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M. Amereh and B. Nadler

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Amplitude-dependent rheological responses of axisymmetric grains

M. AMEREH and B. NADLER[(a)]

Department of Mechanical Engineering, University of Victoria - Victoria, BC, Canada

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Abstract – Oscillatory shear flows of axisymmetric grains exhibit amplitude-dependent rheological responses, which is related to the evolution of the microstructure. In this work, it is shown that the highly ordered configuration of grains at steady-state shear flow undergoes microstructural rearrangement when subjected to shear oscillations. This rearrangement may lead to reduced ordering configurations which give rise to macroscale shear hardening, which can result in shear jamming if the applied shear traction is below the critical shear resistance. On the other hand, it was observed that applying oscillatory shear to the primary condensed shear flow enhances flowability due to microstructure rearrangement. In this study, we investigate the amplitude-dependent rheological responses of axisymmetric grains subjected oscillatory shear flows. First, we look into the evolution of grains alignment subjected to a range of oscillation amplitudes, where we show that the lower oscillation amplitudes have the potential to change the orientation from the ordered steady state to a completely disordered (isotropic) orientation. Next, we study the dependency of the shear flow resistance on the microstructure configuration, and show that the strain hardening and potential jamming have strong dependency on the oscillations amplitude. We also show that, in the case of jamming, the shear strain and the corresponding number of oscillation cycles depend not only on the grains aspect ratio but also on the oscillation amplitude.

Introduction. – The study of granular flows has practical applications in material science, pharmaceutics, agriculture, and food processing industries [1]. Complex rheological properties of such flows have been investigated to understand the role of the microstructure arrangement of the grains on the macro-scale behaviour. For example, the steady-state orientation of axisymmetric grains subjected to a simple shear flow has been widely studied in recent years [2–5]. Researchers have been extensively using the Discrete Element Method (DEM) to understand the effect of grains shape, *e.g.*, degree of asphericity, on the orientation and alignment of grains, and their consequent role in the rheological properties. The transient response, on the other hand, still lacks a theoretical description that can answer fundamental questions regarding the underline physics. The transient responses of granular flow are dominant in nature with notable applications in geophysical phenomena, such as landslides and subsidence due to earthquake [6,7], involving non-steady-state conditions. Also, the dynamics of transient flows observed by DEM simulations and experiments are often nontrivial such that the final configuration is complex and typically difficult to predict by means of simple laws. For instance, the microstructure rearrangement of axisymmetric grains under shear reversal flow can give rise to strain hardening/softening and jamming for sufficiently low shear traction [8]. Oscillatory shear flow is another example of transient flows that give rise to a complex rheological response of axisymmetric grains [9–13]. Studies have shown that the viscosity change in dense granular flows is associated with the packing fraction [14,15], which depends on different parameters such as friction, grains' shape and their interaction [16–22]. In addition, it has been shown that oscillatory shear can reduce the viscosity and hence improve the flowability of pre-sheared granular flows [23,24]. Despite the significance of the results

(a)E-mail: bnadler@uvic.ca (corresponding author)

provided by DEM simulations and experiments, they lack the complementary theory that could give better insight in understanding of the underlying physics. In addition, theory allows to study a wide range of cases and helps interpreting the results. Here, we investigate the amplitude dependency of the rheological response in shear flow of axisymmetric grains, by subjecting the axisymmetric grains to isochoric oscillatory shear for a range of amplitudes. The outline of the letter is as follows. First, we introduce the overall physics, the geometry of the system, and the exerted oscillatory shear. Next, we adopt an evolution law for the orientational tensor of microstructure configuration, which has a key role in the rheology of axisymmetric grains. Finally, we utilize an incompressible anisotropic rheological model which has dependency on the orientational tensor to study the amplitude-dependent rheological responses.

Formulation. – In this section, we present the methodology and model formulation. We consider a system, as depicted in fig. 1, subjected to oscillatory shearing,

$$\gamma(t) = \gamma_0 \sin(\omega t), \quad \dot{\gamma}(t) = \gamma_0 \omega \cos(\omega t), \tag{1}$$

where $\gamma(t)$ and $\dot{\gamma}(t)$ are the oscillatory strain and strain rate, respectively, with amplitude of γ_0, frequency of ω, and t is time.

To obtain a meaningful insight into the transient rheological response, we neglect inertia effects, hence the response is independent of the frequency. It follows that the rate of deformation and the vorticity take the forms $\mathbf{D} = \dot{\gamma}(\boldsymbol{i}\otimes\boldsymbol{j}+\boldsymbol{j}\otimes\boldsymbol{i})/2$ and $\mathbf{W} = \dot{\gamma}(\boldsymbol{i}\otimes\boldsymbol{j}-\boldsymbol{j}\otimes\boldsymbol{i})/2$, respectively. In order to study the the microstructure configuration, we adopted the evolution law for the orientational tensor that was developed for homogeneous field in [25], which has the form

$$\dot{\mathbf{A}} = \mathbf{WA} - \mathbf{AW} + \lambda[\mathbf{AD} + \mathbf{DA} - 2[\mathbf{A}\cdot\mathbf{D}]\mathbf{A}] - \psi D' [\mathbf{A} - \mathbf{I}/3], \tag{2}$$

where $\dot{\mathbf{A}}$ is the material time derivative of the orientational tensor $\mathbf{A}$, and $D' = \sqrt{2\mathbf{D}'\cdot\mathbf{D}'}$ is the magnitude of the deviatoric part of the rate of deformation $\mathbf{D}' = \mathbf{D} - (\mathbf{D}\cdot\mathbf{I})\mathbf{I}/3$. Also, λ and ψ are phenomenological model parameters measuring the tendency of the particles to align with the flow, and the relaxation toward the isotropic orientation, respectively. We also adopt an incompressible anisotropic rheological model proposed in [26],

$$\boldsymbol{\sigma} = -p\mathbf{I} + p\mu(I)[\bar{\mathbf{D}} - \eta(\mathbf{A}\bar{\mathbf{D}} + \bar{\mathbf{D}}\mathbf{A} - 2/3\,(\bar{\mathbf{D}}\cdot\mathbf{A})\,\mathbf{I})], \tag{3}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress, p is the pressure, $\bar{\mathbf{D}} = \mathbf{D}/D$ is the direction of the rate of deformation $\mathbf{D}$, $D = \sqrt{2\mathbf{D}\cdot\mathbf{D}}$, $\mu(I) = \mu_s + \mu_1 I^\beta$ is the inertia rheology, μ_s, μ_1, β are the rheology parameters, $I = D'd/\sqrt{p/\rho_s}$ is the inertia number and d and ρ_s are the dimension and the density of the solid grains, respectively. The phenomenological parameters μ_s, μ_1, β and η are related to the grains shape and the interfacial friction, where η accounts for an anisotropic rheology that gives rise to a reduction in the shear resistance as grains align with flow. These parameters are given by [27] and [26] as

$$\lambda(r_g) = (2/\pi)\tan^{-1}(5.5\,rg), \quad \psi(r_g) = 0.85\exp(-4\,r_g^2),$$
$$\eta(r_g) = 0.746\tan^{-1}(4.266\,r_g),$$

where $r_g = (l-d)/(l+d)$ is the aspect ratio of axisymmetric grains of length l and diameter d.

The dependency of the shear flow resistance on the microstructure configuration gives rise to a complex rheological response during transient shear reversal due to the associated rearrangement of the microstructure. The shear resistance has dependency on the orientational tensor [8] through the shear ordering $A_p = A_{11} + A_{22}$, and its critical value when approaching jamming, *i.e.*, $I \to 0$ and hence $\mu(I) \to \mu_s$, is

$$\tau_{cr} = \frac{1}{\sqrt{2}}\,p\mu_s\,(1 - \eta A_p), \tag{4}$$

where μ_s is the static friction coefficient. Equation (4) identifies a jamming criterion for the applied shear traction and its dependency on the orientational tensor through A_p and the phenomenological parameter η.

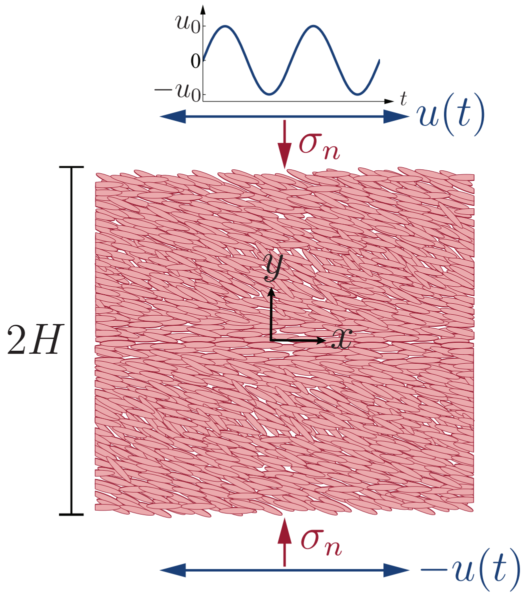

Fig. 1: Schematic representation of an axisymmetric grains flow subjected to oscillatory shear $\gamma(t) = u(t)/H$, with the amplitude $\gamma_0 = u_0/H$, and normal traction σ_n.

Amplitude dependency oscillation. – The amplitude of the oscillatory shear plays a significant role in governing the orientation of grains. In [8], it was shown that in flows of axisymmetric grains subjected to shear reversal, the transient response is terminated at $\gamma \approx 5$, that is, the system returns to steady state. Hence, we only investigate shear strain amplitudes smaller than five. It was also shown that the most significant response, in terms of

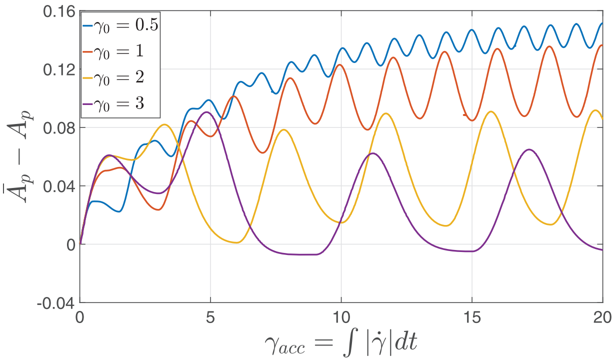

Fig. 2: Evolution of the shear ordering A_p with respect to the steady state $\bar{A}_p$ in oscillatory shear flow for different shear amplitudes $\gamma_0 = \{0.5, 1, 2, 3\}$ and $r_g = 0.6$.

strain hardening, corresponds to the grains with aspect ratio of $r_g = 0.6$, which will be used here. We first investigate the role of the oscillatory amplitude γ_0 on the shear ordering A_p and the critical shear resistance τ_{cr} in oscillatory shear flow. The evolution of A_p in response to different amplitudes $\gamma_0 = \{0.5, 1, 2, 3\}$ is obtained by (2) and shown in fig. 2. As can be seen, large amplitudes of oscillation $\gamma_0 = \{2, 3\}$ yield a change to the microstructure from a steady state, $\bar{A}_p$, to disordered orientation and finally returning to the steady state. We use bar to indicate quantities associated with the steady state. Such reorientation, that occurs in each half-cycle, does not approach disordered state. On the other hand, smaller amplitudes, $\gamma_0 = \{0.5, 1\}$, derive the microstructure away from the ordered steady state approaching a completely disordered (isotropic) state, $A_p = 2/3$, as the microstructure cannot return to the ordered steady state at the end of each half-cycle. Therefore, in each half-cycle the shear ordering, A_p, shifts farther away from the ordered steady state, $\bar{A}_p$, till it converges to a completely isotropic orientation, *i.e.*, $\bar{A}_p - A_p \approx 0.15$. These results suggest that oscillatory shear flow with small amplitudes can give rise to a dynamical arrest due to cyclic shift toward completely disordered (isotropic) orientation state, and hence higher shear resistance. Applying larger amplitude prevents this response as the microstructure undergoes limited disordering, and hence limited strain hardening, by retaining to the steady-state orientation associated with low shear resistance.

Next, we investigate the effect of the aspect ratio of the grains. In shear reversal [8], it was shown that the strain hardening has non-monotonic dependency on the aspect ratio with the maximum value corresponding to $r_g = 0.6$. Such non-monotonic response is due to the different microstructure rearrangement processes during shear reversal of grains of different aspect ratios. The evolution of shear ordering and shear resistance were obtained for oscillatory shear flow with shear amplitudes $\gamma_0 = \{1, 2\}$ and aspect ratios $r_g = \{0.2, 0.4, 0.6, 0.8\}$, shown in figs. 3 and 4. Note that the steady-state shear ordering corresponding to the set of r_g are $\bar{A}_p = \{0.706, 0.763, 0.825, 0.875\}$, respectively. The evolution of the shear ordering has a non-monotonic dependency on r_g for shear strains below a certain value, shown by the blue zone in fig. 3(a). To understand this behaviour, it should be noted that the maximum shear ordering corresponds to $r_g = 0.6$ only for the first few cycles of oscillation with amplitude $\gamma_0 = 2$, after which the response becomes monotonic with respect to r_g. For small r_g, the grains are not well ordered and aligned with the flow even at steady state, hence, oscillatory shear yields relatively insignificant change to the ordering and alignment during the first few cycles during the first few cycles. Also, grains with large r_g are highly ordered and aligned with the flow at steady state and maintain it during the oscillations. However, after approximately six cycles where the accumulative shear strain reaches a certain level, marked by a vertical dashed line, the non-monotonic response is eliminated and the grains with greater aspect ratio undergo larger change in shear ordering, shown by orange zone in the figure. The accumulative shear strain value and the corresponding number of oscillatory cycles depend on the amplitude γ_0. For the smaller shear amplitude, $\gamma_0 = 1$, shown in fig. 3(b), the transition to monotonic response occurs after higher number of oscillatory cycles and hence at higher accumulative shear strain compared to $\gamma_0 = 2$. Smaller amplitudes lead to less microstructure

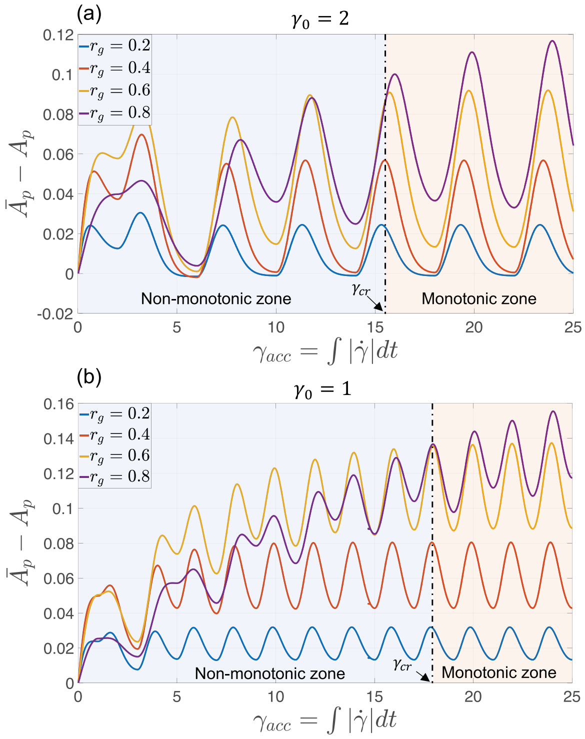

Fig. 3: Evolution of the shear ordering A_p with respect to the corresponding steady-state values in oscillatory shear flow with shear amplitude $\gamma_0 = 2$ and aspect ratios $r_g = \{0.2, 0.4, 0.6, 0.8\}$.

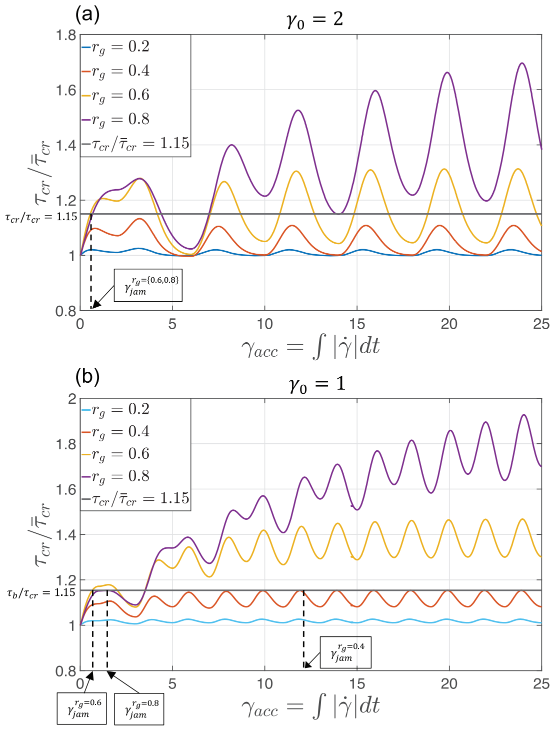

Fig. 4: Evolution of the shear resistance τ_{cr} with respect to the corresponding steady-state values in oscillatory shear flow with shear amplitude $\gamma_0 = 2$ and aspect ratios $r_g = \{0.2, 0.4, 0.6, 0.8\}$.

reorientation during each cycle and therefore requires more cycles to disorder the microstructure.

The evolution of shear resistance, τ_{cr}, for oscillatory shear flows with amplitudes of $\gamma_0 = \{1, 2\}$ and aspect ratios $r_g = \{0.2, 0.4, 0.6, 0.8\}$ is shown in fig. 4. Although the non-monotonic response observed in the evolution of A_p is still observed, it is less significant in the evolution of τ_{cr} due to the interplay of η and A_p in the expression for the critical shear resistance, eq. (4). This evolution presents an explicit jamming criterion, where jamming occurs if the externally applied shear traction is lower than the critical shear traction. The evolution of τ_{cr}, shown in fig. 4, identifies whether different externally applied shear traction, τ_b, can necessarily yield jamming, and also predicts the corresponding required shear strain and cycle number. To better understand the results, as an example, for $\gamma_0 = 2$ and $\tau_b/\bar{\tau}_{cr} = 1.15$, fig. 4(a) predicts that jamming occurs for $r_g = \{0.6, 0.8\}$ during the first cycle, at the shear strain of which is marked on the figure by $\gamma_{jam}^{r_g=\{0.6,0.8\}}$, where the horizontal line $\tau_b/\bar{\tau}_{cr} = 1.15$ crosses the $\tau_{cr}/\bar{\tau}_{cr}$ corresponding to $r_g = \{0.6, 0.8\}$. Also, grains with $r_g = \{0.2, 0.4\}$ never undergo jamming as their critical shear resistances remain below the applied shear resistance $\tau_b/\bar{\tau}_{cr} = 1.15$. For lower oscillatory amplitude $\gamma_0 = 1$, the same value of applied shear traction $\tau_b/\bar{\tau}_{cr} = 1.15$ results in jamming for $r_g = \{0.4, 0.6, 0.8\}$ at different shear strains, shown in fig. 4(b). Jamming takes place at relatively low shear strains for $r_g = \{0.6, 0.8\}$, but larger shear strain, and hence higher number of oscillatory cycles, for $r_g = 0.4$. These results demonstrate that rheological responses of granular materials under oscillatory shear flow is strongly amplitude dependent. It should be noted that in [24] grains of $r_g = 0.5$ were subjected to oscillatory shear with a small amplitude $\gamma_0 = 0.1$ show very similar response, where the microstructure approaches disordered state. However, the accumulative strain, γ_{acc}, takes approximately a three times larger value to reach the disordered state compared to the results of this study. This difference might be related to the fact that the DEM simulations in [24] are two-dimensional and the model used in this work is three-dimensional. The ability of grains to orient normal to the shear plane accelerates the relaxation process relative to the constraint two-dimensional case.

Summary. – In this paper, the rheological responses of axisymmetric grains under oscillatory shear flows are studied using the evolution of the orientational tensor and and incompressible anisotropic inertia rheological model. It is shown that the shear oscillatory flow with low amplitude can give rise to a dynamical arrest due to the cyclic shift toward disordered (isotropic) orientation associated with high shear flow resistance. It is also shown that the evolution of the shear ordering can have a non-monotonic dependency on the aspect ratio of the grains below a certain accumulative shear strain, and its dependency on the oscillation amplitude is identified by the model. Finally, using the jamming criterion, we were able to identify that the state of jamming has strong dependency on the oscillation amplitude, that is, given that the externally applied shear traction is below the critical shear resistance, shear strain and cycle number at which the jamming occurs were predicted for grains with different aspect ratios.

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