles that usually pass through the intersection after the signal turns amber and before the flow of traffic abates. In Table V 'unopposed green' refers to a signal pattern in which traffic going the opposite direction is held back (an advance green for example). The term 'opposed green' refers to a pattern in which traffic does flow in the opposite direction (an ordinary green). Thus, during an opposed green fewer vehicles are able to make left turns than are able to do so during an unopposed green.

Based on the assumptions listed in Table V it is possible to approximate the constants  $c_{ijk}$  fairly easily. For example consider the leg of an intersection illustrated in Figure 28.

<u>SITUATION</u>	<u>VEHICLE STARTS PER SECOND PER LANE</u> (under heavy traffic conditions)
1) Vehicles turning left under:	
a) unopposed green	1/2
b) opposed green	1/10
c) red or amber	1/20
2) Vehicles turning right under:	
a) unopposed green	1/7
b) opposed green	1/7
c) red or amber	1/7
3) Vehicles going straight under:	
a) unopposed green	1/2
b) opposed green	1/2
c) red or amber	1/20

TABLE V. Summary of assumptions upon which the calculation of the constants  $c_{ijk}$  are based.

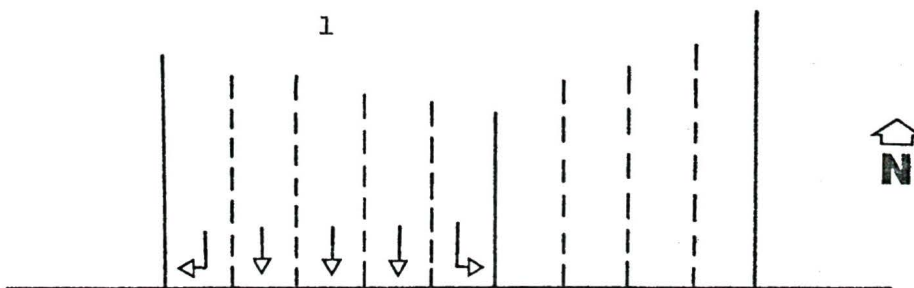


FIGURE 28. A simple leg.

To calculate the number of vehicles per second leaving leg 1 ( $j=1$ ) by turning left ( $i=1$ ) during an advance green for southbound traffic ( $k=1$ ) we need only set  $c_{111}$  equal to  $1/2$  as there is an average of  $1/2$  vehicle per second per lane able to make a left turn during an unopposed green and there is only one left turn lane. If there had been two left turn only lanes then the constant would have the value  $(1/2) \cdot 2$ .

To calculate the number of vehicles per second going straight through the intersection ( $i=3$ ) from leg 1 ( $j=1$ ) during an advance green for southbound traffic ( $k=1$ ) we need only set  $c_{311}$  equal to  $(1/2) \cdot 3$  as there are three lanes reserved for traffic which does not turn and an average of  $1/2$  vehicle per second per lane is able to go through the intersection during an unopposed green.

A more complex situation arises when a lane is used for more than one type of vehicular movement. For example, consider the following leg of an intersection.

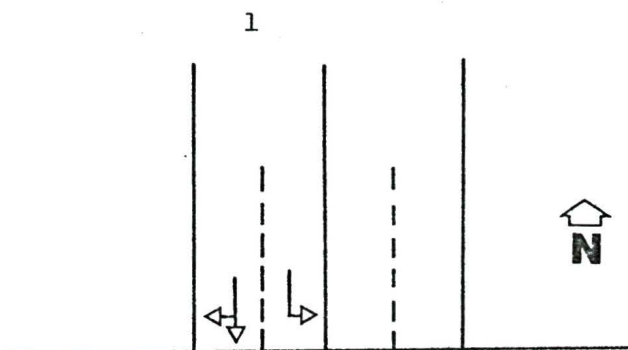


FIGURE 29. A more complex leg.

The calculation of  $c_{211}$  (that is the number of vehicles per second turning right ( $i=2$ ) from leg 1 ( $j=1$ ) during an advance green for southbound traffic ( $k=1$ )) is slightly more involved as we require additional information about the number of vehicles using the curb lane to turn right as compared to the total number using that lane.

For example, suppose 10 vehicles use the lane to turn right in a given period of time and 90 use it to proceed straight through the intersection. The total time required for the 100 vehicles to pass through the intersection is  $7 \times 10 + 2 \times 90 = 250$  seconds (1/7 vehicle per second is able to turn right and 1/2 vehicle per second is able to go straight.) Thus, the rate at which the vehicles turn right is  $c_{211} = 10/250 = 1/25$ .

Using this approach to calculate all the required  $c_{ijk}$ 's for the Cycle Split Problem of a single intersection we were able to apply a standard implementation of the Simplex algorithm to the linear programming problem as stated in Section 2.3.

In a number of cases the present allotment to an amber phase is only 3.0 or 3.6 seconds. At the request of the City of Victoria this was brought up to a standard of 4.2 seconds. Hence in some cases the present allotment does not satisfy the inequalities of the problem as stated. Table VI provides a summary of the results of applying this technique to the intersections of the study area. The first column is the intersection number while the second and third columns give the estimated maximum flow before and after optimization. The fourth column indicates whether the constraints of the problem are presently being broken (by insufficient amber phases) while the last column shows the percentage improvement for those intersections where the present allotment does not break the constraints.

Intersection	Maximum Flow (vehicles per cycle)		Constraints Presently Violated?	% Improvement (if applicable)
	Presently	Optimized		
1	186	189	No	1.66
2	135	139	No	2.41
3	154	144	Yes	
4	105	100	Yes	
5	154	155	No	0.36
6	175	171	Yes	
7	165	161	Yes	
8	163	168	No	2.66
9	122	118	Yes	
10	140	146	No	3.96
11	104	101	Yes	
12	230	239	No	3.83
13	204	209	No	2.28
14	199	187	Yes	
15	169	162	Yes	
16	234	231	Yes	
17	179	176	Yes	
18	157	162	No	3.20
19	118	122	No	3.11
20	148	153	No	3.77
21	92	93	No	1.23
22	139	142	No	1.74
23	97	101	No	4.24
24	80	83	No	4.34
25	166	167	No	0.62
26	134	142	No	5.98
27	177	185	No	4.71
28	64	72	No	11.80
29	93	100	No	7.04
30	91	109	No	20.31
31	116	126	No	9.33
32	95	105	No	10.98
33	162	160	Yes	
34	112	113	No	0.89
35	159	161	No	0.90
36	157	156	Yes	
37	135	137	No	1.37
38	114	116	No	1.53
39	83	95	No	13.59
40	143	139	Yes	

Table VI: Summary of results of applying linear programming techniques to the Cycle Split Problem.

## 5.2 The BOP for the City of Victoria

In the second stage of the project the phases determined in the first stage were used in the computer model described in Chapter 4 to simulate traffic along each leg of the test area. In general the average delay functions appeared similar to the one illustrated in Figure 30.

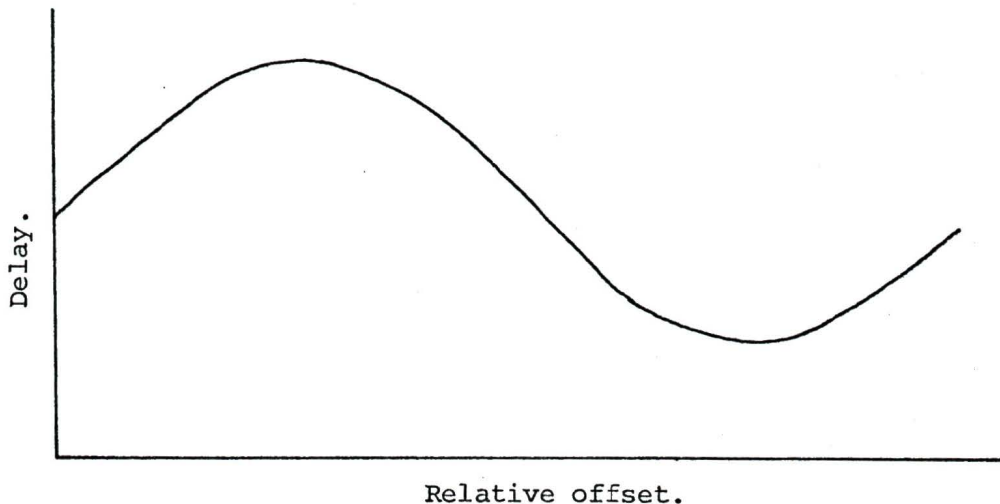


FIGURE 30. A typical average delay function.

Before applying the various minimization techniques outlined in Chapter 3 and in accord with the suggestion of Ross [1] each delay function was approximated by a sinusoidal curve of the form  $g(x) = a \sin\left((b+x)\frac{2\pi}{60}\right) + c$ . The graph of this function is given in Figure 31. Note that this function is periodic with period equal to the cycle length of the signal system for the City of Victoria (i.e. 60 seconds).

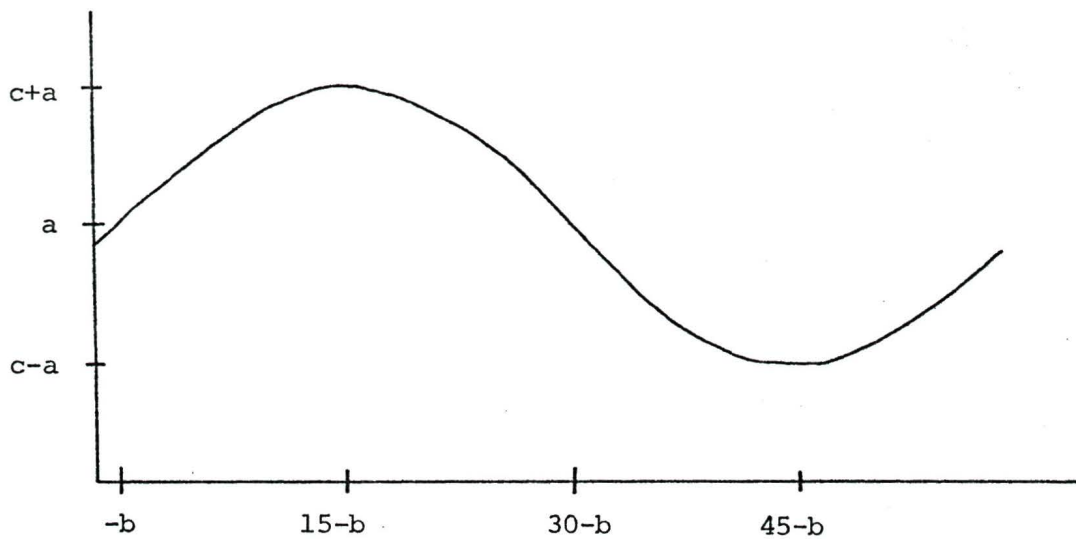


FIGURE 31. The graph of  $g(x) = a \sin\left((b+x)\frac{2\pi}{60}\right) + c$

Results of this approximation are summarized in Table VII. The figures are given for the average delay function - to obtain the delay function for the leg it is necessary to multiply by the number of vehicles.

The root mean square error is defined as:

$$\text{r.m.s.e.} = \sqrt{\frac{\sum_{i=1}^N (g(x_i) - f(x_i))^2}{N}}$$

where:  $g(x)$  is the approximating function,

$f(x)$  is the approximated function, and

$x_1, x_2, \dots, x_N$  are the data points used in the approximation.

$v_i$	$v_j$	number of vehicles traversing leg per hour ( $N_{ij}$ )	$a_{ij}/N_{ij}$	$b_{ij}$	$c_{ij}/N_{ij}$	root mean square error/ $N_{ij}$
1	2	1195	1.17	35.55	29.96	1.37
2	3	2085	1.44	16.96	55.56	1.11
2	13	1299	5.155	17.49	34.15	1.26
3	4	1980	5.19	43.24	21.07	1.05
3	14	291	3.43	13.35	24.88	0.68
4	5	1927	8.99	47.83	17.76	0.99
5	6	1645	5.01	47.02	21.25	0.77
5	15	517	4.14	14.26	25.81	1.68
6	7	1600	7.77	44.98	17.06	1.09
6	16	500	11.30	5.01	29.79	2.32
7	8	1588	7.08	38.68	19.54	1.34
7	17	550	10.54	24.89	31.27	1.63
8	9	1407	8.21	55.93	23.23	1.50
8	18	780	5.77	58.32	23.70	1.09
9	10	1472	7.54	45.68	17.26	0.77
9	19	837	7.61	12.37	38.11	1.44
10	11	1343	6.71	42.15	14.86	0.73
10	20	1001	11.47	22.37	29.87	1.51
11	21	531	4.56	18.34	30.36	1.62
12	13	1856	7.93	11.82	38.11	1.47
12	22	1565	8.36	4.01	36.92	1.38
13	14	2083	3.88	2.61	28.19	0.90
13	23	1252	6.97	7.08	37.21	1.93
14	15	2067	6.64	17.09	26.81	1.44
14	24	277	4.38	16.30	27.29	1.46
15	16	2059	1.65	32.89	39.61	1.36
16	17	1965	3.97	37.87	20.86	0.73
16	25	500	8.50	53.73	27.29	2.18
17	18	1609	7.97	41.11	20.11	1.50
17	26	700	6.39	20.51	22.97	1.70

$v_i$	$v_j$	number of vehicles traversing leg per hour ( $N_{ij}$ )	$a_{ij}/N_{ij}$	$b_{ij}$	$c_{ij}/N_{ij}$	root mean square error/ $N_{ij}$
18	19	1857	6.38	58.27	28.32	1.93
18	27	919	6.05	1.80	24.88	2.28
19	20	1362	5.23	31.57	30.98	1.87
19	28	497	6.18	16.87	35.74	2.09
20	21	1344	6.78	51.63	19.19	0.90
20	29	1326	5.72	27.52	24.74	0.90
22	33	1798	0.39	4.63	56.71	1.12
23	24	1269	4.48	26.76	40.44	2.10
23	34	1297	7.09	30.60	44.91	2.09
24	25	1410	10.00	11.11	41.67	1.37
25	26	1387	10.33	23.66	36.32	2.68
25	35	550	8.11	27.59	39.41	2.82
26	27	840	3.81	3.13	25.94	2.34
26	30	700	3.71	36.15	20.96	1.39
27	28	1009	7.23	0.66	23.71	1.55
27	31	793	6.36	0.56	23.62	1.58
28	29	1113	3.72	22.12	22.76	2.06
29	32	1552	5.27	32.35	23.88	1.04
30	31	377	5.00	55.48	21.58	1.45
30	36	750	2.33	31.80	17.39	1.76
31	32	382	8.93	5.04	33.62	1.97
31	37	792	4.26	53.18	17.49	1.97
32	38	1589	3.93	36.68	16.94	1.59
33	40	1476	3.74	11.64	54.25	0.67
35	36	1330	2.49	14.34	33.25	1.60
36	37	1403	12.09	6.56	44.55	2.01
37	38	1473	1.37	18.34	30.13	1.04
38	29	1533	3.67	47.11	18.72	0.83

TABLE VII: Summary of approximation of delay functions by sinusoidal curves.

Hence, the overall delay function was approximated by:

$$\begin{aligned}
 z_G(\theta_1, \theta_2, \theta_3, \dots, \theta_n) &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} f_{ij}(\theta_i - \theta_j) \\
 &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} g_{ij}(\theta_i - \theta_j) \\
 &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} a_{ij} \sin\left((b_{ij} + \theta_i - \theta_j) \frac{2\pi}{60}\right) + c_{ij}
 \end{aligned}$$

In the third stage the various methods of minimizing the overall delay function were evaluated. The first step was to establish lower and upper bounds on the function.

From Figure 31 it can be seen that the function  $g(x) = a \sin\left((b+x) \frac{2\pi}{60}\right) + c$  takes on its minimum of  $c - a$  when  $x = 45 - b$ . Therefore each function  $g_{ij}$  takes on its minimum of  $c_{ij} - a_{ij}$  when  $\theta_i - \theta_j = 45 - b_{ij}$ .

Hence, a lower bound on the overall delay function can be determined by calculating:

$$\begin{aligned}
 L &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} g_{ij}(45 - b_{ij}) \\
 &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} a_{ij} \sin\left((b_{ij} + 45 - b_{ij}) \frac{2\pi}{60}\right) + c_{ij}
 \end{aligned}$$

$$\begin{aligned}
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} a_{ij} \sin\left(\frac{3}{2} \pi\right) + c_{ij} \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} c_{ij} - a_{ij}
\end{aligned}$$

Similarly an upper bound for the overall delay function can be determined by calculating:

$$\begin{aligned}
U &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} g_{ij} (15 - b_{ij}) \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} a_{ij} \sin\left(\left(b_{ij} + 15 - b_{ij}\right) \frac{2\pi}{60}\right) + c_{ij} \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} a_{ij} \sin\left(\frac{\pi}{2}\right) + c_{ij} \\
&= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} a_{ij} + c_{ij}
\end{aligned}$$

In the case of the City of Victoria, the lower bound was approximately 1.69 million seconds (about 469 hours) while the upper bound was about 2.48 million seconds (approximately 689 hours).

To provide a basis for evaluating the various possible methods of minimizing the overall delay function the mean and average of the function

were calculated. These calculations are summarized below:

i) mean:

$$\begin{aligned} \mu &= E(Z_G(\theta_1, \theta_2, \dots, \theta_{40})) \quad [\text{the expected value of } Z_G] \\ &= \int_0^{60} \int_0^{60} \dots \int_0^{60} \frac{Z_G(\theta_1, \theta_2, \dots, \theta_{40}) d\theta_1 d\theta_2 \dots d\theta_{40}}{60^{40}} \\ &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} c_{ij} \end{aligned}$$

$$\approx 2.08 \text{ million seconds (578 hours)}$$

ii) variance:

$$\begin{aligned} \sigma^2 &= E((Z_G(\theta_1, \theta_2, \dots, \theta_{40}) - \mu)^2) \\ &= \int_0^{60} \int_0^{60} \dots \int_0^{60} \frac{(Z_G(\theta_1, \theta_2, \dots, \theta_{40}) - \mu)^2 d\theta_1 d\theta_2, \dots, d\theta_{40}}{60^{40}} \\ &= \sum_{\substack{i,j \\ [v_i, v_j] \in E(G)}} \frac{a_{ij}^2}{2} \end{aligned}$$

$$\approx 1,888 \text{ million seconds}$$

[hence, the standard deviation is equal to about 0.043 million seconds]

As a further check, ten thousand sets of random offsets were generated and the overall delay function evaluated for each set. The distribution is shown in Figure 32.

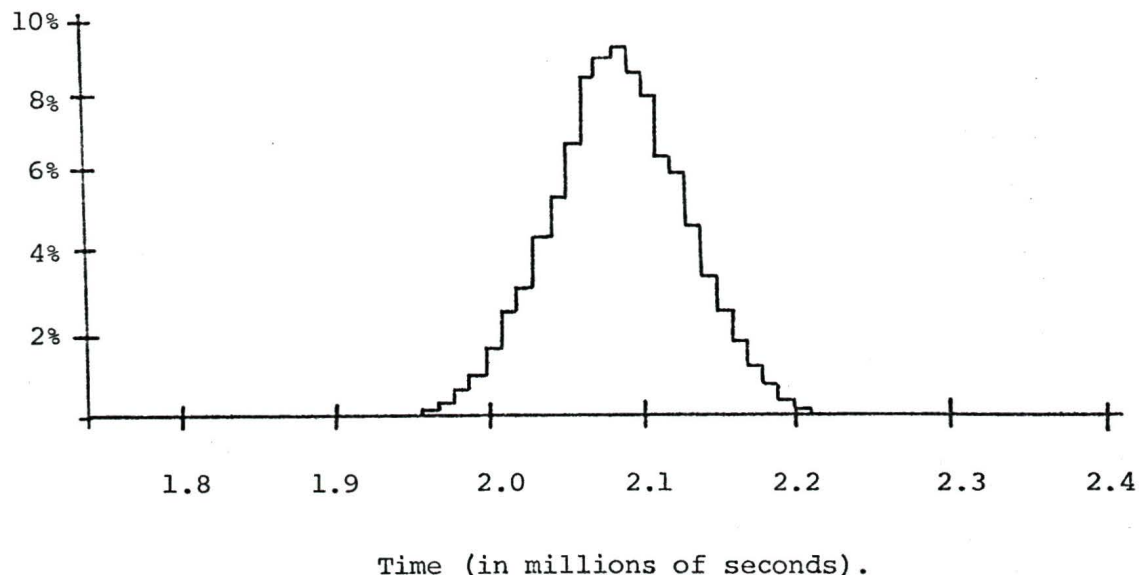


FIGURE 32. Distribution of values of the delay function (based on 10,000 sets of random offsets).

A FORTRAN implementation of the Davidon, Fletcher, Powell method of minimizing a function of several variables was applied to the problem to locate local minima [4]. Various initial points were utilized but all converged to a value of approximately 1.73 million seconds (481 hours). Convergence was within 10 iterations.

The complex method of function minimalization was also applied to the problem [11]. Numerous sets of starting offsets were tried along with various program parameters but all failed to converge. As suggested by C. Bird [2] this failure to converge was probably due to the large number of variables involved.

**To check** the effectiveness of selecting a subset of the edges of the graph associated with the street system which form a tree and contain all the vertices of the graph, a FORTRAN program was written to assign each of the edges a random weight and then to form a minimally weighted

spanning tree utilizing Prim's algorithm [10]. The offsets of the lights were then determined as described in Section 2 of Chapter 3. Since the delay functions for each of the legs were approximated by a function of the form

$$g_{ij}(\theta_i - \theta_j) = a_{ij} \sin\left(\left(b_{ij} + (\theta_i - \theta_j)\right) \frac{2\pi}{60}\right) + c_{ij}$$

the minimal delay occurs when

$$b_{ij} + (\theta_i - \theta_j) = 45. \quad \text{That is,}$$

$$\begin{aligned} m_{ij} &= a_{ij} \sin\left(b_{ij} + (\theta_i - \theta_j) \frac{2\pi}{60}\right) + c_{ij} \\ &= a_{ij} \sin\left(45 * \frac{2\pi}{60}\right) + c_{ij} \\ &= a_{ij} (-1) + c_{ij} \\ &= -a_{ij} + c_{ij}. \end{aligned}$$

Utilizing the offsets obtained in this manner, the total delay function was evaluated. The process of selecting edges to form the tree and subsequent minimization was repeated numerous times to obtain results for a variety of trees.

A tree generating an overall delay of 1.80 million seconds (500 hours) was found within the first minute of CPU time (30 trees examined). After 5 minutes of CPU time (150 trees examined) one generating an overall delay of 1.78 seconds was found. By extending the time limit to 60 minutes (1750 trees generated) a tree yielding an overall delay of 1.77 million seconds (492 hours) was detected. This tree is illustrated in Figure 33.

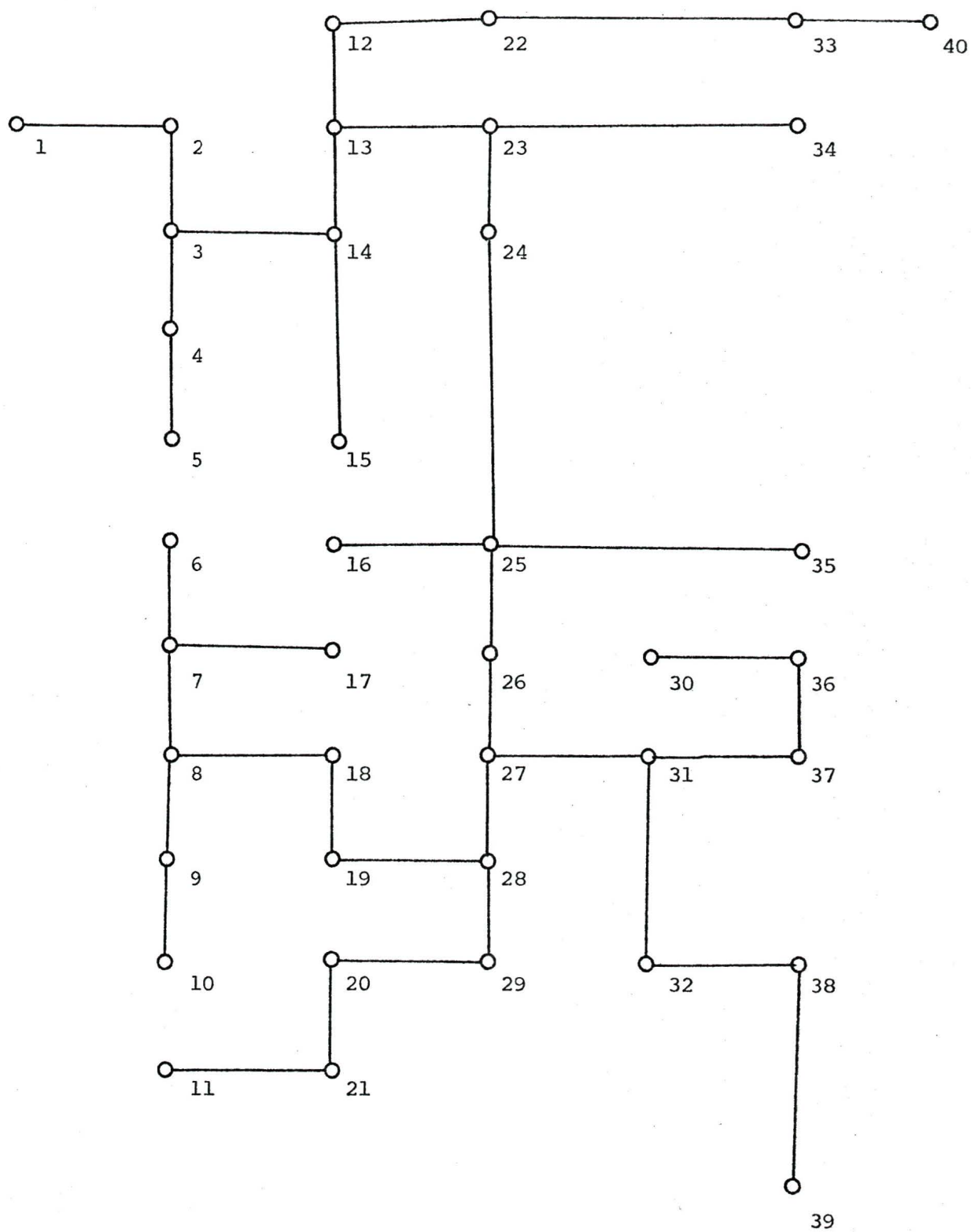


FIGURE 33. Tree yielding an overall delay of 1.77 million seconds.

The present method used by the City of Victoria for determining the offsets of the signals for the system involves selecting a tree (based on the amount of traffic utilizing the associated legs) and then setting the offsets so that a vehicle travelling at a fixed speed (22.5 m.p.h. in the case of Victoria) would have perfect progression through the lights [3]. Figure 34 illustrates the tree presently being used in the City of Victoria. Utilizing this method of setting the offsets resulted in an overall delay of 2.0 million seconds (561 hours). Utilization of the same tree but setting the offsets as described in Section 2 of Chapter 3 resulted in an overall delay of 1.80 million seconds (500 hours). This represents an improvement of about 11%.

The results of the various methods of minimizing the overall delay function are summarized in Figure 35. As can be seen, the Davidon, Fletcher, Powell method resulted in a value quite near the lower limit. However, selecting a tree in the traditional manner and then optimizing over it as in Section 3.2, resulted in an overall delay that is only slightly larger. This method has the advantage that it allows the planner to specify those legs over which the delay is minimal.

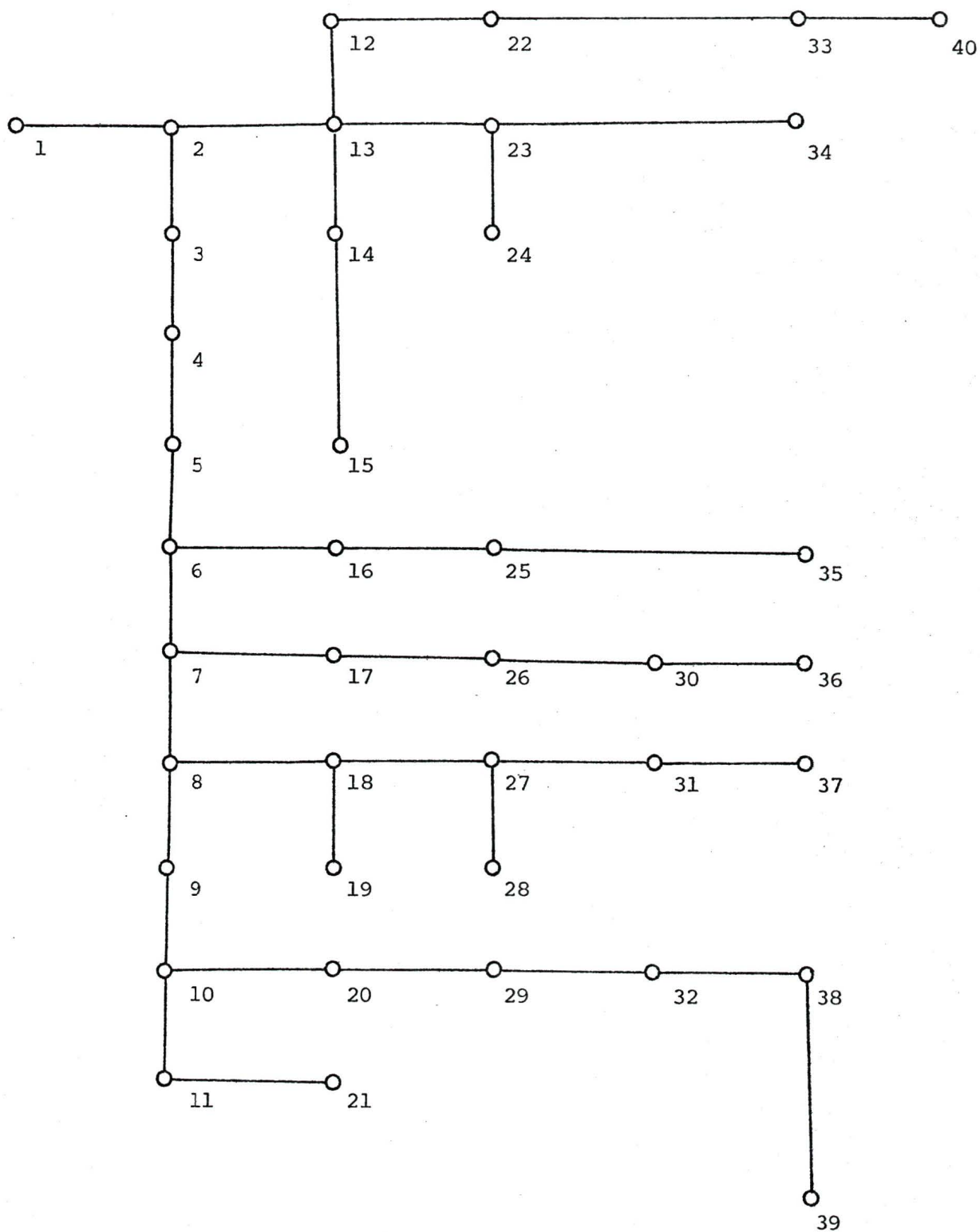


FIGURE 34. Tree presently used by City of Victoria.

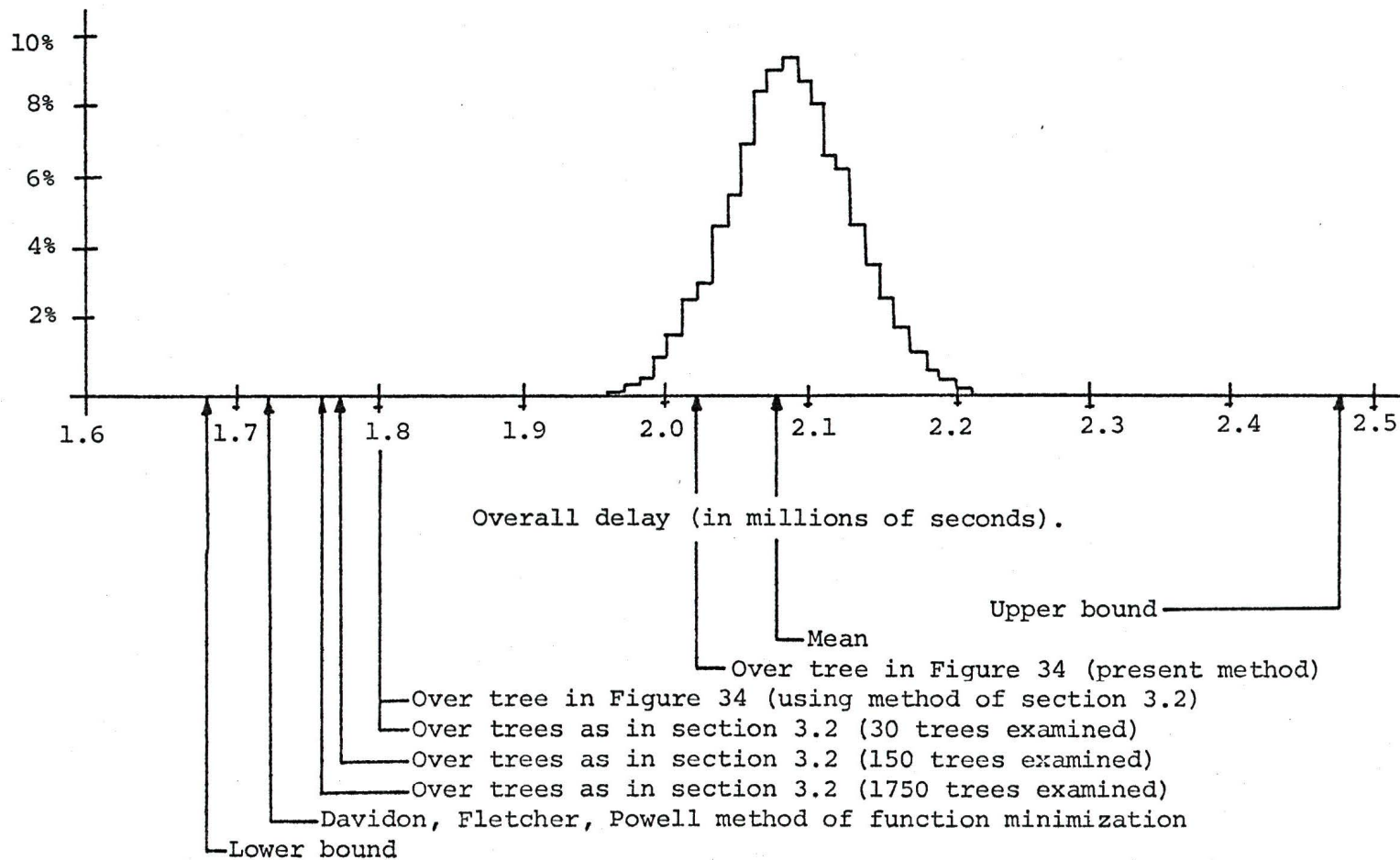


Figure 35. Summary of methods of minimizing the overall delay function.

## CHAPTER 6

### CONCLUSIONS

In the last few years, traffic engineers have shown considerable interest in the minimization of delays at signalized intersections. In almost every urban area this type of delay is the major contributor to traffic delay. This study has emphasized the importance of determining the correct allocation of a cycle to the phases of a signal as well as the optimal synchronization of the signals.

As seen in Chapter 2, the problem of determining how best to split the cycle length among the phases of a light can be formulated as a linear programming problem if we assume that a queue of vehicles is waiting at the intersection (in effect assuming the intersection is working at capacity) and also that there exist constants which describe the possible number of vehicle starts of a particular type from a given leg during a phase of the light. Applying this technique to data for downtown Victoria resulted in projected improvements in flow of between 0.4% and 20.3%. It is noteworthy that the improvements for major intersections were typically less than those for relatively minor intersections. This may be due to the fact that the major intersections have been adjusted more than the minor ones and so were closer to the optimal setting.

The first step in examining how best to synchronize the signals to minimize traffic delay was to create a model to simulate traffic along a given leg. The model developed portrayed traffic conditions fairly accurately as was shown in Section 4.2. By fixing the offset of one light at zero while stepping the other through an offset from zero to the cycle length of the system, it was possible to examine the delay

function for that leg in some detail. In Section 2 of Chapter 5 it was seen that these delay functions could be approximated by sinusoidal curves with period equal to the cycle length.

In Chapter 3 we examined some reductions that could be made to the problem of synchronizing the signals. An algorithm for completely solving the problem was formally stated for the case when the associated graph had components which were either trees or isolated vertices. The last section of Chapter 3 examined possible methods of solving the synchronization problem when no reductions could be made. These methods were then applied to the data for downtown Victoria to evaluate their effectiveness.

The present method of synchronizing the signals for the City of Victoria involves selecting a tree containing all the intersections and then setting the lights so that a vehicle travelling at a fixed rate would have perfect progression over the legs associated with the edges of the tree. This yielded an overall delay of approximately 2.02 million seconds (561 hours). By using the same tree but minimizing as described in Section 4 of Chapter 3, this was reduced to about 1.80 million seconds (500 hours). It is clear that the fixed rate method of synchronizing can be significantly improved upon through the use of simulation to determine the exact nature of the delay functions.

To check if better results could be obtained by a different choice of trees, a number of different trees were selected by application of Prim's algorithm to weights randomly assigned to the edges of the graph. Within the first 60 seconds of CPU time a tree which generated an overall delay of 1.80 million seconds (500 hours) was found. Within 5 minutes, one generating an overall delay of 1.78 million seconds (494 hours) was

found. Increasing the time to 60 minutes of CPU resulted in a slight decrease to about 1.77 million seconds (492 hours). This shows that the present method of selecting a tree over which to minimize delays is probably satisfactory.

The Davidon, Fletcher, Powell method of minimizing a function of a number of variables was also successfully applied to the problem. This resulted in finding a set of offsets which yielded an overall delay of 1.73 million seconds or about 480 hours (only 0.04 million seconds greater than if it were possible to minimize delay over the entire graph). This method, however, has the distinct disadvantage that the planner can not specify those legs over which optimization is to occur.

One problem encountered in this study was the amount of computing time required to simulate traffic along a given leg in order to determine the characteristics of the associated delay function. An avenue of future research would be to develop better analytical methods for determining this function. The use of factor analysis and linear regression techniques upon the results of the present study might prove fruitful. Further, if one is only interested optimizing over a given tree rather than minimizing over the entire graph, one need only determine the relative offset for a leg which will yield the minimal delay over that leg rather than the characteristics of the entire delay function. This approach could lead to significant reductions in the amount of computer time required.

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APPENDIX A

DOCUMENTATION OF SUBROUTINES USED IN PHASE I

SUBROUTINE SPLIT

THIS SUBROUTINE IS COPYRIGHT 1974 BY ROBERT J. TAYLOR  
VICTORIA, BRITISH COLUMBIA  
CANADA

NOV. 1974

PURPOSE: TO OPTIMALLY ALLOCATE THE CYCLE LENGTH OF  
AN INTERSECTION TO ITS PHASES.

METHOD:

THE METHOD INVOLVES SOLVING THE DUAL OF THE  
LINEAR PROGRAMMING PROBLEM FORMALLY STATED  
IN CHAPTER 2 AND DISCUSSED IN DETAIL IN  
CHAPTER 5 OF THE COVERING WORK.

APPROXIMATIONS ARE MADE TO THE CONSTANTS  
C(I,J,K) AS OUTLINED IN CHAPTER 5. BASED  
ON THESE CONSTANTS AND THE D(I,J)'S  
AND A(K)'S PROVIDED, AN INITIAL SIMPLEX  
TABLEAU IS GENERATED. THIS IS SOLVED  
USING AN IMPLEMENTATION OF THE SIMPLEX  
ALGORITHM. IF THE PROBLEM IS INFEASIBLE  
AS STATED, THE RESTRICTIONS ON THE  
AMOUNT OF TRAFFIC FLOW ARE REDUCED AND  
THE PROBLEM RE-FORMULATED. THE CYCLE LENGTH  
IS MAINTAINED AT 60 SECONDS FOR THIS STUDY.

VARIABLES (INCLUDING THOSE IN CALLED ROUTINES):

A(I,J) (I=1,2,....,15); (J=1,2,....,35)  
SIMPLEX TABLEAU.

ALOT(K) (K=1,2,....,20)  
OPTIMAL ALLOTMENT TO PHASE K.

AMIN(K) (K=1,2,....,20)  
MINIMUM ALLOTMENT TO PHASE K.

C(I,J,K) (I=1,2,3); (J=1,2,3,4); (K=1,2,....,20)  
NUMBER OF VEHICLE STARTS PER SECOND OF TYPE  
I FROM LEG J DURING PHASE K.

A 1  
A 2  
A 3  
A 4  
A 5  
A 6  
A 7  
A 8  
A 9  
A 10  
A 11  
A 12  
A 13  
A 14  
A 15  
A 16  
A 17  
A 18  
A 19  
A 20  
A 21  
A 22  
A 23  
A 24  
A 25  
A 26  
A 27  
A 28  
A 29  
A 30  
A 31  
A 32  
A 33  
A 34  
A 35  
A 36  
A 37  
A 38  
A 39  
A 40  
A 41  
A 42  
A 43  
A 44  
A 45  
A 46  
A 47  
A 48  
A 49  
A 50

C	D(I,J)	(I=1,2,3); (J=1,2,3,4)	A	51
C		NUMBER OF VEHICLE STARTS PER HOUR OF TYPE I	A	52
C		FROM LEG J.	A	53
C	E(K)	(K=1,2,...,20)	A	54
C		NUMBER OF VEHICLE STARTS POSSIBLE DURING A	A	55
C		SECOND OF PHASE K.	A	56
C	EBTA	TIME IN CYCLE LIGHT FOR EASTBOUND TRAFFIC TURNS	A	57
C		AMBER.	A	58
C	EBTG	TIME IN CYCLE LIGHT FOR EASTBOUND TRAFFIC TURNS	A	59
C		GREEN.	A	60
C	EBTR	TIME IN CYCLE LIGHT FOR EASTBOUND TRAFFIC TURNS RED.	A	61
C	FLOW	FACTOR BY WHICH ORIGINAL TRAFFIC CONSTRAINTS	A	62
C		ARE MODIFIED TO PRODUCE A FEASIBLE PROBLEM.	A	63
C	I	DO-LOOP PARAMETER.	A	64
C	ICOLJ	CONDITION OF LEG J DURING PHASE K.	A	65
C	ICON(K,J)	(K=1,2,...,20); (J=1,2,3,4)	A	66
C		CONDITION THAT LEG J IS IN DURING PHASE K.	A	67
C		(1: UNOPPOSED GREEN; 2: OPPOSED GREEN;	A	68
C		3: RED OR AMBER)	A	69
C	ID	COMPLETION CODE OF THE SIMPLEX ROUTINE.	A	70
C		(1: OPTIMAL SOLUTION FOUND; 2: ORIGINAL	A	71
C		PROBLEM INFEASIBLE, 3: ORIGINAL PROBLEM UNBOUNDED)	A	72
C	IDENT(I)	(I=1,2,...,6)	A	73
C		ALPHANUMERIC IDENTIFICATION OF INTERSECTION.	A	74
C	IDIR(J)	(J=1,2,3,4)	A	75
C		INTERSECTION NUMBER OF THE NEXT INTERSECTION IN THE	A	76
C		J DIRECTION.	A	77
C	IJK	TEMPORARY STORAGE LOCATION.	A	78
C	IN	INTERSECTION NUMBER.	A	79
C	INT	INTERSECTION NUMBER.	A	80
C	IP	TEMPORARY LOCATION USED IN SELECTING CORRECT	A	81
C		C(I,J,K).	A	82
C	ISUB(K)	(K=1,2,...,20)	A	83
C		PHASE NUMBER OF THE KTH PHASE OF THE SIGNAL.	A	84
C			A	85
C			A	86
C			A	87
C			A	88
C			A	89
C			A	90
C			A	91
C			A	92
C			A	93
C			A	94
C			A	95
C			A	96
C			A	97
C			A	98
C			A	99
C			A	100

C			A 101
C			A 102
C	ITEMP(I)	(I=1,2,....,15) TEMPORARY STORAGE.	A 103
C			A 104
C	ITYP	TYPE OF LANE I.	A 105
C			A 106
C	ITYPE(I)	(I=1,2,....,10) TYPE OF LANE I.	A 107
C		1: LEFT TURN ONLY.	A 108
C		2: RIGHT TURN ONLY.	A 109
C		3: STRAIGHT THROUGH TRAFFIC ONLY.	A 110
C		4: LEFT & STRAIGHT TRAFFIC.	A 111
C		5: RIGHT & STRAIGHT TRAFFIC.	A 112
C		6: ALL TYPES OF TRAFFIC ALLOWED.	A 113
C			A 114
C	J	TYPE OF MOVEMENT.	A 115
C			A 116
C	JP	TEMPORARY LOCATION USED IN SELECTING CORRECT C(I,J,K).	A 117
C			A 118
C			A 119
C			A 120
C	J1	DO-LOOP PARAMETER.	A 121
C			A 122
C	J2	DO-LOOP PARAMETER.	A 123
C			A 124
C	K	DO-LOOP PARAMETER.	A 125
C			A 126
C	KP	TEMPORARY LOCATION USED IN SELECTING CORRECT C(I,J,K).	A 127
C			A 128
C			A 129
C	K1	TEMPORARY STORAGE.	A 130
C			A 131
C	K2	DO-LOOP PARAMETER.	A 132
C			A 133
C	L	ROW OF VARIABLE TO ENTER BASIS.	A 134
C			A 135
C	LABEL(I)	(I=1,2,....,15) LABELS OF THE INITIAL BASIS VECTORS.	A 136
C			A 137
C			A 138
C	LN	LEG NUMBER.	A 139
C			A 140
C	M	NUMBER OF EQUATIONS IN SIMPLEX TABLEAU.	A 141
C			A 142
C	MP1	M + 1	A 143
C			A 144
C	MP2	M + 2	A 145
C			A 146
C	MP3	M + 3	A 147
C			A 148
C	MP4	M + 4	A 149
C			A 150



C	PIVOT	VALUE OF THE PIVOT ELEMENT IN THE GAUSS-JORDAN ELIMINATION.	A 201
C			A 202
C			A 203
C	RED(I,J)	(I=1,2,...,10); (J=1,2,3) NUMBER OF VEHICLE STARTS OF TYPE J FROM LANE I OF THE PRESENT LEG DURING A RED OR AMBER PHASE.	A 204
C			A 205
C			A 206
C	SBTA	TIME IN CYCLE LIGHT FOR SOUTHBOUND TRAFFIC TURNS AMBER.	A 207
C			A 208
C			A 209
C	SBTG	TIME IN CYCLE LIGHT FOR SOUTHBOUND TURNS GREEN.	A 210
C			A 211
C			A 212
C	SBTR	TIME IN CYCLE LIGHT FOR SOUTHBOUND TURNS RED.	A 213
C			A 214
C	SUM	TEMPORARY STORAGE FOR CALCULATING VARIOUS SUMS.	A 215
C			A 216
C	SUMN	TOTAL TIME PRESENTLY ALLOTTED.	A 217
C			A 218
C	TEM	TEMPORARY STORAGE.	A 219
C			A 220
C	TF(J,I,L)	(J=1,2,3,4); (I=1,2,3); (L=1,2,3) NUMBER OF VEHICLE STARTS OF TYPE I FROM LEG J WHEN THAT LEG IS IN A PHASE OF TYPE L.	A 221
C			A 222
C			A 223
C			A 224
C	TOLER	TOLERANCE FOR SIMPLEX ALGORITHM.	A 225
C			A 226
C	TOPP(J)	(J=1,2,3) NUMBER OF VEHICLE STARTS PER SECOND OF TYPE J FROM THE PRESENT LEG DURING AN OPPOSED GREEN PHASE.	A 227
C			A 228
C			A 229
C			A 230
C	TRED(J)	(J=1,2,3) NUMBER OF VEHICLE STARTS PER SECOND OF TYPE J FROM THE PRESENT LEG DURING A RED OR AMBER PHASE.	A 231
C			A 232
C			A 233
C			A 234
C			A 235
C			A 236
C	TUNOPP(J)	(J=1,2,3) NUMBER OF VEHICLE STARTS PER SECOND OF TYPE J FROM THE PRESENT LEG DURING AN UNOPPOSED GREEN PHASE.	A 237
C			A 238
C			A 239
C			A 240
C			A 241
C	UNOPP(I,J)	(I=1,2,...,10); (J=1,2,3) NUMBER OF VEHICLE STARTS OF TYPE J FROM LANE I OF THE PRESENT LEG DURING AN UNOPPOSED GREEN PHASE.	A 242
C			A 243
C			A 244
C			A 245
C			A 246
C	VEL(I)	(I=1,2,...,10) NUMBER OF VEHICLES ENTERING LANE I PER HOUR.	A 247
C			A 248
C			A 249
C	VGSFL(I)	(I=1,2,...,10)	A 250

C		NUMBER OF VEHICLES PER HOUR GOING STRAIGHT	A 251
C		FROM LANE I.	A 252
C	VGSL	VEHICLES GOING STRAIGHT FROM LEG.	A 253
C			A 254
C	VL	VEHICLES GOING LEFT.	A 255
C			A 256
C	VR	VEHICLES GOING RIGHT.	A 257
C			A 258
C	VS	VEHICLES GOING STRAIGHT.	A 259
C			A 260
C	VT	TOTAL VEHICLES IN LEG.	A 261
C			A 262
C	VTLFL(I)	(I=1,2,...,10)	A 263
C		NUMBER OF VEHICLES PER HOUR TURNING LEFT	A 264
C		FROM LANE I.	A 265
C			A 266
C	VTLL	VEHICLES TURNING LEFT FROM LEG.	A 267
C			A 268
C	VTRFL(I)	(I=1,2,...,10)	A 269
C		NUMBER OF VEHICLES PER HOUR TURNING RIGHT.	A 270
C			A 271
C	VRTL	VEHICLES TURNING RIGHT FROM LEG.	A 272
C			A 273
C	W	TEMPORARY STORAGE USED IN CALCULATION OF NUMBER	A 274
C		OF VEHICLES ENTERING EACH LANE.	A 275
C			A 276
C	WBTA	TIME IN CYCLE LIGHT FOR WESTBOUND TRAFFIC TURNS	A 277
C		AMBER.	A 278
C			A 279
C			A 280
C	WBTG	TIME IN CYCLE LIGHT FOR WESTBOUND TRAFFIC TURNS GREEN	A 281
C			A 282
C	WBTR	TIME IN CYCLE LIGHT FOR WESTBOUND TURNS RED.	A 283
C			A 284
C	WG	WHEN EAST/WEST LIGHTS TURN GREEN,	A 285
C			A 286
C	WPS(K)	(K=1,2,...,20)	A 287
C		WHEN IN CYCLE PHASE K STARTS.	A 288
C			A 289
C	Z1	TOTAL NUMBER OF VEHICLE STARTS PER CYCLE WITH	A 290
C		PRESENT ALLOTMENT.	A 291
C			A 292
C	Z2	TOTAL NUMBER OF VEHICLE STARTS PER CYCLE WITH	A 293
C		OPTIMAL ALLOTMENT.	A 294
C			A 295
C	Z3	PERCENTAGE INCREASE OF VEHICLE STARTS PER CYCLE	A 296
C		AFTER OPTIMIZATION.	A 297
C			A 298
C			A 299
C	INPUT:	FOR EACH INTERSECTION, FOUR CARDS DESCRIBING	A 300





C	7-9	NUMBER OF THE FIRST PHASE THE LIGHT ENTERS.	A 401
C	10-12	MINIMUM ALLOTMENT TO THIS PHASE (IN TENTHS OF SECONDS WITH NO DECIMAL POINT).	A 402
C			A 403
C	13-15	PRESENT ALLOTMENT TO THIS PHASE (IN TENTHS OF SECONDS WITH NO DECIMAL POINT).	A 404
C			A 405
C	16-18	NUMBER OF THE SECOND PHASE THE LIGHT ENTERS.	A 406
C	19-21	MINIMUM ALLOTMENT TO THIS PHASE (IN TENTHS OF SECONDS WITH NO DECIMAL POINT).	A 407
C			A 408
C	22-24	PRESENT ALLOTMENT TO THIS PHASE (IN TENTHS OF SECONDS WITH NO DECIMAL POINT).	A 409
C			A 410
C	25-33	DESCRIPTION OF THIRD PHASE OF LIGHT.	A 411
C	34-42	DESCRIPTION OF FOURTH PHASE OF LIGHT.	A 412
C	43-51	DESCRIPTION OF FIFTH PHASE OF LIGHT.	A 413
C	57-80	ALPHANUMERIC IDENTIFICATION OF THE INTERSECTION.	A 414
C			A 415
C			A 416
C	CARD 6: (SECOND CARD DESCRIBING THE PATTERN OF THE LIGHT)		A 417
C			A 418
C	COL.		A 419
C			A 420
C	2-4	INTERSECTION NUMBER.	A 421
C	6	CARD NUMBER (6).	A 422
C	7-9	NUMBER OF THE SIXTH PHASE THE LIGHT ENTERS.	A 423
C	10-12	MINIMUM ALLOTMENT TO THIS PHASE (IN TENTHS OF SECONDS WITH NO DECIMAL POINT).	A 424
C			A 425
C	13-15	PRESENT ALLOTMENT TO THIS PHASE (IN TENTHS OF SECONDS WITH NO DECIMAL POINT).	A 426
C			A 427
C	16-24	DESCRIPTION OF THE SEVENTH PHASE.	A 428
C	25-33	DESCRIPTION OF THE EIGHTH PHASE.	A 429
C	34-42	DESCRIPTION OF THE NINTH PHASE.	A 430
C	43-51	DESCRIPTION OF THE TENTH PHASE.	A 431
C	57-80	ALPHANUMERIC IDENTIFICATION OF THE INTERSECTION.	A 432
C			A 433
C			A 434
C	OUTPUT:	FOR THE INTERSECTION AN OPTIMAL ALLOCATION (IF THE PROBLEM IS FEASIBLE) OF THE CYCLE TO THE VARIOUS PHASES IS PRODUCED. WARNINGS ARE GENERATED IF THE DATA APPEARS INCORRECT. CARDS SUITABLE FOR USE IN THE SUBROUTINE SIMU ARE ALSO PRODUCED. (ONE CARD IS PRODUCED FOR EVERY EDGE INCIDENT WITH THE INTERSECTION.) THE FORMAT OF THE CARDS IS AS FOLLOWS:	A 435
C			A 436
C			A 437
C			A 438
C			A 439
C			A 440
C			A 441
C			A 442
C			A 443
C	COL.		A 444
C			A 445
C	2-4	INTERSECTION NUMBER (I SAY).	A 446
C	5-7	NUMBER OF THE INTERSECTION AT THE OTHER END OF THE LEG (J SAY).	A 447
C			A 448
C	8-17	TIME IN CYCLE THAT LIGHT I TURNS GREEN FOR VEHICLES ENTERING THE LEG (F10.4).	A 449
C			A 450



	DO 33 J1=1,4	A 501
	READ (5,71) IN, LN, IDIR(J1), VGSL, VTLL, VTRL, NLT, NLL, NRL, IDENT	A 502
	WRITE (6,72) IN, LN, IDIR(J1), VGSL, VTLL, VTRL, NLT, NLL, NRL, IDENT	A 503
C		A 504
C	CHECK IF ALL THE INTERSECTIONS HAVE BEEN DONE	A 505
C		A 506
	IF (IN.EQ.0) GO TO 69	A 507
C		A 508
C	CALCULATE THE MINIMUM TRAFFIC FLOW CONSTRAINTS (IE. THE DIJ'S)	A 509
C		A 510
	D(1,J1)=VTLL	A 511
	D(2,J1)=VTRL	A 512
	D(3,J1)=VGSL	A 513
C		A 514
C	CALCULATE THE POSSIBLE NUMBER OF VEHICLE STARTS PER SECOND OF	A 515
C	TYPE I FROM LEG J1 DURING A PHASE OF TYPE K	A 516
C		A 517
	IF (NLT.GT.1) GO TO 5	A 518
	IF (NLT.GT.0) GO TO 4	A 519
	DO 3 I=1,3	A 520
	DO 2 K=1,3	A 521
	TF(J1,I,K)=0.0	A 522
2	CONTINUE	A 523
3	CONTINUE	A 524
	GO TO 33	A 525
C		A 526
C	DETERMINE THE TYPE OF EACH LANE	A 527
C		A 528
4	CONTINUE	A 529
	ITYPE(NLT)=6	A 530
	GO TO 7	A 531
5	CONTINUE	A 532
	DO 6 I=1,NLT	A 533
	ITYPE(I)=3	A 534
6	CONTINUE	A 535
	ITYPE(1)=1	A 536
	IF (NLL.EQ.0) ITYPE(1)=4	A 537
	ITYPE(NLT)=2	A 538
	IF (NRL.EQ.0) ITYPE(NLT)=5	A 539
7	CONTINUE	A 540
C		A 541
C	CALCULATE THE NUMBER OF VEHICLES ENTERING EACH LANE	A 542
C		A 543
	VEL(1)=0	A 544
	VEL(NLT)=0	A 545
	IF (NLL.EQ.1) VEL(1)=VTLL	A 546
	IF (NRL.EQ.1) VEL(NLT)=VTRL	A 547
	NLS=NLT-NLL-NRL	A 548
	VT=VTLL+VTRL+VGSL-VEL(1)-VEL(NLT)	A 549
	W=VT/NLS	A 550

	IF (NLL.NE.1) VEL(1)=W	A 551
	IF (NRL.NE.1) VEL(NLT)=W	A 552
	NLTM1=NLT-1	A 553
	IF (NLTM1.LT.2) GO TO 9	A 554
	DO 8 I=2,NLTM1	A 555
	VEL(I)=W	A 556
8	CONTINUE	A 557
9	CONTINUE	A 558
C		A 559
C	CALCULATE THE NUMBER OF VEHICLES TURNING LEFT FOR EACH LANE	A 560
C		A 561
	DO 10 I=1,NLT	A 562
	VTLFL(I)=0	A 563
10	CONTINUE	A 564
	VTLFL(1)=VTLL	A 565
C		A 566
C		A 567
C	CALCULATE THE NUMBER OF VEHICLES TURNING RIGHT FOR EACH LANE	A 568
C		A 569
	DO 11 I=1,NLT	A 570
	VTRFL(I)=0	A 571
11	CONTINUE	A 572
	VTRFL(NLT)=VTRL	A 573
C		A 574
C		A 575
C	CALCULATE THE NUMBER OF VEHICLES GOING STRAIGHT FOR EACH LANE	A 576
C		A 577
	DO 12 I=1,NLT	A 578
	VGSFL(I)=VEL(I)-VTLFL(I)-VTRFL(I)	A 579
12	CONTINUE	A 580
C		A 581
C	FOR EACH LANE CALCULATE THE POSSIBLE NUMBER OF VEHICLE STARTS	A 582
C	BASED ON THE TYPE OF THE LANE AND THE NUMBER OF VEHICLES	A 583
C	TURNING LEFT, RIGHT OR GOING STRAIGHT.	A 584
C		A 585
	DO 29 I=1,NLT	A 586
	VL=VTLFL(I)	A 587
	VR=VTRFL(I)	A 588
	VS=VGSFL(I)	A 589
C		A 590
C	FOR LEFT TURNS	A 591
C		A 592
	J=1	A 593
	ITYP=ITYPE(I)	A 594
	GO TO (13,14,14,15,14,16), ITYP	A 595
C		A 596
C		A 597
C	FOR LEFT TURN ONLY LANE	A 598
13	UNOPP(I,J)=0.5	A 599
	OPP(I,J)=0.1	A 600
	RED(I,J)=0.05	

	GO TO 17	A 601
C		A 602
C	FOR ANY LANE ALLOWING RIGHT TURNS	A 603
C		A 604
14	UNOPP(I,J)=0.05	A 605
	OPP(I,J)=0.05	A 606
	RED(I,J)=0.05	A 607
	GO TO 17	A 608
C		A 609
C	FOR A LANE ALLOWING LEFT TURNS AND STRAIGHT THROUGH TRAFFIC	A 610
C		A 611
15	UNOPP(I,J)=VL/(2.0*VL+2.0*VS)	A 612
	OPP(I,J)=VL/(10*VL+2*VS)	A 613
	RED(I,J)=0.05	A 614
	GO TO 17	A 615
C		A 616
C	FOR A LANE ALLOWING ALL 3 TYPES OF MOVEMENTS	A 617
C		A 618
16	UNOPP(I,J)=VL/(2.0*VL+2*VS+7*VR)	A 619
	OPP(I,J)=VL/(10.0*VL+2.0*VS+7.0*VR)	A 620
	RED(I,J)=0.05	A 621
17	CONTINUE	A 622
C		A 623
C	FOR VEHICLES TURNING RIGHT	A 624
C		A 625
	J=2	A 626
	GO TO (18,19,18,18,20,21), ITYP	A 627
C		A 628
C	FOR LEFT TURN ONLY LANES, STRAIGHT THROUGH ONLY LANES,	A 629
C	AND LANES ALLOWING LEFT AND STRAIGHT THROUGH TRAFFIC.	A 630
C		A 631
18	UNOPP(I,J)=0.05	A 632
	OPP(I,J)=0.05	A 633
	RED(I,J)=0.05	A 634
	GO TO 22	A 635
C		A 636
C	FOR RIGHT TURN ONLY LANES	A 637
C		A 638
19	UNOPP(I,J)=0.142857	A 639
	OPP(I,J)=0.142857	A 640
	RED(I,J)=0.142857	A 641
	GO TO 22	A 642
C		A 643
C	FOR LANES ALLOWING RIGHT TURNS AND STRAIGHT THROUGH TRAFFIC	A 644
C		A 645
20	UNOPP(I,J)=VR/(7.0*VR+2.0*VS)	A 646
	OPP(I,J)=VR/(7.0*VR+2.0*VS)	A 647
	RED(I,J)=VR/(7*VR+20.0*VS)	A 648
	GO TO 22	A 649
C		A 650

C	FOR LANES ALLOWING ALL TYPES OF MOVEMENTS	A 651
C		A 652
21	UNOPP(I,J)=VR/(7.0*VR+2.0*VS+2.0*VL)	A 653
	OPP(I,J)=VR/(7.0*VR+2.0*VS+10.0*VL)	A 654
	RED(I,J)=VR/(7*VR+20*VS+20*VL)	A 655
22	CONTINUE	A 656
C		A 657
C	FOR VEHICLES GOING STRAIGHT	A 658
C		A 659
	J=3	A 660
	GO TO (23,23,24,25,26,27), ITYP	A 661
C		A 662
C	FOR LANES ALLOWING ONLY LEFT TURNS AND RIGHT TURNS	A 663
C		A 664
23	UNOPP(I,J)=0.05	A 665
	OPP(I,J)=0.05	A 666
	RED(I,J)=0.05	A 667
	GO TO 28	A 668
C		A 669
C	FOR LANES ALLOWING ONLY STRAIGHT THROUGH TRAFFIC	A 670
C		A 671
24	UNOPP(I,J)=0.5	A 672
	OPP(I,J)=0.5	A 673
	RED(I,J)=0.05	A 674
	GO TO 28	A 675
C		A 676
C	FOR A LANE ALLOWING LEFT AND STRAIGHT THROUGH TRAFFIC	A 677
C		A 678
25	UNOPP(I,J)=VS/(2*VS+2*VL)	A 679
	OPP(I,J)=VS/(2*VS+10*VL)	A 680
	RED(I,J)=0.05	A 681
	GO TO 28	A 682
C		A 683
C	FOR LANES ALLOWING RIGHT AND STRAIGHT THROUGH TRAFFIC	A 684
C		A 685
26	UNOPP(I,J)=VS/(2*VS+7*VR)	A 686
	OPP(I,J)=VS/(2*VS+7*VR)	A 687
	RED(I,J)=0.05	A 688
	GO TO 28	A 689
C		A 690
C	FOR LANES ALLOWING ALL TYPES OF MOVEMENTS	A 691
C		A 692
27	UNOPP(I,J)=VS/(2*VS+2.0*VL+7*VR)	A 693
	OPP(I,J)=VS/(2*VS+10*VL+7*VR)	A 694
	RED(I,J)=0.05	A 695
28	CONTINUE	A 696
C		A 697
C	CALCULATE TOTAL NUMBER OF VEHICLE STARTS ALLOWED DURING THE	A 698
C	THREE TYPES OF PHASES	A 699
C		A 700

29	CONTINUE	A 701
	DO 31 J=1,3	A 702
	TUNOPP(J)=0	A 703
	TOPP(J)=0	A 704
	TRED(J)=0	A 705
	DO 30 I=1,NLT	A 706
	TUNOPP(J)=TUNOPP(J)+UNOPP(I,J)	A 707
	TOPP(J)=TOPP(J)+OPP(I,J)	A 708
	TRED(J)=TRED(J)+RED(I,J)	A 709
30	CONTINUE	A 710
31	CONTINUE	A 711
	DO 32 J2=1,3	A 712
C		A 713
C	RECORD THE NUMBER OF VEHICLE STARTS OF TYPE J2 FROM LEG J1	A 714
C	WHEN THAT LEG IS IN A PHASE OF TYPE 1,2, AND 3,	A 715
C		A 716
	TF(J1,1,J2)=TUNOPP(J2)	A 717
	TF(J1,2,J2)=TOPP(J2)	A 718
	TF(J1,3,J2)=TRED(J2)	A 719
32	CONTINUE	A 720
C		A 721
C	END OF DATA MASSAGING FOR ONE LEG INTO AN INTERSECTION	A 722
C		A 723
33	CONTINUE	A 724
C		A 725
C	FOR EACH OF THE POSSIBLE PHASES . . .	A 726
C		A 727
	DO 36 K=1,20	A 728
C		A 729
C	FOR EACH OF THE LEGS GOING INTO AN INTERSECTION . . .	A 730
C		A 731
	DO 35 J=1,4	A 732
C		A 733
C	FOR EACH TYPE OF VEHICLE MOVEMENT . . .	A 734
C		A 735
	DO 34 I=1,3	A 736
C		A 737
C	CALCULATE THE CONSTANTS C(I,J,K)	A 738
C		A 739
	ICOLJ=ICON(K,J)	A 740
	C(I,J,K)=TF(J,ICOLJ,I)	A 741
34	CONTINUE	A 742
35	CONTINUE	A 743
36	CONTINUE	A 744
C		A 745
C	FOR EACH OF THE PHASES SET THE MINIMUM AND PRESENT TIME	A 746
C	ALLOTMENTS TO ZERO	A 747
C		A 748
	DO 37 K=1,20	A 749
	AMIN(K)=0.3	A 750

37	NOW(K)=0.0	A 751
C	CONTINUE	A 752
C	READ IN THE PRESENT AND MINIMUM TIME ALLOTMENTS FOR THE PHASES	A 753
C	WRITE (6,73)	A 754
	DO 39 I=1,2	A 755
	READ (5,74) INT, LN, (ITEMP(J), J=1,15), IDENT	A 756
	WRITE (6,75) INT, LN, (ITEMP(J), J=1,15), IDENT	A 757
	DO 38 J=1,5	A 758
	IJK=(J-1)*3+1	A 759
	IF (ITEMP(IJK).EQ.0) GO TO 38	A 760
	AMIN(ITEMP(IJK))=ITEMP(IJK+1)/10.	A 761
	NOW(ITEMP(IJK))=ITEMP(IJK+2)/10.	A 762
38	CONTINUE	A 763
39	CONTINUE	A 764
	WRITE (6,73)	A 765
C		A 766
C	FOR EACH OF THE PHASES DETERMINE THE TOTAL NUMBER OF VEHICLE	A 767
C	STARTS POSSIBLE PER SECOND	A 768
C		A 769
	DO 42 K=1,20	A 770
	SUM=0.0	A 771
	DO 41 J=1,4	A 772
	DO 40 I=1,3	A 773
	SUM=SUM+C(I,J,K)	A 774
40	CONTINUE	A 775
41	CONTINUE	A 776
	E(K)=SUM	A 777
42	CONTINUE	A 778
C		A 779
C	CALCULATE THE NUMBER OF PHASES THE LIGHT GOES THROUGH	A 780
C	AND RECORD EACH OF THESE	A 781
C		A 782
	NP=0	A 783
	WRITE (6,76)	A 784
C		A 785
C	CALCULATE THE TOTAL TIME PRESENTLY ALOTTED	A 786
C		A 787
	SUMN=0.0	A 788
	DO 43 K=1,20	A 789
	IF (NOW(K).EQ.0) GO TO 43	A 790
	SUMN=SUMN+NOW(K)	A 791
	WRITE (6,77) K, AMIN(K), NOW(K)	A 792
	NP=NP+1	A 793
	ISUB(NP)=K	A 794
43	CONTINUE	A 795
C		A 796
C	CHECK IF THE TOTAL TIME ALLOTTED IS "EQUAL" TO THE CYCLE LENGTH	A 797
C		A 798
C		A 799
		A 800

C	IF (ABS(SUMN-NSIC).GT.0.01) WRITE (6,78)	A 801
C	CHECK IF PRESENT ALLOTMENT FOR EACH PHASE IS INSUFFICIENT	A 802
C	DO 44 K=1,20	A 803
44	IF (NOW(K).LT.AMIN(K)) WRITE (6,79) K	A 804
45	CONTINUE	A 805
C	CONTINUE	A 806
C	FOR EACH TYPE OF VEHICLE MOVEMENT . . .	A 807
C	DO 48 I=1,3	A 808
C	FOR EACH LEG INTO THE INTERSECTION . . .	A 809
C	DO 47 J=1,4	A 810
C	CALCULATE THE TOTAL NUMBER OF VEHICLES PRESENTLY ABLE TO USE THE	A 811
C	INTERSECTION IN EACH PHASE OF THE LIGHT.	A 812
C	SUM=0.0	A 813
	DO 46 K=1,20	A 814
	SUM=SUM+C(I,J,K)*NOW(K)	A 815
46	CONTINUE	A 816
C	CHECK IF INSUFFICIENT FLOW	A 817
C	IF (SUM.LT.D(I,J)*FLOW/3600*NSIC) WRITE (6,80) I,J	A 818
47	CONTINUE	A 819
48	CONTINUE	A 820
C	PREPARE TO SET UP THE SIMPLEX TABLEAU	A 821
C	NPP2=NP+2	A 822
	NPP14=NP+14	A 823
	M=NP	A 824
	N=2*NP+14	A 825
	MP1=M+1	A 826
	MP2=M+2	A 827
	MP3=M+3	A 828
	MP4=M+4	A 829
	NP1=N+1	A 830
	NP2=N+2	A 831
	NP3=N+3	A 832
C	ZERO OUT THE ENTIRE TABLEAU	A 833
C	DO 50 I=1,MP4	A 834
C	DO 49 J=1,NP3	A 835
	A(I,J)=0.0	A 836
		A 837
		A 838
		A 839
		A 840
		A 841
		A 842
		A 843
		A 844
		A 845
		A 846
		A 847
		A 848
		A 849
		A 850

49	CONTINUE	A 851
50	CONTINUE	A 852
C		A 853
C	FILL IN THE REST OF THE TABLEAU	A 854
C		A 855
	DO 52 I=1,M	A 856
	A(I,1)=-1.000	A 857
	A(I,2)=1.000	A 858
	A(I,1+2)=-1.0	A 859
	A(I,NPP14+1)=-1.0	A 860
	A(I,NP1)=E(ISUB(I))	A 861
	A(I,NP2)=-100-I	A 862
	DO 51 J=1,12	A 863
	IP=(J-1)/4+1	A 864
	JP=J-(IP-1)*4	A 865
	KP=ISUB(I)	A 866
	A(I,J+NPP2)=-C(IP,JP,KP)	A 867
51	CONTINUE	A 868
52	CONTINUE	A 869
	DO 53 J=1,NPP14	A 870
	A(MP3,J)=J	A 871
53	CONTINUE	A 872
	A(MP4,1)=-NSIC	A 873
	A(MP4,2)=NSIC	A 874
	DO 54 J=1,NP	A 875
	A(MP4,J+2)=-AMIN(ISUB(J))	A 876
	IP=(J-1)/4+1	A 877
	JP=J-(IP-1)*4	A 878
	A(MP4,J+NPP2)=-D(IP,JP)*FLOW/3600)*NSIC	A 879
54	CONTINUE	A 880
	DO 55 I=1,M	A 881
	LABEL(I)=A(I,NP2)	A 882
55	CONTINUE	A 883
C		A 884
C	PERFORM THE SIMPLEX ROUTINE	A 885
C		A 886
C	CALL SIMPLX	A 887
C		A 888
C	CHECK IF PROBLEMS ENCOUNTERED IN SIMPLEX ROUTINE	A 889
C		A 890
	ID=A(MP4,NP3)	A 891
	IF (ID.NE.1) GO TO 67	A 892
C		A 893
C	FOR EACH PHASE ZERO THE ALLOTMENT	A 894
C		A 895
	DO 56 K=1,20	A 896
	ALOT(K)=0.0	A 897
56	CONTINUE	A 898
C		A 899
C	DETERMINE THE ALLOTMENTS TO THE PHASES	A 900

C	WRITE (6,81)	A 901
	DO 58 I=1,M	A 902
	DO 57 J=1,N	A 903
	IF (A(MP3,J).NE.LABEL(I)) GO TO 57	A 904
	TEM=A(MP1,J)	A 905
	WRITE (6,82) ISUB(I),TEM	A 906
	ALOT(ISUB(I))=TEM	A 907
57	CONTINUE	A 908
58	CONTINUE	A 909
	WRITE (6,73)	A 910
C		A 911
C	CALCULATE THE PRESENT TOTAL FLOW AND THE PROJECTED FLOW	A 912
C		A 913
	Z1=0.0	A 914
	Z2=0.0	A 915
	DO 59 K=1,20	A 916
	Z1=Z1+E(K)*NOW(K)	A 917
	Z2=Z2+E(K)*ALOT(K)	A 918
59	CONTINUE	A 919
C		A 920
C	DETERMINE THE PERCENTAGE INCREASE	A 921
C		A 922
	Z3=Z2/Z1*100-100.0	A 923
	WRITE (6,83) Z1,Z2,Z3	A 924
	TEM=-A(MP1,NP1)	A 925
C		A 926
C	DETERMINE WHEN IN EACH CYCLE EACH OF THE PHASES BEGINS	A 927
C		A 928
	WPS(1)=0.0	A 929
	DO 60 K=2,20	A 930
	WPS(K)=WPS(K-1)+ALOT(K-1)	A 931
60	CONTINUE	A 932
C		A 933
C	DETERMINE WHEN THE VARIOUS LIGHTS CHANGE COLOUR	A 934
C		A 935
	SBTG=0.0	A 936
	NBTG=0.0	A 937
	IF (ALOT(1).NE.0.0) NBTG=WPS(3)	A 938
	IF (ALOT(2).NE.0.0) SBTG=WPS(3)	A 939
	IF (ALOT(8).NE.0.0) GO TO 61	A 940
	IF (ALOT(5).NE.0.0) GO TO 62	A 941
	SBTA=WPS(10)	A 942
	NBTA=SBTA	A 943
	SBTR=SBTA+ALOT(10)	A 944
	NBTR=SBTR	A 945
	GO TO 63	A 946
61	NBTA=WPS(7)	A 947
	NBTR=WPS(8)	A 948
	SBTA=WPS(9)	A 949
		A 950

	SBTR=SBTA+ALOT(9)	A 951
	GO TO 63	A 952
62	SBTA=WPS(4)	A 953
	SBTR=WPS(5)	A 954
	NBTA=WPS(6)	A 955
	NBTR=NBTA+ALOT(6)	A 956
63	CONTINUE	A 957
	WG=NBTR	A 958
	IF (NBTR.LT.SBTR) WG=SBTR	A 959
	EBTG=WG	A 960
	WBTG=WG	A 961
	IF (ALOT(11).NE.0.0) WBTG=WPS(13)	A 962
	IF (ALOT(12).NE.0.0) EBTG=WPS(13)	A 963
	IF (ALOT(18).NE.0.0) GO TO 64	A 964
	IF (ALOT(15).NE.0.0) GO TO 65	A 965
	EBTA=WPS(20)	A 966
	WBTA=EBTA	A 967
	EBTR=EBTA+ALOT(20)	A 968
	WBTR=EBTR	A 969
	GO TO 66	A 970
64	WBTA=WPS(17)	A 971
	WBTR=WPS(18)	A 972
	EBTA=WPS(19)	A 973
	EBTR=EBTA+ALOT(19)	A 974
	GO TO 66	A 975
65	EBTA=WPS(14)	A 976
	EBTR=WPS(15)	A 977
	WBTA=WPS(16)	A 978
	WBTR=WBTA+ALOT(16)	A 979
66	CONTINUE	A 980
C		A 981
C	WRITE OUT THE DATA FOR USE IN SUBROUTINE SIMU	A 982
C		A 983
	WRITE (6,84)	A 984
	IF (IDIR(1).NE.0) WRITE (6,85) IN, IDIR(1), NBTG, NBTA, NBTR, SBTG, SBTA	A 985
	1, SBTR	A 986
	IF (IDIR(2).NE.0) WRITE (6,85) IN, IDIR(2), WBTG, WBTA, WBTR, EBTG, EBTA	A 987
	1, EBTR	A 988
	IF (IDIR(3).NE.0) WRITE (6,85) IN, IDIR(3), SBTG, SBTA, SBTR, NBTG, NBTA	A 989
	1, NBTR	A 990
	IF (IDIR(4).NE.0) WRITE (6,85) IN, IDIR(4), EBTG, EBTA, EBTR, WBTG, WBTA	A 991
	1, WBTR	A 992
	IF (IDIR(1).NE.0) WRITE (7,85) IN, IDIR(1), NBTG, NBTA, NBTR, SBTG, SBTA	A 993
	1, SBTR	A 994
	IF (IDIR(2).NE.0) WRITE (7,85) IN, IDIR(2), WBTG, WBTA, WBTR, EBTG, EBTA	A 995
	1, EBTR	A 996
	IF (IDIR(3).NE.0) WRITE (7,85) IN, IDIR(3), SBTG, SBTA, SBTR, NBTG, NBTA	A 997
	1, NBTR	A 998
	IF (IDIR(4).NE.0) WRITE (7,85) IN, IDIR(4), EBTG, EBTA, EBTR, WBTG, WBTA	A 999
	1, WBTR	A 1000

C	GO TO 1	A1001
C	PROBLEMS IN THE SIMPLEX ROUTINE	A1002
C	CONTINUE	A1003
67	CHECK IF PROBLEM IS INFEASIBLE	A1004
C	IF (ID.EQ.2) GO TO 68	A1005
C	OTHERWISE THE PROBLEM IS UNBOUNDED	A1006
C	WRITE (6,86)	A1007
C	GO TO 1	A1008
C	PROBLEM WAS INFEASIBLE...TRY REDUCING TRAFFIC FLOW	A1009
C	FLOW=FLOW-0.05	A1010
68	WRITE (6,87) FLOW	A1011
C	CHECK IF FLOW TOO LITTLE	A1012
C	IF (FLOW.GT.0.75) GO TO 45	A1013
C	WRITE (6,88)	A1014
C	GO BACK AND DO IT ALL OVER FOR ANOTHER INTERSECTION.	A1015
C	GO TO 1	A1016
C	STOP	A1017
69	FORMAT (1H1,///,1X,'ALLOTMENT OF CYCLE TO PHASE',///)	A1018
70	FORMAT (1X,I3,I4,I4,3F6.0,3I3,17X,6A4)	A1019
71	FORMAT (1X,3I5,3F10.1,3I6,10X,6A4)	A1020
72	FORMAT (//)	A1021
73	FORMAT (1X,I3,1X,I1,5(3I3),5X,6A4)	A1022
74	FORMAT (1X,I5,I5,1X,5(3I3),17X,6A4)	A1023
75	FORMAT (//)	A1024
76	FORMAT (1X,'IN PHASE',I5,' FOR A MINIMUM OF ',F6.2,' SECONDS;	A1025
77	1 NOW ALLOCATED ',F6.2,' SECONDS.')	A1026
78	FORMAT (1X,'WARNING...PRESENT ALLOTMENT DOES NOT SUM TO CYCLE LEN	A1027
79	1GTH.')	A1028
79	FORMAT (1X,'WARNING...PRESENT ALLOTMENT TO PHASE',I4,' IS NOT S	A1029
80	1UFFICIENT')	A1030
	FORMAT (1X,'WARNING...PRESENT ALLOTMENT OF CYCLE DOES NOT ALLOW SU	A1031
		A1032
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	EFFICIENT VEHICLE STARTS OF TYPE ',I5,' FROM LEG ',I5,' .')	A1051
81	FORMAT (//,1X,'AFTER OPTIMIZATION',///)	A1052
82	FORMAT (1X,'IN PHASE ',I5,' FOR ',F8.2,' SECONDS,')	A1053
83	FORMAT (///,1X,'MAXIMUM FLOW OF VEHICLES BEFORE OPTIMIZATION =',F8	A1054
	1.2,' VEHICLES PER CYCLE ',/,1X,'MAXIMUM FLOW OF VEHICLES AFTER	A1055
	2 OPTIMIZATION =',F8.2,' VEHICLES PER CYCLE ',/,1X,'PERCENTAGE I	A1056
	3NCREASE =',F8.2,///)	A1057
84	FORMAT (1X,'THE FOLLOWING CARDS ARE TO USED IN THE SIMULATION',///	A1058
	1)	A1059
85	FORMAT (1X,2I3,6F10.4)	A1060
86	FORMAT (1X,'PROBLEM FORMULATED INCORRECTLY')	A1061
87	FORMAT (1X,'PROBLEM INFEASIBLE AS STATED; TRAFFIC LEVEL REDUCED TO	A1062
	1 ',F8.3)	A1063
88	FORMAT (1X,'PROBLEM INFEASIBLE AS STATED')	A1064
	END	A1065-



5	CONTINUE	B	51
	IF (NAVIB.EQ.0) GO TO 11	B	52
	DO 6 K2=1,N	B	53
	IF (A(MP2,K2).GT.TOLER) GO TO 7	B	54
6	CONTINUE	B	55
	IF (A(MP2,NP1).LT.TOLER) GO TO 11	B	56
C		B	57
C	ORIGINAL PROBLEM IS UNBOUNDED	B	58
C		B	59
	A(MP4,NP3)=3.0D0	B	60
	RETURN	B	61
C		B	62
C	DETERMINE VARIABLE TO ENTER BASIS	B	63
C		B	63
7	DO 8 K2=1,N	B	64
	IF (A(MP2,K2).GT.TOLER.AND.A(MP3,K2).GT.0.0D0) GO TO 9	B	65
8	CONTINUE	B	66
C		B	67
C	ENTER SECOND PHASE OF SIMPLEX ROUTINE	B	68
C		B	69
	GO TO 11	B	70
9	K=K2	B	71
	DO 10 K2=1,N	B	72
	IF (A(MP2,K2).GT.TOLER.AND.A(MP3,K2).GT.0.0D0.AND.A(MP2,K2).GT.A(M	B	73
	1P2,K)) K=K2	B	74
10	CONTINUE	B	75
C		B	76
C	DETERMINE WHICH VARIABLE SHOULD LEAVE THE BASIS	B	77
C		B	78
	CALL MINRAT (K,L)	B	79
	IF (A(MP4,NP3).EQ.2.0D0) RETURN	B	80
	CALL GJELIM (MP2,NP1,L,K)	B	81
	CALL SWAP (L,NP2,MP3,K)	B	82
	CALL SWAP (L,NP3,MP4,K)	B	83
	GO TO 4	B	84
C		B	85
C	DETERMINE IF A VARIABLE CAN ENTER THE BASIS	B	86
C		B	87
11	DO 12 I=1,N	B	88
	IF (DABS(A(MP2,I)).LT.TOLER.AND.A(MP1,I).GT.TOLER.AND.A(MP3,I).GT.	B	89
	10.0D0) GO TO 13	B	90
12	CONTINUE	B	91
C		B	92
C	OPTIMAL SOLUTION FOUND	B	93
C		B	94
	A(MP4,NP3)=1.0D0	B	95
	RETURN	B	96
C		B	97
C	DETERMINE VARIABLE TO ENTER BASIS	B	98
C		B	99

13	K1=I	B 100
	K=K1	B 101
	DO 14 I=K1,N	B 102
	IF (A(MP1,I).GT.A(MP1,K).AND.DABS(A(MP2,I)).LT.TOLER.AND.A(MP3,I).	B 103
	1GT.0.GD0) K=I	B 104
14	CONTINUE	B 105
C		B 106
C	DETERMINE VARIABLE TO LEAVE THE BASIS	B 107
C		B 108
	CALL MINRAT (K,L)	B 109
	IF (A(MP4,NP3).EQ.2.0D0) RETURN	B 110
	CALL GJELIM (MP2,NP1,L,K)	B 111
	CALL SWAP (L,NP2,MP3,K)	B 112
	CALL SWAP (L,NP3,MP4,K)	B 113
	GO TO 11	B 114
	END	B 115-

C	SUBROUTINE GJELIM (NR,NC,L,K)	1
C		2
C	THIS SUBROUTINE IS COPYRIGHT 1974 BY ROBERT J. TAYLOR	3
C	VICTORIA, BRITISH COLUMBIA	4
C	CANADA	5
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SUBROUTINE MINRAT (K,L)
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VICTORIA, BRITISH COLUMBIA
CANADA
NOV. 1974

PURPOSE:      TO DETERMINE WHICH VARIABLE SHOULD LEAVE THE BASIS.

VARIABLES:

                (LISTED IN SUBROUTINE SPLIT)

DOUBLE PRECISION A(15,35),TOLER,LABEL(30)

COMMON A,TOLER,N,NP1,NP2,NP3,NP4,M,MP1,MP2,MP3,MP4
COMMON LABEL

DO 1 I=1,M
  IF (A(I,K).GT.TOLER) GO TO 2
CONTINUE

NONE ARE POSITIVE, THEREFORE ORIGINAL PROBLEM IS INFEASIBLE.

A(MP4,NP3)=2.0D0
RETURN
L1=I
L=L1
DO 3 I=L1,M
  IF (A(I,K).LT.TOLER) GO TO 3
  IF (A(I,NP1)/A(I,K).LT.A(L,NP1)/A(L,K)) L=I
CONTINUE
RETURN
END

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APPENDIX B

DOCUMENTATION OF SUBROUTINES USED IN PHASE II





C		COLOUR L.	A 101
C			A 102
C	CSL(K,L)	(K=1,2,....,10); (L=1,2,3)	A 103
C		TIME IN CYCLE THAT SECONDARY LIGHT FOR LANE K	A 104
C		TURNS COLOUR L.	A 105
C			A 106
C	DLP	DISTANCE THAT LAST CAR IN LANE IS FROM LIGHT.	A 107
C			A 108
C	DNS	ESTIMATED DISTANCE NEEDED FOR VEHICLE AHEAD TO STOP.	A 109
C			A 110
C	DNXT	PRESENT DISTANCE TO VEHICLE AHEAD.	A 111
C			A 112
C	DT	DISTANCE TRAVELLED BY VEHICLE IN TIME-SLICE.	A 113
C			A 114
C	EDN	ESTIMATED DISTANCE TO VEHICLE AHEAD UNDER PRESENT	A 115
C		CONDITIONS.	A 116
C			A 117
C	EPS	TIME INCREMENT.	A 118
C			A 119
C	FA	FUTURE ACCELERATION OF THE VEHICLE.	A 120
C			A 121
C	FDL	FUTURE DISTANCE TO LIGHT.	A 122
C			A 123
C	FL	FUTURE LANE OF VEHICLE.	A 124
C			A 125
C	FLRS	FUTURE LEFT-RIGHT-STRAIGHT FLAG.	A 126
C			A 127
C	FV	FUTURE VELOCITY OF VEHICLE.	A 128
C			A 129
C	GRI	TIME LIGHT TURNS GREEN.	A 130
C			A 131
C	GT	TOTAL GREEN TIME OF LIGHT FOR LANE IN QUESTION.	A 132
C			A 133
C	I	DO-LOOP PARAMETER.	A 134
C			A 135
C	I1	POINTER TO VEHICLE TO BE REMOVED.	A 136
C			A 137
C	I3	DO-LOOP PARAMETER.	A 138
C			A 139
C	I5	DO-LOOP PARAMETER.	A 140
C			A 141
C	IAA	TIME (NEGLECTING OFFSET) LIGHT TURNS AMBER.	A 142
C			A 143
C	IAG	TIME (NEGLECTING OFFSET) LIGHT TURNS GREEN.	A 144
C			A 145
C	IAR	TIME (NEGLECTING OFFSET) LIGHT TURNS RED.	A 146
C			A 147
C	IAT	PRESENT TIME IN CYCLE IF IT HAD TURNED GREEN	A 148
C		AT OFFSET ZERO.	A 149
C			A 150



C	IY	USED IN CALCULATING YFL.	A 201
C	J	DO-LOOP PARAMETER.	A 202
C	J1	DO-LOOP PARAMETER.	A 203
C	JETA	TIME IN CYCLE WHEN LIGHT J TURNS AMBER FOR TRAFFIC ENTERING LEG.	A 204
C	JETG	TIME IN CYCLE WHEN LIGHT J TURNS GREEN FOR TRAFFIC ENTERING LEG.	A 205
C	JETR	TIME IN CYCLE WHEN LIGHT J TURNS RED FOR TRAFFIC ENTERING LEG.	A 206
C	JIN	NUMBER OF INTERSECTION J.	A 207
C	JJ	DO-LOOP PARAMETER (EQUALS JTHETA MINUS 1).	A 208
C	JJ1	JJ-1	A 209
C	JLTA	TIME IN CYCLE WHEN LIGHT J TURNS AMBER FOR TRAFFIC LEAVING LEG.	A 210
C	JLTG	TIME IN CYCLE WHEN LIGHT J TURNS GREEN FOR TRAFFIC LEAVING LEG.	A 211
C	JLTR	TIME IN CYCLE WHEN LIGHT J TURNS RED FOR TRAFFIC LEAVING LEG.	A 212
C	JS	INITIAL OFFSET FOR INTERSECTION J.	A 213
C	JTHETA	OFFSET OF INTERSECTION J.	A 214
C	JXX	TEMPORARY LOCATION FOR NUMBER OF INTERSECTION J.	A 215
C	K	DO-LOOP PARAMETER.	A 216
C	K1 TO K6	TEMPORARY STORAGE LOCATIONS.	A 217
C	KK	DO-LOOP PARAMETER.	A 218
C	L	NUMBER OF LANES GOING IN APPROPRIATE DIRECTION.	A 219
C	NA	NUMBER OF ACCIDENTS THAT OCCURRED.	A 220
C	NALT	NEXT AVAILABLE LOCATION IN ARRAY T.	A 221
C	NBS	NEW BASIC SUBSCRIPT.	A 222
C			A 223
C			A 224
C			A 225
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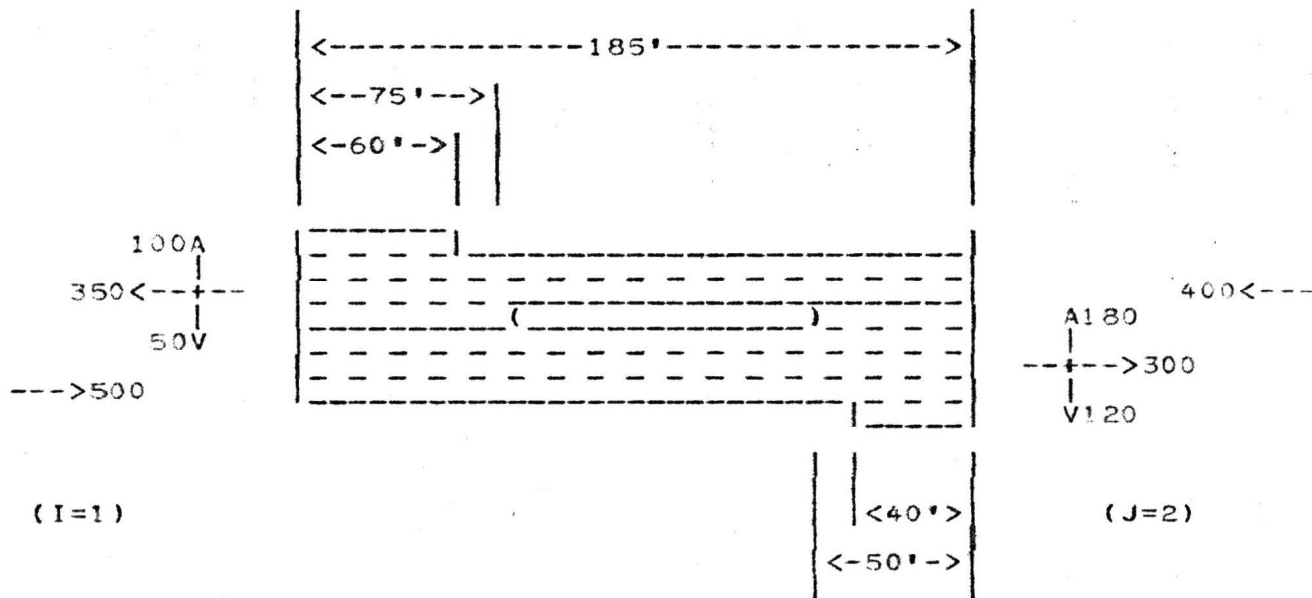


C	PFFV	POINTER TO FIRST FREE VEHICLE.	A 301
C			A 302
C	PFV(I)	(I=1,2,3,....,100) POINTER TO NEXT FREE VEHICLE (0 IMPLIES NO SUCH VEHICLE).	A 303
C			A 304
C			A 305
C	PIC(K)	(K=1,2,3,....,60) USED TO PRINT ROUGH PLOT OF DELAY FUNCTION.	A 306
C			A 307
C			A 308
C	PL	PRESENT LANE OF VEHICLE (0 IMPLIES NOT IN THE SIMULATION AT THE PRESENT TIME).	A 309
C			A 310
C			A 311
C			A 313
C	PLCIL(I,L)	(I=1,2); (L=1,2,3,....,10) POINTER TO LAST VEHICLE IN LANE L (I INDICATES EVEN OR ODD ITERATION).	A 314
C			A 315
C			A 316
C			A 317
C	PLRS	PRESENT LEFT-RIGHT-STRAIGHT FLAG.	A 318
C			A 319
C	PNIL	POINTER TO VEHICLE AHEAD IN SAME LANE.	A 320
C			A 321
C	PNILL	POINTER TO VEHICLE AHEAD IN LEFT TURN ONLY LANE.	A 322
C			A 323
C	PNIRL	POINTER TO VEHICLE AHEAD IN RIGHT TURN ONLY LANE.	A 324
C			A 325
C	PNT	POINTER TO VEHICLE.	A 326
C			A 327
C	PPNT	POINTER TO LAST CAR IN LANE.	A 328
C			A 329
C	PROB	PROBABILITY OF VEHICLE ENTERING LANE FROM SIDE STREET DURING TIME-SLICE.	A 330
C			A 331
C			A 332
C	PROBL	PROBABILITY OF VEHICLE TURNING LEFT.	A 333
C			A 334
C	PROBR	PROBABILITY OF VEHICLE TURNING RIGHT.	A 335
C			A 336
C	PV	PRESENT VELOCITY.	A 337
C			A 338
C	RDI	TIME LIGHT TURNS RED.	A 339
C			A 340
C	SD	STANDARD DEVIATION OF TIME TAKEN TO TRAVERSE LEG.	A 341
C			A 342
C	SDTL	STARTING DISTANCE TO LIGHT.	A 343
C			A 344
C	SUM	TOTAL TIME OF VEHICLES TRAVERSING LEG.	A 345
C			A 346
C	SUMS	SUM OF SQUARED TIMES OF VEHICLES TRAVERSING LEG.	A 347
C			A 348
C			A 349
C	T(I)	(I=1,2,....,1000) TIME THAT VEHICLE I TOOK TO TRAVERSE LEG.	A 350
C			A 351



C	VILL(K)	(K=1,2) VEHICLES PER HOUR ENTERING A LEFT TURN LANE GOING IN DIRECTION K.	A 402
C			A 403
C			A 404
C	VILLX	VEHICLES PER HOUR ENTERING A LEFT TURN LANE IN APPROPRIATE DIRECTION.	A 405
C			A 406
C			A 407
C	VIRL(K)	(K=1,2) VEHICLES PER HOUR ENTERING A RIGHT TURN LANE GOING IN DIRECTION K.	A 408
C			A 409
C			A 410
C			A 411
C	VIRLX	VEHICLES PER HOUR ENTERING A RIGHT TURN LANE IN APPROPRIATE DIRECTION.	A 412
C			A 413
C			A 414
C	VNXT	PRESENT VELOCITY OF VEHICLE AHEAD.	A 415
C			A 416
C	VT	VEHICLES PER HOUR TRAVERSING LEG IN APPROPRIATE DIRECTION.	A 417
C			A 418
C			A 419
C	VTL(K)	(K=1,2) VEHICLES PER HOUR TURNING LEFT OFF LEG (TRAVELLING IN DIRECTION K).	A 420
C			A 421
C			A 422
C			A 423
C	VTLX	VEHICLES PER HOUR TURNING LEFT.	A 424
C			A 425
C			A 426
C	VTOT(K)	(K=1,2) NUMBER OF VEHICLES LEAVING LEG TRAVELLING IN DIRECTION K.	A 427
C			A 428
C			A 429
C			A 430
C	VTR(K)	(K=1,2) VEHICLES PER HOUR TURNING RIGHT OFF LEG (TRAVELLING IN DIRECTION K)	A 431
C			A 432
C			A 433
C	VTRX	VEHICLES PER HOUR TURNING RIGHT.	A 434
C			A 435
C			A 436
C	VW	DESIRED FUTURE VELOCITY OF VEHICLE.	A 437
C			A 438
C	W	TEMPORARY STORAGE LOCATION.	A 439
C			A 440
C	X	TEMPORARY STORAGE LOCATION.	A 441
C			A 442
C	YFL	RANDOM NUMBER BETWEEN 0 AND 1.	A 443
C			A 444
C	Z	AVERAGE TIME BETWEEN VEHICLES ENTERING LANE.	A 445
C			A 446
C			A 447
C	CALLING:	WHEN CALLING THE SUBROUTINE THE FOLLOWING VARIABLES MUST BE DEFINED.	A 448
C			A 449
C			A 450
C		ALENLT(K) (K=1,2)	A 451





FURTHER SUPPOSE THAT THE LIGHTS HAVE A CYCLE LENGTH OF 60 SECONDS, 1000 VEHICLES ARE TO BE TIMED FOR EACH DIFFERENT RELATIVE OFFSET, AND THE LIGHT PATTERNS MAY BE SUMMARIZED AS BELOW:

INTERSECTION #1:

0-7 SECONDS: ADVANCE GREEN TO EASTBOUND  
 7-30 SECONDS: GREEN TO EAST & WESTBOUND  
 30-35 SECONDS: AMBER TO EAST & WESTBOUND  
 35-54 SECONDS: GREEN TO NORTH & SOUTHBOUND  
 54-0 SECONDS: AMBER TO NORTH & SOUTHBOUND

INTERSECTION #2:

0-6 SECONDS: AMBER TO NORTH & SOUTHBOUND  
 6-16 SECONDS: ADVANCE GREEN TO EASTBOUND  
 16-36 SECONDS: GREEN TO EAST & WESTBOUND

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 A 531  
 A 532  
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 A 534

36-42 SECONDS: AMBER TO EAST & WESTBOUND  
42-0 SECONDS: AMBER TO NORTH & SOUTHBOUND

UPON ENTERING THE ROUTINE THE VARIABLES WOULD  
HAVE THE FOLLOWING VALUES:

ALENLT(K)	(K=1,2)	50, 75
ALENRT(K)	(K=1,2)	40, 60
ALENST		185
IETA		30
IETG		0
IETR		35
IIN		1
ILTA		30
ILTG		7
ILTR		35
JETA		36
JETG		36
JETG		16
JETR		42
JIN		2
JLTA		36
JLTG		6
JLTR		42
NLA		5
NLC		4
NSIC		60
NTC		1000
VES(K)	(K=1,2)	500, 400
VGS(K)	(K=1,2)	300, 350
VTL(K)	(K=1,2)	180, 50
VTR(K)	(K=1,2)	120, 100

OUTPUT:

FOR EACH DIFFERENT RELATIVE OFFSET THE  
NUMBERS OF THE INTERSECTIONS AT THE ENDS ARE  
PRINTED ALONG WITH THE DIFFERENCE IN OFFSETS,  
THE AVERAGE TIME TAKEN TO TRAVERSE THE LEG WITH  
THIS RELATIVE OFFSET, AND THE STANDARD DEVIATION  
OF THE TIMES TAKEN TO TRAVERSE THE LEG.  
UPON RETURN FROM THE ROUTINE ALL  
INPUT VARIABLES ARE LEFT UNCHANGED AND THE  
AVERAGE TIMES TAKEN TO TRAVERSE THE LEG ARE  
RETURNED IN THE ARRAY AT. A ROUGH PLOT OF THE  
AVERAGE TIME TAKEN TO TRAVERSE THE LEG IS ALSO  
PRODUCED.

ROUTINES CALLED:

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A 584



C	DO 2 I=1,1000	A 635
	T(I)=0	A 636
2	CONTINUE	A 637
C		A 638
C	CALCULATE OFFSET	A 639
C		A 640
	JTHETA=JJ-1.0	A 641
C		A 642
C	INITIALIZE FOR A SIMULATION RUN WITH THIS OFFSET	A 643
C		A 644
	CALL INTR	A 645
C		A 646
C	SIMULATE USING THIS OFFSET	A 647
C		A 648
	CALL ITER	A 649
C		A 650
C	CALCULATE AVERAGE TIME TO TRAVERSE LEG	A 651
C		A 652
	SUM=0.0	A 653
	SUMS=0.0	A 654
	DO 3 I3=1,NTC	A 655
	SUMS=SUMS+T(I3)*T(I3)	A 656
	SUM=SUM+T(I3)	A 657
3	CONTINUE	A 658
	AVE=SUM/NTC	A 659
	AT(JJ)=AVE	A 660
	TOP=NTC*SUMS-SUM*SUM	A 661
	BOT=NTC*(NTC-1)	A 662
	SD=SQRT(TOP/BOT)	A 663
	WRITE (6,7) IIN,JIN,JTHETA,AVE,SD,VELT,IDENT	A 664
C		A 665
C	END OF SIMULATION WITH THIS OFFSET	A 666
C		A 667
4	CONTINUE	A 668
C		A 669
C	DO ROUGH PLOT OF DELAY FUNCTION	A 670
C		A 671
	WRITE (6,8)	A 672
	DO 6 JJ=1,NSIC	A 673
	DO 5 K=1,100	A 674
	PIC(K)=BLANK	A 675
	IF (K,LT,AT(JJ)) PIC(K)=BLOB	A 676
5	CONTINUE	A 677
	JJM1=JJ-1	A 678
	WRITE (6,9) JJM1,PIC	A 679
6	CONTINUE	A 680
	RETURN	A 681
C		A 682
C		A 683
		A 684

```
C
7  FORMAT (3I4,2F8.2,I6,34X,2(A4,2X))
8  FORMAT (1H1)
9  FORMAT (1X,I3,2X,100A1)
   END
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A 685
A 686
A 687
A 688
A 689-
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C	SUBROUTINE INITR	B	1
C		B	2
C	THIS SUBROUTINE IS COPYRIGHT 1974 BY ROBERT J. TAYLOR	B	3
C	VICTORIA, BRITISH COLUMBIA	B	4
C	CANADA	B	5
C		B	6
C		B	7
C		B	8
C		B	9
	COMMON NA, IX, JTHETA, T, NTC, ALENLT, ALENRT, ALENST, ALL, ALV, AUL, BS	B	10
	COMMON CARDOP, CPL, CSL, EPS, IIN, JIN, NC, NLA, NLT, NSIC, OPTSPD	B	11
	COMMON PARM1, PFFV, SDTL, TE, TIME, TTIME, VGS, VTL, VTR, VES, IRID1, IRID2	B	12
	COMMON PLCIL, IDENT, VELT, IETG, IETA, IETR, ILTG, ILTA, ILTR, JETG, JETA	B	13
	COMMON JETR, JLTG, JLTA, JLTR	B	14
C		B	15
C		B	16
	DIMENSION VGS(2), VTR(2), VTL(2), VES(2), T(1000)	B	17
	INTEGER BLANK/' ', BLOB/'@'/', PIC(100), PLCIL(2, 10), VELT, BS, PFFV	B	18
	REAL ALENLT(2), ALENRT(2), CARDOP(200, 12), CPL(10, 3), CSL(10, 3)	B	19
	REAL TE(1000), AT(1000)	B	20
C		B	21
C	INITIALIZATION SECTION	B	22
C		B	23
	BS=7	B	24
	IX=967532465	B	25
	NA=0	B	26
	SDTL=ALENST+400.0	B	27
	TIME=0.0	B	28
C		B	29
C	CLEAR ALL LANES	B	30
C		B	31
	DO 1 I=1, 10	B	32
	PLCIL(1, I)=0	B	33
	PLCIL(2, I)=0	B	34
1	CONTINUE	B	35
C		B	36
C	CALCULATE TIME IN CYCLE THAT LIGHTS CHANGE COLOURS	B	37
C		B	38
	DO 4 I=1, NLT	B	39
C		B	40
C	TEST WHICH DIRECTION LANE FLOWS	B	41
C		B	42
	IF (I.GT.NLA) GO TO 2	B	43
C		B	44
C	FOR LANES GOING FROM I TO J	B	45
C		B	46
	K1=IETG	B	47
	K2=IETA	B	48
	K3=IETR	B	49
	K4=JLTG+JTHETA	B	50

	K5=JLTA+JTHETA	B	51
	K6=JLTR+JTHETA	B	52
	GO TO 3	B	53
C		B	54
C	FOR LANES GOING FROM J TO I	B	55
C		B	56
2	K1=JETG+JTHETA	B	57
	K2=JETA+JTHETA	B	58
	K3=JETR+JTHETA	B	59
	K4=ILTG	B	60
	K5=ILTA	B	61
	K6=ILTR	B	62
C		B	63
C	RECORD WHEN LIGHTS CHANGE COLOURS	B	64
C		B	65
3	CONTINUE	B	66
	CSL(I,1)=MOD(K1,NSIC)	B	67
	CSL(I,2)=MOD(K2,NSIC)	B	68
	CSL(I,3)=MOD(K3,NSIC)	B	69
	CPL(I,1)=MOD(K4,NSIC)	B	70
	CPL(I,2)=MOD(K5,NSIC)	B	71
	CPL(I,3)=MOD(K6,NSIC)	B	72
	CONTINUE	B	73
4		B	74
C	CLEAR ALL VEHICLES FROM SYSTEM	B	75
C		B	76
	DO 6 I=1,NC	B	77
	TE(I)=-1.0	B	78
	DO 5 J=1,12	B	79
	CARDOP(I,J)=0.0	B	80
5	CONTINUE	B	81
6	CONTINUE	B	82
C		B	83
C		B	84
C	RETURN	B	85
C		B	86
		B	87
	END	B	88-



```

L=NLA
IF (IJK.EQ.2) L=NLT-NLA
C
C
C
DETERMINE VEHICLES USING LEFT LANE, RIGHT LANE, ANY CENTRAL LANE
C
C
C
VTLX=VTL(IJK)
VTRX=VTR(IJK)
VGSX=VGS(IJK)
VT=VTLX+VGSX+VTRX
IF (L.GT.2) GO TO 3
C
C
C
FOR TWO LANES
C
C
C
W=VT/2
VILLX=W
VIRLX=W
VICLX=W
GO TO 6
C
C
C
3
W=VT/(L-2)
VILLX=W
IF (W.LT.VTLX) VILLX=VTLX
IF (L.GT.3) GO TO 4
C
C
C
FOR THREE LANES
C
C
C
VIRLX=W
VICLX=W
GO TO 6
C
C
C
4
W=(VT-VILLX)/(L-3)
VIRLX=W
IF (W.LT.VTRX) VIRLX=VTRX
IF (L.GT.4) GO TO 5
C
C
C
FOR FOUR LANES
C
C
C
VICLX=W
GO TO 6
C
C
C
FOR FIVE OR MORE LANES
C
C
C
5
W=(VT-VILLX-VIRLX)/(L-4)
VICLX=W
C
C
C
6
CONTINUE
VILL(IJK)=VILLX

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	VIRL(IJK)=VIRLX	C	101
	VICL(IJK)=VICLX	C	102
	CONTINUE	C	103
7		C	104
C	BEGINNING OF A NEW TIME-SLICE	C	105
C		C	106
8	TIME=TIME+EPS	C	107
	ITIME=TIME-IFIX(TIME/NSIC)*NSIC	C	108
C		C	109
C	CALCULATE COLOURS OF LIGHTS FOR EACH LANE	C	110
C		C	111
C		C	112
C	FOR FIRST INTERSECTION FOR THE LANE	C	113
		C	114
	DO 9 I=1,NLT	C	115
	GRI=CPL(I,1)	C	116
	AMI=CPL(I,2)	C	117
	RDI=CPL(I,3)	C	118
	IX1=AMI-GRI	C	119
	IX1=IX1+60	C	120
	IAA=MOD(IX1,NSIC)	C	121
	IX1=RDI-GRI	C	122
	IX1=IX1+60	C	123
	IAR=MOD(IX1,NSIC)	C	124
	IX1=ITIME-GRI	C	125
	IX1=IX1+60	C	126
	IAT=MOD(IX1,NSIC)	C	127
	COL=3	C	128
	IF (IAT.LT.IAA) COL=1	C	129
	IF (IAT.GE.IAA.AND.IAT.LT.IAR) COL=2	C	130
	APLIT(I)=COL	C	131
9	CONTINUE	C	132
C		C	133
C	FOR SECOND INTERSECTION FOR THE LANE	C	134
C		C	135
	DO 10 I=1,NLT	C	136
	GRI=CSL(I,1)	C	137
	AMI=CSL(I,2)	C	138
	RDI=CSL(I,3)	C	139
	IX1=AMI-GRI+60	C	140
	IAA=MOD(IX1,NSIC)	C	141
	IX1=RDI-GRI+60	C	142
	IAR=MOD(IX1,NSIC)	C	143
	IX1=ITIME-GRI+60	C	144
	IAT=MOD(IX1,NSIC)	C	145
	COL=3	C	146
	IF (IAT.LT.IAA) COL=1	C	147
	IF (IAT.GE.IAA.AND.IAT.LT.IAR) COL=2	C	148
	ASLIT(I)=COL	C	149
10	CONTINUE	C	150

C		C	151
C		C	152
C	DETERMINE WHICH HALF OF MATRIX CARDOP TO USE	C	153
	IF (BS.EQ.1) GO TO 11	C	154
	BS=1	C	155
	BSS=1	C	156
	NBS=7	C	157
	NBSS=2	C	158
	GO TO 12	C	159
C		C	160
C		C	161
11	BS=7	C	162
	BSS=2	C	163
	NBS=1	C	164
	NBSS=1	C	165
C		C	166
C	SET UP SUBSCRIPTS FOR LATER USE	C	167
C		C	168
12	BSP1=BS+1	C	169
	BSP2=BS+2	C	170
	BSP3=BS+3	C	171
	BSP4=BS+4	C	172
	BSP5=BS+5	C	173
	NBSP1=NBS+1	C	174
	NBSP2=NBS+2	C	175
	NBSP3=NBS+3	C	176
	NBSP4=NBS+4	C	177
	NBSP5=NBS+5	C	178
C		C	179
C	CLEAR POINTER TO LAST VEHICLE IN LANE	C	180
C		C	181
	DO 13 I=1,NLT	C	182
	PLCIL(NBSS,I)=0	C	183
13	CONTINUE	C	184
C		C	185
C	SIMULATE MOVEMENT FOR EACH VEHICLE	C	186
C		C	187
	DO 36 I=1,NC	C	188
C		C	189
C	DETERMINE PRESENT STATUS OF VEHICLE	C	190
C		C	191
	PNIL=CARDOP(I,BS)	C	192
	PL=CARDOP(I,BSP1)	C	193
C		C	194
C	CHECK IF VEHICLE ISN'T IN THE SIMULATION AS YET	C	195
C		C	196
	IF (PL.EQ.0) GO TO 36	C	197
C		C	198
C		C	199
	ABD=1	C	200

	IF (PL.GT.NLA) ABD=2	C 201
	PLRS=CARDOP(I,BSP2)	C 202
	PV=CARDOP(I,BSP3)	C 203
	PA=CARDOP(I,BSP4)	C 204
	PDL=CARDOP(I,BSP5)	C 205
C		C 206
C	CALCULATE VELOCITY REQUIRED DUE TO COLOUR OF LIGHT	C 207
C		C 208
	VEL1=OPTSPD	C 209
C		C 210
C	CHECK IF MAKING RIGHT HAND TURN	C 211
C		C 212
	IF (PL.EQ.NLA.OR.PL.EQ.NLT) GO TO 16	C 213
C		C 214
C		C 215
	PDE=PDL-PV*EPS-4.0	C 216
	IF (PDE.LT.0.0) PDE=0.0	C 217
	PCOL=ASLIT(PL)	C 218
C		C 219
C	CHECK IF APPROACHING FIRST LIGHT FOR LANE	C 220
C		C 221
	IF (PDL.GT.ALENST) GO TO 15	C 222
C		C 223
C	DETERMINE LANE NUMBERS OF LEFT AND RIGHT TURN LANES	C 224
C		C 225
	NLL=1	C 226
	NLR=NLA	C 227
	IF (ABD.EQ.1) GO TO 14	C 228
	NLL=NLA+1	C 229
	NLR=NLT	C 230
C		C 231
C	DETERMINE COLOUR OF LIGHT	C 232
C		C 233
14	PCOL=APLIT(NLL)	C 234
	IF (PLRS.EQ.2) PCOL=APLIT(NLR)	C 235
	IF (PLRS.EQ.3) PCOL=APLIT(PL)	C 236
C		C 237
C	IF LIGHT IS GREEN NO NEED FOR SPEED REDUCTION	C 238
C		C 239
15	IF (PCOL.EQ.1) GO TO 16	C 240
C		C 241
C		C 242
	PDM=PDL-4.0	C 243
	IF (PDL.GT.ALENST) PDM=PDL-ALENST-4.0	C 244
	IF (PDM.LT.0.0) PDM=0.0	C 245
	IF (PDL.GT.ALENST) PDE=PDE-ALENST	C 246
C		C 247
C	IF VEHICLE IS CLOSE ENOUGH TO LIGHT REDUCE SPEED	C 248
C		C 249
	IF (PDE.LE.PARM1) VEL1=OPTSPD*SQRT(PDM/PARM1)	C 250

C		C	251
C		C	252
16	CONTINUE	C	253
	IF (VEL1.LT.0.0) VEL1=0.0	C	254
C		C	255
C	CALCULATE VELOCITY REQUIRED DUE TO MOVEMENT TO BE MADE	C	256
C		C	257
	VEL2=OPTSPD	C	258
C		C	259
C	IF AT FIRST LIGHT OR GOING STRAIGHT NO REDUCTION NECESSARY	C	260
C		C	261
	IF (PLRS.EQ.3.OR.PDL.GT.ALENST) GO TO 17	C	262
C		C	263
C		C	264
	PDE=PDL-PV*EPS-4.0	C	265
	PDM=PDL	C	266
C		C	267
C	IF DISTANCE WARRENTS IT REDUCE SPEED	C	268
C		C	269
	IF (PDE.LE.PARM1) VEL2=OPTSPD*SQRT(PDM/PARM1)	C	270
17	CONTINUE	C	271
C		C	272
C	INSURE VEHICLE STILL "CREEPS" AHEAD	C	273
C		C	274
	IF (VEL2.LT.1.0) VEL2=1.0	C	275
C		C	276
C	CALCULATE VELOCITY REQUIRED DUE TO VEHICLE DIRECTLY AHEAD	C	277
C		C	278
	VEL3=OPTSPD	C	279
C		C	280
C	CHECK IF NO VEHICLE IN LEFT TURN LANE	C	281
C		C	282
	IF (PNIL.EQ.0) GO TO 18	C	283
C		C	284
C	DETERMINE CHARACTERISTICS OF VEHICLE AHEAD	C	285
C		C	286
	VNXT=CARDOP(PNIL,BSP3)	C	287
	ANXT=CARDOP(PNIL,BSP4)	C	288
	DNXT=PDL-CARDOP(PNIL,BSP5)	C	289
C		C	290
C	CHECK IF ACCIDENT HAS OCCURRED	C	291
C		C	292
	IF (DNXT.LE.ALV) GO TO 31	C	293
C		C	294
C		C	295
	EDN=(2.0*VNXT+EPS*ANXT)+DNXT-(2.0*PV+EPS*PA)	C	296
	IF (EDN.LT.0.0) EDN=0.0	C	297
	DNS=0.0	C	298
	IF (VNXT.GT.0.001) DNS=(VNXT/3.50)*(VNXT/2.0)	C	299
	PDE=EDN+DNS-26.0	C	300

	IF (PDE.LT.0.0) PDE=0.0	C 301
	PDM=DNXT-26.0	C 302
	IF (PDM.LT.0.0) PDM=0.0	C 303
C		C 304
C	IF VEHICLE IS CLOSE REDUCE SPEED	C 305
C		C 306
	IF (PDE.LE.PARM1) VEL3=OPTSPD*SQRT(PDM/PARM1)	C 307
C		C 308
C		C 309
18	CONTINUE	C 310
	IF (VEL3.LT.0.0) VEL3=0.0	C 311
C		C 312
C	CALCULATE VELOCITY DUE TO VEHICLE IN LEFT TURN ONLY LANE	C 313
C		C 314
	VEL4=OPTSPD	C 315
C		C 316
C	DETERMINE NUMBER OF LEFT TURN LANE	C 317
C		C 318
	NLL=1	C 319
	IF (ABD.EQ.2) NLL=NLA+1	C 320
C		C 321
C	CHECK IF ALREADY PAST END OF LEFT TURN ONLY LANE	C 322
C		C 323
	IF (PDL.LT.ALENLT(ABD)-ALV) GO TO 20	C 324
C		C 325
C	CHECK IF NECESSARY TO REDUCE SPEED	C 326
C		C 327
	PNILL=PLCIL(BSS,NLL)	C 328
	IF (PL.EQ.NLL) GO TO 20	C 329
	IF (PNILL.EQ.0) GO TO 20	C 330
	IF (ABD.EQ.1.AND.PL.GE.3) GO TO 20	C 331
	IF (ABD.EQ.2.AND.PL.GE.NLA+3) GO TO 20	C 332
	IF (PLRS.EQ.1) GO TO 19	C 333
	IF (CARDOP(PNILL,BSP5)+ALV.LE.ALENLT(ABD)) GO TO 20	C 334
C		C 335
C	DETERMINE STATUS OF VEHICLE IN LEFT TURN LANE	C 336
C		C 337
19	VNXT=CARDOP(PNILL,BSP3)	C 338
	ANXT=CARDOP(PNILL,BSP4)	C 339
	DNXT=PDL-CARDOP(PNILL,BSP5)	C 340
C		C 341
C	CHECK IF ACCIDENT HAS OCCURRED	C 342
C		C 343
	IF (DNXT.LT.ALV) GO TO 31	C 344
C		C 345
	EDN=(2.0*VNXT+EPS*ANXT)+DNXT-(2.0*PV+EPS*PA)	C 346
	IF (EDN.LT.0.0) EDN=0.0	C 347
	DNS=0.0	C 348
	IF (VNXT.GT.0.001) DNS=(VNXT/3.50)*(VNXT/2.0)	C 349
	PDE=EDN+DNS-4.0-ALV	C 350

	IF (PDE.LT.0.0) PDE=0.0	C 351
	PDM=DNXT-26.0	C 352
	IF (PDM.LT.0.0) PDM=0.0	C 353
C		C 354
C	IF VEHICLE IS CLOSE REDUCE SPEED	C 355
C		C 356
	IF (PDE.LE.PARM1) VEL4=OPTSPD*SQRT(PDM/PARM1)	C 357
C		C 358
C	CONTINUE	C 359
20	IF (VEL4.LT.0.0) VEL4=0.0	C 360
C		C 361
C	CALCULATE VELOCITY DUE TO VEHICLE IN RIGHT TURN ONLY LANE	C 362
C		C 363
	VEL5=OPTSPD	C 364
C		C 365
C	DETERMINE NUMBER OF RIGHT TURN ONLY LANE	C 366
C		C 367
	NRL=NLA	C 368
	IF (ABD.EQ.2) NRL=NLT	C 369
C		C 370
C	CHECK IF ALREADY PAST END OF RIGHT TURN LANE	C 371
C		C 372
	IF (PDL.LT.ALENRT(ABD)-ALV) GO TO 22	C 373
C		C 374
C	CHECK IF NECESSARY TO REDUCE SPEED	C 375
C		C 376
	PNIRL=PLCIL(BSS,NRL)	C 377
	IF (PL.EQ.NRL) GO TO 22	C 378
	IF (PNIRL.EQ.0) GO TO 22	C 379
	IF (ABD.EQ.1.AND.PL.LE.NLA-2) GO TO 22	C 380
	IF (ABD.EQ.2.AND.PL.LE.NLT-2) GO TO 22	C 381
	IF (PLRS.EQ.2) GO TO 21	C 382
	IF (CARDOP(PNIRL,BSP5)+ALV.LE.ALENRT(ABD)) GO TO 22	C 383
C		C 384
C	DETERMINE STATUS OF VEHICLE IN RIGHT TURN LANE	C 385
C		C 386
21	VNXT=CARDOP(PNIRL,BSP3)	C 387
	ANXT=CARDOP(PNIRL,BSP4)	C 388
	DNXT=PDL-CARDOP(PNIRL,BSP5)	C 389
C		C 390
C	CHECK IF ACCIDENT HAS OCCURRED	C 391
C		C 392
	IF (DNXT.LT.ALV) GO TO 31	C 393
C		C 394
C		C 395
	EDN=(2.0*VNXT+EPS*ANXT)+DNXT-(2.0*PV+EPS*PA)	C 396
	IF (EDN.LT.0.0) EDN=0.0	C 397
	DNS=0.0	C 398
	IF (VNXT.GT.0.001) DNS=(VNXT/3.50)*(VNXT/2.0)	C 399
		C 400

	PDE=EDN+DNS-4.0-ALV	C 401
	IF (PDE.LT.0.0) PDE=0.0	C 402
	PDM=DNXT-26.0	C 403
	IF (PDM.LT.0.0) PDM=0.0	C 404
C		C 405
C	IF VEHICLE IS CLOSE REDUCE SPEED	C 406
C		C 407
	IF (PDE.LE.PARM1) VEL5=OPTSPD*SQRT(PDM/PARM1)	C 408
		C 409
C		C 410
C	CONTINUE	C 411
22	IF (VEL5.LT.0.0) VEL5=0.0	C 412
		C 413
C	CALCULATE STATUS OF VEHICLE AT NEXT TIME-SLICE	C 414
C		C 415
C		C 416
C	CALCULATE FUTURE ACCELERATION AND VELOCITY	C 417
C		C 418
	VW=AMIN1(VEL1,VEL2,VEL3,VEL4,VEL5)	C 419
	IF (VW.GT.OPTSPD) VW=OPTSPD	C 420
	AW=(VW-PV)/EPS	C 421
	IF (AW.LT.ALL) AW=ALL	C 422
	IF (AW.GT.AUL) AW=AUL	C 423
	FA=AW	C 424
	FV=PV	C 425
	IF (ABS(FA).GT.0.001) FV=PV+FA*EPS	C 426
	IF (FV.LT.0.0) FV=0.0	C 427
C		C 428
C	CHECK IF VEHICLE HAS LEFT SIMULATION AREA	C 429
C		C 430
	DT=(PV+FV)/2.0*EPS	C 431
	IF (DT.GE.PDL) GO TO 27	C 432
C		C 433
C	CALCULATE FUTURE LANE NUMBER	C 434
C		C 435
	FL=PL	C 436
	IF (PLRS.EQ.1.AND.PDL-DT.LE.ALENLT(ABD)) FL=(ABD-1)*NLA+1	C 437
	IF (PLRS.EQ.2.AND.PDL-DT.LE.ALENRT(ABD)) FL=NLT-(NLT-NLA)*(2-ABD)	C 438
C		C 439
C	CALCULATE FUTURE LEFT-RIGHT-STRAIGHT CODE AND DISTANCE TO LIGHT	C 440
C		C 441
	FLRS=PLRS	C 442
	FDL=PDL-DT	C 443
C		C 444
C	CHECK IF VEHICLE HAS JUST ENTERED TIMED AREA	C 445
C		C 446
	IF (FDL.LT.ALENST.AND,TE(I).EQ.-1.0) TE(I)=TIME	C 447
		C 448
C		C 449
C	ADJUST POINTERS TO NEXT VEHICLE ETC.	C 450
C		C 450

	PPNT=PLCIL(NBSS,FL)	C	451
	IF (PPNT.EQ.0) GO TO 24	C	452
	IF (FDL.GE.CARDOP(PPNT,NBSP5)) GO TO 24	C	453
23	PNT=CARDOP(PPNT,NBS)	C	454
	IF (PNT.EQ.0) GO TO 25	C	455
	DLP=CARDOP(PNT,NBSP5)	C	456
	IF (FDL.GT.DLP) GO TO 25	C	457
	PPNT=PNT	C	458
	GO TO 23	C	459
C		C	460
C	THIS (SO FAR) IS THE LAST VEHICLE IN THIS LANE	C	461
C		C	462
24	CARDOP(I,NBS)=PLCIL(NBSS,FL)	C	463
	PLCIL(NBSS,FL)=I	C	464
	GO TO 26	C	465
C		C	466
C	THIS IS NOT THE LAST VEHICLE IN THIS LANE	C	467
C		C	468
25	CARDOP(PPNT,NBS)=I	C	469
	CARDOP(I,NBS)=PNT	C	470
26	CONTINUE	C	471
	GO TO 30	C	472
C		C	473
C	REMOVE VEHICLE FROM SIMULATION	C	474
C		C	475
27	PFV(I)=PFFV	C	476
	PFFV=I	C	477
	IF (PLCIL(NBSS,PL).EQ.I) PLCIL(NBSS,PL)=0	C	478
C		C	479
C	RECORD TIME TAKEN TO TRAVERSE LEG	C	480
C		C	481
	IF (TE(I).EQ.-1.0) GO TO 28	C	482
	X=TIME-TE(I)	C	483
	T(NALT)=X	C	484
	NALT=NALT+1	C	485
C		C	486
C	CHECK IF SIMULATION COMPLETE	C	487
C		C	488
	IF (NALT.GT.NTC) RETURN	C	489
C		C	490
C		C	491
28	CONTINUE	C	492
C		C	493
C	CLEAR RECORD FOR THIS VEHICLE	C	494
C		C	495
	TE(I)=-1.0	C	496
	DO 29 J1=1,12	C	497
	CARDOP(I,J1)=0.0	C	498
29	CONTINUE	C	499
	GO TO 36	C	500

C		C	501
C	RECORD STATUS OF VEHICLE AT NEXT TIME-SLICE	C	502
C		C	503
30	CONTINUE	C	504
	CARDOP(I,NBSP1)=FL	C	505
	CARDOP(I,NBSP2)=FLRS	C	506
	CARDOP(I,NBSP3)=FV	C	507
	CARDOP(I,NBSP4)=FA	C	508
	CARDOP(I,NBSP5)=FDL	C	509
	GO TO 36	C	510
C		C	511
C	REMOVE VEHICLE FROM SIMULATION (ACCIDENT)	C	512
C		C	513
31	CONTINUE	C	514
	I1=I	C	515
	NA=NA+1	C	516
C		C	517
C	CHECK IF LAST VEHICLE IN THIS LANE	C	518
C		C	519
	PL=CARDOP(I1,BSP1)	C	520
	IBP=PLCIL(BSS,PL)	C	521
	IF (IBP.EQ.I1) GO TO 33	C	522
C		C	523
C	FIND VEHICLE IMMEDIATELY BEHIND ONE BEING REMOVED	C	524
C		C	525
32	IBP=CARDOP(IBP,BS)	C	526
	IF (IBP.NE.I1) GO TO 32	C	527
	CARDOP(IBP,BS)=CARDOP(I1,BS)	C	528
	GO TO 34	C	529
C		C	530
C	ADJUST POINTERS AND CLEAR STATUS ARRAY	C	531
C		C	532
33	PLCIL(BSS,PL)=CARDOP(I1,BS)	C	533
34	PFV(I1)=PFFV	C	534
	PFFV=I1	C	535
C		C	536
C	CLEAR RECORD FOR THIS VEHICLE	C	537
C		C	538
	TE(I1)=-1.0	C	539
	DO 35 I5=1,12	C	540
	CARDOP(I1,I5)=0.0	C	541
35	CONTINUE	C	542
36	CONTINUE	C	543
	NLTM1=NLT-1	C	544
C		C	545
C	CHECK TO SEE IF ANY VEHICLES ARE TO BE ADDED TO SIMULATION	C	546
C		C	547
	DO 43 I=2,NLTM1	C	548
C		C	549
C	CHECK IF NO VEHICLE AVAILABLE	C	550

C	IF (PFFV.EQ.0) GO TO 8	C 551
C		C 552
C	CHECK IF ATTEMPTING TO ADD VEHICLES DIRECTLY TO LEFT TURN OR	C 553
C	RIGHT TURN ONLY LANES	C 554
C		C 555
C	IF (I.EQ.NLA.OR.I.EQ.NLA+1) GO TO 43	C 556
	IJF=1	C 557
C		C 558
C	DETERMINE NUMBER OF VEHICLES ENTERING LANE	C 559
C		C 560
	IF (I.GT.NLA) IJF=2	C 561
	VEL=VICL(IJF)	C 562
	IF (I.EQ.2.OR.I.EQ.NLA+1) VEL=VILL(IJF)	C 563
	IF (I.EQ.NLA-1.OR.I.EQ.NLA-1) VEL=VIRL(IJF)	C 564
C		C 565
C	CHECK IF NOT TIME TO ADD ANOTHER VEHICLE YET	C 566
C		C 567
	Z=3600/VEL+TVEL(I)	C 568
	IF (Z.GT.TIME) GO TO 43	C 569
	TVEL(I)=TIME	C 570
C		C 571
C	DETERMINE NUMBER OF VEHICLE TO BE ADDED	C 572
C		C 573
	IPNT=PFFV	C 574
	PFFV=PFV(IPNT)	C 575
C		C 576
C	DETERMINE PROBABILITY THAT VEHICLE ENTERS BY TURNING FROM SIDE	C 577
C	STREET	C 578
C		C 579
	IF (ASLIT(I).EQ.1) GO TO 37	C 580
	GT=CSL(I,2)-CSL(I,1)	C 581
	IF (CSL(I,1).GT.CSL(I,2)) GT=60-CSL(I,1)+CSL(I,2)	C 582
	PROB=(1.0-VES(IJF)/VTOT(IJF))*(NSIC/GT)	C 583
	CALL RANDU (IX,IX,YFL)	C 584
C		C 585
C	CHECK IF VEHICLE ENTERS BY TURNING FROM SIDE STREET	C 586
C		C 587
	IF (YFL.GE.PROB) GO TO 37	C 588
C		C 589
C	VEHICLE TO BE ADDED UPSTREAM	C 590
C		C 591
	FV=0.0	C 592
	FA=0.0	C 593
	FDL=ALENST-26.0	C 594
	TE(IPNT)=TIME	C 595
	GO TO 38	C 596
C		C 597
C	VEHICLE TURNED INTO SIMULATION AREA FROM STREET	C 598
C		C 599
		C 600

37	FV=OPTSPD	C 601
	FDL=SDTL	C 602
	FA=0.0	C 603
	TE(IPNT)=-1.0	C 604
38	CONTINUE	C 605
C		C 606
C	DETERMINE IF VEHICLE IS TO TURN LEFT OR RIGHT	C 607
C		C 608
	FL=I	C 609
	PROBR=0.0	C 610
	PROBL=0.0	C 611
	IF (I.EQ.2.OR.I.EQ.NLA+1) PROBL=VTL(IJF)/VILL(IJF)	C 612
	IF (I.EQ.NLA-1.OR.I.EQ.NLT-1) PROBR=VTR(IJF)/VIRL(IJF)	C 613
	ILRS=3	C 614
	CALL RANDU (IX,IX,YFL)	C 615
	IF (YFL.LE.PROBL) ILRS=1	C 616
	IF (YFL.GE.1.0-PROBR) ILRS=2	C 617
	FLRS=ILRS	C 618
C		C 619
C	ADJUST POINTER TO VEHICLE AHEAD	C 620
C		C 621
	PPNT=PLCIL(NBSS,FL)	C 622
	IF (PPNT.EQ.0) GO TO 40	C 623
	IF (FDL.GE.CARDOP(PPNT,NBSP5)) GO TO 40	C 624
39	PNT=CARDOP(PPNT,NBS)	C 625
	IF (PNT.EQ.0) GO TO 41	C 626
	DLP=CARDOP(PNT,NBSP5)	C 627
	IF (FDL.GT.DLP) GO TO 41	C 628
	PPNT=PNT	C 629
	GO TO 39	C 630
C		C 631
C	VEHICLE ADDED IS LAST IN LANE	C 632
C		C 633
40	CARDOP(IPNT,NBS)=PLCIL(NBSS,FL)	C 634
	PLCIL(NBSS,FL)=IPNT	C 635
	GO TO 42	C 636
C		C 637
C	VEHICLE ADDED IS NOT THE LAST IN THE LANE	C 638
C		C 639
41	CARDOP(PPNT,NBS)=IPNT	C 640
	CARDOP(IPNT,NBS)=PNT	C 641
C		C 642
C	RECORD STATUS OF NEW VEHICLE	C 643
C		C 644
42	CONTINUE	C 645
	CARDOP(IPNT,NBSP1)=FL	C 646
	CARDOP(IPNT,NBSP2)=FLRS	C 647
	CARDOP(IPNT,NBSP3)=FV	C 648
	CARDOP(IPNT,NBSP4)=FA	C 649
	CARDOP(IPNT,NBSP5)=FDL	C 650



APPENDIX C

DOCUMENTATION OF SUBROUTINES USED IN PHASE III

C	SUBROUTINE TREE	A	1
C		A	2
C		A	3
C	THIS SUBROUTINE IS COPYRIGHT 1974 BY ROBERT J. TAYLOR	A	4
C	VICTORIA, BRITISH COLUMBIA	A	5
C	CANADA	A	6
C		A	7
C		A	8
C		A	9
C	PURPOSE: TO CHOOSE A SPANNING TREE OF THE GRAPH ASSOCIATED	A	10
C	WITH A GIVEN STREET SYSTEM AND TO SET THE OFFSETS OF	A	11
C	THE SIGNALS SO THAT THE DELAY IS MINIMAL.	A	12
C		A	13
C		A	14
C	METHOD: RANDOM WEIGHTS BETWEEN 2 AND 3 ARE ASSIGNED TO ALL	A	15
C	EDGES OF THE GRAPH; EDGES THAT ARE NOT IN THE GRAPH	A	16
C	ARE ASSIGNED A LARGE WEIGHT; EDGES THAT ARE TO	A	18
C	BE INCLUDED IN THE TREE ARE ASSIGNED A WEIGHT	A	19
C	OF 1. USING PRIM'S ALGORITHM A MINIMAL SPANNING	A	20
C	TREE IS FORMED. THE OFFSETS OF THE SIGNALS ARE	A	21
C	CALCULATED AS DESCRIBED IN CHAPTER 5 OF THE COVERING	A	22
C	WORK.	A	23
C		A	24
C		A	25
C	VARIABLES:	A	26
C		A	27
C		A	28
C	Q(I,J) (I=1,2,....,100); (J=1,2,....,100)	A	29
C	WEIGHT OF EDGE FROM INTERSECTION I TO J.	A	30
C		A	31
C	A(K) (K=1,2,....,100)	A	32
C	A VALUE FOR KTH LEG.	A	33
C		A	34
C	B(K) (K=1,2,....,100)	A	35
C	B VALUE FOR KTH LEG.	A	36
C		A	37
C	C(K) (K=1,2,....,100)	A	38
C	C VALUE FOR KTH LEG.	A	39
C		A	40
C	CGIJ(K) (K=1,2,....,100)	A	41
C	NUMBER OF VEHICLES GOING FROM I TO J.	A	42
C		A	43
C	CGJI(K) (K=1,2,....,100)	A	44
C	NUMBER OF VEHICLES GOING FROM J TO I.	A	45
C		A	46
C	EI(K) (K=1,2,....,100)	A	47
C	NUMBER OF THE FIRST SIGNAL OF LEG K.	A	48
C		A	49
C	EJ(K) (K=1,2,....,100)	A	50
C	NUMBER OF THE SECOND SIGNAL OF LEG K.	A	51

C			A	52
C	I	DO-LOOP PARAMETER	A	53
C	IEI	NUMBER OF THE FIRST SIGNAL OF LEG K.	A	54
C	IMIN	NUMBER OF INTERSECTION ON ONE END OF EDGE TO BE ADDED.	A	55
C	INTI	NUMBER OF SIGNAL I.	A	56
C	INTJ	NUMBER OF SIGNAL J.	A	57
C	IX	RANDOM NUMBER GENERATOR SEED.	A	58
C	J	DO-LOOP PARAMETER	A	59
C	JEJ	NUMBER OF THE SECOND SIGNAL OF LEG K.	A	60
C	JMIN	NUMBER OF INTERSECTION ON ONE END OF EDGE TO BE ADDED.	A	61
C	K	DO-LOOP PARAMETER.	A	62
C	N	NUMBER OF SIGNALS (VERTICES)	A	63
C	NL	NUMBER OF LEGS (EDGES)	A	64
C	NM1	N - 1	A	65
C	NSIC	NUMBER OF SECONDS IN THE CYCLE.	A	66
C	OFF(I)	(I=1,2,....,100) OFFSET OF SIGNAL I.	A	67
C	OT	OPTIMAL RELATIVE OFFSET FOR THE LEG.	A	68
C	RMIN	MINIMUM WEIGHT OF THE EDGES.	A	69
C	S	DO-LOOP PARAMETER.	A	70
C	TAKEN(J)	(J=1,2,....,100) INDICATES IF JTH INTERSECTION INCLUDED IN TREE 0: NO 1: YES	A	71
C	XIETG(K)	(K=1,2,....,100) TIME THAT SIGNAL I TURNS GREEN FOR VEHICLES ENTERING LEG K.	A	72
C	XILTG(K)	(K=1,2,....,100)	A	73
C			A	74
C			A	75
C			A	76
C			A	77
C			A	78
C			A	79
C			A	80
C			A	81
C			A	82
C			A	83
C			A	84
C			A	85
C			A	86
C			A	87
C			A	88
C			A	89
C			A	90
C			A	91
C			A	92
C			A	93
C			A	94
C			A	95
C			A	96
C			A	97
C			A	98
C			A	99
C			A	100
C			A	101



C	COMMON N,NL,EI,EJ,A,B,C,CGIJ,CGJI,NSIC	A 153
	COMMON XIETG,XILTG,XJETG,XJLTG	A 154
C		A 155
C		A 156
	DIMENSION Q(100,100), OFF(100), A(100), B(100), C(100)	A 157
	DIMENSION CGJI(100), CGIJ(100)	A 158
	DIMENSION XIETG(100), XILTG(100), XJETG(100), XJLTG(100)	A 159
	INTEGER EI(100),EJ(100),TAKEN(100),S	A 160
C		A 161
C		A 162
C	INITIALIZATION SECTION	A 163
C		A 164
C	RMIN=100000.0	A 165
	IX=654231753	A 166
C		A 167
C	SET THE WEIGHTS TO A LARGE VALUE	A 168
C		A 169
	DO 2 I=1,N	A 170
	DO 1 J=1,N	A 171
	Q(I,J)=RMIN	A 172
1	CONTINUE	A 173
2	CONTINUE	A 174
C		A 175
C		A 176
C	FOR THOSE EDGES THAT EXIST SET THE WEIGHTS TO BETWEEN 2 AND 3	A 177
		A 178
	DO 3 K=1,NL	A 179
	IEI=EI(K)	A 180
	JEJ=EJ(K)	A 181
	CALL RANDU (IX,IX,YFL)	A 182
	Q(IEI,JEJ)=YFL+2.0	A 183
	Q(JEJ,IEI)=YFL+2.0	A 184
3	CONTINUE	A 185
C		A 186
C	READ IN AN EDGE THAT SHOULD BE INCLUDED IN THE TREE	A 187
C		A 188
4	READ (5,15,END=16) INTI,JNTI	A 189
C		A 190
C	ASSIGN IT WEIGHT 1	A 191
C		A 192
	Q(INTI,JNTI)=1.0	A 193
	Q(JNTI,INTI)=1.0	A 194
	GO TO 4	A 195
C		A 196
C	SET TAKEN TO INDICATE THAT NONE OF THE INTERSECTIONS HAVE BEEN	A 197
C	INCLUDED IN THE TREE AS YET. SET ALL THE OFFSETS TO 0.	A 198
C		A 199
16	DO 5 I=1,N	A 200
	TAKEN(I)=0	A 201
	OFF(I)=0.0	A 202

5	CONTINUE	A 203
C		A 204
C	CHOOSE THE FIRST INTERSECTION AND GIVE IT OFFSET EQUAL TO 0	A 205
C		A 206
	TAKEN(1)=1	A 207
	OFF(1)=0.0	A 208
C		A 209
C	FOR EACH OF THE EDGES TO BE INCLUDED IN THE TREE	A 210
C		A 211
	NM1=N-1	A 212
	DO 10 S=1,NM1	A 213
C		A 214
C	FIND THE ONE THAT IS CLOSEST	A 215
C		A 216
	RMIN=100000.0	A 217
	DO 7 I=1,N	A 218
	IF (TAKEN(I).EQ.0) GO TO 7	A 219
	DO 6 J=1,N	A 220
	IF (TAKEN(J).EQ.1.OR.Q(I,J).GE.RMIN) GO TO 6	A 221
	IMIN=I	A 222
	JMIN=J	A 223
	RMIN=Q(I,J)	A 224
6	CONTINUE	A 225
7	CONTINUE	A 226
C		A 227
C	DETERMINE THE CORRESPONDING EDGE	A 228
C		A 229
	DO 8 K=1,NL	A 230
	IEI=EI(K)	A 231
	JEJ=EJ(K)	A 232
	IF (IMIN.EQ.IEI.AND.JMIN.EQ.JEJ) GO TO 9	A 233
	IF (JMIN.EQ.IEI.AND.IMIN.EQ.JEJ) GO TO 9	A 234
8	CONTINUE	A 235
9	CONTINUE	A 236
C		A 237
C	WRITE OUT THAT THE EDGE WAS INCLUDED	A 238
C		A 239
	I=IEI	A 240
	J=JEJ	A 241
	WRITE (6,12) I,J	A 242
C		A 243
C	DETERMINE THE OPTIMAL OFFSET FOR THE LEG	A 244
C		A 245
C	OT=45.0+B(K)	A 246
C		A 247
C	DETERMINE THE OFFSET OF THE INTERSECTION JUST ADDED	A 248
C		A 249
	IF (TAKEN(IEI).EQ.1.AND.CGIJ(K).GE.CGJI(K)) OFF(J)=OT+XIETG(K)+OFF	A 250
	1(I)-XJLTG(K)	A 251
	IF (TAKEN(IEI).EQ.1.AND.CGIJ(K).LT.CGJI(K)) OFF(J)=-OT-XJETG(K)+DF	A 252

	1F(I)+XILTG(K)	A 253
	IF (TAKEN(JEJ).EQ.1.AND.CGIJ(K).GE.CGJI(K)) OFF(I)=-OT-XIETG(K)+OF	A 254
	1F(J)+XJLTG(K)	A 255
	IF (TAKEN(JEJ).EQ.1.AND.CGIJ(K).LT.CGJI(K)) OFF(I)=OT+XJETG(K)+OFF	A 256
	1(J)-XILTG(K)	A 257
C		A 258
C	INDICATE THAT THE INTERSECTION IS NOW INCLUDED IN THE TREE	A 259
C		A 260
	TAKEN(JMIN)=1	A 261
10	CONTINUE	A 262
	WRITE (6,14)	A 263
C		A 264
C	OUTPUT THE OFFSETS AND THE % OFFSETS	A 265
C		A 266
	DO 11 I=1,N	A 267
	OFF(I)=AMOD(OFF(I)+10.0*NSIC,NSIC*1.0)	A 268
	Z4=OFF(I)/NSIC*100.0	A 269
	WRITE (6,13) I,OFF(I),Z4	A 270
11	CONTINUE	A 271
C		A 272
C		A 273
	RETURN	A 274
C		A 275
C		A 276
C		A 277
12	FORMAT (1X,'EDGE ',I6,'-',I6,' OPTIMIZED')	A 278
13	FORMAT (I6,5X,F8.1,' SECONDS ',F8.1,' %')	A 279
14	FORMAT (1H1,///,' OPTIMAL OFFSETS',///,' INTERSECTION      OFFSET'//)	A 280
15	FORMAT (I3,2X,I3)	A 281
	END	A 282-

VITA

Surname: TAYLOR Given Names: ROBERT JOHN

Place of Birth: REGINA, SASKATCHEWAN Date of Birth: April 13, 1950

Educational Institutions Attended, with Dates of Entering and Leaving:

UNIVERSITY OF LETHBRIDGE, LETHBRIDGE, ALBERTA 1968 to 1971

UNIVERSITY OF VICTORIA, VICTORIA, B.C. 1972 to 1975

\_\_\_\_\_ to \_\_\_\_\_

\_\_\_\_\_ to \_\_\_\_\_

Degrees, Diplomas, Etc., Awarded, with Dates and Names of Institutions:

Bachelor of Arts and 1971 University of Lethbridge, Lethbridge

Science \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Honors and Awards:

University of Victoria Fellowship, 1972/73

N.R.C. Grants, 1973/1974

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Publications:

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IMPROVING TRAFFIC CONTROL THROUGH SIMULATION AND OPERATIONS RESEARCH

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Author



*Signature*

ROBERT JOHN TAYLOR

*Name*

March 12, 1975

*Date*