THE CALIBRATION OF LUMINOSITY CRITERIA

by

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ABSTRACT

All the information about the distance of a star given by the observational data is contained in the likelihood function. Efficient statistical estimation must, therefore, start with the likelihood. The meaning of the likelihood, and its application to estimation, is reviewed. A Bayesian approach is adopted, and it is shown how nuisance parameters can be eliminated using Bayes's theorem.

The calculation of the likelihood of the distance, given the parallax and the proper motion, is described. The parallax sets a lower limit to the distance, but no upper limit; the proper motion is essential if an upper limit is to be found. The proper motion contribution to the likelihood is obtained from the velocity distribution of stars relative to the sun, using the substitution

tangential velocity = proper motion x distance.

The presence of errors in the proper motion is unimportant for relatively nearby stars with reasonably large proper motions. It is shown how to calculate the likelihood for more distant stars for which the effect of errors in the proper motions is not negligible.

The quoted probable errors of the parallaxes bear little relation to the external errors, judged from comparisons of independent parallaxes for the same star. Estimates are made of the external standard deviations of the parallaxes of nine observatories, based on data given by Dahlgren and by Vasilevskis. The results, in units of 0".001, are:

Allegheny, 12·59 ± 0·45; Cape, 20·39 ± 0·66;
Greenwich, 14·65 ± 1·26; McCormick, 16·39 ± 0·43;
Mt. Wilson, 24·56 ± 1·01; Sproul, 16·12 ± 0·78;
Van Vleck, 13·72 ± 1·16; Yale, 14·70 ± 0·60;
Yerkes, 18·51 ± 0·78.

The practice of selecting stars for observation with large parallaxes leads to a systematic bias, which may be overcome by proper normalization of the likelihood.

The calibration of a luminosity criterion essentially involves the estimation of the parameters of the distribution of the luminosity criterion, conditional on the absolute magnitude and the color. The likelihood of the parameters, based on observations of the luminosity criterion, color, and parallax and proper motion, must be found by eliminating the true values of the absolute magnitude as "nuisance parameters". The estimates of the parameters are found by locating the maximum of the likelihood. This leads to a set of non-linear equations, which can be solved by iteration.

The method is applied to the calibration of the H and K emission line widths and the Fe λ 5250 triplet. The results for the H and K lines are compared to those obtained by a least squares calculation, and it is shown that the least squares result contains a systematic error, so that the luminosity of giant stars is underestimated by 0·6 magnitudes. This supports the suggestion of Hodge and Wallerstein that the distance modulus of the Hyades cluster is in error.

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This thesis started as an attempt to calibrate line ratios originally studied by Wright and Jacobsen, using plates taken at the Dominion Astrophysical Observatory. The attempt was not successful, but it is entirely due to the stimulus provided by the necessity of analyzing the observations that the work of this thesis was carried out. I would like to thank the director, Dr. K. O. Wright, for his generous allotment of observing time; if any of this thesis is of value, it is thanks to the opportunity Dr. Wright gave me to observe.

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CHAPTER I

INTRODUCTION

1.0 It is a measure of the importance of luminosity criteria that the great Canadian astronomer, R. M. Petrie, should have devoted a considerable portion of his life to their study. The most important and most exacting task in the development of luminosity criteria is the gathering of the fundamental data on which their calibration rests. The present work has a more modest aim: to show how to extract from the data as much of the relevant information as possible, so as to obtain as accurate a calibration as the data will allow.

In the course of this work, an entirely unexpected result emerged. It appears that the conventional methods of calibration lead to large systematic errors. If this is true, the entire cosmic distance scale is affected.

1.1 The consistency of the distance scale.

It seems surprising that the cosmic distance scale could be seriously in error, since it is based on many different methods of calibration. Not all of the methods are independent, of course; but there appear to be at least three methods which are truly independent, and which give consistent results. These three methods are:

(a) The moving cluster method, which has been used to estimate the distance of the Hyades cluster. (Heckmann and Lübeck, 1956).

(b) Least squares fitting to absolute magnitudes calculated from trigonometric parallaxes. (e.g., Wilson, 1967).

(c) The method of reduced parallaxes which has been used to calibrate the MK luminosity classes II-V. (Roman, 1952).

Nearly all other methods are based ultimately on these three primary methods. The zero point of the period--luminosity relation for the cepheids has been obtained from cepheids in open clusters, by Kraft (1961); and the distances of the open clusters have been obtained using zero age main sequence fitting, starting with the Hyades cluster. The zero point of the life luminosity relation for novae has been estimated from the expansion parallaxes of galactic novae by Schmidt-Kaler (1957); this is essentially the same as method (b), and is subject to the same type of systematic error.

These three methods give consistent results. Consistency may be taken as evidence for correctness; but consistency also implies that, if one method is in error, then all three methods are in error.

Hodge and Wallerstein (1966) suggested that the distance modulus of the Hyades cluster might be in error, and in need of upward revision by 0.4 magnitudes. They pointed out that small systematic errors in the proper motions (which are known to occur) can lead to a large error in the convergent point. They showed that certain anomalies in the masses of binary stars could be removed by a revision of the distance, and that the accepted distance disagreed with the mean trigonometric parallax.

Their conclusions were strongly criticized by Wilson (1967), who showed that the calibration of the H and K lines, using both trigonometric parallaxes, method (b), and using the Sun-Hyades data, is consistent. In addition, the calibration of the H and K lines is consistent with the absolute magnitudes of the MK luminosity classes.

1.2 What is wrong with least squares?

We shall now see that method (b), least squares fitting to absolute magnitudes calculated from trigonometric parallaxes, produces serious systematic errors. Since Wilson has shown that method (b) gives results consistent with the accepted distance modulus of the Hyades cluster, we may conclude that the Hyades distance modulus is in need of revision.

The estimated absolute magnitudes, $\hat{M}$, are calculated from the logarithm of the observed parallax π_0, and the apparent magnitude m: $\hat{M} = m + 5 + 5 \log_{10}(\pi_0)$.

Naturally, this is not possible when the observed parallax is negative, and in addition the errors are large for small parallaxes. Therefore only those stars are selected whose observed parallaxes are above some limit π_c. Wallerstein (1967) pointed out thatthis practice leads to the preferential selection of those parallaxes with large <u>positive errors</u>: the absolute magnitude is therefore overestimated more often than it is underestimated.

The fact that the absolute magnitude is proportional to the logarithm of the parallax also introduces a systematic error. To see the way in which this works, consider a star with observed parallax $0''.040$. The true parallax is as likely to be $0''.055$ as $0''.025$. Below the distance modulus for all three cases are given.

π_o	$m-\hat{M}$	π_{true}	$m-M_{true}$	$\hat{M}-M_{true}$
$0''.040$	+2·0	$0''.055$	+1·3	-0·7
		$0''.025$	+3·0	+1·0

We see that the absolute magnitude is as likely to be over-estimated by +1·0 magnitudes as underestimated by -0·7: we may expect that, on average, the absolute magnitude is over-estimated.

The method of least squares takes no account of this bias, and so will produce a systematic error. To remedy this, we must first consider how likely the various possible values of the true absolute magnitude are, based on the available data: that is, we must find the likelihood function. We shall see how this is done in Chapter 3.

1.3 What is wrong with reduced parallaxes?

The fact that we cannot justifiably average the logarithms of parallaxes suggests that all would be well if we could average the parallaxes themselves in some way. This is done in the method of reduced parallaxes, used by Roman (1952) to estimate the mean absolute magnitudes of the MK luminosity classes. The observed parallaxes of stars in each class are multiplied by a factor which will reduce them to the value which they would have if the apparent magnitude were +5:

$$\pi_{red} = \pi_0 \exp_{10}\left(\frac{m-5}{5}\right)$$

The reduced parallaxes are then averaged, and the mean absolute magnitude calculated:

$$\overline{M} = 5 \log_{10} \langle \pi_{red} \rangle_{Av} + 10$$

Unfortunately, the parallaxes of faint stars are multiplied by a larger factor than the parallaxes of bright stars. When the reduced parallaxes are averaged, the faint stars are weighted more heavily than the bright stars, and so the mean absolute magnitude of the class is overestimated.

I have been able to show that, if the absolute magnitudes of the class are normally distributed, then the systematic error in the mean absolute magnitude is:

$$\Delta M = \frac{var(M)}{10 \log_{10}(e)}$$

where var (M) is the variance of the absolute magnitudes. (The variance is the square of the dispersion). The dwarf stars, class V, should have a small dispersion in absolute magnitude, and hence the calibration for this class should have only a small error. The giant stars, Class III, have a large dispersion in absolute magnitude, and so the systematic error for Class III should be more serious. If we assume a dispersion of 1·5 magnitudes, we obtain a systematic error of about 0·5 magnitudes.

1.4 Could we use non-empirical calibrations?

We have detected some systematic errors in the statistical analysis, but this is no guarantee that further systematic errors are still undetected. In view of all this uncertainty, it is tempting to think that a theoretical analysis might solve our problems. Thus, a model atmosphere might be fitted to the spectrum to give effective temperature, surface gravity, and abundances: then a stellar structure calculation might be made to give the mass and luminosity. (Strictly, a stellar evolution calculation, since the abundances in the core are changed by burning). Alas, there are far too many uncertainties for this approach to be even remotely possible in the forseeable future.

For the late type stars, we cannot find the helium abundance, and hence we cannot even find the mean molecular weight. For the late type dwarfs, uncertainties in the mixing length theory prevent us from determining the depth of the convection zone. For the extended atmospheres of giant stars, dilution effects are likely to make the assumptions of L.T.E. a poor approximation. Finally, such "simple" data as the equation of state, nuclear reaction rates, and opacities, are far from well known.

Considerations such as these show that theory is simply not powerful enough to predict every astrophysical quantity with assurance. Indeed, if the history of science has one lesson for us, it is this: <u>observation is primary</u>, <u>theory is secondary</u>.

1.5 The way to better estimation.

In practice it is never sufficient to produce just the value of an estimate: it is necessary to know how accurate this estimate is. Ideally, a calibration of a luminosity criterion should tell us how likely any suggested value of the luminosity is, based on the value of the criterion observed. The function that tells us how likely the luminosity is, given the criterion, is the likelihood function. The nature of the likelihood function, and its application to statistical estimation, is discussed in Chapter 2.

In order to find the likelihood of the luminosity based on a luminosity criterion, we must start with the likelihood of the distance, based on the available astrometric data. It is not sufficient just to use trigonometric parallaxes: the proper motions contain essential information, and they should be included in the likelihood of the distance. The method of finding the likelihood based on both the trigonometric parallax and on the proper motion is discussed in Chapter 3.

We then have to find the distribution of the criterion as a function of the luminosity: this gives us the likelihood of the luminosity based on the criterion. The basic idea which we shall use is described in section 2.4, and the detailed methods of calculation are given in Chapter 4. The problem is basically one of regression from error prone observations, and no practical solution has been available previously. Chapter 4 is therefore a contribution to theoretical statistics.

The ideas of Chapters 3 and 4 are applied in Chapter 5 to some examples. In particular, we shall see that there is a systematic error in the method of least squares when applied to the calibration of the H and K emission lines, so that the luminosities of the giant stars are underestimated in the conventional calibration by 0·6 magnitudes.

CHAPTER 2

THE BAYESIAN APPROACH TO STATISTICAL ESTIMATION

2.0 The approach to statistics which is found in most textbooks was developed by Neyman and Pearson with the explicit intention of avoiding any use of initial probabilities and of Bayes's theorem. This imposes a severe limitation on the methods which can be used, so that many problems become difficult to solve. In particular, the problem of eliminating "nuisance parameters", which we discuss in section 2.4, is intractable using the standard approach, but is quite easy once Bayes's theorem is applied.

The Bayesian approach to statistics has a respectable history, going at least back to Laplace. The difficulty has always been that of finding appropriate initial probabilities: Laplace's solution leads to contradictions. A fully consistent theory has been developed by Jeffreys (1961), but it is still considered unorthodox.

The Bayesian approach is likely to be unfamiliar to many astronomers, and so the present chapter reviews the basic ideas which are necessary to an understanding of Chapters 3 and 4.

2.1 The meaning of likelihood.

All the information relevant to a parameter q which is given by the observations x is contained in the likelihood of q based on x, $L(q;x)$. This is the likelihood principle, (Barnard, 1952), and it implies that all statistical inference should properly start from the likelihood.

The likelihood of q based on x is proportional to the probability of x conditional on q:

$$L(q;x) \propto P(x|q).$$

Often x will be a continuous variate, or set of variates. Then we use the probability that the variate is found in the range $(x, x+dx)$, which is

$$P(x < X < x+dx \mid q) = P(dx \mid q)$$

so that

$$L(q;x) \propto P(dx|q).$$

As an example, let us consider the likelihood of the distance modulus m-M of a star, based on the observed parallax π_0. We assume that the observed parallax has a Gaussian distribution, with mean equal to the true parallax, and variance σ_π^2:

$$L(m-M;\pi_0,\sigma_\pi) \propto P(d\pi_0|m-M,\sigma_\pi)$$

$$= \frac{d\pi_0}{\sqrt{2\pi}\,\sigma_\pi} \exp\left[-\frac{1}{2\sigma_\pi^2}\left\{\pi_0 - \exp_{10}\left(-\frac{m-M+5}{5}\right)\right\}^2\right]$$

What are we to do about the factor $d\pi_0$? At this point we notice that the likelihood is defined by proportionality, not by equality: any factor which is independent of the parameter m-M is irrelevant to the likelihood. So we may simply ignore $d\pi_0$ and write

$$L(m-M;\pi_0,\sigma_\pi) \propto \exp\left[-\frac{1}{2\sigma_\pi^2}\left\{\pi_0 - \exp_{10}\left(-\frac{m-M+5}{5}\right)\right\}^2\right]$$

However, we must be careful when we do this: in more complicated cases, if we drop off factors too soon, we may inadvertently drop off something that was not truly independent of the wanted parameter. Thus we shall often carry the differential along in our calculation of the likelihood, abandoning it at a late stage when the final result is near.

2.2 The method of maximum likelihood.

A good estimate of a parameter q is one which is likely to be close to the true value. The maximum likelihood estimate of q is the value which makes the likelihood a maximum. Recall that the likelihood of a parameter is proportional to the probability of the observations conditional on the parameter. Clearly, an estimate which gives the observations a high probability is more plausible than one which gives the observations a low probability; this is the rationale of the method of maximum likelihood.

It may help to consider a simple example. Let us look at the problem of regression in the absence of errors. Suppose we know the values of a luminosity criterion and the absolute magnitude for N stars: $\rho_1 M_1 \ldots \rho_N M_N$. We assume that the luminosity criterion has a Gaussian distribution with parameters h, a, and b, such that

$$P(d\rho \,|\, M; h,a,b) = \left(\frac{h}{2\pi}\right)^{1/2} d\rho \exp\left[-\frac{h}{2}(\rho - a - bM)^2\right].$$

The likelihood of h, a, b is the joint probability of the observations, so

$$L(h,a,b;\rho_1 M_1 \ldots \rho_N M_N) \propto P(d\rho_1 \ldots d\rho_N \,|\, M_1 \ldots M_N; h,a,b)$$

$$= P(d\rho_1 \,|\, M_1; h,a,b) \times \ldots \times P(d\rho_N \,|\, M_N; h,a,b)$$

$$\propto h^{N/2} \exp\left[-\frac{h}{2}\sum_{i=1}^{N}(\rho_i - a - bM_i)^2\right]$$

Since we have a product of probabilities, it is convenient to work with the logarithm of the likelihood:

$$\ln(L) = \frac{N}{2}\ln(h) - \frac{h}{2}\sum_{i=1}^{N}(\rho_i - a - bM_i)^2$$

The maximum likelihood estimates $\hat{h}\ \hat{a}\ \hat{b}$ can be found by setting the derivatives of the log likelihood equal to zero:

$$\frac{\partial \ln L}{\partial a}\bigg|_{\hat{h}\hat{a}\hat{b}} = 0 = \hat{h}\sum_{i=1}^{N}(\rho_i - a - bM_i)$$

$$\frac{\partial \ln L}{\partial b}\bigg|_{\hat{h}\hat{a}\hat{b}} = 0 = \hat{h}\sum_{i=1}^{N}M_i(\rho_i - a - bM_i)$$

$$\frac{\partial \ln L}{\partial h}\bigg|_{\hat{h}\hat{a}\hat{b}} = 0 = \frac{N}{2}\hat{h}^{-1} - \frac{1}{2}\sum(\rho_i - a - bM_i)^2$$

The first two equations are a pair of linear equations which can be solved for $\hat{a}$ and $\hat{b}$; once these are found, $\hat{h}$ can be obtained at once from the last equation.

2.3 Initial and final probabilities: Bayes's theorem.

The method of maximum likelihood gives us an estimate which we hope will lie close to the true value of the parameter, but it does not tell us within what distance from the estimate we may reasonably expect the parameter to be. To answer this sort of question, we need to know the probability distribution of the parameter based on the observation, $P(dq|x)$. What we usually have is the likelihood $L(q;x)$ which is the probability distribution of the observation based on the parameter, $P(dx|q)$. How do we find $P(dq|x)$ from $P(dx|q)$?

When we ask for the probability distribution of a parameter, we cannot hope to base it on the frequency with which the parameter occurs in a given range. A particular star either does or does not have an absolute magnitude between +1 and +2; the parameters of the calibration of a luminosity criterion either do or do not lie within stated limits; the question is a matter of truth, not of frequency. We do not know for certain where the truth lies, but, based on the available evidence, we may hold various degrees of reasonable belief in the various possibilities. When we say that a certain star has a high probability of being a member of the Hyades cluster, we are expressing a high degree of belief in its membership. When we find that the radial velocity of the star is very different from the radial velocity of the Hyades cluster, we change our degree of belief in its membership: we now say it has a low probability of membership.

The last example illustrates very nicely the function of observational evidence: it modifies our initial belief. If the probability of the radial velocity, based on the assumption of cluster membership, is small, then our final belief will be weak compared with our initial belief. If on the other hand, the probability of the observed radial velocity based on cluster membership is large, then our final belief in membership is stronger than our initial belief. Now, the probability of the observed radial velocity, based on cluster membership, is the likelihood of cluster membership. Thus final belief $\propto$ likelihood x initial belief, or, remembering that degrees of beliefs are probabilities:

Final probability $\propto$ likelihood x initial probability.

This is the essential content of Bayes's theorem.

To apply Bayes's theorem to the estimation of a parameter, we have to make some judgement J about the appropriate initial distribution $P(dq|J)$. Then the final distribution is given by

$$P(dq|x,J) \propto L(q;x)P(dq|J).$$

The proportionality constant is found by using the fact that q must have some value,. so that $$\int_q P(dq|x,J) = 1$$

from which we obtain

$$P(dq|x,J) = \frac{L(q;x)P(dq|J)}{\int_q L(q;x)P(dq|J)}$$

Let us illustrate Bayes's theorem with a simple example. Imagine that we have an observed parallax π_0 of standard deviation σ_π so that the likelihood of the distance r of a star is

$$L(r;\pi_0,\sigma_\pi) \propto \exp\left[-\frac{1}{2}\left(\frac{r^{-1}-\pi_0}{\sigma_\pi}\right)^2\right]$$

Otherwise, all that we know is that the distance r is less than the distance to the surface of the galaxy, r_S . Within the galaxy, the initial probability is proportional to the amount of space available, so that

$$P(dr \mid r<r_s) \begin{array}{l} \propto r^2 dr\ , \ r<r_s \\ = 0 \quad , \ r>r_s \end{array} \Bigg\} .$$

Then the final distribution, based on the observed parallax and on our assumption that the star is inside the galaxy, is

$$P(dr \mid \pi_0, \sigma_\pi ; r<r_s) = \frac{L(r;\pi_0,\sigma_\pi)\,P(dr \mid r<r_s)}{\int_r L(r;\pi_0,\sigma_\pi)\,P(dr \mid r<r_s)}$$

$$= \frac{\exp\left[-\frac{1}{2}\left(\frac{r^{-1}-\pi_0}{\sigma_\pi}\right)^2\right] r^2 dr}{\int_{r=0}^{r_s} \exp\left[-\frac{1}{2}\left(\frac{r^{-1}-\pi_0}{\sigma_\pi}\right)^2\right] r^2\, dr} \ , \ r<r_s ,$$

$$= 0 , \ r>r_s .$$

We can see from this example why factors independent of r, such as $d\pi_0 / \sigma_\pi$, are irrelevant to the likelihood. Any factor occurs in both the numerator and the denominator; if it is independent of r, the factor in the denominator can be taken outside the integral, where it cancels the factor in the numerator.

2.4 The elimination of nuisance parameters.

In the example of regression considered in section 2.2, we used the likelihood based on known values of the absolute magnitude M. In practice, we do not know the absolute magnitude; we can only estimate it from observations of the apparent magnitude m, the parallax π_0, and the proper motion μ. Thus we know the form of the distribution

$$P(dp \mid M; h, a, b)$$

and the likelihood

$$L(M; m, \pi_0, \mu)$$

and we need to form the distribution

$$P(dp \mid m, \pi_0, \mu; h, a, b)$$

which is used for forming the likelihood of the parameters based on observations of m, π_0, μ. Essentially, we wish to get rid of M, which is a parameter whose value we do not know and, for our present purposes, we do not want to know. M is a "nuisance parameter"; the elimination of nuisance parameters is a common problem in statistics. In orthodox, non-Bayesian statistics, the problem is extremely difficult; if we allow ourselves to use Bayes's theorem, the problem becomes easy.

It is simpler at this point to put the problem in more general terms. We know the distribution $P(dy \mid \xi)$ of a variate y based on a parameter ξ, but we do not know the value of ξ. We have an observation x which gives information about ξ expressed by the likelihood $L(\xi; x)$. We wish to find the distribution $P(dy \mid x)$ of y conditional on the observation x.

First we make a judgement J about the initial distribution $P(d\xi|J)$ of the parameter ξ. Then we use Bayes's theorem to find the final distribution:

$$P(d\xi\,|\,x,J) = \frac{L(\xi;x)\,P(d\xi|J)}{\int_\xi L(\xi;x)\,P(d\xi|J)}$$

The product of the distribution of y conditional on ξ and the distribution of ξ gives us the joint distribution of y and ξ:

$$P(dy\;d\xi\,|\,x,J) = P(dy\,|\,\xi)\,P(d\xi\,|\,x,J)\,^{*}$$

On integrating over all ξ we obtain the marginal distribution of y conditional on x:

$$P(dy\,|\,x,J) = \int_\xi P(dy\;d\xi|x,J) = \int_\xi P(dy|\xi)\,P(d\xi|x,J)$$

To eliminate the absolute magnitude M, we first make a judgement J about the initial distribution, and then form the final distribution based on J and m, π_0, μ:

$$P(dM\,|\,m,\pi_0,\mu,J) = \frac{L(M;m,\pi_0,\mu)\,P(dM|J)}{\int_M L(M;m,\pi_0,\mu)\,P(dM|J)}$$

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*Here we assume that x is irrelevant to y once ξ is given, as is true in the application we are considering.

We then use the final distribution to eliminate M from the distribution of ρ :

$$P(d\rho \mid m, \pi_0, \mu, J; h, a, b) = \int_M P(d\rho \mid M; h, a, b)\, P(dM \mid m, \pi_0, \sigma_\pi, J)$$

This is the distribution we need for calculating the likelihood of h, a, b.

2.5 <u>The choice of initial distribution</u>.

What judgement J should we make about the appropriate form of the initial distribution $P(dM \mid J)$? Generally, it is only possible to provide a rough and ready guess, and for this reason the use of initial distributions is controversial. Fortunately, the exact form of the initial distribution is not at all critical; two quite different judgements will give very similar results.

In astronomical applications, it is often possible to provide simple distributions which, at least over a limited range, seem to be good representations of reasonable judgement. We have to consider, not only the nature of the objects under study, but also the way in which the sample was obtained.

If we include all stars brighter than some fairly bright apparent magnitude, so that we are sampling local stars, then the initial probability of the distance is proportional to the space available, so that

$$P(dr \mid m > m_0) \propto r^2 dr,$$

$$P(dM \mid m > m_0) \propto \exp_{10}\left(-\tfrac{3}{5}M\right) dM.$$

Usually, when stars are selected for the calibration of a luminosity criterion, a deliberate attempt is made to select stars of widely different absolute magnitudes. This leads to a distribution which is approximately uniform, so that

$$P(dM \mid J) \propto dM$$

is a reasonable initial distribution.

2.6 The asymptotic form of the final distribution.

Bayes's theorem tells us how to find the final distribution from the likelihood and the initial distribution. Once we have the final distribution of a set of parameters, we can in principle calculate the probability that the parameters lie in any given interval. In practice the calculation of the likelihood can be extremely tedious, especially when the amount of data on which the likelihood is based is large. Fortunately, when the amount of data is large, the likelihood approaches an asymptotic distribution of rather simple form. Moreover, as the amount of data is increased, the likelihood becomes a sharper and sharper peak; over the narrow range in which the likelihood is large, the initial distribution scarcely changes at all; and so the final distribution is proportional to the likelihood, independently of what the initial distribution may be.

Suppose we have M parameters $a_1 \ldots a_M$, for which the maximum likelihood estimates are $\hat{a}_1 \ldots \hat{a}_M$. The final distribution based on N data points has the asymptotic form:

$$P(da_1 \ldots da_M \mid N \text{ data points})$$

$$= \left(\frac{\mathrm{Det}(H)}{2\pi}\right)^{1/2} da_1 \ldots da_M \exp\left[-\frac{1}{2}\sum_{k,\ell} H_{k\ell}(a_k-\hat{a}_k)(a_\ell-\hat{a}_\ell) + O(N^{-1/2})\right]$$

By O ($N^{-1/2}$) we mean a quality that is small of the order of $N^{-1/2}$. The matrix H is the precision matrix of the parameters, and can most simply be calculated from the second derivatives of the log likelihood, evaluated at the maximum likelihood values:

$$H_{k\ell} = -\left.\frac{\partial^2 \ln L}{\partial a_k \, \partial a_\ell}\right|_{a=\hat{a}}$$

The dispersion of the parameters can readily be found from the precision matrix--the larger the precision, the smaller the dispersion. The covariance matrix, (which measures the means of the products of the deviations from the mean), is the reciprocal of the precision matrix:

$$\mathrm{cov}(a_k, a_\ell) = E\left[(a_k-\hat{a}_k)(a_\ell-\hat{a}_\ell)\right] = (H^{-1})_{k\ell}$$

In particular, the standard deviation of the k^{th} parameter is:

$$\sigma_k = \sqrt{E\left[(a_k - \hat{a}_k)^2\right]} = \sqrt{(H^{-1})_{kk}}$$

Thus the asymptotic form of the final distribution provides useful estimates of the dispersion of the parameters. From these we may judge how near to our estimates we may reasonably expect the parameters to lie.

Notice that the maximum likelihood estimates lie in the center of the asymptotic final distribution. For small amounts of data, the exact final distribution may be significantly different from the asymtotic final distribution, and the maximum likelihood estimates are not necessarily "best" estimates. But for large amounts of data, when the asymptotic final distribution is a close approximation to the exact final distribution, the maximum likelihood estimates are "best" for any reasonable criterion of goodness. A method of estimation which gives estimates systematically different from maximum likelihood estimates for large amounts of data should be considered systematically in error. Section 5.1 will provide an example.

CHAPTER 3

LIKELIHOOD FUNCTIONS FOR ASTROMETRIC DISTANCES

3.0 The likelihood function $L(\pi_t;\pi_o)$ has the symmetric bell shaped form known as the Gaussian distribution, except that it is truncated for $\pi_t < 0$. (See figure 3.0.1). When we transform the parallax to the distance modulus, we obtain a quite asymmetric curve for the likelihood $L(m-M;\pi_o)$. (See figure 3.0.2). The peak is at the value $(m-M)_o$ corresponding to the observed parallax. Below $(m-M)_o$, the likelihood drops off sharply to zero; above $(m-M)_o$, the likelihood tails off gradually to a finite value which is equal to $L(\pi_t=0;\pi_o)$.

Because the likelihood remains finite even for infinite distance moduli, the parallax does not set any upper limit to the distance. The distance modulus $(m-M)_o$ may well be a gross underestimate, but it is unlikely to be a gross overestimate. Any method of estimation, such as least squares fitting, which fails to take this into account will produce systematic errors; distance moduli will, on average, be underestimated, and absolute magnitudes overestimated.

In order to protect ourselves against gross underestimates of the distance, we need some data which will set

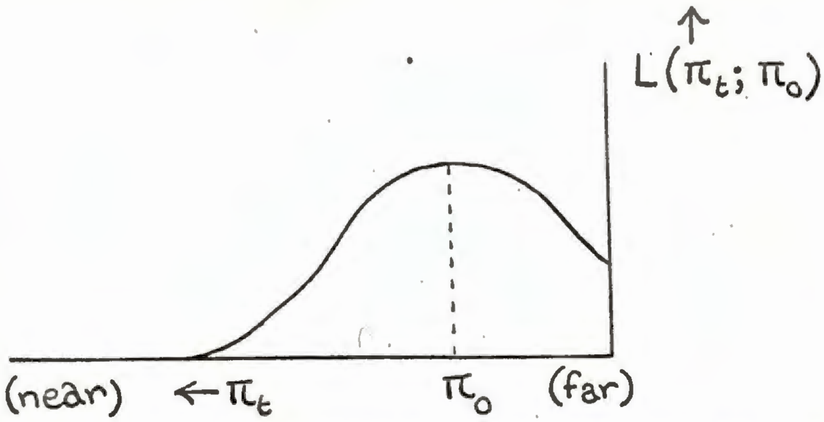

Figure 3.0.1 Likelihood of true parallax, based on observed parallax.

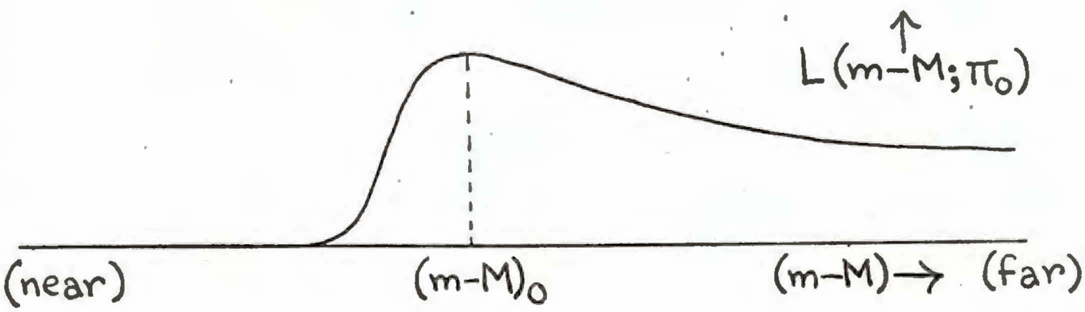

Figure 3.0.2 Likelihood of distance modulus, based on observed parallax.

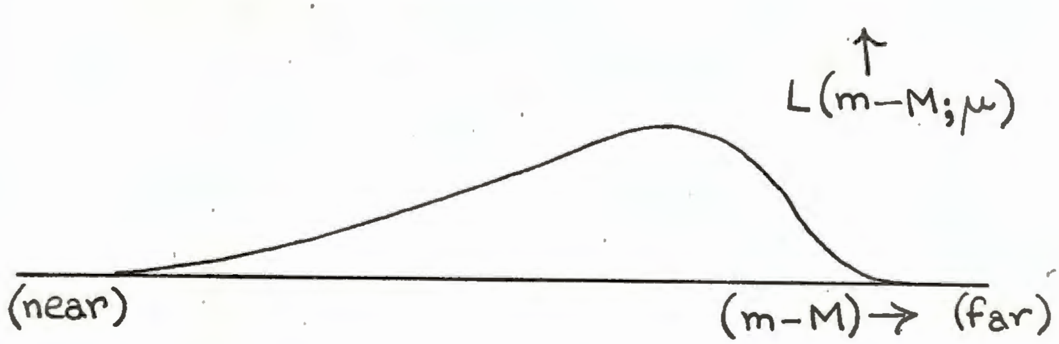

Figure 3.0.3 Likelihood of distance modulus, based on proper motion.

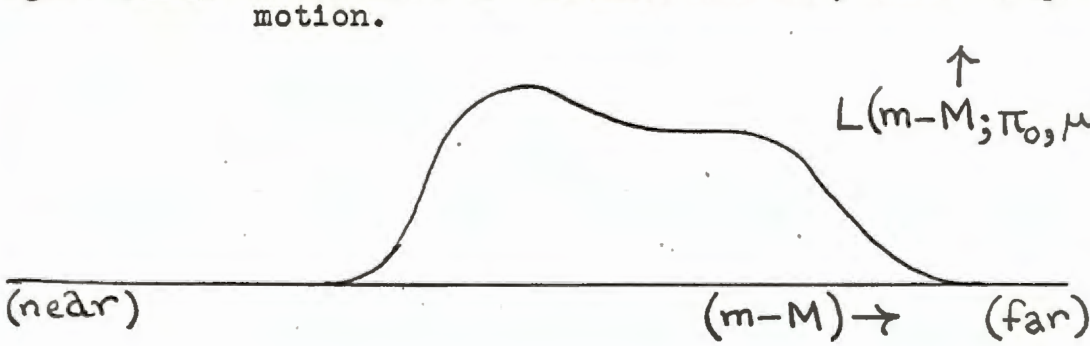

Figure 3.0.4 Likelihood of distance modulus, based on both parallax and proper motion.

an effective upper limit to the distance. Fortunately, an upper limit is provided by the proper motion. A star with a finite proper motion cannot be at an infinite distance, so the likelihood $L(m-M;\mu)$ based on the proper motion falls to zero at large distance. (See figure 3.0.3). The calculation of the proper motion contribution to the likelihood is discussed in section 3.1.

When both the parallax contribution and the proper motion contribution to the likelihood are combined, we obtain a likelihood which falls to zero for both high and low distances. (See figure 3.0.4). This likelihood is generally strongly asymmetric, and the shape will depend on the dispersions of the parallax and proper motion contributions. Any errors in the estimated dispersions will cause systematic errors to appear in estimates of the absolute magnitudes. As we shall see in section 3.2, the quoted probable errors of the parallaxes bear little relation to reality. New values are given for the standard deviations of the parallaxes for nine observatories.

We saw in section 1.2 that the selection of stars with large observed parallaxes leads to a systematic error. This can be overcome by proper adjustment of the likelihood which we discuss in section 3.3

3.1 The proper motion distance.

Let us first imagine that we have the velocity distribution expressed in terms of the components $v_\alpha\ v_\delta\ v_r$ in the directions of increasing right ascension, declination, and radial distance, so that the probability element is $P(dv_\alpha\, dv_\delta\, dv_r \mid \text{velocity data})$. If the star is at distance r then

$$v_\alpha = \mu_\alpha r,^{*} \quad v_\delta = \mu_\delta r\,; \quad dv_\alpha = \mu_\alpha dr,\ dv_\delta = \mu_\delta dr.$$

so that the proper motion contribution to the likelihood is given by

$$\begin{aligned} &L(r;\mu_\alpha \mu_\delta v_r, \text{velocity data}) \\ &\quad \propto P(dv_\alpha\, dv_\delta\, dv_r \mid \text{velocity data}) \\ &\quad = r^2 d\mu_\alpha d\mu_\delta dv_r\, D(v_\alpha = \mu_\alpha r, v_\delta = \mu_\delta r, v_r \mid \text{velocity data}) \end{aligned}$$

The velocity distribution is in fact expressed in terms of the components $V_1\ V_2\ V_3 = V_\Pi\ V_\Theta\ V_Z$ in the direction of the galactic centre Π, the galactic north pole Z, and the galactic rotation Θ. The distribution has been assumed to be Gaussian, so that

$$\begin{aligned} &P(dV_1\, dV_2\, dV_3 \mid U, H) \\ &\quad \propto dV_1\, dV_2\, dV_3 \exp\Big[-\tfrac{1}{2}\sum_{N=1,2,3} H_N (V_N - U_N)^2\Big] \end{aligned}$$

---

*Note that $\mu_\alpha = \mu'_\alpha \cos(\delta)$.

In view of the different ages and kinematic properties of stars of the same spectral type and luminosity class, it is unlikely that a Gaussian form provides a good approximation to the true velocity distribution. However, until we can obtain a reliable estimate of the velocity distribution, we must continue to use the Gaussian form.

The available data on the parameters $U_\pi U_\Theta U_Z$ which are the mean velocity components, relative to the sun, and H_π H_Θ H_Z which are the corresponding precisions, is summarized by Delhaye (1965). The parameters for the late type stars show no significant dependance on spectral type, and so I have taken weighted averages over the spectral types for the various luminosity classes. Table 3.1.1 shows the results. Note that

i) The units are astronomical units per year,

ii) The axis Π is towards the galactic center, whereas Delhaye takes Π to point to the anticentre,

iii) My figures are for the velocity of the stars relative to the sun; Delhaye's velocity of the sun relative to the stars.

iv) The precisions are the reciprocals of the squares of the dispersions: $H_N = 1/\sigma_N^2$.

	Super Giant	Giant	Sub Giant	Dwarf	Sub dwarf
U	-1.667	-1.661	-1.688	-2.098	0.0422
U	-2.468	-3.382	-5.907	-3.632	-28.692
U	-1.371	-1.377	-1.688	-1.353	-2.004
H	0.1329	0.02562	0.01215	0.02523	0.002247
H	0.2773	0.05293	0.03082	0.06916	0.003994
H	0.4585	0.08756	0.03901	0.09231	0.08987

Table 3.1.1: Parameters of the stellar velocity distribution.

To use this distribution, we have to transform the velocity components $V_\pi\ V_\Theta\ V_Z$ into the components $v_\alpha\ v_\delta\ v_r$; this can be done using the direction cosines C_{Nm} where, for example $C_{12} = C_{\pi\delta} = \cos(\widehat{\pi\delta})$.

The calculation of these direction cosines is a straight-forward application of spherical trigonometry; it is discussed in Appendix 3A.

We have

$$V_N = \sum_m C_{Nm} v_m$$

so that the distribution of velocities becomes

$$P(dv_1\, dv_2\, dv_3 \mid U, H)$$

$$\propto dv_1\, dv_2\, dv_3 \exp\left[-\frac{1}{2}\sum_N H_N \left(\sum_m C_{Nm} v_m - U_N\right)^2\right]$$

Substituting $v_1 = \mu_\alpha r$, $v_2 = \mu_\delta r$, $v_3 = v_r$

we have the likelihood in the form

$$L(r; U, H, \mu_\alpha, \mu_\delta, v_r) \propto P(r d\mu_\alpha, r d\mu_\delta, dv_r \mid U, H)$$

$$\propto r^2 \exp\left[-\frac{1}{2}\left\{\begin{array}{l} r^2 \sum_N H_N (C_{N1}\mu_\alpha + C_{N2}\mu_\delta)^2 \\ -2r \sum_N H_N (C_{N1}\mu_\alpha + C_{N2}\mu_\delta)(U_N - C_{N3} v_r) \\ + \text{term independent of } r \end{array}\right\}\right]$$

If we put

$$h_r = \sum_N H_N (C_{N1}\mu_\alpha + C_{N2}\mu_\delta)$$

$$h_r r_\mu = \sum_N H_N (C_{N1}\mu_\alpha + C_{N2}\mu_\delta)(U_N - C_{N3} v_r)$$

then the likelihood becomes simply

$$L(r; h_r, r_\mu)$$

$$\propto r^2 \exp\left[-\tfrac{1}{2} h_r (r - r_\mu)^2\right]$$

Here r_μ is the proper motion distance, and h_r is its precision.

Notice that the expression for $h_r r_\mu$ contains the radial velocity v_r; this is a result of the anisotropy of the velocity distribution. If it happens that the radial velocity is not available, then we must eliminate it from the velocity distribution by integrating

$$P(dv_\alpha\, dv_\delta \mid U, H) = \int_{v_r} P(dv_\alpha\, dv_\delta\, dv_r \mid U, H)$$

The resulting distribution has a slightly lower precision. We obtain for h_r and $h_r r_\mu$

$$h_r = \sum_N H_N \left(C_{N1}\mu_\alpha + C_{N2}\mu_\delta\right)^2 - \frac{\left\{\sum_N H_N C_{N3}\left(C_{N1}\mu_\alpha + C_{N2}\mu_\delta\right)\right\}^2}{\sum_N H_N C_{N3}^2}$$

$$h_r r_\mu = \sum_N H_N U_N \left(C_{N1}\mu_\alpha + C_{N2}\mu_\delta\right) - \frac{\sum_L H_L U_L C_{L3} \sum_M H_M C_{M3}\left(C_{M1}\mu_\alpha + C_{M2}\mu_\delta\right)}{\sum_N H_N C_{N3}^2}$$

There is a small correction for the effect of galactic rotation: this is described in Appendix 3B. For small proper motions we should also take into account the random errors; this is discussed in Appendix 3C. (As the likelihood then takes a very much more complex form, we shall generally ignore the effect of random errors. For nearby class III stars, this is permissible, but not for class II or I stars.)

3.2 The errors of trigonometric parallaxes.

The likelihood of the true parallax π given an observed parallax π_0 of precision h_π is $L(\pi;\pi_0, h_\pi) \propto \exp\left[-\frac{h_\pi}{2}(\pi-\pi_0)^2\right]$. The parallaxes given on the right hand pages of the Yale parallax catalogue are relative parallaxes obtained by various observatories. The several relative parallaxes for each star must be averaged, and the result reduced to absolute. The absolute parallaxes given on the left hand pages of the Yale catalogue are not the best possible estimates; we consider how to obtain better estimates in this section.

The Yale catalogue weights the parallaxes according to the quoted "probable errors". However, these internal errors bear little relation to the external errors, and it is the external errors we should consider.

Dahlgren (1960) has compared the r.m.s. differences between independent parallax measurements of the same star by different observatories. The remarkable fact emerges from his figures that there is very little correlation between the quoted probable errors and the r.m.s. differences. It is difficult to understand why this should be so, but the figures seem indisputable. The following table, quoted from Dahlgren's paper, gives the r.m.s. difference between McCormick and Allegheny parallaxes as a function of the quoted probable error.

p.e	r.m.s. difference	
5-7	16.8	±2.8
8	17.6	±2.0
9	18.0	±1.2
10	19.7	±1.6
11	19.3	±1.1
12	23.8	±1.6
13	20.1	±1.4
14	21.0	±1.7
15	17.9	±2.4
16	21.0	±2.7
17-22	20.0	±3.1

Table 3.2.1 R.m.s. differences, McCormick-Allegheny

A least squares fit to these data gives
$(\text{r.m.s.})^2 = 388.7 + 0.47\ (\text{p.e.}^2 - 136)$. This shows that for all practical purposes the quoted probable errors can be ignored; they have no predictive value.

Dahlgren has tabulated the r.m.s. differences between the parallaxes obtained by 10 observatories up to about the time of the 2nd edition of the Yale catalogue. In addition, Vasilevskis (1964) has tabulated the root mean square differences between parallaxes in the supplement to the Yale catalogue and the values in the 2nd edition.

Using these data, I have obtained maximum likelihood estimates of the variances of the parallaxes of nine observatories. (Dearborn parallaxes show a strong magnitude effect, and so were ignored in the solution). The method of solution is described in Appendix 3D; the results are tabulated in Table 3.2.2 below. The standard deviations are given in units of 0".001, while the precisions, which are used in actual computations, are quoted in units of $('')^{-2}$.

These precisions should be used to weight the relative parallaxes of L different observatories;

$$h_\pi = h_1 + \dots + h_L$$
$$\pi_0 = (h_1\pi_1 + \dots + h_L\pi_L)/h_\pi$$

Observatory	Standard deviation (units of 0".001)	Precision (units of $('')^{-2}$)
Allegheny	12.59 ± 0.45	6311
Cape	20.39 ± 0.66	2405
Greenwich	14.65 ± 1.26	4660
McCormick	16.39 ± 0.43	3724
Mt. Wilson	24.56 ± 1.01	1658
Sproul	16.12 ± 0.78	3851
Van Vleck	13.72 ± 1.16	5316
Yale	14.70 ± 0.60	4629
Yerkes	18.51 ± 0.78	2918

Table 3.2.2: Standard deviations of parallaxes

To convert the relative parallaxes to absolute, we have to add the mean parallax of the comparison stars. For most parallax determinations, the mean magnitude of the comparison stars is 11.2; for Allegheny stars 1451 and later (marked with an asterisk in the Yale catalogue) it is 12.2. Binnendijk (1943) has determined, from proper motions, the mean parallax of stars as a function of magnitude and galactic latitude b. I find that the following gives an adequate fit to his data:

$$\overline{p}_{11.2} = \exp(-6.106 + 0.8866 \sin(b)),$$

$$\overline{p}_{12.2} = \overline{p}_{11.2} / 1.303.$$

3.3 <u>The treatment of selection bias.</u>

There are systematic differences between the parallaxes of Northern and Southern observatories, as Dahlgren has shown. This indicates that parallaxes are subject to systematic errors. It is not possible to remove them, since we do not know where the zero point of the errors lies. Until we can find the zero point, it will be best to select those stars with the largest parallaxes.

In so doing, however, we also select those parallaxes with the largest positive errors, and so introduce a selection bias. Proper account of this must be taken by the likelihood in order to avoid the introduction of new systematic errors.

We shall often select only those stars with parallaxes greater than some critical value π_c. Then the probability element $P(d\pi_0 | \pi, h_\pi, \pi_c)$ must be normalized, not over the range $-\infty < \pi_0 < +\infty$ but over the range $\pi_c < \pi_0 < +\infty$:

$$P(d\pi_0 | \pi, h_\pi, \pi_c) = \frac{d\pi_0 \exp\left[-\frac{1}{2} h_\pi (\pi_0 - \pi)^2\right]}{\int_{\pi_0' > \pi_c} d\pi_0' \exp\left[-\frac{1}{2} h_\pi (\pi_0' - \pi)^2\right]}$$

Recalling the definition of the complementary error function

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{y > x} \exp(-y^2)\,dy$$

we obtain for the likelihood

$$L(\pi; \pi_0, h_\pi, \pi_c) \propto P(d\pi_0 | \pi, h_\pi, \pi_c)$$

$$\propto \frac{\exp\left[-\frac{1}{2} h_\pi (\pi_0 - \pi)^2\right]}{\operatorname{erfc}\left[\left(\frac{h_\pi}{2}\right)^{1/2} (\pi_c - \pi)\right]}$$

The effect of the term involving π_c is to skew the likelihood towards smaller values of π or larger distances.

It may happen that a selection procedure S is used which does not have a sharp cut off, but which is such that the probability of accepting a star for observation, $P(\text{accept} | \pi_0, S)$ is larger for large values of observed parallax than for small values. Then the likelihood is given by

$$L(\pi;\pi_0,h_\pi,S) \propto \frac{P(\text{accept}\,|\,\pi_0,S)\exp\left[-\frac{1}{2}h_\pi(\pi_0-\pi)^2\right]}{\int_{\pi_0'} P(\text{accept}\,|\,\pi_0',S)\exp\left[-\frac{1}{2}h_\pi(\pi_0'-\pi)^2\right]d\pi_0'}$$

If we are using someone else's observations, we may not have any initial knowledge of the selection procedure used. Then we must compare the frequency density of observed parallaxes in the sample, $D(\pi_0|S)$ with that obtained by random sampling of stars of the appropriate type, $D(\pi_0|R)$. The acceptance probability is given by

$$P(\text{accept}\,|\,\pi_0,S) \propto \frac{D(\pi_0|S)}{D(\pi_0|R)}$$

Appendix 3A Calculation of the direction cosines.

The angle between two directions $\alpha_1 \delta_1$ and $\alpha_2 \delta_2$ is found as the third side of a spherical triangle; the first two sides are $\pi/2 - \delta_1$ and $\pi/2 - \delta_2$, and the included angle is $\alpha_1 - \alpha_2$. The cosine formula gives

$$\cos\left(\widehat{\alpha_1\delta_1 \quad \alpha_2\delta_2}\right) = \cos(\pi/2-\delta_1)\cos(\pi/2-\delta_2) + \sin(\pi/2-\delta_1)\sin(\pi/2-\delta_2)\cos(\alpha_1-\alpha_2)$$
$$= \sin(\delta_1)\sin(\delta_2) + \cos(\delta_1)\cos(\delta_2)\cos(\alpha_1-\alpha_2).$$

To find the direction cosines C_{Nm} we need the right ascension and declination of the three galactic axes $\Pi\,\Theta\,Z$, and the three directions of motion $\sigma_\alpha\,\sigma_\delta\,\sigma_r$. These are given in the table below.

	α	δ
Π	4.622066	-0.504400
Θ	5.233810	+0.836313
Z	3.344922	+0.482874
σ_α	$\alpha + \pi/2$	$\alpha+\pi,\ \delta > 0$ $\alpha,\ \delta < 0$
σ_δ	0	$\pi/2 - \lvert\delta\rvert$
σ_r	α	δ

Table 3.A.1: Equatorial coordinates of galactic axes and directions of motion

Appendix 3B <u>Correction for galactic rotation</u>

Galactic rotation alters the mean stellar velocity by an amount proportional to the distance: thus we should replace U_N by $U_N + q_N r$. Since

$$V_N - (U_N + q_N r)$$
$$= (C_{N1}\mu_\alpha + C_{N2}\mu_\delta)r + C_{N3}\sigma_r - (U_N + q_N r)$$

the net effect is to replace

$$(C_{N1}\mu_\alpha + C_{N2}\mu_\delta) \quad \text{by} \quad (C_{N1}\mu_\alpha + C_{N2}\mu_\delta - q_N)$$

The values of q_N are given in terms of the Oort constants A and B by

$$q_\pi = (A-B)\sin(\ell)\cos(b) = (A-B)C_{23}$$
$$q_\theta = -(A+B)\cos(\ell)\cos(b) = (A-B)C_{13}$$
$$q_z = 0$$

In terms of the FK4 system, Fricke (1967) has obtained the following values of the Oort constants:

A = +0.0030 "/yr.

B = -0.0025 "/yr.

Appendix 3C <u>Effect of errors in the proper motions</u>.

So far we have neglected the possibility of errors in the proper motions. This is reasonable when the proper motions are considerably larger than their dispersions, as is true for the nearby giant stars. For stars which have annual proper motions of 0".010 or less, the effect of errors is important. Unfortunately, the proper motion then provides no effective upper limit to the distance, so in the present investigation we have largely avoided using stars with small proper motions.

However, it is at least worth considering how to obtain the likelihood when errors are appreciable, in order to be able to apply it when the need arises.

The likelihood which we derived in section 3.1 is based on the <u>true</u> values $\vec{\nu} = \{\nu_\alpha, \nu_\delta\}$ of the proper motion. We shall write it in the form:

$$L(r;\nu,\varpi,A) \propto r^2 \exp\left[-\tfrac{1}{2}(\vec{\nu}r-\vec{\varpi}).A.(\vec{\nu}r-\vec{\varpi})\right],$$

where the components of the matrix A are

$$A_{kl} = \sum_N H_N C_{Nk} C_{Nl},^*$$

and the components of the vector $\vec{\varpi}$ are given by

$$\sum_l A_{kl}\varpi_l = \sum_N H_N (U_N - C_{N3} v_r) C_{Nl}.^*$$

We assume that the final distribution of the values of the proper motion given the observed values $\vec{\mu}$ is a Gaussian distribution with precision matrix B:

---

* These relations serve to <u>define</u> A and $\vec{\varpi}$.

$$P(d\vec{\nu}\,|\,\vec{\mu},B) = \frac{d\nu_\alpha d\nu_\delta}{2\pi}\exp\left[-\tfrac{1}{2}(\overleftarrow{\nu}-\overleftarrow{\mu}).B.(\vec{\nu}-\vec{\mu})\right].$$

We want the likelihood based on the observed values $\vec{\mu}$; the true values $\vec{\nu}$ are nuisance parameters. We apply the method of section 2.4 for eliminating nuisance parameters;

$$L(r;\vec{\mu},\vec{\omega},A,B) \propto \int_\nu L(r;\vec{\nu},\vec{\omega},A)P(d\nu\,|\,\mu,B)$$

$$\propto r^2\int_\nu d\nu_\alpha d\nu_\delta \exp\left[-\tfrac{1}{2}\left\{(\overleftarrow{\nu}r-\overleftarrow{\omega}).A.(\vec{\nu}r-\vec{\omega})+(\overleftarrow{\nu}-\overleftarrow{\mu}).B.(\vec{\nu}-\vec{\mu})\right\}\right].$$

After performing the integration, we obtain the desired result:

$$L(r;\vec{\mu},\vec{\omega},A,B)$$

$$\propto r^2\exp\left[+\tfrac{1}{2}(\overleftarrow{\mu}B+r\overleftarrow{\omega}A).(B+r^2A)^{-1}.(A\vec{\omega}r+B\vec{\mu})\right].$$

Appendix 3D Estimation of the variances of trigonometric parallaxes.

If two independent measurements of the same parallax differ by $\Delta_{\ell m} = \pi_\ell - \pi_m$, and the variances of the two measurements are σ_ℓ^2 and σ_m^2, then the corresponding density is*

$$D(\Delta_{\ell m} | \sigma_\ell, \sigma_m) = \{2\pi(\sigma_\ell^2 + \sigma_m^2)\}^{-1/2} \exp\left[-\frac{1}{2}\frac{\Delta_{\ell m}^2}{\sigma_\ell^2 + \sigma_m^2}\right]$$

If there are $N_{\ell m}$ such independent measurements $\Delta_{\ell m 1} \dots \Delta_{\ell m N}$, their contribution to the likelihood of σ_ℓ, σ_m is given by

$$\ln L(\sigma_\ell, \sigma_m ; N_{\ell m}, \Delta_{\ell m 1} \dots \Delta_{\ell m N})$$
$$= -\frac{1}{2} \sum_{i=1}^{N_{\ell m}} \left\{ \ln(\sigma_\ell^2 + \sigma_m^2) + \frac{\Delta_{\ell m i}^2}{\sigma_\ell^2 + \sigma_m^2} \right\}$$
$$= -\frac{1}{2} \left\{ N_{\ell m} \ln(\sigma_\ell^2 + \sigma_m^2) + \frac{S_{\ell m}^2}{\sigma_\ell^2 + \sigma_m^2} \right\}$$

where

$$S_{\ell m}^2 = \sum_i \Delta_{\ell m i}^2$$

The total likelihood for all the parallax stations is given by

$$\ln L(\sigma_1 \dots \sigma_L ; S_{12} \dots, N_{12} \dots)$$
$$= -\frac{1}{2} \sum_{\ell \leq m} \left\{ N_{\ell m} \ln(\sigma_\ell^2 + \sigma_m^2) + \frac{S_{\ell m}^2}{\sigma_\ell^2 + \sigma_m^2} \right\}$$

---

* To obtain this density, write down the joint distribution of π_ℓ and π_m, and eliminate the true value π_t of the parallax by the method of section 2.4 .

On differentiating with respect to σ_ℓ^2, we obtain the maximum likelihood equations

$$-2\left.\frac{\partial \ln L}{\partial \sigma_\ell^2}\right|_{\sigma=\hat{\sigma}} = 0$$

$$= \sum_{m\neq\ell}\left\{\frac{N_{\ell m}}{\hat{\sigma}_\ell^2+\hat{\sigma}_m^2} - \frac{S_{\ell m}^2}{(\hat{\sigma}_\ell^2+\hat{\sigma}_m^2)^2}\right\} + \frac{N_{\ell\ell}}{\hat{\sigma}_\ell^2} + \frac{1}{2}\,\frac{S_{\ell\ell}^2}{\hat{\sigma}_\ell^4}\ .$$

These equations can be solved by the following iteration

$$\left\{N_{\ell\ell} + \sum_{m\neq\ell} N_{\ell m}\left(\frac{\hat{\sigma}_\ell^2(r)}{\hat{\sigma}_\ell^2(r)+\hat{\sigma}_m^2(r)}\right)\right\}\hat{\sigma}_\ell^2(r+1)$$

$$= S_{\ell\ell}^2 + \sum_{m\neq\ell} S_{\ell m}^2\left(\frac{\hat{\sigma}_\ell^2(r)}{\hat{\sigma}_\ell^2(r)+\hat{\sigma}_m^2(r)}\right)^2.$$

The precision of the resulting estimates can be found from the second derivatives of the log likelihood

$$h(\sigma_\ell) = -\left.\frac{\partial^2 \ln L}{\partial(\sigma_\ell^2)^2}\right|_{\sigma=\hat{\sigma}}$$

$$= \sum_{m\neq\ell}\left\{\frac{S_{\ell m}^2}{(\hat{\sigma}_\ell^2+\hat{\sigma}_m^2)^3} - \frac{\frac{1}{2}N_{\ell m}}{(\hat{\sigma}_\ell^2+\hat{\sigma}_m^2)^2}\right\}$$

$$+\frac{1}{2}\left\{\frac{S_{\ell\ell}^2}{(\hat{\sigma}_\ell^2)^3} + \frac{\frac{1}{2}N_{\ell\ell}}{(\hat{\sigma}_\ell^2)^2}\right\}.$$

CHAPTER 4

REGRESSION FROM ERROR PRONE OBSERVATIONS

4.0 The likelihood of the absolute magnitude M, based on the color index I and the luminosity criterion ρ , is proportional to the probability element of ρ given I and M:

$$L(M;\rho,I) \propto P(d\rho|M,I)$$

The task of calibrating a luminosity criterion is the task of estimating the parameters of the distribution $P(d\rho|M,I)$. For the purposes of the present work, we shall assume that the distribution takes the particularily simple form

$$P(d\rho|M,I;h,a) \propto d\rho \exp\left[-\tfrac{1}{2}h\left\{\rho-\sum_{n=0}^{N}a_n Q_n(M,I)\right\}^2\right]$$

The restriction here is that, for given M and I, the distribution of ρ is normal. The mean of ρ can be any reasonable function of M and I, which is represented in a series of orthogonal basis functions $Q_0(M,I) \ldots Q_N(M,I)$. The variance of ρ is independent of M and I; this last restriction could be removed, at the cost of increasing the complexity of the computation.

Ideally, we should like to use a much more general form, which would be capable of representing, arbitrarily closely, any continuous density. Unfortunately, the computation involved is extremely difficult; although considerable progress has been made, not all the problems have been solved.

4.1 The likelihood of the parameters.

The likelihood we need is proportional to the distribution of the luminosity criterion based on the parameters and on the observations relevant to the absolute magnitude, $\vartheta = \{m, \pi_o, h_\pi, r_\mu, h_r\}$:

$$L(h,a;\rho,I,\vartheta) \propto \prod_{k=1}^{K} P(d\rho_k \mid I_k \vartheta_k; h,a)$$

What we know is the distribution of the luminosity criterion based on the true value of the absolute magnitude

Here the true value M is a nuisance parameter, and must be eliminated using the device explained in section 2.4:

$$P(d\rho \mid I,\vartheta;h,a) = \int_M P(d\rho \mid I,M;h,a) \frac{L(M;\vartheta)dM}{\int_{M'} L(M';\vartheta)dM'}$$

The normalizing integral over M' is irrelevant to the likelihood, and so may be omitted. The likelihood is

$$L(h,a;\rho,I,\vartheta) \propto \prod_{k=1}^{K} \int_M P(d\rho_k \mid I_k, M; h,a)\, L(M;\vartheta_k)dM$$

Substituting for the distribution of the luminosity criterion, we find

$$L(h,a;\rho,I,\vartheta) \propto \prod_{k=1}^{K} h^{1/2} \int_M \exp\left[-\frac{h}{2}\left\{\rho_k - \sum_n a_n Q_n(M,I_k)\right\}^2\right] L(M,\vartheta_k)\,dM .$$

It is convenient to work with the logarithm of the likelihood. Instad of a product, we then have a sum of the contributions made by the data for each star:

$$\ln L(h,a;\rho,I,\vartheta) = \sum_{k=1}^{K} \ln L_k$$

$$\ln L_k = \ln\left\{h^{1/2} \int_M \exp\left[-\frac{h}{2}\left\{\rho_k - \sum_n a_n Q_{nk}\right\}^2\right] L(M,\vartheta_k)\,dM\right\}$$

where we have written $Q_{nk} = Q_n(M,I_k)$.

When both components of a binary star are included in the sample of stars, we have to take into account the information that both components are at the same distance. This is discussed in Appendix 4A.

4.2 Deriving the maximum likelihood equations.

The maximum likelihood equations are obtained by setting the derivatives of the log likelihood equal to zero:

$$\frac{\partial \ln L}{\partial a_m} = \sum_k \frac{\partial \ln L_k}{\partial a_m} = 0 \; ; \qquad \frac{\partial \ln L}{\partial h} = \sum_k \frac{\partial \ln L_k}{\partial h} = 0 .$$

Substituting for $\ln L_k$, we find

$$\frac{\partial \ln L_k}{\partial a_m} = \frac{\partial}{\partial a_m} \ln\left\{h^{1/2} \int_M \exp\left[-\frac{h}{2}\left(\rho_k - \sum_n a_n Q_{nk}\right)^2\right] L(M,\vartheta_k)\,dM\right\}$$

$$= -h \frac{\int_M Q_{mk}\left(\rho_k - \sum_n a_n Q_{nk}\right) \exp\left[-\frac{h}{2}\left(\rho_k - \sum_n a_n Q_{nk}\right)^2\right] L(M;\vartheta_k)\, dM}{\int_M \exp\left[-\frac{h}{2}\left(\rho_k - \sum_n a_n Q_{nk}\right)^2\right] L(M;\vartheta_k)\, dM}$$

It is convenient to abbreviate this using the notation of an <u>expected</u> <u>value</u>:

$$E\left[f(x) \mid \alpha\right] = \frac{\int_x f(x)\, P(dx \mid \alpha)}{\int_x P(dx \mid \alpha)}$$

Putting $P(dM \mid \Theta_k, h, a) \propto \exp\left[-\frac{h}{2}\left(\rho_k - a_n Q_{nk}\right)^2\right] L(M;\vartheta_k)\, dM$

with $\Theta_k = \{\vartheta_k\ \rho_k\ I_k\}$, we have

$$\frac{\partial \ln L_k}{\partial a_m} = -h\, E\left[Q_{mk}\left(\rho_k - \sum_n a_n Q_{nk}\right) \Big| \Theta_k, h, a\right]$$

In exactly the same way, we can find

$$\frac{\partial \ln L_k}{\partial h} = -\frac{1}{2} h^{-1} - \frac{1}{2} E\left[\left(\rho_k - \sum_n a_n Q_{nk}\right)^2 \Big| \Theta_k, h, a\right]$$

On taking the sum over k, we find the maximum likelihood equations are

$$\sum_{k=1}^{K} E\left[Q_{mk}\left(\rho_k - \sum_n a_n Q_{nk}\right) \Big| \Theta_k, h, a\right] = 0$$

and

$$\sum_{k=1}^{K} E\left[\left(\rho_k - \sum_n a_n Q_{nk}\right)^2 \Big| \Theta_k, h, a\right] = K h^{-1}$$

4.3 <u>Solving the maximum likelihood equation.</u>

Because of the dependence of the expected values on the parameters, the equations we have just derived are non-linear;

if it were not for this dependence, they would form a linear set of equations whose solution would be simple. We can make this clearer by rewriting the equations in the form

$$\sum_n a_n \left\{ \sum_k E\left[Q_{mk} Q_{nk} \mid \Theta_k, h, a\right] \right\} = E\left[\rho_k Q_{mk} \mid \Theta_k, h, a\right]$$

$$h^{-1} = \frac{1}{K} \sum_k E\left[\left(\rho_k - \sum_n a_n Q_{nk}\right)^2 \mid \Theta_k, h, a\right]$$

This suggests the following iterative scheme. Start with any suitable approximation $h^{(0)}, a^{(0)}$. Calculate the expected values. Solve for a new and improved approximation $h^{(1)}, a^{(1)}$. Calculate new and improved expected values, and solve for a still better approximation $h^{(2)}, a^{(2)}$. Repeat the cycle until two successive approximations agree to within the required accuracy. If the r^{th} approximation is $h^{(r)}, a^{(r)}$, the $(r+1)^{th}$ approximation is given by

$$\sum_n a_n^{(r+1)} \left\{ \sum_k E\left[Q_{mk} Q_{nk} \mid \Theta_k, h^{(r)}, a^{(r)}\right] \right\} = E\left[\rho_k Q_{mk} \mid \Theta_k, h^{(r)}, a^{(r)}\right]$$

$$\left(h^{(r+1)}\right)^{-1} = K^{-1} \sum_k E\left[\left(\rho_k - \sum_n a_n^{(r+1)} Q_{nk}\right)^2 \mid \Theta_k, h^{(r)}, a^{(r)}\right]$$

The convergence of the iteration is linear; after each iteration, the error of the approximation is reduced by a roughly constant factor. In practice, I found that the factor was about 3, so that convergence was reasonably rapid.

4.4 The estimated absolute magnitude and its dispersion.

Once we have obtained an estimate of the parameters, we can find an estimate of the absolute magnitude of any star for which the luminosity criterion and the color is known. The likelihood of M is

$$L(M;\rho,I;h,a) \propto P(d\rho|M,I;h,a)$$
$$\propto \exp\left[-\frac{h}{2}\left\{\rho-\sum_n a_n Q_n(M,I)\right\}^2\right]$$

The maximum likelihood estimate, $\hat{M}$, is given by the solution of the equation

$$\rho-\sum_n a_n Q_n(\hat{M},I) = 0$$

It is important to estimate the dispersion of this estimate. The dispersion has two components, which we shall call the extrinsic and the intrinsic dispersion.

The intrinsic dispersion is due to the fact that the luminosity criterion is not completely determined by the absolute magnitude and the color; for a given M and I, the variance of ρ is given by

$$var(\rho) = h^{-1}$$

and the intrinsic variance of M is

$$var(\hat{M};in) = var(\rho)\left(\frac{\partial\hat{M}}{\partial\rho}\right)^2 = h^{-1}\Big/\left(\sum a_n \frac{\partial Q_n(\hat{M},I)}{\partial\hat{M}}\right)^2$$

The extrinsic dispersion of M is due to the fact that our estimates of the parameters are not exact, but themselves have a dispersion. The extrinsic dispersion of M is given in terms of the covariance matrix $cov(a_l,a_m)$ of the parameters $a_0 \ldots a_N$ by

$$\mathrm{var}(M;ex) = \sum_{\ell,m=0}^{N} \mathrm{cov}(a_\ell, a_m) \frac{\partial \hat{M}}{\partial a_\ell} \frac{\partial \hat{M}}{\partial a_m}$$

$$= \sum_{\ell,m=0}^{N} \mathrm{cov}(a_\ell, a_m)\, Q_\ell(\hat{M},I)\, Q_m(\hat{M},I) \left\{ \sum_n a_n \frac{\partial Q_n(\hat{M},I)}{\partial \hat{M}} \right\}^{-2}$$

Recall from section 2.6 that the covariance matrix is the reciprocal of the precision matrix, and that the precision matrix is found from the second derivatives of the likelihood

$$\mathrm{cov}(a_\ell, a_m) = (H^{-1})_{\ell m}$$

$$H_{\ell m} = - \left. \frac{\partial^2 \ln L}{\partial a_\ell\, \partial a_m} \right|_{a=\hat{a}}$$

We have already obtained the first derivative of the log likelihood. Differentiating again, we can find

$$-\frac{\partial^2 \ln L}{\partial a_\ell\, \partial a_m} = h \sum_k E\left[Q_{\ell k} Q_{mk} \mid \Theta_k, h, a \right]$$

$$- h^2 \sum_k E\left[Q_{\ell k} Q_{mk} \left(\rho_k - \sum_n a_n Q_{nk}\right)^2 \middle| \Theta_k, h, a \right]$$

$$+ h^2 \sum_k E\left[Q_{\ell k} \left(\rho_k - \sum a_n Q_{nk}\right) \middle| \Theta_k, h, a \right] E\left[Q_{mk} \left(\rho_k - \sum a_n Q_{nk}\right) \middle| \Theta_k, h, a \right]$$

The first term is due to the limited amount of data available; as the amount of data increases, the precision due to the first term increases. The second and third terms are due to the errors in the data; in the absence of errors, the second and third terms cancel.

An additional use for the covariance matrix is as an aid in deciding how many terms are needed in the expansion $\sum_{n=0}^{N} a_n Q_{nk}$. In general, we should find that for some integer N, the coefficients a_n are not much larger than their standard derivations when $n > N$. Then we include in the expansion only those terms with $n \leqslant N$. The standard deviation of a_n is given by

$$\sigma(a_n) = \sqrt{\mathrm{cov}(a_n, a_n)} \; .$$

Appendix 4A <u>The inclusion of binary stars.</u>

The absolute magnitudes M_A and M_B of the components of a binary star are connected by the relation

$$M_A - m_A = M_B - m_B \quad \text{or} \quad M_B = M_A + \Delta m$$

The contribution to the log likelihood is

$$\ln L_{AB}$$

$$= \ln\left\{\int_{M_A} P(\rho_A | M_A I_A ; h, a) P(\rho_B | M_B = M_A + \Delta m, I_B ; h, a) L(M_A ; \vartheta_A) dM_A\right\}$$

$$= \ln\left\{h \int_M \exp\left[-\frac{h}{2}\left\{(\rho_A - \sum_n a_n Q_{nA})^2 + (\rho_B - \sum_n a_n Q_{nB})^2\right\}\right] L(M ; \vartheta_A) dM\right\}$$

where $Q_{nA} = Q_n(M, I_A)$, $Q_{nB} = Q_n(M + \Delta m, I_B)$.

On differentiating, we obtain the contribution to the maximum likelihood equations. This amounts to adding a contribution to each of the sums of expected values.

To $\sum_k E[Q_{\ell k} Q_{mk} | \Theta_k, h, a]$ add $E[Q_{\ell A} Q_{mA} + Q_{\ell B} Q_{mB} | \Theta_{AB}, h, a]$

To $\sum_k E[\rho_k Q_{\ell k} | \Theta_k, h, a]$ add $E[\rho_A Q_{\ell A} + \rho_B Q_{\ell B} | \Theta_{AB}, h, a]$

To $\sum_k E[(\rho_k - \sum_n a_n Q_{nk})^2 | \Theta_k, h, a]$

add $\sum_k E[(\rho_A + \sum_n a_n Q_{nA})^2 + (\rho_B + \sum a_n Q_{nB})^2 | \Theta_k, h, a]$

The contribution to the precision matrix is obtained by differentiating twice.

CHAPTER 5

RESULTS AND CONCLUSIONS

5.0 We shall now see how the methods of Chapters 3 and 4 can be applied to two particular luminosity criteria, developed by Wilson and Bappu and by Scarfe. A re-analysis of a luminosity criterion should not be construed as being in any way a criticism of those who originally developed the criterion. On the contrary, the statistician must always be in debt to those who provide the data which he uses. Particularly is this so in astronomy, where observational data require so much skill and patience to collect.

5.1 The H and K emission line widths.

Probably the best available spectroscopic luminosity criterion is the mean width of the CaII H and K emission lines, discovered by Wilson and Bappu (1957) and calibrated by them using spectroscopic and trigonometric parallaxes. They found a linear relation between the absolute magnitude and the logarithm of the width, which is independent of color. Wilson (1959) fitted a straight line to the data given by the sun and the four Hyades giants.

The latter calibration depends upon the distance modulus adopted for the Hyades cluster. When Hodge and

Wallerstein (1966) suggested that the distance modulus of the Hyades cluster might be in error, they pointed out that a consequence would be an error in the calibration of the H and K emission line widths. Wilson (1967) recalibrated the H and K emission line widths using data for 65 stars of large trigonometric parallaxes. Two stars of the original 67 were rejected as giving deviant results; when this was done, the new calibration, entirely independent of the Hyades cluster distance, was found to be in excellent agreement with the old. Wilson therefore concluded that the Hyades distance modulus is probably correct.

We saw in section 1.2 that the method of least squares fitting to absolute magnitudes calculated from trigonometric parallaxes, which was the method used by Wilson, gives rise to systematic errors. In order to find out how large the systematic errors are, I calibrated the H and K emission line widths using both the method of least squares and the method of maximum likelihood. We saw in Section 2.6 that the maximum likelihood estimates of a set of parameters lie in the center of the final distribution, provided that the amount of data was reasonably large. Consequently any difference between the least squares estimates and the maximum likelihood estimates which is in excess of one or two standard deviations should be ascribed to a systematic error in the least squares estimates.

I used data for the 67 stars in Wilson's list, and also for the sun. For the 67 stars, I took the relative parallaxes from the Yale catalog, and calculated the absolute parallaxes and their precisions by the method of section 3.2. I took the proper motions and the apparent magnitudes and colors from the Bright Star catalogue; the proper motions were used in the maximum likelihood calculation but not in the least squares calculation. The luminosity criterion $\rho = \log_{10}(W)$ turned out to have no significant dependence on color or on terms quadratic in absolute magnitude; I therefore assumed a frequency distribution of the following simple form:

$$P(d\rho | M; h, a) \propto h^{1/2} d\rho \exp\left[-\frac{h}{2}(\rho - a_0 - a_1 x)^2\right]; \quad x = \frac{M_V - 3{\cdot}7065}{4{\cdot}5025}$$

The results obtained by the two methods are as follows:

	Maximum Likelihood	Least Squares
h	731·1	393·7
a_0	1·5853±0·0052	1·6030
a_1	-0·2581±0·0087	-0·2829

The covariance matrix for the maximum likelihood estimates of a_0, a_1 is

$$\text{var}(a_0) = \text{cov}(a_0, a_0) = 0{\cdot}00002687$$

$$\text{cov}(a_0, a_1) = 0{\cdot}00001271$$

$$\text{var}(a_1) = \text{cov}(a_1, a_1) = 0{\cdot}00007608$$

Notice first that the least squares estimate of the precision h is much smaller than the maximum likelihood estimate. The least squares method implicitly assumes that the likelihood of the absolute magnitude is Gaussian, whereas the correct likelihood has a much more extended "tail". The least squares method therefore underestimates the dispersion of the absolute magnitudes, and thus attributes more of the scatter to the luminosity criterion, (or underestimates the precision of the luminosity criterion).

The estimates of a_0, a_1 differ by more than three standard deviations, a difference which is statistically significant. The estimates for M_V are calculated as

$$\hat{M}_v = \frac{4 \cdot 5025}{a_1}\left(\log_{10}(W) - a_0\right) + 3 \cdot 7065$$

and are given below for two typical values of $\rho = \log_{10}(W)$

	M_V Maximum Likelihood	M_V Least Squares
$\rho = \log_{10}(W)$		
1·51	+5·02	+5·19
1·80	-0·04	+0·57

The value $\log_{10}(W) = 1 \cdot 51$ is typical of a dwarf star, and for this value the estimates of M_V are in reasonable agreement. The value $\log_{10}(W) = 1 \cdot 80$ is typical of a giant star, and the estimates of M_V differ by 0·61 magnitudes.

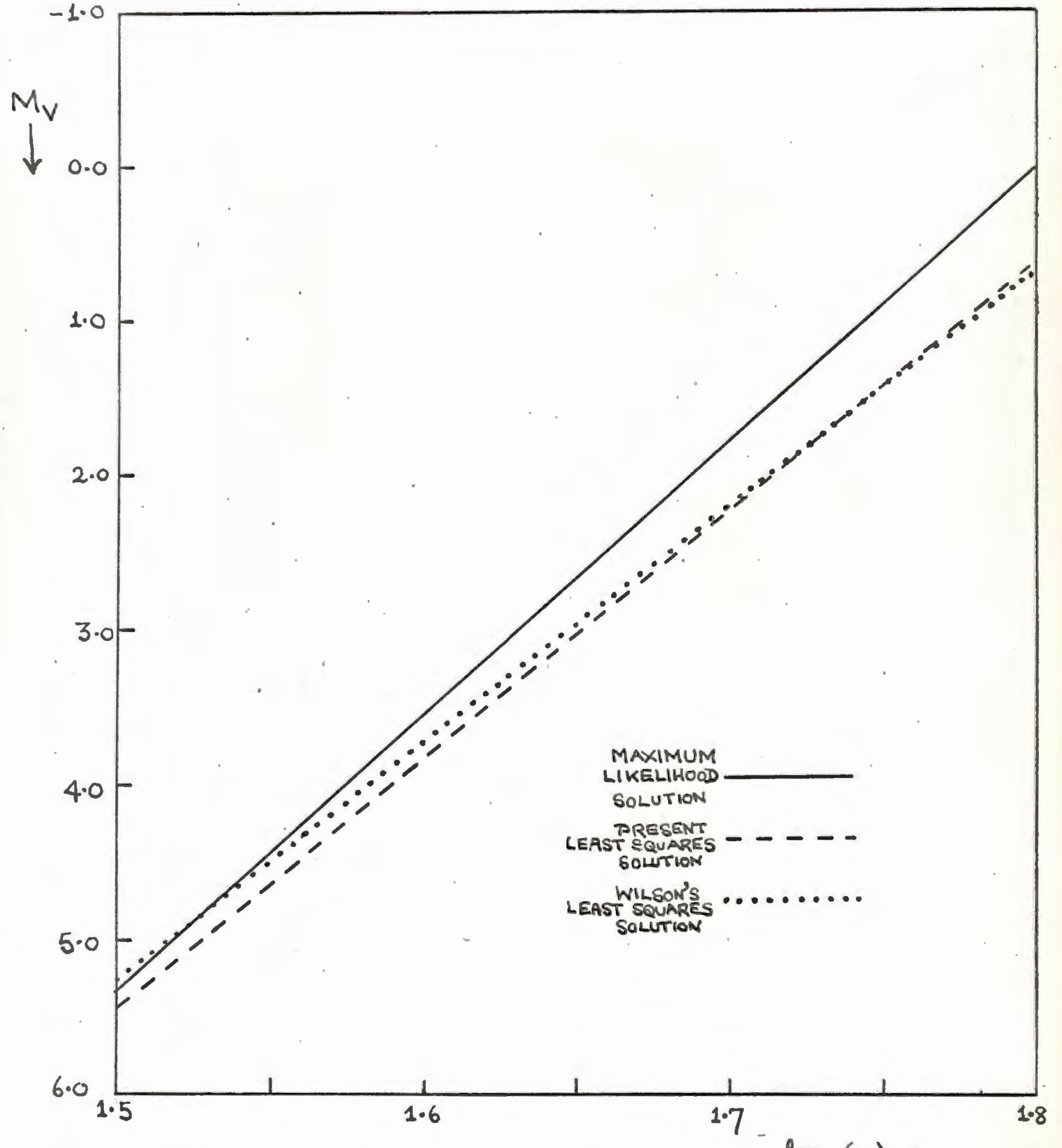

Figure 5.1.1 Plots of the M_V - log (W) relationship.

In figure 5.1.1 the M_V - $\log_{10}(W)$ relations shown are for the maximum likelihood solution, the present least squares solution, and Wilson's 1967 least squares solution. We should expect the two least squares solutions to differ somewhat, for several reasons. Wilson's solution is for the regression of M_V on $\log_{10}(W)$, while the present solution is for the regression of $\log_{10}(W)$ on M_V; the absolute parallaxes and their precisions have slightly different values in the present solution; and the present solution uses data for the sun and the two deviant stars omitted from Wilson's solution. In spite of these differences, the two solutions agree reasonably well.

The four Hyades giants have a $\log_{10}(W)$ of about 1·80, and so the maximum likelihood estimate of their mean absolute magnitude differs by about 0·61 magnitudes from the least squares estimate and from the estimate based on the accepted value of the Hyades distance modulus.

The standard deviation is calculated from the precision h and the covariance matrix of a_0, a_1. The intrinsic variance for one star is:

$$\mathrm{var}(M_V; \mathrm{in}) = \left(\frac{\partial M_V}{\partial \rho}\right)^2 \mathrm{var}(\rho) = \left(\frac{4 \cdot 5025}{a_1}\right)^2 h^{-1} = 0 \cdot 416 .$$

For four stars the variance is one quarter of this, or 0·104. The extrinsic variance, which is the same for four stars as it is for one, is:

$$\mathrm{var}(M_V;ex) = \left(\frac{\partial \hat{M}_V}{\partial a_0}\right)^2 \mathrm{var}(a_0) + 2\left(\frac{\partial \hat{M}_V}{\partial a_0}\right)\left(\frac{\partial \hat{M}_V}{\partial a_1}\right)\mathrm{cov}(a_0, a_1) + \left(\frac{\partial \hat{M}_V}{\partial a_1}\right)^2 \mathrm{var}(a_1)$$

Using $\dfrac{\partial \hat{M}_V}{\partial a_0} = -\dfrac{4\cdot 5}{a_1}$, $\dfrac{\partial \hat{M}_V}{\partial a_1} = -\dfrac{4\cdot 5}{a_1}(\log_{10}(W) - a_0)$,

we find the extrinsic variance to be 0·027. The total variance is 0·131, which gives a standard deviation of 0·362.

The difference of 0·61 magnitudes is therefore about 1.7 standard deviations. This evidence supports, but does not prove, the suggestion of Hodge and Wallerstein that the accepted value of the Hyades distance modulus is in error.

5.2 The FeI λ5250 triplet.

This triplet is one of several photoelectric criteria developed at the Cambridge observatories. Properly, it should not be considered in isolation, but should be calibrated in conjunction with the other Cambridge criteria. However, its behavior is somewhat complex, and it is of interest to see what the present methods of analysis can reveal.

Observational data on the FeI λ5250 triplet were obtained by Scarfe (1966). I used the data for 253 stars from his list which were in the bright star catalogue and for which B-V colors were àvailable. I assumed a frequency distribution of the following form:

$$P(d\rho \mid M_v, B-V) \propto h^{1/2} \exp\left[-\frac{h}{2}\left\{\rho - f(M_v, B-V)\right\}^2\right]$$

$$f(M_v, B-V) = a_0 + a_1 x + a_2 y + a_3\left(\frac{3}{2}x^2 - \frac{1}{2}\right) + a_4 xy + a_4\left(\frac{3}{2}y^2 - \frac{1}{2}\right)$$

$$x = \frac{M_v - 0.75066}{4.8867}, \quad y = \frac{B-V - 1.0656}{0.24076}$$

Here ρ is the natural logarithm of the ratio of the strengths of the central line to the sidebands.

The maximum likelihood estimates are

$h = 4775$

$a_0 = +0.0445 \pm 0.0011$

$a_1 = -0.0273 \pm 0.0019$

$a_2 = +0.0170 \pm 0.0013$

$a_3 = -0.0161 \pm 0.0013$

$a_4 = -0.0189 \pm 0.0017$

$a_5 = -0.0019 \pm 0.0007$

The covariance matrix, in units of 10^{-6}, is

	0	1	2	3	4	5
0	1·16	0·07	0·16	0·00	0·12	-0·22
1	0·07	3·72	1·32	0·87	-0·94	-0·27
2	0·16	1·32	1·59	-0·31	-0·74	-0·37
3	0·00	-0·87	-0·31	1·73	0·20	-0·15
4	0·12	-0·94	-0·74	0·20	2·73	0·66
5	-0·22	-0·27	-0·37	-0·15	0·66	0·44

Here the figure in row m, column n, is $10^6 \operatorname{cov}(a_m, a_n)$.

The extrinsic variance, calculated from the covariance matrix, is generally much smaller than the intrinsic variance. The intrinsic variance is given by

$$\mathrm{var}(M_v; in) = \mathrm{var}(\rho)\left(\frac{\partial\bar{\rho}}{\partial M_v}\right)^{-2}$$

$$= 6{\cdot}7\left\{1 + 0{\cdot}36\,(M_v - 0{\cdot}75) + 2{\cdot}93\,(B\text{-}V - 1{\cdot}066)\right\}^{-2}$$

It will be seen that the intrinsic variance is small for red stars and large for blue stars. For stars with B-V bluer than 1·066 it is even possible for the variance to become infinite; this is because the criterion ρ may have a zero derivative $\partial\bar{\rho}/\partial M_v$. The intrinsic standard deviations of the absolute magnitude for some typical values of M_v and B-V are given below.

Type	M_v	B-V	Standard deviation of M_v
G5III	1·00	0·81	6·27
KoIII	1·20	0·99	1·95
K5III	0·00	1·50	1·06
MoIII	-0·10	1·54	1·02

A higher precision, and hence a smaller standard deviation, might be obtained by including more terms in the regression function $f(M_v,\ B\text{-}V)$.

5.3 Ratios of line depths.

I attempted to calibrate the 14 line ratios used by Wright and Jacobsen (1966). This attempt was a failure.

In order to measure the ratios speedily, I used line depths; this was probably a mistake. It is quite possible that the major effect of surface gravity is to alter the width of the lines; the old classification of c, g, and d was based on line widths, and c stars had very narrow lines. If one must use line depths, it is essential to overexpose the plates; this I failed to do, so that the line depths on some of my plates ran into the bottom of the calibration curve, and had to be rejected. However, as Wright has pointed out, the use of one measurement of a spectral line should be avoided; equivalent widths, which involve the area of a line, are to be preferred.

One important lesson to be learned from this attempt is the importance of considering extrinsic as well as intrinsic dispersions. I observed 56 stars; this was too small a number and the uncertainties of the parallaxes were high. Consequently the extrinsic dispersion was much larger than the intrinsic dispersion. Typically, the intrinsic standard deviation of a line ratio is about 2 magnitudes; by combining 14 line ratios, one might hope to get an estimate with a standard deviation of about half a magnitude.

But such an estimate would be meaningless; the extrinsic standard deviation, which represents a systematic error, is about ten magnitudes.

5.4 Conclusions.

The original aims of the present work were two: to develop methods of calibrating luminosity criteria which made effective use of as much information as possible, and so have a higher precision; and to develop estimates of the dispersion of the calibrations. Both of these aims have been achieved.

In addition, systematic errors which were present in previous methods of calibration have been eliminated. These systematic errors led to an error in the cosmic distance scale. It is now important to develop a method for the efficient estimation of the distance of the Hyades cluster. Perhaps the data is insufficient to allow an accurate distance; but at least we should endeavor to find within what limits we may expect the distance to lie.

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COMPUTER TABULATIONS

For reference purposes, the Fortran programs and the data used are given. The raw data on the stars are reduced by the program "REDUCE ASTROMETRIC DATA". The parameters of the luminosity calibration were estimated from the reduced data by the program "READ DATA AND CALCULATE REGRESSION COEFFICIENTS". The starting point would be the least squares estimation by the subroutine "LSQR". The results would then be used as an initial approximation for the solution of the maximum likelihood equations by the subroutine "RGRSN".

CT1 Explanation of raw data

The raw data for each single star is punched onto three IBM cards, and for each binary star onto four cards. The computer reads in the data for the binary stars on sets of four cards, and then the data for the single stars on sets of three cards. Each card starts with the HD number of the star, and the computer checks that this is the same for each card in the set of three, (or the first three in a set of four).

The first card contains data on the apparent magnitude, color, and the luminosity criterion. The first card for the first star of the H and K line stars has:

131156 4·54 0·76 1·45

This tells us that HD 131156 has

$V = 4{\cdot}54$, $B\text{-}V = 0{\cdot}76$, and $\log_{10}(W) = 1{\cdot}45$.

The second card contains the data for calculating the proper motion distance and its precision. The second card for the first star has

131156 V 14 46·8 19 31 0·134 -0·107 4.

This tells us that HD 131156 belongs to luminosity class V; is at right ascension $14^h\ 46^m{\cdot}8$ and declination 19° 31'; has proper motion of 0·134 seconds of arc per year in right ascension and -0·107 in declination; and finally that the radial velocity is 4 km/sec.

The third card contains the parallax data. The third card for the first star has

131156 30 4 147 A 157 M 165 Y 126 S

This tells us that HD 131156 would not have been selected if its parallax had been less than $0\overset{''}{.}030$; that there are 4 measurements of the parallax; $0\overset{''}{.}147$, by Allegheny; $0\overset{''}{.}157$, by McCormick; $0\overset{''}{.}165$, by Yale Southern Station; and $0\overset{''}{.}126$, by Sproul. The criterion used by Wilson for selecting stars depended on the ratio of the observed parallax to the quoted probable error, and so the value of π_c, $0\overset{''}{.}030$ in the example, differs for different stars.

The fourth card contains the same information for the fainter component of a binary star as the first card does for the brighter component.

CT2 Raw data for H and K line stars

131156			4.54		0.76	1.45			
131156		V			14 46.8	19 31	0.134	-0.107	4.
131156	30	4	147	A	157 M	165 Y 126 S			
F131156			6.44		1.06	1.30			
165341			4.02		0.88	1.48			
165341		V			18 0.4	2 31	0.256	-1.097	-7.
165341	30	5	184	A	185 M	211 Y 212YK	172 S		
F165341			6.02		1.26	1.38			
201091			5.19		1.19	1.40			
201091		V			21 2.4	38 15	4.120	3.179	-64.
201091	30	6	285	A	307 M	271YK 322 W	272 S	299 V	
201092			6.02		1.38	1.28			
155886			5.29		0.81	1.40			
155886		V			17 9.2	-26 27	-0.497	-1.137	-1.
155886	50	1	186	Y					
155885			5.33		0.85	1.40			
3546			4.37		0.88	1.74			
3546		III			0 33.3	28 46	-0.232	-0.249	-84.
3546	30	4	33	A	17 M	42YK 16 S			
3651			5.84		0.86	1.46			
3651		V			0 34.2	20 43	-0.466	-0.369	-34.
3651	36	2	96	A	101 M				
4128			2.04		1.02	1.79			
4128		III			0 38.6	-18 32	0.230	0.040	13.
4128	48	2	69	M	52 Y				
4628			5.76		0.88	1.32			
4628		D			0 43.1	4 46	0.752	-1.142	-13.
4628	30	2	137	A	155 M				
6805			3.44		1.16	1.77			
6805		III			1 3.6	-10 43	0.213	-0.132	12.
6805	30	2	29	A	29 Y				
6860			2.03		1.63	1.90			
6860		III			1 4.1	35 5	0.177	-0.113	0.
6860	36	3	33	A	58 M	45 W			
8512			3.61		1.06	1.79			
8512		III			1 19.0	-8 42	-0.080	-0.215	17.
8512	30	4	31	A	58 M	13 Y 27 C			
10476			5.23		0.83	1.42			
10476		V			1 37.1	19 47	-0.296	-0.671	-34.
10476	36	2	125	A	143 M				

69.

10780			5.59	0.82		1.48				
10780		V		1 40.5		63 22	0.584	-0.246		2.
10780	36	3	108 M	108YK	118 W					
12929			2.00	1.15		1.77				
12929		III		2 1.5		22 59	0.192	-0.146		-14.
12929	36	3	29 A	67 M	44 W					
13974			4.87	0.61		1.60				
13974		V		2 11.0		33 46	1.155	-0.240		-6.
13974	30	3	62 A	123 M	94 S					
16160			5.82	0.97		1.38				
16160		V		2 30.6		6 25	1.807	1.459		23.
16160	30	3	145 A	137 M	153 Y					
17925			6.05	0.87		1.46				
17925		C		2 47.7		-13 11	0.395	-0.170		19.
17925	36	3	133 M	113 Y	131 C					
20630			4.82	0.68		1.53				
20630		V		3 14.1		3 0	0.267	0.096		19.
20630	36	3	112*A	89 M	104 Y					
22049			3.73	0.89		1.42				
22049		V		3 28.2		-9 48	-0.975	0.022		15.
22049	30	5	293 A	302 M	297 Y	312 S	306 C			
23249			3.55	0.92		1.53				
23249		IV		3 38.4		-10 6	-0.092	0.744		-6.
23249	30	4	110 A	134 M	96 Y	97 C				
26965			4.42	0.82		1.48				
26965		V		4 10.7		-7 49	-2.225	-3.418		-42.
26965	30	5	198 A	198 M	200 Y	212 V	188 C			
29139			0.86	1.53		1.83				
29139		III		4 30.2		16 18	0.069	-0.190		54.
29139	30	5	37 A	35 M	47YK	63 W	75 V			
30495			5.51	0.64		1.53				
30495		C		4 43.1		-17 7	0.133	0.174		17.
30495	36	2	71 M	64 Y						
32147			6.21	1.06		1.45				
32147		C		4 55.8		-5 52	0.557	-1.089		27.
32147	30	7	100 A	121 M	104 Y	107 W	103 S	104 V	88 C	

37394			6.23	0.84		1.49			
37394		D		5 33.2		53 26	0.010	-0.521	1.
37394	36	2	83 A	102 M					
47205			3.95	1.06		1.67			
47205		IV		6 32.3		-19 10	0.064	-0.076	3.
47205	42	2	56 M	51 Y					
62509			1.15	1.00		1.75			
62509		III		7 39.2		28 16	-0.623	-0.052	3.
62509	30	2	88*A	107 M					
82885			5.41	0.77		1.54			
82885		IV-V		9 29.7		36 16	-0.705	-0.251	13.
82885	30	4	117 A	106 M	66 W	114 S			
95689			1.79	1.06		1.85			
95689		II-III		10 57.6		62 17	-0.119	-0.070	-9.
95689	30	3	33 A	16 W	24 G				
101501			5.35	0.72		1.51			
101501		V		11 35.8		34 46	-0.014	-0.390	-5.
101501	48	1	105 A						
103095			6.45	0.75		1.38			
103095		VI		11 47.2		38 26	3.994	-5.800	-98.
103095	30	2	108 A	106 M					
104979			4.12	0.99		1.76			
104979		III		12 0.1		9 17	-0.221	0.042	-30.
104979	30	3	34 A	46 M	27 Y				
109345			6.21	1.07		1.76			
109345		III		12 28.9		33 56	0.001	-0.016	-43.
109345	42	1	48 A						
114710			4.28	0.57		1.52			
114710		V		13 7.2		28 23	-0.799	0.876	6.
114710	36	3	111 A	119 M	133 S				
116976			4.75	1.09		1.83			
116976		III		13 22.1		-15 27	-0.121	0.018	-14.
116976	50	1	53 Y						
117176			4.98	0.71		1.61			
117176		V		13 23.5		14 19	-0.237	-0.583	4.
117176	36	2	32 A	45 M					

122742			6.34	0.73	1.51			
122742		V		13 58.6	11 16	0.080	-0.314	-17.
122742	42	1	58*A					
123139			2.05	1.02	1.80			
123139		III-IV		14 0.8	-35 53	-0.521	-0.522	1.
123139	48	2	78 Y	42 C				
124897			0.06	1.23	1.81			
124897		III		14 11.1	19 42	-1.098	-2.003	-5.
124897	30	4	86 A	76 M	95YK 92 V			
131511			6.01	0.83	1.53			
131511		D		14 48.9	19 33	-0.453	0.209	-34.
131511	42	2	81 A	79 M				
131873			2.08	1.47	1.85			
131873		III		14 51.0	74 34	-0.032	0.007	17.
131873	30	3	26 A	47 M	18 G			
131977			5.80	1.11	1.42			
131977		V		14 51.6	-20 58	1.041	-1.745	20.
131977	36	3	199 M	158 Y	160 C			
140573			2.65	1.17	1.76			
140573		III		15 39.4	6 44	0.134	0.039	3.
140573	30	2	46 A	38 M				
146791			3.24	0.96	1.73			
146791		III		16 13.0	-4 27	0.082	0.035	-10.
146791	30	3	30 A	46 M	38 Y			
148387			2.74	0.91	1.70			
148387		III		16 22.6	61 44	-0.023	0.058	-14.
148387	30	2	48 A	2 M				
149661			5.74	0.81	1.49			
149661		V		16 31.1	-2 7	0.451	-0.317	-15.
149661	30	4	83 A	49 M	104 Y 122 C			
150997			3.50	0.92	1.76			
150997		III-IV		16 39.5	39 7	0.035	-0.090	8.
150997	36	2	53 A	47 M				
160269			5.23	0.61	1.60			
160269		V		17 34.0	61 57	0.253	-0.513	-13.
160269	30	5	46 A	70 M	78YK 52 G	80 S		

169916			2.84	1.04	1.74				
169916		III		18 21.8	-25 29		-0.047	-0.188	-43.
169916	42	2	30 M	67 Y					
175751			4.82	1.08	1.76				
175751		III		18 51.7	-5 59		0.061	-0.037	-93.
175751	30	2	34 A	40 Y					
184406			4.44	1.16	1.72				
184406		III		19 29.2	7 10		0.211	-0.157	-24.
184406	36	3	16 A	51 M	53 Y				
185144			4.68	0.79	1.46				
185144		V		19 32.6	69 29		0.575	-1.745	27.
185144	30	3	177 A	181 M	165 G				
188376			4.70	0.76	1.67				
188376		D		19 49.7	-26 34		0.205	0.079	-21.
188376	50	1	58 Y						
188512			3.71	0.86	1.59				
188512		IV		19 50.4	6 9		0.039	-0.483	-40.
188512	30	5	78 A	66 M	82 Y	0YK	67 S		
190406			5.80	0.61	1.53				
190406		D		19 59.6	16 48		-0.402	-0.415	4.
190406	30	3	69 A	29 M	64 S				
191026			5.33	0.85	1.66				
191026		IV		20 2.6	35 42		-0.232	-0.438	-34.
191026	30	4	22 A	31 M	45YK	30 S			
192310			5.73	0.88	1.51				
192310		V		20 9.0	-27 20		1.241	-0.182	-55.
192310	42	3	147 M	114 Y	94 S				
192947			3.55	0.94	1.77				
192947		III		20 12.5	-12 51		0.060	0.005	0.
192947	30	4	26 A	-17 M	52 Y	50 C			
197989			2.45	1.03	1.76				
197989		III		20 42.2	33 36		0.355	0.325	-10.
197989	30	3	52*A	39 M	2YK				
198149			3.43	0.92	1.65				
198149		IV		20 43.3	61 27		0.090	0.820	-87.
198149	30	4	66 A	64 M	66YK	78 S			

219134		5.57	1.01	1.38						
219134		V	23 8.5	56 37	2.072	0.299	-18.			
219134	30	3	137 A	158 M	181 S					
222107		3.88	1.02	1.76						
222107		III-IV	23 32.7	45 55	0.158	-0.421	7.			
222107	36	3	44 A	26 M	44 S					
222404		3.22	1.03	1.66						
222404		IV	23 35.2	77 4	-0.065	0.154	-42.			
222404	30	4	63 A	62 M	66YK	57 G				

CT3 Raw data for FeI 5250 stars

89484	2.61	1.14	1.0570			
89484	III	10 14.5	20 21	0.307	-0.152	-37.
89484	2 9 A	28 M				
89485	3.80	0.91	1.0270			
135101	6.68	C.68	1.0290			
135101	V	15 8.2	19 39	-C.591	0.284	-37.
135101	2 25 A	6 M				
F135101	7.58	0.73	1.0270			
201091	5.57	1.18	0.9370			
2C1C91	V	21 2.4	38 15	4.120	3.179	-64.
201091	6 285 A	3C7 M	271YK 322 W	272 S	299 V	
2C1C92	6.C2	1.38	0.9320			
166	6.14	0.75	1.CC60			
166	V	0 1.4	28 28	0.376	-0.180	-8.
166	2 71 A	41 M				
3546	4.38	0.87	1.0360			
3546	III	0 33.3	28 46	-0.232	-0.249	-84.
3546	4 33 A	17 M	42YK 16 S			
3627	3.28	1.28	1.C620			
3627	III	C 34.C	30 19	0.133	-0.090	-7.
3627	2 15 A	34 M				
3651	5.86	C.85	1.0020			
3651	V	0 34.2	20 43	-0.466	-0.369	-34.
3651	2 96 A	1C1 M				
3712	2.23	1.17	1.C840			
3712	II-III	C 34.8	55 59	0.C50	-0.029	-4.
3712	4 4 A	25 M	-4YK 14 W			
4614	3.44	0.58	C.9460			
4614	D	C 43.C	57 17	1.101	-0.523	9.
4614	3 173 A	179 M	19C S			
4628	5.76	0.88	0.9720			
4628	D	0 43.1	4 46	0.752	-1.142	-13.
4628	2 137 A	155 M				
4656	4.44	1.50	1.0800			
4656	III	C 43.5	7 2	0.083	-0.047	32.
4656	2 12 A	16 Y				
5234	4.84	1.22	1.0840			
5234	III	C 49.1	58 26	-0.033	-0.040	-23.
5234	1 3 A					

5395	4.64	0.96	1.0410			
5395	III-IV	0 50.7	58 38	-0.092	-0.043	-47.
5395	2 68 A	-3 M				
5516	4.43	0.93	1.0140			
5516	III-IV	0 51.9	22 53	-0.037	-0.040	-10.
5516	2 6 A	-11 M				
6186	4.28	0.96	1.0440			
6186	III	0 57.8	7 21	-0.082	0.029	7.
6186	3 23 A	22 M	38 Y			
6582	5.18	0.69	0.9960			
6582	VI	1 1.6	56 26	3.430	-1.575	-97.
6582	3 143 A	138 M	105YK			
7087	4.66	1.03	1.0470			
7087	III	1 6.1	20 30	0.035	-0.011	16.
7087	2 8 A	18 M				
7106	4.51	1.10	1.0570			
7106	III-IV	1 6.2	29 34	0.069	-0.036	30.
7106	1 34 M					
7318	4.66	1.03	1.0390			
7318	III	1 8.3	24 3	0.012	-0.030	6.
7318	2 -10 A	21 M				
8207	4.90	1.08	1.0520			
8207	III-IV	1 16.4	45 0	0.035	0.008	-12.
8207	3 21*A	36 M	25YK			
8491	4.74	1.04	1.0500			
8491	III	1 18.9	67 36	0.077	0.030	-12.
8491	2 11*A	6 M				
9138	4.86	1.37	1.0700			
9138	III	1 25.0	5 38	0.291	-0.043	35.
9138	3 45 A	1 M	4 Y			
9270	3.62	0.97	1.0320			
9270	III	1 26.1	14 50	0.026	-0.006	15.
9270	2 -15 A	46 M				
9408	4.72	1.00	1.0400			
9408	III	1 27.4	58 43	0.036	-0.017	6.
9408	2 15 A	18 M				

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9927	3.56	1.28	1.0780			
9927	III	1 31.8	48 7	0.063	-0.112	16.
9927	2 29 A	-7 M				
10380	4.43	1.37	1.0830			
10380	III	1 36.2	4 59	-0.024	0.005	0.
10380	3 50 A	34 M	2 Y			
10476	5.23	0.83	0.9890			
10476	V	1 37.1	19 47	-0.296	-0.671	-34.
10476	2 125 A	143 M				
10761	4.26	0.96	1.0470			
10761	III	1 40.1	8 39	0.071	0.053	14.
10761	2 12 A	23 Y				
10780	5.59	0.82	0.9900			
10780	V	1 40.5	63 22	0.584	-0.246	2.
10780	3 108 M	108YK	118 W			
12533	2.10	1.21	1.0920			
12533	II	1 57.8	41 51	0.046	-0.050	-12.
12533	4 -4 A	15 M	4 W 11 S			
12929	2.00	1.15	1.0490			
12929	III	2 1.5	22 59	0.192	-0.146	-14.
12929	3 29 A	67 M	44 W			
15694	5.25	1.27	1.0660			
15694	III	2 26.3	1 49	0.022	-0.003	26.
15694	1 36 Y					
16160	5.82	0.97	0.9720			
16160	V	2 30.7	6. 25	1.807	1.459	23.
16160	3 145 A	137 M	155 Y			
17361	4.52	1.11	1.0620			
17361	III	2 41.9	28 50	0.151	-0.121	-15.
17361	2 22 A	17 M				
19476	3.81	0.98	1.0460			
19476	III	3 2.8	44 29	0.181	-0.155	29.
19476	2 32 A	10 M				
19656	4.64	1.11	1.0650			
19656	III	3 4.8	39 14	-0.022	0.006	7.
19656	1 19 A					

19787	4.37	1.03	1.0450			
19787	III	3 5.9	19 21	0.152	-0.007	25.
19787	2 21 A	26 M				
20630	4.82	0.68	1.0030			
20630	V	3 14.1	3 0	0.267	0.096	19.
20630	3 112*A	89 M	104 Y			
22484	4.29	0.57	1.0050			
22484	V	3 31.8	0 5	-0.234	-0.479	28.
22484	3 55 A	43 M	60 Y			
25975	6.09	0.95	1.0130			
25975	III	4 1.6	37 28	-0.107	-0.192	-40.
25975	1 19 A					
27697	3.76	0.98	1.0590			
27697	III	4 17.2	17 18	0.110	-0.031	38.
27697	2 13 A	14 M				
28100	4.69	0.98	1.0520			
28100	III	4 20.9	14 29	0.0	-0.033	32.
28100	1 10 A					
28305	3.54	1.02	1.0630			
28305	III	4 22.8	18 58	0.112	-0.038	39.
28305	2 23 A	-8YK				
28307	3.85	0.96	1.0380			
28307	III	4 22.9	15 44	0.105	-0.028	40.
28307	1 32 M					
29139	0.86	1.53	1.0970			
29139	III	4 30.2	16 18	0.069	-0.190	54.
29139	5 37 A	35 M	47YK 63 W	75 V		
31398	2.66	1.57	1.1230			
31398	II	4 50.5	33 0	0.008	-0.019	18.
31398	2 16 A	3 M				
31421	4.06	1.15	1.0600			
31421	III	4 50.7	13 21	-0.073	-0.048	1.
31421	2 12 A	15 M				
37160	4.09	0.94	1.0310			
37160	III	5 31.4	9 14	0.093	-0.305	99.
37160	4 34 A	18 Y	10YK 27 S			

37394	6.23	0.84	1.0080			
37394	D	5 33.2	53 26	0.010	-0.521	1.
37394	2 83 A	102 M				
37984	4.92	1.16	1.0580			
37984	III	5 37.3	1 26	-0.054	-0.014	88.
37984	2 14 A	-1 Y				
38529	5.96	0.78	1.0280			
38529	D	5 41.4	1 8	-0.073	-0.148	29.
38529	1 27*A					
39003	3.97	1.14	1.0680			
39003	III	5 44.6	39 7	0.0	0.008	10.
39003	2 17 A	6 M				
39881	6.60	0.65	1.0060			
39881	D	5 50.3	13 56	0.389	-0.468	-2.
39881	2 83 A	35 M				
40035	3.69	0.99	1.0560			
40035	III	5 51.3	54 17	0.085	-0.128	8.
40035	4 23 A	2 M	18YK 29 S			
40801	6.10	0.97	1.0370			
40801	III	5 56.1	42 55	0.119	-0.146	38.
40801	1 11 A					
42807	6.46	0.68	1.0180			
42807	D	6 7.6	10 40	0.394	-0.286	3.
42807	2 54*A	53 M				
43039	4.34	1.03	1.0650			
43039	III	6 9.0	29 32	-0.067	-0.264	20.
43039	2 10 A	21 M				
45410	5.88	0.94	1.0360			
45410	IV	6 22.1	58 14	-0.019	-0.337	36.
45410	2 14 A	37 W				
46480	5.91	0.90	1.0390			
46480	SG	6 28.6	61 34	-0.199	-0.279	-46.
46480	1 23 A					
49293	4.47	1.12	1.0600			
49293	III	6 42.6	2 31	-0.016	-0.013	11.
49293	3 19 A	14 M	0 Y			

49878		4.55	1.36	1.0740			
49878	III		6 45.5	77 6	0.079	-0.011	-26.
49878	2	9 M	24 G				
52497		5.16	0.93	1.0440			
52497	II		6 56.3	24 21	-0.004	-0.003	-9.
52497	1	10 A					
54563		6.33	0.88	1.0140			
54563	D		7 4.2	21 25	-0.161	-0.482	-15.
54563	3	22 A	20 M	19 S			
55280		5.22	1.09	1.0380			
55280	III		7 7.2	59 49	-0.091	-0.261	24.
55280	1	31 A					
57727		4.97	0.89	1.0370			
57727	III		7 17.4	25 15	-0.065	-0.026	6.
57727	1	19 A					
58207		3.80	0.99	1.0920			
58207	III		7 19.5	28 0	-0.117	-0.089	8.
58207	2	35 A	17 M				
59148		4.92	1.12	1.0660			
59148	III		7 23.6	28 7	-0.030	-0.027	36.
59148	1	13 A					
59294		4.55	1.29	1.0870			
59294	III		7 24.2	12 13	0.0	-0.019	-15.
59294	2	19 A	30 M				
60522		4.07	1.49	1.0860			
60522	III		7 29.8	27 7	-0.033	-0.109	-21.
60522	2	10 A	9 M				
62345		3.57	0.93	1.0470			
62345	III		7 38.4	24 38	-0.027	-0.054	21.
62345	2	25 A	14 M				
62509		1.15	1.00	1.0510			
62509	III		7 39.2	28 16	-0.623	-0.052	3.
62509	2	88*A	107 M				
62613		6.56	0.73	1.0220			
62613	C		7 39.8	80 31	-0.474	0.076	-8.
62613	2	75*A	53 G				

62721	4.90	1.45	1.0790			
62721	III	7 40.3	18 45	-0.075	-0.062	81.
62721	2 7 A	36 M				
64307	5.42	1.42	1.1000			
64307	III	7 48.2	74 11	-0.008	-0.038	35.
64307	1 18 G					
65345	5.28	C.91	1.0310			
65345	III	7 53.2	2 29	-0.160	0.097	46.
65345	2 24 A	23 Y				
66141	4.40	1.25	1.0570			
66141	III	7 57.1	2 37	-0.031	0.102	71.
66141	3 15 A	11 Y	35 S			
66216	4.89	1.12	1.C520			
66216	III	7 57.1	28 4	-C.021	-0.045	-11.
66216	1 11 A					
67767	5.73	0.83	1.C17C			
67767	D	8 4.4	25 49	-0.065	-0.352	-43.
67767	2 31 A	17YK				
69267	3.52	1.58	1.C870			
69267	III	8 11.1	9 30	-C.046	-0.051	22.
69267	3 -3 A	10 M	36 Y			
7C272	4.25	1.55	1.0940			
7C272	III	8 16.0	43 31	-0.014	-0.103	24.
7C272	2 25 A	-2 M				
71115	5.12	0.94	1.0450			
71115	II	8 20.5	7 53	-0.034	-0.008	15.
71115	2 3 A	28 Y				
72292	5.35	1.24	1.C570			
72292	III	8 26.9	20 47	-0.045	-0.052	24.
72292	1 12*A					
73108	4.61	1.17	1.0720			
73108	III	8 31.5	64 41	-0.052	0.023	15.
73108	2 13 A	8 G				
73471	4.44	1.21	1.C740			
73471	III	8 33.5	3 42	-C.021	-0.020	25.
73471	2 14 A	42 Y				

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73593	5.52	0.99	1.0410			
73593	IV	8 34.1	46 11	0.029	0.084	-37.
73593	1 12*A					
74739	4.20	1.00	1.0570			
74739	II	8 40.8	29 8	-0.020	-0.047	16.
74739	2 15 A	28 M				
75732	6.06	0.86	1.0020			
75732	D	8 46.8	28 43	-0.481	-0.240	27.
75732	2 69 A	76 M				
76294	3.30	1.00	1.0510			
76294	II-III	8 50.1	6 20	-0.100	0.011	23.
76294	3 24 A	0 M	61 Y			
77912	4.71	1.04	1.0610			
77912	I-II	9 0.2	38 51	-0.028	-0.023	17.
77912	1 16 A					
79096	6.40	0.74	1.0050			
79096	D	9 6.8	15 24	-0.521	0.240	45.
79096	3 65 A	42 M	71 S			
80493	3.30	1.55	1.0850			
80493	III	9 15.0	34 49	-0.217	0.013	38.
80493	2 22 A	7 M				
82308	4.48	1.59	1.0940			
82308	III	9 26.0	23 25	-0.023	-0.044	27.
82308	2 14 A	23 M				
82885	5.48	0.77	1.0990			
82885	IV-V	9 29.7	36 16	-0.705	-0.251	13.
82885	4 117 A	106 M	66 W 114 S			
83805	5.50	0.94	1.0470			
83805	III	9 35.8	40 13	-0.052	-0.046	30.
83805	1 15 A					
85503	4.10	1.23	1.0760			
85503	III	9 47.1	26 29	-0.218	-0.059	14.
85503	2 20 A	15 M				
86728	5.60	0.66	1.0250			
86728	V	9 55.2	32 25	-0.522	-0.436	56.
86728	3 69 A	18 M	42 S			

89758	3.21	1.58	1.0970			
89758	III	10 16.4	42 0	-0.082	0.025	-21.
89758	2 29 A	23 M				
94247	5.36	1.36	1.0890			
94247	III	10 47.5	55 7	-0.068	-0.015	1.
94247	1 13*A					
94264	3.92	1.C6	1.0460			
94264	III-IV	10 47.7	34 45	0.090	-0.286	16.
94264	2 7 A	30 M				
95689	1.95	1.07	1.0600			
95689	II-III	1C 57.6	62 17	-0.119	-0.070	-9.
95689	3 33 A	16 W	24 G			
96436	5.66	0.96	1.0240			
96436	SG	11 1.8	2 30	-0.384	-0.088	55.
96436	2 32 A	29 M				
98262	3.71	1.39	1.C940			
98262	III	11 13.1	33 38	-C.025	0.021	-9.
98262	2 3 A	24 M				
98839	5.06	1.01	1.C860			
98839	II	11 17.3	44 2	-0.035	-0.016	3.
98839	1 -7 A					
99491	6.19	C.79	1.0010			
99491	D	11 21.7	3 33	-C.722	0.177	-3.
99491	3 34 M	67 Y	47 W			
99648	5.18	1.00	1.0500			
99648	II-III	11 22.8	3 24	0.018	-0.017	-9.
99648	2 30 A	26 Y				
1C1501	5.46	0.71	0.9970			
1C1501	V	11 35.8	34 46	-0.014	-0.390	-5.
1C15C1	1 105 A					
1C2224	3.85	1.20	1.0720			
1C2224	III	11 4C.8	48 20	-0.138	0.019	-9.
102224	2 8 A	18 M				
103C95	6.46	C.75	0.9850			
1C3C95	VI	11 47.2	38 26	3.994	-5.800	-98.
103095	2 1C8 A	1C6 M				

104979	4.24	0.99	1.0490			
104979	III	12 0.1	9 17	-0.221	0.042	-30.
104979	3 34 A	46 M	27 Y			
106760	5.08	1.14	1.0820			
106760	III	12 11.5	33 37	-0.049	-0.122	-42.
106760	1 19*A					
107328	5.10	1.15	1.1000			
107328	III	12 15.3	3 52	-0.292	-0.072	35.
107328	3 -11 A	2 M	-8 Y			
107950	4.97	0.89	1.0410			
107950	III	12 19.2	52 7	0.011	0.007	-13.
107950	1 32 A					
108225	5.22	0.94	1.0450			
108225	III-IV	12 20.9	39 34	-0.075	-0.038	-4.
108225	1 24 A					
108381	4.56	1.13	1.0690			
108381	III-IV	12 22.0	28 49	-0.084	-0.088	4.
108381	1 -3 A					
109317	5.43	1.02	1.0530			
109317	III	12 28.7	33 48	0.018	-0.041	-20.
109317	1 26 A					
111028	5.86	0.99	1.0380			
111028	IV	12 41.3	10 6	0.275	-0.452	52.
111028	4 31 A	15 M	5 W -18 S			
111812	5.07	0.67	1.0150			
111812	III	12 46.8	28 5	-0.016	-0.016	-1.
111812	1 5 A					
112033	5.10	0.90	1.0620			
112033	III	12 48.4	21 47	-0.048	-0.029	-6.
112033	1 17 A					
112989	5.08	1.18	1.0960			
112989	II-III	12 55.5	31 19	-0.021	-0.014	-13.
112989	2 10 A	17 S				
113226	2.95	0.94	1.0490			
113226	II-III	12 57.2	11 30	-0.274	0.016	-14.
113226	1 31 A					

113996	4.90	1.50	1.0980			
113996	III	13 2.4	28 10	0.030	-0.078	-16.
113996	2 -3 A	32 M				
115004	5.05	1.05	1.0790			
115004	III	13 9.2	40 41	-0.049	0.007	-21.
115004	2 2*A	-9 M				
117176	5.16	0.71	1.0110			
117176	V	13 23.5	14 19	-0.237	-0.583	4.
117176	2 32 A	45 M				
119425	5.62	1.11	1.0840			
119425	III	13 38.0	4 3	-0.293	-0.077	-42.
119425	3 5 A	-2 Y	-12 W			
120477	4.28	1.56	1.0920			
120477	III	13 44.6	16 18	-0.095	0.034	-6.
120477	2 13 A	-21 M				
121370	2.80	0.59	1.0080			
121370	IV	13 49.9	18 54	-0.063	-0.365	0.
121370	2 94 A	109 M				
122742	6.43	0.72	1.0170			
122742	V	13 58.6	11 16	0.080	-0.314	-17.
122742	1 58*A					
124897	0.24	1.23	1.0700			
124897	III	14 11.1	19 42	-1.098	-2.003	-5.
124897	4 86 A	76 M	95YK 92 V			
127665	3.78	1.29	1.0830			
127665	III	14 27.5	30 49	-0.101	0.115	-14.
127665	2 31 A	3 M				
128750	2.98	1.10	1.0400			
128750	G	14 33.6	18 44	-0.033	-0.081	-14.
128750	2 11 M	55 W				
129312	5.03	1.01	1.0650			
129312	III	14 36.7	8 35	-0.006	-0.004	-22.
129312	2 6 A	8 Y				
129972	4.69	0.94	1.0450			
129972	III	14 40.6	17 23	-0.060	-0.058	-9.
129972	1 39 A					

131111	5.50	1.03	1.0410			
131111	III-IV	14 46.6	37 41	-0.213	0.087	-66.
131111	1 23*A					
131511	5.98	0.79	0.9970			
131511	D	14 48.9	19 33	-0.453	0.209	-34.
131511	2 81 A	79 M				
131156	4.64	0.74	0.9940			
131156	V	14 46.8	19 31	0.134	-0.107	4.
131156	4 147 A	157 M	165 Y 126 S			
133165	4.62	1.05	1.0560			
133165	III	14 57.9	2 29	-0.057	0.005	-16.
133165	2 16 A	18 Y				
133208	3.63	0.95	1.0520			
133208	III	14 58.2	40 47	-0.044	-0.039	-20.
133208	2 13 A	37 M				
133582	4.67	1.26	1.0830			
133582	III	15 0.2	27 20	-0.179	-0.014	-26.
133582	1 13*A					
134190	5.21	0.97	1.0570			
134190	III	15 3.4	54 56	0.053	0.006	29.
134190	1 25 A					
135482	5.44	1.11	1.0830			
135482	G	15 10.2	5 19	-0.021	-0.002	-34.
135482	2 14 A	26 Y				
135722	3.54	0.96	1.0530			
135722	III	15 11.5	33 41	0.085	-0.121	-12.
135722	2 26 A	20 M				
137759	3.47	1.17	1.0620			
137759	III	15 22.7	59 19	-0.008	0.009	-11.
137759	2 20 A	47 M				
138481	5.15	1.59	1.1440			
138481	III	15 27.3	41 10	0.010	-0.015	-9.
138481	2 17 A	12 M				
139195	5.40	0.95	1.0530			
139195	G	15 31.7	10 21	0.041	-0.136	8.
139195	1 26 A					

140538		5.80	0.68	1.0330			
140538	D		15 39.0	2 50	-0.084	-0.157	14.
140538	1	43 A					
140573		2.75	1.17	1.0700			
140573	III		15 39.4	6 44	0.134	0.039	3.
140573	2	46 A	38 M				
141477		4.28	1.70	1.1120			
141477	G		15 44.2	18 27	-0.048	-0.092	-39.
141477	2	24 A	-7 M				
141680		5.33	1.03	1.0460			
141680	III		15 45.2	2 30	0.027	-0.056	-4.
141680	2	21*A	31 Y				
148387		2.89	0.92	1.0350			
148387	III		16 22.6	61 44	-0.023	0.058	-14.
148387	2	48 A	2 M				
148856		2.31	0.95	1.0530			
148856	III		16 25.9	21 42	-0.103	-0.022	-26.
148856	3	9 A	24 M	31 S			
148897		5.29	1.29	1.0720			
148897	II		16 26.2	20 42	-0.080	-0.070	18.
148897	1	-3 A					
150275		6.39	1.00	1.0620			
150275	III		16 34.9	77 39	-0.100	0.271	-32.
150275	2	16*A	8 G				
150449		5.44	1.08	1.0630			
150449	III		16 36.0	56 13	0.0	0.065	-19.
150449	1	12*A					
150997		3.61	0.92	1.0420			
150997	III-IV		16 39.5	39 7	0.035	-0.090	8.
150997	2	53 A	47 M				
153210		3.42	1.14	1.0690			
153210	III		16 52.9	9 32	0.293	-0.014	-56.
153210	3	5 A	58 M	51 Y			
154733		5.72	1.31	1.0850			
154733	III		17 2.1	22 13	-0.101	-0.047	-96.
154733	1	10 A					

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155410	5.12	1.28	1.0860			
155410	III	17 6.3	40 54	-0.054	0.004	-56.
155410	1 15 A					
156283	3.36	1.44	1.1190			
156283	II	17 11.6	36 55	-0.029	-0.001	-26.
156283	2 12 A	27 M				
157999	4.44	1.52	1.1210			
157999	II	17 21.6	4 14	-0.001	0.004	-27.
157999	3 2 A	-14 M	17 Y			
158633	6.31	0.76	1.0020			
158633	D	17 25.0	67 23	-0.529	0.0	-40.
158633	3 84 A	64 M	65 G			
158899	4.48	1.43	1.1090			
158899	III	17 26.7	26 11	0.018	0.015	-26.
158899	2 7 A	17 M				
159181	2.99	0.97	1.0480			
159181	II	17 28.2	52 23	-0.017	0.008	-20.
159181	2 14 A	-6 M				
159222	6.54	0.64	1.0030			
159222	D	17 28.4	34 21	-0.238	0.046	-52.
159222	1 49*A					
161074	5.59	1.45	1.0780			
161074	III	17 38.4	24 37	-0.060	-0.108	-27.
161074	1 20*A					
161096	2.94	1.17	1.0700			
161096	III	17 38.5	4 37	-0.043	0.154	-12.
161096	5 18 A	21 M	40 Y 23YK	15 S		
161797	3.48	0.75	1.0000			
161797	IV	17 42.6	27 47	-0.313	-0.748	-16.
161797	2 104 A	108 M				
162211	5.34	1.16	1.0520			
162211	III	17 44.8	25 39	-0.008	-0.043	-26.
162211	1 13 A					
163588	3.90	1.18	1.0350			
163588	III	17 51.8	56 53	0.093	0.074	-26.
163588	2 21 A	41 M				

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163770	3.99	1.36	1.1190			
163770	II	17 52.8	37 16	0.004	0.002	-27.
163770	2 -5 A	-3 M				
163993	3.82	0.94	1.0410			
163993	III	17 53.9	29 16	0.085	-0.019	-2.
163993	2 18 A	7 M				
164058	2.42	1.53	1.0900			
164058	III	17 54.3	51 30	-0.011	-0.024	-28.
164058	3 11 A	19 M	20 W			
165341	4.28	0.86	0.9870			
165341	V	18 0.4	2 31	0.256	-1.097	-7.
165341	5 184 A	185 M	211 Y 212YK	172 S		
165760	4.73	0.97	1.0360			
165760	III-IV	18 2.5	8 43	0.006	0.029	-3.
165760	3 16*A	9 M	29 Y			
166208	5.11	0.91	1.0410			
166208	G	18 4.5	43 27	-0.002	-0.064	-16.
166208	1 7 A					
166229	5.67	1.17	1.0450			
166229	III	18 4.6	36 23	-0.100	-0.187	-7.
166229	1 8*A					
166620	6.40	0.87	0.9840			
166620	V	18 6.3	38 27	-0.310	-0.475	-19.
166620	2 95 A	106 M				
167042	5.94	0.95	1.0130			
167042	III	18 8.5	54 15	0.113	0.248	-16.
167042	2 7 A	39 M				
168532	5.49	1.53	1.0800			
168532	II	18 15.1	24 24	0.008	0.0	-14.
168532	3 -13*A	21 M	0 W			
168656	4.92	0.93	1.0410			
168656	III	18 15.9	3 20	-0.004	0.009	5.
168656	3 19 A	-5 M	14 Y			
168775	4.34	1.18	1.0580			
168775	III	18 16.4	36 1	0.023	0.042	-22.
168775	2 7 A	0 M				

169191	5.48	1.26	1.0690			
169191	III	18 18.4	17 47	0.066	0.018	-19.
169191	1 20 A					
169414	3.92	1.18	1.0400			
169414	III	18 19.4	21 43	0.194	-0.250	-58.
169414	2 6 A	32 M				
170829	6.59	0.79	0.9910			
170829	IV	18 26.4	20 45	0.006	-0.261	-59.
170829	1 26*A					
171635	4.95	0.62	1.0010			
171635	IB	18 30.8	56 58	-0.009	-0.007	-12.
171635	1 5 A					
171779	5.42	1.11	1.0620			
171779	III	18 31.7	52 16	-0.006	0.003	-24.
171779	1 0 A					
173780	4.92	1.20	1.0580			
173780	III	18 42.0	26 33	0.015	0.018	-17.
173780	2 18 A	26 M				
175225	5.62	0.84	1.0110			
175225	D	18 49.4	52 51	-0.048	0.268	2.
175225	2 43 A	9 M				
175306	4.78	1.20	1.0510			
175306	II-III	18 49.7	59 16	0.079	0.024	-20.
175306	3 3 A	-10 M	3 G			
175535	4.97	0.90	1.0600			
175535	III	18 50.8	50 35	0.0	-0.029	8.
175535	1 21*A					
176411	4.21	1.07	1.0730			
176411	III	18 55.1	14 56	-0.056	-0.075	-48.
176411	2 24 A	17 M				
176527	5.28	1.24	1.0710			
176527	III	18 55.7	26 6	0.083	-0.010	-24.
176527	1 -5 A					
176670	5.11	1.47	1.1180			
176670	II	18 56.2	32 0	0.008	0.009	-16.
176670	1 -6 A					

177808	5.80	1.54	1.0810			
177808	III	19 1.2	31 36	0.070	-0.071	6.
177808	2 5 M	24 W				
178428	5.99	0.71	1.0100			
178428	D	19 3.5	16 42	0.053	-0.310	14.
178428	2 58 A	53 M				
179094	5.93	1.09	1.0400			
179094	IV	19 6.1	52 16	-0.104	-0.062	4.
179094	1 1 A					
179958	6.62	0.65	1.0110			
179958	V	19 9.5	49 40	-0.210	0.632	-38.
179958	5 37 A	44 M	37YK 43 W	43 S		
180006	5.24	1.02	1.0480			
180006	III	19 9.8	56 41	0.036	0.044	-16.
180006	1 7 A					
180610	5.28	1.68	1.0390			
180610	III	19 12.1	57 32	-0.014	-0.072	-27.
180610	1 21 A					
180711	3.24	1.00	1.0550			
180711	III	19 12.5	67 29	0.094	0.090	25.
180711	3 30 A	19 M	21 G			
180809	4.46	1.27	1.1060			
180809	II	19 12.9	37 57	-0.005	0.0	-31.
180809	2 -15 A	9 M				
181276	3.98	0.95	1.0450			
181276	III	19 14.8	53 11	0.057	0.122	-29.
181276	2 27 A	6 M				
182488	6.50	0.81	0.9790			
182488	V	19 19.8	33 1	0.080	0.164	-21.
182488	1 55*A					
182572	5.23	0.78	1.0130			
182572	IV	19 20.2	11 44	0.719	0.636	-100.
182572	3 48 A	74 M	101YK			
182762	5.31	1.02	1.0510			
182762	III	19 21.1	19 36	0.089	-0.067	1.
182762	1 -6 A					

184406	4.65	1.17	1.0520			
184406	III	19 29.2	7 10	0.211	-0.157	-24.
184406	3 16 A	51 M	53 Y			
185144	4.78	0.79	0.9910			
185144	V	19 32.6	69 29	0.575	-1.745	27.
185144	3 177 A	181 M	165 G			
185734	4.79	1.01	1.0790			
185734	III-IV	19 35.4	29 55	-0.003	0.039	6.
185734	2 6 A	-2 M				
185958	4.45	1.05	1.0790			
185958	II	19 36.6	17 15	0.004	-0.037	-22.
185958	2 33 A	-14 M				
186486	5.45	0.94	1.0310			
186486	III	19 39.6	25 32	0.004	0.018	-9.
186486	1 20 A					
186675	5.02	0.96	1.0260			
186675	III	19 40.7	37 7	0.068	0.031	-24.
186675	1 15 A					
188310	4.86	1.01	1.0560			
188310	III	19 49.4	8 12	0.095	-0.082	-42.
188310	3 20 A	34 M	3 Y			
188512	3.90	0.87	1.0120			
188512	IV	19 50.4	6 9	0.039	-0.483	-40.
188512	5 78 A	66 M	82 Y	OYK 67 S		
188947	4.03	1.04	1.0400			
188947	III	19 52.6	34 49	-0.036	-0.029	-27.
188947	3 7 A	4 M	15 S			
189276	5.13	1.58	1.1250			
189276	II-III	19 54.0	58 35	-0.012	-0.021	5.
189276	1 1 A					
189319	3.71	1.54	1.0990			
189319	III	19 54.3	19 13	0.061	0.024	-33.
189319	2 3 A	14 M				
190147	5.28	1.13	1.0670			
190147	II-III	19 58.5	49 50	0.014	0.002	1.
190147	1 11 A					

190360	5.68	0.72	0.9940			
190360	IV	19 59.5	29 38	0.676	-0.530	-46.
190360	2 44 A	42 M				
192806	4.73	1.27	1.0660			
192806	III	20 11.6	27 30	-0.041	0.008	3.
192806	2 2 A	11 M				
193092	5.50	1.64	1.1500			
193092	II	20 13.4	40 3	-0.001	-0.004	-20.
193092	1 4 A					
194317	4.60	1.33	1.0750			
194317	III	20 19.9	31 52	0.037	-0.002	-15.
194317	2 8 A	7 M				
196755	5.23	0.74	1.0090			
196755	IV	20 34.3	9 44	0.313	0.015	-52.
196755	4 14 A	19 M	36 Y 15YK			
197076	6.40	0.64	1.0060			
197076	D	20 36.2	19 34	0.119	0.303	-37.
197076	1 50 A					
197752	5.13	1.18	1.0740			
197752	III	20 40.6	24 55	-0.035	-0.178	31.
197752	2 4 A	-7 M				
197964	4.45	1.04	1.0180			
197964	IV	20 42.0	15 46	-0.037	-0.198	-7.
197964	4 20 A	43 M	5YK 2 S			
197989	2.64	1.04	1.0520			
197989	III	20 42.2	33 36	0.355	0.325	-10.
197989	3 52*A	39 M	2YK			
198134	5.20	1.31	1.0740			
198134	III	20 43.2	34 0	0.043	0.002	-23.
198134	1 1 A					
198149	3.59	0.93	1.0090			
198149	IV	20 43.2	61 27	0.090	0.820	-87.
198149	4 66 A	64 M	66YK 78 S			
198387	6.34	0.91	0.9960			
198387	D	20 44.9	52 3	0.064	-0.159	-41.
198387	1 16*A					

200905	3.92	1.64	1.1570			
200905	IB	21 1.3	43 32	0.003	0.002	-20.
200905	2 -8 A	14 M				
203344	5.82	1.05	1.0630			
203344	III-IV	21 16.6	23 26	0.235	-0.126	-89.
203344	1 -8 A					
203504	4.24	1.10	1.0600			
203504	III	21 17.5	19 23	0.105	0.065	-76.
203504	3 20 A	-11 M	12YK			
204724	4.76	1.65	1.1150			
204724	III	21 25.4	23 12	0.015	0.003	-19.
204724	2 4 A	21 M				
205512	4.98	1.04	1.0630			
205512	III	21 30.7	38 5	0.120	0.098	-66.
205512	3 12 A	-5 M	13 S			
206778	2.54	1.58	1.1590			
206778	IB	21 39.3	9 25	0.025	0.002	5.
206778	3 -22 A	17 M	8 Y			
206859	4.52	1.18	1.0920			
206859	IB	21 39.8	16 53	0.007	-0.013	-22.
206859	2 -1 A	6 M				
206952	4.85	1.14	1.0490			
206952	III	21 40.4	70 51	0.116	0.101	-37.
206952	3 11 A	0 M	-8 G			
209747	4.90	1.42	1.0840			
209747	III	22 0.6	4 34	0.105	0.103	-16.
209747	3 10 A	12 M	3 Y			
212943	4.93	1.05	1.0470			
212943	III-IV	22 22.8	4 12	0.078	-0.308	54.
212943	3 28 A	6 M	17 Y			
215665	4.14	0.95	1.0690			
215665	II-III	22 41.7	23 2	0.052	-0.012	-4.
215665	2 32 A	41 M				
216228	3.68	1.06	1.0630			
216228	III	22 46.1	65 40	-0.067	-0.122	-12.
216228	2 27 A	48 M				

218031	4.91	1.04	1.0510			
218031	III	22 59.7	49 30	0.161	0.168	-35.
218031	2 -9 A	5 M				
218101	6.45	0.83	1.0180			
218101	IV	23 0.2	16 2	-0.190	-0.193	-27.
218101	1 29 M					
218658	4.56	0.80	1.0310			
218658	III	23 4.7	74 51	0.010	-0.025	-19.
218658	3 3 A	5 G	-20 S			
219134	5.65	1.00	0.9790			
219134	V	28 8.5	56 37	2.072	0.299	-18.
219134	3 137 A	158 M	181 S			
219615	3.85	0.92	1.0330			
219615	III	23 12.0	2 44	0.756	0.022	-14.
219615	3 18 A	37 M	20 Y			
219916	4.90	0.82	1.0250			
219916	III	23 14.5	67 34	0.058	0.020	-18.
219916	2 27 A	11 G				
220954	4.45	1.05	1.0800			
220954	III	23 22.9	5 50	-0.127	-0.043	6.
220954	3 15 A	-6 M	10 Y			
221345	5.32	1.02	1.0420			
221345	III	23 26.4	38 41	0.285	-0.079	-59.
221345	1 11 A					
222107	4.00	1.02	1.0080			
222107	III-IV	23 32.7	45 55	0.158	-0.421	7.
222107	3 44 A	26 M	44 S			
222404	3.42	1.03	1.0490			
222404	IV	23 35.2	77 4	-0.065	0.154	-42.
222404	4 63 A	62 M	66YK 57 G			

CT4 Explanation of reduced data

After reduction, the data for each star is punched onto a single card. The stack of data cards is preceded by two preliminary cards.

The first preliminary card contains the number of stars and pairs of stars. The first card in the reduced data set for the H and K line stars has

63 4

which tells us that there are 63 stars or pairs of stars, and 4 pairs of stars, so that altogether there are 67 = 63 + 4 stars.

The second preliminary card gives the sample mean and standard deviation of the absolute magnitude and the color. The second card of the H and K line reduced data set has

0·37065E01 0·45025E01 0·95298E00 0·44900E00

This tells us that the sample mean and standard deviation of the absolute magnitude are

$\overline{M_V} = 3{\cdot}7065$, $s(M_V) = 4{\cdot}5025$; and for the color

$\overline{B-V} = 0{\cdot}95298$, $s(B-V) = 0{\cdot}44900$.

These values are used by the regression program as scaling factors.

The data cards for each star contain the HD number, the apparent magnitude, the color, the luminosity criterion, the proper motion distance and its precision, the parallax and its precision, the minimum sampling parallax, and upper and lower limits on the absolute magnitude. The print out for the H and K line data set also has the mean absolute magnitude on the right hand side; this is not punched on the cards.

Refer to the third card on the H and K line data set. The first four numbers are

131156 4·54 0·76 1·4500

This tells us that HD131156 has V = 4·54, B-V = 0·76, and log (W) = 1·4500. The next seven numbers are

0·196E02 0·587E-03 0·1540 18515 30 5·78 5·10

This tells us that r = 19·6 parsecs, with precision h_r = 0·000587 psc^{-2}; that the parallax is $\pi_0 = 0''154$, with precision h_r = 18515 $('')^{-2}$; that the minimum sampling parallax $\pi_c = 0''030$; and that outside the interval $5.78 < M_V < 5.10$ the likelihood of the absolute magnitude is so small that it can be ignored.

The fourth card gives data on the fainter companion to 131156 for V, B-V, and log (W).

97.

CT5 Reduced data for H and K lines

63 4

0.37065E 01 0.45025E 01 0.95298E 00 0.44900E 00

131156	4.54	0.76	1.4500	0.196E 02	0.587E-03	0.1540	18515	30	5.78	5.10	5.460
131156	6.44	1.06	1.3000								
165341	4.02	0.88	1.4800	0.294E 01	0.866E-01	0.1943	21435	30	5.69	5.19	5.456
165341	6.02	1.26	1.3800								
201091	5.19	1.19	1.4000	0.445E 01	0.288E 00	0.2924	23785	30	7.67	7.35	7.515
201092	6.02	1.38	1.2800								
155886	5.29	0.81	1.4000	0.304E 01	0.121E 00	0.1885	4630	50	7.14	6.02	6.641
155885	5.33	0.85	1.4000								
3546	4.37	0.88	1.7400	0.283E 02	0.617E-02	0.0285	16805	30	2.99	0.19	1.571
3651	5.84	0.86	1.4600	0.746E 01	0.149E-01	0.0991	10035	36	6.42	4.94	5.751
4128	2.04	1.02	1.7900	0.166E 02	0.160E-02	0.0605	8360	48	1.93	-1.04	0.655
4628	5.76	0.88	1.3200	0.285E 01	0.138E 00	0.1447	10035	30	6.99	6.00	6.568
6805	3.44	1.16	1.7700	0.172E 02	0.302E-02	0.0300	10935	30	2.34	-0.93	0.710
6860	2.03	1.63	1.9000	0.203E 02	0.198E-02	0.0442	11695	36	1.36	-2.32	-0.094
8512	3.61	1.06	1.7900	0.200E 01	0.228E-02	0.0324	17082	30	2.37	-0.56	1.078
10476	5.23	0.83	1.4200	0.647E 01	0.416E-01	0.1329	10035	36	6.31	5.23	5.810
10780	5.59	0.82	1.4800	0.101E 02	0.139E-01	0.1123	8310	36	6.43	4.99	5.763
12929	2.00	1.15	1.7700	0.108E 02	0.269E-02	0.0446	11695	36	1.34	-2.28	0.006
13974	4.87	0.61	1.6000	0.233E 01	0.659E-01	0.0888	13885	30	5.19	4.04	4.670
16160	5.82	0.97	1.3800	0.180E 01	0.324E 00	0.1466	14665	30	7.01	6.49	6.707
17925	6.05	0.87	1.4600	0.133E 02	0.124E-01	0.1250	10777	36	7.01	5.89	6.481
20630	4.82	0.68	1.5300	0.171E 02	0.514E-02	0.1047	14665	36	5.40	4.26	4.866
22049	3.73	0.89	1.4200	-0.489E 01	0.640E-01	0.3016	20932	30	6.28	5.96	6.125
23249	3.55	0.92	1.5300	-0.774E 01	0.149E-01	0.1108	17082	30	4.20	3.21	3.760

26565	4.42	0.82	1.4800	0.203E 01	0.103E 01	0.2020	22402	30	6.16	5.86	5.980
29139	0.86	1.53	1.8300	0.241E 02	0.254E-02	0.0520	19935	30	0.22	-1.86	-0.722
30495	5.51	0.64	1.5300	0.628E 01	0.272E-02	0.0685	8360	36	5.58	3.05	4.496
32147	6.21	1.06	1.4500	0.395E 01	0.923E-01	0.1055	27912	30	6.68	5.87	6.336
37394	6.23	0.84	1.4900	0.630E 01	0.191E-01	0.0927	10035	36	6.70	5.11	6.004
47205	3.95	1.06	1.6700	0.525E 02	0.192E-03	0.0551	8360	42	3.71	0.32	2.230
62509	1.15	1.00	1.7500	0.250E 01	0.292E-01	0.0978	10035	30	1.71	0.20	1.078
82885	5.41	0.77	1.5400	0.968E 01	0.152E-01	0.1125	15545	30	6.11	5.08	5.625
95689	1.79	1.06	1.8500	0.267E 02	0.936E-03	0.0318	12855	30	-0.67	-3.78	-1.248
101501	5.35	0.72	1.5100	0.101E 02	0.995E-02	0.1102	6305	48	6.23	4.53	5.454
103095	6.45	0.75	1.3800	0.914E 01	0.836E-01	0.1125	10035	30	7.24	5.95	6.646
104979	4.12	0.99	1.7600	-0.298E 01	0.105E-02	0.0399	14665	30	3.22	-0.39	1.865
109345	6.21	1.07	1.7600	0.400E 03	0.111E-04	0.0534	6305	42	6.06	1.53	4.076
114710	4.28	0.57	1.5200	-0.134E 02	0.511E-02	0.1246	13885	36	5.18	4.20	4.732
116976	4.75	1.09	1.8300	0.242E 02	0.437E-03	0.0572	4630	50	4.83	-0.61	2.652
117176	4.98	0.71	1.6100	0.619E 01	0.335E-01	0.0421	10035	36	4.32	3.13	3.621
122742	6.34	0.73	1.5100	0.593E 01	0.418E-02	0.0619	6305	42	6.38	2.85	5.053
123139	2.05	1.02	1.8000	0.716E 01	0.203E-01	0.0689	7047	48	2.19	-0.29	1.194
124897	0.06	1.23	1.8100	0.130E 01	0.370E 00	0.0922	18275	30	0.38	1.00	0.416
131511	6.01	0.83	1.5300	0.204E 02	0.743E-02	0.0851	10035	42	6.34	4.59	5.521
131873	2.08	1.47	1.8500	-0.571E 02	0.372E-04	0.0326	14925	30	0.91	-4.24	-1.167
131977	5.80	1.11	1.4200	0.272E 01	0.293E 00	0.1762	10777	36	7.38	6.60	7.040
140573	2.65	1.17	1.7600	-0.114E 02	0.131E-02	0.0472	10035	30	2.13	-1.61	0.766
146791	3.24	0.96	1.7300	-0.239E 02	0.538E-03	0.0401	14665	30	2.34	-1.24	0.966

148387	2.74	0.91	1.7000	-0.704E 02	0.232E-04	0.0349	10035	30	1.85	-6.01	-1.137
149661	5.74	0.81	1.4900	-0.206E 00	0.220E-01	0.0902	17082	30	6.03	4.80	5.506
150997	3.50	0.92	1.7600	0.874E 02	0.750E-04	0.0548	10035	36	3.18	0.18	1.858
160269	5.23	0.61	1.6000	-0.110E 01	0.687E-02	0.0654	21695	30	4.92	3.39	4.269
169916	2.84	1.04	1.7400	0.916E 01	0.217E-02	0.0525	8360	42	2.54	-1.11	1.131
175751	4.82	1.08	1.7600	0.619E 02	0.464E-03	0.0386	10935	30	4.01	-0.98	1.852
184406	4.44	1.16	1.7200	0.102E 02	0.640E-02	0.0386	14665	36	3.50	0.96	2.360
185144	4.68	0.79	1.4600	0.163E 00	0.153E 00	0.1772	14925	30	6.22	5.56	5.928
188376	4.70	0.76	1.6700	0.114E 02	0.326E-02	0.0595	4630	50	4.83	0.58	3.146
188512	3.71	0.86	1.5900	0.108E 02	0.605E-02	0.0661	21435	30	3.42	1.90	2.737
190406	5.80	0.61	1.5300	0.839E 01	0.783E-02	0.0589	13885	30	5.46	3.24	4.553
191026	5.33	0.85	1.6600	0.105E 02	0.727E-03	0.0321	16805	30	4.09	-0.20	2.471
192310	5.73	0.88	1.5100	0.756E 01	0.119E 00	0.1192	12210	42	6.58	5.48	6.081
192947	3.55	0.94	1.7700	0.191E 02	0.232E-03	0.0286	17082	30	2.16	-3.23	0.116
197989	2.45	1.03	1.7600	0.713E 01	0.205E-02	0.0388	12955	30	1.57	-1.98	0.163
198149	3.43	0.92	1.6500	0.402E 00	0.801E-02	0.0710	16805	30	3.33	1.71	2.638
219134	5.57	1.01	1.3800	0.164E 01	0.148E 00	0.1570	13885	30	6.89	6.12	6.555
222107	3.88	1.02	1.7600	0.614E 01	0.779E-02	0.0409	13885	36	3.04	0.77	1.987
222404	3.22	1.03	1.6600	0.196E 01	0.103E-02	0.0645	17845	30	2.95	1.21	2.164

CT6 Reduced data for FeI 5250 stars

Processed data for 253 stars: Fe 1 5250 triplet

100.

250 3

0.75066E 00 0.48867E 01 0.10656E 01 0.24072E 00

89484	2.61	1.14	0.0554	0.253E 02	0.329E-02	0.0207	10035	1.20	-1.92
89485	3.80	0.91	0.0266						
135101	6.68	0.68	0.0286	0.147E 02	0.150E-01	0.0226	10035	5.35	3.64
F135101	7.58	0.73	0.0266						
201091	5.57	1.18	-0.0651	0.445E 01	0.288E 00	0.2924	23785	8.05	7.73
201092	6.02	1.38	-0.0704						
166	6.14	0.75	0.0060	0.985E 01	0.992E-02	0.0612	10035	5.98	3.39
3546	4.38	0.87	0.0354	0.283E 02	0.617E-02	0.0285	16805	3.00	0.20
3627	3.28	1.28	0.0602	0.232E 02	0.132E-02	0.0235	10035	1.98	-1.93
3651	5.86	0.85	0.0020	0.746E 01	0.149E-01	0.0991	10035	6.44	4.96
3712	2.23	1.17	0.0807	0.630E 02	0.139E-03	0.0109	14615	0.08	-5.37
4614	3.44	0.58	-0.0555	0.518E 01	0.643E-01	0.1814	13885	5.03	4.37
4628	5.76	0.88	-0.0284	0.285E 01	0.138E 00	0.1447	10035	6.99	6.00
4656	4.44	1.50	0.0770	0.493E 02	0.427E-03	0.0148	10935	2.71	-2.09
5234	4.84	1.22	0.0807	0.213E 02	0.232E-03	0.0051	6305	3.10	-1.96
5395	4.64	0.96	0.0402	0.279E 02	0.285E-03	0.0437	10035	4.02	-0.23
5516	4.43	0.93	0.0139	0.344E 02	0.790E-04	0.0010	10035	1.99	-3.53
6186	4.28	0.96	0.0431	-0.539E 02	0.323E-03	0.0286	14665	2.97	-1.29
6582	5.18	0.69	-0.0040	0.269E 01	0.432E-01	0.1350	12955	6.24	5.30
7087	4.66	1.03	0.0459	0.181E 03	0.484E-04	0.0130	10035	2.90	-4.36
7106	4.51	1.10	0.0554	0.138E 03	0.136E-03	0.0354	3730	4.21	-3.55
7218	4.66	1.03	0.0383	0.785E 02	0.766E-04	0.0028	10035	2.34	-3.56
8207	4.90	1.08	0.0507	0.543E 02	0.248E-04	0.0277	12955	3.62	-4.29
8491	4.74	1.04	0.0488	0.202E 01	0.385E-03	0.0112	10035	2.90	-1.32

9138	4.86	1.37	0.0677	0.312E 02	0.330E-02	0.0219	14665		3.27	0.18
9270	3.62	0.97	0.0315	0.260E 03	0.248E-04	0.0088	10035		1.65	-6.14
9408	4.72	1.00	0.0392	0.170E 03	0.460E-04	0.0182	10035		3.21	-4.30
9927	3.56	1.28	0.0751	0.259E 02	0.110E-02	0.0174	10035		2.01	-1.86
10380	4.43	1.37	0.0797	-0.144E 03	0.265E-04	0.0319	14665		3.25	-2.32
10476	5.23	0.83	-0.0111	0.647E 01	0.416E-01	0.1329	10035		6.31	5.23
10761	4.26	0.96	0.0459	0.290E 02	0.364E-03	0.0178	10935		2.67	-2.19
10780	5.59	0.82	-0.0101	0.101E 02	0.139E-01	0.1123	8310		6.43	4.99
12533	2.10	1.21	0.0880	0.395E 02	0.230E-03	0.0068	15545		-0.36	-4.87
12929	2.00	1.15	0.0478	0.108E 02	0.269E-02	0.0446	11695		1.34	-2.28
15694	5.25	1.27	0.0639	0.376E 03	0.215E-04	0.0371	4630		4.86	-4.87
16160	5.82	0.97	-0.0284	0.180E 01	0.324E 00	0.1473	14665		7.02	6.49
17361	4.52	1.11	0.0602	0.936E 01	0.170E-02	0.0216	10035		3.15	-0.15
19476	3.81	0.98	0.0450	0.394E 02	0.231E-02	0.0257	10035		2.60	-1.30
19656	4.64	1.11	0.0667	-0.190E 03	0.243E-04	0.0208	6305		3.55	-3.77
19787	4.37	1.03	0.0440	0.471E 02	0.120E-02	0.0242	10035		3.10	-1.33
20630	4.82	0.68	0.0030	0.171E 02	0.514E-02	0.1047	14665		5.40	4.26
22484	4.29	0.57	0.0050	-0.296E 01	0.194E-01	0.0548	14665		3.83	2.79
25975	6.09	0.95	0.0129	0.128E 02	0.479E-02	0.0209	6305		5.01	2.25
27697	3.76	0.98	0.0573	0.661E 02	0.778E-03	0.0150	10035		2.10	-2.51
28100	4.69	0.98	0.0507	0.463E 02	0.824E-04	0.0116	6305		3.24	-3.29
28305	3.54	1.02	0.0611	0.640E 02	0.825E-03	0.0148	9225		1.94	-2.66
28307	3.85	0.96	0.0373	0.695E 02	0.721E-03	0.0336	3730		3.51	-2.51
29139	0.86	1.53	0.0926	0.241E 02	0.254E-02	0.0520	19935		0.22	-1.86

31398	2.66	1.57	0.1160	0.174E 03	0.287E-04	0.0132	10035	0.91	-6.76
31421	4.06	1.15	0.0583	0.961E 01	0.658E-03	0.0148	10035	2.39	-1.56
37160	4.09	0.94	0.0305	0.200E 02	0.577E-02	0.0262	17705	2.58	0.14
37394	6.23	0.84	0.0080	0.630E 01	0.191E-01	0.0927	10035	6.70	5.11
37984	4.92	1.16	0.0564	-0.121E 03	0.259E-03	0.0094	10935	2.91	-0.29
38529	5.96	0.78	0.0276	0.792E 01	0.213E-02	0.0284	6305	5.13	1.54
39003	3.97	1.14	0.0658	-0.538E 03	0.361E-05	0.0154	10035	2.33	-6.45
39881	6.60	0.65	0.0060	0.436E 01	0.262E-01	0.0672	10035	6.57	4.71
40035	3.69	0.99	0.0545	0.185E 02	0.126E-02	0.0216	16805	2.01	-1.47
40801	6.10	0.97	0.0363	0.209E 02	0.206E-02	0.0136	6305	4.74	1.32
42807	6.46	0.68	0.0178	0.458E 01	0.170E-01	0.0554	10035	6.16	4.16
43039	4.34	1.03	0.0630	0.128E 02	0.547E-02	0.0165	10035	2.75	0.62
45410	5.88	0.94	0.0354	0.243E 02	0.309E-02	0.0218	7965	4.67	1.33
46480	5.91	0.90	0.0383	0.410E 00	0.321E-02	0.0261	6305	5.01	2.16
49293	4.47	1.12	0.0583	0.820E 02	0.370E-04	0.0140	14665	2.48	-4.43
49878	4.55	1.36	0.0714	0.282E 02	0.510E-03	0.0208	8620	3.25	-1.58
52497	5.16	0.93	0.0431	0.279E 03	0.226E-05	0.0127	6305	3.76	-6.72
54563	6.33	0.88	0.0139	0.397E 01	0.209E-01	0.0234	13885	4.83	4.27
55280	5.22	1.09	0.0373	0.212E 02	0.349E-02	0.0343	6305	4.57	0.85
57727	4.97	0.89	0.0363	0.307E 02	0.395E-03	0.0219	6305	3.92	-1.42
58207	3.80	0.99	0.0880	0.200E 02	0.169E-02	0.0313	10035	2.79	-1.13
59148	4.92	1.12	0.0639	0.144E 03	0.125E-03	0.0160	6305	3.66	-3.23
59294	4.55	1.29	0.0834	0.112E 03	0.219E-04	0.0258	10035	3.35	-4.93
60522	4.07	1.49	0.0825	0.108E 02	0.848E-03	0.0127	10035	2.30	-1.32

62345	3.57	C.93	0.C459	C.860E 02	0.254E-03	0.0240	10035	2.29	-3.70
625C9	1.15	1.00	C.C497	C.250E 01	0.292E-01	0.0978	10035	1.71	0.20
62613	6.56	C.73	C.C218	-C.116E 01	0.196E-01	0.0684	11195	6.52	4.90
62721	4.90	1.45	0.C76C	C.1C8E 03	0.756E-03	0.0208	10035	3.50	-1.84
643C7	5.42	1.42	C.C953	C.978E 02	0.467E-04	0.0215	4890	4.54	-3.32
65345	5.28	C.91	C.C3C5	C.75CE 01	0.160E-02	0.0264	10935	4.05	0.59
66141	4.40	1.25	C.C554	-C.972E 02	0.442E-03	0.0218	14785	2.80	-0.17
66216	4.89	1.12	C.C5C7	C.400E 02	0.162E-03	0.0143	63C5	3.56	-2.41
67767	5.73	0.83	C.C169	-C.155E 01	0.947E-02	0.0299	9225	4.72	3.27
69267	3.52	1.58	C.C834	C.758E 02	0.374E-03	0.0158	14665	1.63	-3.38
7C272	4.25	1.55	C.C898	C.482E 02	0.575E-03	0.0186	10035	2.75	-2.03
71115	5.12	0.94	0.C44C	C.1C6E 03	0.904E-04	0.0168	10935	3.48	-3.09
72292	5.35	1.24	0.0554	C.935E 02	0.342E-03	0.0147	6305	4.03	-1.76
731C8	4.61	1.17	0.0695	C.5C9E 02	0.243E-03	0.0146	11195	2.85	-2.41
73471	4.44	1.21	0.0714	C.197E 03	0.684E-04	0.0291	10935	3.31	-4.38
73593	5.52	0.99	0.0402	-C.461E 02	0.224E-03	0.0149	6305	4.21	-0.63
74739	4.20	1.00	0.0554	C.111E 03	0.161E-03	0.0236	10035	2.91	-3.58
75732	6.06	0.86	0.0020	0.143E 02	0.209E-01	0.0754	10035	6.21	4.19
76294	3.30	1.00	0.0497	0.511E 02	0.560E-03	0.0331	14665	2.16	-2.93
77912	4.71	1.04	0.0592	C.174E 03	0.778E-04	0.0200	6305	3.60	-3.92
79C96	6.4C	0.74	0.0050	0.122E 02	0.149E-01	0.0643	13885	6.20	4.19
8C493	3.30	1.55	0.0816	C.458E 02	0.263E-02	0.0206	10035	1.89	-1.85
823C8	4.48	1.59	0.0898	0.153E 03	0.159E-03	0.0215	10035	3.10	-3.55
82885	5.48	0.77	0.0944	0.968E 01	0.152E-01	0.1125	15545	6.18	5.15

83805	5.50	0.94	0.0459	0.118E 03	0.275E-03	0.0194	6305	4.36	-1.95
85503	4.10	1.23	0.0732	0.264E 02	0.295E-02	0.0225	10035	2.77	-0.54
86728	5.60	0.66	0.0247	0.177E 02	0.330E-01	0.0523	13885	5.09	2.87
89758	3.21	1.58	0.0926	-0.413E 02	0.302E-03	0.0314	10035	2.21	-2.60
94247	5.36	1.36	0.0853	0.555E 02	0.216E-03	0.0165	6305	4.12	-1.80
94264	3.92	1.06	0.0450	0.166E 02	0.202E-02	0.0205	10035	2.50	-0.78
95689	1.95	1.07	0.0583	0.267E 02	0.936E-03	0.0318	12855	0.83	-3.62
96436	5.66	0.96	0.0237	0.426E 02	0.405E-02	0.0355	10035	4.79	0.83
98262	3.71	1.39	0.0898	-0.952E 02	0.207E-04	0.0159	10035	2.09	-5.20
98839	5.06	1.01	0.0825	0.964E 02	0.817E-04	-0.0020	6305	2.95	-3.19
99491	6.19	0.79	0.0010	0.601E 01	0.115E-01	0.0562	10020	5.91	3.43
99648	5.18	1.00	0.0488	0.944E 02	0.598E-05	0.0331	10935	4.19	-4.18
101501	5.46	0.71	-0.0030	0.101E 02	0.995E-02	0.1102	6305	6.34	4.64
102224	3.85	1.20	0.0695	0.189E 02	0.569E-03	0.0167	10035	2.27	-2.05
103095	6.46	0.75	-0.0151	0.914E 01	0.836E-01	0.1125	10035	7.25	5.96
104979	4.24	0.99	0.0478	-0.298E 01	0.105E-02	0.0399	14665	3.34	-0.27
106760	5.08	1.14	0.0788	0.291E 02	0.117E-02	0.0231	6305	4.07	-0.34
107328	5.10	1.15	0.0953	0.289E 02	0.453E-02	-0.0018	14665	2.04	0.70
107950	4.97	0.89	0.0402	-0.328E 03	0.969E-05	0.0370	6305	4.40	-4.37
108225	5.22	0.94	0.0440	0.619E 02	0.239E-03	0.0293	6305	4.42	-1.91
108381	4.56	1.13	0.0667	0.412E 02	0.598E-03	0.0024	6305	2.69	-1.60
109317	5.43	1.02	0.0516	0.174E 03	0.391E-04	0.0314	6305	4.69	-3.73
111028	5.86	0.99	0.0373	0.590E 02	0.208E-02	0.0174	15545	4.02	0.32
111812	5.07	0.67	0.0149	0.127E 03	0.385E-04	0.0104	6305	3.58	-3.95

112033	5.10	0.90	0.0602	0.517E 02	0.214E-03	0.0224	6305	4.07	-2.04
112989	5.08	1.18	0.0917	0.125E 03	0.457E-04	0.0181	10155	3.55	-3.79
113226	2.95	0.94	0.0478	0.136E 02	0.227E-02	0.0362	6305	2.36	-1.57
113996	4.90	1.50	0.0935	0.728E 02	0.126E-03	0.0154	10035	3.26	-2.85
115004	5.05	1.05	0.0760	0.125E 03	0.633E-04	0.0024	10035	2.71	-3.54
117176	5.16	0.71	0.0109	0.619E 01	0.335E-01	0.0421	10035	4.50	3.31
119425	5.62	1.11	0.0807	0.737E 01	0.494E-02	0.0051	12595	3.23	2.03
120477	4.28	1.56	0.0880	0.398E 02	0.187E-03	0.0056	10035	2.13	-2.89
121370	2.80	0.59	0.0080	0.160E 02	0.421E-02	0.1048	10035	3.47	2.08
122742	6.43	0.72	0.0169	0.593E 01	0.418E-02	0.0619	6305	6.47	2.94
124897	0.24	1.23	0.0677	0.130E 01	0.370E 00	0.0922	18275	0.56	1.18
127665	3.78	1.29	0.0797	0.877E 01	0.131E-03	0.0257	10035	2.57	-3.49
128750	2.98	1.10	0.0392	0.355E 02	0.488E-03	0.0295	5390	2.28	-3.28
129312	5.03	1.01	0.0630	0.425E 03	0.305E-05	0.0116	10935	3.14	-6.72
129972	4.69	0.94	0.0440	0.350E 02	0.514E-03	0.0439	6305	4.30	-1.52
131111	5.50	1.03	0.0402	0.107E 03	0.581E-03	0.0268	6305	4.62	-1.39
131511	5.98	0.79	-0.0030	0.204E 02	0.743E-02	0.0851	10035	6.31	4.56
131156	4.64	0.74	-0.0060	0.196E 02	0.587E-03	0.1540	18515	5.88	5.20
133165	4.62	1.05	0.0545	0.564E 02	0.156E-03	0.0212	10935	3.18	-2.83
133208	3.63	0.95	0.0507	0.772E 02	0.272E-03	0.0267	10035	2.46	-3.52
133582	4.67	1.26	0.0797	0.316E 02	0.184E-02	0.0167	6305	3.43	-0.44
134190	5.21	0.97	0.0554	0.805E 02	0.174E-03	0.0295	6305	4.42	-2.32
135482	5.44	1.11	0.0797	0.227E 03	0.240E-04	0.0234	10935	4.09	-4.26
135722	3.54	0.96	0.0516	0.134E 02	0.213E-03	0.0285	10035	2.43	-3.27

137759	3.47	1.17	0.0602	0.135E 04	0.351E-06	0.0344	10035	2.57	-6.02
138481	5.15	1.59	0.1345	0.203E 03	0.250E-05	0.0197	10035	3.70	-6.57
139195	5.40	0.95	0.0516	0.393E 02	0.811E-03	0.0303	6305	4.63	-0.49
140538	5.80	0.68	0.0325	0.254E 02	0.254E-02	0.0470	6305	5.49	1.07
140573	2.75	1.17	0.0677	-0.114E 02	0.131E-02	0.0472	10035	2.23	-1.51
141477	4.28	1.70	0.1062	0.381E 02	0.628E-03	0.0168	10035	2.70	-1.80
141680	5.33	1.03	0.0450	0.418E 02	0.191E-03	0.0287	10935	4.18	-1.83
148387	2.89	0.92	0.0344	-0.704E 02	0.232E-04	0.0349	10035	2.00	-5.86
148856	2.31	0.95	0.0516	0.350E 02	0.785E-03	0.0231	13885	0.80	-3.54
148897	5.29	1.29	0.0695	0.880E 01	0.709E-03	0.0009	6305	3.34	-0.24
150275	6.39	1.00	0.0602	-0.371E 01	0.956E-03	0.0157	11195	4.68	1.42
150449	5.44	1.08	0.0611	-0.149E 03	0.573E-04	0.0150	6305	4.14	-1.91
150997	3.61	0.92	0.0411	0.874E 02	0.750E-04	0.0548	10035	3.29	0.29
153210	3.42	1.14	0.0667	-0.147E 02	0.631E-02	0.0365	14665	2.40	1.21
154733	5.72	1.31	0.0816	0.771E 02	0.824E-03	0.0136	6305	4.36	-0.64
155410	5.12	1.28	0.0825	0.120E 03	0.215E-03	0.0188	6305	3.96	-2.51
156283	3.36	1.44	0.1124	0.984E 02	0.627E-04	0.0213	10035	1.97	-5.13
157999	4.44	1.52	0.1142	-0.482E 03	0.151E-05	0.0057	14665	1.95	-7.21
158633	6.31	0.76	0.0020	0.972E 01	0.238E-01	0.0764	14925	6.36	4.77
158899	4.48	1.43	0.1035	-0.154E 03	0.298E-04	0.0141	10035	2.78	-3.76
159181	2.99	0.97	0.0469	0.893E 02	0.221E-04	0.0102	10035	1.09	-6.42
159222	6.54	0.64	0.0030	0.176E 02	0.489E-02	0.0517	6305	6.35	2.55
161074	5.59	1.45	0.0751	0.365E 02	0.496E-03	0.0225	6305	4.56	-0.67
161096	2.94	1.17	0.0677	-0.158E 02	0.153E-02	0.0263	21435	1.34	-1.17

161797	3.48	0.75	0.0	0.992E 01	0.988E-02	0.1088	10035	4.22	2.87
162211	5.34	1.16	0.0507	0.105E 03	0.468E-04	0.0162	6305	4.08	-3.43
163588	3.90	1.18	0.0344	-0.346E 02	0.646E-03	0.0319	10035	2.91	-0.98
163770	3.99	1.36	0.1124	-0.710E 03	0.139E-05	-0.0009	10035	1.42	-7.58
163993	3.82	0.94	0.0402	0.486E 01	0.617E-03	0.0171	10035	2.26	-1.79
164058	2.42	1.53	0.0862	0.389E 03	0.967E-05	0.0183	11695	0.80	-8.32
165341	4.28	0.86	-0.0131	0.294E 01	0.866E-01	0.1943	21435	5.95	5.45
165760	4.73	0.97	0.0354	-0.229E 03	0.209E-04	0.0208	14665	3.09	-3.76
166208	5.11	0.91	0.0402	0.113E 03	0.474E-04	0.0103	6305	3.61	-3.68
166229	5.67	1.17	0.0440	0.233E 02	0.881E-03	0.0105	6305	4.18	0.10
166620	6.40	0.87	-0.0161	0.965E 01	0.846E-02	0.1023	10035	7.03	5.60
167042	5.94	0.95	0.0129	-0.279E 02	0.841E-03	0.0223	10035	4.59	1.30
168532	5.49	1.53	0.0770	-0.580E 02	0.523E-05	0.0022	11695	2.98	-5.12
168656	4.92	0.93	0.0402	-0.282E 03	0.774E-05	0.0139	14665	2.93	-4.82
168775	4.34	1.18	0.0564	-0.122E 03	0.432E-04	0.0075	10035	2.30	-3.52
169191	5.48	1.26	0.0667	-0.659E 01	0.311E-03	0.0228	6305	4.46	-0.71
169414	3.92	1.18	0.0392	-0.511E 00	0.691E-02	0.0185	10035	2.42	1.05
170829	6.59	0.79	-0.0090	0.131E 02	0.117E-02	0.0281	6305	5.75	1.47
171635	4.95	0.62	0.0010	0.391E 02	0.224E-04	0.0082	6305	3.36	-4.30
171779	5.42	1.11	0.0602	-0.483E 01	0.374E-05	0.0032	6305	3.59	-5.64
173780	4.92	1.20	0.0564	-0.129E 03	0.144E-04	0.0237	10035	3.63	-4.35
175225	5.62	0.84	0.0109	-0.105E 02	0.169E-02	0.0334	10035	4.68	1.48
175306	4.78	1.20	0.0497	-0.899E 01	0.404E-03	0.0029	14925	2.08	-1.08
175535	4.97	0.90	0.0583	0.190E 03	0.920E-05	0.0233	6305	3.97	-5.48

176411	4.21	1.07	0.0705	-0.997E 01	0.233E-03	0.0238	10035	2.93	-2.27
176527	5.28	1.24	0.0686	0.974E 01	0.584E-03	-0.0024	6305	3.15	-0.46
176670	5.11	1.47	0.1115	-0.254E 03	0.339E-05	-0.0033	6305	2.93	-5.74
177808	5.80	1.54	0.0779	0.169E 02	0.792E-03	0.0135	5390	4.56	0.24
178428	5.99	0.71	0.0099	0.147E 02	0.529E-02	0.0585	10035	5.76	3.01
179094	5.93	1.09	0.0392	0.471E 01	0.234E-03	0.0040	6305	4.14	-0.70
179958	6.62	0.65	0.0109	-0.128E 01	0.182E-01	0.0431	18465	5.73	4.89
180006	5.24	1.02	0.0469	-0.399E 02	0.306E-04	0.0100	6305	3.73	-3.40
180610	5.28	1.68	0.0383	0.107E 03	0.194E-04	0.0240	6305	4.31	-4.30
180711	3.24	1.00	0.0535	0.512E 02	0.302E-03	0.0275	14925	1.88	-3.60
180809	4.46	1.27	0.1007	-0.343E 03	0.220E-05	-0.0034	10035	1.72	-6.84
181276	3.98	0.95	0.0440	-0.189E 03	0.259E-04	0.0221	10035	2.63	-4.34
182488	6.50	0.81	-0.0212	-0.305E 02	0.391E-03	0.0569	6305	6.43	2.41
182572	5.23	0.78	0.0129	0.821E 01	0.154E-01	0.0696	12955	5.17	3.27
182762	5.31	1.02	0.0497	0.145E 02	0.113E-02	-0.0037	6305	3.11	0.13
184406	4.65	1.17	0.0507	0.102E 02	0.640E-02	0.0386	14665	3.71	1.17
185144	4.78	0.79	-0.0090	0.163E 00	0.153E 00	0.1772	14925	6.32	5.66
185734	4.79	1.01	0.0760	-0.159E 03	0.263E-04	0.0054	10035	2.63	-3.61
185958	4.45	1.05	0.0760	0.867E 02	0.613E-04	0.0177	10035	2.91	-4.00
186486	5.45	0.94	0.0305	-0.296E 03	0.760E-05	0.0223	6305	4.42	-4.29
186675	5.02	0.96	0.0257	0.241E 02	0.204E-03	0.0175	6305	3.81	-1.93
188310	4.86	1.01	0.0545	0.302E 02	0.144E-02	0.0201	14665	3.18	-0.42
188512	3.90	0.87	0.0119	0.108E 02	0.605E-02	0.0661	21435	3.61	2.09
188947	4.03	1.04	0.0392	-0.927E 02	0.368E-04	0.0108	13885	1.91	-4.17

189276	5.13	1.58	0.1178	-0.797E 03	0.473E-06	0.0038	6305	3.33	-7.81
189319	3.71	1.54	0.0944	0.636E 02	0.180E-03	0.0091	10035	1.76	-3.67
190147	5.28	1.13	0.0649	0.800E 02	0.111E-04	0.0136	6305	3.92	-4.78
190360	5.68	0.72	-0.0060	0.368E 01	0.310E-01	0.0455	10035	5.11	4.00
192806	4.73	1.27	0.0639	-0.310E 02	0.147E-03	0.0074	10035	2.69	-2.09
193092	5.50	1.64	0.1398	-0.547E 03	0.422E-06	0.0063	6305	3.82	-7.69
194317	4.60	1.33	0.0723	0.606E 02	0.952E-04	0.0098	10035	2.68	-3.33
196755	5.23	0.74	0.0090	0.264E 02	0.270E-02	0.0227	17585	3.57	0.53
197076	6.40	0.64	0.0060	-0.239E 01	0.232E-02	0.0518	6305	6.21	2.39
197752	5.13	1.18	0.0714	0.360E 02	0.832E-03	0.0018	10035	2.75	-0.69
197964	4.45	1.04	0.0178	0.371E 02	0.628E-03	0.0201	16805	2.69	-1.61
197989	2.64	1.04	0.0507	0.713E 01	0.205E-02	0.0388	12955	1.76	-1.79
198134	5.20	1.31	0.0714	0.800E 02	0.105E-03	0.0030	6305	3.36	-2.75
198149	3.59	0.93	0.0090	0.405E 00	0.800E-02	0.0710	16805	3.49	1.87
198387	6.34	0.91	-0.0040	0.918E 01	0.214E-02	0.0179	6305	5.15	1.89
200905	3.92	1.64	0.1458	-0.521E 03	0.830E-06	0.0023	10035	1.57	-8.46
203344	5.82	1.05	0.0611	0.326E 02	0.277E-02	-0.0063	6305	3.48	0.99
203504	4.24	1.10	0.0583	0.197E 03	0.278E-03	0.0109	12955	2.18	-3.70
204724	4.76	1.65	0.1089	0.373E 03	0.748E-05	0.0120	10035	2.95	-6.16
205512	4.98	1.04	0.0611	0.142E 03	0.226E-03	0.0096	13885	2.79	-2.75
206778	2.54	1.58	0.1476	0.605E 02	0.138E-03	-0.0012	14665	-0.48	-5.05
206859	4.52	1.18	0.0880	0.226E 03	0.864E-04	0.0031	10035	2.22	-4.24
206952	4.85	1.14	0.0478	0.283E 02	0.353E-03	0.0048	14925	2.28	-1.62
209747	4.90	1.42	0.0807	0.167E 02	0.405E-03	0.0096	14665	2.66	-1.30

212943	4.93	1.05	0.0459	0.115E 02	0.318E-02	0.0202	14665	3.26	0.78
215665	4.14	0.95	0.0667	0.673E 02	0.147E-03	0.0368	10035	3.31	-3.05
216228	3.68	1.06	0.0611	-0.239E 01	0.907E-03	0.0373	10035	2.87	-1.38
218031	4.91	1.04	0.0497	-0.330E 01	0.167E-02	-0.0019	10035	2.28	0.56
218101	6.45	0.83	0.0178	0.365E 01	0.811E-03	0.0303	3730	6.02	1.15
218658	4.56	0.80	0.0305	0.106E 02	0.624E-04	0.0005	15045	1.66	-3.51
219134	5.65	1.00	-0.0212	-0.275E 00	0.362E 00	0.1572	13885	6.97	7.16
219615	3.85	0.92	0.0325	0.763E 01	0.202E-01	0.0246	14665	2.37	1.47
219916	4.90	0.82	0.0247	0.377E 02	0.104E-03	0.0225	11195	3.50	-2.81
220954	4.45	1.05	0.0770	-0.183E 02	0.473E-03	0.0092	14665	2.19	-1.09
221345	5.32	1.02	0.0411	0.142E 02	0.416E-02	0.0126	6305	3.92	1.32
222107	4.00	1.02	0.0080	0.614E 01	0.779E-02	0.0409	13885	3.16	0.89
222404	3.42	1.03	0.0478	0.196E 01	0.103E-02	0.0645	17845	3.15	1.41

CT8 Program for reduction of data

The master program reads in data cards of the form described in CT1; interprets the literal data; passes the data to the two subroutines LKPM and LKPAR; uses the results to calculate limits on M_V, and the sample means and standard deviations of M_V and B-V; and punches out data of the form described in CT4.

The subroutine LKPM calculates the proper motion distance RMU, and also the sine of the galactic latitude SB. It uses the means U(N) and precisions H(N) of the stellar velocity distribution; the right ascension RA and declination DEC expressed in radians; the radial velocity RV; and an index I which is zero if no radial velocity is available.

The subroutine LKPAR calculates the absolute parallax PIO and its precision HPI. It uses the N relative parallaxes RPI(J), and indeces IP(J) of the observatory, and also the sine of the galactic latitude SB.

```
                C REDUCE ASTROMETRIC DATA
0001                  INTEGER PI,RAH,DECD, DECM
0002                  INTEGER HPI,PIC
0003                  LOGICAL WAST,WARH,WNHD,WNLC
0004                  REAL*8 HD1,HD2,HD3,LC(19), LCS,HD(500) ,HDD(20)
0005                  DIMENSION OBS(10),UV(3,5),HV(3,5),L(3),H(3),IP(10),PI(10),NLC(19)
0006                  DIMENSION V(500),BV(500),RHO(500),RMU(500),HMU(500),PIO(500),
                     1          HPI(500),PIC(500),AVMAX(500),AVMIN(500),AVMEAN(500),
                     2          CB(10),RP(10) ,
                     3 FMTMAG(18),FMTPMN(18),FMTPAR(18)
0007                  DIMENSION V2(20),BV2(20),RHO2(20)
                C
                C UV(1,2)=UV(PI,G) 1ST SUBS=GALACTIC AXIS, 2ND SUBS=LUMINOSITY CLASS
                C C=1,G=2,SG=3,D=4,SD=5  LUMINOSITY CLASSES
                C PI=1,TH=2,Z=3          GALACTIC AXES
                C
0008                  DATA OBS/' Y',' M',' C',' A',' G',' W','YK',' S',' V','*A'/
0009                  DATA UV/ -1.6667, -2.4684, -1.3713,
                     1        -1.6613, -3.3819, -1.3770,
                     2        -1.6878, -5.9072, -1.6878,
                     3        -2.0984, -3.6324, -1.3527,
                     4        +0.0422,-28.6920, -2.0042/
0010                  DATA HV/  0.1329,  0.2774,  0.4585,
                     1         0.0256,  0.0529,  0.0876,
                     2         0.0122,  0.0308,  0.0390,
                     3         0.0252,  0.0692,  0.0923,
                     4         0.00224, 0.00399, 0.00899/
0011                  DATA LC/'C','I','IA','IAB','IB',
                     1        'G','I-II','II','II-III','III',
                     2        'SG','III-IV','IV','IV-V',
                     3        'D','V',
                     4        'SD','V-VI','VI'/
0012                  DATA NLC/5*1,5*2,4*3,2*4,3*5/
0013                  DATA NORV/'N'/
0014                  NTLC=19
0015                  NPT=25
0016                  PT=NPT-1
0017                  E10=ALOG(10.0)
0018                  WRITE(6,2000)
0019            2000  FORMAT('1')
                C READ FOUR PRELIMINARY CARDS
0020                  READ (5,100)NSTAR,NDBL
0021                  WRITE(6,100)NSTAR,NDBL
0022             100  FORMAT(2X,2I8)
0023                  READ(5,300)FMTMAG
0024                  WRITE(6,300)FMTMAG
0025                  READ(5,300)FMTPMN
0026                  WRITE(6,300)FMTPMN
0027                  READ(5,300)FMTPAR
```

113.

```
0028              WRITE(6,300)FMTPAR
0029          300 FORMAT(18A4)
            C READ NSTAR SETS OF 3 CARDS
0030              K=0
0031              WRITE(6,2000)
0032            1 K=K+1
0033           11 IF(K.GT.NSTAR)GOTO99
0034              READ(5,FMTMAG)HD1,V(K),BV(K),RHO(K)
0035              WRITE(6,FMTMAG)HD1,  V(K),BV(K),RHO(K)
            C  INSERT TRANSFORMATION OF RHO HERE IF NEEDED
0036              RHO(K)=ALOG(RHO(K))
0037              READ(5,FMTPMN)HD2,LCS,RAH,RAM,DECD,DECM,PMRA,     PMDEC,RV,CRV
0038              WRITE(6,FMTPMN)HD2,LCS,RAH,RAM,DECD,DECM,PMRA,     PMDEC,RV,CRV
0039              READ (5,FMTPAR)HD3,PIC(K),NPI,(PI(J),OB(J),J=1,NPI)
0040              WRITE(6,FMTPAR)HD3,PIC(K),NPI,(PI(J),OB(J),J=1,NPI)
0041              IF(K.GT.NCBL)GOTO 5
0042              READ (5,FMTMAG)HDD(K),V2(K),BV2(K),RHO2(K)
0043              WRITE(6,FMTMAG)HDD(K),V2(K),BV2(K),RHO2(K)
            C  INSERT TRANSFORMATION OF RHO2(K) HERE IF NEEDED
0044              RHO2(K)=ALOG(RHO2(K))
0045            5 CONTINUE
            C RECOGNISE LUMINOSITY CLASS
0046              ILC=0
0047              DO 2 I=1,NTLC
0048              IF(LCS.EQ.LC(I))ILC=NLC(I)
0049            2 CONTINUE
            C RECOGNISE PARALLAX STATIONS AND GENERATE WARNING OF NON RECOGNITION
0050              WNST=.FALSE.
0051              DO 3 J=1,NPI
0052              IP(J)=0
0053              DO 4 I=1,10
0054              IF(OB(J).EQ.OBS(I))IP(J)=I
0055            4 CONTINUE
0056              IF(IP(J).EQ.0)WNST=.TRUE.
0057            3 CONTINUE
            C GENERATE WARNING SIGNS
            C NO WARNING FOR NULL RHO
0058              WNRH=.FALSE.
0059              WNHD=HD2.NE.HD1.OR.HD3.NE.HD1
0060              WNLC=ILC.EQ.0
0061              IF(WNST.OR.WNRH.OR.WNHD.OR.WNLC)GOTO 3000
            C PROCESS DATA FOR KTH STAR
0062              HD(K)=HD1
            C SELECT APPROPRIATE U,H
0063              DO 6 N=1,3
0064              U(N)=UV(N,ILC)
0065              H(N)=HV(N,ILC)
0066            6 CONTINUE
            C PUT RA,DEC INTO RADIANS
```

```
0067              RA=3.1415926*(FLOAT(RAH)+RAM/60.)/12.
0068              IF(DECD.LT.0)DECM=-IABS(DECM)
0069              DEC=3.1415926*(FLOAT(DECD)+FLOAT(DECM)/60.)/180.
0070              DO 7 J=1,NPI
0071              RP(J)=FLOAT(PI(J))/1000.
0072            7 CONTINUE
          C SET RADIAL VELOCITY INDEX
0073              IRV=1
0074              IF(CRV.EQ.NORV)IRV=0
          C CALL LIKELIHOOD PARAMETER SUBROUTINES
0075              CALL LKPM(U,H,RA,DEC,PMRA,PMDEC,RV,IRV,RMU(K),HMU(K),SB)
0076              CALL LKPAR(NPI,IP,RP,SB,PIO(K),HP)
0077              HPI(K)=HP
0078              WRITE(6,1300)
0079              IF(MOD(K,12).EQ.0)WRITE(6,2000)
0080         1300 FORMAT(' ')
0081              GOTO 1
          C
          C ENTER WARNING
0082         3000 WRITE(6,900)HD1,HD2,HD3
0083          900 FORMAT('    CARD WARNING',3A8)
0084              IF(WNLC)WRITE(6,500)
0085              IF(WNST)WRITE(6,600)
0086              IF(WNRH)WRITE(6,700)
0087              IF(WNHD)WRITE(6,800)
0088          500 FORMAT('  LUMINOSITY CLASS NOT RECOGNISED')
0089          600 FORMAT('  PARALLAX STATION NOT RECOGNISED')
0090          700 FORMAT('  RHD IS NULL')
0091          800 FORMAT('  HD NUMBERS UNEQUAL')
          C REDUCE NSTAR AND RETURN
0092              NSTAR=NSTAR-1
0093              GOTO 11
0094           99 CONTINUE
          C CALCULATE AVMAX,AVMIN
0095              DO 21 K=1,NSTAR
0096              IF(RMU(K).GE.0.0)RMAX=RMU(K)+SQRT(10./HMU(K))
0097              IF(RMU(K).LT.0.0)RMAX=RMU(K)+SQRT(RMU(K)**2+10./HMU(K))
0098              IF(PIO(K).GE.0.0)PMAX=PIO(K)+SQRT(10./HPI(K))
0099              IF(PIO(K).LT.0.0)PMAX=PIO(K)+SQRT(PIO(K)**2+10./HPI(K))
0100              PMIN=PIO(K)- SQRT(11./HPI(K))
0101              IF(PMIN.GT.0.0)RMAX=AMIN1(RMAX,1.0/PMIN)
0102              AVMAX(K)=V(K)+5.0+5.0*ALOG10(PMAX)
0103              AVMIN(K)=V(K)+5.0-5.0*ALOG10(RMAX)
          C INTEGRATE TO FIND AVMEAN(K)=INTGRL(AV*DLK)/INTGRL(DLK)
0104              SDLK=0.0
0105              SAVDLK=0.0
0106              DMAX=ALOG(RMAX)
0107              DMIN=-ALOG(PMAX)
0108              DD=(DMAX-DMIN)/PT
```

```
                  DO 22 I=1,NPT
                  D=DMIN+FLOAT(I-1)*DD
                  R=EXP(D)
                  AV=V(K)-5.0*ALOG10(R/10.0)
                  P=1.0/R
                  DLK=EXP(-0.5*(HPI(K)*(P-PIO(K))**2+HMU(K)*(R-RMU(K))**2))
                  DLK=DLK*R**2
                  SDLK=SDLK+DLK
                  SAVDLK=SAVDLK+AV*DLK
               22 CONTINUE
                  AVMEAN(K)=SAVDLK/SDLK
               21 CONTINUE
      C FIND AVM,BVM
                  AVM=0.0
                  BVM=0.0
                  DO 23 K=1,NSTAR
                  AVM=AVM+AVMEAN(K)
                  BVM=BVM+BV(K)
                  IF(K.GT.NDBL)GOTO 23
                  AVM=AVM+AVMEAN(K)+V2(K)-V(K)
                  BVM=BVM+BV2(K)
               23 CONTINUE
                  STAR=NSTAR+NDBL
                  AVM=AVM/STAR
                  BVM=BVM/STAR
      C   FIND AVS,BVS
                  AVS=0.0
                  BVS=0.0
                  DO 24 K=1,NSTAR
                  AVS=AVS+(AVMEAN(K)-AVM)**2
                  BVS=BVS+(BV(K)-BVM)**2
                  IF(K.GT.NDBL)GOTO 24
                  AVS=AVS+(AVMEAN(K)+V2(K)-V(K)-AVM)**2
                  BVS=VBS+(BV2(K)-BVM)**2
               24 CONTINUE
                  AVS=SQRT(3.0*AVS/STAR)
                  BVS=SQRT(3.0*AVS/STAR)
      C PUNCH DATA
                  WRITE(6,2000)
                  WRITE(6,1000)NSTAR,NDBL
                  WRITE(7,1000)NSTAR,NDBL
             1000 FORMAT(2X,2I8)
                  WRITE(6,1500)AVM,AVS,BVM,BVS
                  WRITE(7,1500)AVM,AVS,BVM,BVS
             1500 FORMAT(1X,4E13.5)
                  DO 31 K=1,NSTAR
                  WRITE(6,1400)HD(K),V(K),BV(K),RHO(K),RMU(K),HMU(K),PIO(K),
                 1             HPI(K),PIC(K),AVMAX(K),AVMIN(K),AVMEAN(K)
                  WRITE(7,1410)HD(K),V(K),BV(K),RHO(K),RMU(K),HMU(K),PIO(K),
```

```
              1         HPI(K),PIC(K),AVMAX(K),AVMIN(K)
                IF(K.GT.NDBL)GOTO 32
                WRITE(6,1600)HDD(K),V2(K),BV2(K),RHO2(K)
                WRITE(7,1610)HDD(K),V2(K),BV2(K),RHO2(K)
             32 CONTINUE
                IF(MOD(K+NDBL,24).EQ.0)WRITE(6,2000)
             31 CONTINUE
           1400 FORMAT('0',A8,2F6.2,F8.4,2E10.3,F7.4,I6,1X,A3,2F7.2,20X,F8.3)
           1410 FORMAT(    A8,2F6.2,F8.4,2E10.3,F7.4,I6,1X,A3,2F7.2)
           1600 FORMAT(1X, A8,2F6.2,F8.4)
           1610 FORMAT(     A8,2F6.2,F8.4)
                STOP
                END
```

```
0001              SUBROUTINE LKPM(U,H,RA,DEC,PMRA,PMDC,RV,I,RMU,HMU,SB)
            C PURPOSE
            C    TO CALCULATE PROPER MOTION DISTANCE RMU,PRECISION HMU, AND
            C    SINE OF GALACTIC LATITUDE SB
            C USAGE
            C    U,H ARE MEAN AND PRECISION OF STELLAR VELOCITY DN.IN A.U./YR
            C    RA,DEC ARE RIGHT ASCENSION, DECLINATION IN RADIANS
            C    PMRA,PMDC ARE PROPER MOTIONS IN SECONDS OF ARC / YEAR
            C    RV IS RADIAL VELOCITY IN KM/SEC, I=0 IF NO RADIAL VELOCITY AVAILABLE
            C    GALACTIC AXES FOR U,H ARE PI=1,THETA=2,Z=3
            C
0002              DIMENSION U(3),H(3),ALPHA(3),A(3),DELTA(3),D(3),C(3,3),SCMU(3),
                 1  UCR(3)
            C GALACTIC AXES IN EQUATORIAL COORDINATES
0003              DATA ALPHA/4.6220665,5.5338097,3.3449221/
0004              DATA DELTA/-0.5043998,0.8363128,0.482874/
            C OORT CONSTANTS (A+B) AND (A-B) IN SECONDS PER YEAR
0005              DATA APB,AMB/0.0005,0.0055/
            C PLACE DIRECTIONS OF PMRA,PMDC,RV IN A,D
0006              A(1)=RA+1.570796
0007              D(1)=0.0
0008              A(2)=RA
0009              IF(DEC.GT.0.0)A(2)=A(2)+3.14159
0010              D(2)=1.570796-ABS(DEC)
0011              A(3)=RA
0012              D(3)=DEC
            C DIRECTION COSINES OF PMRA,PMDC,RV REFERRED TO GALACTIC AXES C(IGA,IPM)
0013              DO 1 N=1,3
0014              DO 1 M=1,N
0015              C(N,M)=SIN(DELTA(N))*SIN(D(M))+COS(DELTA(N))*COS(D(M))
                 1                                          *COS(ALPHA(N)-A(M))
0016              C(M,N)=C(N,M)
0017            1 CONTINUE
            C FIND SUM(C(N,M)*MU(M))
0018              DO 2 N=1,3
0019              SCMU(N)=C(N,1)*PMRA+C(N,2)*PMDC
0020            2 CONTINUE
            C CORRECT FOR GALACTIC ROTATION WITH OORT CONSTANTS
0021              SCMU(1)=SCMU(1)+AMB*C(2,3)
0022              SCMU(2)=SCMU(2)-APB*C(1,3)
            C  SUBTRACT RADIAL VELOCITY FROM MEAN STELLAR VELOCITY (IF RV AVAILABLE)
0023              DO 3 N=1,3
0024              UCR(N)=U(N)
0025              IF(I.NE.0)UCR(N)=UCR(N)-C(N,3)*RV/4.74
0026            3 CONTINUE
            C  FORM HMU=SUM(HN*SCMU**2),HRMU=SUM(HN*UCR*SCMU)
0027              HMU=0.0
0028              HRMU=0.0
0029              DO 4 N=1,3
```

```
      HMU=HMU+H(N)*SCMU(N)**2
      HRMU=HRMU+H(N)*UCR(N)*SCMU(N)
    4 CONTINUE
      IF(I.NE.0)GOTO 5
C CORRECT FOR ABSENCE OF RADIAL VELOCITY
      HC3CMU=0.0
      HC3C3 =0.0
      HUC3  =0.0
      DO 6 N=1,3
      HC3CMU=HC3CMU+H(N)*C(N,3)*SCMU(N)
      HC3C3 =HC3C3 +H(N)*C(N,3)**2
      HUC3  =HUC3  +H(N)*U(N)*C(N,3)
    6 CONTINUE
      HMU=HMU-(HC3CMU**2)/HC3C3
      HRMU=HRMU-HUC3*HC3CMU/HC3C3
C PLACE RMU,SB AND RETURN
    5 RMU=HRMU/HMU
      SB=C(3,3)
      RETURN
      END
```

119.

```
0001              SUBROUTINE LKPAR(N,IP,RPI,SB,PIO,HPI)
            C
            C PURPOSE
            C    TO FIND ABSOLUTE PARALLAX PIO AND PRECISION HPI
            C USAGE
            C    N IS NUMBER RELATIVE PARALLAXES
            C    IP(J) IS INDEX OF JTH PARRALLAX
            C    IP=1,Y,2,M,3,C,4,A,5,G,6,W,7,YK,8,S,9,V,10,*A.
            C    RPI(J) IS JTH RELATIVE PARALLAX
            C    SB IS SINE OF GALACTIC LATITUDE
0002              DIMENSION IP(10),RPI(10),HST(10)
0003              DATA HST/4630.,3730.,2417.,6305.,4890.,1660.,2920.,3850.,5320.,
                 1          6305./
            C    HST(J) IS PRECISION FOR JTH STATION
            C
            C TRANSFORM TO ABSOLUTE PARALLAXES
0004              CP11=EXP(-6.106+0.8866*SB)
0005              CP12=CP11/1.303
0006              DO 1 J=1,N
0007              IF(IP(J).NE.10)RPI(J)=RPI(J)+CP11
0008              IF(IP(J).EQ.10)RPI(J)=RPI(J)+CP12
0009            1 CONTINUE
            C FORM HPI,PIO
0010              HPI=0.
0011              PIO=0.
0012              DO 2 J=1,N
0013              HPI=HPI+HST(IP(J))
0014              PIO=PIO+HST(IP(J))*RPI(J)
0015            2 CONTINUE
0016              PIO=PIO/HPI
0017              RETURN
0018              END
```

CT9 Program for regression

The master program reads in data in the form described in CT4 and calculates estimates H2,A2(L) of the parameters of the density

$$D(\rho|M_V, B-V) \propto \exp\left[-\frac{H}{2}\sum_{\ell} A_\ell Q_\ell(x,y)\right]$$

where $x = (M_V - \bar{M}_V)/\sigma_M$, $y = (B-V-\overline{B-V})/\sigma_{BV}$,

$$Q_\ell(x,y) = P_{\ell x}(x)\,P_{\ell y}(y).$$

The index arrays LX(L) and LY(L) are generated by the subroutine LXY. A first approximation H1,A1 is calculated by the subroutine LSQR using the method of least squares.

Using the values H1,A1 the sums of expected values derived in section 4.3 are calculated by the subroutine RGRSN and placed in S, B, C:

$$S = \sum_k E\left[\left(\rho_k - \sum_\ell A_\ell Q_{\ell k}\right)^2 \Big| H1, A1\right],$$

$$B_\ell = \sum_k E\left[\rho_k Q_{\ell k} \,|\, H1, A1\right], \quad C_{\ell m} = \sum_k E\left[Q_{\ell k} Q_{mk} \,|\, H1, A1\right].$$

The next approximation H2,A2 is then found, and the iteration repeated until the desired accuracy, measured by CRTH and CRTA, is obtained.

The covariance matrix is then calculated and the results printed. As a check on the results, the expected value of the criterion is found for M_V, B-V typical of selected MK types.

```
                  C READ DATA AND CALCULATE REGRESSION COEFFICIENTS
0001                    REAL*8 HD,HD2
0002                    DIMENSION PX(10),PY(10)
0003                    DIMENSION SP(7),LC(4),AVMK(7,4),BVMK(7,4),RHMK(7,4)
0004                    DIMENSION A1(20),A2(20),B(20),C(20,20),LWK(20),MWK(20)
0005                    DIMENSION COV(20,20)
0006                    DIMENSION ALSQ(20)
0007                    COMMON/LKDPAR/V(500),BV(500),RH(500),RMU(500),HMU(500),
                      1             PIO(500),HPI(500),PIC(500),AVMAX(500),AVMIN(500),
                      2            AVM,AVS,BVM,BVS,RHM,RHS,NSTAR
0008                    COMMON/DBLPAR/V2(20),BV2(20),RH2(20),NDBL
0009                    COMMON/SUN/NSUN,AVSUN,BVSUN,RHSUN
0010                    COMMON/SBSCT2/LX(20),LY(20),NX1,NY1,NT,NA
0011                    DATA SP/'F0','F5','G0','G5','K0','K5','M0'/,
                      1      LC/'V','III','II','IB'/
0012                    DATA AVMK/ 2.7, 3.6, 4.6, 5.2, 5.8, 7.5, 8.9,
                      1            1.5, 1.7, 1.0, 1.0, 1.2, 0.0, -0.1,
                      2           -2.5,-2.3,-2.1,-2.0,-2.0,-2.0,-2.5,
                      3           -4.8,-4.6,-4.6,-4.6,-4.5,-4.6,-4.8/
0013                    DATA BVMK/ 0.29, 0.42, 0.58, 0.68, 0.81, 1.15, 1.40,
                      1            0.27, 0.42, 0.66, 0.81, 0.99, 1.50, 1.54,
                      2            0.25, 0.38, 0.72, 0.90, 1.10, 1.50, 1.58,
                      3            0.20,0.37, 0.72, 1.00, 1.25, 1.56, 1.70/
0014                    E105=ALOG(10.0)/5.0
0015                    NSUN=0
0016                    AVSUN=4.79
0017                    BVSUN=0.62
0018                    RHSUN=1.523
0019                    READ (5,100)NSTAR,NDBL
0020                    WRITE(6,101)NSTAR,NDBL
0021                100 FORMAT(2X,2I8)
0022                101 FORMAT('1   ',2I8)
0023                    READ (5,110)AVM,AVS,BVM,BVS
0024                    WRITE(6,110)AVM,AVS,BVM,BVS
0025                110 FORMAT(1X,4E13.5)
0026                    DO 1 K=1,NSTAR
0027                    READ (5,200)HD,V(K),BV(K),RH(K),RMU(K),HMU(K),PIO(K),HPI(K),PIC(K)
                      1,AVMAX(K),AVMIN(K)
0028                    IF(PIC(J).LE.0.0)PIC(J)=-0.99
0029                    PIC(K)=PIC(K)/1000.0
0030                    WRITE(6,201)HD,V(K),BV(K),RH(K),RMU(K),HMU(K),PIO(K),HPI(K),PIC(K)
                      1,AVMAX(K),AVMIN(K)
0031                    IF(K.GT.NDBL)GOTO 1
0032                    READ (5,210)HD2,V2(K),BV2(K),RH2(K)
0033                    WRITE(6,211)HD2,V2(K),BV2(K),RH2(K)
0034                  1 CONTINUE
0035                200 FORMAT(    A8,2F6.2,F8.4,2E10.3,F7.4,F6.0,  F4.0,2F7.2)
0036                201 FORMAT('0',A8,2F7.2,F9.4,2E13.3,F9.4,F9.0,F8.3,2F7.2)
0037                210 FORMAT(    A8,2F6.2,F8.4)
```

```
  211 FORMAT(1X,A8,2F6.2,F8.4)
      WRITE(6,4400)
 4400 FORMAT('1')
      NX1=3
      NY1=3
      NT=3
      CALL LXY
      NX=NX1-1
      NY=NY1-1
      CALL LSQR(H1,A1)
      H2=H1
      HLSQ=H1
      DO 10 L=1,NA
      ALSQ(L)=A1(L)
      A2(L)=A1(L)
   10 CONTINUE
      WRITE(6,300)H1
      WRITE(6,310)(L,A1(L),L=1,NA)
  300 FORMAT('0    H1=',E14.6)
  310 FORMAT('     L=',I2,'    A1=',E14.6)
C CALL REGRESSION SUBROUTINE
    7 CALL RGRSN(H1,A1,S,B,C)
C CALCULATE NEW REGRESSION COEFFICIENTS
      H2=   FLOAT(NSTAR+NSUN+NDBL)/(2.0*S)
      CALL ARRAY(2,NA,NA,20,20,C,C)
      CALL MINV(C,NA,DET,LWK,MWK)
      CALL ARRAY(1,NA,NA,20,20,C,C)
      DO 4 L=1,NA
      A2(L)=0.0
      DO 4 M=1,NA
      A2(L)=A2(L)+C(L,M)*B(M)
    4 CONTINUE
      SDA=0.0
      SA=0.0
      DO 5 L=1,NA
      SA=SA+ABS(A2(L))
      SDA=SDA+ABS(A2(L)-A1(L))
    5 CONTINUE
      CRTA=SDA/SA
      CRTH=ABS(H2-H1)/ABS(H2)
      H1=H2
      DO 6 L=1,NA
      A1(L)=A2(L)
    6 CONTINUE
C WRITE RESULTS
      WRITE(6,400)H2,CRTH,CRTA,DET
  400 FORMAT('0    H2=',E14.6,'    CRTH=',E14.6,'    CRTA=',E14.6,'  DET=',
     1 E14.6)
      WRITE(6,410)(L,A2(L),L=1,NA)
```

```
  410 FORMAT('    L=',I3,'    A2=',E14.6)
      IF(CRTA.GT.NA*1.E-5.OR.CRTH.GT.1.E-5)GOTO 7
      WRITE(6,500)
  500 FORMAT('   END OF ITERATION')
      WRITE(6,420)((L,M,C(L,M),M=1,NA),L=1,NA)
  420 FORMAT('    L=',I3,'    M=',I3,'    C=',E14.6)
      CALL CVRNC(H2,A2,S,B,C,COV)
      WRITE(6,1000)H2,HLSQ
 1000 FORMAT('1   H2=',E13.6,'    HLSQ=',E13.6)
      WRITE(6,1100)
 1100 FORMAT('0 LX LY',7X,'A2',7X,'ALSQ')
      WRITE(6,1200)(LX(L),LY(L),A2(L),ALSQ(L),L=1,NA)
 1200 FORMAT('0',2I3,2E18.6)
      WRITE(6,1300)
 1300 FORMAT('0  COVARIANCE MATRIX')
      DO 46 L=1,NA
      WRITE(6,1400)L,(COV(L,M),M=1,NA)
   46 CONTINUE
 1400 FORMAT('0',I3,6X,9E11.3)
      WRITE(6,4000)
 4000 FORMAT('1SPLC         MV           B-V           RH')
      DO 41 I=1,7
      DO 41 J=1,4
      X=(AVMK(I,J)-AVM)/AVS
      Y=(BVMK(I,J)-BVM)/BVS
      CALL LEP(PX,X,NX)
      CALL LEP(PY,Y,NY)
      Z=0.0
      DO 42 L=1,NA
      Z=Z+A1(L)*PX(LX(L))*PY(LY(L))
   42 CONTINUE
      RHMK(I,J)=Z
      WRITE(6,4100)  SP(I),LC(J),AVMK(I,J),BVMK(I,J),RHMK(I,J)
 4100 FORMAT(1X,A2,A4,2F8.2,4X,E14.6)
   41 CONTINUE
      STOP
      END
```

```
0001            SUBROUTINE RGRSN(H,A,S,B,C)
0002            DIMENSION A(20),B(20),C(20,20),BK(20),CK(20,20),
               1          PX(10),PY(10),P(20),PX2(20),PY2(20),P2(20)
0003            DIMENSION PZAP(20),PZAPSC(20,20),COV(20,20),LWK(20),MWK(20)
0004            COMMON/LKDPAR/V(500),BV(500),RH(500),RMU(500),HMU(500),
               1              PIO(500),HPI(500),PIC(500),AVMAX(500),AVMIN(500),
               2              AVM,AVS,BVM,BVS,RHM,RHS,NSTAR
0005            COMMON/DBLPAR/V2(20),BV2(20),RH2(20),NDBL
0006            COMMON/SUN/NSUN,AVSUN,BVSUN,RHSUN
0007            COMMON/SESCT2/LX(20),LY(20),NX1,NY1,NT,NA
          C
          C GIVEN APPROXIMATE VALUES H,A CALCULATES REGRESSION COEFFICIENTS S,B,C
          C S=SUM(SK/EK),B=SUM(BK/ED),C=SUM(CK/EK)
          C EK=INTEGRAL(EXP(-H*(Z-SUM(A*P(X,Y))**2)*LKHD(X,DATA))
          C BK(L)=INTEGRAL(P(L,X,Y)*EXP(    )*LKHD(X,DATA)*Z)
          C CK(L,M)=INTEGRAL(P(L,X,Y)*P(M,W,Y)*EXP(     )*LKHD(X,DATA))
          C SK=INTEGRAL((Z-SUM(A*P(X,Y))**2*EXP(     )*LKHD(X,DATA))
          C FIRST TERM IS SOLAR DATA,LIKELIHOOD IS SHARP
          C NEXT NDBL TERMS ARE FOR DOUBLE STARS
          C REMEMBER H IS HALF THE PRECISION
          C
          C  SET INITIAL VALUES
0008            INDEX=1
0009            GOTO 4
0010            ENTRY CVRNC(H,A,S,B,C,COV)
0011            INDEX=2
0012          4 CONTINUE
0013            NX=NX1-1
0014            NY=NY1-1
0015            E105=ALOG(10.0)/5.0
0016            NPT=36
0017            S=0.0
0018            DO 1 L=1,NA
0019            B(L)=0.0
0020            DO 1 M=1,L
0021            C(L,M)=0.0
0022            IF(INDEX.EQ.2)COV(L,M)=0.0
0023          1 CONTINUE
0024            IF(NSUN.EQ.0)GOTO10
          C  START WITH SOLAR DATA FOR H AND K LINES
0025            X=(AVSUN-AVM)/AVS
0026            Y=(BVSUN-BVM)/BVS
0027            Z= RHSUN
0028            CALL LEP(PX,X,NX)
0029            CALL LEP(PY,Y,NY)
0030            SK=Z
0031            DO 2 L=1,NA
0032            P(L)=PX(LX(L))*PY(LY(L))
0033            SK=SK-A(L)*P(L)
```

```
            B(L)=B(L)+Z*P(L)
            DO 2 M=1,L
            C(L,M)=C(L,M)+P(L)*P(M)
            IF(INDEX.EQ.2)COV(L,M)=COV(L,M)+P(L)*P(M)
          2 CONTINUE
            S=S+SK**2
         10 CONTINUE
C START SUMMATION OVER NSTAR POINTS
C  FIRST NDBL POINTS ARE DOUBLE STARS
            DO 11 K=1,NSTAR
C SET INITIAL VALUES TO ZERO
            EK=0.0
            SK=0.0
            DO 12 L=1,NA
            BK(L)=0.0
            IF(INDEX.EQ.2)PZAP(L)=0.0
            DO 12 M=1,L
            CK(L,M)=0.0
            IF(INDEX.EQ.2)PZAPSQ(L,M)=0.0
         12 CONTINUE
C  COLLECT LIKELIHOOD PARAMETERS PRIOR TO INTEGRATION
            Y=(BV(K)-BVM)/BVS
            Z= RH(K)
            CALL LEP(PY,Y,NY)
            DAV=(AVMAX(K)-AVMIN(K))/FLOAT(NPT-1)
            AVC=AVMIN(K)
            RV=V(K)+5.0
            RM=RMU(K)
            HR=HMU(K)/2.0
            PI=PIC(K)
            HP=HPI(K)/2.0
            PC=PIC(K)
            SP=SQRT(HP)
            IF(K.GT.NDBL)GOTO 13
C  DOUBLE STAR PARAMETERS
            Y2=(BV2(K)-BVM)/BVS
            Z2=RH2(K)
            DV21=V2(K)-V(K)
            CALL LEP(PY2,Y2,NY)
         13 CONTINUE
C  START INTEGRATION W.R.T. DISTANCE MODULUS
            DO 21 I=1,NPT
            AV=AVC+FLOAT(I-1)*DAV
C  CALCULATE LIKELIHOOD
            R=EXP(E105*(RV-AV))
            Q=1.0/R
            DLK=R*R*EXP(-HR*(R-RM)**2-HP*(Q-PI)**2)
            IF(PC.GT.-0.08)DLK=DLK/ERFC(SP*(PC-Q))
C  CALCULATE PX
```

```
0075            X=(AV-AVM)/AVS
0076            CALL LEP(PX,X,NX)
0077            SI=Z
0078            IF(K.GT.NDBL)GOTO 22
      C CALCULATE PX2 FOR DOUBLE STAR
0079            AV2=AV+DV21
0080            X2=(AV2-AVM)/AVS
0081            CALL LEP(PX2,X2,NX)
0082            SI2=Z2
0083         22 CONTINUE
      C CALCULATE P,SI AND,FOR DOUBLES,P2,SI2
0084            DO 23 L=1,NA
0085            P(L)=PX(LX(L))*PY(LY(L))
0086            SI=SI-A(L)*P(L)
0087            IF(K.GT.NDBL)GOTO 23
0088            P2(L)=PX2(LX(L))*PY2(LY(L))
0089            SI2=SI2-A(L)*P2(L)
0090         23 CONTINUE
0091            ZAP=SI
0092            ZAP2=SI2
0093            SI=SI**2
0094            IF(K.LE.NDBL)SI=SI+SI2**2
      C MULTIPLY LIKELIHOOD BY CONDITIONAL DENSITY
0095            IF(H.GT.0.0)DLK=DLK*EXP(-H*SI)
      C ADD CONTRIBUTIONS TO EK,SK,BK,CK
0096            EK=EK+DLK
0097            SK=SK+SI*DLK
0098            DO 24 L=1,NA
0099            PDLK=P(L)*DLK
0100            BK(L)=BK(L)+PDLK*Z
0101            IF(K.LE.NDBL)BK(L)=BK(L)+P2(L)*DLK*Z2
0102            DO 24 M=1,L
0103            CK(L,M)=CK(L,M)+PDLK*P(M)
0104            IF(K.LE.NDBL)CK(L,M)=CK(L,M)+P2(L)*P2(M)*DLK
0105         24 CONTINUE
      C FOR INDEX=2, ADD CONTRIBUTIONS TO PZAP, PZAPSQ
0106            IF(INDEX.NE.2)GOTO 30
0107            DO 31 L=1,NA
0108            PZAP(L)=PZAP(L)+P(L)*ZAP*DLK
0109            IF(K.GT.NDBL)PZAP(L)=PZAP(L)+P2(L)*ZAP2*DLK
0110            DO 31 M=1,L
0111            IF(K.GT.NDBL)PZAPSQ(L,M)=PZAPSQ(L,M)+P(L)*P(M)*(ZAP**2)*DLK
0112            IF(K.LE.NDBL)PZAPSQ(L,M)=PZAPSQ(L,M)
               1 + (P(L)*ZAP+P2(L)*ZAP2)*(P(M)*ZAP+P2(M)*ZAP2)*DLK
0113         31 CONTINUE
0114         30 CONTINUE
      C  END OF INTEGRATION
0115         21 CONTINUE
      C  ADD CONTRIBUTIONS TO S,B,C,
```

```
0116           S=S+SK/EK
0117           DO 14 L=1,NA
0118           B(L)=B(L)+BK(L)/EK
0119           DO 14 M=1,L
0120           C(L,M)=C(L,M)+CK(L,M)/EK
0121           IF(INDEX.EQ.2)COV(L,M)=COV(L,M)+CK(L,M)/EK
              1 - 2.0*H*(PZAPSQ(L,M)/EK-(PZAP(L)/EK)*(PZAP(M)/EK))
0122        14 CONTINUE
         C  END OF SUMMATION
0123        11 CONTINUE
0124           DO 3 L=1,NA
0125           DO 3 M=1,L
0126           C(M,L)=C(L,M)
0127           IF(INDEX.EQ.2)COV(M,L)=COV(L,M)
0128         3 CONTINUE
0129           IF(INDEX.NE.2)RETURN
0130           CALL ARRAY(2,NA,NA,20,20,COV,COV)
0131           CALL MINV(COV,NA,DET,LWK,MWK)
0132           CALL ARRAY(1,NA,NA,20,20,COV,COV)
0133           DO 32 L=1,NA
0134           DO 32 M=1,NA
0135           COV(L,M)=COV(L,M)/(2.0*H)
0136        32 CONTINUE
0137           RETURN
0138           END
```

128.


```
      SUBROUTINE LSQR(H,A)
      DIMENSION A(20),B(20),C(20,20)
      DIMENSION P(20),PX(10),PY(10)
      COMMON/LKDPAR/V(500),BV(500),RH(500),RMU(500),HMU(500),
     1              PIO(500),HPI(500),PIC(500),AVMAX(500),AVMIN(500),
     2              AVM,AVS,BVM,BVS,RHM,RHS,NSTAR
      COMMON/DBLPAR/V2(20),BV2(20),RH2(20),NDBL
      COMMON/SUN/NSUN,AVSUN,BVSUN,RHSUN
      COMMON/SBSCT2/LX(20),LY(20),NX1,NY1,NT,NA
      NX=NX1-1
      NY=NY1-1
      E105=ALOG(10.0)/5.0
      IF(NDBL.LT.0)NDBL=0
      IF(NSUN.LT.0)NSUN=0
      KSUN=NSTAR+NDBL+NSUN
   30 INDEX=1
      SDMAG=0.5
      SW=0.0
      DO 29 L=1,NA
      B(L)=0.0
      DO 29 M=1,L
      C(L,M)=0.0
   29 CONTINUE
      VARMAG=SDMAG**2
      IF(NDBL.EQ.0)GOTO 32
C PLACE DOUBLE STAR DATA AND SOLAR DATA
      DO 31 K=1,NDBL
      HPI(NSTAR+K)=HPI(K)
      PIO(NSTAR+K)=PIO(K)
      BV(NSTAR+K)=BV2(K)
      V(NSTAR+K)=V2(K)
      RH(NSTAR+K)=RH2(K)
   31 CONTINUE
   32 IF(NSUN.EQ.0)GOTO 34
      V(KSUN)=AVSUN
      PIO(KSUN)=0.1
      BV(KSUN)=BVSUN
      RH(KSUN)=RHSUN
   34 CONTINUE
C COMPUTE LEAST SQUARE COEFFICIENTS
      DO 35 K=1,KSUN
      IF(PIO(K).LE.0.0)GOTO 35
      VAR=VARMAG
      IF(K.NE.KSUN)VAR=VAR+1.0/(HPI(K)*(E105*PIO(K)))**2)
      W=1.0/VAR
      AV=V(K)+5.0+5.0*ALOG10(PIO(K))
      X=(AV-AVM)/AVS
      Y=(BV(K)-BVM)/BVS
      Z=RH(K)
```

```
      CALL LEP(PX,X,NX)
      CALL LEP(PY,Y,NY)
      IF(INDEX.GT.1)GOTO 36
C FIRST ENTRY, INDEX=1, COMPUTE B(L)=SUM(W*Z*P(L)), C(L,M)=SUM(W*PL*PM)
      DO 37 L=1,NA
      P(L)=PX(LX(L))*PY(LY(L))
      B(L)=B(L)+Z*P(L)*W
      DO 37 M=1,L
      C(L,M)=C(L,M)+P(L)*P(M)*W
   37 CONTINUE
      GOTO 35
C SECOND ENTRY,INDEX=2, COMPUTE S=SUM(W*(Z-SUM(A*P))**) AND SW=SUM(W)
   36 CONTINUE
      SK=Z
      SW=SW+W
      DO 38 L=1,NA
      SK=SK-A(L)*PX(LX(L))*PY(LY(L))
   38 CONTINUE
      S=S+W*SK**2
   35 CONTINUE
C END OF SUMMATION
      IF(INDEX.GT.1)GOTO 40
      DO 41 L=1,NA
C PLACE B IN A PRIOR TO CALLING SIMQ
      A(L)=B(L)
      DO 41 M=1,L
C SYMMETRISE C
      C(M,L)=C(L,M)
   41 CONTINUE
C SOLVE FOR A
      CALL ARRAY(2,NA,NA,20,20,C)
      CALL SIMQ(C,A,NA,ID)
      INDEX=2
      IF(ID.EQ.0)GOTO 34
C IF EQUATIONS SINGULAR, SET A ZERO, AND RETURN
      DO 42 L=1,NA
      A(L)=0.0
   42 CONTINUE
      RETURN
C SECOND ENTRY, COMPUTE H AND RETURN
   40 H=SW/(2.0*S)
      RETURN
      END
```

```
          SUBROUTINE LXY
          COMMON/SBSCT2/LX(20),LY(20),NX,NY,NT,NA
          L=0
          DO 1 MT=1,NT
          DO 1 MX=1,NX
          MY=MT-MX+1
          IF(MY.LT.1.OR.MY.GT.NY)GOTO 1
          L=L+1
          LX(L)=MX
          LY(L)=MY
        1 CONTINUE
          NA=L
          WRITE(6,100)(L,LX(L),LY(L),L=1,NA)
          WRITE(6,200)NA
      100 FORMAT('    L=',I2,'  LX=',I2,'   LY=',I2)
      200 FORMAT('  NA=',I2)
          RETURN
          END
```

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