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Stackelberg–Nash game approach for price-based demand response in retail electricity trading

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ABSTRACT

This paper studies the price-based demand response problem in a deregulated retail electricity trading, aiming to coordinate the energy consumption behavior of end-users under dynamic retail prices. The challenge here is that in addition to the hierarchical decision-making process between utility company and end-users considered in existing works, the non-cooperative and competitive interdependence among end-users cannot be ignored. To address this issue, we first construct a novel Stackelberg–Nash game, in which the Stackelberg game is used to capture the hierarchical decision-making process between utility company and end-users, while the Nash game is dedicated to describing the interdependence among end-users. Then the existence and uniqueness of the Stackelberg–Nash equilibrium is provided along with theoretical analysis. On the basis of the analysis of equilibrium, we propose a distributed iterative algorithm with an adaptive step size, which is benchmarked with a fixed step-size algorithm. The comparison results on a real-life residential retail electricity market show that our proposed algorithm has better performance in terms of effectiveness and scalability.

1. Introduction

With the rapid development of information and communication technologies (ICTs), the traditional power system is undergoing a dramatic evolutionary transition to the smart grid (SG). One of the main characteristics of SG is the introduction of two-way information and energy flows [1], which enables the flexible energy usage patterns and the active participation of loads. As a result, demand response (DR) has become a research hotspot in the field of SG, which provides a promising means to improve the stability, reliability, and efficiency of power grids [2,3]. Since the electricity price plays a crucial role in market scheduling, the price-based DR becomes the focus of the research in DR, the purpose of which is to promote end-users to change their energy usage patterns in response to the dynamic electricity price, such as time-of-use pricing [4] and real-time pricing (RTP) [5], so as to achieve the cost reduction [6], peak load shift [7] or other interests.

Recently, there have been many studies on price-based DR, including the coordination of energy consumption in grid-connected or islanded microgrids [8], demand management for a community of energy buildings [9], differential reliability demand of power system [10], and the coordination of charging/discharging in electric vehicle fleets [11], mainly from a social or individual perspective. On the one hand, from a social viewpoint, one hopes to maximize the revenues of all market participants or maintain a smoother load profile. For example, in [12], the authors study the DR management with the aim of maximizing the social welfare, that is, the comfort (or utility) of users minus the energy cost of utility company (UC). In addition to focusing on the social welfare/benefit, some other works expect to smooth the load profile to avoid the grid fluctuations. For instance, the work in [13] aims to reduce the peak-to-average ratio on demand side to make the load profile of multi-microgrid system as smooth as possible. On the other hand, from an individual perspective, UC and end-users are selfish. That is, all stakeholders seek benefits for themselves, resulting in the electricity price changing with the bidding actions of all market participants [14]. In [15], the authors study a deterministic DR problem under a RTP mechanism, aiming to minimize the energy cost of customers. Some other studies focus on designing DR programs to maximize the profit of UC, see, for example, [16,17].

However, all of the above works focus on modeling the problem from an optimization perspective and assume that all participants

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make decision simultaneously. In other words, the works mentioned above ignore the structural characteristics of hierarchical/sequential decision-making in electricity trading. Considering that game theory is a powerful tool used to analyze the strategic interactions among multiple decision makers [18], an increasing attention is paid to model the DR problems based on game theory. For example, in [19], the DR problem is formulated as two non-cooperative games to maximize the UCs' profit and customers' payoff, respectively. However, the hierarchical/sequential decision-making process between UCs and customers is ignored. Then, Stackelberg game has received further attention as it a typical and specialized model for analyzing the hierarchical decision-making process among multiple decision makers. Some representative works are presented here [20–23]. In [20,21], the interactions between retailer (or SG) and EVs are modeled as a Stackelberg game. Specifically, the leader (i.e., retailer or SG) decides the price first to optimize its revenue, then the followers (i.e., plug-in electric vehicle groups (PEVGs)) determine their charging strategies according to the received price information to maximize their payoffs. Followed by [20], the authors in [17,22] adopt the Stackelberg game to model the DR problem in a network of multiple UCs and end-users. A similar framework is also designed in [23], but it models the two-layer energy management problem in a multi-energy industrial park, aiming at maximizing the profits of distributed generator operators and industrial users.

Nevertheless, it is worth noting that the most existing Stackelberg game-based studies, including above-mentioned works, only focus on the hierarchical decision-making process between UC/retailer and end-users, while ignoring the potential impact and interaction among end-users. In fact, in retail electricity trading, the non-cooperative and competitive interdependence among consumers/end-users is usually non-negligible, otherwise the power supply may be insufficient or the peak valley difference of the load may increase due to the competition between end-users and the differentiation in load demand, thus cannot meet the needs of end-users. For example, although the work [20] provides a comprehensive analytical framework for capturing the interactions between a SG and a number of PEVGs, the optimization problems of lower-level PEVGs are mutually independent when known the upper-level SG's strategy, which is unreasonable as the strategies of all end-users are not completely independent. Moreover, the decision-making of PEVGs cannot in turn guide the timely adjustment of electricity prices, thereby failing to stimulate the enthusiasm of EVs to participate in grid dispatch. The same issue appears in [23,24]. Some other works, like [25] only focus on the communication between active loads but ignoring the hierarchical interactive architecture between different participants. In addition, to approximately characterize the impact of environmental changes on individual behavior, the concept of environmental cost is first proposed in [26]. Then, researchers in [27,28] further investigate environmental cost-based model for economic dispatch of combined heat and power (CHP) generation units.

Motivated by the above discussions, this paper investigates the price-based DR in a residential retail electricity trading, in which both the hierarchical decision-making process between UC and end-users, as well as the potential impact and interaction among end-users are considered. Compared with the existing works, the main contributions of this paper are threefold:

(1) This paper investigates the hierarchical price-based DR, in which the non-cooperative and competitive interdependence among end-users is considered simultaneously. Different from the existing works [23,24] that the energy consumption of lower-level end-users are mutually independent when given the retail price of upper-level UC. An environmental impact cost is introduced in this paper to show the interdependence among the energy consumption of all end-users.

(2) Compared with the existing Stackelberg game-based DR models [20–22] which only focus on the hierarchical decision-making between UC and end-users, we construct a novel Stackelberg–Nash game (SNG) model to capture the strategic interactions among all market participants. Specifically, Stackelberg game is dedicated to depicting the hierarchical decision-making process between UC and end-users, while Nash game is used to describe the non-cooperative and competitive interdependence among end-users.

(3) By utilizing the backward induction, the existence and uniqueness of Stackelberg–Nash equilibrium (SNE) is proved with rigorously theoretical analysis. Moreover, to prevent the end-users' private information being disclosed to UC, a distributed iterative algorithm with an adaptive step size is developed to seek the unique SNE, which shows superior performance in terms of convergence than the benchmarked one with fixed step size [24].

The rest of this paper is arranged as follows. Section 2 introduces system model and Section 3 reformulates the price-based DR problem as a SNG. A distributed iterative algorithm is proposed in Section 4. Section 5 presents the simulation results and Section 6 draws the conclusions.

2. System model

In this section, we introduce the system model including the basic framework of the system and the mathematical models.

2.1. Basic framework of the system

Generally speaking, the power system consists of generation, distribution, and consumption, which correspond to three layers, namely the generators, distribution system operators (i.e., aggregators/retails or UCs), and end-users (see Fig. 1). This paper mainly focuses on the residential retail electricity trading (see the shaded area in Fig. 1), which spans the last two layers, i.e., upper-level UC and lower-level end-users. Note that there is a two-way communication flow between UC and end-users. To be specific, the UC broadcasts the dynamic retail price information to end-users while the end-users deliver the energy consumption information to UC. Since the location of residential users is often fixed, resulting in their market value being almost the same, thus here we assume that all end-users receive the same retail price at the same time. In a traditional *regulated retail market*, all market participants have no ability to influence the retail price, limiting the flexibility of the market. By contrast, in a *deregulated retail market*, the energy usage patterns of end-users and the retail price of UC are mutually influenced. Consequently, the aim of price-based DR in a deregulated retail market is to coordinate the energy consumption behavior of a finite set $\mathcal{N} = \{1, 2, \dots, N\}$ of residential end-users within time period $\mathcal{T} = \{1, 2, \dots, T\}$ in response to dynamic retail prices, thereby maximizing the revenues of all market participants.

2.2. Mathematical models

2.2.1. Utility company model

As shown in Fig. 1, UC acts as a mediator between end-users and power producers. On one hand, UC takes an active participation in wholesale market and purchases electricity from power producers. On the other hand, UC sells the purchased electricity to end-users in a retail market. Therefore, the goal of UC is to maximize the revenue by trading in the wholesale and retail markets. Specifically, the revenue of UC at time slot t is defined as [22]

$$U_{uc,t} = r_t \sum_{i=1}^{N} P_{i,t} - \omega_t \sum_{i=1}^{N} P_{i,t}, \tag{1}$$

where r_t and ω_t are the retail and wholesale prices at time slot t, $P_{i,t}$ is the energy consumption of end-user i at time slot t. Since this paper mainly focuses on the operation of retail market, the wholesale prices set by the grid operator (GO) are assumed to be prior knowledge. Then

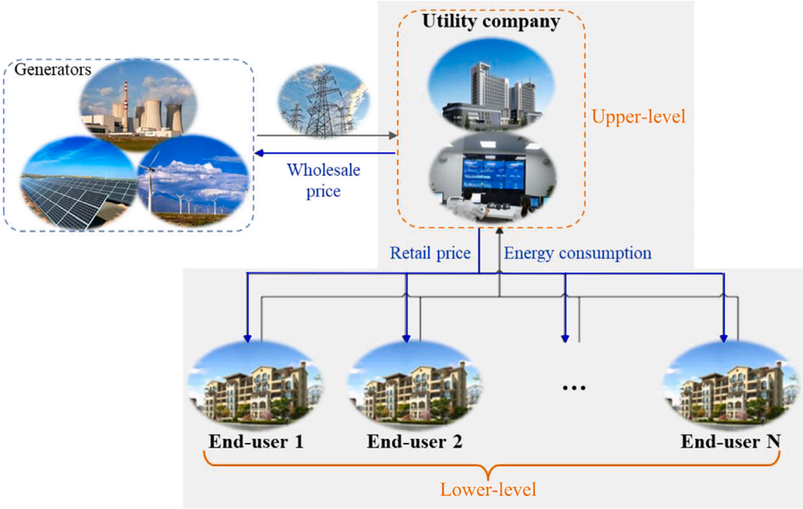

Fig. 1. General framework of power system including a retail electricity trading in residential area.

the objective of UC is formulated as the following optimization problem

$$\begin{cases} \max\limits_{\{r_t, t\in\mathcal{T}\}} \ U_{uc} = \sum_{t=1}^{T} U_{uc,t}, \\ \text{s.t.} \quad r_t^{min} \le r_t \le r_t^{max}, \ \forall t \in \mathcal{T}, \end{cases} \tag{2}$$

where r_t^{min} and r_t^{max} are the allowable lower and upper bounds of retail price at time slot t. The intuition behind the constraint is to guarantee the UC is profitable or at least break-even. Then, the feasible strategy set of UC is defined as

$$\Omega_{uc} = \left\{ r = (r_1, r_2, \dots, r_T) | \omega_t \le r_t \le r_t^{max}, \forall t \in \mathcal{T} \right\}. \tag{3}$$

2.2.2. End-users model

In practice, the end-users always expect to improve their profits as much as possible, which inevitably increases the amount of consumed energy. That is to say, the end-users prefer to improve their satisfaction by consuming more power energy. To this end, we define a satisfaction function which is increasing w.r.t. the consumed energy while decreasing w.r.t. the marginal profit and takes the following form [29]:

$$\phi_{i,t}(P_{i,t}) = \alpha_{i,t} P_{i,t} - \frac{\beta_i}{2} P_{i,t}^2, \ \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T}, \tag{4}$$

where $\alpha_{i,t}$ and β_i are two positive user-dependent parameters. Specifically, $\alpha_{i,t}$ is the marginal profit indicating that end-user i shall gain/lose profit when increasing/decreasing a unit quantity of energy consumption at time slot t. β_i is the sensitivity index reflecting the end-users' attitude towards the changes in energy consumption. For example, a larger β_i shows that the end-user i holds a more conservative attitude towards energy consumption, and vice versa. Moreover, the concept of environmental cost is first proposed in [26], in which the authors introduced a parameter to approximately characterize the impact of environmental changes on individual behavior. Then, researchers in [27, 28] further investigate environmental cost-based model for economic dispatch of combined heat and power (CHP) generation units. Inspired by the existing works, an environmental impact cost with the following form

$$\psi_{i,t}\Big(\sum_{j\in\mathcal{N}\setminus\{i\}} P_{j,t}\Big) = \gamma_{i,t}\Big(\sum_{j\in\mathcal{N}\setminus\{i\}} P_{j,t}\Big)^2, \tag{5}$$

is introduced to indicate the impact of the total energy consumption of all other end-users on the revenue of end-user i, where $\gamma_{i,t}$ is an impact factor. The intuition behind the environmental impact cost is that the energy consumed by other end-users has direct impact on the behavior of end-user i. Note that the condition under which this model can be applied precisely is that all end-users are active in retail electricity market and can interact with each other. Then, the revenue of end-user i at time slot t is defined as

$$\begin{aligned} U_{i,t} = & \phi_{i,t}(P_{i,t}) - r_t P_{i,t} \\ & - \psi_{i,t}\Big(\sum_{j\in\mathcal{N}\setminus\{i\}} P_{j,t}\Big), \ \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T}, \end{aligned} \tag{6}$$

where the first term represents the satisfaction gain of end-user i at time slot t; the second term shows the electricity cost of end-user i for purchasing the needed energy from UC; and the last term is the environmental impact cost of end-user i caused by all other end-users. Therefore, for each end-user $i \in \mathcal{N}$, he needs to solve the following optimization problem to determine the optimal energy consumption strategy

$$\begin{cases} \max\limits_{\{P_{i,t}, t\in\mathcal{T}\}} \ U_i = \sum_{t=1}^{T} U_{i,t}, \\ \text{s.t.} \quad P_i^{min} \le P_{i,t} \le P_i^{max}, \ \forall t \in \mathcal{T}, \\ \qquad \sum_{i=1}^{N} P_{i,t} \le E_t, \ \forall t \in \mathcal{T}. \end{cases} \tag{7}$$

The first constraint indicates that the energy consumption of each end-user is bounded by the minimum energy consumption P_i^{min} and maximum energy consumption P_i^{max}. The second one shows that at any time $t \in \mathcal{T}$, the total energy consumption of all end-users cannot exceed the available energy E_t from UC. Here note that the two constraints in (7) are power flow constraints and the model does not involve reactive power and other network constraints. The main reasons are twofold: (1) It can be obviously seen from Fig. 1 that we mainly focus on the price-based demand response in retail electricity trading, in which the price refers to the retail price of a residential area rather than the locational marginal price (LMP).[1] (2) Considering that household electricity is only metered with active power and not reactive power, this work focuses only on the constraints related to active power.

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[1] The locational marginal price is defined as the cost of serving an additional unit of power demand at network nodes [30], so LMP is usually tied to the power network.

Without loss of generality, the feasible strategy set of end-user i is defined by

$$\Omega_i = \left\{ P_i = (P_{i,1}, P_{i,2}, \dots, P_{i,T}) \mid P_i^{min} \le P_{i,t} \le P_i^{max}, \sum_{i=1}^{N} P_{i,t} \le E_t,\ \forall t \in \mathcal{T} \right\}. \tag{8}$$

Remark 1. In most existing works [20–22,24], the optimization problems of lower-level end-users are mutually independent when known the upper-level UC's strategy. For example, in [24], when leader's strategy (i.e., incentives offered by GO and service provider) is given, the optimization objective and feasible strategy set of each customer only depend on its own strategy, which cannot reflect the non-cooperative and competitive interdependence among end-uses. By contrast, in this paper, as shown in (5) and the second constraint in Eq. (7), when knowing the retail price r of the upper-level UC, the objective and feasible strategy set of each lower-level end-user are still coupled by the energy consumption behavior of all end-users. Thus, this work effectively models the coupling and interdependence between the lower-level end-users.

It is worth noting that since the objective and feasible strategy set of each end-user depends on the strategies of all other end-users, the system model is a dynamic model and the optimization problems of all end-users cannot be directly decoupled into several independent problems to solve. Additionally, the above mathematical models cannot visually show the hierarchical decision-making relationship between UC and end-users. To address these issues, we next propose a game framework to characterize the interactive relationships among all participants.

3. Stackelberg-Nash game formulation and analysis

This section first constructs a SNG framework to model the interaction among all market participants and then analyzes its corresponding solution concept.

In the residential retail market, UC acts as the leader and N end-users are regarded as the followers. Then a *one-leader, N-follower SNG* is formulated and represented as

$$\mathcal{G} = \left\{ \{UC \cup \mathcal{N}\}, \{\Omega_{uc} \cup \{\Omega_i\}_{i\in\mathcal{N}}\}, \{U_{uc} \cup \{U_i\}_{i\in\mathcal{N}}\} \right\}, \tag{9}$$

in which each component is described as follows:

- *Players*: UC and end-users[2] in set $\mathcal{N}$;
- *Strategy sets*: The union of feasible strategy sets Ω_{uc} of UC and $\{\Omega_i\}_{i\in\mathcal{N}}$ of all end-users;
- *Payoff functions*: The union of revenue functions U_{uc} of UC and $\{U_i\}_{i\in\mathcal{N}}$ of all end-users.

It can be seen from (1) that the revenue function of UC depends not only on the retail price, but also on the energy consumption strategies of all end-users. Therefore, hereafter, to visually show the mutual influence between the strategies of UC and end-users, we rewrite U_{uc} in the form $U_{uc}(r, P)$, where $P = (P_1, P_2, \dots, P_N)$ represents the strategies of all end-users. Considering that the strategies of all end-users are interdependent, U_i is analogously rewritten as $U_i(r, P_i, P_{-i})$, where P_{-i} denotes the strategies of all other $N-1$ end-users except i. In game model, it is usually assumed that all players are rational, which means that all players are able to grasp the current situation, identify the possibilities of all situations, and maximize expected utility. In this way, rational players will eventually reach Nash equilibrium by gradually eliminating strictly dominated strategies. Therefore, the equilibrium solution of the proposed Stackelberg–Nash game model (which shall be introduced later) can balance the behavior of all participants. Note that the aim of this work is to achieve a win-win situation, rather than unilaterally improving individual benefits, thus the game-theoretic modeling approach is reasonable.

3.1. Stackelberg-Nash game process

For the formulated one-leader, N-follower SNG $\mathcal{G}$, once the strategy sets and payoff functions are given by (1), (3), (6), and (8), the game is played by the following steps:

(S1) Leader selects a strategy from its strategy set and announces it to all followers. Since the leader has the priority to determine the strategy, the game process starts when the leader releases its strategy, namely the retail price $r = (r_1, r_2, \dots, r_T)$, to all followers.

(S2) Followers determine their best-response strategies in reaction to leader's strategy. Specifically, when followers receiving the leader's strategy r, they play a non-cooperative Nash game to reach the NE, which is also denoted as the best-response strategy $P_i(r)$ and can be obtained by

$$P_i(r) = \arg\max_{P_i\in\Omega_i} U_i(r, P_i, P_{-i}), \forall i \in \mathcal{N}. \tag{10}$$

(S3) Leader determines its optimal strategy based on the best-response strategies that have been identified by all followers. According to the identified best-response strategies $P_1(r), P_2(r), \dots, P_N(r)$ obtained in (S2), the leader further selects an optimal strategy from its strategy set Ω_{uc}, which is denoted as r^* and is calculated by

$$r^* = \arg\max_{r\in\Omega_{uc}} U_{uc}\left(r, P_1(r), P_2(r), \dots, P_N(r)\right). \tag{11}$$

(S4) Repeat (S1)-(S3) to seek the optimal strategies of both leader and followers.

Specifically, the process from (S1) to (S3) is repeated until all players no longer change their strategies, indicating that the optimal strategies have been found.

3.2. Stackelberg-Nash Equilibrium (SNE)

In game theory, a solution of Nash game is often defined as Nash equilibrium (NE), in which no player can unilaterally derive from the NE to increase his utility. Similarly, a solution of Stackelberg game is usually defined as Stackelberg equilibrium (SE), and upon which no player (i.e., leader and all followers) has incentive to unilaterally change its strategy [23]. Inspired by this fact, it is natural to define a Stackelberg–Nash equilibrium (SNE) to characterize the solution concept of SNG [31].

Definition 1. Consider the SNG $\mathcal{G}$ defined in Section 3.1. A set of strategies $(r^*, P_1^*, P_2^*, \dots, P_N^*)$ constitutes a Stackelberg–Nash equilibrium (SNE) if and only if for $\forall r \in \Omega_{uc}$, $\forall i \in \mathcal{N}$, $\forall P_i \in \Omega_i$, the following two inequalities are satisfied:

$$\begin{cases} U_{uc}\left(r^*, P_i^*(r^*), P_{-i}^*(r^*)\right) \ge U_{uc}\left(r, P_i^*(r), P_{-i}^*(r)\right), \\ U_i(r^*, P_i^*(r^*), P_{-i}^*(r^*)) \ge U_i(r^*, P_i(r^*), P_{-i}^*(r^*)), \end{cases} \tag{12}$$

where $P_i^*(\cdot)$ is the best-response of follower i coming from the NE of the Nash game played by all followers. $P_{-i}^*(\cdot)$ denotes the best-response of all other followers to leader's strategy except that of i.

The first inequality in (12) indicates that the leader cannot further improve his revenue by choosing other strategies rather than the strategy r^*. The second one in (12) shows that after the leader choosing the optimal strategy r^*, no follower can increase his revenue by unilaterally deriving from the equilibrium strategy $P_i^*(r^*)$. That is to say, at SNE, the revenues of both UC and end-users are guaranteed to be maximized.

For illustration, the information flow and framework of the constructed SNG are shown in Fig. 2.

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[2] For ease of understanding, in the following we shall use “leader” and “UC”, as well as “follower(s)” and “end-user(s)” interchangeably.

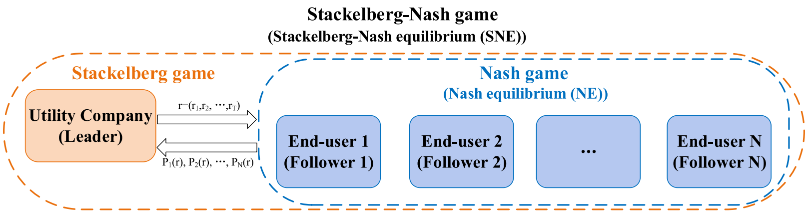

Fig. 2. Information flow and framework of the constructed SNG.

3.3. Existence and uniqueness of SNE

Next, we analyze the properties of SNE. The following theorem shows the existence and uniqueness of the SNE.

Theorem 1. *For the formulated SNG $\mathcal{G}$, there always exists a unique SNE that satisfies* (12).

Proof. Due to the characteristics of hierarchical and sequential decision-making of the formulated SNG, the backward induction, an effective method used to obtain a sequence of optimal strategies in sequential games [18], is employed here to derive the SNE. Specifically, we first identify that each follower has a unique optimal best-response strategy when receiving leader's strategy, and then trace back to verify that the leader admits a unique best strategy when provided the identified followers' best-response strategies. The details are shown below:

(1) Identify that the best-response strategies of all followers are unique. Note that when informed of the leader's strategy r, all followers essentially play a non-cooperative Nash game (see Sections 3.2 and 3.3). Therefore, identifying that the followers have unique best-response strategies is equivalent to verifying that the non-cooperative Nash game has a unique NE. As a result, we next proceed to prove that a unique NE exists in the non-cooperative Nash game played by all followers. As it has been shown in [32] that a *strictly concave N-person game* has a unique NE. So we then verify that the non-cooperative Nash game played by all followers is a strictly concave N-person game.[3]

First, as shown in (8), the strategy set Ω_i is bounded by a finite set of linear inequalities, so it can readily conclude that the strategy set Ω_i of each player is nonempty, compact, and convex. Then from the definition of revenue function $U_i(r, P_i, P_{-i})$, one can see that U_i is twice continuously differentiable in Ω_i. By taking the first-order derivative of U_i w.r.t $P_{i,t}$, one obtains

$$\frac{\partial U_i(r, P_i, P_{-i})}{\partial P_{i,t}} = \alpha_{i,t} - \beta_i P_{i,t} - r_t. \tag{13}$$

Furthermore, taking the second-order derivative, one has $\frac{\partial^2 U_i(r,P_i,P_{-i})}{\partial P_{i,t}^2} = -\beta_i < 0$, which shows that the Hessian matrix of U_i is diagonal and negative definite. We thereby conclude that the revenue function of each follower is strictly concave. Moreover, one can easily see from (4)–(8) that both the strategy set and payoff function of each follower depend on the strategies of all followers. Finally, the number of followers playing the non-cooperative game is N. Through the above analysis, one can see that the non-cooperative Nash game played by all followers is a strictly concave N-person game. Therefore, there exists a unique NE among all followers. By setting (13) to zero, the unique best-response strategy of follower i at time slot t in response to the leader's strategy r_t can be obtained by

$$P_{i,t}^*(r_t) = \frac{\alpha_{i,t} - r_t}{\beta_i}, \forall i \in \mathcal{N}. \tag{14}$$

(2) Verify that the leader has a unique best strategy when given the identified best-response strategies of all followers. When obtaining the identified best-response strategies of all followers, substitute them (i.e., (14)) into the leader's revenue function, then one has

$$U_{uc,t} = r_t \sum_{i=1}^{N} \frac{\alpha_{i,t} - r_t}{\beta_i} - \omega_t \sum_{i=1}^{N} \frac{\alpha_{i,t} - r_t}{\beta_i}. \tag{15}$$

Taking the first-order derivative w.r.t. r_t, one obtains

$$\frac{\partial U_{uc,t}}{\partial r_t} = \sum_{i=1}^{N} \frac{\alpha_{i,t}}{\beta_i} - 2\sum_{i=1}^{N} \frac{r_t}{\beta_i} + \omega_t \sum_{i=1}^{N} \frac{1}{\beta_i}. \tag{16}$$

Moreover, the second-order derivative is $\frac{\partial^2 U_{uc,t}}{\partial r_t^2} = -2\sum_{i=1}^{N} \frac{1}{\beta_i} < 0$, which indicates that the revenue function of leader is strictly concave w.r.t. its price strategy. Therefore, the optimization problem (2) has a unique global optimal solution and can be calculated by setting (16) to zero, that is,

$$r_t^* = \frac{\sum_{i=1}^{N} \frac{1}{\beta_i}(\omega_t + \alpha_{i,t})}{\sum_{i=1}^{N} \frac{2}{\beta_i}}. \tag{17}$$

Upon determining the unique best strategy r_t^* of leader at time slot t, the best-response strategies of all followers to r_t^* can be uniquely determined by (14). From the above analysis and derivation, one can see that the best strategies of both leader and followers (i.e., r^* and $\{P_i^*\}_{i\in\mathcal{N}}$) satisfy the conditions in (12). Therefore, a unique SNE always exists in the formulated one-leader, N-follower SNG. ■

4. Distributed SNE-seeking algorithm

Many efforts have been devoted to obtaining the SNE, for example, the authors in [33] develops a sensitivity-formulation based approach to determine Nash solutions in multiobjective problems which, however, needs to perform a sensitivity analysis to approximate the reaction functions. In this work, it can be seen from the proof of Theorem 1 that the SNE can be directly derived which, however, is in a centralized manner. In addition, it can be seen from (17) that calculating the leader's optimal price strategy requires the private parameter information (i.e., $\alpha_{i,t}$ and β_i) of all followers. To solve the above two issues, we next propose a distributed iterative algorithm to seek the SNE, which does not need to disclose the followers' parameter information to leader. Moreover, each follower can determine its best-response strategy locally. The details of the proposed algorithm are summarized in Algorithm 1.

To begin with, input the wholesale price $\omega = (\omega_1, \omega_2, \dots, \omega_T)$ and a set of predefined parameters. Then initialize the initial retail price r^0, UC's optimal strategy r^*, and UC's maximum revenue U_{uc}^*. Next, the SNE is obtained by the following steps:

(S1) Update the price strategy r^k by

$$r^k = r^{k-1} + \Delta r^k, \tag{18}$$

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[3] A game is called a strictly concave N-person game if the following conditions are satisfied [32]: (1) the strategy set Ω_n of each player $n \in \mathcal{N}$ is nonempty, compact, and convex; (2) the payoff function $f_n(x_n, x_{-n}), \forall n \in \mathcal{N}$, is strictly concave w.r.t. x_n; (3) the strategy set and payoff function of each player depend on the strategies of all players; and (4) $|\mathcal{N}| = N$.

Algorithm 1 Distributed Iterative Algorithm for Seeking SNE.

```
Input: The wholesale price $\omega$ and a set of predefined parameters $\delta$, $\sigma$, $\epsilon$, $r_t^{max}$, $P_i^{min}$, $P_i^{max}$, and $E_t$ for $\forall i \in \mathcal{N}$, $\forall t \in \mathcal{T}$
Output: The SNE $(r^*, P_1^*, P_2^*, \cdots, P_N^*)$
Initialize: Let $r^* = r^0 = \omega$ and $U_{uc}^* = U_{uc}^0 = 0,\ k = 0$
Iteration: $k \leftarrow k + 1$
    Step 1: Update price $r^k$ by $r^k = r^{k-1} + \Delta r^k$
    Step 2: UC broadcasts $r^k$ to all end-users
    Step 3: Each end-user calculates its best-response strategy through (19) and sends it back to UC
    Step 4: UC checks the feasibility of the energy consumption strategies of all end-users:
        1) if $P_{i,t}^k(r_t^k) < P_i^{min}$ then $r_t^k = r_t^{k-1}$
        2) if $\sum_{i=1}^N P_{i,t}^k(r_t^k) > E_t$ then
                $\Delta r^k = \delta(e^{\sigma(\sum_{i=1}^N P_{i,t}^k(r_t^k) - E_t)} - 1)$
    Step 5: UC calculates its revenue at time slot $t$ by (20) and updates the total revenue by $U_{uc}^k = \sum_{t=1}^T U_{uc,t}^k$
    Step 6: Whether UC has found a better price strategy:
        if $U_{uc}^k \geq U_{uc}^*$ and $r^k \leq r^{max}$ then
            $r^* = r^k,\ \ U_{uc}^* = U_{uc}^k$
        else
            $U_{uc}^*$ remains unchanged
        end if
    Step 7: Check the stopping criterion:
        if $|U_{uc}^k - U_{uc}^{k-1}| \leq \epsilon$
            break
        end if
Return: The optimal price and energy consumption profile
```

where $\Delta r^k = \delta(e^{\sigma|U_{uc}^k - U_{uc}^{k-1}|} - 1), k > 1$, is an adaptive step size with system-related parameters δ and σ. Note that in the first iteration, we need to choose a fixed and small step size (i.e., Δr^1) to trigger the iteration process, which has no specific restrictions, as long as it is the same order of magnitude as the electricity price.

(S2) The UC broadcasts the updated price strategy r^k to all end-users.

(S3) Once receiving the UC's strategy r_t^k at time slot t, each end-user locally calculates its best-response strategy by

$$P_{i,t}^k(r_t^k) = \frac{\alpha_{i,t} - r_t^k}{\beta_i}, \forall i \in \mathcal{N}. \tag{19}$$

Note that the strategy obtained by (19) is the optimal strategy in response to the current updated price r_t^k, rather than the final optimal one. Then, all end-users send the best-response strategies back to UC.

(S4) UC checks the feasibility of the best-response strategies calculated in (S3), that is, whether the two constraints in (7) (corresponding to the physical limitations of the practical retail electricity trading) are satisfied: (a) If $P_{i,t}^k(r_t^k) < P_i^{min}$, it means that $P_{i,t}^k(r_t^k)$ cannot be reduced anymore, so r_t^k cannot continue to increase. Therefore, the current price strategy r_t^k is set to the value at last iteration. Since $P_{i,t}^k$ is decreasing w.r.t. r_t^k, it is not necessary to check $P_{i,t}^k(r_t^k) > P_i^{max}$. (b) If $\sum_{i=1}^N P_{i,t}^k(r_t^k) > E_t$, it indicates that the end-users have higher energy needs than the UC can provide. Therefore, UC needs to increase the retail price to prompt end-users to reduce the energy consumption requirement.

(S5) Once receiving the feasible best-response strategies of all end-users, UC computes his revenue by

$$U_{uc,t}^k = (r_t^k - \omega_t) \sum_{i=1}^N P_{i,t}^k(r_t^k). \tag{20}$$

(S6) Judge whether the updated price strategy r_t^k is better than the previous one. If $U_{uc}^k \geq U_{uc}^*$ and $r^k \leq r^{max}$ are both met, let $r^* = r^k$ and $U_{uc}^* = U_{uc}^k$; otherwise, U_{uc}^* remains unchanged.

(S7) Check the stopping criterion. Specifically, judge whether the error between U_{uc}^k and U_{uc}^{k-1} is less than ϵ. If not, go to (S1) and proceed to the next iteration; otherwise, output the optimal price and energy consumption strategies.

Note that since we incrementally update the price strategy by (18) rather than directly calculating it by (17) and all end-users locally update their own energy consumption strategies by (19), the proposed iterative algorithm is thereby implemented in a distributed manner without sharing the global parameter information. Therefore, the proposed distributed iterative algorithm does not disclose the followers' parameter information to leader. In other words, the proposed method is privacy-preserving. Moreover, compared with the methods of passing the parameters, such as the centralized methods in this paper, the proposed distributed iterative algorithm effectively reduces the communication and computing burden as a powerful central controller is needed to process and exchange a vast amount of information with all the followers when passing the global parameters. However, when applying the proposed distributed iterative algorithm, the disadvantage is that each follower needs to be equipped with a smart processor to locally update their own energy consumption strategies. Therefore, the upfront investment cost is higher. In addition, we have shown that the revenue function of UC is strictly concave w.r.t. the price strategy. Therefore, continuously incrementing the price strategy through (18) from the smallest feasible solution shall eventually lead to the maximum revenue value. That is, the optimal price strategy can be uniquely found in an iterative way. Accordingly, the optimal energy consumption strategies of all end-users can be locally and uniquely determined by (19). In this way, the convergence of the proposed distributed iterative algorithm is naturally and easily guaranteed.

Remark 2. In updating the price strategy of UC, we choose an adaptive step size to accelerate the convergence speed of the iterative algorithm. Specifically, we denote $|U_{uc}^k - U_{uc}^{k-1}|$ as the gap of UC's revenue between two consecutive iterations. It is clearly seen that a larger gap leads to a larger step size. Therefore, with UC's revenue becoming saturated, the step size shall be smaller and smaller, and eventually tend to zero.

5. Simulation results

5.1. Simulation setup

We conduct the simulations based on a real system model and parameter information from a residential retail electricity market in Texas, USA [34], whose operation mode is shown in Fig. 3, in which the utility company broadcasts the retail price information to end-users and the end-users deliver the energy consumption information to utility company. The wholesale price depicted in Fig. 4 is determined in advance by GO and comes from Commonwealth Edison Company [35]. The time period covers a whole day and is divided into 24 time slots. The user-dependent satisfaction parameters $\alpha_{i,t}$ and β_i, as well as other related parameters are from [24] and listed in Table 1. Here, three end-users have different α but the same β and γ. It is because that they have different marginal profits but with the same satisfaction and environmental cost sensitivity when faced with the changes in energy consumption. Let $\delta = 0.818$, $\sigma = 0.0013$, and $\epsilon = 10^{-4}$. It is necessary to declare that all parameters can be set to other values according to different retail electricity market which, however, cannot affect the analysis and evaluation of the proposed algorithm. The simulation results are shown below.

5.2. Case study 1: Effectiveness verification

5.2.1. Performance of the proposed algorithm

This case scenario considers a residential retail electricity trading with one UC and three end-users. After employing the proposed SNG-based distributed algorithm, the retail prices during the entire time horizon is depicted in Fig. 4. It shows that the retail price at any

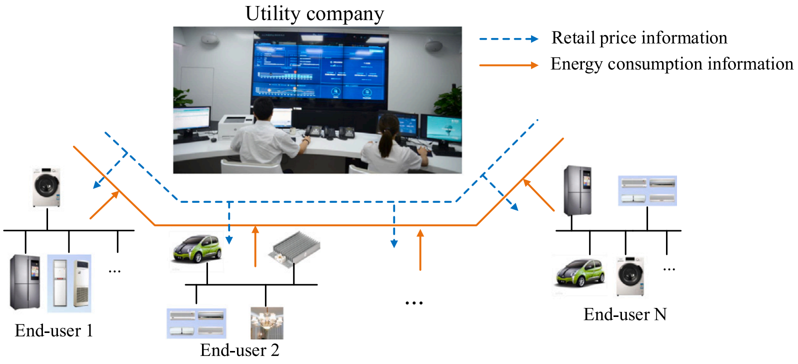


Fig. 3. Operation mode of residential retail electricity market.

Table 1
Parameter settings of UC and end-users.

Parm. Unit	$\alpha_{i,t}$	β_i	$\gamma_{i,t}$	P_i^{min}	P_i^{max}	r_t^{max}
User 1	5.0	0.1	0.05	4	19	–
User 2	5.5	0.1	0.05	8	25	–
User 3	6.0	0.1	0.05	10	29	–
UC	–	–	–	–	–	$2.1\omega_t$

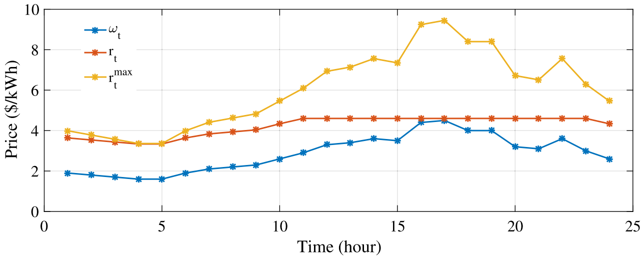


Fig. 4. Comparison of wholesale price and retail price in a day.

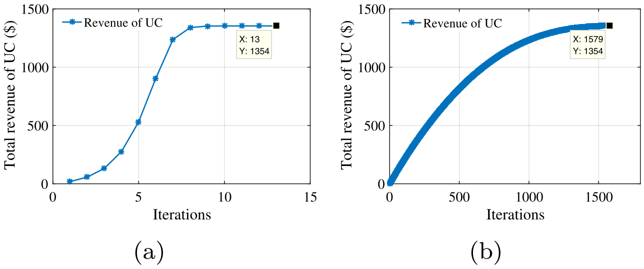


Fig. 5. UC's revenue under (a) proposed algorithm and (b) general iterative algorithm.

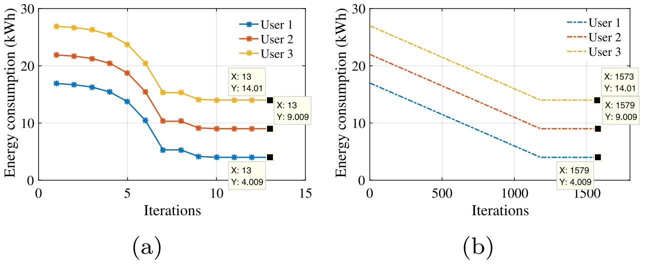


Fig. 6. Energy consumption of all end-users under (a) proposed algorithm and (b) general iterative algorithm at 12:00.

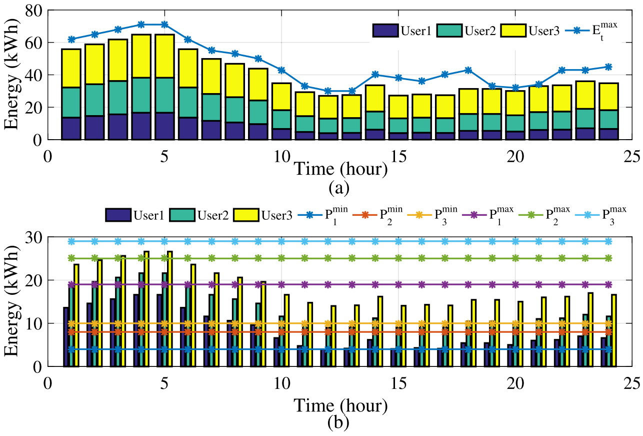
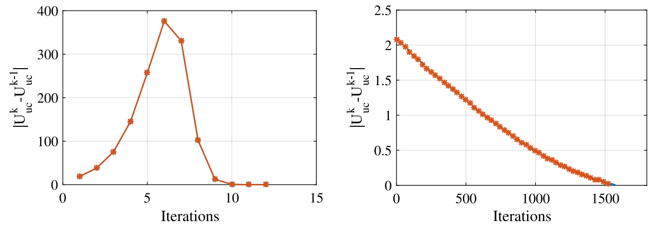


Fig. 7. Energy consumption strategies of all end-users in a day.

time is within the feasible price range. Then we compare our proposed distributed algorithm with adaptive step size and a general iterative method with fixed step size ($\Delta r^k = 5.5 \times 10^{-4}$\$/kWh) [24] to verify the performance of the proposed distributed algorithm. Here note that a larger fixed step size would fail to find the optimal solution, because the price increases too fast to largely miss the optimal solution. The comparison results are shown in Figs. 5 and 6. Specifically, Fig. 5 plots the UC's total revenue under two compared algorithms, which shows that both algorithms can find the maximum total revenue of UC, but our proposed algorithm only takes 13 iterations while the fixed- step-size one needs more than 1500 iterations. The main reason is that the fixed step size is chosen relatively small to avoid missing the optimal value which, however need more iterations, while the adaptive step size can be adjusted in real time according to the revenue difference. Fig. 6 illustrates the energy consumption trajectories of three end-users at 12:00, which shows that both algorithms can obtain the same energy consumption strategies consistent with the SNE. The specific energy consumption strategy of all end-users are illustrated in Fig. 7. It can be observed from Fig. 7(a) that at any time, the total energy consumption of all three end-users does not exceed the available energy from UC. Fig. 7(b) shows that the energy consumption of all end-users falls within their feasible ranges. Therefore, the two inequalities in (7) are both satisfied. In addition, Fig. 8 plots the error trajectories (i.e., $|U_{uc}^k - U_{uc}^{k-1}|$) of the two compared algorithms. As can be seen, the error of the algorithm with fixed step size is regularly reduced to zero while that of the proposed algorithm can be adjusted flexibly and approach to zero at a faster speed. From the above comparison, it can be seen that the proposed distributed algorithm can effectively solve the price-based DR in residential retail market and find the optimal price and energy consumption strategies with a faster convergence speed,



(a) (b)

Fig. 8. The convergence of (a) proposed algorithm and (b) general iterative algorithm.

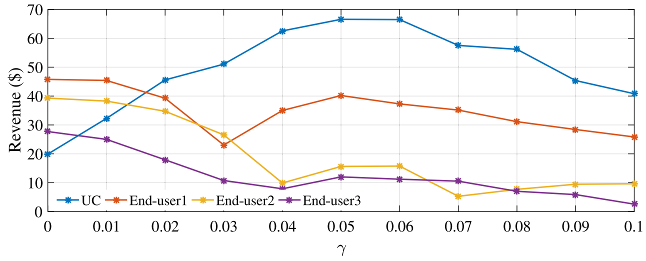


Fig. 9. Impact of $\gamma_{i,t}$ on UC and end-users' revenue (t=12:00).

Table 2
The running time (s) for two case studies.

	Case 1	Case 2				
Number of end-users	3	10	20	30	40	50
Fixed step size	0.210	1.54	1.921	2.224	3.067	4.618
Adaptive step size	0.022	0.024	0.042	0.056	0.084	0.127

5.2.2. Impact of the impact factor

We next show the impact of $\gamma_{i,t}$ on UC and end-users' revenue. For illustration, we consider a single time slot, i.e., 12:00. As shown in Fig. 9, as $\gamma_{i,t}$ varying from 0 to 0.1 with a step of 0.01, the end-users' revenue decreases first and then increases, while UC's revenue increases first and then decreases. It is because the increase in $\gamma_{i,t}$ means that the total energy consumption of all other end-users has an increasing impact on the revenue of a single end-user, thereby resulting in a decrease in end-users' revenue. In addition, as $\gamma_{i,t}$ increases, retail prices gradually decrease, so UC's revenue also deceases. However, when $\gamma_{i,t}$ is greater than 0.05, the degree of mutual influence between end-users is too large, resulting in the decrease in the revenue of both UC and end-users, which is often undesirable.

5.3. Case study 2: Scalability verification

In this scenario, a large-scale retail market with more end-users (i.e., from 10 to 50) is further considered to verify the scalability of the proposed algorithm. Since the trajectory trends of the retail price, energy consumption, and UC's total revenue are similar to that of three-users case, they are omitted for space limitations. In addition, the running time required for two compared algorithms under different number of end-users is listed in Table 2. It shows that (1) for the same number of end-users, our proposed algorithm takes less time than the one with fixed step size; (2) with the increase of the number of end-users, the running time of the iterative algorithm with fixed step size increases faster than the one with adaptive step size. The reason is that the price update of the algorithm with fixed step size increases regularly with small increments, while that of the adaptive one can be adjusted flexibly by the UC's revenue gap between two consecutive iterations. Thus, unnecessary iteration overhead is reduced.

Moreover, to verify the superiority of the proposed SNG-based model, a benchmark without the SNG-based DR is designed. in which all end-users selfishly maximize their own revenues by consuming as much energy as possible. Fig. 10 shows the total energy consumption of 10 end-users with/without (w/o) SNG-based DR. It can be observed that without the SNG-based DR, all end-users tend to consume the maximum energy during daytime, resulting in the total energy consumption exceeds the available energy from UC at peak periods. By contrast, when employing the proposed SNG-based DR model, all end-users coordinate their energy consumption behavior such that the available energy constraint of UC cannot be violated. Therefore, the case of no DR has much higher energy consumption than the case with DR during some peak periods. In other words, the proposed DR approach reduces the energy consumption of end-users in some peak periods. So the total energy consumption of all end-users are smoother in the case with DR than the one without DR. But the total energy consumption of all end-users are the same throughout the day under these two cases, so the satisfaction of all end-user can be still guaranteed.

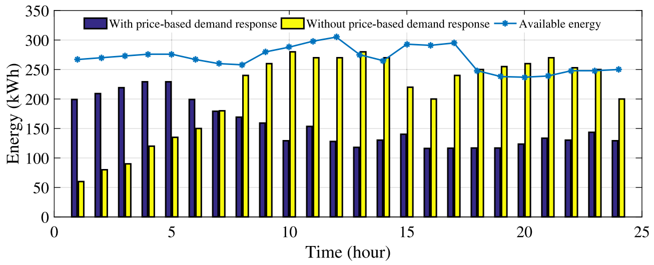


Fig. 10. Total energy consumption of 10 end-users w/o SNG-based DR.

6. Conclusion

This paper studies a novel price-based demand response problem in retail electricity market, which not only considers the hierarchical decision-making process between utility company and end-users, but also analyzes the inherent interdependence among end-users. To characterize the strategic interaction among all market participants, we construct a novel Stackelberg–Nash game and proof the existence and uniqueness of the Stackelberg–Nash equilibrium by means of backward induction. Furthermore, in order to protect the privacy information of end-users, we develop a distributed iterative algorithm with an adaptive step size, which converges faster than the general one with fixed step size. Considering that the uncertainties of wholesale prices, consumers' demands, and the associated risks play an important role in wholesale market trading, we shall further investigate uncertainty in wholesale electricity trading and its associated risks by means of model-free methods, such as stochastic programming or learning-based approaches in the future.

CRediT authorship contribution statement

Yanni Wan: Develop the main idea of the paper, Analyze and explain the main result, Draft the manuscript, Reviewed the manuscript. **Jiahu Qin:** Establish the framework and structure of the paper, Make critical revision of the manuscript regarding important intellectual content, Reviewed the manuscript. **Yang Shi:** Establish the framework and structure of the paper, Make critical revision of the manuscript regarding important intellectual content, Reviewed the manuscript. **Weiming Fu:** Polish the presentation, Study supervision, Reviewed the manuscript. **Feng Xiao:** Polish the presentation, Study supervision, Reviewed the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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