

Variable-Rate $\pi/4$ -APSK Modulation

Rudolf Schnorr von Carolsfeld
B.Eng., University of Victoria

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to the required standard


Dr. Vijay K. Bhargava (Department of Electrical and Computer Engineering)


Dr. Qiang Wang (Department of Electrical and Computer Engineering)


Dr. Hausi Müller (Department of Computer Science)


Dr. Paul Fisher (School of Health Information Science)

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University of Victoria

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SUPERVISOR: V. K. BHARGAVA


Abstract

A new method of modulation for digital communication that supports variable data transfer rates over an additive white Gaussian noise communication channel is presented. The proposed modulation technique was developed for use in a non-mobile data telemetry system. Based on the $\pi/4$ -QPSK modulation scheme adopted for the North American digital cellular system, the technique offers the increased spectral efficiency of APSK signaling and is called $\pi/4$ -APSK signaling.


Amplitude scaling of the $\pi/4$ -QPSK channel signal provides an additional dimension in the modulation domain. In this thesis, expressions for the theoretical error performance of the $\pi/4$ -APSK signaling technique are determined, and optimum receiver decision regions and signal constellations are derived. Results of computer simulation of the proposed system subject to an ideal AWGN channel are presented. It is shown that this new modulation technique may be used to increase the data transfer rate of the system, to support error correcting codes for improved data reliability, or to support an enhanced network protocol.

Using a $\pi/4$ -APSK signal with two optimally-spaced signal points per quadrant (3 bits per symbol) provides a symbol error probability of 10^{-5} at a signal-to-noise ratio of approximately 19 dB; the same probability is attained at about 25 dB using 4-level $\pi/4$ -APSK signaling (4 bits per symbol), and at about 31 dB using 8-level $\pi/4$ -APSK (5 bits per symbol).


Examiners:



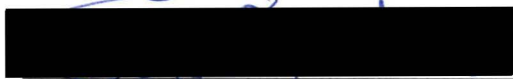
Dr. Vijay K. Bhargava (Department of Electrical and Computer Engineering)



Dr. Qiang Wang (Department of Electrical and Computer Engineering)



Dr. Hausi Müller (Department of Computer Science)



Dr. Paul Fisher (School of Health Information Science)

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To Tracel and Maia

Chapter 1: Introduction

1.1. Justification of Research

1.1.1. A Data Telemetry System

The research described in this thesis was part of a project designed to fulfill a practical requirement for a flexible, low cost modem for stationary remote monitoring and telemetry systems. Such a telemetry system is depicted in Figure 1. Data is collected by a large number of remote terminal units (RTU) and transferred via radio link to repeaters which in turn are linked via radio to other repeaters or to a central monitoring facility.

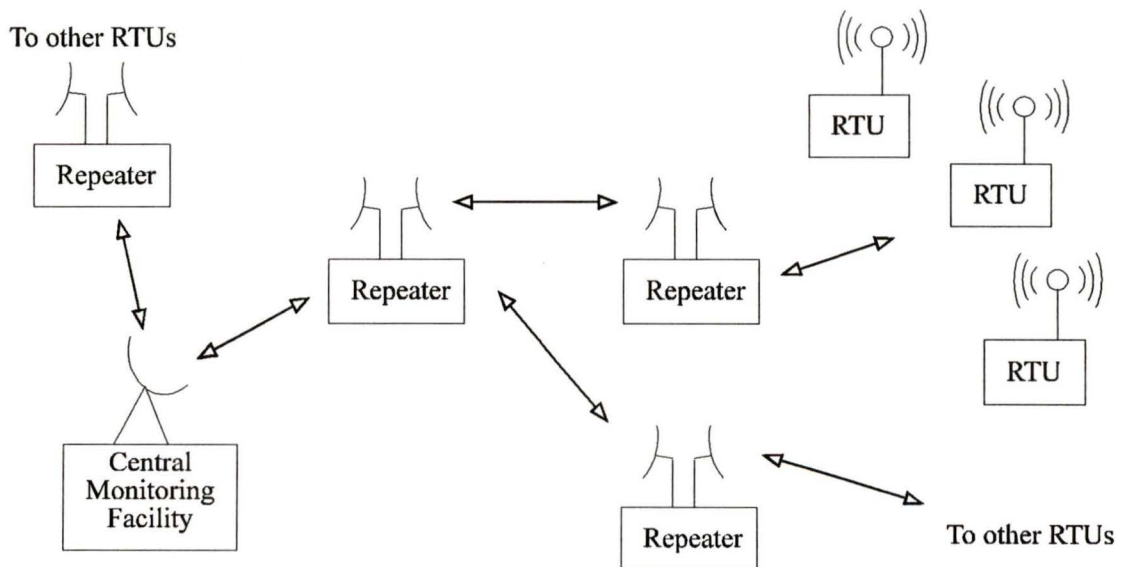


Figure 1. A remote monitoring and telemetry system

The RTUs transmit packets of collected data at infrequent intervals (perhaps once every minute) at a relatively slow rate (1200 bps) using a multiple access network protocol. These data packets are then sent at the same data rate via a limited number of repeaters to a central monitoring facility. In some implementations of this system, there are many RTUs for each repeater, and the transmission load on the repeaters is significant. All components in this network rely on modem circuits originally intended for data transfer over conventional telephone circuits. These circuits require a relatively long time to become synchronized for data transfer (lockup) and offer a low data rate. As a result, growth of the network through addition of more RTUs is restricted, since a potential bottleneck could result if too many RTUs or other repeaters attempt to communicate with one repeater.

In an effort to enhance the system described above, the company that instigated this project wished to design repeater stations capable of retransmission at a higher data rate. The required rate of retransmission would depend on the number of RTUs associated with the repeater and the frequency with which data is received from each. This leads to the requirement for a repeater modem that can handle infrequent low speed reception of data from the RTUs, and high speed retransmission to subsequent repeaters or the central monitoring facility.

Another aspect of this telemetry system to be addressed was the transmission of control information together with the telemetry data. This can either be accomplished through an enhancement of the network protocol (placing control information in a packet header, for example), or by providing a side-channel for control information (communicating control information and data simultaneously). The chip currently in use in the telemetry system does not allow modification of the modulation scheme to

support side-channel transmission. If the modem were capable of transmitting control information and data simultaneously and independently, this would offer much greater flexibility in the design of the network protocol.

These points lead to the desire for a particular kind of modem:

- If conditions allow, the modulation technique to be implemented should have the ability either to switch from a low speed to high speed transmission, or be able to transmit two independent messages simultaneously (such as control and data messages), perhaps using a secondary channel that can be enabled or disabled as required.
- From an economic perspective, a uniform modularity in the system is highly desirable, meaning that a single type of modem used throughout the network is preferred to two or more specialized types.
- Each modem should use primarily off-the-shelf components to minimize cost.

1.1.2. Digital Cellular Telephony

In addition to the desire that the modem for the above telemetry system be low-cost, relatively uncomplicated, and modular in construction, it should also have the potential for use in other applications with future market potential. In this regard, the company had also been interested in researching aspects of digital cellular communication techniques, since the growth potential in this market is considered to be quite high [14, 23].

Considering the above requirements, we decided that the augmentation of $\pi/4$ -shifted Differential Quaternary Phase Shift Keying ($\pi/4$ -QPSK) [1] should be investigated.

$\pi/4$ -QPSK was chosen for mobile radio because of its suitability to the mobile channel and reasonable efficiency. Since the data telemetry system is stationary, the need for a linear modulation method¹ does not exist as it does for mobile fading channels. The impact that multipath and shadow fading has on amplitude modulation techniques is not a serious consideration for a non-mobile channel. This makes the use of multiple signal amplitude levels a possibility, and offers a method for implementing variable-rate transmission or a transparent secondary channel.

The goal of this thesis is to investigate the use of $\pi/4$ -QPSK signaling with amplitude variations to enhance system capabilities. The resulting modem should be capable of using $\pi/4$ -QPSK signaling for low rate data transfer, and of introducing amplitude variations for an increased symbol alphabet. This thesis determines the suitability of such a modulation technique to the data telemetry system described above.

The increased symbol alphabet can be used in a number of ways with a variety of network protocols, including provision of variable rate data transfer, a side channel capability for control information, or perhaps to transmit information to repeaters while simultaneously communicating with RTUs. In fact, if this system offers satisfactory performance, a variety of coding schemes could be implemented for data security or reliability. In other non-mobile or slow fading mobile situations, this technique could also be used to enhance throughput or data security of mobile cellular communications.

1 Linearly modulated signals require a linear channel for perfect reception; linear modulation is discussed in Chapter 2.

1.1.3. Proprietary Versus Available Technology

A commercially available QAM or APSK telephone modem may at first appear to be acceptable for the data telemetry link described above.² However, the following points explain the desire for a fresh approach:

- Communication over the data telemetry channel is carried out in one direction at a time (simplex or half duplex). Many inexpensive telephone modems support full duplex operation, splitting the available bandwidth into two sub-bands and limiting the data transfer rate by these reduced bandwidths. Not being optimized for one-way (simplex) communications, the unused bandwidth translates into an underutilized channel capacity.
- Data transmitted over the telemetry channel described above is not subject to adjacent channel interference, unlike data communication over conventional telephone channels. Rayleigh fading, shadowing, and other similar degrading influences common in mobile radio channels or cellular telephone applications also do not apply, since the telemetry channel is fixed. Additional data may be transferred if these effects are not present. Channel signal-to-noise ratios up to 30 dB can be expected in the above telemetry system.
- Standards for digital cellular communication equipment have been defined for $\pi/4$ -QPSK signaling, and the possibility of increasing the data rate of cellular channels can only be accomplished if this same modulation scheme is used. A technique capable of doubling the data rate of $\pi/4$ -QPSK signaling would be beneficial.

² Acronyms and abbreviations appearing in this thesis are defined in the appendix.

- A modulation scheme capable of supporting error correction coding techniques with no reduction in the data transfer rate was desired. Some mechanism to provide protection against errors with a probability of error up to 10^{-5} should be supported.

Thus, although an increased data rate could be achieved by simply installing a higher speed modem, the cost of these modems, the nature of the data telemetry system, and the desire for faster synchronization, extra transmission capability, and compatibility with cellular telephony equipment suggested that a proprietary modem design was worth considering.

1.2. Thesis Overview

The balance of this thesis has the following layout. In Chapter 2, some aspects of linear modulation techniques are reviewed: encoding, modulation, demodulation, detection mechanisms, and theoretical error performance are described. These concepts are then extended to $\pi/4$ -QPSK modulation, and focused on the modulation mechanism that is studied in the rest of the thesis: $\pi/4$ -APSK. The chapter concludes with a survey of the literature with respect to $\pi/4$ -QPSK systems and amplitude modulation used in conjunction with it.

In Chapter 3, a definition of $\pi/4$ -APSK is given, and its use in fixed-link (AWGN channel) applications is discussed. An expression for its error performance is derived and presented, based on the receiver decision regions. Optimal signal constellations and decision thresholds are determined. System enhancement resulting from an increased data rate are discussed and comments are made on encoding possibilities.

Chapter 4 is a brief description of a proposed system implementation (block diagram) and includes results of a computer simulation of the modem system. Chapter 5 summarizes the findings of this research and offers suggestions for further study.

Chapter 2: Theory and Literature Review

This thesis describes a type of amplitude-phase shift keying (APSK) based on the $\pi/4$ -shifted quadriphase shift keying ($\pi/4$ -QPSK) modulation used in digital cellular applications in North America. In Section 2.1, aspects of modem operation using linear signaling techniques are reviewed, with emphasis on PSK, APSK and $\pi/4$ -QPSK modulation.³ Section 2.2 summarizes demodulation mechanisms and performance measures for PSK signaling. The literature review of Section 2.3 comments on published research in the areas of digital modulation techniques applicable to this thesis.

2.1. Linear Modem Systems

In this section, the general characteristics of linear modulation and demodulation techniques are reviewed. Linear modulation techniques [6] are those signaling schemes, including shaped PSK and QAM signals, which require low distortion (high linearity) both during frequency translation from baseband to carrier frequencies and during amplification of the channel signal. Correctly transmitted linearly modulated signals can then be demodulated by the receiver using an appropriate method. Without a sufficiently linear channel, the correct relationship between signal points is lost, making the demodulation and detection process ineffective. The spectral efficiency depends on the number of modulation levels used, but an increased number of levels

3 For a thorough discussion of modulation theory, a number of excellent texts can be recommended, including Haykin [13], Lindsey & Simon [15], Stremler [18] or Sklar [74].

must be traded-off against a higher bit error rate. A number of commercially available modems use linear signaling over satellite and microwave channels, or over the PSTN (public switched telephone network).

Nonlinear modulation techniques, also referred to as constant envelope or continuous phase modulation (CPM) techniques, are techniques in which the demodulation mechanism is largely unaffected by nonlinear amplification, such as multilevel FSK (e.g. MSK, GMSK, FFSK, etc.). Their constant envelopes allow the use of inexpensive non-linear class C power amplifiers at the cost of spectral efficiency, which is limited to approximately 1 bps/Hz. These techniques are also used in a number of systems, including some satellite communications systems and the European cellular radio system.

Since the modem discussed in this thesis is based upon an amalgamation of APSK and QPSK modulation, APSK and PSK modulation schemes are described in more detail in the following.

2.1.1. PSK and APSK Modulation Techniques

During the initial discussions of the new digital radio technology for North America in 1986, MSK and other CPM techniques were given the greatest attention [6]. This was because the nonlinearity of inexpensive transmitter power amplifiers in general tends to spread the spectrum of the signal, negating the bandwidth efficiency gained through the use of linear techniques [1]. It was thought that the spectral characteristics of the amplified channel signal would be sufficiently poor to make linearly modulated signals unusable in a mobile environment. But since 1987, researchers have recognized that the better bandwidth efficiency that can be obtained by the use of linear modulation

[47, 49] may in fact offset the apparently inferior spectral characteristics if appropriate signal conditioning is used.

The most common form of linear modulation is phase shift keying (PSK). A set of $M=2^p$ equiprobable, equal energy signals are used in the signaling technique known as M -ary phase-shift-keying (MPSK). M is the size of the symbol alphabet (and the number of distinct phase values used), and p is the number of bits required for binary encoding and decoding of the signal phase. An MPSK signal may be defined by [5]:

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos(\omega_c t + \phi_i) \quad (1)$$

where E_s is the energy of the signal, T_s the symbol period, and the phase ϕ_i depends on the source stream and is adjusted once each symbol period in accordance with the symbol mapping, or encoding rules. In this manner, the information is transmitted in the phase of the channel signal. Examples of such signaling techniques include binary PSK (BPSK; $p=1$), quadriphase shift keying or quaternary PSK (QPSK; $p=2$), octaphase shift keying or 8-ary PSK (8-PSK; $p=3$), and so on. Typical signal constellations⁴ are shown in Figure 2, which also indicates the symbol mapping rules for the transmitter and the decision regions for the receiver.

4 Signal constellations are a geometric representation of the structure of a composite signal, showing the relative energy levels of the in-phase and quadrature components. An in-depth treatment is given by Stremmer [19]. The axes may be arbitrarily assigned to a reference phase and represent signal energy or amplitude, and are unlabeled throughout this thesis.

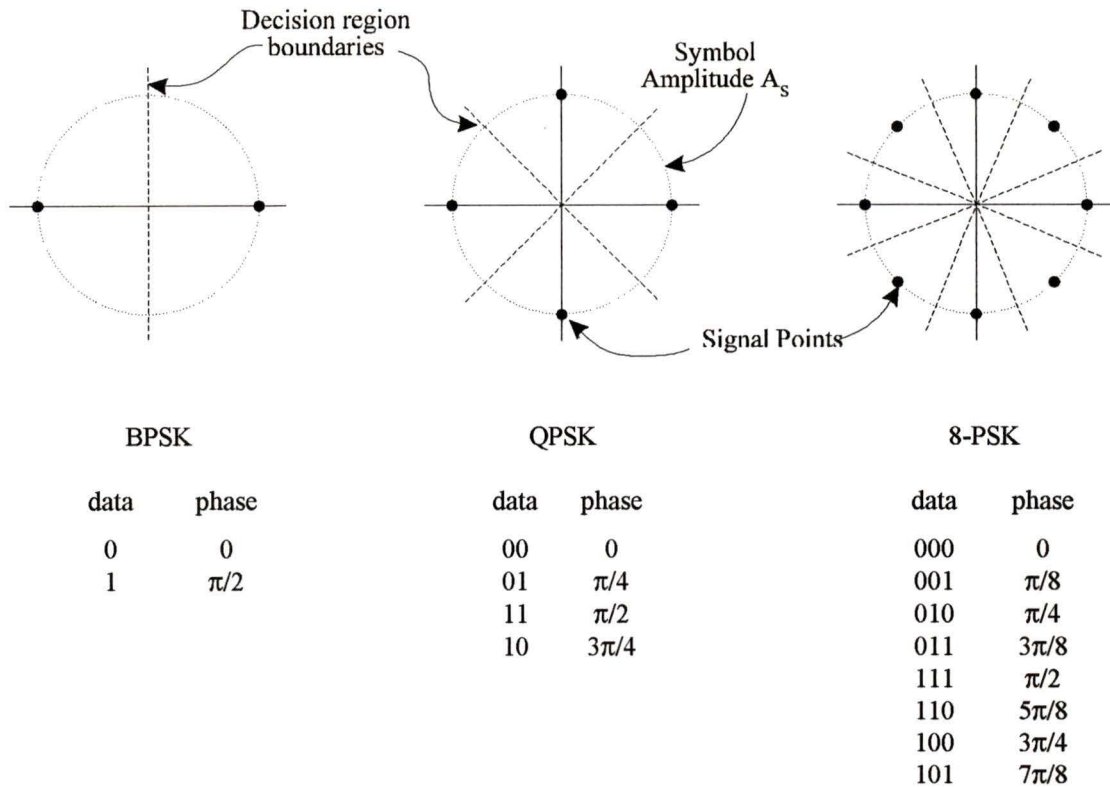


Figure 2. Binary encoding, signal constellation and decision regions for MPSK signaling.

For MPSK signaling, an absolute phase transition is made at each symbol period; that is, the phase representation of each binary symbol may be measured relative to an absolute phase reference. The receiver must measure the magnitude of this phase, and then decode the binary message associated with it. Clearly, the receiver will be able to do this correctly whenever the phase of the received signal s_i is found to fall within the decision region, I_i , associated with that signal point. A receiver error occurs if the phase is outside the correct decision region. Note that the receiver is not basing its decision on the amplitude (energy) of the signal, but only on the absolute phase.

Greater spectral efficiency can be obtained by increasing the size of the MPSK symbol alphabet. In doing so, however, the detection process becomes increasingly difficult,

since the greater number of phase states results in smaller decision regions. The receiver must then be able to distinguish smaller phase values, and is more susceptible to noise and interference [15]. One way the detection process can be simplified is to alter the modulation process slightly, using differentially encoded PSK (DPSK).

Using DPSK, as with MPSK, the information is transmitted in the signal phase. Unlike MPSK signaling, however, it is not the absolute signal phase, but the phase transition relative to the previous signal point that identifies the transmitted symbol. The signal constellation for M-ary DPSK may be the same as that shown for M-ary QPSK in Figure 2, with the 0-radian reference axis rotated to coincide with the phase of the previous symbol. The simplification resulting from differential encoding is that the demodulator no longer needs to establish an absolute reference phase against which the received signal phase is compared. Still, the decoding process is completely dependent on the phase, and not on the amplitude (energy) of the received signal.

Differential encoding may simplify the receiver, but offers no improvement in spectral efficiency over MPSK. More effective use of the signal space can be attained by increasing the signal spacing efficiency (minimizing the mean Euclidean distance between signal points) using amplitude-phase shift keying (APSK). In APSK signals, the amplitude as well as the phase of the carrier are varied according to the encoding rules. An APSK signal may be expressed as:

$$s_{jk}(t) = \sqrt{\frac{2E_j}{T_s}} \cos(\omega_c t + \phi_k) \quad (2)$$

Examples of APSK signaling include QAM (16-ary APSK), 64-ary QAM (64-ary APSK), etc., signal constellations for which are shown in Figure 3.

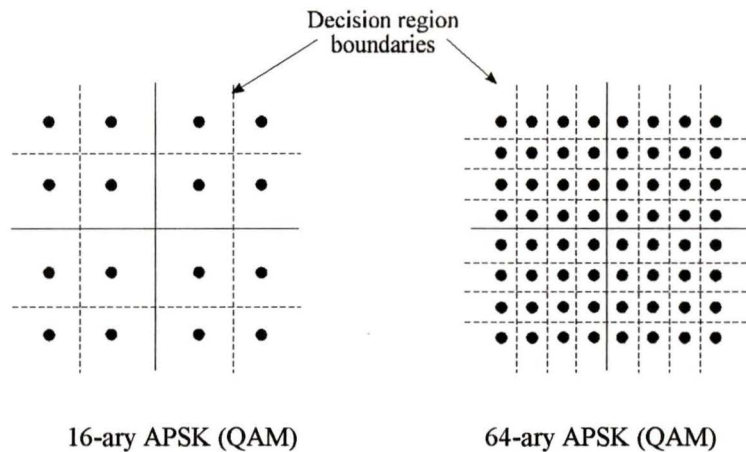


Figure 3. Signal constellation diagrams for M-ary APSK signaling.

Significant effort has been made by researchers to determine optimal signal constellations for APSK signaling, since the most efficient packing scheme will offer the greatest energy efficiency. Nonetheless, the simplest systems to implement may be suboptimal, in the sense that the signal points are not positioned to attain the minimum Euclidean distance between points. Examples of modem systems using suboptimal signal constellations are not uncommon. In these systems, the signal points may be skewed away from the optimum constellation to allow simpler phase detection, for example, independently from amplitude detection.

Recent results published by Webb [51] in 1992 illustrate this approach to the definition of a suboptimal APSK signal constellation. Webb's "star QAM" is a reapplication of IAPSK (Independent APSK) described by Weber [53] in 1978. IAPSK is a form of amplitude-shifted 8-ary PSK: two concentric rings of 8-ary PSK define the signal constellation. A skewed APSK and an IAPSK signal constellation are shown in Figure 4. A similar approach to the description of a suboptimal signal constellation is taken in later chapters of this thesis.

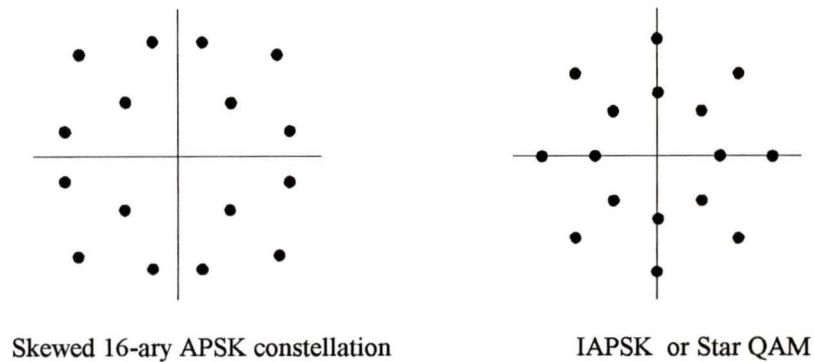


Figure 4. Suboptimal APSK signal constellations [37, 53].

2.1.2. Modem Techniques for Cellular Radio

In this section, the specific $\pi/4$ -QPSK modulation technique used in some cellular radio applications is introduced, and its evolution and general characteristics are reviewed. The discussion clarifies why $\pi/4$ -QPSK signaling was chosen as the basis of study in this thesis.

The project described in Section 1.1 calls for a modem design that can be used for remote telemetry over a bandlimited radio channel, using a signaling technique that offers high spectral efficiency and allows uncomplicated implementation. Digital cellular communication systems have been built to exacting bandwidth and performance restrictions, and were intended to service the consumer market; in these respects the design requirements for digital cellular communication and for the modem described in this thesis are similar. In the design decisions for this modem we can use many of the discussions of modulation techniques made in the evolution of the digital cellular communication system, which has been described by a number of authors [1, 4, 6, 14, 31]. The essential elements in this evolution are described in the following.

The recently adopted modulation scheme for the North American digital cellular system has been designed to upgrade the existing Advanced Mobile Phone System (AMPS), adding digital voice and data transmission capability to new subscriber equipment. When the AMPS was designed, constant envelope continuous phase frequency shift keying (CPFSK) was selected for a number of reasons. First, in this type of multilevel FM signal, the carrier frequency has no abrupt phase transitions, which cause significant sidelobe regeneration when passed through an inexpensive non-linear power amplifier [1]. A direct consequence of sidelobe regeneration is the radiation of out-of-band power, which in turn causes adjacent channel interference in a frequency division multiplexed radio system. Secondly, a limiter-discriminator detector could be used, allowing simple circuit design and high noise immunity.

CPFSK was found to be very well suited to the channel restrictions and the available technology. Unfortunately, as can be seen in the following table, M-ary FSK signaling is spectrally inefficient for $M \geq 8$ compared to linear modulation techniques (MPSK).

M	2	4	8	16	32	64
PSK	0.5	1	2	4	8	16
FSK	1	1	0.75	0.5	0.3125	0.1875

Table 1. Bandwidth (spectral) efficiency of M-ary signals (bps/Hz) [27]

The implementation of digital cellular radio required a modulation scheme with higher spectral efficiency than the multilevel FM scheme used in the AMPS. The combination of limited transmission bandwidth and high spectral efficiency requirements had lead to three linear modulation techniques being considered by 1987: QPSK, offset QPSK

(OQPSK), and $\pi/4$ -shifted QPSK [1]. These are summarized briefly to give an appreciation for the choice of $\pi/4$ -QPSK.⁵

The theoretical communication channel bandwidth required for all three techniques is the same. In practice, however, the spectrum of the QPSK signal increases the most when passed through a non-linear system. This is because of the possibility that instantaneous π -radian phase transitions may occur at the symbol rate, or every two bits [13]. When such a QPSK signal undergoes non-linear amplification, the resulting transmitted signal has a non-constant envelope, the filtered sidelobes (filtered before amplification) are restored, and in-phase to quadrature in-band crosstalk is generated [14]. This can cause severe co-channel interference and adjacent channel interference, and the spectral efficiency gained by using linear modulation may be lost to spectral spreading after non-linear amplification.

The problem of sidelobe regeneration can be reduced using OQPSK, in which the phase transitions occur at the bit rate (i.e. twice the symbol rate) and are limited to $\pi/2$ radians. Since a π -radian phase shift is thereby reduced to two $\pi/2$ -radian shifts, the sidelobes are not regenerated to the same extent by non-linear amplification. However, the OQPSK receiver requires a more complicated demodulator with a coherent phase reference because of the extra phase transitions, which introduce a $\pi/2$ phase ambiguity [41].

A third approach is to use $\pi/4$ -shifted differentially encoded QPSK ($\pi/4$ -QPSK), which has been adopted by the Telecommunications Industry Association's Interim Standard

5 A good comparative evaluation is given by Austin, et al. [2] and by Oetting [39]

(IS-54) for the North American digital cellular telephone system [12]. IS-54 specifies the use of $\pi/4$ -QPSK modulation and narrowband time division, allowing each 30 kHz channel to support six users instead of the single user supported by the first generation AMPS.

In $\pi/4$ -QPSK signaling, as for DPSK, the signal phase that defines the current symbol is measured relative to the preceding symbol (i.e., the signal phase corresponding to the preceding symbol). According to some encoding rule, two data bits ($p=2$) define a channel symbol; each symbol belongs to a $\pi/2$ -spaced quadriphase signal set. Signal points are selected from two such signal sets in alternation, where these two sets are shifted by $\pi/4$ radians with respect to each other as shown in Figure 5.

As this figure shows, possible phase transitions for the subsequent symbol are restricted to $\pm\pi/4$ or $\pm3\pi/4$ radians. As an example, the previous symbol is indicated in the figure by s_{i-1} , which has a reference phase of 0 radians for convenience, and the current symbol is indicated by s_i , showing the transition that would occur with a $\pi/4$ -radian phase shift.

As was the case for OQPSK, restricting the possible phase transitions to less than $\pm\pi$ radians reduces the spreading of the spectrum caused by non-linear amplification. Unlike OQPSK, however, these transitions occur at the symbol rate instead of at the bit rate. This makes $\pi/4$ -QPSK very suitable for narrow bandwidth channels, and the differential encoding allows the receiver to be readily implemented using either coherent or differential detection [22].

Akaiwa [1] identified three major factors that made $\pi/4$ -QPSK the preferred choice for digital mobile radio applications: (1) the limited effect of non-linear amplification, (2) its robustness against fading, and (3) the relative ease of receiver implementation.

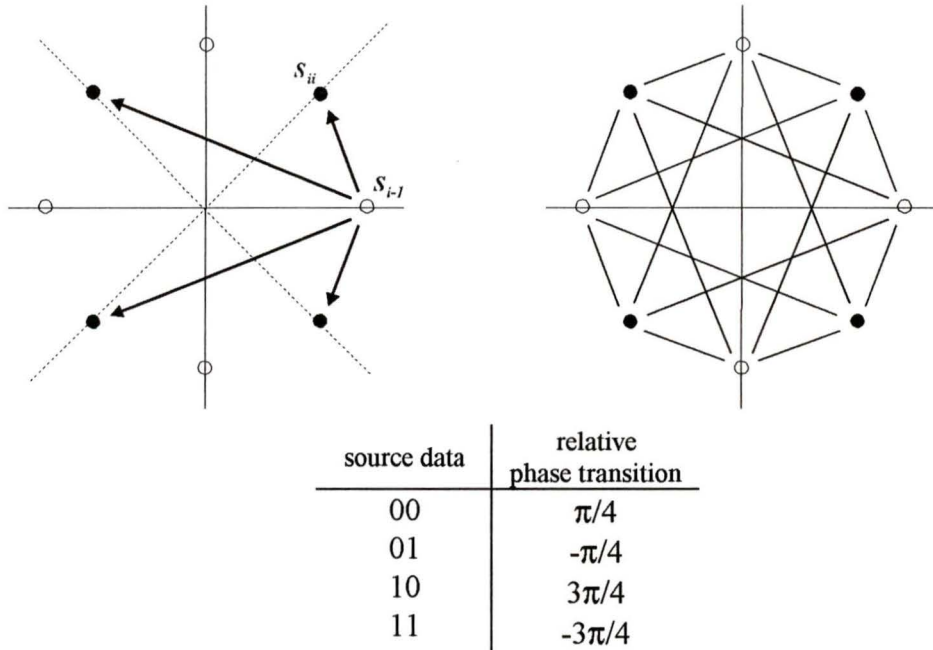


Figure 5. Signal constellation and phase transitions for $\pi/4$ -QPSK signaling.

Envelope fluctuations and sidelobe regeneration can also be reduced using other modulation methods, such as QAM signaling schemes. But the need for coherent modulation using these techniques makes them more costly to implement than CPFSK or $\pi/4$ -QPSK systems. Also, the inability of QAM systems to track the absolute phase during fades in a Rayleigh fading channel results in long error bursts [51, 52]. No alternative signaling techniques appear to be compatible with the now accepted $\pi/4$ -QPSK signaling standard. Implementation of any of them would require

development of new standards if they were to be used for future digital cellular applications.

Some approaches to improve the spectral efficiency of linear modulation schemes have been described in the literature. These include various baseband pulse shaping techniques that could be applied to $\pi/4$ -QPSK, such as shaped PSK [10], sinusoidally-shaped PSK [30], controlled transition PSK [46], and others. These processes can often be accomplished through digital input and output filter mapping, and allow the channel signal to be matched to the channel conditions, resulting in very little out-of-band power radiation.

The $\pi/4$ -APSK signaling technique described in this thesis makes use of variations in the phase and amplitude (energy) of the signal in the same way that other APSK techniques do, and also has a suboptimal signal constellation with some similarity to the technique described by Weber and Webb. The difference is that in a suitably designed modem system, this technique is compatible with the $\pi/4$ -QPSK signaling method used for digital cellular radio. The contribution of this thesis is the investigation of this suboptimal APSK signaling technique.

2.2. Demodulation

Fines *et al.* [17] have stated that "more than 90% of the signal processing required in a modem is performed at the demodulator," and there is little reason to doubt this assertion. To simplify the analysis of a modem system, many authors assume AWGN and an optimal receiver, and the application of the proposed modulation scheme to a fixed radio link allows similar assumptions to be made in this thesis. This section gives a brief overview of receiver operation for MPSK and $\pi/4$ -QPSK, particularly as

related to implementation for $\pi/4$ -APSK signals. For an in-depth discussion of this topic, a number of excellent works are available [27, 33, 41, 45, 55].

2.2.1. Detection and Decoding of DPSK

The traditional approach to demodulation with an MPSK receiver is to measure the phase relative to an absolute phase reference (coherent detection [14]); this requires near perfect carrier phase and frequency synchronization. In practice, the receiver is often unable to attain perfect synchronism, although implementing complex AGC and PLL circuitry may give a reasonable approximation. Such circuitry, however, may result in a π -radian phase ambiguity in the received signal and a corresponding error in the decoded data stream.

Differential encoding of the PSK signal allows the receiver to resolve this phase ambiguity, and obviates the need for a phase coherent receiver. Differential detection (also referred to as noncoherent detection or differentially coherent detection) also allows for the design of simple and inexpensive systems capable of very fast synchronization and resynchronization.

An ideal receiver could have the correlation structure shown in Figure 6. The local receiver oscillator is brought into phase and frequency coherence with the equalized received signal. This then allows detection using a correlation circuit (or matched filter) to determine the in-phase and quadrature components of the received signal. The data stream is then recovered by the decoder using a predetermined mapping rule, which could be stored in a lookup table, for example.

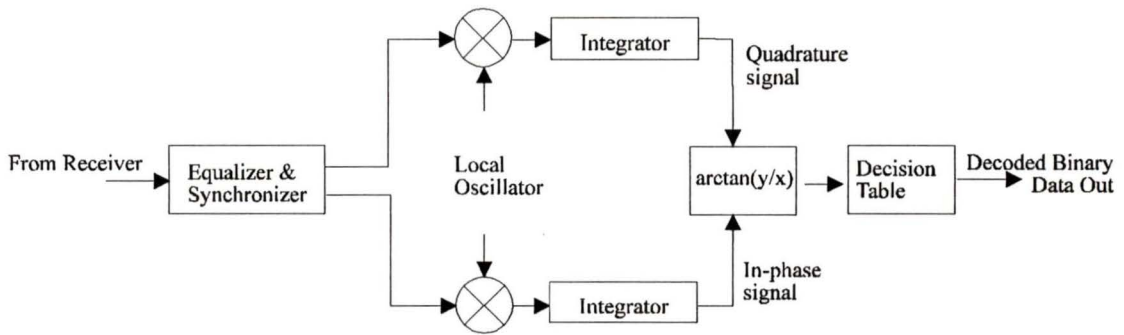


Figure 6. Ideal PSK receiver.

DPSK signals, including $\pi/4$ -QPSK, may be detected using either a coherent reference or a differential detector. As a form of DPSK signaling, the $\pi/4$ -APSK signals described in this thesis are also detectable using coherent or differential detection methods. Coherent and differential (noncoherent) detectors for $\pi/4$ -QPSK systems have been described by a number of authors [17, 34, 36, 47, 48]. A block diagram of a modem system employing differential encoding and detection is shown in Figure 7.

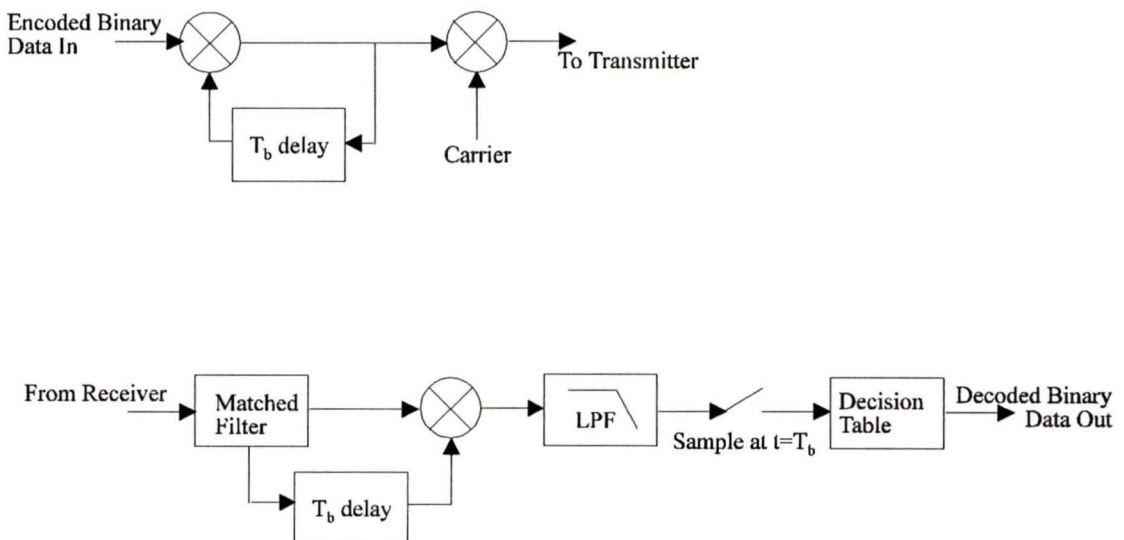


Figure 7. Block diagram of a DPSK modem [41]

In the above figure, an encoded binary data stream is multiplied first by a delayed version of itself and then by the baseband carrier. The signal is then transmitted through the channel and applied to the input of the matched filter. The output from this filter is multiplied with a delayed version of itself, and the high frequency components are removed by the low pass filter (LPF). The values of samples taken at the symbol rate reference a lookup table to determine the decoded binary output data [41].

2.2.2. Performance of Linear Modulation Techniques

A commonly used figure of merit for any modulation technique is the probability of making a channel symbol error (symbol or word error probability P_e); this probability depends on the signal energy to noise energy ratio (SNR). The noise energy observed in the channel will naturally depend on the application. For typical mobile radio applications, the noise is assumed to be Rician or Rayleigh distributed because of random fading, shadowing or multipath interference. In this project, however, the fixed radio link allows us to assume additive white Gaussian noise (AWGN).

Expressions for the symbol error probability P_e using coherent M-ary QPSK or M-ary APSK signaling in an AWGN channel may be found in a number of texts, and are relatively easy to obtain. They are included here for future reference without derivation, for a signal of symbol energy E_s and noise energy N_0 (average symbol energy $E_{avg}=E_s$) [27]:

$$P_e \cong \operatorname{erfc}\left(\sqrt{\frac{E_s}{N_0}} \sin \frac{\pi}{M}\right) \quad (\text{QPSK}) \quad (3)$$

$$P_e \cong 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{3E_{avg}}{2(M-1)N_0}}\right) \quad (\text{APSK}) \quad (4)$$

Coherent detection offers a power efficiency advantage over differential detection. The derivation of an expression for the error performance using differential detection is much more complex than for coherent detection; a summary is given by Lindsey & Simon [33]. The performance of differentially detected M-ary QPSK at a probability of symbol error around 10^{-5} is 2~3 dB worse than if coherently detected [33]. It is approximated by:

$$P_e \cong \operatorname{erfc} \left(\sqrt{\frac{2E_s}{N_0}} \sin \frac{\pi}{2M} \right) \quad (\text{differential QPSK}) \quad (5)$$

Another common figure of merit, the probability of a specific bit being in error (bit error probability P_b), depends to some extent on the symbol definition. Gray encoding [27, 33] is a symbol encoding method that makes a symbol error most likely to result in at most one bit error. The binary input data is mapped to M-ary symbols such that two adjacent symbols in the signal constellation differ in only one bit position. The approximate relationship between P_b and P_e if Gray encoding is used is given by [27, 43]:

$$P_b \approx \frac{P_e}{\log_2 M} \quad M \geq 2 \quad (6)$$

2.3. Review of Literature Relating to $\pi/4$ -QPSK

A review of available literature has not resulted in the discovery of any previous attempts of intentionally using amplitude scaling in conjunction with $\pi/4$ -QPSK modulation. $\pi/4$ -QPSK was rediscovered by Akaiwa and Nagata [1] specifically for use in the mobile radio environment of second generation digital cellular communication systems, largely because of its inherent ability to remain unaffected by

non-linear amplification and its relatively constant signal envelope. Having no severe amplitude variations in practice, $\pi/4$ -QPSK is particularly useful in a mobile environment subjected to various types of fading. The application of $\pi/4$ -QPSK to a non-mobile channel, with the resulting potential for superimposed amplitude modulation does not appear to have been studied, as it is in this thesis.

Much of the literature describing research in APSK modulation schemes is based on a search for higher bandwidth efficiency (a well-packed signal constellation) or improved performance in non-linearly amplified channels [30, 51]. These improvements are attained primarily through the development of new baseband pulse shaping schemes to reduce intersymbol interference and out-of-band spectral radiation [2, 10, 29, 30, 46], multiple symbol detection [11, 54], multidimensional constellations [19, 20], and error correction and encoding schemes [56].

Noticeably absent from most of these research efforts, however, is an attempt to augment the actual $\pi/4$ -QPSK signal constellation as proposed in this thesis. The increased symbol alphabet resulting from augmentation would allow the system to offer a higher data rate or secondary channel communication capability as required by this project, or to support error control coding (using TCM codes as described by Ungerboeck [50] for example), and a number of other possibilities.

The recent research carried out by Webb et al. [51, 52] on variable-rate star QAM has a common ancestry with the research presented in this thesis. Webb's scheme is not compatible with the established $\pi/4$ -QPSK signaling mechanism, however, and therefore could not be adapted for use in this project. Specifically, the use of amplitude shifting to augment the signal constellation and increase the data transfer rate using $\pi/4$ -QPSK signaling does not appear to have been addressed by any of this literature.

2.4. Chapter Summary

Various aspects of data encoding and signal modulation in relation to the modem under consideration have been presented in this chapter. M-ary PSK offers increased bandwidth efficiency over FSK signaling. OQPSK offers lower sidelobe regeneration than QPSK. QAM provides the greatest bandwidth efficiency, but at the cost of added receiver complexity. $\pi/4$ -QPSK offers good bandwidth efficiency, low out of band radiation, and simple receiver implementation.

Available literature, as it applies to $\pi/4$ -QPSK signaling and its variations, has been reviewed. Comments have been made regarding aspects of modulation and demodulation, encoding and decoding, and error performance of a modulation scheme which expands the symbol alphabet of a $\pi/4$ -QPSK signal constellation through amplitude shifting. A review of the literature has shown no previous research to consider this new signaling technique.

Using amplitude scaling in conjunction with $\pi/4$ -QPSK, as discussed in this thesis, requires that a number of parameters be taken into consideration. The error performance may be compromised due to variations in power levels, but this must be measured against the increase in data rate. The augmentation of the signal space allows encoding to be considered for encryption or data reliability. For any static (non-mobile) or quasi-static communication channel subjected to slow and shallow fading, this new modulation technique may offer a significant increase in communication capability. The prospects for this technique are discussed in the coming chapters.

Chapter 3: Variable-Rate $\pi/4$ -APSK Modulation

This chapter discusses aspects of the modem under consideration. The $\pi/4$ -APSK modulation scheme is defined in Section 3.1, with a description of data encoding and symbol mapping as it is applied to the system. An expression for its theoretical error performance is derived and presented in Section 3.2. This is then used in Section 3.3 to determine the best signal constellation, depending on the number of levels used. Decision thresholds are also described, followed by a discussion of resulting system enhancement in Section 3.4.

3.1. Modulation

3.1.1. Definition

As discussed in Section 2.1.2, when $\pi/4$ -QPSK modulation is used, information is contained in the $\pm\pi/4$ radian or $\pm3\pi/4$ radian phase transitions of the signal. If the communication channel is not subject to deep fading, the signal energy (the amplitude) may be varied as well as the phase. In this manner, additional information can be transferred, increasing the amount of information carried by the signal. This combination of $\pi/4$ -QPSK signaling with amplitude scaling is referred to as $\pi/4$ -APSK ($\pi/4$ -shifted Amplitude Phase Shift Keying) in this thesis.

We can describe the $\pi/4$ -APSK signal using the definition of $\pi/4$ -QPSK from Section 2.1, by simply augmenting Equation (1) to include signal amplitude scaling as follows:

$$s_{jk}(t) = \sqrt{\frac{2E_j}{T_s}} \cos(\omega_c t + \phi_k) \quad (7)$$

where E_j is the energy of the signal point, T_s is the symbol time, and ϕ_k is the phase. The subscript j will depend on the number of energy levels in the signal: $j \in (1, 2)$ for 2 levels, $j \in (1 \dots 4)$ for four levels, and so on. As with $\pi/4$ -QPSK signaling, the $\pi/4$ -APSK signal phase that defines the current symbol is measured relative to the preceding symbol. Furthermore, the signal points also alternate between two $\pi/2$ -spaced quadriphase signal sets, where the two signal sets are shifted by $\pi/4$ radians with respect to each other. In a number of other respects, however, $\pi/4$ -APSK signaling is markedly different from $\pi/4$ -QPSK.

As is the case for $\pi/4$ -QPSK signaling, $p=2$ bits determine the signal phase (the signal may have one of $2^p=4$ phases) and identify the quadrant of the signal constellation. Unlike $\pi/4$ -QPSK signaling, however, the $\pi/4$ -APSK signal point can also have any one of up to 2^q energy levels, as specified by another independent set of q bits. In each quadrant of the two dimensional signal space, there may be 2^q signal points as shown in Figure 8 below. Clearly, with $q=0$, we have the special and familiar case of $\pi/4$ -QPSK signaling previously shown in Figure 5.

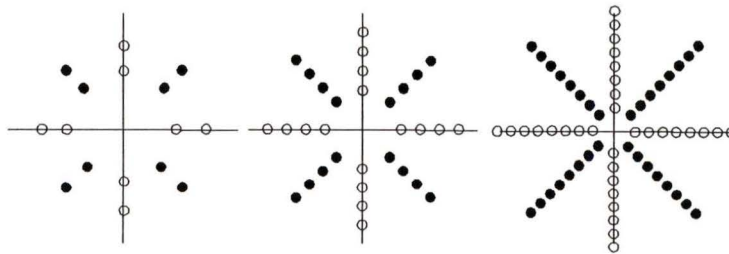


Figure 8. Signal constellation for $\pi/4$ -APSK with $q = 1, 2$ and 3 .

Each increment in q allows one extra bit of information to be transmitted in each symbol, for a total of $M=pq$ bits per symbol. Varying q in accordance with the channel

conditions provides an opportunity for variable rate data transmission using $\pi/4$ -APSK signaling.

The variable rate capability of this modulation technique will naturally depend on the channel conditions. A noisy communication channel (with a low SNR) may be able to support no more than a standard $\pi/4$ -QPSK signal for acceptable error performance. Section 3.2 shows that with reasonably good channel conditions, however, low error rate transmission is theoretically possible using two, four, eight or even more signal energy levels. For the purposes of this project, performance is determined under the assumption that $q \in (1, 2, 3)$.

3.1.2. Symbol Mapping and Encoder Structure

The symbol mapping of the data bits onto the channel signal points can affect the performance of the system. Gray encoding may be applied to both streams of source data as illustrated in Figure 9, the p bits defining the signal phase, and the q bits defining the signal amplitude.

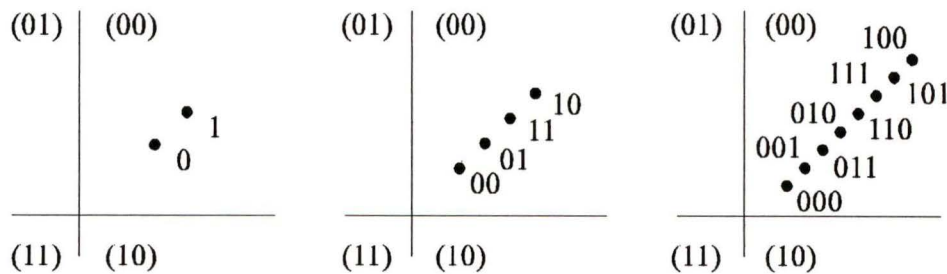


Figure 9. $\pi/4$ -APSK symbol mapping using a Gray code.
(Phase bits in parentheses)

The encoder structure for this signaling method is quite straightforward; a fully digital implementation has been described by Hansen [25]. The encoder could also be implemented around a conventional $\pi/4$ -QPSK modulator; an example is illustrated in Figure 10.

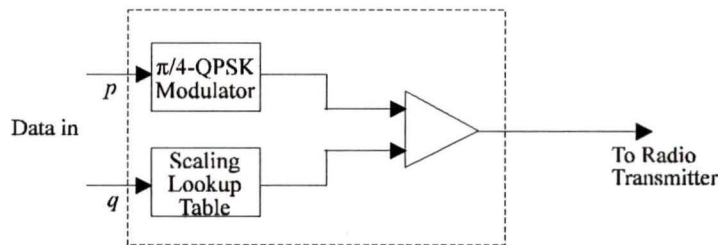


Figure 10. $\pi/4$ -APSK Encoder Structure

In this implementation, the encoder comprises a standard $\pi/4$ -QPSK modulator and an amplitude scaler. The p bits that define the channel signal phase are fed into the $\pi/4$ -QPSK modulator. The output of this modulator is then scaled by an amount that depends on the q bits that define the channel signal amplitude. This scaling may be accomplished using a simple amplifier configuration, with the reference to the amplifier being derived from a lookup table that encodes the q amplitude bits.

3.2. Error Performance

The ideal receiver for $\pi/4$ -APSK can obtain complete information about the phase and amplitude of the received signal, as described by the matched filter receiver of Section 2.2. Its error performance depends only on the relative interference energy caused by channel noise, and may be determined analytically from the signal constellation. The receiver required for this project is to make use of conventional

$\pi/4$ -QPSK receiver circuitry in its implementation, as described in Section 3.2.3. The theoretical error performance of this "practical" receiver may also be obtained using the signal constellation, but its constellation is slightly different than for the ideal case.

This section discusses the error performance for both situations. Appropriate decision regions are defined, and by integrating the joint probability density function (pdf) of the noise over the decision region boundaries, the error probability for $\pi/4$ -APSK signaling is determined.

3.2.1. Ideal Receiver Decision Regions

Given an independent, random data source and a memoryless non-mobile channel subject to Additive White Gaussian Noise (AWGN), the probability of occurrence of any received symbol in the data stream can be assumed to be independent and identically distributed. The matched filter receiver shown in Figure 7 will choose a symbol using a decision rule that maximizes the *a posteriori* probability of the symbol having been transmitted [33]. This probability will of course depend on the phase and energy of the received signal during the symbol period. Therefore, since each symbol is transmitted with equal probability, the optimal decision regions are everywhere equidistant from the two closest signal points. The optimal decision regions for four-level $\pi/4$ -APSK signaling are shown using dotted lines in Figure 11. In this figure, t_{i-1} and t_i indicate the lower and upper receiver decision thresholds for the amplitude of signal point s_j , respectively.

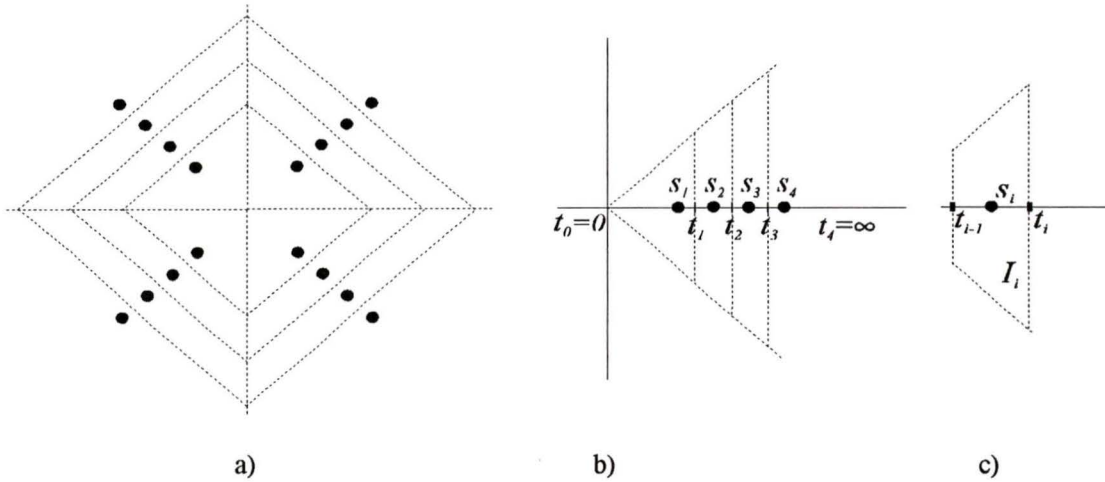


Figure 11. Optimal decision region boundaries for 4-level $\pi/4$ -APSK signaling, showing a) all decision regions, b) rotational equivalence, and c) simplified decision region.

Clearly, the probability of correct reception of a symbol transmitted using $\pi/4$ -APSK is the probability of simultaneously choosing the correct signal phase and the correct signal amplitude over the corresponding decision region. The complete symmetry of the decision regions for this signal constellation allows us to treat each quadrant identically, using the 0-radian reference axis as depicted in Figure 11b) for simplicity. The i th decision region for the signal point s_i may therefore be depicted as in Figure 11c), which shows the signal point and the corresponding decision region I_i . We can see that the signal point $s_i = m_i + n_i$ falls within the decision region I_i whenever:

$$(t_{i-1} < |m_i + n_i| < t_i) \text{ and } (-\pi/4 < \varphi_i < \pi/4), \quad (8)$$

where φ_i is the phase of the message signal m_i plus the noise vector n_i .

3.2.2. Probability of Error Expression for an Ideal Receiver

A matched filter receiver will correctly interpret a received signal point if the condition in Equation (8) is satisfied. For AWGN, the in-phase and quadrature components of

the noise are identical and statistically independent. For AWGN, the probability distribution functions of the in-phase and quadrature components of the noise are identical and statistically independent, have variances σ_x^2 , σ_y^2 and means η_x , η_y , and are given by [40]:

$$f_n(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-(x-\eta_x)^2/2\sigma_x^2} \quad f_n(y) = \frac{1}{\sigma_y \sqrt{2\pi}} e^{-(y-\eta_y)^2/2\sigma_y^2} \quad (9)$$

The probability of correctly receiving a transmitted message m_i may thus be expressed as the product of the probability that the amplitude of the signal plus noise is within the thresholds, and the probability that the signal phase is within the correct quadrant:

$$P[C|m_i] = P[t_{i-1} < |m_i + n_i| < t_i] P[-\pi/4 < \phi_i < \pi/4] \quad (10)$$

The average probability of correct reception $P[C]$ can be expressed as the average of the probabilities of correct reception given that a specific symbol m_i was transmitted:

$$\begin{aligned} P[C] &= \frac{1}{M} \sum_{i=1}^M P[C|m_i] \\ &= \frac{1}{M} \sum_{i=1}^M P[t_{i-1} < |m_i + n_i| < t_i] P[-\pi/4 < \phi_i < \pi/4] \end{aligned} \quad (11)$$

If the decision region is simplified through translation to the 0-radian reference axis as in Figure 11c), the joint p.d.f. of the in-phase and quadrature components of the noise $f_n(x,y)$ may be integrated to determine the probability of correct reception by a matched filter receiver for a message m_i :

$$\begin{aligned} P[C|m_i] &= \iint_{I_i} f_n(x,y) dy dx \\ &= \int_{x=t_{i-1}}^{x=t_i} \int_{y=-x}^{y=x} f_n(x,y) dy dx \\ &= 2 \int_{x=t_{i-1}}^{x=t_i} \int_{y=0}^{y=x} f_n(x,y) dy dx \end{aligned} \quad (12)$$

The last integral in Equation (12) follows from the symmetry of the decision region around the x -axis. The joint pdf of the noise phase and noise amplitude in rectangular coordinates is given by [40]:

$$f_N(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{(x-\eta_x)^2}{\sigma_x^2} + \frac{(y-\eta_y)^2}{\sigma_y^2}\right)\right]. \quad (13)$$

Since $\eta_y=0$ and $\sigma_x^2=\sigma_y^2=N_0$, we can combine Equations (12) and (13) to form Equation (14):

$$P[C] = \frac{1}{M} \sum_{i=1}^M \left\{ \frac{1}{\pi N_0} \int_{t_{i-1}}^{t_i} \int_0^x e^{-(x-\eta_x)^2/2N_0} e^{-y^2/2N_0} dy dx \right\} \quad (14)$$

At first glance, the exponentials in the integrands of this expression suggest straightforward evaluation using error functions, which are defined by:

$$\text{erf}\left(\frac{a}{\sqrt{2}}\right) = \frac{1}{\sqrt{\pi}} \int_0^a e^{-y^2/2} dy. \quad (15)$$

Unfortunately, a closed form solution for this expression cannot be found [24], because of the upper limit of x on the second integral in Equation (14). The probability may nonetheless be determined by numerical approximation. One straightforward approach that gives a reasonable approximation is to break the decision region I_i into J smaller integrable regions as depicted in Figure 12.

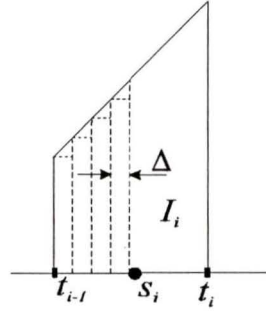


Figure 12. Approximation of decision regions for $\pi/4$ -APSK signaling

The approximation to the decision region shown in Figure 12 allows us to obtain an expression for a lower bound on the average probability of correct reception that can be readily computed. This is the sum of the probabilities of the i th signal point falling within the small area defined by $x=(x_i, x_i+\Delta)$, $y=(0, x_i)$, and is given by the following expression:

$$P[C] \geq \frac{1}{M} \sum_{i=1}^M \left\{ \frac{1}{\sqrt{\pi N_0}} \sum_{j=0}^{J-1} \left(\Delta e^{-(x_j - \eta_i)^2 / 2N_0} \right) \frac{1}{\sqrt{\pi N_0}} \int_0^{x_j} e^{-y^2 / 2N_0} dy \right\}, \quad (16)$$

where $x_j = t_{i-1} + j\Delta$ is the lower bound of the i th decision region plus the j th offset. Using the definition of the error function in Equation (15), the integration in Equation (16) may now be performed, giving the following upper bound on the probability of symbol error for M -ary $\pi/4$ -APSK signaling with an ideal receiver:

$$P_e = 1 - P[C]$$

$$P_e \leq 1 - \frac{1}{M\sqrt{\pi N_0}} \sum_{i=1}^M \left\{ \sum_{j=0}^{J-1} \left(\Delta e^{-(x_j - \eta_i)^2 / 2N_0} \right) \operatorname{erf}\left(\frac{x_j}{\sqrt{2}}\right) \right\}. \quad (17)$$

This expression is useful in determining the optimal locations of the signal points and decision thresholds in a $\pi/4$ -APSK signal constellation, as described in Section 3.3.

3.2.3. Practical Receiver Structure

The ideal receiver described in earlier sections can fully identify the in-phase and quadrature signal components, and use this knowledge to determine the transmitted information. Recall that for $\pi/4$ -APSK signaling, this data includes p bits described by the relative signal phase and q bits described by the signal amplitude. The ideal receiver can determine the value of all $p+q$ bits using one receiver circuit.

One goal for this project, however, is to suggest a straightforward receiver implementation using conventional $\pi/4$ -QPSK technology. This requirement implies that the amplitude decoding portion of the receiver could "piggyback" the $\pi/4$ -QPSK receiver described in Figure 7 above. This receiver structure is illustrated in Figure 13.

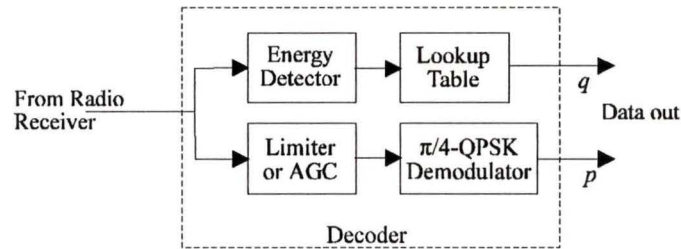


Figure 13. $\pi/4$ -APSK Decoder Structure

The decoder comprises an energy detector and a standard $\pi/4$ -QPSK demodulator; the received signal is simultaneously processed by both of these blocks. The energy level of the received signal may be compared to values stored in a lookup table or some other reference to decode the symbol with the maximum likelihood of occurrence. A conventional $\pi/4$ -QPSK demodulator obtains the phase information. If necessary, the signal fed to this demodulator may first be passed through a hard limiter or AGC circuit, since amplitude variations are not useful to the phase detection process.

3.2.4. Practical Receiver Decision Regions

For the practical receiver, the decision regions are no longer everywhere equidistant from the two closest signal points. Since the received signal is processed for phase and amplitude information independently, the boundaries of the decision regions and the decision thresholds are as shown in Figure 14 (for 4-level $\pi/4$ -APSK).

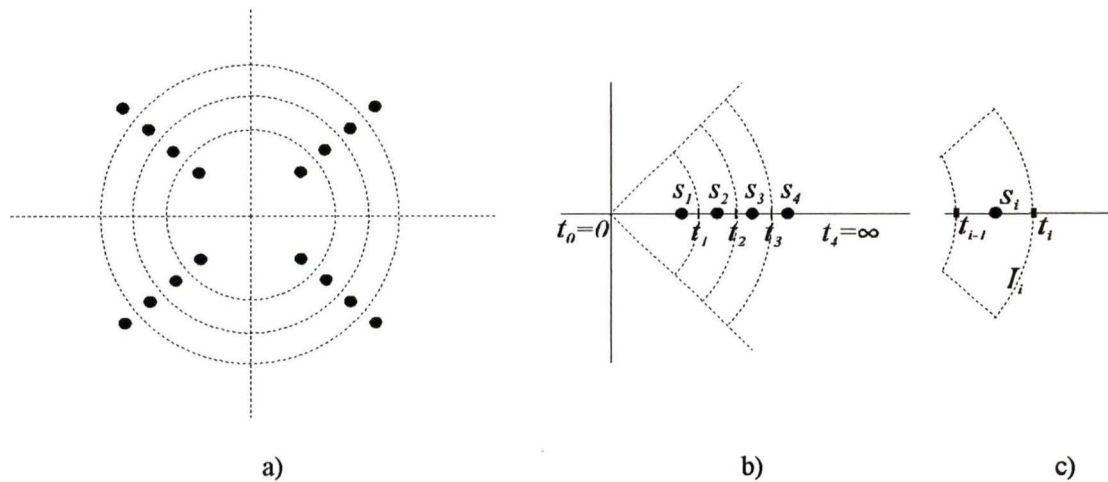


Figure 14. Practical decision region boundaries for 4-level $\pi/4$ -APSK signaling, showing a) all decision regions, b) rotational equivalence, and c) simplified decision region.

3.2.5. Probability of Error Expression for a Practical Receiver

For the perfect practical receiver, the probability of correctly interpreting a received signal point affected by AWGN is still given by Equation (12), with a change of variables to reflect the circular nature of the decision regions:

$$\begin{aligned}
 P[C|m_i] &= \iint_{I_i} f_{n_i}(x, \theta) dx d\theta \\
 &= \int_{x=t_{i-1}}^{x=t_i} \int_{\theta=-\pi/4}^{\theta=\pi/4} f_{n_i}(x, \theta) dx d\theta \\
 &= \frac{1}{\sqrt{2\pi}\sigma_x} \int_{x=t_{i-1}}^{x=t_i} e^{-(x-\eta_x)^2/2\sigma_x^2} dx \frac{1}{\sqrt{2\pi}\sigma_\theta} \int_{y=-\pi/4}^{y=\pi/4} e^{-\theta^2/2\sigma_\theta^2} d\theta
 \end{aligned} \tag{18}$$

The first integral of this expression may be evaluated as the difference of two error functions, as follows:

$$\frac{1}{\sqrt{2\pi}\sigma_x} \int_{x=t_{i-1}}^{x=t_i} e^{-(x-\eta_x)^2/2\sigma_x^2} dx = \text{erf}(t_i - \eta_x) - \text{erf}(t_{i-1} - \eta_x) \quad (19)$$

The second integral in (18) is more difficult to evaluate. A derivation and the following approximation for it is given by Lindsey & Simon [33].

$$\frac{1}{\sqrt{2\pi}\sigma_\theta} \int_{y=-\pi/4}^{y=\pi/4} e^{-\theta^2/2\sigma_\theta^2} d\theta \cong \text{erf}\left(\sqrt{\frac{2E_i}{N_0}} \sin \frac{\pi}{M}\right) \quad (20)$$

The resulting approximation to the probability of symbol error for a practical $\pi/4$ -APSK receiver is:

$$\begin{aligned} P_e &= 1 - \frac{1}{M} \sum_i P[C|m_i] \\ &\cong 1 - \frac{1}{M} \sum_i \left[\{\text{erf}(t_i - \eta_x) - \text{erf}(t_{i-1} - \eta_x)\} \text{erf}\left(\sqrt{\frac{2E_i}{N_0}} \sin \frac{\pi}{M}\right) \right] \end{aligned} \quad (21)$$

3.3. Optimal Signal Constellation

The location of signal points in the signal constellation is critical to effective modem operation using $\pi/4$ -APSK signaling, since their positioning defines the decision regions, and the previous section has shown that the error probabilities are a direct function of these. This section describes the requirements placed on the positioning of these signal points within the constellation, namely constant energy and minimum Euclidean distance between points. It also gives some expressions for evaluating the signal scaling coefficients, suggests values that result in optimal constellations (giving the minimum error rate), and provides suitable expressions and values for the

corresponding decision thresholds. The calculations assume an ideal receiver and corresponding decision regions, but have also been shown to be valid at large signal-to-noise ratios (SNR>5dB) for the practical receiver structure and decision regions.

3.3.1. Signal Point Spacing Considerations

There are two basic conditions that restrict the location of the signal points in an $\pi/4$ -APSK constellation. First, to allow communication equipment built for $\pi/4$ -QPSK signaling to be used with $\pi/4$ -APSK signaling, the average energy content for a random data signal must remain constant with respect to $\pi/4$ -QPSK signaling. That is:

Condition 1:

$$\frac{1}{k} \sum_{j=1}^k E_j = \frac{1}{k} \sum_{j=1}^k \gamma_j E_s = E_s \quad (22)$$

where E_j is the signal energy of the j th signal point, γ_j is the scaling coefficient — a multiplicative constant — and E_s is the symbol energy of a $\pi/4$ -QPSK signal point.

The second requirement that we may make is that the Euclidean distance between two adjacent signal points in any one quadrant must always be less than or equal to the Euclidean distance to a signal point in any other quadrant. This requirement reduces the possibility of a noise vector causing the matched receiver to select the wrong quadrant when decoding the lowest-energy signal point in the quadrant. Thus:

Condition 2:

$$E_{j,\phi} - E_{j-1,\phi} \leq E_{j-1,\phi} - E_{j-1,\phi'} \quad (23)$$

This second condition allows us to place a lower limit on the minimum Euclidean distance that a lowest-energy signal point, s_j , may be from the origin of the

constellation (i.e., the minimum energy level of a $\pi/4$ -APSK signal), and the distance between s_1 and the closest point in the same quadrant, s_2 . Figure 15 illustrates this situation for a 2-level $\pi/4$ -APSK signal constellation: the Euclidean distance a from the lowest-energy signal point, $s_{1,1}$, to the next higher point in the same quadrant, $s_{1,2}$, must never be greater than b , the Euclidean distance to either lowest-energy point in the other two quadrants (i.e., $s_{2,1}$ or $s_{4,1}$).

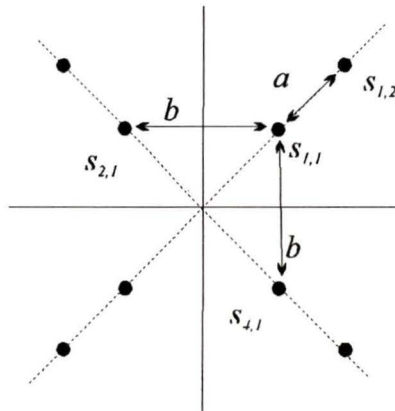


Figure 15. Lower limit on lowest-energy signal point in $\pi/4$ -APSK constellation.

For more than two levels of amplitude scaling ($q > 1$) there is no need to place a restriction similar to Condition 2 on higher energy signal points, since these points will always be closer to a point in the same quadrant than a point in another quadrant. As a result, only the first condition restricts the location of these higher energy signal points, which may be chosen arbitrarily.

Using the two conditions specified above and some algebraic manipulation, we may write an expression describing the signal point locations. First we recall that the energy

E_j of an amplitude-scaled signal point and the Euclidean distance of the signal point from the origin d_j are related by:

$$E_j = (d_j)^2. \quad (24)$$

Letting $(d_{j-1,j})$ be the Euclidean distance between two adjacent signal points s_{j-1} and s_j in the same quadrant, we can see that Condition 2 above allows us to express the Euclidean distances from the origin to the first signal point s_1 , from the first point to the second point s_2 , and so on, as follows:

$$\begin{aligned} d_1 &= \frac{d_{1,2}}{\sqrt{2}} \\ d_2 &= \frac{d_{1,2}}{\sqrt{2}} + d_{1,2} \\ d_3 &= \frac{d_{1,2}}{\sqrt{2}} + d_{1,2} + d_{2,3} \\ &\vdots \end{aligned} \quad (25)$$

Recalling from Equation (22) that $\sum_{j=1}^k \gamma_j = k$, we get a system of solvable equations in $d_{j-1,j}$:

$$\frac{1}{k} \sum_{j=1}^k \gamma_j = \left(\frac{d_{1,2}}{\sqrt{2}}\right)^2 + \left(\frac{d_{1,2}}{\sqrt{2}} + d_{1,2}\right)^2 + \left(\frac{d_{1,2}}{\sqrt{2}} + d_{1,2} + d_{2,3}\right)^2 + \dots + \left(\frac{d_{1,2}}{\sqrt{2}} + d_{1,2} + d_{2,3} + \dots + d_{k-1,k}\right)^2 \quad (26)$$

where $d_{1,2}$ is determined from Condition 2, and:

$$\begin{aligned} d_{2,3} &= \beta_1 d_{1,2} \\ d_{3,4} &= \beta_2 d_{1,2} \\ &\vdots \\ d_{j-1,j} &= \beta_{j-2} d_{1,2}. \end{aligned}$$

The multipliers β_i may be assigned arbitrarily, as long as the equal energy requirement of Condition 1 is satisfied. This system of equations can be used to determine the scaling coefficients γ_i for any 2^q levels in the $\pi/4$ -APSK constellation. Note that for 2-

level $\pi/4$ -APSK, only the first two terms in Equation (26) are required; for 4-level $\pi/4$ -APSK, four terms are required, and so on.

3.3.2. Optimal Signal Scaling Coefficients

Using the expression for the error performance of $\pi/4$ -APSK from Section 3.2 and the expression for the signal point spacing given above, we can determine the optimum spacing between signal points in terms of error performance. The values of the decision thresholds for the receiver can then be determined.

The first step in determining the optimal signal spacing is to choose values for the scaling coefficients γ_j . Each of the 2^q signal points in a quadrant is located at a distance $\sqrt{\gamma_j E_s}$ from the origin, where E_s is again the symbol energy of a $\pi/4$ -QPSK signal. We may choose any values for γ_j , as long as the two conditions given in Section 3.3.1 are satisfied. Table 2 lists three such sets of values; the first row of values are determined using the equality in Equation (23), the other values satisfy only the inequality. The values of γ_1 and γ_2 in row ① are referred to as the *minimum energy scaling coefficients*. This name reflects the fact that these values give the minimum permissible energy for the lowest-energy signal point (i.e., the energy level for equality in Condition 2). The remaining two rows give scaling coefficients that result in signal points with less space between them, yet still satisfy both conditions. The spacing coefficients may take on any values, as long as the two conditions specified by Equations (22) and (23) are satisfied. The values in rows ② and ③ have been selected to satisfy these conditions, but have no special significance.

	$d_{1,2}$	γ_1	γ_2
①	1.082	0.292	1.708
②	1.183	0.350	1.650
③	1.414	0.500	1.500

Table 2. Some scaling coefficients for 2-level $\pi/4$ -APSK.

In this table, $d_{1,2}$ is the Euclidean distance between s_1 and s_2 . As $d_{1,2}$ increases, the scaling coefficients γ_1 must come closer together to maintain a constant average energy level. Using these scaling coefficients and the expression for the probability of error in Equation (17), the error curves shown in Figure 16 may be obtained.

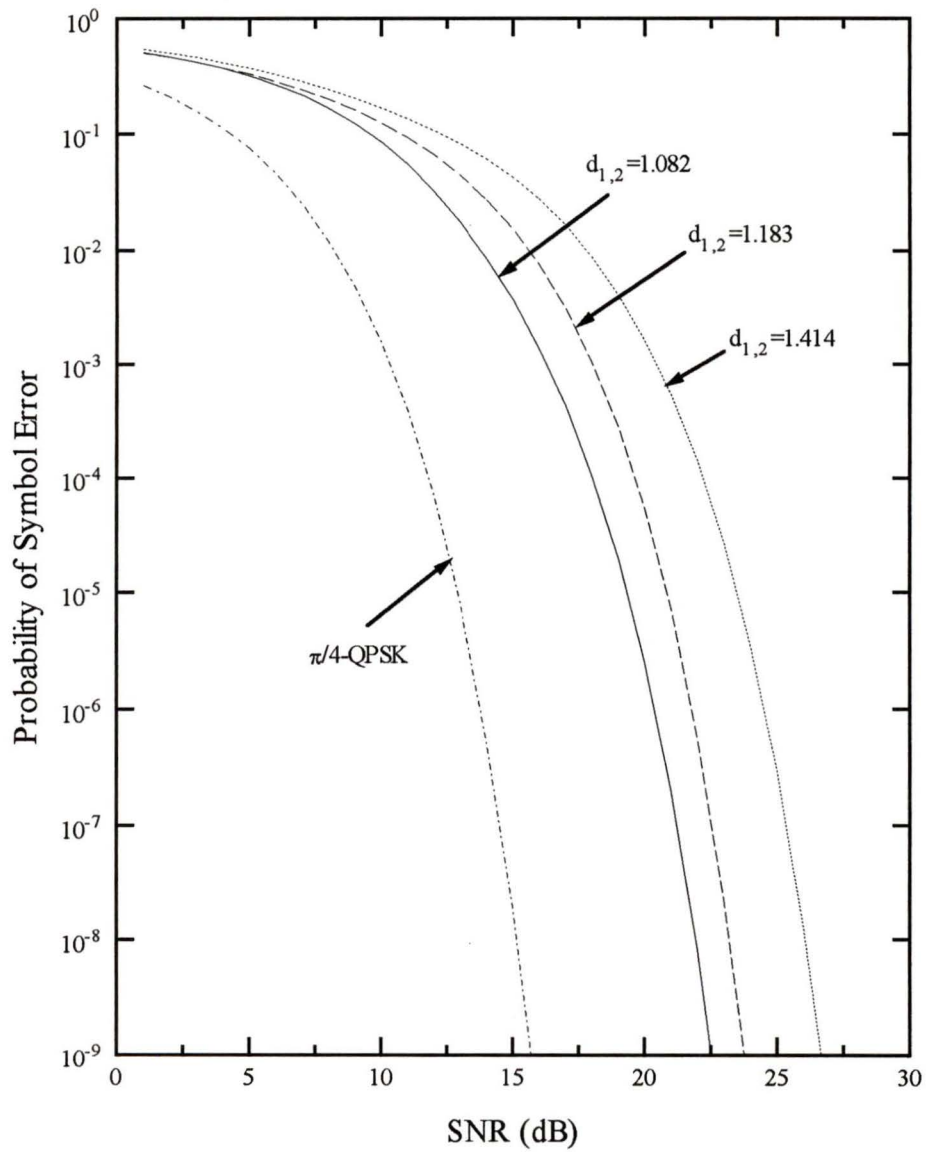


Figure 16. Probability of symbol error for 2-level $\pi/4$ -APSK signaling.

Figure 16 shows that the best error performance is obtained using the minimum energy scaling coefficients (Case ①). Reducing the spacing between adjacent signal points (Cases ② and ③) increases the probability of symbol error, since the receiver is more likely to make an error in decoding the signal amplitude if the signal points are closer together. The minimum energy scaling coefficients specify the optimal signal constellation for 2-level $\pi/4$ -APSK signaling.

The scaling coefficients for 4-level $\pi/4$ -APSK can be determined in a similar manner. Some coefficients are given in Table 3:

	$d_{1,2}$	γ_1	γ_2	γ_3	γ_4	β_1	β_2
①	0.571	0.082	0.475	1.197	2.243	1	1
②	0.451	0.051	0.296	0.989	2.664	$\sqrt{2}$	$2\sqrt{2}$
③	0.671	0.113	0.658	1.314	1.915	$\sqrt{2}/2$	$\sqrt{2}/4$
④	0.707	0.250	0.750	1.250	1.750	1.677	1.4
⑤	0.894	0.400	0.800	1.200	1.600	1.5	1.142

Table 3. Some scaling coefficients for 4-level $\pi/4$ -APSK.

In this table, the first three rows contain minimum energy scaling coefficients. This is possible because the multiplier for the third and fourth signal points (β_1 and β_2 in Equation 26) may be chosen arbitrarily, subject to the restriction of Condition 1. For Case ①, these two multipliers are equal, giving a signal constellation with points that are symmetrically and equally spaced. In Case ② the point spacing increases for the

higher energy signal points, and in Case ③ it decreases for the higher energy signal points. These variations in the constellation are illustrated in Figure 17, with a reference signal amplitude level indicated by the dotted circle.

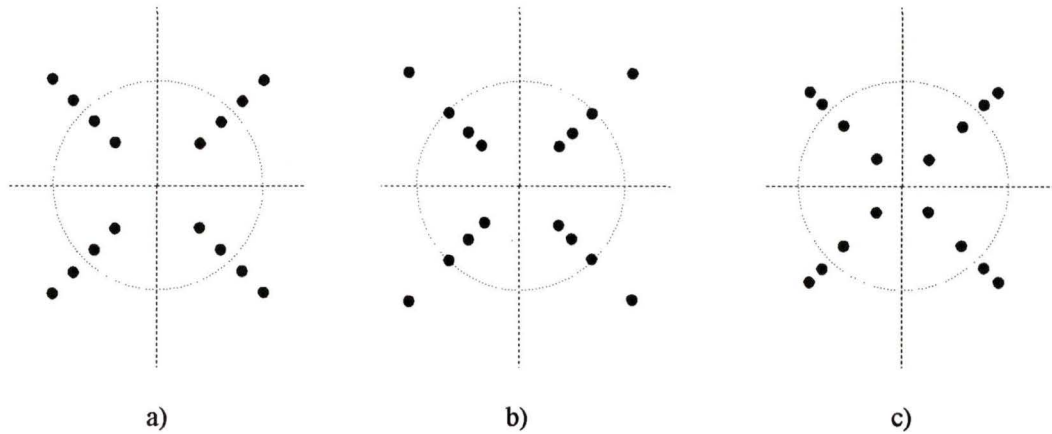


Figure 17. Varying the spacing of signal points for a 4-level $\pi/4$ -APSK constellation: a) equal spacing, b) increasing spacing, and c) decreasing spacing.

The arbitrarily chosen scaling coefficients in the final two rows of Table 3 (Cases ④ and ⑤) are not minimum energy scaling coefficients — they satisfy only the inequality of Equation (23) and were chosen for convenience. The error curves obtained using the values in Table 3 are shown in Figure 18.

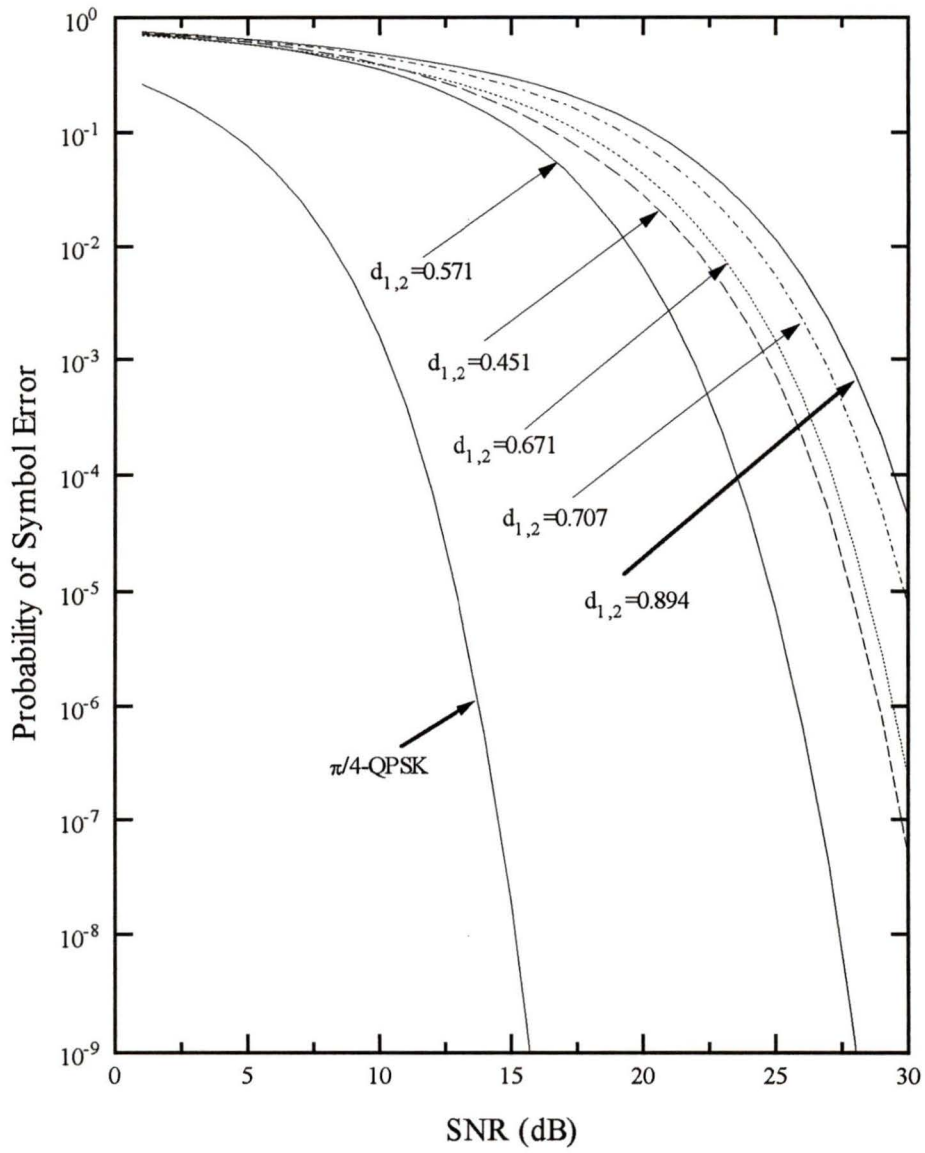


Figure 18. Probability of symbol error for 4-level $\pi/4$ -APSK signaling.

Figure 18 again shows that reducing the spacing between adjacent signal points increases the probability of symbol error (Cases ④ and ⑤) — the error performance deteriorates if the minimum energy scaling coefficients are not used. The error rate is lowest using the equally-spaced scaling coefficients (Case ①), and increases for both Cases ② and ③ in which the point spacing varies for the higher energy signal points. The equally-spaced minimum energy scaling coefficients specify the optimal signal constellation for 4-level $\pi/4$ -APSK signaling. Simulations have also shown that for SNR levels over 5 dB, the theoretical performance is nearly equivalent using either ideal or practical receiver decision boundaries.

In theory, any 2^q signal points could be specified for each quadrant, gaining a higher data rate at the cost of degraded error performance. However, the anticipated channel conditions (maximum ~ 30 dB SNR) and acceptable error rate (around 10^{-5}) for the application of this system, as described in Section 1.1, place a practical limit on the number of levels that may be supported. Furthermore, receiver implementation becomes increasingly complex for very closely packed signal constellations because of the need for highly accurate equalization to compensate for intersymbol interference [9]. For this project, no more than 4 levels in the $\pi/4$ -APSK signal constellation are considered necessary, since 4 levels can provide as much as twice the throughput over standard $\pi/4$ -QPSK, and ease of receiver implementation is an important consideration for a low cost receiver. For completeness, however, 8-level $\pi/4$ -APSK signaling is briefly considered in the following.

The error performance of 4-level $\pi/4$ -APSK indicates that equally-spaced minimum energy scaling coefficients result in the optimum signal constellation and the best error performance. Similar results are expected for signaling techniques using a greater

number of signal points. Table 4 gives the scaling coefficients for an equally-spaced 8-level $\pi/4$ -APSK constellation:

$d_{1,2}$	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8
0.295	0.022	0.128	0.321	0.602	0.970	1.426	1.970	2.601

Table 4. Scaling coefficients for 8-level $\pi/4$ -APSK.

The error curve for equally-spaced 8-level $\pi/4$ -APSK is shown in Figure 19, which indicates that even 8 levels could provide error rates near the desired limit of 10^{-5} at very high SNR values. The curves for 4-level $\pi/4$ -APSK, 2-level $\pi/4$ -APSK, and unscaled $\pi/4$ -QPSK signaling (using Equation 5) are shown for comparison.

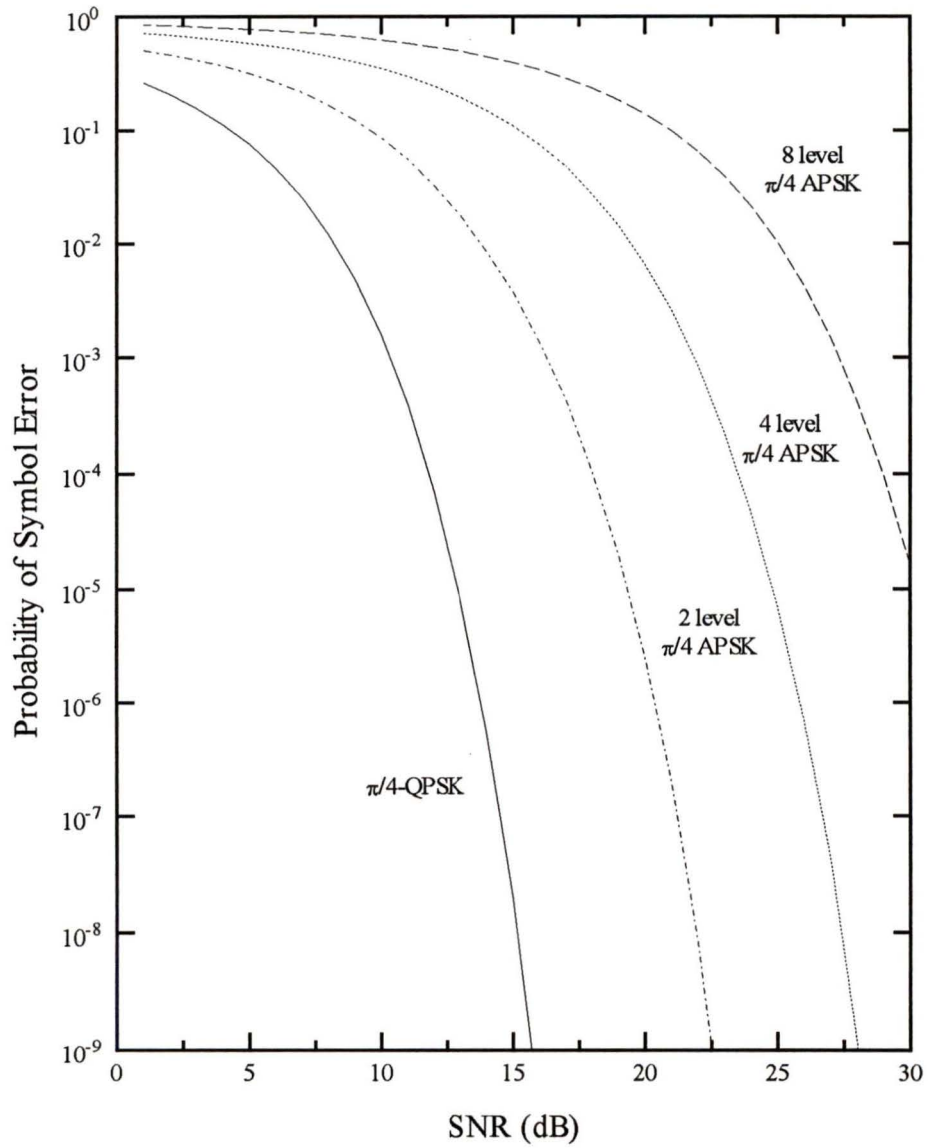


Figure 19. Error performance of optimally spaced $\pi/4$ -APSK signaling.

3.3.3. Optimal Decision Thresholds

The amplitude decision thresholds for multilevel $\pi/4$ -APSK signaling can be obtained from the scaling coefficients determined above. Assuming an equally-spaced signal constellation for optimal performance, the decision thresholds are simply the Euclidean midpoints between any two adjacent signal points. The following table lists the amplitude decision thresholds that may be used by the receiver if the transmitted signal constellation is generated using optimal minimum energy scaling coefficients:

2-level ($q=1$)		4-level ($q=2$)		8-level ($q=3$)	
t_0	0	t_0	0	t_0	0
t_1	1.414	t_1	0.690	t_1	0.258
t_2	∞	t_2	1.261	t_2	0.654
		t_3	1.833	t_3	0.949
		t_4	∞	t_4	1.245
				t_5	1.541
				t_6	1.837
				t_7	2.133
				t_8	∞

Table 5. Decision thresholds for $\pi/4$ -APSK signaling with minimum energy scaling coefficients.

3.3.4. Error Decomposition

The error curves in the previous sections represent the symbol error probability for $\pi/4$ -APSK signaling. Recall that the source data for this system comprises two independent data streams: p bits encoded into the phase and q bits encoded into the

amplitude. The purpose of these data streams is not generally known, and one stream may require a higher or lower bit error probability than the other. Either or both streams could be subject to independent error-correction codes to satisfy more stringent data reliability requirements.

It is informative to decompose the symbol error probability for $\pi/4$ -APSK given above into the bit error rates for the two completely independent data streams. In this section, the bit error rates for the two streams are determined, assuming that the practical receiver described in Section 3.2.3 is used.

First, we recall the relationship between the bit error rate P_B and the symbol error rate P_e for MPSK if Gray coding is used, given by [43]:

$$P_b = \frac{P_e}{\log_2 M} \quad M \geq 2 \quad (27)$$

The expression for probability that a symbol will be correctly received was given in Equation (18) and is repeated here:

$$P[C|m_i] = \frac{1}{\sqrt{2\pi\sigma_x}} \int_{x=t_{i-1}}^{x=t_i} e^{-(x-\eta_x)^2/2\sigma_x^2} dx \frac{1}{\sqrt{2\pi\sigma_\theta}} \int_{y=-\pi/4}^{y=\pi/4} e^{-\theta^2/2\sigma_\theta^2} d\theta. \quad (28)$$

Recall that this expression is the product of the probability distribution functions of the amplitude noise and the phase noise. From this, we can see that the probability P_{ea} of the receiver making an error in decoding the signal amplitude as a result of noise, assuming that the phase is correct, is:

$$\begin{aligned} P_{ea} &= 1 - \frac{1}{\sqrt{2\pi\sigma_x}} \int_{x=t_{i-1}}^{x=t_i} e^{-(x-\eta_x)^2/2\sigma_x^2} dx \\ &= 1 - \left(\operatorname{erf}\left(\frac{t_i}{\sqrt{2}}\right) - \operatorname{erf}\left(\frac{t_{i-1}}{\sqrt{2}}\right) \right) \end{aligned} \quad (29)$$

The probability P_{ep} that the receiver will make an error when determining the phase of the received signal, assuming that the amplitude is correct, is then simply:

$$P_{ep} = P_e - P_{ea} \quad (30)$$

The corresponding expressions for the bit error rates are given by:

$$P_{ba} = \frac{1}{\log_2 M} \left[1 - \left(\operatorname{erf}\left(\frac{t_i}{\sqrt{2}}\right) - \operatorname{erf}\left(\frac{t_{i-1}}{\sqrt{2}}\right) \right) \right] \quad (31)$$

$$P_{bp} = \frac{1}{\log_2 M} [P_e - P_{ea}]$$

The error curves resulting from this decomposition of the symbol error probability for 2-level and 4-level $\pi/4$ -APSK are given in Figures 20 and 21.

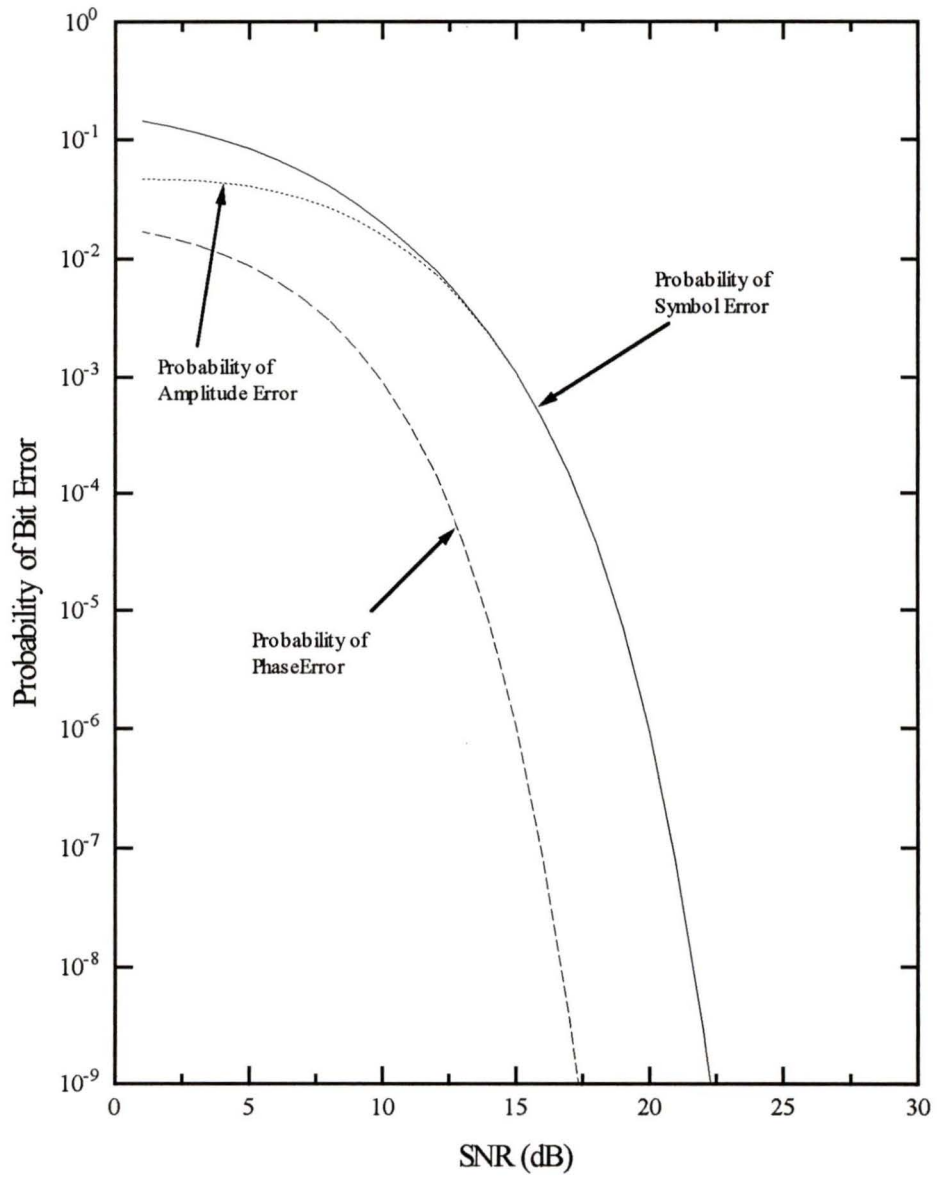


Figure 20. Components of Probability of Bit Error for 2-level $\pi/4$ -APSK .

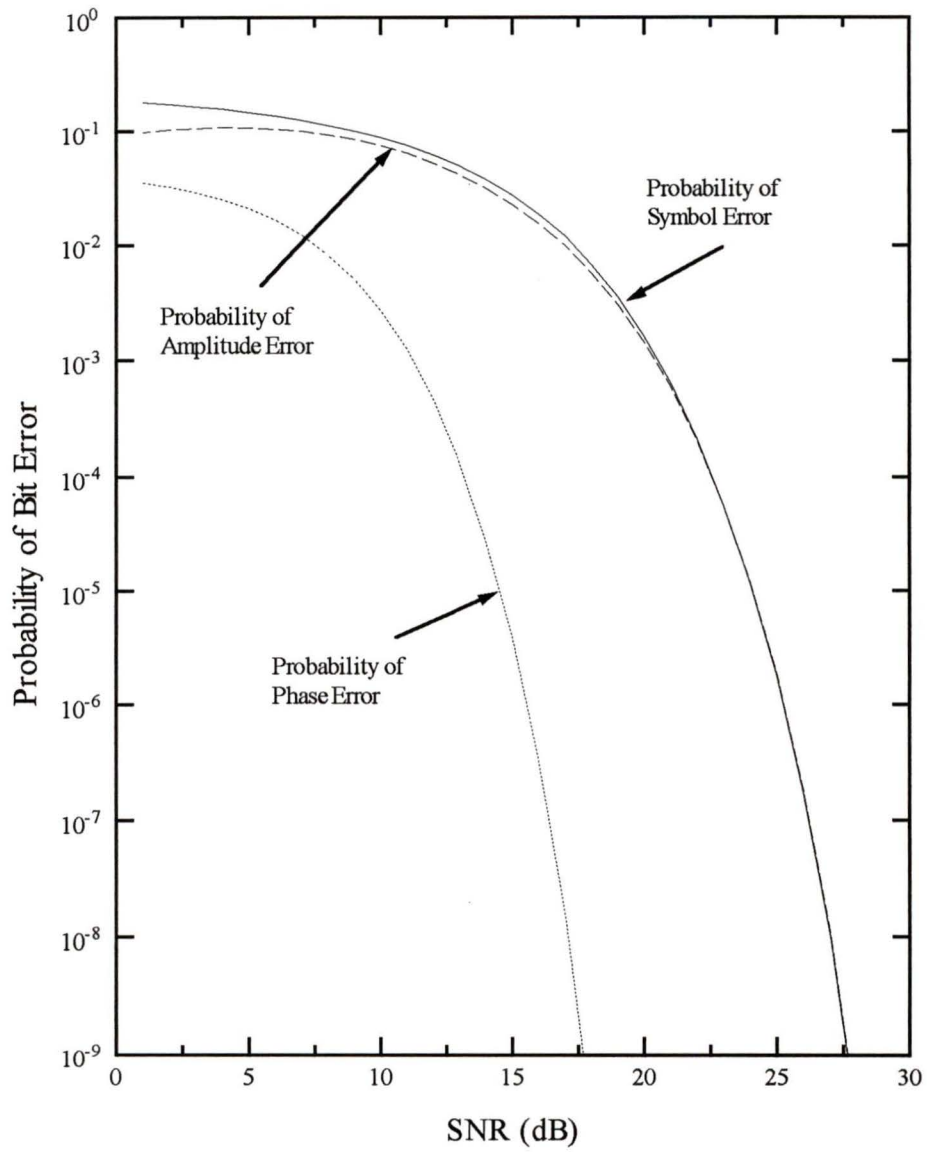


Figure 21. Components of Probability of Bit Error for 4-level $\pi/4$ -APSK.

Clearly, the greater contribution to the symbol error probability for 2-level or 4-level $\pi/4$ -APSK is caused by the demodulating receiver selecting the incorrect amplitude for any given phase. This is intuitively satisfying, since a look at the signal constellation shows that, on average, a smaller noise vector is required to distort the transmitted signal to a point with a different amplitude than to a point with a different phase at the same power level. In fact, this situation is ensured by placing the restriction on the placement of the signal points described by Condition 2 in Section 3.3.1.

Since the bit error rate for the amplitude-encoded bits is greater than that for the phase-encoded bits, the system can be designed to either take advantage of the greater reliability of one stream, or compensate for the poorer performance of the other. For example, speech data may tolerate a higher bit error rate than control information, so the speech data could be transferred using the q amplitude-coded bits, and the control information on the p phase-coded bits. An appropriate choice of error correcting code can also be used to bring the bit stream error rate of either channel to acceptable levels.

3.4. System Performance

Having determined the optimal signal spacing and decision thresholds for $\pi/4$ -APSK signaling, this section briefly considers some of the system enhancements that may be expected through its implementation. The data transfer rate is the first, most obvious aspect that may be improved by an increase in the size of the symbol alphabet, but the ability for this modulation scheme to support encoding to provide error correction or encryption capabilities is also noteworthy. These points are discussed in this section.

3.4.1. Data Rate Enhancement

Results in the previous sections clearly indicate that the data transfer rate over a non-mobile radio channel may be enhanced using $\pi/4$ -APSK signaling, depending on the acceptable error rate. At an SNR of 25 dB, for example, the error performance of 4-level $\pi/4$ -APSK is still better than 10^{-6} , which is an acceptable rate for this project, and indeed for many other applications. Under these circumstances, the effective data rate can be double that obtained using conventional $\pi/4$ -QPSK.

Even a channel with an SNR of 20 dB can support communication with an error rate better than 10^{-5} if 2-level $\pi/4$ -APSK is employed. This illustrates the *variable rate* nature of this signaling technique. The expected channel characteristics may be determined before communication commences, and then two, four or even more levels of amplitude scaling may be used for data transfer.

For the data telemetry project, the data rate enhancement can be used in a number of ways, as has been mentioned in the introductory chapter. For example, the data transfer rate on the repeater backbone can be increased using 4-level $\pi/4$ -APSK (if the channel is good enough) to support up to twice the number of RTUs on the network at the same system load. Or, the communication protocol can be changed to take advantage of the side-channel capability offered by this modulation technique.

An interesting extension of the capabilities of this new modulation technique is its potential use in an *adaptive rate transceiver* in digital cellular applications. If a mobile radio is temporarily not moving (a parked or stopped car, for example), the radio channel may under certain circumstances provide high enough signal-to-interference ratios to allow dynamic variation of the data transfer rate.

3.4.2. Coding

The differences in error probability for the phase-encoded bit stream and the amplitude-encoded bit stream was seen in Section 3.3.4. With some added transmitter and receiver complexity, the less reliable bits may be encoded for greater error protection at the cost of some throughput. This could be accomplished using any error control code, such as convolutional codes or Reed-Solomon codes, and could be accomplished in software.

Alternatively, the augmented signal constellation can be used to support trellis-coded modulation (TCM) as described by Ungerboeck [50]. Chevillat [9] indicates that using a rate- $r/(r+1)$ -trellis encoder with a 4-level $\pi/4$ -APSK signal, for example, could provide a 3-dB coding gain over an uncoded signal. This would, however, improve the average bit error rate, and not necessarily the bit error rate of the amplitude-encoded or phase-encoded bit streams independently.

3.5. Chapter Summary

This chapter has provided a thorough discussion of $\pi/4$ -APSK signaling. The signal, its constellation and the symbol mapping were defined. The receiver design was considered, and the appropriate decision regions described. Expressions for the error performance have been derived, and these were used to determine the optimal signal point spacing and decision thresholds, for both the ideal receiver and a practical implementation. Error curves were presented to show the theoretical system performance over an ideal AWGN channel. System enhancements under certain channel conditions were shown to be attainable using this method of modulation.

Chapter 4: System Simulation

This chapter proposes a system implementation based on the findings of the previous chapters. Results of computer simulation to confirm the signal constellation and the error rates are also given.

4.1. System Implementation

The modulation technique described in Chapter 3 may be implemented in a system that uses the encoder and decoder blocks introduced previously, as described by the block diagram shown below:

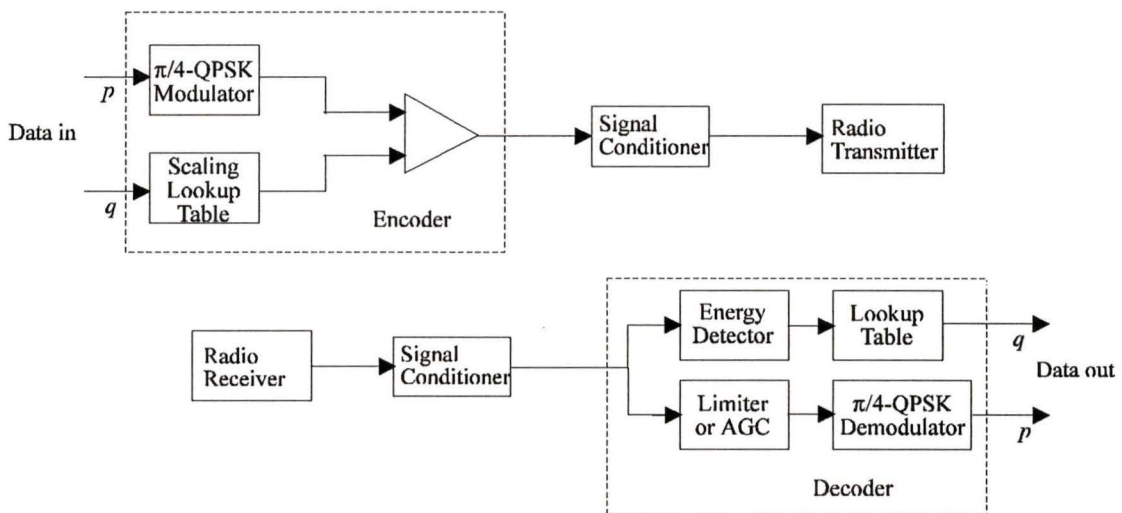


Figure 22. $\pi/4$ -APSK modem block diagram.

In this implementation, the encoder comprises a standard $\pi/4$ -QPSK modulator and an amplitude scaler, as described in Section 3.1.2. The encoded signal may be filtered by the radio transmitter to ensure bandwidth integrity using a Nyquist filter, or any of the

baseband filtering techniques mentioned in Chapter 2, as long as the receive filters return the signal to near its transmitted state. The receiver may also require channel equalization and synchronization mechanisms, but none of these requirements would be in addition to the mechanisms necessary for conventional $\pi/4$ -QPSK signaling. The decoder comprises an energy detector and a standard $\pi/4$ -QPSK demodulator as described in Section 3.2.3.

Prototype development is the traditional approach to testing and verifying the above implementation, but computer simulation can also provide proof of concept and a good indication of the expected performance of a practical system. Results of such a simulation are given in the next section.

4.2. Simulation Results

Computer simulation offers a convenient method of investigating various aspects of $\pi/4$ -APSK signaling, and of estimating its performance using representative signals. The encoder described in Figure 22 above was modeled in software, and the resultant signals subjected to computer-generated Gaussian noise to verify the amplitude-scaling nature of the $\pi/4$ -APSK signal.

The constellation shown in Figure 23 for a 4-level signal with an SNR of 25 dB illustrates the nature of the ideal channel signal. The figure assumes no transmit filtering, nor baseband pulse shaping. The figure confirms the additive nature of the Gaussian noise with the optimally spaced 4-level signal constellation, and permits easy visualization of the ideal decision regions.

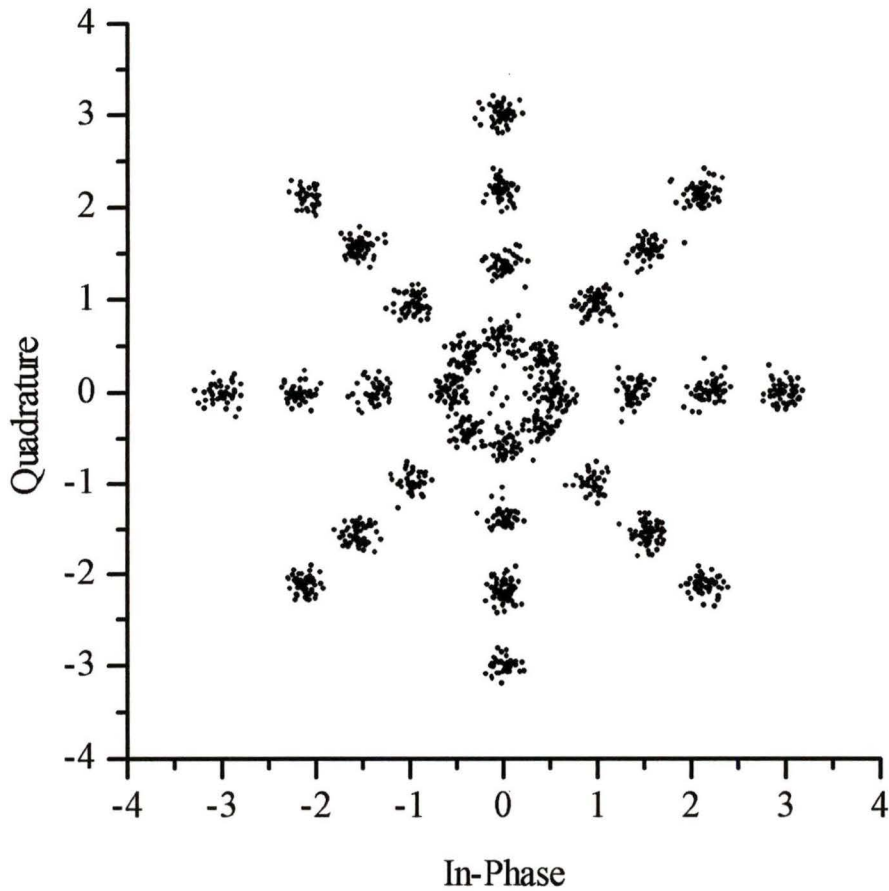


Figure 23. Simulated signal constellation for 4-level $\pi/4$ -APSK (SNR=25 dB).

Using the simulation model and assuming a matched filter receiver, the error performance of 2-level and 4-level $\pi/4$ -APSK signaling was determined. The results are given in Figures 24 and 25 with the theoretical performance determined in Chapter 3 included for comparison. The simulation confirms that actual $\pi/4$ -APSK signals may be expected to be received with approximately the same error probability

as theory suggests. Any difference between theoretical and simulated curves may be attributed to the approximation used to obtain the theoretical curve, which gives a worst-case symbol error probability. At high values of SNR, this bound is decreasingly tight.

4.3. Chapter Summary

This chapter has provided a suggested implementation for the $\pi/4$ -APSK modem. A conventional $\pi/4$ -QPSK modulator and demodulator with associated hardware may be used to process the phase bits. A simple amplifying circuit in the encoder and a signal energy detector in the decoder provides the necessary capability to process the amplitude bits. Computer simulation indicates that the signal constellation and error performance of the proposed system approach the expected values. Assuming appropriate channel filtering, the spectral characteristics of the $\pi/4$ -APSK signal may be such that $\pi/4$ -QPSK hardware can be used for this modem.

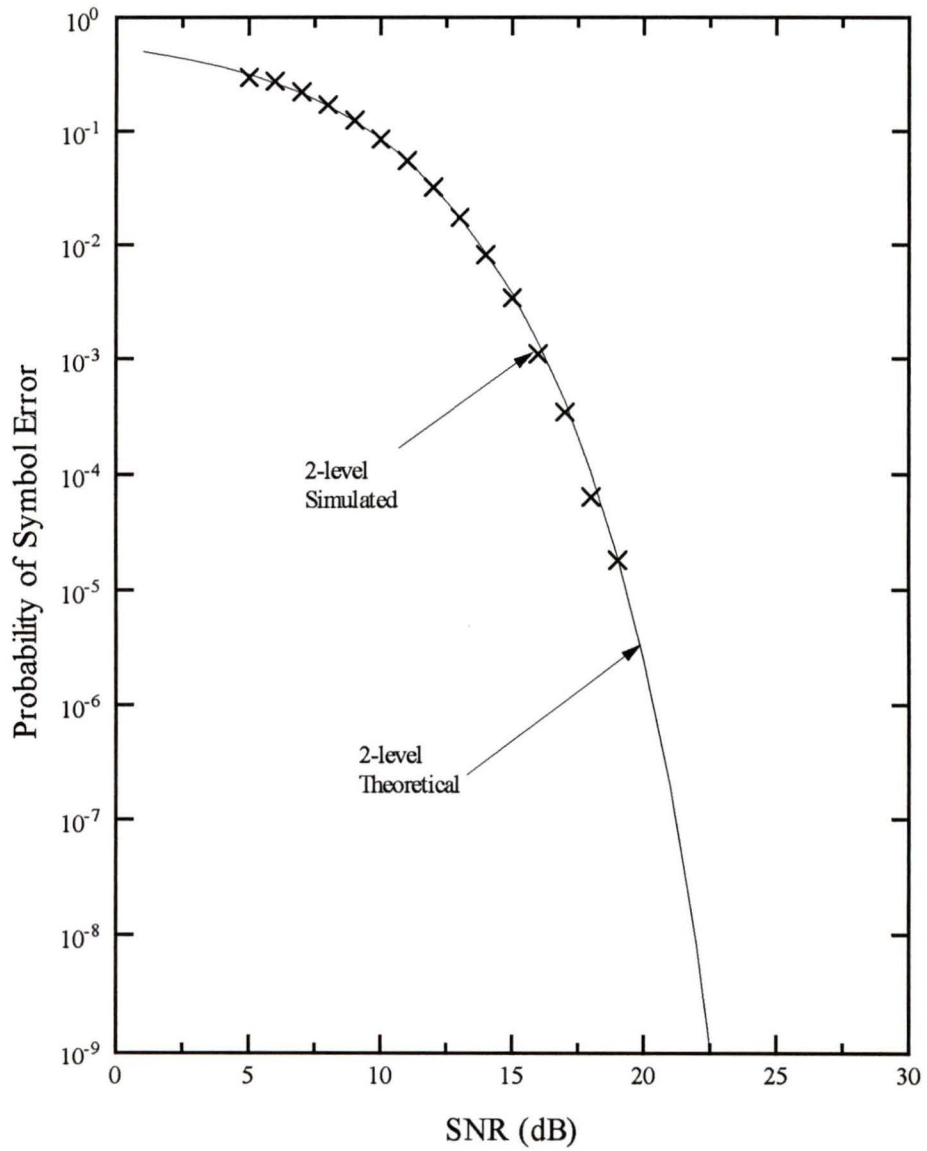


Figure 24. Probability of symbol error for simulated 2-level $\pi/4$ -APSK signaling

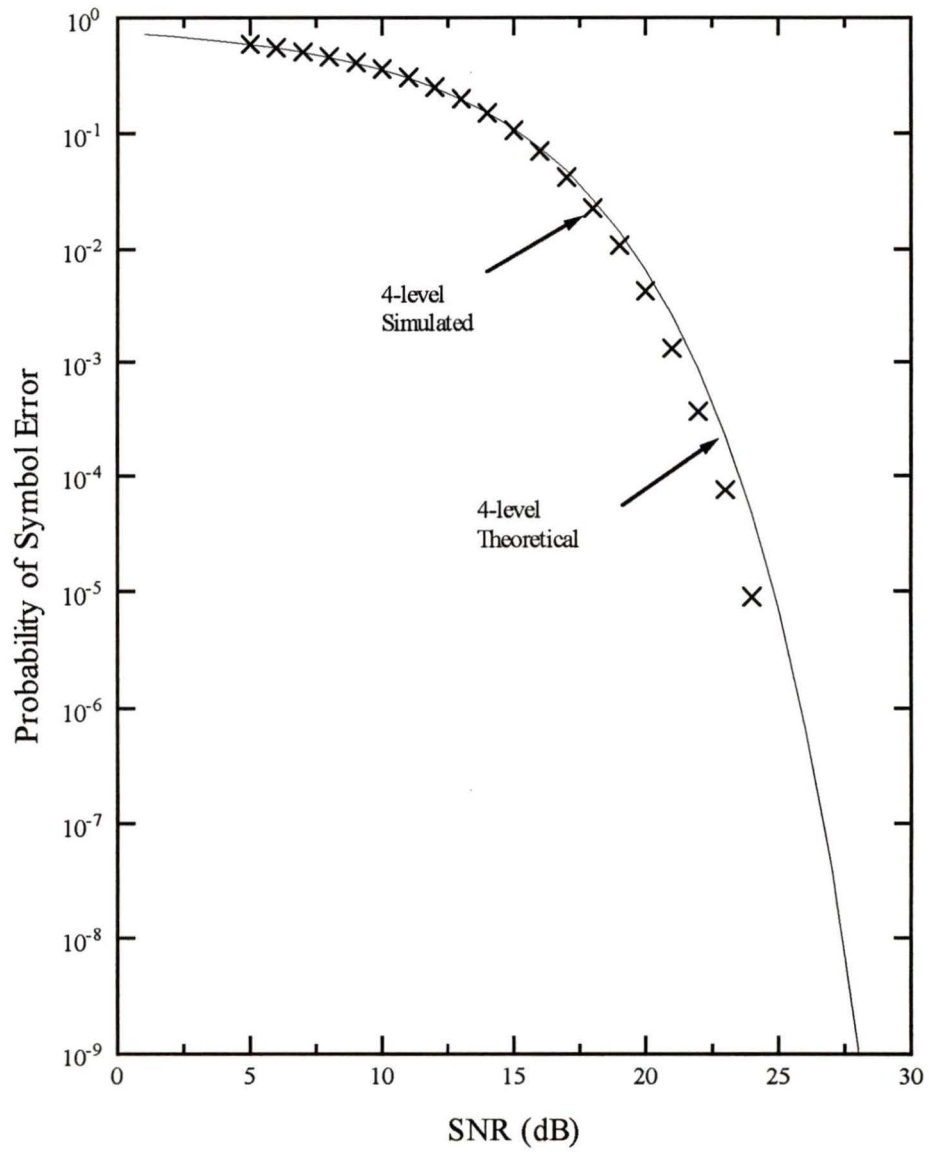


Figure 25. Probability of symbol error for simulated 4-level $\pi/4$ -APSK signaling.

Chapter 5: Conclusions

5.1. Research Completed

A new modulation technique called $\pi/4$ -APSK, based on $\pi/4$ -QPSK signaling with amplitude scaling, has been presented. This technique offers an increased data transfer rate over a non-mobile radio channel, and is largely compatible with conventional $\pi/4$ -QPSK systems.

Following a review of theory and available literature, the signal constellation and symbol mapping for $\pi/4$ -APSK were described. The decision regions for an ideal matched filter receiver and a more practical matched filter receiver were presented. Based on these decision regions, expressions for the error performance were derived and used to determine the optimal signal point spacing.

The signal spacing coefficients required to give an optimal signal constellation (in terms of the error performance) under certain conditions were derived. Error curves were presented to show the theoretical system performance in an AWGN channel. Values for the receiver decision thresholds were given for multilevel $\pi/4$ -APSK, and the error performance of the independent bit streams was illustrated.

System enhancements in terms of data transfer rate and overall system flexibility were seen to be attainable using this method of modulation. The use of $\pi/4$ -APSK signaling was shown to support variable data rates communication as well as a number of other communication strategies that would be unavailable using conventional $\pi/4$ -QPSK modulation.

5.2. Future Research

The research described in this thesis has emphasized the theoretical capabilities of $\pi/4$ -APSK signaling under ideal channel conditions. The multifaceted area of baseband pulse shaping and channel filtering necessary to combat intersymbol interference experienced in a real channel and to provide a mechanism for the practical implementation of the "practical receiver" presented in Chapter 3 has not been investigated. Research completed thus far indicates that with an appropriate choice of transmit and receive filters, the modulation technique would be useful.

As indicated in Chapter 2, extensive literature is available [2, 10, 18, 29, 30, 44, 46] on various aspects of baseband pulse shaping techniques and other methods to combat intersymbol interference for $\pi/4$ -APSK signaling. A thorough investigation of the effect of these techniques on the spectral characteristics of $\pi/4$ -APSK under various channel filtering conditions is required to obtain a clearer appreciation of its compatibility with conventional $\pi/4$ -QPSK hardware.

Also related to the practical reception of $\pi/4$ -APSK signals is a consideration of symbol timing recovery using differentially encoded data streams, and published results of this field may be useful, including [16, 21, 26, 38]. Decoding reliability may be improved using a decoding window that spans multiple symbols, as described by Wilson [54], or using Viterbi or block decoding [28, 32, 42]. A further area of study is digital implementation of the modem, using the techniques and structures described by Fines [17], Chevillat [9] or Chalmers [8], for example. Implementation using a coherent receiver may offer a simpler mechanism for obtaining signal energy information, making a separate energy detector circuit unnecessary. The trade-offs

between a differential detection scheme and a coherent detection scheme should be investigated

As mentioned in Chapter 3, the enhanced symbol alphabet permits the use of error correcting codes; their implementation and anticipated coding gains assuming various values for the minimum free distance is a topic of some interest. For example, Makrakis [35] has employed trellis coded modulation with noncoherent QAM, and a similar technique may be useful with $\pi/4$ -APSK.

The promise of a variable rate data transmission capability using $\pi/4$ -APSK also points toward an investigation of its application to mobile cellular radio. The North American digital cellular system uses a TDMA protocol, wherein 260 bits of user information can be transmitted in each slot of 324 bits — the remainder are control bits. Using a variable-rate $\pi/4$ -APSK modulation technique to transmit the user information bits may lead to a significant increase in user data throughput under appropriate channel conditions (the rate could be doubled to 520 bits per slot using 4-level $\pi/4$ -APSK, for example). The technique may be applied directly for non-moving mobile stations, but some investigation into the effect of fading is necessary if it is to be applied to a fully mobile channel. Some recent research by Webb [52] has shown differentially encoded amplitude-shifted QAM can be applied to mobile channels; Castle [7] has also introduced a modem that uses multilevel differential signaling. Differential amplitude encoding shows promise for $\pi/4$ -APSK signaling.

For the data telemetry system described in Chapter 1, or any non-mobile or slow moving communication channel experiencing only slow fading, the new modulation technique described in this thesis offers a significant increase in communication throughput.

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Appendix

Abbreviations used in this Thesis

AGC	Automatic Gain Control
AWGN	Additive White Gaussian Noise
AMPS	Advanced Mobile Phone System
APSK	Amplitude Phase Shift Keying
CPFSK	Continuous Phase FSK
CPM	Continuous Phase Modulation
DPSK	Differential PSK
FFSK	Fast FSK
FSK	Frequency Shift Keying
GMSK	Gaussian MSK
IAPSK	Independent APSK
MPSK	M-ary PSK or Multi-Phase Shift Keying
MSK	Minimum Shift Keying
OQPSK	Offset QPSK
PLL	Phase Lock Loop
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QASK	Quadrature Amplitude Shift Keying (also QAM)
QPSK	Quadrature Phase Shift Keying
TCM	Trellis-Coded Modulation
TDMA	Time Division Multiple Access

Vita

Surname: Schnorr von Carolsfeld **Given Names:** Veit Rudolf
Place of Birth: Victoria, B.C. Canada **Date of Birth:** 7. March, 1964

Educational Institutes Attended:

University of Victoria.

Degrees Awarded:

B.Eng. University of Victoria 1988

Honours and Awards:

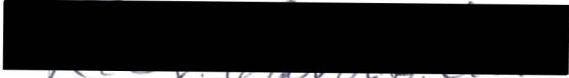
British Columbia Science Council Graduate Research Engineering and Technology Award	1991-1993
Microtel Pacific research Limited Scholarship	1986
British Columbia Government Scholarship	1985-1986
British Columbia Provincial Scholarship	1983

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Author:


Rudolf Schnorr von Carolsfeld

Date:

28. April 1993