

Broadband Acoustical Superresolution Imaging of Breaking Ocean Waves

by

Rex Kelley Andrew
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Dr. R. Lynn Kirlin, Co-supervisor (Dept. of Electrical and Computer Engineering)

Dr. David M. Farmer, Co-supervisor (Dept. of Elec. & Comp. Eng.)

Dr. Dale Shpak, Departmental Member (Dept. of Elec. & Comp. Eng.)

Dr. Charles Konzelman, Outside Member (Dept. of Mechanical Eng.)

Dr. Rolf G. Lueck, Outside Member (School of Earth and Ocean Sciences)

Dr. Michael J. Buckingham, External Examiner (Marine Physical Laboratory, Scripps
Institute of Oceanography)

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University of Victoria

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Co-supervisors: Dr. R. Lynn Kirilin and Dr. David M. Farmer

ABSTRACT

An acoustic array was deployed in the nearsurface layer in Saanich Inlet, B.C., to image breaking waves using only the naturally occurring acoustical radiation from the breaking region over the band [160 Hz, 2000 Hz]. The 15-element array was configured as a horizontal cross with an 8 m aperture, bottom-moored, and positioned nominally 3 m beneath the surface.

Due to sensor sparseness, the array PSF at any particular frequency was badly contaminated by grating lobes. A novel broadband scheme was devised to combine information at multiple independent frequencies to yield unambiguous images with resolution of about 0.2 m at the sea surface.

The broadband scheme assumed space-time separability in the source mutual spectral density. This is only considered valid for breaking waves above about 400 Hz. Nonstationarity and time-bandwidth constraints yielded at most six independent frequency bands within the system passband.

A parametric image analysis showed that the images align closely with the wind and can be observed moving downwind with a speed about two-thirds the phase speed of the dominant component of the wind waves.

Absolute power levels were found to be consistent with previously published results. The absolute power levels were parameterized by $S_0 e^{20+\lambda(f)}$ where $S_0 = 1 \mu\text{Pa}^2/\text{Hz}$ and $\lambda(f)$ is well-described by a simple first order relation $\lambda = b_0 + b_1 \ln f$, where b_0 varied depending on the size of the wave but b_1 appeared to be a more universal constant estimated at -4.55 ± 0.47 .

The source mechanism for frequencies below about 400 Hz was modeled two ways: (1) as a point source (which would follow if an acoustically compact "collective oscillation" region had formed), and (2) as due to off-peak spectral contributions from bubbles resonant at 400 Hz. Neither model achieved a satisfactory fit to the observed data. This seems to imply that the mechanism below about 400 Hz was acoustically extended and radiating as energetically as any resonant bubbles.

Dr. R. Lynn ~~Kyflin~~, Co-supervisor (Dept. of Electrical and Computer Engineering)

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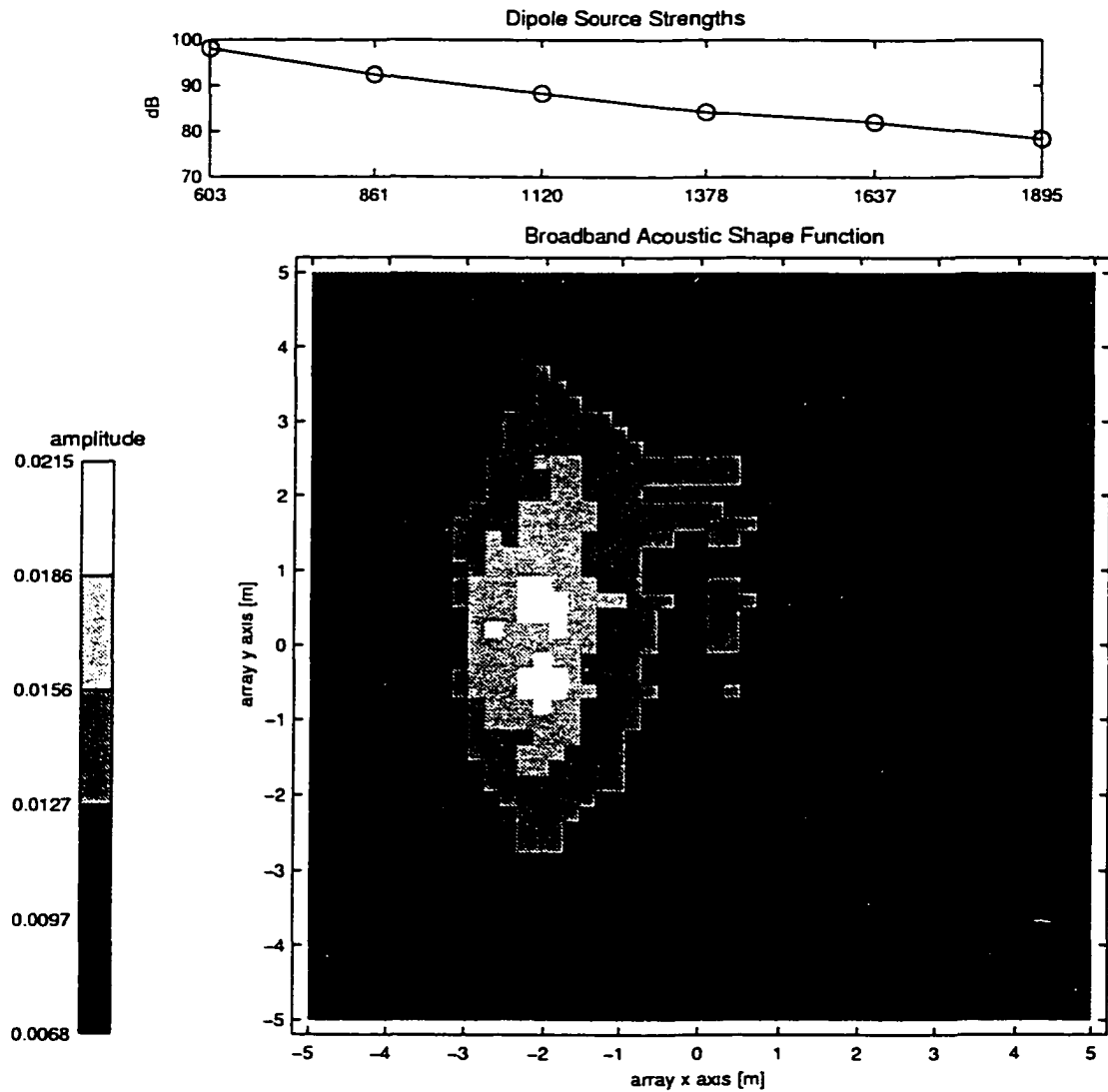
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Broadband Acoustical Superresolution Imaging of Breaking Ocean Waves



Rex K. Andrew

List of Acronyms

Acronym	Expansion	Defined
AORG	Acoustical Oceanography Research Group	Section E.5
CSDM	Cross-Spectral Density Matrix	pg 61
HWHM	Half Width at Half Maximum	pg 49
IOS	Institute of Ocean Science	Section E.5
MCF	Mutual Coherence Function	pg 30
MEM	Maximum Entropy Method	pg 7
MSD	Mutual Spectral Density	pg 30
NAH	Nearfield Acoustical Holography	pg 43
POCS	Projection Onto Convex Sets	pg 91
PSF	Point Spread Function	pg 49
SURFEX	SURFace EXperiment	pg 6
ULSAS	Upward Looking Sonar Array System	pg 72

Often, the things which are most familiar to us turn out to be the hardest to understand.

— *Edwin T. Jaynes*

Take what is useful and develop from there.

— *Bruce Lee*

$2 + 2 = 5$ for sufficiently large values of 2.

— *Net.wisdom from Usenet newsgroup sci.math.num-analysis*

If we don't succeed, we run the risk of failure.

— *Dan Quayle*

Daddy, I overdid it again!

— *his son*

Chapter 1

Introduction

1.1 Background

Breaking wind waves are now believed to play a fundamental rôle in the coupling of the atmosphere and the ocean. They are the primary mechanism for transferring atmospheric momentum into surface currents (Melville [96]), which in turn drive ocean circulation patterns. Breaking waves, both large and small, provide a “damping factor” that dissipates mechanical energy and moderates the growth of waves against the influence of wind forcing and wave-wave interaction (Komen *et al.*, [69]). Breakers entrain atmospheric gases into subsurface bubbles, producing an inhomogeneous layer of bubbly fluid, which may affect sound propagation (Farmer and Lemon [46]) and may significantly increase acoustic backscatter from the surface at near-grazing angles (McDonald [93]). The turbulence generated by breaking waves is suspected of enhancing the dissipation in the near-surface sublayer (Craig and Banner [26]) which in turn greatly enhances heat and gas transfer from the bottom of the atmospheric boundary layer into the ocean (Thorpe [131]).

Our ability to monitor the contribution of breakers to the coupling process requires measurements during windy and often stormy conditions, which are not ideal and sometimes hazardous for human observers. For these reasons, oceanographers are seeking

alternative methods for remotely monitoring the breaking activity of the sea surface in all possible sea states.

One of the more accessible aspects of breaking wind-driven waves is the “crashing” sound they radiate. Earlier this century, during the development of military sonar systems, the sound radiating from breaking waves was considered “noise”, and had a significant although dubious rôle as a degrading factor in sonar system performance.

More recently, however, we have greatly expanded our ability to use the sound radiated by breaking waves to actually interpret the state of the marine boundary layer.

Early descriptions of the deep-water ambient noise field ascribed a dominant component of the ambient noise to a continuous surface “sheet” of dipoles (Urick [137], chapter 7). The dipole sheet was, of course, a statistical model for a randomly rough sea surface randomly populated by random-sized breakers.

Later, Anderson [1] used a high-gain acoustic array mounted on an underwater blimp to acquire directional sea surface noise. A complicated statistical argument was then used (Anderson [1], Shang and Anderson [120]) to infer that the sound sources at the sea surface were actually discrete sources, as opposed to a continuum.

This more-or-less intuitive finding was verified by Farmer and Vagle [47] who used simultaneous audio and visual observations to show that an individual breaking wave indeed radiates a broadband acoustic signal into the water.

This radiated signal was subsequently used by Ding and Farmer [37] to locate and track breaking waves. Using a four-element array, they determined the position for individual sources (i.e., breakers) by cross-correlating the signals received on each sensor to compute the time-difference of arrival (Farmer and Ding [45]).

These results, combined with recent advances made in the laboratory, showed that the sound radiated by surface sources, specifically breaking waves, could in fact be used to measure sea surface properties. The mechanical energy dissipated by a breaking wave appears to be directly related to the intensity of the radiated sound (Loewen and

Melville [82]): this admits the potential for inferring the surface wave field dissipation via an acoustical measure. The radiated autospectrum also appears to be related to simple models of the bubble size distribution within the breaking region (Loewen and Melville [83], Medwin and Daniel [95]), possibly providing an inverse problem for the bubble size distribution and hence the volume of gas entrained by the breaker. The spatial statistics of waves tracked across the ocean surface provide key parameters needed in models of surface wave field growth (Ding and Farmer [36], Phillips [111]). The sea surface ambient sound and whitecapping activity also correlate well with wind strength (Knudsen *et al.* [65], Monahan and O'Muircheartaigh [102]) and now ambient sound is used to monitor ocean surface wind speed (Kerman *et al.* [64]).

These important acoustical measures of breaking waves, and the inferences they provide on the state of the marine boundary layer, have all been achieved with relatively simple sensors. Our understanding of sea surface processes, however, is far from complete, and we must therefore turn to more sophisticated instruments. As an example, if breaking waves can be detected, measured and tracked by the naturally-occurring sound they radiate, can their individual spatial dimensions also be imaged by their radiated sound? If so, what would this tell us?

To date, two experiments have been conducted to image breaking waves. The first (Crowther and Hansla [29]) utilized a seven-beam sonar system configured like an insect-eye, bottom-moored in 85 m of water. The seven beams provided seven overlapping footprints at the surface, with footprint diameter approximately 9.7 m at 24 kHz. They used a two-tone image model and an entropy-type function to regularize the inverse problem and produced images of acoustically loud shapes moving across the acoustically quiet background field-of-view. Simultaneous visual observations identified whitecaps more-or-less within the perimeters of these shapes. Their results showed the acoustically active region to be considerably larger than the apparent visual size of the whitecap.

The second experiment (Epifanio and Buckingham [44]) used a sonar system containing 126 beams at a slant range from the surface of 45 m, operating in the range

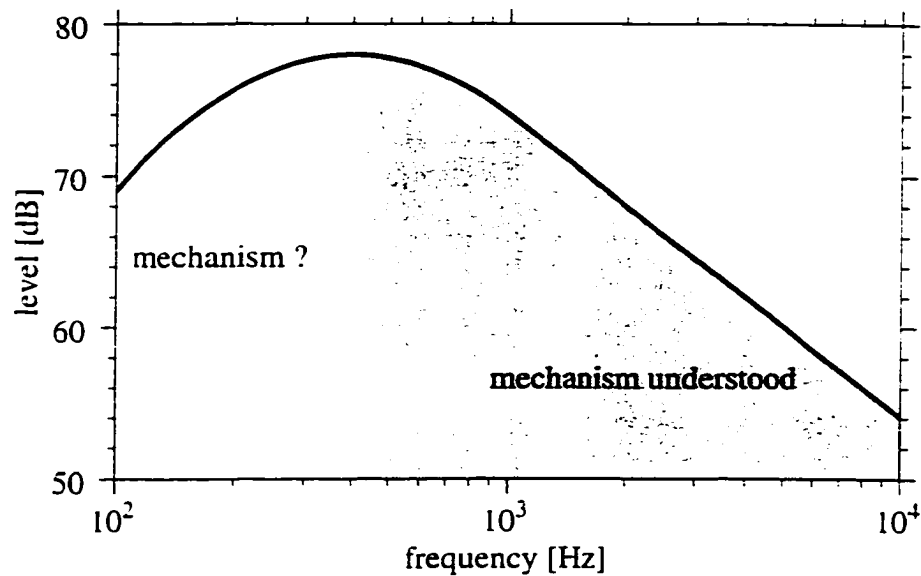


FIGURE 1.1: A typical ambient noise spectrum.

8 kHz to 80 kHz. Each beam had a footprint diameter of approximately 0.75 m at 80 kHz. These results show that the spatial structure of the acoustically active region evolves quite rapidly, on time scales at least as short as 40 ms.

Both of these experiments imaged the breakers at frequencies well above the peak of their radiated spectral signature. The portion of the background ambient sound due to natural surface sources has a well-known shape as shown in figure 1.1. The autospectrum has a broad peak at about 200 - 500 Hz, and a slope of about -5 or -6 dB per octave above. Measurements have generally shown that the radiated signatures of individual breakers tend, although louder, to have the same shape. The source mechanism for frequencies above the peak is fairly well-established to be the incoherent “ringing” of individual bubbles generated by entrainment in the breaker. This region is shaded in figure 1.1.

The source mechanism for spectral energy below the peak is not as well-established. There are, in the author’s opinion, no entirely convincing theories at the present time.

One of the central problems is the lack of quality data at these frequencies. The two imaging experiments described above, for instance, were conducted at frequencies well-above the peak.

In summary, the radiated breaker signature is dominated by spectral contributions from the decade below and the decade above the peak, and *this* is the signal that appears to be directly correlated with several of the most interesting oceanographic parameters, such as mechanical wavefield dissipation and total gas entrainment.

In this project, we attempt to image breaking waves at these frequencies near the peak of the ambient sound autospectrum.

1.2 This Work

The imaging problem at these low-audio frequencies is formidable. The wavelength λ at 500 Hz is approximately 3 m, and a conventional imaging system will have a diffraction-limited spatial resolution bounded by the “Rayleigh resolution” of about $\lambda/2$. In practice, this implies that the surface footprint of a conventional imaging system, positioned dozens of meters below the surface as in the two previous imaging experiments, will be far too large to resolve the breaking region of all but the most gargantuan wave. The spatial resolution available through such an approach is utterly inadequate for the size of whitecaps typically encountered in coastal locations.

A principle feature of this work is therefore the design and development of a technique that would provide satisfactory spatial resolution at these lower audio frequencies. Numerous trade-offs were required. The hydrophone array was moved from the far-field to the nearfield, at the cost of subjecting the system to the harsher nearsurface environment and thoroughly contaminating the data with mooring noise. The system’s upper frequency was set at 2 kHz, yielding a spatial resolution on the surface of less than 0.2 m, at the cost of a uniformly spaced sensor pattern. Information across two octaves in frequency were combined to eliminate spatial aliasing due to the sensor sparseness, at

the cost of foregoing true single frequency images.

The system was deployed three times, in experiments labeled herein as SURFEX I, II and III. The first two deployments were essentially “shake-down” trials: the imaging data come from SURFEX III.

The experiment ultimately yielded never-before-seen image-like information about breaking regions at frequencies near the peak of the radiated autospectrum. This information clearly shows an acoustically active shape with roughly the same elongation displayed by whitecaps, moving downwind, and oriented consistently with the wind direction. The technique used here also provides an estimate of the frequency-dependent source strength levels of the breaking region, which had not been achieved in prior imaging experiments.

The technique, however, falls short of imaging at frequencies below the peak of the radiated autospectrum. That problem is *really* tough! Adequate spatial resolution simply could not be achieved at wavelengths many times greater than the expected dimensions of features of interest. The data can be used, however, to make some inferences regarding the poorly understood source mechanism in this regime

1.3 Superresolution

The term “superresolution” in the title is certainly an eyecatcher, but it also carries the cachet of a buzzword. It is therefore prudent to argue why its usage here is appropriate.

A quick survey of the literature does not provide a precise definition of superresolution. It appears to have been used originally to describe the process of analytically extending the Fourier transform of a bandlimited image beyond the band limits (Blackledge *et al.* [15]). The imaging techniques pursued here do not fall under that strict definition. The term “superresolution”, however, has come to signify a much wider class of image enhancement techniques. Richardson and Marsh ([115]) suggest that superresolution defines any imaging process that yields a “better” image than some

corresponding “conventional” imaging process. The imaging techniques used in this project satisfy this criteria three ways:

1. Positioning the array so that sources are located in the nearfield of the array. As discussed in chapter four, the point spread function (PSF) of the nearfield array in SURFEX III has better resolution than the PSF of the array used in SURFEX II and provides much better resolution than would an array deployed at the conventional depths used by Ding and Farmer [37], Crowther and Hansla [29] or Epifanio and Buckingham [44] .

Thus, the choice alone of a nearfield array over a far-field array confers a better resolving capability.

2. Using image priors. The use of image priors is described more fully in chapter four, but, briefly, an entropy prior (that is, a data-independent model enforcing a monotone image) permits the resolution of closely spaced point sources which cannot otherwise be resolved using conventional diffraction-limited techniques. Reconstructions using entropy (the “maximum entropy method” or MEM) seem to be automatically qualified for superresolution status (Press *et al.* [114]). MEM reconstructions have been very successful in radio astronomy.

It is not clear, though, that entropy is the best prior for smooth extended sources that lack the vast differences in luminosity that are inherent in astronomy. Very recently, Briggs [21] has suggested the use of non-negative least-squares – i.e., essentially a prior that enforces non-negativity – as an adequate prior for high-fidelity imaging of extended sources. Inasmuch as his results are comparable to MEM images in terms of resolution, non-negativity constraints, which are used in this project, may provide images with “resolution” higher than that available with conventional processing.

3. Using a parametric scheme that permits a “better fitting” image solution than can be achieved otherwise. Haacke, Liang and Izen [55] demonstrated that a piecewise

linear model achieved a better fit to a piecewise linear source than a conventional (nonparametric) model. The improvement was clearly related to the use of a model that modeled the source very well. This was also one of few approaches that defined resolution in terms of an overall fit to an extended image. In this sense, a source is better resolved by a parametric image when the model is an accurate representation of the original source. A parameterized source model is used to accumulate source configuration statistics in chapter seven.

1.4 Thesis Organization

The essential background for the imaging algorithm used in this work is developed over three chapters. Chapter two sets forth the nature of the phenomena to be imaged, and reviews the accepted and suggested source mechanisms. Chapter three establishes a mathematical model for the source, including four key assumptions, and briefly considers the signal propagation issues. Chapter four merges the source and signal propagation models to arrive at a broadband imaging statement for producing resolved unambiguous image-like information over the frequency band 600 Hz to 2000 Hz. Chapter four also has a brief discussion of conventional imaging techniques and why these techniques fail in this particular case.

Chapter five provides a brief description of the salient features of the SURFEX III deployment. Attention is directed towards the data collected during a day-long storm on 4/13/95.

Chapter six outlines the basic signal processing needed to generate broadband images and provides examples from the data set of 4/13/95. Chapter seven extends the analysis of chapter six by interpreting the images in terms of oceanographic parameters such as wind direction and breaker source strength level. Some additional models of the lower-frequency mechanism are considered.

Chapter eight provides a brief conclusion and suggestions for future work.

There are numerous appendices, involving mostly signal processing or hardware details. Of note is appendix I, which contains a “gallery” of stripchart-like plots and a compendium of imaging results for every wave analyzed in chapter seven.

Chapter 2

Source Physics

Before tackling the imaging problem, it would be prudent to consider the phenomena we expect to observe via the imaging process. The mathematical machinery of passive imaging requires *sources*: upon further examination, however, the definition of a “source” proves to be an elusive matter. This is not an issue in electromagnetic radiometry or spectroscopy, where radiant sources have been used for more than a century and are quite well understood. In contrast, the acoustical radiation from breaking waves has attracted significant research interest only within the last decade. Furthermore, the use of acoustical radiation to study the breaking process itself is even more recent. And while the acoustical properties of the breaking region are anticipated to be related to the hydrodynamics of the breaking process, this relationship is still, despite a flurry of recent investigation, not yet fully understood.

This therefore motivates a brief review of the sound generation mechanism of breaking waves. The rôle of bubbles is addressed in more detail in section 2.1, including an examination of “collective oscillations”. In section 2.2, the key features of the breaking process that apply to sound generation are reviewed. In section 2.3, the essential features of breaking waves and subsurface bubbles are combined to produce a definitive picture of the source physics — that is, the principle nature of the phenomena to be imaged.

2.1 Bubbles

Bubbles play an essential rôle in virtually every aspect of sea surface sound; therefore, any consideration of sources must necessarily involve a consideration of the bubbles themselves.

An individual bubble acts like a simple mass-spring oscillator, with the highly compressible internal gas acting like the spring, and the fluid external to the bubble providing the mass. If somehow “struck” by an impulsive force, the bubble will settle into free decaying *radial* (or *volume*) oscillations at a resonant frequency f_0 inversely related to its quiescent radius R_0 . Arguing that the internal gas behaves adiabatically, Minnaert [99] derived the relationship

$$f_0 = \frac{1}{2\pi R_0} \sqrt{\frac{3\gamma P_{i0}}{\rho}} \quad (2.1)$$

which is now generally known as the Minnaert formula (Leighton [78], section 3.2). Here, γ is the polytropic gas constant. For ordinary water ($\rho = 1000 \text{ kg/m}^3$) and atmospheric pressure P_{i0} , this equation specifies the relationship shown in figure 2.1.

The damping forces that cause bubble oscillations to decay were originally investigated by Devin ([33]) who showed the damping coefficient to be the sum of viscous dissipation, acoustic radiation resistance and thermal dissipation. Experimental observations by Strasberg [125] and Updegraff [134] suggested decay constants of order of at most tens of milliseconds for the sizes of bubbles ordinarily encountered under natural conditions.

While bubbles vibrating in the radial oscillation mode — which is the dominant radiating mode — can therefore be identified as simple monopole sources of damped outgoing spherical harmonic sound waves, it is also relevant to consider how such bubbles are initially forced. Two significant ways are discussed in sections 2.1.1 and 2.1.2 below. A related problem is the acoustical interaction of multiple bubbles in, e.g., a swarm. In this case, the bubbles no longer oscillate independently. This issue constrains the choice of analysis band and is therefore of direct concern to this work, and so is

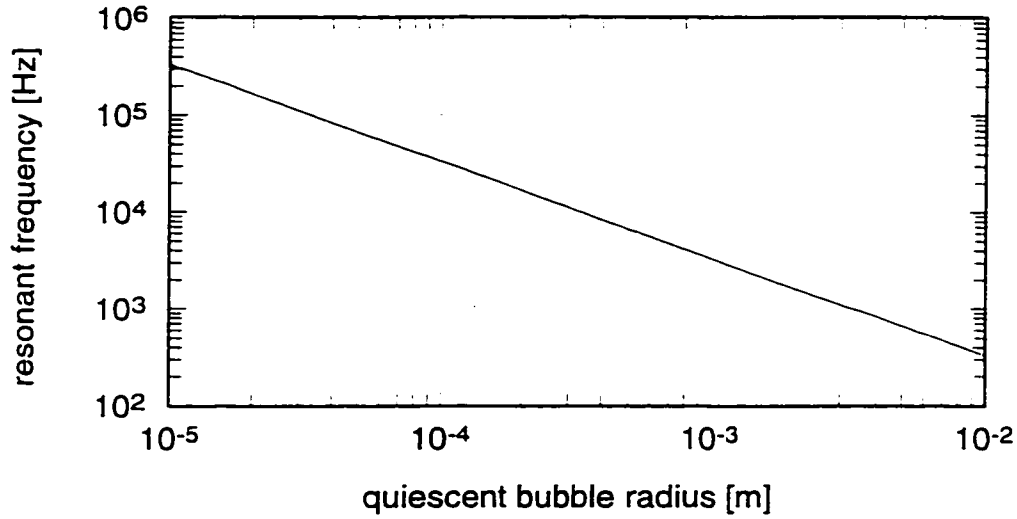


FIGURE 2.1: The Minnaert relationship. Resonant frequency versus bubble radius. 1 atmosphere, nominal density of water.

treated in more detail in section 2.1.3.

2.1.1 Birthing Wails

Minnaert and later Strasberg provided experimental confirmation of Eq.(2.1) by listening to the sound produced by oscillating bubbles. Moreover, by using a combination of high-speed photography and acoustic measurements, Strasberg was able to infer that, at the moment of pinch-off (i.e., when the bubble detaches from the inflation nozzle to become an independent body), uneven surface forces over the bubble wall are suddenly freed to induce a variety of vibrational modes. He then showed that the zeroth-order or radial mode was the most efficient, so that the bubble will essentially “ring” at the frequency predicted by Eq.(2.1).

The acoustic signature of a newly-created (or “newborn”) bubble is now generally known as the “birthing wail” (Crum [30]).

A newly-created bubble at position $\mathbf{r}_0$ therefore launches an acoustic field $p(\mathbf{r}, t)$

governed by the inhomogeneous wave equation

$$\nabla^2 p(\mathbf{r}, t) - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} p(\mathbf{r}, t) = f(t) \delta(\mathbf{r} - \mathbf{r}_0) \quad (2.2)$$

where c_0 is the phase speed in the surrounding fluid. Eq.(2.2) is valid everywhere except within some small neighborhood of $\mathbf{r}_0$ (i.e., except within the bubble itself). The inhomogeneous term, which includes the source “amplitude” $f(t)$, represents the forcing due to the bubble.

2.1.2 Adult Screams

According to Lighthill ([80], section 1.10), the unsteady pressure fluctuations characteristic of turbulence are also sources of sound. These fluctuations appear as an inhomogeneous term in the governing wave equation and are mathematically equivalent to a volume distribution of quadrupole sources. Crighton and Ffowcs-Williams [27] suggested that the presence of bubbles can “enhance” (i.e., amplify) the radiated sound: the quadrupole mechanism is inherently inefficient, but might couple energy into the bubbles, which then radiate using the more efficient monopolar mode. Theoretically, the bubbles could enhance the turbulent pressure autospectrum by a factor of $(c_0/c_{eff})^4$, where c_0 is the sound speed in the pure liquid and c_{eff} is the effective or “bulk” sound speed in the bubbly two-phase flow (see section 2.1.3). As an example, a 1% volume fraction of air (i.e., a void fraction of 1%) would amplify the pressure autospectrum by 50 dB.

Investigators (Korman, Roy and Crum [71], Hughes and Korman [61], Korman and Crum [70], Kolaini and Goumilevski [66], Kolaini and Hobbs [67], Kolaini, private communication) have sought to verify this prediction by using submerged jets in laboratory tanks. A hydrophone measures the pressure autospectrum radiated from the jet region before and after bubbles are released into the flow. Results indeed indicate that the radiated autospectrum is amplified by bubbles, although not to the extent expected.

Nevertheless, this provides a second mechanism for sound generation: mature, or

“adult”, bubbles, which have existed long enough for their birthing wails to die away (requiring at most tens of milliseconds), may be stimulated into re-emission by the local pressure fluctuations characteristic of a turbulent flow. Crum [30] has termed these acoustic signatures “adult screams”. The radiated acoustic field $p(\mathbf{r}, t)$ of a turbulent two-phase flow is then governed by [49]

$$\nabla^2 p(\mathbf{r}, t) - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} p(\mathbf{r}, t) = -m(\mathbf{r}, t) - d(\mathbf{r}, t) - \frac{\partial^2 T_{i,j}}{\partial x_i \partial x_j}. \quad (2.3)$$

The inhomogeneous terms are: $m(\mathbf{r}, t)$, the monopole sources, representing the bubbles, $d(\mathbf{r}, t)$, the dipole sources, and $\partial^2 T_{i,j} / \partial x_i \partial x_j$, the quadrupole term, involving the excess momentum flux tensor $T_{i,j}$. The monopole sources, however, overwhelm the dipole and quadrupole sources, so that the equation governing the acoustic field is essentially the same as that in section 2.1.1.

2.1.3 Gang Rumbles (Collective Oscillations)

While a theory involving the “birthing wails” of individually resonating newly-created bubbles seems to account satisfactorily for the sound generated by breaking waves at frequencies above about 400 Hz, it appears inadequate at lower frequencies: measurements of the radii of bubbles produced in spilling flows in laboratory tanks, under gentle waterfalls and at-sea generally do not show evidence of bubbles with radii large enough to possess a resonant frequency below about 400 Hz.¹ Two possibilities exist:

- Large bubbles of these sizes are in fact created during entrainment but have thus far evaded detection;
- Some alternative mechanism is generating sound below about 400 Hz.

The first possibility has largely been discounted because the rise times of large bubbles are quite short, implying that they would not be submerged long enough to

¹A bubble with a resonant frequency near 200 Hz, for example, would have by Eq.(2.1) a radius of 16 mm: near 100 Hz, a radius of 32 mm.

make a major contribution to the radiated sound. In addition, broadband measurements by Hollett [60] of acoustic autospectra radiated from breaking waves show that the low-frequency contribution reaches its maximum late in the development stage of the breaker, and this too is inconsistent with the short expected lifespans of large bubbles.

A popular theory for an alternative mechanism involves *collective oscillations* of the entrained bubble cloud. In this theory, the individual bubbles in a compact region of high bubble density oscillate in-phase (i.e., “collectively”), but at a much lower frequency than the natural resonant frequencies of the individual bubbles. In the spirit of earlier nomenclature, this acoustical signature might be termed a “gang rumble”.

While this theory seems to account for a number of features observed in low-frequency acoustic radiation, it makes the identification of the “source” a bit more subjective. And since the primary goal of a passive imaging system is to identify the *source*, it is perhaps worthwhile to examine the problem of collective oscillations a bit more closely.

The bubble column experiment Yoon *et al.* [143] performed the paradigmatic collective oscillation experiment: a regular column of bubbles was generated in a water tank by pumping air through a number of hollow needles at the bottom of the tank, as shown in figure 2.2. The radii of the individual bubbles were measured optically. A sensor located in the bubble-free fluid outside the bubble column measured a radiating sound autospectrum with resonances well below those of the individual bubbles — the column was “rumbling”. Analysis showed that the rumble resonances corresponded to the eigenmodes of an acoustic standing wave bounded by (i.e., trapped in) the approximate geometric volume of the bubble column. Indeed, when moving an internal sensor vertically along the axis of the column, Yoon *et al.* were able to produce the definitive patterns of acoustic standing wave modes at the rumble frequencies.

Wave propagation in a bubbly medium Since the bubbles are randomly located in the bubble cloud, the behavior of the acoustic wave in the bubbly region may be

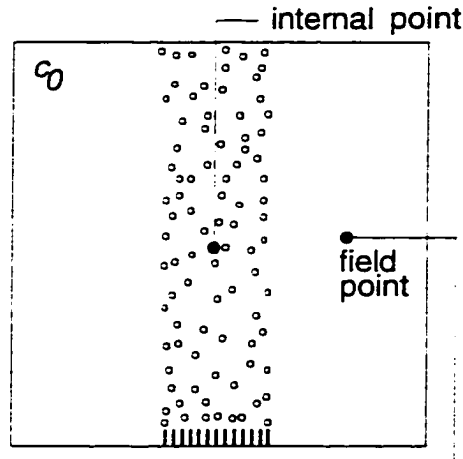


FIGURE 2.2: The bubble column experiment. Bubbles are released from an array of hollow needles shown at the bottom center of the tank. The sound speed in the tank is c_0 . The experiment is monitored with two hydrophones, one outside the bubble column measuring the radiating acoustic field, and another inside the column for sampling the internal acoustic field.

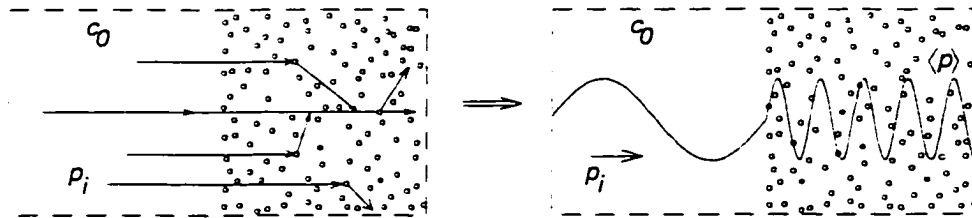


FIGURE 2.3: Propagation of a plane wave into a swarm of bubbles. Left: an incident plane wave p_i , represented by rays, propagates to the right through a fluid with sound speed c_0 . Right: the coherent wave $\langle p \rangle$ inside the bubbly fluid has the same radian frequency but a different wavelength than the incident plane wave p_i .

viewed as wave propagation in a random medium. Consider the propagation of a plane wave $p_i(\mathbf{r})e^{-i\omega t}$ from a bubble-free fluid halfspace into a swarm of bubbles, as shown in figure 2.3. Each bubble will scatter a portion of the acoustic field incident on it. The formal solution for the acoustic field $p(\mathbf{r})$ at a point $\mathbf{r}$ is (Morse and Ingard [103], section 8.2)

$$p(\mathbf{r}) = p_i(\mathbf{r}) + \sum_i^N \iint_{S_i} (p(\mathbf{r}_0) \nabla_0 g(\mathbf{r}_0, \mathbf{r}) - g(\mathbf{r}_0, \mathbf{r}) \nabla_0 p(\mathbf{r}_0)) \cdot d\mathbf{S}_0 \quad (2.4)$$

where S_i is the surface of the i^{th} bubble, $i = 1 \dots N$. This equation is valid everywhere, in the pure fluid or bubbly fluid, except inside or on the surface of a bubble. The term $g(\mathbf{r}_0, \mathbf{r})$ is the Green function satisfying the associated inhomogeneous equation

$$\nabla^2 g(\mathbf{r}_0, \mathbf{r}) + k_0^2 g(\mathbf{r}_0, \mathbf{r}) = -\delta(\mathbf{r}_0 - \mathbf{r})$$

and the Sommerfeld radiation boundary condition at infinity. The term $k_0 = \omega/c_0$.

Eq.(2.4) is an inhomogeneous integral equation; the inhomogeneous term is $p_i(\mathbf{r})$, representing a field launched by a source located at negative infinity. Considering only the bubbly region, Eq.(2.4) corresponds to the problem of solving for the field obeying the homogeneous Helmholtz equation

$$\nabla^2 p(\mathbf{r}) + k_0^2 p(\mathbf{r}) = 0 \quad (2.5)$$

with prescribed boundary conditions on the surface of each bubble.

For the limiting case where the bubbles can be considered isotropic point scatterers, and for interbubble distances much less than an acoustic wavelength, Foldy [50] showed that the ensemble average of the sum of scattered contributions from a large number of randomly located scatters yields a "coherent wave" $\langle p(\mathbf{r}) \rangle$ in the bubbly region obeying

$$\nabla^2 \langle p(\mathbf{r}) \rangle + k_{eff}^2 \langle p(\mathbf{r}) \rangle = 0 \quad (2.6)$$

where k_{eff} is the effective wavenumber given by $k_{eff} = \omega \sqrt{\rho_{eff} \kappa_{eff}}$. Using terminology from Morse and Ingard [103], ρ_{eff} is the effective density and κ_{eff} is the effective compressibility of the bubbly fluid. These are essentially local properties averaged over the

entire bubble swarm. For two-phase flows with a void fraction of about 1% or less, $\rho_{eff} \approx \rho_0$, the density of the pure fluid, but $\kappa_{eff} \gg \kappa_0$.

The physical picture is as follows: a deterministic wave incident on a random bubbly medium scatters sound from every bubble. The bubbles are not themselves *primary* sources of sound, but *secondary* sources: the scattered radiation is therefore secondary radiation. These scattered waves interfere destructively within the random regime in all directions except one: in the forward direction, they interfere constructively and sum to produce a stochastic wave $p(\mathbf{r})$. When appropriately averaged, $\langle p(\mathbf{r}) \rangle$ appears to be a wave with the *same frequency* as the deterministic incident wave, but a lower phase speed, and therefore a shorter wavelength.

Note that *neither* Eq.(2.5) nor Eq.(2.6) have inhomogeneous terms — there are no sources in the bubbly region.

While the acoustic field in the bubble swarm is actually a stochastic wave, in practice the finite surface area of an actual hydrophone provides a degree of spatial averaging that yields an approximate measure $p_{\text{measured}}(\mathbf{r})$ of the coherent field $\langle p(\mathbf{r}) \rangle$.

The effective sound speed The bulk or effective sound speed properties of a two-phase flow were described by Wood [140], who showed that the presence of the bubbles induces a significant increase in the “bulk compressibility” over the compressibility of the pure fluid. Using a simple isothermal relation for the bulk density ρ_{eff} , he derived the relation

$$\frac{1}{c_{eff}^2} = \frac{d\rho_{eff}}{dp} = \frac{(1-\beta)^2}{c_0^2} + \frac{\beta^2}{c_g^2} + \frac{\rho_0\beta(1-\beta)}{P_{i0}} \quad (2.7)$$

where β is the *void fraction*, the volume fraction occupied by the gas and c_g is the sound speed in the gas. This is now known as Wood’s equation. As an example, the effective sound speed for various void fractions is shown in figure 2.4 for one atmosphere, standard seawater density, and $c_0 = 1500 \text{ ms}^{-1}$. The effective sound speed can clearly be much less than that in the pure fluid — in fact, less than the sound speed in the pure gas too!

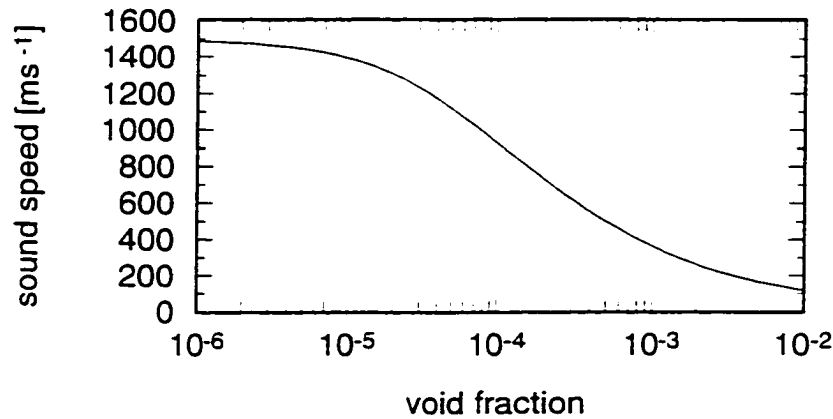


FIGURE 2.4: Effective sound speed (Wood's equation).

This effective sound speed is significant when considering propagation conditions in the ocean because void fractions can range from very small (10^{-9} - 10^{-8} , see table 2.1) in the background bubble layer to quite high (up to 100%, Lamarre and Melville [73, 74]) in the core of breaking waves.

Since in practice there is little difference between $\langle p(\mathbf{r}) \rangle$ and $p_{\text{measured}}(\mathbf{r})$, it is customary when dealing with problems of propagation in a bubbly fluid to consider the “analogous sound speed problem”. In the analog problem, the two-phase flow is replaced by a pure fluid possessing the bulk sound speed c_{eff} , as in figure 2.5. This simplifies the treatment to that of a first-year physics problem in wave propagation.

So where's the source? The bubble column experiment can therefore be analyzed by considering the sound speed analog problem, as in figure 2.6: this is the method used by Yoon *et al.* They solved for the acoustic field subject to

$$\nabla^2 p(\mathbf{r}) + k_0^2 p(\mathbf{r}) = 0 \quad \text{external fluid} \quad (2.8a)$$

and

$$\nabla^2 p(\mathbf{r}) + k_1^2 p(\mathbf{r}) = 0 \quad \text{“inner fluid”} \quad (2.8b)$$

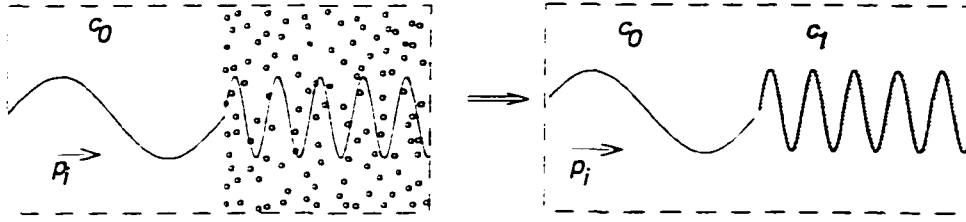


FIGURE 2.5: Bubbly fluid and its sound speed analog. Right: a deterministic plane wave p_i propagates from a region of pure fluid with sound speed c_0 into a bubble swarm with sound speed c_0 : scattering gives rise to a coherent wave $\langle p \rangle$ that propagates with an effective speed of c_1 . Right: a problem considered to be equivalent. A plane wave propagates from a region of fluid with sound speed c_0 into a region of fluid with sound speed c_1 .

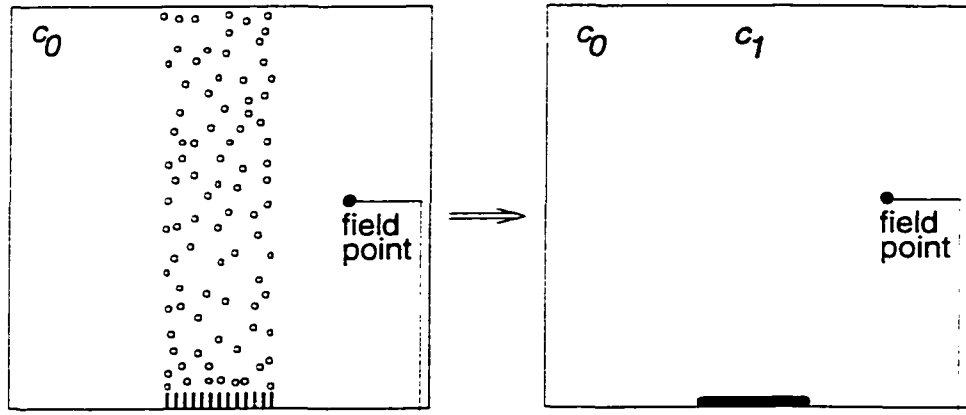


FIGURE 2.6: The bubble column experiment and its sound speed analog. Left: the experiment. Right: a problem considered equivalent. The bubble column has been replaced with a column of fluid with sound speed c_1 . The primary sources (the bubbles launching off the tips of the needles) are represented by an inhomogeneous boundary condition at the bottom of the c_1 region, shown as a heavier black line.

with some appropriate boundary conditions. ($k_1 = \omega/c_1$.) Although the standard approach for handling Eqs.(2.8a) and (2.8b) is to expand the fields into eigenmodes, *neither* equation specifies a source. So where's the source?

Nicholas *et al.* handled the problem mathematically by introducing an inhomogeneous boundary condition. Physically, in the bubble column experiment, this would imply that bubbles detaching from the needles are the primary sources. There is, of course, a discrepancy with this interpretation. Since the problem has been formulated within the approximations of linear acoustics, the low-frequency gang rumbles radiating from the column must be excited by low-frequency birthing wails of detaching bubbles. Visual observations, however, showed that the bubbles leaving the needle have, in fact, radii far too small to resonate, according to the Minnaert formula, at gang rumble frequencies. Their resonant frequencies are at least an order of magnitude higher.

The identification of "the source" in this and similar problems remains one of Crum's unresolved issues in bubble-related ambient noise [30]. Therefore, as it is unclear how to pose the problem mathematically, the imaging problem is best restricted to frequencies where collective oscillations are not expected to be the dominant mechanism.

2.2 Breaking Waves

Banner and Peregrine [7] and Melville [97] provide good reviews of breaking waves: they emphasize that oceanic breakers are unsteady two-phase flows which investigators have found difficult to describe quantitatively.

As a consequence, the following sections provide only qualitative discussions of the key features of interest to this work.

2.2.1 Spillers versus Plungers

Investigators currently differentiate between the following two types of breaking waves (Peregrine [110]):

spilling Using data from wind-generated laboratory waves acquired by Ebuchi [40].

Longuet-Higgins *et al.* in a series of papers (Longuet-Higgins and Cleaver [88], Longuet-Higgins, Cleaver and Fox [89], Longuet-Higgins [86]) have suggested that a shear instability mechanism at the crest of a growing wave generates intense vorticity partway down the forward face of the wave. Small capillary waves are radiated forward from this point. For less energetic waves, the vorticity associated with these advance waves is advected back into the wave crest where it induces “vortex ripples”. These ripples grow and eventually collapse into a turbulent two-phase flow (sometimes called a “roller” or a “bore”) that rides down the forward face of the wave. The lower boundary of the bore can thereafter be treated as a classical mixing layer (Battjes [10]).

There is evidence that newly-generated bubbles enhance turbulent kinetic energy production (Lance and Bataille [76]) and possibly dissipation (Wang *et al.* [138]), although bubble generation via gas entrainment in turbulent bores is not yet well understood (Battjes [9], Lemos [79]).

plunging Several types of hydrodynamic instabilities exist for traveling finite amplitude waves. Benjamin and Feir [12] identified a subharmonic instability that modulated the envelope of a group of small amplitude waves. As the envelope grows, the waves passing the maximum of the envelope become big enough to break. Longuet-Higgins [85] investigated a superharmonic instability that developed near the crest of steeper deep-water waves: this mechanism was shown to form a jet near the crest. When the jet falls forward and intercepts the smooth upstream fluid to trap air, vorticity is generated impulsively in the flow around the trapped air volume: the jet itself also entrains air as it submerges, driving bubbles substantially below the surface. When the jet collapses, it is “pinched-off”, detaching the submerged mass of bubbles in a “bubble cloud”. The trapped air volume at the surface collapses to form a turbulent two-phase flow much like

a spilling wave.

Longuet-Higgins and Cokelet [90] surmised that waves that steepen initially under the subharmonic mechanism eventually break due to the superharmonic mechanism.

As there is currently no clear criterion to indicate even *when* a wave will break (Banner and Peregrine, [7]), there is even less evidence to determine whether a wave will develop as a spiller or as a plunger. Indeed, there is also no exact operational procedure for distinguishing between spillers and plungers: one can imagine that at-sea breakers will typically possess characteristics of both. Nevertheless, visual observations of waves at the Patricia Bay experiment site suggest that vigorously plunging breakers are in fact quite rare, and this notion agrees with other observations (Banner and Peregrine [7], Longuet-Higgins [87]). Nearsurface conductivity measurements in surrounding coastal waters similarly indicate that typical fetch-limited breaking waves inject bubbles down to typically 10 or 20 cm, depths too shallow to ordinarily be associated with plunging waves (Gemmrich, private communication). We therefore expect the waves at the experiment site to possess predominantly spiller characteristics, and that high void fraction flows will be confined to a thin region at the surface.

2.2.2 Whitecap Evolution

Breaking waves are the primary mechanism for entraining bubbles at the ocean surface. Entrainment appears to begin at windspeeds much less than the commonly accepted threshold speed associated with the onset of whitecapping.² Updegraff and Anderson [134, 136, 135] found that “microbreakers” form at windspeeds as low as 1 to 2 m/sec and typically inject one or two or several bubbles. In contrast, the breaking waves under study here entrain millions of bubbles per cubic metre — so many that albedo of the breaking region can increase as much as tenfold over that of adjacent

²Typically about 3 to 3.5 ms⁻¹. [102]

non-breaking regions (Monahan [102]), creating the well-known “whitecap”.

Monahan [100] suggested the terms “Stage A” and “Stage B” to classify the visual signatures of breakers. Stage A features were due to actively breaking waves, while Stage B features consisted of the “fossil foam” or “foam rafts” that remained in the wake of a Stage A breaker. Crowther and Hansla’s measurements [29] suggest that Stage B structures do not radiate sound, so the imaging source description need only consider the Stage A breaker.

Monahan [101] also provides a succinct terminology for the *subsurface* bubble structure associated with breaking waves that will be useful in section 2.3. He distinguishes between four phases of development:

α -plume The subsurface extension of the turbulent bore of a spiller, or the entraining jet of a plunger, or both. The α -plume contains all of the active entrainment. The Stage A breaker is the visible manifestation of the α -plume.

β -plume The remnants of an α -plume once the turbulence of the entrainment flow has subsided. Bubbles structure evolves in the β -plume mainly due to buoyancy as larger bubbles rise back to the surface. Monahan and O’Muircheartaigh [102] assumed the Stage B whitecap was primarily related to the β -plume.

γ -plume A very mature β -plume, at least one wave period old. Nearsurface currents, most notably Langmuir circulations (Thorpe [130]), begin to advect γ -plumes away from their places of origin.

η -layer The persistent nearsurface “background” bubble layer devoid of any structure related to individual breakers. Very old γ -plumes lose their identity and merge into the η -layer.

Monahan provides a descriptive summary of the relevant features of these four phases: details pertinent to this work are shown in table 2.1. The most important features here are (a) the α -plume is quite thin vertically, (b) the effective sound speed

	α -plumes	β -plumes	γ -plumes	η -layer
horizontal scale	0.2 - 1 m ²	8 - 50 m ²	1 - 500 m ²	—
vertical scale	—	0.8 m	0.42 - 0.75 m	—
time scale	~ 1 s	3.5 - 4.3 s	$10^2 - 10^3$ s	—
void fraction	3 - 30 %	$1 - 2 \times 10^{-4}$	$10^{-7} - 10^{-6}$	$10^{-9} - 10^{-8}$

TABLE 2.1: Summary of characteristic features of Monahan's bubble formations. (Different original sources.)

in the β -plume may only be 60% to 70% of c_0 , and (c) the effective sound speed in the η -layer is essentially c_0 .

2.2.3 Bubble Size Distribution

Oceanographers typically characterize the population of subsurface bubbles via the *bubble size distribution*, defined as the number of bubbles per unit volume with a radius between R and $R + \Delta R$, commonly written as $n(R) dR$. Wu [141] provides a good review of attempts to quantify bubble size distributions. Experiments of, for example, Kolovayev [68], Su, Ling and Cartmill [126] and Ling and Pao [81] used optical techniques and found that the distribution peaked for radii around 50 to 80 μm and dropped steeply to either side. Distribution estimates from these experiments found essentially no bubbles with radii $R \gtrsim 300 \mu\text{m}$.³

More recently, Updegraff [134], Medwin and Beaky [94] and Medwin and Daniel [95] used the "birthing wails" notion to hunt for larger bubbles. Updegraff deployed a four-hydrophone array in the ocean at a nominal depth of 1 meter and listened to the sounds generated by gently spilling breakers in a light wind. Medwin and co-workers used several hydrophones positioned beneath the spill-zone of a laboratory wave tank filled with tap water to listen to the sounds generated by spilling artificial breakers.

³By Eq.(2.1), a bubble with $R_0 = 300 \mu\text{m}$ resonates at 10.8 kHz.

In all cases, signatures of individually resonating bubbles were detected and generally located at the breaking event and very close to the surface. More importantly, Medwin's experiments showed individual damped sinusoids oscillating to frequencies as low as 440 Hz.⁴

These observations strongly suggest that the sound radiated from a breaking wave above about 400 Hz may be attributed to the "birthing wails" of individual bubbles.

2.3 The Source

Based on the fundamental features of bubbles and breaking waves presented in sections 2.1 and 2.2, the "source" is now be defined as the population of radiating bubbles distributed throughout the α -plume. These bubbles are known to act as primary sources and correspond to inhomogeneous monopole source terms in the governing wave equation. The excitation mechanism could be either the uneven forces experienced at pinch-off or a fluctuating pressure field due to turbulence within the plume.

Each bubble radiates a decaying oscillatory acoustic field at a frequency related to its radius. Under classical theory, the acoustic radiation from bubbles created in the bore will decay away in tens of milliseconds, long before the bubbles themselves are whisked off into the wake.

The bubble size distribution over the α -plume population may be a broad function of radius, implying a population composed of many different sizes and radiating acoustic signatures at many different resonant frequencies: but bubbles with radii large enough to have resonant frequencies below about 400 Hz are not expected. The superposition of all the acoustic signatures from all the radiating bubbles in each unit volume will give rise to a random acoustic signal.

The spatial correlation of the excitation mechanism — either entrainment or turbulence — within the α -plume will depend on the turbulence in the flow: vertical

⁴Implying a bubble radius of 7.4 mm.

turbulent scales are bounded above by the plume depth and below by (most likely) the Kolmogorov microscale. Horizontal scales are probably similar. Correlation lengths within the plume probably scale with either the local Weber number (Kerman [63]), involving the fluid surface curvature (mediated by surface tension), or bubble radii (i.e., neighboring bubbles may influence each other, but bubbles separated by more than, say, a few dozen bubble radii should be essentially uncoupled hydrodynamically.) Since it is difficult to imagine a mechanism that would be correlating bubble creation over significant portions of an extended source region, it seems fair to assume that the correlation length scale of the excitation mechanism is comparable to either the correlation length scale of the turbulent flow or the wavelengths of the small scale capillary waves advecting through the bore (i.e., see section 2.2.1). This implies that the bubble birth-times and sizes should be essentially uncorrelated over all but the smallest distances in the active entrainment region.

Furthermore, the majority of breaking waves at the site are expected to be spillers. This implies that the typical depths of the α -plume will be much less than an acoustic wavelength over the range of the imaging system passband (see chapter 5 and appendix E): the source region would therefore be acoustically “thin” for the majority of waves observed during the experiment.

The breaking waves will also generate β - and γ -plumes and the η -layer, as shown in figure 2.7. These bubbles fields will provide secondary sources, scattering the primary radiation that originated in the α -plume. Since the information of interest is the image of the primary sources, only the coherent field propagating through the β -, γ and η -fields is relevant. The influence of these three “mature” bubble fields can then be handled by assuming an analogous sound speed field $c(\mathbf{r})$, as shown in figure 2.8.

This particular source model is therefore a straightforward application of well-established principles, and has the additional advantage of possessing features that can easily be translated into relevant signal processing quantities, as is shown in the next chapter.

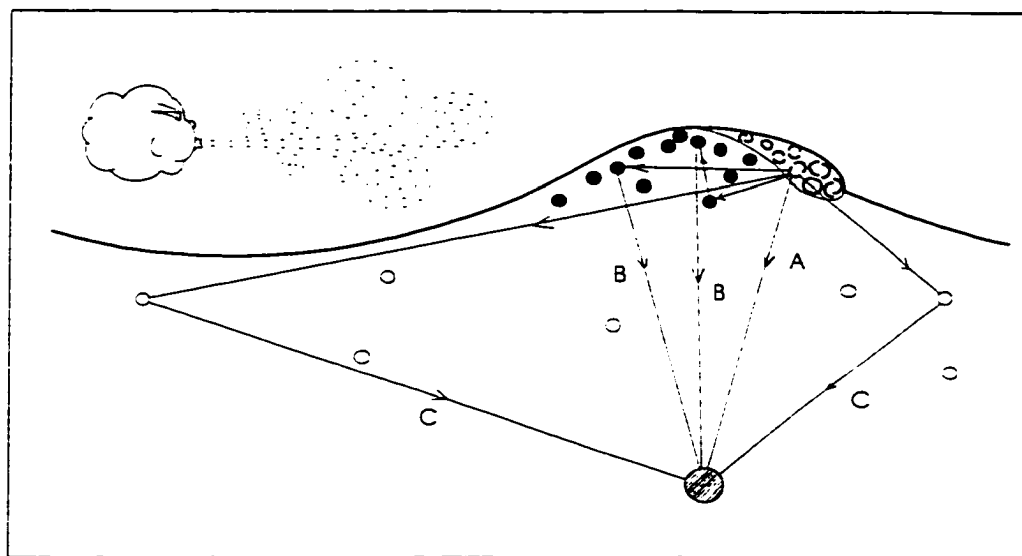


FIGURE 2.7: Primary and secondary radiation for a breaking wave. The α bubble field is represented by the broken bubbles; β field by the filled circles; γ and η fields by the unfilled circles. Path A is direct radiation: B and C secondary or scattered radiation. Hatch-filled circle is a sensor.

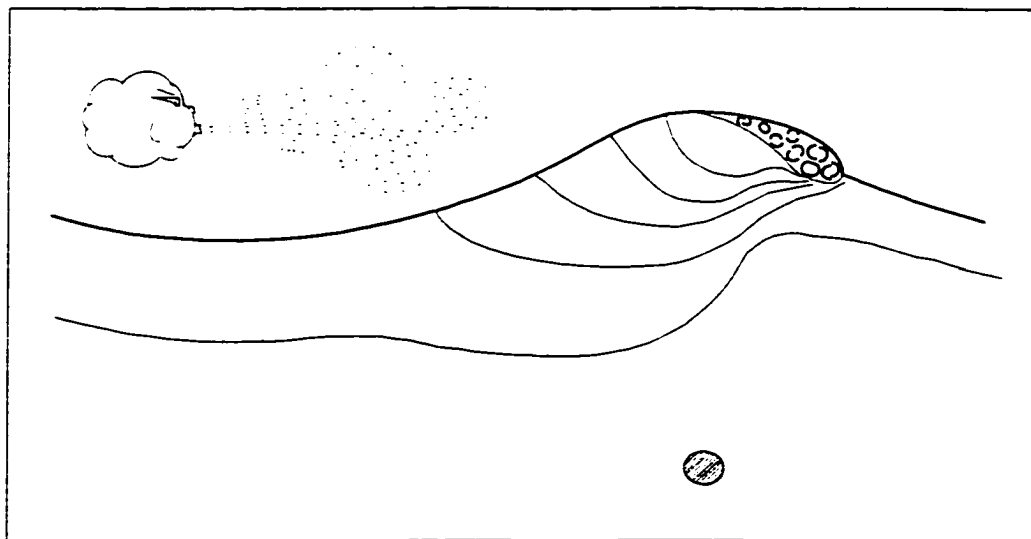


FIGURE 2.8: Bulk Sound Speed Analog for a Breaking Wave. A bulk or effective sound speed field $c(\mathbf{r})$, suggested here by representative isovelocity contours in an $x-z$ "slice", is substituted for the β , γ and η bubble fields.

Chapter 3

Signal Physics

In passive imaging, the problem is to utilize measurements of an acoustic field $p_t(\mathbf{r}, t)$ governed by the inhomogeneous wave equation

$$\nabla^2 p_t(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} p_t(\mathbf{r}, t) = f(\mathbf{r}, t)$$

to determine the nature of the inhomogeneous term $f(\mathbf{r}, t)$. The influence of a differential source element $f(\mathbf{r}', t)d\mathbf{r}'$ at $\mathbf{r}'$ is, of course, propagated out to a field point $\mathbf{r}$ by an appropriate Green function. The source function, however, might be a random function of time and space, in which case the field is also a random function. Moreover, since an individual breaking wave has finite spatial dimension, it must also be considered as an *extended source*. Both factors imply that the features of interest in the radiated acoustic field will be the higher-order statistics: this has a variety of implications for signal processing algorithms.

Therefore, section 3.1 introduces some background terminology regarding extended sources, and includes a discussion of the relevance of these terms to breaking waves. Section 3.2 presents the primary assumptions required for the broadband imaging problem and follows with a derivation of the integral equation relating source information to field quantities: this is the fundamental result used in the remainder of this work. Two lesser issues are covered for completeness in section 3.3: a discussion of Green functions

and the related wavenumber bandwidth of a propagated field.

3.1 Extended Sources

Although individual bubble vibration has been shown in chapter two to be a deterministic damped sinusoid, the cumulative acoustic effect of the superposition of damped sinusoids from millions of bubbles distributed over an extended source region is more properly treated within the theory of partial coherence (Beran and Parrent [13]). This is the most general approach for handling sources which are extended in either time or space. Under this paradigm, source and field information depend on second-order moments of $p(\mathbf{r})$. The deterministic problem is then a special case.

Consider source acoustic pressure disturbances $p(\mathbf{r}'_S, t_1)$ and $p(\mathbf{r}''_S, t_2)$, where $\mathbf{r}'_S$ and $\mathbf{r}''_S$ are located within the source region. The *mutual coherence function* (MCF) is then

$$\Gamma_t(\mathbf{r}'_S, \mathbf{r}''_S, t_1, t_2) = \langle p^*(\mathbf{r}'_S, t_1) p(\mathbf{r}''_S, t_2) \rangle. \quad (3.1)$$

(Here the notation $\langle \cdot \rangle$ denotes ensemble averaging and $*$ complex conjugation.) Many random processes are wide sense-stationary, for which the mutual coherence depends only on the temporal difference $\tau = t_2 - t_1$; when this is true, the *mutual spectral density* (MSD) is defined via the Wiener-Khinchin theorem as the Fourier transform of the MCF:

$$\Gamma(\mathbf{r}'_S, \mathbf{r}''_S, \omega) = \mathcal{F} \{ \Gamma_t(\mathbf{r}'_S, \mathbf{r}''_S, \tau) \} \quad (3.2)$$

Some standard terminology follows from Eqs.(3.1) and (3.2): when $\mathbf{r}''_S = \mathbf{r}'_S$, the MSD is known as the source intensity $I(\mathbf{r}'_S, \omega) = \Gamma(\mathbf{r}'_S, \mathbf{r}'_S, \omega)$ at $\mathbf{r}'_S$; the MSD can then be normalized by the intensity to yield the *degree of spectral coherence*

$$\gamma(\mathbf{r}'_S, \mathbf{r}''_S, \omega) = \frac{\Gamma(\mathbf{r}'_S, \mathbf{r}''_S, \omega)}{\sqrt{I(\mathbf{r}'_S, \omega)} \sqrt{I(\mathbf{r}''_S, \omega)}} \quad (3.3)$$

The degree of spectral coherence may be complex: its modulus lies in $[0, 1]$. When the modulus equals one, the source is described as *coherent*. When the modulus equals

zero, the source is described as *incoherent*. For all intermediate values, the source is said to be *partially coherent*.

(N.B. It is important to distinguish between sources and acoustic waves in this definition. Bracewell ([18], chapter 12) emphasizes that the waves generated by sources may well propagate within the source region itself (setting up an “interior field”) before radiating into the exterior region. *Primary* sources are assumed to be uncoupled to this interior field and therefore provide a forcing independent of the interior field. The source MSD involves only the cross-statistics of these independent forcing functions. If, on the other hand, the sources couple to the interior field, the sources become *secondary* sources, the problem can be treated under the sound speed analog, and it is no longer appropriate to consider the source MSD.)

The physical significance of these three types of sources for breaking waves is discussed below:

coherent This represents the surface of a loudspeaker or the hull of the ship and is not likely to be applicable to natural sources. A fully coherent source produces a fully coherent field with constructive and destructive interference patterns that provide directivity.

incoherent Incoherent source elements radiate fields unrelated to one another. Incoherent sources, however, produce a partially coherent field. A familiar example is that of background ambient noise in the ocean (Cron and Sherman [28]). If the noise is modeled as originating from a distribution of equi-power incoherent sources distributed uniformly over 4π incident angles, the noise is called “isotropic noise” and $|\gamma|^2$ (which here equals the normalized MSD) has the well-known shape of a spherical Bessel function, which is non-zero at nearly every two-point separation distance.

For breaking waves, freshly entrained bubbles immediately commencing freely oscillating birthing wails may represent an incoherent distribution of primary

sources.

partially coherent Individual source elements are partially correlated: one example recently given in the optics literature would be a Bessel-correlated source (Foley, Kim and Nussenzveig [51]). In this and similar models, correlation is high between adjacent elements and decreases with increasing separation distance. Partially coherent sources yield partially coherent fields which also exhibit in general some degree of directivity.

Partial coherence may be a significant issue for breaking waves within the frequency range [~ 5 Hz, ~ 400 Hz]. This, however, is a range that does not correspond to any individual bubbles so far detected: this was the motivation for the theory of collective oscillations discussed in section 2.1.3. If the secondary sources within the two-phase flow — that is, the bubbles — are considered “sources”, then they would define a partially coherent source.

The *secondary* sources, however, are not *primary* sources, and so cannot be treated in the same manner as primary sources in this imaging application. Since the primary forcing below about 400 Hz remains unclear, it is not possible to make a statement regarding the coherence of this mechanism in this frequency range.¹

3.2 Source Information Propagation

We begin the derivation of the broadband integral equation for the breaking wave image by stating the principle assumptions of the signal model:

Assumption 1 *The source excitation at frequencies above about 400 - 500 Hz is due to individual bubbles radiating at their Minnaert frequency as they reconfigure into spheroidal shapes.*

¹Nicholas *et al.* [105], in their analysis of the bubble column experiment, and Oğuz [107], in his model of bubble-cloud mediated ambient sound generation, have implicitly regarded the forcing to be an incoherent wideband source.

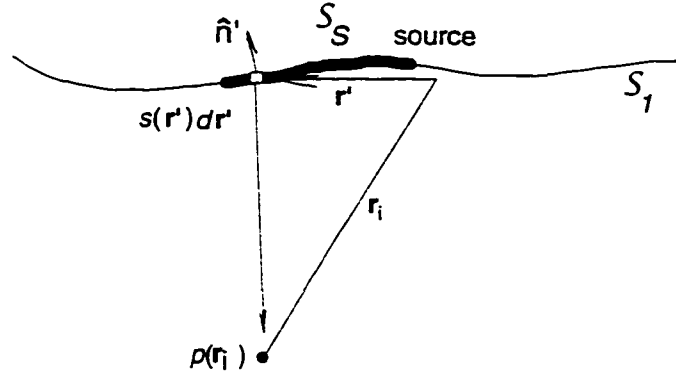


FIGURE 3.1: Radiation from a source element

Assumption 2 *Bubble forcing is mostly oscillatory in character and will time-average to zero.*

Assumption 3 *Bubbles of different sizes are created in equal proportions within each unit volume in the source, i.e., the bubble size distribution of freshly entrained bubbles is geometrically similar throughout the entrainment region.*

Assumption 4 *Bubbles contribute at frequencies above about 400 Hz independently to the sound field, so that, as sources, they are statistically independent in space and frequency.*

Consider the acoustic field pressure at field point $\mathbf{r}_i$ due to a time-domain excitation $s(t; \mathbf{r}') d\mathbf{r}'$ from a source element at $\mathbf{r}'$ as shown in figure 3.1. The source element is assumed to be located on a boundary S_1 where homogeneous Dirichlet conditions hold everywhere except over the region S_s . The source element contributes a component

$$s(t; \mathbf{r}') * \frac{\partial}{\partial n'} g_t(\mathbf{r}_i, \mathbf{r}', t) dS' \quad (3.4)$$

to the total acoustic field, where $g_t(\mathbf{r}_i, \mathbf{r}', t)$ is the time-domain Green function appropriate for the problem. $*$ denotes convolution, and $\partial/\partial n'$ is the normal derivative with respect to the primed quantity at $\mathbf{r}'$ (Barton [8], ch. 10).

The total acoustic field at the field point is the sum of contributions from each elemental source within the source volume. For an extended source, this “sum” takes the form of an integral over the 3-D source volume. Thus the full acoustic field is

$$p_t(\mathbf{r}_i, t) = \int_{S_s} s(t; \mathbf{r}') * \frac{\partial}{\partial n'} g_t(\mathbf{r}_i, \mathbf{r}', t) dS' \quad (3.5)$$

Each elemental source excitation $s(t; \mathbf{r}')$ in a breaking wave is a random function, so it is convenient to write $s(t; \mathbf{r}')$ using the generalized Cramér representation

$$s(t; \mathbf{r}) = \int_{-\infty}^{\infty} e^{i\omega t} dZ(\omega; \mathbf{r}). \quad (3.6)$$

The principle assumptions impose the following mathematical and statistical properties on the orthogonal increments $dZ(\omega; \mathbf{r})$:

$$\langle dZ(\omega; \mathbf{r}) \rangle = 0 \quad (\text{Assumption 2}) \quad (3.7a)$$

$$\langle dZ^*(\omega; \mathbf{r}) dZ(\omega'; \mathbf{r}') \rangle = \Gamma_{ss}(\mathbf{r}, \mathbf{r}'; \omega) \delta(\omega - \omega') d\omega d\omega' \quad (\text{Assumption 4}) \quad (3.7b)$$

$$\Gamma_{ss}(\mathbf{r}, \mathbf{r}'; \omega) = S_{ss}(\omega) \mathcal{I}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \quad (\text{Assumptions 3 and 4}) \quad (3.7c)$$

That is, considering only frequencies $\gtrsim 400$ Hz, the orthogonal increments $dZ(\omega; \mathbf{r})$ have zero mean and are statistically independent at different frequencies. The function $\Gamma_{ss}(\mathbf{r}, \mathbf{r}'; \omega)$ is the MSD of the source region and is modeled by a delta spatial correlation. In addition, the source is considered to be separable, the product of a frequency-dependent autospectral density source strength $S_{ss}(\omega)$ and a broadband acoustic shape function $\mathcal{I}(\mathbf{r})$. The shape function defines the acoustic shape of the breaking wave: this function will be non-negative anywhere within the acoustically active region and zero outside.

The intention in this work is to use $\mathcal{I}(\mathbf{r})$ as a surrogate for the areal extent of the breaking wave.

Using Eq.(3.4) ,Eq.(3.5) and Eq.(3.6), the full acoustic field at field point $\mathbf{r}_i$ is

$$\begin{aligned}
 p_t(\mathbf{r}_i, t) &= p_{t,i}(t) \\
 &= \int_{S_s} s(t; \mathbf{r}') * \frac{\partial}{\partial n'} g_t(\mathbf{r}_i, \mathbf{r}', t) dS' \\
 &= \int_{-\infty}^{\infty} dt' \int_{S_s} dS' s(t'; \mathbf{r}') \frac{\partial}{\partial n'} g_t(\mathbf{r}_i, \mathbf{r}', t - t') \\
 &= \int_{-\infty}^{\infty} dt' \int_{S_s} dS' \frac{\partial}{\partial n'} g_t(\mathbf{r}_i, \mathbf{r}', t - t') \int_{-\infty}^{\infty} dZ(\omega; \mathbf{r}') e^{i\omega t'} \\
 &= \int_{-\infty}^{\infty} dZ(\omega; \mathbf{r}') \int_{S_s} dS' \frac{\partial}{\partial n'} g(\mathbf{r}_i, \mathbf{r}'; \omega) e^{i\omega t}. \tag{3.8}
 \end{aligned}$$

where $g(\mathbf{r}_i, \mathbf{r}'; \omega)$ is the Fourier transform of the time-domain Green function.

To connect the available time-domain data with the intensity profile, we must consider the cross-statistics $r_{i,j}(\tau)$ between sensors i and j : by Eq.(3.1), this is the MCF

$$\Gamma_t(\mathbf{r}_i, \mathbf{r}_j, \tau) = \langle p_t(\mathbf{r}_i, t) p_t(\mathbf{r}_j, t + \tau) \rangle = \langle p_t^*(\mathbf{r}_i, t) p_t(\mathbf{r}_j, t + \tau) \rangle.$$

Using Eq.(3.8) this becomes

$$\begin{aligned}
 \Gamma_t(\mathbf{r}_i, \mathbf{r}_j, \tau) &= \iint_{S_s} dS' dS'' \int_{-\infty}^{\infty} \langle dZ^*(\omega; \mathbf{r}') dZ(\omega'; \mathbf{r}'') \rangle \\
 &\quad \times \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n''} g(\mathbf{r}_j, \mathbf{r}''; \omega') \right) e^{i(\omega' - \omega)t + i\omega' \tau} \\
 &= \iint_{S_s} dS' dS'' \int_{-\infty}^{\infty} d\omega d\omega' S_{ss}(\omega) \mathcal{I}(\mathbf{r}') \delta(\omega' - \omega) \delta(\mathbf{r}' - \mathbf{r}'') \\
 &\quad \times \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n''} g(\mathbf{r}_j, \mathbf{r}''; \omega') \right) e^{i(\omega' - \omega)t + i\omega' \tau} \\
 &= \int_{S_s} dS' \mathcal{I}(\mathbf{r}') \int_{-\infty}^{\infty} d\omega S_{ss}(\omega) \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) e^{i\omega \tau} \\
 &= \int_{-\infty}^{\infty} d\omega \left[S_{ss}(\omega) \int_{S_s} dS' \mathcal{I}(\mathbf{r}') \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) \right] e^{i\omega \tau} \\
 &= \mathcal{F}^{-1} \left\{ S_{ss}(\omega) \int_{S_s} dS' \mathcal{I}(\mathbf{r}') \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) \right\}
 \end{aligned}$$

or, ultimately,

$$\Gamma_t(\mathbf{r}_i, \mathbf{r}_j, \tau) = \mathcal{F}^{-1} \left\{ S_{ss}(\omega) \int_{S_s} dS' \mathcal{I}(\mathbf{r}') \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) \right\} \tag{3.9}$$

However, by Eq.(3.2), this can be written as

$$\begin{aligned}\Gamma_i(\mathbf{r}_i, \mathbf{r}_j, \tau) &= \mathcal{F}^{-1}\{\Gamma(\mathbf{r}_i, \mathbf{r}_j, \omega)\} \\ &= \mathcal{F}^{-1}\{\Gamma_{i,j}(\omega)\}\end{aligned}\quad (3.10)$$

i.e., the inverse Fourier transform of the MSD between sensors i and j . Comparing Eq.(3.9) with Eq.(3.10), a simpler relation can be derived for the shape function:

$$\Gamma_{i,j}(\omega) = S_{ss}(\omega) \int_{S_s} dS' \mathcal{W}(\mathbf{r}') \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right)$$

Since the broadband shape function $\mathcal{W}(\mathbf{r})$ is zero outside the source region, the domain of integration may be extended over all space without altering the results, yielding

$$\Gamma_{i,j}(\omega) = S_{ss}(\omega) \int_{-\infty}^{\infty} dS' \mathcal{W}(\mathbf{r}') \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) \quad (3.11)$$

This is the fundamental result: Eq.(3.11) shows that the MSD between two points in space is in fact a functional of the shape function, and nearly has the form of a general inverse problem. If the source strength autospectrum $S_{ss}(\omega)$ were known, Eq.(3.11) would be a Fredholm integral equation of the first kind for the shape function. However, the source autospectrum is not known, so the problem is a bit more complicated.

Image solutions based on Eq.(3.11) are the subject of chapter four.

Discussion of the assumptions There are two primary assumptions made in this section: temporal/spatial independence, and MSD separability.

temporal/spatial independence This model has both experimental and theoretical support for the sound generated above about 400 Hz. Using acoustical techniques, Medwin and co-workers (discussed in section 2.2.3) detected bubbles with sizes that corresponded to resonant frequencies of 400+ Hz. Loewen and Melville [83] then conducted Monte Carlo computer simulations to produce synthetic time series and autospectra from source regions randomly populated by

bubbles drawn from Medwin's empirical bubble size distribution. Their results match the autospectral characteristics of oceanic ambient sound (e.g., that published by Wenz [139] quite well in both level and shape above about 400 - 500 Hz.

It is difficult to conceive of a mechanism that might "orchestrate" the bubble pinch-off times and resulting radii over substantial portions of the entrainment region: as discussed in section 2.3, the bubble birth times and sizes might only have significant correlation over distances much smaller than typical length scales of breakers. In addition, since a model incorporating an incoherent source region has features that are consistent with ambient oceanic sound, it appears that temporal and spatial independence is, by Occam's razor, a satisfactory model.

MSD separability There is no direct evidence that Assumption 3 is valid: in fact, there is very little evidence whatsoever regarding the dependence of $n(R; \mathbf{r}, t)$ on space and time within the α -plume. Kerman [63] advanced a theory regarding the evolution of $n(R; \mathbf{r}, t)$. He speculated that the total number of bubbles entrained may be a similar function of the strength of the breaker (and hence a function of wind stress u_*); but the distribution of bubbles, following principles of fracture mechanics, depended mostly on the size of each bubble, i.e., a property with strong local dependence.

If this assumption was not valid, one might expect the acoustic image to be more intense at some locations at certain frequencies than at others. To make this comparison, it would be necessary to form an image at each frequency. Unfortunately, as will be discussed in chapter four, such "single frequency" images cannot be reliably constructed from the data acquired during this project, and, therefore, such comparisons cannot be made.

3.3 Propagation Issues

3.3.1 Green Functions

In the real ocean, radiation that occurs in the α -plume region located near a randomly rough, negligible impedance boundary propagates down through the β - and γ -plumes and the η -layer to the sensors. These β , γ and η bubble fields may be treated simply as regions with a spatially-variant sound speed.

The effects of the anomalous sound speed and rough, low-impedance boundary could theoretically be subsumed into the Green function for the problem, and this would allow Eq.(3.11) to be an *exact* representation of the propagation of the MSD in the true oceanic nearsurface layer.

Furthermore, at the frequencies under consideration, the α -plume is acoustically thin, and the distance of the radiating bubbles from the surface acoustically negligible. This implies that, to a good approximation, the entire source region can be treated as an inhomogeneous boundary condition on the surface.

The actual problem would then have the geometry of figure 3.2: a volume $\mathcal{V}$ with sound speed $c(\mathbf{r})$ is bounded above at $z = \eta(x, y)$ by a surface $\mathcal{S}_1$ and below by a hemispherical surface $\mathcal{S}_2$. Homogeneous Dirichlet boundary conditions hold everywhere on $\mathcal{S}_1$ except for a “patch” of finite support $\mathcal{S}_S$ which has an inhomogeneous boundary condition $p(\mathbf{r}) = \sigma(\mathbf{r})$, say. The acoustic field at a field point $p(\mathbf{r})$ is then specified by

$$\nabla^2 p(\mathbf{r}) + k^2(\mathbf{r})p(\mathbf{r}) = 0 \quad (3.12)$$

(that is, no sources within the fluid medium) with boundary conditions

$$p(\mathbf{r}) = \begin{cases} 0 & \text{for } \mathbf{r} \in \mathcal{S}_1 \text{ and } \mathbf{r} \notin \mathcal{S}_S \\ \sigma(\mathbf{r}) & \text{for } \mathbf{r} \in \mathcal{S}_S. \end{cases}$$

The field and its normal derivative on $\mathcal{S}_2$ need not be specified. (See below.)

This problem is treated in the standard way (Morse and Ingard [103]) using the (Dirichlet) Green function $G_D(\mathbf{r}, \mathbf{r}_0)$ which solves the auxiliary partial differential equa-

tion

$$\nabla^2 G_D(\mathbf{r}, \mathbf{r}_0) + k^2(\mathbf{r})G_D(\mathbf{r}, \mathbf{r}_0) = -\delta(\mathbf{r} - \mathbf{r}_0)$$

and in addition obeys the homogeneous Dirichlet boundary condition $G_D(\mathbf{r}, \mathbf{r}_0) = 0$ for either $\mathbf{r}$ or $\mathbf{r}_0$ on the boundary $\mathcal{S}_1$. This, of course, converts, using the Helmholtz integral (Andrew [3]) to

$$\begin{aligned} p(\mathbf{r}) &= - \oint\!\!\!\oint_{\mathcal{S}_1 \cup \mathcal{S}_2} [G_D(\mathbf{r}, \mathbf{r}_0) \nabla_0 p(\mathbf{r}_0) - p(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0)] \cdot d\mathbf{S}_0 \\ &= \iint_{\mathcal{S}_1} \sigma(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0) \cdot d\mathbf{S}_0 - \iint_{\mathcal{S}_2} [G_D(\mathbf{r}, \mathbf{r}_0) \nabla_0 p(\mathbf{r}_0) - p(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0)] \cdot d\mathbf{S}_0 \\ &= \iint_{\mathcal{S}_S} \sigma(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0) \cdot d\mathbf{S}_0 - \iint_{\mathcal{S}_2} [G_D(\mathbf{r}, \mathbf{r}_0) \nabla_0 p(\mathbf{r}_0) - p(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0)] \cdot d\mathbf{S}_0 \end{aligned} \quad (3.13)$$

where the surface normals point into $\mathcal{V}$. The first integral in the last line above follows because the contribution from the surface is zero everywhere except over $\mathcal{S}_S$. The second integral in the last line above will be shown below to be zero.

In practice, neither the surface geometry nor the sound speed anomaly $c(\mathbf{r})$ versus $\mathbf{r}$ are known very well. A model that approximates the actual conditions but allows for a greatly simplified Green function is shown in figure 3.3: the surface $\mathcal{S}_1$ is assumed to be the $\eta = z = 0$ plane, and the halfspace $\mathcal{V}$ is assigned a constant sound speed c_0 .

The function G_D which satisfies this simpler model is

$$G_D = \frac{e^{ik\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}}}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}} - \frac{e^{ik\sqrt{(x-x_0)^2+(y-y_0)^2+(z+z_0)^2}}}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+z_0)^2}}. \quad (3.14)$$

Clearly, using a simplified Green function in an imaging problem will result in a degradation of the resulting image. One obvious way, therefore, of improving image quality is to develop a more accurate Green function for the actual nearsurface layer. As an example, Oğuz [107] modeled the subsurface bubble clouds as hemispherical volumes of fluid, with an alternate sound speed c_{eff} , attached to the ocean surface.

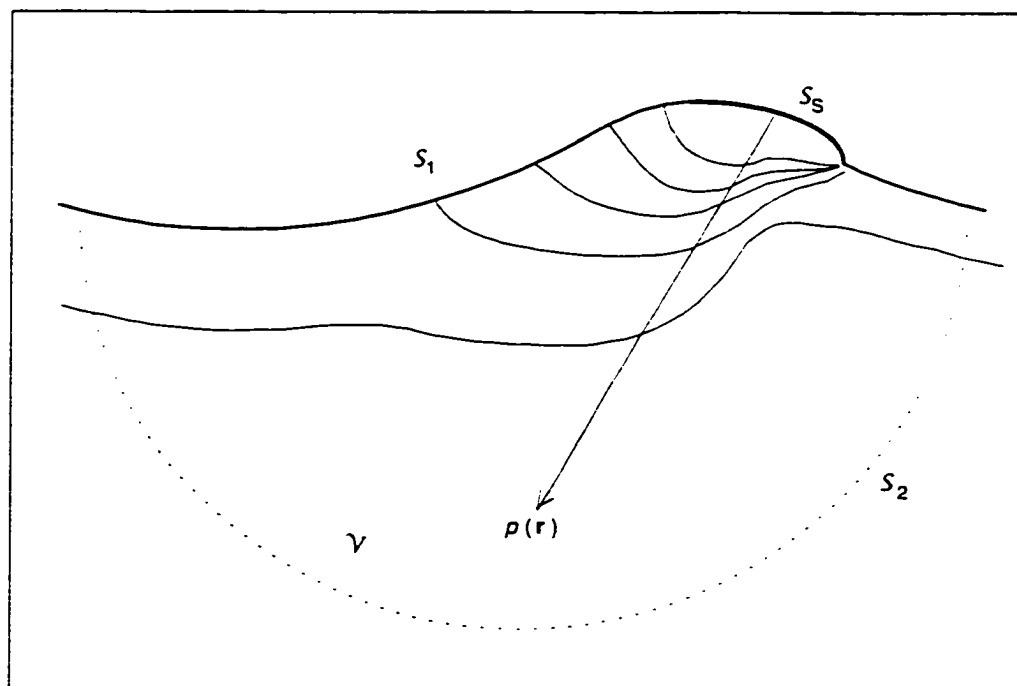


FIGURE 3.2: Actual propagation problem

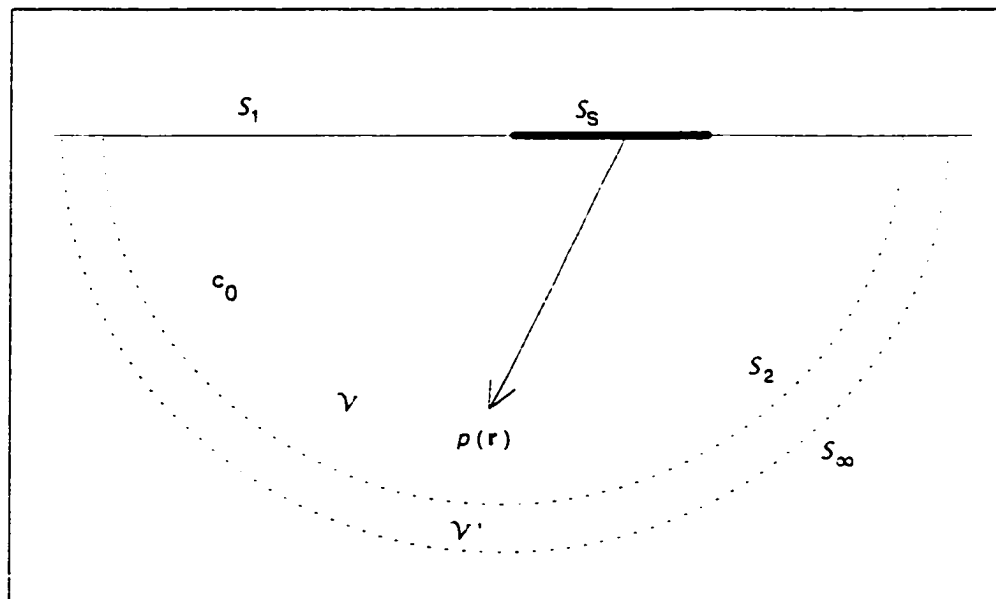


FIGURE 3.3: Idealized propagation problem.

These hemispherical fluid “inclusions” can be considered resonant cavities or “acoustic filters” or even acoustic “lenses”.

The second integral in Eq.(3.13) is zero according to the following argument. Consider the volume $\mathcal{V}'$ shown in figure 3.3: this is a hemispherical “shell” bounded on the inside by surface $\mathcal{S}_2$, on the outside by surface $\mathcal{S}_\infty$, and on the top by a portion of the surface $\mathcal{S}_S$, call it $\mathcal{S}_{SS}$. There are no sources within $\mathcal{V}'$ so the Helmholtz integral equation for the field $p(\mathbf{r})$ at field point $\mathbf{r}$ within $\mathcal{V}$ is

$$p(\mathbf{r}) = \oint\limits_{\mathcal{S}_2 \cup \mathcal{S}_\infty \cup \mathcal{S}_{SS}} [G_D(\mathbf{r}, \mathbf{r}_0) \nabla_0 p(\mathbf{r}_0) - p(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0)] \cdot d\mathbf{S}_0 = 0$$

where the surface normal points out of $\mathcal{V}'$. This integral is zero because the evaluation point, $\mathbf{r}$ is outside $\mathcal{V}'$.

As before, the integral over $\mathcal{S}_{SS}$ is zero because either G_D or p is zero on that boundary. The integral over $\mathcal{S}_\infty$ will be zero in the limit when the radius of that surface goes to infinity (assuming both G_D and p satisfy the Sommerfeld radiation condition (Junger and Feit,[62])). This implies that

$$\iint\limits_{\mathcal{S}_2} [G_D(\mathbf{r}, \mathbf{r}_0) \nabla_0 p(\mathbf{r}_0) - p(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0)] \cdot d\mathbf{S}_0 = 0$$

and therefore, Eq.(3.13) becomes

$$p(\mathbf{r}) = \iint\limits_{\mathcal{S}_S} \sigma(\mathbf{r}_0) \nabla_0 G_D(\mathbf{r}, \mathbf{r}_0) \cdot d\mathbf{S}_0 \quad (3.15)$$

($d\mathbf{S}_0$ here has direction $-\hat{\mathbf{z}}$.)

Discussion of the propagation model assumptions The two key assumptions made in this section, the flat surface and a uniform sound speed, are critiqued below.

flat surface model The appropriate way to determine the error caused by substituting a flat surface for the curved surface is to do a perturbation expansion of the field in terms of some surface roughness parameter ζ . This could indeed be done if the

frequency	larger sea	smaller sea
500 Hz	1.80	0.33
2000 Hz	7.21	1.34

TABLE 3.1: Rayleigh roughness parameters for the larger and smaller sea forecasts

surface S_1 were known. However, rather than invoke all this mathematical machinery, we might gauge the problem utilizing as a first approximation the Rayleigh roughness parameter (Brekhovskikh and Lysanov [20], section 9.1)

$$\zeta = 2k\eta_{\text{rms}} \cos \theta. \quad (3.16)$$

The surface is considered rough if $\zeta \gg 1$ and smooth if $\zeta \ll 1$. This parameter usually arises in forward scattering problems, where θ is the “angle of incidence”. This has no meaning here, so let us assume the worst case value of $\theta = 0$.

The surface roughness η_{rms} is calculated for the two sinusoidal seas forecast in Appendix A. The roughness parameters at low and high passband frequencies are shown in table 3.1 for these two seas.

It is seen that the flat surface is not a good approximation at higher frequencies, though perhaps it is adequate at the lower end.

In SURFEX III, the time-varying 2-D surface over the array was not measured, so corrections for surface roughness cannot be made. More accurate imaging would require knowledge of this surface, especially at the higher frequencies.

uniform velocity Table 2.1 suggests that the γ and η bubble fields modify the bulk sound speed by an amount $\Delta c \ll c_0$. Lamarre and Melville [75] recently confirmed these values with direct sound speed measurements at depths from 0.5 m to 2.0 m in winds of about 8 ms^{-1} . They found they were unable to detect sound speed anomalies Δc greater than about 0.1% of c_0 below about 1 m.

This would imply that the sound speed in the fluid surrounding the array is, to a

good approximation, c_0 , and that this approximation remains valid above the array to within about 1 m of the surface.

Lamarre and Melville detected anomalies $\Delta c \lesssim 1 - 3\%$ of c_0 in the γ field within 0.5 m of the surface. This too may be approximated as c_0 . But they also noticed that the β -plumes could have anomalies as large as $O(0.5c_0)$.

This suggests that the β -plume establishes a fluid “inclusion” with a significantly different sound speed over a vertical scale of about a wave height. For forecast wave-heights (see Appendix A) this fluid inhomogeneity is acoustically small at the lower passband frequencies (where $\lambda \approx O(2 - 5\text{m})$) but becomes significant at the upper passband frequency (where $\lambda \approx O(0.7 - 0.8\text{m})$).

Unfortunately, the space-time subsurface bubble field above the array was not measured during SURFEX III, so propagation anomalies caused by β -plumes cannot be corrected. More accurate imaging must include this knowledge and use a more appropriate Green function.

3.3.2 Propagation as a Bandlimiting Process

Nearfield acoustical holography (NAH, Maynard *et al.* [92]), using an exact mathematical description of scalar wave propagation, predicts a property of the measured wavefield that will prove to be useful in chapter six. If an acoustic field $p(x, y, z_0)$ is known or specified on the plane $z = z_0$, then the field $p(x, y, z_A)$ on a parallel plane at $z = z_A$ is given by

$$p(x, y, z_A) = p(x, y, z_0) ** G_1(x, y; z_0, z_A) \quad (3.17)$$

where $**$ denotes 2-D convolution and $G_1(x, y; z_0, z_A)$ is the normal derivative with respect to z_0 of the Dirichlet Green function Eq.(3.14). Propagation can therefore be viewed as a (spatial) filtering operation. This filtering operation essentially bandlimits the propagating field: consider the 2-D Fourier transform of Eq.(3.17):

$$\tilde{p}(k_x, k_y, z_A) = \tilde{p}(k_x, k_y, z_0) \tilde{G}_1(k_x, k_y; z_0, z_A);$$

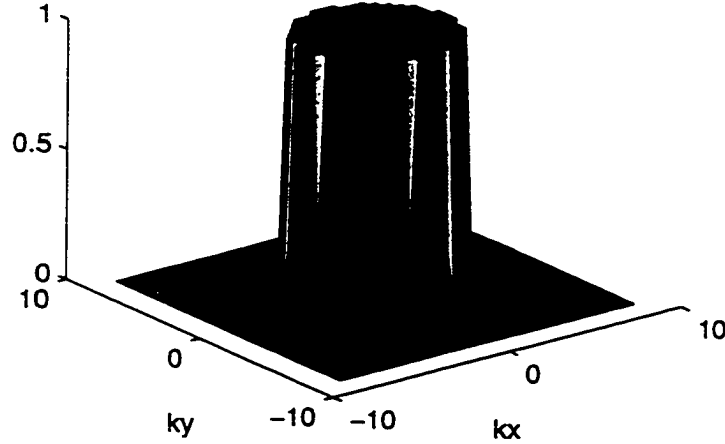


FIGURE 3.4: Magnitude of the k space Green function, $k = 4.2$. The vertical axis is $|\tilde{G}_1(k_x, k_y; z_a, z_0)|$.

here $\tilde{G}_1(k_x, k_y; z_0, z_A)$ is known as the k space Green function which in this case is

$$\tilde{G}_1(k_x, k_y; z_0, z_A) = -e^{i|z_A - z_0|\sqrt{k^2 - k_x^2 - k_y^2}} \quad (3.18)$$

If the source plane is considered the ocean surface at $z_0 = 0$ and z_A is taken to be a typical array depth of 3 m, then the magnitude of the k space Green function at $k = 4.2 \text{ m}^{-1}$ (this corresponds to a wavelength of 1.5 m, or half the array depth) has the shape shown in figure 3.4: $\tilde{G}_1$ acts as a low-pass filter in the wavenumber domain, with a magnitude equal to one over the support $k_x^2 + k_y^2 < k^2$ but dropping off outside.

At interplane separation distances greater than about one wavelength, the measured data are essentially bandlimited at the propagating wavenumber, so that the best resolution achievable is the Rayleigh limit of about half a wavelength. For smaller interplane separation, the measurements are in the extreme nearfield of the source, and contain additional information via non-propagating or evanescent wavefield components.

For SURFEX III, sources in the frequency range considered valid for incoherent source mechanisms are essentially in the Fresnel zone of the array and little evanescent

information is available. This implies that the wavenumber spectrum of the field at the measurement plane will be bandlimited to wavenumbers less than $k = 2\pi/\lambda$.

This notion is directly applicable to the propagation of the MSD, and implies that the field intensity $\Gamma(\mathbf{r}, \mathbf{r}; \omega)$, which physically represents the autospectral power at each hydrophone, is a bandlimited spatial function.

Bandlimitedness is used in chapter six to provide a crude initial guess of the centroid location of the breaking wave over the array.

Chapter 4

Image Reconstruction

4.1 Introduction

The imaging process transforms source information $O(\mathbf{x})$ with an instrument, or system, possessing some kind of “impulse response” $H(\mathbf{x}, \mathbf{x}')$, to produce an image $M(\mathbf{x})$. The relationship between the object and the image can usually be described quite generally as

$$M(\mathbf{x}) = \int H(\mathbf{x}, \mathbf{x}') O(\mathbf{x}') d\mathbf{x}'. \quad (4.1)$$

The function $H(\mathbf{x}, \mathbf{x}')$ is called the *kernel* in the theory of integral equations. In addition, the available image $D(\mathbf{x})$ (the “data”) usually consists of the image contaminated by noise, so that

$$D(\mathbf{x}) = M(\mathbf{x}) + N(\mathbf{x}) \quad (4.2)$$

where $N(\mathbf{x})$ is some noise process.

A conventional acoustical “imaging system”, as shown in figure 4.1, can be described in this framework. Information about the source (the “object”) propagates from the source region to an array of L sensors. The sensor data are then “steered” in some fashion using weighted time delays w_i (these would be weighted phase factors in a

frequency-domain implementation) and then summed to produce the output image. This is, of course, simply a beamforming system, and the kernel is shown in section 4.3 to be defined by the weighting and summing operation. The data steering, summation and image production could be implemented in a real-time "imaging system" or, as in the present application, performed off-line on raw recorded sensor outputs. In either case, the "image plane" is not a physical location, as in optical imaging, but rather a mathematical abstraction of the domain of the resulting image.

As the beamformer steer point is rastered over the source plane, the system output produces a noisy image $D(\mathbf{x}) = M(\mathbf{x}) + N(\mathbf{x})$. This implementation evidently requires a set of weights $\{w_i\}$: in section 4.2, a simple choice of weights is shown to provide

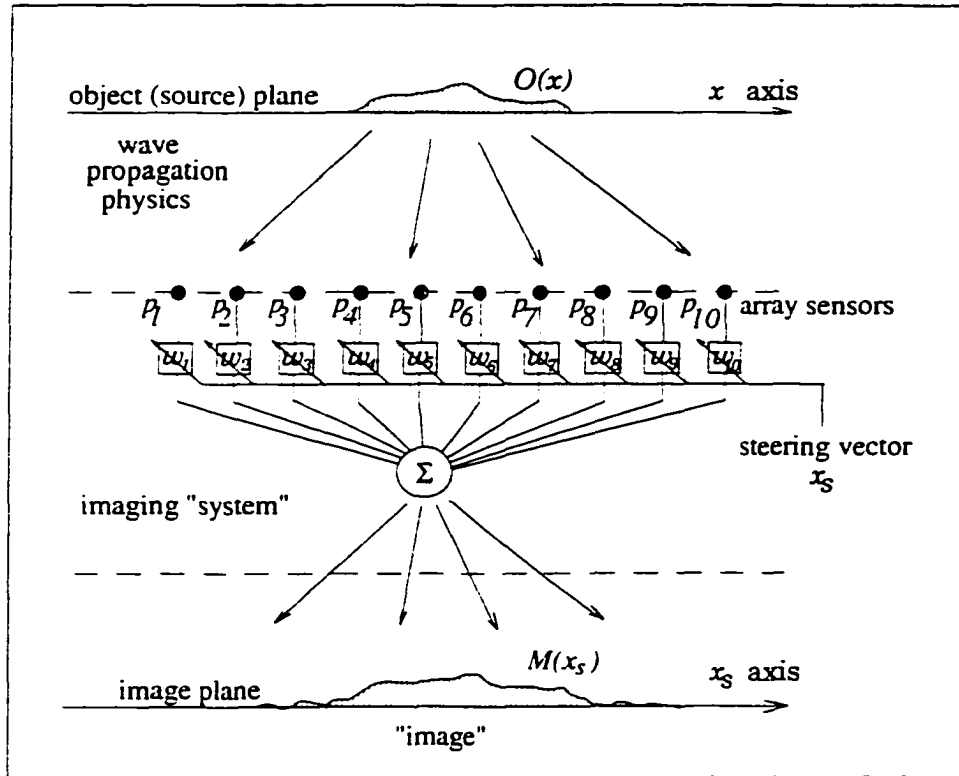


FIGURE 4.1: Schematic diagram of a 1-D imaging problem: information from a 1-D source at $z = 0$ propagates to an array of sensors.

a useful measure of resolution. If, given some choice of weights, the resulting noisy image $D(\mathbf{x})$ is judged to be a satisfactory representation of the features of interest in the object $O(\mathbf{x})$, then $D(\mathbf{x})$ could be used in further analysis as a surrogate for the object.

In section 4.3, however, the sparse sensor constellation of the *actual* SURFEX III array is shown to cause an unacceptably degraded kernel in any conventional beam-forming imaging system.

Therefore, given that any available image $D(\mathbf{x})$ would not be a satisfactory representation of the object information $O(\mathbf{x})$, the problem is then to “reconstruct” the object from the data and knowledge of the kernel. DeMoment [32] gives an excellent overview of image reconstruction problems, and shows that they are a typical class of inverse problem. Hansen [57] and Press *et al.* ([114], chapter 18) both provide good introductions to inverse problems: the discussion below follows DeMoment’s but uses Press *et al.*’s notation which is a bit simpler.

The standard approach to an inverse problem is to find a solution $\mathcal{O}$ that minimizes the combination of two seminorms $\mathcal{A}$ and $\mathcal{B}$. The former is usually some measure $\mathcal{A}(D, \mathcal{O})$ of the “distance” between the data D and the “model data” $M(\mathcal{O})$, which will depend explicitly on $\mathcal{O}$. A typical measure is the goodness-of-fit, which is small when the model data nearly matches the real data and becomes larger as the fit worsens.

The latter is a regularization functional, which provides a measure of the “distance” between $\mathcal{O}$ and some *a priori* notion of an ideal solution. Inverse problems are well-known to be ill-posed, in the sense that a wide class of solutions minimize $\mathcal{A}(D, \mathcal{O})$ alone: a unique solution can only be obtained by singling out from that class the one solution that most closely fits some secondary criterion, represented by $\mathcal{B}(\mathcal{O})$. In general, the prescription is

$$\mathcal{O}_\mu = \arg \min_{\mathcal{O}} [\mathcal{A}(D, \mathcal{O}) + \mu \mathcal{B}(\mathcal{O})] \quad (4.3)$$

where μ is called the *regularization parameter*.

A direct solution for the image reconstruction problem requires two features: first, the dependence on the weights is eliminated in section 4.4 by reformulating the problem in a probabilistic framework; second, the field-of-view is partitioned in section 4.5 into a discrete number of cells, with the source shape function considered to be constant over each cell. No other assumptions are made regarding the shape of the shape function, so, largely in comparison to the next approach, this will be called the *nonparametric approach*, and is described in section 4.5.1.

An alternative approach, developed in section 4.5.2, is to approximate the image with a simple shape involving only a few controlling parameters. While this *parametric approach* produces less satisfactory images, it will prove highly useful as a tool for image interpretation and quantitative analysis in chapters six and seven.

4.2 Resolution and Superresolution

One of the often-examined properties of an imaging system is its resolution. A kernel $H(\mathbf{x}, \mathbf{x}')$ with “good resolution” will peak sharply and narrowly, e.g., like a Dirac delta function, near $\mathbf{x} = \mathbf{x}'$ and so will produce output closely resembling the source. Conversely, a kernel with “poor resolution” will be quite broad and smooth, and will produce a smoothed output that might not resemble the source at all.

There are two situations for array-based imaging systems. For arrays operating on incoherent far-field data (section 4.2.1), the kernel is known as the *beam pattern* and the typical measure of resolution is the *beamwidth* (Urick [137], section 3.9). For arrays operating on nearfield data (section 4.2.2), the kernel is known as the *point spread function* or PSF, and a measure of the resolution is the *half-width at half-maximum* or HWHM (Stoughton and Strait [124]).

The resolution issue in this project requires consideration of both the far-field and nearfield cases, as follows.

(N.B. Beam patterns and PSFs are usually frequency-dependent, so the rest of this

section considers the imaging problem “at some frequency ω ”. This might be effected by filtering each channel with a narrow bandpass filter centered around ω : alternatively, each channel could be transformed into the Fourier domain and only the coefficient corresponding to frequency ω passed on for subsequent processing.)

4.2.1 Far-Field Imaging

As a simple example, consider using a beamforming imaging system with unity gain weights (a “phased array”) and strictly far-field sources, as shown in figure 4.2. The far-field impulse response of the array for waves incident at angle θ when the array is steered to angle θ_S is now the beam pattern $B(\theta, \theta_S)$.

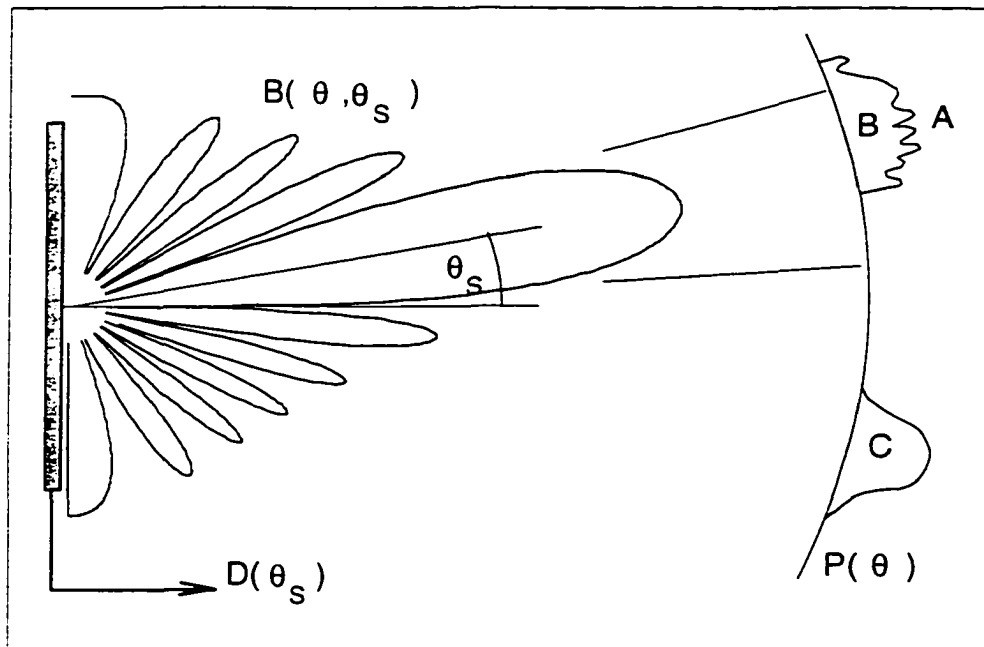


FIGURE 4.2: Conceptual drawing of “resolution”. Beampattern $B(\theta, \theta_S)$ of a line array steered to $\theta_S = 10^\circ$ from broadside. System output $D(\theta_S)$ is a function of steering angle. The far field incoherent intensity $P(\theta_S)$ has two main features at B and C: feature B has in addition some “ripples”. The approximate main lobe beamwidth is indicated.

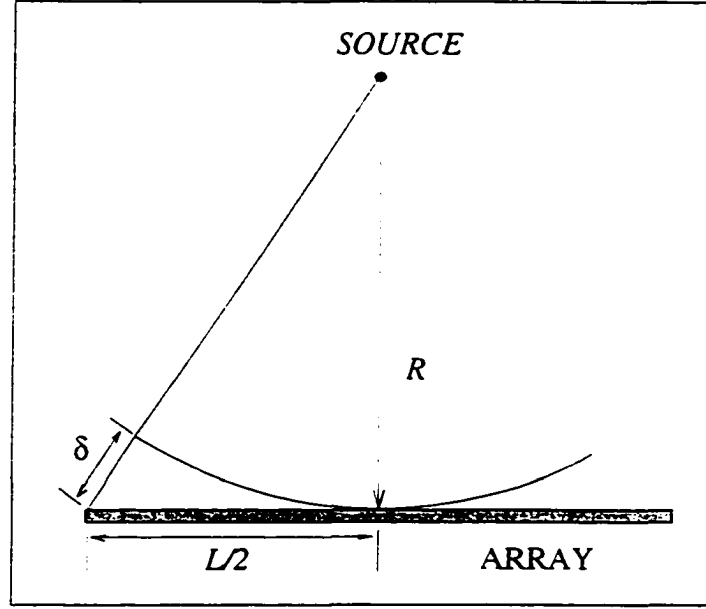


FIGURE 4.3: Geometrical considerations for defining the far field of an array.

The resolution of the system is pictorially and intuitively related to the width of the main lobe. Rapid source oscillations like those at A are not well-resolved because they occur over a sector much narrower than the main lobe beamwidth. On the other hand, source features B and C are several beamwidths apart and will be well resolved. Better resolution therefore corresponds to narrower main lobe beamwidth. One possible measure of beamwidth is the -3 dB point (i.e., the beam “half-width”), which Urick ([137], section 3.9) gives approximately (for continuous line arrays steered broadside) as

$$\theta_{-3 \text{ dB}} [^\circ] = 25.3^\circ (\lambda/L) \quad (4.4)$$

where λ is the wavelength and L is the array aperture. Thus, at a given frequency, better resolution requires a narrower beam, which in turn requires a larger aperture.

The far-field approximation breaks down when sources are located closer than the far-field, or Fraunhofer zone, of the array. There is no definitive boundary between the Fraunhofer and Fresnel zones; different authors have used various criteria, usually

in relation to transmitting arrays. For a receiving array, it is convenient to adopt an array design rule-of-thumb (Steinberg [123], section 13.3) that requires the error in path length between the true curved wavefront and a hypothetical planar wavefront be less than one-tenth of a wavelength anywhere along the array. According to figure 4.3, the worst case path length error occurs at the end of an array receiving a broadside signal. By geometry,

$$(\delta + R)^2 = R^2 + L^2/4.$$

Thus

$$\frac{\delta}{\lambda} = \frac{R}{\lambda} \left(\sqrt{1 + L^2/4R^2} - 1 \right).$$

The Fraunhofer zone of the array would then be restricted to ranges R such that δ/λ is less than or equal to 1/10. Thus, setting $\delta/\lambda = 1/10$,

$$R = \frac{5}{4\lambda} (L^2 - \lambda^2/25) = \frac{L^2}{\lambda} \left(\frac{5}{4} - \frac{\lambda^2}{20L^2} \right).$$

The second term in the parentheses is a correction for apertures shorter than the wavelength (long wavelengths). When this term is neglected (i.e., when the aperture is at least a wavelength across) $R \approx 5/4 L^2/\lambda \approx L^2/\lambda$, a standard approximation.

Experience at IOS has shown that the largest practical structure deployable in coastal waters from available oceanographic vessels is limited to a maximum aperture of about $L = 8$ metres. Table 4.1 provides for several frequencies the resulting depths required to place surface sources in the Fraunhofer zone of such an aperture. In addition, given the array depth and -3 dB half beamwidth, the linear dimension of the main lobe footprint at the surface can be computed: half this quantity is an essential measure of resolution, and is also listed in table 4.1.

Table 4.1 shows that the phase error incurred by the Fraunhofer approximation increases with increasing frequency: the only way to retain the validity of the approximation at higher frequencies is to increase the distance between the array and the

frequency	wavelength	far-field boundary	-3 dB half width	resolution
[Hz]	[m]	[m]	[degrees]	[m]
250	6	13	19.0	4.5
500	3	26	9.5	4.4
1000	1.5	53	4.7	4.4
2000	0.8	107	2.4	4.5

TABLE 4.1: Onset of far field, -3 dB point and half footprint size] (i.e., resolution at the sea surface) for various frequencies for an array with an aperture of 8 m.

source. This increasing distance might be expected to decrease resolution at the surface, but in fact the beamwidth becomes narrower with increasing frequencies, so that, in this case, this simple far-field array system provides a nearly-constant resolution of about 4.5 metres at the ocean surface.

As described in chapter one, surface sources in Patricia Bay are expected to be roughly this size or a bit smaller, so a configuration with better resolution was required.

4.2.2 Nearfield Imaging

If the sources are located closer to the array than indicated in table 4.1, the sources fall either into the Fresnel zone or into the nearfield of the array. In either case, the concept of a beam pattern must be replaced by that of a PSF. The PSF is essentially the response on the “image plane” due to a point source at a particular position in the field of view (as opposed to a point source “at infinity” as is assumed in far-field imaging.) In general, the 2-D PSF may not be spatially invariant, that is, it will not merely translate when the point source is translated, but will change shape as well.

There are some approximations for finding the PSFs for simple array geometries, but for a practical application the PSF must be investigated numerically. Figure 4.4 shows the -3 dB halfwidth, using unity gain weights, of nearfield PSFs over the system

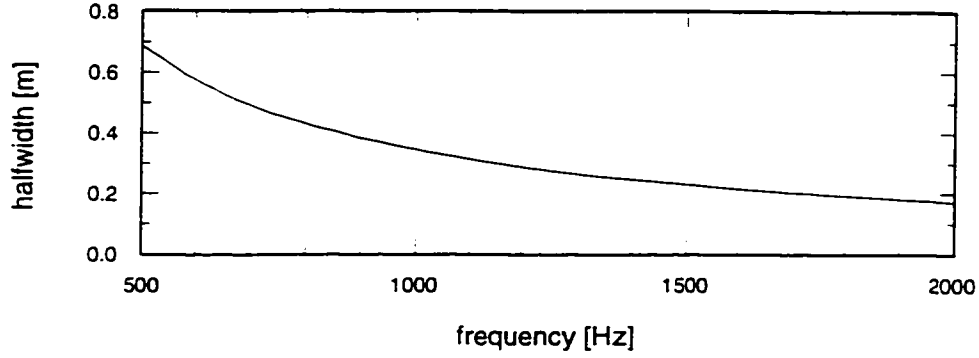


FIGURE 4.4: PSF half width (“resolution”) on a focal plane 3 metres from the array plane, versus frequency. SURFEX III sensor constellation, focal point is at (0,0).

passband. The geometry is that shown in figure 4.1: the array is “focused” at the broadside point of a source plane 3 metres above the array plane. The array aperture is again 8 m.

Thus, this simple example demonstrates that the nearfield array has better (sometimes significantly better) resolution than a “far-field” system, particularly at the upper frequency limit of the system.

4.3 Beamforming Imaging

Assume for the moment a set of weights $\{w_i\}$ and the beamforming system represented in figure 4.1. The beamformed output at steer point $\mathbf{x}_s$ and frequency ω is then

$$b(\mathbf{x}_s, \omega) = \sum_{l=1}^L w_l(\mathbf{x}_s, \omega) p(\mathbf{r}_l, \omega) \quad (4.5)$$

where $p(\mathbf{r}_l, \omega)$ is the Fourier component of the acoustic field at sensor position $\mathbf{r}_l$ at frequency ω .

Since the source in this problem is a stochastic source, the output of interest here

is the averaged square magnitude of the beamformer output

$$\begin{aligned} m(\mathbf{x}_s, \omega) &= \langle |b(\mathbf{x}_s, \omega)|^2 \rangle \\ &= \sum_{k=1}^L \sum_{l=1}^L w_k^*(\mathbf{x}_s, \omega) w_l(\mathbf{x}_s, \omega) \langle p^*(\mathbf{r}_k, \omega) p(\mathbf{r}_l, \omega) \rangle \end{aligned}$$

Now $\langle p^*(\mathbf{r}_k, \omega) p(\mathbf{r}_l, \omega) \rangle$ is the MSD $\Gamma(\mathbf{r}_k, \mathbf{r}_l, \omega)$ between the points $\mathbf{r}_k$ and $\mathbf{r}_l$, and is given by Eq.(3.11). Thus

$$m(\mathbf{x}_s, \omega) = \sum_{k=1}^L \sum_{l=1}^L w_k^*(\mathbf{x}_s, \omega) w_l(\mathbf{x}_s, \omega) \Gamma(\mathbf{r}_k, \mathbf{r}_l, \omega) \quad (4.6)$$

$$\begin{aligned} &= \sum_{k=1}^L \sum_{l=1}^L w_k^*(\mathbf{x}_s, \omega) w_l(\mathbf{x}_s, \omega) S_{SS}(\omega) \\ &\quad \times \int_{-\infty}^{\infty} d\mathbf{r} \text{III}(\mathbf{r}) \left(\frac{\partial}{\partial n} g^*(\mathbf{r}_k, \mathbf{r}; \omega) \right) \left(\frac{\partial}{\partial n} g(\mathbf{r}_l, \mathbf{r}; \omega) \right) \\ &= S_{SS}(\omega) \int_{-\infty}^{\infty} d\mathbf{r} H(\mathbf{x}_s, \mathbf{r}; \omega) \text{III}(\mathbf{r}). \end{aligned} \quad (4.7)$$

The kernel of this integral is the real PSF

$$H(\mathbf{x}_s, \mathbf{r}; \omega) = \sum_{k=1}^L \sum_{l=1}^L w_k^*(\mathbf{x}_s, \omega) w_l(\mathbf{x}_s, \omega) \left(\frac{\partial}{\partial n} g^*(\mathbf{r}_k, \mathbf{r}; \omega) \right) \left(\frac{\partial}{\partial n} g(\mathbf{r}_l, \mathbf{r}; \omega) \right) \quad (4.8)$$

and depends on the weights $\{w(\omega)\}$. Note that, if $S_{SS}(\omega)$ were known, Eq.(4.7) would provide a linear integral equation for the shape function.

Array PSF An intuitive feel for the “quality” of beamformed images can be gained by considering some PSFs for the actual SURFEX III array. The geometry is shown in figure 4.1: the PSF was focused at (0,0) on the source plane, the source plane - array plane separation was 3 m, and the source excitation at $\mathbf{r}_0$ was taken to the boundary, so that the field is described by

$$p(\mathbf{r}|\mathbf{r}_0) = \frac{1}{4\pi} \frac{e^{ikR}}{R} \frac{ikR - 1}{R} \frac{\mathbf{R} \cdot \mathbf{z}}{R}$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}_0$ and $\mathbf{z}$ is the unit normal to the surface at $\mathbf{r}_0$. The actual array sensor constellation is shown in figure 4.5: the sensor clusters at the ends of each arm have

$\lambda/2$ spacing at the highest frequency in the system passband. The array is sparse along each arm and *really* sparse in the four adjacent quadrants!

The PSFs for this array, assuming simple unity gain phase weights $w_j(\mathbf{x}, \omega) = e^{-ik|\mathbf{x}-\mathbf{r}_j|}$, are shown in figures 4.6-4.11 for frequencies 603, 861, 1120, 1378, 1637 and 1895 Hz and a fluid sound speed of 1475 ms^{-1} .

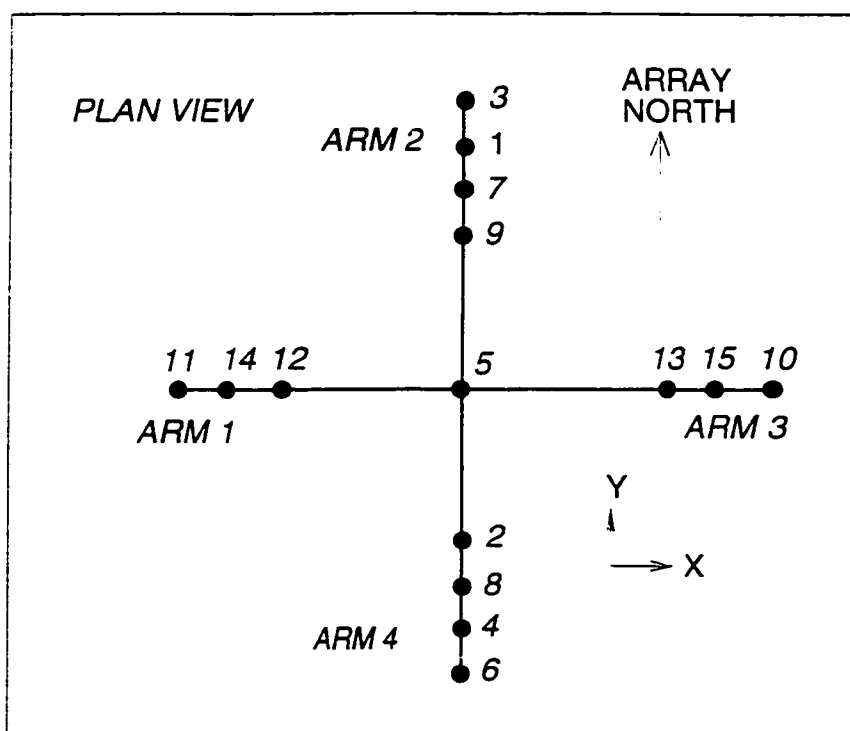


FIGURE 4.5: ULSAS sensor constellation, plan view, SURFEX III. Interelement spacing is 37 cm. Full arm length is 4 m. Not drawn to scale: a scale drawing is provided in Appendix E.

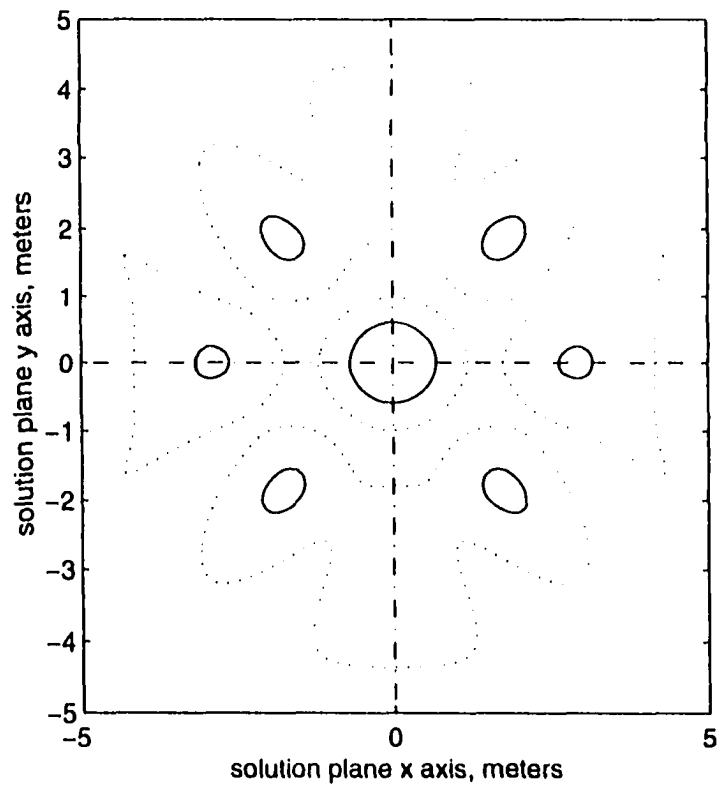


FIGURE 4.6: Array PSF at 603 Hz. Solid contour at -3 dB, dotted contour at -10 dB.

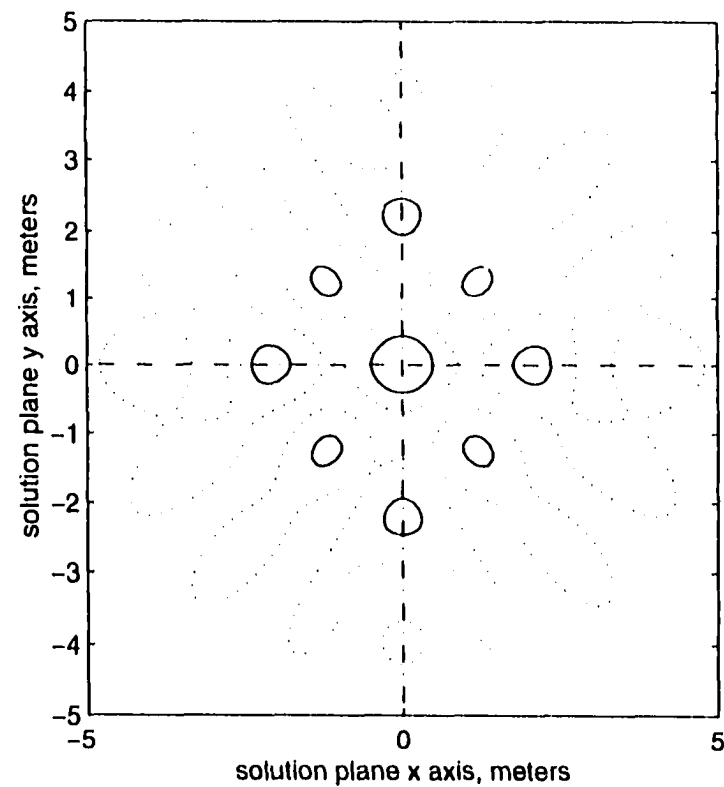


FIGURE 4.7: Array PSF at 861 Hz. Solid contour at -3 dB, dotted contour at -10 dB.

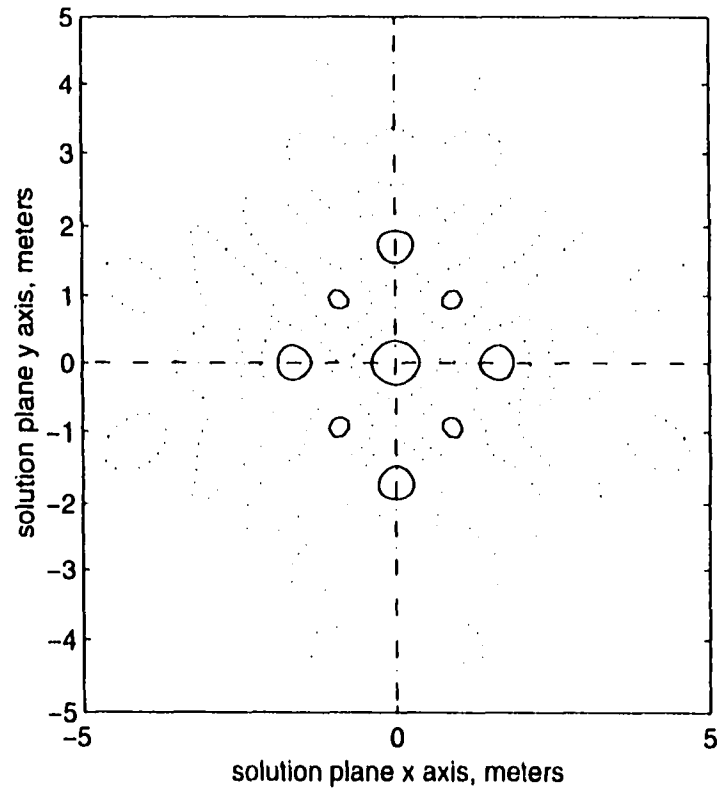


FIGURE 4.8: Array PSF at 1120 Hz. Solid contour at -3 dB, dotted contour at -10 dB.

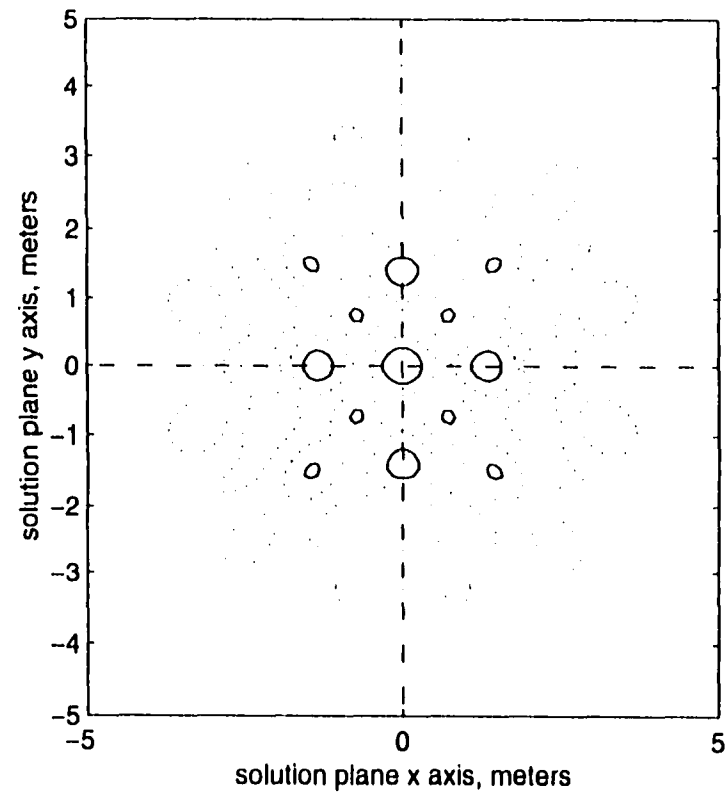


FIGURE 4.9: Array PSF at 1378 Hz. Solid contour at -3 dB, dotted contour at -10 dB.

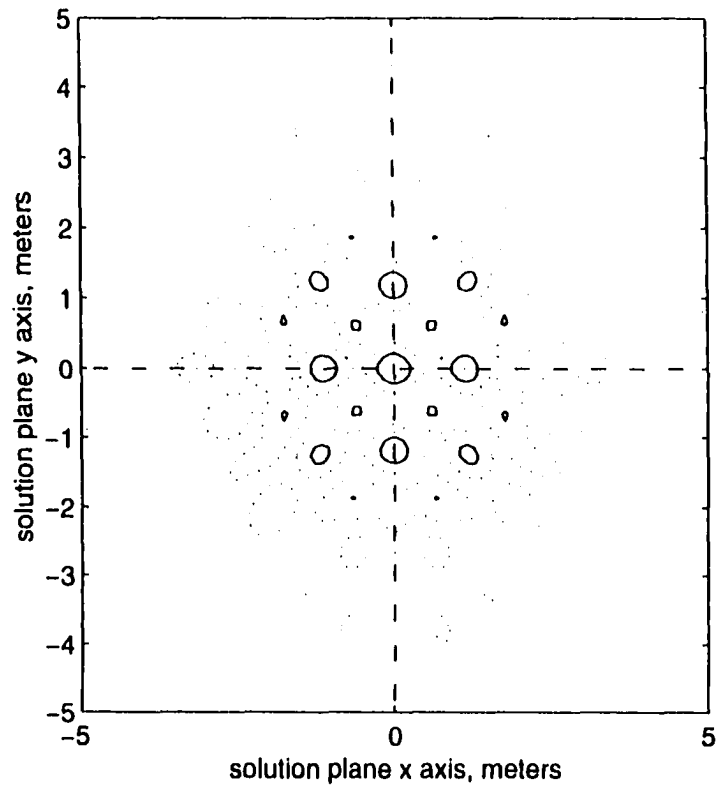


FIGURE 4.10: Array PSF at 1637 Hz. Solid contour at -3 dB, dotted contour at -10 dB.

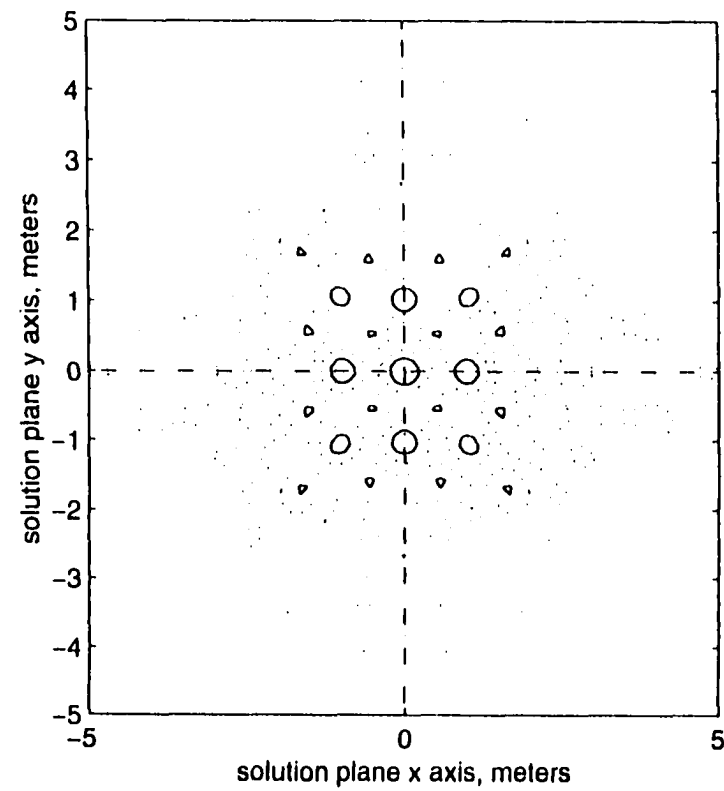


FIGURE 4.11: Array PSF at 1895 Hz. Solid contour at -3 dB, dotted contour at -10 dB.

In general, the central peak of the PSF can be seen to sharpen with increasing frequency: the halfwidth of this peak at half maximum (the -3 dB point) is the quantity plotted in figure 4.4. The array sparseness, however, creates numerous grating lobes: these grating lobes introduce non-uniqueness into the beamformer output by aliasing information from lobes other than the main lobe into the main lobe output. Different weights could control the PSF sidelobes, but the grating lobes will still persist because the wavefield is fundamentally undersampled.

Thus, if this array were used as a conventional rasterizing beamforming imager, the sparseness of the sensor constellation would cause severe aliasing in the output image.

Using Broadband Information Note, however, that the grating lobes in figures 4.6-4.11 occur in different places on the source plane for different frequencies. This suggests that, if the source plane information has a frequency-independent component, then beamformed images at the different frequencies could possibly be used to sort out this unique component.

To pursue this notion, let us raster the steer point $\mathbf{x}_s$ in Eq.(4.7) to N distinct points over the source plane and collect the N outputs into a column vector

$$\mathbf{m}(\omega) = S_{SS}(\omega) \int d\mathbf{r} \mathbf{h}(\mathbf{r}, \omega) III(\mathbf{r})$$

where $\mathbf{h}$ is a column vector with each element equal to the PSF function Eq.(4.8) steered to the corresponding steer point. Now define the “broadband image” by integrating over ω :

$$\mathbf{m}_{BB} = \int_{-\infty}^{\infty} \left[\int d\omega S_{SS}(\omega) \mathbf{h}(\mathbf{r}, \omega) \right] III(\mathbf{r}) d\mathbf{r}. \quad (4.9)$$

The range of integration here is over the frequencies where the source model is considered valid. We will assume the integral exists.

Again, if the source autospectrum were known, Eq.(4.9) might provide a broadband integral equation for the shape function. One could discretize III into N cells and steer the beamformer to N different points, thereby producing N linear equations in N

unknowns. The term in the brackets $[-]$ could then be considered a “broadband kernel” and could in principle be inverted to solve for the N cell values of the shape. One would then hope that the PSF grating lobes would be in different places at different frequencies, so that the broadband effect might provide a “well-behaved” kernel without the non-uniqueness imposed by the grating lobe problem.

The broadband kernel, however, still contains the channel weighting functions, and the intent here is to eliminate the need for them. To that end, identify the k, l^{th} element of the “signal” cross-spectral density matrix (CSDM) $\mathbf{R}_{SS}(\omega)$ as

$$[\mathbf{R}_{SS}(\omega)]_{k,l} = \Gamma(\mathbf{r}_k, \mathbf{r}_l, \omega),$$

that is, the k, l^{th} element is the field MSD between $\mathbf{r}_k$ and $\mathbf{r}_l$. Then, using Eq.(4.6),

$$\mathbf{m}(\omega) = \text{diag} \{ \mathbf{W}^H(\omega) \mathbf{R}_{SS}(\omega) \mathbf{W}(\omega) \}$$

and the broadband image is

$$\mathbf{m}_{BB} = \int d\omega \text{diag} \{ \mathbf{W}^H(\omega) \mathbf{R}_{SS}(\omega) \mathbf{W}(\omega) \} \quad (4.10)$$

The ambient noise entering through the sensors can be handled similarly: the output due solely to ambient noise is

$$\mathbf{n}(\omega) = \text{diag} \{ \mathbf{W}^H(\omega) \mathbf{N}(\omega) \mathbf{W}(\omega) \}$$

where $\mathbf{N}(\omega)$ is the ambient noise CSDM. The broadband output noise is then

$$\mathbf{n}_{BB} = \int d\omega \text{diag} \{ \mathbf{W}^H(\omega) \mathbf{N}(\omega) \mathbf{W}(\omega) \} \quad (4.11)$$

The actual broadband data is constructed by operating on realizations of array data according to Eq.(4.5):

$$\mathbf{b}(\omega) = \mathbf{W}^H(\omega) \mathbf{p}(\omega) \quad (4.12)$$

where $\mathbf{p}(\omega)$ is the vector of narrowband acoustic fields. Then the data vector will be, from Eqs.(4.5) and (4.12),

$$\begin{aligned} \mathbf{d}_{BB} &= \int d\omega \frac{1}{N_s} \sum_{n=1}^{N_s} \mathbf{b}_n(\omega) \odot \mathbf{b}_n^*(\omega) \\ &= \int d\omega \frac{1}{N_s} \sum_{n=1}^{N_s} \left(\mathbf{W}^H(\omega) \mathbf{p}_n(\omega) \right) \odot \left(\mathbf{W}^H(\omega) \mathbf{p}_n(\omega) \right)^* \end{aligned} \quad (4.13)$$

where the symbol $\odot$ denotes the Hadamard product of two vectors and arithmetic averaging has been substituted for ensemble averaging. Terms with subscript $(\cdot)_j$ are the j^{th} realization of $(\cdot)$. The Hadamard product can be expanded into

$$\begin{aligned} &\frac{1}{N_s} \sum_{j=1}^{N_s} \left(\mathbf{W}^H(\omega) \mathbf{p}_j(\omega) \right) \odot \left(\mathbf{W}^H(\omega) \mathbf{p}_j(\omega) \right)^* \\ &= \text{diag} \left\{ \frac{1}{N_s} \sum_{j=1}^{N_s} \mathbf{W}^H(\omega) \mathbf{p}_j(\omega) \mathbf{p}_j^H(\omega) \mathbf{W}(\omega) \right\} \\ &= \text{diag} \{ \mathbf{W}^H(\omega) \mathbf{S}(\omega) \mathbf{W}(\omega) \} \end{aligned}$$

where $\mathbf{S}(\omega)$ is the sample cross-spectral density matrix at frequency ω . Using this expansion in the broadband data vector expression of Eq.(4.13), one could now identify the goodness of fit measure $\mathcal{A}$ with the L_2 norm of the error vector

$$\mathcal{A} = |\mathbf{d}_{BB} - (\mathbf{m}_{BB} + \mathbf{n}_{BB})|^2$$

which, using Eqs.(4.10) and (4.11), is

$$\mathcal{A} = \left| \int d\omega \text{diag} \left[\mathbf{W}^H(\omega) \left(\mathbf{S}(\omega) - \mathbf{R}_{SS}(\omega) - \mathbf{N}(\omega) \right) \mathbf{W}(\omega) \right] \right|^2.$$

From the last expression above, it can be seen that $\mathcal{A}$ will be minimized if the sample CSDM is “close” to the model CSDM: this will be true if, for example, the Frobenius norm

$$\|\mathbf{S}(\omega) - \mathbf{R}_{SS}(\omega) - \mathbf{N}(\omega)\|_F$$

is jointly minimized at each frequency. This is an equivalent statement of the image reconstruction problem, but the the weighting coefficients have been completely eliminated. This is possible in this experiment because the raw channel outputs $p_i(t)$ have

been recorded, so that the underlying sample cross-spectral density matrices can be estimated. The CSDMs in this problem contain all the source information available to the array: further beamforming could produce “images”, but the beamforming process itself would introduce no new information concerning the source.

4.4 An Alternative Probabilistic Approach

Consider using the Frobenius norm of the error matrix

$$\mathcal{A} = \|\mathbf{S}(\omega) - \mathbf{R}_{SS}(\omega) - \mathbf{N}(\omega)\|_F$$

as a measure of goodness of fit between the observed CSDM $\mathbf{S}(\omega)$ and the “model” CSDM $\mathbf{R}_{ss}(\omega)$, which is a function of $S_{SS}(\omega)$ and $\mathbf{III}(\mathbf{r})$. Let us suppose that the shape function $\mathbf{III}$ can be represented by some vector $\boldsymbol{\theta}$. A common way to formulate the minimizer might be to realign the elements of the matrices into vectors.

$$\mathbf{r}(\boldsymbol{\theta}) = \text{vec}\{\mathbf{R}(\omega|\boldsymbol{\theta})\}$$

$$\mathbf{s} = \text{vec}\{\mathbf{S}(\omega)\}$$

and then pose a “least squares” problem:

$$\boldsymbol{\theta} = \arg \min_{\boldsymbol{\theta}} \|\mathbf{s} - \mathbf{r}(\boldsymbol{\theta})\|^2$$

If it can be assumed that the errors $[\mathbf{s} - \mathbf{r}(\boldsymbol{\theta})]_m$ are all uncorrelated, this reduces to

$$\boldsymbol{\theta} = \arg \min_{\boldsymbol{\theta}} \sum_m \frac{([\mathbf{s}]_m - [\mathbf{r}(\boldsymbol{\theta})]_m)^2}{\sigma_m^2}$$

If the problem could be posed this way, various powerful optimization schemes, such as Gauss-Newton or Levenberg-Marquardt techniques, could be applied.

Unfortunately, the errors are not at all uncorrelated: in fact, it’s somewhat difficult to even quantify the errors. First, $\mathbf{s}$ is merely an estimate of $\mathbf{r}(\boldsymbol{\theta})$, so there will be some estimation error simply due to the finite averaging problem. Secondly, ambient noise

enters the system through the sensors and supplies a noise term which will in general be partially correlated with other noise terms.

Given these two issues, it should be apparent that it is not appropriate to simply vectorize the elements of the two CSDMs and apply a standard least-squares minimizer.

A better approach is to simply go probabilistic!

Consider the sample narrowband CSDM $\mathbf{S}(\omega)$ at a particular frequency ω . Previous statistical analysis (Andrew [2]) has demonstrated that the acoustic pressure radiated from large breaking waves (i.e., not “microbreakers”) can be considered to have a Gaussian distribution over short time intervals. Therefore, assume the sample narrowband CSDM has a complex Wishart distribution (Goodman [53])

$$f_{CW}(\mathbf{S}|\mathbf{R}, N_{samp}; \omega) = C|\mathbf{R}(\omega)|^{-N_{samp}} \exp(-N_{samp} \text{tr} \mathbf{R}^{-1}(\omega) \mathbf{S}(\omega)),$$

where C contains terms unrelated to $\mathbf{R}(\omega)$ (but possibly a function of $\mathbf{S}(\omega)$). The mean $E\{\mathbf{S}(\omega)\} = \mathbf{R}(\omega) = \mathbf{R}_{ss}(\omega) + \mathbf{N}(\omega)$. The narrowband log-likelihood function L_ω is then

$$L_\omega = L(\mathbf{R}|\mathbf{S}, N_{samp}; \omega) = C - N_{samp} \ln |\mathbf{R}(\omega)| - N_{samp} \text{tr} \mathbf{R}^{-1}(\omega) \mathbf{S}(\omega) \quad (4.14)$$

where C is a different constant, though still not a function of $\mathbf{R}(\omega)$.

This provides an alternative “goodness-of-fit” function based directly on a distributional argument: the narrowband log-likelihood will reach a maximum (or equivalently, the negative of the log-likelihood will reach a minimum) when the model CSDM $\mathbf{R}(\omega)$ attains the best fit to the available data CSDM $\mathbf{S}(\omega)$.

Assume now that measurements are made at N_f *different* frequencies. Under Assumption 4, chapter three, the information at each of these different frequencies is independent, so the total “broadband density” is formally

$$f_{CW}(\mathbf{S}(\omega_1), \mathbf{S}(\omega_2), \dots, \mathbf{S}(\omega_{N_f})) = \prod_{i=1}^{N_f} f_{CW}(\mathbf{S}(\omega_i)) \quad (4.15)$$

and the “broadband log-likelihood” is now

$$L_{BB} = \sum_{i=1}^{N_f} L_{\omega_i}(\mathbf{R}|\mathbf{S}, N_{samp}; \omega_i) \quad (4.16)$$

where it has been assumed that N_{samp} samples are collected at each frequency.

The ambient noise CSDM $\mathbf{N}(\omega)$ applicable at the time of breaking is not known. However, it is probably valid to assume that the background ambient noise is temporally invariant over scales of 10 to 20 seconds, so that, following Paulraj and Kailath [108], the unknown noise CSDM may be replaced by the noise CSDM from a time adjacent to the breaking event when there are no sources in the field of view.

In practice, only an estimate $\hat{\mathbf{N}}$ of the adjacent noise CSDM is available: its use formally converts the log-likelihood function into a *conditional* log-likelihood function.

The goodness-of-fit function $\mathcal{A}$ is therefore

$$\mathcal{A}(D, III, S_{SS} | \hat{\mathbf{N}}_1, \hat{\mathbf{N}}_2, \dots, \hat{\mathbf{N}}_{N_f}) = -L_{BB} \quad (4.17)$$

where D represents the data, which here is $\{\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_{N_f}\}$. The subscript notion refers to the discretized frequency, i.e., $\mathbf{S}_1 = \mathbf{S}(\omega_1)$. The goodness of fit function is therefore the negative broadband log-likelihood conditioned on the adjacent noise CSDMs.

4.5 (Non)Parameterizing the Solution

In the previous section, it was shown that the broadband conditional log-likelihood is an appropriate measure, based on a distributional argument, of goodness-of-fit between the sample CSDMs and the model CSDMs. The separability assumption allows us to write the model CSDM as

$$\mathbf{R}(\omega) = S_{SS}(\omega) \mathbf{R}_s(III). \quad (4.18)$$

Without loss of generality, we assume that the shape function is everywhere scaled to $O(1)$, and write the source autospectral strength as

$$S_{ss}(\omega) = S_0 e^{20 + \lambda(\omega)} \quad (4.19)$$

where $S_0 = 1 \mu\text{Pa}^2/\text{Hz} @ 1 \text{ m}$ as required for dimensional accuracy. This parameterization is convenient because it has been found experimentally that the typical values of λ are

of order one, similar to the shape values. (This improves the performance in typical optimization algorithms, see e.g. Press *et al.* [114].)

The elements of the nondimensionalized CSDM $\mathbf{R}_*(\mathcal{I})$ are, from Eq.(3.11),

$$[\mathbf{R}_*(\mathcal{I})]_{i,j} = \int_{-\infty}^{\infty} d\mathbf{r}' \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) \mathcal{I}(\mathbf{r})' \quad (4.20)$$

i.e., functionals of the broadband shape function $\mathcal{I}(\mathbf{r})$. The geometry for this application is shown in figure 3.3: the integral is a surface integral, $\mathcal{I}(\mathbf{r})$ is an extended 2-D source function, and the function $g(\cdot)$ is the normal derivative of Dirichlet Green function, Eq.(3.14).

To make further progress, it will be necessary to evaluate Eq.(4.20) for various trial solutions $\mathcal{I}(\mathbf{r})$. For the present, assume the range of integration is limited to an overhead “field of view”, with surface sources outside this region considered part of the background noise. (That is, the integration over the rest of the source plane yields zero.) Eq.(4.20) can be in fact done analytically for some special source-sensor positions $\{\mathbf{r}_i\}$, but this greatly restricts the set of admissible sensor positions. A more general methodology that allows for arbitrary $\{\mathbf{r}_i\}$ (i.e., for sensors on an array platform having arbitrary pitch, roll and yaw angles) requires numerical estimation of Eq.(4.20). Two approaches may be used:

point collocation If $\mathcal{I}(\mathbf{r})$ is treated as a generalized distribution, it may have any arbitrary shape. However, as shown in section 3.3.2, propagation is a spatially bandlimiting operation, so the fine structure details of $\mathcal{I}(\mathbf{r})$ on scales smaller than the highest resolution are essentially unrecoverable. Therefore, there is little additional error in discretizing the field-of-view into $N_x \times N_y$ “cells”, with each cell of dimensions comparable to the best system resolution, as shown in figure 4.4, and then considering $\mathcal{I}(\mathbf{r})$ to be constant over each cell. This is usually known as point collocation.

Let the surface integral Eq.(4.20) be equivalently rewritten as the sum of $N = N_x N_y$ integrals, one over each cell, and then let the value of $\mathcal{I}(\mathbf{r})$ in cell (m, n)

at $\mathbf{r} = (m\Delta x, n\Delta y)$ be some constant $III_{m,n}$. Then Eq.(4.20) becomes

$$[\mathbf{R}_s(III)]_{i,j} = \sum_{\substack{m=-N_x/2 \\ (N_x \text{ even})}}^{+N_x/2} \sum_{\substack{n=-N_y/2 \\ (N_y \text{ even})}}^{+N_y/2} III_{m,n} \times \int_{\text{cell } m,n} d\mathbf{r}' \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) \quad (4.21)$$

The 2-D integrals can now be estimated to high accuracy by a standard cubature technique such as DCUTRI (Berntsen and Espelid [14]).

In the present application, the field-of-view is defined as a 10 m by 10 m square over the array, and $N_x = N_y = 50$, resulting in $N = 2500$ cells, each measuring 0.20 m on a side. 40KB of disk space are required to store the precomputed double precision integrals needed for the evaluation of a single CSDM element: 4.8MB of disk space are needed for the full 15 element array at each frequency.

Timing tests on an HP735 RISC workstation with a 99MHz CPU showed that a single CSDM could be computed in about 2 seconds if the cell cubatures were available as precomputed disk files.

direct numerical integration If $III(\mathbf{r})$ can be treated as a piece-wise continuous function with a small number $N_p = \dim \boldsymbol{\theta}$ of controlling parameters $\boldsymbol{\theta}$, then Eq.(4.20) could be estimated directly using a standard cubature algorithm such as DCUTRI. Some details regarding the evaluation and simulation of extended sources using this approach are provided in Appendix B.

Actual timing tests, however, showed that the computation of one CSDM element using the adaptive cubature algorithm DCUTRI on the HP735 RISC workstation required about 30 seconds: by extrapolation, the computation for the entire 15×15 element CSDM would take over 60 minutes.

Direct cubature of Eq.(4.20) is therefore much more computationally expensive than point collocation, and results in at best a minor increase in accuracy.

At a single frequency, the sample CSDM provides L^2 degrees of freedom (except for some highly unlikely source structures.) If the source is examined at N_f different and independent frequencies, the total number of degrees of freedom will be $N_f L^2$.

If the shape function is treated as a generalized distribution, it will have an infinite number of degrees of freedom: the discretized shape function will then have N independent variables, plus the N_f source strength parameters. In typical image resoration problems, $N + N_f$ will be much larger than $N_f L^2$, so the problem is underdetermined.

If the shape function is treated as a function $\mathbf{III}(\boldsymbol{\theta})$ of a small number $N_p = \dim \boldsymbol{\theta}$ of parameters, with $N_p + N_f$ less than $N_f L(L + 1)/2$, the problem is overdetermined, and can be attacked directly by minimizing a goodness-of-fit measure. This is the *parametric* approach and leads to the well-known maximum likelihood method.

These two approaches are described in the following sections.

4.5.1 Nonparametric Images

When the problem is underdetermined, the reconstruction must be stabilized, or *regularized*, by adding additional constraints in the form of the functional $\mathcal{B}(\mathbf{III})$ to the goodness-of-fit function $\mathcal{A}$. The goodness-of-fit function has been given as $\mathcal{A}(D, \mathbf{III}, \boldsymbol{\lambda})$ where $\mathbf{III}$ is the vectorized cell values $\mathbf{III} = \text{vec}\{\mathbf{III}_{m,n}\}^T$ and $\boldsymbol{\lambda}$ is the vector of source strength parameters $\boldsymbol{\lambda} = [\lambda(\omega_1), \lambda(\omega_2), \dots, \lambda(\omega_{N_f})]^T = [\lambda_1, \lambda_2, \dots, \lambda_{N_f}]^T$. The complete vector of unknowns may be written as $\boldsymbol{\theta}^T = [\mathbf{III}^T, \boldsymbol{\lambda}^T]$ showing explicitly the discretized shape values and the source strength parameters, the latter of which may be considered here to be nuisance parameters.

$\mathcal{B}(\mathbf{III})$ can be shown (DeMoment [32]) to represent *a priori* knowledge on the broadband shape function $\mathbf{III}(\mathbf{r})$. In this application, $\mathbf{III}(\mathbf{r})$ is assumed to be positive over its region of support and zero everywhere else. This type of prior is also used for celestial intensity maps in radioastronomy (Briggs [21]) and for directional wave spectra in oceanography (Long and Hasselmann, [84]).

In radioastronomy, the regularizer is often taken to be the negative of the entropy.

also known as the *negentropy* (Press *et al.* [114]) This has been defined in various ways, all of which seem to provide nearly the same results (Press *et al.* [114]): a form closely associated with information theory is

$$\mathcal{B} = \sum_{m,n} \mathcal{I}_{m,n} \ln \mathcal{I}_{m,n}$$

where $\mathcal{B}$ is the negentropy (and thus $-\mathcal{B}$ would be the entropy.) This form has been found to be useful in radioastronomy because the source map is assumed to consist of strong point-like sources superimposed over much weaker extended features. The logarithm is then considered to be a particularly useful nonlinearity in that it reduces the dynamic range of the solution (Cornwell, personal communication).

The regularizer in oceanography is essentially a penalty function for negative values:

$$\mathcal{B} = \sum_{m,n} (\mathcal{I}_{m,n} - |\mathcal{I}_{m,n}|)^2.$$

In either case, the prescription for computing a nonparameterized image is then to compute

$$\mathcal{I}_{\mu} = \arg \min_{\mathcal{I}, \lambda} (\mathcal{A}(D, \mathcal{I}, \lambda) + \mu \mathcal{B}(\mathcal{I})) \quad (4.22)$$

More recently, Briggs [21] considered optimizers that incorporated a non-negativity constraint into the structure of the algorithm, rather than into the “cost function”. In his work, he utilized non-negative least squares, but another approach is to simply use a constrained optimizer that enforces $\{\mathcal{I}_{m,n} \geq 0 | \forall m, n\}$. The corresponding prescription in this case would be to compute

$$\mathcal{I}_{\mu} = \arg \min_{\mathcal{I}, \lambda} \mathcal{A}(D, \mathcal{I}, \lambda) \quad \text{with } \{\mathcal{I}_{m,n} \geq 0 | \forall m, n\}. \quad (4.23)$$

The first form requires the choice of regularization parameter μ , a subject which alone has generated a significant literature. The second form in fact does not need a regularization parameter, which is equivalent to setting $\mu = 0$ in the first form. For the images produced here, however, it was found that the choice of μ (typically

in the range $(0, 1]$) primarily affects the dynamic range of $III_{m,n}$ but not the relative details. A constrained optimization solution contained details that seemed essentially identical compared to those of unconstrained optimization solutions with just about any regularization parameter that would still yield a solution above the numerical noise floor of the algorithm.

As a standard graphics colormap algorithm can easily compensate for an altered dynamic range, the choice of μ did not prove to have a significant effect on the appearance of the nonparametric images.

Details concerning the implementation of this approach are contained in Appendix C.

The advantage of the nonparametric approach is that it is applicable regardless of the shape of the source within the field-of-view. Indeed, it remains a valid approach when there are more than one extended sources, or even none at all, within the field-of-view (as long as any source element obeys the separability criterion Eq.(3.7c).)

The disadvantage of this approach is the complexity of the computation, due primarily to the number of cells needed to grid the field-of-view: this problem utilized $N = 2500$ cells, and while this is considered to be a “small” image reconstruction problem (Cornwell, personal communication), it is large enough to move the optimization problem out of the range of many standard codes.

A second disadvantage is that a large dimensional image is difficult to quantify and interpret, especially if it is desired to accumulate statistical information on the images.

4.5.2 Parametric Images

If the imaging problem is restricted to times when there is a single source within the field of view, the profile could be approximated by a function controlled by a small number of “shape parameters”. This approach could vastly reduce the number of free variables and eliminate the need for regularization. Therefore, assume the source has a

“Gaussian shape”, i.e., that the intensity profile is given by

$$III(u, v) = \exp \left\{ - \left(\frac{u^2}{w^2} + \frac{v^2}{l^2} \right) \right\} \quad (4.24)$$

where (w, l) are, following Wu [142], the cross-wind dimension and down-wind dimension scales, respectively, and where (u, v) are transformed source plane coordinates

$$\begin{aligned} u &= (x - x_0) \cos \phi + (y - y_0) \sin \phi \\ v &= -(x - x_0) \sin \phi + (y - y_0) \cos \phi. \end{aligned} \quad (4.25)$$

Here, (x_0, y_0) are the x-axis and y-axis centroids (first moments) of the profile. The angle ϕ is the angle between the u axis and the x -axis.

Therefore, the complete parameter vector for the broadband Gaussian shape function is $\theta = [x_0, y_0, w, l, \phi, \lambda_1, \lambda_2, \dots, \lambda_{N_f}]^T$. The parameter vector for each additional frequency merely adds one additional term $\lambda(\omega_i)$, so the “price” of the broadband formulation is one additional parameter per frequency. The number of degrees of freedom in the model is $N_f + 5$.

The maximum likelihood parameter vector is then

$$\theta_{ML} = \arg \max_{\theta} L_{BB} = \arg \min_{\theta} \mathcal{A}(D|\theta). \quad (4.26)$$

and this yields the parameterized image. Details on the implementation of this approach are provided in appendix D.

The advantage of the parametric approach is that it reduces a large-dimensional image to a few parameters, which can be easily accumulated and compared over many images.

The disadvantage of this approach is that it does not produce a “true” image of the source and so cannot reveal any interesting or useful structure. The approach also becomes invalid if more than one source appears simultaneously in the field-of-view.

Chapter 5

The Experiment

The instrument “ULSAS” (Upward Looking Sonar Array System) was made available for this experiment and was extensively modified in order to acquire multi-channel acoustic data from breaking waves. The pertinent details of the instrument are outlined in appendix E.

ULSAS was deployed three times in Patricia Bay in the summer of 1994, the fall of 1994 and the spring of 1995. These deployments, each lasting several months, are tentatively labeled SURFEX I, II and III, respectively. SURFEX I and II served to refine the array measurement techniques: only the data from SURFEX III will be discussed in this work.

The principle features of SURFEX III are presented in section 5.1. A single storm event during that deployment was selected for imaging: a preliminary assessment of the acoustic data during that event is provided in section 5.2. A comprehensive imaging analysis of the breakers during this storm is reserved for chapter six.

5.1 SURFEX III Deployment

We deployed ULSAS for SURFEX III on 3/19/95 for 57 days in Patricia Bay, located on the northeastern side of Saanich Inlet on the east side of Vancouver Island, as shown

in figure 5.1. The deployment site, shown in greater detail in figure 5.2, has a fetch of about 5 km to the west and a water depth of about 80 m. The actual mooring scheme is diagrammed in figure 5.3. A Coastal Climate Systems meteorological buoy was moored nearby (at the position labeled "ORE buoy") to provide near-surface environmental parameters. In addition, 6 metre "Halibut" poles were attached to the end of each arm of the array in order to aid in visual sighting and warn off local boaters.

Following the discussion of section 4.2, our approach for a high-resolution technique led to the design and construction of a "nearfield" array. Caution and practical considerations persuaded us from deploying the array in the extreme nearfield: the proximity to the surface might have subjected the array to downward washes of bubbles from the breakers, placing the sensors inside the source region. In addition, the surface drift currents and orbital motion in high winds might have combined to heave portions of the array through the actual fluid surface into the air. We therefore set the nominal depth of the array for SURFEX III at approximately 3 metres.

At this depth, surface drift currents had a considerable effect upon the array: on-board tilt sensors recorded angles up to $\pm 8^\circ$. Strumming due to aerodynamic flow around the Halibut poles and hydrodynamic flow around the array structure itself introduced considerable acoustic contamination. In addition, the wave orbital motion subjected the array arms to stresses greater than those for which they were designed, causing further radiated and structure-borne mechanical noise ("squeaks"). The hydrophone mounts provided enough attenuation against the strum to prevent preamplifier saturation, but the contamination is still dominant in the acoustic data, which usually require additional pre-processing digital filtering. The broadband "squeaks" cannot be so removed, but they are usually easily identified and avoided.

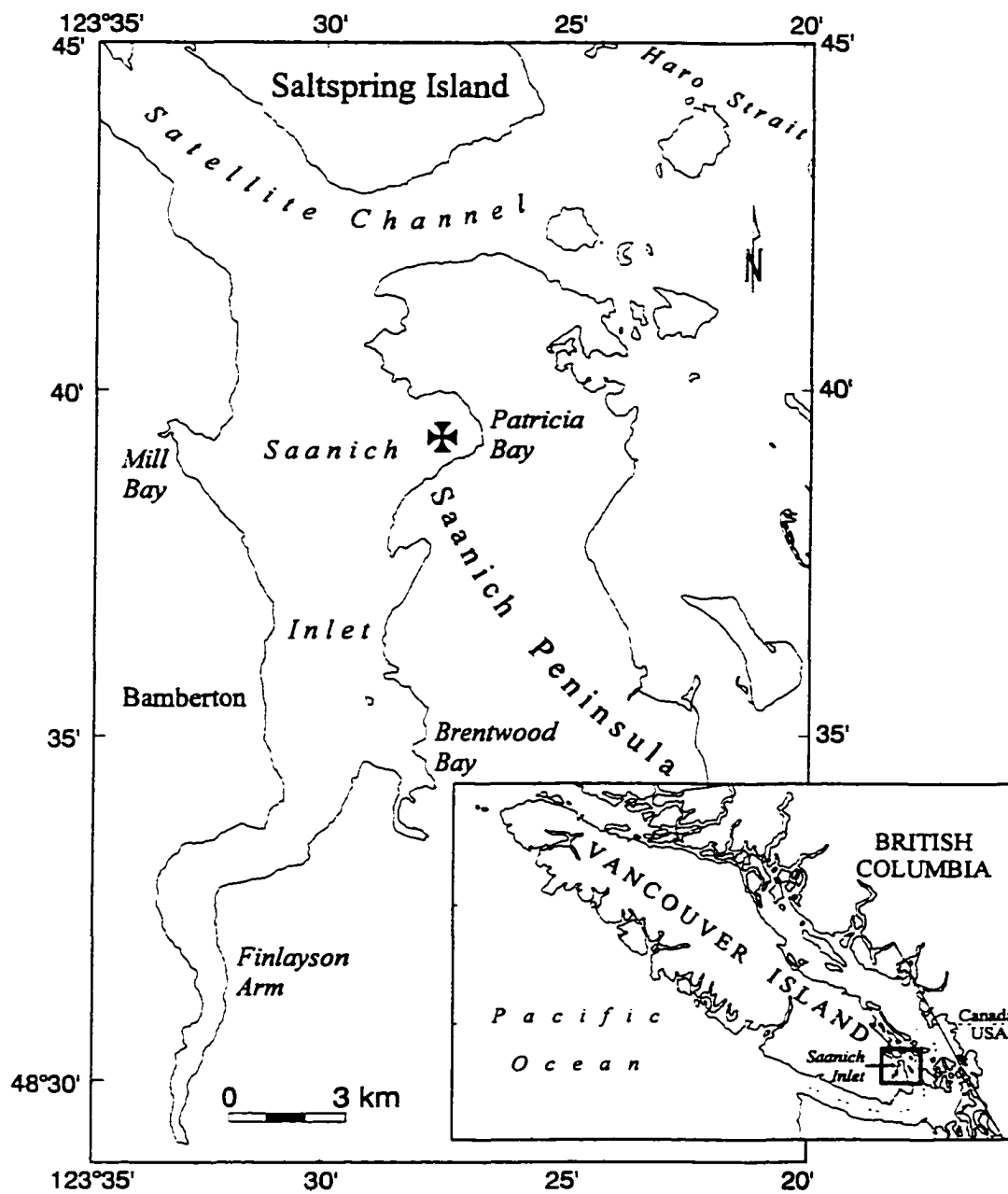


FIGURE 5.1: Deployment geography. SURFEX III.

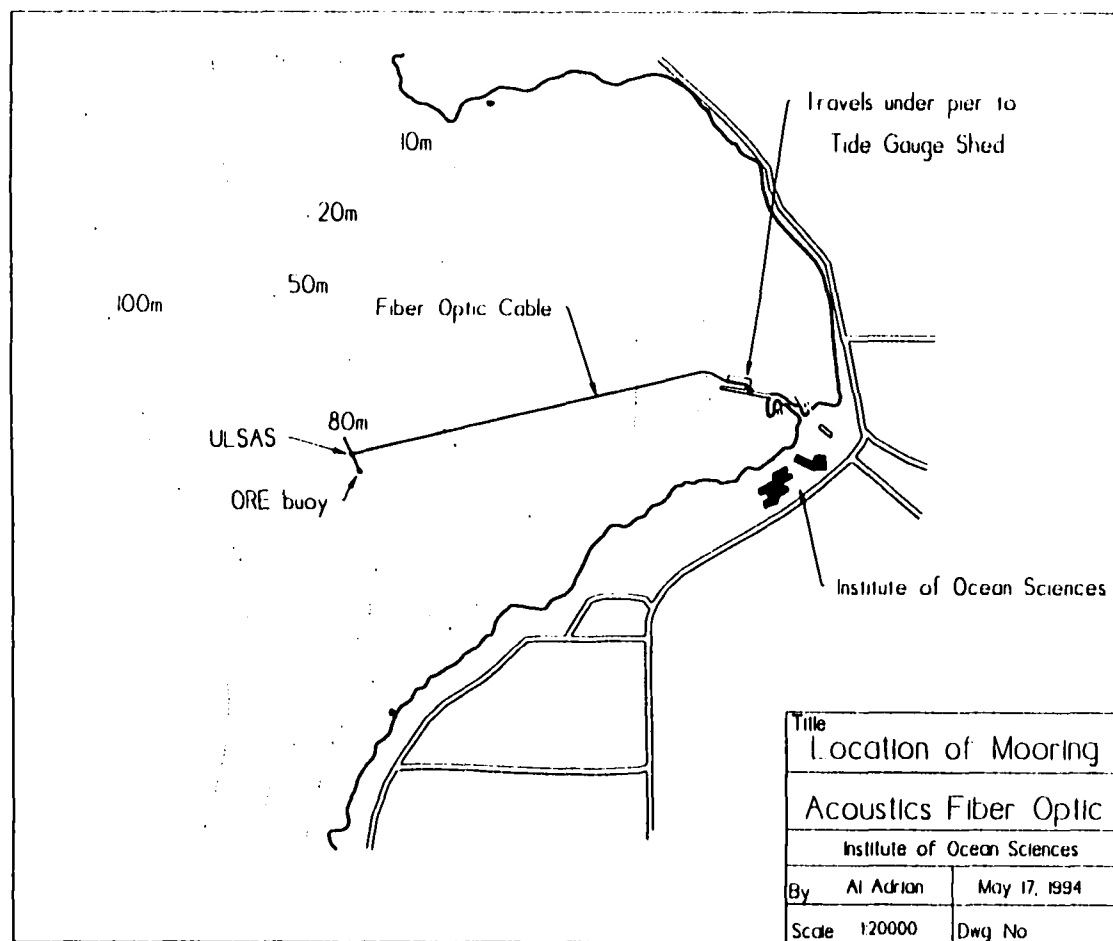


FIGURE 5.2: Deployment site, SURFEX III. The ULSAS system was about 1 km from the nearest shoreline in about 80 m of water with a sloping bottom.

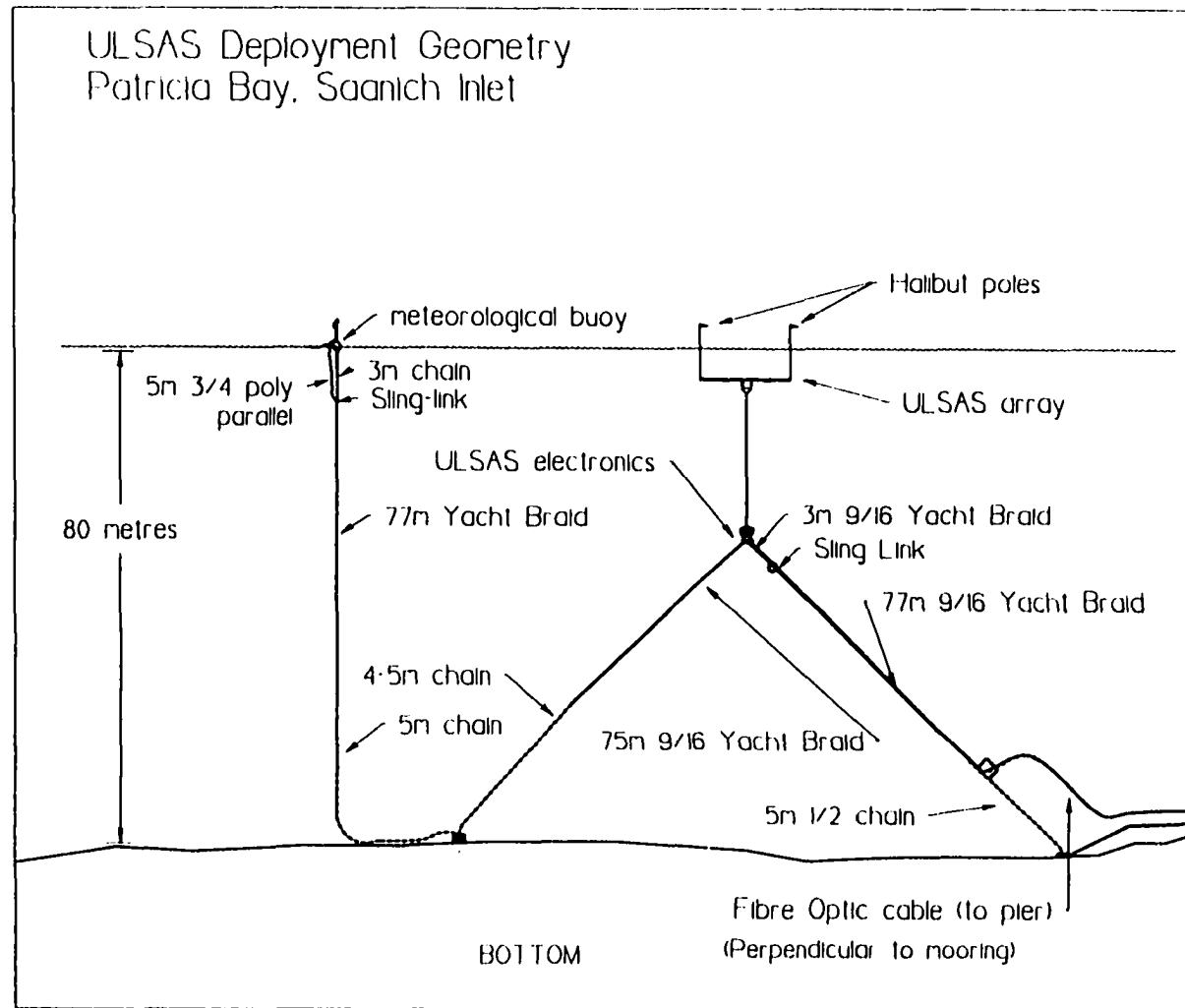


FIGURE 5.3: Mooring scheme for SURFEX III.

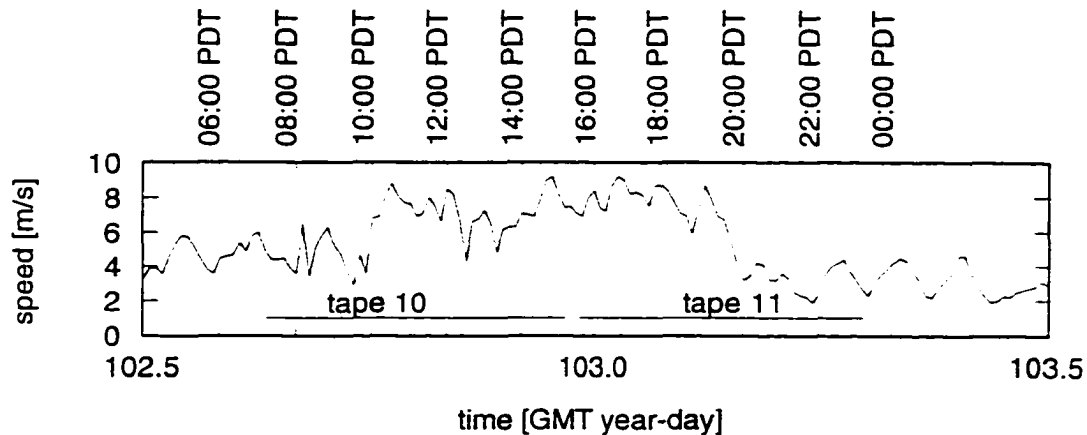


FIGURE 5.4: Vector-averaged wind speed. SURFEX III. 4/13/95. Measured at a height of 3m.

The most interesting event of SURFEX III was the storm of 4/13/95, acquired on tapes 10 and 11. The vector-averaged wind speed during this event is shown in figure 5.4, the air-sea temperature in figure 5.5, and the wind direction in figure 5.6. The wind direction was quite consistent for Patricia Bay (which experiences some strongly veering winds) at about 260° : this corresponds to a fetch of about 5 km.

The depth and x and y -axis tilts of the array during this event are shown in figures 5.8 and 5.7. The depth varied throughout the event from 2 metres to about 3.5 metres, and the maximum array tilt was about 8° . The array bearing is shown in figure 5.9 at a sequence of sample points.

5.2 Data Description

Single Channel Broadband Power A qualitative assessment of the data can be deduced from inspection of the broadband power from a single channel. After conversion to in-water units of μPa and high-pass filtering to remove strum components, the data from channel 5 (the center hydrophone) are squared and smoothed over a 68 msec

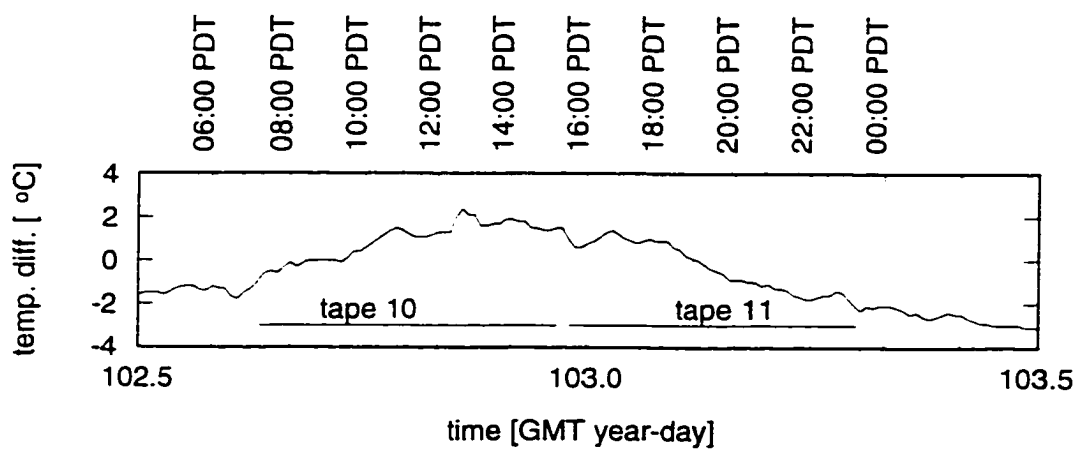
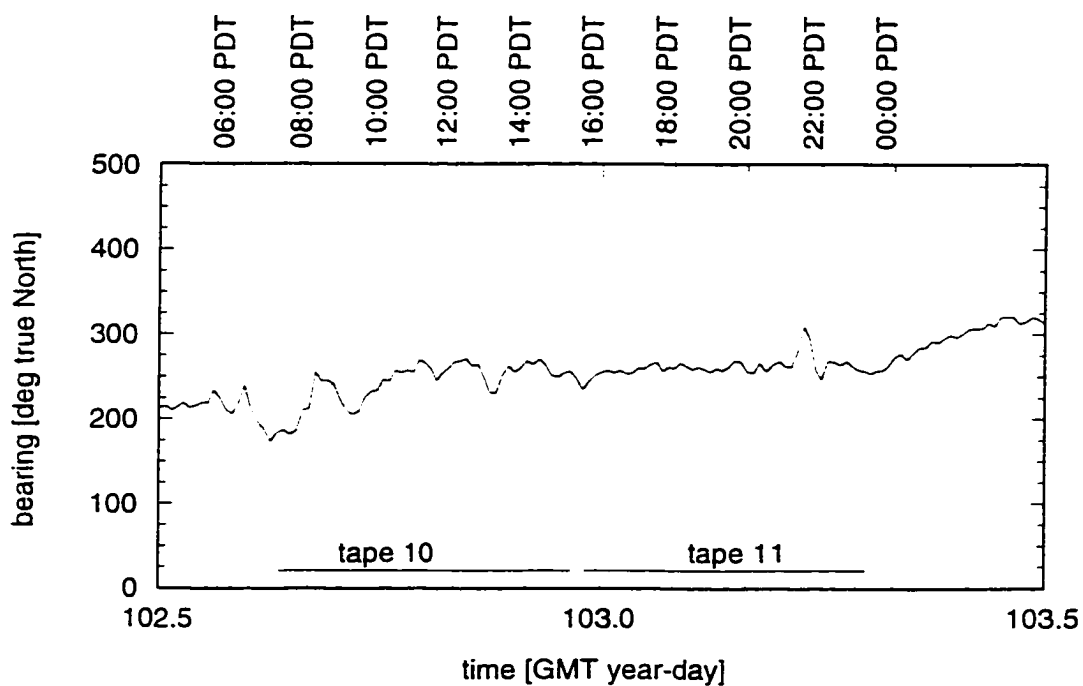


FIGURE 5.5: Air-Sea Temperature Difference, SURFEX III, 4/13/95.

FIGURE 5.6: Wind Direction, SURFEX III, 4/13/95. Solid line is the bearing averaged over 10 minute intervals, the dotted lines represent $\pm$ one standard deviation.

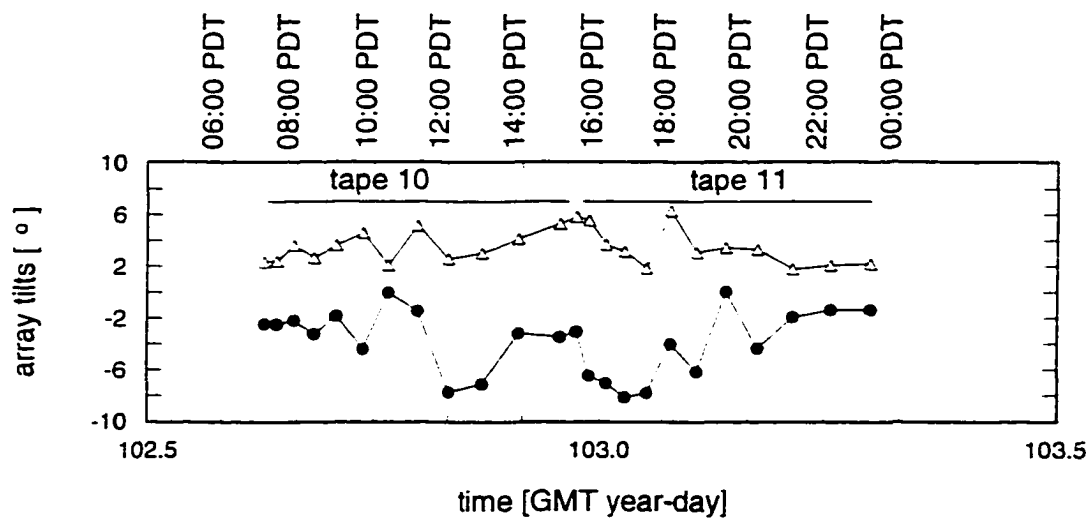


FIGURE 5.7: Array x - and y -axis array tilts. SURFEX III, 4/13/95. X -axis tilts are shown as solid dots, y -axis tilts are shown as triangles.

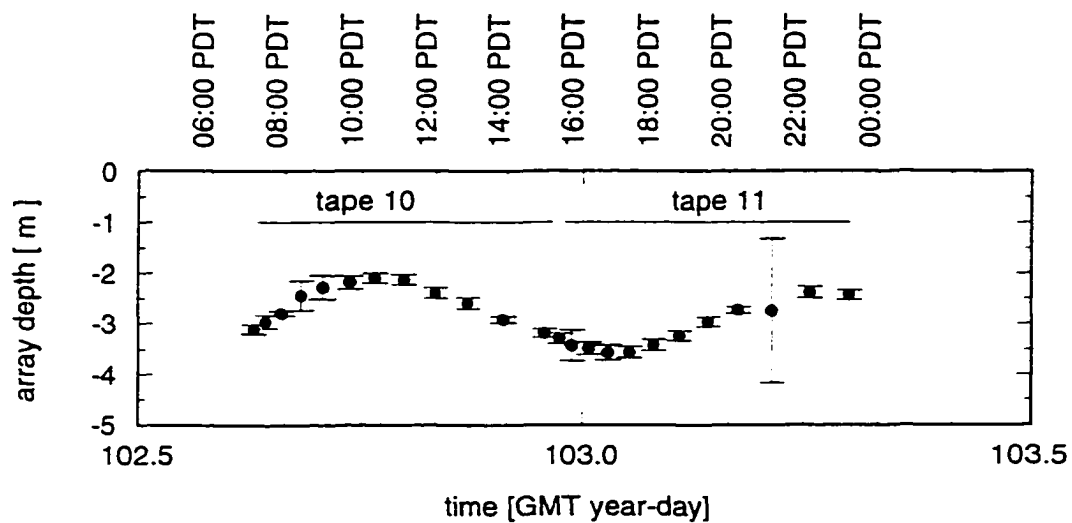


FIGURE 5.8: Array depth. SURFEX III, 4/13/95. Error bars represent $\pm$ one standard deviation.

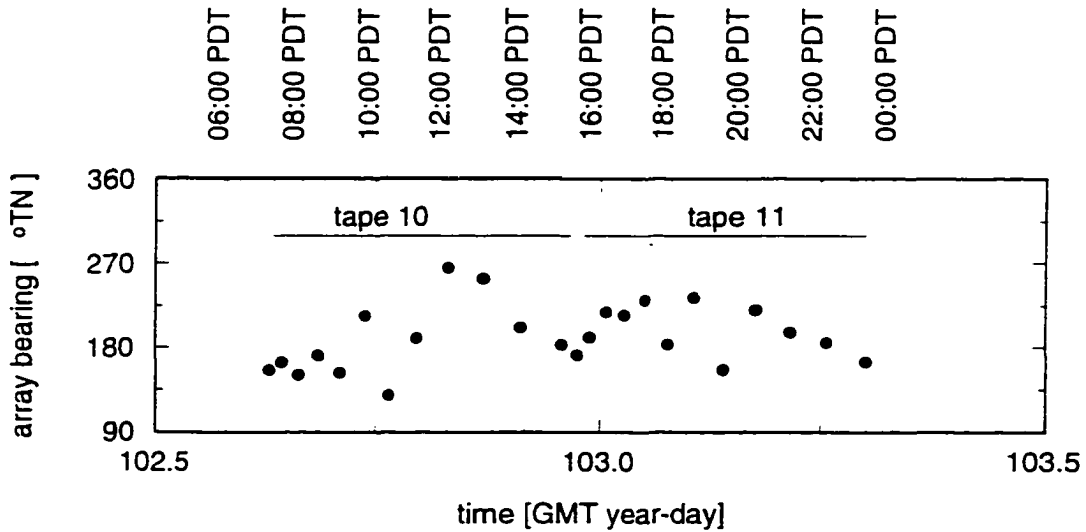


FIGURE 5.9: Array bearing, SURFEX III. 4/13/95.

window. Typical characteristics of the data are revealed in figure 5.10:

- figure 5.10A shows a segment believed to be predominantly “background noise”, from prior to the peak of the storm :
- figure 5.10B shows the signature of a single wave, achieving a peak SNR of about 10 dB;
- figure 5.10C shows data from stormier conditions: the background level is about 7 dB higher than in figure 5.10A, and multiple “squeaks” permeate the data, including at least one during the apparent wave break.

Single Channel Narrowband Power Hollett [60] found that the sound radiated from a plunging wave is initially concentrated at higher frequencies and then migrates to lower frequencies as a collective oscillation mechanism forms. The wave from figure 5.10B can be analyzed in a similar fashion. The data were partitioned into consecutive 11.6 msec (64 point) blocks and the autospectral density estimated over each block.

using a multi-taper technique [109] with a bandwidth resolution of 258 Hz. (This required 5 tapers.) Figure 5.11 shows the resulting series of estimated narrowband levels for 7 mostly non-overlapping bands for the duration of this breaking event.

There is no clear migration of spectral energy during this event, which would indicate that this event is more likely a spilling wave than a plunging wave. This, of course, is consistent with the notion (see section 2.2) that coastal waves are more likely to be spillers. The spectral energy seems to rise and fall almost simultaneously across all bands, even at the lower frequencies (where contributions from individual bubbles are theorized to be insignificant.) In fact, the “spike” at about 1.5 s (which has been broadened slightly by smoothing) is evident only in the lower bands, and could possibly represent a single bubble resonant somewhere between 300 and 600 Hz.

This qualitative narrowband spectral analysis does not conclusively resolve the low frequency sound generation mechanism, but it does suggest that this event was a spiller. While this type of analysis was not pursued with other waves, there is no reason to suspect that this wave was in any way unique, and therefore reinforces the source model assumptions made in chapter three.

Multi-Channel Observation of a Breaking Wave A multi-channel broadband acoustic signal trace can provide a “first-guess” of the juxtaposition of the breaking wave relative to the array. Figure 5.12 shows the signature of the wave of figure 5.10B on each channel. Judging solely by power levels, the wave appears to have broken somewhere between the center sensor and arms 3 (which contains channels 2, 8, 4, and 6) and 4 (which contains channels 13, 15, and 10.) The dipole character of the acoustic radiation attenuates the contribution to arms 1 and 2 to the extent that the broadband signal is hardly noticeable above the background noise.

A quantitative analysis of the power levels at each sensor is presented in section 6.3 for making an initial estimate of the x - and y -axis positions of the breaker centroid.

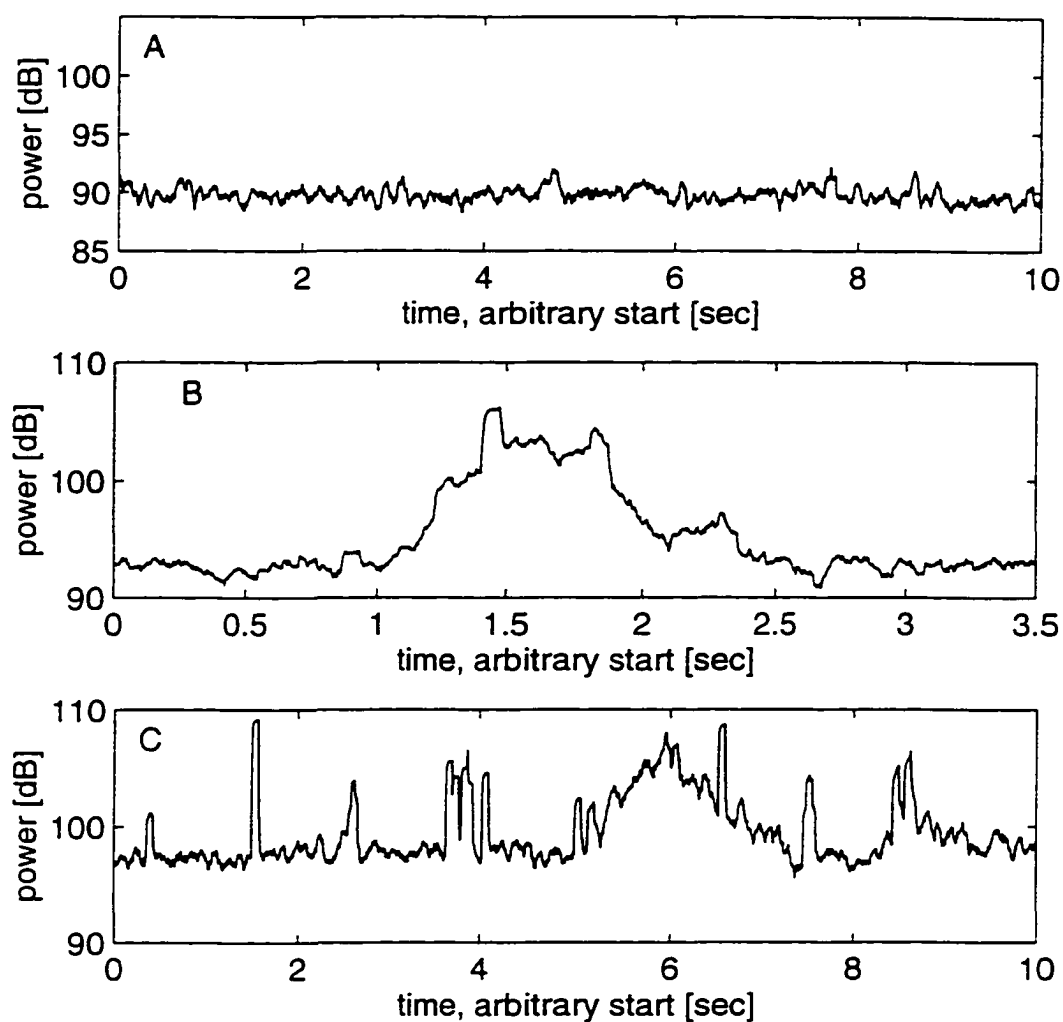


FIGURE 5.10: Example data from channel 5 from the wind event of 4/13/95. Vertical units are dB re $1 \mu\text{Pa}^2$ in the passband $[160, 2000]\text{Hz}$. The data have been prefiltered with a high-pass filter, squared and then smoothed over a 68 msec window. A (top): “background noise”, about 09:19 PDT, additionally decimated 4-to-1 for plotting; B (middle): a typical wave, 08:03 PDT, wind about 5 m/s; C (bottom): background plus a wave plus “squeaks” (spikes), 15:23 PDT, wind about 7 m/s.

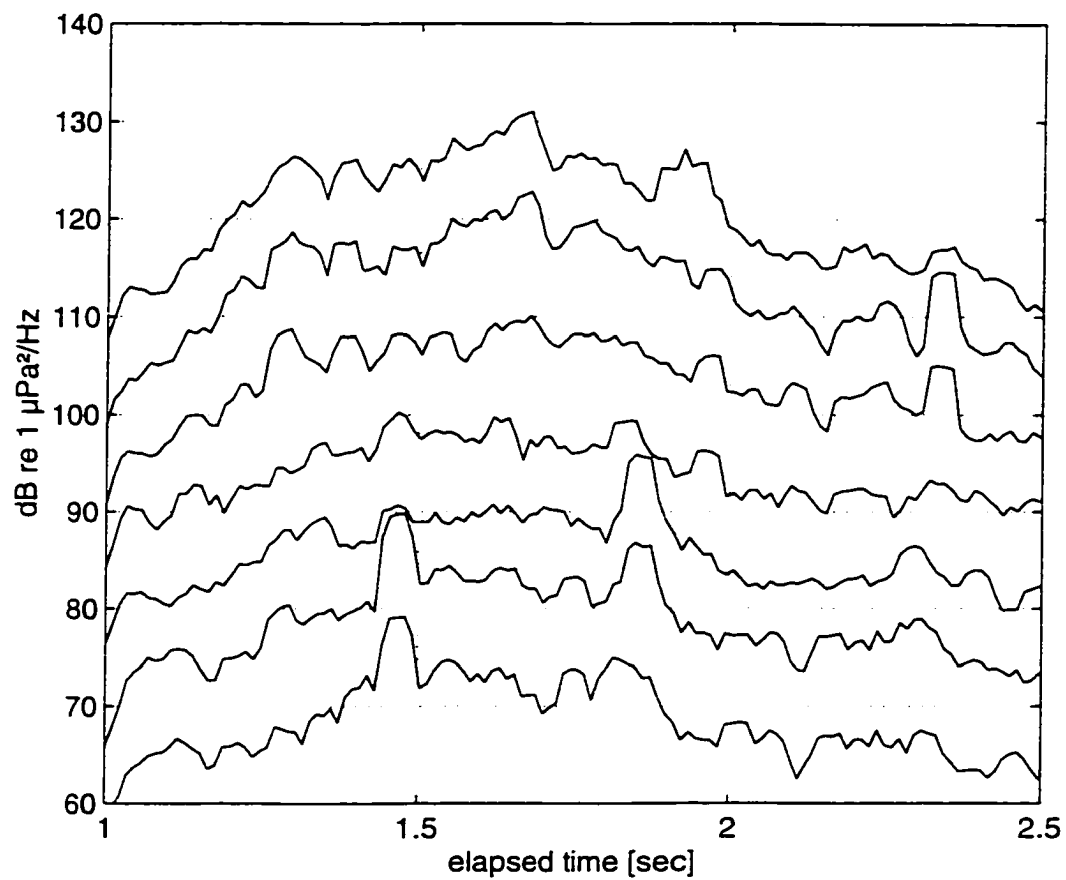


FIGURE 5.11: Narrowband levels versus time for a sample breaking event, channel 5. for bands centered at (bottom to top) 344, 603, 861, 1120, 1378, 1637 and 1895 Hz. Narrowband bandwidth is 258 Hz. Smoothed over 46 msec. Ordinate scale applies to lowest curve: all others displaced consecutively by 10 dB.

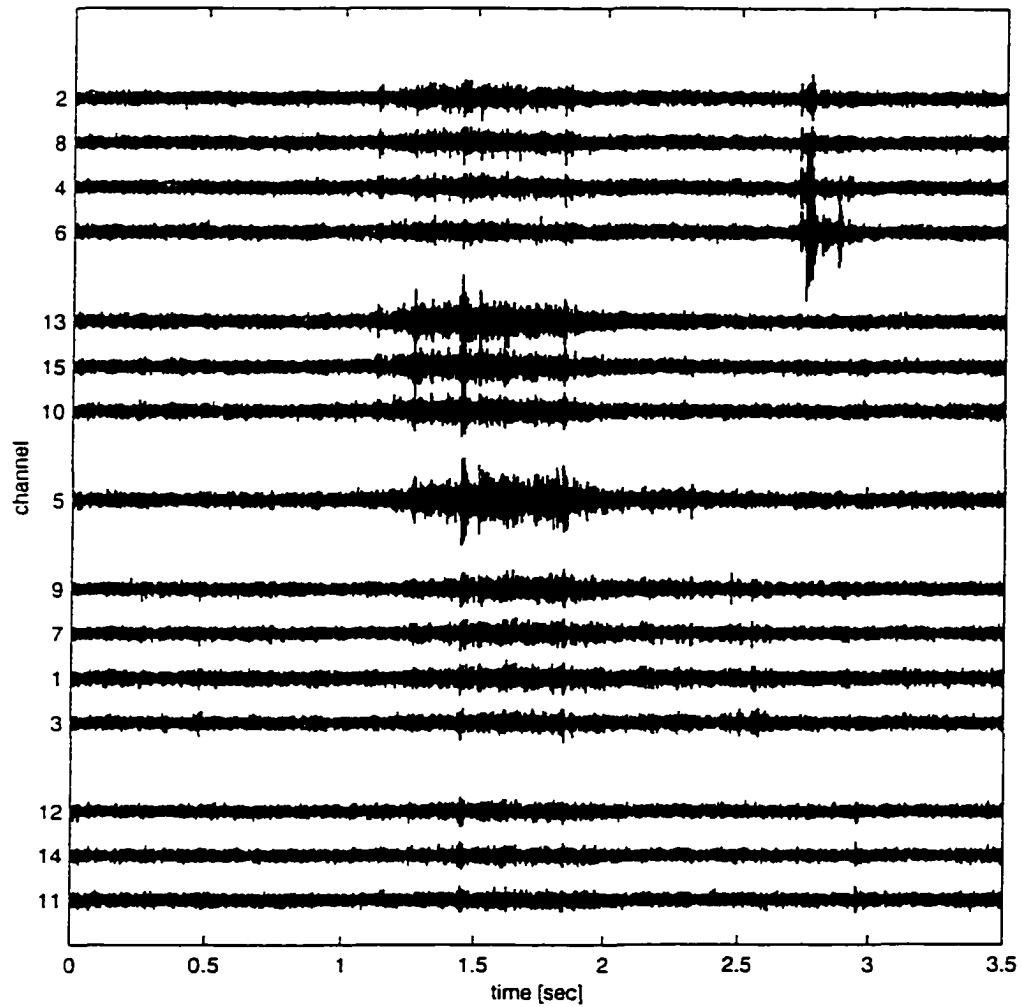


FIGURE 5.12: Broadband acoustic signals per channel for a typical wave. The signals have been corrected to μPa and prefiltered to remove vibrational contamination. The channels are grouped according to arm.

Chapter 6

Signal Processing

The principle data analysis consists of two steps: first, a nonparametric or parametric image is formed, and second, a further analysis is conducted based on the image solutions. The description of the signal processing through the formation of images is provided in this chapter. An analysis of the data is given in chapter seven.

The program for computing an acoustic image requires identification of sections of the multichannel acoustic record that contain (1) a representative sample of ambient sound, and (2) a quasi-stationary section of breaker sound. These data are then supplied to a routine that estimates the ambient and ambient+signal CSDMs. The diagonal components of the ambient+signal CSDM are then used to make a “first guess” at the centroid of the source. The CSDMs and the “first guess” are then supplied to the optimizers for image computation.

Each step above is briefly outlined in the following sections.

Wave W3 provided “well-behaved” data, and so will be used as an example of the signal and image processing. It was not uncommon, however, for waves to provide “ill-behaved” data. Ill-behaved data often limited the available bandwidth over which a broadband image could be formed, and sometimes prevented the formation of any image whatsoever. Some examples of this behavior are noted in the final section.

6.1 Data Record Identification

Even though the VCR tapes contained multiplexed digital data, playback through the digital audio interface provided an acceptable capability of monitoring the recorded signal via the headphones. Data from half the channels appeared on the right audio channel and the other half (including the annotation) on the left. The annotation produced only annoying digital noise so that side was usually turned off.

This audio monitoring allowed considerable portions of the tapes to be reviewed at real-time playback speed. Shipping and air traffic (the site is near an airport) can sometimes be identified. Mooring noise, as described in section 5.2, is also heard. Large breaking waves produce a transient increase in signal power, while smaller “micro-breakers” can sometimes be identified by the resonant ringing of the entrained bubble.

Promising sections of the tape are acquired onto computer disk. This results in more than 10 MB of data per minute. Data review is further facilitated by extracting channel 5 data. These data were recorded by the array center hydrophone: loud signals on this channel are more likely to be “overhead” and within the array field-of-view. In windy conditions, such as the storm event of 4/13/95, this single channel record generally shows about one breaking event per minute on average. Using the position of the event on the signal channel record, the full multichannel data segment containing the breaker and a sample of accompanying ambient sound is extracted and written into a much smaller “wave” file.

These wave files provide the data for a *single* wave. For example, figure 6.1 shows the short-time averaged high-pass filtered passband power from the center hydrophone for wave W3. The data have been filtered with a fifth-order Butterworth high pass filter with corner frequency at 160 Hz to remove the strum component and then converted to units of μPa , squared, and averaged over consecutive 11 msec blocks. The record shows the onset of the acoustic signal at about 5.5 seconds, peaking at about 6.3 seconds, and dying away by about 8 seconds. Several spikes can also be seen, probably due to

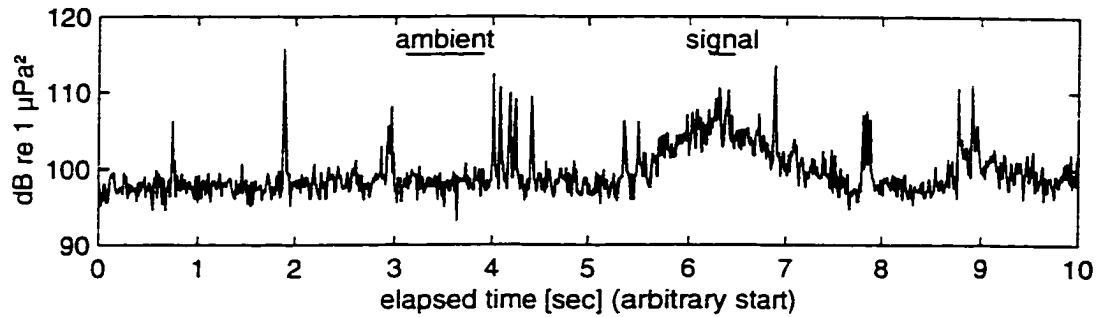


FIGURE 6.1: Wave W3: 15:24:01 PDT.

mooring noise. The background ambient level can be estimated by the signal level prior to the event onset: this is about 97 dB. The breaker signal has bursts to about 109 dB, so this event had, at sensor 5, a maximum SNR of about 10 dB.

Figure 6.1 is also annotated with lines indicating the selection of ambient and ambient+signal (labeled “signal” for brevity). The selection process was performed by eye. The ambient segment was intended to be as large as possible without including any audible breaker precursor, in order for the ambient-only CSDM to be well-estimated. The position and duration of the signal segment attempted to balance the inherent non-stationarity of the signal against the statistical variability of short-time estimators. A previous statistical examination of breaking waves (Andrew, [2]) showed that the data could be considered quasi-stationary over time scales of about a half second. This quasi-stationarity, however, breaks down if the signal power appears to take an obvious jump, or contains some “bursts” (which could be mooring noise). Therefore, signal segment selection generally tried to use sections of the signature with a nearly flat “local” average, near the peak (so as to have the best SNR), of length half a second or less, and which avoided bursty components.

The ambient and signal+ambient selection for each wave are shown in appendix I.

This example also demonstrates the typical non-stationary behavior of the acoustic signal. The breaking signal is transient and random, with a short-time power that rises

unsteadily from zero to a peak and then dies away to zero again.

6.2 CSDM Estimation

CSDM estimation uses the standard approach. The data set is segmented into disjoint blocks containing N samples for each channel m , $m = 1, 2, \dots, M$. The mean is removed from each channel, and then the demeaned data are transformed into K Fourier sets by windowing the data with K tapers $k = 1, 2, \dots, K$ to obtain K windowed blocks and then transforming each block with a DFT. The complex coefficient for bin f , taper k from channel m is then extracted, producing, say $X_{f,k,m}$. A “raw” estimate of CSDM element (i, j) is then formed via

$$X_{f,k,i}^* X_{f,k,j}.$$

These raw estimates are arithmetically averaged over the entire data set. If the data set contains L blocks, the number of accumulants will be KL .

The choice of transform size N , the desired analysis bandwidth and the taper shape are all interrelated. The multiple taper technique (Thomson, [128], Percival [109]) produces (1) tapers that are optimally concentrated in frequency, (2) auto- and cross-spectral estimates that are efficient, and (3) terminology that is convenient. These tapers are N -point digital prolate spheroidal sequences $v^{(k)}(NW)$. The taper shape is controlled by the “time-bandwidth” product NW and the taper index k . For a given time-bandwidth product, the number of tapers which remain concentrated mostly within the resolution bandwidth is about $K \approx 2NW - 1$.

The choice of sample size (N) and resolution bandwidth (W) must be balanced against statistical variability. A large sample size and narrow analysis bandwidth will yield fine frequency resolution, but at the cost of high variance because only a few samples will be accumulated over the available data set. Conversely, a small sample size will result in many samples and thus estimators with low variance, but at the cost of a broad analysis bandwidth.

parameter	value
f_s [Hz]	5512.5
N	64
NW	3
bandwidth [Hz]	258
bin centers [Hz]	603, 861, 1120, 1378, 1637, 1895

TABLE 6.1: CSDM estimation parameters.

At least M realizations must be averaged before the CSDM estimator matrices become invertible. While this imaging application does not invert the sample covariance matrices, this is a good guideline for a lower bound on the number of blocks that a data set should provide. The maximum size of the data set is limited by the duration of time in which the signal can be considered to be “quasi-stationary”: this is generally half a second or less.

The parameters chosen for this work are shown in table 6.1. The time-bandwidth product $NW = 3$ produces a resolution bandwidth of about 258 Hz, which is about 4 DFT bins. Roughly orthogonal coverage of the frequency band [474 Hz, 2024 Hz] can be obtained with the 6 bands centered at the frequencies given in table 6.1. There are 5 tapers with $NW = 3$ that have good frequency concentration: these are shown in figure 6.2.

The tapers must be normalized by their “eigenvalues”, which represent the proportion of energy concentration of the taper transform in the analysis bandwidth. Values near 1.000 are desirable, values below about 0.9000 (90% concentration, i.e., 10% spectral leakage) are generally avoided. The eigenvalues for the 5 tapers used here are shown in table 6.2.

(N.B. It is not possible to apply a normalization, such as the one suggested by Thomson and Chave [129] for robust estimation of CSDMs, because the necessary signal-

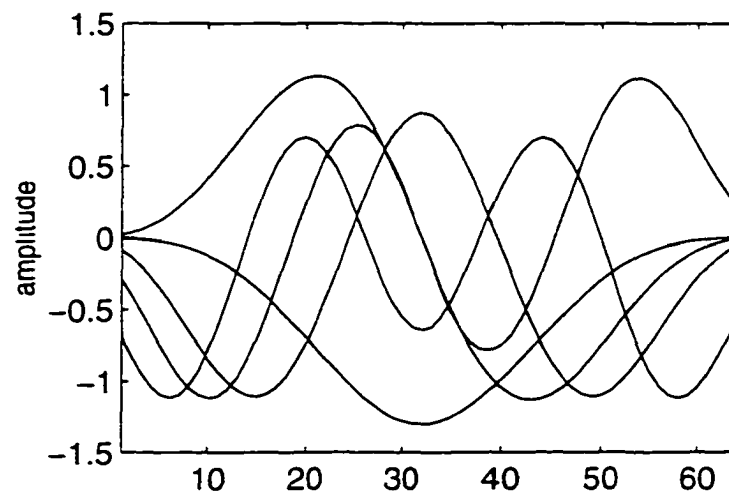


FIGURE 6.2: Digital prolate spheroidal sequences. 64-point Tapers $k = 1, \dots, 5$. Taper k has $k - 1$ zero crossings. The $k = 1$ taper shown here is non-positive, and differs only from a conventional taper by a factor of -1 .

index	eigenvalue
1	0.9999998651
2	0.9999907546
3	0.9997149849
4	0.9949143842
5	0.9461378891

TABLE 6.2: Taper eigenvalues

to-noise ratio information connecting the signal and the background noise would be lost.)

The number of accumulants in both the ambient and ambient+signal CSDM estimators are given for each wave in table I.1 in appendix I.

6.3 POCS

The power levels across the array represent spatial samples of the field intensity, which according to section 3.3.2, is a bandlimited function. If this function were known, a reasonable guess for the initial values of the source centroid (x_0, y_0) might be the position where this function peaks. In principle, a hydrophone at this position would output the largest signal power.

Of course, the field intensity is not known *a priori* except at a few sparse points. The POCS (Projection Onto Convex Sets) algorithm (see Press *et al.* [114], section 3.5) allows one to reconstruct the original bandlimited function given only samples of the function at discrete points (and the band limit.) Unfortunately, the sparseness of the sample points (i.e., the available sensor positions) in the (x, y) plane precludes reconstructing over the whole (x, y) plane. *That* problem is just too sparse! But an adequate compromise reconstructs the received power over just the x-axis (y-axis) and then chooses the peak of the function as x_0 (y_0).

The POCS algorithm basically iterates between two alternate representations of the data, reinforcing known constraints at each iteration. In this scheme, the two representations are the spatial data and its Fourier (wavenumber) transform. The algorithm is implemented (along the x-axis, for example) by assuming a sampling grid along the x-axis such that the actual hydrophones correspond to nodes of the grid. In the “space-data” representation, the space-data are constrained to be positive, and the values at the sensor nodes are constrained to be equal to the available data points. In the wavenumber “k-data” representation, the k-data are constrained to be bandlimited.

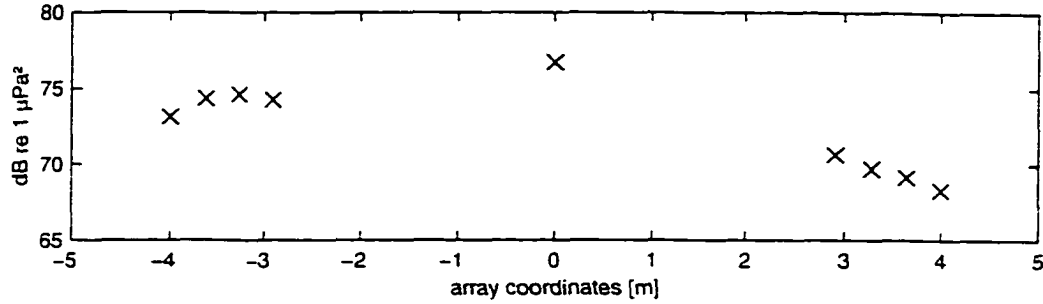


FIGURE 6.3: Field intensity versus sensor position, x-axis, wave W3.

The iteration starts with a space-data vector which has zeros at each node except the sensor nodes, where the node value is the data value. A typical example would be the power distribution shown in figure 6.3, which is the distribution for wave W3 at $f = 603\text{Hz}$. The sensor output power is greatest from the center hydrophone but slightly skewed to the left, so one might expect x_0 to be near $x = 0-$.

The space-data vector is transformed to the k-data vector, and all the data values above the band limit $k = \omega/c$ are set to zero.¹ The modified k-data vector is inverse transformed back to the space domain. This space-data vector is bandlimited, but is not the correct solution because the data at the sensor nodes will in general be incorrect. Therefore, the data values at the sensor nodes are discarded and replaced by the true values. In addition, any negative value is reset to zero, enforcing positivity. The iteration process then repeats.

A sequence of space-data solutions for the field intensity of wave W3 shown in figure 6.3 is shown in figure 6.4. The POCS algorithm was allowed to run for 100 iterations. An automated peak-picking routine selected the maximum at -0.3636 : this then would be the initial guess for x_0 .

¹The sparse array spacing provides adequate spatial sampling of the wavefield for frequencies less than about 750 Hz. Above this frequency, the samples are spatially aliased and POCS reconstructions are seriously degraded. For automation purposes, therefore, the POCS reconstruction is always applied to the data from the lowest frequency band available, in this case, at 603 Hz.

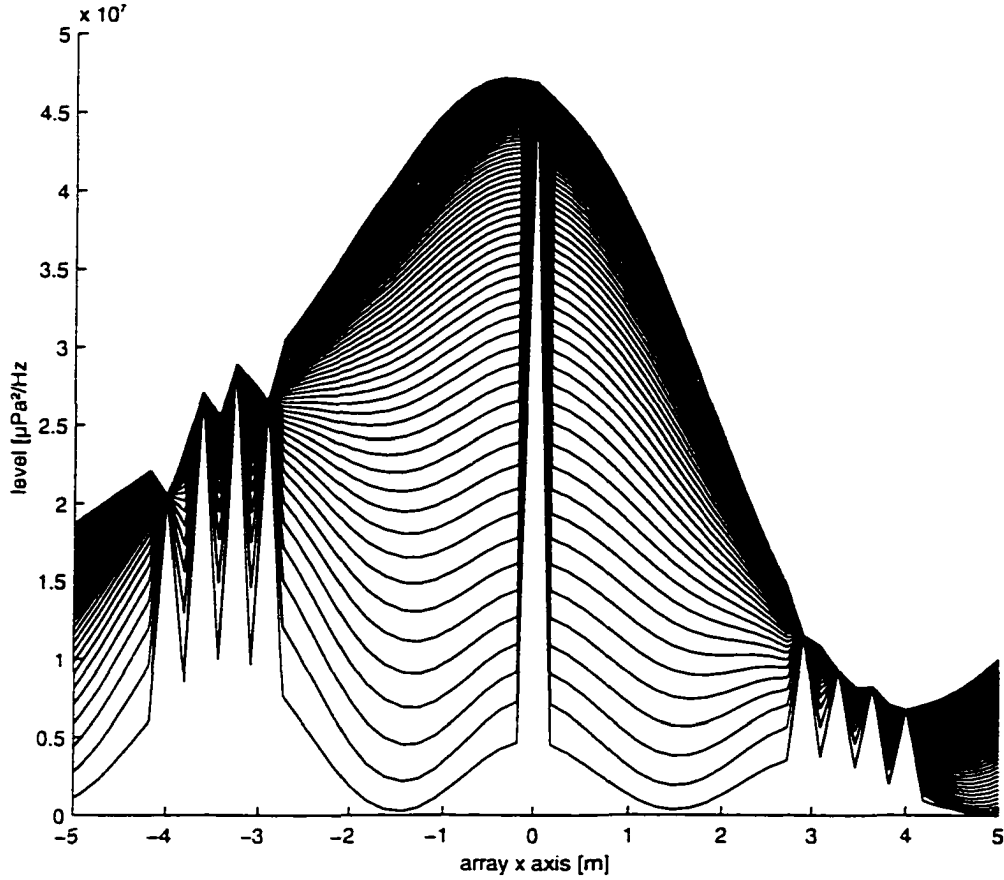


FIGURE 6.4: Sequence of POCS solutions for the Field intensity, x-axis, wave W3.

6.4 A Well-behaved Image

A successful image will generally follow when the incoherent power peaks near the center of each arm, as in figure 6.3, at each of the six different frequency bands given in table 6.1. The ambient and signal+ambient CSDMs are supplied to the optimizers described in Appendices C and D, which converge after a few hours.

Wave W3 is an example of a successful imaging process. At convergence, the non-parametric solution provides the broadband pixel image $\mathbf{I}$ and the set of frequency parameters $\{\lambda_1, \lambda_2, \dots, \lambda_6\}$. These are shown together in figure 6.5. The regularization parameter has been set to zero for this image, as explained in Appendix C, and the

broadband shape function quantized into four bands that span the dynamic range of the solution. The frequency parameters are converted to units of dB re $1 \mu\text{Pa}^2$ (see the discussion of appendix J regarding source strength units.)

At convergence, the parametric image has the shape values given in table 6.3 and 6.4. These parameters provide a full description of the image model but do not, by themselves, have the same impact as an actual image. The parameters can be converted into a image using the mapping of Eq.(4.24): the result is shown in figure 6.6.

The nonparametric image reveals an extended noisy area downwind of the region of primary activity, and essentially no wake. The acoustically active region is long and narrow, with the long axis aligned roughly perpendicular to the wind direction, which is similar to the typical configuration of whitecaps. The parametric image captures the essential position and orientation information, but the source strength levels are quite different than the nonparametric source strength levels.

This difference in source strength levels seems to be related to the dynamic range of the nonparametric solution. Convergent solutions for nonparametric solutions would not fully span the range $[0,1]$. One possible cause for this behavior might be the variability in the CSDM estimators. The ambient background CSDM may be well-approximated by the estimator $\hat{\mathbf{N}}$, which is derived from a relatively long sample of ambient sound, but may not be well-represented in the ambient+signal CSDM $\mathbf{S}$ because the latter is derived from a very short record. Even if $\hat{\mathbf{N}}$ is a good approximation of $\mathbf{N}$, statistical variability (in addition to model misfit) will cause misfit errors between $\mathbf{S}$ and $\mathbf{R}_{ss} + \hat{\mathbf{N}}$. The optimizer may be trying to minimize these errors by elevating the levels of the peripheral pixels.

Not every wave produces such a successful solution. A compendium of problems is presented in the next section.

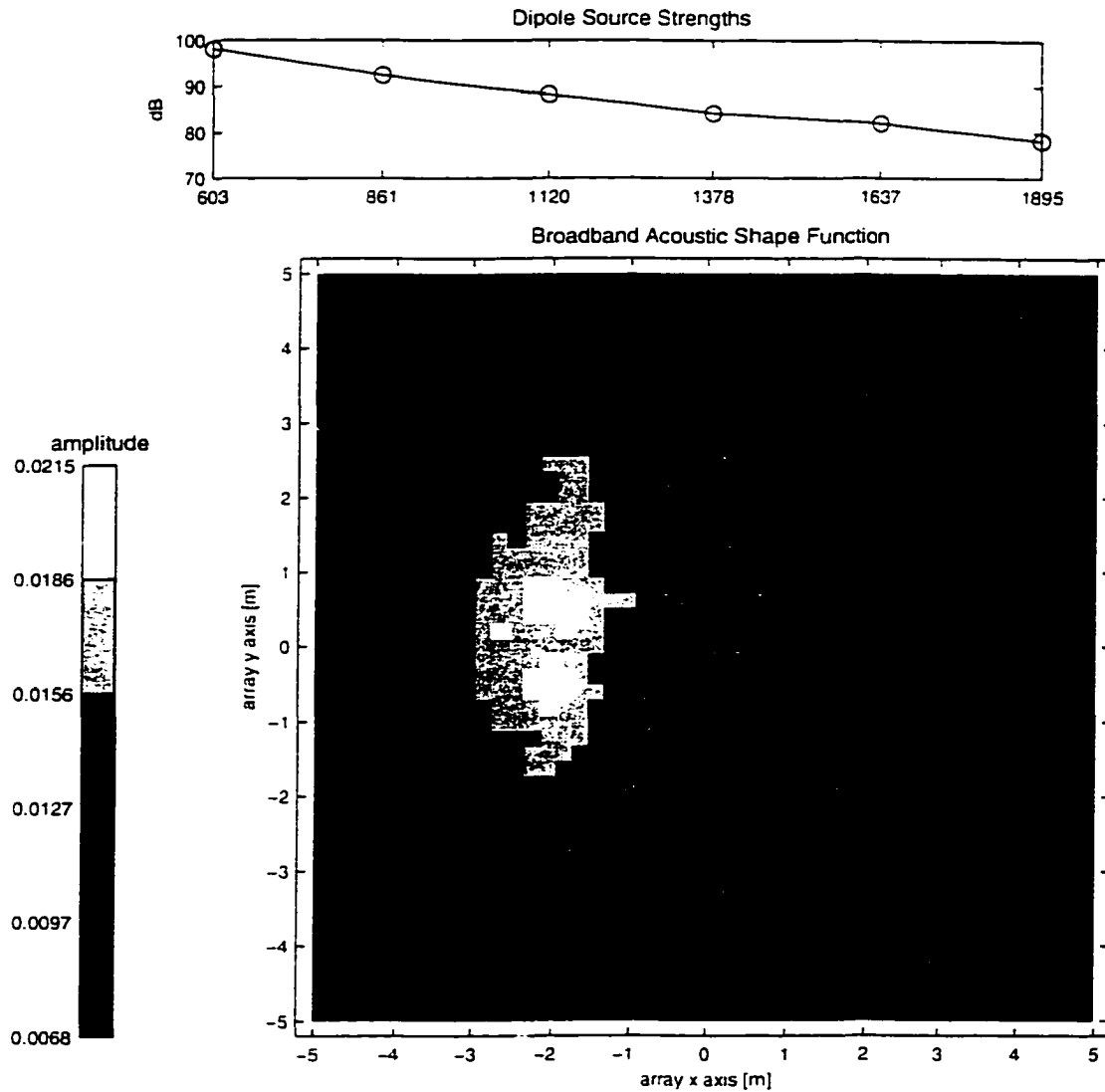


FIGURE 6.5: Nonparametric solution, wave W3. The upper plot shows the source strength in units of dB re $1 \mu\text{Pa}^2$. The lower plot shows the shape function in which the third dimension, broadband level, is quantized into one of four grey levels. The mapping of the grey levels versus actual function values is shown on the sidebar to the left.

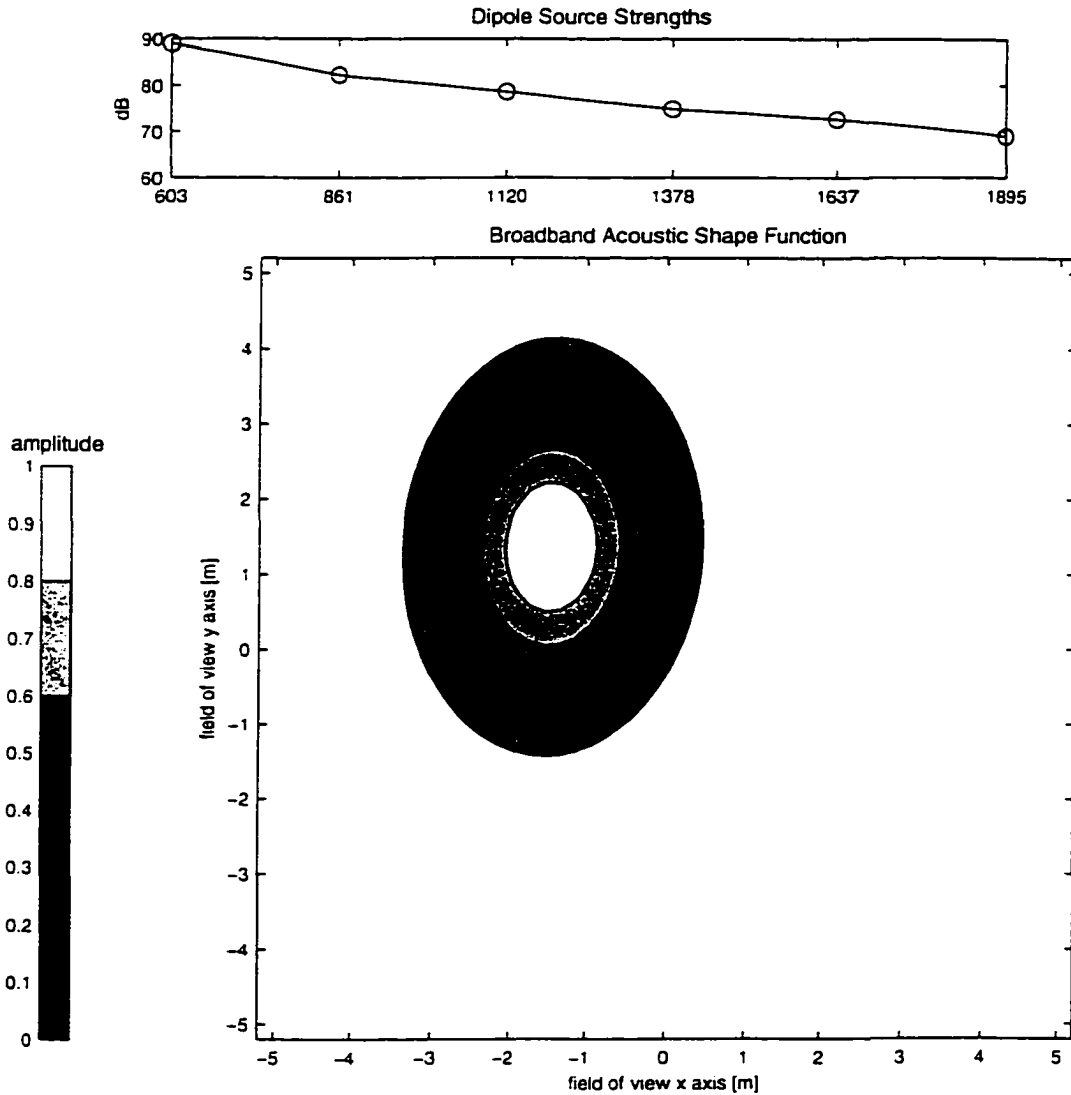


FIGURE 6.6: Parametric solution, wave W3. The upper plot shows the source strength in units of dB re $1 \mu\text{Pa}^2$. The lower plot shows the shape function in which the third dimension, broadband level, is quantized into one of four grey levels. By construction, the grey levels map to the range $[0.1]$, as shown on the sidebar to the left.

frequency:	603	861	1120	1378	1637	1895
λ	0.50	-1.07	-1.90	-2.76	-3.27	-4.11

TABLE 6.3: Source strength parameter $\lambda(\omega)$ solution for wave W3.

$N_{ambient}$	N_{signal}	$x_0[m]$	$y_0[m]$	$w[m]$	$l[m]$	$\phi [rad]$
340	105	-1.43	1.33	1.80	1.23	87.1

TABLE 6.4: Maximum likelihood parameter solution for wave W3. All frequencies used. The first and second columns provide the number of accumulants in the estimation of the ambient-only and ambient+signal CSDMs, respectively. The third and fourth columns provide the source centroid (units of metres) in flat array frame coordinates at the nominal fluid surface. Columns 5 and 6 provide the major and minor axis scales (units of metres). Column 7 provides the orientation of the wave major axis relative to the array frame of reference (units of degrees).

6.5 What Can Go Wrong

There are apparently two primary problems that prevent the program outlined in previous sections from achieving successful broadband images.

1. The wave may not be fully within the field-of-view, or,
2. arm-end noise contaminates the signal.

These problems are described next.

Half-observed waves Solutions are not formed for actively radiating surface regions outside the array field-of-view. Such regions, if they are adjacent to the field-of-view, may contribute a partially coherent signal across the array which is unlike that due to the background ambient sound. The optimizer will then alter the solution within the field-of-view to try to account for the extra energy. This will invariably bias the solution within the field of view.

This is most evident when attempting to fit a parametric shape to an energy source at the edge of the field-of-view. A considerable portion of the parametric solution may fall outside the field-of-view. This outer portion is ignored, as if the gaussian shape was truncated. The optimizer must then alter the model to match the remaining model energy with the received signal energy. It generally attempts to do this by increasing the width and length scales, thus broadening the gaussian. This broadened model achieves a poor fit to the data, so the optimizer will tend to move the gaussian centroid further out. And so the cycle keeps repeating. It is not uncommon in these situations to see intermediate solutions with source centroids tens of metres away, and wave shapes six to ten metres across.

Therefore, waves that seem to be on the edge of the field-of-view must be discarded. The incoherent power levels across each arm, as described in section 6.3, are used to determine when this problem exists: if the incoherent power peaks on the last two sensors of an arm, the wave centroid is probably located on or near the edge of the field-of-view, and the wave is not processed.

Arm-end noise A significant amount of low-frequency noise is present on the outer sensors of the arms during some of the waves. Evidence of this problem is shown in figure 6.7: this is a plot of the incoherent levels at $f = 603$ Hz on both x - and y -axes for wave W6. The levels along the x -axis show abnormally high values at the end sensors. The POCS algorithm will clearly fail to find an x -value for the source centroid within the field-of-view: the algorithm, in fact, places the x_0 on the field-of-view boundary. Attempts to fit a model with data like this will invariably require the optimizer to put a solution near the edge of the field-of-view, and all the difficulties associated with this problem will arise again.

The acoustic signals on channels 6, 5 and 3 (leftmost, center and rightmost x -axis sensors, respectively, in figure 6.7) are shown in figure 6.8. The signal on channel 5 may be taken to be ambient+signal. Channels 6 and 3 are contaminated by several noise

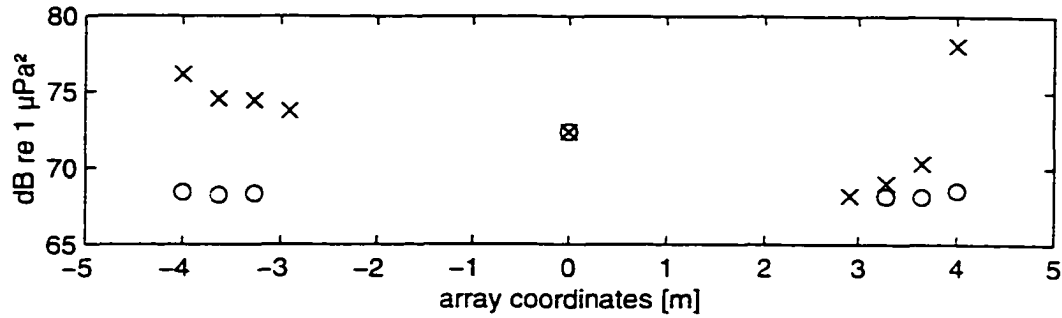


FIGURE 6.7: Field intensity versus sensor position, wave W6. x axis levels are indicated by $\times$ and the y axis levels by o . Frequency $f = 603$ Hz.

bursts. These bursts resemble resonant “ringing” with frequencies as high as 800 Hz.

The source of these noise bursts is most likely unsteady vibrations originating in the Halibut poles. This vibration then travels down into the arms, where it is greatest at the ends (which are closest to the attachment point of the poles) and then attenuates as the vibration moves inward along the arms.

This contamination, when present, is usually the most severe at the lowest frequencies. If the effect is seen at 1120 Hz, it will also be present at 861 Hz and 603 Hz.

This type of contamination is present in roughly half the waves analyzed. For each wave which appears to be fully within the field-of-view (see above), the incoherent levels across each arm are examined at each frequency. If, at a particular frequency, the incoherent levels do not peak near the center (i.e., the levels peak near the ends as in figure 6.7), the information at that frequency is not used.

Solutions are only sought if three or more bands of information pass this criterion. (These are usually the upper frequency bands.)

Discarded bands are documented as necessary in Appendix I for each wave.

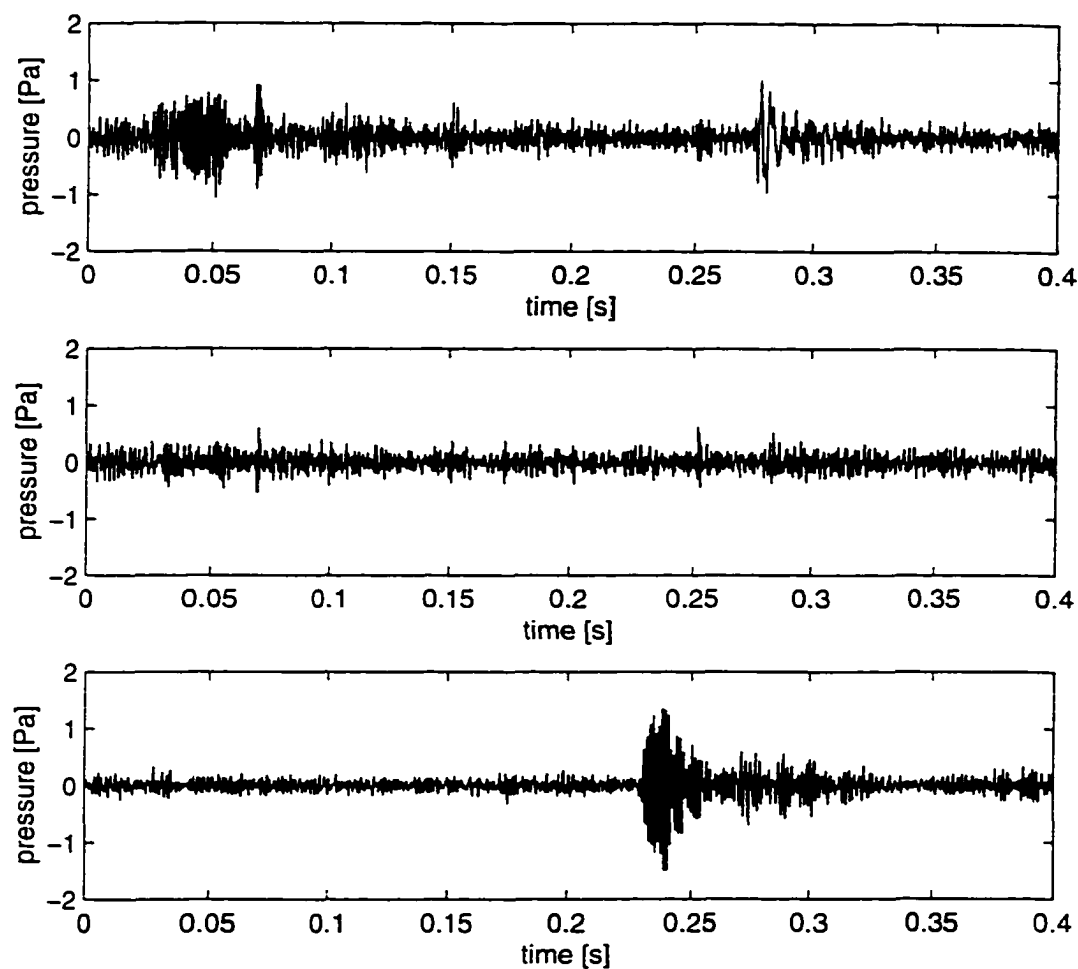


FIGURE 6.8: Acoustic pressure, channels 6 (top), 5 and 3 (bottom), wave W6. Vertical units are Pa in the passband [160 Hz, 2000 Hz]. The raw data have been prefiltered with a high-pass filter with corner frequency at 160 Hz, then converted to in-water pressure units.

Chapter 7

Data Analysis

The essential signal processing leading up to the formation of nonparametric and parametric images was described in chapter six. In this chapter, a further analysis is conducted based on the image solutions. Sections 7.1 through 7.3 involve analysis of the image solutions of 30 waves, and provide inferences related to the wind and source strength levels. Section 7.4 explores the interpretation of the sound generation mechanism at frequencies below about 400 Hz (where the imaging technique is considered invalid) using information gained via imaging solutions above about 400 Hz.

7.1 Event Motion

The time evolution of a single breaker should show a radiant region moving roughly downwind. To investigate this notion, waves W3 and W5 were chosen for a sequence analysis. These waves were chosen because they allowed full broadband solutions (i.e., all measurement bands contributed to the solutions and should therefore yield more stable estimates), had significant SNR over their duration, and had minimal simultaneous contamination.

In each case, the acoustic record was partitioned more-or-less “by eye” into segments approximately 0.2 - 0.4 s long, and a solution computed for each segment. The results

for each wave are presented in the following sections.

7.1.1 Wave W3

Wave W3 contained four segments, A through D, that yielded stable solutions. Attempts to produce solutions prior to segment A or later than segment D yielded unstable results, due most likely to insufficient SNR. The segmentation plan is shown in figure 7.1.

A graphic presentation of the sequence of shape solutions is provided in figure 7.2 through figure 7.5. These are nonparametric images involving all 6 frequency bands, and clearly show the active acoustic region propagating to the right. The concentration of the source region clearly diminishes in figure 7.5, which is possibly related to the collapse of the breaker into a β -plume, although it is not clear that the MSD remains separable at this stage of entrainment.

A parametric solution over each segment produces the solutions in tables 7.1 and 7.2. The ML shape parameter estimators are shown in table 7.1 and the ML source strength parameter estimators in table 7.2. Segment C is the “main” segment, whose parameters are used in Appendix I.

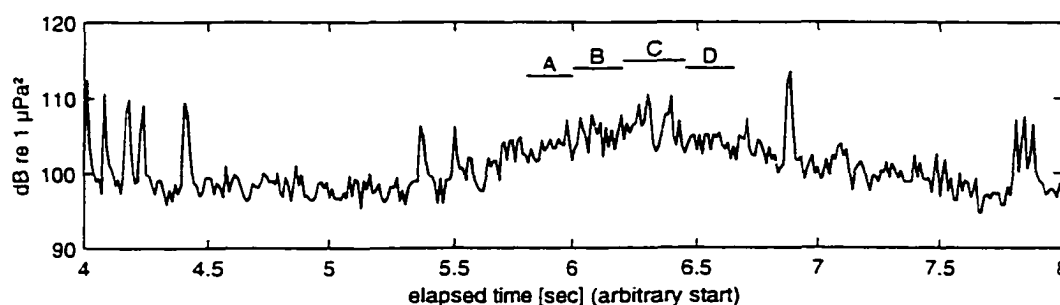


FIGURE 7.1: Short-time segmentation of wave W3. End times, relative seconds: A, [5.8, 6.0]; B, [6.0, 6.2]; C, [6.2, 6.45]; D, [6.45, 6.65]. Time axis referenced to 15:24:01 PDT (see wave W3 in Appendix I).

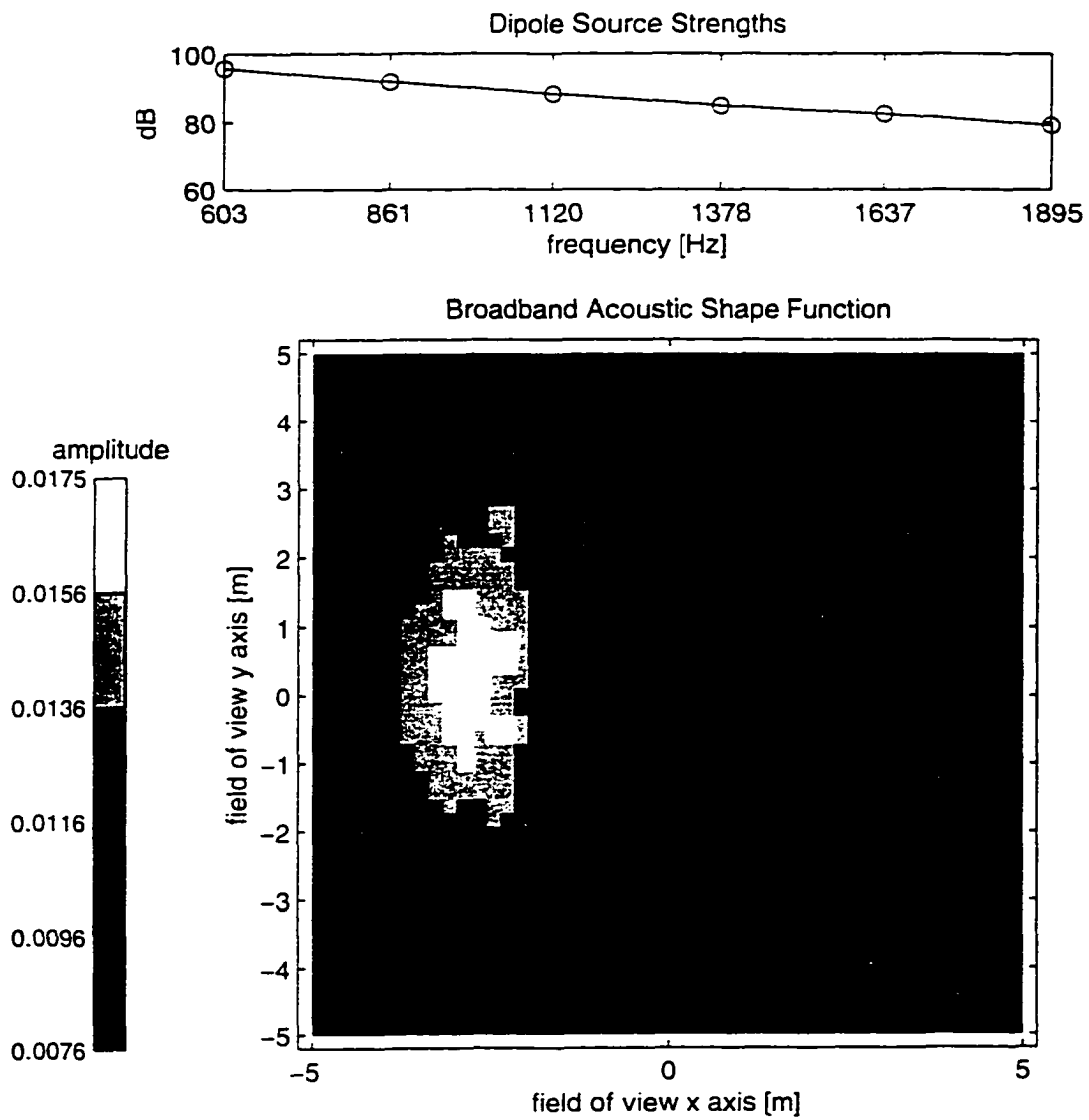


FIGURE 7.2: Nonparametric solution, wave W3, segment A

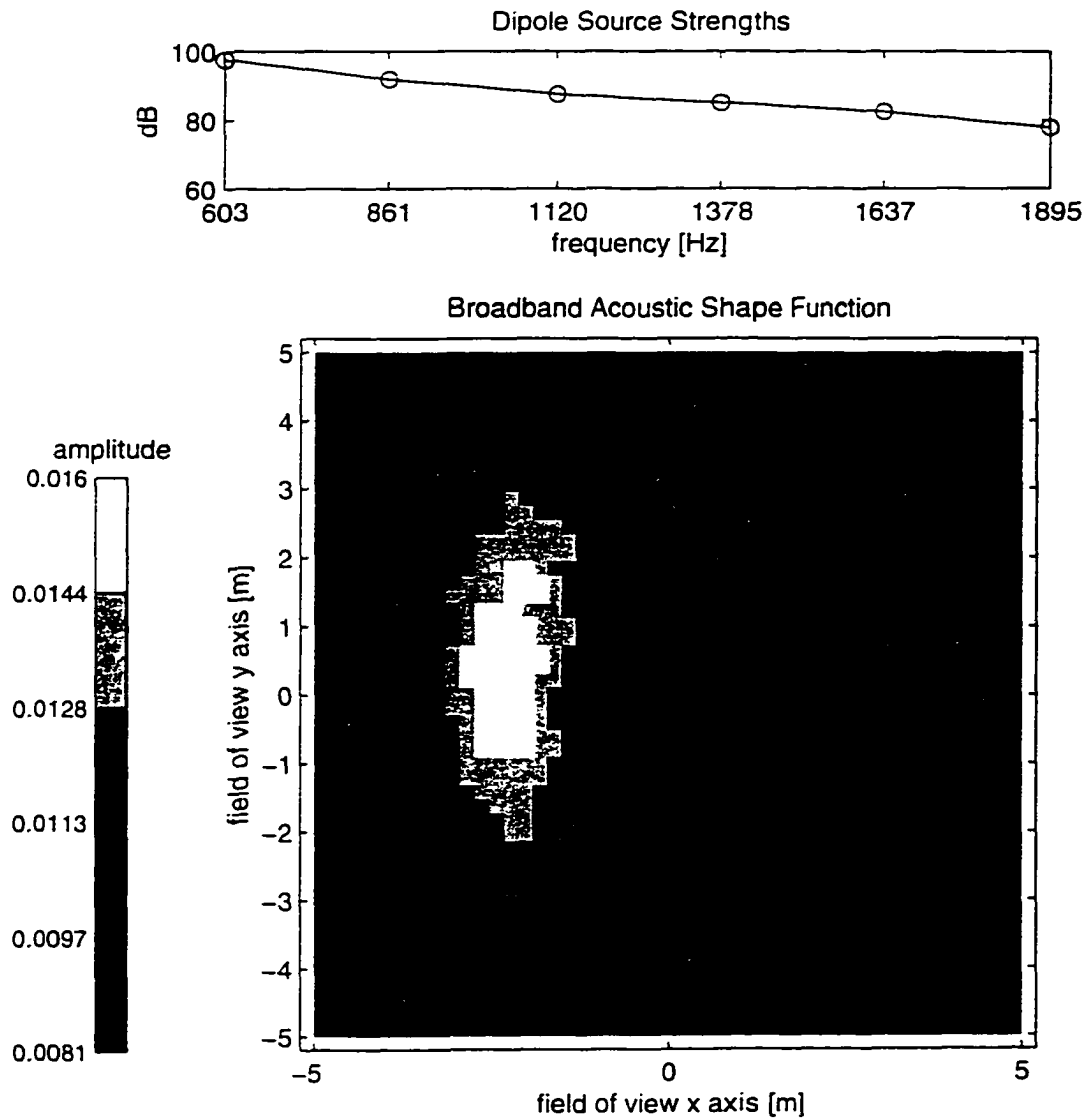


FIGURE 7.3: Nonparametric solution, wave W3, segment B

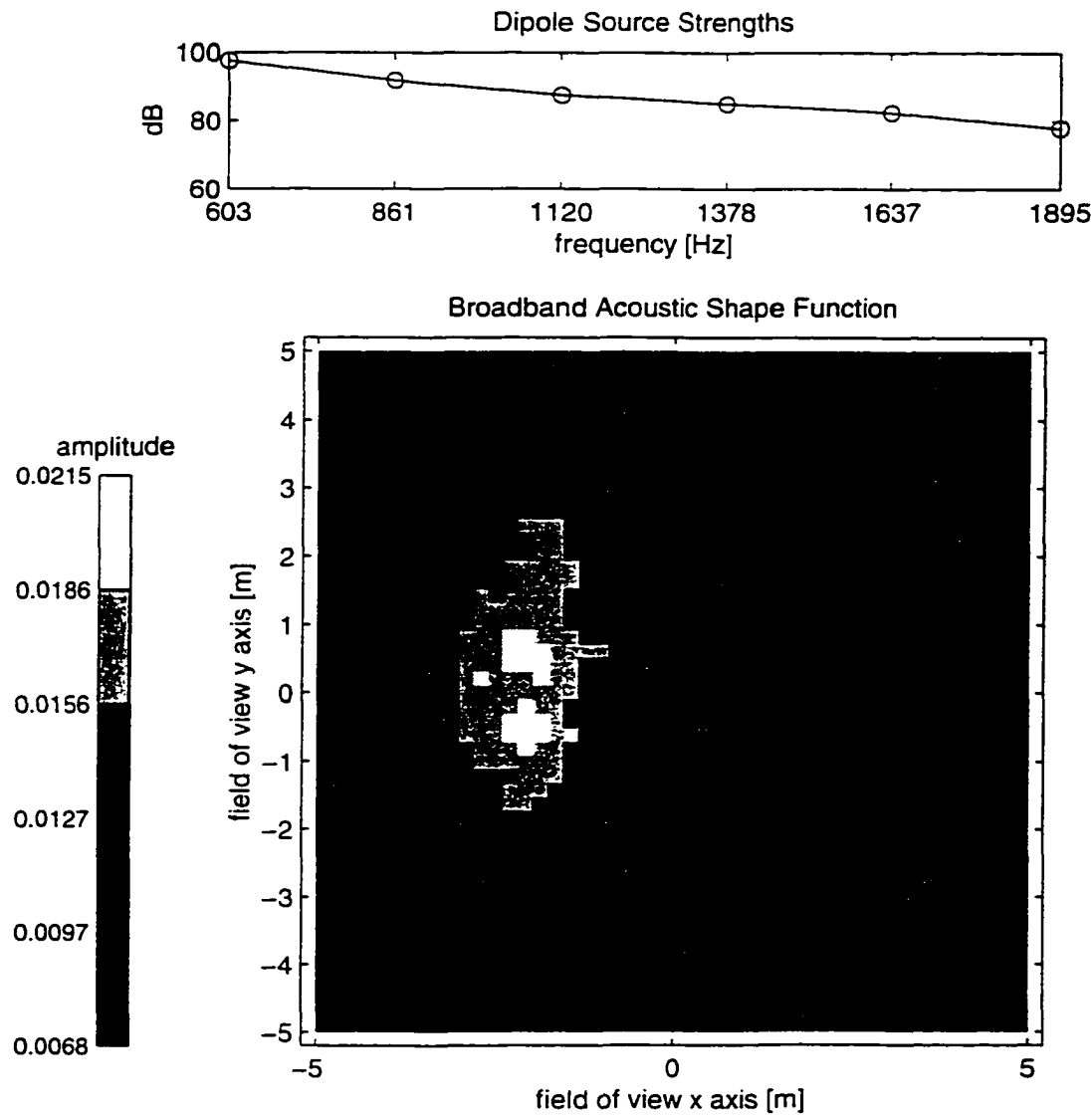


FIGURE 7.4: Nonparametric solution, wave W3, segment C

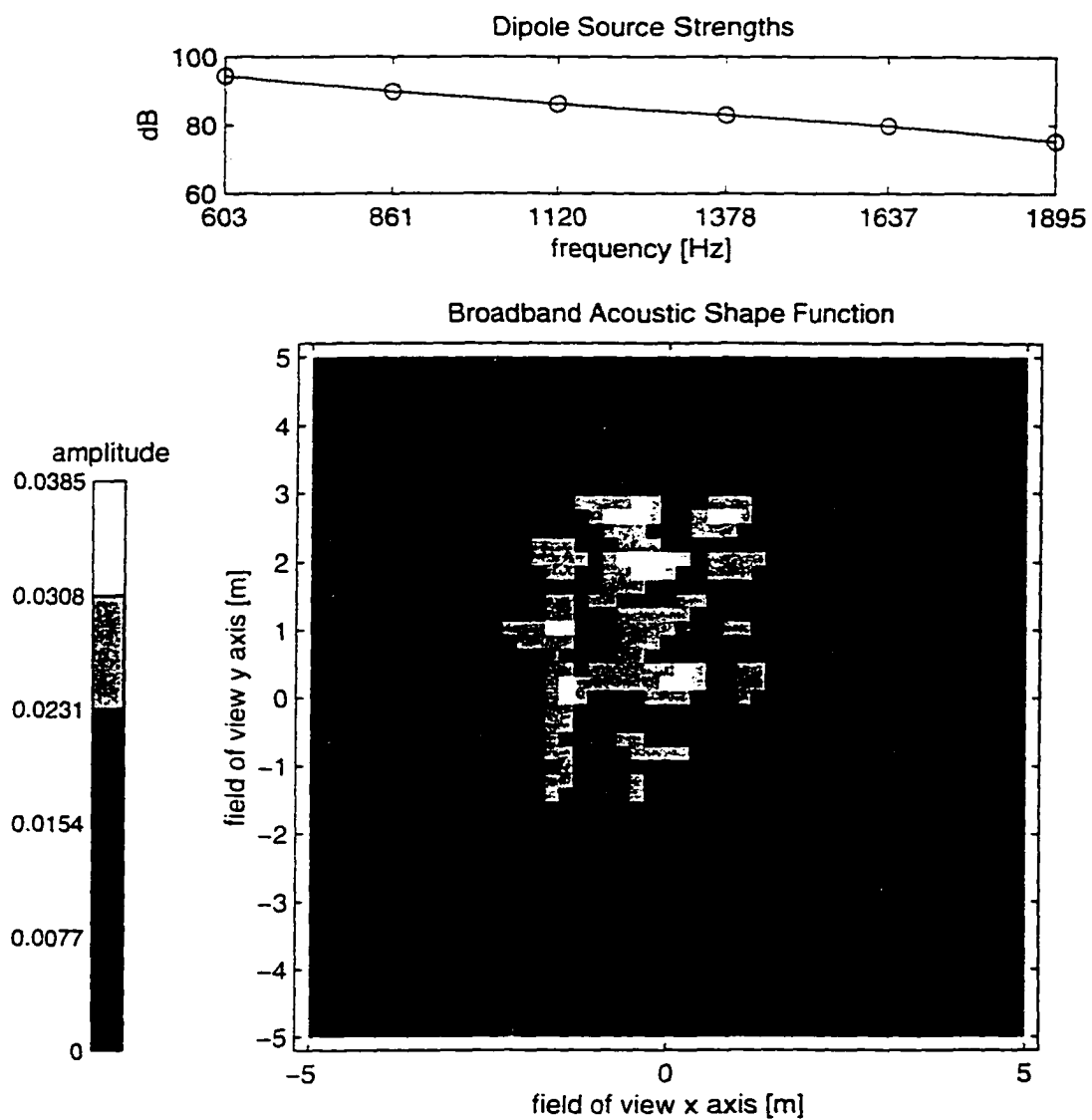


FIGURE 7.5: Nonparametric solution, wave W3, segment D

A graphic presentation of the sequence of shape solutions is provided in figure 7.6. The wind direction, corrected to the array field-of-view frame-of-reference, is also shown. The radiant region is clearly seen to be moving almost directly downwind. These parametric solutions correspond quite closely with the nonparametric images as expected.

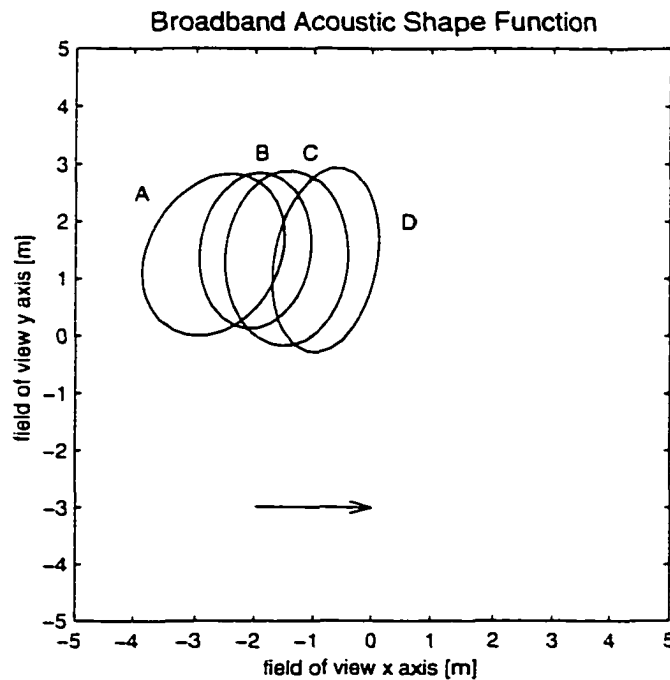


FIGURE 7.6: Sequence of solutions for wave W3. Solution evolves from A to D: only the half-power contour (intensity ≈ 0.5) solution is drawn. The approximate wind direction is also indicated with the arrow.

wave	$N_{ambient}$	N_{signal}	$x_0[m]$	$y_0[m]$	$w[m]$	$l[m]$	$\phi [^\circ]$
3A	340	85	-2.62	1.39	1.74	1.30	64.2
3B	340	85	-1.95	1.46	1.61	1.11	83.8
3C	340	105	-1.43	1.33	1.80	1.23	87.1
3D	340	85	-0.77	1.30	1.92	1.03	80.2

TABLE 7.1: Maximum likelihood parameter evolution for wave W3. All frequencies used. The second and third columns provide the number of accumulants in the estimation of the ambient-only and ambient+signal CSDMs, respectively. The fourth and fifth columns provide the source centroid (units of metres) in flat array frame coordinates at the nominal fluid surface. Columns 6 and 7 provide the major and minor axis scales (units of metres). Column 8 provides the orientation of the wave major axis relative to the array frame of reference (units of degrees).

frequency:	603	861	1120	1378	1637	1895
wave						
3A	-0.13	-1.09	-1.86	-2.61	-3.11	-3.88
3B	0.72	-0.91	-1.82	-2.47	-3.01	-4.05
3C	0.50	-1.07	-1.90	-2.76	-3.27	-4.11
3D	-0.41	-1.48	-2.26	-3.00	-3.69	-4.70

TABLE 7.2: Source strength parameter $\lambda(\omega)$ evolution for wave W3.

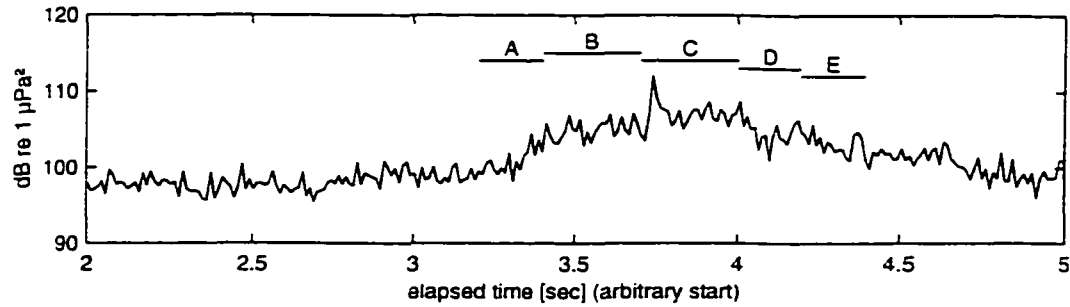


FIGURE 7.7: Short-time segmentation of wave W5. End times, relative seconds: A. [3.2, 3.4]; B. [3.4, 3.7]; C. [3.7, 4.0]; D. [4.0, 4.2]; E. [4.2, 4.4]. Time axis referenced to 15:25:16 PDT (see wave W5 in Appendix I.)

7.1.2 Wave W5

Wave W5 contained five segments, A through E, that yielded stable solutions. Attempts to produce solutions prior to segment A or later than segment E yielded unstable results, due most likely to insufficient SNR. The segmentation plan is shown in figure 7.7.

The ML shape parameter estimators are shown in table 7.3 and the ML source strength parameter estimators in table 7.4. Segment B is the “main” segment, whose parameters are used in Appendix I.

A graphic presentation of the sequence of shape solutions is provided in figure 7.8. The wind direction, corrected to the field-of-view frame-of-reference, is also shown. The radiant region is again clearly seen to be moving almost directly downwind.

7.1.3 Motion Analysis

In both cases, it appears that the radiant region is wider along the cross-wind direction, which conforms to visual observations that stage A breakers have a wider cross-wind dimension.

wave	$N_{ambient}$	N_{signal}	$x_0[m]$	$y_0[m]$	$w[m]$	$l[m]$	$\phi [^\circ]$
5A	470	85	-1.00	1.50	1.84	1.00	90.4
5B	470	125	-1.17	1.54	1.77	1.20	92.2
5C	470	125	0.01	1.55	1.64	1.22	59.5
5D	470	85	0.67	1.44	1.57	1.32	72.1
5E	470	85	1.42	1.30	1.70	1.22	130.0

TABLE 7.3: Maximum likelihood parameter evolution for wave W5. All frequencies used. The second and third columns provide the number of accumulants in the estimation of the ambient-only and ambient+signal CSDMs, respectively. The fourth and fifth columns provide the source centroid (units of metres) in flat array frame coordinates at the nominal fluid surface. Columns 6 and 7 provide the major and minor axis scales (units of metres). Column 8 provides the orientation of the wave major axis relative to the array frame of reference (units of degrees).

frequency:	603	861	1120	1378	1637	1895
wave						
5A	1.05	-0.12	-1.52	-2.36	-3.05	-4.18
5B	0.85	-0.23	-1.64	-2.49	-3.24	-4.39
5C	0.76	-0.46	-1.28	-2.20	-2.81	-3.83
5D	-0.17	-1.12	-2.11	-2.80	-3.45	-4.29
5E	-0.51	-1.65	-2.43	-3.18	-3.98	-4.92

TABLE 7.4: Source strength parameter $\lambda(\omega)$ evolution for wave W5.

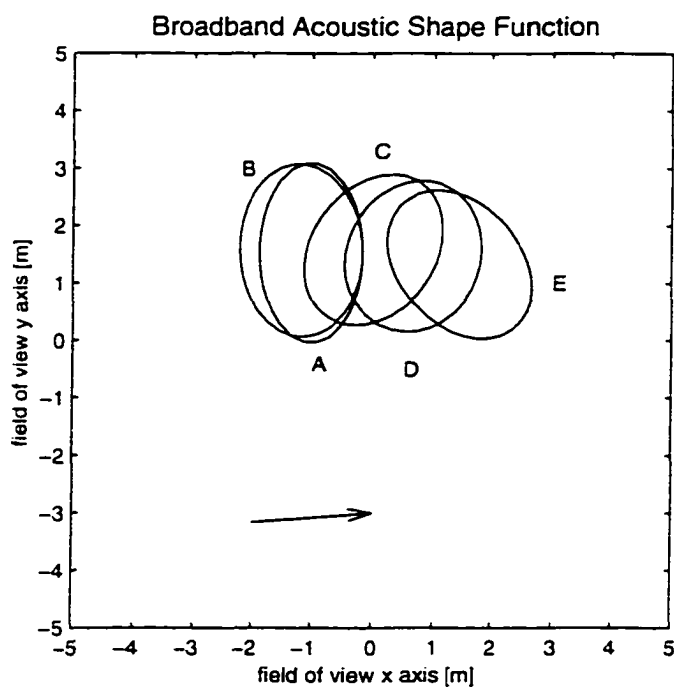


FIGURE 7.8: Sequence of solutions for wave W5. Solution evolves from A to E: only the half-power contour (intensity = 0.5) solution is drawn. The approximate wind direction is also indicated with the arrow.

The mean speed of each breaker can be computed by measuring the Euclidean distance between the centroids of the first and last shape functions and dividing by the total segmentation duration. Using this formula, the mean speed $\bar{c}_{br}$ of the wave W3 breaker is 2.35 ms^{-1} , and the mean speed of the wave W5 breaker is 2.43 ms^{-1} . The mean speed of wave W5 would be greater than 3 ms^{-1} if measured over segments B - E. In contrast, the phase speed of the dominant wind waves is from Appendix A $c_p = 3.65 \text{ m s}^{-1}$. This is consistent with measurements of mean breaking speed made by Ding and Farmer ([36]) which showed that $\bar{c}_{br} \approx 45\%$ to 75% of c_p .

7.2 Inferences on Wind Direction

The event motion was shown in the previous section to indicate the direction of the prevailing surface wind. As whitecaps are generally elongated along the crests of waves, and the waves generally align with the wind, the orientation of the whitecaps should be another indicator of wind direction. It is revealing to investigate whether the orientation of the acoustic source regions provides similar information.

The wind direction and velocity are seen from figures 5.6 and 5.4 to be roughly stationary during the period from which the waves of appendix I were selected. The orientation solutions for these waves therefore comprise samples from approximately the same distribution. These samples are shown in figure 7.9 as triangles, plotted versus time: the estimation errors on each point are, as described in Appendix D, too small to be seen on this scale. Also plotted in figure 7.9 are the vector-averaged wind direction estimates generated every 10 minutes by the nearby Minimet buoy, along with standard error bars, also provided by the buoy.

There are 21 orientation solutions for the time period considered: the arithmetic mean orientation using all solutions is $\bar{\phi} = 280.1^\circ$. It is not clear that these samples are drawn from any known distribution, and, in fact, the appearance of outliers suggests the distribution of $\bar{\phi}$ is somewhat leptokurtic. The standard error of the estimator $\bar{\phi}$

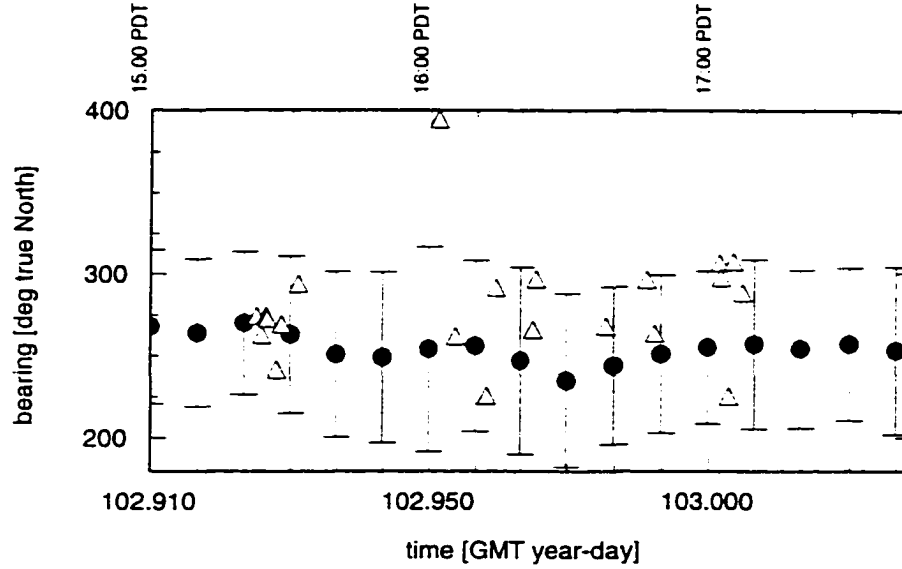
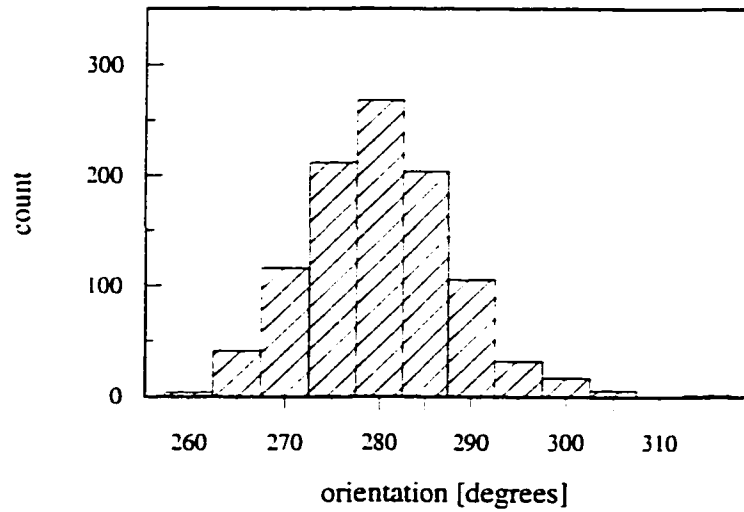


FIGURE 7.9: Sequence of inferred wind directions. The triangles are derived from the orientation ϕ of the wave shape function, and the circles are from the nearby Minimet buoy. The vertical error bars on the circles represent 1 standard error.

is therefore estimated by bootstrapping (Efron and Tibshirani [42]). In this case, 1000 bootstrap replicates were generated, and the bootstrap standard error was estimated as $\widehat{se}_B(\bar{\phi}) = 7.6^\circ$. A histogram of the 1000 bootstrap replicates is shown in figure 7.10.

Inspection of figure 7.9 shows that there is clearly one significant outlier at about 16:02. This is the solution for wave W9. As described in Appendix I, some of the lower frequency information had to be disregarded for this wave, and there is some evidence that the model fit was not good over the remaining frequencies, but it was otherwise not more unusual than other waves possessing similar “ill-behaved” data. If $\bar{\phi}$ and $\widehat{se}_B(\bar{\phi})$ are recomputed with this point deleted, the resulting value is $\bar{\phi}_{(9)} \pm \widehat{se}_{B,(9)} = 272.3 \pm 4.6^\circ$. This is well within the standard error of the surface wind direction measurements.

These results reveal that the acoustically active source regions are oriented cross-wise to the wind, and, by inference, elongated along the crests of the waves just as are

FIGURE 7.10: Histogram of $\bar{\phi}$ replicates.

whitecaps. Measurements of the source region orientation are therefore quite consistent with surface measurements of wind direction.

7.3 Source Strength

7.3.1 Data Reduction

Using the same argument presented in the previous section, the sequence of source strength solutions for the waves analyzed in Appendix I may be considered samples from a single underlying distribution. In this case, the distribution of source strengths is assumed to be associated with a single wind speed: from figure 5.4, this is approximately 7 to 8 m s⁻¹.

All available solutions are plotted together in figure 7.11: solution points at *different* frequencies for the *same* wave are connected by simple line segments. The most prominent feature of this figure is the appearance that the frequency dependence of the source strength parameters, over the available frequency band, is predominantly linear

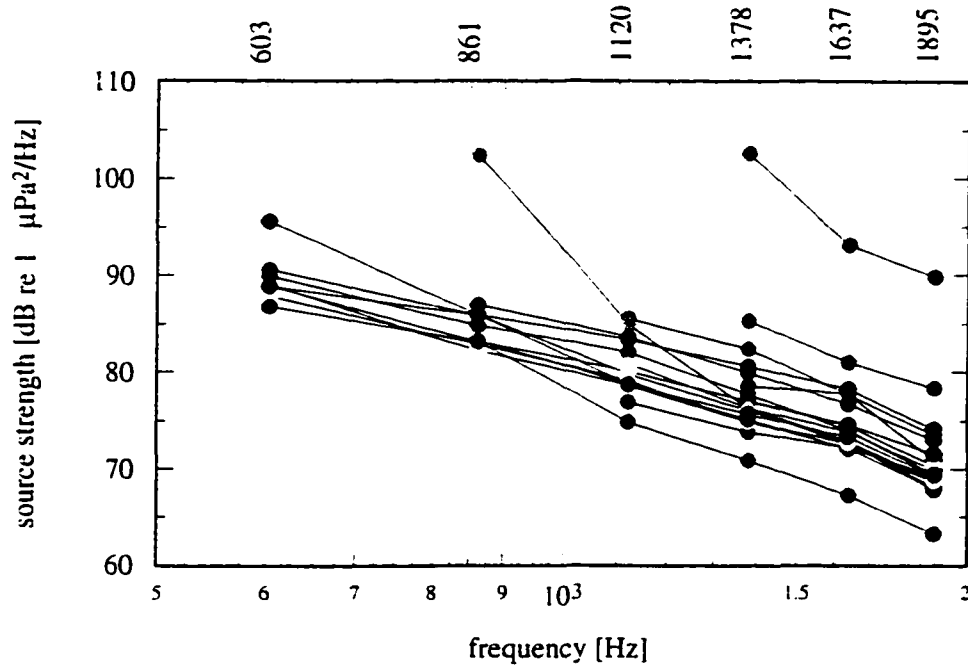


FIGURE 7.11: Source strength parameters. 21 waves with 3 or more solutions.

in log-frequency. This suggests that the source strength parameter be modeled as

$$\lambda(f) = b_0 + b_1 \ln f \quad (7.1)$$

where b_0 will vary from wave to wave as each varies in size and breaking intensity. (One might suppose the distribution of b_0 to be strongly wind dependent.) The parameter b_1 , however, might be considered a more universal function, so it is reasonable to combine all available solutions for the estimation of b_1 .

The estimation proceeds as follows. A linear regression is performed on each available solution against a model which is first-order in log-frequency. This produces 21 "raw" values of $\hat{b}_0$ and $\hat{b}_1$, as shown in table 7.5.

TABLE 7.5: Source parameter regression fit, each wave

wave	points	$\hat{b}_0$	$\hat{b}_1$	$\hat{se}(\lambda \cdot f)$	$\hat{cd}$
1					
2	3	30.7	-4.57	0.12	0.96
3	6	25.3	-3.88	0.10	1.00
4	3	37.1	-5.36	0.36	0.79
5	6	30.0	-4.51	0.22	0.98
6	4	20.1	-3.19	0.07	0.99
7	6	23.1	-3.52	0.15	0.99
8	3	37.9	-5.58	0.02	1.00
9	4	37.0	-5.29	0.15	0.98
10					
11	3	36.1	-5.04	0.06	0.99
12					
13	6	26.3	-3.96	0.27	0.97
14	6	25.4	-3.90	0.17	0.99
15					
16					
17	3	71.3	-9.39	0.28	0.95
18	3	26.2	-3.94	0.05	0.99
19					
20	5	68.7	-9.74	0.61	0.95
21	4	35.0	-5.17	0.06	1.00
22	6	36.4	-5.41	0.19	0.99
23	6	18.7	-2.81	0.23	0.96
continued on next page					

TABLE 7.5: (continued)

<i>continued from previous page</i>					
wave	points	$\hat{b}_0$	$\hat{b}_1$	$\hat{se}(\lambda \cdot f)$	$\hat{cd}$
24	3	34.5	-5.12	0.06	1.00
25					
26					
27	5	27.4	-4.03	0.14	0.98
28	5	25.4	-3.89	0.11	0.99
29					
30	6	31.7	-4.90	0.28	0.98

TABLE 7.5: Linear regression fit for source parameter for the model $\lambda(f) = b_0 + b_1 \ln f$, for each wave. Waves with less than 3 frequency points are shown as blank lines. Number of frequency points shown in column 2. Column 5 provides the estimated standard error of the regression of λ on $\ln f$, and column 6 provides the estimated coefficient of determination.

Table 7.5 also gives estimates of the standard error $\hat{se}(\lambda \cdot f)$ of the estimate of λ on $\ln f$ and the coefficient of determination $\hat{cd}$. These are computed by the standard formulae (Dowdy and Wearden [38], chapter 9)

$$\hat{se}(\lambda \cdot f) = s_{\lambda \cdot f}$$

$$\hat{cd} = r^2$$

which use

$$\begin{aligned}
 s_{\lambda, f}^2 &= s_{\lambda}^2(1 - r^2) \\
 s_{\lambda}^2 &= \left(\sum_i^n (\lambda_i - \bar{\lambda})^2 \right) / n \\
 r^2 &= 1 - \frac{\sum_i^n (\lambda_i - \hat{\lambda}(f_i))^2}{n s_{\lambda}^2} \\
 \hat{\lambda}(f_i) &= \hat{b}_0 + \hat{b}_1 \ln f_i.
 \end{aligned}$$

The process mean $\bar{b}_1$ is to be computed from the set of raw estimates $\{\hat{b}_1\}$. The histogram (not shown) of this set contains evidence of a few outliers, so the process mean is computed using

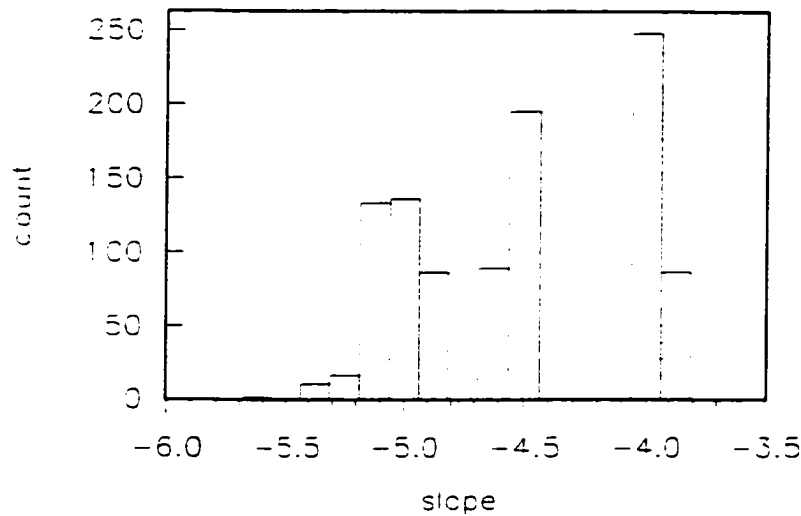
$$\bar{b}_1 = \text{median}(\{\hat{b}_1\}) \quad (7.2)$$

The distribution of the estimator $\bar{b}_1$ is not known, so the estimator standard error is again approximated with the bootstrap standard error. A histogram of the 1000 bootstrap replicates is shown in figure 7.12. The value of $\bar{b}_1$ is estimated at $\bar{b}_1 \pm \hat{se} = -4.55 \pm 0.47$.

As mentioned above, the values of $\hat{b}_0$ should have a distribution which is a function of wind speed. If these source strength curves are assumed to have a slope of $\bar{b}_1$, then raw estimates of b_0 can be made for each available curve. This sample set also shows evidence of outliers (histogram not shown), so the median is again used to make a robust estimate of b_0 given $b_1 = \bar{b}_1$: $\bar{b}_0|(b_1 = \bar{b}_1) = 30.4$.

7.3.2 Source Strength Analysis

According to Urick ([137], section 7.2), the surface component of the ambient sound spectrum from about 500 Hz to about 50 kHz has a shape known as the *Knudsen spectrum*. The slope of the Knudsen spectrum is nearly constant at about -6 dB per octave, or about -20 dB per decade. (Sometimes this is given as $f^{-5/3}$, or a slope of about -16.7 dB per decade.)

FIGURE 7.12: Histogram of $\bar{b}_1$ replicates.

This surface component is generally attributed to natural sea-surface processes, and, in particular, breaking waves. There must therefore be a connection between the spectral shape of the sound radiated from individual breaking waves and the Knudsen spectrum.

Several investigators have in fact had good success in modeling the shape of the Knudsen ambient sound curves by considering the ambient curve to have a shape similar to the radiated spectrum of individual breaking waves and then assuming that the latter is due to the incoherent contributions from a population of damped ringing bubbles within the breaker. Medwin and Daniel ([95]) and Medwin and Beaky ([94]) placed a hydrophone under small breaking waves in laboratory tanks and measured acoustic autospectra with an approximate Knudsen slope from about 500 Hz to about 50 kHz. Loewen and Melville ([83]) used Monte Carlo simulations to predict the radiated acoustic field from a region of active bubbles and also achieved a curve with similar slope and amplitude from about 400 Hz to above 50 kHz.

The source levels estimated in the previous section are related *by construction* to

the radiated spectra of individual breaking waves: the two are connected by a dipole propagation model. If the radiated spectra have shapes similar to that of the ambient autospectrum, then the mean source level and the ambient autospectrum should be similarly connected by the dipole propagation model. This can be checked directly.

The one-sided ambient autospectrum was estimated using the multiple-taper technique on W3 channel 5 data with $N = 256$ and $NW = 3$ with the first 5 tapers (Percival and Walden [109].) This estimator is shown in figure 7.13 with 90% jackknifed confidence intervals. Urick ([137], section 7.3) suggests that a typical level at 1000 Hz for a wind speed of about 7 m s^{-1} at a coastal location is about 67 - 68 dB re $1 \mu\text{Pa}^2/\text{Hz}$, so the estimator levels appear accurate. The anti-alias filter cut-off is quite evident near 2000 Hz. Below about 500 Hz, the ambient level plateaus at about 70 dB. This plateau is a well-known feature of ambient sound spectra (Urlick [137], section 7.2) and marks the region where man-made (shipping) noise overwhelms the Knudsen shape.

A Knudsen curve fixed at a level of 65 dB at 1000 Hz is also shown in figure 7.13: it is not extended below about 500 Hz as it is not clear whether it remains a valid model below this point. The ambient autospectrum follows this Knudsen curve in general, although there is evidence of some statistically significant fine structure.

Two model radiated spectra are shown in figure 7.13: both were computed for a fictitious sensor located 3 m directly below the centroid of a patch of acoustically active fluid with the parametric broadband shape function of wave W3. The source region was energized with (1) a regression fit of Eq.(7.1) on to wave W3 source strength parameters alone, and (2) the mean source strength model involving $\bar{b}_0$ and $\bar{b}_1$. The actual solutions for W3, propagated to the fictitious sensor position, are also shown: the radiated model (1) fits quite well to these actual (propagated) solutions at the six analysis frequencies.

There is, as expected, good agreement between the slope of both models after dipole propagation and the slope of the ambient autospectrum. The solution for W3 alone fits very well, and the median solution fits within the estimated confidence intervals.

This supports the idea that the mean source strength parameterization, when propagated out with a dipole radiation model, yields a mean radiated spectra which has the same spectral shape as the surface component of the background ambient noise.

Another feature of interest is the low-frequency cut-off in source-level: in principle, there ought not be individual bubbles with resonance frequencies below some appropriate low-frequency. The analysis here does not extend down into that regime, but over the available two octaves, the source level shows no sign whatsoever of a low-frequency

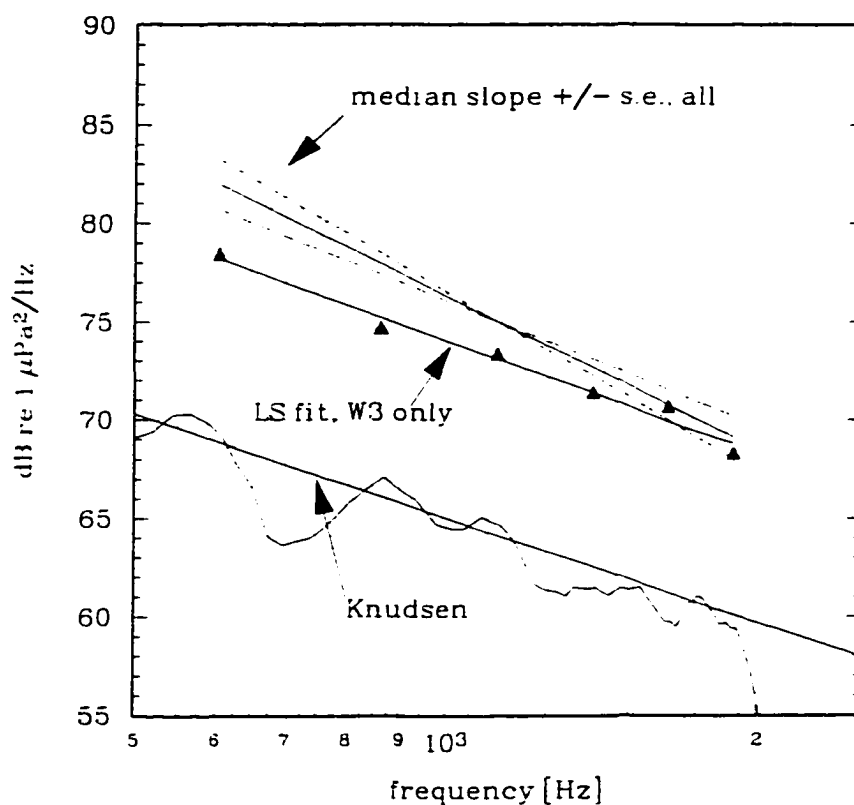


FIGURE 7.13: Comparison of W3 source level versus ambient and Knudsen levels. Channel 5 data. The triangles represent the actual solutions for the source level. The 90% confidence interval on the ambient autospectrum estimator is shown as the dotted lines. Two regression lines are shown: the one labeled “LS fit, W3 only” uses the slope $\hat{b}_1$ from wave W3. The other, labeled “mean slope” uses $\bar{b}_1$.

cut-off.

Analysis of the radiated signal in this lower frequency regime is pursued in the next section.

7.4 Modeling Low-Frequency ($f < 400$ Hz) Emissions

Although the data below about 400 Hz is often contaminated, there are a few waves where quality data were acquired both above 400 Hz, where the dominant mechanism is believed to be individual bubbles ringing at their resonance frequencies, and below 400 Hz, where the dominant mechanism is not as well defined. Of course, any analysis at the lower frequencies that involves imaging is inherently disadvantaged because resolved images simply cannot be obtained at the lower frequencies. Nevertheless, an attempt is made in this section to infer the nature of the source mechanism at the lower frequencies based on imaging solutions obtained at the upper frequencies. The first investigation, a “spatial” procedure, models the source region as a point and compares the location of the point with the image-based solution. The second investigation, a “temporal” procedure, compares a bubble-based spectral model against the actual radiated low-frequency signature.

7.4.1 Point Source Model

If the acoustically active region is much smaller than an acoustic wavelength, the source may be considered to be acoustically compact and treated essentially as a point source (Morse and Ingard [103], section 7.1). This suggests modeling the low-frequency mechanism as a finite dipole, with the true source at some depth z and the image source at $-z$. Such a configuration would generate a distinct signature on the array CSDM. Assuming monochromatic radiation at frequency ω , the resulting dipole field will be

$$p(\mathbf{r}_1; \omega) = \frac{P_0 e^{ikr_{1a}}}{4\pi r_{1a}} - \frac{P_0 e^{ikr_{1b}}}{4\pi r_{1b}} \quad (7.3)$$

where the point source is located by $\mathbf{r}_a = [x_0, y_0, z_0]^T$ ($z < 0$), the mirror image at $\mathbf{r}_b = [x_0, y_0, -z_0]^T$ and $r_{1a(b)} = |\mathbf{r}_1 - \mathbf{r}_{a(b)}|$. The second-order field moments across the array will then be

$$\begin{aligned} \langle p^*(\mathbf{r}_1)p(\mathbf{r}_2) \rangle &= \left(\frac{P_0^* e^{-ikr_{1a}}}{4\pi r_{1a}} - \frac{P_0^* e^{-ikr_{1b}}}{4\pi r_{1b}} \right) \left(\frac{P_0 e^{ikr_{2a}}}{4\pi r_{2a}} - \frac{P_0 e^{ikr_{2b}}}{4\pi r_{2b}} \right) \\ &= \frac{|P_0|^2}{16\pi^2} \left(\frac{e^{ik(r_{2a}-r_{1a})}}{r_{1a}r_{2a}} + \frac{e^{ik(r_{2b}-r_{1b})}}{r_{1b}r_{2b}} - \frac{e^{ik(r_{2a}-r_{1b})}}{r_{1b}r_{2a}} - \frac{e^{ik(r_{2b}-r_{1a})}}{r_{1a}r_{2b}} \right) \end{aligned} \quad (7.4)$$

The unknowns here are the source strength $|P_0|^2$ and the 3-element position vector $[x_0, y_0, z_0]$. As before, the source strength is parameterized by $|P_0|^2 = S_0 e^{20+\lambda}$.

These second order moments are estimated by the elements of the sample CSDM $\mathbf{S}(\omega)$. A simple model-fitting procedure models the elements of the sample CSDM as

$$[\mathbf{S}]_{l,m} = \langle p^*(\mathbf{r}_l)p(\mathbf{r}_m) \rangle + [\mathbf{N}]_{l,m} + \epsilon_{l,m} \quad (7.5)$$

where $\mathbf{N}$ is the co-existent ambient noise CSDM and $\epsilon_{l,m}$ is the misfit error term. The CSDM $\mathbf{N}$ is however unknown, and must therefore be replaced by the estimated ambient noise CSDM $\hat{\mathbf{N}}$. Then, using

$$\begin{aligned} \mathbf{s} &= \text{vec}\{\mathbf{S}\} \\ \mathbf{p} &= \text{vec}\{\langle p^*(\mathbf{r}_l)p(\mathbf{r}_m) \rangle\} \\ \mathbf{n} &= \text{vec}\{\hat{\mathbf{N}}\} \\ \mathbf{e} &= \text{vec}\{\epsilon_{l,m}\}. \end{aligned}$$

the model error is

$$\mathbf{e} = \mathbf{s} - \mathbf{p} - \mathbf{n}$$

and the least-squares solution for $\boldsymbol{\theta} = [x_0, y_0, z_0, \lambda]^T$ is

$$\hat{\boldsymbol{\theta}}_{LS} = \arg \min_{\boldsymbol{\theta}} \mathbf{e}^H \mathbf{e}. \quad (7.6)$$

This algorithm was coded as a Matlab M file and optimized with the function `leastsq()`. The function requires an initial guess $\boldsymbol{\theta}_0$ which was generally supplied as

$[x_0, y_0, -2.0, 2.0]^T$ where (x_0, y_0) were the centroid solutions from the associated image problem.

The low-frequency CSDM was estimated at $f = 344$ Hz: this is the center of the first fully orthogonal analysis band with time-bandwidth product $NW = 3$ below the band centered at $f = 603$ Hz. As shown in table 6.1, this band covers the range $[215 \text{ Hz}, 473 \text{ Hz}]$.

Eight waves had “well-behaved” solutions down to the lowest frequency of 603 Hz and therefore appeared to offer space-time data below 603 Hz that was uncontaminated by structural noise. The finite dipole model was run on each: the results are shown in table 7.6.

These low-frequency solutions are best contrasted with their corresponding image solutions. These are shown in figures 7.14 through 7.19. Two of these waves, W14 and W30, did not have convergent solutions within the field-of-view. The remainder have two distinct features: the source is located more than a metre below the surface, and located laterally outside the half-power contour of the image.

The available solution points are so deep and so far removed laterally from the image centroid that it is difficult to attribute to the low-frequency radiation any source mechanism that could be related in any way to the higher frequency mechanism. As it seems more likely that the lower and higher frequency mechanisms should be closely co-located in space, it seems that either the point source model or the finite-dipole model may be suspect. Future work should possibly try a “true” dipole model and compare the model fit to that of the finite dipole model presented here.

wave	$N_{ambient}$	N_{signal}	$x_0[m]$	$y_0[m]$	$z_0[m]$	λ	level [dB]
3	340	85	-1.02	3.70	-1.06	2.40	97.3
5	470	125	-0.68	0.89	-2.70	1.72	94.3
7	1720	85	0.50	0.79	-2.38	4.71	107.3
13	860	215	-0.04	-2.07	-2.56	4.20	105.1
14	535	150					
22	430	170	-0.54	0.90	-2.29	4.41	106.0
23	815	170	-0.43	-2.06	-2.39	3.21	100.8
30	860	215					

TABLE 7.6: Fit of low-frequency finite-dipole model to waves 3, 5, 7, 13, 14, 22, 23 and 30. Fit frequency is 344 Hz. The second and third columns provide the number of accumulants in the estimation of the ambient-only and ambient+signal CSDMs, respectively. The fourth, fifth and sixth columns provide the source position (units of metres). Column 7 provides the source strength parameter λ and column the corresponding source strength in units of dB re $1 \mu\text{Pa}^2/\text{Hz}$. No convergent solutions were found for waves W14 and W30.

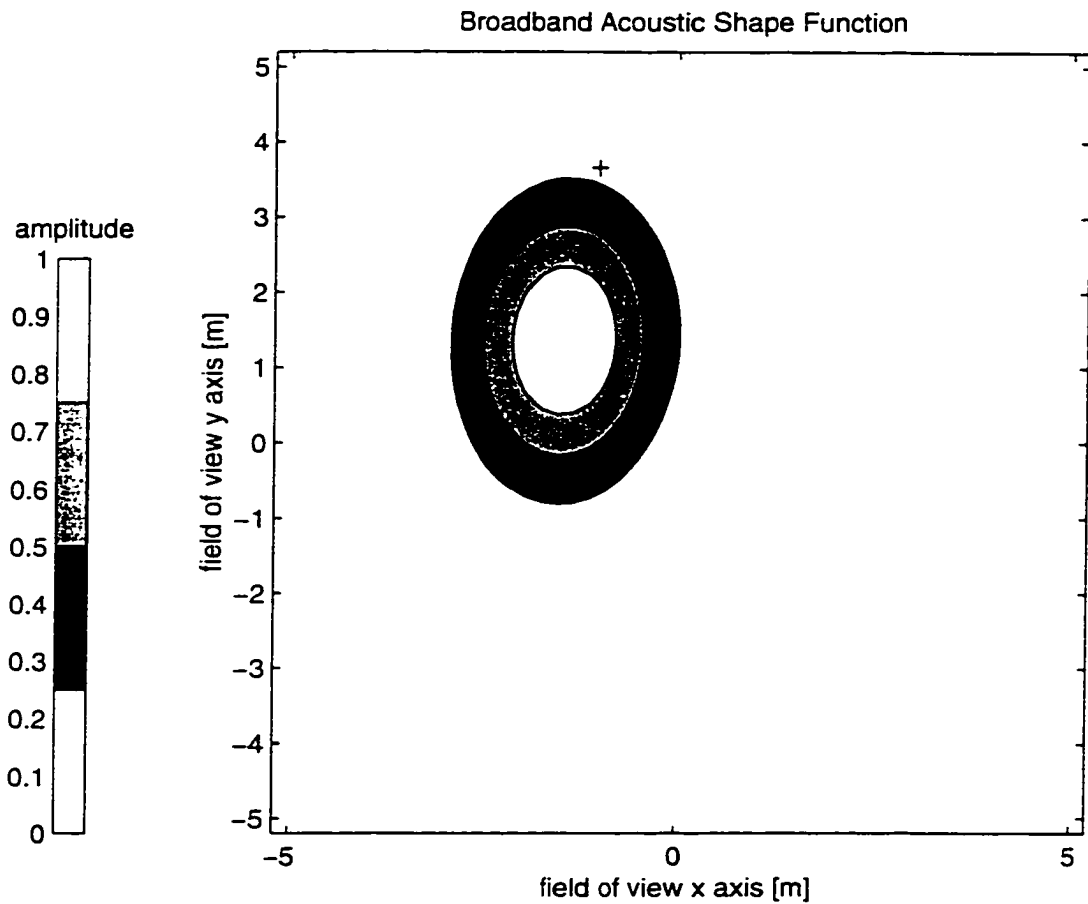


FIGURE 7.14: Comparison of W3 image and finite dipole low-frequency solutions. (x, y) position of point source solution shown with '+'.

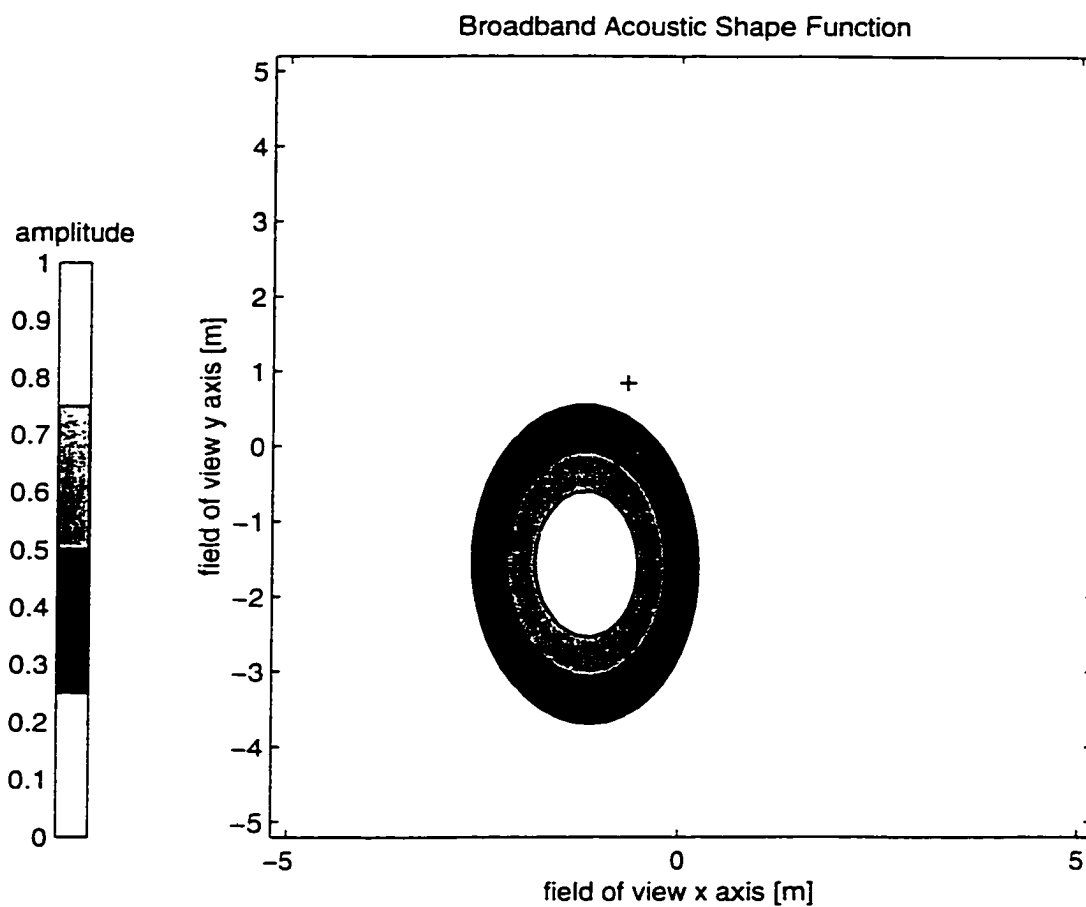


FIGURE 7.15: Comparison of W5 image and finite dipole low-frequency solutions. (x, y) position of point source solution shown with '+'.

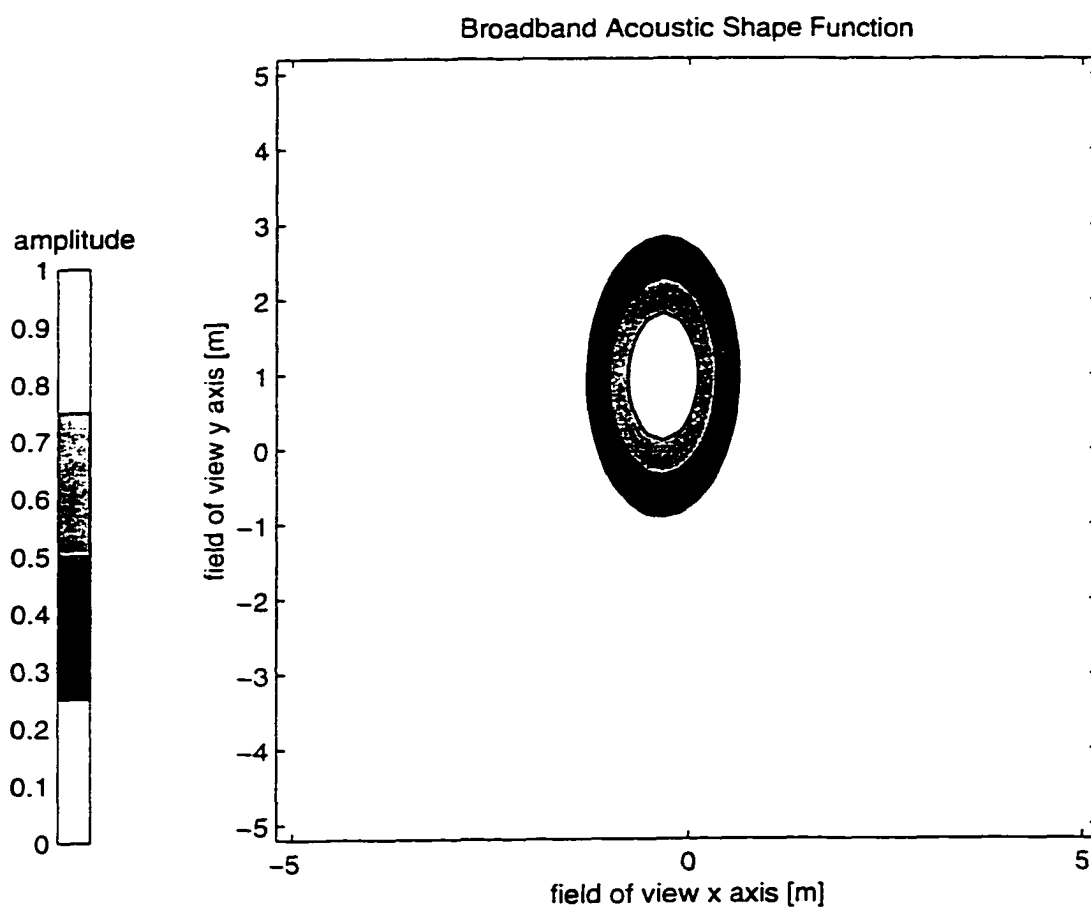


FIGURE 7.16: Comparison of W7 image and finite dipole low-frequency solutions. (x, y) position of point source solution shown with '+'.

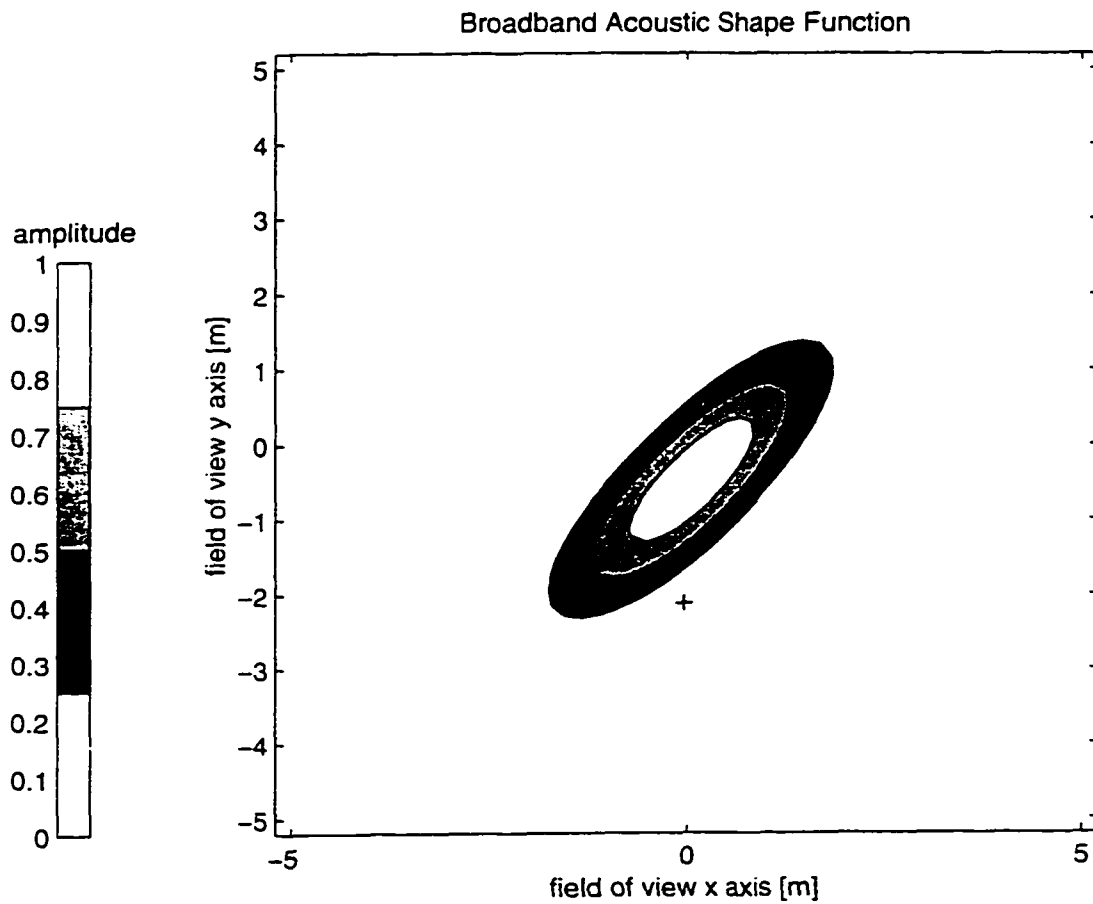


FIGURE 7.17: Comparison of W13 image and finite dipole low-frequency solutions. (x, y) position of point source solution shown with '+'.

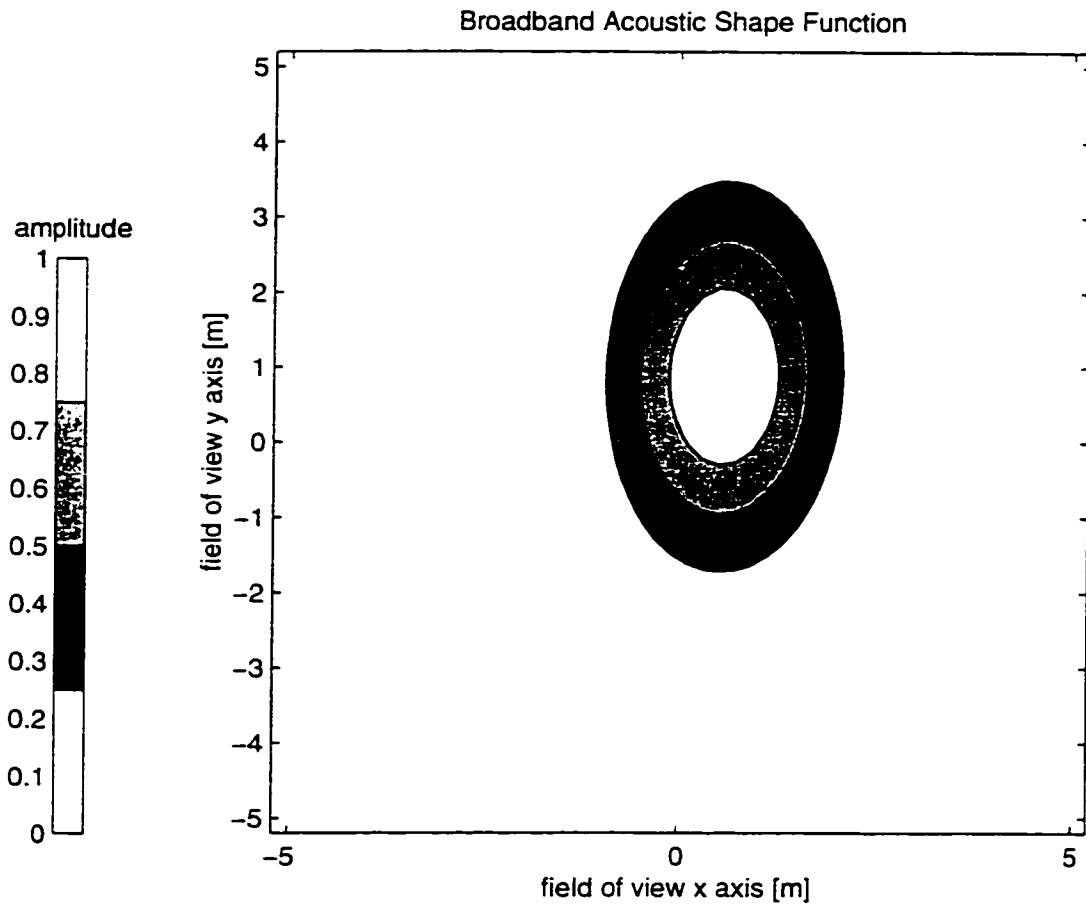


FIGURE 7.18: Comparison of W22 image and finite dipole low-frequency solutions. (x, y) position of point source solution shown with '+'.

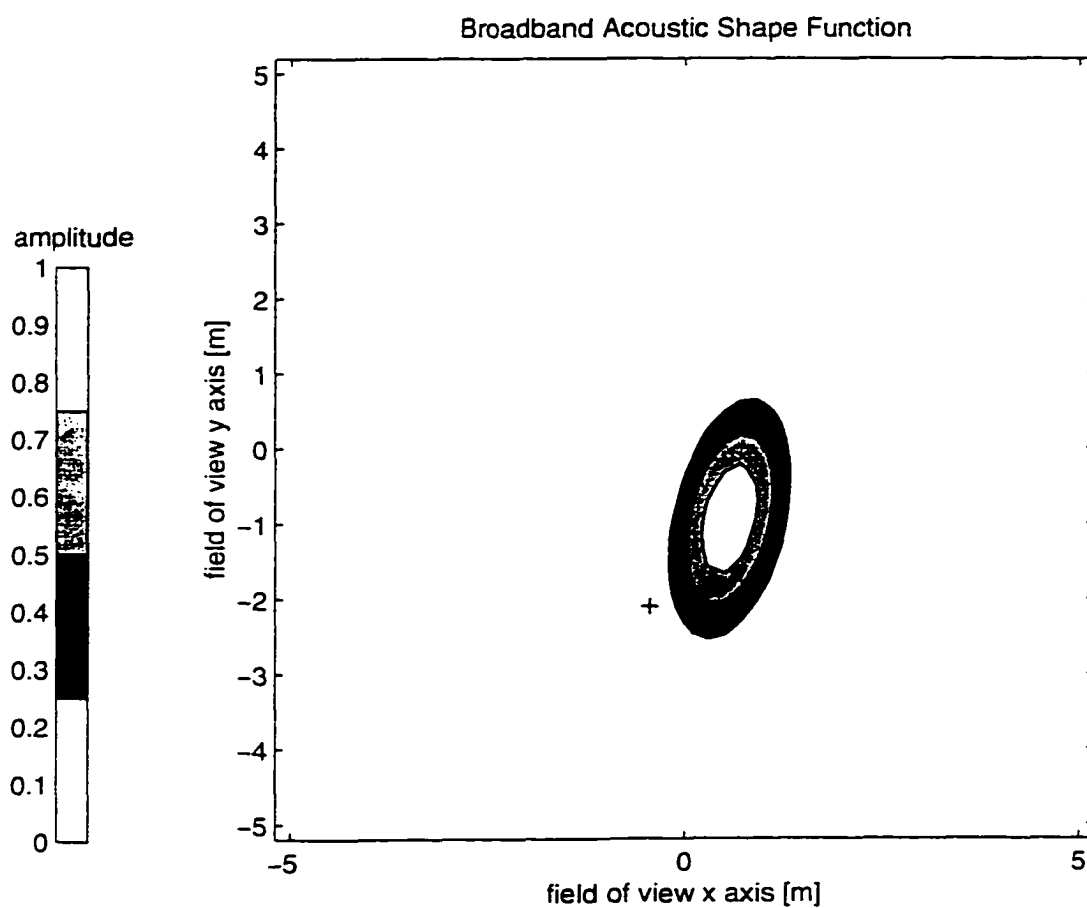


FIGURE 7.19: Comparison of W23 image and finite dipole low-frequency solutions. (x, y) position of point source solution shown with '+'.

7.4.2 Radiated Low-Frequency Spectrum

Although, as described in section 7.3.2, investigators have been fairly successful in reproducing the ambient autospectrum above 400 - 500 Hz with a source model involving individually ringing bubbles, these efforts did not directly address the mechanism and radiated signature at lower frequencies. Loewen and Melville ([83]) computed a simulated autospectrum down to below 200 Hz, and indeed predicted a fall-off with decreasing frequency, yielding a peak at about 400 Hz, as is seen in actual data. They did not, however, compare their results in this lower frequency regime to any actual data.

It is therefore of interest to perform a similar comparison to the low-frequency data acquired in this experiment. The simple model under test is that of individually radiating bubbles. The bubble distribution at radii greater than about 4 mm (corresponding to about 820 Hz) is not well established: the largest bubbles reported remain those of Medwin and Daniel ([95]) at about 7.4 mm. Let the model under test therefore contain bubbles with radii up to about 7 mm, thus attributing the radiated autospectrum down to about 400 Hz to resonant bubble oscillations. Does the model explain the autospectrum below 400 Hz?

Assume the time-dependence of the bubble radius $a(t)$ goes as $e^{-\beta t} e^{-i\omega_\beta t}$ where ω_β is the damped resonance frequency equal to $\sqrt{\omega_0^2 - \beta^2}$, ω_0 is the resonance frequency given by Eq.(2.1), and β is related to a dimensionless damping parameter δ by

$$\delta = 2\beta/\omega_\beta. \quad (7.7)$$

The squared magnitude of the Fourier transform is then proportional to $(1 + (\omega_\beta - \omega)^2/\beta^2)^{-1}$. This shows that a damped sinusoid has spectral energy at frequencies other than resonance — energy, in fact, from $\omega = 0$ to ∞ .

The damping parameter is described in section 2.1: for “big” bubbles with radii of ≈ 1 cm, the damping is primarily due to thermal dissipation δ_T and radiation resistance δ_r . Using the relations provided by Clay and Medwin ([24], section A6.1.2), the thermal

dissipation and radiation resistance at 400 Hz are

$$\delta_T = 9.02 \times 10^{-3}$$

$$\delta_r = 1.36 \times 10^{-2}$$

so that $\delta \approx 2.26 \times 10^{-2}$.

The source level will depend on the size of the wave. Assume the source excitation down to 400 Hz continues to follow the model parameterization determined in section 7.3. For a quantitative comparison, consider wave W3 (parametric solution.) Using the mean slope $\bar{b}_1 = -0.0036 \text{ Hz}^{-1}$ but the specific wave estimate $\hat{b}_0 = 2.11$ from table 7.5, $\lambda(f)$ at $f = 400 \text{ Hz}$ can be predicted to be 0.67. The spectral contribution at frequencies below 400 Hz due to damped bubbles vibrating at 400 Hz is then

$$S_{SS}(\omega) = \frac{S_0 e^{20+0.67}}{1 + (\omega_3 - \omega)^2 / \beta^2} \quad (7.8)$$

The total signal received at some field point $\mathbf{r}$ follows from Eq.(3.11). The result at hydrophone 5 (the center hydrophone) is shown in figure 7.20.

Figure 7.20 also includes the estimated autospectra for the ambient segment and the signal+ambient segment. The signal+ambient autospectrum was estimated using $N = 256$, $NW = 3$ and $K = 5$ tapers. The ambient only autospectrum was estimated using $N = 1024$, $NW = 3$ and $K = 5$ tapers: the resulting curve was additionally interpolated 16:1 for a smoother appearance. Both estimators had about the same number of degrees of freedom, and so have similar 90 % confidence intervals.

Also shown in figure 7.20 is the system transfer function (biased upward by 10 dB for plotting purposes) as described in appendix E.

As is immediately apparent from figure 7.20, the bubble spectrum decreases with decreasing frequency far too quickly to account for the spectral energy in the signal. The discrepancy cannot be attributed to the low-frequency roll-off of the system high-pass filters. There must therefore be a mechanism other than individual bubbles radiating at their resonance frequencies which is radiating primarily at these low frequencies.

There is also the suggestion of a spectral “hump” from about 180 Hz to about 300 Hz, peaking at roughly 245 Hz, which is mirrored to some degree in the ambient-only autospectrum below.

Could this “hump” be in fact evidence of a collective oscillation? This is difficult to ascertain, as the data record is quite short. The peak is only about 3 dB higher than the adjacent minimum, and this excess is not significant at the 10 % level.

The point-source model can be used again at the peak frequency of about 245 Hz

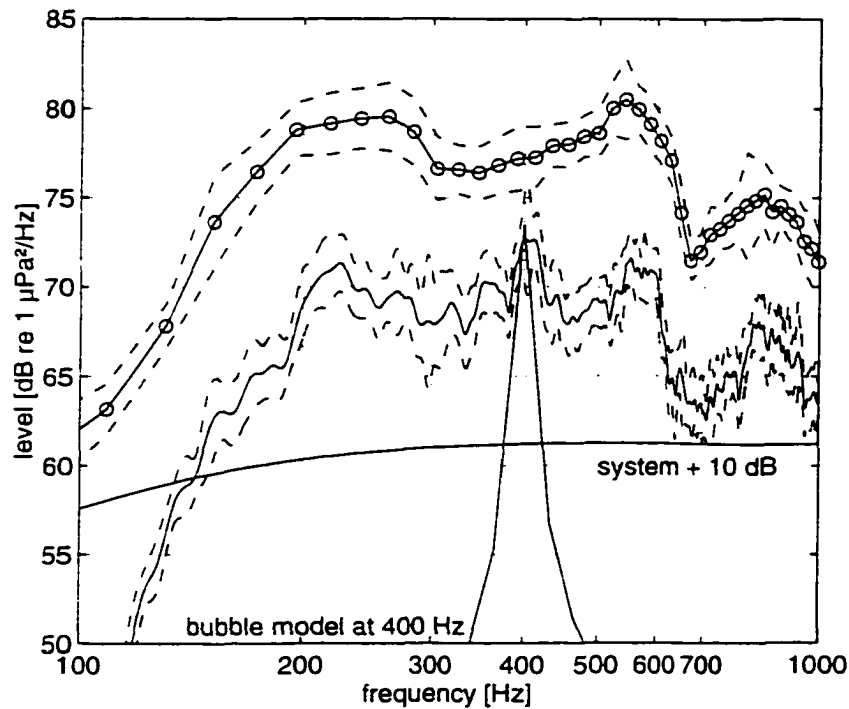


FIGURE 7.20: Comparison of the spectrum of a bubble resonant at 400 Hz (“bubble model”) with the ambient and signal+ambient autospectra from wave W3. Evaluated for hydrophone 5. The upper curve is the estimated signal+ambient autospectrum, the lower the estimated ambient only autospectrum. 90 % confidence intervals. The empty circles locate the bin-centered estimates. The system transfer function is also shown (units of dBV) biased upward by 10 dBV.

to determine the position of the radiation source assuming it is acoustically compact. This procedure produces the least-squares source location $[-1.55, -2.10, -1.98]^T$: this position is marked with a “+” on figure 7.21. The solution is clearly incorrect, which, as before, is likely an indicator of model-mismatch.

7.4.3 Low-Frequency Modeling Conclusions

Attempts to model the low-frequency mechanism as a point source (which would be an obvious model for an acoustically compact subsurface volume) or the off-resonance contribution of smaller bubbles were not satisfactory. Part of the reason may be the mismatch between the true mechanism and the models used.

These results imply that the simple models proposed here are probably not accurate and suggest that the low frequency energy is being generated at the surface by either large bubbles or a more complex mechanism such as collective oscillation or turbulence amplification.

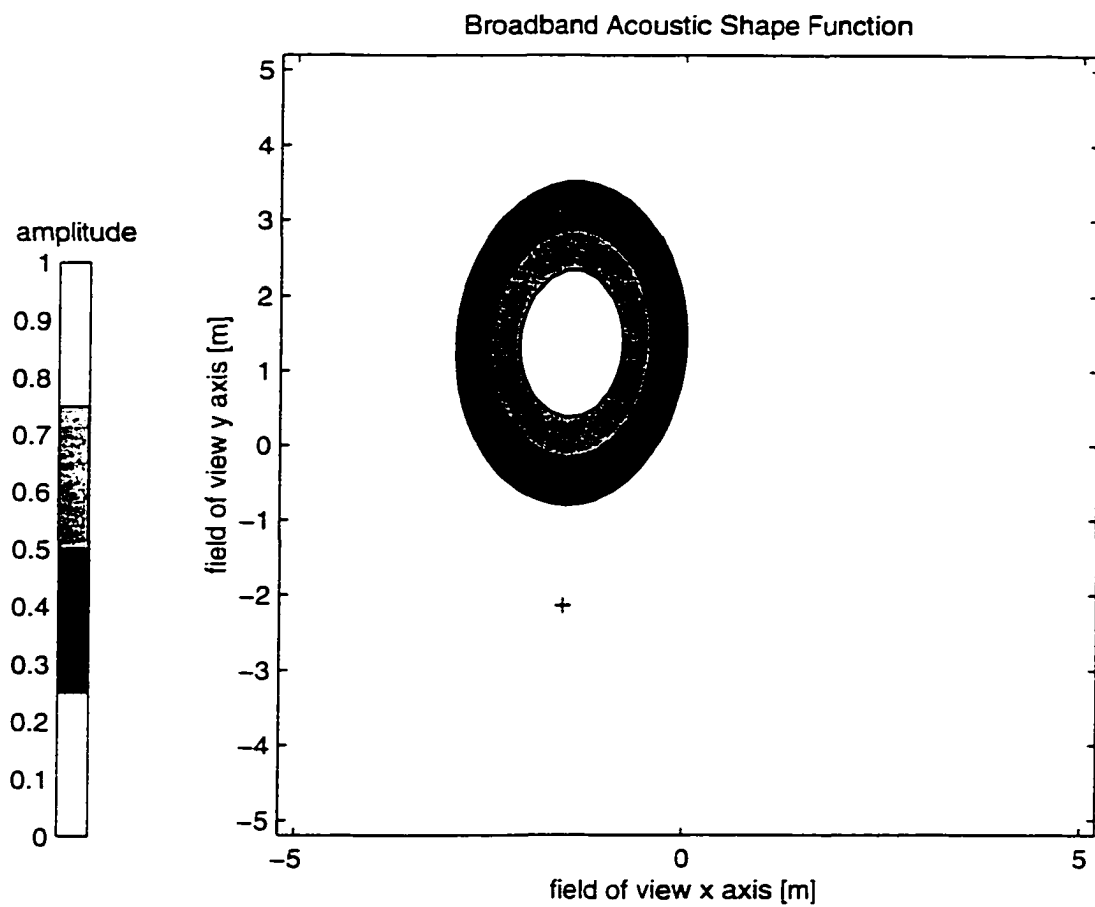


FIGURE 7.21: Comparison of W3 image and finite dipole low-frequency solutions. $f = 245$ Hz. (x, y) position of point source solution shown with '+'.

Chapter 8

Conclusions

8.1 Results

Acoustic data for breaking ocean waves at a fetch-limited coastal site were acquired at essentially a single wind speed (about 7 ms^{-1}) over frequencies that included the peak of the radiated autospectrum. A total of 30 breaking events were selected for image analysis, spanning approximately 1.5 hours during the peak of a sustained wind storm on 4/13/95. Nonparametric imaging of one typical wave reveals a radiant region moving downwind and spreading slightly. 21 events yielded solutions via the parametric procedure, demonstrating that the radiant regions were elongated, with the orientation of the shorter axis quite consistent with the wind direction. This orientation was cross-checked by tracking two breaking regions; these also translate downwind, along the direction of their shorter axes.

The absolute acoustic source levels of the breaking region are estimated over the frequency range [603 Hz, 1895 Hz]. The values seem to correlate well with the function $e^{20+b_0+b_1 \ln f}$ (in units of $\mu\text{Pa}^2/\text{Hz}$). Values of b_0 are of $O(30)$, and depend on the size of the breaker: bigger breakers had larger values. The value of b_1 seemed more consistent across the available data set, and had an aggregate median value of -4.55 ± 0.47 .

The source mechanism for frequencies below about 400 Hz (the “collective oscillation” regime) is modeled two ways: (1) as a point source (which would follow if an acoustically compact “collective oscillation” volume had formed), and (2) as off-peak spectral contributions from bubbles resonant at 400 Hz. Neither model achieves a satisfactory fit to the observed data. This seems to imply that the mechanism below about 400 Hz is acoustically extended and radiating as energetically as any resonant bubbles.

8.2 Achievements

No one had imaged breaking waves at these particular frequencies before, so the imaging task alone involved two major achievements: the development of an acoustic imaging system and the development of a novel broadband approach for combining multi-frequency information. These are reviewed below.

1. **The Nearfield Array** The need for an imaging system with an adequate theoretical resolution eliminated conventional far-field arrays. Only a nearfield array, with a point spread function (PSF) now instead of a beam pattern, seemed to offer enough resolution to resolve the expected dimensions of the source regions. It is quite likely that, had the system been constrained to depths of 10 or 20 m or deeper, (the standard deployment depths of many AORG instruments), source resolution would have been impossible at this site.

Deployment in the nearfield brings a new portfolio of problems: the array package will now be subjected to substantial heave due to wave orbital motion, which will apply greater stresses on the array frame. Stronger nearsurface currents will induce stronger strum-related vibrations. Both of these effects will produce more intense structure-borne vibrational contamination which can ruin acoustic measurements involving the low audio range. The highly-compliant hydrophone mounts designed for this application provided considerable attenuation of vibrational noise at the frequencies of interest: the presence of strong noise compo-

nents in the data suggests that, without these particular hydrophone mounts, the acoustic data would clearly have been saturated by mooring noise contamination and rendered useless.

Calibration of a large aperture discrete element array is at best an awkward affair, and at worst such an ugly undertaking that the entire calibration process itself is largely ignored. An experimental technique was developed on this project to simplify the process by using the *in situ* background ambient sound field for the calibration signal. Even this simple notion was not a straightforward textbook application, as the proximity to the surface and the localized contribution of breaking waves required a more careful treatment. (This is the only situation known to the author in which the background ambient sound was considered to be the “signal” and the localized sources in the nearfield field-of-view considered to be “noise”!)

2. **Broadband Imaging** Bandwidth and budget constraints prohibited the construction of an array with a fully populated sensor constellation, leaving instead a “sparse” array. There were simply too few sensors for the available aperture, and no sensor constellation or weighting scheme could have eliminated the unavoidable grating lobes. The grating lobes introduced an unacceptable amount of spatial aliasing at the upper passband frequencies, where the resolution was about 0.2 m and otherwise adequate for resolving surface sources. At the low end of the passband, the grating lobe problem disappeared, but the resolving power of the system approached the dimensions of the sources themselves.

In order to retain the resolving power at the higher frequencies while avoiding the ambiguities introduced by spatial aliasing, a novel approach was developed to combine the information at different frequencies into a broadband imaging statement. This solution turned out to be valid only when the space-time source information is separable, i.e., the spatial information does not depend on fre-

quency. This assumption, in the case of breaking waves, may not be valid “from DC to daylight”, as the source mechanism, and therefore the spatial profile, below about 400 Hz is likely to be different than the source mechanism at higher frequencies. The broadband scheme was ultimately considered valid only above about 400 to 500 Hz.

While simulations showed that the broadband imaging scheme was indeed effective, the computational burden, which can be large for a single-frequency imaging system, is multiplied directly by the number of frequencies involved. Fortunately, computers are just barely keeping pace with our computational requirements.

This work has established the fundamental parameters for improving the detection of breaking waves with tracking with long-baseline underwater correlators. Previous attempts (Farmer and Ding, [45], Ding and Farmer, [36]) experienced at best meager SNR, with the result that smaller breaking waves may have eluded detection and tracking. This leads to a bias in breaking statistics towards large breakers. The statistical contribution of these smaller scale waves is unknown, but, in order to determine this contribution, information about these smaller waves must be acquired. This kind of information will likely require some hybridized array / correlator approach. The basic inputs to the design of such an approach are (1) the source level and (2) the source size at the frequencies that dominate the spectral signature. These are the primary factors influencing SNR which in turn directly influence the detectability of the smaller scale breakers. With this work these parameters now become available.

8.3 Recommendations

Acoustical imaging of breaking waves at low audio frequencies is a tough problem, tough enough that it detracts from the associated effort of measuring and understanding ocean surfaces processes, which is also a tough problem. Recommendations therefore fall into two categories: improving oceanographic capabilities *with* and *without* acoustic imaging

systems.

Recommendations Involving Imaging The most glaring assumption made in this work is that of MSD space-time separability. The assumption was necessary to achieve unambiguous resolved spatial images, but is aesthetically unattractive for two reasons: first, it requires the simultaneous optimization of pixel levels and source levels, which in practice seemed to be a time-consuming balancing act, and, far more importantly, second, it may be an invalid model. The sequence of nonparametric images in figure 7.2 through 7.5 seems to show a blob of fluid with an acoustically active core translating left to right and spreading: the final image, figure 7.5, is certainly suggestive of the later stages of breaking, when the two-phase bore loses its identity, becomes fully entrained in the underlying fluid and begins to form a “foam raft” that signals the beginning of the J plume. Of course this happens: we know this from common observation. But the MSD separability is probably not a valid model at this stage, so that any similarity between the collapsing bore and the smeared-out image of figure 7.5 is purely in the mind of the reader.

This issue surfaces again in a much more interesting and important area: the acoustical activity of the non-breaking surface. Some theories predict that during wave breaking, the ocean surface is not simply a smooth fluid interface with the occasional whitecap, but is populated with a nearly-continuous distribution of breakers of all scales. The largest scale breakers would be the familiar whitecaps, but the smallest scales could be “microbreakers” that are too small to be measured visually. The rôle of these microbreakers in wave growth is largely unknown, but, if they exist, they would conceivably induce a speckle-like acoustical image on the fluid surface between the large scale breakers. A model of such a source would definitely not have MSD space-time separability.

The separability issue would again be in question for lower windspeeds, when *only* microbreaking is taking place.

Rather than examining the separability issue, future efforts would be well-advised to skip it entirely and build conventional, fully-populated imaging systems *and see what's out there!*

As addressed at the end of chapter three, there are some additional assumptions on the propagation model that need to be addressed, specifically the "flat-surface" model and the assumption of a uniform sound velocity. Neither are correct. Future imaging experiments must involve a larger suite of oceanographic measurements in order to estimate, even to first order, the surface height displacement and the sound speed in the nearsurface layer. Furthermore, for wave growth studies, the directional wave height spectrum must be measured directly, and not estimated indirectly via the wind record, as was done here. The coupling is not perfect, and there is considerable physics involved in the discrepancy between the two (and the dissipation due to breaking). The *directional* spectrum is required here because the distribution of breaker lengths, which is a predicted quantity in Phillips' theory of wave growth (Phillips [111]), depends on the shape of the directional spectrum.

Another important feature of the propagation problem is the influence of the α and β plumes. Deane [31] has very recently predicted that an entrained volume of high-void-fraction bubbly fluid has considerable acoustic absorption. His measurements were taken in shoaling surf, where he measured void fractions of 10% to 30%, but the principle should hold wherever a considerable volume of gas is entrained. In these cases, the volumetric absorption prevents the transmission of any acoustic waves launched from within (or, equivalently, incident from the far side): the only acoustic radiation that escapes the "cloud" is that radiated by bubbles in a thin shell on the outer periphery of the cloud.

This has considerable consequences for the type of acoustic imaging attempted here. Either the source may be heavily masked by an intervening entrainment of bubbles, or the source is not where we expect, i.e., the source is not at the surface but on some bubble cloud boundary below the surface.

This problem suggests that future experiments seek to obtain a simultaneous measure of the space-time void fraction structure beneath the breaker wave. This can currently be done by interspersing conductivity sensors throughout the bubbly region, but a more comprehensive and less invasive procedure, possibly using active sonar, is needed.

Recommendations Not Involving Imaging A plethora of surface-related oceanographic processes can be studied using acoustics without having to design and build a costly imaging system. Many of these issues arose during the course of this work but had to be ignored in order to maintain a narrow focus on the primary objective.

As an example, several wave breaking statistics can be collected through the use of a single nearsurface hydrophone. Breakers appear in the acoustic record as power transients superimposed on the background ambient level. This, of course, is the kind of data provided in appendix I.

These time series often revealed other more interesting phenomena. In the “quiet” intervals between the overhead breakers, the acquired sound sometimes appeared to be statistically stationary (this, presumably is the true background ambient sound) but occasionally contained a persistent low-level burble, like the sound made in small creeks. This latter sound was very likely due to a distribution of microbreakers across the field-of-view. The signal-to-background ambient sound was very low at 3 m depth for this feature, and so these microbreakers are likely to escape acoustic detection altogether at depths of 10 m or more.

This suggests the possibility of quantifying the scale of breaking waves by using a vertical combination of hydrophones, with the top sensor very near the surface (of $O(10\text{ cm})$), the next at say 3 m and the another at say 20 m.

Another possibility is to string several hydrophones at a very shallow depth along the surface, as the dipole effect will provide a spatial filter in the lateral direction.

There is no need to make only remote passive measurements of breaking regions. A

projector at the surface transmitting a signal from within a breaker to a distant receiver (or, better yet, an *array* of distant receivers!) might very well provide a key acoustic probe of bubble cloud structure.

Recommendations Involving the Low-Frequency Mechanism There remains this problem of the source mechanism at frequencies below about 400 Hz. At these frequencies, the wavelength in ordinary seawater is larger, perhaps much larger than the α plume, and an approach involving imaging is bound to be very difficult indeed. It should be possible to develop models more complete than the simple ones attempted in chapter seven and compare them to acoustic data. This, however, is complicated business.

Any future approach should certainly make maximum use of all other acoustic and oceanographic variables. Of particular importance would be the space-time void fraction field. This information has the added attraction of quantifying the vague distinction between “spillers” and “plungers”. Kolaini (private communication) has suggested that a necessary, but probably not sufficient, auxiliary measurement would be a census of the bubble size distribution within the breaking region.

In principle, a collective oscillation sets up a standing acoustic wave within the high-void fraction core, and it might be possible to detect such an acoustic wave using one or multiple sensors (not necessarily hydrophones) inside the core. As mentioned in the previous section, one or more sensors in the external fluid would also help in quantifying how and how much internal energy escapes as acoustic radiation.

This low-frequency issue is important to resolve, not only for academic interest, but because the low-frequency emanation is intimately coupled with the large void-fraction region in the bubble cloud core, and it is this region that provides the dominant contribution to the total amount of atmospheric gas entrained during a breaking event.

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Appendix A

Wave Forecast

A key element in understanding the oceanographic processes at the experiment site is a wave field forecast. The surface wave field is a *fetch-limited* wavefield and depends on local weather conditions in a very complicated way not fully understood. Nevertheless, the amplitude and frequency of the dominant wind-wave component can be estimated using an approximate procedure described by the Coastal Engineering Research Center [25]. The procedure requires the fetch F and wind stress factor U_A .

The fetch for the ULSAS deployment site in Patricia Bay depends on wind direction: the greatest fetch is approximately 12 km for winds from the NW, and 5 km for winds from the W or SW. (The fetch is negligible for winds from other directions.) Winds from these directions have the best chance of generating the breaking waves needed for this study.

Typical wind stress (or “adjusted wind speed”) can be estimated using measurements made at the site with a Coastal Research Minimet meteorological buoy. This instrument records a variety of environmental parameters, including vector-averaged wind speed at a height of 3 m. The CERC procedure for calculating the adjusted wind speed is then as follows:

- apply an elevation correction, based on boundary layer theory, to adjust the wind speed measured at 3 m to that which would have been measured at the standard

10 m height (their Eq. 3.26);

$$U_{10} = U_3 \left(\frac{10}{3} \right)^{1/7}$$

- Apply a stability correction

$$U = R_T U_{10}$$

to account for inhibited (enhanced) momentum transfer that can occur when the sea surface is cooler (warmer) than the air immediately above. Both air and sea temperatures are measured by the Minimet buoy, and the correction factor R_T can be determined using the measured air-sea temperature difference and CERC figure 3-14.

- Convert to “adjusted wind speed” using their equation 3-28a:

$$U_A = 0.71 U^{1.23}.$$

The adjusted wind speed, fetch and period T and height H of the dominant spectral component are then approximately related through the non-dimensional relations

$$\frac{gH}{U_A^2} = 1.6 \times 10^{-3} \left(\frac{gF}{U_A^2} \right)^{1/2}$$

and

$$\frac{gT}{U_A} = 2.857 \times 10^{-1} \left(\frac{gF}{U_A^2} \right)^{1/3}$$

For deep water, the CERC provides an approximate relation between H and the amplitude A of a corresponding model sinusoid via their Eq. 3-13:

$$A = 2^{-3/2} H$$

Largest sea wave forecast It is of interest to calculate the worst-case wave fields that may occur at the experiment site. There are two definitions here of “worst-case”: consider first the largest sea. A storm on 30 November 1994 generating the sort of vigorous breaking wave field required in this study produced wind speeds of about 16 ms^{-1} at the site. The worst case air-sea temperature difference measured during SURFEX II was about -5°C . This difference produces a correction factor $R_T = 1.13$; the adjusted wind speed for a 16 ms^{-1} wind at 3 m given an unstable (i.e., worst-case) atmosphere is then 30.9 ms^{-1} .

Using the greatest fetch of 12 km (i.e., winds from the NW), the period of the dominant spectral component is $T = 4.48 \text{ s}$, yielding a frequency of 0.22 Hz. The height of the dominant spectral component is $H = 1.73 \text{ m}$, which yields a sinusoidal amplitude of 0.61 m.

Smallest sea wave forecast The second kind of “worst-case” sea is the smallest sea. Of course, the smallest sea is a flat-calm, but consider here a fully developed sea for “lesser” conditions, i.e., a stable atmosphere and shorter fetch. Vigorous breaking is still observed at 8 ms^{-1} (see below), so choose $U_3 = 8 \text{ ms}^{-1}$. Let the temperature difference be the opposite of the case used above, or about $+5^\circ\text{C}$. This difference produces a correction factor $R_T = 0.85$; the adjusted wind speed for an 8 ms^{-1} wind at 3 m given an stable atmosphere is then 9.27 ms^{-1} .

Using the shorter fetch of 5 km (i.e., winds from the W or SW), the period of the dominant spectral component is $T = 2.24 \text{ s}$, yielding a frequency of 0.45 Hz. The height of the dominant spectral component is $H = 0.34 \text{ m}$, which yields a sinusoidal amplitude of 0.12 m.

SURFEX III Wave Hindcast These relations may also be used to estimate the dominant wave field parameters in effect during the storm of 4/13/95 during SURFEX III. The measured windspeed, from figure 5.4, is about 7.5 ms^{-1} . The air-sea temperature difference, figure 5.5 went from negative to positive (i.e., unstable to stable)

		units	larger forecast	smaller forecast	storm hindcast
period	T	s	4.48	2.24	2.34
frequency	f	Hz	0.22	0.45	0.43
amplitude	A	m	0.61	0.12	0.13
wavenumber	k	m^{-1}	0.19	0.82	0.74

TABLE A.1: Estimated Wavefield Parameters for Patricia Bay Site

during the storm as a front moved through, so an average value of 0° may be assumed for the entire storm, which corresponds to $R_T = 1.0$. The adjusted wind speed is then 10.4 ms^{-1} .

The wind during this storm was roughly from the West, so this would indicate a fetch of 5 km. Therefore, the period of the dominant spectral component would be $T = 2.34 \text{ s}$, yielding a frequency of 0.43 Hz. The height of the dominant spectral component would be $H = 0.38 \text{ m}$, which yields a sinusoidal amplitude of 0.13 m.

Summary Wave forecasting algorithms were used to predict the worst case waves expected at the Patricia Bay experiment site, and again in a hindcast to estimate the size of the waves during the SURFEX III storm of 4/13/95. These parameters are shown in table A.1, where the wavenumber has been calculated using the deep-water dispersion relationship

$$\omega^2 = gk \quad (\text{A.1})$$

where g is the acceleration due to gravity.

Appendix B

Simulations with Extended Sources

It is not in general easy to model or simulate extended sources accurately. Throughout this work, extended sources are represented implicitly by the CSDMs they would generate: this representation, being an ensemble average, serves to compress the essentially infinite dimensional source information into a finite rank matrix. The assumption that the source has an incoherent MSD further simplifies the problem, reducing the associated $2m$ -D integrals to 2-D integrals.

As a starting point, consider the basic simulation of the field MSD due to a plane wave field $A(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}$ between sensors at $\mathbf{r}_m$ and $\mathbf{r}_n$. The field MSD is

$$\begin{aligned}\Gamma(\mathbf{r}_m, \mathbf{r}_n) &= \langle p^*(\mathbf{r}_m), p(\mathbf{r}_n) \rangle \\ &= \iint \langle A^*(\mathbf{k})A(\mathbf{k}') \rangle e^{i(\mathbf{k}\cdot\mathbf{r}_n - \mathbf{k}'\cdot\mathbf{r}_m)} d\mathbf{k}d\mathbf{k}'\end{aligned}$$

The incoherent assumption allows $\langle A^*(\mathbf{k})A(\mathbf{k}') \rangle = N(\mathbf{k})\delta(\mathbf{k} - \mathbf{k}')$ so that

$$\Gamma(\mathbf{r}_m, \mathbf{r}_n) = \int N(\mathbf{k})e^{i\mathbf{k}\cdot(\mathbf{r}_n - \mathbf{r}_m)} d\mathbf{k}. \quad (\text{B.1})$$

This is merely a form of the well-known van Cittert-Zernike theorem in radioastronomy (e.g., see Thompson, Moran and Swenson [127]), and shows that $\Gamma(\mathbf{r}_m - \mathbf{r}_n)$ is the

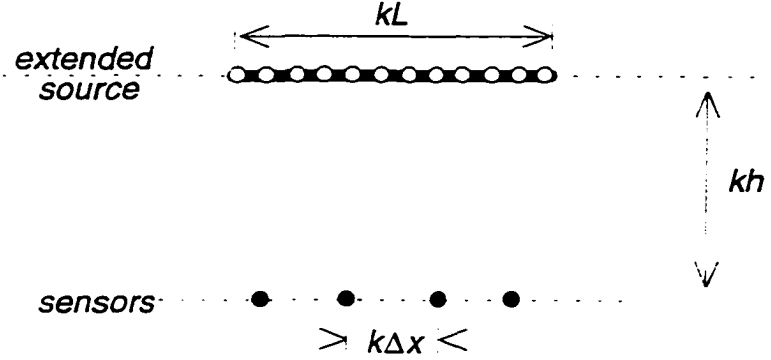


FIGURE B.1: Simulation source-array geometry

Fourier transform of $N(k)$. In this case, simulations using simple models for $N(k)$ can yield analytic expressions for $\Gamma(\mathbf{r}_m, \mathbf{r}_n)$.

Simulations of extended near-field sources are more complicated because the integrals can rarely be evaluated analytically. Assuming wave propagation is governed by the Green function $g(\mathbf{r}, \mathbf{r}_m)$, the field MSD is, following Eq.(B.1)

$$\Gamma(\mathbf{r}_m, \mathbf{r}_n) = \int I(\mathbf{r}') g^*(\mathbf{r}', \mathbf{r}_m) g(\mathbf{r}', \mathbf{r}_n) d\mathbf{r}'. \quad (\text{B.2})$$

where $I(\mathbf{r}')$ is the source intensity (see section 3.1.) For arbitrary $\mathbf{r}_m, \mathbf{r}_n$, this integral will have to be estimated numerically.

It is of interest to show that a numerical approximation of the CSDM represented by Eq.(B.2) is indeed accurate. Towards this end, consider the geometry of figure B.1. A 1-D extended source of length $kL = 1$, shown by the thick black line, radiates information to a parallel line of four sensors a distance $kh = 1$ away. The sensors are equi-spaced with an interelement separation distance of $k\Delta x = 0.5$. The Green function in this situation is conveniently replaced by a proportional nondimensionalized form

$$g_*(x, h, x_m, 0) = \frac{e^{i\sqrt{(kx - kx_m)^2 + kh^2}}}{\sqrt{(kx - kx_m)^2 + kh^2}}. \quad (\text{B.3})$$

In addition,

$$I(\mathbf{r}') = \begin{cases} 1 & |x| \leq kL/2 \\ 0 & \text{otherwise} \end{cases} \quad (\text{B.4})$$

Using Eq.(B.3) and Eq.(B.4) in Eq.(B.2) results in a definite 1-D integral

$$\Gamma(\mathbf{r}_m, \mathbf{r}_n) = \int_{-kL/2}^{kL/2} g_{+}^{*}(x, h, x_m, 0) g_{+}(x, h, x_n, 0) dx. \quad (\text{B.5})$$

This is the integral that must be evaluated by some quadrature scheme for each element of the CSDM.

Alternatively, the extended source could be thought of as a collection of M incoherent point sources distributed over the length $[-kL/2, kL/2]$. These are represented in figure B.1 by the empty circles superimposed onto the black line. A point source at kx_j produces an MSD of

$$\Gamma_j(x_m, x_n) = \frac{1}{M} g_{+}^{*}(x_j, h, x_m, 0) g_{+}(x_j, h, x_n, 0)$$

where $1/M$ is a convenient normalization. The effect of all M point sources is

$$\Gamma_{(M)}(x_m, x_n) = \sum_{j=1}^M \Gamma_j(x_m, x_n). \quad (\text{B.6})$$

Ideally, as $M \rightarrow \infty$, the CSDM $\mathbf{R}_{(M)}$ with elements defined by Eq.(B.6) should converge to the CSDM $\mathbf{R}$ with elements given by estimating Eq.(B.5) numerically.

To investigate this notion, the four eigenvalues $\lambda_1, \dots, \lambda_4$ of $\mathbf{R}_{(M)}$ were computed for $M = 1, 2, \dots, 50$ and plotted in figures B.2 through B.5. The eigenvalues of $\mathbf{R}$ using Eq.(B.5) were also computed, using adaptive recursive Newton-Cotes 8 panel quadrature with a tolerance of 1×10^{-12} . These eigenvalues are shown as dotted lines in figures B.2 through B.5.

This shows that numerical evaluation of the functionals for the elements of a simulated CSDM produces a CSDM which is consistent with the CSDM generated by a distribution of point sources in the limit as the distribution becomes a continuum.

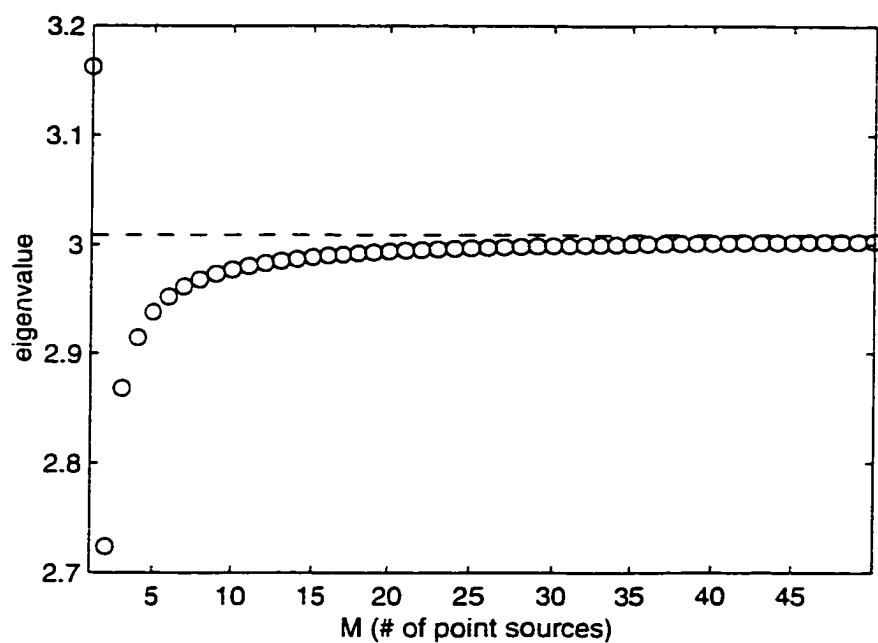


FIGURE B.2: Eigenvalue λ_1 versus number of point sources. Dotted line is the value of λ_1 computed from direct numerical integration of the extended source.

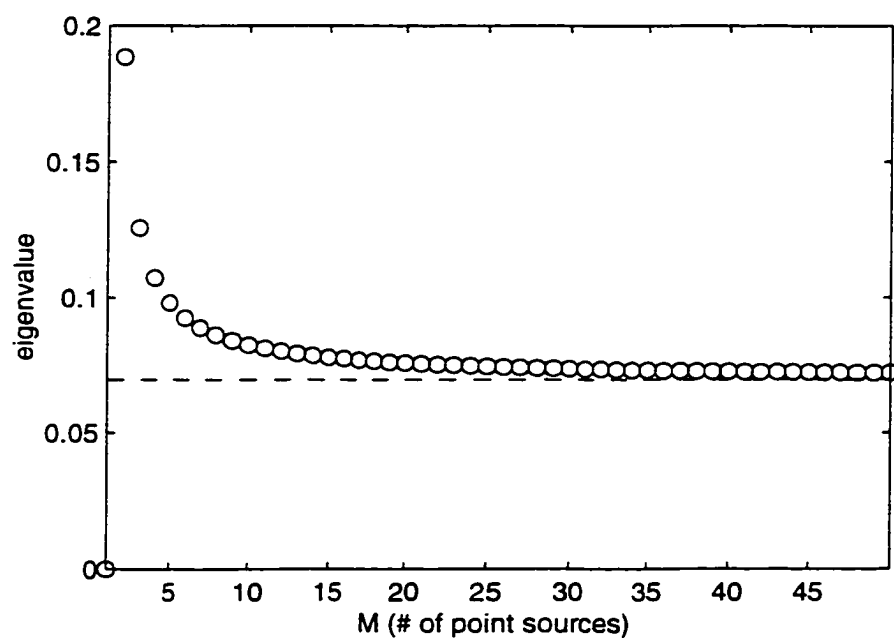
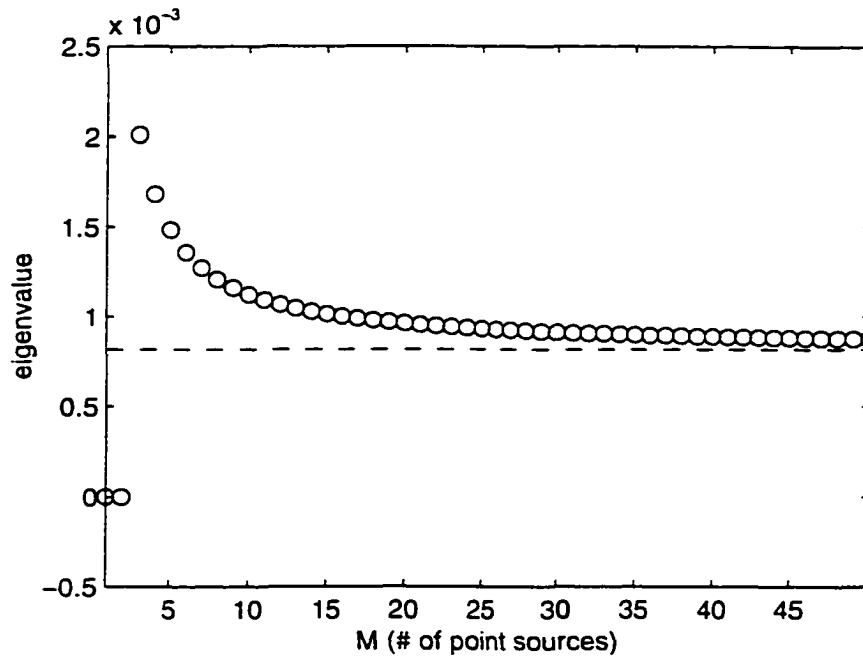
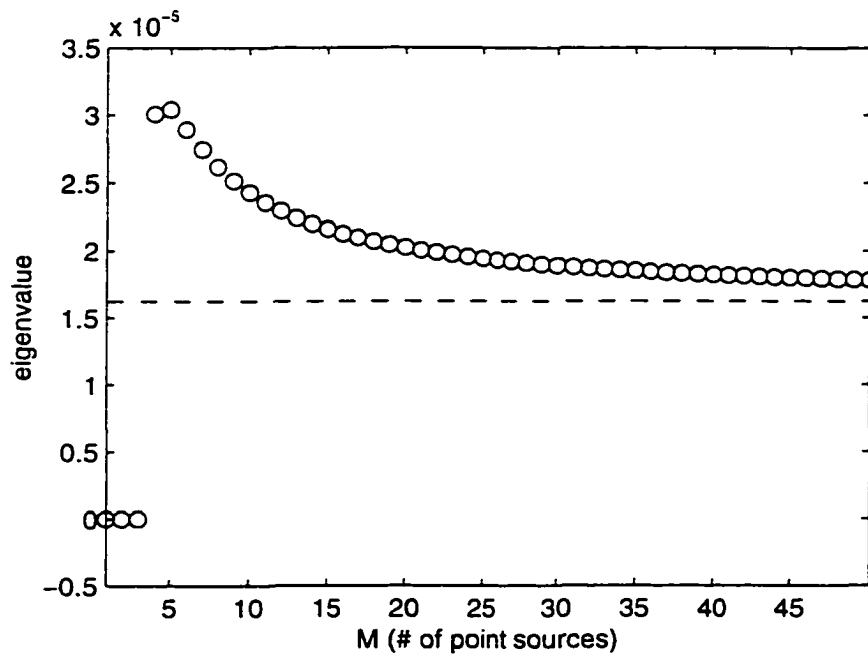


FIGURE B.3: Eigenvalue λ_2 versus number of point sources.

FIGURE B.4: Eigenvalue λ_3 versus number of point sources.FIGURE B.5: Eigenvalue λ_4 versus number of point sources.

This same technique is applied to simulating 2-D incoherent sources. The source is constrained to be non-zero over some finite region of support and zero elsewhere. The 2-D integral corresponding to Eq.(B.2) then becomes a definite cubature problem.

As an example, a rectangular source with uniform intensity might be defined over the region shown in figure B.6. A good cubature routine is DCUTRI (Berntsen and Espelid [14]), which allows for a vector integrand. The source must be partitioned into an initial set of triangles, as shown. The coordinates of the triangle vertices and the vector integrand function are passed to DCUTRI, which returns the vector result

$$\begin{bmatrix} \text{Re}\{\Gamma\} \\ \text{Im}\{\Gamma\} \end{bmatrix}.$$

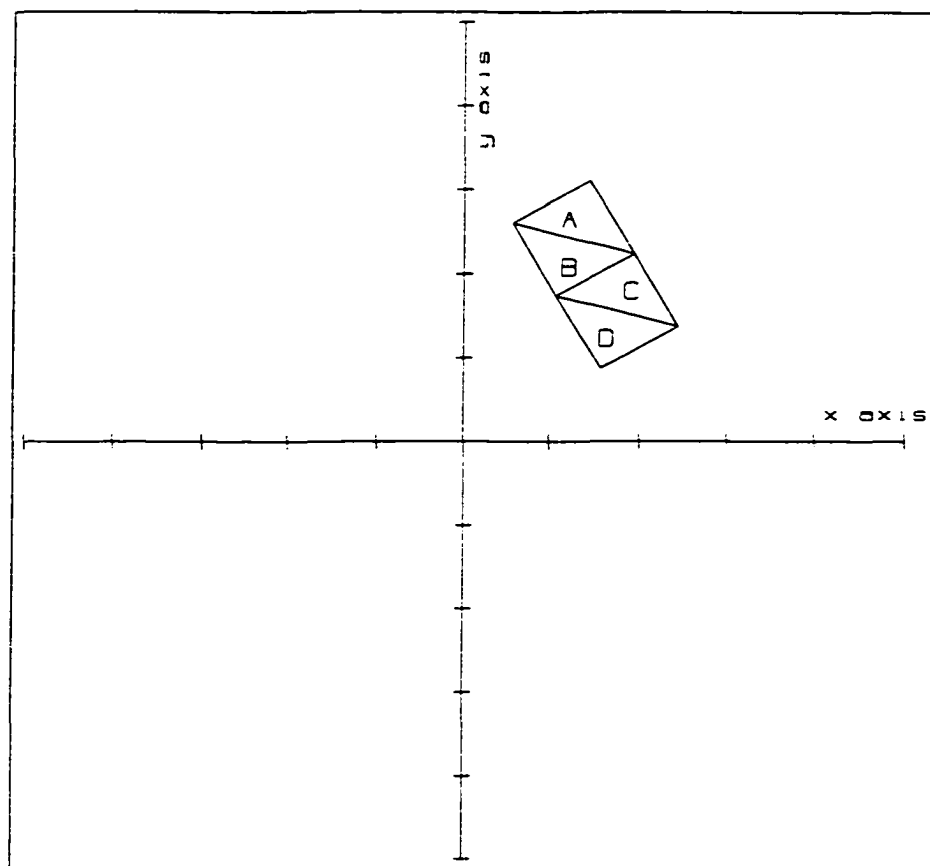


FIGURE B.6: Geometric set-up for 2-D simulation. This would be an extended source supported over a 1 m by 2 m rectangular region. The axes span the typical field-of-view of the ULSAS array. The support region is partitioned into four triangles. A, B, C and D.

Appendix C

Nonparametric Algorithm Details

The “nonparametric image” is the discrete (or pixelized) shape $\mathbb{I}_{m,n}$ that minimizes the regularized “cost” function

$$\mathcal{A}(D, \mathbb{I}, \lambda) + \mu \mathcal{B}(\mathbb{I})$$

that combines a function $\mathcal{A}$, which measures the “goodness-of-fit” between the data D and the model data represented by $\mathbb{I}$ and λ , and the function $\mathcal{B}$, which measures the goodness-of-fit of $\mathbb{I}$ against some *a priori* knowledge on $\mathbb{I}$. The symbols $\mathbb{I} = \text{vec}(\mathbb{I}_{m,n})$ and $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_{N_f}]^T$. The prescription for computing a nonparametric image is then

$$\mathbb{I}_\mu = \arg \min_{\mathbb{I}, \lambda} (\mathcal{A}(D, \mathbb{I}, \lambda) + \mu \mathcal{B}(\mathbb{I})). \quad (\text{C.1})$$

Details of the implementation of this approach are presented in this appendix.

C.1 “Cost” Function

The “cost” function here is the linear combination of $\mathcal{A}$ and $\mathcal{B}$. Consider first $\mathcal{A}$. Following the arguments in chapter four, $\mathcal{A}$ is the negative broadband log-likelihood,

given by

$$-L_{BB} = -\sum_{i=1}^{N_f} L_{\omega_i}(\mathbf{R}_i(\boldsymbol{\theta})|\mathbf{S}_i, N_{samp}; \omega_i) \quad (\text{C.2})$$

where it is assumed that N_{samp} sample outer products (rank one covariance matrix realizations) are collected at each of N_f frequencies and $\boldsymbol{\theta} = [\mathbf{III}^T, \boldsymbol{\lambda}^T]^T$. As nearly everything is frequency dependent here, the explicit frequency dependence (using either ω or subscript i) will be omitted in the following development unless it is needed for clarification. Thus, the sample CSDM at frequency ω_i is $\mathbf{S}_i - \mathbf{S}$ and the total model CSDM $\mathbf{R}_i(\boldsymbol{\theta}) - \mathbf{R}(\boldsymbol{\theta})$.

The total model CSDM is the sum

$$\mathbf{R}(\boldsymbol{\theta}) = \mathbf{R}_{ss}(\boldsymbol{\theta}) + \hat{\mathbf{N}} \quad (\text{C.3})$$

of a “signal” CSDM $\mathbf{R}_{ss}(\boldsymbol{\theta})$, due solely to the extended source, and the estimate of the background noise $\hat{\mathbf{N}}$. The CSDM $\mathbf{R}_{ss}(\boldsymbol{\theta})$ can further be decomposed into

$$\mathbf{R}_{ss}(\boldsymbol{\theta}) = S_0 e^{20+\lambda} \mathbf{R}_*(\mathbf{III}) \quad (\text{C.4})$$

where $\mathbf{R}_*(\mathbf{III})$ is a nondimensionalized CSDM.

The elements of $\mathbf{R}_*(\mathbf{III})$ require the evaluation of a 2-D integral over the field of view. A practical numerical scheme for doing this has been described already in section 4.5: the field of view is defined as a 10 m by 10 m square over the array and subdivided into 2500 cells, each of which measures 0.20 m on a side. The 2-D integral over the field-of-view is replaced by a sum of 2-D integrals, one over each cell.

Under the point collocation assumption, an exact solution is sought for $\mathbf{III}(\mathbf{r})$ at a single point in cell, say at the cell center $\mathbf{r}_0^{(m,n)}$. The true solution is then approximated everywhere else over the cell by assuming the shape function to be equal to $\mathbf{III}(\mathbf{r}_0^{(m,n)}) = \mathbf{III}_{m,n}$. The value of each element of $\mathbf{R}_*$ can now be formally approximated by

$$[\mathbf{R}_*]_{i,j} = \sum_m \sum_n \mathbf{III}_{m,n} \int_{\text{cell } m,n} d\mathbf{r}' (\nabla' g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \cdot \mathbf{n}') (\nabla' g(\mathbf{r}_j, \mathbf{r}'; \omega) \cdot \mathbf{n}') \quad (\text{C.5})$$

where the approximate shape function, being constant over the cell, has been brought out of the integral. Some implementations make the further interpretation that the entire cell contribution originates with a single point source located at the cell center: this replaces the integral in Eq.(C.5) with a simpler term $(\nabla' g^*(\mathbf{r}_i, \mathbf{r}_0^{(m,n)}; \omega) \cdot \mathbf{n}')(\nabla' g(\mathbf{r}_j, \mathbf{r}_0^{(m,n)}; \omega) \cdot \mathbf{n}')$. In the present application (one involving *extended* sources), it seems fundamentally inappropriate to reduce an extended source to a distribution of point sources, so the integral in Eq.(C.5) is retained. This integral, by itself, then represents the narrowband contribution of a uniform layer of sources over the cell surface. Let this contribution be

$$C_{m,n}^{i,j} = \int_{\text{cell } m,n} d\mathbf{r}' (\nabla' g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \cdot \mathbf{n}') (\nabla' g(\mathbf{r}_j, \mathbf{r}'; \omega) \cdot \mathbf{n}'). \quad (\text{C.6})$$

The term $C_{m,n}^{i,j}$ is an element of a “cell cubature” matrix $\mathbf{C}^{i,j}$. These are formally 2-D integrals, but in practice must be estimated using a cubature technique, such as the algorithm DCUTRI (Berntsen and Espelid [14]). These matrices are indexed by (i, j) , corresponding to the number of hydrophones: since $\mathbf{R}_*$ is Hermitian, only $15(15+1)/2$ cell cubature matrices are required, corresponding, for example, to $j = 1 \dots 15$ and $i = j \dots 15$. (This would load the diagonal and upper triangle of $\mathbf{R}_*$.) Each element of $\mathbf{C}^{i,j}$ is the cubature over one cell: $\mathbf{C}^{i,j}$ is therefore a 50×50 complex matrix. These cubature matrices are precomputed and stored as double precision binary files: each requires 40 kB of disk storage. 120 matrices are needed at a single frequency, requiring about 4.8 MB of disk storage, and total storage for 6 frequencies is nearly 30 MB.

The value of each element of $\mathbf{R}_*$ is therefore

$$[\mathbf{R}_*]_{i,j} = \sum_m \sum_n C_{m,n}^{i,j} \mathcal{W}_{m,n} \quad (\text{C.7})$$

Each narrowband log-likelihood is

$$L = C - N_{\text{samp}} (\ln |\mathbf{R}| + \text{tr} \mathbf{R}^{-1} \mathbf{S}). \quad (\text{C.8})$$

C is a constant independent of θ : neither it nor the value of N_{samp} affect the optimizer.

so both will henceforth be ignored. This leaves a modified log-likelihood

$$L = -(\ln |\mathbf{R}| + \text{tr} \mathbf{R}^{-1} \mathbf{S}). \quad (\text{C.9})$$

as the applicable “cost” function.

Possible regularizers $\mathcal{B}$ are discussed in section 4.5.1. A common form used in image processing is the negentropy (Press *et al.* [114], section 18.7)

$$\mathcal{B} = \sum_{m,n} \mathcal{I}_{m,n} \ln \mathcal{I}_{m,n}. \quad (\text{C.10})$$

Skilling and Bryan ([121]) proposed a similar measure:

$$\mathcal{B} = \sum_{m,n} \mathcal{I}_{m,n} [\ln(\mathcal{I}_{m,n}/D) - 1] \quad (\text{C.11})$$

where D is some default “background” value of the image. Eq.(C.10) penalizes departures of $\mathcal{I}$ from a uniform prior of 1, while Eq.(C.11) penalizes departures from a uniform value of D . In practice, Eq.(C.10) acted like Eq.(C.11), so the only difference appeared to be the choice of this “background” level. These regularizers lead to maximum entropy image solutions.

A regularizer used in oceanography (Long and Hasselmann, [84]) is

$$\mathcal{B} = \sum_{m,n} (\mathcal{I}_{m,n} - |\mathcal{I}_{m,n}|)^2. \quad (\text{C.12})$$

This is simply an L_2 penalty function for negative pixels, with a smooth gradient at zero. In principle, this regularizer could allow an optimizer to set some pixels negative during solution.

There are perhaps many more possible regularizers, and this is what makes solving inverse problems more of an art. In practice, for this application, the choice of regularizer (and regularization parameter) was rather unimportant: more on this in section C.3.

C.2 “Gradient” Function

Optimizers can use finite differences to estimate the gradient of the cost function, but in this case, analytical gradients are easy to compute and require less computational time.

The gradient of the cost function is the gradient of $\mathcal{A}$ and the regularizer $\mathcal{B}$. The gradient $\nabla_{\theta}\mathcal{A}$ is $-\nabla_{\theta}L_{BB}$, which by Eq.(C.2) is the negative sum of the gradients of the modified narrowband log-likelihoods

$$-\nabla_{\theta}L_{BB} = -\sum_{\omega} \nabla_{\theta}L. \quad (\text{C.13})$$

At a particular frequency, the term

$$\nabla_{\theta}L = \begin{bmatrix} \nabla_{m,n}L \\ \frac{\partial L}{\partial \lambda} \end{bmatrix} \quad (\text{C.14})$$

where the term $\nabla_{m,n}L$ results in a 2500×1 vector with elements $\text{vec}\{\partial L/\partial \mathbf{I}_{m,n}\}$. These partial derivatives are easy to compute: using Eq.(C.3), Eq.(C.4), Eq.(C.7) and the modified narrowband log-likelihood Eq.(C.9) and the following expressions from matrix theory¹

$$\begin{aligned} \frac{\partial}{\partial \theta_i} |\mathbf{A}| &= |\mathbf{A}| \text{tr} \mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial \theta_i}, \\ \frac{\partial}{\partial \theta_i} \mathbf{A}^{-1} &= -\mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial \theta_i} \mathbf{A}^{-1}, \\ \frac{\partial}{\partial \theta_i} \ln |\mathbf{A}| &= |\mathbf{A}^{-1}| \frac{\partial |\mathbf{A}|}{\partial \theta_i} = \text{tr} \mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial \theta_i}, \end{aligned}$$

the partial derivatives are

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{I}_{m,n}} &= \text{tr} \left(\mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \mathbf{I}_{m,n}} (\mathbf{I} - \mathbf{R}^{-1} \mathbf{S}) \right) \\ &= e^{20+\lambda} \text{tr} \mathbf{R}^{-1} \mathbf{G}^{m,n} (\mathbf{I} - \mathbf{R}^{-1} \mathbf{S}) \end{aligned} \quad (\text{C.15})$$

¹These relations were the topic of a thread on the Usenet newsgroup `sci.math.num-analysis`. Suggested there but not used in this work were two “very good references” on matrix calculus: Rogers [116] and Graham [54].

where the elements of

$$[\mathbf{G}^{m,n}]_{i,j} = C_{m,n}^{i,j}. \quad (\text{C.16})$$

These matrices $\mathbf{G}^{m,n}$ represent the “impulse response” on each element of the nondimensionalized CSDM due to cell (m,n) : they are different than the matrices $\mathbf{C}^{i,j}$ introduced earlier. These matrices are indexed by (m,n) , so there are 50×50 or 2500 $\mathbf{G}$ matrices: each is a 15×15 element matrix. As their elements are just shuffled combinations of the elements of the $\mathbf{C}$ matrices, the $\mathbf{G}$ matrices are not *also* computed: this would require an additional 30 MB of RAM storage! Rather, $\mathbf{G}$ matrix elements are drawn from $\mathbf{C}$ matrix storage according to Eq.(C.16) as needed.

The partial derivative with respect to λ is also simple, yielding

$$\frac{\partial L}{\partial \lambda} = \text{tr} \mathbf{R}^{-1} \mathbf{R}_{ss} (\mathbf{I} - \mathbf{R}^{-1} \mathbf{S}). \quad (\text{C.17})$$

The analytic gradients of regularizers Eq.(C.10) through Eq.(C.12) are simple to compute. They were in fact computed, coded, used and judged not have a significant impact on solutions, and so will not be given explicitly here.

C.3 Implementation

Optimizers Eq.(C.1) is essentially an unconstrained nonlinear optimization problem, with the added complexity that the solution space may have a very large number of variables. In radioastronomy, problems with more than 1 million pixels are not uncommon. In this work the problem involves 2500 pixels plus up to six frequency-dependent coefficients. This is not considered a “large” image reconstruction problem (Cornwell, personal communication) but large enough to severely restrict the choice of available optimizers. The Hessian in a quasi-Newton algorithm, for example, would be a 2506×2506 element matrix, which is much too large for the available computational resources. This effectively eliminates the class of quasi-Newton optimizers.

A standard alternative for large dimensional problems is to use a conjugate-gradient (CG) algorithm. For quadratic problems of N variables, a CG algorithm will achieve the solution in N steps (Antoniou [5]). For the present problem, however, the CG algorithm has several drawbacks that make it an undesirable approach.

First, the CG algorithm accumulates information about the curvature of the error function as the trial solution moves each iteration. However, as noted by Skilling and Bryan ([121]), the error function can be highly nonlinear, so that curvature information acquired at recent solution points can be quite different than that computed at earlier points. The accumulating curvature information is therefore never quite accurate and can send the trial points on a highly suboptimal trajectory through solution space, requiring far more than N iterations to converge.

Second, the evaluation of the cost and gradient functions for each trial solution was computationally intensive, so that an algorithm executing at best $O(2500)$ iterations (or, at worst, many more) was expected to require a prohibitively long time to converge.

Skilling and Bryan ([121]) proposed an alternative technique that essentially forms a local quadratic model of the error surface, disregards old curvature information and simply makes a step that minimizes the local quadratic model. They report this algorithm to be highly effective in solving problems like Eq.(C.1).

The drawback with this approach is that their algorithm is difficult to code for the general case. The coding has already been done for the radioastronomy problem, and Fortran source for several different implementations is widely available on the Internet. Unfortunately, the present problem has a different structure than the radioastronomy problem, so the pre-existing code is of no use.

A third alternative uses a very small class of quasi-Newton-like optimizers which are designed for large dimensional problems. One such example is L-BFGS-B (Zhu *et al.* [144]), which implements a limited memory model of the BFGS quasi-Newton optimizer using bound constraints. The appellation “limited memory” implies that the full Hessian is not stored, but rather only its main diagonal components, with the hope

that this limited subset of elements captures enough curvature information to guide the trial iterates on a “more optimal” trajectory towards the solution.

Implementation Fortran source for L-BFGS-B is available over the Internet and was downloaded, compiled, and tested. Interfacing the optimizer with the cost and gradient functions, denoted f and g respectively, required some coding trade-offs. After some hacking on pre-existing code, f and g were implemented as Matlab M files with the intention of using Matlab as a computational engine (a procedure described more fully in appendix D. This implementation was convenient but seemed slow.

Functions f and g were then implemented in Fortran using LAPACK [77] and the BLAS [16] from Netlib [104] for matrix computations. All the C matrix information for all frequencies was loaded into static variables to eliminate time-consuming disk access. The code was compiled without any compiler optimization.

Benchmarks The numerical accuracy of the routines was verified by comparing the Matlab routines to hand calculations, and then comparing the Fortran output to the Matlab output.

Timing benchmarks compared the computation speed of f and g for a single frequency on an HP735 RISC-based Unix workstation running HP-UX on a 99 MHz CPU. The results in table C.1 show the Fortran implementation to be clearly superior. Results could be improved further using almost any code optimization strategy, since none at all were used in this development phase.

	f	g
M file	0.56 s	219.7 s
Fortran	0.32 s	11.0 s

TABLE C.1: Benchmarks for cost and gradient functions, Matlab and Fortran, single frequency.

Using L-BFGS-B and the Fortran subroutines for f and g , a nonparametric image utilizing all six frequency bands can be produced in about 2 - 3 hours and about 20 - 50 iterations. The trial solution after only about 5 solutions will already bear a strong resemblance to the final solution, except that its dynamic range is usually quite small. The remaining iterations seem to be primarily adjusting the balance between the pixel levels and the source strength parameters.

Regularizers The negentropy regularizer was used with several values of the regularization parameter μ . For μ large, the solution was biased towards a uniform level, with the image compressed into a very small dynamic range about that background level. As μ decreased towards zero, the solution dynamic range around the background level increased. As a typical graphics package colormap algorithm can be rescaled to any dynamic range, the difference in image solutions for different values of μ can be rendered inconsequential. The resulting images in this problem are therefore rather insensitive to the value of μ .

Furthermore, as μ went to zero, no new image structure appeared. The $\mu = 0$ case represents an ill-posed or underdetermined problem, and the data noise in such a problem often introduces artifacts into the solution which are unrelated to the true solution. In this case, however, the full broadband problem appears to have been only mildly underdetermined, so there seemed little penalty in choosing $\mu = 0$.

The negentropy regularizer also enforces positivity through the logarithmic term. Positivity, or, more correctly, non-negativity, can also be achieved by the penalty function regularizer of Eq.(C.12), or by applying a constrained optimizer. This latter approach was used with L-BFGS-B: the pixels were constrained to be non-negative. In practice, no constraint was active at the solution points for images shown in this work.

Oddities There are two unusual and persistent features of this implementation.

First, L-BFGS-B usually stopped before full convergence. Full convergence is defined here as $|g| \leq 1.0d-9$. As described above, the essential image is apparent after only a few

iterations, so this premature stopping does not seem to impose any loss of image content. This propensity for stopping optimizers prior to full convergence is a common occurrence in the maximum entropy reconstruction community (Sande, personal communication).

Second, the optimizer does not seem to produce source strength levels that are consistent for different values of the *a priori* background. For example, the optimizer does not force the pixels to span the full range $[0, 1]$, as is required by construction in the parametric model. The slopes of the source level parameters versus frequency are consistent with the slopes obtained using the parametric model, but depend in level on the background level.

For the images presented in this work, the *a priori* background level was set to 0.10. This choice yielded a fair dynamic range in the pixel values, which is always shown on the sidebar accompanying each image.

It is not clear why the optimizer is exhibiting this behavior. This problem might be related to the premature stopping described above: consistent values of pixel level and source strength level may not have “settled out” yet. On the other hand, there is no *a priori* reason to expect the image solution to fully span $[0, 1]$. It is possible that the use of specialized regularizers could control this.

In the present case, absolute levels determined by weighting the pixel level with the source strength parameter must be viewed with some suspicion. (But probably no more or less suspicion than a model based on a Gaussian lump!)

Conclusion The image reconstruction algorithm developed here proved to be a remarkably robust tool with respect to μ and exhibited interesting properties. There is considerable room for refinement in terms of algorithm structure, code optimization, incorporation of *a priori* information, and selection of the regularization parameter μ .

Appendix D

Maximum Likelihood Details

The “maximum likelihood image” is the shape function $\mathcal{I}_{ML}(\theta)$ based on the parameter vector $\hat{\theta}_{ML}$,

$$\hat{\theta}_{ML} = \arg \max_{\theta} L_{BB},$$

which maximizes the broadband log-likelihood L_{BB} (or alternatively minimizes $-L_{BB}$.) This is in general a constrained nonlinear optimization problem, where the constraints require that the shape dimensions (w, l) be positive.¹

Details of the implementation of this optimizer are presented in this appendix.

D.1 “Cost” Function

The “cost” function here is the negative broadband log-likelihood, given by

$$-L_{BB} = - \sum_{i=1}^{N_f} L_{\omega_i}(\mathbf{R}_i(\theta) | \mathbf{S}_i, N_{samp}; \omega_i). \quad (\text{D.1})$$

This function has already been described in Appendix C: as developed there, the total model CSDM (dropping the explicit i index) is the sum

$$\mathbf{R}(\theta) = \mathbf{R}_{ss}(\theta) + \hat{\mathbf{N}} \quad (\text{D.2})$$

¹It would certainly be a surprise if either constraint were active at the solution point!

of a “signal” CSDM $\mathbf{R}_{ss}(\boldsymbol{\theta})$, due solely to the extended source, and the estimate of the background noise $\hat{\mathbf{N}}$. The CSDM $\mathbf{R}_{ss}(\boldsymbol{\theta})$ can further be decomposed into

$$\mathbf{R}_{ss}(\boldsymbol{\theta}) = S_0 e^{20+\lambda} \mathbf{R}_s(\boldsymbol{\theta}) \quad (\text{D.3})$$

where $\mathbf{R}_s(\boldsymbol{\theta})$ is a nondimensionalized CSDM.

The elements of $\mathbf{R}_s(\boldsymbol{\theta})$ require the evaluation of a 2-D integral over the field of view. A practical numerical scheme for doing this has been described already in section 4.5: the field of view is defined as a 10 m by 10 m square over the array and subdivided into 2500 cells, each of which measures 0.20 m on a side. The 2-D integral over the field-of-view is replaced by a sum of 2-D integrals, one over each cell. The shape function is further assumed to be constant over each cell, and is therefore brought outside the integral. The value of each element of $\mathbf{R}_s$ is now approximated by

$$\begin{aligned} [\mathbf{R}_s]_{i,j} &= \sum_m \sum_n \mathcal{W}_{m,n}(\boldsymbol{\theta}) \int_{\text{cell } m,n} d\mathbf{r}' \left(\frac{\partial}{\partial n'} g^*(\mathbf{r}_i, \mathbf{r}'; \omega) \right) \left(\frac{\partial}{\partial n'} g(\mathbf{r}_j, \mathbf{r}'; \omega) \right) \\ &= \sum_m \sum_n \mathcal{W}_{m,n}(\boldsymbol{\theta}) C_{m,n}^{i,j}. \end{aligned} \quad (\text{D.4})$$

The term $\mathcal{W}_{m,n}(\boldsymbol{\theta})$ represents the value of $\mathcal{W}(\boldsymbol{\theta})$ computed at the center of cell (m, n) . The term $C_{m,n}^{i,j}$ is an element of a “cell cubature” matrix $\mathbf{C}^{i,j}$. These matrices are indexed by (i, j) , corresponding to the number of hydrophones: since $\mathbf{R}_s$ is Hermitian, only $15(15+1)/2$ cell cubature matrices are required, corresponding, for example, to $j = 1 \dots 15$ and $i = j \dots 15$. (This would load the diagonal and upper triangle of $\mathbf{R}_s$.) Each element of $\mathbf{C}^{i,j}$ is the cubature over one cell: $\mathbf{C}^{i,j}$ is therefore a 50×50 complex matrix. These cubature matrices are precomputed and stored as double precision binary files: each requires 40 kB of disk storage. 120 matrices are needed at a single frequency, requiring about 4.8 MB of disk storage, and total storage for 6 frequencies is nearly 30 MB.

Each narrowband log-likelihood is

$$L = C - N_{\text{samp}} (\ln |\mathbf{R}| + \text{tr} \mathbf{R}^{-1} \mathbf{S}). \quad (\text{D.5})$$

C is a constant independent of θ : neither it nor the value of N_{samp} affect the optimizer, so both will henceforth be ignored. This leaves a modified log-likelihood

$$L = -(\ln |\mathbf{R}| + \text{tr} \mathbf{R}^{-1} \mathbf{S}). \quad (\text{D.6})$$

as the applicable “cost” function.

D.2 “Gradient” Function

Optimizers can use finite differences to estimate the gradient of the cost function, but analytical expressions were developed for this implementation.

The gradient of Eq.(D.1) clearly involves the sum of the gradients of each narrow-band log-likelihood function, Eq.(D.6):

$$\nabla_{\theta} L_{BB} = \sum_{\omega} \nabla_{\theta} L \quad (\text{D.7})$$

The term $\nabla_{\theta} L$ is

$$\nabla_{\theta} L = \left[\frac{\partial L}{\partial x_0}, \frac{\partial L}{\partial y_0}, \frac{\partial L}{\partial w}, \frac{\partial L}{\partial l}, \frac{\partial L}{\partial \phi}, \frac{\partial L}{\partial \lambda_1}, \frac{\partial L}{\partial \lambda_2}, \dots, \frac{\partial L}{\partial \lambda_{N_f}} \right]. \quad (\text{D.8})$$

These terms require the expressions $\partial |\mathbf{A}| \partial \theta_i$, $\partial \mathbf{A}^{-1} \partial \theta_i$ and $\partial \ln |\mathbf{A}| \partial \theta_i$ already introduced in appendix C. Using these expressions each term

$$\frac{\partial L}{\partial \theta_j} = -\text{tr} \left(\mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \theta_j} (\mathbf{I} - \mathbf{R}^{-1} \mathbf{S}) \right). \quad (\text{D.9})$$

This expression further requires

$$\begin{aligned} \frac{\partial \mathbf{R}}{\partial \theta_j} &= \frac{\partial}{\partial \theta_j} (\mathbf{R}_{ss} + \hat{\mathbf{N}}) = \frac{\partial}{\partial \theta_j} \mathbf{R}_{ss} = \frac{\partial}{\partial \theta_j} S_0 e^{20+\lambda} \mathbf{R}_* \\ &= \begin{cases} S_0 e^{20+\lambda} \mathbf{R}_* = \mathbf{R}_{ss} & \text{if } \theta_j = \lambda. \\ S_0 e^{20+\lambda} \frac{\partial \mathbf{R}_*}{\partial \theta_j} & \text{otherwise.} \end{cases} \end{aligned} \quad (\text{D.10})$$

using Eq.(D.2) and Eq.(D.3).

The term $\partial \mathbf{R}_\bullet / \partial \theta_j$ is a matrix with each element defined using Eq.(D.4):

$$\frac{\partial}{\partial \theta_j} [\mathbf{R}_\bullet]_{i,j} = \sum_m \sum_n \frac{\partial \mathcal{I}_{m,n}(\boldsymbol{\theta})}{\partial \theta_j} C_{m,n}^{i,j}. \quad (\text{D.11})$$

The derivatives versus the shape function follow from Eq.(4.24) and Eq.(4.25):

$$\frac{\partial \mathcal{I}}{\partial x_0} = -2\mathcal{I} \left(-\frac{u \cos \phi}{w^2} + \frac{v \sin \phi}{l^2} \right) \quad (\text{D.12a})$$

$$\frac{\partial \mathcal{I}}{\partial y_0} = -2\mathcal{I} \left(-\frac{u \sin \phi}{w^2} - \frac{v \cos \phi}{l^2} \right) \quad (\text{D.12b})$$

$$\frac{\partial \mathcal{I}}{\partial w} = 2 \frac{u^2 \mathcal{I}}{w^3} \quad (\text{D.12c})$$

$$\frac{\partial \mathcal{I}}{\partial l} = 2 \frac{v^2 \mathcal{I}}{l^3} \quad (\text{D.12d})$$

$$\begin{aligned} \frac{\partial \mathcal{I}}{\partial \phi} = -2\mathcal{I} \left[\frac{u}{w^2} (-(x - x_0) \sin \phi + (y - y_0) \cos \phi) \right. \\ \left. + \frac{v}{l^2} (-(x - x_0) \cos \phi - (y - y_0) \sin \phi) \right] \end{aligned} \quad (\text{D.12e})$$

where (x, y) are the coordinates of the center of cell (m, n) .

The analytical gradient of each narrowband cost function is therefore found by substituting Eqs.(D.9,D.10,D.11) and Eq.(D.12) into Eq.(D.8). The full gradient is then determined as in Eq.(D.7) by summing the individual “narrowband” gradients over frequency.

D.3 Implementation

The maximum likelihood solution is determined by supplying an optimizer with the cost function f and its gradient $\mathbf{g}$, both of which are described in full in the previous section. An initial guess $\boldsymbol{\theta}_0$ is required: choices for good $\boldsymbol{\theta}_0$ are discussed in chapter six.

Timing tests were conducted on an HP735 RISC workstation with a 99MHz CPU. The cost and cost gradient function were coded into Matlab M files, and timed using the Matlab timer functions. The cost function required 12.7 s and the cost gradient function required 39.2 s, both involving summation over six frequencies.

By comparison, a finite difference scheme for estimating the gradient would have required $11 + 1$ function evaluations, which would take over 144 seconds. Some efficiency could be gained by judicious coding of the cost function versus λ_i , but the finite-difference gradient would still remain more computationally intensive than the analytical gradient.

The two optimizers used are discussed in the following paragraphs.

Second-Order quasi-Newton Optimizer A quasi-Newton optimizer with a Fletcher switch and an inexact line search was used as the primary numerical workhorse. The line search algorithm used quadratic interpolation to improve the efficiency of the stepsize calculation. All routines, including both the cost and gradient function, were coded into Matlab M files. The optimizer used either a BFGS type algorithm or a DFP type algorithm, depending on the Fletcher switch at each iteration. This is a second-order approach in that the highest-order derivatives required (in this case estimated) were the second-order terms needed for the Hessian (Antoniou [5]).

Fourth-Order Tensor Optimizer Since the cost function is inherently nonlinear, faster convergence can sometimes be achieved by computing (in this case, by estimation) third- and higher-order derivatives. This is the approach used by Chow, Eskow and Schnabel [23] with the TENSOR fourth-order optimizer.

This code is available as FORTRAN source from Netlib [104]. In order to apply it to the cost and gradient functions, which were already written in Matlab M files (and not easy to convert to FORTRAN), a driver program was written in FORTRAN that started Matlab as an computational engine before initializing and starting the optimizer. Then, whenever the optimizer required the evaluation of the cost or gradient function, a request was sent to the Matlab engine to perform the needed computation and return the answer. This was a “quick and dirty” programming solution that eliminated substantial coding time.

Performance TENSOR was not as verbose with diagnostics as was the quasi-Newton optimizer, so the latter was generally used first to fine tune the initial guess θ_0 . This would require 20 to 100 iterations, or about 1 - 3 hours of wallclock time on an HP735 RISC workstation with a typical user load, once a “good” initial guess was found. When the quasi-Newton optimizer seemed to find a promising sequence of solution iterates leading more or less “down hill”, the TENSOR optimizer was started with one of the intermediate iterates and run until convergence using a iterate step tolerance STEPTL of 1×10^{-4} and a gradient tolerance GRADTL of 1×10^{-6} . Convergence was generally reached within about 10 iterations, or about 1 - 2 hours of wallclock time.

In practice, reasonably-behaved solutions did not force either w or l to zero or to negative values. This feature of the cost function eliminated the need of coding overhead to handle constraints and permitted the use of unconstrained optimizers for this problem.

D.4 Simulation

A critical step in the development of this technique is the characterization of the algorithm through simulation. Simulation at a single frequency involves the following steps:

- * set the true parameters θ_{true} ;
- * compute $\mathbf{R}_{ss}(\theta_{true})$;
- * assume a noise matrix $\mathbf{N}$ and compute the sample CSDM $\mathbf{S}$:
- * compute a sample noise matrix $\hat{\mathbf{N}}$;
- * compute the model CSDM as $\mathbf{R} = \mathbf{R}_{ss}(\theta_{true}) + \hat{\mathbf{N}}$;
- * set θ_0 as the first guess;
- * optimize.

Each step is described below.

True Parameters The true source was defined for the simulation as a Gaussian hump with parameters $[-1, -2, 2, 0.6, 1.0472, \lambda^T]^T$, with source strength parameters $\lambda = [1.07, 1.07, 1.07, 1.07, 1.07, 1.07]^T$.

Compute Signal CSDM The signal CSDM $\mathbf{R}_{ss}(\boldsymbol{\theta}_{true})$ was computed via Eq.(D.3) and Eq.(D.4).

Compute Noise CSDM Let the noise CSDM be $\mathbf{N} = N_L \mathbf{I}$. The "noise level" N_L was adjusted so that

$$10 \log_{10} N_L = 10 \log_{10} (\max\{\text{diag}\{\mathbf{R}_{ss}\}\}) - 10.$$

i.e., the SNR in decibels of the signal power from the loudest channel was 10 dB.

Compute the Sample CSDM Assume now that the sample CSDM $\mathbf{S}$ has a complex Wishart distribution $CW(\mathbf{S}|\mathbf{R}, N_{samp})$ with $E\{\mathbf{S}\} = \mathbf{R} = \mathbf{R}_{SS}(\boldsymbol{\theta}_{true}) + \mathbf{N}$ and $N_{samp} = 50$ (50 consecutive snapshots corresponds to about one half second of averaging with 64 point transforms.) A realization of $\mathbf{S}$ was then computed using the following M code fragment:

```
function A = wishart(R,N)
% A = wishart(R,N) generates a matrix A with a complex Wishart
% distribution. R = E{A}, number of averaged outer products = N.
% Dividing by N yields the "sample covariance matrix".

p = size(R);
[U,oops] = chol(R); % invokes Cholesky decomposition

Z = zeros(p,p);
```

```

for n=1:N,
    z = randn(p,1) + i*randn(p,1);
    % generates complex vector of iid Gaussian variates
    Z = Z + z*z';
end

A = U'*Z*U/2;    % the factor 2 is required due to the complexity
tmp = diag(A);
A = A - diag(tmp) + real(diag(tmp)); % fix to ensure real diagonal

```

Compute Sample Noise CSDM Assume now that only an estimate $\hat{\mathbf{N}}$ of the true noise CSDM $\mathbf{N}$ is available. Assume that $\hat{\mathbf{N}}$ has a complex Wishart distribution $CW(\hat{\mathbf{N}}|\mathbf{N}, N_V)$, with $N_V = 200$ corresponding to about 2 s of averaging for consecutive 64-point transforms. This simulates the fact that $\mathbf{N}$ is not known. A realization of $\hat{\mathbf{N}}$ was computed using the M code fragment above.

Compute Model CSDM The total “model” CSDM $\mathbf{R}$ was then $\mathbf{R}_{SS}(\boldsymbol{\theta}_{true}) + \hat{\mathbf{N}}$.

Initial Guess The initial guess for $\boldsymbol{\theta}$ was $\boldsymbol{\theta}_0 = [0, -2, 1, 1, \pi/4, 1, 1, 1, 1, 1, 1]^T$.

Optimization The quasi-Newton optimizer was applied to the narrowband log-likelihood at $f = 603$ and 1895 Hz. Broadband simulation required generating a set $\mathbf{R}_{ss}$, $\hat{\mathbf{N}}$ and $\mathbf{S}$ for each of six frequencies. The algorithm was coded to terminate if

$$|\mathbf{g}| < \epsilon_g \text{ and } |\boldsymbol{\theta}_n - \boldsymbol{\theta}_{n-1}| < \epsilon_\theta$$

where ϵ_g and ϵ_θ are two user-adjustable parameters generally set to 1×10^{-5} and 1×10^{-3} .

Results The results of the simulations are shown in tables D.1 and D.2. The narrowband estimates are clearly inferior to the broadband estimates. The broadband shape

parameter estimator are within about 10% of the true values or less, and the broadband source strength parameters are within about 1 dB.

D.5 Error Estimates

Estimates for errors on $\hat{\theta}_{ML}$ could be approached by bootstrapping or jackknifing (Efron and Gong [43]): this technique in principle could yield not only the standard error for each parameter, but even confidence intervals. Bootstrapping is, however, impractical in this situation because the time required for each optimization run ($O(\text{hours})$) would make the overall time, assuming thousands of bootstrap replications, quite impractical.

An alternative approach is to exploit the result (Efron and Tibshirani [42], chapter 21, pg 305) that the maximum likelihood estimator has the limiting Gaussian distribution (for large samples) of

$$\hat{\theta}_{ML} \sim N(\theta_{true}, \mathbf{J}^{-1})$$

where $\mathbf{J}$ is the Fisher information matrix (FIM) with elements given by

$$[\mathbf{J}]_{j,k} = E \left\{ \frac{\partial^2 L_{BB}}{\partial \theta_j \partial \theta_k} \right\}. \quad (\text{D.13})$$

The FIM must be evaluated at the true value θ_{true} . However, neither θ_{true} nor $\mathbf{J}$ are known. Therefore, the distribution must be approximated: one possibility is $N(\hat{\theta}_{ML}, \mathbf{J}^{-1}(\hat{\theta}_{ML}))$, where the FIM is evaluated at the ML estimator. As will be shown below, it is not clear how to take the expectation of the information, so a second possibility is $N(\hat{\theta}_{ML}, \hat{\mathbf{J}}^{-1}(\hat{\theta}_{ML}))$, where $\hat{\mathbf{J}}$ is

$$[\hat{\mathbf{J}}]_{j,k} = \frac{\partial^2 L_{BB}}{\partial \theta_j \partial \theta_k}. \quad (\text{D.14})$$

When $\hat{\mathbf{J}}$ is evaluated at $\hat{\theta}_{ML}$, Eq.(D.14) is known as the *observed information* (Edwards [41], chapter 5, pg 72) or the *stochastic Fisher matrix* (Scharf [118], chapter 7). Geometrically, the observed information is curvature information on the log-likelihood evaluated at the maximum.

freq	$N_{ambient}$	N_{signal}	$x_0[m]$	$y_0[m]$	$w[m]$	$l[m]$	$\phi [rad]$
guess			-0.00	-2.00	1.00	1.00	0.79
true			-1.00	-2.00	2.00	0.30	1.05
603	200	50	-1.04	-2.05	2.31	0.15	1.01
1895	200	50	-0.95	-1.96	1.86	0.21	1.07
BB(6)	200	50	-1.02	-2.00	2.20	0.33	1.06

TABLE D.1: Simulation results, shape parameters. The second and third columns provide the number of accumulants in the estimation of the ambient-only and ambient+signal CSDMs, respectively. The fourth and fifth columns provide the source centroid (units of metres) in flat array frame coordinates. Columns 6 and 7 provide the major and minor axis scales (units of metres). Column 8 provides the orientation of the wave major axis relative to the array frame of reference (units of radians).

frequency:	603	861	1120	1378	1637	1895
guess	1.00	1.00	1.00	1.00	1.00	1.00
true	1.07	1.07	1.07	1.07	1.07	1.07
603 Hz	1.51					
1895 Hz						1.53
BB(6)	0.79	0.948	0.79	0.90	0.94	0.92

TABLE D.2: Simulation results, source strength parameters.

A reasonable approach to error estimation is then to estimate the standard error of $\hat{\theta}_j$ by the square root of the variance of $\hat{\theta}_j$, which is

$$\text{Var}\{\hat{\theta}_j\} \approx [\hat{\mathbf{J}}^{-1}(\hat{\boldsymbol{\theta}}_{ML})]_{j,j}. \quad (\text{D.15})$$

Efron and Tibshirani go on to establish $1 - \alpha$ confidence intervals based on the limiting Gaussian distribution, but we shall settle here for estimates of standard error.

The second partial derivative of a narrowband log-likelihood function follows directly from Eq.(D.9) on Eq.(D.5):²

$$\begin{aligned} \frac{\partial^2 L}{\partial \theta_k \partial \theta_j} = & -\text{tr} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \theta_k} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \theta_j} (\mathbf{I} - \mathbf{R}^{-1} \mathbf{S}) \\ & + \text{tr} \mathbf{R}^{-1} \frac{\partial^2 \mathbf{R}}{\partial \theta_k \partial \theta_j} (\mathbf{I} - \mathbf{R}^{-1} \mathbf{S}) \\ & + \text{tr} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \theta_j} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \theta_k} \mathbf{R}^{-1} \mathbf{S} \end{aligned} \quad (\text{D.16})$$

The expected information would require the expectation of Eq.(D.16), which in turn requires $E\{\mathbf{S}\}$. If the parametric model is accurate, one could assume that $E\{\mathbf{S}\} = \mathbf{R}$. However, this assumption is unlikely to be valid, as the true shape functions are not nice, neat Gaussian lumps. This would argue against trying to form the FIM.

The observed information would therefore require full evaluation of the terms in Eq.(D.16). While this is possible, a simple approximation is to use the analytical representations for the first-order derivatives already developed in section D.2 and finite differences to approximate the second-order derivatives.

As an example, if $\partial L(\hat{\boldsymbol{\theta}}_{ML})/\partial \theta_j \equiv D_j(\hat{\boldsymbol{\theta}}_{ML})$, then

$$\frac{\partial^2 L}{\partial \theta_k \partial \theta_j} \approx \frac{D_k(\hat{\boldsymbol{\theta}}_{ML} + \boldsymbol{\delta}_k) - D_k(\hat{\boldsymbol{\theta}}_{ML})}{\delta} \quad (\text{D.17})$$

where $\boldsymbol{\delta}_k$ is a vector of zeros except for the k th element, which is δ . The perturbation δ was generally chosen to be $\theta_j/100$ if $\theta_j \neq 0$ or 0.01 if $\theta_j = 0$.

The observed information matrix should be a real symmetric matrix, and the finite-difference approximations tend to bear this out: therefore, computation time can be nearly halved by assuming $[\hat{\mathbf{J}}]_{j,k} = [\hat{\mathbf{J}}]_{k,j}$.

²Eq.(D.5) is required here because N_{samp} is now important.

parameter	x_0 [m]	y_0 [m]	w [m]	l [m]	ϕ [rad]
value	-1.43	1.33	1.80	1.23	1.52
s.e.	0.02	0.03	0.02	0.015	0.06

TABLE D.3: Standard error estimates for wave W3 shape parameters.

frequency	603	861	1120	1378	1637	1895
value	0.50	-1.07	-1.90	-.276	-3.27	-4.11
s.e.	0.04	0.04	0.03	0.03	0.03	0.03

TABLE D.4: Standard error estimates for wave W3 source strength parameters.

This computation was performed on wave W3 as an example and yielded the results shown in tables D.3 and D.4.

Appendix E

ULSAS System Description

This appendix describes the instrument “ULSAS” developed during this project, including the system performance parameters applicable for the SURFEX III deployment. An overview of the instrument is provided in section E.1, the instrument subsystems in subsequent sections. The last section is a glossary of frequently used acronyms.

E.1 System Block Diagram

ULSAS has a long history at IOS, passing through several projects and multiple deployments, undergoing minor and sometimes major modifications each time. Hill [59] discusses the origins of the system. Essentially every subsystem was modified yet again for this project. A system diagram applicable at the time of SURFEX III is shown in figure E.1. A system block diagram is shown in figure E.2.

There were two primary assemblies, the *array package* and the *electronics package*. Both were positively buoyant. They were interconnected by a 20 m Glenair multi-conductor cable and a 1/2” nylon rope: the rope acted as the strain member, and the cable was taped loosely (i.e., incorporating “service loops”) to the rope every 50 to 80 cm. Attachment bridles at both packages used nylon fittings to minimize mooring noise (“clanks” and “bangs”) during instrument motion.

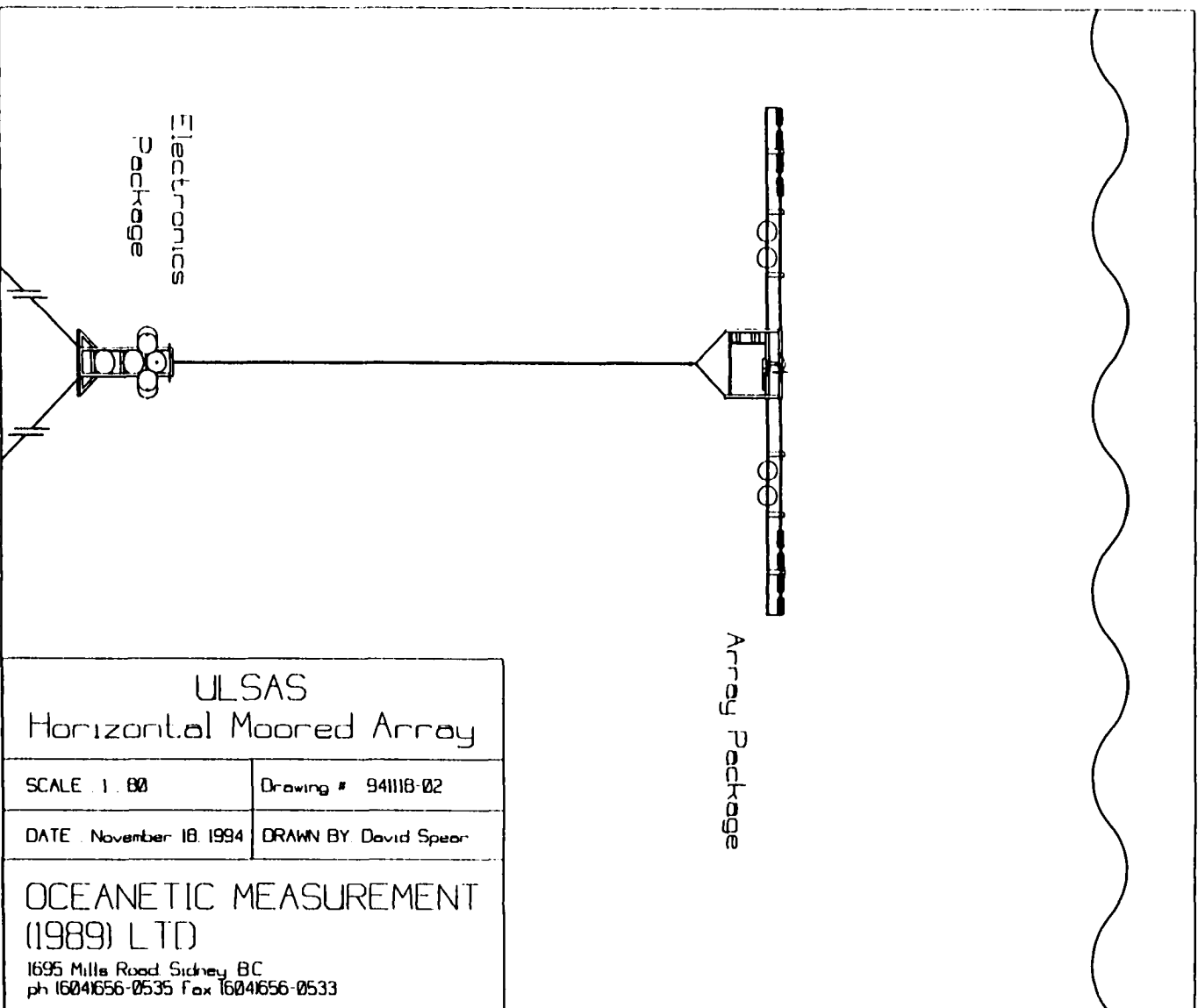


FIGURE E.1: ULSAS system diagram.

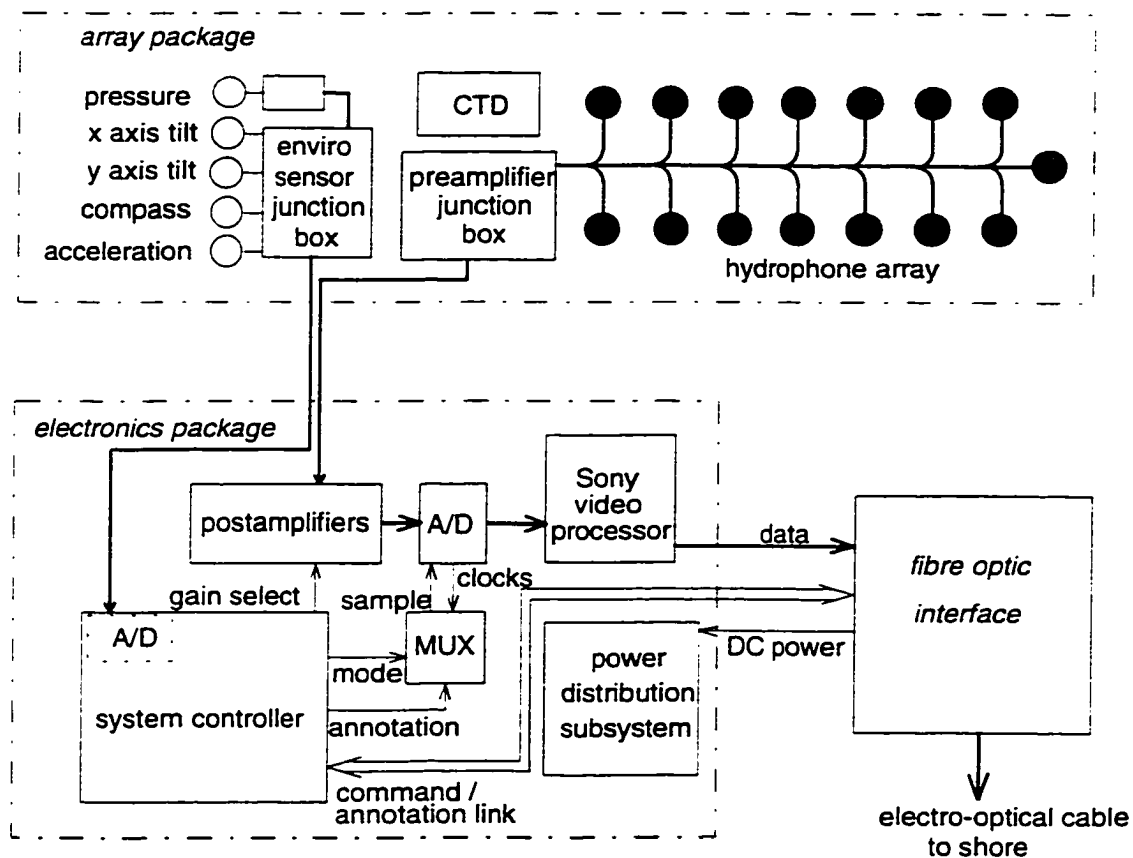


FIGURE E.2: ULSAS system block diagram.

The array subsystem consisted of the hydrophones, their mounts, the array structure, including the arms and the central frame, and canisters containing environmental sensors. The primary environmental canister housed x -axis and y -axis tiltmeters, compass, accelerometer and a pressure sensor to determine the array orientation in 3-D space. An additional "high-precision" pressure sensor was installed on the frame for SURFEX III. These sensors are discussed in section E.2.1.

During SURFEX II, a Seabird SeaCat CTD was attached to the array frame to allow us to infer the sound speed in the vicinity of the array. This was a stand-alone instrument which did not communicate with the rest of ULSAS: its measured data was only available after instrument recovery.

Further details of the array subsystem are provided in section E.2.

The electronics package consisted of two canisters and several attached Viny floats. One canister housed the main instrument electronics, and the smaller canister housed the fibre-optic interface (FOIF).

The analog subsystem consisted of hydrophone preamplifiers (which were actually located in a junction box on the array package), and post-amplifiers, located in a card cage in the main electronics canister. The subsystem provided 15 channels with identical design specifications. This subsystem is described in section E.3.

The digital subsystem consisted of a microcontroller with associated on-board peripherals, the multiplexer (MUX), sample-and-hold circuitry, the analog-to-digital converter (ADC) and a digital-to-video converter. System clocks were generated in the ADC and provided to the MUX, which orchestrated the continuous digitization of the multi-channel acoustic data. The digitized data was passed to a video processor board that converted the digital data to NTSC video format.

Environmental sensor data was acquired via a second ADC on the microcontroller unit (MCU) and then passed to both the communications uplink and to the MUX, where the environmental data was multiplexed with the digital acoustic data. The MCU sampled the environmental sensors every half-second, under interrupt control

from an on-board real-time clock. The MCU also conducted the power-on instrument set-up and afterwards could also respond to a small set of simple commands to change system settings, return instrument status and so on.

Further details of the digital subsystem are provided in section E.4.

The communications subsystem involved the hybrid underwater electro-optical system "MIDlink 1", developed by Oceanetic Measurement, Ltd., (OML) for another IOS project. (For further details, see the OML documentation, [106].) This system provided a wideband data uplink, a bi-directional command channel, and supply power to the instrument. The instrument-end of this system, the FOIF, multiplexed NTSC video and 9600 baud annotation data (from the MCU serial port) together and transmitted it up the fibre at 1300 nm. Downlink commands, transmitted from the shore unit at 850 nm, were converted to 9600 baud and sent to the MCU serial port.

The 2 km Kevlar-reinforced electro-optical cable connected the deployment site with a blockhouse on the IOS jetty. The shore-end of the link is shown in figure E.3. Optical uplink data were decoded and demultiplexed into the NTSC video signal and the 9600 baud annotation channel. The video signal was routed to a commercial off-the-shelf JVC VCR, where the data was recorded on standard super high grade VCR tapes. When recording at the "extended play (EP)" setting (which was the norm), more than eight hours of data could be recorded on one tape, yielding approximately 5 GB of data.

The shore-unit command link could interface directly with the communications port of a PC. (Typically, this was a laptop PC.) Communications over the command channel were controlled with standard commercial communications software running in full-duplex mode. Commands could be sent to the instrument, and status and/or annotation could be received from the instrument. Full details concerning the available command set and shore-end hardware and software configuration are provided by Andrew [4].

The standard shore-end system included a custom modulator unit in the command channel. This unit converted the 9600 baud uplink data to an audio FSK signal, which was recorded on the audio channel of the VCR tape. The downlink commands were

passed directly to the MIDlink shore unit.

The MIDlink shore-unit converted wall-power from the jetty blockhouse to 240 VAC 60 Hz, which was sent down the electrical conductors of the cable. The FOIF converted this to 24 VDC, which was passed to the ULSAS power distribution subsystem. This subsystem used DC-to-DC converters to provide the different levels required by the instrument electronics. The subsystem used modules developed by OML for several generations of IOS acoustic instruments. These types of converters generate considerable system noise related to the chopper frequency: system tests showed that the converters produced less noise when they were synchronized.

ULSAS digital VCR data tapes were played back in the laboratory on the system shown in figure E.4. This consisted of a commercial off-the-shelf JVC VCR, a Sony digital audio interface and a 486 PC. The VCR played the video data to the Sony unit, which converted it to a digital TTL signal. This signal, along with several related bit- and word-clocks, was then routed to a custom "PCM board" in the PC: this card, developed in-house at IOS, framed the serial data into two-byte words, converted the serial data to parallel data, put the bytes onto the bus and sent an interrupt for each byte.

A quadruple buffering program on the PC (written by R. Teichrob at IOS) set up a DMA channel from the bus to one of four 4 KB RAM buffers, and then wrote each

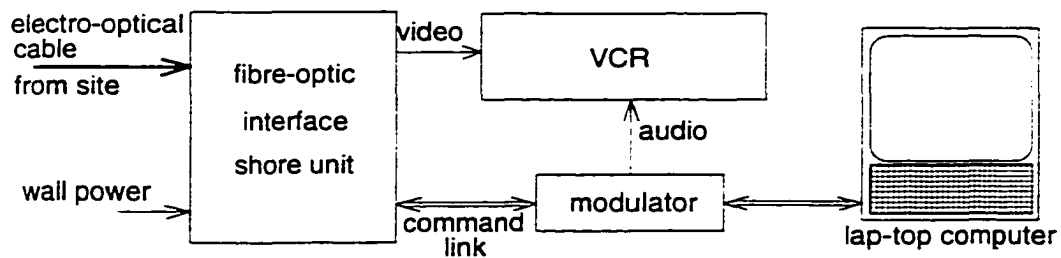


FIGURE E.3: ULSAS shore-end system block diagram.

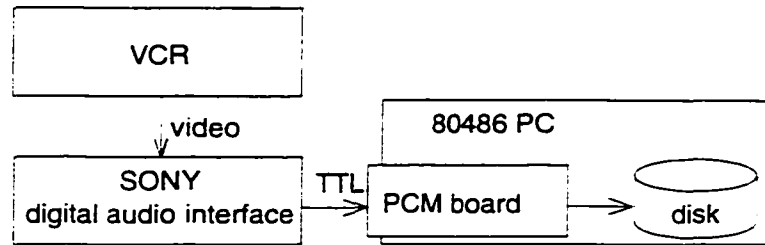


FIGURE E.4: ULSAS laboratory playback system block diagram.

buffer to disk as it was filled. This program required all of the system resources: when run under Windows, it occasionally missed data when it was forced to wait for bus or disk time.

E.2 Array Subsystem

A plan view of the hydrophone array is shown in figure E.5, which corresponds to the sensor constellation shown in figure 4.5. The array arms were roughly four metres long, and are shown in further detail in figure E.6: they were designed to be light and easy-to-deploy. They were specified to flex less than one-tenth of the shortest passband wavelength under typical hydrodynamic flows. The flow loading was computed from a computer model of the dynamic motion of the instrument in wave orbital motion. The structural tubing was further sheathed in an anti-strum netting to reduce strum-induced vibrations in the arms.

The array hydrophones were of two types: nine transducers were ITC 1001 hydrophones, and dated back to the original ULSAS projects. The six additional transducers were ITC 4127 hydrophones, borrowed from the Acoustics Division of the Naval Undersea Warfare Center, Keyport Detachment, Keyport, Washington.

Special attention was devoted to the design and construction of the hydrophone mounts. Previous experience had shown that the hydrophones would be subjected to

considerable vibration- and acceleration-induced signals at the low end of the system passband, signals strong enough saturate the front-end electronics. It was therefore desirable to design hydrophone mounts that would be compliant enough to attenuate the contribution of these structure-borne signals. The resulting design suspended each phone in an elastic web inside a PVC “collar”. The theoretical compliance of this design is described more fully in appendix G. There was also a concern regarding the contamination of the incident signal due to scattering from the mount itself. This effect is examined in appendix H. In summary, the mechanical and acoustic properties of the mounts appeared to have been satisfactory.

The array was not calibrated as an array: the nearfield response would have been particularly tedious to determine. Instead, the individual hydrophones and their associated channels were calibrated with an *in situ* scheme that utilized ambient sound. This procedure is detailed in appendix F. The extension of these calibration results to an array calibration assumes that the sensors are not perturbed (by array frame flexure) more than one-tenth of the shortest passband wavelength from their design positions.

E.2.1 Attitude Sensors

The array package had five location and orientation sensors, measuring x - and y -axis tilt, orientation with respect to magnetic North, depth and acceleration. With the exception of the depth sensor, these sensors were the same sensors used on previous versions of ULSAS. The array acceleration was too small to register a measurable signal on the accelerometer: the remaining sensors are described in the following sections.

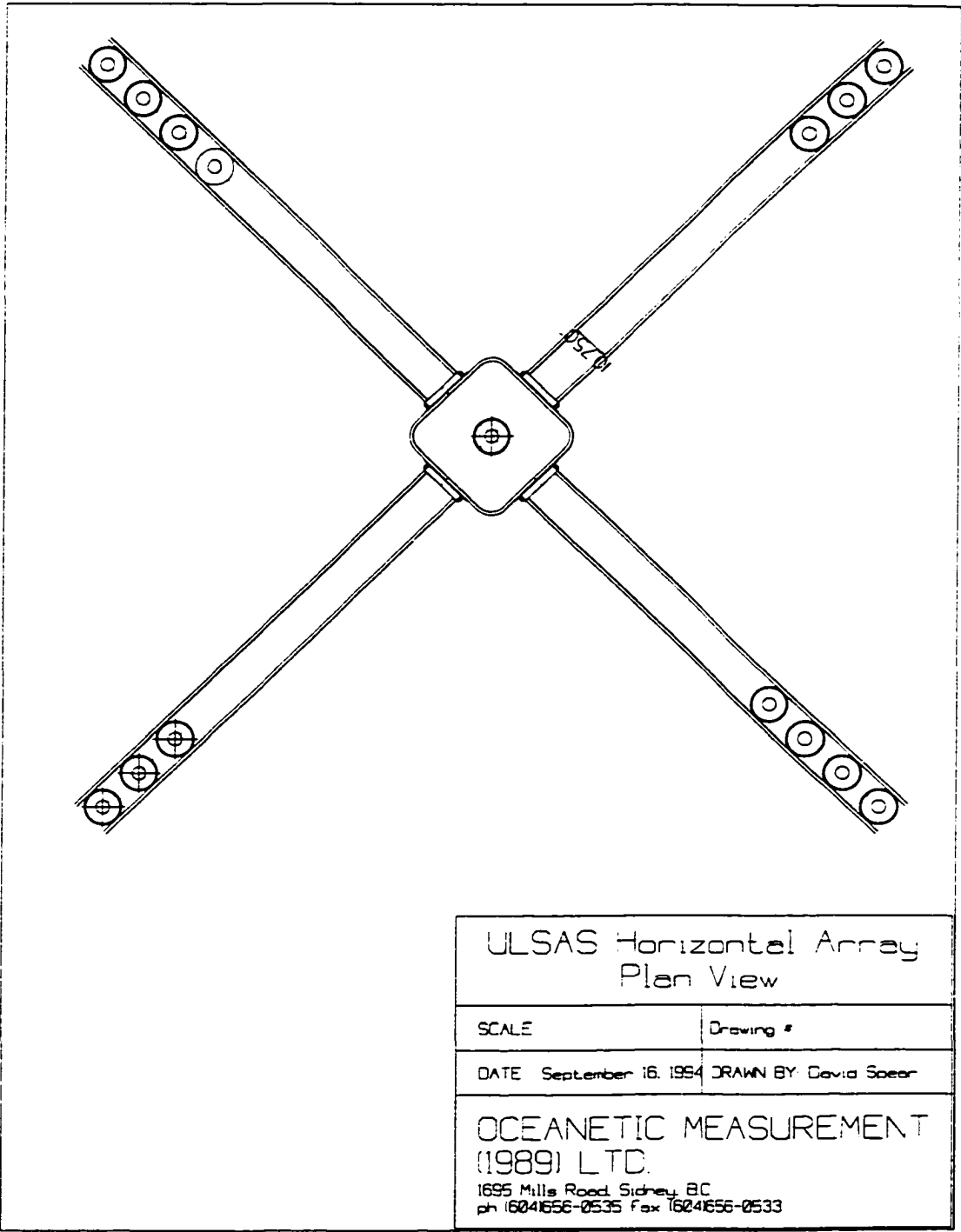


FIGURE E.5: ULSAS hydrophone array plan view.

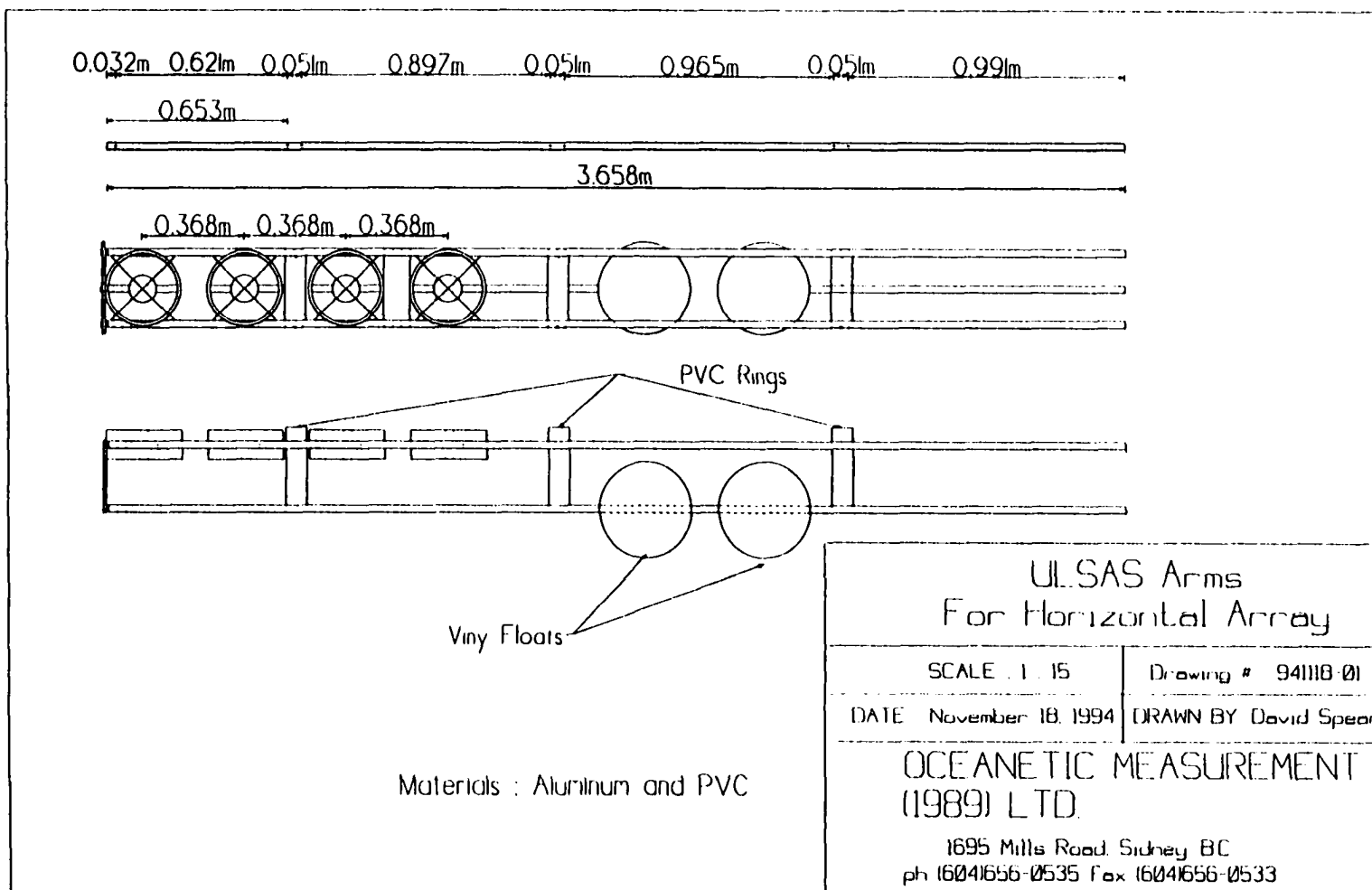


FIGURE E.6: Array "arms".

E.2.1.1 Tilt Sensors

x - and y -axis tilts were measured using Humphrey model CP-17-0601-1 plumb-bob-type tiltmeters. These sensors have a resolution of 0.2° over the range $\pm 45^\circ$. The nominal output is an analog voltage in the range $[0.5]$ volts, with 0° corresponding roughly to 2.5V. Both sensors were calibrated by performing single-axis tilt tests in the laboratory through a range of measured angles, measuring the resulting end-to-end output, and forming a linear regression on the measurements.

E.2.1.2 Compass

The array orientation was measured using KVH Industries solid stat fluxgate compass. This sensor has a resolution of $\pm 1^\circ$ over tilts up to $\pm 45^\circ$ from vertical. The nominal output is an analog voltage in the range $[0, 3.59]$ volts, or roughly 10 mv to 1° . The compass was calibrated by performing flat rotations of the array package in the laboratory through a range of measured angles, measuring the resulting end-to-end output, and forming a linear regression on the measurements.

E.2.1.3 High Precision Pressure Sensor

In an effort to measure the non-directional gravity wave spectrum above the array with a cheap and simple sensor, we investigated the possibility of using hydrostatic pressure measurements at the array. Our solution required the development and integration of a high-precision ULSAS subsystem not previously documented. The following sections detail our solution and its performance.

Sensor Requirements Sinusoidal variations in surface height produce variations in the hydrostatic ("water-column") pressure that attenuate with increasing depth: the depth dependence of this pressure field is a fundamental result in hydrodynamics (c.f., Pond and Pickard [112], section 12.33.) If the surface height at a single point is given

by

$$\eta(t) = A \sin \omega t$$

(where $\eta = 0$ specifies the nominal mean sea surface), then the pressure at depth z is given by

$$p(z, t) = \rho g z + A \rho g e^{-kz} \cos \omega t \quad (\text{E.1})$$

where ρ is the density of water, g is the acceleration due to gravity, depth is positive downwards, ω is the frequency of the gravity wave sinusoid and k is the wavenumber given by the deep-water dispersion relationship

$$\omega^2 = gk$$

The first term in Eq.(E.1) is the hydrostatic pressure and the second contains the desired information regarding the surface height. The e-folding depth of the surface height information is the wavelength of the dominant spectral component (in this model, the only spectral component) divided by 2π .

The surface height, on the other hand, depends on the state of evolution of the wave field, which in turns depends on local weather conditions in a very complicated way not fully understood. Nevertheless, it is possible to use standard approximations based on theoretical and empirical relations to predict the sea state for a given set of environmental parameters. This is done in appendix A for several cases: the one of interest here is the forecast for the “smaller” sea state. This may be considered to be representative of the smallest waves that might still be of interest in this work.

Given these “smaller sea” values for wave parameters, the pressure signal at depth z can now be calculated via Eq.(E.1). For example, the excess pressure (beyond that of atmospheric pressure) at a depth of 3 m would be

$$p(3, t) = 30,000 + 100.5 \cos \omega t \quad ([\text{Pa}])$$

implying that the surface height information is contained in a sinusoid with amplitude 100.5 Pa riding on a background level of 30 kPa. Figure E.7 shows these amplitudes as a function of depth over the expected depth range of the array. These are the pressure levels the proposed system must be able to measure.

The array is expected to operate in depths of 0 - 7 m, which specifies a background hydrostatic pressure of up to 70 kPa.

The Sensor A high precision Paroscientific sensor was available which had barely enough sensitivity, according to specifications, for this purpose. It was rated for depths to 130 m and has a measurement hysteresis of 70 Pa. This hysteresis effectively limits the sensor sensitivity: pressure fluctuations less than 70 Pa could not be reliably sensed.

The sensor converted the sensed pressure to an oscillator frequency. The sensor was modified to include electronics that square the sinusoid (making essentially a digital signal) and drive it through its cable to the preamplifier junction box. The signal was boosted there with another driver and then sent down the Glenair cable to the main electronics canister where it was routed to the pressure interface circuit on the computer interface board. This interface circuit consisted of an Altera EPLD which implemented

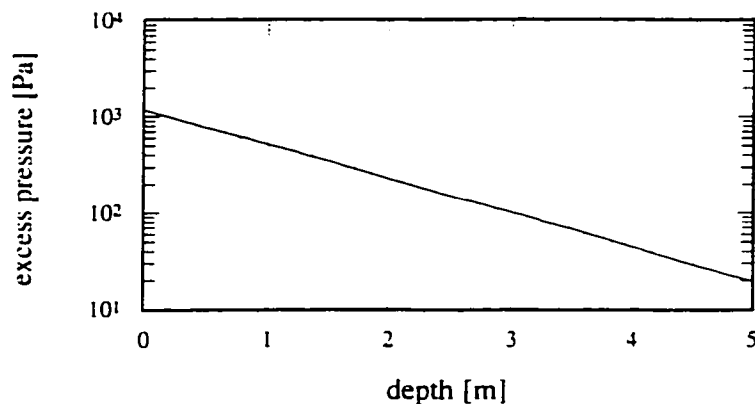


FIGURE E.7: Excess hydrodynamic pressure signal due to surface waves.

a counter that counted the number of cycles of the system clock ECLK in a window equal to 16384 cycles of the pressure sensor period. Thus the number of counts Q registered by the interface is $Q = 16384 \times T / \text{ECLK}$, where T is the period of the sensor signal and ECLK is 2Mhz. Thus,

$$T = \frac{Q}{16384 \times \text{ECLK}}$$

The resultant “pressure” was determined using a calibration formula provided by Paroscientific (Paroscientific, 1991) for this particular sensor.

To achieve the necessary precision over the expected dynamic range (i.e., about 10 parts in 70,000), the Altera implemented a 24-bit counter. For example, a pressure of about 34 kPa (a depth of about 3.5 metres) would generate a sensor frequency of 39522 Hz with a period of 25.3024 μ s. The window, equal to 16384 sensor cycles, would then be 0.4146 sec long, during which about 829107 cycles of ECLK will be accumulated. The computer interface then would output the 24-bit value 0CA6B3H (equal to 829107 decimal).

Calibration The sensor was calibrated in the water tank at the IOS Tunnel Room in order to develop a relation between true pressure and “sensor pressure” (i.e., the “pressure” resulting from converting the measured counts to Pascals using the manufacturer’s formula.) As a first test, the sensor was lowered underwater in measured increments and the pressure read via the standard ULSAS annotation uplink. The pressure at the sensor was modeled as

$$p_{true}(z) = \rho g z + 1 \text{ atm}$$

where 1 atm. was assumed to be 101 kPa. The pressure as a function of counts Q based on the Paroscientific formula is

$$p_{paro} = \text{paro}(Q)$$

where `paro()` is a Matlab M file derived from the Paroscientific documentation. The raw measurements of p_{paro} versus p_{true} are shown in figure E.8. The calibration to first order is therefore

$$p_{\text{true}} = A \text{paro}(Q) + B$$

Using a linear regression, A was found to be essentially 1 and B to be -6.9×10^{-3} Pa, so that

$$p_{\text{true}} = \text{paro}(Q) - 6.9 \times 10^{-3} \quad ([\text{Pa}])$$

A second test was performed to investigate the precision of the sensor subsystem. The sensor was raised by the smallest amount practical (one cog tooth) on the winch: this turned out to be an increment of 3 mm. This is a change in hydrostatic pressure of approximately 30 Pa. For the nominal test depth, a change of 1 count (i.e., Q to $Q + 1$) results in a change in calculated pressure (using `paro()`) of about 15 Pa. Even measurements in still water showed some degree of variance, so sensor measurements

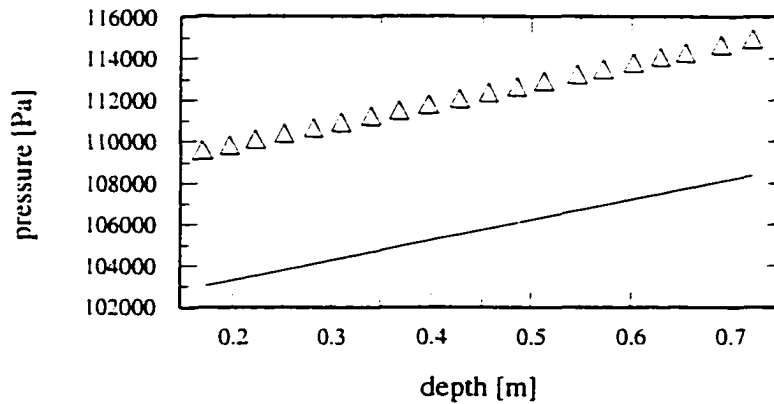


FIGURE E.8: Pressure sensor calibration. IOS Tunnel Room dunk test, Paroscientific sensor S/N 45899. The top data are “raw” pressure values computed from the Paroscientific equation, and the solid line is the “true” pressure.

were made continuously for about a minute at each depth. The acquired data are summarized in figure E.9. This analysis shows that the subsystem can indeed measure below the hysteresis level of the sensor, but that measurements of changes less than about 30 Pa are subject to standard errors of about ± 15 Pa. This is probably the fundamental limit in precision of the ULSAS pressure sensor subsystem.

Performance While analysis indicated that the depth sensor was sensitive enough to provide measures of both depth and local instantaneous water column height, the subsystem did not perform according to expectations during deployment. The problem was determined to arise from digital noise leaking through air into the counter EPLD: the counter circuit was too close to the microcontroller board. This had the effect of destroying the accuracy of the low order byte, which substantially reduced the precision of the measurements.

However, while the subsystem failed to provide local instantaneous water column depth, the sensor system output could still be averaged over several seconds to provide a

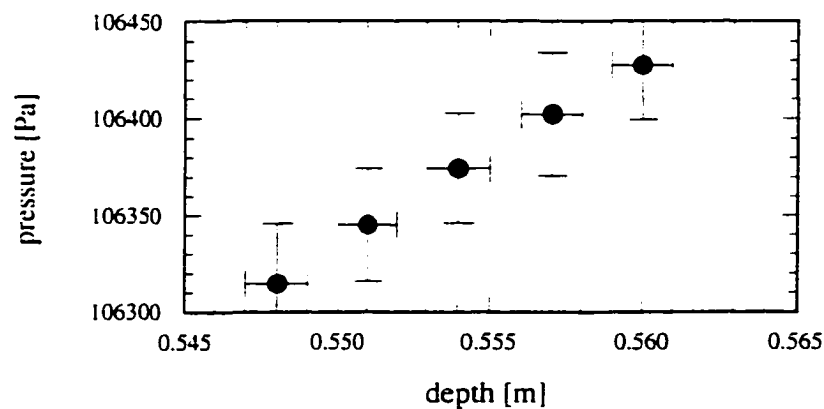


FIGURE E.9: Pressure sensor hysteresis. Paroscientific sensor S/N 45899. The error bars represent vertically one standard deviation and horizontally the estimated measurement error.

usually useful measure of nominal array depth. As an example, figure 5.8 clearly shows the diurnal depth variation of the moored package resulting from local tides. The error bars also show how several points are effected by the digital subsystem noise.

E.3 Analog Subsystem

The analog subsystem consisted of preamplifiers located in a junction box on the array package and second stage “postamplifiers” located in a chassis inside the electronics canister. A schematic diagram of the system components is shown in figure E.10. The preamplifier subsystem components were:

- HPF1: a one-pole high-pass filter to protect the front end of the FET preamplifier A1 from the DC offset produced by the transducer. It had a corner frequency f_c at 23 Hz;
- A1: a low-noise OEI AH0013 wideband FET preamplifier configured for a gain of 32 dB;
- HPF2: a second one-pole high-pass filter with corner frequency f_c at 117 Hz;
- A2: a TLE2141 precision op-amp, configured for a gain of 19.8 dB.

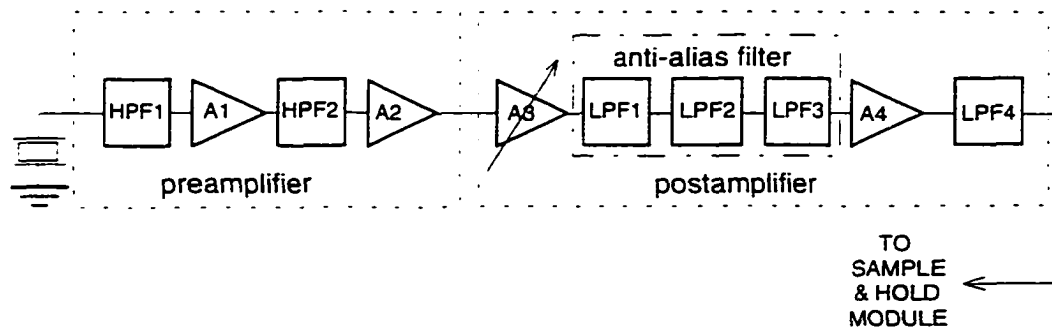


FIGURE E.10: Analog signal conditioning schematic diagram

The postamplifier subsystem components were:

- A3: an AD625 programmable gain instrumentation amplifier, configured for gains of 0, 12, 24 and 36 dB: only the 0 dB setting was used during this experiment, as the acoustic signal was strong enough to saturate this stage at any higher gain. This op-amp was also determined to be susceptible to occasional CMOS lock-up during extremely strong signals, a problem that has plagued IOS near-surface acoustic measurements for years. This problem was not seen in any of the SURFEX III data used here:
- anti-alias filter: the anti-alias filter used a E. G. & G. Reticon RF6609A switched-capacitor filter. This kind of filter samples the input signal internally, and therefore introduces its own sampling error. To mitigate this error, the RF6609A must be accompanied by two additional filters. This entire assembly consisted of:
 - LPF1: a two-pole Butterworth low-pass filter with corner frequency f_c at 3.167 kHz: this filter was required to attenuate any input signals near the internal clock frequency of the RF6609A, which would be folded into the passband output of that chip:
 - LPF2: the RF6609A, a seven-pole elliptic low-pass switched-capacitor filter with corner frequency f_c at 2.0 kHz, maximum passband ripple of 0.5 dB:
 - LPF3: a one-pole low-pass filter with corner frequency f_c at 768 kHz to smooth the output of the RF6609A.
- A4: a unity-gain follower to drive the signal off-board:
- LPF4: a one-pole high-pass filter with corner frequency f_c at 23 Hz.

The magnitude of the design analog subsystem transfer function, $|H(\omega)|$, is shown in figure E.11. Almost all of the data results in this report utilized the passband of [160,2000]Hz, and figure E.11 shows that the system transfer function was nearly constant within ± 1 dB over this band.

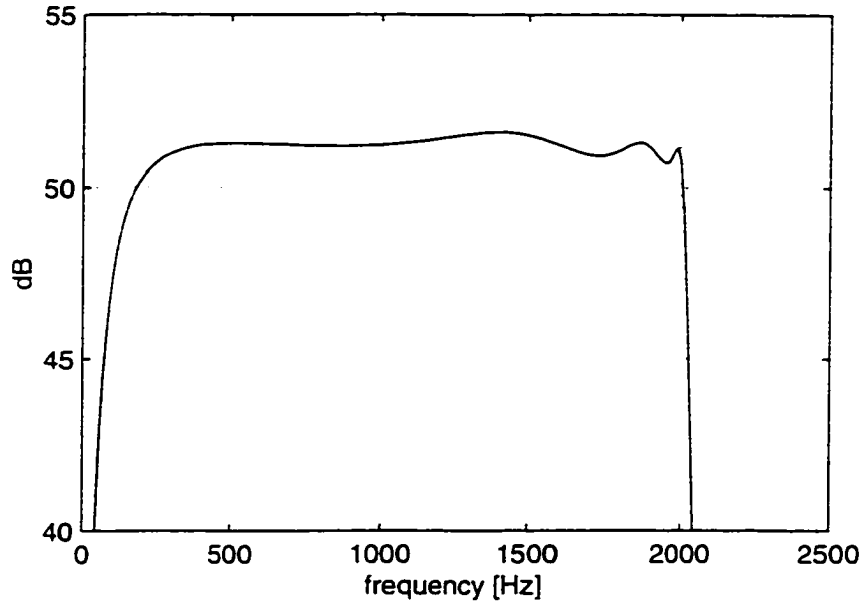


FIGURE E.11: System transfer function

E.3.1 Gain and THD

A sinusoid with nominal frequency of 330 Hz was injected into each channel via pins 2 and 3 of the preamplifier connectors. The maximum likelihood estimators of frequency f_{MLE} and amplitude A_{MLE} were computed from the recovered time series data, and the channel gain computed using $20 \log 2A_{MLE}/V_{in}[\text{dB}]$ where V_{in} is the applied peak-to-peak voltage. The nominal value is 51.1 dB. The discrepancies, about 2 dB, are most likely due to line losses along the Glenair cable and insertion losses at the voltage followers. The total harmonic distortion (THD) was estimated using the ratio of the sum of the squared maximum likelihood amplitudes of the first 3 overtones to A_{MLE}^2 . Both gain and THD are shown in table E.1.

E.3.2 System Noise Floor

The system “noise floor” was investigated by terminating the preamplifier inputs with either 47 pF capacitors or shorting leads while the system was assembled on the bench.

The DC bias and broadband noise density are shown in table E.2. (The broadband noise power was normalized by the system passband, estimated at 2 kHz.) The autospectra are shown in figure E.12. Tonals are quite evident: contributions at multiples of 60 Hz are related to the omnipresent EMI in the laboratory environment, and the tones at multiples of about 800 Hz are generated internally by system clocks. The internal tonals were acceptably below the expected levels of typical waves breaking within the array field-of-view.

E.3.3 Crosstalk

Crosstalk between channel i and channel j was computed by determining the maximum likelihood amplitude of the signal on channel j ($A_{MLE,j}$) at f_{MLE} (as found from channel i). Then the crosstalk is given as $20 \log A_{MLE,j}/A_{MLE,i}$. Crosstalk results are shown in table E.3.

E.4 Digital Subsystem

The ULSAS digital subsystem required an upgrade for SURFEX II and III in order to make maximum use of the VCR tape storage bandwidth. This necessitated changes to the signal acquisition hardware, the multiplexer and the system controller.

The digital subsystem had two modes: array mode, in which all 15 channels were acquired, and wideband mode, in which only two channels were acquired, but at a much higher sampling rate. The data presented in this work utilized array mode exclusively.

A subsystem block diagram is shown in figure E.13, and the pertinent capabilities of the subsystem are outlined below.

E.4.1 Analog to digital path

The essential elements of the analog-to-digital conversion process are detailed in the following paragraphs.

Sample and hold The analog signals were passed to a board of sample and hold (S&H) modules built by Oceanprobe Systems for the original ULSAS. The S&H modules were strobed in parallel by the MUX, so all channels were sampled simultaneously.

chan	gain [dB]	THD
1	48.0	0.4%
2	49.3	0.3%
3	49.1	0.4%
4	48.9	0.3%
5	49.6	0.2%
6	49.3	0.2%
7	49.3	0.3%
8	49.5	0.2%
9	48.9	0.6%
10	49.3	0.3%
11	49.4	0.3%
12	49.3	0.3%
13	49.2	0.2%
14	49.2	0.2%
15	49.3	0.3%

TABLE E.1: Gain and THD per channel

	DC bias	noise density
chan	[mV]	[$\mu\text{V}_{\text{rms}}/\sqrt{\text{Hz}}$]
1	-4.0	210
2	-4.2	402
3	-6.1	496
4	-6.9	309
5	-9.4	409
6	5.0	335
7	-11.3	279
8	-7.1	438
9	-7.6	483
10	-10.6	1402
11	-9.6	2296
12	-6.8	1145
13	-9.2	1682
14	-4.4	722
15	-18.0	2674

TABLE E.2: DC bias and broadband noise density per channel.

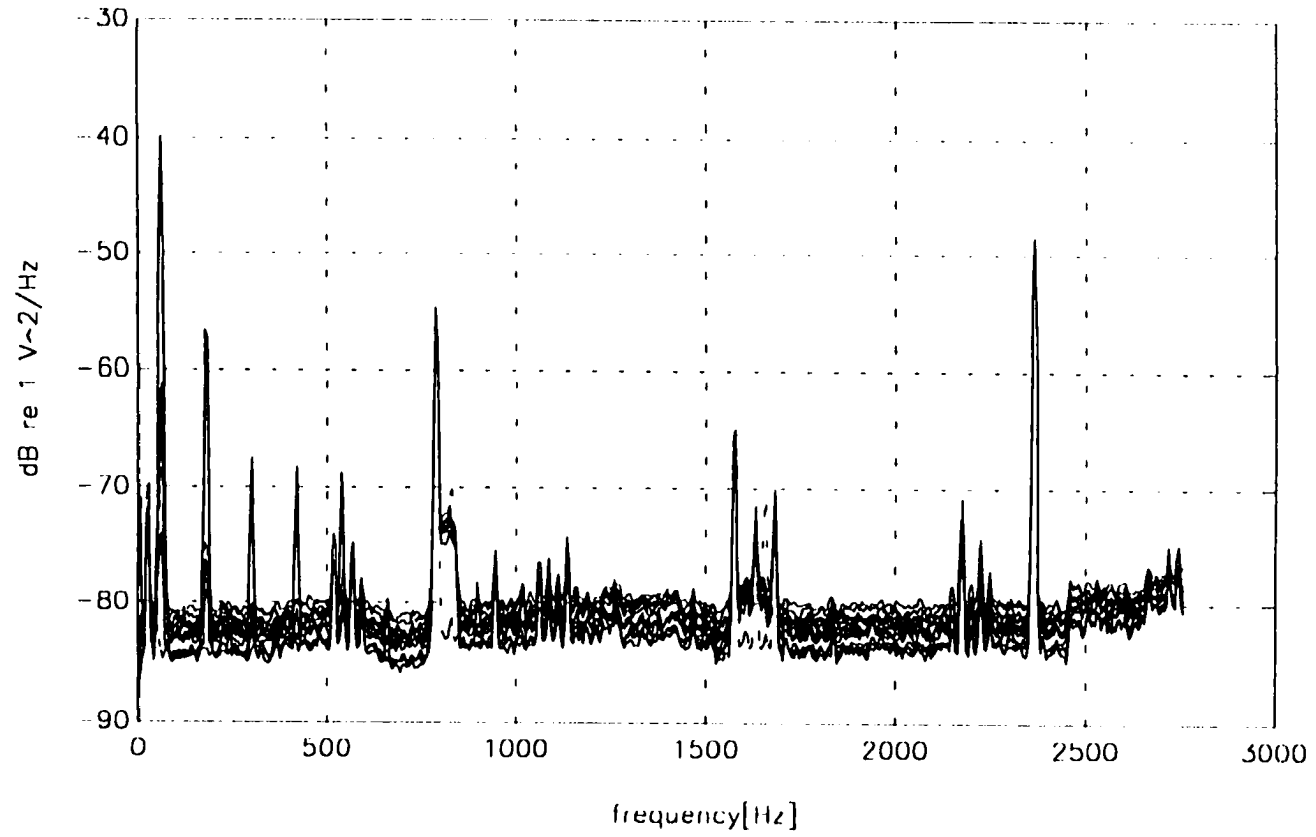


FIGURE E.12: System noise autospectral densities, channels 1 -15. 11 seconds, 60 disjoint blocks, Hanning window.

inject chan	cross channel														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	-59.2	-59.5	-57.5	-59.6	-57.3	-59.3	-57.6	-105.5	-65.5	-65.3	-65.2	-65.8	-65.7	-111.6
2	-59.3	0	-59.0	-58.6	-60.2	-57.8	-60.1	-57.9	-119.5	-66.3	-66.1	-65.9	-66.3	-66.5	-110.6
3	-59.8	-59.2	0	-60.5	-60.5	-57.7	-59.8	-57.8	-99.5	-65.9	-65.8	-65.6	-66.0	-66.1	-104.2
4	-59.2	-57.1	-59.0	0	-59.7	-57.5	-59.2	-57.5	-114.7	-65.4	-65.2	-65.2	-65.5	-65.6	-112.0
5	-59.9	-57.5	-59.8	-60.8	0	-60.3	-60.5	-57.9	-94.1	-66.1	-66.0	-66.0	-66.4	-66.5	-105.6
6	-59.7	-57.5	-59.5	-58.5	-60.0	0	-60.3	-57.8	-112.8	-66.1	-66.1	-66.4	-42.8	-40.5	-99.1
7	-59.6	-57.4	-59.5	-58.5	-60.1	-60.1	0	-60.3	-77.8	-66.2	-65.8	-65.7	-66.1	-66.2	-101.2
8	-59.5	-57.4	-59.3	-58.3	-59.7	-57.6	-60.5	0	-86.5	-65.3	-65.6	-65.6	-66.0	-66.8	-104.7
9	-59.5	-57.3	-59.3	-58.3	-59.8	-57.5	-59.5	-59.8	0	-65.1	-65.3	-65.5	-65.7	-66.0	-100.3
10	-68.1	-65.1	-67.8	-66.1	-67.9	-65.5	-68.8	-65.8	-103.8	0	-54.5	-54.6	-54.7	-54.8	-104.9
11	-67.2	-65.4	-66.4	-66.4	-66.3	-65.8	-67.0	-66.3	-103.9	-54.6	0	-54.7	-44.0	-55.0	-92.8
12	-68.2	-65.1	-67.8	-65.9	-68.1	-65.4	-68.6	-65.9	-111.2	-54.5	-54.5	0	-54.8	-54.8	-111.4
13	-68.8	-64.9	-68.0	-66.1	-68.0	-65.5	-68.6	-66.0	-109.1	-54.6	-54.6	-54.5	0	-54.8	-78.5
14	-68.1	-65.2	-67.6	-66.3	-67.7	-65.8	-68.4	-66.4	-63.3	-54.6	-54.6	-54.6	-54.8	0	-55.9
15	-71.0	-65.2	-67.9	-66.2	-67.8	-65.7	-68.2	-66.3	-63.1	-54.6	-54.6	-54.6	-54.8	-54.9	0

TABLE E.3: Crosstalk results. Sinusoid injected into channel i down the left side and output measured on channel j across.

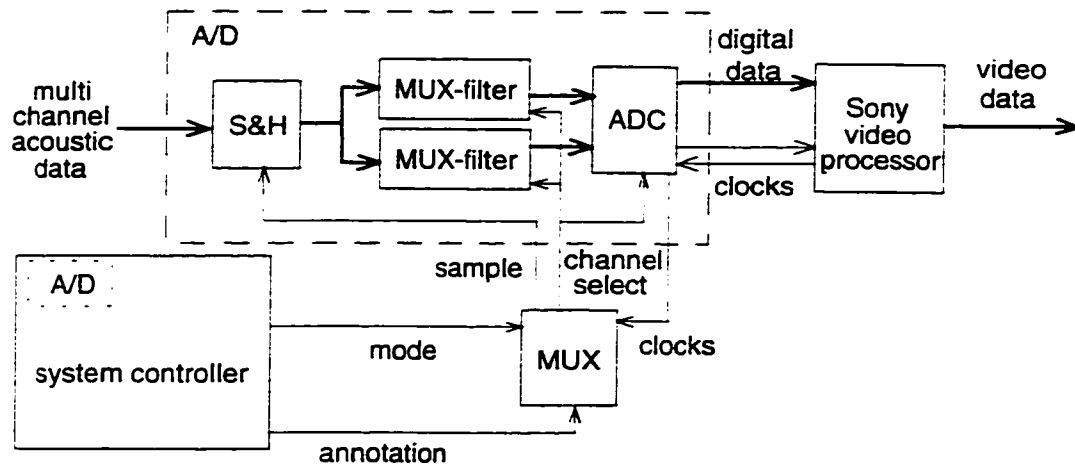


FIGURE E.13: Digital subsystem block diagram

MUX-filters The sampled levels were channeled through two “MUX-filter” cards: these cards were originally designed for other AORG instruments. Here, *sans* filters, they merely acted as analog switches to route the appropriate signal under control of the multiplexer to the A/D.

ADC The analog-to-digital converter (ADC) was an “ADC1” module developed originally by OML for other AORG instruments. The module used a two-channel 16-bit converter. The module was configured as a daughterboard on the video processor board and provided the latter with the master clock.

Video converter The digital data was converted from digital format to NTSC video format by a board scavenged from a commercial Sony digital audio processor. The board also divided down the master clock from the ADC1 daughterboard to produce bit, word and frame clocks used throughout the rest of the system.

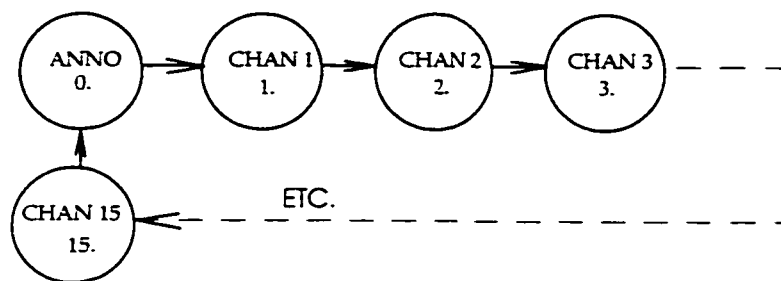


FIGURE E.14: Finite State Machine for the multiplexer

E.4.2 MUX

The multiplexer was an Altera programmable EPLD designed in-house by R. Teichrob. All 15 channels were sampled simultaneously at a rate of $88.200 \text{ kHz} / 16 = 5512.5 \text{ Hz}$. The finite state machine diagram for the multiplexer is shown in figure E.14.

The resulting tape data format is shown in table E.4. The word LSB is set for multiplexer state 0, and cleared for all other states. At state 0, the annotation word low byte is always 01H and the high byte may be a character from the annotation packet, or 00H.

The MUX also provided the clock for the anti-alias filters.

E.4.3 System Controller

The microcontroller was an off-the-shelf "Eagle" system controller board built by Omnex Engineering. The board used the Motorola MC68HC11 microcontroller unit (MCU) and provided the MCU with on-board peripherals such as RAM, ROM, an RS232 in-

...	A	D1	D2	D3	...	D14	D15	A	D1	D2	...
-----	---	----	----	----	-----	-----	-----	---	----	----	-----

TABLE E.4: Digital data format. A is an annotation word, D_i is a word of digital data from channel i . i increments from 1 to 15.

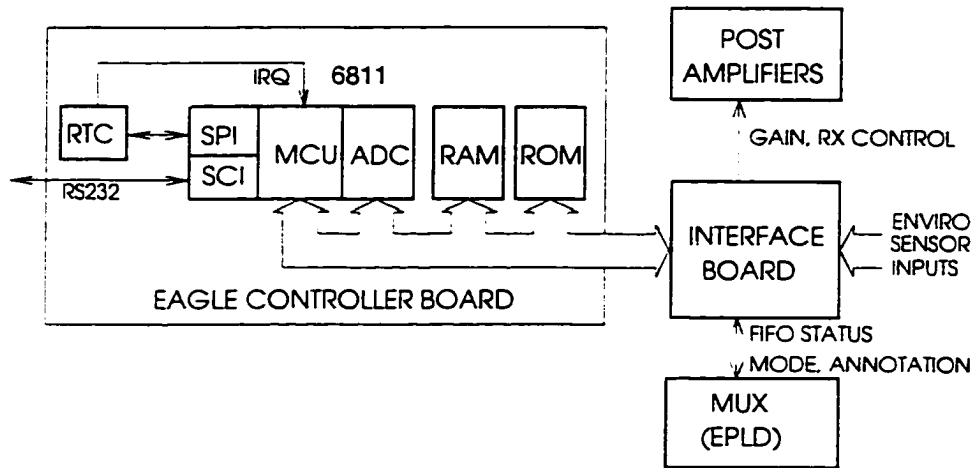


FIGURE E.15: Level 0 context diagram for the system controller

terface, a parallel interface and a battery-backed real-time clock (RTC). The board was mounted as a daughterboard on a 3U computer interface board, which provided the mechanical interface with the main chassis and held all the electronic interfaces for instrument signals.

A custom operating system (ULSOS) was written for the MCU. This was coded in C and compiled into Motorola S files using the Avocet/HiTech C compiler. The operating system performed the power-up sequence, executed a small set of commands sent from shore, and transferred annotation information periodically to (a) the MUX (for inclusion in digital data stream) and (b) to the uplink (for shore monitoring.) A complete description of ULSOS and its configuration in the digital subsystem can be found in Andrew [4]. The level 0 context diagram from that document is reproduced in figure E.15 for reference. The most essential feature of concern here is the annotation.

Annotation The environmental inputs were sampled by the MCU on-chip 8 channel 8-bit ADC. The sampling times were driven by hardware interrupts every 0.5 seconds from the RTC. At each interrupt, the MCU would sample the 8 ADC channels, read

element	format	size
RTC	RTC:hh:mm:ss,mm/dd/yy,	22 bytes
state byte	A B NL	3 bytes
A/D	ADC:A1,A2,A3,A4,PPPPPNLCR	24 bytes
total: (formatted)		49 bytes

TABLE E.5: Annotation format.

the pressure sensor interface, retrieve the time from the RTC, and retrieve a "status" byte from the computer interface board. The raw data would be reformatted into a human-readable character stream, bundled into an annotation "block" and written to the MUX via a FIFO. The MUX would insert the information one byte at a time into the high byte of the annotation time slot word.

The annotation information is formatted as shown in figure E.5, where each character represents 1 ASCII byte except NL which is 0AH and CR which is 0DH.

E.5 Glossary

Acronyms are ubiquitous in hardware descriptions. Some of the more significant ones used in this appendix are defined below.

AORG Acoustical Oceanography Research Group.

FOIF Fibre-optic interface.

HPF High pass filter.

IOS Institute of Ocean Sciences.

ITC International Transducer Corporation, manufacturer of hydrophones.

JVC Manufacturer of commercial electronics.

LPF Low pass filter.

MCU Microcontroller unit.

MUX Multiplexer.

OEI Manufacturer of electronic devices.

OML Oceanetic Measurement (1989) Ltd.

ULSAS Upward Looking Sonar Array System.

ULSOS ULSAS operating system.

Appendix F

In-Situ Ambient Sound Calibration

F.1 Introduction

Typical calibration schemes for acoustic systems involve measuring the absolute receiving sensitivity of the transducer in a standard calibration tank, and end-to-end bench measurements of channel gain and phase characteristics. Calibration for each channel in an N -element array can therefore be expensive, costing as much as N times the calibration of a single sensor.

It is therefore desirable to investigate alternative *in situ* techniques. Also known in the literature as *self-calibration* techniques, these schemes are generally of three distinct types:

far-field narrowband point-source Friedlander and Weiss [52] assumed the general case of multiple simultaneous far-field sources at unknown angles and used eigenstructure decomposition to estimate relative sensor gains and phases at single frequencies. Brown *et al* [22] present a similar approach for determining just the relative phases.

nearfield narrowband point-source Sotirin [122] used a narrowband point-source radiating a known level and located at a known position in the nearfield of an array to estimate relative phases and absolute gains at the source frequency: a propagation model was used to compute the resulting field at each sensor location.

broadband extended source Sotirin also used background ambient sound, which is essentially an extended source field incident from all directions, for a relative gain calibration.

Each of these methods relies on an *a priori* assumption that some critical feature of the calibration “signal” is identical. In the far-field method, the signal amplitude is assumed constant across the array. In the nearfield method, the projector level and propagation path must be known. In the broadband method, the background ambient sound is assumed to be a spatially homogeneous process so that the acoustic pressure autospectrum is the same at each sensor location.

For an absolute calibration, the complete transfer function for one channel must also be known *a priori* for the far-field and broadband methods.

Narrowband methods have the disadvantage that the measured acoustic field at each sensor is the coherent combination of the incident field and the scattered field diffracting off the surrounding elements and array structure. The narrowband methods must perforce incorporate this effect.

On the other hand, the scattered field contribution for the ambient sound is arguably much less severe: it is spatially averaged because the incident field can be decomposed into plane waves arriving simultaneously from a continuum of directions, and temporally because the constituent plane waves are all mutually uncorrelated. The possibility exists that nearby structures could “shadow” a sensor, but the effect should be much less than that for single narrowband wavefields. Indeed, Potter [113] has recently shown that shadowing does not exist for perfect reflectors in an isotropic sound field.

To use the ambient sound, however, the assumption of spatial homogeneity must

also be carefully examined. Hamson [56] has investigated the sound field due to a distribution of surface sources: she shows that boundary conditions at the top and bottom of the water column have a significant influence on the autospectral levels in the vertical plane. These factors, however, have much less impact on the horizontal variability, so that the assumption of spatial homogeneity may be a good approximation for a *horizontal* array far from the boundaries.

Unfortunately, the ULSAS array did not maintain a horizontal aspect during the SURFEX III deployment: deployed at a nominal depth of -3 metres, the array was subjected to near-surface currents particularly during noisy sea states, which were the most interesting for both calibration and imaging purposes. These currents induced x and y -axis tilts of nearly 10° , tilts that positioned the hydrophones throughout a 2 metre vertical span in the water column. Thus it will be necessary to revisit the issue of spatial homogeneity in order to devise the best possible *in situ* calibration scheme under the deployment circumstances.

In the calibration scheme presented below, the gain of each channel is a function of pairs of autospectral densities. The estimation of autospectra is normally a standard task, but not so for a near-surface array. It is not possible to get a sufficient measurement of the background ambient process that is not contaminated by nonstationary breaking events in the nearfield of the array. This presents an additional signal processing problem.

In section F.2 we develop the theory for the *in situ* calibration equations. In section F.3 we examine the consequences of the proximity of the array to the sea surface. We present a more robust solution for separating the influence of the nearfield breaking events from the background sound data in section F.4, and finally, based on the considerations of the previous sections, conclude with a table of the best possible calibration estimates for the array.

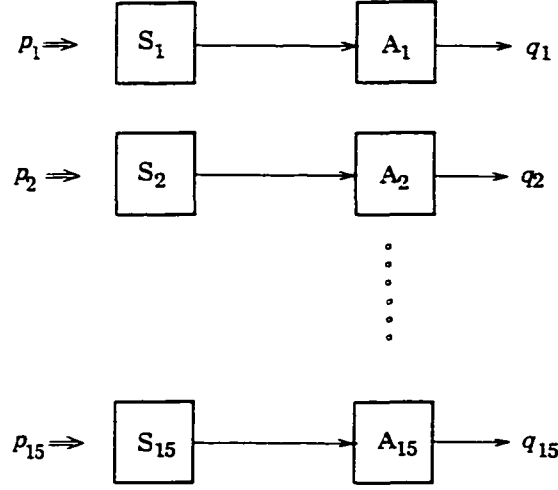


FIGURE F.1: Essential Signal Conditioning Components for Hydrophone Calibration

F.2 Calibration Equations

The essential signal path elements are shown in figure F.1: the electronic signal conditioning circuitry consists of a transducer with receiving sensitivity $S_m(f)$ and an amplifier/filter circuit with frequency response function $A_m(f)$. In the frequency domain, the resulting output signal $Q_m(f)$ (in units of raw A/DC bits) is

$$Q_m = A_m S_m P_m = H_m P_m. \quad (\text{F.1})$$

where the frequency notation has been dropped and P_m is the in-water acoustic pressure at hydrophone m . The combined effects of the transducer and amplifier are lumped together in the second equation into the single transfer function $H_m(f)$.

Using Eq.(F.1), the one-sided output autospectrum $G_{m,m} = \langle Q_m^* Q_m \rangle$ is

$$\begin{aligned} G_{m,m} &= \langle Q_m^* Q_m \rangle \\ &= \langle A_m^* S_m^* P_m^* A_m S_m P_m \rangle \\ &= |A_m|^2 |S_m|^2 \langle |P_m|^2 \rangle \\ &= |H_m|^2 G_{PP}. \end{aligned} \quad (\text{F.2})$$

The term $\langle |P_m|^2 \rangle$ is the acoustic pressure autospectrum at hydrophone m : the critical feature of this *in situ* calibration scheme is the assumption that this value is the same, represented here by G_{PP} , everywhere in the vicinity of the array.

We assume that the hydrophone receiving sensitivities $S_m(f)$ are essentially constant over the system passband: this model is a reasonably accurate representation based on prior hydrophone calibrations. This reduced the complexity of the current problem to that of determining one value per channel.

The sensitivity for the channel 5 hydrophone is known from an in-water acoustic calibration performed at USRD/NRL in Orlando [17]. The gain of channel 5, A_5 , is known from system integration tests and is presented in Appendix E. Therefore, the one-sided acoustic pressure autospectrum is

$$G_{PP} = \frac{G_{5,5}}{|H_5|^2} = \frac{G_{5,5}}{|A_5|^2 |S_5|^2} \quad (\text{F.3})$$

If the one-sided output autospectra $G_{m,m}$ were known, then Eqs(F.2) and (F.3) could be combined to yield transfer function magnitudes for the remaining channels:

$$|H_m|^2 = \frac{G_{PP}}{G_{m,m}} \quad (\text{F.4})$$

In practice, the one-sided output autospectra $G_{m,m}$ are unknown and must be replaced by their estimates.

These relations can also be manipulated to yield channel-to-channel phase mismatch. Unfortunately, past experience has shown that *in situ* calibration techniques based on the ambient sound field are too inaccurate to provide cross-channel phase calibrations that are statistically different from zero.

F.3 Sound Fields Near the Surface

In order to gauge the effect of the proximity to the surface, we consider the geometry of figure F.2. A sensor is located in a semi-infinite isovelocity halfspace a distance z below the plane $z = 0$ where Dirichlet boundary conditions hold. Sources are located

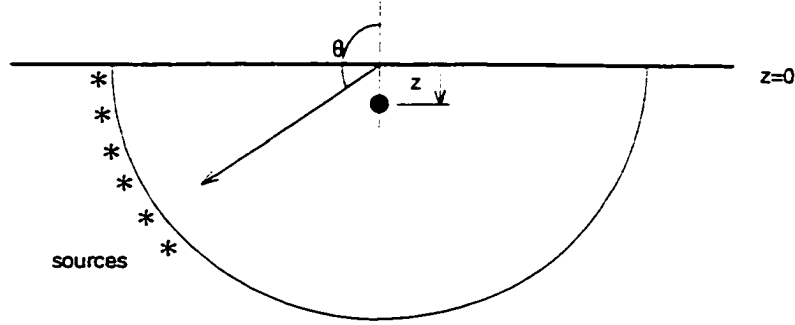


FIGURE F.2: Geometry for near-surface sound field calculations. All sources are far from the sensor.

on an imaginary hemispherical surface far enough from the sensor that their radiated acoustic fields are essentially planar in the vicinity of the sensor. The sound speed is c .

A source excitation $s_1(t, \phi, \theta)$ will therefore be associated with a plane wave with wavenumber vector $\mathbf{k} = -[k \cos \phi \sin \theta, k \sin \phi \sin \theta, k \cos \theta]^T$, where $k = \omega/c$. The situation near the sensor is shown in figure F.3. The direct path supplies a contribution $s_1(t)$ and the boundary will give rise to a secondary contribution $s_2 = -s_1(t - \tau)$. The Fourier transforms of these fields are S_1 and $S_2 = -e^{-i\omega\tau} S_1$. From figure F.3, the phase delay $\omega\tau$ is $2kz \cos \theta \equiv 2\xi(\theta)$. Therefore, $S_2 = -e^{-2i\xi(\theta)} S_1$.

The total pressure at the sensor is the sum of all contributions over the entire hemispherical surface,

$$p = \iint_{\text{lower hemisphere}} S(\phi, \theta) (1 - e^{-2i\xi(\theta)}) d\Omega,$$

and the autospectral density at the sensor is then

$$\langle |p|^2 \rangle = \iiint \iint \langle S(\phi, \theta) S^*(\phi', \theta') \rangle (1 - e^{-2i\xi(\theta)}) (1 - e^{2i\xi(\theta')}) d\Omega d\Omega'. \quad (\text{F.5})$$

If we assume that all the distant sources are mutually uncorrelated, then

$$\langle S(\phi, \theta) S^*(\phi', \theta') \rangle = N(\phi, \theta) \delta(\phi - \phi') \delta(\cos \theta - \cos \theta'). \quad (\text{F.6})$$

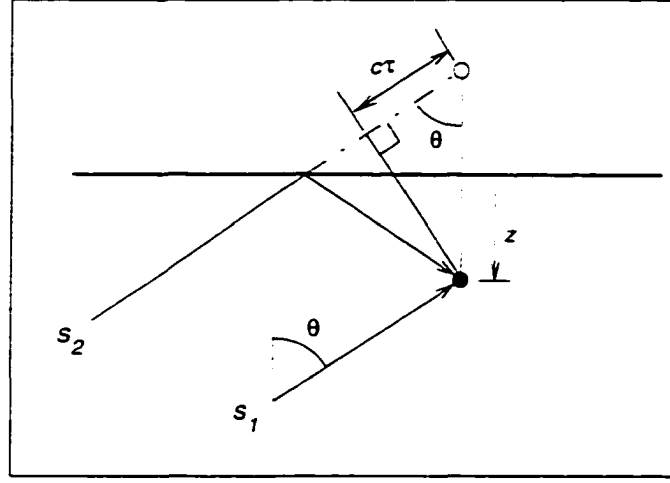


FIGURE F.3: Reflection geometry for a nearsurface sensor. The direct path contribution s_1 and the surface bounce path contribution s_2 are due to a distant source s .

where $N(\phi, \theta)$ is the angular spectral density of the incident plane waves. Inserting Eq.(F.6) into Eq.(F.5) and integrating out the terms in ϕ' and θ' leaves

$$\begin{aligned}
 \langle |p|^2 \rangle &= \iint_{\text{lower hemisphere}} N(\phi, \theta) \left(1 - e^{-2i\xi(\theta)}\right) \left(1 - e^{2i\xi(\theta)}\right) d\Omega \\
 &= 4 \iint_{\text{lower hemisphere}} N(\phi, \theta) \sin^2(kz \cos \theta) d\Omega \\
 &= S_{PP}(kz),
 \end{aligned} \tag{F.7}$$

that is, a function of kz .

There are three possibilities:

1. $kz \gg 1$: this corresponds to a deep sensor, and the model is no longer appropriate because (surface) sources will begin to contribute from the upper hemisphere:
2. $kz \ll 1$: the term $\sin(kz \cos \theta) \approx kz \cos \theta$ so that

$$\begin{aligned}
 S_{PP} &= 4 \iint N(\phi, \theta) (kz)^2 \cos^2 \theta d\Omega \\
 &= 4(kz)^2 \iint N(\phi, \theta) \cos^2 \theta \sin \theta d\theta d\phi.
 \end{aligned}$$

Four features are immediately evident:

- $S_{PP}(kz) \rightarrow 0$ as $k \rightarrow 0$ (i.e., at low frequencies):
- $S_{PP}(kz) \rightarrow 0$ as $z \rightarrow 0$ (i.e., as the sensor approaches the Dirichlet boundary):
- Sources near $\theta = \pi/2$ do not contribute (along the null of the source dipole):
- Sources near $\theta = 0$ do not contribute (due to destructive interference at the sensor).

3. $kz \approx 1$: This is the configuration that involves ULSAS during SURFEX III: the range of applicable kz is $(3/2, 24)$. This case is therefore examined more carefully below.

In order to proceed, we need the angular spectral density $N(\phi, \theta)$. This is unknown, but the representative features of the problem can be demonstrated by assuming the isotropic hemisphere

$$N(\phi, \theta) = 1. \quad (\text{F.8})$$

Substituting Eq.(F.8) into Eq.(F.7) leaves

$$S_{PP} = 4 \int \int_{\text{lower hemisphere}} \sin^2(kz \cos \theta) \sin \theta \, d\theta \, d\phi.$$

This can be evaluated analytically to yield

$$S_{PP} = \frac{1}{2} \left(1 - j_0(2kz) \right) \quad (\text{F.9})$$

where $j_0(x)$ is the spherical Bessel junction of order zero. A plot of Eq.(F.9) is shown in figure F.4 for $c = 1485 \text{ ms}^{-1}$: the autospectral level varies considerably quite near the surface but levels out with increasing frequency, reaching the asymptotic value of $1/2$, or -3 dB , for $kz \rightarrow \infty$.

Although the true angular spectral density is not known, this problem suggests that, for a given span of sensor depths, there is less error in assuming spatial homogeneity

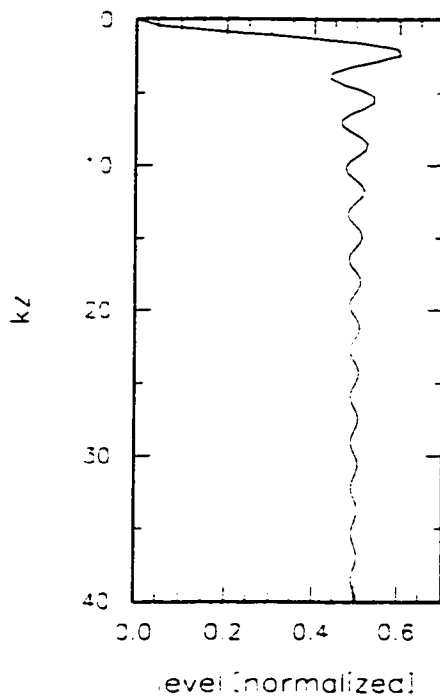


FIGURE F.4: Variation of autospectral level with depth.

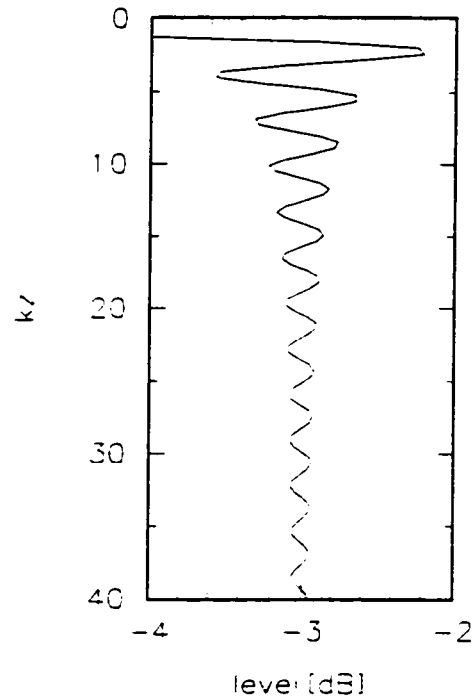


FIGURE F.5: Variation of autospectral level with depth, dB.

at higher frequencies than at lower frequencies. During windy events, the sensors of the ULSAS array might easily span depths of -2 metres to -4 metres. Based on the isotropic hemispherical noise model, table F.1 shows that at the lower end of the system passband, at 603 Hz, the autospectral level can vary from about -3.2 dB to -2.7 dB, a difference of almost 0.5 dB. At the upper end of the system passband, at 1895 Hz, the difference is at most 0.15 dB.

This example shows that the sound field is not spatially homogeneous near the surface. However, we can also infer that, even though we do not know the true angular spectral density, the error in assuming spatial homogeneity is only of the order of a tenth of a decibel at the upper end of the system passband. This error is not overwhelming compared to other measurement inaccuracies in the system, and so it is not

freq	depth [m]		
	-2	-3	-4
603	-2.72	-3.12	-3.23
1895	-3.09	-2.94	-3.08

TABLE F.1: Autospectral levels [dB] versus depth.

unreasonable to pretend that the sound field is indeed homogeneous at some upper passband frequency and carry out the *in situ* calibration technique *at that frequency* using ambient sound.

F.4 Estimating the Output Autospectra

The application of the calibration equations Eqs.(F.4) and (F.3) requires estimating the autospectral densities at 1895 Hz. This would normally be a straightforward task given a sample of background ambient sound. Unfortunately, it was not possible to find a sample long enough to produce a estimate with a sufficiently small sample variance that was not also contaminated by breaking events in the nearfield of the array. The available data are in fact a mixture of background ambient sound and a nonstationary bursty noise process due to the nearfield breaking events. The bursty noise can seriously bias the spectral estimators, so we would like a technique that could distinguish and eliminate its influence.

We assume a processing scheme that partitions the data into a series of M disjoint data blocks, each of size N , that are windowed and then projected onto a complex exponential corresponding to radian frequency ω . This produces two Fourier coefficients which are then squared, summed and scaled to produce one autospectral level datum, say $|Q_i(\omega)|^2$, in units of bits^2/Hz .

The background ambient sound is, by definition, due essentially to a large number of *distant* sources: by the Central Limit Theorem, it should have a Gaussian distribu-

tion. Were it to be acquired alone, without any contamination, the Fourier coefficients $|Q_i(\omega)|^2$ would be distributed as $G_{i,i}(\omega)\chi_2^2$: we desire to estimate $G_{i,i}(\omega)$.

The actual distribution of the mixture data $|Q_i(\omega)|^2$ is therefore $G_{i,i}\chi_2^2 + \text{"other"}$. Since the bursty events introduce large levels, the effects of the contaminating process are largely confined to the upper tail of the χ^2 distribution. Thus, a robust way of minimizing the influence of the bursty noise is to consider only the data near the mode of the empirical distribution.

It will be convenient to transform the Fourier coefficients to the decibel scale according to the monotone transform

$$s = 10 \log_{10} |Q_i(\omega)|^2 :$$

this transforms a $\sigma^2\chi^2$ distribution to the *dB distribution* (Dyer, [39]) which has the probability density function (PDF)

$$f_s(\xi) = \frac{1}{\alpha} \frac{\alpha}{\sigma^2} e^{\alpha\xi} e^{-e^{\alpha\xi}/2\sigma^2}$$

where $\alpha = 0.1 \ln 10$.

An empirical PDF could be constructed from the resulting sequence of decibel values using a simple histogram and we could attempt to estimate $\sigma^2 = G_{i,i}$ by fitting a model PDF to the histogram. Manipulations involving histograms, however, invariably require the subjective choice of bin widths, a difficulty avoided entirely by using cumulative distribution functions (CDF).

The dB distribution CDF is

$$F_s(\xi) = 1 - e^{-e^{\alpha\xi}/2\sigma^2} = 1 - e^{-e^{\alpha(\xi-\mu)}/2}$$

where $\mu = 10 \log_{10} \sigma^2$. This form shows that the CDF is located solely by the parameter μ .

The procedure is then to build the empirical CDF $\hat{F}(\xi_j)$ on a discretized grid on j over a fit region that included the lower tail and the mode but excluded the upper tail.

An additional nuisance parameter β is required because the normalization of the empirical CDF involves the nonstationary contributions. The estimator $\hat{\mu}^2 = 10 \log_{10}(\hat{G}_{i,i})$ is then

$$\arg \min_{\mu, \beta} \sum_i \left| \hat{F}(\xi_i) - \beta F(\xi_i | \mu) \right|^2. \quad (\text{F.10})$$

Eq.(F.10) poses a two-parameter nonlinear least squares problem which can be minimized using, for example, the Matlab command `fminu()`. This procedure must be applied to each channel to generate a robust estimate of the output autospectral density for use in Eqs.(F.4) and (F.3).

F.5 Calibration Processing

In order to apply this *in situ* technique, we had to select a desirable data set. Desirable calibration data sets required moderately windy conditions without excessive wave motion. Quieter conditions allowed distant shipping and internal noise to dominate the data, neither of which were deemed as useful for calibration purposes. More energetic conditions introduced more nearfield breaking events and greater mooring noise, either of which further contaminated the data. Tape 7 was selected because it was acquired near midnight, when local shipping traffic was low, and because there was a light wind generating moderately-sized but fairly infrequent breakers. A 40-second excerpt was extracted starting at 01:01:55 on 04/09/95. The data was partitioned into $M = 3445$ blocks and then windowed with the zeroth order $N = 64$ -point digital prolate spheroidal sequence with time-bandwidth product $NW = 3$. The windowed data were then processed at the Fourier coefficient corresponding to $f = 1895$ Hz and the squared and scaled Fourier coefficients transformed into decibels.

A histogram of the resulting power data from channel 5 is shown in figure F.6. The histogram clearly shows that the data has a dominant contribution obeying the underlying χ^2 distribution: furthermore, the influence of the nonstationary nearfield sources is confined to the upper tail, demonstrated by the large uppermost bin.

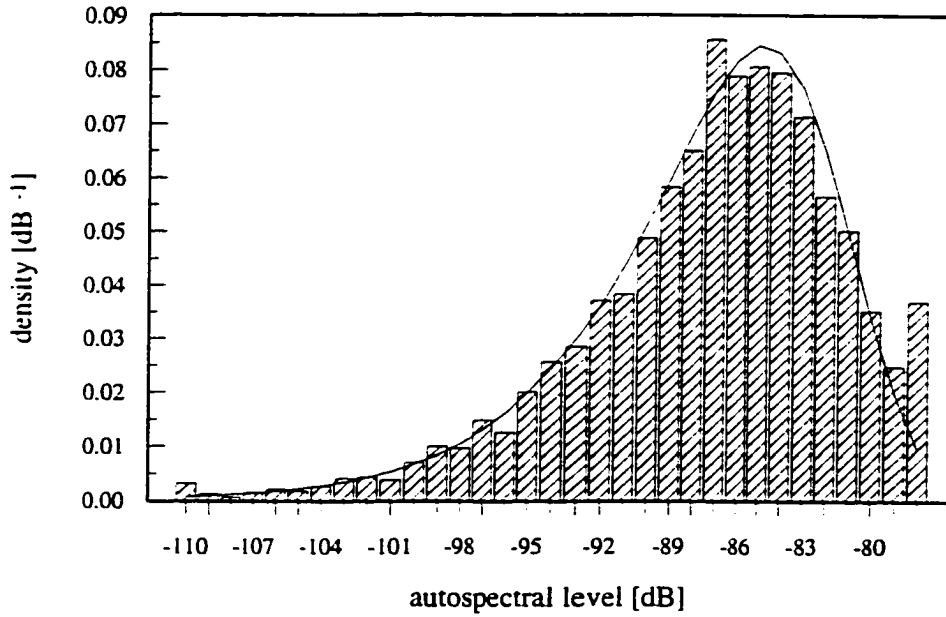


FIGURE F.6: Histogram of channel 5 output power levels. The levels have been scaled to units of dB re $1 \text{ V}^2/\text{Hz}$. The best fit dB distribution PDF is superimposed for comparison.

These data were then processed to produce an empirical CDF, and the two model CDF parameters μ and β estimated via the nonlinear least-squares prescription of Eq.(F.10) using a tolerance of 1×10^{-7} on either parameter. The best-fit PDF for these data is shown superimposed on the histogram of figure F.6.

The output autospectral densities for the remaining channels were calculated in a similar manner, and the calibration correction computed according to Eqs.(F.4) and (F.3). The results are presented in table F.2, showing the estimated output autospectral density, the corresponding gain of the channel, and the gain in dB. The autospectral levels have been scaled to units of V^2/Hz using the known A/DC scaling. The reference channel was channel 5. Table F.2 reveals that $|H_i(f)|$ varied by almost 4 dB over the array.

channel	level [dB re 1 V ² /Hz]	gain [bits/ μ Pa]	gain [dB re 1 bit/ μ Pa]
1	-86.9	1.19×10^{-3}	-58.5
2	-84.9	1.50	-56.5
3	-84.4	1.60	-55.9
4	-85.9	1.34	-57.5
5	-87.9	1.07	-59.5
6	-84.6	1.55	-56.2
7	-84.8	1.53	-56.3
8	-84.7	1.55	-56.2
9	-85.8	1.37	-57.4
10	-88.4	1.01	-59.9
11	-88.0	1.05	-59.6
12	-87.7	1.09	-59.3
13	-87.6	1.10	-59.2
14	-88.2	1.02	-59.8
15	-87.5	1.11	-59.1

TABLE F.2: Calibration values for each channel, computed at $f = 1895\text{Hz}$. The second column is $10 \log_{10} \hat{G}_{i,i}(f)$, the estimated output autospectral density in decibels. The third column is $|\hat{H}_i(f)|$. The fourth column is $20 \log_{10} |\hat{H}_i(f)|$.

Appendix G

Hydrophone Mounts: Vibration Transmissibility

The elastic suspension and hydrophone mass in each hydrophone mount form a compliant three-dimensional mechanical mass-spring system which will couple structural vibrations into the hydrophone transducer. Such a system will generally have one or possibly several resonant modes of vibration. In order to minimize the transfer of vibration energy into the acoustic data, it is therefore important to design the frequency-dependent transfer function (also known as the *transmissibility function* [11]) of these mounts so that the “eigenfrequencies” of these modes do not occur in the system pass-band.

The mount mechanical system can be analyzed in the small-amplitude approximation using the linearized theory of small oscillations [48]. A typical mount is shown in figure G.1: a simplified model is shown in figure G.2. The hydrophone is held at the center of a “collar” of radius R by a two-dimensional mesh of elastic cords. Each elastic cord has spring constant κ , and an unstretched length of l_0 , and the hydrophone has radius a and virtual mass M . The virtual mass is equal to the *in vacuo* mass M_H plus an “added mass” due to the mass of the surrounding fluid which must be displaced by the hydrophone as it moves. Milne-Thompson ([98], §9.221) suggests an added mass of

ρV_H where ρ is the density of the fluid and V_H is the volume of the hydrophone.

The Cartesian coordinates (x, y, z) form suitable generalized coordinates, with the hydrophone equilibrium position being $(0, 0, 0)$. The equations of motion are therefore determined by Lagrange's equation

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = - \frac{\partial V}{\partial q} \quad (\text{G.1})$$

where q represents any one of $\{x, y, z\}$, T is the kinetic energy and V is the potential energy. The kinetic energy for this system is obviously

$$\begin{aligned} T &= \frac{1}{2} M (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \\ &= \frac{1}{2} M \sum_{\sigma} \dot{q}_{\sigma}^2 \\ &= \frac{1}{2} \sum_{\sigma} \sum_{\lambda} M \delta_{\sigma\lambda} \dot{q}_{\sigma}^2 \end{aligned} \quad (\text{G.2})$$

where $\delta_{\sigma\lambda}$ is the Kronecker delta and the last term will be useful further on.

The three-dimensional potential energy of the simplified system is

$$V = \frac{1}{2} \kappa [(l_1 - l_0)^2 + (l_2 - l_0)^2 + (l_3 - l_0)^2 + (l_4 - l_0)^2] \quad (\text{G.3})$$

where

$$\begin{aligned} l_1 &= \sqrt{(R - a - x)^2 + y^2 + z^2}, \\ l_2 &= \sqrt{x^2 + (R - a - y)^2 + z^2}, \\ l_3 &= \sqrt{(R - a + x)^2 + y^2 + z^2}, \\ l_4 &= \sqrt{x^2 + (R - a + y)^2 + z^2}. \end{aligned}$$

To use this in Lagrange's equation, the potential is expressed as a Taylor series expansion around equilibrium which through second-order is

$$V(x, y, z) \approx V(0, 0, 0) + \sum_{\sigma} q_{\sigma} \left(\frac{\partial V}{\partial q} \right)_{(0,0,0)} + \frac{1}{2} \sum_{\sigma} \sum_{\lambda} q_{\sigma} q_{\lambda} \left(\frac{\partial^2 V}{\partial q_{\sigma} \partial q_{\lambda}} \right)_{(0,0,0)}.$$

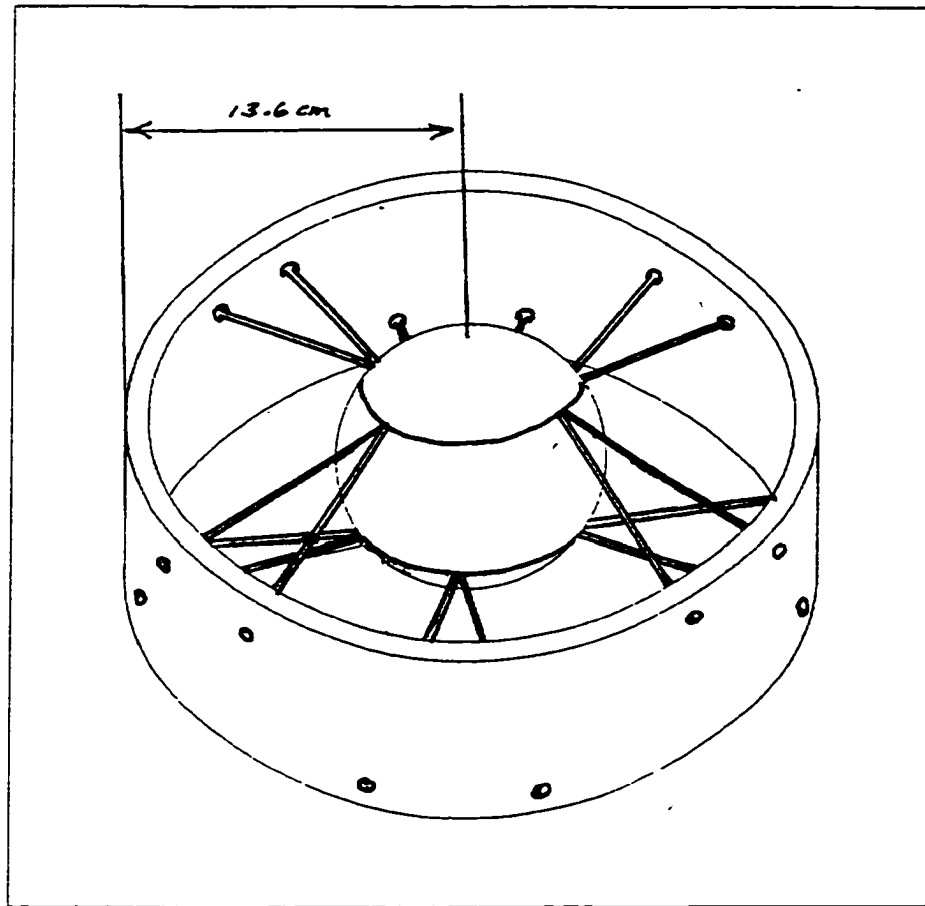


FIGURE G.1: ULSAS array hydrophone mount: the hydrophone was “tied in” to a complicated 3-dimensional web of elastic cords. Drawn to scale for the ITC1001 hydrophones: the ITC4127 hydrophones are slightly smaller.

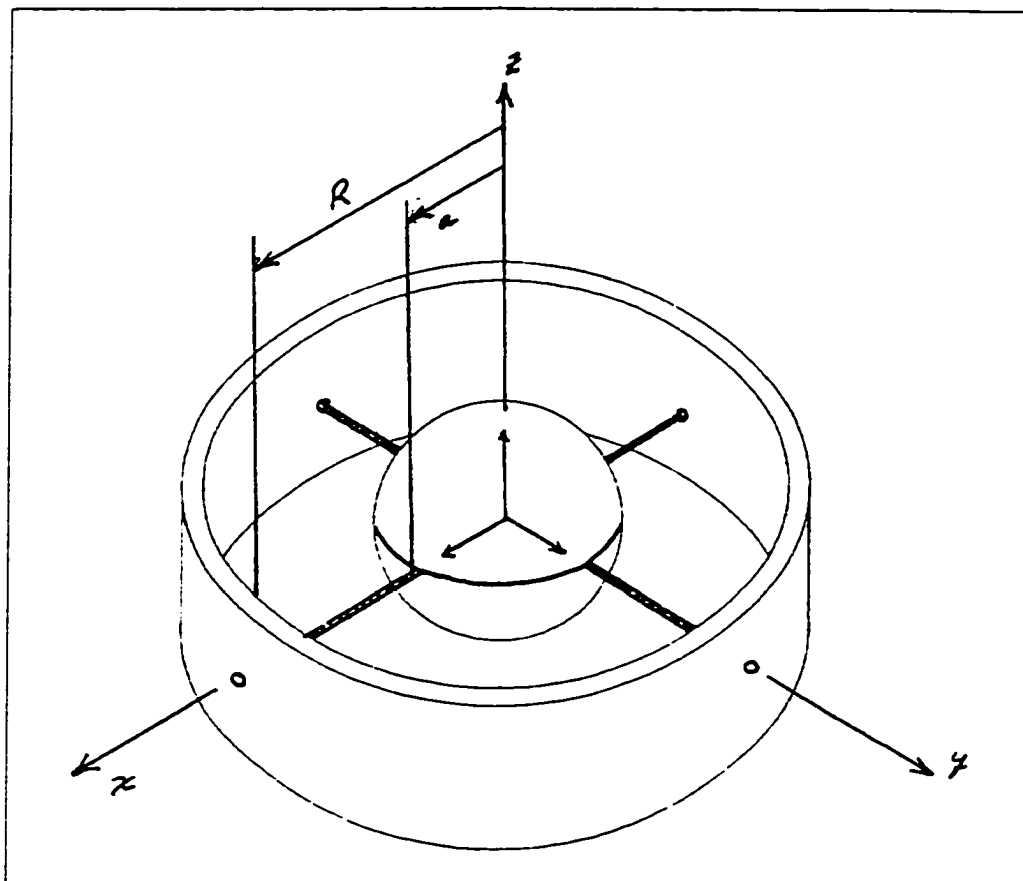


FIGURE G.2: Simplified hydrophone mount for the analysis of this appendix: a is the hydrophone radius and R is the "collar" radius. The hydrophone equilibrium position is $(0,0,0)$.

By definition, the term $\partial V/\partial q$ is zero at equilibrium, so the expansion reduces to

$$\begin{aligned} V(x, y, z) &\approx V(0, 0, 0) + \frac{1}{2} \sum_{\sigma} \sum_{\lambda} q_{\sigma} q_{\lambda} \left(\frac{\partial^2 V}{\partial q_{\sigma} \partial q_{\lambda}} \right)_{(0,0,0)} \\ &= V(0, 0, 0) + \frac{1}{2} \sum_{\sigma} \sum_{\lambda} v_{\sigma\lambda} q_{\sigma} q_{\lambda} \end{aligned} \quad (\text{G.4})$$

where

$$v_{\sigma\lambda} = \left(\frac{\partial^2 V}{\partial q_{\sigma} \partial q_{\lambda}} \right)_{(0,0,0)}. \quad (\text{G.5})$$

Substituting Eqs. (G.2) and (G.4) for the kinetic and potential energy into Lagrange's equation, Eq.(G.1), yields the equations of motion

$$\sum_{\lambda} (M \delta_{\sigma\lambda} \ddot{q}_{\lambda} + v_{\sigma\lambda} q_{\lambda}) = 0 \quad (\sigma = 1, 2, 3). \quad (\text{G.6})$$

These equations of motion can be written more concisely in matrix notation using $\mathbf{q} = [x, y, z]^T$. Then Eq.(G.6) becomes

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{V} \mathbf{q} = \mathbf{0} \quad (\text{G.7})$$

where the elements of $\mathbf{V}$ are given in Eq.(G.5) and $\mathbf{M}$ is called the *mass matrix*.

For small oscillations, $\mathbf{q}$ can be written as $\mathbf{p} e^{i\omega t}$ where $\mathbf{p}$ represents the amplitudes. Then Eq.(G.7) becomes

$$(\mathbf{V} - \omega^2 \mathbf{M}) \mathbf{p} = \mathbf{0} \quad (\text{G.8})$$

which is a generalized eigenvalue equation for the normal modes of the mass-spring system.

To investigate the eigenmodes, both matrices $\mathbf{V}$ and $\mathbf{M}$ need to be determined. The mass matrix from Eq.(G.2) is

$$\mathbf{M} = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix}. \quad (\text{G.9})$$

The elements in $\mathbf{V}$ are, after some extensive calculations,

$$\mathbf{V} = \begin{bmatrix} \frac{2\kappa}{R-a}(2R-2a-l_0) & 0 & 0 \\ 0 & \frac{2\kappa}{R-a}(2R-2a-l_0) & 0 \\ 0 & 0 & \frac{4\kappa}{R-a}(R-a-l_0) \end{bmatrix}.$$

Thus, the cross-terms in $\mathbf{V}$ are zero, and the equations of motion in the x , y and z directions decouple. There are three eigenmodes, but the modes along x and y have the same eigenfrequencies: that for z is different. By inspection, these eigenfrequencies are:

$$\omega_{x,y} = \sqrt{\frac{2\kappa}{M(R-a)}(2R-2a-l_0)},$$

$$\omega_z = \sqrt{\frac{4\kappa}{M(R-a)}(R-a-l_0)}.$$

The real mount, figure G.1, uses four elastic cords at each “corner” instead of one as used in the simplified model, so the actual eigenfrequencies will be instead

$$\omega_{x,y} = \sqrt{\frac{8\kappa}{M(R-a)}(2R-2a-l_0)}, \quad (\text{G.10})$$

$$\omega_z = \sqrt{\frac{16\kappa}{M(R-a)}(R-a-l_0)}. \quad (\text{G.11})$$

The masses and radii for these hydrophones are shown in table G.1. The spring constant was measured in a separate experiment to be 92.8 N/m, the unstretched length of elastic was 0.054 m, and the radius of the collar was 0.127 m. The resulting eigenfrequencies for the two types of hydrophones used are shown in table G.2.

It is educational to model the mounts as mass-spring systems and determine the frequency response of the system to externally supplied (i.e., structure-borne) vibration. Since the system can be decoupled into x , y and z modes, the analysis requires only a 1-D model. Consider the simplified system model of figure G.3. Structure-borne vibration forces the mount collar, modeled here as a plate initially at $x = 0$, to execute some displacement $x_1(t)$. The collar is connected via the elastic suspension, modeled

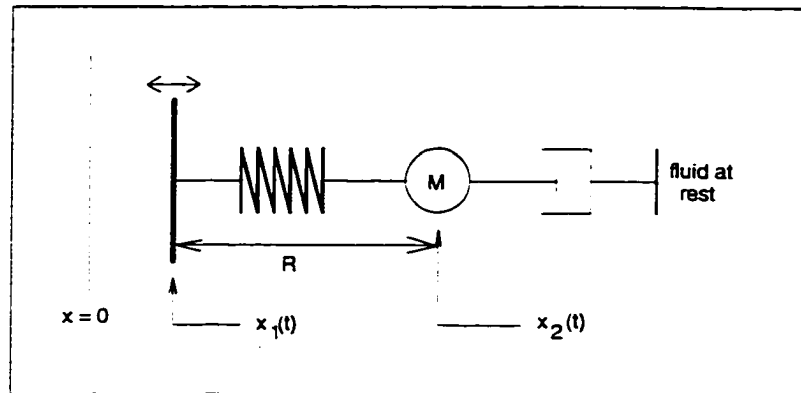


FIGURE G.3: Simplified 1-D spring-mass-dashpot mount model.

here as a spring with “effective” spring constant K_{eff} , to the hydrophone with mass M , which will respond to the forcing with displacement $x_2(t)$. The hydrophone will experience a drag force as it moves through the surrounding fluid (considered to be at rest): at the small velocities expected, this drag force can be approximated by Stokes’s law of resistance [72] to be

$$F_D = -6\pi\mu a\dot{x}_2 \quad (G.12)$$

where μ is the dynamic viscosity. This drag is modeled in figure G.3 as a dashpot.

hydrophone	mass [kg]	radius [m]
ITC1001	1.23	0.054
ITC4127	0.33	0.032

TABLE G.1: Masses and radii of ULSAS Hydrophones

hydrophone	(x, y) eigenfrequencies [Hz]	z eigenfrequency [Hz]
ITC1001	3.5	2.2
ITC4127	6.6	5.0

TABLE G.2: Mount Eigenfrequencies

The mass therefore obeys the differential equation

$$M\ddot{x}_2 = K_{eff} (R - (x_2 - x_1)) - 6\pi\mu a\dot{x}_2$$

or

$$M\ddot{x}_2 + K_{eff}(x_2 - R) + 6\pi\mu a\dot{x}_2 = K_{eff}x_1. \quad (G.13)$$

The quantity of interest here is not the absolute position $x_2(t)$ of the hydrophone, but its relative displacement, $q(t) = x_2(t) - R$. Then $\dot{x}_2 = \dot{q}$ and $\ddot{x}_2 = \ddot{q}$. Then Eq.(G.13) becomes

$$M\ddot{q}_2 + K_{eff}q + 6\pi\mu a\dot{q}_2 = K_{eff}x_1.$$

Taking Fourier transforms of both sides yields

$$[-\omega^2 M + K_{eff} + i\omega 6\pi\mu a] Q(\omega) = K_{eff} X_1(\omega).$$

The displacement frequency response is then

$$H(\omega) = \frac{Q(\omega)}{X_1(\omega)} = \frac{1}{1 + i\omega 6\pi\mu a / K_{eff} - \omega^2 M / K_{eff}}.$$

The “effective” spring constant K_{eff} is related to the eigenfrequencies of table G.2 by $K_{eff} = \omega_0^2 M$ and includes the effects of all contributing elastic cords for each component. The resulting transmissibility functions for these mounts are shown in figures G.4 and G.5 for the ITC1001 and ITC4127 hydrophones, respectively.

Even with compliant mounts with out-of-band resonances, resonant oscillations are nevertheless seen throughout the data: a typical example is shown in figure G.6. The frequency of these oscillations is approximately 38 Hz, so this “ringing” is not likely to be the primary eigenmodes of the hydrophone mounts. There are several candidate mechanisms that might have produced these vibrations:

- flow-induced vibrations in the tensioned strain member between the array and the electronics package has been shown [2] to produce vibrations around 40 Hz, an effect seen also in acoustic data from other IOS instruments;

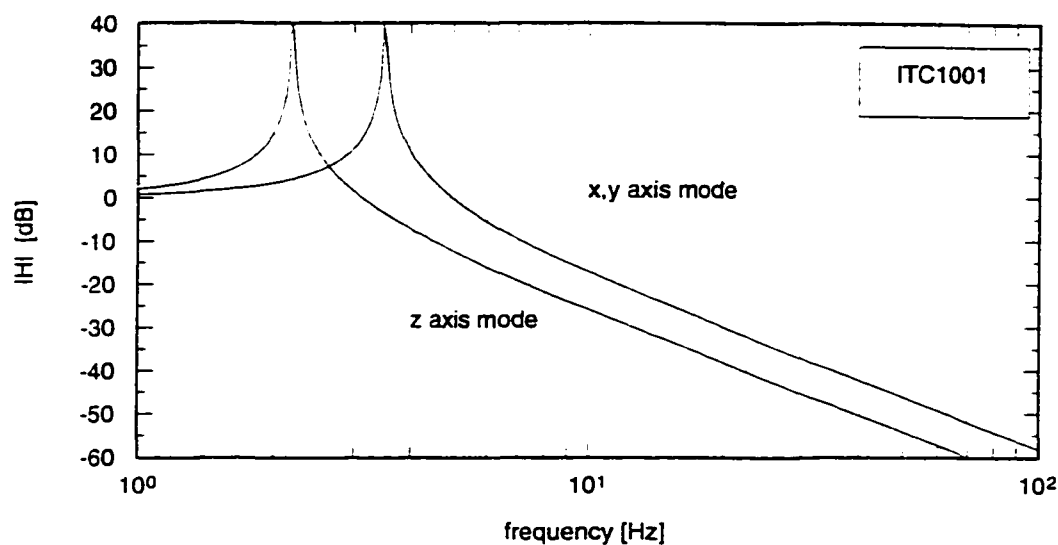


FIGURE G.4: Transmissibility functions for the ITC1001 for horizontal (x and y axes) and vertical (z axis) vibrations.

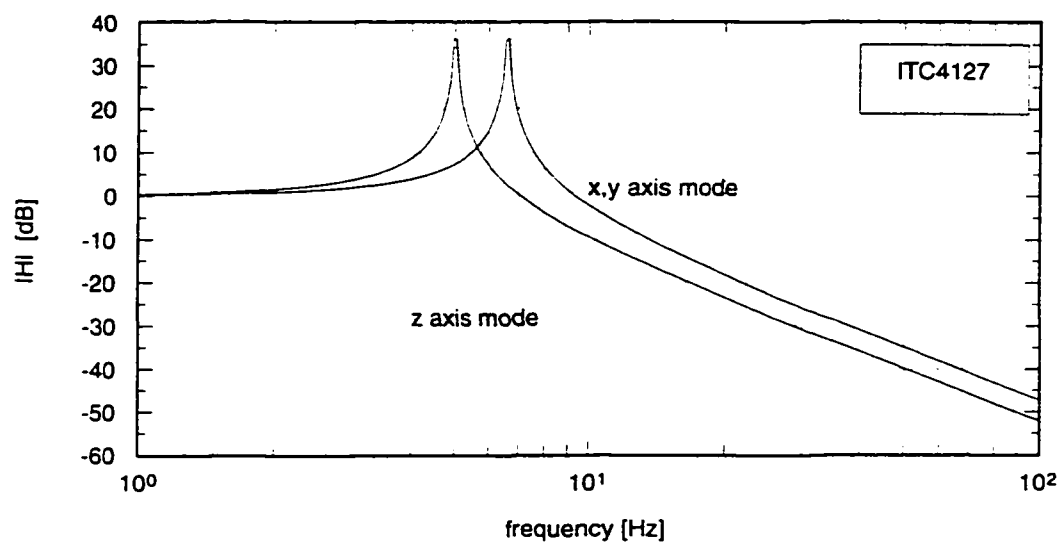


FIGURE G.5: Transmissibility functions for the ITC4127 for horizontal (x and y axes) and vertical (z axis) vibrations.

- halibut poles affixed to the endplates of the array arms may also have conducted strum-induced vibration from their above-surface lengths, where they were subjected to direct wind forcing, down into the array arms;
- vigorous forcing by wave action on the positively buoyant array, which was constrained by the strain member from below, may have induced bending mode vibrations into the array arms.

Regardless of the source mechanism, figures G.4 and G.5 show that vibrational energy at or above 40 Hz was attenuated by at least 25 dB.

In conclusion, using a Lagrangian formulation and the linearized theory of small amplitude oscillations, the ULSAS elastic hydrophone mounts have been shown to have three decoupled eigenmodes with eigenfrequencies below about 10 Hz. Since the ULSAS system passband is [160 Hz, 500 Hz], these resonances in the mount systems are well below the passband. Using a simple 1-D spring-mass-dashpot model, further calculations show that the mounts provide at least 25 dB attenuation at the typical strum frequency of about 40 Hz.

While the effects of structure-borne vibrations on the hydrophones were not, of course, *completely* eliminated by the ULSAS elastic mounts, the mounts appear to have successfully attenuated contaminating vibrational energy arising elsewhere on the array

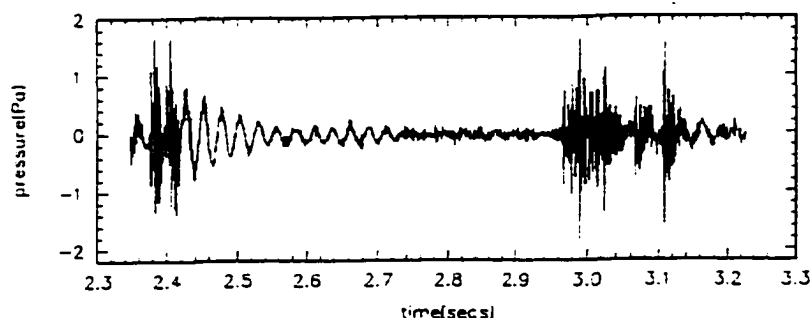


FIGURE G.6: Resonant “ringing” in the data

*APPENDIX G. HYDROPHONE MOUNTS: VIBRATION TRANSMISSIBILITY*²⁴³

structure. This prevented saturation of the preamplifiers from acceleration-induced signals and thereby maintained the linearity of the signal conditioning electronics and the quality of the acquired data. The remaining effects were easily removed from the data using a high-order Butterworth high pass filter.

Appendix H

Hydrophone Mounts: Diffraction

H.1 Introduction

Any structure, frame member or support around the hydrophone will diffract or scatter an incident acoustic field and generate a secondary signal at the hydrophone. This secondary signal can be represented as noise and may be coherent with the incident field, a feature of some consequence in beamforming systems. Of primary importance is the hydrophone mount itself, as it is the closest structural element to the hydrophone.

The mount itself will generate two types of fields. The first is a simple scattered field due to the impedance mismatch at the mount surface between the surrounding fluid and the mount material. This depends primarily on the 3-D geometry of the mount. The second arises as the incident field induces elastic waves in the mount itself. This depends both on the 3-D geometry and the mechanical properties of the mount material.

The ideal approach to mount design would involve both the mount geometry and the material properties: in principle, these determine the elastic response of the structure and the boundary conditions between the fluid and the solid. Thus, by varying the mount material and geometry, a design could be chosen that minimizes the scattered signal.

While the mathematics of the interactions are relatively simple, the consideration of irregular geometries produces problems that are, in general, far too complicated to be attacked analytically. In practice, the only practical alternative is to apply a *computational mechanics* approach. The 3-D elastic waves in the mount can be determined using the finite-element method (FEM), and the surface pressure and normal velocity on the mount surface, which give rise the scattered signal, can be propagated to the hydrophone location using the boundary-element method (BEM). Since the fluid loading on the surface modifies the elastic waves within the structure, the problem involves a coupling between the interior and exterior fields (Mathews and Hitchings, [91].)

This comprehensive approach was deemed unnecessary for the essential reason that available mount materials were extremely limited. Thus, for the design of the mount, PVC was selected for two reasons. First, its specific acoustic impedance ρc is close to that of water. The density of PVC is listed as $\rho = 1300 - 1580 \text{ kg/m}^3$ and its elastic modulus $E = 2.4 - 4.1 \times 10^9 \text{ Pa}$ ([58].) Therefore, using the means of these values, its bulk sound speed would be $c = \sqrt{E/\rho} = 1497 \text{ m/s}$. Its specific acoustic impedance is therefore $\approx 2.2 \times 10^6 \text{ rayls}$, which compares to that of sea water, $\rho_0 c_0 \approx 1.54 \times 10^6 \text{ rayls}$. At least for a fluid-fluid boundary, as the ratio $\rho c / \rho_0 c_0 \rightarrow 1$, the scattered wave amplitude should go to zero (because the impedance mismatch vanishes.) Here, this ratio is 1.4. Second, PVC is easily machined, permitting some latitude in choice of mount geometry.

PVC, however, is not a completely rigid material and will support elastic waves, so a fully coupled FEM-BEM analysis would still be required for a comprehensive design. A simpler approach was adopted here: the material was considered to be a perfectly rigid “hard scatterer”. This simplification reduces the calculation of the diffracted signal to a standard application of BEM, as explained in the next section.

H.2 BEM

The time-harmonic acoustic field $p(\mathbf{r})$ (with $e^{-i\omega t}$ suppressed) at some field point $\mathbf{r}$ due to an incident field $p_i(\mathbf{r})$ scattered from some 3-D surface S is given by the Helmholtz-Kirchhoff integral (Baker and Copson, [6], chapter 1. Clay and Medwin, [24], appendix A.10):

$$p(\mathbf{r}) = p_i(\mathbf{r}) + \int_S (p(\mathbf{r}') \nabla' g(\mathbf{r}|\mathbf{r}') - g(\mathbf{r}|\mathbf{r}') \nabla' p(\mathbf{r}')) \cdot d\mathbf{S}' \quad (\text{H.1})$$

where $g(\mathbf{r}|\mathbf{r}')$ is the 3-D free-space Green function

$$g(\mathbf{r}|\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}. \quad (\text{H.2})$$

and $k = \omega/c$. The surface normal here is considered to point away from the scatterer.

As suggested by Eq.(H.1), the total field pressure consists of contributions from the incident field and the scattered field. We are interested in the latter. Similar to Eq.(H.1), the scattered field $p_s(\mathbf{r})$ is

$$p_s(\mathbf{r}) = \int_S (p_s(\mathbf{r}') \nabla' g(\mathbf{r}|\mathbf{r}') - g(\mathbf{r}|\mathbf{r}') \nabla' p_s(\mathbf{r}')) \cdot d\mathbf{S}'. \quad (\text{H.3})$$

This has the form of a radiation problem: if the scattered pressure and its normal gradient were known on the surface of the scatterer, the pressure at any field point could be computed via Eq.(H.3).

The boundary condition for a hard scatterer specifies that the total fluid velocity normal the surface be zero. This normal fluid velocity consists of a contribution due to the incident field and a second component, which gives rise to the scattered acoustic field. This second component must be exactly opposite that due to the incident field in order for their sum to be zero.

Using the linearized Euler equation for time-harmonic fields,

$$\mathbf{v}_i = \frac{1}{i\omega\rho_0} \nabla p_i = -\mathbf{v}_s = -\frac{1}{i\omega\rho_0} \nabla p_s, \quad (\text{H.4})$$

that is, the “incident” fluid velocity is proportional to the negative of the “scattered” fluid velocity. Thus, if the incident field p_i is specified, the scattered fluid velocity, and hence the gradient of the scattered pressure, is known.

The scattered surface pressure is not, however, independent of the normal component of its gradient on the surface. If the field point $\mathbf{r}$ is taken to the surface $\mathcal{S}$ in Eq.(H.3), the following *surface boundary integral equation* (SBIE) results for the scattered surface pressure p_s (Brebbia and Dominguez [19], chapter 2):

$$\frac{1}{2}p_s(\mathbf{r}) = \int_{\mathcal{S}} (p_s(\mathbf{r}')\nabla'g(\mathbf{r}|\mathbf{r}') - g(\mathbf{r}|\mathbf{r}')\nabla'p_s(\mathbf{r}')) \cdot d\mathbf{S}' \quad \mathbf{r} \in \mathcal{S}. \quad (\text{H.5})$$

This is a Fredholm integral equation of the second kind (Tricomi [133], section 1.2) for the surface pressure p_s given its normal gradient. Once p_s is known, it and its normal gradient can be substituted back into Eq.(H.3) and the scattered component of the field pressure evaluated.

The BEM approximates this computation by discretizing the boundary surface $\mathcal{S}$ into a finite number of planar elements. The scheme used for the hydrophone mounts is shown in figure H.1, where 32 elements have been fitted around the perimeter of the mount. This results, in this case, in a total of 128 elements (including top, bottom, inner and outer faces.)

The surface integral in Eq.(H.5) is then approximated by the sum of the integrals over each element:

$$\frac{1}{2}p_s(\mathbf{r}) = \sum_j \int_{S_j} (p_s(\mathbf{r}')\nabla'g(\mathbf{r}|\mathbf{r}') - g(\mathbf{r}|\mathbf{r}')\nabla'p_s(\mathbf{r}')) \cdot d\mathbf{S}' \quad \mathbf{r} \in \mathcal{S}. \quad (\text{H.6})$$

where S_j is the surface of element j . To further evaluate Eq.(H.6), the point collocation assumption that the pressure or its normal gradient are approximately constant over each element. This allows them to be brought outside each integral. This reduces Eq.(H.6) to an algebraic relation (Sarkissian [117])

$$\mathbf{A}\mathbf{p} = \mathbf{B}\mathbf{v} \quad (\text{H.7})$$

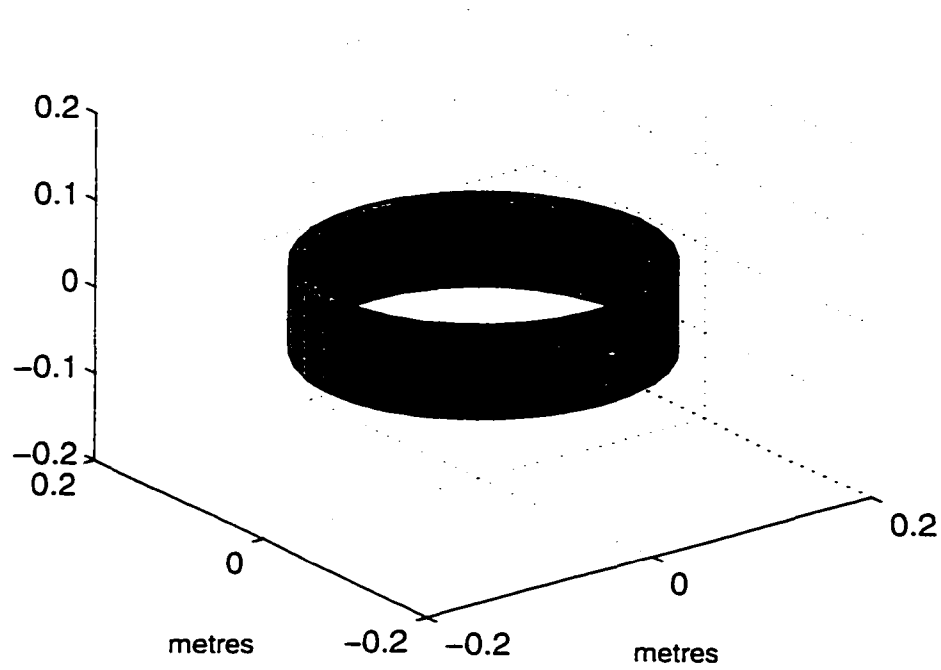


FIGURE H.1: Element mesh for hydrophone mount

where

$$A_{ij} = \delta_{ij} - \int_{S_j} \nabla' g(\mathbf{r}_i | \mathbf{r}') \cdot \mathbf{n}_j dS'$$

$$B_{ij} = - \int_{S_j} g(\mathbf{r}_i | \mathbf{r}') dS'$$

$$p_i = p_s(\mathbf{r}_i)$$

$$v_i = \nabla p_s(\mathbf{r}_i) \cdot \mathbf{n}_i.$$

Here, $\mathbf{r}_i$ is a vector to the center and $\mathbf{n}_i$ the normal of element i , respectively.

Matrix $\mathbf{A}$ may not be invertible at certain characteristic wavenumbers (Schenk [119]). At these wavenumbers, $-k^2$ is an eigenvalue of the Laplacian for the interior Dirichlet problem (Sarkissian [117]). It is unclear, for complicated geometries, whether such characteristic wavenumbers exist or not — in this case, whether the mount structure will possess any interior eigenmodes. As the probability cannot be ruled out, it is ad-

visible to use a standard procedure to circumvent the problem. Additional equations are generated for the acoustic field at a number of *interior* points, i.e., points $\mathbf{r}$ inside the structure. The field $p(\mathbf{r})$ at these interior points is zero because the scattered field and the incident field cancel everywhere within the volume of the scatterer, so that

$$p_s(\mathbf{r}) = \sum_j \int_{S_j} (p_s(\mathbf{r}') \nabla' g(\mathbf{r}|\mathbf{r}') - g(\mathbf{r}|\mathbf{r}') \nabla' p_s(\mathbf{r}')) \cdot d\mathbf{S}' = 0 \quad \mathbf{r} \in \mathcal{V}$$

where $\mathcal{V}$ is the finite volume bounded by S . After discretization, these additional relations serve to overdetermine Eq.(H.7), so that the least-squares solution for the surface pressure is

$$\mathbf{p} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{B} \mathbf{v}. \quad (\text{H.8})$$

Once $\mathbf{p}$ is known, it and $\mathbf{v}$ can be inserted in a discretized version of Eq.(H.3).

$$p(\mathbf{r}) = \mathbf{c}^T \mathbf{p} - \mathbf{d}^T \mathbf{v} \quad (\text{H.9})$$

where

$$c_i = \int_{S_i} \nabla' g(\mathbf{r}|\mathbf{r}') \cdot \mathbf{n}_i dS'$$

$$d_i = \int_{S_i} g(\mathbf{r}|\mathbf{r}') dS'.$$

This overdetermined combination of surface and interior points for the solution of the surface pressure is known as the CHIEF (combined Helmholtz integral equation formulation) algorithm (Schenk [119].)

H.3 Results

The CHIEF algorithm was applied to the mount mesh shown in figure H.1. The mount shell had an inner radius of 5.0", a width of 0.5", and a height of 4.0". A single interior point was chosen at midheight midway between the inner and outer surfaces along the x -axis. The mount structure was assumed to be ensonified by an incident plane

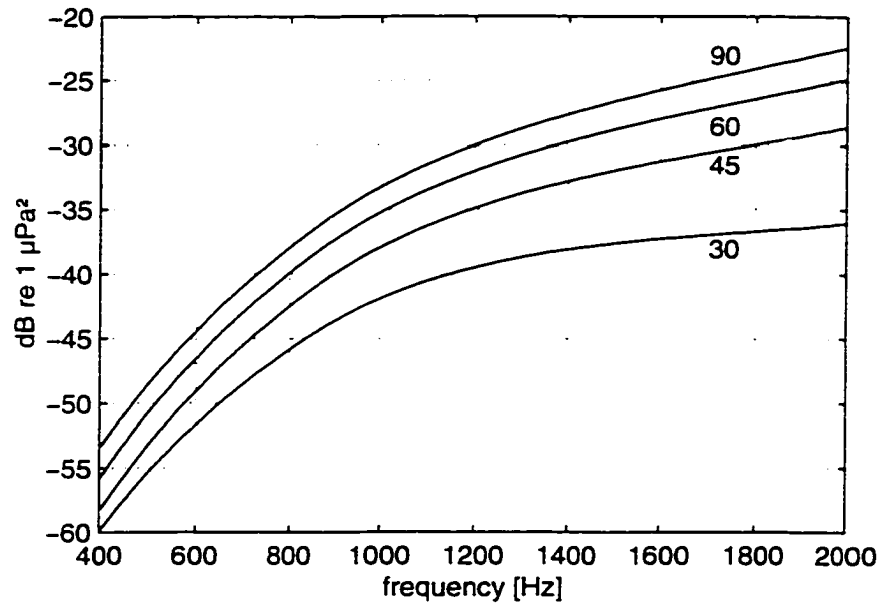


FIGURE H.2: Scattered field versus frequency. Incident angle is shown as angle in degrees relative to the symmetry axis.

wave propagating at angle θ relative to the z axis: hence, $\theta = 0$ represents a vertically incident wave, and $\theta = \pi/2$ a horizontally incident wave. This was thought to represent the effect of sources within the field of view over the array. The scattered field $p(\mathbf{r})$ was computed at the center of the mount at frequencies from 400 Hz to 2000 Hz. Figure H.2 shows the scattered field in decibels.

These computations show that the levels of the scattered field are weaker than the background ambient sound, which, as discussed in chapters 5 and 6, is typically 10 dB or less below the signal strength. Thus, the PVC mounts used on this array do not contribute significant acoustical contamination to the received signal.

Appendix I

Wave Processing Summary

Details for each identified breaking wave event for the Patricia Bay event of 4/13/95 are presented here. Section 1 contains representative data plots, section 2 the actual parameter estimators, and section 3 any additional processing details.

I.1 Wave Data

This section contains one representative data plot for each wave event processed. Each plot shows the short-time averaged high-pass filtered passband power from the center hydrophone. The data are filtered with a fifth-order Butterworth high pass filter with the corner frequency at 160Hz to remove the strum component. The data are then converted to units of μPa , squared, and averaged over consecutive 11 msec blocks. The plots therefore show broadband power (units of dB re 1 μPa^2 in the band [160,2000]Hz) versus time. The plots are annotated to show which portions were selected for background ambient and image processing (called, respectively, “ambient” and “signal”).

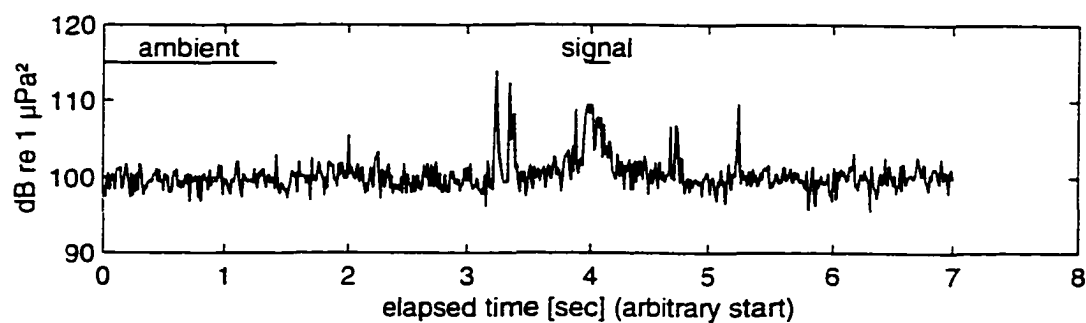


FIGURE I.1: Wave W1: 15:20:01 PDT.

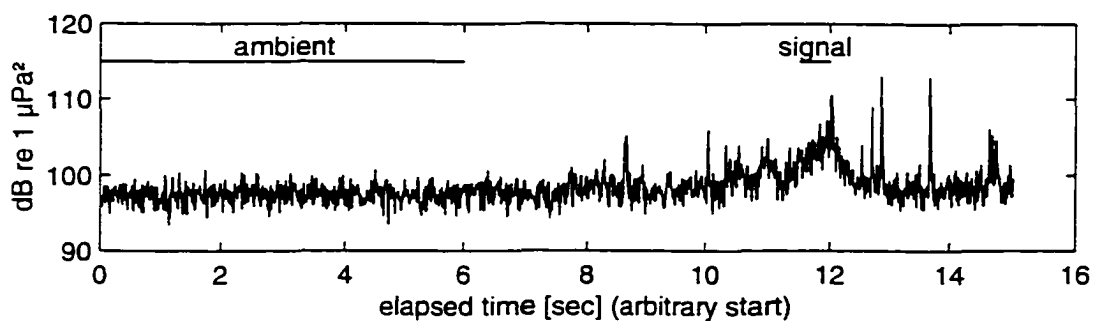


FIGURE I.2: Wave W2: 15:22:46 PDT.

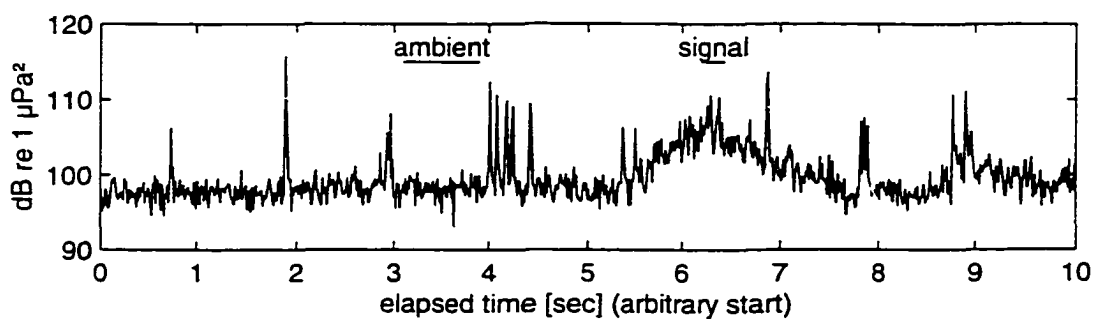


FIGURE I.3: Wave W3: 15:24:01 PDT.

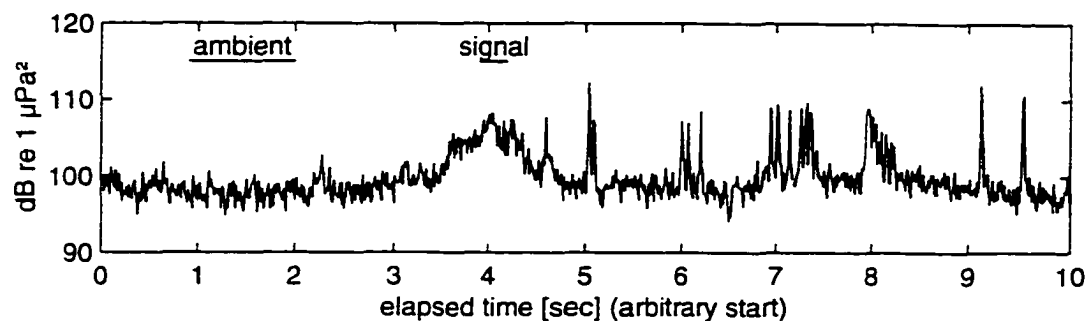


FIGURE I.4: Wave W4: 15:24:56 PDT.

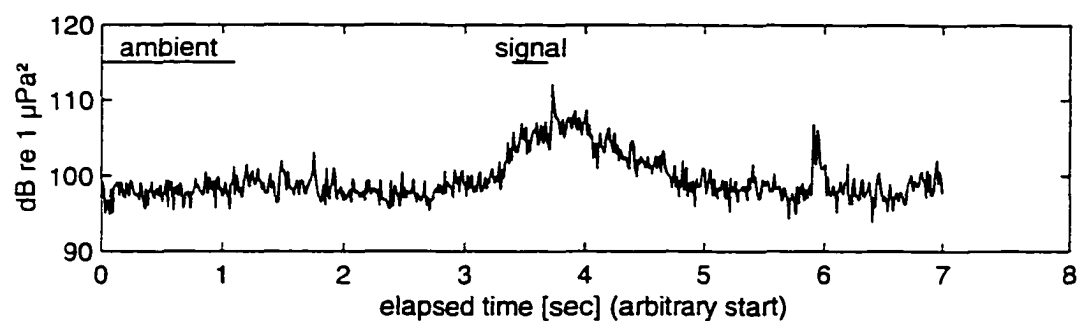


FIGURE I.5: Wave W5: 15:25:16 PDT.

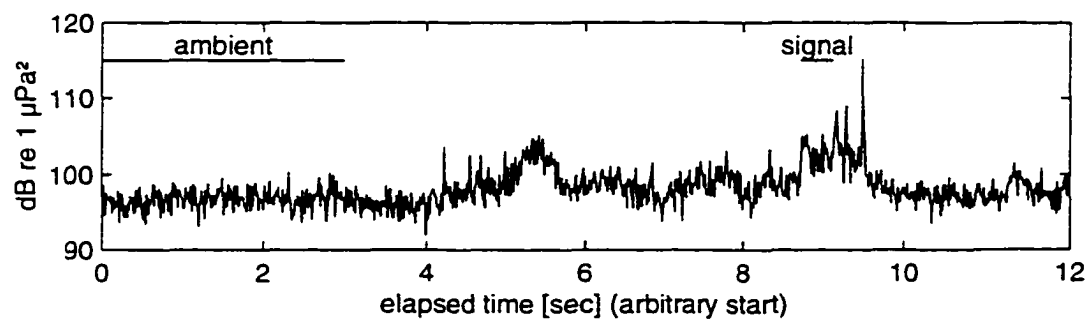


FIGURE I.6: Wave W6: 15:27:04 PDT.

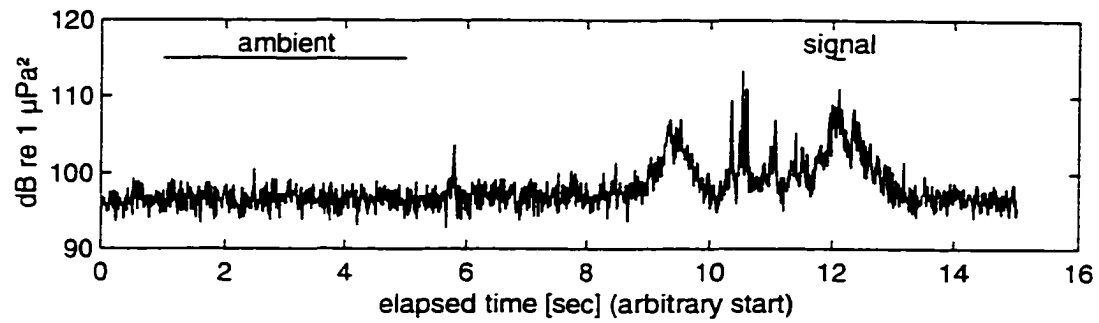


FIGURE I.7: Wave W7: 15:28:07 PDT.

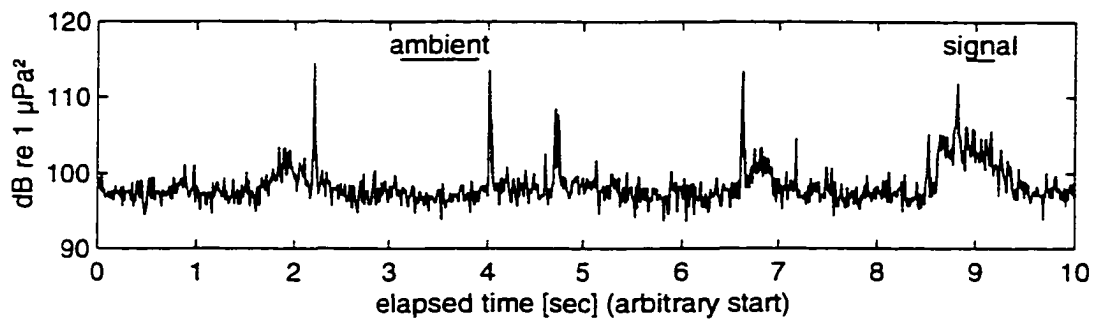


FIGURE I.8: Wave W8: 15:31:52 PDT.

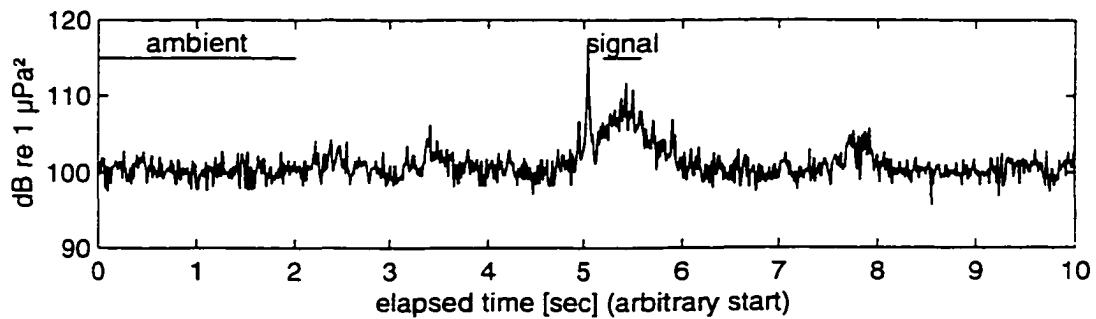


FIGURE I.9: Wave W9: 16:02:05 PDT.

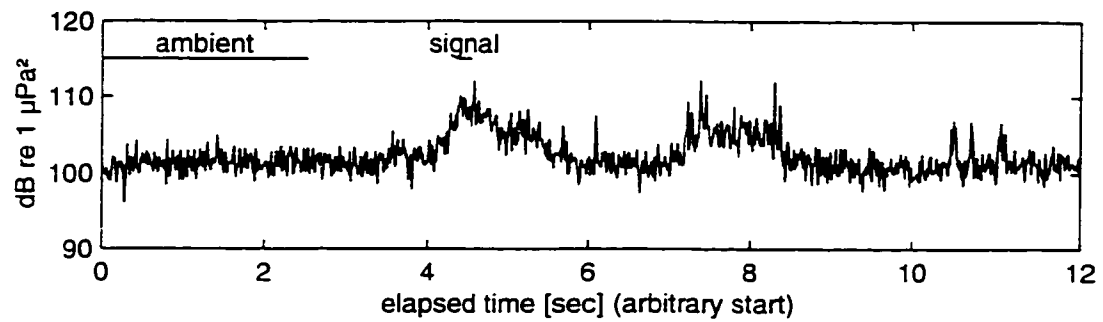


FIGURE I.10: Wave W10: 16:05:03 PDT.

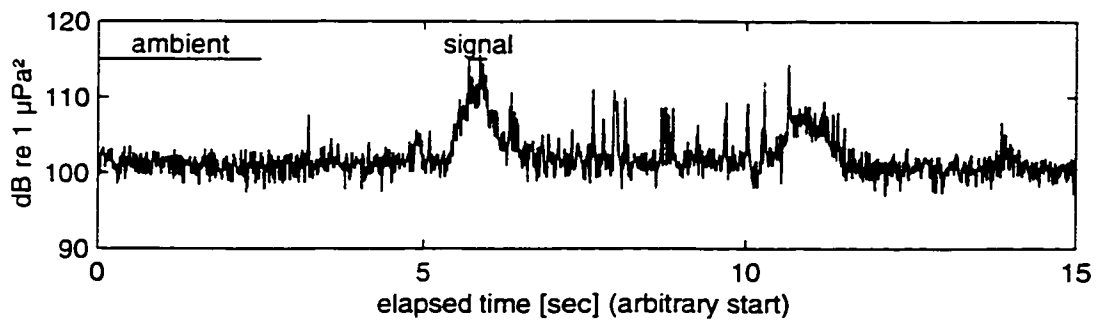


FIGURE I.11: Wave W11: 16:05:52 PDT.

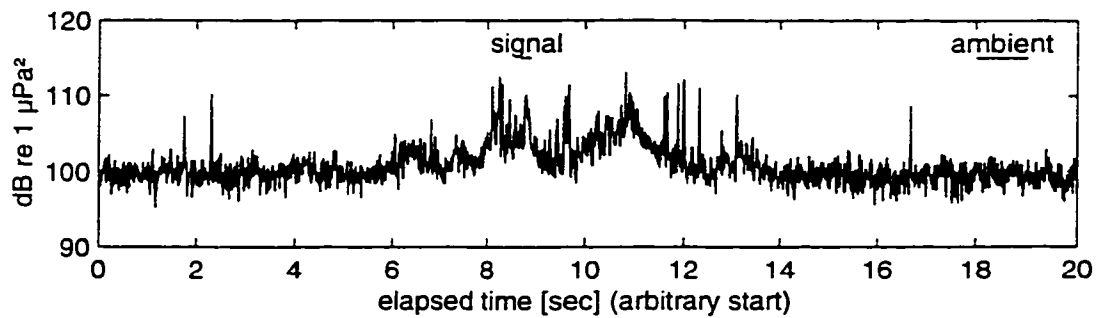


FIGURE I.12: Wave W12: 16:07:37 PDT.

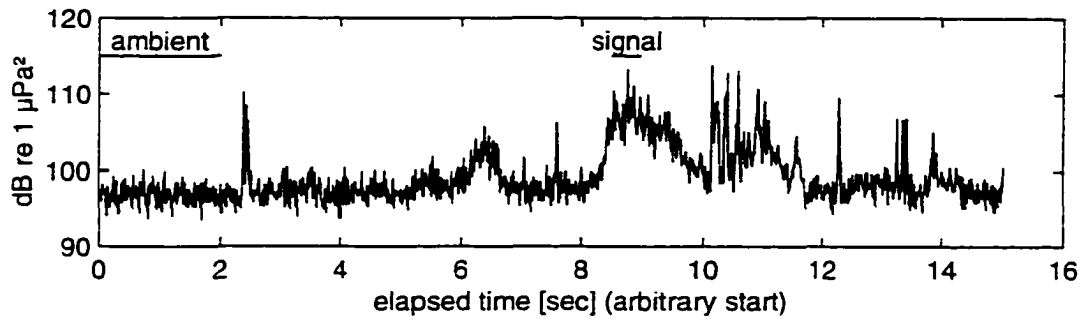


FIGURE I.13: Wave W13: 16:12:22 PDT.

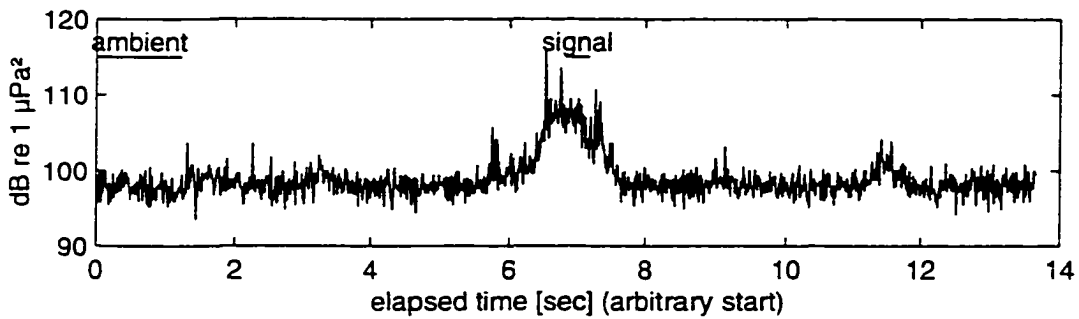


FIGURE I.14: Wave W14: 16:14:30 PDT.

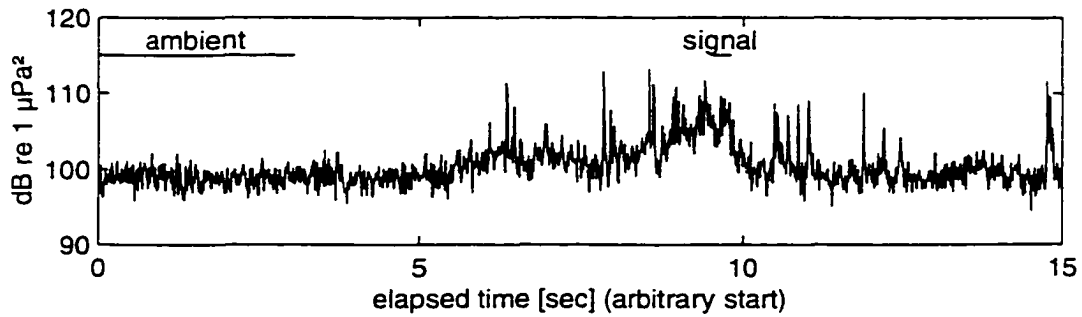


FIGURE I.15: Wave W15: 16:21:42 PDT.

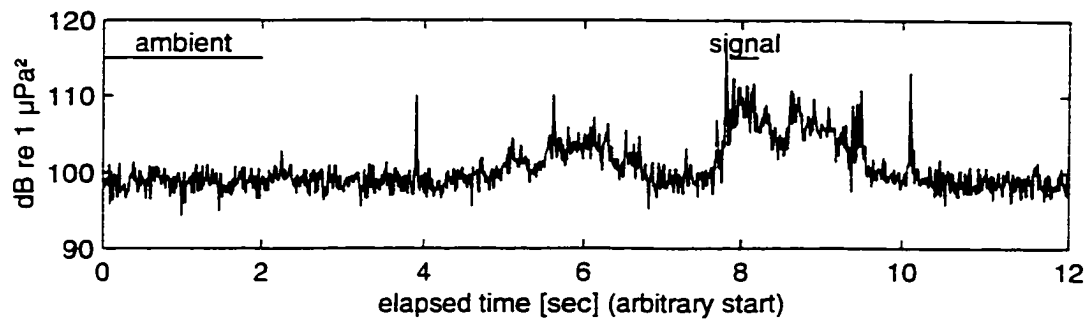


FIGURE I.16: Wave W16: 16:22:43 PDT.

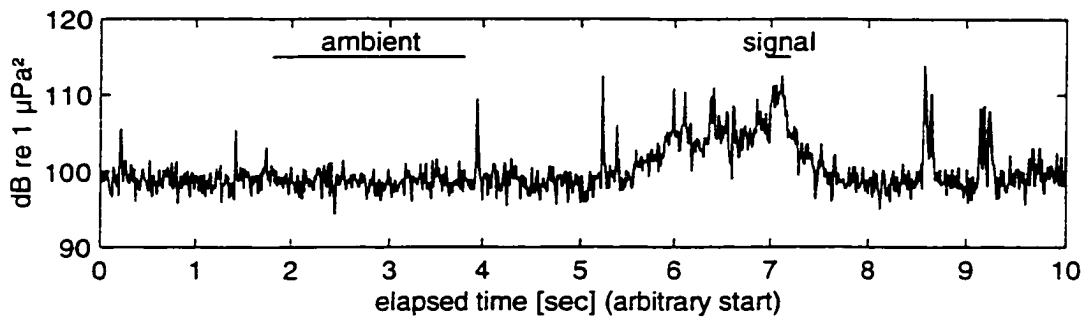


FIGURE I.17: Wave W17: 16:22:57 PDT.

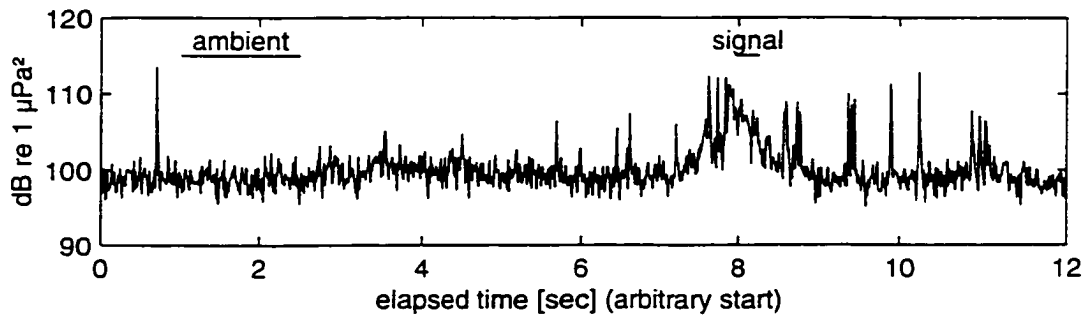


FIGURE I.18: Wave W18: 16:23:31 PDT.

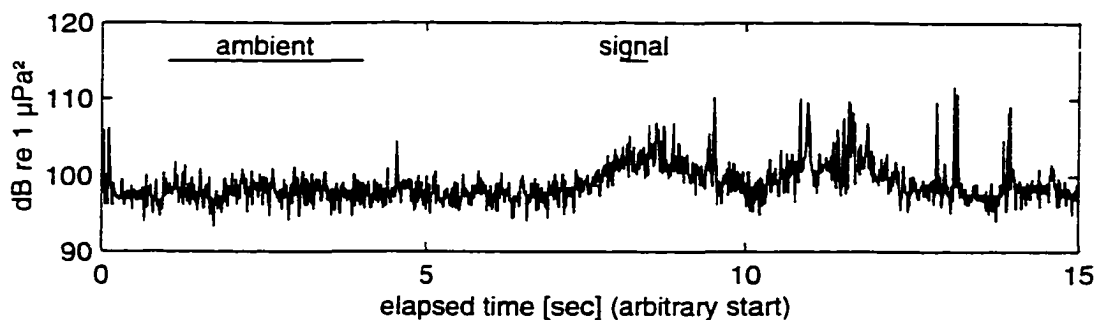


FIGURE I.19: Wave W19: 16:36:51 PDT.

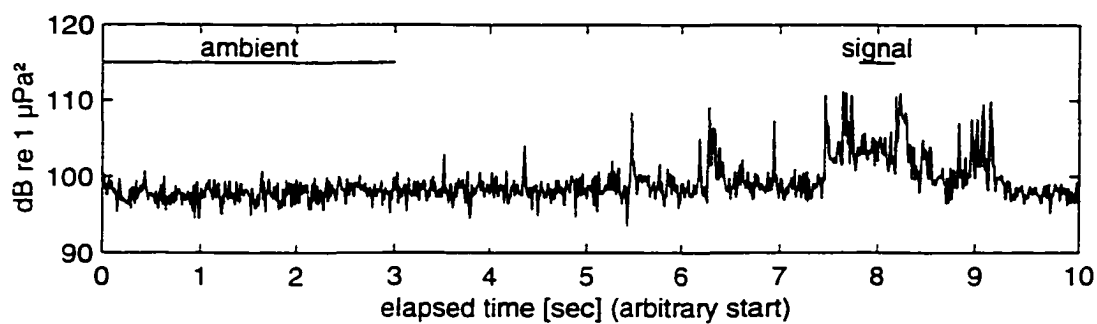


FIGURE I.20: Wave W20: 16:38:20 PDT.

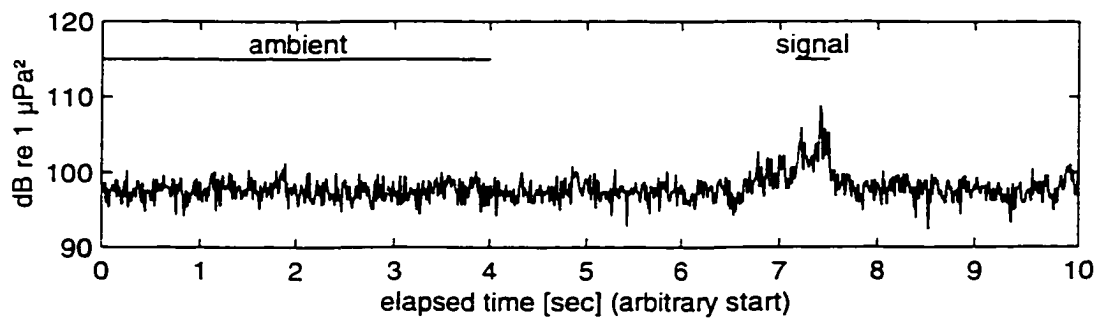


FIGURE I.21: Wave W21: 16:46:53 PDT.

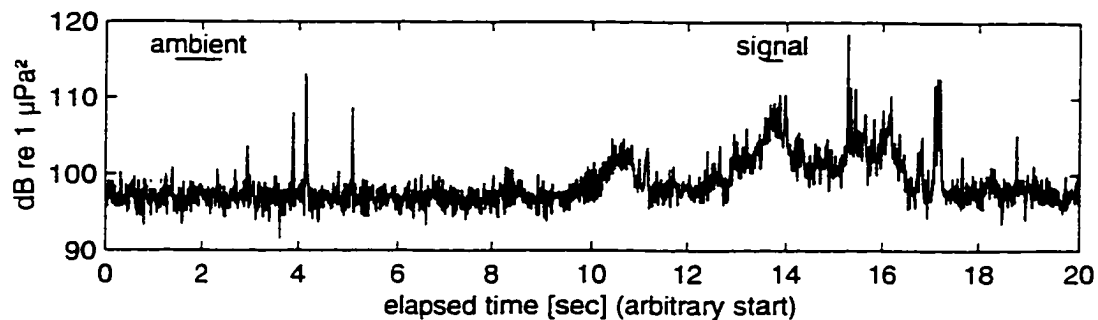


FIGURE I.22: Wave W22: 16:48:33 PDT.

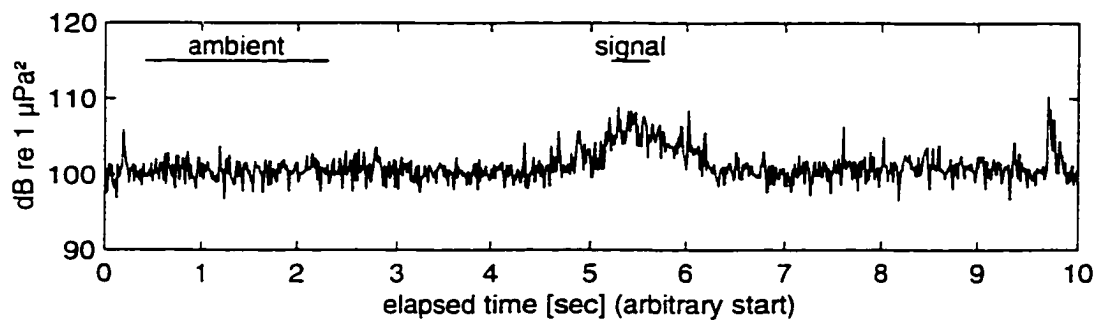


FIGURE I.23: Wave W23: 17:02:33 PDT.

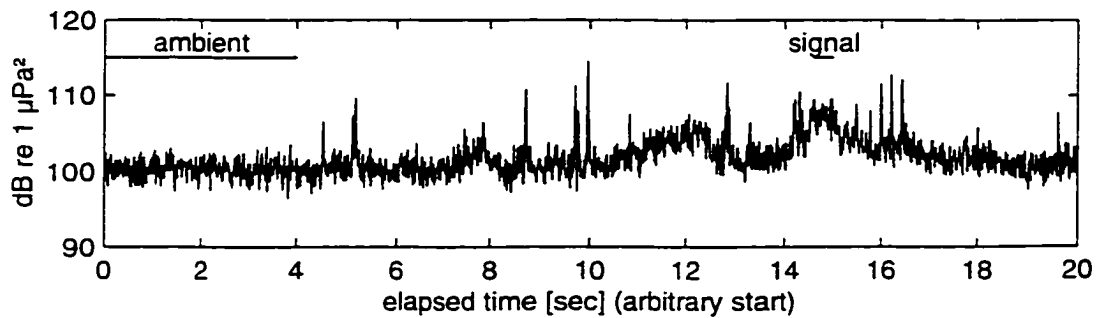


FIGURE I.24: Wave W24: 17:02:49 PDT.

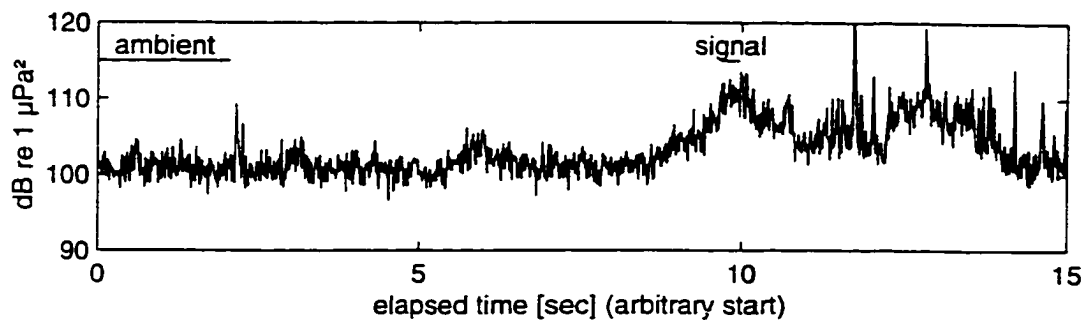


FIGURE I.25: Wave W25: 17:03:40 PDT.

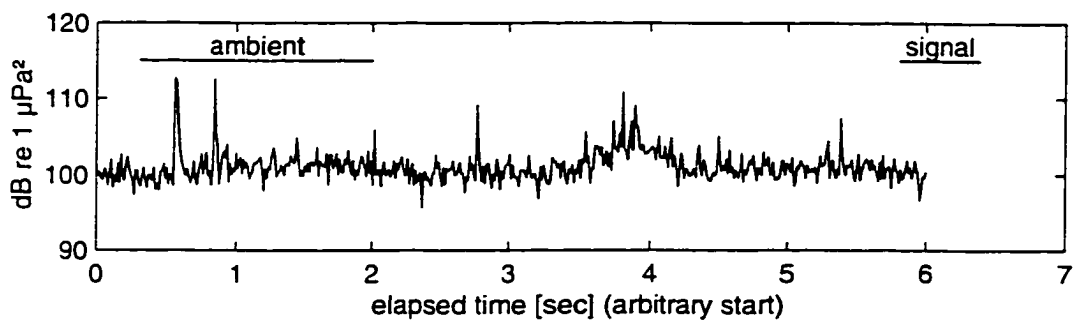


FIGURE I.26: Wave W26: 17:05:05 PDT.

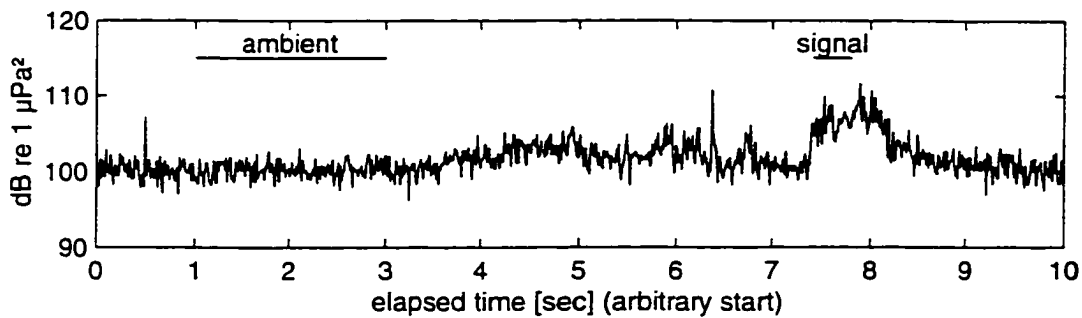


FIGURE I.27: Wave W27: 17:05:13 PDT.

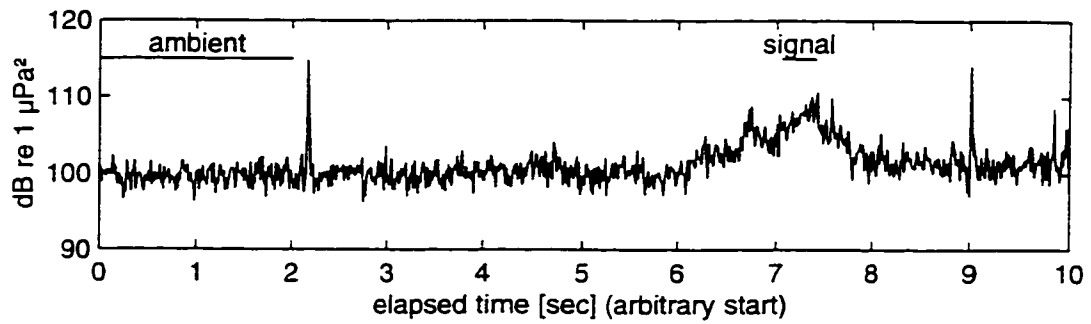


FIGURE I.28: Wave W28: 17:05:35 PDT.

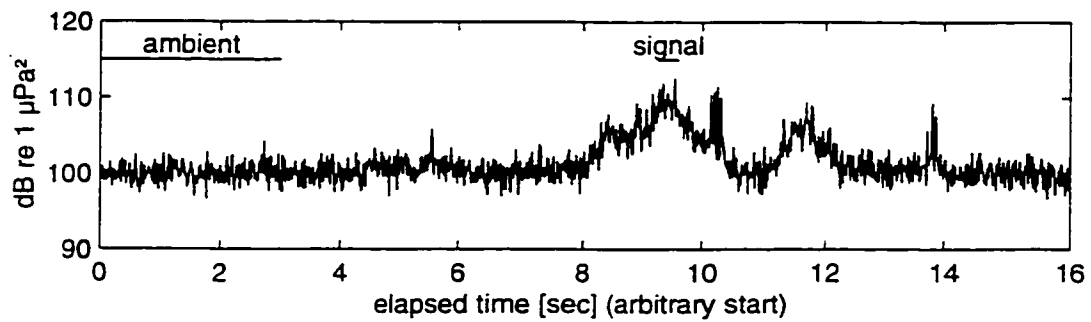


FIGURE I.29: Wave W29: 17:05:47 PDT.

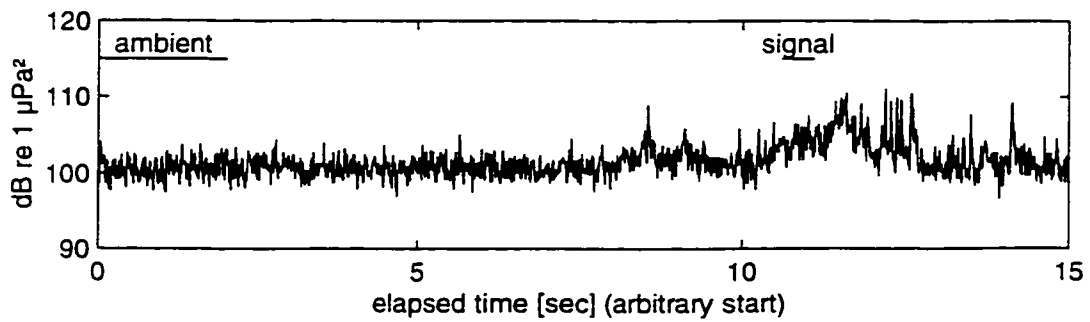


FIGURE I.30: Wave W30: 17:07:40 PDT.

I.2 Wave Model Parameters

This section contains in tables I.1 and I.2 the actual maximum likelihood parameter estimators for the broadband shape function (Eqs.(4.24) and (4.25)) and the frequency-dependent source strengths (Eq.(4.19)) for each wave, respectively. The raw estimators have been reduced in some cases to a more consistent paradigm: the solution $[x_0, y_0, w, l, \phi]^T$ so example, is equivalent to $[x_0, y_0, w, l, \phi + \pi]^T$, which is also equivalent to $[x_0, y_0, l, w, \phi + \pi/2]^T$ and $[x_0, y_0, l, w, \phi + 3\pi/2]^T$. Since the optimizer may find any of these (identical) solutions, the results tabulated in table I.1 have had ϕ adjusted so that $w > l$ and $\phi + \pi/2 \in [\pi, 2\pi]$.

TABLE I.1: Maximum likelihood parameters for the broadband wave shape.

wave	$N_{ambient}$	N_{signal}	$x_0[m]$	$y_0[m]$	$w[m]$	$l[m]$	$\phi + \pi/2 [^\circ TN]$
1	340	115					
2	2580	215	0.53	-0.85	1.99	0.55	274.4
3	340	105	-1.43	1.33	1.80	1.23	262.5
4	470	130	-3.67	-2.61	3.81	1.02	274.5
5	470	125	-1.17	-1.54	1.77	1.20	272.7
6	1290	170	-3.07	-0.44	2.90	2.28	241.8
7	1720	85	-0.31	0.94	1.56	0.78	269.3
8	340	115	1.24	-2.40	2.70	0.91	293.9
9	860	170	-0.70	0.20	1.19	0.46	394.8
10	1075	85					
11	1075	125	0.00	-1.20	1.74	0.49	261.8
12	430	85					
13	860	215	0.08	-0.47	2.05	0.67	225.7

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wave	$N_{ambient}$	N_{signal}	$x_0[m]$	$y_0[m]$	$w[m]$	$l[m]$	$\phi [^\circ TN]$
14	535	150	1.37	-2.85	3.42	1.44	291.6
15	1290	125					
16	860	150					
17	860	105	2.15	-1.62	3.28	0.06	265.9
18	645	125	0.05	0.27	2.04	0.87	296.8
19	1290	190					
20	1290	160	1.27	-2.59	3.64	0.57	268.1
21	1720	150	-2.51	0.45	1.22	0.73	296.5
22	430	170	0.56	0.86	2.16	1.27	263.4
23	815	170	0.56	-0.93	1.35	0.58	306.8
24	1720	170	1.32	2.61	9.47	1.44	297.7
25	860	125					
26	730	85					
27	860	170	-0.51	-0.43	1.49	0.62	225.5
28	860	150	1.06	-2.14	3.60	3.00	307.2
29	1290	150					
30	860	215	0.74	-2.30	5.60	1.28	288.5

The columns in table I.1 have the following meanings. The second and third columns provide the number of accumulants in the estimation of the ambient-only and ambient+signal CSDMs, respectively. The fourth and fifth columns provide the source centroid (units of metres) in flat array frame coordinates at the nominal fluid surface. Columns 6 and 7 provide the major and minor axis scales (w, l , units of metres). Column 8 provides the orientation of the wave minor axis relative to true North (units of degrees).

TABLE I.2: Maximum likelihood parameters for the wave source strength parameter $\lambda(\omega)$.

wave	frequency					
	603 Hz	861 Hz	1120 Hz	1378 Hz	1637 Hz	1895 Hz
1						
2				-2.44	-2.99	-3.91
3	0.50	-1.07	-1.90	-2.76	-3.27	-4.11
4				-1.91	-2.09	-3.66
5	0.85	-0.23	-1.64	-2.49	-3.24	-4.39
6			-2.30	-3.00	-3.38	-4.04
7	0.47	-0.86	-1.57	-2.27	-2.85	-3.80
8				-2.41	-3.33	-4.19
9			-0.31	-1.03	-2.11	-3.07
10						
11				-0.36	-1.35	-1.96
12						
13	0.70	-0.47	-1.11	-2.10	-2.96	-4.01
14	0.23	-0.92	-1.79	-2.75	-3.39	-4.34
15						
16						
17				3.63	1.43	0.67
18				-2.26	-2.83	-3.52
19						
20		3.58	-0.44	-2.44	-3.25	-4.31
21			-1.38	-2.43	-3.23	-4.14

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wave	603 Hz	861 Hz	1120 Hz	1378 Hz	1637 Hz	1895 Hz
22	2.01	-0.22	-1.88	-2.70	-3.40	-4.33
23	0.46	-0.23	-0.81	-1.45	-1.98	-2.93
24			-1.45		-3.28	-4.18
25						
26						
27		0.01	-0.75	-1.60	-2.34	-3.20
28		-0.84	-1.87	-2.58	-3.13	-4.04
29						
30	-0.02	-0.86	-2.77	-3.68	-4.52	-5.45

I.3 Additional Processing Details

This section lists further details associated with the processing of each breaking wave.

W1: Although autospectral levels seemed reasonable across both arms at all frequencies, no convergent solution was found using either all frequencies or only 1378, 1637 and 1895 Hz. Possible problems: bad choice of ambient, signal excerpt too short, or model violated (more than one breaker in the field of view).

W2: Channel 11, the outermost sensor on arm 1, had abnormally high autospectral levels at 603, 861 and 1120 Hz, so processing excluded these frequencies.

W3: No unusual features noted.

W4: Wave was apparently on the edge of the field of view and therefore difficult to fit: no plausible convergent solutions using data at 603, 861 or 1120 Hz, either alone or in conjunction with data at higher frequencies. Given solution used data only 1378, 1637 and 1895 Hz, but is probably not reliable.

W5: No unusual features noted.

W6: Abnormal autospectral levels from the sensors at the ends of arms 2 and 4 at 603 and 861 Hz: processing excluded these frequencies. Autospectral levels not significantly peaked along arms 2 and 4 at higher frequencies either.

W7: No unusual features noted.

W8: Abnormal autospectral levels from the sensors at the ends of arms 2 and 4 at 603, 861 and 1120 Hz: processing excluded these frequencies.

W9: Abnormal autospectral levels from the sensors at the ends of arm 2 at 603 and 861 Hz: processing excluded these frequencies. Convergence was otherwise difficult, with the optimizer stopping when the relative gradient was near zero.

W10: No reasonable convergent solution: wave perhaps too close to the edge of the field of view.

W11: Stable solution using 1378, 1637 and 1895 Hz: including lower frequencies either pulled the solution off towards the edge of the field of view and/or produced no reasonable convergent solution.

W12: No reasonable convergent solution: wave possibly too close to the edge of the field of view.

W13: No unusual features noted.

W14: No unusual features noted.

W15: No reasonable convergent solution: wave possibly too close to the edge of the field of view.

W16: No reasonable convergent solution: wave possibly too close to the edge of the field of view.

- W17:** Stable solution using 1378, 1637 and 1895 Hz. although the size and power parameters do not fit well with expected values. Centroid position estimate using 603 Hz was (0.2, -3.6), quite some distance from the stable solution estimate using higher frequencies.
- W18:** Stable solution using 1378, 1637 and 1895 Hz: including lower frequencies either pulled the solution off towards the edge of the field of view and/or produced no reasonable convergent solution.
- W19:** No reasonable convergent solution: wave possibly too close to the edge of the field of view.
- W20:** Stable solution using all frequencies except 603Hz: the autospectral densities at 603 Hz indicated a wave outside the field of view.
- W21:** Stable solution using all frequencies except 603 and 861 Hz: the autospectral densities at those frequencies indicated a wave outside the field of view.
- W22:** Stable solution using all frequencies, but autospectra at lower frequencies seemed to indicate a different centroid position than those for higher frequencies.
- W23:** No unusual features noted.
- W24:** Autospectral densities at 603, 861 and 1378 Hz seemed to suggest a breaker at the edge of the field of view, but the densities at the remaining frequencies suggested a breaker within the field of view. Solution excluded 603, 861 and 1378 Hz.
- W25:** No reasonable convergent solution: wave too close to the edge of the field of view.
- W26:** No reasonable convergent solution in either lower or upper frequency regime: possibly not enough signal-to-noise ratio.

W27: Stable solution using all frequencies except 603Hz: the autospectral densities at 603 Hz indicated a wave outside the field of view.

W28: Stable solution using all frequencies except 603Hz: the autospectral densities at 603 Hz indicated a wave outside the field of view.

W29: The center hydrophone power was abnormally low for the *y* axis profile at all frequencies. No convergent solution resulted within the field-of-view.

W30: No unusual features noted.

Appendix J

Source Strength Considerations

There are several conventions for representing the “strength” of an acoustic source. In this work, the situation is complicated further by the interpretation of the “source” as a continuous random boundary condition. While this is merely a convenient mathematical formalism, it precipitates additional nomenclature which requires clarification.

This appendix therefore presents a discussion of the source strength units used in this work.

J.1 Point Sources

The most physical approach considers the forcing provided directly by the acceleration of the bubble wall. Assume the bubble is spherical, of radius a , with time-harmonic radial velocity $u_r(t) = U_0 e^{-i\omega t}$. The acoustic pressure in the fluid (henceforth suppressing the $e^{-i\omega t}$ dependence) is then (Morse and Ingard [103], section 7.1)

$$p = -(ik\rho c S/4\pi r) e^{ikr} \quad (\text{J.1})$$

where ρ and c are the density and sound speed of the surrounding fluid, respectively, $k = \omega/c$. S is the *volume velocity* $S = 4\pi a^2 U_0$, with units of $\text{m}^3 \text{s}^{-1}$. The volume velocity is sometimes considered the “source strength” of simple pulsating sources.

Eq.(J.1) has the form of a monopole wave. Consider the inhomogeneous Helmholtz equation

$$\nabla^2 p + k^2 p = -A\delta(\mathbf{r} - \mathbf{r}_0)$$

for a point source located in unbounded space at $\mathbf{r}_0$. The formal solution follows with the use of the free-space Green function

$$\begin{aligned} p_M(\mathbf{r}) &= \iiint A\delta(\mathbf{r}' - \mathbf{r}_0)g(\mathbf{r}|\mathbf{r}') dV' \\ &= Ae^{ikR}/4\pi R \end{aligned} \quad (\text{J.2})$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}_0$. Since the 3-D Dirac delta function has units of m^{-3} (Barton [8], section 1.3) and g has units of m^{-1} , A must have units of Pa m . By comparing Eq.(J.1) with Eq.(J.2), A is seen to be $-ik\rho cS$, which indeed has units of Pa m .

Eq.(J.2) is the equation for a monopole point source. Conventional terminology would call A the “strength” of the monopole, but this would not be the same as the source strength S . Although the acoustic field of the bubble and the monopole point source are the same, the definition of source strength is different. A monopole is only a mathematical model, whereas a bubble is a physical entity with a mechanism for generating sound.

A bubble at $z = -h$ below the ocean surface induces an antiphase source at $z = h$, so that the bubble and its image form a “dipole pair”. For $kh \ll 1$, this field is (Morse and Ingard [103], section 7.1)

$$p = -k^2 \rho c D \cos \theta e^{ikr} (1 + i/kr) / 4\pi r \quad (\text{J.3})$$

where θ is the angle relative to the dipole pair axis. The term D in Eq.(J.3) is the “dipole source strength” with units of $\text{m}^4 \text{s}^{-1}$, and is

$$D_{\text{BUB}} = 2Sh = 8\pi a^2 U_0 h. \quad (\text{J.4})$$

Eq.(J.3) has the form of a dipole wave. Consider the solution of the homogeneous Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \quad (\text{J.5})$$

in the halfspace $z < 0$ with boundary conditions

$$p = A\delta^{(2)}(\mathbf{r} - \mathbf{r}_0)$$

on the $z = 0$ plane. This represents essentially a point source on the boundary. The superscript (2) on the Dirac function indicates that it is a 2-D delta function. The formal solution follows from section 3.3:

$$\begin{aligned} p_D(\mathbf{r}) &= \iint A\delta(\mathbf{r}' - \mathbf{r}_0) \frac{\partial}{\partial z'} G_D(\mathbf{r}|\mathbf{r}') dS' \\ &= ikA \cos\theta e^{ikR} (1 + i/kR) / 2\pi R \end{aligned} \quad (\text{J.6})$$

where G_D is the Dirichlet Green function. Dimensional analysis shows that A must have units of Pa m^2 . A may also be called the strength of the dipole source, and by comparing Eq.(J.3) and Eq.(J.6) provides a second definition

$$|D_{\text{DIP}}| = k\rho c D_{\text{BUB}}/2. \quad (\text{J.7})$$

The factor of 1/2 arises from the use of the Dirichlet Green function. As before, this is different than the dipole source strength D_{BUB} .

Note also that, in this case, a *point source* on the boundary is characterized by a form involving D , the dipole source strength, rather than S , the volume velocity.

Leighton points out (Leighton [78], section 3.3.2) that the bubble wall velocity (in m s^{-1}) is generally not measured directly, but rather the acoustic field (in pascals) at some distant point $\mathbf{r}$ from the source. At distances where $kr \gg 1$, the field is proportional to $1/r$, so a convention common in the cavitation literature is to define the dipole source strength as the product of the field p and the distance r . Using Eq.(J.3), this yields

$$D_{\text{CAV}} = k^2 \rho c D_{\text{BUB}} / 4\pi = 2\rho c k^2 a^2 U_0 h. \quad (\text{J.8})$$

Another definition sometimes used (e.g., Medwin and Beaky [94]) is to refer to the radiated pressure at a standard distance. This is similar to the definition of D_{CAV} : in the far field on the axis of a dipole, the pressure magnitude from Eq.(J.3) will be $|p(r)| = k^2 \rho c D / 4\pi$ in pascals. The pressure magnitude referenced to $r = 1\text{ m}$ is then

$$D_{\text{PRS}} = |p(r)|r / (r = 1 \text{ m}). \quad (\text{J.9})$$

This is numerically identical to Eq.(J.8) but the units are "pascals at 1 m".

This latter approach is used by Ding and Farmer ([34, 35]) to yield the "source level intensity" of breaking waves. Acoustic intensity is the proper measure of power flux and is (Tichy [132])

$$I(\mathbf{r}) = \langle \text{Re}\{p(\mathbf{r})\} \text{Im}\{\mathbf{u}(\mathbf{r})\} \rangle. \quad (\text{J.10})$$

for stochastic wavefields. Acoustic intensity has units of W m^{-2} . In the source near field, the reactive intensity can be quite large, representing considerable stored energy. This energy does not propagate away into the far field. The only energy reaching the far field is the active intensity. The far field active intensity along the dipole axis of a dipole field is (Morse and Ingard [103], section 7.1)

$$I_r = |p|^2 / \rho c \quad (\text{J.11})$$

which is proportional to r^{-2} . Intensities measured in the far field (their measurements were made with $kr \gg 1$) at distance r are then converted to a reference intensity for a distance 1 m by applying a spherical spreading correction. Their source levels are not reported in units of acoustic intensity, however, but use the device

$$10 \log_{10} I_r(r = 1\text{ m}) / I_{\text{ref}} = 20 \log_{10} |p(r = 1\text{ m})| / |p_{\text{ref}}|$$

such that the units are "dB re $1 \mu\text{Pa}^2$ at 1 metre". This is in effect identical to convention Eq.(J.9).

J.2 Source Layer

Consider now the solution of the homogeneous Helmholtz equation Eq.(J.5) with a inhomogeneous "layer" boundary condition

$$p = \sigma$$

on the $z = 0$ plane. The formal solution is

$$p(\mathbf{r}) = \iint \sigma(\mathbf{r}') \frac{\partial}{\partial z'} G_D(\mathbf{r}|\mathbf{r}') dS'. \quad (\text{J.12})$$

Dimensional analysis shows that σ must have units of pascals: comparing Eq.(J.12) with Eq.(J.3) shows that

$$\sigma = k\rho c D'_{\text{BUB}} \quad (\text{J.13})$$

where D'_{BUB} is "the dipole source strength per unit area". The total dipole source strength would then be

$$D_{\text{total}} = (k\rho c)^{-1} \iint \sigma dS$$

and, of course, the total dipole field is Eq.(J.12).

J.3 Random Source Layer

Sources in this work are represented mathematically as a layer of random sources on the boundary. A physical picture of this layer would be a continuum of bubbles, each located at some random depth $-h$, $|kh| \ll 1$, and pulsating with a random radial velocity u . Consider these velocities and depths as comprising two random fields over the surface, and their product to define a random field of dipole source strengths. In the continuum limit, this field must, as in the previous section, be considered a "dipole source strength per unit area".

The spatial cross-correlation of this field is

$$\langle D^*(\mathbf{r}) D(\mathbf{r} + \mathbf{q}) \rangle = D_{\text{rms}}^2 f(\mathbf{r}, \mathbf{r} + \mathbf{q}) \quad (\text{J.14})$$

where $D_{\text{rms}}^2 = \text{Var}\{D(\mathbf{r})\}$ is the ensemble average of the square of the dipole source strength per unit area. The function $f(\mathbf{r}, \mathbf{r} + \mathbf{q})$ in Eq.(J.14) will generally decrease with increasing separation distance $|\mathbf{q}|$: if the field is spatially homogeneous, this function will be $f(\mathbf{r} - (\mathbf{r} + \mathbf{q})) = f(\mathbf{q})$. This function peaks at $\mathbf{q} = \mathbf{0}$ where it attains the value 1.

The field cross-correlations used in this work require terms $\langle p^*(\mathbf{r}_i)p(\mathbf{r}_j) \rangle$. From Eq.(J.12), this kind of term yields an expression

$$\langle p^*(\mathbf{r}_i)p(\mathbf{r}_j) \rangle = \iint dS' \iint dS'' \langle \sigma^*(\mathbf{r}')\sigma(\mathbf{r}'') \rangle \frac{\partial}{\partial z'} G_D^*(\mathbf{r}_i|\mathbf{r}') \frac{\partial}{\partial z''} G_D(\mathbf{r}_j|\mathbf{r}''). \quad (\text{J.15})$$

By dimensional analysis, the term $\langle \cdot \rangle$ in the integrand must have units of Pa^2 . More specifically, when the term $\langle p^*(\mathbf{r}_i)p(\mathbf{r}_j) \rangle$ represents a spectral density, as it does throughout this work, the units must be $\text{Pa}^2 \text{ Hz}^{-1}$. This is the convention for the units of $S_{SS}(\omega)$.

The layer cross-correlation can be related to the dipole source strength as follows. The value of $\langle \sigma^*(\mathbf{r}')\sigma(\mathbf{r}'') \rangle$ is determined at each frequency band. From Eq.(J.13), this term can be rewritten as

$$\langle \sigma^*(\mathbf{r}')\sigma(\mathbf{r}'') \rangle = k^2 \rho^2 c^2 \langle D'^*(\mathbf{r}')D'(\mathbf{r}'') \rangle.$$

Using Eq.(J.14) and the assumption of spatial homogeneity,

$$\langle \sigma^*(\mathbf{r})\sigma(\mathbf{r} + \mathbf{q}) \rangle = k^2 \rho^2 c^2 D_{\text{rms}}^2 f(\mathbf{q}). \quad (\text{J.16})$$

This provides the relation between the layer value and the mean-square dipole source strength per unit area. The relation depends critically on the dimensionless function f . This function is modeled in the main body of this work by the “delta-correlation” $f = \delta(\mathbf{q})$. This is only a convenient device: no real-world random process has a true singular correlation no process has infinite bandwidth. The correlation “width” may be exceedingly small, but for real-world processes, the function f should remain bounded by 1 at $\mathbf{q} = \mathbf{0}$. By this convention, Eq.(J.16) should only be “inverted” for D_{rms}^2 at

$\mathbf{q} = \mathbf{0}$, the operation of dividing through by $f(\mathbf{q})$ being undefined for all other values of $\mathbf{q}$.

This definition prevents further complications involving the units of $\langle \sigma^*(\mathbf{r}')\sigma(\mathbf{r}'') \rangle$ and $\langle D'^*(\mathbf{r}')D'(\mathbf{r}'') \rangle$ from arising.

Note that, as in Eq.(J.7), a single layer has been characterized by a dipole source strength parameter.