

Identification and Utilization of Loss of Motion Capabilities of Robotic Manipulators

by

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Abstract

A novel reciprocity-based method for identifying 1-DOF (degree-of-freedom) loss velocity-degenerate (singular) configurations of kinematically-redundant manipulators is presented. The developed methodology uses the properties of reciprocal screws to determine the 1-DOF-loss velocity-degenerate configurations. A by-product of the methodology is that a reciprocal screw related to the lost motion DOF for each degenerate configuration is determined. The methodology is successfully applied to determine the 1-DOF-loss velocity-degenerate configurations of four redundant serial manipulators: the 7-joint spherical-revolute-spherical manipulator; the 7-joint double-elbow manipulator; the twin 7-joint CSA/ISE STEAR Testbed Manipulator (STM) arms; and the 8-joint NASA Advanced Research Manipulator II (ARMII).

The reciprocity-based method for identifying 1-DOF-loss velocity-degenerate configurations is then extended to the case of identifying multi-DOF-loss velocity-degenerate configurations of kinematically-redundant manipulators. As with the 1-DOF-loss methodology, a by-product of the multi-DOF-loss methodology is that reciprocal screws related to the lost motion DOFs for each degenerate configuration are determined. The methodology is successfully applied to determine the 2-DOF-loss velocity-degenerate configurations of two redundant serial manipulators: the 7-joint spherical-revolute-spherical manipulator and the 7-joint double-elbow manipulator.

The utilization of velocity-degenerate configurations to optimize the pose of either non-redundant or redundant serial manipulators to sustain desired wrenches is

considered. An algorithm is developed that determines a desirable start point for the optimization of a serial manipulator's pose. The start-point algorithm (SPA) uses the knowledge of the velocity-degenerate configurations of a serial manipulator to determine a pose that would be best suitable to sustain a desired wrench. The SPA is tested on three manipulators: the 7-joint spherical-revolute-spherical manipulator, the 6-joint zero-offset PUMA-type manipulator, and the 7-joint STM-1 manipulator. The results for all three examples show that by using the SPA with the optimization routine, the resulting poses obtained usually required less effort from the actuators when compared to the poses obtained without using the SPA.

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¹The results contained in this section have been presented in Nokleby and Podhorodeski (2000a, 2000b, and 2001a).

²The results contained in this section have been presented in Nokleby and Podhorodeski (2000a, 2000b, and 2001a).

³The results contained in this section have been presented in Nokleby and Podhorodeski (2000b

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In Memory of Andrew and Jessie Grant

Chapter 1

Introduction

1.1 Overview

In this chapter, a brief introduction to robotic manipulators is presented. This is followed by the motivation and background for the research presented in this dissertation. The chapter finishes with an outline of the rest of the dissertation.

1.2 Robotic Manipulators

A robotic manipulator is a mechanical device consisting of links connected together by joints. The joints of a manipulator normally consist of all revolute (rotational) joints or a combination of revolute and prismatic (translational) joints. For the purpose of this work, the links of a manipulator are considered to be rigid bodies. Note that elastic effects of the materials comprising the links should be taken into account for high-speed or highly-loaded devices (Tsai, 1999).

Robotic manipulators can be divided into three main types: serial manipulators;

parallel manipulators; and hybrid manipulators. A serial manipulator is an open-chain device consisting of multiple links connected in series. All joints of a serial manipulator must be actuated. A parallel manipulator is a closed-chain device consisting of multiple serial branches acting on a common payload. Unlike a serial manipulator, a parallel manipulator does not have to have all of its joints actuated, but must have at least one joint actuated per branch. A hybrid manipulator is a manipulator that is a combination of serial and parallel manipulators.

1.2.1 Redundant Manipulators

A serial manipulator is redundant if the number of joints exceeds the task degrees-of-freedom (DOFs). For general spatial motion requiring 6-DOF, a serial manipulator with more than six joints is considered redundant.

Redundancy in parallel manipulators is more complex. For a parallel manipulator performing 6-DOF spatial motion, each serial branch of the parallel manipulator must have six joints. For a non-redundant parallel manipulator, only six joints of the parallel manipulator must be actuated, with at least one joint actuated in each branch. Redundancy in parallel manipulators takes on three forms: actuation redundancy; sensing redundancy; and joint redundancy. An actuation-redundant parallel manipulator has more than six of its joints actuated. A sensing-redundant parallel manipulator has only six joints actuated, but has the displacement of additional non-actuated joints measured. A joint-redundant parallel manipulator has one or more branches that contain additional actuated joints, i.e., some of the branches have more than six joints.

1.3 Motivation

Canada's contribution to the International Space Station (ISS) is the Mobile Servicing System (MSS) built by MD Robotics (see Figure 1.1). The MSS (MD Robotics, 2002) is comprised of the Mobile Base System (MBS), the Space Station Remote Manipulator System (SSRMS or Canadarm2), and the Special Purpose Dextrous Manipulator (SPDM or Canada Hand). The MBS is a small mobile base unit that can travel along the main truss of the ISS. The Canadarm2 is a large 7-revolute joint serial manipulator. The SPDM consists of two 7-revolute joint serial manipulators attached to a base containing an additional revolute joint (see Figure 1.2).

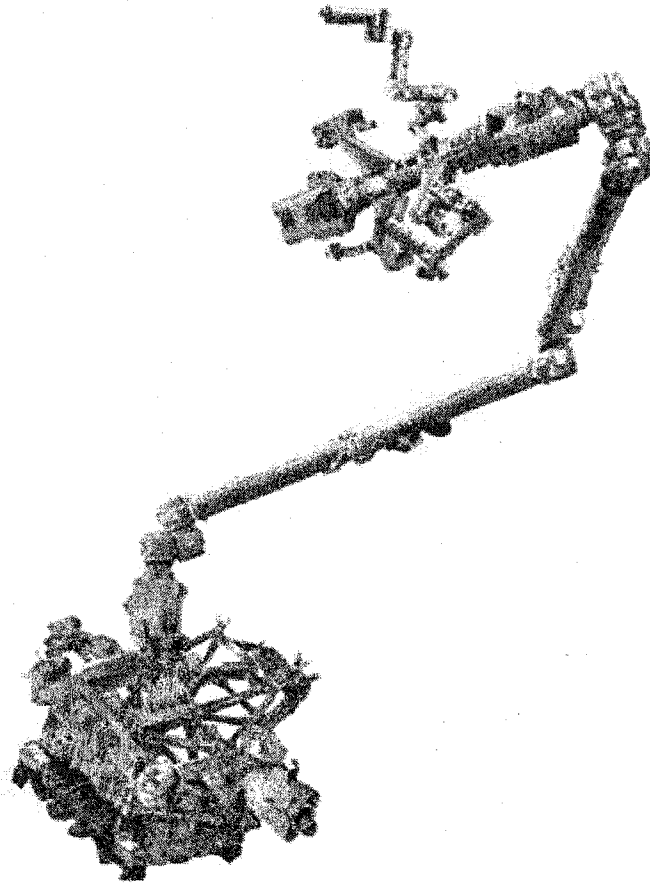


Figure 1.1: Mobile Servicing System (MSS) Comprised of the Mobile Base System (MBS), the Space Station Remote Manipulator System (SSRMS or Canadarm2), and the Special Purpose Dexterous Manipulator (SPDM or Canada Hand) (MD Robotics: <http://www.mdrobotics.ca>)

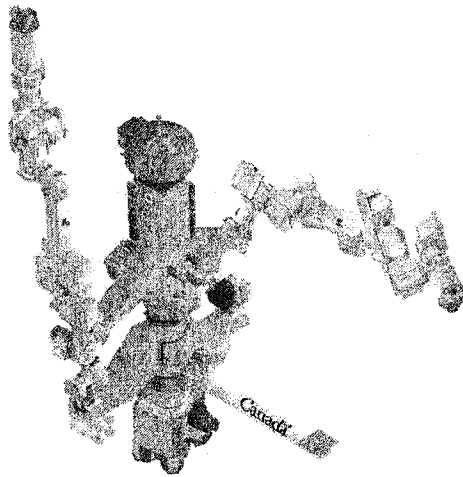


Figure 1.2: Special Purpose Dextrous Manipulator (SPDM or Canada Hand) (MD Robotics: <http://www.mdrobotics.ca>)

The Canadian Space Agency (CSA), under a Strategic Technologies for Automation and Robotics (STEAR) project, contracted International Submarine Engineering (ISE) to design and manufacture the CSA/ISE STEAR Testbed Manipulator (STM). The STM is the main component of the CSA Automation and Robotics Testbed (CART) (see Figure 1.3). The STM is a ground-based manipulator with arms kinematically similar to the arms of the SPDM. The STM is comprised of two 7-revolute joint serial manipulators attached to a fixed base. The original motivation for this research was to find the velocity-degenerate (singular) configurations of the STM.



Figure 1.3: CSA Automation and Robotics Testbed (CART) (Photo Courtesy of Ryan Fisher)

1.4 Background

1.4.1 Velocity-Degenerate Configurations of Robotic Manipulators

A velocity-degenerate (singular) configuration is a configuration in which a robotic manipulator has lost at least one motion DOF. In such a configuration, the manipulator is unable to execute an arbitrary instantaneous motion, i.e., the joints of the manipulator do not span the 6-system of spatial motion. These degenerate or singular configurations are often referred to as special configurations (Hunt, 1986 and 1987a). The determination of velocity-degenerate configurations is critical to understanding a robotic manipulator's kinematics and can be important in the implementation of a controller for the manipulator.

1.4.2 Velocity-Degenerate Configurations of Non-Redundant Serial Manipulators

Inverse Velocity Solution of Non-Redundant Serial Manipulators

The inverse velocity problem of a manipulator, given the desired velocity of the end-effector what are the twist amplitudes (joint rates) required to achieve the desired velocity, can be solved using screws¹. For 6-DOF motion, assuming the manipulator is in a non-degenerate configuration, the inverse velocity solution can be expressed as:

$$\dot{\mathbf{t}} = [\mathcal{J}]^{-1}\mathbf{V} \quad (1.1)$$

where $\dot{\mathbf{t}}$ is the vector of twist amplitudes, $[\mathcal{J}]$ is the 6x6 matrix of unit joint screw coordinates (also referred to in the literature as the Jacobian matrix), and \mathbf{V} is the desired end-effector velocity.

Some non-redundant serial manipulator controllers rely on the inverse of the matrix of unit joint screw coordinates to compute the twist amplitudes of the manipulator. If the manipulator is in a velocity-degenerate configuration, the matrix $[\mathcal{J}]$ goes singular, the controller will fail, and the joint rates will become infinite (Wang and Waldron, 1987). It is therefore critical to know the velocity-degenerate configurations of robotic manipulators.

¹See Appendix A for a review of manipulator kinematics using screws.

Identification of Velocity-Degenerate Configurations of Non-Redundant Serial Manipulators

Several methodologies exist for the determination of velocity degeneracies of non-redundant manipulators. The most common method is setting the determinant of the matrix of unit joint screw coordinates to zero ($|\$| = 0$) to determine the degenerate configurations (Gorla, 1981; Lipkin and Duffy, 1982; Paul and Stevenson, 1983; Waldron, Wang, and Bolin, 1985; and Hunt, 1987b). This method only works for non-redundant manipulators. For kinematically redundant manipulators, the matrix of unit joint screw coordinates is not square and thus taking the determinant of $\$$ is not possible.

Other methods have been proposed to determine velocity-degenerate configurations of non-redundant manipulators. Lai and Yang (1986) developed a method that is based on the concepts of service sphere, free service region, and dextrous wrist to determine the velocity-degenerate configurations of simple robots. The authors define a simple robot as robot with a closed-form solution to the inverse kinematic problem.

Ahmad and Luo (1988) developed a method to solve the singularities of the general manipulator with a spherical-wrist by analyzing the triangular equations associated with the manipulator's inverse kinematics.

Lipkin and Pohl (1991) proposed a methodology for enumerating all the velocity-degenerate configurations of a manipulator using vector quantities instead of using a joint angle approach.

Tourassis and Ang, Jr. (1992) developed a method for determining velocity-degenerate configurations of a manipulator based on the singularities of the main-arm

and wrist sub-assemblies.

1.4.3 Velocity-Degenerate Configurations of Redundant Serial Manipulators

Methods for Resolving the Inverse Kinematics of Redundant Manipulators

For redundant manipulators, an infinity of possible solutions exist to the inverse kinematics problem. For a redundant manipulator the matrix of unit joint screw coordinates is non-square ($[\$]_{6 \times n}$ where $n > 6$), therefore, equation (1.1) cannot be used to solve for the joint rates of a redundant manipulator. Whitney (1969) proposed using the Moore-Penrose generalized (pseudo) inverse of $[\$]$ to solve the inverse velocity problem of redundant serial manipulators. The pseudo-inverse of the matrix of unit joint screw coordinates, $[\$]^+$, is given by:

$$[\$]^+ = [\$]^T ([\$][\$]^T)^{-1} \quad (1.2)$$

The twist amplitudes can then be found from:

$$\dot{\mathbf{t}} = [\$]^+ \mathbf{V} \quad (1.3)$$

Using the pseudo inverse of the matrix of unit joint screw coordinates to solve the inverse velocity problem approximately minimizes the kinetic energy of the manipulator (Whitney, 1969).

Numerous other methods exist for resolving redundancy. The following represent a sample of the works that can be found in the literature.

Yoshikawa (1984) developed a manipulability measure for serial manipulators and used this measure to develop an algorithm to control redundant manipulators that

avoided singularities. Yoshikawa defined his measure of manipulability, w , as:

$$w = \sqrt{|[\$][\$]^T|} \quad (1.4)$$

Baillieul (1985) proposed the extended Jacobian technique to resolve the kinematics of redundant manipulators. As the name implies, the technique works by adding additional conditions to extend the Jacobian to make it square and thus invertible (provided the extended Jacobian is non-singular). Extending the Jacobian introduces the possibilities of algorithmic singularities. An algorithmic singularity is a singularity of the extended Jacobian which is not associated with the non-extended Jacobian losing rank.

Angeles and Habib (1985) proposed two numerical schemes for solving the kinematics of redundant manipulators as a nonlinear programming problem. The first method proposed formulating the problem as an overdetermined nonlinear algebraic system and using a weighted least-square approximation to solve the problem. The second method proposed consisted of first solving the existing undetermined nonlinear algebraic problem using a Newton-type method and then searching along the projection of the gradient of the performance index on the null space of the Jacobian.

Hollerbach and Suh (1987) resolved the kinematics of redundant manipulators through torque optimization.

Klein and Blaho (1987) compared four different measures to quantify the dexterity of a redundant manipulator and showed how they can be used to compute the optimal pose and optimal working points of redundant manipulators. The four measures they looked at were: determinant of the Jacobian; condition number of the Jacobian; minimum singular value of the Jacobian; and the joint range availability.

Ghosal and Roth (1988) showed how redundancy can be used to alter the first-

order properties of point trajectories and alter the transmission ratio measure $\sqrt{|[\$]^T[\$]|}$. Note that Ghosal and Roth's transmission ratio measure differs from Yoshikawa's manipulability measure (equation (1.4)) in the order in which the matrices are multiplied.

Kazerounian and Wang (1988) developed global optimization formulations of the joint rates and the kinetic energy of redundant manipulators.

Long and Paul (1992) proposed a 8-revolute joint manipulator that consists of a 4-revolute joint positioning sub-system and a 4-revolute joint orientating sub-system. In primary mode, the controllers Long and Paul developed for each sub-system use only three of the four joints. If a sub-system cannot track the desired motion due to a singularity, the controller for the sub-system experiencing difficulty switches to secondary mode and the fourth joint of the sub-system is called in to action. The system ensures that the manipulator can always span the 6-system of spatial motion.

Chen and Lin (1998) formulated a constrained optimization problem to plan the motions of redundant manipulators.

Ding, Ong, and Poo (2000) maximized the shortest distance to obstacles to resolve the kinematics of redundant manipulators at the joint position level.

Carignan and Howard (2000) proposed a partitioned redundancy management scheme for 8-revolute joint manipulators that incorporate a spherical wrist of four intersecting joints.

Identification of Velocity-Degenerate Configurations of Redundant Serial Manipulators

For a redundant manipulator, singularities of the pseudo inverse of $[\$]$ can be examined to resolve velocity-degenerate configurations of redundant manipulators. Velocity-

degenerate configurations occur when the determinant of the $[\$][\$]^T$ portion of $[\$]^+$ is equal to zero (Luh and Gu, 1985). Although the matrix formed by $[\$][\$]^T$ is a square matrix, the form of expressions for its elements can be unwieldy. The resulting expression for $|\$[\$]^T|$ can be difficult to simplify and analytical solutions to the velocity-degeneracy problem can be hard to find.

Other methods for dealing with the problem of resolving velocity-degenerate configurations of redundantly actuated serial manipulators have been proposed. Litvin and Parenti Castelli (1985) and Litvin et al. (1985 and 1986) used derivatives of displacement functions to form Jacobian matrices of manipulators and considered singularity of the determinants of the Jacobians to identify special configurations. The methodology works for both non-redundant and redundant manipulators.

Podhorodeski, Fenton, and Goldenberg (1989) and Podhorodeski, Goldenberg, and Fenton (1991 and 1993) applied a decomposition method to identify the degeneracies of redundant manipulators. The method requires multiple Gram-Schmidt type decompositions to identify all singularities of a redundant manipulator. The proposed method is difficult to apply beyond kinematically-simple (spherical-wristed) redundant manipulators.

Duffy and Crane III (1989), Nokleby and Podhorodeski (2000a), and Podhorodeski, Nokleby, and Wittchen (2000) used 6-joint sub-groups of $[\$]$ to determine the velocity-degenerate configurations of redundant manipulators performing a 6-DOF task. Configurations that cause the determinants of all possible 6-joint sub-groups to simultaneously equal zero are velocity-degenerate configurations (Sugimoto, Duffy, and Hunt, 1982). This methodology works well for 7-joint manipulators since only seven unique 6-joint sub-groups exist. For an 8-joint manipulator, 28 6-joint sub-groups exist and for a 9-joint manipulator, 84 6-joint sub-groups exist. It is clear that the method-

ology does not work well for manipulators with higher degrees of redundancy due to the large number of conditions that must be checked to ensure that all the 6-joint sub-group determinants are simultaneously zero.

Kreutz-Delgado, Long, and Seraji (1990 and 1992) used a combination of finding conditions that cause a vector of cofactors of the Jacobian to be zero and looking for row and column dependencies of the Jacobian to determine the velocity-degenerate configurations of 7-joint manipulators.

Burdick (1995) developed a recursive algorithm that identifies all singular configurations of revolute-only redundant manipulators. This methodology does not require the formulation of the determinant of $[\$]$. The methodology is based on reciprocity of screws. This is a substantial work, but it has been reported that implementation of the methodology for the symbolic (analytical) case rapidly becomes complex and that identification of velocity-degenerate configurations using numerical results from the algorithm is difficult (Williams II, 1998).

Royer, Bidard, and Androit (1998) used kinematic geometry to find the velocity-degenerate configurations of a 7-joint anthropomorphic manipulator.

Cheng and Kazerounian (2000) determined the singular configurations of the 7-joint anthropomorphic manipulator by studying the manipulator geometrically. They state that a singularity will occur when two revolute joint axes become collinear. This statement is true for non-redundant manipulators, but is not always true for redundant manipulators. The basis of the author's analysis is fundamentally flawed and leads to erroneous statements about the nature of singular configurations of redundant manipulators.

Dupuis (2001) and Dupuis, Papadopoulos, and Hayward (2001) developed a singular vector method for computing the rank-deficiency loci of rectangular Jacobians.

This is a reformulation of the reciprocity-based method presented in Chapter 2 into linear algebra terms. The authors note that the method has an advantage over the reciprocity-based methodology because, in addition to dealing with the case of a Jacobian with more columns than rows (i.e., a redundantly-actuated manipulator), it can handle the case where the Jacobian has more rows than columns. This latter case concerns under-actuated manipulators, i.e., manipulators that have less than the six joints required for 6-DOF spatial motion. An under-actuated manipulator is a special case that, in general, is not very commonly encountered.

1.4.4 Pose Optimization of Serial Manipulators Using Knowledge of Their Velocity-Degenerate Configurations

There has been a great deal of work concerning how to deal with velocity-degenerate configurations. Whitney (1972) proposed two methods for dealing with velocity-degenerate configurations. The first method was when a manipulator is at or near a velocity-degenerate configuration to use only the non-degenerate portion of the Jacobian to calculate a solution for the obtainable motions and neglect the unobtainable motions. Whitney's other proposal was to add additional joints to overcome velocity degeneracies when the original six joints of the manipulator become degenerate.

Wampler II (1986) developed a damped least-squares formulation for the inverse kinematics of a manipulator near a velocity-degenerate configuration. Nakamura and Hanafusa (1986) developed a weighted least-squares formulation as an alternative to using the pseudo-inverse at or near velocity-degenerate configurations.

Hunt (1986 and 1987a) showed how motions to escape from velocity-degenerate

configurations can be planned using knowledge of the reciprocal screw² quantities associated with a velocity-degenerate configuration. Podhorodeski (1993) also used knowledge of the reciprocal screw quantities of a velocity-degenerate configuration to plan best feasible motions from velocity-degenerate configurations.

Angeles et al. (1988) developed an algorithm for solving numerically the inverse kinematics of a manipulator at velocity-degenerate configurations. The developed algorithm eliminates the problem of branch switching associated with velocity-degenerate configurations and thus the algorithm is suitable for continuous path applications.

Gutman, Lee, and D'Costa (1991) proposed a phantom DOF strategy. The phantom DOF exists only in the manipulator model and is activated at singular configurations to eliminate singularity problems associated with the controller.

Chevallereau (1998) developed a method for determining the feasible trajectories of a manipulator starting from a 1-DOF motion loss singularity.

There has been less work with how to exploit velocity-degenerate configurations to achieve useful benefits. Hunt (1978 and 1986) and Wang and Waldron (1987) were the first to present the idea of exploiting the structural loading characteristics of a manipulator near velocity-degenerate configurations. In a velocity-degenerate configuration a manipulator becomes structural to a wrench or wrenches acting in a certain direction or directions (the number of directions is equivalent to the number of DOF of motion loss).

Wang and Waldron (1987) note that for an application that requires a drill to be

²Sugimoto, Duffy, and Hunt (1982) showed that in a velocity-degenerate configuration there exists a screw that is reciprocal to all joint screws (see Section A.1.2 for a definition of a reciprocal screw). This concept is germane to finding twist and wrench annihilators as proposed by Angeles (1994).

held in place while a piece is fed into the drill, it would be desirable to align the axis of the drill with the axis of the reciprocal screw. With the drill and reciprocal screw axes aligned, the required joint torques to hold the drill in place would be reduced to zero provided the pitch of the reciprocal screw is the ratio of the drilling torque to the feed force.

Kieffer and Lenarčič (1994) did an analysis of how humans use singular configurations of their limbs to exploit mechanical advantage of near degenerate configurations. They show that minimization of the joint torques in a redundant manipulator leads to behavior similar to humans in the exploitation of mechanical advantage of near degenerate configurations. Kieffer and Lenarčič note that the common thinking on velocity-degenerate configurations is that they should be avoided (e.g., see Yoshikawa, 1984; Luh and Gu, 1985; Stanišić and Pennock, 1985; Gutman, Lee, and D'Costa, 1991; Chiaverini, Siciliano, and Egeland, 1991; Long and Paul, 1992; Tchoń and Matuszok, 1995; Beiner, 1997; and Chen and Lin, 1998), but question this conclusion based on their examples of humans using velocity-degenerate configurations to gain mechanical advantage.

Researchers have considered various methods for optimizing the pose of serial manipulators. Togai (1986) used the condition number of the Jacobian as a measure of how close a manipulator is to a velocity-degeneracy and used this measure to determine optimal poses of a manipulator for kinematic manipulability.

Chiu (1987) presented an index for measuring the compatibility of a redundant serial manipulator's pose with respect to fine and coarse manipulation tasks. The author states that the optimization of this index is an effective way of utilizing redundancy. The author also notes that the resulting motions using the index for a manipulator performing a task are similar to the motions a human makes to perform

a similar task.

Papadopoulos and Gonthier (1999) used a min-max optimization scheme to plan redundant manipulator postures during large force tasks.

Zha (2002) used a genetic algorithm to optimize the pose of a manipulator to achieve good kinematics and dynamics performance.

None of the above methods exploit the knowledge of the manipulator's velocity-degenerate configurations to improve the solution to the problem of optimizing the pose of the manipulator to sustain a desired wrench.

1.5 Outline of Dissertation

The following is a brief outline of the remaining chapters of the dissertation.

1.5.1 Chapter 2: Identification of 1-DOF-Loss Velocity-Degenerate Configurations of Kinematically-Redundant Manipulators

In Chapter 2, a novel reciprocity-based method for identifying 1-DOF-loss velocity-degenerate configurations of kinematically-redundant manipulators is presented. The developed methodology uses the properties of reciprocal screws to determine the 1-DOF-loss velocity-degenerate configurations of kinematically-redundant manipulators. A by-product of the methodology is that a reciprocal screw related to the lost motion DOF for each degenerate configuration is determined.

1.5.2 Chapter 3: Identification of Multi-DOF-Loss Velocity-Degenerate Configurations of Kinematically-Redundant Manipulators

In Chapter 3, the reciprocity-based method for identifying 1-DOF-loss velocity degeneracies presented in Chapter 2 is extended to the case of identifying multi-DOF-loss velocity-degenerate configurations of kinematically-redundant manipulators. The developed methodology uses the properties of reciprocal screws to determine the degenerate configurations of kinematically-redundant manipulators. As with the 1-DOF-loss methodology, a by-product of the multi-DOF-loss methodology is that reciprocal screws related to the lost motion DOFs for each degenerate configuration are determined.

1.5.3 Chapter 4: Pose Optimization of Serial Manipulators Using Knowledge of Their Velocity-Degenerate Configurations

In Chapter 4, the utilization of velocity-degenerate configurations to optimize the pose of either non-redundant or redundant serial manipulators to sustain desired wrenches is considered. An algorithm is developed that determines a desirable start point for the optimization of a serial manipulator's pose. The start-point algorithm (SPA) uses the knowledge of the velocity-degenerate configurations of a serial manipulator to determine a pose that would be best suitable to sustain a desired wrench.

1.5.4 Chapter 5: Conclusions and Recommendations for Future Work

In Chapter 5, conclusions about the dissertation will be presented. In addition, recommendations will be made about possible areas where future research could be conducted.

Chapter 2

Identification of 1-DOF-Loss

Velocity-Degenerate

Configurations of

Kinematically-Redundant

Manipulators

2.1 Overview

In this chapter, a reciprocity-based method for identifying 1-DOF (degree-of-freedom) loss velocity-degenerate (singular) configurations of kinematically-redundant manipulators is presented. The developed methodology uses the properties of reciprocal screws to determine the 1-DOF-loss velocity-degenerate configurations of kinematically-redundant manipulators. A by-product of the methodology is that a reciprocal screw

related to the lost motion DOF for each degenerate configuration is determined. Numerous examples are presented to demonstrate the effectiveness of the reciprocity-based methodology. The chapter finishes with a discussion of the new method for determining the 1-DOF-loss velocity-degenerate configurations of kinematically-redundant manipulators.

2.2 Methodology for Identification of 1-DOF-Loss Velocity-Degenerate Configurations¹

Assuming a spatial task, i.e., 6-DOF, a serial manipulator is redundant if the number of joints (n) is greater than six ($n > 6$). Six joint screws ($\$_{sub_1}, \$_{sub_2}, \dots, \$_{sub_6}$) are chosen to form a 6-joint sub-group matrix of unit joint screw coordinates, $[\$]_{sub}$. Note that these six joints are chosen such that they are not inherently linearly dependent. This leaves $n - 6$ joint screws that can be considered as redundant joint screws $\$_{r_1}, \$_{r_2}, \dots, \$_{r_{(n-6)}}$. Setting the determinant of $[\$]_{sub}$ to zero ($|[\$]_{sub}| = 0$) allows all conditions (say α in total) that cause the 6-joint sub-group to become velocity degenerate to be identified (Gorla, 1981 and Waldron, Wang, and Bolin, 1985).

In a velocity-degenerate configuration there exists a screw (\mathbf{W}_{recip}) that is reciprocal to all joint screws (Sugimoto, Duffy, and Hunt, 1982), i.e.:

$$\mathbf{W}_{recip} \otimes \$_j = 0, \text{ for } j = 1 \text{ to } n \quad (2.1)$$

where \otimes denotes a reciprocal product between two screws (see equation (A.5) of Appendix A), $\$_j$ is the j^{th} joint screw and n is the total number of joints. Applying

¹The results contained in this section have been presented in Nokleby and Podhorodeski (2000a, 2000b, and 2001a).

this to the six joints that comprise $[\$]_{sub}$, reciprocal screws, $\mathbf{W}_{recip_1}, \mathbf{W}_{recip_2}, \dots, \mathbf{W}_{recip_\alpha}$, can be found for each of the α velocity-degeneracy conditions using linear algebra techniques. The reciprocal screw, \mathbf{W}_{recip_k} , is reciprocal to the six joints that comprise $[\$]_{sub}$ when the i^{th} $[\$]_{sub}$ degeneracy condition is true, but will not necessarily be reciprocal to the redundant joints $\$_{r_1}, \$_{r_2}, \dots, \$_{r_{(n-6)}}$. The redundant joints may still allow the manipulator to span the 6-system of general spatial motion. In general, additional conditions will be required to cause \mathbf{W}_{recip_i} to be reciprocal to all of the redundant joints $\$_{r_1}, \$_{r_2}, \dots, \$_{r_{(n-6)}}$.

Taking reciprocal products of \mathbf{W}_{recip_1} and each redundant joint $\$_{r_1}, \$_{r_2}, \dots, \$_{r_{(n-6)}}$ and setting the results to zero:

$$\begin{aligned} \mathbf{W}_{recip_1} \otimes \$_{r_1} &= 0 \\ \mathbf{W}_{recip_1} \otimes \$_{r_2} &= 0 \\ &\vdots \\ \mathbf{W}_{recip_1} \otimes \$_{r_{(n-6)}} &= 0 \end{aligned} \tag{2.2}$$

yields all additional conditions necessary for \mathbf{W}_{recip_1} to be reciprocal to all of the redundant joints $\$_{r_1}, \$_{r_2}, \dots, \$_{r_{(n-6)}}$, simultaneously. The first condition causing $[\$]_{sub}$ to be degenerate combined with the additional conditions identified through the reciprocal products of equations (2.2) defines sets of conditions causing the redundant manipulator to be velocity degenerate.

The above procedure is repeated for $\mathbf{W}_{recip_2}, \mathbf{W}_{recip_3}, \dots, \mathbf{W}_{recip_\alpha}$. The procedure identifies all sets of conditions (say β in total) that result in the redundant-manipulator joints $\$_{sub_1}, \$_{sub_2}, \dots, \$_{sub_6}, \$_{r_1}, \$_{r_2}, \dots, \$_{r_{(n-6)}}$ becoming degenerate. As a by-product, a reciprocal screw related to the lost motion DOF is generated for each of the β velocity degeneracies, i.e., \mathbf{W}_1 to \mathbf{W}_β are identified.

2.3 Examples

To illustrate the developed methodology being applied to the problem of determining 1-DOF-loss velocity-degenerate configurations for kinematically-redundant manipulators, four different manipulators will be used. The first two manipulators are both 7-joint manipulators with spherical wrist layouts (a spherical-revolute-spherical manipulator and a double-elbow manipulator). The next manipulator is the CSA/ISE STEAR Testbed Manipulator (STM). Specifically the 7-joint STM-1 arm will be used. The last manipulator will be the 8-joint NASA Advanced Research Manipulator II (ARMII).

2.3.1 Spherical-Revolute-Spherical Manipulator²

Hollerbach (1985) demonstrated that the “optimal” layout of a 7-joint manipulator for the elimination of singularities is R⊥R⊥R⊥R⊥R⊥R⊥R, where ⊥ refers to two successive joints being perpendicular to one another. Note that the first three joints and the last three joints both form spherical groups. The Denavit and Hartenberg (D&H) parameters (Denavit and Hartenberg, 1955) used to model the manipulator are presented in Table 2.1. The parameters of Table 2.1 correspond to the link transformations:

$${}^j_{j-1}\mathbf{T} = Rot_{\hat{\mathbf{z}}_{j-1}}(\theta_j) Trans_{\hat{\mathbf{z}}_{j-1}}(d_j) Trans_{\hat{\mathbf{x}}_j}(a_j) Rot_{\hat{\mathbf{x}}_j}(\alpha_j) \quad (2.3)$$

where ${}^j_{j-1}\mathbf{T}$ is a homogeneous transformation describing the location and orientation of link-frame F_j with respect to link-frame F_{j-1} , $Rot_{\hat{\mathbf{z}}_{j-1}}(\theta_j)$ denotes a rotation about

²The results contained in this section have been presented in Nokleby and Podhorodeski (2000a, 2000b, and 2001a).

the $\hat{\mathbf{z}}_{j-1}$ axis by θ_j , $Trans_{\hat{\mathbf{z}}_{j-1}}(d_j)$ denotes a translation along the $\hat{\mathbf{z}}_{j-1}$ axis by d_j , $Trans_{\hat{\mathbf{x}}_j}(a_j)$ denotes a translation along the $\hat{\mathbf{x}}_j$ axis by a_j , and $Rot_{\hat{\mathbf{x}}_j}(\alpha_j)$ denotes a rotation about the $\hat{\mathbf{x}}_j$ axis by α_j (Paul, 1981). Figure 2.1 shows the zero-displacement configuration of the manipulator.

Table 2.1: Denavit and Hartenberg Parameters for the Spherical-Revolute-Spherical Manipulator

F_{j-1}	θ_j	d_j	a_j	α_j	F_j
F_0	θ_1	0	0	$\frac{\pi}{2}$	F_1
F_1	θ_2	0	0	$-\frac{\pi}{2}$	F_2
F_2	θ_3	g	0	$\frac{\pi}{2}$	F_3
F_3	θ_4	0	0	$-\frac{\pi}{2}$	F_4
F_4	θ_5	h	0	$\frac{\pi}{2}$	F_5
F_5	θ_6	0	0	$-\frac{\pi}{2}$	F_6
F_6	θ_7	0	0	0	F_7

Waldron, Wang, and Bolin (1985) demonstrated that the choice of an appropriate translational velocity reference point and reference orientation can greatly simplify the joint screw coordinate terms. Soylu and Duffy (1998) identified frames of reference for a manipulator that yielded the minimum total number of terms of the elements of the matrix of unit joint screw coordinates. Choosing a reference frame that is located at the intersection of the wrist spherical group and oriented with z_{ref} in the direction of \mathcal{S}_5 and y_{ref} in the opposite direction to that of \mathcal{S}_4 allows the joint screws

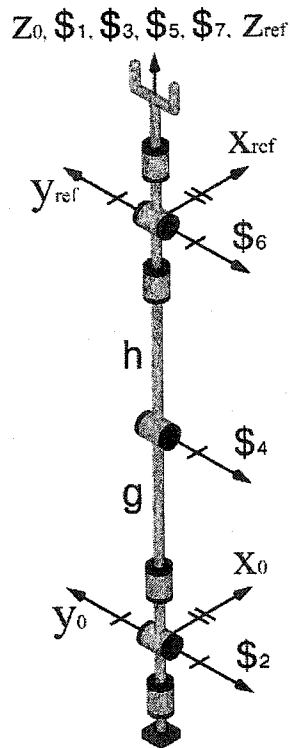


Figure 2.1: Zero-Displacement Configuration of the Spherical-Revolute-Spherical Manipulator

to be found as:

$$\begin{aligned}
 {}^{ref}\$1 &= \left\{ \begin{array}{c} s_2 c_3 c_4 + c_2 s_4 \\ -s_2 s_3 \\ -s_2 c_3 s_4 + c_2 c_4 \\ -s_2 s_3 (c_4 g + h) \\ -s_2 c_3 (g + c_4 h) - c_2 s_4 h \\ s_2 s_3 s_4 g \end{array} \right\} \\
 {}^{ref}\$2 &= \left\{ -c_4 s_3, -c_3, s_3 s_4; -c_3 (c_4 g + h), s_3 (g + c_4 h), c_3 s_4 g \right\}^T \\
 {}^{ref}\$3 &= \left\{ s_4, 0, c_4; 0, -s_4 h, 0 \right\}^T \\
 {}^{ref}\$4 &= \left\{ 0, -1, 0; -h, 0, 0 \right\}^T \\
 {}^{ref}\$5 &= \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\
 {}^{ref}\$6 &= \left\{ s_5, -c_5, 0; 0, 0, 0 \right\}^T \\
 {}^{ref}\$7 &= \left\{ -c_5 s_6, -s_5 s_6, c_6; 0, 0, 0 \right\}^T
 \end{aligned} \tag{2.4}$$

where $c_{ij} = \cos(\theta_i + \theta_j)$ and $s_{ij} = \sin(\theta_i + \theta_j)$. The matrix of unit joint screw coordinates for the manipulator is:

$${}^{ref}[\$] = {}^{ref} \left[\begin{array}{cccccc} \$1 & \$2 & \$3 & \$4 & \$5 & \$6 & \$7 \end{array} \right] \tag{2.5}$$

Select $\$2, \$3, \$4, \$5, \$6,$ and $\$7$ to form $[\$]_{sub}$:

$${}^{ref}[\$]_{sub} = {}^{ref} \left[\begin{array}{cccccc} \$2 & \$3 & \$4 & \$5 & \$6 & \$7 \end{array} \right] \tag{2.6}$$

with the redundant joint being $\$1$.

The determinant of $[\$]_{sub}$ is:

$$|{}^{ref}[\$]_{sub}| = -c_3 s_4^2 s_6 g h^2 \tag{2.7}$$

Therefore, if a) $s_4 = 0$, b) $c_3 = 0$, or c) $s_6 = 0$, then the six joints comprising $[\$]_{sub}$ define a degenerate sub-group of screws. Degenerate configurations of the 7-joint arm will include one of these three conditions. Additional conditions required can be found by enforcing reciprocity of $\$1$ with screws characterizing the lost motion DOF for each of the $[\$]_{sub}$ degenerate conditions.

a) Setting $s_4 = 0$ in equation (2.6) yields:

$${}^{ref}[\$]_{sub_a} = \begin{bmatrix} -c_4 s_3 & 0 & 0 & 0 & s_5 & -c_5 s_6 \\ -c_3 & 0 & -1 & 0 & -c_5 & -s_5 s_6 \\ 0 & c_4 & 0 & 1 & 0 & c_6 \\ -c_3(c_4 g + h) & 0 & -h & 0 & 0 & 0 \\ s_3(g + c_4 h) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.8)$$

The reciprocal screw for the six joints comprising $[\$]_{sub}$ with $s_4 = 0$ can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_a} = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \quad (2.9)$$

Note that \mathbf{W}_{recip_a} is not unique. In a 1-DOF-loss degenerate configuration the joint screws span a 5-system and therefore there is an infinity of possible reciprocal screw quantities. These reciprocal screw quantities are all scalar multiples of one another, the one of equation (2.9) being the case of unit screw coordinates.

Taking the reciprocal product between \mathbf{W}_{recip_a} and $\$1$ and setting the result to zero yields:

$${}^{ref}\mathbf{W}_{recip_a} \oplus {}^{ref}\$1 = s_2 s_3 s_4 g = 0 \quad (2.10)$$

Since $s_4 = 0$, no further conditions are necessary to make \mathbf{W}_{recip_a} reciprocal to joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$. Therefore, $s_4 = 0$ defines a 1-condition family of degenerate configurations.

b) Setting $c_3 = 0$ in equation (2.6) yields:

$${}^{ref}\$]_{sub_b} = \begin{bmatrix} -c_4 s_3 & s_4 & 0 & 0 & s_5 & -c_5 s_6 \\ 0 & 0 & -1 & 0 & -c_5 & -s_5 s_6 \\ s_3 s_4 & c_4 & 0 & 1 & 0 & c_6 \\ 0 & 0 & -h & 0 & 0 & 0 \\ s_3 (g + c_4 h) & -s_4 h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.11)$$

The reciprocal screw for the six joints comprising $[\$]_{sub}$ with $c_3 = 0$ can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \quad (2.12)$$

Taking the reciprocal product between \mathbf{W}_{recip_b} and $\$1$ and setting the result to zero yields:

$${}^{ref}\mathbf{W}_{recip_b} \otimes {}^{ref}\$1 = s_2 s_3 s_4 g = 0 \quad (2.13)$$

Thus, if $s_2 = 0$ & $c_3 = 0$ or $c_3 = 0$ & $s_4 = 0$, \mathbf{W}_{recip_b} is reciprocal to joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$. It was shown in Section 2.3.1a that $s_4 = 0$ alone results in a degenerate configuration, therefore, $c_3 = 0$ & $s_4 = 0$ does not represent a new degenerate configuration. However, $s_2 = 0$ & $c_3 = 0$ defines a new 2-condition family of degenerate configurations.

c) Setting $s_6 = 0$ in equation (2.6) yields:

$${}^{ref}\mathbb{[\$]}_{sub_c} = \begin{bmatrix} -c_4s_3 & s_4 & 0 & 0 & s_5 & 0 \\ -c_3 & 0 & -1 & 0 & -c_5 & 0 \\ s_3s_4 & c_4 & 0 & 1 & 0 & c_6 \\ -c_3(c_4g + h) & 0 & -h & 0 & 0 & 0 \\ s_3(g + c_4h) & -s_4h & 0 & 0 & 0 & 0 \\ c_3s_4g & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.14)$$

Let $\mathbf{W}_{recip_c} = \{ L, M, N; P, Q, R \}^T$. Setting ${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{[\$]}_j = 0$, for $j = 2$ to 7, with $s_6 = 0$ yields:

$$\begin{aligned} & {}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{[\$]}_2 \\ & = -Lc_3(c_4g + h) + Ms_3(g + c_4h) + Nc_3s_4g - Pc_4s_3 - Qc_3 + Rs_3s_4 = 0 \\ & {}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{[\$]}_3 = -Ms_4 + Ps_4 + Rc_4 = 0 \\ & {}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{[\$]}_4 = -Lh - Q = 0 \\ & {}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{[\$]}_5 = R = 0 \\ & {}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{[\$]}_6 = Ps_5 - Qc_5 = 0 \\ & {}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{[\$]}_7 = Rc_6 = 0 \end{aligned} \quad (2.15)$$

A free choice exists for one of the values of the reciprocal screw \mathbf{W}_{recip_c} since an infinity of linearly dependent reciprocal screws exists for a 1-DOF velocity-degenerate configuration. Choosing $Q = s_5h$ and solving the system of equations defined in (2.15) allows \mathbf{W}_{recip_c} to be found as:

$${}^{ref}\mathbf{W}_{recip_c} = \left\{ -s_5, c_5, \frac{-c_3c_4s_5 - s_3c_5}{c_3s_4}; c_5h, s_5h, 0 \right\}^T \quad (2.16)$$

Taking the reciprocal product between \mathbf{W}_{recip_c} and $\mathbb{[\$]}_1$ and setting the result to zero

yields:

$${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathcal{S}_1 = -s_2 s_4 c_5 g = 0 \quad (2.17)$$

Thus, if $s_2 = 0$ & $s_6 = 0$, $s_4 = 0$ & $s_6 = 0$, or $c_5 = 0$ & $s_6 = 0$, \mathbf{W}_{recip_c} is reciprocal to joints \mathcal{S}_1 , \mathcal{S}_2 , \mathcal{S}_3 , \mathcal{S}_4 , \mathcal{S}_5 , \mathcal{S}_6 , and \mathcal{S}_7 . It was shown in Section 2.3.1a that $s_4 = 0$ alone results in a degenerate configuration, therefore, $s_4 = 0$ & $s_6 = 0$ does not represent a new family of degenerate configurations. However, $s_2 = 0$ & $s_6 = 0$ and $c_5 = 0$ & $s_6 = 0$ define two new 2-condition families of degenerate configurations.

Examining all of the degenerate configurations yields four sets of conditions (one set requiring the satisfaction of a single condition and three sets requiring the satisfaction of a pair of conditions) defining families of degenerate configurations resulting in a 1-DOF-loss for the $(R \perp R \perp R)^{sph} \perp R \perp (R \perp R \perp R)^{sph}$ manipulator. These degenerate configurations and their associated reciprocal screws can be summarized as:

- 1) $s_4 = 0$

$${}^{ref}\mathbf{W}_1 = {}^{ref}\mathbf{W}_{recip_a} = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T$$
- 2) $s_2 = 0$ & $c_3 = 0$

$${}^{ref}\mathbf{W}_2 = {}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T$$
- 3) $s_2 = 0$ & $s_6 = 0$

$${}^{ref}\mathbf{W}_3 = {}^{ref}\mathbf{W}_{recip_c} = \left\{ -s_5, c_5, \frac{-c_3 c_4 s_5 - s_3 c_5}{c_3 s_4}; c_5 h, s_5 h, 0 \right\}^T$$
- 4) $c_5 = 0$ & $s_6 = 0$

$${}^{ref}\mathbf{W}_4 = {}^{ref}\mathbf{W}_{recip_c} = \left\{ -s_5, c_5, \frac{-c_3 c_4 s_5 - s_3 c_5}{c_3 s_4}; c_5 h, s_5 h, 0 \right\}^T$$

(2.18)

Note that the condition sets outlined in (2.18) are identical to those obtained by Podhorodeski, Goldenberg, and Fenton (1991) using a sequential decomposition technique.

2.3.2 Double-Elbow Manipulator³

The double-elbow 7-joint manipulator layout is $R\perp R\parallel R\parallel R\perp R\perp R$, where \parallel refers to two successive joints being parallel to one another. Note that the first two joints form a pointer group (universal joint) and the last three joints form a spherical group. The D&H parameters used to model the manipulator are presented in Table 2.2. The parameters of Table 2.2 correspond to the link transformations defined in equation (2.3). Figure 2.2 shows the zero-displacement configuration of the manipulator.

Table 2.2: Denavit and Hartenberg Parameters for the Double-Elbow Manipulator

F_{j-1}	θ_j	d_j	a_j	α_j	F_j
F_0	θ_1	0	0	$\frac{\pi}{2}$	F_1
F_1	θ_2	$-f$	g	0	F_2
F_2	θ_3	0	h	0	F_3
F_3	θ_4	0	i	0	F_4
F_4	θ_5	0	0	$-\frac{\pi}{2}$	F_5
F_5	θ_6	0	0	$\frac{\pi}{2}$	F_6
F_6	θ_7	0	0	0	F_7

Choosing a reference frame that is located at the intersection of the wrist spherical

³The results contained in this section have been presented in Nokleby and Podhorodeski (2000b and 2001a).

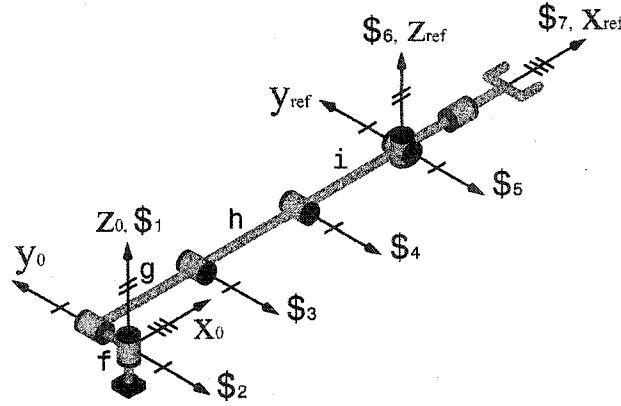


Figure 2.2: Zero-Displacement Configuration of the Double-Elbow Manipulator

group and oriented with x_{ref} in the direction of the final forearm and y_{ref} in the opposite direction of $\$5$ allows the joint screws to be found as:

$$\begin{aligned}
 {}^{ref}\$1 &= \left\{ s_{234}, 0, c_{234}; -c_{234}f, c_2g + c_{23}h + c_{234}i, s_{234}f \right\}^T \\
 {}^{ref}\$2 &= \left\{ 0, -1, 0; s_{34}g + s_4h, 0, c_{34}g + c_4h + i \right\}^T \\
 {}^{ref}\$3 &= \left\{ 0, -1, 0; s_4h, 0, c_4h + i \right\}^T \\
 {}^{ref}\$4 &= \left\{ 0, -1, 0; 0, 0, i \right\}^T \\
 {}^{ref}\$5 &= \left\{ 0, -1, 0; 0, 0, 0 \right\}^T \\
 {}^{ref}\$6 &= \left\{ -s_5, 0, c_5; 0, 0, 0 \right\}^T \\
 {}^{ref}\$7 &= \left\{ c_5s_6, -c_6, s_5s_6; 0, 0, 0 \right\}^T
 \end{aligned} \tag{2.19}$$

The matrix of unit joint screw coordinates for the manipulator is:

$${}^{ref}[\$] = {}^{ref} \begin{bmatrix} \$1 & \$2 & \$3 & \$4 & \$5 & \$6 & \$7 \end{bmatrix} \tag{2.20}$$

Select $\$1$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$ to form $[\$]_{sub}$:

$${}^{ref}[\$]_{sub} = {}^{ref} \begin{bmatrix} \$1 & \$3 & \$4 & \$5 & \$6 & \$7 \end{bmatrix} \quad (2.21)$$

with the redundant joint being $\$2$. Note that joints $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$ could not be selected to form $[\$]_{sub}$ because they are linearly dependent, i.e., those six joints are inherently velocity degenerate.

The determinant of $[\$]_{sub}$ is:

$$|{}^{ref}[\$]_{sub}| = -s_4 s_6 h i (c_{2g} + c_{23}h + c_{234}i) \quad (2.22)$$

Therefore, if a) $s_4 = 0$, b) $s_6 = 0$, or c) $c_{2g} + c_{23}h + c_{234}i = 0$, then the six joints comprising $[\$]_{sub}$ define a degenerate sub-group of screws. Degenerate configurations of the 7-joint arm will include one of these three conditions. Additional conditions required can be found by enforcing reciprocity of $\$2$ with screws characterizing the lost motion DOF for each of the $[\$]_{sub}$ degenerate conditions.

a) Setting $s_4 = 0$ in equation (2.21) yields:

$${}^{ref}[\$]_{sub_a} = \begin{bmatrix} s_{234} & 0 & 0 & 0 & -s_5 & c_5 s_6 \\ 0 & -1 & -1 & -1 & 0 & -c_6 \\ c_{234} & 0 & 0 & 0 & c_5 & s_5 s_6 \\ -c_{234}f & 0 & 0 & 0 & 0 & 0 \\ c_{2g} + c_{23}h + c_{234}i & 0 & 0 & 0 & 0 & 0 \\ s_{234}f & \pm h + i & i & 0 & 0 & 0 \end{bmatrix} \quad (2.23)$$

Setting ${}^{ref}\mathbf{W}_{recip_a} \otimes {}^{ref}\$j = 0$, for $j = 1, 3$ to 7 , with $s_4 = 0$ allows \mathbf{W}_{recip_a} to be found as:

$${}^{ref}\mathbf{W}_{recip_a} = \left\{ 1, \frac{c_{234}f}{c_{2g} + c_{23}h + c_{234}i}, 0; 0, 0, 0 \right\}^T \quad (2.24)$$

Taking the reciprocal product between \mathbf{W}_{recip_a} and $\$2$ and setting the result to zero yields:

$${}^{ref}\mathbf{W}_{recip_a} \otimes {}^{ref}\$2 = s_{34}g + s_4h = \pm s_3g = 0 \quad (2.25)$$

where the \pm is the result of the fact that for $s_4 = 0$, θ_4 can be equal to 0 or π . Thus, if $s_3 = 0$ & $s_4 = 0$, \mathbf{W}_{recip_a} is reciprocal to all of the joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$. Therefore, $s_3 = 0$ & $s_4 = 0$ defines a 2-condition family of degenerate configurations.

b) Setting $s_6 = 0$ in equation (2.21) yields:

$${}^{ref}[\$]_{sub_b} = \begin{bmatrix} s_{234} & 0 & 0 & 0 & -s_5 & 0 \\ 0 & -1 & -1 & -1 & 0 & \mp 1 \\ c_{234} & 0 & 0 & 0 & c_5 & 0 \\ -c_{234}f & s_4h & 0 & 0 & 0 & 0 \\ c_2g + c_{23}h + c_{234}i & 0 & 0 & 0 & 0 & 0 \\ s_{234}f & c_4h + i & i & 0 & 0 & 0 \end{bmatrix} \quad (2.26)$$

Setting ${}^{ref}\mathbf{W}_{recip_b} \otimes {}^{ref}\$j = 0$, for $j = 1, 3$ to 7 , with $s_6 = 0$ allows \mathbf{W}_{recip_b} to be found as:

$${}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, 1, 0; -\frac{c_5(c_2g+c_{23}h+c_{234}i)}{s_{2345}}, 0, -\frac{s_5(c_2g+c_{23}h+c_{234}i)}{s_{2345}} \right\}^T \quad (2.27)$$

Taking the reciprocal product between \mathbf{W}_{recip_b} and $\$2$ finds:

$${}^{ref}\mathbf{W}_{recip_b} \otimes {}^{ref}\$2 = 0 \quad (2.28)$$

Thus, if $s_6 = 0$, \mathbf{W}_{recip_b} is reciprocal to joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$ without any further conditions being required. Therefore, $s_6 = 0$ defines a 1-condition family of degenerate configurations.

c) Setting $c_2g + c_{23}h + c_{234}i = 0$ in equation (2.21) yields:

$${}^{ref}\$_{subc} = \begin{bmatrix} s_{234} & 0 & 0 & 0 & -s_5 & c_5s_6 \\ 0 & -1 & -1 & -1 & 0 & -c_6 \\ c_{234} & 0 & 0 & 0 & c_5 & s_5s_6 \\ -c_{234}f & s_4h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ s_{234}f & c_4h + i & i & 0 & 0 & 0 \end{bmatrix} \quad (2.29)$$

The reciprocal screw for the six joints comprising ${}_{subc}\$,$ with $c_2g + c_{23}h + c_{234}i = 0,$ can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_c} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (2.30)$$

Taking the reciprocal product between \mathbf{W}_{recip_c} and $\$_2$ and setting the result to zero yields:

$${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\$_2 = 0 \quad (2.31)$$

Thus, if $c_2g + c_{23}h + c_{234}i = 0,$ \mathbf{W}_{recip_c} is reciprocal to joints $\$_1, \$_2, \$_3, \$_4, \$_5, \$_6,$ and $\$_7$ without any further conditions being required. Therefore, $c_2g + c_{23}h + c_{234}i = 0$ defines a 1-condition family of degenerate configurations.

Examining all of the degenerate configurations yields three sets of conditions (two sets requiring the satisfaction of a single condition and one set requiring the satisfaction of a pair of conditions) defining families of degenerate configurations resulting in a single motion DOF loss for the $(R \perp R)^{pt} \parallel R \parallel (R \perp R \perp R)^{sph}$ manipulator. These degenerate configurations and their respective reciprocal screws can be summarized

as:

$$\begin{aligned}
 & 1) \quad s_6 = 0 \\
 & \quad \mathit{ref} \mathbf{W}_1 = \mathit{ref} \mathbf{W}_{recip_b} = \left\{ 0, 1, 0; -\frac{c_5(c_{2g}+c_{23}h+c_{234}i)}{s_{2345}}, 0, -\frac{s_5(c_{2g}+c_{23}h+c_{234}i)}{s_{2345}} \right\}^T \\
 & 2) \quad c_{2g} + c_{23}h + c_{234}i = 0 \\
 & \quad \mathit{ref} \mathbf{W}_2 = \mathit{ref} \mathbf{W}_{recip_e} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \\
 & 3) \quad s_3 = 0 \ \& \ s_4 = 0 \\
 & \quad \mathit{ref} \mathbf{W}_3 = \mathit{ref} \mathbf{W}_{recip_a} = \left\{ 1, \frac{c_{234}f}{c_{2g}+c_{23}h+c_{234}i}, 0; 0, 0, 0 \right\}^T
 \end{aligned} \tag{2.32}$$

Note that the condition sets outlined in (2.32) are identical to those obtained by Podhorodeski, Goldenberg, and Fenton (1993).

2.3.3 CSA/ISE STEAR Testbed Manipulator (STM)⁴

The Canadian Space Agency (CSA), under a Strategic Technologies for Automation and Robotics (STEAR) project, contracted International Submarine Engineering (ISE) to design and manufacture the STEAR Testbed Manipulator (STM). The STM is a ground-based manipulator with arms kinematically similar to the arms of the Special Purpose Dextrous Manipulator (SPDM) which is being built for the International Space Station (ISS). The analysis presented is for the 7-joint STM-1 or “right” arm, however, the analysis applies to the 7-joint STM-2 or “left” arm since

⁴The results contained in this section have been presented in Nokleby and Podhorodeski (2000c).

they are mirror images of one another.

The layout of the STM arms is $R\perp R\perp R\parallel R\parallel R\perp R\perp R$. The D&H parameters used to model the STM-1 are presented in Table 2.3. The parameters of Table 2.3 correspond to the link transformations defined in equation (2.3). Figure 2.3 shows the zero-displacement configuration of the manipulator.

Table 2.3: Denavit and Hartenberg Parameters for the STM-1 Manipulator

F_{j-1}	θ_j	d_j	a_j	α_j	F_j
F_0	θ_1	0	a	$\frac{\pi}{2}$	F_1
F_1	θ_2	b	c	$-\frac{\pi}{2}$	F_2
F_2	θ_3	0	d	0	F_3
F_3	θ_4	0	e	0	F_4
F_4	θ_5	f	g	$\frac{\pi}{2}$	F_5
F_5	θ_6	$-h$	k	$\frac{\pi}{2}$	F_6
F_6	θ_7	0	0	0	F_7

Choosing a reference frame to be a frame coincident with F_5 of the STM-1 was found to maximize the number of zero elements in the joint screws (Podhorodeski,

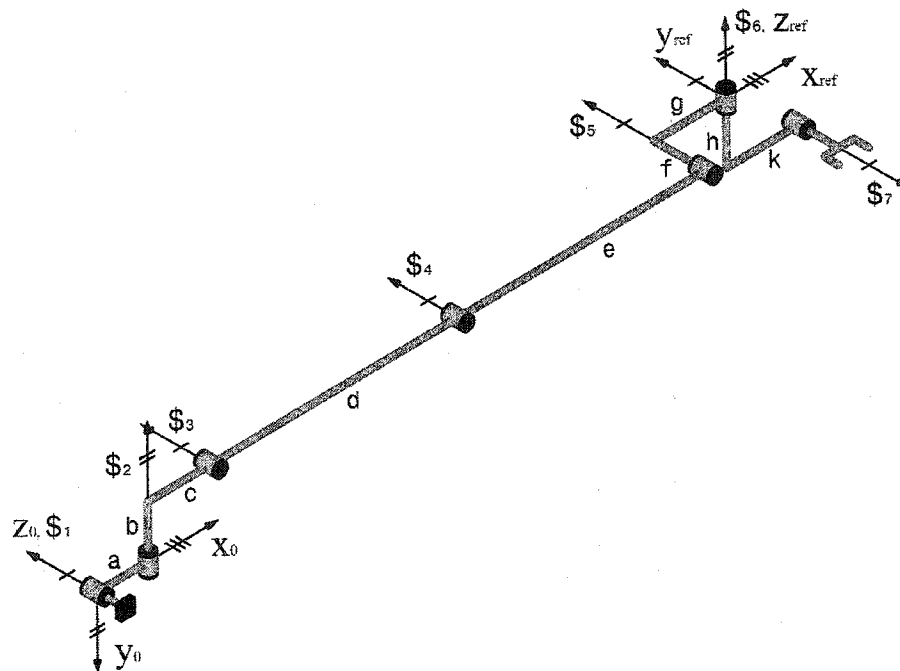


Figure 2.3: Zero-Displacement Configuration of the STM-1 Manipulator

Notash, and Pittens, 1992), yielding:

$$\begin{aligned}
 {}^{ref}\$1 &= \left\{ \begin{array}{c} s_2 c_{345} \\ c_2 \\ s_2 s_{345} \\ s_{345} (a - f s_2) + c_2 (b c_{345} + c s_{345} + d s_{45} + e s_5) \\ s_2 (-b + d s_3 + e s_{34} + g s_{345}) \\ c_{345} (-a + f s_2) + c_2 (-g + b s_{345} - c c_{345} - d c_{45} - e c_5) \end{array} \right\} \\
 {}^{ref}\$2 &= \left\{ -s_{345}, 0, c_{345}; -f c_{345}, c + d c_3 + e c_{34} + g c_{345}, -f s_{345} \right\}^T \\
 {}^{ref}\$3 &= \left\{ 0, 1, 0; d s_{45} + e s_5, 0, -d c_{45} - e c_5 - g \right\}^T \\
 {}^{ref}\$4 &= \left\{ 0, 1, 0; e s_5, 0, -e c_5 - g \right\}^T \\
 {}^{ref}\$5 &= \left\{ 0, 1, 0; 0, 0, -g \right\}^T \\
 {}^{ref}\$6 &= \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\
 {}^{ref}\$7 &= \left\{ s_6, -c_6, 0; -h c_6, -h s_6, -k \right\}^T
 \end{aligned} \tag{2.33}$$

The matrix of unit joint screw coordinates for the manipulator is:

$${}^{ref}[\$] = {}^{ref} \left[\begin{array}{cccccc} \$1 & \$2 & \$3 & \$4 & \$5 & \$6 & \$7 \end{array} \right] \tag{2.34}$$

Select $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$ from equation (2.33) to form $[\$]_{sub}$:

$${}^{ref}[\$]_{sub} = {}^{ref} \left[\begin{array}{cccccc} \$2 & \$3 & \$4 & \$5 & \$6 & \$7 \end{array} \right] \tag{2.35}$$

with the redundant joint being $\$1$. The determinant of $[\$]_{sub}$ is:

$$|{}^{ref}[\$]_{sub}| = -d e s_4 s_6 (c + d c_3 + e c_{34} + g c_{345} - h s_{345}) \tag{2.36}$$

Therefore, if a) $s_4 = 0$, b) $s_6 = 0$, or c) $c + d c_3 + e c_{34} + g c_{345} - h s_{345} = 0$, then the six joints comprising $[\$]_{sub}$ define a degenerate sub-group of screws. Degenerate

configurations of the 7-joint arm will include one of these three conditions. Additional conditions required can be found by enforcing reciprocity of $\$1$ with screws characterizing the lost motion DOF for each of the $[\$]_{sub}$ degenerate conditions.

a) Setting $s_4 = 0$ in equation (2.35) yields:

$${}^{ref}[\$]_{sub_a} = \begin{bmatrix} -s_{345} & 0 & 0 & 0 & 0 & s_6 \\ 0 & 1 & 1 & 1 & 1 & -c_6 \\ c_{345} & 0 & 0 & 0 & 0 & 0 \\ -fc_{345} & ds_{45} + es_5 & es_5 & 0 & 0 & -hc_6 \\ c + dc_3 + ec_{34} + gc_{345} & 0 & 0 & 0 & 0 & -hs_6 \\ -fs_{345} & -dc_{45} - ec_5 - g & -ec_5 - g & -g & 0 & -k \end{bmatrix} \quad (2.37)$$

With $s_4 = 0$, equating ${}^{ref}\mathbf{W}_{recip_a} \otimes {}^{ref}\$j = 0$, for $j = 2$ to 7 , allows \mathbf{W}_{recip_a} to be found as:

$${}^{ref}\mathbf{W}_{recip_a} = \left\{ c_5, M, s_5; P, gs_5, 0 \right\}^T \quad (2.38)$$

where

$$M = \frac{fc_{34}s_6 + s_{345}(hc_5c_6 + ks_5 + gs_5c_6)}{s_6(c + dc_3 + ec_{34} + gc_{345} - hs_{345})}$$

$$P = \frac{hc_5c_6 + ks_5 + gs_5c_6 + Mhs_6}{s_6}$$

Taking the reciprocal product between \mathbf{W}_{recip_a} and $\$1$ and simplifying yields:

$$\begin{aligned} {}^{ref}\mathbf{W}_{recip_a} \otimes {}^{ref}\$1 &= s_2 [gs_5c_6 + hc_5c_6 + ks_5] [-bs_{345} + cc_{345} + dc_{45} + ec_5 + g] \\ &+ s_6 [as_{34} + bc_2c_{34} + cc_2s_{34}] [c + dc_3 + ec_{34} + gc_{345} - hs_{345}] \\ &+ fs_2s_6 [-bc_{34} - cs_{34} + gs_5 + hc_5] \end{aligned} \quad (2.39)$$

Thus, if $s_4 = 0$ & equation (2.39) = 0, \mathbf{W}_{recip_a} is reciprocal to joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$. Therefore, $s_4 = 0$ & equation (2.39) = 0 defines a 2-condition family of degenerate configurations.

b) Setting $s_6 = 0$ in equation (2.35) yields:

$${}^{ref}[\$]_{sub_b} = \begin{bmatrix} -s_{345} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & -c_6 \\ c_{345} & 0 & 0 & 0 & 0 & 0 \\ -fc_{345} & ds_{45} + es_5 & es_5 & 0 & 0 & -hc_6 \\ c + dc_3 + ec_{34} + gc_{345} & 0 & 0 & 0 & 0 & 0 \\ -fs_{345} & -dc_{45} - ec_5 - g & -ec_5 - g & -g & 0 & -k \end{bmatrix} \quad (2.40)$$

With $s_6 = 0$, setting ${}^{ref}\mathbf{W}_{recip_b} \otimes {}^{ref}\$j = 0$, for $j = 2$ to 7 , allows \mathbf{W}_{recip_b} to be found as:

$${}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, 1, 0; \frac{c+dc_3+ec_{34}+gc_{345}}{s_{345}}, 0, 0 \right\}^T \quad (2.41)$$

Taking the reciprocal product between \mathbf{W}_{recip_b} and $\$1$ and setting the result to zero yields:

$${}^{ref}\mathbf{W}_{recip_b} \otimes {}^{ref}\$1 = s_2 (-bs_{345} + cc_{345} + dc_{45} + ec_5 + g) = 0 \quad (2.42)$$

Thus, if $s_2 = 0$ & $s_6 = 0$ or $s_6 = 0$ & $-bs_{345} + cc_{345} + dc_{45} + ec_5 + g = 0$, \mathbf{W}_{recip_b} is reciprocal to joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$. Therefore, $s_2 = 0$ & $s_6 = 0$ and $s_6 = 0$ & $-bs_{345} + cc_{345} + dc_{45} + ec_5 + g = 0$ define two 2-condition families of degenerate configurations.

c) Setting $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$ in equation (2.35) yields:

$${}^{ref}[\$]_{sub_c} = \begin{bmatrix} -s_{345} & 0 & 0 & 0 & 0 & s_6 \\ 0 & 1 & 1 & 1 & 1 & -c_6 \\ c_{345} & 0 & 0 & 0 & 0 & 0 \\ -fc_{345} & ds_{45} + es_5 & es_5 & 0 & 0 & -hc_6 \\ hs_{345} & 0 & 0 & 0 & 0 & -hs_6 \\ -fs_{345} & -dc_{45} - ec_5 - g & -ec_5 - g & -g & 0 & -k \end{bmatrix} \quad (2.43)$$

With $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$, setting ${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\$j = 0$, for $j = 2$ to 7 , allows \mathbf{W}_{recip_c} to be found as:

$${}^{ref}\mathbf{W}_{recip_c} = \left\{ 0, 1, 0; h, 0, 0 \right\}^T \quad (2.44)$$

Taking the reciprocal product between \mathbf{W}_{recip_c} and $\$1$ and setting the result to zero yields:

$${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\$1 = s_2 (-b + ds_3 + es_{34} + gs_{345} + hc_{345}) = 0 \quad (2.45)$$

Thus, if $s_2 = 0$ & $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$ or $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$ & $-b + ds_3 + es_{34} + gs_{345} + hc_{345} = 0$, \mathbf{W}_{recip_c} is reciprocal to joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, and $\$7$. Therefore, $s_2 = 0$ & $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$ and $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$ & $-b + ds_3 + es_{34} + gs_{345} + hc_{345} = 0$ define two 2-condition families of degenerate configurations.

Examining all of the degenerate configurations yields five families of 2-condition configurations resulting in a single DOF loss for manipulators having geometries kinematically equivalent to the STM-1. These degenerate configurations and their asso-

ciated reciprocal screws can be summarized as:

$$\begin{aligned}
 1) \quad & s_4 = 0 \ \& \ s_2 [gs_5c_6 + hc_5c_6 + ks_5] [-bs_{345} + cc_{345} + dc_{45} + ec_5 + g] \\
 & \quad + s_6 [as_{34} + bc_2c_{34} + cc_2s_{34}] [c + dc_3 + ec_{34} + gc_{345} - hs_{345}] \\
 & \quad + fs_2s_6 [-bc_{34} - cs_{34} + gs_5 + hc_5] = 0 \\
 & \quad {}^{ref}\mathbf{W}_1 = {}^{ref}\mathbf{W}_{recip_a} = \left\{ c_5, \ M, \ s_5; \ P, \ gs_5, \ 0 \right\}^T \\
 & \quad \text{where } M = \frac{fc_{34}s_6 + s_{345}(hc_5c_6 + ks_5 + gs_5c_6)}{s_6(c + dc_3 + ec_{34} + gc_{345} - hs_{345})} \ \& \ P = \frac{hc_5c_6 + ks_5 + gs_5c_6 + Mhs_6}{s_6} \\
 2) \quad & s_2 = 0 \ \& \ s_6 = 0 \\
 & \quad {}^{ref}\mathbf{W}_2 = {}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, \ 1, \ 0; \ \frac{c + dc_3 + ec_{34} + gc_{345}}{s_{345}}, \ 0, \ 0 \right\}^T \\
 3) \quad & s_2 = 0 \ \& \ c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0 \\
 & \quad {}^{ref}\mathbf{W}_3 = {}^{ref}\mathbf{W}_{recip_c} = \left\{ 0, \ 1, \ 0; \ h, \ 0, \ 0 \right\}^T \\
 4) \quad & s_6 = 0 \ \& \ -bs_{345} + cc_{345} + dc_{45} + ec_5 + g = 0 \\
 & \quad {}^{ref}\mathbf{W}_4 = {}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, \ 1, \ 0; \ \frac{c + dc_3 + ec_{34} + gc_{345}}{s_{345}}, \ 0, \ 0 \right\}^T \\
 5) \quad & c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0 \ \& \ -b + ds_3 + es_{34} + gs_{345} + hc_{345} = 0 \\
 & \quad {}^{ref}\mathbf{W}_5 = {}^{ref}\mathbf{W}_{recip_c} = \left\{ 0, \ 1, \ 0; \ h, \ 0, \ 0 \right\}^T
 \end{aligned} \tag{2.46}$$

The condition sets outlined in (2.46) are identical to those obtained by Podhorodeski, Nokleby, and Wittchen (2000) using the technique of setting the determinants of all possible 6-joint sub-groups to zero to determine the velocity-degenerate configurations of the STM-1. Note that Podhorodeski, Nokleby, and Wittchen (2000)

showed that for the actual STM-1 dimensions and joint limits, only the first four degenerate configurations can be encountered by the manipulator.

2.3.4 NASA Advanced Research Manipulator II (ARMII)

The National Aeronautics and Space Administration (NASA) Advanced Research Manipulator II (ARMII) is an 8-joint manipulator with a layout of R⊥R⊥R⊥R⊥R⊥R⊥R⊥R. Note that the first three joints and the last four joints both form spherical groups. The layout of the ARMII is similar to the spherical-revolute-spherical manipulator except the wrist spherical group for the ARMII consists of four joints instead of the three used in the spherical-revolute-spherical manipulator.

D&H parameters for the ARMII using Craig's frame assignment convention (Craig, 1989) are presented in Table 2.4. The parameters of Table 2.4 correspond to the link transformations:

$${}^j{}_{j-1}\mathbf{T} = Rot_{\hat{\mathbf{x}}_{j-1}}(\alpha_{j-1}) Trans_{\hat{\mathbf{x}}_{j-1}}(a_{j-1}) Trans_{\hat{\mathbf{z}}_j}(d_j) Rot_{\hat{\mathbf{z}}_j}(\theta_j) \quad (2.47)$$

where ${}^j{}_{j-1}\mathbf{T}$ is a homogeneous transformation describing the location and orientation of link-frame F_j with respect to link-frame F_{j-1} , $Rot_{\hat{\mathbf{x}}_{j-1}}(\alpha_{j-1})$ denotes a rotation about the $\hat{\mathbf{x}}_{j-1}$ axis by α_{j-1} , $Trans_{\hat{\mathbf{x}}_{j-1}}(a_{j-1})$ denotes a translation along the $\hat{\mathbf{x}}_{j-1}$ axis by a_{j-1} , $Trans_{\hat{\mathbf{z}}_j}(d_j)$ denotes a translation along the $\hat{\mathbf{z}}_j$ axis by d_j , and $Rot_{\hat{\mathbf{z}}_j}(\theta_j)$ denotes a rotation about the $\hat{\mathbf{z}}_j$ axis by θ_j (Craig, 1989). Figure 2.4 shows the zero-displacement configuration of the manipulator.

Choosing a reference frame to be a frame coincident with F_4 of the ARMII allows

Table 2.4: Denavit and Hartenberg Parameters for the ARMII Manipulator

F_{j-1}	α_{j-1}	a_{j-1}	d_j	θ_j	F_j
F_0	0	0	0	θ_1	F_1
F_1	$\frac{\pi}{2}$	0	0	θ_2	F_2
F_2	$-\frac{\pi}{2}$	0	g	θ_3	F_3
F_3	$\frac{\pi}{2}$	0	0	θ_4	F_4
F_4	$-\frac{\pi}{2}$	0	h	$\theta_5 - \frac{\pi}{2}$	F_5
F_5	$-\frac{\pi}{2}$	0	0	$\theta_6 + \frac{\pi}{2}$	F_6
F_6	$\frac{\pi}{2}$	0	0	$\theta_7 - \frac{\pi}{2}$	F_7
F_7	$\frac{\pi}{2}$	0	0	θ_8	F_8

the joint screws to be found as (Williams II, 1992):

$$\begin{aligned}
 \text{ref}\$1 &= \left\{ \begin{array}{l} c_2s_4 + s_2c_3c_4 \\ c_2c_4 - s_2c_3s_4 \\ s_2s_3 \\ -s_2s_3(c_4g + h) \\ s_2s_3s_4g \\ s_2c_3g + h(c_2s_4 + s_2c_3c_4) \end{array} \right\} \\
 \text{ref}\$2 &= \left\{ -s_3c_4, s_3s_4, c_3; -c_3(c_4g + h), c_3s_4g, -s_3(g + c_4h) \right\}^T \\
 \text{ref}\$3 &= \left\{ s_4, c_4, 0; 0, 0, s_4h \right\}^T \\
 \text{ref}\$4 &= \left\{ 0, 0, 1; -h, 0, 0 \right\}^T \\
 \text{ref}\$5 &= \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \\
 \text{ref}\$6 &= \left\{ c_5, 0, -s_5; 0, 0, 0 \right\}^T \\
 \text{ref}\$7 &= \left\{ s_5c_6, -s_6, c_5c_6; 0, 0, 0 \right\}^T \\
 \text{ref}\$8 &= \left\{ -c_5s_7 + s_5s_6c_7, c_6c_7, s_5s_7 + c_5s_6c_7; 0, 0, 0 \right\}^T
 \end{aligned} \tag{2.48}$$

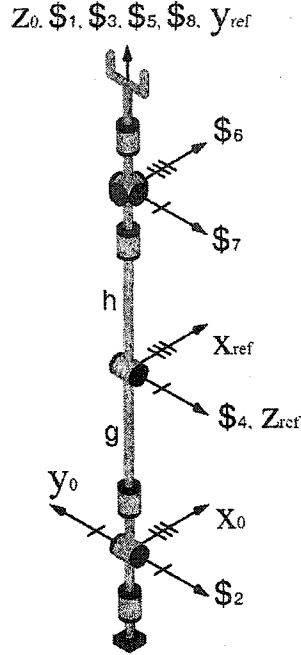


Figure 2.4: Zero-Displacement Configuration of the ARMII Manipulator

The matrix of unit joint screws for the manipulator is:

$${}^{ref}[\$] = {}^{ref} \begin{bmatrix} \$1 & \$2 & \$3 & \$4 & \$5 & \$6 & \$7 & \$8 \end{bmatrix} \quad (2.49)$$

Select $\$2, \$3, \$4, \$5, \$6,$ and $\$7$ from equation (2.48) to form $[\$]_{sub}$:

$${}^{ref}[\$]_{sub} = {}^{ref} \begin{bmatrix} \$2 & \$3 & \$4 & \$5 & \$6 & \$7 \end{bmatrix} \quad (2.50)$$

with the redundant joints being $\$1$ and $\$8$. The determinant of $[\$]_{sub}$ is:

$$|{}^{ref}[\$]_{sub}| = -c_3 s_4^2 c_6 g h^2 \quad (2.51)$$

Therefore, if a) $s_4 = 0$, b) $c_3 = 0$, or c) $c_6 = 0$, then the six joints comprising $[\$]_{sub}$ define a degenerate sub-group of screws. Degenerate configurations of the 8-joint arm will include one of these three conditions. Additional conditions required can be

found by enforcing reciprocity of $\$1$ and $\$8$ with screws characterizing the lost motion DOF for each of the $[\$]_{sub}$ degenerate conditions.

a) Setting $s_4 = 0$ in equation (2.50) yields:

$${}^{ref}[\$]_{sub_a} = \begin{bmatrix} -s_3 c_4 & 0 & 0 & 0 & c_5 & s_5 c_6 \\ 0 & c_4 & 0 & 1 & 0 & -s_6 \\ c_3 & 0 & 1 & 0 & -s_5 & c_5 c_6 \\ -c_3 (c_4 g + h) & 0 & -h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -s_3 (g + c_4 h) & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.52)$$

The reciprocal screw for the six joints comprising $[\$]_{sub}$ with $s_4 = 0$ can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_a} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (2.53)$$

Setting the reciprocal products between \mathbf{W}_{recip_a} & $\$1$ and \mathbf{W}_{recip_a} & $\$8$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_a} \otimes {}^{ref}\$1 = s_2 s_3 s_4 g = 0 \quad (2.54)$$

$${}^{ref}\mathbf{W}_{recip_a} \otimes {}^{ref}\$8 = 0 \quad (2.55)$$

Since $s_4 = 0$, no further conditions are necessary to make \mathbf{W}_{recip_a} reciprocal to joints $\$1$, $\$2$, $\$3$, $\$4$, $\$5$, $\$6$, $\$7$, and $\$8$. Therefore, $s_4 = 0$ defines a 1-condition family of degenerate configurations.

b) Setting $c_3 = 0$ in equation (2.50) yields:

$${}^{ref}\mathcal{S}_{sub_b} = \begin{bmatrix} -s_3c_4 & s_4 & 0 & 0 & c_5 & s_5c_6 \\ s_3s_4 & c_4 & 0 & 1 & 0 & -s_6 \\ 0 & 0 & 1 & 0 & -s_5 & c_5c_6 \\ 0 & 0 & -h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -s_3(g + c_4h) & s_4h & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.56)$$

The reciprocal screw for the six joints comprising \mathcal{S}_{sub_b} with $c_3 = 0$ can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (2.57)$$

Setting the reciprocal products between \mathbf{W}_{recip_b} & \mathcal{S}_1 and \mathbf{W}_{recip_b} & \mathcal{S}_8 to zero yields:

$${}^{ref}\mathbf{W}_{recip_b} \otimes {}^{ref}\mathcal{S}_1 = s_2s_3s_4g = 0 \quad (2.58)$$

$${}^{ref}\mathbf{W}_{recip_b} \otimes {}^{ref}\mathcal{S}_8 = 0 \quad (2.59)$$

Thus, if $s_2 = 0$ & $c_3 = 0$ or $c_3 = 0$ & $s_4 = 0$, \mathbf{W}_{recip_b} is reciprocal to joints $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4, \mathcal{S}_5, \mathcal{S}_6, \mathcal{S}_7$, and \mathcal{S}_8 . It was shown in Section 2.3.4a that $s_4 = 0$ alone results in a degenerate configuration, therefore, $c_3 = 0$ & $s_4 = 0$ does not represent a new family of degenerate configurations. However, $s_2 = 0$ & $c_3 = 0$ defines a new 2-condition family of degenerate configurations.

c) Setting $c_6 = 0$ in equation (2.50) yields:

$${}^{ref}\mathbb{S}_{subc} = \begin{bmatrix} -s_3c_4 & s_4 & 0 & 0 & c_5 & 0 \\ s_3s_4 & c_4 & 0 & 1 & 0 & -s_6 \\ c_3 & 0 & 1 & 0 & -s_5 & 0 \\ -c_3(c_4g + h) & 0 & -h & 0 & 0 & 0 \\ c_3s_4g & 0 & 0 & 0 & 0 & 0 \\ -s_3(g + c_4h) & s_4h & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.60)$$

Setting ${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{S}_j = 0$, for $j = 2$ to 7 , with $c_6 = 0$ allows \mathbf{W}_{recip_c} to be found as:

$${}^{ref}\mathbf{W}_{recip_c} = \left\{ c_5, \frac{c_3c_4c_5 - s_3s_5}{c_3s_4}, -s_5; s_5h, 0, c_5h \right\}^T \quad (2.61)$$

Setting the reciprocal products between \mathbf{W}_{recip_c} & \mathbb{S}_1 and \mathbf{W}_{recip_c} & \mathbb{S}_8 to zero yields:

$${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{S}_1 = -s_2s_4s_5g = 0 \quad (2.62)$$

$${}^{ref}\mathbf{W}_{recip_c} \otimes {}^{ref}\mathbb{S}_8 = c_3s_4s_6c_7h = 0 \quad (2.63)$$

Note that $s_4 = 0$ or $s_2 = 0$ & $c_3 = 0$ already result in degenerate configurations (see Sections 2.3.4a and 2.3.4b, respectively) and that $c_3 = 0$, $s_5 = 0$, & $c_6 = 0$ causes ${}^{ref}\mathbf{W}_{recip_c}$ to collapse into a zero screw (${}^{ref}\mathbf{W}_{recip_c} = \mathbf{0}_{6 \times 1}$), therefore, these conditions do not define new families of degenerate configurations. However, if $s_2 = 0$, $c_6 = 0$, & $c_7 = 0$ or $s_5 = 0$, $c_6 = 0$, & $c_7 = 0$, \mathbf{W}_{recip_c} is reciprocal to joints \mathbb{S}_1 , \mathbb{S}_2 , \mathbb{S}_3 , \mathbb{S}_4 , \mathbb{S}_5 , \mathbb{S}_6 , \mathbb{S}_7 , and \mathbb{S}_8 . These sets of conditions define two further 3-condition families of degenerate configurations.

Examining all of the degenerate configurations yields four sets of conditions (one set requiring the satisfaction of a single condition, one set requiring the satisfaction

of two conditions, and two sets requiring the satisfaction of three conditions) defining families of degenerate configurations resulting in a single motion DOF loss for manipulators having geometries kinematically equivalent to the ARMII. These degenerate configurations and their respective reciprocal screws can be summarized as:

$$1) \quad s_4 = 0$$

$${}^{ref}\mathbf{W}_1 = {}^{ref}\mathbf{W}_{recip_a} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T$$

$$2) \quad s_2 = 0 \ \& \ c_3 = 0$$

$${}^{ref}\mathbf{W}_2 = {}^{ref}\mathbf{W}_{recip_b} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T$$

(2.64)

$$3) \quad s_2 = 0, \ c_6 = 0, \ \& \ c_7 = 0$$

$${}^{ref}\mathbf{W}_3 = {}^{ref}\mathbf{W}_{recip_c} = \left\{ c_5, \frac{c_3 c_4 c_5 - s_3 s_5}{c_3 s_4}, \ -s_5; \ s_5 h, \ 0, \ c_5 h \right\}^T$$

$$4) \quad s_5 = 0, \ c_6 = 0, \ \& \ c_7 = 0$$

$${}^{ref}\mathbf{W}_4 = {}^{ref}\mathbf{W}_{recip_c} = \left\{ c_5, \frac{c_3 c_4 c_5 - s_3 s_5}{c_3 s_4}, \ -s_5; \ s_5 h, \ 0, \ c_5 h \right\}^T$$

The above results were confirmed using the symbolic math package Maple 6 to determine the rank of the matrix of unit screw coordinates, ${}^{ref}[\$]$. For each set of conditions, the rank of ${}^{ref}[\$]$ was found to be five, i.e., the manipulator had lost 1-DOF of motion capability.

Williams II (1992, 1994a, and 1994b) attempted to derive the velocity-degenerate configurations of the ARMII by looking at a partitioned Jacobian (matrix of unit joint screw coordinates). Since the ARMII has a spherical wrist, Williams II partitioned the main-arm joints (the joints responsible for translation of the wrist centre) from

the spherical-wrist joints (the joints responsible for orientation). The two partitioned groups of joints were analyzed separately to attempt to determine conditions that result in velocity degeneracies for the manipulator.

As Lipkin and Duffy (1982) show for a non-redundant manipulator with three revolute-joints forming a spherical group (i.e., intersecting at a common point such as a spherical-wrist centre) that the terms of the screw-coordinate matrix (Jacobian) can be simplified if an appropriate location is used for the reference frame origin. In particular, if the location of the frame of reference is selected to be the intersection point of the three revolute-joint axes, the matrix of unit joint screw coordinates takes on the form:

$$[\$]_{6 \times 6} = \begin{bmatrix} [\mathbf{s}_{arm}]_{3 \times 3} & [\mathbf{s}_{wrist}]_{3 \times 3} \\ [\mathbf{s}_{o arm}]_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix} \quad (2.65)$$

where it has been assumed that the final three joints form a spherical wrist. In equation (2.65), the first three columns are the joint screw coordinates of the main-arm and the last three columns are the joint screw coordinates of the spherical-wrist.

The determinant of $[\$]_{6 \times 6}$ is:

$$|[\$]| = \left| \begin{bmatrix} [\mathbf{s}_{arm}] & [\mathbf{s}_{wrist}] \\ [\mathbf{s}_{o arm}] & \mathbf{0}_{3 \times 3} \end{bmatrix} \right| = |[\mathbf{s}_{o arm}]| |[\mathbf{s}_{wrist}]| \quad (2.66)$$

Therefore, for a non-redundant serial manipulator, a degeneracy exists if the main-arms joints go degenerate ($|[\mathbf{s}_{o arm}]| = 0$) or if the wrist joints go degenerate ($|[\mathbf{s}_{wrist}]| = 0$).

Williams II (1992, 1994a, and 1994b) tries to apply the same principle to redundant manipulators with a spherical-wrist. It should be apparent that this is not possible simply by looking at the matrix of unit joint screw coordinates for the

ARMII. For the chosen frame of reference, the ARMII matrix of unit joint screw coordinates is of the form:

$$[\$]_{6 \times 8} = \begin{bmatrix} [\mathbf{s}_{arm}]_{3 \times 4} & [\mathbf{s}_{wrist}]_{3 \times 4} \\ [\mathbf{s}_{o arm}]_{3 \times 4} & \mathbf{0}_{3 \times 4} \end{bmatrix} \quad (2.67)$$

where the first four columns are the joint screw coordinates of the main-arm and the last four columns are the joint screw coordinates of the spherical-wrist. The matrix of unit joint screw coordinates for the ARMII is not square, therefore, no determinant can be taken. Noting that the pseudo inverse of the matrix of unit joint screw coordinates is $[\$]^+ = [\$]^T ([\$][\$]^T)^{-1}$, the determinant of $[\$][\$]^T$ can be used to determine velocity-degenerate configurations (Luh and Gu, 1985). The matrix $[\$][\$]^T$ is of the form:

$$\begin{aligned} [\$][\$]^T &= \begin{bmatrix} [\mathbf{s}_{arm}] & [\mathbf{s}_{wrist}] \\ [\mathbf{s}_{o arm}] & \mathbf{0}_{3 \times 4} \end{bmatrix} \begin{bmatrix} [\mathbf{s}_{arm}]^T & [\mathbf{s}_{o arm}]^T \\ [\mathbf{s}_{wrist}]^T & \mathbf{0}_{4 \times 3} \end{bmatrix} \\ &= \begin{bmatrix} [\mathbf{s}_{arm}][\mathbf{s}_{arm}]^T + [\mathbf{s}_{wrist}][\mathbf{s}_{wrist}]^T & [\mathbf{s}_{arm}][\mathbf{s}_{o arm}]^T \\ [\mathbf{s}_{o arm}][\mathbf{s}_{arm}]^T & [\mathbf{s}_{o arm}][\mathbf{s}_{o arm}]^T \end{bmatrix} \end{aligned} \quad (2.68)$$

Comparing the matrices of equations (2.65) and (2.68), the $[\$][\$]^T$ matrix does not have the sub-matrix of zeros that is found in the matrix of unit joint screw coordinates for the non-redundant spherical-wristed manipulator. It is this sub-matrix of zeros that allows the partitioning of the main-arm and wrist joints to allow the determination of velocity-degenerate configurations of non-redundant spherical-wristed manipulators. It is clear from the form of $[\$][\$]^T$ that looking at conditions that cause only the main-arm or the wrist to go singular for redundant spherical-wristed manipulators does not guarantee that the whole manipulator is in a velocity-degenerate configuration.

The degeneracy conditions Williams II (1994a and 1994b) found can be summarized as: a) $s_4 = 0$, b) $s_2 = 0$ & $c_3 = 0$, c) $s_2 = 0$ & $s_4 = 0$, and d) $c_6 = 0$ & $c_7 = 0$. Williams II (1994a) points out that condition (c) is a subset of (a), i.e., condition (c) is a special case of the more general condition (a). Therefore, Williams II identified three unique sets of conditions resulting in velocity degeneracies for the ARMI: i) $s_4 = 0$, ii) $s_2 = 0$ & $c_3 = 0$, iii) $c_6 = 0$ & $c_7 = 0$. Conditions (i) and (ii) agree with the results obtained using the reciprocity-based methodology. Condition (iii) does not agree with the results obtained using the reciprocity-based methodology. Williams II (1994a and 1994b) claims that condition (iii) is a degeneracy for the full Jacobian, but this is not correct. It can be shown that with $c_6 = 0$ & $c_7 = 0$ the Jacobian retains full rank (this was confirmed using Maple 6). The results of the reciprocity-based methodology show that the condition $c_6 = 0$ & $c_7 = 0$ alone does not result in a degeneracy, but requires either $s_2 = 0$ or $s_5 = 0$ to be true in addition. The results show that partitioning the matrix of unit joint screw coordinates to identify singular configurations does not work for redundant spherical-wristed manipulators.

2.4 Discussion

2.4.1 The Results Found

From the examples presented in the previous section it is clear that the developed methodology is capable of determining the 1-DOF-loss velocity-degenerate configurations of kinematically-redundant manipulators. The spherical-revolute-spherical and double-elbow manipulator examples verify that the methodology works. The results obtained in both examples are identical to results previously reported in the

literature by Podhorodeski, Goldenberg, and Fenton (1991 and 1993).

The STM results confirmed the results obtained by Podhorodeski, Nokleby, and Wittchen (2000).

The STM example shows that the developed methodology can be applied to manipulators involving layouts of additional kinematic complexity, i.e., manipulators that have multiple length and offset parameters and do not have spherical wrists. With the exception of its three parallel joints, all joints of the STM are separated by both an offset and link length. To determine the degenerate configurations of the STM, the developed methodology is simpler to apply than the sub-matrix determinant method employed by Podhorodeski, Nokleby, and Wittchen (2000). For the reciprocity-based methodology, the number of expressions requiring investigation is linear in terms of the number of redundant joints, i.e., $\alpha(n - 6)$. In addition, one 6x6 determinant must be taken. Comparing this with the sub-matrix determinant method used by Duffy and Crane III (1989), Nokleby and Podhorodeski (2000a), and Podhorodeski, Nokleby, and Wittchen (2000) which requires seven 6x6 determinants (for a 7-joint manipulator), it is clear that the developed methodology is more efficient.

The final example featuring the ARMII with $n = 8$, demonstrates that the methodology works for higher levels of redundancy. The example also shows that partitioning the matrix of unit joint screw coordinates to identify singular configurations does not work for redundant spherical-wristed manipulators.

2.4.2 Reciprocal Screws

As illustrated in the examples, the reciprocity-based methodology yields reciprocal screws for each of the degenerate configurations. The sub-matrix determinant method does not yield these reciprocal screws. The reciprocal screw quantity, \mathbf{W}_i , characterizes the lost instantaneous motion (velocity) DOF for the i^{th} degenerate configuration. Within the degenerate configuration, the manipulator will not be able to produce a motion that would do work subject to a force spanned by the reciprocal screw. Allowable motions in the i^{th} degenerate configuration are defined by the reciprocal product equation:

$$\mathbf{W}_i \otimes \mathbf{V}_{feasible} = 0 \quad (2.69)$$

where $\mathbf{V}_{feasible} = \{\boldsymbol{\omega}^T; \mathbf{v}^T\}_{feasible}^T$ represents the possible instantaneous motions. Knowledge of these reciprocal screw quantities can be exploited for planning motions to escape degenerate configurations (Hunt, 1986 and 1987a), for finding best feasible motions (Podhorodeski, 1993), and for exploiting structural loading characteristics of near degenerate configurations (Hunt, 1986 and Wang and Waldron, 1987).

2.4.3 Joint-Redundant Parallel Manipulators

Although the methodology presented in this chapter was applied to redundant serial manipulators, the methodology can be used to identify velocity-degenerate configurations of joint-redundant parallel manipulators. The methodology can be applied to identify the velocity-degenerate configurations of the branches of a joint-redundant parallel manipulator. If a branch of a parallel manipulator enters a velocity-degenerate configuration, then the entire manipulator is in a velocity-degenerate

configuration because the movement of the platform of the parallel manipulator is restricted by the degenerate branch.

Chapter 3

Identification of Multi-DOF-Loss

Velocity-Degenerate

Configurations of

Kinematically-Redundant

Manipulators

3.1 Overview

In this chapter, the reciprocity-based method for identifying 1-DOF-loss velocity degeneracies presented in Chapter 2 is extended to the case of identifying multi-DOF-loss velocity-degenerate (singular) configurations of kinematically-redundant manipulators. The developed methodology uses the properties of reciprocal screws to determine the degenerate configurations of kinematically-redundant manipulators. Similar

to the 1-DOF-loss methodology, a by-product of the multi-DOF-loss methodology is that reciprocal screws related to the lost motion DOFs for each degenerate configuration are determined. Examples are presented to demonstrate the effectiveness of the reciprocity-based methodology. The chapter finishes with a discussion of the new method for determining the multi-DOF-loss velocity-degenerate configurations.

3.2 Methodology for Identification of Multi-DOF-Loss Velocity-Degenerate Configurations¹

3.2.1 Identification of 2-DOF-Loss Velocity-Degenerate Configurations

Applying the methodology for identifying 1-DOF-loss velocity degeneracies from Chapter 2 allows all sets of conditions (say β in total) that result in all of a redundant-manipulator's joints $\$_{sub_1}, \$_{sub_2}, \dots, \$_{sub_6}, \$_{r_1}, \$_{r_2}, \dots, \$_{r_{(n-6)}}$ becoming degenerate. As a by-product, a reciprocal screw related to the lost motion DOF is generated for each of the β velocity degeneracies, i.e., \mathbf{W}_1 to \mathbf{W}_β are identified.

Set the 1st set of 1-DOF-loss degeneracy conditions to be true. Define \mathbf{W}_1^* to be a complement of the reciprocal screw \mathbf{W}_1 such that for the chosen frame of reference ${}^{ref}\mathbf{W}_1^* = {}^{ref}\{\mathbf{w}_{1o}^T; \mathbf{w}_1^T\}^T$ where ${}^{ref}\mathbf{W}_1 = {}^{ref}\{\mathbf{w}_1^T; \mathbf{w}_{1o}^T\}^T$. Since \mathbf{W}_1 , by definition, cannot be a zero screw ($\mathbf{W}_1 \neq \mathbf{0}_{6 \times 1}$), the reciprocal product between \mathbf{W}_1 and its complement can never be zero ($\mathbf{W}_1 \otimes \mathbf{W}_1^* \neq 0$). Choosing five joint screws along

¹The results contained in this section have been presented in Nokleby and Podhorodeski (2001b and 2003a).

with \mathbf{W}_1^* , allows a new sub-group matrix of “joint screw coordinates” to be defined as:

$$[\$]_{sub_1^*} = \begin{bmatrix} \$_{sub_{1_1}} & \$_{sub_{2_1}} & \$_{sub_{3_1}} & \$_{sub_{4_1}} & \$_{sub_{5_1}} & \mathbf{W}_1^* \end{bmatrix} \quad (3.1)$$

The only condition on the choice of the screws for $[\$]_{sub_1^*}$ is that $\$_{sub_{1_1}}, \$_{sub_{2_1}}, \$_{sub_{3_1}}, \$_{sub_{4_1}}, \$_{sub_{5_1}}$ and \mathbf{W}_1^* cannot be inherently linearly dependent. This leaves $(n-6)+1 = n-5$ screws, $\$_{r_{1_1}}, \$_{r_{2_1}}, \dots, \$_{r_{(n-5)_1}}$, that can be thought of as redundant screws. Note that the incorporation of \mathbf{W}_1^* in the sub-group matrix of “joint screw coordinates” embodies knowledge of the 1st 1-DOF-loss velocity-degenerate configuration. The reciprocity-based method used to find 1-DOF-loss velocity degeneracies can now be used to find all sets of conditions that lead to 2-DOF-loss velocity degeneracies, provided the 1st set of degeneracy conditions are also true.

The above procedure is repeated for each set of 1-DOF-loss velocity-degenerate configurations and allows all unique sets of conditions (say γ in total) that result in a 2-DOF-loss velocity degeneracy to be identified. As a by-product of the methodology, two reciprocal screws are generated for each of the γ 2-DOF-loss velocity degeneracies, i.e., \mathbf{W}_{1_1} & \mathbf{W}_{2_1} to \mathbf{W}_{1_γ} & \mathbf{W}_{2_γ} are identified. The two reciprocal screws found for each of the 2-DOF-loss degenerate configurations form a basis of the motion loss for that particular degenerate configuration.

3.2.2 Identification of Higher-DOF-Loss Velocity-Degenerate Configurations

Set the 1st set of 2-DOF-loss degeneracy conditions to be true. Choosing four joint screws along with $\mathbf{W}_{1_1}^*$ and $\mathbf{W}_{2_1}^*$, allows a new sub-group matrix of “joint screw

coordinates” to be defined as:

$$[\$]_{sub_1^*} = \begin{bmatrix} \$_{sub_{1_1}} & \$_{sub_{2_1}} & \$_{sub_{3_1}} & \$_{sub_{4_1}} & \mathbf{W}_{1_1}^* & \mathbf{W}_{2_1}^* \end{bmatrix} \quad (3.2)$$

The only condition on the choice of the screws for $[\$]_{sub_1^*}$ is that $\$_{sub_{1_1}}, \$_{sub_{2_1}}, \$_{sub_{3_1}}, \$_{sub_{4_1}}, \mathbf{W}_{1_1}^*$, and $\mathbf{W}_{2_1}^*$ cannot be inherently linearly dependent. This leaves $(n - 6) + 2 = n - 4$ screws, $\$_{r_{1_1}}, \$_{r_{2_1}}, \dots, \$_{r_{(n-4)_1}}$, that can be thought of as redundant screws. Again, the same method used to find 1-DOF-loss velocity degeneracies can now be used to find all sets of conditions that lead to a 3-DOF-loss velocity degeneracy, provided the 1st set of 2-DOF-loss degeneracy conditions are also true.

Repeating the procedure for each set of 2-DOF-loss velocity-degenerate configurations allows all sets of conditions (say δ in total) that result in a 3-DOF-loss velocity degeneracy to be identified. As a by-product of the methodology, three reciprocal screws are generated for each of the δ 3-DOF-loss velocity degeneracies, i.e., $\mathbf{W}_{1_1}, \mathbf{W}_{2_1}, \& \mathbf{W}_{3_1}$ to $\mathbf{W}_{1_\delta}, \mathbf{W}_{2_\delta}, \& \mathbf{W}_{3_\delta}$ are identified. The three reciprocal screws found for each of the 3-DOF-loss degenerate configurations form a basis of the motion loss for that particular degenerate configuration.

Based on the fact that to enter a q -DOF-loss velocity-degenerate configuration, a manipulator must first enter a $(q - 1)$ -DOF-loss velocity-degenerate configuration, the same procedure can be extended to yield all sets of conditions resulting in velocity degeneracies of higher-DOF-loss. In addition, reciprocal screws characterizing the motion loss for each degenerate configuration are found.

3.3 Examples

To illustrate the developed methodology being applied to the problem of determining 2-DOF-loss velocity-degenerate configurations for kinematically-redundant manipulators, two different manipulators will be used: the 7-joint spherical-revolute-spherical manipulator and the 7-joint double-elbow manipulator.

3.3.1 Spherical-Revolute-Spherical Manipulator²

As presented in Section 2.3.1, the joint screws for the spherical-revolute-spherical manipulator are:

$$\begin{aligned}
 {}^{ref}\$1 &= \left\{ \begin{array}{c} s_2 c_3 c_4 + c_2 s_4 \\ -s_2 s_3 \\ -s_2 c_3 s_4 + c_2 c_4 \\ -s_2 s_3 (c_4 g + h) \\ -s_2 c_3 (g + c_4 h) - c_2 s_4 h \\ s_2 s_3 s_4 g \end{array} \right\} \\
 {}^{ref}\$2 &= \left\{ -c_4 s_3, -c_3, s_3 s_4; -c_3 (c_4 g + h), s_3 (g + c_4 h), c_3 s_4 g \right\}^T \\
 {}^{ref}\$3 &= \left\{ s_4, 0, c_4; 0, -s_4 h, 0 \right\}^T \\
 {}^{ref}\$4 &= \left\{ 0, -1, 0; -h, 0, 0 \right\}^T \\
 {}^{ref}\$5 &= \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\
 {}^{ref}\$6 &= \left\{ s_5, -c_5, 0; 0, 0, 0 \right\}^T \\
 {}^{ref}\$7 &= \left\{ -c_5 s_6, -s_5 s_6, c_6; 0, 0, 0 \right\}^T
 \end{aligned} \tag{3.3}$$

As derived in the same section (Section 2.3.1), the 1-DOF-loss velocity degeneracies, along with their associated reciprocal screws, for the spherical-revolute-spherical manipulator were shown to be:

²The results contained in this section have been presented in Nokleby and Podhorodeski (2001c and 2003a).

1) $s_4 = 0$

$${}^{ref}\mathbf{W}_1 = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T$$

2) $s_2 = 0$ & $c_3 = 0$

$${}^{ref}\mathbf{W}_2 = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T$$

3) $s_2 = 0$ & $s_6 = 0$

$${}^{ref}\mathbf{W}_3 = \left\{ -s_5, c_5, \frac{-c_3 c_4 s_5 - s_3 c_5}{c_3 s_4}; c_5 h, s_5 h, 0 \right\}^T$$

4) $c_5 = 0$ & $s_6 = 0$

$${}^{ref}\mathbf{W}_4 = \left\{ -s_5, c_5, \frac{-c_3 c_4 s_5 - s_3 c_5}{c_3 s_4}; c_5 h, s_5 h, 0 \right\}^T$$

(3.4)

Condition (1): $s_4 = 0$

Setting $s_4 = 0$ and ${}^{ref}\mathbf{W}_1^* = \{0, 0, 0; 0, 0, 1\}^T$ allows $[\$]_{sub_1^*}$ to be defined as:

$${}^{ref}[\$]_{sub_1^*} = {}^{ref} \left[\begin{array}{cccccc} \$2 & \mathbf{W}_1^* & \$4 & \$5 & \$6 & \$7 \end{array} \right]$$

$$= \left[\begin{array}{cccccc} -s_3 c_4 & 0 & 0 & 0 & s_5 & -c_5 s_6 \\ -c_3 & 0 & -1 & 0 & -c_5 & -s_5 s_6 \\ 0 & 0 & 0 & 1 & 0 & c_6 \\ -c_3 (c_4 g + h) & 0 & -h & 0 & 0 & 0 \\ s_3 (g + c_4 h) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array} \right] \quad (3.5)$$

with the redundant joints being $\$1$ and $\$3$. The determinant of $[\$]_{sub_1^*}$ is:

$$\left| {}^{ref}[\$]_{sub_1^*} \right| = -s_3 s_6 h (g + c_4 h) \quad (3.6)$$

Therefore, if either a) $s_3 = 0$, b) $s_6 = 0$, or c) $g + c_4 h = 0$, then the six “joints” comprising $[\$]_{sub_1^*}$ are degenerate.

a) Setting $s_3 = 0$ in $[\$]_{sub_1^*}$ (with $s_4 = 0$) yields:

$${}^{ref}[\$]_{sub_{1a}^*} = \begin{bmatrix} 0 & 0 & 0 & 0 & s_5 & -c_5 s_6 \\ -c_3 & 0 & -1 & 0 & -c_5 & -s_5 s_6 \\ 0 & 0 & 0 & 1 & 0 & c_6 \\ -c_3 (c_4 g + h) & 0 & -h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.7)$$

The reciprocal screw for the six “joints” comprising $[\$]_{sub_{1a}^*}$, with $s_3 = 0$ & $s_4 = 0$, can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_{1a}} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (3.8)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{1a}}$ & $\$1$ and $\mathbf{W}_{recip_{1a}}$ & $\$3$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{1a}} \otimes {}^{ref}\$1 = -s_2 c_3 (g + c_4 h) - c_2 s_4 h = 0 \quad (3.9)$$

$${}^{ref}\mathbf{W}_{recip_{1a}} \otimes {}^{ref}\$3 = 0 \quad (3.10)$$

Since s_3 and s_4 are assumed to equal zero, if in addition $g + c_4 h = 0$ or $s_2 = 0$, then $\mathbf{W}_{recip_{1a}}$ is reciprocal to joints $\$1$ to $\$7$. Therefore, $s_3 = 0$, $s_4 = 0$, & $g + c_4 h = 0$

and $s_2 = 0$, $s_3 = 0$, & $s_4 = 0$ define two 3-condition families of 2-DOF-loss velocity-degenerate configurations.

b) Setting $s_6 = 0$ in $[\$]_{sub_1^*}$ (with $s_4 = 0$) yields:

$${}^{ref}[\$]_{sub_1^*} = \begin{bmatrix} -s_3 c_4 & 0 & 0 & 0 & s_5 & 0 \\ -c_3 & 0 & -1 & 0 & -c_5 & 0 \\ 0 & 0 & 0 & 1 & 0 & c_6 \\ -c_3(c_4 g + h) & 0 & -h & 0 & 0 & 0 \\ s_3(g + c_4 h) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.11)$$

Equating ${}^{ref}\mathbf{W}_{recip_{1b}} \otimes {}^{ref}\$ _j = 0$, for $j = 2, 4$ to 7 , and ${}^{ref}\mathbf{W}_{recip_{1b}} \otimes {}^{ref}\mathbf{W}_1^* = 0$ allows $\mathbf{W}_{recip_{1b}}$ to be found as:

$${}^{ref}\mathbf{W}_{recip_{1b}} = \left\{ -s_5, \frac{s_3 c_4 c_5 h - c_3 c_4 s_5 g}{s_3(g + c_4 h)}, 0; c_5 h, s_5 h, 0 \right\}^T \quad (3.12)$$

Setting the reciprocal products between ${}^{ref}\mathbf{W}_{recip_{1b}}$ & $\$ _1$ and ${}^{ref}\mathbf{W}_{recip_{1b}}$ & $\$ _3$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{1b}} \otimes {}^{ref}\$ _1 = s_2 c_4 s_5 g = 0 \quad (3.13)$$

$${}^{ref}\mathbf{W}_{recip_{1b}} \otimes {}^{ref}\$ _3 = 0 \quad (3.14)$$

Since s_4 and s_6 are assumed to equal zero, if in addition $s_2 = 0$ or $s_5 = 0$, then $\mathbf{W}_{recip_{1b}}$ is reciprocal to joints $\$ _1$ to $\$ _7$. Therefore, $s_2 = 0$, $s_4 = 0$, & $s_6 = 0$ and $s_4 = 0$, $s_5 = 0$, & $s_6 = 0$ define two 3-condition families of 2-DOF-loss velocity-degenerate configurations.

c) Setting $g + c_4h = 0$ in $[\$]_{sub_1^*}$ (with $s_4 = 0$) yields:

$${}^{ref}[\$]_{sub_{1c}^*} = \begin{bmatrix} -s_3c_4 & 0 & 0 & 0 & s_5 & -c_5s_6 \\ -c_3 & 0 & -1 & 0 & -c_5 & -s_5s_6 \\ 0 & 0 & 0 & 1 & 0 & c_6 \\ -c_3(c_4g + h) & 0 & -h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.15)$$

The reciprocal screw for the six “joints” comprising $[\$]_{sub_1^*}$, with $s_4 = 0$ & $g + c_4h = 0$, can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_{1c}} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (3.16)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{1c}}$ & $\$1$ and $\mathbf{W}_{recip_{1c}}$ & $\$3$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{1c}} \otimes {}^{ref}\$1 = -s_2c_3(g + c_4h) = 0 \quad (3.17)$$

$${}^{ref}\mathbf{W}_{recip_{1c}} \otimes {}^{ref}\$3 = 0 \quad (3.18)$$

Note that with $s_4 = 0$ & $g + c_4h = 0$, no further conditions are necessary to make $\mathbf{W}_{recip_{1c}}$ reciprocal to joints $\$1$ to $\$7$. Therefore, $s_4 = 0$ & $g + c_4h = 0$ defines a 2-condition family of 2-DOF-loss velocity-degenerate configurations. Note that the condition $s_3 = 0$, $s_4 = 0$, & $g + c_4h = 0$ found in Section 3.3.1(1)a is a special case of the more general condition $s_4 = 0$ & $g + c_4h = 0$.

Condition (2): $s_2 = 0$ & $c_3 = 0$

Setting $s_2 = 0$ & $c_3 = 0$ and ${}^{ref}\mathbf{W}_2^* = \{0, 0, 0; 0, 0, 1\}^T$ allows $[\$]_{sub_2}^*$ to be defined as:

$${}^{ref}[\$]_{sub_2}^* = {}^{ref} \begin{bmatrix} \mathbf{W}_2^* & \$3 & \$4 & \$5 & \$6 & \$7 \end{bmatrix} = \begin{bmatrix} 0 & s_4 & 0 & 0 & s_5 & -c_5 s_6 \\ 0 & 0 & -1 & 0 & -c_5 & -s_5 s_6 \\ 0 & c_4 & 0 & 1 & 0 & c_6 \\ 0 & 0 & -h & 0 & 0 & 0 \\ 0 & -s_4 h & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.19)$$

with the redundant joints being $\$1$ and $\$2$. The determinant of $[\$]_{sub_2}^*$ is:

$$\left| {}^{ref}[\$]_{sub_2}^* \right| = -s_4 s_6 h^2 \quad (3.20)$$

Therefore, if either a) $s_4 = 0$ or b) $s_6 = 0$, then the six “joints” comprising $[\$]_{sub_2}^*$ are degenerate.

a) Setting $s_4 = 0$ in $[\$]_{sub_2}^*$ (with $s_2 = 0$ & $c_3 = 0$) yields:

$${}^{ref}[\$]_{sub_{2a}}^* = \begin{bmatrix} 0 & 0 & 0 & 0 & s_5 & -c_5 s_6 \\ 0 & 0 & -1 & 0 & -c_5 & -s_5 s_6 \\ 0 & c_4 & 0 & 1 & 0 & c_6 \\ 0 & 0 & -h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.21)$$

The reciprocal screw for the six “joints” comprising $[\$]_{sub_2^*}$, with $s_2 = 0$, $c_3 = 0$, & $s_4 = 0$, can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_{2a}} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (3.22)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{2a}}$ & $\$1$ and $\mathbf{W}_{recip_{2a}}$ & $\$2$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{2a}} \otimes {}^{ref}\$1 = -s_2 c_3 (g + c_4 h) - c_2 s_4 h = 0 \quad (3.23)$$

$${}^{ref}\mathbf{W}_{recip_{2a}} \otimes {}^{ref}\$2 = s_3 (g + c_4 h) = 0 \quad (3.24)$$

Since s_2 , c_3 , and s_4 are assumed to equal zero, if in addition $g + c_4 h = 0$, then $\mathbf{W}_{recip_{2a}}$ is reciprocal to joints $\$1$ to $\$7$. It was shown in Section 3.3.1(1)c that $s_4 = 0$ & $g + c_4 h = 0$ results in a degenerate configuration, therefore, no new conditions are identified.

b) Setting $s_6 = 0$ in $[\$]_{sub_2^*}$ (with $s_2 = 0$ & $c_3 = 0$) yields:

$${}^{ref}[\$]_{sub_{2b}^*} = \begin{bmatrix} 0 & s_4 & 0 & 0 & s_5 & 0 \\ 0 & 0 & -1 & 0 & -c_5 & 0 \\ 0 & c_4 & 0 & 1 & 0 & c_6 \\ 0 & 0 & -h & 0 & 0 & 0 \\ 0 & -s_4 h & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.25)$$

Equating ${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\$j = 0$, for $j = 3$ to 7 , and ${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\mathbf{W}_2^* = 0$ allows $\mathbf{W}_{recip_{2b}}$ to be found as:

$${}^{ref}\mathbf{W}_{recip_{2b}} = \left\{ -s_5, c_5, 0; c_5 h, s_5 h, 0 \right\}^T \quad (3.26)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{2b}}$ & $\$1$ and $\mathbf{W}_{recip_{2b}}$ & $\$2$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\$1 = 0 \quad (3.27)$$

$${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\$2 = s_3 c_5 g = 0 \quad (3.28)$$

Since s_2 , c_3 , and s_6 are assumed to equal zero, if in addition $c_5 = 0$, then $\mathbf{W}_{recip_{2b}}$ is reciprocal to joints $\$1$ to $\$7$. Therefore, $s_2 = 0$, $c_3 = 0$, $c_5 = 0$, & $s_6 = 0$ defines a 4-condition family of 2-DOF-loss velocity-degenerate configurations.

Condition (3): $s_2 = 0$ & $s_6 = 0$

Setting $s_2 = 0$ & $s_6 = 0$ and ${}^{ref}\mathbf{W}_3^* = \{c_5 h, s_5 h, 0; -s_5, c_5, \frac{-c_3 c_4 s_5 - s_3 c_5}{c_3 s_4}\}^T$ allows $[\$]_{sub_3^*}$ to be defined as:

$${}^{ref}[\$]_{sub_3^*} = {}^{ref} \begin{bmatrix} \$2 & \$3 & \$4 & \$5 & \$6 & \mathbf{W}_3^* \end{bmatrix} = \begin{bmatrix} -s_3 c_4 & s_4 & 0 & 0 & s_5 & c_5 h \\ -c_3 & 0 & -1 & 0 & -c_5 & s_5 h \\ s_3 s_4 & c_4 & 0 & 1 & 0 & 0 \\ -c_3 (c_4 g + h) & 0 & -h & 0 & 0 & -s_5 \\ s_3 (g + c_4 h) & -s_4 h & 0 & 0 & 0 & c_5 \\ c_3 s_4 g & 0 & 0 & 0 & 0 & \frac{-c_3 c_4 s_5 - s_3 c_5}{c_3 s_4} \end{bmatrix} \quad (3.29)$$

with the redundant joints being $\$1$ and $\$7$. The determinant of $[\$]_{sub_3^*}$ is:

$$\left| {}^{ref}[\$]_{sub_3^*} \right| = -s_4 g h K \quad (3.30)$$

where

$$K = -c_3^2 c_4^2 c_5^2 + 2c_3 s_3 c_4 c_5 s_5 + c_5^2 - c_3^2 c_5^2 + c_3^2 + c_3^2 h^2 - c_3^2 c_4^2 h^2 \quad (3.31)$$

Therefore, if either a) $K = 0$ or b) $s_4 = 0$, then the six “joints” comprising $[\$]_{sub_3^*}$ are degenerate. It was already shown in Section 3.3.1(1)b that $s_2 = 0$, $s_4 = 0$, & $s_6 = 0$ is a 2-DOF-loss degeneracy, therefore, the only condition that needs to be investigated is whether or not $s_2 = 0$, $s_6 = 0$, & $K = 0$ results in a 2-DOF-loss degeneracy.

a) Assume $s_2 = 0$, $s_6 = 0$, & $K = 0$. Equating ${}^{ref}\mathbf{W}_{recip_{3a}} \otimes {}^{ref}\$\mathbf{j} = 0$, for $j = 2$ to 6, and ${}^{ref}\mathbf{W}_{recip_{3a}} \otimes {}^{ref}\mathbf{W}_3^* = 0$ allows $\mathbf{W}_{recip_{3a}}$ to be found as:

$${}^{ref}\mathbf{W}_{recip_{3a}} = \left\{ -s_5, c_5, \frac{-c_3c_4s_5 - s_3c_5}{c_3s_4}; c_5h, s_5h, 0 \right\}^T \quad (3.32)$$

Note that ${}^{ref}\mathbf{W}_{recip_{3a}}$ is the same as ${}^{ref}\mathbf{W}_3$, therefore, $s_2 = 0$, $s_6 = 0$, & $K = 0$ is a special case of the 1-DOF-loss degeneracy $s_2 = 0$ & $s_6 = 0$.

Condition (4): $c_5 = 0$ & $s_6 = 0$

Setting $c_5 = 0$ & $s_6 = 0$ and ${}^{ref}\mathbf{W}_4^* = c_3s_4 * \{c_5h, s_5h, 0; -s_5, c_5, \frac{-c_3c_4s_5 - s_3c_5}{c_3s_4}\}^T = \{0, c_3s_4s_5h, 0; -c_3s_4s_5, 0, -c_3c_4s_5\}^T$ allows $[\$]_{sub_4}^*$ to be defined as:

$${}^{ref}[\$]_{sub_4}^* = \begin{bmatrix} \$2 & \$3 & \$4 & \$5 & \$6 & \mathbf{W}_4^* \end{bmatrix} = \begin{bmatrix} -s_3c_4 & s_4 & 0 & 0 & s_5 & 0 \\ -c_3 & 0 & -1 & 0 & 0 & c_3s_4s_5h \\ s_3s_4 & c_4 & 0 & 1 & 0 & 0 \\ -c_3(c_4g + h) & 0 & -h & 0 & 0 & -c_3s_4s_5 \\ s_3(g + c_4h) & -s_4h & 0 & 0 & 0 & 0 \\ c_3s_4g & 0 & 0 & 0 & 0 & -c_3c_4s_5 \end{bmatrix} \quad (3.33)$$

with the redundant joints being $\$1$ and $\$7$. The determinant of $[\$]_{sub_4}^*$ is:

$$\left| {}^{ref}[\$]_{sub_4}^* \right| = c_3^2s_4s_5gh(1 + s_4^2h^2) \quad (3.34)$$

Therefore, if a) $s_4 = 0$ or b) $c_3 = 0$, then the six “joints” comprising $[\$]_{sub_4^*}$ are degenerate. Noting that $c_3 = 0$, $c_5 = 0$, & $s_6 = 0$ causes ${}^{ref}\mathbf{W}_4^*$ to collapse into a zero screw (${}^{ref}\mathbf{W}_4^* = \mathbf{0}_{6 \times 1}$), the only condition that needs to be investigated is whether or not $s_4 = 0$, $c_5 = 0$, & $s_6 = 0$ results in a 2-DOF-loss degeneracy.

a) Setting $s_4 = 0$ in $[\$]_{sub_4^*}$ (with $c_5 = 0$ & $s_6 = 0$) yields:

$${}^{ref}[\$]_{sub_{4a}^*} = \begin{bmatrix} -s_3c_4 & 0 & 0 & 0 & s_5 & 0 \\ -c_3 & 0 & -1 & 0 & 0 & 0 \\ 0 & c_4 & 0 & 1 & 0 & 0 \\ -c_3(c_4g + h) & 0 & -h & 0 & 0 & 0 \\ s_3(g + c_4h) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_3c_4s_5 \end{bmatrix} \quad (3.35)$$

Equating ${}^{ref}\mathbf{W}_{recip_{4a}} \otimes {}^{ref}\$_j = 0$, for $j = 2$ to 6 , and ${}^{ref}\mathbf{W}_{recip_{4a}} \otimes {}^{ref}\mathbf{W}_4^* = 0$ allows $\mathbf{W}_{recip_{4a}}$ to be found as:

$${}^{ref}\mathbf{W}_{recip_{4a}} = \left\{ s_3(g + c_4h), c_3c_4g, 0; 0, -s_3h(g + c_4h), 0 \right\}^T \quad (3.36)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{4a}}$ & $\$_1$ and $\mathbf{W}_{recip_{4a}}$ & $\$_7$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{4a}} \otimes {}^{ref}\$_1 = -s_2c_4g(g + c_4h) = 0 \quad (3.37)$$

$${}^{ref}\mathbf{W}_{recip_{4a}} \otimes {}^{ref}\$_7 = 0 \quad (3.38)$$

Since s_4 , c_5 , and s_6 are assumed to equal zero, if in addition $g + c_4h = 0$ or $s_2 = 0$, then $\mathbf{W}_{recip_{4a}}$ is reciprocal to joints $\$_1$ to $\$_7$. It was shown in Section 3.3.1(1)c that $s_4 = 0$ & $g + c_4h = 0$ results in a 2-DOF motion loss degenerate configuration and it

was shown in Section 3.3.1(1)b that $s_2 = 0$, $s_4 = 0$, & $s_6 = 0$ results in degenerate configuration, therefore, no new conditions are identified.

Examining all of the degenerate configurations yields five sets of conditions (one set requiring the satisfaction of two conditions, three sets requiring the satisfaction of three conditions, and one set requiring the satisfaction of four conditions) defining families of 2-DOF-loss degenerate configurations for the spherical-revolute-spherical manipulator. These degenerate configurations and their associated reciprocal screws

can be summarized as:

$$1) \quad s_4 = 0 \ \& \ g + c_4h = 0$$

$$\begin{aligned} {}^{ref}\mathbf{W}_{1_1} &= {}^{ref}\mathbf{W}_1 = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\ {}^{ref}\mathbf{W}_{2_1} &= {}^{ref}\mathbf{W}_{recip1c} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \end{aligned}$$

$$2) \quad s_2 = 0, \ s_3 = 0, \ \& \ s_4 = 0$$

$$\begin{aligned} {}^{ref}\mathbf{W}_{1_2} &= {}^{ref}\mathbf{W}_1 = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\ {}^{ref}\mathbf{W}_{2_2} &= {}^{ref}\mathbf{W}_{recip1a} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \end{aligned}$$

$$3) \quad s_2 = 0, \ s_4 = 0, \ \& \ s_6 = 0$$

$$\begin{aligned} {}^{ref}\mathbf{W}_{1_3} &= {}^{ref}\mathbf{W}_1 = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\ {}^{ref}\mathbf{W}_{2_3} &= {}^{ref}\mathbf{W}_{recip1b} = \left\{ -s_5, \frac{s_3c_4c_5h - c_3c_4s_5g}{s_3(g+c_4h)}, 0; c_5h, s_5h, 0 \right\}^T \end{aligned}$$

$$4) \quad s_4 = 0, \ s_5 = 0, \ \& \ s_6 = 0$$

$$\begin{aligned} {}^{ref}\mathbf{W}_{1_4} &= {}^{ref}\mathbf{W}_1 = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\ {}^{ref}\mathbf{W}_{2_4} &= {}^{ref}\mathbf{W}_{recip1b} = \left\{ -s_5, \frac{s_3c_4c_5h - c_3c_4s_5g}{s_3(g+c_4h)}, 0; c_5h, s_5h, 0 \right\}^T \end{aligned}$$

$$5) \quad s_2 = 0, \ c_3 = 0, \ c_5 = 0, \ \& \ s_6 = 0$$

$$\begin{aligned} {}^{ref}\mathbf{W}_{1_5} &= {}^{ref}\mathbf{W}_2 = \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\ {}^{ref}\mathbf{W}_{2_5} &= {}^{ref}\mathbf{W}_{recip2b} = \left\{ -s_5, c_5, 0; c_5h, s_5h, 0 \right\}^T \end{aligned}$$

(3.39)

Note that condition (1) is only possible if $g = h$ and $\theta_4 = \pi$, i.e., the condition is link

length dependent.

3.3.2 Double-Elbow Manipulator

As presented in Section 2.3.2, the joint screws for the double-elbow manipulator are::

$$\begin{aligned}
 {}^{ref}\$1 &= \left\{ s_{234}, 0, c_{234}; -c_{234}f, c_2g + c_{23}h + c_{234}i, s_{234}f \right\}^T \\
 {}^{ref}\$2 &= \left\{ 0, -1, 0; s_{34}g + s_4h, 0, c_{34}g + c_4h + i \right\}^T \\
 {}^{ref}\$3 &= \left\{ 0, -1, 0; s_4h, 0, c_4h + i \right\}^T \\
 {}^{ref}\$4 &= \left\{ 0, -1, 0; 0, 0, i \right\}^T \\
 {}^{ref}\$5 &= \left\{ 0, -1, 0; 0, 0, 0 \right\}^T \\
 {}^{ref}\$6 &= \left\{ -s_5, 0, c_5; 0, 0, 0 \right\}^T \\
 {}^{ref}\$7 &= \left\{ c_5s_6, -c_6, s_5s_6; 0, 0, 0 \right\}^T
 \end{aligned} \tag{3.40}$$

As derived in the same section (Section 2.3.2), the 1-DOF-loss velocity degeneracies, along with their associated reciprocal screws, for the double-elbow manipulator were

shown to be:

$$1) \quad s_6 = 0$$

$${}^{ref}\mathbf{W}_1 = \left\{ 0, 1, 0; -\frac{c_5(c_{2g}+c_{23}h+c_{234}i)}{s_{2345}}, 0, -\frac{s_5(c_{2g}+c_{23}h+c_{234}i)}{s_{2345}} \right\}^T$$

$$2) \quad c_{2g} + c_{23}h + c_{234}i = 0$$

$${}^{ref}\mathbf{W}_2 = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (3.41)$$

$$3) \quad s_3 = 0 \ \& \ s_4 = 0$$

$${}^{ref}\mathbf{W}_3 = \left\{ 1, \frac{c_{234}f}{c_{2g}+c_{23}h+c_{234}i}, 0; 0, 0, 0 \right\}^T$$

Note, to simplify the derivation of the 2-DOF-loss configurations, degeneracy condition (2) will be used first, followed by degeneracy condition (3) and degeneracy condition (1).

Condition (2): $c_2g + c_{23}h + c_{234}i = 0$

Setting $c_2g + c_{23}h + c_{234}i = 0$ and ${}^{ref}\mathbf{W}_2^* = \{0, 0, 0; 0, 1, 0\}^T$ allows $[\$]_{sub_2^*}$ to be defined as:

$${}^{ref}[\$]_{sub_2^*} = {}^{ref} \begin{bmatrix} \mathbf{W}_2^* & \$3 & \$4 & \$5 & \$6 & \$7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -s_5 & -c_5s_6 \\ 0 & -1 & -1 & -1 & 0 & -c_6 \\ 0 & 0 & 0 & 0 & c_5 & s_5s_6 \\ 0 & s_4h & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_4h + i & i & 0 & 0 & 0 \end{bmatrix} \quad (3.42)$$

with the redundant joints being $\$1$ and $\$2$. The determinant of $[\$]_{sub_2^*}$ is:

$$\left| {}^{ref}[\$]_{sub_2^*} \right| = -s_4s_6hi \quad (3.43)$$

Therefore, if either a) $s_4 = 0$ or b) $s_6 = 0$, then the six ‘‘joints’’ comprising $[\$]_{sub_2^*}$ are degenerate.

a) Setting $s_4 = 0$ in $[\$]_{sub_2^*}$ (with $c_2g + c_{23}h + c_{234}i = 0$) yields:

$${}^{ref}[\$]_{sub_{2a}^*} = \begin{bmatrix} 0 & 0 & 0 & 0 & -s_5 & -c_5s_6 \\ 0 & -1 & -1 & -1 & 0 & -c_6 \\ 0 & 0 & 0 & 0 & c_5 & s_5s_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_4h + i & i & 0 & 0 & 0 \end{bmatrix} \quad (3.44)$$

The reciprocal screw for the six joints comprising $[\$]_{sub_2^*}$, with $s_4 = 0$ & $c_2g + c_{23}h + c_{234}i = 0$, can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_{2a}} = \left\{ 1, 0, 0; 0, 0, 0 \right\}^T \quad (3.45)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{2a}}$ & $\$1$ and $\mathbf{W}_{recip_{2a}}$ & $\$2$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{2a}} \otimes {}^{ref}\$1 = -c_{234}f = 0 \quad (3.46)$$

$${}^{ref}\mathbf{W}_{recip_{2a}} \otimes {}^{ref}\$2 = -s_{34}g + s_4h = 0 \quad (3.47)$$

Since s_4 and $c_2g + c_{23}h + c_{234}i$ are assumed to equal zero, if in addition $c_{234} = 0$ & $s_{34} = 0$, then $\mathbf{W}_{recip_{2a}}$ is reciprocal to joints $\$1$ to $\$7$. Note that if $s_4 = 0$ and $s_{34} = 0$, then $s_3 = 0$ must be true. Therefore, $s_3 = 0$, $s_4 = 0$, $c_{234} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$ defines a 4-condition family of 2-DOF-loss velocity-degenerate configurations.

b) Setting $s_6 = 0$ in $[\$]_{sub_2^*}$ (with $c_2g + c_{23}h + c_{234}i = 0$) yields:

$${}^{ref}[\$]_{sub_{2b}^*} = \begin{bmatrix} 0 & 0 & 0 & 0 & -s_5 & 0 \\ 0 & -1 & -1 & -1 & 0 & -c_6 \\ 0 & 0 & 0 & 0 & c_5 & 0 \\ 0 & s_4h & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_4h + i & i & 0 & 0 & 0 \end{bmatrix} \quad (3.48)$$

Equating ${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\$j = 0$, for $j = 3$ to 7 , and ${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\mathbf{W}_2^* = 0$ allows $\mathbf{W}_{recip_{2b}}$ to be found as:

$${}^{ref}\mathbf{W}_{recip_{2b}} = \left\{ 0, 0, 0; c_5, 0, s_5 \right\}^T \quad (3.49)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{2b}}$ & $\$1$ and $\mathbf{W}_{recip_{2b}}$ & $\$2$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\$1 = s_{2345} = 0 \quad (3.50)$$

$${}^{ref}\mathbf{W}_{recip_{2b}} \otimes {}^{ref}\$2 = 0 \quad (3.51)$$

Since s_6 and $c_2g + c_{23}h + c_{234}i$ are assumed to equal zero, if in addition $s_{2345} = 0$, then $\mathbf{W}_{recip_{2b}}$ is reciprocal to joints $\$1$ to $\$7$. Therefore, $s_6 = 0$, $s_{2345} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$ defines a 3-condition family of 2-DOF-loss velocity-degenerate configurations.

Condition (3): $s_3 = 0$ & $s_4 = 0$

Setting $s_3 = 0$ & $s_4 = 0$ and ${}^{ref}\mathbf{W}_3^* = c_2g + c_{23}h + c_{234}i * \{0, 0, 0; 1, \frac{c_{234}f}{c_2g + c_{23}h + c_{234}i}, 0\}^T = \{0, 0, 0; c_2g + c_{23}h + c_{234}i, c_{234}f, 0\}^T$ allows $[\$]_{sub_3^*}$ to be defined as:

$$\begin{aligned} {}^{ref}[\$]_{sub_3^*} &= {}^{ref} \begin{bmatrix} \$1 & \mathbf{W}_3^* & \$4 & \$5 & \$6 & \$7 \end{bmatrix} \\ &= \begin{bmatrix} s_{234} & 0 & 0 & 0 & -s_5 & -c_5s_6 \\ 0 & 0 & -1 & -1 & 0 & -c_6 \\ c_{234} & 0 & 0 & 0 & c_5 & s_5s_6 \\ -c_{234}f & c_2g + c_{23}h + c_{234}i & 0 & 0 & 0 & 0 \\ c_2g + c_{23}h + c_{234}i & c_{234}f & 0 & 0 & 0 & 0 \\ s_{234}f & 0 & i & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (3.52)$$

with the redundant joints being $\$2$ and $\$3$. The determinant of $[\$]_{sub_3^*}$ is:

$$\left| {}^{ref}[\$]_{sub_3^*} \right| = s_6i \left((c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2 \right) \quad (3.53)$$

Note that in order for $(c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2$ to equal zero, both $c_{234} = 0$ and $c_2g + c_{23}h + c_{234}i = 0$ must be true, therefore, if either a) $s_6 = 0$ or b) $c_{234} = 0$ & $c_2g + c_{23}h + c_{234}i = 0$, then the six “joints” comprising $[\$]_{sub_3^*}$ are degenerate.

a) Setting $s_6 = 0$ in $[\$]_{sub_3^*}$ (with $s_3 = 0$ & $s_4 = 0$) yields:

$${}^{ref}[\$]_{sub_{3a}^*} = \begin{bmatrix} s_{234} & 0 & 0 & 0 & -s_5 & 0 \\ 0 & 0 & -1 & -1 & 0 & -c_6 \\ c_{234} & 0 & 0 & 0 & c_5 & 0 \\ -c_{234}f & c_2g + c_{23}h + c_{234}i & 0 & 0 & 0 & 0 \\ c_2g + c_{23}h + c_{234}i & c_{234}f & 0 & 0 & 0 & 0 \\ s_{234}f & 0 & i & 0 & 0 & 0 \end{bmatrix} \quad (3.54)$$

Equating ${}^{ref}\mathbf{W}_{recip_{3a}} \otimes {}^{ref}\$j = 0$, for $j = 1, 4$ to 7 , and ${}^{ref}\mathbf{W}_{recip_{3a}} \otimes {}^{ref}\mathbf{W}_3^* = 0$ allows $\mathbf{W}_{recip_{3a}}$ to be found as:

$${}^{ref}\mathbf{W}_{recip_{3a}} = \left\{ \frac{s_{2345}c_{234}f}{(c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2}, \frac{-s_{2345}(c_2g + c_{23}h + c_{234}i)}{(c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2}, 0, c_5, 0, s_5 \right\}^T \quad (3.55)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{3a}}$ & $\$2$ and $\mathbf{W}_{recip_{3a}}$ & $\$3$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{3a}} \otimes {}^{ref}\$2 = \frac{s_{2345}c_{234}f(s_{34}g + s_4h)}{(c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2} = 0 \quad (3.56)$$

$${}^{ref}\mathbf{W}_{recip_{3a}} \otimes {}^{ref}\$3 = \frac{s_{2345}c_{234}f(s_4h)}{(c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2} = 0 \quad (3.57)$$

Note that with $s_3 = 0$, $s_4 = 0$, & $s_6 = 0$, no further conditions are necessary to make $\mathbf{W}_{recip_{3a}}$ reciprocal to joints $\$1$ to $\$7$. Therefore, $s_3 = 0$, $s_4 = 0$, & $s_6 = 0$ defines a 3-condition family of 2-DOF-loss velocity-degenerate configurations.

b) Setting $c_{234} = 0$ & $c_2g + c_{23}h + c_{234}i = 0$ in $[\$]_{sub_3^*}$ (with $s_3 = 0$ & $s_4 = 0$) yields:

$${}^{ref}[\$]_{sub_3^*} = \begin{bmatrix} s_{234} & 0 & 0 & 0 & -s_5 & -c_5s_6 \\ 0 & 0 & -1 & -1 & 0 & -c_6 \\ 0 & 0 & 0 & 0 & c_5 & s_5s_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ s_{234}f & 0 & i & 0 & 0 & 0 \end{bmatrix} \quad (3.58)$$

The reciprocal screw for the six joints comprising $[\$]_{sub_3^*}$, with $s_3 = 0$, $s_4 = 0$, $c_{234} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$ can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_{3b}} = \left\{ \sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}, 0, 0, 0, 0 \right\}^T \quad (3.59)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{3b}}$ & $\$2$ and $\mathbf{W}_{recip_{3b}}$ & $\$3$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{3b}} \otimes {}^{ref}\$2 = \sqrt{\frac{1}{2}}(s_{34}g + s_4h) = 0 \quad (3.60)$$

$${}^{ref}\mathbf{W}_{recip_{3b}} \otimes {}^{ref}\$3 = \sqrt{\frac{1}{2}}s_4h = 0 \quad (3.61)$$

Note that with $s_3 = 0$, $s_4 = 0$, $c_{234} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$, no further conditions are necessary to make $\mathbf{W}_{recip_{3b}}$ reciprocal to joints $\$1$ to $\$7$. It was shown in Section 3.3.2(2)a that $s_3 = 0$, $s_4 = 0$, $c_{234} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$ results in degenerate configuration, therefore, no new conditions are identified.

Condition (1): $s_6 = 0$

Setting $s_6 = 0$ and ${}^{ref}\mathbf{W}_1^* = s_{2345} * \left\{ -\frac{c_5(c_2g + c_{23}h + c_{234}i)}{s_{2345}}, 0, -\frac{s_5(c_2g + c_{23}h + c_{234}i)}{s_{2345}}; 0, 1, 0 \right\}^T = \left\{ -c_5(c_2g + c_{23}h + c_{234}i), 0, -s_5(c_2g + c_{23}h + c_{234}i); 0, s_{2345}, 0 \right\}^T$ allows $[\$]_{sub_1^*}$

to be defined as:

$$\begin{aligned}
 & {}^{ref} [\$]_{sub_1^*} \\
 &= {}^{ref} \left[\begin{array}{cccccc} \$1 & \$2 & \$3 & \$4 & \mathbf{W}_1^* & \$6 \end{array} \right] \\
 &= \left[\begin{array}{cccccc} s_{234} & 0 & 0 & 0 & -c_5 (c_2g + c_{23}h + c_{234}i) & -s_5 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ c_{234} & 0 & 0 & 0 & -s_5 (c_2g + c_{23}h + c_{234}i) & c_5 \\ -c_{234}f & s_{34}g + s_4h & s_4h & 0 & 0 & 0 \\ c_2g + c_{23}h + c_{234}i & 0 & 0 & 0 & s_{2345} & 0 \\ s_{234}f & c_{34}g + c_4h + i & c_4h + i & i & 0 & 0 \end{array} \right]
 \end{aligned} \tag{3.62}$$

with the redundant joints being $\$5$ and $\$7$. The determinant of $[\$]_{sub_1^*}$ is:

$$\left| {}^{ref} [\$]_{sub_1^*} \right| = -s_3gh \left((c_2g + c_{23}h + c_{234}i)^2 + s_{2345}^2 \right) \tag{3.63}$$

Note that in order for $(c_2g + c_{23}h + c_{234}i)^2 + s_{2345}^2$ to equal zero, both $s_{2345} = 0$ and $c_2g + c_{23}h + c_{234}i = 0$ must be true, therefore, if either a) $s_3 = 0$ or b) $s_{2345} = 0$ & $c_2g + c_{23}h + c_{234}i = 0$, then the six “joints” comprising $[\$]_{sub_1^*}$ are degenerate.

a) Setting $s_3 = 0$ in $[\$]_{sub_1^*}$ (with $s_6 = 0$) yields:

$$\begin{aligned}
 & {}^{ref} [\$]_{sub_{1a}^*} \\
 &= \left[\begin{array}{cccccc} s_{234} & 0 & 0 & 0 & -c_5 (c_2g + c_{23}h + c_{234}i) & -s_5 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ c_{234} & 0 & 0 & 0 & -s_5 (c_2g + c_{23}h + c_{234}i) & c_5 \\ -c_{234}f & s_{34}g + s_4h & s_4h & 0 & 0 & 0 \\ c_2g + c_{23}h + c_{234}i & 0 & 0 & 0 & s_{2345} & 0 \\ s_{234}f & c_{34}g + c_4h + i & c_4h + i & i & 0 & 0 \end{array} \right]
 \end{aligned} \tag{3.64}$$

Equating ${}^{ref}\mathbf{W}_{recip1a} \otimes {}^{ref}\mathcal{S}_j = 0$, for $j = 1$ to $4, 6$, and ${}^{ref}\mathbf{W}_{recip1a} \otimes {}^{ref}\mathbf{W}_1^* = 0$ allows $\mathbf{W}_{recip1a}$ to be found as:

$${}^{ref}\mathbf{W}_{recip1a} = \left\{ \begin{array}{c} c_4 ((c_2g + c_{23}h + c_{234}i)^2 + s_{2345}^2) \\ c_{23}f (c_2g + c_{23}h + c_{234}i) \\ -s_4 ((c_2g + c_{23}h + c_{234}i)^2 + s_{2345}^2) \\ c_5 c_{23} s_{2345} f \\ -s_4 ((c_2g + c_{23}h + c_{234}i)^2 + s_{2345}^2) i \\ s_5 c_{23} s_{2345} f \end{array} \right\} \quad (3.65)$$

Setting the reciprocal products between $\mathbf{W}_{recip1a}$ & \mathcal{S}_5 and $\mathbf{W}_{recip1a}$ & \mathcal{S}_7 to zero yields:

$${}^{ref}\mathbf{W}_{recip1a} \otimes {}^{ref}\mathcal{S}_5 = s_4 ((c_2g + c_{23}h + c_{234}i)^2 + s_{2345}^2) i = 0 \quad (3.66)$$

$${}^{ref}\mathbf{W}_{recip1a} \otimes {}^{ref}\mathcal{S}_7 = s_4 c_6 ((c_2g + c_{23}h + c_{234}i)^2 + s_{2345}^2) i = 0 \quad (3.67)$$

Since s_3 and s_6 are assumed to equal zero, if in addition $s_4 = 0$ or $s_{2345} = 0$ & $c_2g + c_{23}h + c_{234}i = 0$, then $\mathbf{W}_{recip1a}$ is reciprocal to joints \mathcal{S}_1 to \mathcal{S}_7 . It was shown in Section 3.3.2(3)a that $s_3 = 0$, $s_4 = 0$, & $s_6 = 0$ results in degenerate configuration and it was shown in Section 3.3.2(2)b that $s_6 = 0$, $s_{2345} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$ results in a degenerate configuration, therefore, no new conditions are identified.

b) Setting $s_{2345} = 0$ & $c_2g + c_{23}h + c_{234}i = 0$ in $[\$]_{sub_1^*}$ (with $s_6 = 0$) yields:

$${}^{ref}[\$]_{sub_{1b}^*} = \begin{bmatrix} s_{234} & 0 & 0 & 0 & 0 & -s_5 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ c_{234} & 0 & 0 & 0 & 0 & c_5 \\ -c_{234}f & s_{34}g + s_4h & s_4h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ s_{234}f & c_{34}g + c_4h + i & c_4h + i & i & 0 & 0 \end{bmatrix} \quad (3.68)$$

The reciprocal screw for the six joints comprising $[\$]_{sub_1^*}$, with $s_6 = 0$, $s_{2345} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$, can be found from inspection to be:

$${}^{ref}\mathbf{W}_{recip_{1b}} = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T \quad (3.69)$$

Setting the reciprocal products between $\mathbf{W}_{recip_{1b}}$ & $\$5$ and $\mathbf{W}_{recip_{1b}}$ & $\$7$ to zero yields:

$${}^{ref}\mathbf{W}_{recip_{1b}} \otimes {}^{ref}\$5 = 0 \quad (3.70)$$

$${}^{ref}\mathbf{W}_{recip_{1b}} \otimes {}^{ref}\$7 = 0 \quad (3.71)$$

Note that with $s_6 = 0$, $s_{2345} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$, no further conditions are necessary to make ${}^{ref}\mathbf{W}_{recip_{1b}}$ reciprocal to joints $\$1$ to $\$7$. It was shown in Section 3.3.2(2)b that $s_6 = 0$, $s_{2345} = 0$, & $c_2g + c_{23}h + c_{234}i = 0$ results in a degenerate configuration, therefore, no new conditions are identified.

Examining all of the degenerate configurations yields three sets of conditions (two sets requiring the satisfaction of three conditions and one set requiring the satisfaction of four conditions) defining families of 2-DOF-loss degenerate configurations for the double-elbow manipulator. These degenerate configurations and their associated

reciprocal screws can be summarized as:

$$1) \quad s_3 = 0, \quad s_4 = 0, \quad \& \quad s_6 = 0$$

$${}^{ref}\mathbf{W}_{1_1} = {}^{ref}\mathbf{W}_3 = \left\{ 1, \frac{c_{234}f}{c_2g + c_{23}h + c_{234}i}, 0; 0, 0, 0 \right\}^T$$

$${}^{ref}\mathbf{W}_{2_1} = {}^{ref}\mathbf{W}_{recip_{3a}} = \left\{ \begin{array}{c} \frac{s_{2345}c_{234}f}{(c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2} \\ \frac{-s_{2345}(c_2g + c_{23}h + c_{234}i)}{(c_2g + c_{23}h + c_{234}i)^2 + c_{234}^2f^2} \\ 0 \\ c_5 \\ 0 \\ s_5 \end{array} \right\}$$

(3.72)

$$2) \quad s_6 = 0, \quad s_{2345} = 0, \quad \& \quad c_2g + c_{23}h + c_{234}i = 0$$

$${}^{ref}\mathbf{W}_{1_2} = {}^{ref}\mathbf{W}_2 = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T$$

$${}^{ref}\mathbf{W}_{2_2} = {}^{ref}\mathbf{W}_{recip_{2b}} = \left\{ 0, 0, 0; c_5, 0, s_5 \right\}^T$$

$$3) \quad s_3 = 0, \quad s_4 = 0, \quad c_{234} = 0, \quad \& \quad c_2g + c_{23}h + c_{234}i = 0$$

$${}^{ref}\mathbf{W}_{1_3} = {}^{ref}\mathbf{W}_2 = \left\{ 0, 1, 0; 0, 0, 0 \right\}^T$$

$${}^{ref}\mathbf{W}_{2_3} = {}^{ref}\mathbf{W}_{recip_{2a}} = \left\{ 1, 0, 0; 0, 0, 0 \right\}^T$$

3.4 Discussion

3.4.1 Characteristics of Lost Motion DOF

The examples presented in the previous section demonstrate that the reciprocity-based methodology can be extended to find higher DOF loss degenerate configurations

of kinematically-redundant manipulators.

Like the 1-DOF-loss methodology, the multi-DOF-loss methodology identifies reciprocal screws that characterize the loss DOF. The reciprocal screws found for each of the degenerate configurations form a basis of the motion loss for that particular degenerate configuration. Within the degenerate configuration, the manipulator will not be able to produce a motion that would do work subject to a force spanned by the reciprocal screws. Allowable motions in the i^{th} q -DOF degenerate configuration are defined by the reciprocal product equations:

$$\begin{aligned} \mathbf{W}_{1i} \otimes \mathbf{V}_{feasible} &= 0 \\ \vdots & \\ \mathbf{W}_{qi} \otimes \mathbf{V}_{feasible} &= 0 \end{aligned} \tag{3.73}$$

where $\mathbf{V}_{feasible} = \{\boldsymbol{\omega}^T; \mathbf{v}^T\}_{feasible}^T$ represents the possible instantaneous motions (velocities).

3.4.2 Drawback of Using Screw Complements

A drawback of the developed methodology for finding multi-DOF-loss velocity-degenerate configurations is the incorporation of the complement screws \mathbf{W}^* 's. For identifying 2-DOF-loss velocity-degenerate configurations only one complement screw is added to the sub-matrix of "joint screw coordinates". For >2 DOF loss, more than one complement screw is added. The addition of complement screws into the sub-matrix of "joint screw coordinates" can make taking the determinant of the sub-matrix of "joint screw coordinates" more difficult. For example, suppose one wanted to find any 3-DOF-loss velocity-degenerate configurations that are derived from degeneracy condition number 3 in equation (3.39) for the spherical-revolute-spherical manipula-

tor. To apply the methodology would require the incorporation of ${}^{ref}\mathbf{W}_{23}^*$ in the sub-matrix of “joint screw coordinates”. The complexity of ${}^{ref}\mathbf{W}_{23}^*$ would make finding simple expressions for the determinant difficult. Also, in general, the more complement screws added to the sub-matrix of “joint screw coordinates”, the more difficult it is to find simple expressions for the determinant.

3.4.3 On the Non-Frame Invariance of \mathbf{W}^*

Note that the complement of a screw (\mathbf{W}^*) is a non-frame invariant quantity (Truesdell, 1966). Although \mathbf{W}^* is non-frame invariant, the final conditions identified leading to ≥ 2 -DOF-loss are frame invariant. If for any reason one wanted to work in a different frame of reference, one could use the screw transformation defined in equation (A.3) to first transform all the joint screws and reciprocal screws to the desired frame of reference and then apply the methodology presented in this Chapter, i.e., one would take the complement of the reciprocal screws after the reciprocal screws have been transformed to the desired frame of reference.

Chapter 4

Pose Optimization of Serial Manipulators Using Knowledge of Their Velocity-Degenerate Configurations

4.1 Overview

In this chapter, the utilization of velocity-degenerate configurations to optimize the pose of either non-redundant or redundant serial manipulators to sustain desired wrenches is considered. An algorithm is developed that determines a suitable start point for the optimization of a serial manipulator's pose. The start-point algorithm (SPA) uses the knowledge of the velocity-degenerate (singular) configurations of a serial manipulator to determine a pose that would be best suitable to sustain a desired wrench. In a velocity-degenerate configuration a manipulator becomes structural to

a wrench or wrenches acting in a certain direction or directions (Hunt, 1978 and 1986 and Wang and Waldron, 1987). Near a velocity-degenerate configuration a manipulator gains high mechanical advantage. Results for several examples are presented. The chapter finishes with a discussion.

4.2 Formulation of the Pose Optimization Problem¹

Assume that one wants to sustain a known wrench $\mathbf{F} = \{\mathbf{f}^T; \mathbf{m}^T\}^T$ (where \mathbf{f} is the force and \mathbf{m} is the moment). Equation (A.17) of Appendix A, $\boldsymbol{\tau} = -[\$]^T [\Delta] \mathbf{F}$, can be used to solve for the necessary actuated joint torques and forces for this problem. If the pose of the manipulator is allowed to change, the necessary actuated joint torques for revolute joints and forces for prismatic joints to sustain the wrench can be optimized.

For the inverse static force problem of serial manipulators, it would be desirable to minimize the required actuated joint torques and forces $\boldsymbol{\tau}$. Mathematically this can be done by minimizing the p -norm of the vector $\boldsymbol{\tau}$:

$$\|\boldsymbol{\tau}\|_p = \left(\sum_{j=1}^n \tau_j^p \right)^{1/p} \quad (4.1)$$

where p is an even positive number, τ_j is the actuated torque or force of joint j , and n is the number of joints. Setting p to two and optimizing would result in a minimum Euclidean norm solution of the actuated joint torques and forces. Setting p to some

¹The results contained in this section have been presented in Nokleby and Podhorodeski (2003b).

large value (say 100), would allow the minimization of the maximum actuated joint torque or force value.

If all the joints are revolute and the limits on the joint torques are identical, equation (4.1) can be used with no problem. If the manipulator is a combination of revolute and prismatic joints or the limits on all the joint torques are not the same, a vector of normalized joint torques and forces should be used to ensure that the optimization is equally weighting each joint and to ensure that the different units associated with revolute and prismatic joints do not affect the optimization.

To make a normalized actuated torque and force vector ($\hat{\tau}$), each element of τ can be divided by the maximum allowable actuation torque or force for each joint (Papadopoulos and Gonthier, 1999):

$$\hat{\tau} = \left\{ \dots \hat{\tau}_j \dots \right\}^T \quad \text{where } \hat{\tau}_j = \frac{\tau_j}{\tau_{j_{\max}}} \text{ for } j = 1 \text{ to } n \quad (4.2)$$

where $\hat{\tau}_j$ is the normalized actuation torque or force, τ_j is the required actuator torque or force, and $\tau_{j_{\max}}$ is the maximum allowable actuation torque or force of joint j , respectively.

The dimensionless actuated torque and force vector ($\hat{\tau}$) can now be used in place of τ in equation (4.1):

$$\|\hat{\tau}\|_p = \left(\sum_{j=1}^n \hat{\tau}_j^p \right)^{1/p} \quad (4.3)$$

There are three sets of inequality constraints involved in the optimization. Since actuators do not have unlimited load capacity, inequality constraints on the available actuated joint torques and forces are necessary. The inequality constraints for the inverse static force problem can be expressed as:

$$\tau_{lower} \leq \tau \leq \tau_{upper} \quad (4.4)$$

where $\boldsymbol{\tau}_{lower}$ is a vector containing the lower limits on the actuated joint torques and forces and $\boldsymbol{\tau}_{upper}$ is a vector containing the upper limits on the actuated joint torques and forces. The inequality constraint of equation (4.4) can be rewritten as two separate sets of constraints:

$$\boldsymbol{\tau} - \boldsymbol{\tau}_{upper} \leq \mathbf{0} \quad (4.5)$$

$$\boldsymbol{\tau}_{lower} - \boldsymbol{\tau} \leq \mathbf{0} \quad (4.6)$$

where $\boldsymbol{\tau} = -[\$]^T [\Delta] \mathbf{F}$ (see equation (A.17) of Appendix A). Nonlinear inequality constraints must be of the form of equations (4.5) and (4.6) to work in the chosen optimization routine. The optimization routine used in this work is Sequential Quadratic Programming (SQP). Specifically, the SQP routine used is a built-in routine in the MATLAB Optimization Toolbox Version 2.1 (MathWorks, 2001).

A second set of inequality constraints comes from the limits on the joint angles. This set of constraints on the joint limits can be expressed as:

$$\boldsymbol{\theta}_{lower} \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}_{upper} \quad (4.7)$$

where $\boldsymbol{\theta}_{lower}$ is a vector containing the lower limits on the joint angles and lengths for revolute and prismatic actuators and $\boldsymbol{\theta}_{upper}$ is a vector containing the upper limits on the joint angles and lengths for revolute and prismatic actuators. Note that the inequality constraints outlined in equation (4.7) do not need to be split into two separate sets of inequality constraints, as was done for the constraints on $\boldsymbol{\tau}$, because the joint displacements are the search variables of the optimization and can be incorporated directly into the MATLAB SQP routine as lower and upper bounds.

The final inequality constraint is optional. The desired position of the end-effector

can be specified within some tolerance. This constraint can be written as:

$$\|\mathbf{p}_{actual} - \mathbf{p}_{desired}\| - p_{tolerance} \leq 0 \quad (4.8)$$

where \mathbf{p}_{actual} is the actual position of the end-effector, $\mathbf{p}_{desired}$ is the desired position of the end-effector, and $p_{tolerance}$ is a tolerance on the position. Note that $p_{tolerance}$ must be greater than or equal to zero. Setting $p_{tolerance} = 0$ results in the constraint turning into an equality constraint since a 2-norm of a vector can never be less than zero.

The pose optimization problem for a serial manipulator can thus be expressed as:

$$\begin{aligned} \text{minimize} \quad & f(\boldsymbol{\theta}) = \|\hat{\boldsymbol{\tau}}\|_p = \left(\sum_{j=1}^n \hat{\tau}_j^p \right)^{1/p} \\ \text{subject to:} \quad & \boldsymbol{\tau} - \boldsymbol{\tau}_{upper} \leq \mathbf{0} \\ & \boldsymbol{\tau}_{lower} - \boldsymbol{\tau} \leq \mathbf{0} \\ & \boldsymbol{\theta}_{lower} \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}_{upper} \\ & \|\mathbf{p}_{actual} - \mathbf{p}_{desired}\| - p_{tolerance} \leq 0 \end{aligned} \quad (4.9)$$

where $\boldsymbol{\tau} = -[\$]^T [\Delta] \mathbf{F}$ and $\hat{\boldsymbol{\tau}}$ is defined by equation (4.3). For a manipulator with n joints, there are n search variables ($\boldsymbol{\theta}$) and $4n + 1$ inequality constraints ($2n$ for the $\boldsymbol{\tau}$ constraints plus $2n$ for the $\boldsymbol{\theta}$ constraints plus 1 for the position constraint). Note the number of inequality constraints is only $4n$ if there is no constraint on the position, i.e., the last constraint in the problem outlined in equation (4.9) is dropped.

4.3 Start-Point Algorithm (SPA)²

The optimization problem presented in equation (4.9) is non-linear and multi-modal, i.e., there are numerous local minima. The start-point algorithm (SPA) introduced here is designed to find a start point for the optimization routine so that the optimization converges to a better minimum (although it does not guarantee that the minimum found is the global minimum). The basic idea of the SPA is to use the knowledge of the velocity-degenerate configurations of a manipulator to determine a pose that would be best suitable to sustain a desired wrench. In a velocity-degenerate configuration a manipulator becomes structural to a wrench or wrenches acting in a certain direction or directions (Hunt, 1978 and 1986 and Wang and Waldron, 1987). The idea is to find the velocity-degenerate configuration that is closest to becoming structural to the wrench that needs to be sustained. For a required wrench that needs to be sustained a good solution to the pose optimization problem would most likely be near to the velocity-degenerate configuration closest to becoming structural to the wrench.

Knowledge of the velocity-degenerate configurations of the manipulator being considered is critical for the implementation of the SPA. Specifically, formulas for the solutions for each family of velocity-degenerate configurations being considered must be known.

To determine which velocity-degenerate configuration is closest to becoming structural to a given wrench, sets of joints values, θ_1 to θ_{Num} , where Num is the total number of families of velocity-degenerate configurations, are calculated that put the

²The results contained in this section have been presented in Nokleby and Podhorodeski (2003b).

manipulator into configurations belonging to one of each of the families of velocity-degenerate configurations. A velocity-degenerate configuration “closeness” measure, κ , based on the sum of the squares of the reciprocal products between the joint screws and the normalized wrench to be sustained is calculated for each set of joint values (including the original set of joint values θ_0) as:

$$\kappa(\theta) = \sum_{j=1}^n \left(\$_j \circledast \frac{\mathbf{F}}{\|\mathbf{f}\|} \right)^2 \quad (4.10)$$

The smaller the “closeness” measure κ , the closer the manipulator is to a velocity-degenerate configuration that is structural to the wrench $\mathbf{F} = \{\mathbf{f}^T, \mathbf{m}^T\}^T$. For the wrench to be sustained, the configuration with the lowest κ value is “best” for sustaining the wrench \mathbf{F} , i.e., it requires the least amount of effort from the actuators. If $\kappa = 0$, the manipulator is structural to the desired wrench and the actuators are unloaded. The configuration with the lowest velocity-degenerate configuration “closeness” measure is used as the start point for the optimization to find a pose to sustain a given wrench.

The SPA can be summarized as follows:

Start-Point Algorithm (SPA)

Step 1:

Input θ_0 (the manipulator’s original configuration).

Step 2:

For $i = 1$ to Num (the total number of families of velocity-degenerate configurations):

2.1) Calculate a set of joint values (θ_i) such that the manipulator is in a configuration that falls in the i^{th} family of velocity-degenerate configurations. Note that all joint

values are kept constant as the original joint values (θ_0) except those that need to be modified to put the manipulator in a velocity-degenerate configuration.

2.2) Calculate the joints screws $[\$(\theta_i)]$.

2.3) Calculate the velocity-degenerate configuration “closeness” measure κ_i using equation (4.10).

Step 3:

Calculate κ_0 for the manipulator in its original configuration (θ_0) using equation (4.10).

Step 4:

Find the minimum κ_i , $i = 0$ to Num . The configuration corresponding to the lowest κ value is used as the start point for the optimization.

End of Algorithm

In Step 2.1, if multiple solutions exist to put the manipulator into a particular family of velocity-degenerate configurations, the solution that is closest to the existing joint value is chosen. For example, if a velocity-degenerate configuration required $\cos(\theta_3) = 0$, both $\theta_3 = -\frac{\pi}{2}$ and $\theta_3 = \frac{\pi}{2}$ are valid. In this example, the existing value of θ_3 would be used to determine which case the manipulator is currently closer to and that case would be used.

4.4 Examples

4.4.1 Presented Examples and Chosen Parameters

To demonstrate the SPA and its utilization in the pose optimization of serial manipulators for sustaining desired wrenches, three different manipulators will be used: the 7-joint spherical-revolute-spherical manipulator; the 6-joint zero-offset PUMA-type manipulator; and the 7-joint CSA/ISE STEAR Testbed Manipulator-1 (STM-1).

MATLAB Version 6.0 was used as the environment for running the examples. A series of MATLAB m-files (script files used in MATLAB) were created that calculated all the necessary kinematics, ran the SPA and optimization routine, and displayed the results for the example manipulators. Error checks were put in to confirm that the optimization routine converged to a feasible minimum.

As mentioned in Section 4.2, a SQP routine from the MATLAB Optimization Toolbox Version 2.1 was used to perform the optimization. The specific function used was `fmincon` which was set to run the medium-scale algorithm (MathWorks, 2001). For the optimization, the convergence or termination tolerance for the search variables, objective function, and constraint violation were set to 1×10^{-4} , 1×10^{-6} , and 1×10^{-6} , respectively.

For the examples, a p -norm value of 100 was used in equation (4.3).

4.4.2 Spherical-Revolute-Spherical Manipulator

Manipulator Model

Figure 4.1 shows the 7-joint R₁L₁R₁L₁R₁L₁R₁L₁R₁ spherical-revolute-spherical manipulator. The D&H parameters used to model the manipulator are presented in

Table 4.1. The parameters of Table 4.1 correspond to the link transformations defined in equation (2.3).

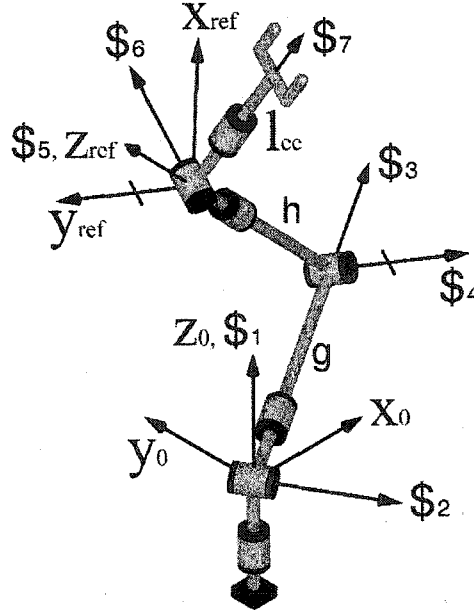


Figure 4.1: Spherical-Revolute-Spherical Manipulator ($\theta_1 = 30^\circ$, $\theta_2 = -80^\circ$, $\theta_3 = -20^\circ$, $\theta_4 = 120^\circ$, $\theta_5 = 120^\circ$, $\theta_6 = 60^\circ$, $\theta_7 = 10^\circ$)

Referring to Figure 4.1, a reference frame that is located at the intersection point of the spherical-wrist group formed by joints 5 through 7 and oriented with z_{ref} in the direction of $\$5$ and y_{ref} in the opposite direction to that of $\$4$ was chosen. The homogeneous transformations describing the position and orientation of F_j , $j = 0$ to 8, with respect to F_{ref} , are defined by:

$${}_{j}^{ref}\mathbf{T} = \begin{bmatrix} {}^{ref}\hat{\mathbf{x}} & {}^{ref}\hat{\mathbf{y}} & {}^{ref}\hat{\mathbf{z}} & {}^{ref}\mathbf{p}_{o_{ref} \rightarrow o_j} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad j = 0 \text{ to } 8 \quad (4.11)$$

Table 4.1: Denavit and Hartenberg Parameters for the Spherical-Revolute-Spherical Manipulator

F_{i-1}	θ_i	d_i	a_i	α_i	F_i
$F_0 (F_{base})$	θ_1	0	0	$\frac{\pi}{2}$	F_1
F_1	θ_2	0	0	$-\frac{\pi}{2}$	F_2
F_2	θ_3	g	0	$\frac{\pi}{2}$	F_3
F_3	θ_4	0	0	$-\frac{\pi}{2}$	F_4
F_4	θ_5	h	0	$\frac{\pi}{2}$	F_5
F_5	θ_6	0	0	$-\frac{\pi}{2}$	F_6
F_6	θ_7	0	0	0	F_7
F_7	0	l_{ee}	0	0	$F_8 (F_{ee})$

with

$${}_{base}^{ref} \mathbf{T} = {}_0^{ref} \mathbf{T} = ({}_1^0 \mathbf{T} {}_2^1 \mathbf{T} {}_3^2 \mathbf{T} {}_4^3 \mathbf{T} {}_{ref}^4 \mathbf{T})^{-1} = {}_4^{ref} \mathbf{T}^{-1} {}_3^4 \mathbf{T}^{-1} {}_2^3 \mathbf{T}^{-1} {}_1^2 \mathbf{T}^{-1} {}_0^1 \mathbf{T}^{-1}$$

$${}_1^{ref} \mathbf{T} = ({}_2^1 \mathbf{T} {}_3^2 \mathbf{T} {}_4^3 \mathbf{T} {}_{ref}^4 \mathbf{T})^{-1} = {}_4^{ref} \mathbf{T}^{-1} {}_3^4 \mathbf{T}^{-1} {}_2^3 \mathbf{T}^{-1} {}_1^2 \mathbf{T}^{-1}$$

$${}_2^{ref} \mathbf{T} = ({}_3^2 \mathbf{T} {}_4^3 \mathbf{T} {}_{ref}^4 \mathbf{T})^{-1} = {}_4^{ref} \mathbf{T}^{-1} {}_3^4 \mathbf{T}^{-1} {}_2^3 \mathbf{T}^{-1}$$

$${}_3^{ref} \mathbf{T} = ({}_4^3 \mathbf{T} {}_{ref}^4 \mathbf{T})^{-1} = {}_4^{ref} \mathbf{T}^{-1} {}_3^4 \mathbf{T}^{-1}$$

$${}_4^{ref} \mathbf{T} = {}_4^{ref} \mathbf{T}^{-1}$$

$${}_5^{ref} \mathbf{T} = {}_4^{ref} \mathbf{T} {}_5^4 \mathbf{T}$$

$${}_6^{ref} \mathbf{T} = {}_4^{ref} \mathbf{T} {}_5^4 \mathbf{T} {}_6^5 \mathbf{T}$$

$${}_7^{ref} \mathbf{T} = {}_4^{ref} \mathbf{T} {}_5^4 \mathbf{T} {}_6^5 \mathbf{T} {}_7^6 \mathbf{T}$$

$${}_{ee}^{ref} \mathbf{T} = {}_8^{ref} \mathbf{T} = {}_4^{ref} \mathbf{T} {}_5^4 \mathbf{T} {}_6^5 \mathbf{T} {}_7^6 \mathbf{T} {}_8^7 \mathbf{T}$$

and

$${}^4_{ref}\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In equation (4.11), ${}^{ref}\hat{\mathbf{x}}$, ${}^{ref}\hat{\mathbf{y}}$, and ${}^{ref}\hat{\mathbf{z}}$ are the three unit vectors of the rotation matrix describing the orientation of F_j with respect to F_{ref} and ${}^{ref}\mathbf{p}_{o_{ref} \rightarrow o_j}$ is a vector from the origin of F_{ref} to the origin of F_j .

The joint screws for the spherical-revolute-spherical manipulator are:

$$\begin{aligned} {}^{ref}\mathcal{S}_1 &= \left\{ \begin{array}{c} s_2 c_3 c_4 + c_2 s_4 \\ -s_2 s_3 \\ -s_2 c_3 s_4 + c_2 c_4 \\ -s_2 s_3 c_4 g - s_2 s_3 h \\ -s_2 c_3 g - s_2 c_3 c_4 h - c_2 s_4 h \\ s_2 s_3 s_4 g \end{array} \right\} \\ {}^{ref}\mathcal{S}_2 &= \left\{ -c_4 s_3, -c_3, s_3 s_4; -c_3 c_4 g - c_3 h, s_3 g + s_3 c_4 h, c_3 s_4 g \right\}^T \\ {}^{ref}\mathcal{S}_3 &= \left\{ s_4, 0, c_4; 0, -s_4 h, 0 \right\}^T \\ {}^{ref}\mathcal{S}_4 &= \left\{ 0, -1, 0; -h, 0, 0 \right\}^T \\ {}^{ref}\mathcal{S}_5 &= \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\ {}^{ref}\mathcal{S}_6 &= \left\{ s_5, -c_5, 0; 0, 0, 0 \right\}^T \\ {}^{ref}\mathcal{S}_7 &= \left\{ -c_5 s_6, -s_5 s_6, c_6; 0, 0, 0 \right\}^T \end{aligned} \quad (4.12)$$

Table 4.2: Joint Parameters for the Spherical-Revolute-Spherical Manipulator

Joint	Motor Torque (N·m)	Gear Ratio	Gear Box Efficiency	Joint Torque (N·m)	Joint Range
1	±15	100:1	0.85	±1275	-170° to 170°
2	±10	100:1	0.85	±850	-170° to 170°
3	±10	100:1	0.85	±850	-170° to 170°
4	±10	100:1	0.85	±850	-170° to 170°
5	±2	150:1	0.85	±255	-170° to 170°
6	±2	150:1	0.85	±255	-170° to 170°
7	±2	150:1	0.85	±255	-170° to 170°

The matrix of joint screw coordinates for the manipulator is:

$${}^{ref}[\$] = {}^{ref}[\$1 \ \$2 \ \$3 \ \$4 \ \$5 \ \$6 \ \$7] \quad (4.13)$$

Manipulator Parameters

For the spherical-revolute-spherical manipulator example, the link lengths for the manipulator were $g = 1.000$ m, $h = 1.000$ m, and $l_{ee} = 0.150$ m. Table 4.2 shows the joint parameters used for the spherical-revolute-spherical manipulator example. Note that the joint torque listed in Table 4.2 represents the torque post gear box, i.e., the joint torque is equal to the motor torque times the gear ratio times the gear box efficiency.

Velocity-Degenerate Configurations and the SPA

Podhorodeski, Goldenberg, and Fenton (1991) showed that there are four families of 1-DOF velocity-degenerate configurations for the spherical-revolute-spherical manipulator: i) $s_4 = 0$, ii) $s_2 = 0$ & $c_3 = 0$, iii) $s_2 = 0$ & $s_6 = 0$, and iv) $c_5 = 0$ & $s_6 = 0$ (these configurations were also proven in Section 2.3.1). It was shown in Section 3.3.1 that there are five 2-DOF velocity-degenerate configurations for the spherical-revolute-spherical manipulator: I) $s_4 = 0$ & $g + c_4h = 0$, II) $s_2 = 0$, $s_3 = 0$, & $s_4 = 0$, III) $s_2 = 0$, $s_4 = 0$, & $s_6 = 0$, IV) $s_4 = 0$, $s_5 = 0$, & $s_6 = 0$, and V) $s_2 = 0$, $c_3 = 0$, $c_5 = 0$, & $s_6 = 0$. Based on the joint limits listed in Table 4.2, the 2-DOF-loss degenerate configuration I) $s_4 = 0$ & $g + c_4h = 0$ cannot be encountered, therefore, only the last four 2-DOF-loss degenerate configurations are considered.

For the spherical-revolute-spherical manipulator example, two cases were considered for the SPA. The first case utilized only the 1-DOF-loss configurations in the SPA ($Num = 4$ in the SPA). This case is denoted SPA-1. The second case involved using both the 1-DOF-loss and 2-DOF-loss configurations ($Num = 8$ in the SPA). This case is denoted SPA-2.

The solutions used for the velocity-degenerate configurations were:

$$\begin{aligned}
 \text{i) } s_4 = 0: & \quad \theta_4 = 0 \\
 \text{ii) } s_2 = 0 \ \& \ c_3 = 0: & \quad \theta_2 = 0 \ \& \ \theta_3 = \pm \frac{\pi}{2} \\
 \text{iii) } s_2 = 0 \ \& \ s_6 = 0: & \quad \theta_2 = 0 \ \& \ \theta_6 = 0 \\
 \text{iv) } c_5 = 0 \ \& \ s_6 = 0: & \quad \theta_3 = \pm \frac{\pi}{2} \ \& \ \theta_6 = 0 \\
 \text{II) } s_2 = 0, \ s_3 = 0, \ \& \ s_4 = 0: & \quad \theta_2 = 0, \ \theta_3 = 0, \ \& \ \theta_4 = 0 \\
 \text{III) } s_2 = 0, \ s_4 = 0, \ \& \ s_6 = 0: & \quad \theta_2 = 0, \ \theta_4 = 0, \ \& \ \theta_6 = 0 \\
 \text{IV) } s_4 = 0, \ s_5 = 0, \ \& \ s_6 = 0: & \quad \theta_4 = 0, \ \theta_5 = 0, \ \& \ \theta_6 = 0 \\
 \text{V) } s_2 = 0, \ c_3 = 0, \ c_5 = 0, \ \& \ s_6 = 0: & \quad \theta_2 = 0, \ \theta_3 = \pm \frac{\pi}{2}, \ \theta_5 = \pm \frac{\pi}{2}, \ \& \ \theta_6 = 0
 \end{aligned} \tag{4.14}$$

Results

Two sets of results are presented to compare the effects of using SPA-1, SPA-2, and no SPA (SPA-0). The first set of results used no position constraint on the end-effector. The second set of results used a position constraint on the end-effector with $p_{tolerance}$ in equation (4.8) set to 0.050 m.

No End-Effector Position Constraint This set of optimization runs utilized no constraint on the position of the end-effector. The runs consisted of the manipulator sequentially sustaining either nine or 17 wrenches, i.e., the optimization routine was executed nine or 17 times for each run. The wrenches to be sustained in the runs were defined as:

$${}^{base}\mathbf{F}_i = \left\{ \begin{array}{l} {}^{base}\mathbf{f}_i \\ {}^{base}\mathbf{p}_{0 \rightarrow p_{ee}} \times {}^{base}\mathbf{f}_i \end{array} \right\}, \text{ for } i = 1 \text{ to } 9 \text{ or } 17 \tag{4.15}$$

where

$${}^{base}\mathbf{f}_i = \left\{ \begin{array}{c} 10 \cos\left(\frac{\pi}{4}(i-1)\right) \\ 10 \sin\left(\frac{\pi}{4}(i-1)\right) \\ 10 \end{array} \right\}^T \mathbf{N}$$

and ${}^{base}\mathbf{p}_{0 \rightarrow p_{ee}}$ is the position of the end-effector for the specified initial configuration $\boldsymbol{\theta}_{0_{initial}}$ of the manipulator. Note that each wrench to be sustained was transformed from the base frame to the reference frame using a screw transformation (see equation (A.3) in Appendix A) since the calculation of κ in equation (4.10) must be done in a common frame.

Five different initial configurations ($\boldsymbol{\theta}_{0_{initial}}$) were considered:

$$\begin{aligned} \boldsymbol{\theta}_{0_{initial_1}} &= \{0, -45^\circ, 45^\circ, -45^\circ, 45^\circ, -45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_2}} &= \{0, 45^\circ, -45^\circ, 45^\circ, -45^\circ, 45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_3}} &= \{0, 45^\circ, 0, 45^\circ, 0^\circ, 45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_4}} &= \{0, -45^\circ, 0, -45^\circ, 0^\circ, -45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_5}} &= \{0, 45^\circ, 0, -45^\circ, 0^\circ, 45^\circ, 0\}^T \end{aligned} \quad (4.16)$$

Note that after the first wrench, the solution from the previous wrench became the $\boldsymbol{\theta}_0$ value for the next wrench.

The results for the runs with nine wrenches to be sustained are presented in Table 4.3 and the results for the runs with 17 wrenches to be sustained are presented in Table 4.4. In Tables 4.3 and 4.4, $\boldsymbol{\theta}_{0_{initial}}$ is the initial configuration for each run and is used to calculate the force in equation (4.15), k is the total number of iterations, $\sum \|\hat{\boldsymbol{\tau}}\|_{100}$ is the sum of the final objective function values for each run, and *Starts* is the total number of times that the SPA start point was used as opposed to $\boldsymbol{\theta}_0$.

Table 4.3: Results for the Spherical-Revolute-Spherical Manipulator Sequentially Sustaining Nine Wrenches with No Constraint on the Position of the End-Effector

$\theta_{0_{initial}}$	SPA-0		SPA-1			SPA-2		
	k	$\sum \ \hat{\tau}\ _{100}$	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>
1	363	0.1062	391	0.0930	6	264	0.0923	6
2	284	0.1395	257	0.1056	5	212	0.1211	4
3	257	0.1187	143	0.1047	5	95	0.1042	4
4	193	0.0941	231	0.0918	6	246	0.0919	5
5	200	0.0867	178	0.0877	6	144	0.0929	6

Table 4.4: Results for the Spherical-Revolute-Spherical Manipulator Sequentially Sustaining 17 Wrenches with No Constraint on the Position of the End-Effector

$\theta_{0_{initial}}$	SPA-0		SPA-1			SPA-2		
	k	$\sum \ \hat{\tau}\ _{100}$	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>
1	504	0.2112	468	0.1686	10	364	0.1631	11
2	535	0.2273	354	0.1764	8	271	0.1919	9
3	457	0.2037	297	0.1849	9	184	0.1776	9
4	421	0.2172	346	0.1687	10	327	0.1662	10
5	275	0.1846	323	0.1724	11	210	0.1786	10

Referring to Tables 4.3 and 4.4, several trends in the results can be noted. In general, when the SPA was utilized, the total objective function value, $\sum \|\hat{\tau}\|_{100}$, was lower for the runs that used the SPA than the runs that did not use the SPA. Usually, utilizing the SPA resulted in a lower total number of iterations than not using the SPA. Comparing the runs using SPA-1 and SPA-2 shows that, in general, the SPA-2 resulted in fewer iterations with a total objective function value comparable to that obtained using SPA-1.

Tables 4.5, 4.6, and 4.7 show the details of the runs with $\theta_{0_{initial_3}} = \{0, 45^\circ, 0, 45^\circ, 0^\circ, 45^\circ, 0\}^T$ for SPA-0, SPA-1, and SPA-2, respectively. In Tables 4.5, 4.6, and 4.7, F_i is the force defined by equation (4.15), *Initial* κ is the velocity-degenerate configuration “closeness” measure for the initial manipulator configuration θ_0 , *Final* κ is the velocity-degenerate configuration “closeness” measure for the optimized pose, k is the number of iterations, and $\|\hat{\tau}\|_{100}$ is the final objective function value.

Table 4.5: Results for $\theta_{0_{initial_3}}$ with SPA-0

F_i	<i>Initial κ</i>	<i>Final κ</i>	k	$\ \hat{\tau}\ _{100}$
1	3.4142	3.4142	1	0.02840
2	4.0451	1.3859	22	0.01006
3	3.0458	2.5068	6	0.01422
4	1.7243	1.3609	7	0.01006
5	1.5175	0.4843	37	0.00780
6	4.7210	3.2092	83	0.01541
7	5.4184	4.9786	16	0.01729
8	3.8544	1.2728	51	0.01006
9	2.5542	0.2915	34	0.00541

Table 4.6: Results for $\theta_{0_{initial_3}}$ with SPA-1

F_i	<i>Initial κ</i>	<i>Final κ</i>	k	$\ \hat{\tau}\ _{100}$
1	3.4142	3.4142	1	0.02840
2	4.0451	1.3859	22	0.01006
3	3.0458	2.5068	6	0.01422
4	1.7032	1.1484	6	0.01006
5	0.9626	0.4320	26	0.00690
6	3.2533	1.2272	28	0.01006
7	2.5289	2.0912	2	0.01422
8	1.0702	0.9560	2	0.01006
9	0.4978	0.0065	50	0.00076

Table 4.7: Results for $\theta_{0_{initial_3}}$ with SPA-2

F_i	<i>Initial κ</i>	<i>Final κ</i>	k	$\ \hat{\tau}\ _{100}$
1	3.4142	3.4142	1	0.02840
2	4.0451	1.3859	22	0.01006
3	3.0458	2.5068	6	0.01422
4	1.7032	1.1484	6	0.01006
5	0.8770	0.3311	18	0.00674
6	2.6556	1.4584	7	0.01006
7	2.9565	2.7701	2	0.01422
8	1.5746	1.3263	6	0.01006
9	0.0881	0.0009	27	0.00038

Referring to Tables 4.5, 4.6, and 4.7, it is clear that the *Initial* κ values for the runs using SPA-1 and SPA-2 were the same or lower for all the wrenches than the run not using the SPA. In addition, the *Final* κ value for the runs using SPA-1 and SPA-2 were the same or lower (with the exception of \mathbf{F}_8 for the run with SPA-2) for all the wrenches than the run not using the SPA. Also, the runs using SPA-1 and SPA-2 generally required less iterations than the run not using the SPA.

Constraint on the End-Effector Position This set of optimization runs utilized a constraint on the position of the end-effector. The tolerance on the position ($p_{tolerance}$) was set to 0.050 m. The runs consisted of the manipulator sequentially sustaining nine wrenches, i.e., the optimization routine was executed nine times for each run. The wrenches to be sustained in the runs were defined as:

$${}^{base}\mathbf{F}_i = \left\{ \begin{array}{c} {}^{base}\mathbf{f}_i \\ {}^{base}\mathbf{p}_{0 \rightarrow p_{ee}} \times {}^{base}\mathbf{f}_i \end{array} \right\}, \text{ for } i = 1 \text{ to } 9 \quad (4.17)$$

where

$${}^{base}\mathbf{f}_i = \left\{ \begin{array}{c} 10 \cos\left(\frac{\pi}{4}(i-1)\right) \\ 10 \sin\left(\frac{\pi}{4}(i-1)\right) \\ 0 \end{array} \right\} \mathbf{N}$$

and ${}^{base}\mathbf{p}_{0 \rightarrow p_{ee}}$ is the position of the end-effector for the specified initial configuration $\theta_{0_{initial}}$ of the manipulator..

Four different initial configurations ($\theta_{0_{initial}}$) were considered:

$$\begin{aligned} \theta_{0_{initial_1}} &= \{0, 0, 0, -45^\circ, 0, 0, 0\}^T \\ \theta_{0_{initial_2}} &= \{0, -45^\circ, 0, -45^\circ, 0, -45^\circ, 0\}^T \\ \theta_{0_{initial_3}} &= \{0, -45^\circ, 0, 0, 0^\circ, -45^\circ, 0\}^T \\ \theta_{0_{initial_4}} &= \{0, 10^\circ, 0, 10^\circ, 0^\circ, 10^\circ, 0\}^T \end{aligned} \quad (4.18)$$

Table 4.8: Results for the Spherical-Revolute-Spherical Manipulator Sequentially Sustaining Nine Wrenches with a Constraint on the Position of the End-Effector

$\theta_{0_{initial}}$	SPA-0		SPA-1			SPA-2		
	k	$\sum \ \hat{\tau}\ _{100}$	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>
1	239	0.0931	332	0.0713	8	296	0.0727	6
2	9	0.0899	14	0.0899	2	14	0.0899	2
3	189	0.1074	233	0.1074	8	162	0.1076	5
4	231	0.1083	297	0.0798	8	294	0.0886	7

Note that after the first wrench, the solution from the previous wrench became the θ_0 value for the next wrench.

The results for the runs with a constraint on the end-effector position are presented in Table 4.8.

Referring to Table 4.8, several trends in the results can be noted. In general, when the SPA was utilized, the total objective function value, $\sum \|\hat{\tau}\|_{100}$, was the same or lower for the runs that used the SPA than the runs that did not use the SPA. The results also show that the SPA required more iterations than not using the SPA.

4.4.3 Zero-Offset PUMA-Type Manipulator

Manipulator Model

Figure 4.2 shows the 6-joint R.LR||R.L.R.L.R.L.R zero-offset PUMA-type manipulator. Note that the last three joints form a spherical group. The zero-offset PUMA-type manipulator is modelled after the Unimation PUMA (Craig, 1989), but without the offset between the second and third joints and the link length between the third and

fourth joints that are incorporated in the actual PUMA. D&H parameters for the manipulator using Craig's frame assignment convention (Craig, 1989) are presented in Table 4.9. The parameters of Table 4.9 correspond to the link transformations defined in equation (2.47).

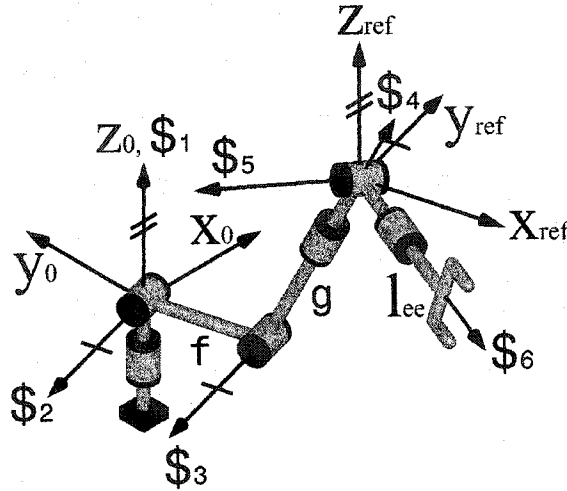


Figure 4.2: Zero-Offset PUMA-Type Manipulator ($\theta_1 = -75^\circ$, $\theta_2 = 0^\circ$, $\theta_3 = 155^\circ$, $\theta_4 = -35^\circ$, $\theta_5 = -75^\circ$, $\theta_6 = 0^\circ$)

Referring to Figure 4.2, a reference frame that is located at the intersection point of the spherical-wrist group formed by joints 4 through 6 and oriented with \mathbf{z}_{ref} parallel to $\$1$ and \mathbf{y}_{ref} parallel to $\$2$ and $\$3$ was chosen. The homogeneous transformations describing the position and orientation of F_j , $j = 0$ to 7, with respect to F_{ref} are defined by:

$${}_{ref}^j \mathbf{T} = \begin{bmatrix} {}_{ref} \hat{\mathbf{x}} & {}_{ref} \hat{\mathbf{y}} & {}_{ref} \hat{\mathbf{z}} & {}_{ref} \mathbf{p}_{o_{ref} \rightarrow o_j} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad j = 0 \text{ to } 7 \quad (4.19)$$

Table 4.9: Denavit and Hartenberg Parameters for the Zero-Offset PUMA-Type Manipulator

F_{j-1}	α_{j-1}	a_{j-1}	d_j	θ_j	F_j
$F_0 (F_{base})$	0	0	0	θ_1	F_1
F_1	$\frac{\pi}{2}$	0	0	θ_2	F_2
F_2	0	f	0	θ_3	F_3
F_3	$\frac{\pi}{2}$	0	g	θ_4	F_4
F_4	$-\frac{\pi}{2}$	0	0	θ_5	F_5
F_5	$\frac{\pi}{2}$	0	0	θ_6	F_6
F_6	0	0	l_{ee}	0	$F_7 (F_{ee})$

with

$${}_{base}^{ref} \mathbf{T} = {}_0^{ref} \mathbf{T} = ({}_1^0 \mathbf{T} {}_2^1 \mathbf{T} {}_3^2 \mathbf{T} {}_{ref}^3 \mathbf{T})^{-1} = {}_{ref}^3 \mathbf{T}^{-1} {}_3^2 \mathbf{T}^{-1} {}_2^1 \mathbf{T}^{-1} {}_1^0 \mathbf{T}^{-1}$$

$${}_{1}^{ref} \mathbf{T} = ({}_2^1 \mathbf{T} {}_3^2 \mathbf{T} {}_{ref}^3 \mathbf{T})^{-1} = {}_{ref}^3 \mathbf{T}^{-1} {}_3^2 \mathbf{T}^{-1} {}_2^1 \mathbf{T}^{-1}$$

$${}_{2}^{ref} \mathbf{T} = ({}_3^2 \mathbf{T} {}_{ref}^3 \mathbf{T})^{-1} = {}_{ref}^3 \mathbf{T}^{-1} {}_3^2 \mathbf{T}^{-1}$$

$${}_{3}^{ref} \mathbf{T} = {}_{ref}^3 \mathbf{T}^{-1}$$

$${}_{4}^{ref} \mathbf{T} = {}_{3}^{ref} \mathbf{T} {}_4^3 \mathbf{T}$$

$${}_{5}^{ref} \mathbf{T} = {}_{3}^{ref} \mathbf{T} {}_4^3 \mathbf{T} {}_5^4 \mathbf{T}$$

$${}_{6}^{ref} \mathbf{T} = {}_{3}^{ref} \mathbf{T} {}_4^3 \mathbf{T} {}_5^4 \mathbf{T} {}_6^5 \mathbf{T}$$

$${}_{ee}^{ref} \mathbf{T} = {}_{7}^{ref} \mathbf{T} = {}_{3}^{ref} \mathbf{T} {}_4^3 \mathbf{T} {}_5^4 \mathbf{T} {}_6^5 \mathbf{T} {}_7^6 \mathbf{T}$$

and

$${}_{ref}^3 \mathbf{T} = \begin{bmatrix} -c_\beta & 0 & -s_\beta & 0 \\ s_\beta & 0 & -c_\beta & -g \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ where } \beta = -\pi + \theta_2 + \theta_3$$

The joint screws for the zero-offset PUMA-type manipulator are:

$$\begin{aligned}
 {}^{ref}\$1 &= \left\{ 0, 0, 1; 0, s_{23}g + c_2f, 0 \right\}^T \\
 {}^{ref}\$2 &= \left\{ 0, -1, 0; c_{23}g - s_2f, 0, s_{23}g + c_2f \right\}^T \\
 {}^{ref}\$3 &= \left\{ 0, -1, 0; c_{23}g, 0, s_{23}g \right\}^T \\
 {}^{ref}\$4 &= \left\{ -s_\beta, 0, c_\beta; 0, 0, 0 \right\}^T \\
 {}^{ref}\$5 &= \left\{ c_\beta s_4, -c_4, s_\beta s_4; 0, 0, 0 \right\}^T \\
 {}^{ref}\$6 &= \left\{ -c_\beta c_4 s_5 - s_\beta c_5, -s_4 s_5, -s_\beta c_4 s_5 + c_\beta c_5; 0, 0, 0 \right\}^T
 \end{aligned} \tag{4.20}$$

where $\beta = -\pi + \theta_2 + \theta_3$. The matrix of joint screw coordinates for the manipulator is:

$${}^{ref}[\$] = {}^{ref}[\$1 \ \$2 \ \$3 \ \$4 \ \$5 \ \$6] \tag{4.21}$$

Manipulator Parameters

For the zero-offset PUMA-type manipulator example, the link lengths for the manipulator were $f = 1.000$ m, $g = 1.000$ m, and $l_{ee} = 0.150$ m. Table 4.10 shows the joint parameters used for the spherical-revolute-spherical manipulator example. Note that the joint torque listed in Table 4.10 represents the torque post gear box.

Velocity-Degenerate Configurations and the SPA

Setting the determinant of the matrix of unit joint screws ${}^{ref}[\$]$ to zero yields the velocity degenerate configurations of the zero-offset PUMA-type manipulator (Gorla, 1981, Waldron, Wang, and Bolin, 1985, and Hunt, 1987b):

$$|{}^{ref}[\$]| = c_3 s_5 f g (s_{23}g + c_2f) = 0$$

Table 4.10: Joint Parameters for the PUMA-Type Manipulator

Joint	Motor Torque (N·m)	Gear Ratio	Gear Box Efficiency	Joint Torque (N·m)	Joint Range
1	±15	100:1	0.85	±1275	-170° to 170°
2	±10	100:1	0.85	±850	-170° to 170°
3	±10	100:1	0.85	±850	-170° to 170°
4	±2	150:1	0.85	±255	-170° to 170°
5	±2	150:1	0.85	±255	-170° to 170°
6	±2	150:1	0.85	±255	-170° to 170°

Therefore, if i) $c_3 = 0$, ii) $s_5 = 0$, or iii) $s_{23}g + c_2f = 0$ the manipulator is a configuration that causes it to lose instantaneously 1-DOF of motion. Only the 1-DOF-loss velocity-degenerate configurations were used in the SPA ($Num = 3$ in the SPA). This case is denoted SPA-1.

The solutions used for the velocity-degenerate configurations were:

$$\begin{aligned}
 \text{i) } c_3 = 0: & \quad \theta_3 = \pm \frac{\pi}{2} \\
 \text{ii) } s_5 = 0: & \quad \theta_5 = 0 \\
 \text{iii) } s_{23}g + c_2f = 0: & \quad \theta_2 = \text{atan2}(s_3g + f, -c_3g) \text{ or } \text{atan2}(-s_3g - f, c_3g)
 \end{aligned} \tag{4.22}$$

where atan2 denotes a quadrant corrected arctangent function (Craig, 1989).

Results

This set of optimization runs utilized no constraint on the position of the end-effector. The runs consisted of the manipulator sequentially sustaining nine wrenches, i.e., the optimization routine was executed nine times for each run. The wrenches to be

sustained in the runs were defined as:

$${}^{base}\mathbf{F}_i = \left\{ \begin{array}{c} {}^{base}\mathbf{f}_i \\ {}^{base}\mathbf{p}_{0 \rightarrow p_{ee}} \times {}^{base}\mathbf{f}_i \end{array} \right\}, \text{ for } i = 1 \text{ to } 9 \quad (4.23)$$

where

$${}^{base}\mathbf{f}_i = \left\{ \begin{array}{c} 10 \cos\left(\frac{\pi}{4}(i-1)\right) \\ 10 \sin\left(\frac{\pi}{4}(i-1)\right) \\ 10 \end{array} \right\} \mathbf{N}$$

and ${}^{base}\mathbf{p}_{0 \rightarrow p_{ee}}$ is the position of the end-effector for the specified initial configuration $\boldsymbol{\theta}_{0_{initial}}$ of the manipulator.

Nine different initial configurations ($\boldsymbol{\theta}_{0_{initial}}$) were considered:

$$\begin{aligned} \boldsymbol{\theta}_{0_{initial_1}} &= \{0, 45^\circ, 45^\circ, 45^\circ, 45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_2}} &= \{0, -45^\circ, -45^\circ, -45^\circ, -45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_3}} &= \{0, -45^\circ, 45^\circ, 45^\circ, -45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_4}} &= \{0, 45^\circ, -45^\circ, -45^\circ, 45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_5}} &= \{0, 45^\circ, -45^\circ, 45^\circ, -45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_6}} &= \{0, 45^\circ, 0, 0, 45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_7}} &= \{0, -45^\circ, 0, 0, -45^\circ, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_8}} &= \{0, 0, 45^\circ, 45^\circ, 0, 0\}^T \\ \boldsymbol{\theta}_{0_{initial_9}} &= \{0, 0, -45^\circ, -45^\circ, 0, 0\}^T \end{aligned} \quad (4.24)$$

Note that after the first wrench, the solution from the previous wrench became the $\boldsymbol{\theta}_0$ value for the next wrench.

The results for the runs are presented in Table 4.11.

Referring to Table 4.11, when the SPA was utilized, the total objective function value was lower for the runs that used the SPA than the runs that did not use the

Table 4.11: Results for the PUMA-Type-Spherical Manipulator Sequentially Sustaining Nine Wrenches with No Constraint on the Position of the End-Effector

$\theta_{0_{initial}}$	SPA-0		SPA-1		
	k	$\sum \ \hat{\tau}\ _{100}$	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>
1	142	0.1185	210	0.0857	5
2	172	0.0389	273	0.0230	7
3	90	0.1042	177	0.0831	4
4	212	0.0508	162	0.0499	8
5	243	0.0640	232	0.0560	4
6	14	0.1603	151	0.0817	6
7	312	0.0541	200	0.0517	3
8	184	0.1358	103	0.1007	8
9	172	0.0582	181	0.0366	4

SPA. Comparing the total number of iterations shows no real pattern. For some runs, using the SPA resulted in less iterations, while for others it resulted in more iterations.

The results for $\theta_{0_{initial_6}}$ show clearly the advantage of using the SPA. Only 14 iterations were required for the run without the SPA. Using the SPA resulted in a total of 151 iterations. However, if you compare the total objective function values you see that the SPA converged to poses that required significantly less effort from the actuators (49 % less effort). The low number of iterations shows that the run without the SPA was stuck in a search region and was unable to move away to a region with better solutions. The SPA allows the optimization routine to start in regions that more likely have better minima.

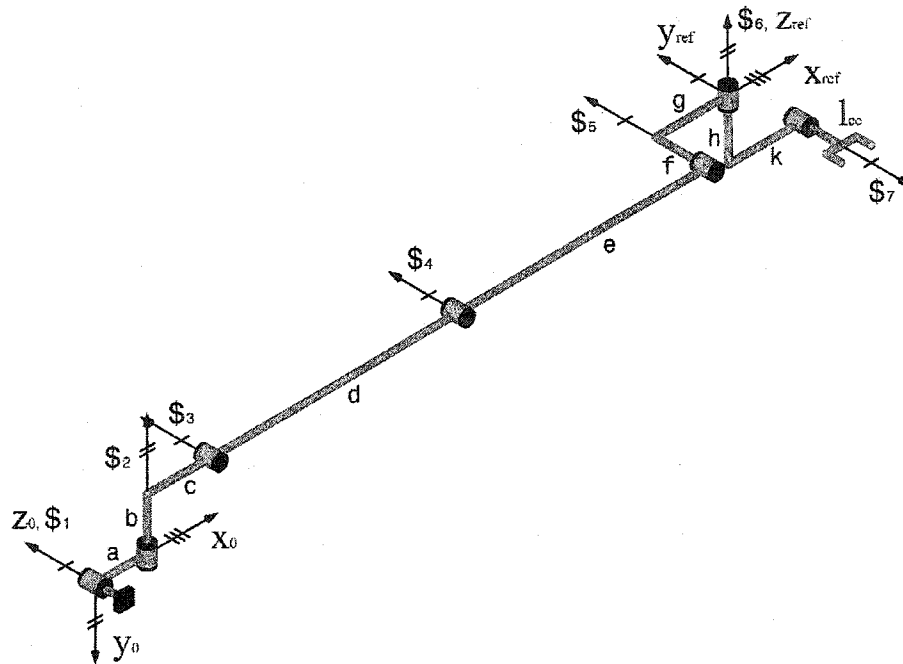


Figure 4.3: Zero-Displacement Configuration of the STM-1 Manipulator

4.4.4 CSA/ISE STEAR Testbed Manipulator-1 (STM-1)

Manipulator Model

Figure 4.3 shows the 7-joint R₁L₁R₁L₁R₁||R₁||R₁L₁R₁L₁R₁ STM-1 manipulator. The D&H parameters used to model the manipulator are presented in Table 4.12. The parameters of Table 4.12 correspond to the link transformations defined in equation (2.3).

A reference frame coincident with F_5 of the STM-1 was chosen. The homogeneous transformations describing the position and orientation of F_j , $j = 0$ to 8, with respect

Table 4.12: Denavit and Hartenberg Parameters for the STM-1 Manipulator

F_{j-1}	θ_j	d_j	a_j	α_j	F_j
F_0	θ_1	0	a	$\frac{\pi}{2}$	F_1
F_1	θ_2	b	c	$-\frac{\pi}{2}$	F_2
F_2	θ_3	0	d	0	F_3
F_3	θ_4	0	e	0	F_4
F_4	θ_5	f	g	$\frac{\pi}{2}$	F_5
F_5	θ_6	$-h$	k	$\frac{\pi}{2}$	F_6
F_6	θ_7	0	0	0	F_7
F_7	0	l_{ee}	0	0	$F_8 (F_{ee})$

to F_{ref} are defined by:

$${}_{j}^{ref}\mathbf{T} = \begin{bmatrix} {}^{ref}\hat{\mathbf{x}} & {}^{ref}\hat{\mathbf{y}} & {}^{ref}\hat{\mathbf{z}} & {}^{ref}\mathbf{p}_{o_{ref} \rightarrow o_j} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad j = 0 \text{ to } 8 \quad (4.25)$$

with

$${}_{base}^{ref}\mathbf{T} = {}_0^5\mathbf{T} = ({}_1^0\mathbf{T}_2^1\mathbf{T}_3^2\mathbf{T}_4^3\mathbf{T}_5^4\mathbf{T})^{-1} = {}_5^4\mathbf{T}^{-1} {}_4^3\mathbf{T}^{-1} {}_3^2\mathbf{T}^{-1} {}_2^1\mathbf{T}^{-1} {}_1^0\mathbf{T}^{-1}$$

$${}_1^{ref}\mathbf{T} = {}_1^5\mathbf{T} = ({}_2^1\mathbf{T}_3^2\mathbf{T}_4^3\mathbf{T}_5^4\mathbf{T})^{-1} = {}_5^4\mathbf{T}^{-1} {}_4^3\mathbf{T}^{-1} {}_3^2\mathbf{T}^{-1} {}_2^1\mathbf{T}^{-1}$$

$${}_2^{ref}\mathbf{T} = {}_2^5\mathbf{T} = ({}_3^2\mathbf{T}_4^3\mathbf{T}_5^4\mathbf{T})^{-1} = {}_5^4\mathbf{T}^{-1} {}_4^3\mathbf{T}^{-1} {}_3^2\mathbf{T}^{-1}$$

$${}_3^{ref}\mathbf{T} = {}_3^5\mathbf{T} = ({}_4^3\mathbf{T}_5^4\mathbf{T})^{-1} = {}_5^4\mathbf{T}^{-1} {}_4^3\mathbf{T}^{-1}$$

$${}_4^{ref}\mathbf{T} = {}_4^5\mathbf{T} = {}_5^4\mathbf{T}^{-1}$$

$${}_5^{ref}\mathbf{T} = {}_5^5\mathbf{T} = \mathbf{I}_{4 \times 4}$$

$${}_6^{ref}\mathbf{T} = {}_6^5\mathbf{T}$$

$${}_7^{ref}\mathbf{T} = {}_7^5\mathbf{T} = {}_6^5\mathbf{T} {}_7^6\mathbf{T}$$

$${}_{ee}^{ref}\mathbf{T} = {}_8^5\mathbf{T} = {}_6^5\mathbf{T} {}_7^6\mathbf{T} {}_8^7\mathbf{T}$$

The joint screws for the STM-1 manipulator are:

$$\begin{aligned}
 {}^{ref}\$1 &= \left\{ \begin{array}{c} s_2 c_{345} \\ c_2 \\ s_2 s_{345} \\ s_{345} (a - f s_2) + c_2 (b c_{345} + c s_{345} + d s_{45} + e s_5) \\ s_2 (-b + d s_3 + e s_{34} + g s_{345}) \\ c_{345} (-a + f s_2) + c_2 (-g + b s_{345} - c c_{345} - d c_{45} - e c_5) \end{array} \right\} \\
 {}^{ref}\$2 &= \left\{ -s_{345}, 0, c_{345}; -f c_{345}, c + d c_3 + e c_{34} + g c_{345}, -f s_{345} \right\}^T \\
 {}^{ref}\$3 &= \left\{ 0, 1, 0; d s_{45} + e s_5, 0, -d c_{45} - e c_5 - g \right\}^T \\
 {}^{ref}\$4 &= \left\{ 0, 1, 0; e s_5, 0, -e c_5 - g \right\}^T \\
 {}^{ref}\$5 &= \left\{ 0, 1, 0; 0, 0, -g \right\}^T \\
 {}^{ref}\$6 &= \left\{ 0, 0, 1; 0, 0, 0 \right\}^T \\
 {}^{ref}\$7 &= \left\{ s_6, -c_6, 0; -h c_6, -h s_6, -k \right\}^T
 \end{aligned} \tag{4.26}$$

The matrix of unit joint screw coordinates for the manipulator is:

$${}^{ref}[\$] = {}^{ref} \left[\begin{array}{cccccc} \$1 & \$2 & \$3 & \$4 & \$5 & \$6 & \$7 \end{array} \right] \tag{4.27}$$

Manipulator Parameters

Table 4.13 shows the actual link and offset lengths for the STM-1 manipulator. For the example it was assumed that $l_{ee} = 0.1727$ m. Table 4.14 shows the actual joint parameters for the STM-1 manipulator (with the exception of the gear box efficiency which is an assumed value). Note that the joint torque listed in Table 4.14 represents the torque post gear box.

Table 4.13: Link and Offset Lengths for the STM-1 Manipulator

j	d_i (m)	a_i (m)
1	0.0000	$a = 0.1727$
2	$b = 0.1727$	$c = 0.2134$
3	0.0000	$d = 0.8382$
4	0.0000	$e = 0.8382$
5	$f = 0.1727$	$g = 0.2441$
6	$h = 0.2007$	$k = 0.2441$

Velocity-Degenerate Configurations and the SPA

It was shown in Section 2.3.3 that there are five 1-DOF-loss velocity-degenerate configurations for the STM-1 manipulator: i) $s_4 = 0$ & equation (2.39) = 0, ii) $s_2 = 0$ & $s_6 = 0$, iii) $s_2 = 0$ & $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$, iv) $s_6 = 0$ & $-bs_{345} + cc_{345} + dc_{45} + ec_5 + g = 0$, and v) $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$ & $-b + ds_3 + es_{34} + gs_{345} + hc_{345} = 0$. Podhorodeski, Nokleby, and Wittchen (2000) showed that for the actual STM-1 dimensions and joint limits, only the first four degenerate configurations can be encountered by the manipulator. Therefore, only the first four 1-DOF-loss velocity-degenerate configurations were used in the SPA ($Num = 4$ in the SPA). This case is denoted SPA-1.

The solutions used for the velocity-degenerate configurations were (Podhorodeski,

Table 4.14: Joint Parameters for the STM-1 Manipulator

Joint	Motor Torque (N·m)	Gear Ratio	Gear Box Efficiency	Joint Torque (N·m)	Joint Range
1	±6.42	120:1	0.85	±654.84	-135° to 90°
2	±6.42	120:1	0.85	±654.84	-30° to 180°
3	±6.42	120:1	0.85	±654.84	-120° to 120°
4	±2.96	120:1	0.85	±301.92	-130° to 130°
5	±0.63	160:1	0.85	±85.68	-120° to 120°
6	±0.63	160:1	0.85	±85.68	-30° to 210°
7	±0.63	160:1	0.85	±85.68	-175° to 175°

Nokleby, and Wittchen, 2000):

i) $s_4 = 0$ & equation(2.39) = 0:

$$\theta_4 = 0 \text{ \& } \theta_2 = \text{atan2}(B, A) \pm \text{atan2}\left((A^2 + B^2 - C^2)^{\frac{1}{2}}, C\right)$$

ii) $s_2 = 0$ & $s_6 = 0$:

$$\theta_2 = 0 \text{ or } \pi \text{ \& } \theta_6 = 0 \text{ or } \pi$$

(4.28)

iii) $s_2 = 0$ & $c + dc_3 + ec_{34} + gc_{345} - hs_{345} = 0$:

$$\theta_2 = 0 \text{ or } \pi \text{ \& } \theta_3 = \text{atan2}(E, D) \pm \text{atan2}\left((D^2 + E^2 - F^2)^{\frac{1}{2}}, F\right)$$

iv) $s_6 = 0$ & $-bs_{345} + cc_{345} + dc_{45} + ec_5 + g = 0$

$$\theta_6 = 0 \text{ or } \pi \text{ \& } \theta_5 = \text{atan2}(H, G) \pm \text{atan2}\left((G^2 + H^2 - I^2)^{\frac{1}{2}}, I\right)$$

where

$$\begin{aligned}
A &= s_6 (bc_3 + cs_3) (c + (d + e) c_3 + gc_{35} - hs_{35}) \\
B &= (gs_5c_6 + hc_5c_6 + ks_5) (-bs_{35} + cc_{35} + (d + e) c_5 + g) \\
&\quad + fs_6 (-bc_3 - cs_3 + gs_5 + hc_5) \\
C &= -as_3s_6 (c + dc_3 + ec_{34} + gc_{345} - hs_{345}) \\
D &= -bs_{45} + cc_{45} \\
E &= -bc_{45} - cs_{45} \\
F &= -dc_{45} - ec_5 - g \\
G &= gc_{34} + hs_{34} \\
H &= -gs_{34} - hc_{34} \\
I &= -c - dc_3 - ec_{34}
\end{aligned}$$

Results

This set of optimization runs utilized no constraint on the position of the end-effector. The runs consisted of the manipulator sequentially sustaining nine wrenches, i.e., the optimization routine was executed nine times for each run. The wrenches to be sustained in the runs were defined as:

$${}^{base}\mathbf{F}_i = \left\{ \begin{array}{c} {}^{base}\mathbf{f}_i \\ {}^{base}\mathbf{p}_{0 \rightarrow pee} \times {}^{base}\mathbf{f}_i \end{array} \right\}, \text{ for } i = 1 \text{ to } 9 \quad (4.29)$$

where

$${}^{base}\mathbf{f}_i = \left\{ \begin{array}{c} 10 \cos \left(\frac{\pi}{4}(i - 1) \right) \\ 10 \sin \left(\frac{\pi}{4}(i - 1) \right) \\ 10 \end{array} \right\} \text{ N}$$

and ${}^{base}\mathbf{p}_{0 \rightarrow pee}$ is the position of the end-effector for the specified initial configuration $\theta_{0_{initial}}$ of the manipulator.

Table 4.15: Results for the STM-1 Manipulator Sequentially Sustaining Nine Wrenches with No Constraint on the Position of the End-Effector

$\theta_{0_{initial}}$	SPA-0		SPA-1		
	k	$\sum \ \hat{\tau}\ _{100}$	k	$\sum \ \hat{\tau}\ _{100}$	<i>Starts</i>
1	167	0.2277	160	0.1501	6
2	127	0.2510	92	0.1644	3
3	104	0.2089	125	0.2104	4
4	195	0.1972	179	0.1652	3
5	163	0.1924	82	0.1935	4
6	243	0.1917	103	0.1574	4
7	253	0.1514	154	0.1467	5

Seven different initial configurations ($\theta_{0_{initial}}$) were considered:

$$\begin{aligned}
\theta_{0_{initial_1}} &= \{0, 45^\circ, 0, 45^\circ, 0, 45^\circ, 0\}^T \\
\theta_{0_{initial_2}} &= \{0, 45^\circ, 45^\circ, 0, 45^\circ, 45^\circ, 0\}^T \\
\theta_{0_{initial_3}} &= \{0, 45^\circ, -45^\circ, 0, -45^\circ, 45^\circ, 0\}^T \\
\theta_{0_{initial_4}} &= \{0, 45^\circ, 0, 0, 0, 45^\circ, 0\}^T \\
\theta_{0_{initial_5}} &= \{0, 0, 0, 45^\circ, 0, 0, 0\}^T \\
\theta_{0_{initial_6}} &= \{0, 135^\circ, 0, 45^\circ, 0, 135^\circ, 0\}^T \\
\theta_{0_{initial_7}} &= \{0, 135^\circ, 45^\circ, 45^\circ, 45^\circ, 135^\circ, 0\}^T
\end{aligned} \tag{4.30}$$

Note that after the first wrench, the solution from the previous wrench became the θ_0 value for the next wrench.

The results for the runs are presented in Table 4.15.

Referring to Table 4.15, the results show that in general using the SPA resulted in the optimization generating a set of poses that had a lower total objective function

value compared with not using the SPA. With the exception of the run with $\theta_{0_{initial}}$, using the SPA resulted in a lower total number of iterations than not using the SPA.

4.5 Discussion

4.5.1 The Results Found

The spherical-revolute-spherical example was performed to show how the SPA works with a redundant manipulator. The zero-offset PUMA-type manipulator example was done to show how the developed methodology works with a non-redundant serial manipulator. The STM-1 example was conducted to show how the methodology works for a real world manipulator. The results for all three examples show that by using the SPA with the optimization routine, the resulting poses obtained usually require less effort from the actuators when compared to the poses obtained without using the SPA.

Referring to results for the spherical-revolute-spherical manipulator and STM-1 examples shows that using the SPA, when no constraint was imposed on the position of the end-effector, usually resulted in a decrease in the number of iterations required. This trend was not evident in the results for the non-redundant zero-offset PUMA-type manipulator.

The addition of a constraint on the position of the end effector had an adverse effect on the number of iterations required for the spherical-revolute-spherical example. This decrease in the performance in the number of iterations is most likely due to the fact that the SPA sometimes pushes the position of the end-effector away from the desired end-effector position, i.e., the initial configuration of the manipula-

tor for the optimization violates the constraint of $\|\mathbf{p}_{actual} - \mathbf{p}_{desired}\| - p_{tolerance} \leq 0$. Although the initial configuration sometimes did not satisfy the position constraint, the optimized pose did and as stated above, the poses obtained using the SPA were better than the poses obtained without the SPA in terms of required actuator output. For off-line path planning, the difference in the number of iterations required is inconsequential.

4.5.2 Velocity-Degenerate Configuration “Closeness” Measure

Referring to Tables 4.5, 4.6, and 4.7, the results show how the optimization routine for all three cases (SPA-0, SPA-1, and SPA-2) converges to poses that have lower velocity-degenerate configuration “closeness” measures than the initial pose. Although the objective function of the optimization was to minimize the p -norm of the normalized torque vector, minimizing the p -norm of the normalized torque vector also results in reducing the velocity-degenerate configuration “closeness” measure. The results confirm that the proposed velocity-degenerate configuration “closeness” measure is a suitable measure for determining how well a given pose will sustain a wrench and that it is an appropriate measure for determining a suitable pose to start the optimization.

4.5.3 Manipulator Link Weights

Note that the weight of the links for the manipulators was neglected in the examples. Incorporation of the weight of the links could be done by adding additional wrenches representing the effect of the weight of the links to the wrench to be sustained. These additional wrenches would be dependent on the manipulator’s pose.

Chapter 5

Conclusions and Recommendations for Future Work

5.1 Overview

In this chapter, conclusions about the dissertation will be presented. In addition, recommendations will be made about possible areas where future research could be conducted.

5.2 Conclusions

5.2.1 Chapter 2: Identification of 1-DOF-Loss Velocity-Degenerate Configurations of Kinematically-Redundant Manipulators

In Chapter 2, a novel methodology based on the properties of reciprocal screws was presented for the identification of 1-DOF (degree-of-freedom) loss velocity-degenerate (singular) configurations of kinematically-redundant manipulators. A by-product of the developed methodology is that reciprocal screws characterizing the lost motion of the manipulator are generated for each family of velocity-degenerate configurations.

Although the reciprocity methodology was applied only to redundant serial manipulators, the methodology can be used to identify velocity-degenerate configurations of joint-redundant parallel manipulators. The methodology can be applied to identify the velocity-degenerate configurations of the branches of a joint-redundant parallel manipulator. If a branch of a parallel manipulator enters a velocity-degenerate configuration, then the entire manipulator is in a velocity-degenerate configuration because the movement of the platform of the parallel manipulator is restricted by the degenerate branch.

The developed methodology was successfully tested using two spherical-wristed 7-joint manipulators: the spherical-revolute-spherical manipulator and the double-elbow manipulator. Four sets of conditions were identified for the spherical-revolute-spherical manipulator and three sets of conditions were identified for the double-elbow manipulator. The results obtained for both manipulators were identical to results reported in the literature using other methods.

The methodology was also applied to determine the velocity-degenerate configurations of the kinematically-complex 7-joint CSA/ISE STEAR Testbed Manipulator (STM). This task was the original motivation for this research. The reciprocity-based methodology successfully identified five sets of conditions causing a 1-DOF-loss velocity degeneracy. The results obtained were identical to those obtained by Podhorodeski, Nokleby, and Wittchen (2000) using the method of analyzing all possible 6-joint sub-groups of the matrix of unit joint screw coordinates to identify velocity-degenerate configurations. Due to the complexity of the joint screws for the STM, the methodology employed by Podhorodeski, Nokleby, and Wittchen (2000) was very difficult to use with the STM. Although applying the reciprocity-based methodology was not trivial, it was much easier to apply to the STM than the 6-joint sub-group method used by Podhorodeski, Nokleby, and Wittchen (2000).

Lastly, the reciprocity-based methodology was applied to the 8-joint NASA Advanced Research Manipulator II (ARMII). Four sets of 1-DOF-loss velocity-degenerate configurations were correctly identified. The results also showed that partitioning the matrix of unit joint screw coordinates to identify singular configurations does not work for redundant spherical-wristed manipulators.

5.2.2 Chapter 3: Identification of Multi-DOF-Loss Velocity-Degenerate Configurations of Kinematically-Redundant Manipulators

In Chapter 3, the reciprocity-based methodology for identifying 1-DOF-loss velocity-degenerate configurations was extended to determine multi-DOF loss velocity-degenerate configurations of kinematically-redundant manipulators. Like the 1-DOF-loss method-

ology, a by-product of the multi-DOF loss methodology is that reciprocal screws characterizing the lost motion of the manipulator are generated for each family of velocity-degenerate configurations.

The developed methodology was successfully applied to determine the 2-DOF loss velocity-degenerate configurations of two spherical-wristed 7-joint manipulators: the spherical-revolute-spherical manipulator and the double-elbow manipulator. Five sets of conditions were identified that caused a 2-DOF loss for the spherical-revolute-spherical manipulator. Three sets of conditions were identified that caused a 2-DOF loss for the double-elbow manipulator.

It was also shown that the developed multi-DOF loss methodology is frame variant due to the use of complements of the reciprocal screws in the methodology. However, it was shown that the frame variance is not a problem since if one wanted to work in a different frame of reference, one could use a screw transformation to first transform all the joint screws and reciprocal screws to the desired frame of reference and then apply the developed methodology.

5.2.3 Chapter 4: Pose Optimization of Serial Manipulators Using Knowledge of Their Velocity-Degenerate Configurations

In Chapter 4, a velocity-degenerate configuration “closeness” measure was proposed. This measure was used to develop a Start Point Algorithm (SPA) to improve the results to the non-linear and multi-modal optimization problem associated with optimizing the pose of a manipulator to sustain a desired wrench. The SPA uses knowledge of the velocity-degenerate configurations of a serial manipulator to determine a

suitable start point for the optimization.

The SPA was tested on three manipulators: the 7-joint spherical-revolute-spherical manipulator, the 6-joint PUMA-type manipulator, and the 7-joint STM-1 manipulator. The results for all three examples showed that by using the SPA with the optimization routine, the resulting poses obtained usually required less effort from the actuators when compared to the poses obtained without using the SPA. The redundant manipulator examples also showed that using the SPA, when no constraint was imposed on the position of the end-effector, usually resulted in a decrease in the number of iterations required by the optimization routine. This decrease in iterations did not occur for the non-redundant manipulator example.

The results showed that the addition of a constraint on the position of the end effector had an adverse effect on the number of iterations required. This decrease in the performance in the number of iterations is most likely due to the fact that the SPA sometimes pushes the position of the end-effector away from the desired end-effector position. Although the initial configuration sometimes did not satisfy the position constraint, the optimized pose did and the poses obtained using the SPA were better than the poses obtained without the SPA in terms of required actuator output. For off-line path planning, the difference in the number of iterations required is inconsequential.

The results also showed how the optimization routine converged to poses that had lower velocity-degenerate configuration "closeness" measures than the initial pose of the manipulator. Although the objective function of the optimization was to minimize the p -norm of the normalized torque vector, minimizing the p -norm of the normalized torque vector also resulted in reducing the velocity-degenerate configuration "closeness" measure. The results confirmed that the velocity-degenerate configura-

tion “closeness” measure is a suitable measure for determining how well a given pose will sustain a wrench and that it is an appropriate measure for determining a suitable pose to start the optimization.

5.3 Recommendations for Future Work

5.3.1 Identification of Multi-DOF Loss Velocity-Degenerate Configurations of Kinematically-Redundant Manipulators

In the area of identification of velocity-degenerate configurations of kinematically redundant manipulators, the main area that could be explored further is the development of an improved method for identifying multi-DOF loss velocity-degenerate configurations. The developed methodology for identifying 1-DOF-loss velocity-degenerate configurations is efficient and effective. Although the developed methodology for identifying multi-DOF-loss velocity-degenerate configurations works, it is limited by the use of the complement of screws. The complexity of some of the complements of the screws makes finding simple expressions for the determinant difficult. Also, in general, the more screw complements added to the sub-matrix of “joint screw coordinates”, the more difficult it is to find simple expressions for the determinant.

5.3.2 Pose Optimization of Serial and Parallel Manipulators

Further areas of research for the pose optimization of serial manipulators could focus on the optimization routine used. The examples presented in this dissertation were done to compare the effects of using the SPA on the optimization results. The purpose

of the examples was not to determine the effectiveness of the optimization routine. Using a prepackaged MATLAB routine was done for ease of programming. Using a purpose coded SQP routine may improve the effectiveness of the optimization. In addition, global optimization techniques, such as genetic algorithms, might be considered. No matter what optimization routine is chosen, using the SPA should lead to improved results.

An area of research that should be considered is extending the SPA to the pose optimization problem of parallel manipulators. The SPA for serial manipulators could be used as a model for the development of a new SPA for parallel manipulators.

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Appendix A

Manipulator Kinematics Using Screws

This appendix presents an overview of the use of screws in manipulator kinematics. The reader is referred to the works of Ball (1900), Hunt (1978), and Bottema and Roth (1979) for more detailed information on screw theory.

A.1 Screws

A screw (S) is a line in space having an associated linear pitch. Screws can be represented as:

$$S = \begin{Bmatrix} s \\ s_o \end{Bmatrix} = \lambda \begin{Bmatrix} l \\ l_o + pl \end{Bmatrix} \quad (\text{A.1})$$

where s and s_o are the screw coordinates, l and l_o are the Plücker coordinates of the line, λ is an associated magnitude, and p is the pitch of the screw (Hunt, 1978 and

1987b). The pitch of a screw can be found from:

$$p = \frac{\mathbf{s} \cdot \mathbf{s}_o}{\|\mathbf{s}\|}$$

A screw is said to be a unit or normalized screw ($\$$) if $\|\mathbf{s}\| = 1$ or in the case where $\mathbf{s} = \mathbf{0}_{3 \times 1}$, if $\|\mathbf{s}_o\| = 1$ (Hunt, 1987b).

For manipulators, a revolute joint can be represented by a zero-pitch unit screw $\$_{rev} = \{\mathbf{1}^T; \mathbf{1}_o^T\}^T$ and a prismatic joint can be represented by an infinite-pitch unit screw which when normalized to the ∞ -pitch gives $\$_{pris} = \{\mathbf{0}_{3 \times 1}^T; \mathbf{1}^T\}^T$.

The screw coordinates of the joints of a manipulator can be found from:

$$\mathit{ref} \$_j = \left\{ \begin{array}{c} \mathit{ref} \hat{\mathbf{z}}_j \\ \mathit{ref} \hat{\mathbf{z}}_j \times \mathit{ref} \mathbf{r}_{j \rightarrow p_{ee}} \end{array} \right\}_{revolute} \quad \text{or} \quad = \left\{ \begin{array}{c} \mathbf{0}_{3 \times 1} \\ \mathit{ref} \hat{\mathbf{z}}_j \end{array} \right\}_{prismatic} \quad (\text{A.2})$$

where $\hat{\mathbf{z}}_j$ denotes the unit vector of the j^{th} joint axis direction, p_{ee} denotes a point coincident with the origin of F_{ref} and attached to the end-effector, and $\mathbf{r}_{j \rightarrow p_{ee}}$ denotes a vector from a point on the axis of joint j to the point p_{ee} . Note that for zero-pitch and infinite-pitch unit screws, the screw coordinates are equivalent to the Plücker coordinates.

A.1.1 Screw Transformations

A screw transformation is defined as:

$$\mathit{ref}_a \mathbf{T}_S = \begin{bmatrix} \mathit{ref}_a \mathbf{R} & \mathbf{0}_{3 \times 3} \\ \mathit{ref}_b \tilde{\mathbf{p}}_{o_a \rightarrow o_b} & \mathit{ref}_a \mathbf{R} \\ \mathit{ref}_b \tilde{\mathbf{p}}_{o_a \rightarrow o_b} & \mathit{ref}_b \mathbf{R} \end{bmatrix} \quad (\text{A.3})$$

where $\mathit{ref}_a \mathbf{R}$ is a 3x3 rotation matrix and $\mathit{ref}_a \tilde{\mathbf{p}}_{o_a \rightarrow o_b}$ is a 3x3 cross product skew-symmetric matrix based on the components of a vector, $\mathit{ref}_a \tilde{\mathbf{p}}_{o_a \rightarrow o_b} = \mathit{ref}_a \{p_{x_{o_a \rightarrow o_b}}\}$,

$p_{y_{o_a \rightarrow o_b}}, p_{z_{o_a \rightarrow o_b}}\}^T$, from the origin of F_{ref_a} to the origin of F_{ref_b} . The cross product skew-symmetric matrix ${}^{ref_a}\tilde{\mathbf{p}}_{o_a \rightarrow o_b}$ is:

$${}^{ref_a}\tilde{\mathbf{p}}_{o_a \rightarrow o_b} = \begin{bmatrix} 0 & -p_{z_{o_a \rightarrow o_b}} & p_{y_{o_a \rightarrow o_b}} \\ p_{z_{o_a \rightarrow o_b}} & 0 & -p_{x_{o_a \rightarrow o_b}} \\ -p_{y_{o_a \rightarrow o_b}} & p_{x_{o_a \rightarrow o_b}} & 0 \end{bmatrix} \quad (\text{A.4})$$

A.1.2 Reciprocal Screws

Let the screw quantity $\mathbf{A} = \{\mathbf{a}^T; \mathbf{a}_o^T\}^T$ represent the velocity of a body and the screw quantity $\mathbf{B} = \{\mathbf{b}^T; \mathbf{b}_o^T\}^T$ represent a wrench acting on the body. If the power developed by wrench \mathbf{B} acting on the body moving with velocity \mathbf{A} is zero, the screw quantities \mathbf{A} and \mathbf{B} are said to be reciprocal to one another (Hunt, 1978). Mathematically, the two screws, \mathbf{A} and \mathbf{B} , are reciprocal if their reciprocal product is zero:

$$\mathbf{A} \otimes \mathbf{B} = \mathbf{a} \cdot \mathbf{b}_o + \mathbf{a}_o \cdot \mathbf{b} = 0 \quad (\text{A.5})$$

where \otimes denotes a reciprocal product between two screws.

A.2 Serial Manipulator Kinematics Using Screws

A.2.1 Velocity Solutions

The end-effector velocity (\mathbf{V}) of a manipulator is defined as:

$$\mathbf{V} = \begin{Bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{Bmatrix} \quad (\text{A.6})$$

where ω is the angular velocity and \mathbf{v} is the translational velocity. The forward velocity solution of a serial manipulator, given the twist amplitudes (joint rates) what is the end-effector velocity, can be described using screws. Letting $\$j$ denote the screw coordinates of joint j allows \mathbf{V} to be determined by:

$$\mathbf{V} = \begin{Bmatrix} \omega \\ \mathbf{v} \end{Bmatrix} = \sum_{j=1}^n \$j \dot{t}_j \quad (\text{A.7})$$

where n is the total number of joints and \dot{t}_j is the twist amplitude (joint rate) of joint j . In matrix form this can be expressed as:

$$\mathbf{V} = \begin{Bmatrix} \omega \\ \mathbf{v} \end{Bmatrix} = [\$] \dot{\mathbf{t}} \quad (\text{A.8})$$

where $[\$] = [\$_1 \ \$_2 \ \dots \ \$_n]$ is the matrix of unit joint screw coordinates commonly referred to as the Jacobian matrix and $\dot{\mathbf{t}} = [\dot{t}_1 \ \dot{t}_2 \ \dots \ \dot{t}_n]^T$ is the vector of twist amplitudes (joint rates).

For 6-DOF (degree-of-freedom) motion, assuming a non-velocity-degenerate and non-redundant manipulator (i.e., $n = 6$), the inverse velocity solution can be expressed as:

$$\dot{\mathbf{t}} = [\$]^{-1} \mathbf{V} \quad (\text{A.9})$$

A.2.2 Force Solutions

The wrench (\mathbf{F}) applied by the end-effector of a manipulator is defined as:

$$\mathbf{F} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{m} \end{Bmatrix} \quad (\text{A.10})$$

where \mathbf{f} is the force applied and \mathbf{m} is the moment applied. The inverse static force problem of a serial manipulator, given the wrench being applied by the end-effector what are the joint torques (or forces for prismatic joints), can be solved using conservation of power (power in equals power out):

$$\boldsymbol{\tau}^T \dot{\mathbf{t}} = \{\mathbf{f}^T; \mathbf{m}^T\} \begin{Bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{Bmatrix} = \mathbf{F}^T ([\Delta] \mathbf{V}) \quad (\text{A.11})$$

where $\boldsymbol{\tau}$ is a vector of joint torques and forces and the matrix $[\Delta]$ is an interchange operator that transforms screws between axis-coordinate order to ray-coordinate order and is defined as:

$$[\Delta] = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \\ \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix} \quad (\text{A.12})$$

Substituting the forward velocity solution $\mathbf{V} = [\$]\dot{\mathbf{t}}$ into equation (A.11) yields:

$$\boldsymbol{\tau}^T \dot{\mathbf{t}} = \mathbf{F}^T [\Delta] [\$]\dot{\mathbf{t}} \quad (\text{A.13})$$

Equation (A.13) is true for all $\dot{\mathbf{t}}$, therefore:

$$\boldsymbol{\tau}^T = \mathbf{F}^T [\Delta] [\$] \quad (\text{A.14})$$

Transposing both sides of equation (A.14) yields the inverse static force problem:

$$\boldsymbol{\tau} = [\$]^T [\Delta] \mathbf{F} \quad (\text{A.15})$$

where the fact that $[\Delta]^T = [\Delta]$ has been utilized.

The forward static force solution for a non-redundant manipulator can be expressed as:

$$\mathbf{F} = \left([\$]^T [\Delta] \right)^{-1} \boldsymbol{\tau} = [\Delta] [\$]^{-T} \boldsymbol{\tau} \quad (\text{A.16})$$

where the fact that $[\Delta]^{-1} = [\Delta]$ has been utilized.

If one wants to sustain a wrench (as opposed to applying a wrench) with a serial manipulator, equations (A.15) and (A.16) become, respectively:

$$\boldsymbol{\tau} = -[\$]^\text{T} [\Delta] \mathbf{F} \quad (\text{A.17})$$

$$-\mathbf{F} = [\Delta] [\$]^{-\text{T}} \boldsymbol{\tau} \quad (\text{A.18})$$

A.3 Parallel Manipulator Kinematics Using Screws

A.3.1 Velocity Solutions

Assume a parallel manipulator has m branches. For the i^{th} branch, it is assumed that there are l_i joints of which n_i joints are actuated. For the actuated joint j of the i^{th} branch, a unit screw, $\$'_{j_i}$, reciprocal to all joints of branch i except for the actuated joint j can be found, i.e.:

$$\$_{k_i} \circledast \$'_{j_i} = 0, \text{ for } k = 1 \text{ to } l_i, j \neq k \quad (\text{A.19})$$

Noting that for the i^{th} branch:

$$\mathbf{V} \circledast \$'_{j_i} = \sum_{k=1}^{l_i} \dot{t}_{k_i} \$_{k_i} \circledast \$'_{j_i} = \dot{t}_{j_i} \$_{j_i} \circledast \$'_{j_i}, \text{ for } j = 1 \text{ to } n_i \quad (\text{A.20})$$

the joint rate of the actuated joint j of the i^{th} branch can be found as:

$$\dot{t}_{j_i} = \frac{\mathbf{V} \circledast \$'_{j_i}}{\$_{j_i} \circledast \$'_{j_i}} \quad (\text{A.21})$$

Define the reciprocal screw matrix $[\$']$ as:

$$[\$'] = \begin{bmatrix} \dots & \$'_{j_i} & \dots \end{bmatrix} \quad (\text{A.22})$$

where $i = 1$ to m and $j = 1$ to n_i and define the diagonal matrix \mathbf{D} of the inverses of the reciprocal products of the actuated joints and their associated reciprocal screws as:

$$\mathbf{D} = \begin{bmatrix} \ddots & & \mathbf{0} \\ & \frac{1}{\mathcal{S}_{j_i} \otimes \mathcal{S}'_{j_i}} & \\ \mathbf{0} & & \ddots \end{bmatrix} \quad (\text{A.23})$$

The inverse velocity solution of a parallel manipulator can thus be expressed in matrix form as:

$$\dot{\mathbf{t}} = \mathbf{D} ([\Delta] [\mathcal{S}'])^T \mathbf{V} = \mathbf{D} [\mathcal{S}']^T [\Delta] \mathbf{V} \quad (\text{A.24})$$

Solving for \mathbf{V} in equation (A.24) allows the forward velocity solution of a non-redundantly-actuated parallel manipulator to be expressed in matrix form as:

$$\mathbf{V} = \begin{Bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{Bmatrix} = \left(\mathbf{D} [\mathcal{S}']^T [\Delta] \right)^{-1} \dot{\mathbf{t}} = [\Delta] [\mathcal{S}']^{-T} \mathbf{D}^{-1} \dot{\mathbf{t}} \quad (\text{A.25})$$

A.3.2 Force Solutions

The force (\mathbf{F}) applied by a parallel manipulator is the sum of the forces applied by each actuated joint of the manipulator. Therefore, the forward static force solution for a parallel manipulator is given by:

$$\mathbf{F} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{m} \end{Bmatrix} = \sum_{i=1}^m \left(\sum_{j=1}^{n_i} \mathcal{S}'_{j_i} w_{j_i} \right)$$

where w_{j_i} is the wrench intensity applied by the j^{th} joint of the i^{th} branch. In matrix form, the forward static force solution can be expressed as:

$$\mathbf{F} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{m} \end{Bmatrix} = [\mathcal{S}'] \mathbf{w} \quad (\text{A.26})$$

where \mathbf{w} is a vector of wrench intensities acting on the screws of $[\$']$.

The relationship between the wrench intensity w_{j_i} and the joint torque/force τ_{j_i} for joint j of the i^{th} branch is:

$$w_{j_i} = \frac{\tau_{j_i}}{\$'_{j_i} \otimes \$'_{j_i}} \quad (\text{A.27})$$

In matrix form, the relationship between the wrenches and the joint torques/forces for a parallel manipulator can be expressed as:

$$\mathbf{w} = \mathbf{D}\boldsymbol{\tau} \quad (\text{A.28})$$

Substituting equation (A.28) into equation (A.26) yields the forward static force solution relating the joint torques/forces to the force being applied by the manipulator:

$$\mathbf{F} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{m} \end{Bmatrix} = [\$']\mathbf{D}\boldsymbol{\tau} \quad (\text{A.29})$$

The inverse static force solution for a non-redundantly-actuated parallel manipulator is:

$$\boldsymbol{\tau} = ([\$']\mathbf{D})^{-1}\mathbf{F} = \mathbf{D}^{-1}[\$']^{-1}\mathbf{F} \quad (\text{A.30})$$