THE DELTA SCUTI STAR 20 CANUM VENATICORUM

by

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ABSTRACT

The Cassegrain spectrograph attached to the 72-inch reflecting telescope at the Dominion Astrophysical Observatory, Victoria was used to obtain spectra of the Delta Scuti star 20 CVn (HD 115604). Altogether 230 spectra were taken on five nights using the 21121 camera of the spectrograph.

Only 216 spectra, representing four night's observations were measured for radial velocity on the B. G. & Z. (Brower, Grant and Zeiss) Oscilloscope Measuring machine at the Observatory. The same twelve stellar lines were measured on all the spectra. Results obtained indicated a variable radial velocity with an amplitude of approximately 2 km/sec. A statistical analysis of the result confirmed this variability.

Thirty-two spectra of the non-variable star 30 LMi (HD 90277) were also taken for comparison purposes. This star was chosen as being as similar in properties as possible to 20 CVn. These spectra were taken and measured under the same conditions as those of 20 CVn. The results were treated in the same way, and then compared with those obtained for 20 CVn. No variation was detected in the case of 30 LMi.

An attempt to determine the period of 20 CVn using the radial velocity observations resulted in two estimates of the period, one = $0^{d}.135$ and another in the vicinity of $0^{d}.176$. The former value appears to fit the data slightly better, and is suggested here as the true period.

Simultaneous photometric and spectroscopic observations were taken on one night in an attempt to determine the phase relation-

ship between the light and velocity curves of 20 CVn. Maximum light apparently occurs somewhere on the rising branch of the velocity curve. This places this star in the same class as the Delta Scuti stars δ Delphini and ρ Puppis.

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CHAPTER 1

INTRODUCTION

1.1 The Delta Scuti Stars

The Delta Scuti variables are described in the General Catalogue of Variable Stars (Kukarkin, et al, 1958) as pulsating stars of spectral type F. In actual fact spectral types vary between about A2 and F6. The general properties of this group of stars can perhaps best be described by calling them short-period, small amplitude pulsating variable stars, since both light and velocity variations are very small, and the period of variation is very short. As an example consider the star δ Scuti itself. It has a period of $0\overset{d}{.}19$, an amplitude in V light of approximately $0\overset{d}{.}30$, and an amplitude of velocity variation ranging from 7.4 km/sec to 12.4 km/sec (Fath, 1935, 1937; Colacevich, 1935). The Delta Scuti stars are fainter than the RR Lyrae variable stars, and range in absolute magnitude, M_V, from about $+1\overset{m}{.}0$ to $+2\overset{m}{.}6$. This places them in a region of the instability strip, just above the main sequence in the Hertzsprung-Russell diagram.

Apart from the difficulties in detection caused by the small amplitude in both light and velocity variation, matters are further complicated by the fact that the light and velocity curves frequently exhibit beat phenomena. This was first shown by Colacevich (1935) who found that, although the period was constant, the form of the velocity curve of δ Scuti was variable. At the same time Fath (1935, 1937) noticed that the amplitude of the light curve of this star varied with a period 26 times that of the light period. Later work has shown that this multiple periodicity is a property of virtually all the Delta Scuti stars. So far the only known exception to this fact is ρ Puppis. This multiple periodicity is not unique to the Delta Scuti stars, since many of the RR Lyrae variables also exhibit a similar phenomenon (Preston, 1964).

One explanation for this modulation effect has been put forward by Fitch (1966) who suggested that the modulation is produced by the tidal effect of a faint companion in a very eccentric orbit about the primary. In connection with this suggestion Bessell (1969) has shown that δ Scuti, ρ Puppis and δ Delphini have high microturbulent velocities as well as appearing to be overabundant in the iron-group elements, and underabundant in calcium and scandium. These properties are similar to those of the metallic-line stars, all of which, according to Abt (1961) are probable spectroscopic binaries. Further, Preston (1965) has suggested δ Delphini is a double-lined spectroscopic binary with a highly elliptical orbit. He has not, to the writer's knowledge, however, published any data on the orbit, and no evidence for the binary nature of δ Delphini has been found on any of the spectra of this star taken at the Dominion Astrophysical Observatory (v. Preston, 1965).

Breger (1970) has undertaken a survey of Am stars in the Delta Scuti Instability Strip and has shown them to be stable against pulsation. He discusses the possibility that (a) Am characteristics tend to inhibit pulsation or (b) pulsation destroys the observable Am characteristics.

At this stage, therefore, any conclusions reached about either the binary nature of the Delta Scuti stars or any similarity between the spectra of these stars and the Am stars must be considered as preliminary.

The light and velocity curves of both Cepheid and RR Lyrae variables are approximately mirror images of each other in that minimum light corresponds approximately with maximum velocity and vice-versa. The first simultaneous photometry and spectroscopy of Delta Scuti stars was performed by Fath (1935), and Colacevich (1935) who obtained two nights of simultaneous observations of δ Scuti and showed that the phase relationship for this star is the same as that for the Cepheid and RR Lyrae stars, since minimum light occurs within 0.10P of maximum velocity. More recently Wilson and Walker (1956) demonstrated that the same relationship exists for CC Andromedae. Since then, however, further investigation has shown that for the Delti Scuti stars the phase relationship between light and velocity curves is not as rigid as for the longer period pulsating variables. For ρ Puppis (Struve, Sahade and Zebergs,

1956) minimum brightness occurs halfway along the descending branch of the velocity curve, while in the case of DQ Cephei (Sahade, Struve, Wilson and Zebergs, 1956) minimum brightness occurs about halfway along the ascending branch of the velocity curve. Thus for the Delta Scuti stars it would appear as if minimum brightness occurs within 0.25P on either side of maximum velocity.

1.2 Previous Observations of 20 CVn

Photometric observations of 20 CVn ($\alpha = 13^h\ 15^m\!.3$; $\delta = 40^\circ\ 50'$ (1950); $V = 4^m\!.74$; $B-V = +0.30$) have previously been reported by Danziger and Dickens (1967) and Breger (1969b). Danziger and Dickens report a range of $0^m\!.031$ in V in $0^d\!.092$ on JD 2,439,149. From their light curve it appears that these observations covered a little more than half a complete cycle, and as a result they suggest a period of approximately $0^d\!.14$. More recently Breger has reported a variation of $0^m\!.022$ in V in $0^d\!.096$ on JD 2,439,302. He made no attempt to determine the period, but his results also appear to cover a little more than half a period.

At the time of writing no detailed radial velocity observations had been reported.

The aim of this investigation was to obtain a large number of spectra of 20 CVn over a short period of time in order to :- (a) confirm the presence of a variable radial velocity; (b) examine the nature of any variation; and (c) attempt to determine the period of this variation. Simultaneous photometric and spectroscopic observations were taken with the aim of determining the phase relationship between the light and velocity curves.

CHAPTER 2

THE RADIAL VELOCITY MEASURES

2.1 The Observations

The variation of the Delta Scuti stars is fast and as a result exposure times have to be short and a fairly fast photographic emulsion used. Fortunately 20 CVn is a bright star and is thus ideal for this type of work. Further, it is a sharp-lined star, with a projected rotational velocity (i.e. vsini) of less than 10 km/sec (Danziger and Dickens, 1967), and thus it was not necessary to use plates with a very fine-grained emulsion. The type of photographic plate best satisfying the above criteria is the Kodak IIaO plate, and all spectra were taken using this type of plate.

Several spectrograph combinations were available for this project. These combinations are listed in Table 2.1.1.

TABLE 2.1.1

SPECTROGRAPH COMBINATIONS

Telescope	Camera	Dispersion	Exposure Time
		A/mm	min
72-inch (Cassegrain)	21121	15	2
	2161	30	1
48-inch (Coude)	32121	10	10
	3282	6.5	15
	9682M	2.4	60

Notes:- (i) Exposure time is the approximate exposure time for 20 CVn when using IIaO plates;

(ii) For a description of the spectrographs themselves, see Richardson (1968).

The system used for naming the spectrograph cameras is perhaps best described by means of an example. The 21121 camera, for instance, derives its name from the following:-

(a) The focal length of the camera is 21 inches; (b) The diffraction grating is ruled with 1200 lines/mm; and (c) the first order spectrum is used, hence 21121. Further, a mosaic grating is used with the 9682M camera, hence the M. This is a combination of four gratings, each ruled with about 800 lines/mm.

A multiple-exposure plateholder can be used with both telescopes. This device enables more than one spectrum to be exposed on the same photographic plate by moving the plate in a direction perpendicular to the dispersion after each exposure. In the case of the 72-inch telescope eight or nine spectra per plate can be obtained in this way, while with the 48-inch telescope only three spectra per plate can be obtained. This device increases efficiency, since the process of moving a photographic plate within the plateholder takes far less time than removing a plate from the plateholder after each exposure and replacing it with another plate.

After consideration of the above information it was decided that the 21121 camera was best suited for providing the combination of accuracy and speed needed for this project, and this camera was used for all observations.

The slow motion controls of the telescope were used to trail the image of the star back and forth along the slit of the spectrograph during each exposure. This is a departure from the normal technique of allowing the image of the star to drift along the length of the slit by introducing a slight error in the drive rate, and is necessary because the short exposure time does not allow sufficient time for the image of the star to drift the whole length of the slit with the result that unevenly exposed spectra are produced.

The iron arc spectrum used as a comparison was generally exposed midway through each exposure.

Altogether 230 spectra of 20 CVn were taken during 1968 and 1969 by various observers as described in Table 2.1.2.

Initial results indicated that the variation in velocity of 20 CVn was rather small. Therefore, for comparison purposes it was decided to take a series of spectra of a non-variable star under similar conditions. 30 LMi (HD 90277; $\alpha = 10^h\ 23^m.1$; $\delta = 34^\circ\ 03'$ (1950); $V = 4^m.73$; $B-V = +0.26$), a star with properties similar to those of 20 CVn was chosen for this purpose. The properties of the two stars are compared in Table 2.1.3. The author and Dr. C. D. Scarfe obtained a run of 32 spectra of 30 LMi on 1970 March 5.

TABLE 2.1.2

OBSERVATIONS OF 20 CVn DURING 1968 AND 1969

DATE	No OF SPECTRA	OBSERVER
1968 April 16	56	Dr. D. Crampton
1969 April 8	31	Dr. C. D. Scarfe, and the author
April 12	14	Dr. D. Crampton
April 15	97	Dr. D. Crampton
May 21	32	Dr. C. D. Scarfe

TABLE 2.1.3

PROPERTIES OF 30 LMi AND 20 CVn

Visual magnitude	$4^{\mathrm{m}}.73$	(i)	$4^{\mathrm{m}}.74$	(i)
Spectral type	F0 V	(i)	F0 II-IIIp	(i)
Rotational velocity (vsini)	30 km/sec	(ii)	$<$ 10km/sec	(iii)
Radial velocity	13 km/sec	(i)	variable	

References: (i) Bright Star Catalogue (Hoffleit, 1964);
(ii) Slettebak (1955);
(iii) Kraft as quoted by Danziger and Dickens (1967).

2.2 The Choice of Lines

To obtain some idea of the range of velocities involved, and the accuracy which could be expected ten spectra were first measured on the Zeiss Abbé Comparator visual measuring machine. The stellar wavelengths and settings used were the same as those used by Niehaus (1970) for the F0 V star μ^1 Bootis. Twelve stellar lines were measured on each plate. Line velocities were calculated in each case. Even at this stage it was obvious that some of the lines measured were going to be very unsatisfactory. For instance the line λ3933.682 Å (the K line of Ca) was very broad and, as a result, very difficult to set on visually. This produced a large scatter in the velocities measured using this line. Results obtained from these measures are summarized in Table 2.2.1. Differences in mean velocities for different lines are substantial, indicating the unsuitability of these wavelengths and settings for this star.

To choose twelve stellar lines suitable for measuring on all the spectra the following procedure was carried out. One of the above spectra, which appeared to be of average quality, was chosen and measured visually for a second time. On this occasion the positions of many stellar and iron arc comparison lines were measured. Including the original twelve lines, the positions of a total of 41 stellar lines were measured. The displacement corresponding to the mean velocity of this plate, as measured earlier, was removed from all the readings for the stellar lines. This provided 'zero velocity' settings for all the lines. Using the iron arc comparison lines as standards the dispersion at various positions on the plate was calculated. This enabled approximate wavelengths to be calculated for each line. All the lines measured were then identified using the Second Revised Edition of Rowland's Solar Wavelengths (Moore, et al, 1966), and a table of wavelengths for the lines in the F5 Ib star α Persei (Wright, 1951). Of the 41 lines measured nine were rejected because of doubtful identification or blending. The wavelengths provided in the relevant tables were then used to calculate settings for the remaining thirty-two stellar lines.

TABLE 2.2.1

INITIAL RESULTS USING THE ABBÉ COMPARATOR

DATE OF OBSERVATION, 1968 APRIL 16

Heliocentric velocities (km/sec)

Plate Number, 64,000 +

	755	759	762	766	773	777	781	785	789b	793	Mean	σ
3933.7	-3.1	12.2	7.8	10.9	8.6	6.0	13.7	-7.5	-3.7	17.1	6.2	8.3
4005.3	-4.9	-1.5	-0.1	-2.5	-4.0	-2.1	-1.5	-0.4	-7.2	-0.8	-2.5	2.2
4030.8	4.4	5.9	6.3	4.3	3.0	3.7	6.1	2.8	4.7	8.7	5.0	1.8
4045.8	2.2	8.2	4.7	5.4	3.6	4.9	6.2	5.8	3.3	10.6	5.5	2.5
4063.6	5.6	2.4	6.4	1.9	2.5	4.3	6.2	3.6	0.7	10.2	4.4	2.8
4101.7	5.5	8.4	11.4	7.8	9.7	1.4	13.4	11.1	7.9	12.0	8.9	3.5
4254.3	6.1	8.5	13.9	7.0	9.3	7.4	9.0	7.6	8.4	12.0	8.9	2.4
4271.5	8.6	7.6	14.7	5.8	10.7	10.4	5.0	11.3	7.9	15.0	9.7	3.4
4325.8	7.2	8.0	8.7	7.5	9.8	9.1	8.5	9.0	8.5	12.7	8.9	1.5
4340.5	7.3	10.1	10.8	7.2	14.0	13.8	8.1	8.6	5.5	8.2	9.4	2.8
4351.8	6.2	5.7	7.0	5.2	10.3	7.4	7.1	8.6	8.2	7.8	7.4	1.5
4404.8	7.9	8.3	7.6	7.2	7.0	6.3	10.4	9.3	7.8	4.3	7.6	1.7
Mean	4.4	7.0	8.3	5.6	7.0	6.0	7.7	5.8	4.3	11.2		
σ	4.3	3.6	4.1	3.4	4.9	4.2	4.0	5.5	5.2	4.4		

σ = The standard deviation of a single observation about the mean.

This was done by direct, linear interpolation using the known wavelengths and standard settings of the iron arc comparison lines. The dispersion is not quite constant along the plate, so a check was made to ensure that the errors introduced by this method were not too large. Typically the difference in setting between successive iron arc lines of 0.0003 mm. Since the internal error of measurement is larger than this, this error cannot be considered to be significant. In addition to the above 32 lines, wavelengths and settings were obtained for a further 25 lines using a table for F-type stars provided by Dr. D. Crampton of the Dominion Astrophysical Observatory. Thus there was a total of 57 lines from which to choose.

The ten spectra which were originally measured visually were then remeasured using the B. G. & Z. (Brower, Grant and Zeiss) oscilloscope measuring machine at the Dominion Astrophysical Observatory (see Appendix B for some notes on the use of this measuring machine). The rejection of lines with very asymetrical profiles accounted for twenty-one lines. The remaining 36 lines were measured on each of the ten spectra. The results are given in Table 2.2.2.

Once all ten plates had been measured a residual in the sense (line velocity-plate mean velocity) was calculated for each line. A mean residual and a standard deviation about this mean were calculated for each of the 36 lines (see Table 2.2.3). Twelve of the 36 lines were then chosen according to the following criteria:- (a) Smallest standard deviation about the mean residual; (b) The lines should be reasonably uniformly distributed throughout the region of the spectrum to be measured

(Approximately $\lambda 3800A-\lambda 4600A$); and (c) Symmetrical lines, i.e. the lines should be easy to set on using the oscilloscope.

In an attempt to obtain greater accuracy adjustments were made to the settings of some of the lines. For those lines where the mean residual was considerably larger than the standard deviation about the mean, the setting was adjusted so that the mean residual was zero (see Table 2.2.4). It was decided not to use a t-test at this stage since it was apparent that fewer than half the lines needed adjusting.

At the same time twelve comparison lines were chosen which were reasonably uniformly distributed throughout the region of the spectrum to be measured. A list of all lines used, their wavelengths, final settings and rVs factors (where applicable) is given in Table 2.2.5.

2.3 <u>The Measurements</u>

All spectra were measured using the same stellar and comparison lines. In calculating plate mean velocities, unit weight was given to each line on the plate. In an effort to eliminate incorrectly measured lines it was decided to reject all those lines with $|v_i - \bar{v}| > 2.5\sigma$

where, v_i = the individual line velocity;

$\bar{v}$ = the plate mean velocity;

σ = standard deviation of a single observation.

Given a set of values v_i distributed normally about a mean $\bar{v}$, the probability of a value v_i occurring by chance such that $|v_i - \bar{v}| \geqslant 2.58\sigma$ is 0.01. Therefore the occurrence of such a

TABLE 2.2.2

MEASUREMENT OF 36 LINES PER PLATE, B. G. & Z.

DATE OF OBSERVATION, 1968 APRIL 16

Heliocentric velocities (km/sec)

Plate Number, 64,000 +

λ	755	759	762	766	773	777	781	785	789b	793
3759.3	4.6	5.5	7.7	7.7	8.1	7.0	6.6	7.6	5.5	5.9
3865.5	8.2	8.7	11.2	9.6	12.7	9.4	9.4	10.7	11.3	11.1
3913.5	9.5	8.7	10.8	16.2	11.9	9.3	15.1	10.0	9.6	9.5
3922.9	8.4	6.4	9.9	8.6	7.2	7.8	7.5	10.2	9.2	7.2
3933.7	-3.2	8.1	14.0	1.9	-4.1	3.6	8.9	4.9	5.2	5.3
3944.0	4.0	6.1	11.1	6.9	7.0	6.4	9.1	10.1	9.6	8.1
4005.3	-1.6	3.4	6.2	2.8	4.0	2.1	1.0	3.9	1.6	0.9
4012.4	6.9	5.4	8.5	8.7	9.2	9.5	9.1	7.6	7.6	8.4
4030.7	5.2	5.0	9.5	5.9	5.6	4.4	5.4	4.8	4.8	6.5
4045.8	3.9	4.8	7.1	6.8	6.0	7.3	6.4	5.5	6.8	6.8
4063.6	7.2	3.1	6.9	9.6	6.2	6.0	5.7	6.5	4.5	7.0
4077.7	3.2	4.9	4.8	6.0	5.7	7.7	5.2	4.2	6.6	6.8
4101.7	6.8	5.1	8.6	9.1	9.9	8.0	11.1	8.5	7.8	8.4
4149.4	-1.5	0.8	1.1	-1.3	0.1	1.6	1.0	-3.0	1.5	2.1
4163.7	6.8	5.4	8.2	6.4	5.4	8.2	7.6	9.9	6.6	7.0
4181.8	11.0	11.0	13.7	11.7	9.2	10.7	12.3	15.3	13.7	11.5
4215.5	8.4	8.3	9.0	8.3	8.5	9.0	8.6	11.4	8.5	9.5
4226.7	4.7	2.4	5.4	4.4	5.0	4.1	5.9	7.0	5.6	2.5
4235.9	3.0	3.8	6.9	4.7	5.9	4.3	7.9	7.2	9.2	3.7
4254.3	6.6	8.1	11.5	10.4	8.9	8.4	12.4	10.2	9.1	9.2
4271.5	7.6	4.1	8.7	8.7	8.1	8.0	9.3	10.8	9.9	10.9
4307.9	4.3	5.3	4.7	3.4	6.0	7.0	6.8	7.8	5.9	5.5
4314.1	11.9	12.8	14.9	12.9	15.5	11.8	14.8	15.6	16.0	14.6
4315.1	1.8	1.1	2.8	0.5	7.0	1.8	1.9	3.0	1.6	2.3
4325.8	5.0	5.5	7.5	4.3	7.5	6.4	7.2	8.9	6.5	5.2
4340.5	3.7	8.3	8.4	5.6	8.8	12.1	9.8	11.2	4.9	9.7
4351.8	6.2	5.6	7.6	6.2	6.9	6.3	7.6	6.0	7.2	6.7
4404.8	6.9	6.7	9.6	7.4	5.7	9.7	8.9	8.5	9.6	7.5
4468.5	8.2	6.6	11.6	7.2	10.0	9.9	12.2	8.8	10.4	9.8
4476.0	10.6	11.5	11.9	-	9.9	10.9	11.2	6.8	8.2	9.8
4481.2	-0.8	7.1	-	6.2	8.0	6.2	7.4	6.3	6.2	7.2
4501.4	-4.7	-3.2	-3.3	-2.7	-2.7	-2.6	-3.0	-2.3	-3.7	-1.6
4508.3	7.6	5.6	7.9	9.1	7.8	7.5	8.8	8.4	9.0	8.4
4528.6	7.6	6.0	8.6	7.2	7.4	5.8	8.3	9.3	7.0	8.7
4563.8	5.9	6.0	9.5	8.6	7.7	5.4	9.3	9.4	8.8	8.0
4572.0	5.9	9.8	11.3	15.1	8.7	6.9	8.9	10.3	9.5	9.2
Mean	5.3	5.9	8.4	7.0	7.1	6.9	7.9	7.7	7.3	7.2

TABLE 2.2.3

RESIDUALS (km/sec)

Velocities from Table 2.2.2

Plate Number, 64,000 +

λ	755	759	762	766	773	777	781	785	789b	793	Mean	$\sigma_{M.R.}$
3759.3	-0.7	-0.4	-0.7	0.7	1.0	1.1	-1.3	-0.1	-1.8	-1.3	-0.35	1.01
3865.5	2.9	2.8	2.8	2.6	5.6	2.5	1.5	3.0	4.0	3.9	3.16	1.11
3913.5	4.2	2.8	1.6	9.2	4.8	2.4	7.2	2.3	2.3	2.3	3.91	2.50
3922.9	3.1	0.5	1.5	1.6	0.1	0.9	-0.4	2.5	1.9	0.0	1.17	1.15
3933.7	-8.5	2.2	5.7	-5.1	-11.2	-3.3	1.0	-2.8	-2.1	-1.9	-2.60	4.96
3944.0	-1.3	0.2	2.8	-0.1	-0.1	-0.5	1.2	2.4	2.3	0.9	0.78	1.26
4005.3	-6.9	-2.4	-2.2	-4.2	-3.1	-4.8	-6.9	-3.8	-5.7	-6.3	-4.63	1.78
4012.4	1.6	-0.5	0.1	1.7	2.1	2.6	1.2	-0.1	0.3	1.2	1.02	1.03
4030.7	-0.1	-0.9	1.1	-1.1	-1.5	-2.5	-2.5	-2.9	-2.5	-0.7	-1.36	1.28
4045.8	-1.4	-1.1	-1.3	-0.2	-1.1	0.4	-1.5	-2.2	-0.5	-0.4	-0.93	0.75
4063.6	1.9	-2.8	-1.4	1.6	-0.9	-2.2	-1.2	-2.8	-0.9	-0.2	-0.89	1.63
4077.7	-2.1	-1.0	-3.6	-1.0	-1.4	0.8	-2.7	-3.5	-0.7	-0.4	-1.56	1.41
4101.7	1.5	0.8	0.2	2.1	2.8	1.1	3.2	0.8	0.5	1.2	1.26	1.21
4149.4	-6.8	-6.7	-7.3	-8.3	-7.0	-5.3	-6.9	-10.7	-5.8	-5.1	-6.99	1.62
4163.7	1.5	-0.5	-0.1	-0.6	-1.7	1.3	-0.3	2.2	-0.7	-0.2	0.09	1.19
4181.8	5.7	6.0	5.3	4.7	2.1	3.8	4.9	7.7	6.4	4.3	5.08	1.52
4215.5	3.1	2.4	0.6	1.3	1.4	2.1	0.7	3.7	1.2	2.2	1.87	1.02
4226.7	-0.6	-3.5	-3.0	-2.6	-2.1	-2.8	-2.0	-0.7	-1.7	-4.7	-2.37	1.24
4235.9	-2.3	-2.1	-1.4	-2.3	-1.2	-2.6	0.0	-0.5	1.9	-3.5	-1.40	1.55
4254.3	1.3	2.2	3.1	3.4	1.8	1.5	4.5	2.5	1.8	2.0	2.41	0.99
4271.5	2.3	-1.8	0.3	1.7	1.0	1.2	1.3	3.1	2.6	3.7	1.54	1.56
4307.9	-1.0	-0.6	-3.7	-2.6	-1.1	0.1	-1.1	0.1	-1.4	-1.7	-1.30	1.16
4314.1	6.6	6.9	6.6	5.9	8.4	4.9	6.9	7.9	8.7	7.4	7.02	1.14
4315.1	-3.5	-4.8	-5.6	-6.5	-0.1	-5.1	-6.0	-4.7	-5.7	-4.9	-4.69	1.81
4325.8	-0.3	-0.4	-0.9	-2.7	0.4	-0.5	-0.7	1.2	-0.8	-2.0	-0.67	1.10
4340.5	-1.6	2.4	0.0	-1.4	1.7	5.2	1.9	3.5	-2.4	2.5	1.18	2.45
4351.8	0.9	-0.3	-0.7	-0.8	-0.2	-0.6	-0.3	-1.7	-0.1	-0.5	-0.43	0.65
4404.8	1.6	0.8	1.3	0.4	-1.4	2.8	1.0	0.8	2.3	0.3	0.99	1.16
4468.5	2.9	3.2	3.2	0.2	2.8	3.0	4.4	1.1	3.1	2.6	2.64	1.16
4476.0	5.3	5.6	3.5	-	2.8	4.0	3.3	-0.9	0.9	2.6	3.01	2.04
4481.2	-6.1	1.2	-	0.8	0.9	-0.7	-0.5	-1.4	-1.1	0.0	-0.94	2.12
4501.4	-10.0	-9.1	-11.6	-9.7	-9.8	-9.5	-10.9	-10.0	-11.0	-8.8	-10.04	0.88
4508.3	2.3	-0.3	-0.5	2.1	0.7	0.6	0.9	0.7	1.7	1.2	0.94	0.92
4528.6	2.3	0.1	0.2	0.2	0.3	-1.1	0.4	1.6	-0.3	1.5	0.52	1.00
4563.8	0.6	0.1	1.1	1.6	0.6	1.5	1.4	1.7	1.5	0.8	0.79	0.96
4572.0	0.6	3.9	2.9	8.1	1.6	0.0	1.0	2.6	2.2	2.0	2.49	2.28

$\sigma_{M.R.}$ = Standard deviation about the mean residual.

TABLE 2.2.4

CORRECTION TO THE SETTINGS OF THE STELLAR LINES USED

λ	First Setting	Mean Residual	$\sigma_{M.R.}$	rVs Factor	Correction	Final Setting
Å	mm	km/sec			mm	mm
3865.5	78.1447	3.16	1.11	1196.4	0.0026	78.1473
4012.4	87.6731	1.02	1.03	1151.4	-	87.6731
4045.8	89.8427	-0.93	0.75	1140.8	-0.0008	89.8419
4101.7	93.4774	1.26	1.21	1124.5	-	93.4774
4163.7	97.5029	0.09	1.19	1107.6	-	97.5029
4215.5	100.8797	1.87	1.02	1093.0	0.0017	100.8814
4254.3	103.4067	2.41	0.99	1081.7	0.0022	103.4089
4351.8	109.7626	-0.43	0.65	1055.7	-	109.7626
4404.8	113.2190	0.99	1.16	1042.1	-	113.2190
4501.4	119.5386	-10.04	0.88	1017.8	-0.0099	119.5287
4508.3	119.9867	0.94	0.92	1016.1	-	119.9867
4563.8	123.6192	0.79	0.96	1002.7	-	123.6192

TABLE 2.2.5

FINAL COMPARISON (C) AND STELLAR (S) LINES USED

Identification	λ	Setting	rVs Factor
3C	3849.969	77.1365	
S2	3865.526	78.1473	1196.4
4C	3930.298	82.3445	
5C	3969.260	84.8726	
S8	4012.390	87.6731	1151.4
9C	4045.815	89.8427	
S10	4045.803	89.8419	1140.8
10C	4063.596	90.9978	
S13	4101.737	93.4774	1124.5
13C	4132.060	95.4477	
S15	4163.654	97.5029	1107.6
15C	4202.031	100.0000	
S17	4215.545	100.8814	1093.0
17C	4235.943	102.2080	
S20	4254.346	103.4089	1081.7
21C	4325.765	108.0620	
S27	4351.833	109.7626	1055.7
24C	4404.752	113.2173	
S28	4404.778	113.2190	1042.1
28C	4494.568	119.0884	
S32	4501.449	119.5287	1017.8
S33	4508.283	119.9867	1016.1
29C	4528.619	121.3169	
S35	4563.761	123.6192	1002.7

value is probably caused by an error in measurement. First a mean velocity and a σ were calculated for each plate. Then all those lines with $|v_i - \bar{v}| > 2.5\sigma$ were rejected, and a new mean velocity and standard deviation calculated.

The spectra taken on the nights of 1968 April 16, and 1969 April 8, April 15, and May 21 were all measured once by the author. The fourteen spectra taken before cloud intervened on 1969 April 12 covered too short a period of time (about 40 minutes) to be of any value, and were not measured. The spectra of 30 LMi taken on 1970 March 5 were also measured once each by the author.

All results were treated in a similar way wherever possible. Normal points were calculated from each night's observations by combining all unrejected lines on four successive spectra. The interval between these normal points is generally 10-20 minutes (0.05P - 0.10P, assuming the period is about $0^{d}.14$ as suggested by Danziger and Dickens, 1967). Normal points calculated using less than four spectra produced a rather large scatter, which tended to mask the variation. On the other hand, normal points from more than four spectra, although showing an even smaller scatter, covered too long a period of time (up to 0.20P). and also produced too few points on some nights to show anything conclusive.

The method described above for calculating mean velocities helps to reduce the random scatter of observations, but introduces two difficulties in the estimation of errors. The presence of a variable radial velocity means that the velocity varies from plate to plate and thus the mean velocities and standard deviations about these means are calculated using four sets of twelve observations rather than 48 independent observations. This results in an underestimate of the true error. Further, the velocities over a period of ten to twenty minutes are averaged and during this time the velocity may change by up to 1 km/sec; this is assuming a sinusoidal variation with a period of about $3^{h}_{\cdot}5$, and an amplitude of about 2 km/sec. This latter effect causes an overestimate of the true error. The first effect is dominant if the individual lines on four successive plates are used to calculate the standard deviation of the mean. Alternatively, if the plate mean velocities over four successive plates are used the first effect is no longer present and the standard deviation of the mean is an overestimate of the true error. A better, though arbitrary, estimate of the true error for each night's observations is obtained by taking the root-mean-square (R.M.S.) value of the standard deviations obtained during the night by each method, and then taking a mean of these two R.M.S. values.

Results of measurements of both 20 CVn and 30 LMi are given in Tables 2.3.1 - 2.3.6, and velocity curves are plotted in Figures 2.3.1 - 2.3.6.

The 32 spectra obtained on 1969 May 21 were also measured by Dr. C. D. Scarfe, who used the same measuring instrument and settings as the author. The results obtained by Scarfe are given in Table 2.3.5, and Figure 2.3.5. The individual plate mean velocities obtained by both measurers after lines with $|v-\bar{v}| > 2.5\sigma$ were rejected, are plotted against each other in Figure 2.3.7.

TABLE 2.3.1

HELIOCENTRIC RADIAL VELOCITIES OF 20 CVn FOR $JD_{\odot}$ 2,439,962.0 + t
(April 16, 1968)

t days	Velocity km/sec	$(S.E.M.)_1$ km/sec	$(S.E.M.)_2$ km/sec	n
0.844	6.06	0.18	0.39	48
0.852	6.51	0.21	0.48	48
0.864	6.91	0.14	0.10	48
0.873	7.56	0.21	0.25	48
0.883	6.92	0.15	0.21	48
0.892	6.82	0.19	0.50	48
0.901	7.18	0.18	0.36	48
0.908	6.68	0.20	0.44	48
0.917	7.56	0.23	0.61	48
0.924	6.95	0.22	0.71	48
0.939	7.49	0.21	0.47	48
0.949	6.84	0.25	0.70	48
0.962	7.20	0.17	0.39	48
0.972	8.18	0.23	0.70	48

R.M.S. value = 0.20   0.49

Estimated error = ± 0.35 km/sec

$(S.E.M.)_1$ is the standard error of the mean velocity using all the line velocities.

$(S.E.M.)_2$ is the standard error of the mean velocity using the plate mean velocities.

n is the number of lines involved in taking the mean.

TABLE 2.3.2

HELIOCENTRIC RADIAL VELOCITIES OF 20 CVn FOR $JD_{\odot}$ 2,440,319.0 + t
(1969 April 8)

t days	Velocity km/sec	$(S.E.M.)_1$ km/sec	$(S.E.M.)_2$ km/sec	n
0.776	5.25	0.20	0.12	48
0.784	4.95	0.22	0.28	48
0.842	7.00	0.28	0.34	45
0.851	5.30	0.27	0.24	46
0.904	4.87	0.25	0.43	48
0.915	5.80	0.19	0.15	48
0.933	6.07	0.34	0.58	48
0.944	6.25	0.36	0.29	35

R.M.S. value = 0.27   0.33

Estimated error = ± 0.30 km/sec

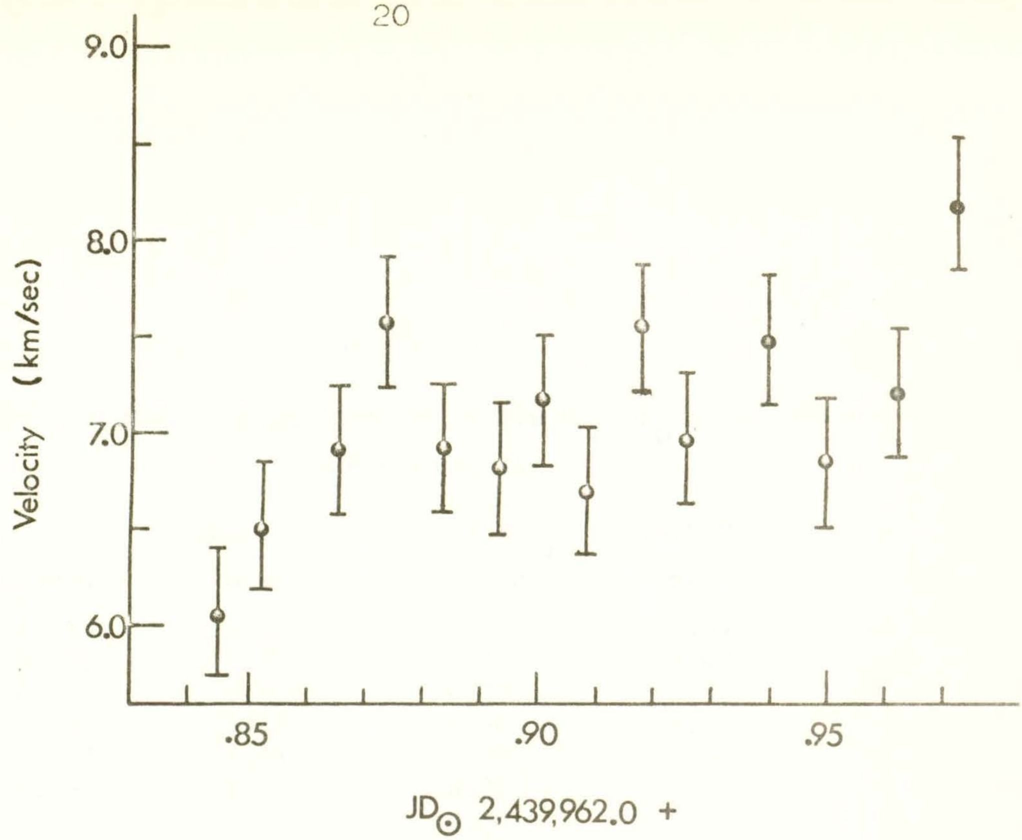

Figure 2.3.1 Heliocentric Radial Velocities of 20 CVn for 1968 April 16.

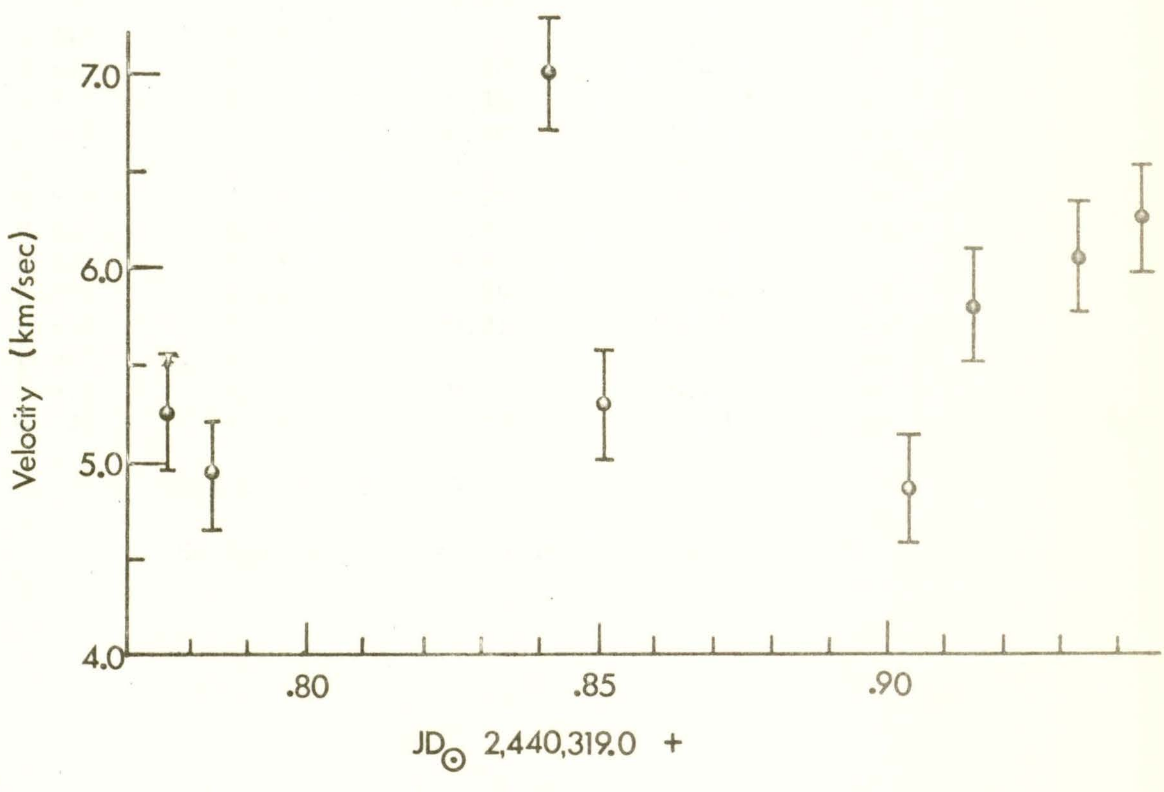

Figure 2.3.2 Heliocentric Radial Velocities of 20 CVn for 1969 April 8.

TABLE 2.3.3

HELIOCENTRIC RADIAL VELOCITIES OF 20 CVn FOR $JD_{\odot}$ 2,440,326.0 + t

(1969 April 15)

t days	Velocity km/sec	$(S.E.M.)_1$ km/sec	$(S.E.M.)_2$ km/sec	n
0.766	5.93	0.32	0.34	48
0.772	6.85	0.30	0.40	48
0.779	6.04	0.27	0.32	46
0.785	6.60	0.30	0.35	48
0.792	7.23	0.22	0.32	47
0.797	6.43	0.28	0.26	46
0.804	6.66	0.30	0.47	47
0.811	6.19	0.31	0.73	47
0.817	5.41	0.24	0.18	48
0.823	5.66	0.24	0.41	46
0.829	6.50	0.27	0.12	47
0.838	6.25	0.32	0.49	47
0.843	6.23	0.25	0.33	47
0.850	6.72	0.28	0.42	46
0.857	6.72	0.32	0.50	47
0.865	7.04	0.26	0.27	48
0.871	6.91	0.30	0.75	48
0.879	7.52	0.28	0.31	48
0.885	6.80	0.22	0.50	46
0.892	7.25	0.32	0.56	48
0.901	7.25	0.29	0.68	47
0.909	7.35	0.21	0.19	46
0.907	7.35	0.21	0.19	46
0.915	6.99	0.21	0.40	46
0.922	7.10	0.23	0.53	58
	R.M.S. value =	0.27	0.44	

Estimated error = ± 0.35 km/sec

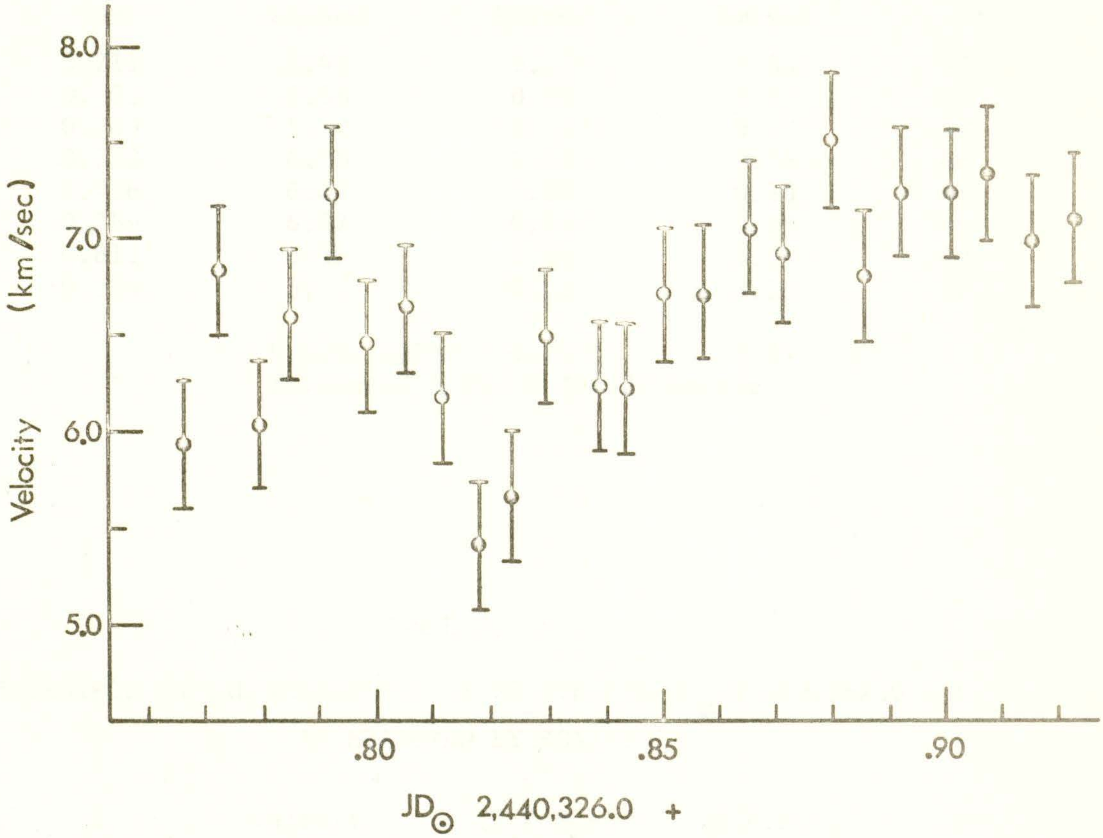

Figure 2.3.3 Heliocentric Radial Velocities of 20 CVn for 1969 April 15.

TABLE 2.3.4

HELIOCENTRIC RADIAL VELOCITIES OF 20 CVn FOR $JD_{\odot}$ 2,440,362.0 + t
(1969 May 21)

t days	Velocity km/sec	$(S.E.M.)_1$ km/sec	$(S.E.M.)_2$ km/sec	n
0.711	5.45	0.25	0.30	47
0.723	5.68	0.21	0.11	48
0.743	5.72	0.20	0.17	46
0.758	6.28	0.23	0.16	48
0.776	6.03	0.20	0.40	46
0.789	6.87	0.22	0.23	46
0.811	6.16	0.23	0.24	48
0.826	5.81	0.21	0.18	47

R.M.S. value = 0.22   0.24

Estimated error = ± 0.23 km/sec

TABLE 2.3.5

HELIOCENTRIC RADIAL VELOCITIES OF 20 CVn FOR $JD_{\odot}$ 2,440,362.0 + t
AS MEASURED BY SCARFE

t days	Velocity km/sec	$(S.E.M.)_1$ km/sec	$(S.E.M.)_2$ km/sec	n
0.711	5.89	0.30	0.27	47
0.723	5.88	0.27	0.18	48
0.743	6.02	0.27	0.23	48
0.758	6.40	0.27	0.29	48
0.776	6.40	0.27	0.33	47
0.789	7.42	0.28	0.38	47
0.811	6.40	0.26	0.44	46
0.826	6.16	0.19	0.13	48

R.M.S. value = 0.27   0.31

Estimated error = ± 0.29 km/sec

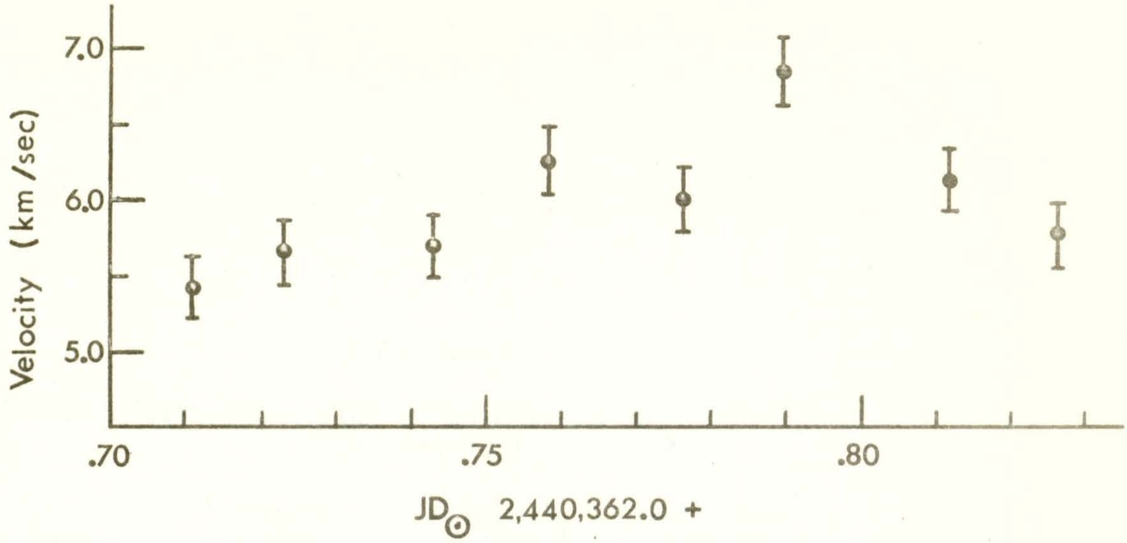

Figure 2.3.4 Heliocentric Radial Velocities of 20 CVn for 1969 May 21

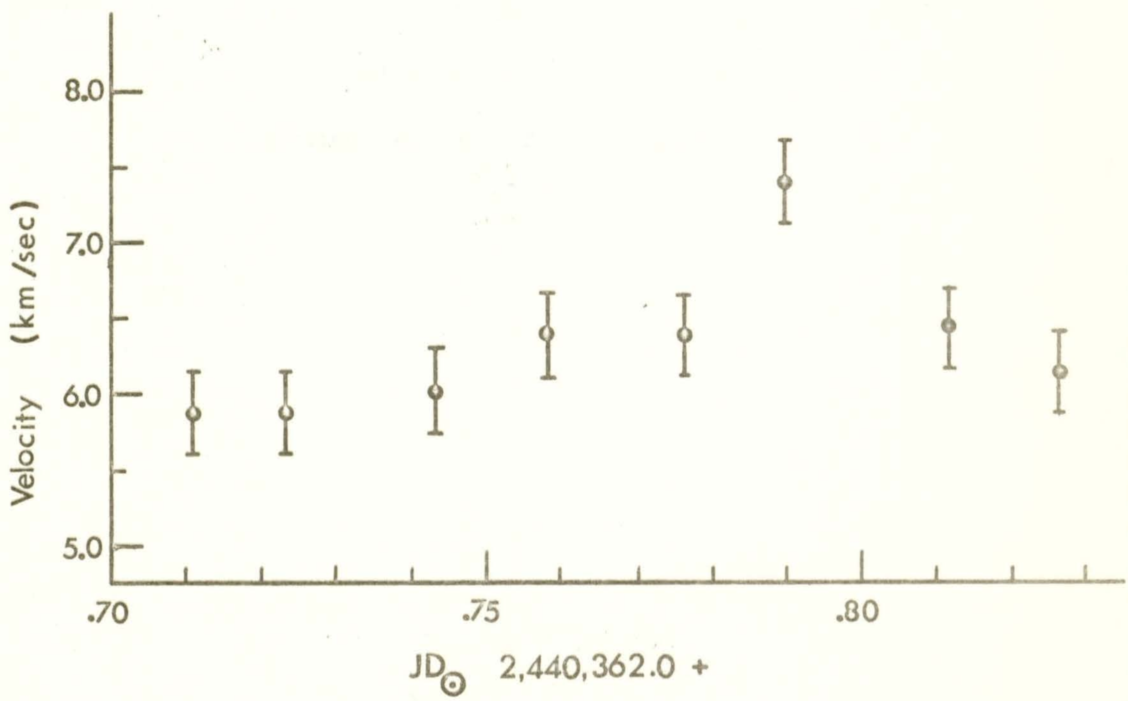

Figure 2.3.5 Heliocentric Radial Velocities of 20 CVn for 1969 May 21, as measured by Scarfe.

TABLE 2.3.6

HELIOCENTRIC RADIAL VELOCITIES OF 30 LMi FOR $JD_{\odot}$ 2,440,650.0 + t
(1970 March 5)

t days	Velocity km/sec	$(S.E.M.)_1$ km/sec	$(S.E.M.)_2$ km/sec	n
0.843	13.32	0.85	0.41	48
0.853	13.98	0.78	0.74	48
0.871	13.27	0.70	0.29	48
0.885	13.39	0.68	0.41	48
0.903	13.33	0.73	0.32	48
0.927	14.11	0.56	0.25	48
0.957	14.22	0.52	0.42	48
0.978	13.07	0.64	0.23	48
	R.M.S. value =	0.69	0.41	

Estimated error = ± 0.55 km/sec

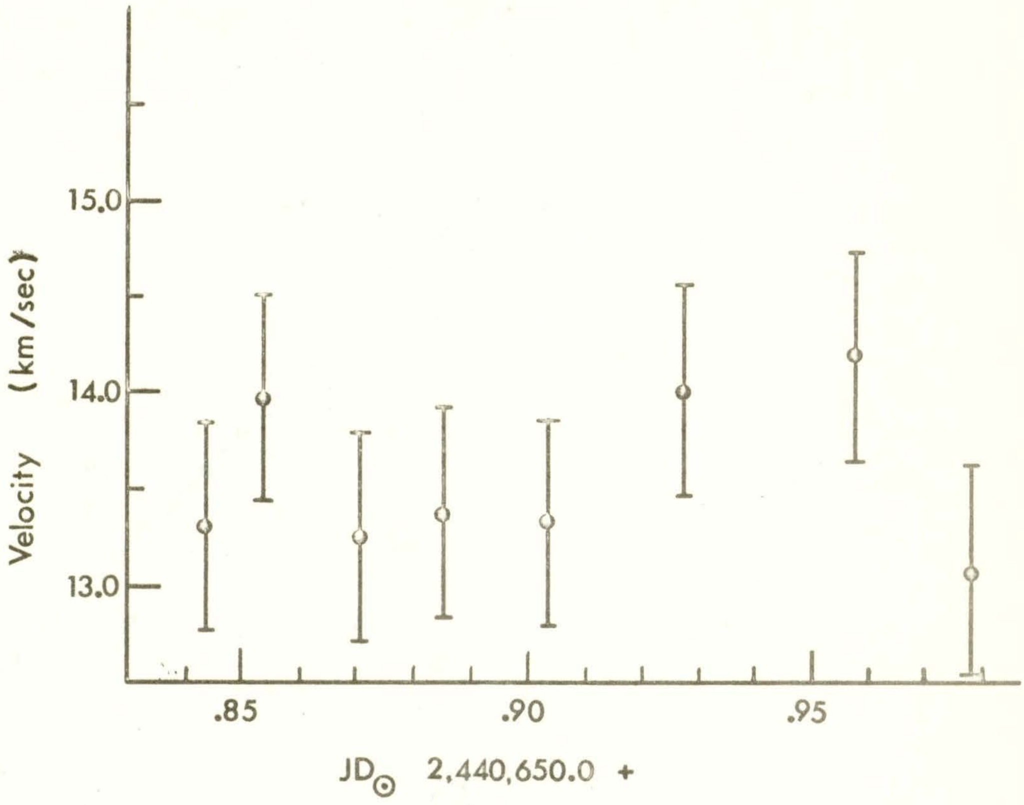

Figure 2.3.6 Heliocentric Radial Velocities of 30 LMi for 1970 March 5.

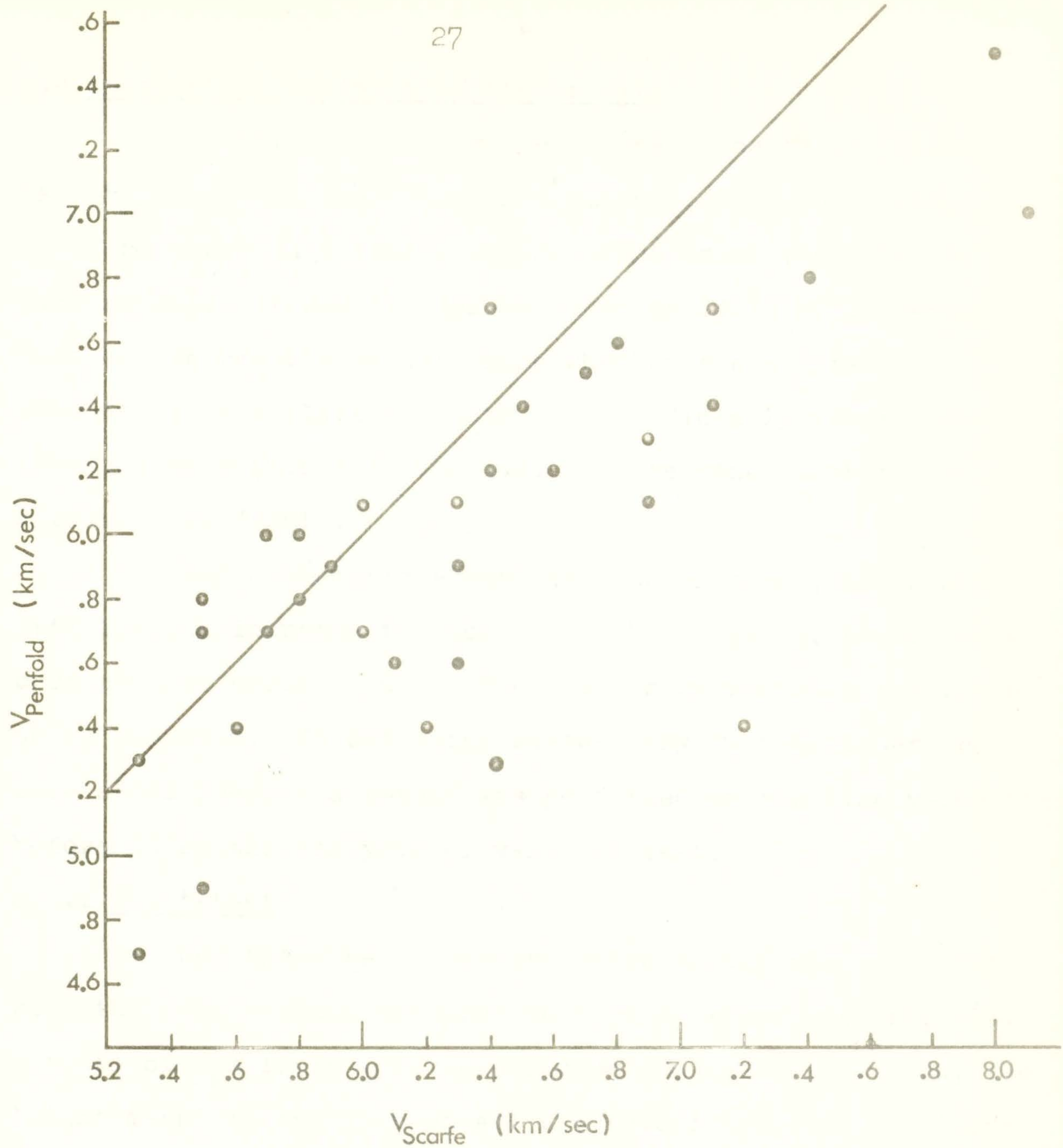

Figure 2.3.7 Comparison of the Velocities of 20 CVn for 1969 May 21, as measured by Penfold and Scarfe. The line drawn represents $V_{Penfold} = V_{Scarfe}$.

2.4 Statistical Analysis of the Results

In all cases the range of the velocity variation is very small (of the order of 2 km/sec), while a typical standard error in a single normal point is of the order of 0.33 km/sec. It was decided to do an analysis of variance test on the results to determine whether the scatter in velocities from plate to plate is significantly greater than the scatter within a plate, i.e. to determine whether the variation is "real".

Any systematic errors in line settings, which may artificially increase the scatter within a plate, should have been removed earlier during the process of choosing the lines to be measured. To determine whether any systematic errors were still present a t-test was performed on the line velocities before doing the analysis of variance test.

2.4.1 The t-test

Let there be N_k spectra taken during any one night, with the same m lines measured on each of these spectra, i.e. a total of $N_k m$ lines were measured on any one night. Now, the 'night mean' is the mean of all $N_k m$ velocities; and the 'line mean' is the mean velocity of the N_k velocities for a particular line.

Any of the line means may differ from the night mean because the setting from which the line velocity is calculated is systematically too high or too low. However, if there is a large scatter in the line velocities for that particular line a difference between line mean and night mean may not be significant. It is to determine whether this difference is significant that the t-test is used.

Define, $$t_{ik} = \frac{|\bar{v}_{ik} - \mu_k|}{S_{ik}} \sqrt{N_k - 1}$$

where, $\bar{v}_{ik}$ = mean velocity of the i^{th} line on the k^{th} night;

μ_k = the night mean;

whence, $\bar{v}_{ik} - \mu_k$ = the mean residual of the i^{th} line on the k^{th} night, $\bar{R}_{ik}$;

also, $$S_{ik} = \sqrt{\frac{\sum_{j=1}^{N_k} (R_{ijk} - \bar{R}_{ik})^2}{N_k - 1}}$$

S_{ik} is the standard deviation of the residuals about the above mean residual;

R_{ijk} is the residual of the i^{th} line on the j^{th} plate on the k^{th} night from the plate mean.

Note the use of residuals in the t-test; this eliminates any effect a possible variable radial velocity may have on t.

The above discussion deals with each night separately. A more reliable result can be obtained by combining the results of all four nights and performing a t-test on these combined results.

In this case, $$t_i = \frac{\bar{R}_i}{S_i} \sqrt{\sum_{k=1}^{4} N_k - 1}$$

where, $\bar{R}_i = \dfrac{\sum_{k=1}^{4} \sum_{j=1}^{N_k} R_{ijk}}{\sum_{k=1}^{4} N_k}$ the mean residual of the i^{th} line over all nights;

$$S_i = \sqrt{\frac{\sum_{k=1}^{4} \sum_{j=1}^{N_k} \left(R_{ijk} - R_i\right)^2}{\sum_{k=1}^{4} N_k - 1}}$$

S_i is the standard deviation of the residuals about this mean.

Goodman (1960) gives tables of the significant values of t at various levels, and for various numbers of degrees of freedom. If t_i is significant then each velocity for that particular line has to be adjusted by $-\bar{R}_i$. When this has been done for all m lines new night means, new plate means and standard deviation have to be calculated before the analysis of variance test can be performed.

Results of the t-test on both stars are given in Table 2.4.1. Only velocities measured by the author were used for this part of the work. The values of t underlined are those which are significant at the 99% level, i.e. the mean residual has 99% probability of being significantly different from zero. As a result of this test the adjustments indicated in Table 2.4.2 were made before performing the analysis of variance test. These adjustments were made to the calculated line velocities directly and not to the settings of each line. New night means, new plate means and standard deviations were calculated for each night before proceeding to the analysis of variance.

TABLE 2.4.1

t-TEST RESULTS FOR 20 CVn AND 30 LMi

Line	20 CVn	30 LMi
3865.5	3.16	0.76
4012.4	2.28	1.19
4045.8	7.10	0.62
4101.7	2.01	3.04
4163.7	3.23	2.58
4215.5	2.87	8.86
4254.3	1.13	1.15
4351.8	6.34	0.12
4404.8	4.26	3.87
4501.4	0.43	1.83
4508.3	7.62	1.32
4563.8	8.26	1.90

Significant values of t are:-

(i) 2.60 for 215 degrees of freedom (20 CVn);

(ii) 2.74 for 31 degrees of freedom (30 LMi).

TABLE 2.4.2

ADJUSTMENTS TO LINE VELOCITIES FOR 20 CVn AND 30 LMi

Line	20 CVn km/sec	30 LMi km/sec
3865.5	+0.25	-
4012.4	-	-
4045.8	+0.47	-
4101.7	-	+2.08
4163.7	+0.30	-
4215.5	-0.18	-3.70
4254.3	-	-
4351.8	+0.51	-
4404.8	-0.34	+1.97
4501.4	-	-
4508.3	-0.67	-
4563.8	-0.98	-

2.4.2 The Analysis of Variance

Once systematic effects have been removed the analysis of variance test can be used to determine whether the variance between plates is significantly greater than the variance within a plate.

The estimate of the variance between plates, $V_{bk} = \dfrac{m \sum_{j=1}^{N_k} (V_{jk} - \mu_k)^2}{N_k - 1}$

where all the symbols are as defined previously.

Since there are N_k plates there are N_k-1 degrees of freedom $= \nu_{bk}$

The estimate of variance within a plate, $V_{wk} = \dfrac{\sum_{j=1}^{N_k} \sum_{i=1}^{m} (V_{ijk} - \bar{V}_{jk})^2}{N_k (m-1)}$

In this case there are $N_k(m-1)$ degrees of freedom $= \nu_{wk}$

Finally the variance ratio, $F_k = \dfrac{V_{bk}}{V_{wk}}$

Goodman (1960) gives a table of significant values of F at the 5% and 1% levels, and for various numbers of degrees of freedom.

As was the case with the t-test, a more reliable result is obtained for 20 CVn by combining the analysis of variance results of all four nights and performing an analysis of variance test on these combined results. In this case, the estimate of variance between plates,

$$V_{bc} = \frac{m \sum_{k=1}^{4} \sum_{j=1}^{N_k} \left(\bar{V}_{jk} - \bar{V}\right)^2}{\sum_{k=1}^{4} N_k - 1}$$

where, $\bar{V}$ = the mean velocity over all nights

and the other symbols are as defined earlier,

and, $\nu_{bc} = \sum_{k=1}^{4} N_k - 1$ = total number of plates - 1.

The estimate of variance within a plate,

$$V_{wc} = \frac{\sum_{k=1}^{4} \sum_{j=1}^{N_k} \sum_{i=1}^{m} \left(V_{ijk} - \bar{V}_{jk}\right)^2}{\sum_{k=1}^{4} N_k (m-1)}$$

and, $\nu_{wc} = \sum_{k=1}^{4} N_k (m-1)$ = total number of lines - number of plates.

Finally the variance ratio, $F_c = \dfrac{V_{bc}}{V_{wc}}$

The results of the analysis of variance test on both stars are given in Table 2.4.3. Also given in this Table are the significant values of the variance ratio at both the 5% and 1% levels (Goodman, 1960).

TABLE 2.4.3

RESULTS OF THE ANALYSIS OF VARIANCE TEST

	20CVn	30 LMi
V_{bc} $(km/sec)^2$	12.43	8.40
ν_{bc}	215	31
V_{wc} $(km/sec)^2$	3.31	21.78
ν_{wc}	2376	352
F_c	3.76	0.39
$F_{5\%}$	1.10	1.48
$F_{1\%}$	1.14	1.74

2.5 The Period of 20 CVn

An attempt was made to determine the period of 20 CVn using a computer program provided by Dr. G. Hill of the Dominion Astrophysical Observatory. This program fits a curve of the form:

$$V = V_o + A_1 \cos \frac{2\pi t}{P} + A_2 \sin \frac{2\pi t}{P}$$

to the data, where,

V = Observed velocity,

V_o = 'Systemic' velocity,

A_1, A_2 = Constants,

t = Time elapsed from some arbitrary zero point; usually the first observation.

P = Trial period.

Starting at P_o and proceeding in steps of ΔP to P_1 a curve of the above form is fitted to the data and the constants V_o, A_1, A_2, calculated. A theoretical curve is calculated and at each step the sum of the squares of the deviations from this curve (where deviation = observed-calculated velocity) is calculated. The period at which this sum is the smallest is assumed to be the correct period, and a mesh with ΔP smaller by a factor of ten is then constructed about this point, and the above procedure repeated to obtain a final estimate of the period.

It was assumed that the period of 20 CVn was of the order of $0^d.14$ as suggested by Danziger and Dickens (1967). A step-size of $0^d.005$ was used with the period range from $0^d.090$ to $0^d.200$, and all the radial velocity observations were read in.

Using the above method a period of $0^d.176$ was obtained. It was noted, however, that in the vicinity of the correct period several good period fits should be obtained. This means that if the sum of the deviations squared is plotted against the value of the period, a broad minimum should occur in the vicinity of the true period. A plot of this nature using the above data showed a very narrow minimum at $P = 0^d.176$, but a broader and shallower minimum about $P = 0^d.135$. Initial step-sizes of $0^d.05$, $0^d.01$ and $0^d.001$ were also tried, and in each case, except for $\Delta P = 0^d.05$, a very narrow minimum was obtained in the range $0^d.164 < P < 0^d.176$, but never at the same value, while the minimum obtained about $0^d.135$ was always broader but shallower. For a step-size of $0^d.05$ the deepest and broadest minimum does occur at $P = 0^d.135$, however. Phases were calculated using periods of $0^d.176$ and $0^d.135$, and diagrams of velocity against phase were plotted in each case. These results are given in Table 2.5.1 and Figures 2.5.1 and 2.5.2.

TABLE 2.5.1

COMPLETE RADIAL VELOCITY OBSERVATIONS OF 20 CVn

t JD$_\odot$	Velocity km/sec	Phase P=0^d.135	Phase P=0^d.176	t JD$_\odot$	Velocity km/sec	Phase P=0^d.135	Phase P=0^d.176
2,439,962.844	6.06	0.00	0.00	2,440,326.797	6.43	0.95	0.88
.852	6.51	.06	.05	.804	6.66	.00	.92
.864	6.91	.15	.11	.811	6.19	.05	.96
.873	7.56	.21	.16	.817	5.41	.10	.00
.883	6.92	.29	.22	.823	5.66	.14	.03
.892	6.82	.36	.27	.829	6.50	.19	.06
.901	7.18	.42	.32	.838	6.25	.25	.12
.908	6.68	.47	.36	.843	6.23	.29	.14
.917	7.56	.54	.41	.850	6.72	.34	.18
.924	6.95	.59	.45	.857	6.72	.39	.22
.939	7.49	.70	.54	.865	7.04	.45	.27
.949	6.84	.78	.60	.871	6.91	.50	.30
.962	7.20	.87	.67	.879	7.52	.56	.35
.972	8.18	.95	.73	.885	6.80	.60	.38
2,440,319.776	5.25	.95	.00	.892	7.25	.65	.42
.784	4.95	.01	.04	.901	7.25	.72	.47
.842	7.00	.44	.37	.907	7.35	.76	.51
.851	5.30	.50	.42	.915	6.99	.82	.55
.904	4.87	.90	.72	.922	7.10	.88	.59
.915	5.80	.98	.79	2,440,362.711	5.45	.99	.95
.933	6.07	.11	.89	.723	5.68	.08	.02
.944	6.25	.19	.95	.743	5.72	.23	.13
2,440,326.766	5.93	.72	.71	.758	6.28	.34	.21
.772	6.85	.76	.74	.776	6.03	.47	.32
.779	6.04	.82	.78	.789	6.87	.57	.39
.785	6.60	.86	.81	.811	6.16	.73	.51
.792	7.23	.91	.86	.826	5.81	.85	.60

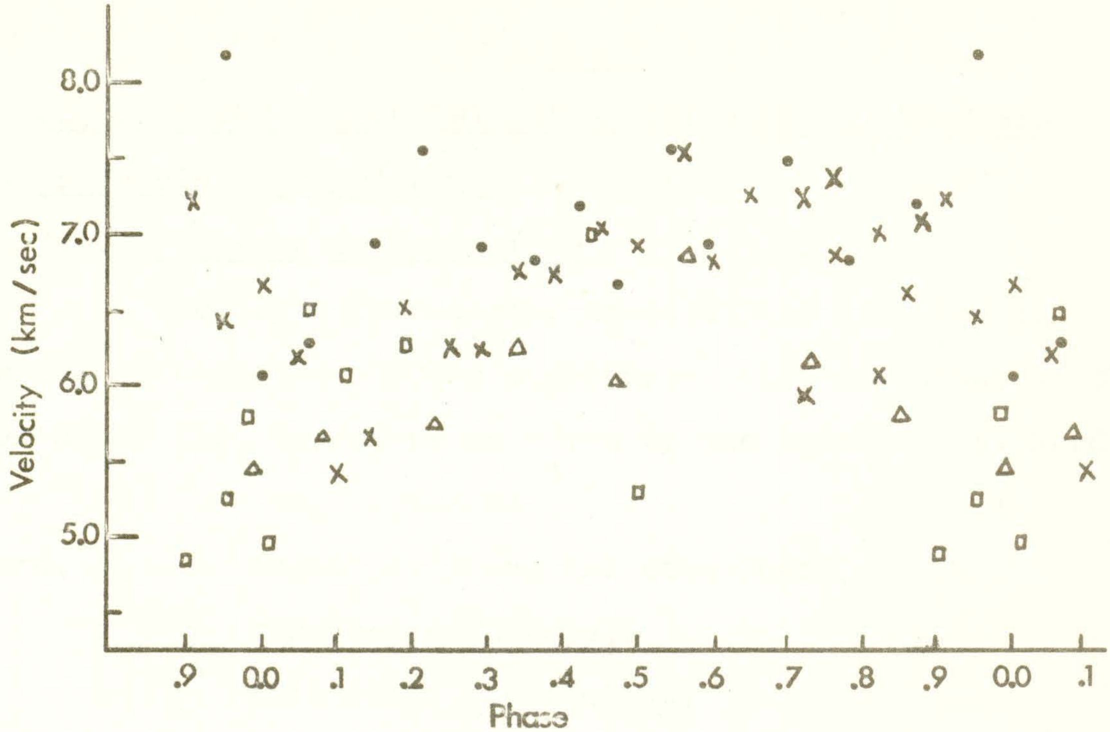

Figure 2.5.1 Phase Diagram for Velocities of 20 CVn (Period = $0^{d}.135$).

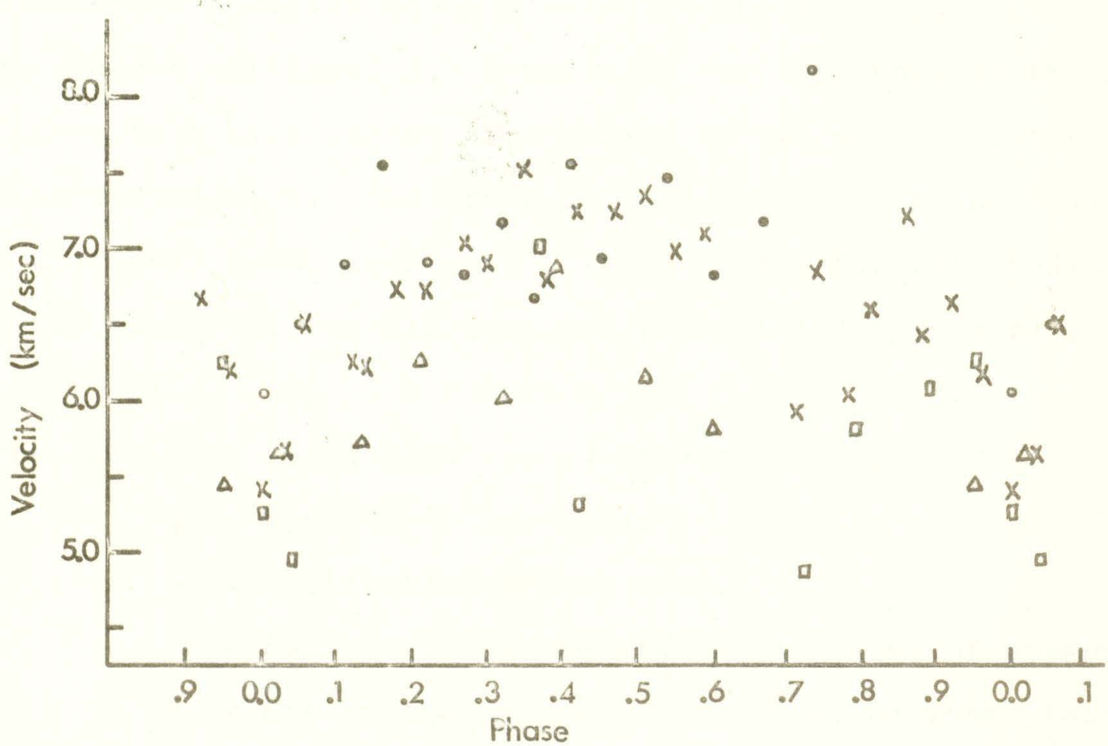

Figure 2.5.2 Phase Diagram for Velocities of 20 CVn (Period = $0^{d}.176$).

For both Diagrams:

• = 1968 April 16;
□ = 1969 April 8;
x = 1969 April 15;
Δ = 1969 May 21.

CHAPTER 3

SIMULTANEOUS PHOTOMETRY AND RADIAL VELOCITY MEASURES

3.1 Photo-electric Reductions

3.1.1 Atmospheric Extinction

The light from a star is partially absorbed in passing through the earth's atmosphere. The magnitude of the star above the atmosphere is given by the equation (v. Hardie, 1962)

$$m_o = m - kX \qquad 3.1$$

where, m_o = magnitude above the atmosphere,

m = observed magnitude,

k = extinction coefficient,

and, X is the path length, given in terms of the air mass at the observer's zenith (i.e. $X = 1.0$ at the zenith).

The relative air mass, X, in units of the thickness at the zenith is given to a high degree of accuracy by the secant of the zenith distance, z. The error introduced by replacing X with sec z is only 0.005 at $z = 60^o$. Thus except for zenith distances greater than 60^o, the air mass can be replaced by sec z.

i.e. $$m_o = m - k\sec z \qquad 3.2$$

where, $\sec z = (\sin\phi \sin\delta + \cos\phi \cos\delta \cos h)^{-1}$,

ϕ = Observer's latitude,

δ = Declination of the star,

h = Hour angle of the star at the time of observation.

Thus k is the slope of the straight line obtained when plotting the observed magnitude, m, against sec z.

By taking a number of observations of the same star on one night the slope of the line can be determined and a value obtained for the extinction coefficient. However, the extinction over the whole sky is not uniform. Further, the extinction in one part of the sky varies during any one night. Thus, to minimize the effect of any unusual atmospheric phenomena, it is better to make a number of independent determinations of k, either during one night or, even better, over a number of nights to obtain an average value.

3.1.2 Extinction as a Function of Colour Index

In the empirical determination of the extinction for the various wavelength bands it is found that the values of these coefficients are dependent on the colour index of the star. This arises from the fact that the shorter wavelengths are attenuated more than the longer wavelengths in the atmosphere (the scattering of light by molecules is roughly inversely proportional to the fourth power of the wavelength).

When working in a system of several bands at once it is convenient to work in terms of a single magnitude and one or more colour indices, and to treat extinction in a differential manner for the colour indices,

i.e. $$C_o = C - k_c X \qquad 3.3$$

where, C_o = Colour index above the atmosphere,

C = Observed colour index,

and, k_c, the colour index extinction coefficient, is merely the difference between corresponding magnitude extinction coefficients.

Generally it is possible to determine the manner in which the extinction varies with the colour index by using a number of stars. The result can be expressed as a linear function of the magnitude coefficient, k, thus:-

$$k = k' + k''C \qquad 3.4$$

where, k' is the magnitude extinction coefficient for a star of zero colour index,

k" is the increment in this coefficient for a star of colour index, C = 1.0.

Similarly, for the colour index extinction coefficient,

$$k_c = k_c' + k_c''C \qquad 3.5$$

where, k_c' is the colour index extinction coefficient for a star of zero colour index,

k_c'' is the increment in this coefficient for a star of colour index, C = 1.0.

Thus, $$m_o = m - k'X - k''CX \qquad 3.6$$

and, $$C_o = C - k_c'X - k_c''CX \qquad 3.7$$

therefore, $$C_o = C(1 - k_c''X) - k_c'X \qquad 3.8$$

To determine all the coefficients two stars of widely differing colour indices, but close together in the sky, can be observed through a number of different air masses. If the two stars have essentially the same position then differential measures of magnitude and colour will be given by,

$$\Delta m_o = \Delta m - k''(\Delta C)X \qquad 3.9$$

and, $$\Delta C_o = \Delta C - k_c''(\Delta C)X \qquad 3.10$$

Thus from the observations plots of (i) Δm vs $(\Delta C)X$, and (ii) ΔC vs $(\Delta C)X$, will produce straight lines with slopes k'' and k''_c respectively. Once the second-order coefficients have been obtained the first-order coefficients can be determined from the same observations by plotting

(i) $(m - k''CX)$ vs X, and (ii) $C(1 - k''_c CX)$ vs X.

3.1.3. Measures of Variable Stars

When observing variable stars a constant comparison star is chosen, and differential measures of magnitudes and colours are used. It is customary to choose a comparison star close enough to the variable star in position so that their air masses are virtually identical. Further, if the two stars have almost the same brightness and colour index then, to a first approximation, it can be assumed that all the colour extinction effects are eliminated between the two. Second order terms need only be considered in work requiring a very high degree of accuracy (of the order of thousandths of a magnitude). The equipment used for this project is capable of measuring stellar magnitudes to the nearest hundredth of a magnitude. Thus second order and higher effects need not be considered.

From equation (3.6),

$$V_o = V - k'_v X - k''_v CX$$

for each star, where,

V = observed magnitude in the V-band,

V_o = magnitude in the V-band above the atmosphere,

and, the other symbols are as defined earlier.

With the use of (3.4) this becomes,

$$V_o = V - k_v X$$

and, since differential extinction effects are assumed to be eliminated, the differential measures are given by,

$$\Delta V_o = \Delta V - k_v \Delta X \qquad 3.11$$

Similarly, using (3.7) and (3.5) and applying them to the two stars to obtain differential measures,

$$\Delta C_o = \Delta C - k_c \Delta X$$

or, $$\Delta(B - V)_o = \Delta(B - V) - k_{BV} \Delta X \qquad 3.12$$

where, $\Delta(B - V)$ = Observed difference in colour indices,

$\Delta(B - V)_o$ = Difference in colour indices above the atmosphere,

and, k_{BV} = Colour index extinction coefficient for (B-V).

3.2 <u>The Observation</u>

Simultaneous photometric and spectroscopic observations of 20 CVn were obtained on the night of 1969 May 21. The photometric observations were taken using the twelve-inch reflecting telescope at the University of Victoria. This telescope is equipped with an Astrometrics AP-4 Photometer, which includes an EMI 6256SA photomultiplier. Auxiliary equipment consists of an Astrometrics RR/P-2 D.C. amplifier, a Heath Servo-recorder (Model EU-20B), and a Heath Multi-speed Chart Drive (Model EU-20-26). The high voltage is provided by a Model 3K10 High Voltage Power Source by Power Designs Pacific, Inc.

The photometric observations were taken in the yellow and blue regions of the spectrum using Corning glass filters numbers 3384 and 5030 respectively. Hereinafter the observations using these filters will be referred to as V_P and B_P respectively, and the colour index as $(B - V)_P$. The yellow filter is the same as the V filter used in the standard UBV system of Johnson and Morgan (1953). Only one part of the Johnson and Morgan B filter was used (viz. Corning glass filter no. 5030). The other part of the filter consists of a 2mm thickness of Schott GG13 glass.

The star HR 5110 ($\alpha = 13^h\ 32^m.6$; $\delta = 37°\ 26'$ (1950); $V = 5^m.01$; $B - V = +0.38$) was used as a comparison star. Danziger and Dickens (1967) used this same star as a comparison for their work on 20 CVn and found it to be satisfactory for this purpose.

Although differential measures were used it is still necessary to take readings of sky brightnesses. Different regions of the sky are observed, and there is no reason to assume that sky brightnesses are the same in these regions. Observations were made in the order Vsky, Vstar, Bstar, Bsky, Vsky, etc. Each observation lasted for approximately thirty seconds. The observations were reduced by hand. To 'integrate' the readings straight lines were drawn through the tracings of both sky and star readings at what was judged to be the mean reading (v. Fig. 3.2.1). The differences between sky and star readings were then read off in centimetres.

Then, $$\Delta V_P = V_c - V_s = 2.5 \log \frac{I_s}{I_c}$$

where, V_c, V_s = the magnitudes of comparison and program stars respectively,

I_c, I_s = intensity readings in V from the chart recorder.

Similarly, for the B observations:-

$$\Delta B_P = B_c - B_s = 2.5 \log \frac{I_s'}{I_c'}$$

whence, $\Delta(B - V)_P = (B - V)_c - (B - V)_s$

$$= \Delta B_P - \Delta V_P$$

In this way ΔV_P, ΔB_P, and $\Delta(B - V)_P$ were calculated for each program star observation. A set of three program star readings was taken between each comparison star reading. For the second observation in each set the compariosn star intensity reading was taken to be the mean of the readings taken before and after that set. A total of twenty-five program star observations were taken in nine sets (two each in the first and last sets). Each set covered a period of about seven minutes (approximately 0.03 Period), and thus it is reasonable to take a mean for each set, giving a total of nine 'normal observations'. The time of each normal observation was taken to be the time of the second observation in each set. The results of the radial velocity observations taken at the same time are tabulated along with these photometric results in Table 3.2.1.

As mentioned earlier the spectra obtained at the same time as the photometric observations were measured by both the author and Dr. C. D. Scarfe. There is an obvious correlation between the two sets of results (v. Fig. 2.3.7), and this justified combining them in calculating normal points for this part of the work. The velocities in Table 3.2.1 are derived by taking the mean of all unrejected lines on two successive spectra as measured by the author along with the same two spectra measured by Scarfe.

The aim of the photometric observations was to

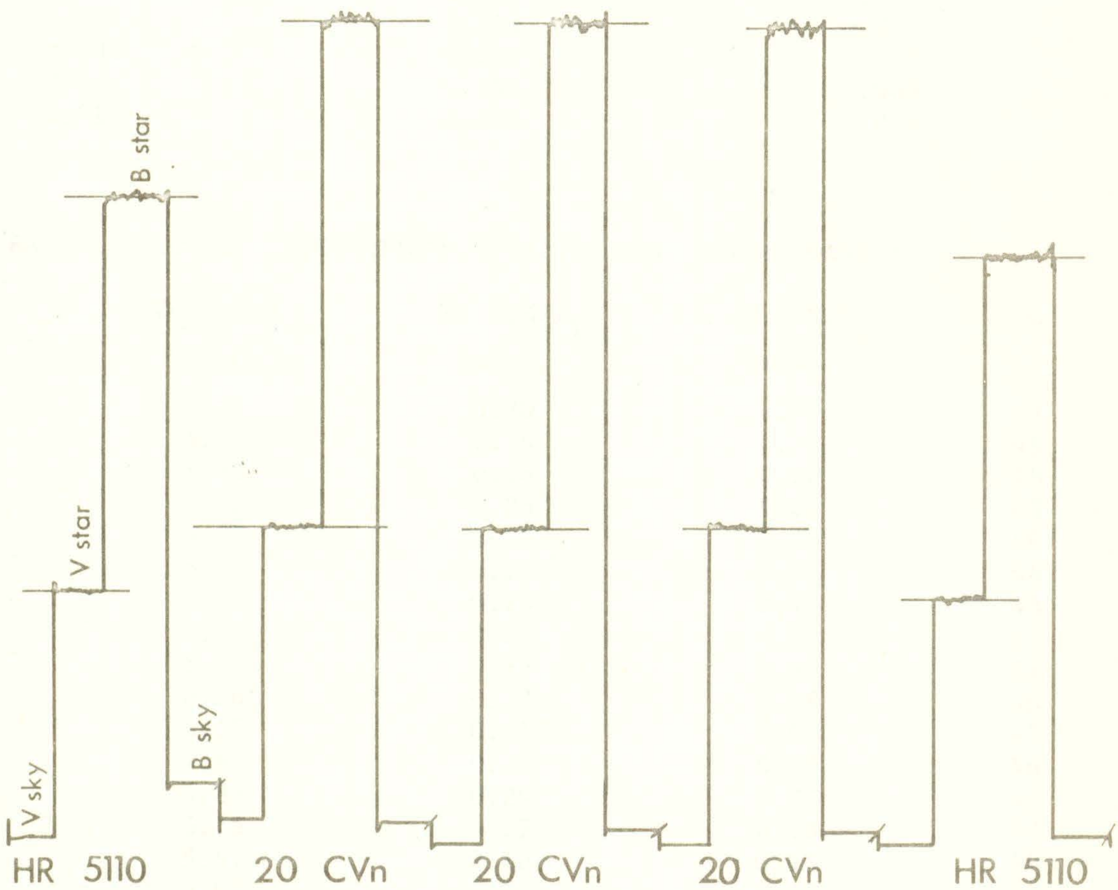

Figure 3.2.1 Example of Output from the Chart Recorder

TABLE 3.2.1

OBSERVATIONS OF 20 CVn FOR JD$_{\odot}$ 2,440,362.0 + t

(a) PHOTOMETRIC OBSERVATIONS

t days	ΔV_P m	$\Delta(B - V)_P$ m
0.727	0.265	0.095
0.737	0.270	0.080
0.749	0.250	0.073
0.759	0.247	0.073
0.770	0.250	0.073
0.781	0.243	0.077
0.791	0.247	0.073
0.805	0.247	0.083
0.814	0.235	0.095

(b) RADIAL VELOCITY OBSERVATIONS (Penfold and Scarfe combined)

t days	Velocity km/sec	$(S.E.M.)_1$ km/sec	$(S.E.M.)_2$ km/sec	n
0.708	5.61	0.30	0.54	47
0.715	5.85	0.26	0.10	46
0.720	5.81	0.26	0.20	48
0.726	5.75	0.23	0.10	48
0.740	5.92	0.25	0.38	46
0.746	5.83	0.23	0.10	48
0.753	6.13	0.23	0.18	48
0.759	6.54	0.26	0.22	48
0.772	5.88	0.25	0.41	47
0.779	6.56	0.22	0.19	46
0.786	6.67	0.21	0.06	47
0.792	7.63	0.27	0.25	46
0.807	6.83	0.24	0.23	46
0.816	5.75	0.22	0.10	48
0.822	5.91	0.18	0.15	48
0.829	6.07	0.23	0.22	47
	R.M.S. value =	0.24	0.25	

Estimated error = $\pm$ 0.25 km/sec

determine the phase relationship between the light and velocity curves. As this was the only information needed from the photometric observations no attempt was made to reduce the photometric results from the natural system of the instrument to the Johnson and Morgan UBV system. According to Johnson (1963) this transformation may not be linear and it was felt that errors introduced in any transformation would be of the same order as, or larger than the expected amplitude of light variation of the star, and might very well mask the variation.

Preliminary observations of HR 5110, the comparison star, and a selection of other stars of similar brightness and colour index to 20 CVn were taken on different nights to determine values of the extinction coefficients (Equations (3.2 and (3.3)). Values obtained for the magnitude extinction coefficient and the colour index extinction coefficient were -0.0014 and -0.0026 respectively. On the night that the simultaneous observations were taken the maximum difference in sec z between comparison and program stars was 0.02 at any one time. This indicated maximum corrections to ΔV_P and $\Delta(B - V)_P$ (Equations (3.11) and (3.12)) of $0^m.003$ and $0^m.005$ respectively. Since observations were taken to the nearest $0^m.01$ these corrections were neglected.

To determine internal errors the standard deviation of a single observation was found for both the V_P and $(B - V)_P$ observations of the comparison star during the night the simultaneous observations were obtained. In addition observations of two eclipsing variables, 44i Boo and VW Cep (Scarfe and

Brimacombe, 1970) and the comparison stars used in observing them were used to obtain an upper limit to the estimate of internal errors in ΔV_P. A good night's observations of each variable was chosen, ΔV_P light curves were plotted and smooth curves drawn through the points. The root-mean-square deviation (i.e. ΔV_P(observed)-ΔV_P (from the curve)) of a single observation was calculated for each night. All results indicated that internal errors were of the order of $\pm$ $0^m.01$ and less. These results are given below.

HR 5110:- Standard deviation of a single observation in V_P = $0^m.007$

Standard deviation of a single observation in $(B-V)_P$ = $0^m.007$

44i Boo:- R.M.S. deviation of a single observation in V_P = $0^m.007$

VW Cep:- R.M.S. deviation of a single observation in V_P = $0^m.009$

Internal errors in both V_P and $(B-V)_P$ were assumed to be $0^m.008$.

The V_P and $(B-V)_P$ light curves are given in Figures 3.2.2 and 3.2.3 respectively. Also shown in these figures are the light curves obtained by Danziger and Dickens (1967) and Breger (1969b), and the (B-V) curve of Danziger and Dickens. Figure 3.2.4 shows the simultaneous photometric and radial velocity observations plotted on the same time scale.

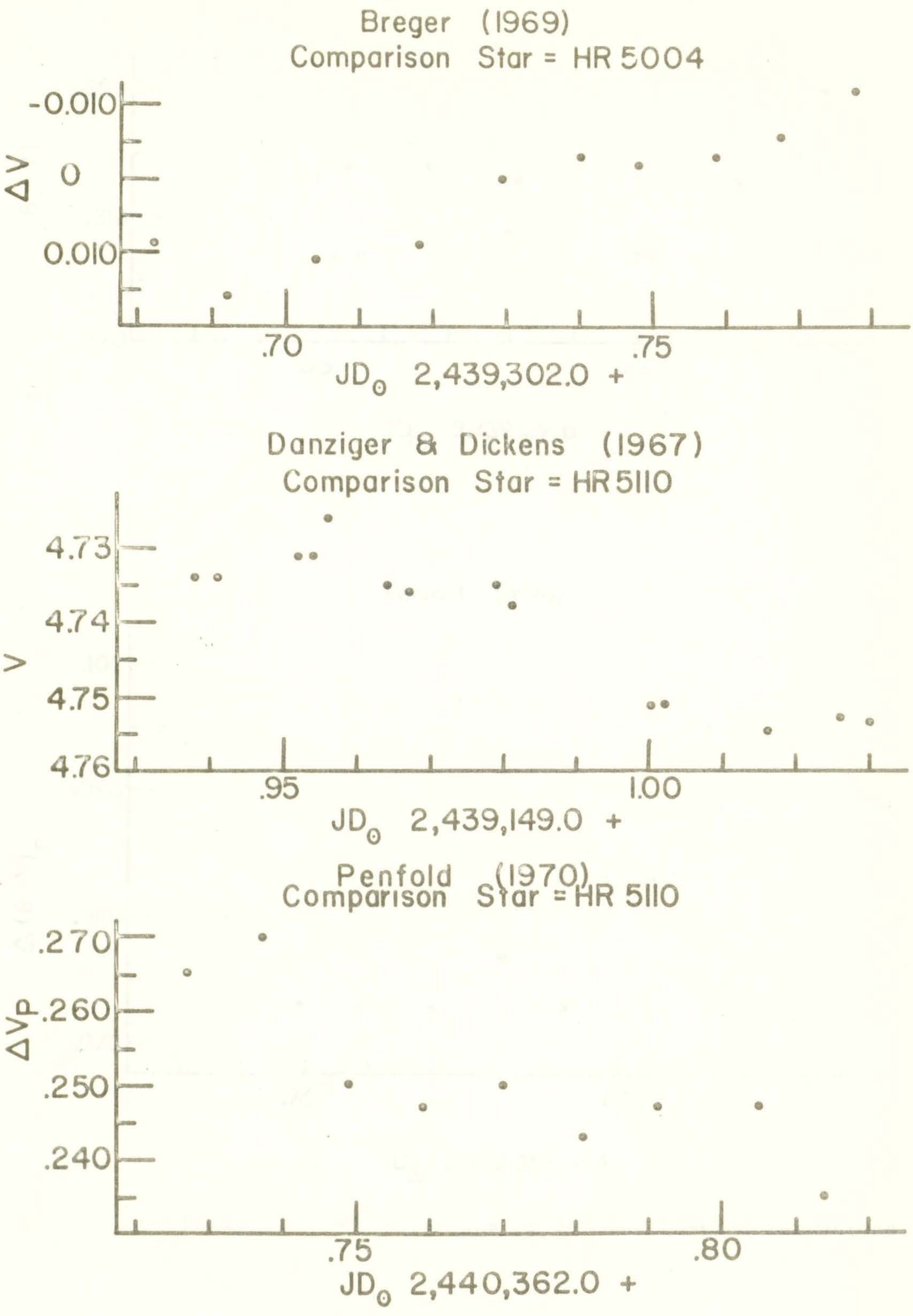

Figure 3.2.2 Photometric Observations of 20 CVn; V Observations.

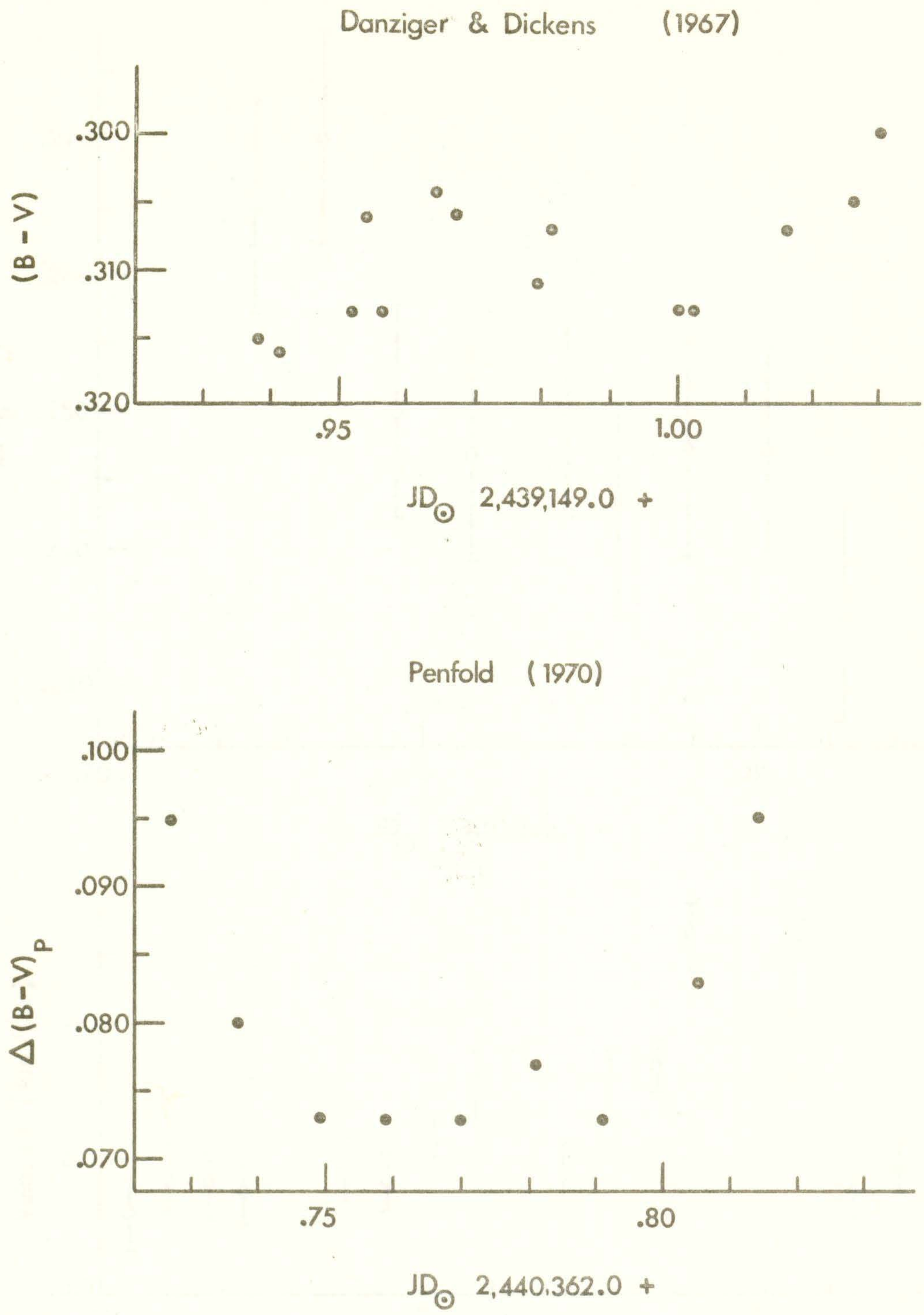

Figure 3.2.3 Photometric Observations of 20 CVn; (B-V) Observations.

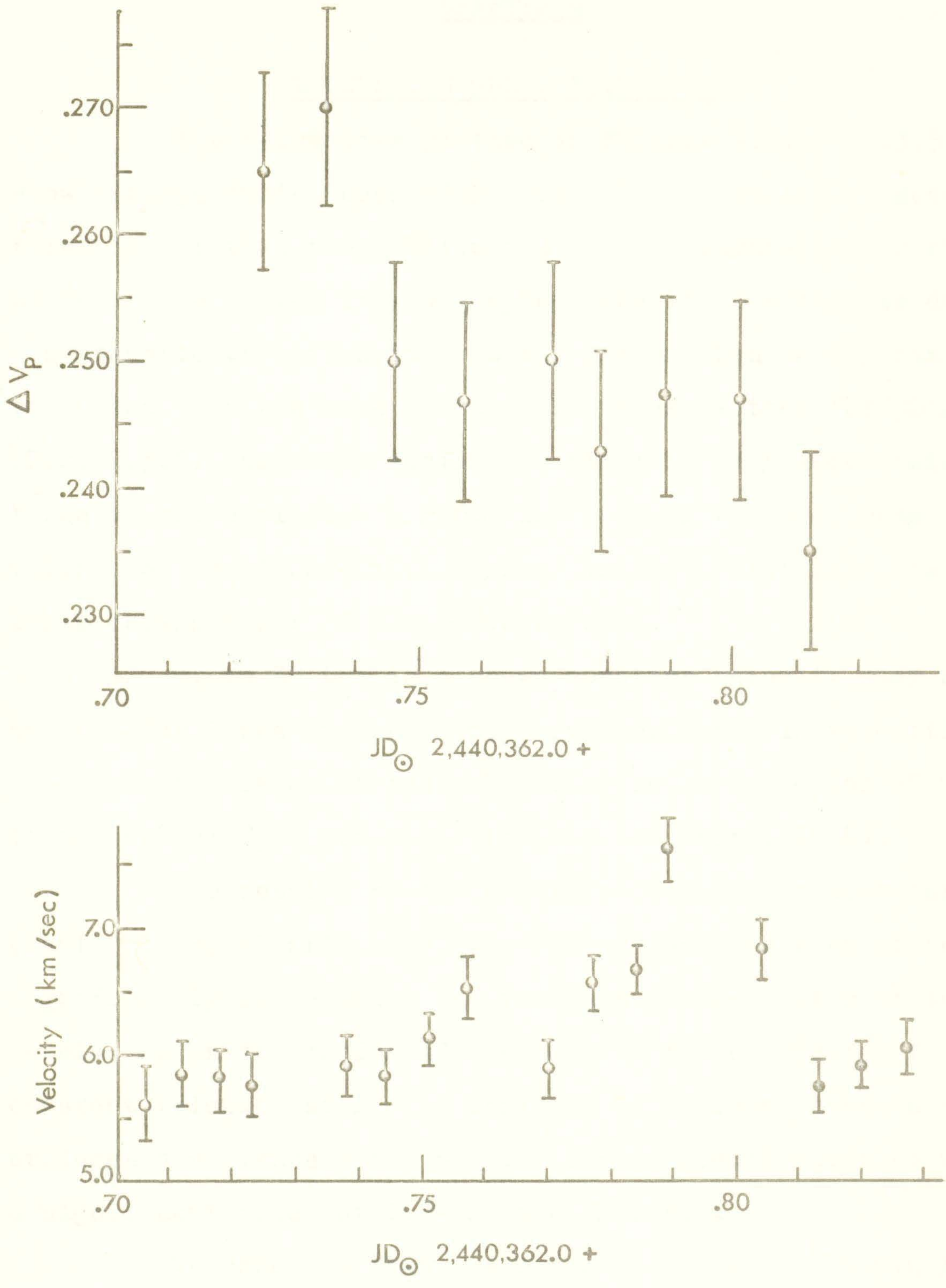

Figure 3.2.4 Simultaneous Photometric and Radial Velocity Observations of 20 CVn, 1969 May 21.

CHAPTER 4

DISCUSSION AND CONCLUSIONS

The velocities plotted in Figures 2.3.1 - 2.3.5 all show a range during each night which is approximately seven times larger than their estimated error. Further, from the shape of the curves it would appear that this effect is due to a systematic variation in velocity rather than to any random variation. In the case of the velocities plotted for 30 LMi (Fig. 2.3.6) the range during the night is only about twice as large as the estimated error. The overall range of mean velocities for 30LMi (~1.1 km/sec) is also about half that of those obtained for 20 CVn (~2.0 km/sec).

Since 30 LMi is a known constant velocity star and has broader lines than 20 CVn, a smaller range in velocities, or at worst a range of the same order is expected for 20 CVn if the velocity is not variable (Abt and Smith, 1969).

The results of the analysis of variance test for 20 CVn (Table 2.4.3) clearly show that the variance between plates is significantly larger than the variance within plates at the 1% level. As an indication of the results to be expected from a constant velocity star, the analysis of variance test on 30 LMi produces a variance ratio of 0.39 (Table 2.4.3) compared with a significant value of 1.74 at the 1% level.

We conclude that the radial velocity of 20 CVn is variable, as suggested by the earlier photometric results (Danziger & Dickens, 1967; Breger, 1969b), which indicate that this star is a variable of the Delta Scuti type.

On the average the range in velocities is about 2.0 km/sec, which would suggest an amplitude of the order of 1.0 - 1.5 km/sec.

It is interesting to note that if individual analysis of variance tests are performed on each night's observations of 20 CVn, the test on the velocities of 1969 May 21 ($JD_{\odot}$ 2,440,362) produces a result which is barely significant at the 1% level. This would suggest that the variation in velocity is very small on that night. A glance at the relevant velocity curve (Figure 2.3.4) shows that this is not necessarily so. There is apparently a systematic effect which on this night affected the two lines λ4508.3 and λ4563.8 causing these two mean line velocities to be vastly different from the population mean compared with other nights (+1.17 km/sec and +2.46 km/sec respectively). It is this which helps to produce the large values of t for these two lines (v. Table 2.4.1); it also increases the internal variance, and hence the low value of F on this night. This effect was seen by both measurers, and since there is a correlation between the results of the two (v. Fig. 2.3.7) this effect must be real. Whatever the effect, it has apparently affected only the iron arc comparison lines or only the stellar lines in this region of the spectrum. If the effect had been the same on both sets of lines no abnormality would have been noticed. The effect would merely have been 'corrected out' when applying corrections from the calculated correction curve. This shows the advisability of doing a 'combined' t-test and analysis of variance test to minimize nightly effects such as this.

Velocity curves of some Delta Scuti type variable stars are given in Figures 4.1 and 4.2. The arrows in Figure 4.2 indicate the times of alternate maxima and minima as calculated by the authors of the paper from which these data were taken. Comparison of these curves with those obtained for 20 CVn show that the velocity curves of 20 CVn are typical of stars of the Delta Scuti type, viz. variable amplitude, and irregular curves in general.

The technique used to obtain the velocities of 20 CVn appears, in general, to be suitable for the observation of Delta Scuti variables provided the amplitude of velocity variation is larger than about 1.5 km/sec. Further, it is obvious from the observations of 30 LMi that such small amplitudes will not be detectable in the broader lined stars (i.e. fast rotators), where errors in measurement are larger. This apparently applies even for moderate projected rotational velocities (i.e. vsini) of the order of 30 km/sec and more.

The period estimated using Hill's program appears, in this case, to be rather dependent on the step-size chosen. From the two phase diagrams (Figs 2.5.1 and 2.5.2) it would appear that a period of $0\overset{d}{.}135$ fits the data slightly better than a period in the vicinity of $0\overset{d}{.}176$, and as a result we suggest that the period of 20 CVn is $0\overset{d}{.}135$ as determined from the radial velocity observations. Unfortunately the combination of very short period and widely spaced photometric observations taken by various observers does not allow these observations to be used to confirm the above period. Further, none of the sets of photometric observations covers a large enough part of the cycle for them to be used individually to confirm the period.

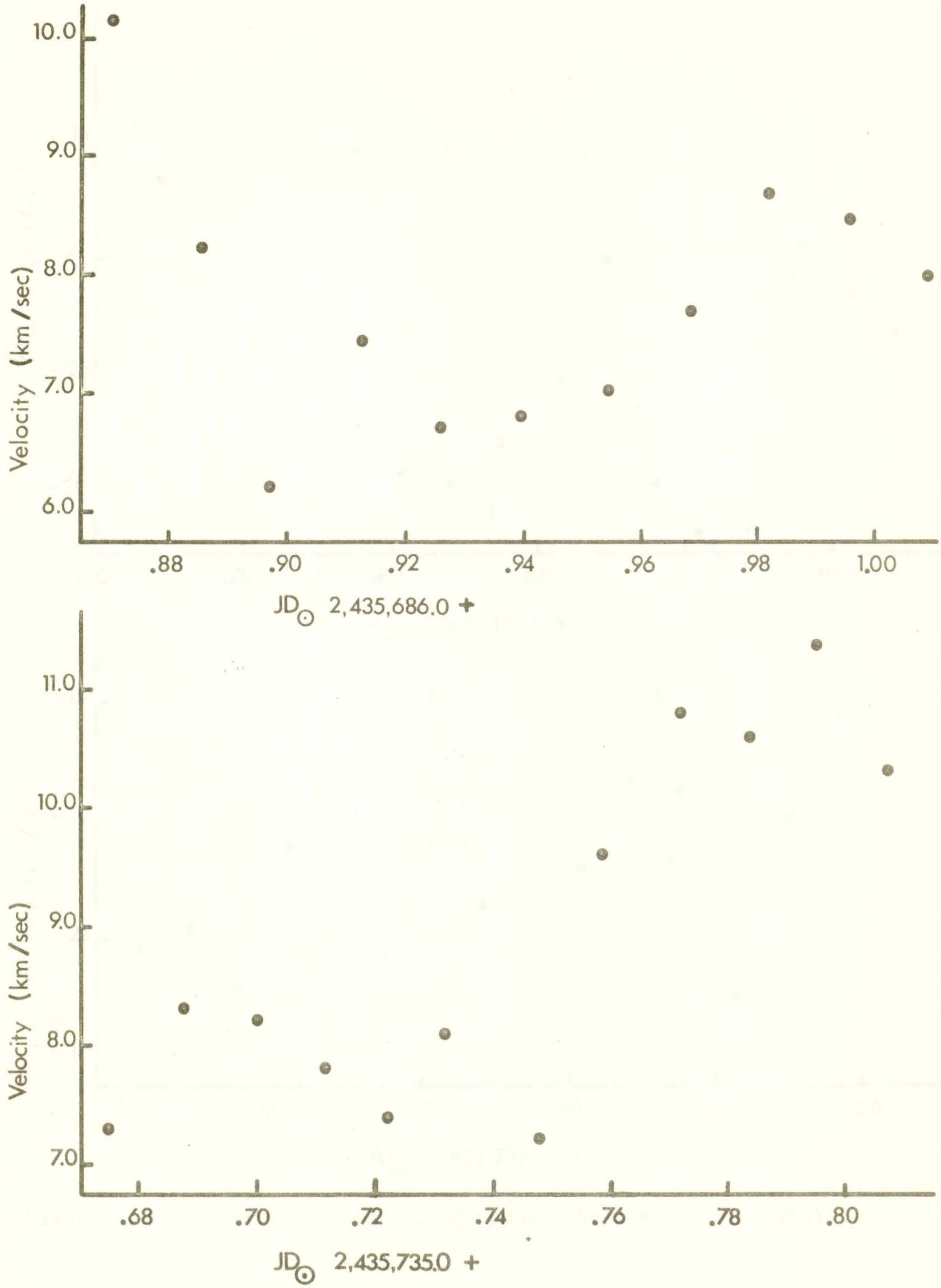

Figure 4.1 Velocity Curves of δ Delphini (Struve, et al, 1957).

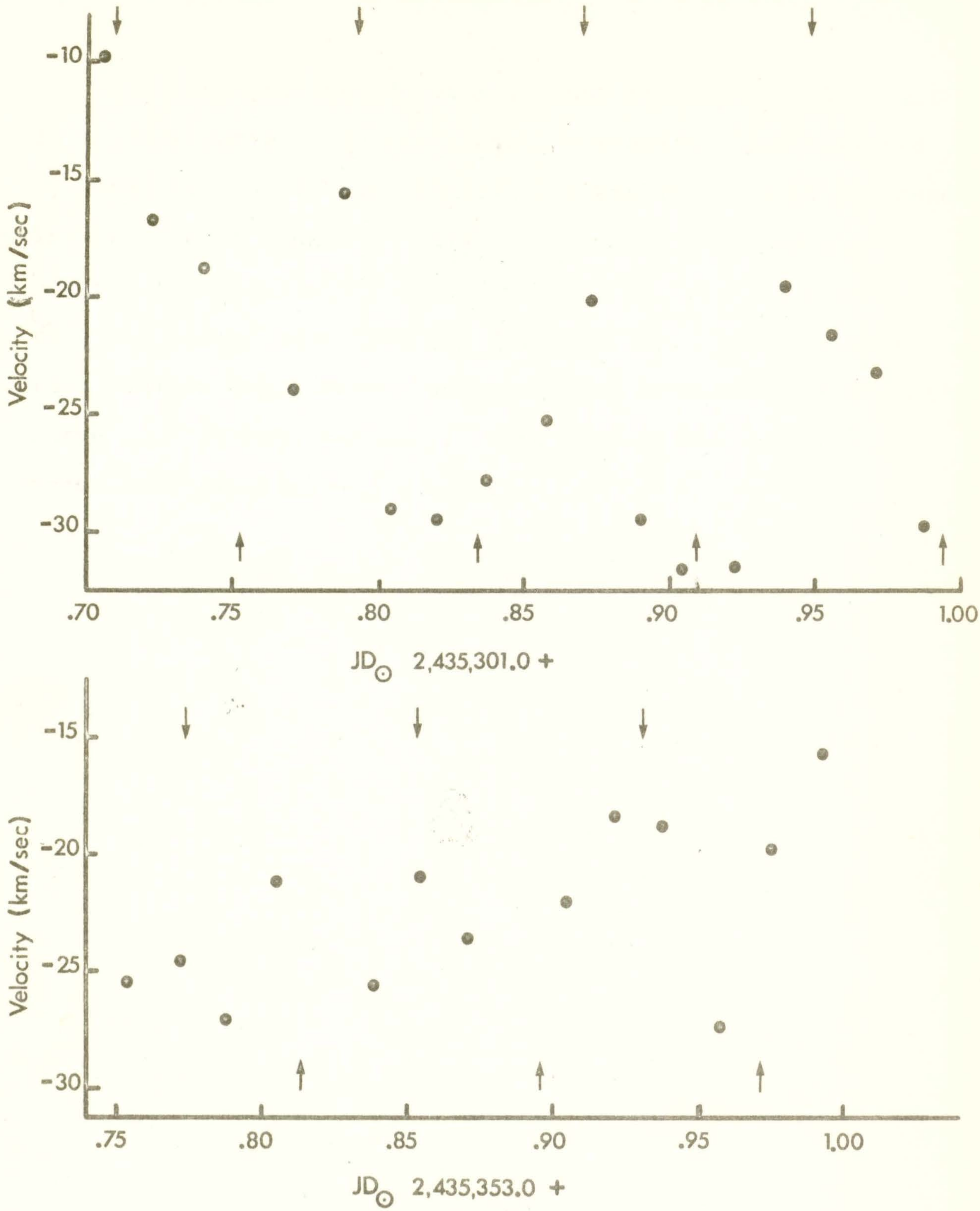

Figure 4.2 Velocity Curves of DQ Cephei (Sahade, et al, 1956).

The photometric observations of 1969 May 21 result in a light curve which has a similar shape to that obtained by Danziger and Dickens (1967). The amplitude of $0^{m}.035$ found is the largest found for this star so far. Maximum light occurs at approximately 1969 May 21.23 (U.T.), while maximum velocity occurs near 1969 May 21.29 (U.T.). This suggests that maximum light occurs on the rising branch of the velocity curve near minimum velocity. This puts 20 CVn in the same group as δ Delphini and ρ Puppis.

APPENDIX A

THE COMPUTER PROGRAM FOR THE REDUCTION OF RADIAL VELOCITIES

In its original form this program was written by Dr. D. Crampton of the Dominion Astrophysical Observatory, and was solely for the purpose of reducing radial velocities. Crampton's version of the program was a general one and it could be used to calculate radial velocities of a number of different stars during the same run on the computer. However, for this project radial velocities of only two stars were being measured, and a few modifications were made to the original program so that standard data such as wavelengths and standard settings only had to be read in once, since these were the same for both stars. Co-ordinates of each star also had to be read in only once each.

This basic program was later modified further by the author to calculate the parameters for performing the t-test (Chapter 2.4.1) and to calculate the variance ratio in the analysis of variance test (Chapter 2.4.2).

'Comment' cards within the program itself describe the operations performed.

The data are read in the following form:-

Block 1:- Curvature correction, the number of comparison lines and the number of stellar lines for which standard data are to be read in (Note: Curvature correction = 0.0 km/sec for spectra taken using a grating).

Block 2:- (a) Wavelengths of all comparison lines;

(b) Standard settings of these comparison lines;

(c) Wavelengths of all stellar lines;

(d) Standard settings of the stellar lines;

(e) rVs factors for the stellar lines.

Block 3:- HD number (or some other identification), right ascension hours, right ascension minutes, declination degrees, declination minutes.

All the above need only be read in once. Thereafter for each plate:-

Block 4:- Plate number, time of observation (hours and minutes), hour angle hours, hour angle minutes (both positive for west), longitude of the sun (degrees and minutes, for 1950), and precession for that year.

Block 5:- Number of comparison lines measured, number of stellar lines measured, correction for reducing the velocity to the sun (if this is read in as 0.0 the correction will be calculated), and check (a parameter which is zero if there is another plate on the same night still to be reduced; negative if this is the last plate for a particular night but there is another night's observations still to be reduced; positive if there is no more data after this plate.).

Block 6:- Observed settings of comparison lines.

Block 7:- Observed setting and weight of each stellar line.

Note: Each data block starts on a new card irrespective of where the last block finished.

The final version of the program is listed below.

```
C      RADIAL VELOCITY REDUCTION PROGRAM
C     THE FIRST PART OF THE DATA IS THE STANDARD SETTINGS FOR EACH
C     DISPERSION AND SPECTRAL CLASS.  THE VERY FIRST CARD OF DATA MUST
C     GIVE THE CURVATURE CORRECTION  IE 0.0 FOR GRATING PLATES
C     THE WAVELENGTHS MUST BE READ IN IN ORDER
C     CW=COMPARISON WAVELENGTH, CC,ITS STANDARD SETTING
C     SW=STELLAR WAVELENGTH, SC,ITS SETTING, RVS,ITS RVS FACTOR
C     IA=INDEX NO OF CW, CO,ITS OBSERVED SETTING
C     IO=INDEX NO OF SW,SO THE OBSERVED SETTING, WT THE ASSIGNED WEIGHT
C     FIRST CARD FOR EACH PLATE CONTAINS HD, RIGHT ASCENSION AND DEC.,
C     THE HD NUMBER AND COORDINATES ONLY HAVE TO BE READ IN ONCE
C     SECOND CARD, PLATE NUMBER, U.T. , HOUR ANGLE (+VE FOR STARS WEST)
C     LONGITUDE OF SUN AND PRECESSION FOR THAT YEAR  ( -41.9 USUALLY)
C     THIRD CARD, THE NUMBER OF COMPARISON LINES ,N,  AND THE NUMBER
C     OF STAR LINES M, CORRECTION FOR REDUCTION TO THE SUN, CHECK
C
      DIMENSION X(200),Y(200),C(8),A(200,8),CW(200),CC(200),DEX(200),
     1HVEL(200),SW(200),SC(200),RVS(200),IA(200),CO(200),OSW(200),SO(20
     20),RV(200),SET(200),VEL(200),D(200),YY(200),VELL(200),DEL(200),
     3WT(200),WTT(200),HD(3),AY(200),ARR(1000),IPIV(8),AUX(16),PN(2),IO(
     4100),SDEX(200),ALLVEL(100,12),SIG(100),PLMEAN(100),B(5),AAA(100,12
     5),RES(100,12)
      READ(5,99) APN,NC,NS
   99 FORMAT(F5.2,I4,I4)
      READ(5,100)(CW(J),J=1,NC),(CC(J),J=1,NC),(SW(K),K=1,NS),(SC(K),K=1
     1,NS),(RVS(K),K=1,NS)
  100 FORMAT(16F5.1)
    1 READ(5,2)HD,RTH,RTM,DECD,DECM
    2 FORMAT(3A4,F3.0,F5.1,F5.0,F3.0)
 7000 IPLNO=0
      POPME=0.
      I111=0
      BANG=0.
      WATE=0.
    3 READ(5,4)PN,GCTH,GCTM,HRH,HRM,SUND,SUNM,SUNC,APREC,N,M,SOL,CHECK
    4 FORMAT(2A3,F4.0,F3.0,F5.0,F3.0,F5.0,F5.1,F5.0,F7.1/2I3,F8.2,I3)
C     CHECK NOT ZERO, THEN END OF DATA. START STATISTICAL ANALYSIS.
C     CHECK NEGATIVE, ANOTHER NIGHT'S OBSERVATIONS TO COME.
      READ(5,104)(IA(J),CO(J),J=1,N)
  104 FORMAT( 8(I2,1X,F4.1,3X))
      READ(5,102)(IO(L),SO(L),WT(L),L=1,M)
  102 FORMAT( 8(I2,F5.1,F2.0,1X))
      WRITE(6,1001)HD,PN
 1001 FORMAT(1H1 , 3A4,5X,13HPLATE NUMBER ,2A3)
      WRITE(6,2000)GCTH,GCTM
 2000 FORMAT(1H ,2X,'UT=',F4.0,F3.0)
      IF(SOL)440,400,440
  440 WRITE(6,441)SOL
  441 FORMAT(26H0REDUCTION TO SUN GIVEN AS ,F7.2//)
      RED=SOL
      GO TO 450
C
C     COMPUTE REDUCTION TO SUN
C
  400 RTA=(RTH*15.+RTM/4.)/57.2958
      IF(DECD) 407,408,408
  407 DECM=-DECM
  408 DEC=(DECD+DECM/60.0)/57.2958
```

```
      GCT=GCTH+GCTM/60.
      SUN=SUND+SUNM/60.
      SUNP=(SUN+GCT*SUNC/3600.+APREC/60.)/57.2958
      IF(HRH) 409,412,412
 409  HRM=-HRM
 412  HRA=(HRH*15.+HRM/4.)/57.2958
      EPS=0.409199
      PHI=0.762011
      PERI=1.76662
      SBET=SIN(DEC)*COS(EPS)-COS(DEC)*SIN(EPS)*SIN(RTA)
      BETA=ATAN(SBET/SQRT(1.-SBET**2))
      CCELON=(COS(DEC)*COS(RTA))/COS(BETA)
      SCELON=(SIN(DEC)*SIN(EPS)+COS(DEC)*COS(EPS)*SIN(RTA))/COS(BETA)
      CELON=ATAN(SCELON/CCELON)
      IF(SCELON) 401,402,402
 401  IF(CCELON) 403,404,404
 402  IF(CCELON) 403,406,406
 403  CELON=CELON+3.14159
      GO TO 406
 404  CELON=CELON+6.28319
 406  VELA=(29.76*SIN(SUNP-CELON)-0.500*SIN(PERI-CELON))*COS(BETA)
      VELD=-0.47*COS(PHI)*COS(DEC)*SIN(HRA)
      RED=VELA+VELD
      CELM=(CELON-CELD)*60.
      CELON=CELON*57.2958
      CELD=AINT(CELON)
      BETA=BETA*57.2958
      BETD=AINT(BETA)
      BETM=(BETA-BETD)*60.
      IF(BETD)413,414,413
 413  BETM=ABS(BETM)
 414  ICELD=INT(CELD)
      ICELM=INT(CELM)
      IBETD=INT(BETD)
      IBETM=INT(BETM)
 420  RED=RED+APN
      WRITE(6,411)ICELD,ICELM,IBETD,IBETM,RED
 411  FORMAT(7H0LAMBDA ,I4,I3,5H BETA ,2I3,2X,18HREDUCTION TO SUN =,F7.2
 4111//)

      COMPUTE OBSERVED - STANDARD SETTINGS

 450  DO 9 J=1,N
      IJ=IA(J)
      Y(J)=CC(IJ)-CO(J)
      IF (ABS (Y(J))-500.)10,12,12
  12  IF(Y(J))13,13,14
  13  Y(J)=Y(J)+1000.
      GO TO 10
  14  Y(J)=Y(J)-1000.
  10  X(J)=CW(IJ)*1.000E-3
      AY(J)=Y(J)
   9  CONTINUE
      WRITE(6,1006)(Y(I),I=1,N)
1006  FORMAT(23H0COMPARISON CORRECTIONS /(8F8.1))

      FIT A 3RD DEGREE POLYNOMIAL TO CORRECTION CURVE
```

```
   11 DO 20 J=1,N
      DO 20 I=1,4
      A(J,I)=X(J)**(I-1)
   20 CONTINUE
      CALL ARRAY(2,N,4,200,8,ARR,A)
      EPS=1.0E-7
      CALL LLSQ(ARR,AY,N,4,1,C,IPIV,EPS,IER,AUX)
      WRITE(6,1002)C(1),C(2),C(3),C(4)
 1002 FORMAT(26HOLEAST SQUARE COEFFICIENTS /4X,4E13.5)
      SSR=AUX(1)
      BN=N
      ERMS=SQRT(SSR/(BN-4.))
      WRITE(6,2222)ERMS,IER
 2222 FORMAT(1X,5HSIGMA  ,F7.2,6X,4HIER  ,I3)
C     IF ONE COMPARISON LINE IS MORE THAN 3 SIGMA FROM LINE IT IS REJECTED
      DO 117 L=1,N
      XX=C(1)+C(2)*X(L)+C(3)*X(L)**2+C(4)*X(L)**3
      SDEX(L)=XX-Y(L)
      DEX(L)=ABS(SDEX(L))
      IF(DEX(L)-3.*ERMS)117,117,16
  117 CONTINUE
      IF(ERMS-5.) 19,118,118
  118 DO 119 L=1,N
      IF(DEX(L)-1.5*ERMS)119,119,16
  119 CONTINUE
      DO 120 L=1,N
      IF(DEX(L)-10.0)120,120,16
  120 CONTINUE
      GO TO 19
   16 N=N-1
      WRITE (6,1102)L
 1102 FORMAT(1X,I3,25H COMPARISON LINE REJECTED  )
      IF(L-N) 18,18,11
   18 DO 17 I=L,N
      X(I)=X(I+1)
      Y(I)=Y(I+1)
   17 CONTINUE
      DO 15 I=1,N
   15 AY(I)=Y(I)
      GO TO 11
   19 LN=0
      WRITE(6,1103)(SDEX(L),L=1,N)
 1103 FORMAT(10HORESIDUALS /(8F8.1))
C
C     COMPUTE LINE VELOCITIES
C
      IPLNO=IPLNO+1
      DO25 L=1,M
      K=IO(L)
   30 RV(L)=RVS(K)
      X(K)=SW(K)*1.000E-3
      YY(L)=C(1)+C(2)*X(K)+C(3)*X(K)**2+C(4)*X(K)**3
      SET(L)=SO(L)+YY(L)-1000.
      IF(SO(L)-1000.)38,38,41
   41 SET(L)=SET(L)+1000.
   38 IF(SET(L))37,36,36
   37 SET(L)=SET(L)+1000.
      GO TO 38
```

```
   36 D(L)=SET(L)-SC(K)
      IF(D(L))40,40,39
   40 D(L)=D(L)+1000.
   39 VEL(L)=D(L)*RV(L)*1.0E-03
      HVEL(L)=VEL(L)+RED
      OSW(L)=SW(K)
      IF(WT(L ))65,25,65
   65 LN=LN+1
      WTT(LN)=WT(L)
      VELL(LN)=HVEL(L)
   25 CONTINUE
      WRITE(6,1003)
 1003 FORMAT(1H0,50X,8HVELOCITY/66H WAVELENGTH SETTING  CORR SETTING DIS
 10031PL    RVS      GEO     HELIO     WT     )
      WRITE(6,1004)(OSW(L),SO(L),YY(L),SET(L),D(L),RV(L),VEL(L),HVEL(L),
     1WT(L),L=1,M)
 1004 FORMAT(3X,F7.1,2X,F6.1,F7.1,1X,F6.1,1X,F6.1,1X,F6.0,2X,F6.1,2X,
 10041F6.1,F4.0)
C
C     COMPUTE MEAN VELOCITY AND SIGMA, DISCARD VELOCITIES GREATER THAN
C     2.50 SIGMA FROM MEAN, COMPUTE NEW MEAN,SIGMA AND S.E. OF MEAN
C     N.B.   NO LINES SHOULD BE REJECTED WHEN DOING THE STATISTICAL ANALYSI
C
      NO=0
  620 M=LN
  622 TOT=0.
      SVL=0.
      SWT=0.
      IF(M-1) 511,511,90
  511 ISWT=WTT(1)
      SVL=VELL(1)
      AVE=SVL
      SM=0.
      GO TO 512
   90 DO 75 LN=1,M
      SVL=SVL+VELL(LN)*WTT(LN)
      SWT=SWT+WTT(LN)
   75 CONTINUE
      AVE=SVL/SWT
      DO 76 LN=1,M
      DEL(LN)=ABS (AVE-VELL(LN))
      TOT=TOT+WTT(LN)*DEL(LN)**2.
   76 CONTINUE
      SM=M
      S=SQRT(TOT/(SM-1.))
      SM=S/(SQRT(SWT))
      ISWT=SWT
  512 WRITE(6,1010)M,ISWT,AVE,SM,S
 1010 FORMAT(8H0MEAN OF ,I3,10H LINES, WT ,I3,2H = ,F6.1,3H ME,F5.1,
 101014H SIG, F5.1)
      PLMEAN(IPLNO)=AVE
      L=0
      IF(NO)9998,77,9998
   77 NO=NO+1
      NUM=0
      DO 80 LN=1,M
      IF(DEL(LN)-2.5*S)78,79,79
   79 WRITE(6,1011)VELL(LN)
```

```
 1011 FORMAT(9X,F7.1,9H REJECTED )
      WTT(LN)=0.
   78 ALLVEL(IPLNO,LN)=VELL(LN)
      I112=IFIX(WTT(LN))
      I111=I111+I112
   80 CONTINUE
      DO 9997 I=1,M
      POPME=POPME+ALLVEL(IPLNO,I)
      RES(IPLNO,I)=ABS(PLMEAN(IPLNO)-ALLVEL(IPLNO,I))
      BANG=BANG+RES(IPLNO,I)**2.
      WATE=WATE+1.
 9997 CONTINUE
      GO TO 622
 9998 IF(CHECK.EQ.0.)GO TO 3
C
C     START OF STATISTICAL ANALYSIS
C     THIS IS DONE SEPARATELY FOR EACH NIGHT'S OBSERVATIONS
C     CALCULATE POPULATION MEAN AND DO T-TEST.
C
      WRITE(6,7010)
 7010 FORMAT(1H1,3X,43H WAVELENGTH RESIDUAL VARIANCE    T    DEGREES)
 9000 POPME=POPME/WATE
      DO 9010 IABC=1,M
      STARME=0.
      DO 9008 IDEF=1,IPLNO
      STARME=STARME+ALLVEL(IDEF,IABC)
 9008 CONTINUE
      STARME=STARME/IPLNO
      ESS=0.
      DO 9009 IGHI=1,IPLNO
      RESLIN=ABS(STARME-POPME)
      APE=ABS(RES(IGHI,IABC)-RESLIN)
      ESS=ESS+APE**2.
 9009 CONTINUE
      ESS=(ESS/(IPLNO-1))**0.5
      TEE=((STARME-POPME)*(IPLNO-1)**0.5)/ESS
      TEE=ABS(TEE)
      MUNCH=IPLNO-1
      A111=STARME-POPME
      WRITE(6,7020)OSW(IABC),A111,ESS,TEE,MUNCH
 7020 FORMAT(1H ,5X,F7.1,3X,F6.2,3X,F6.2,2X,F6.2,3X,I2)
 9010 CONTINUE
C
C     ESTIMATE OF VARIANCE.
C
      WRITE(6,9001)POPME
 9001 FORMAT(1H1,'POPULATION MEAN = ',F6.2)
      FINK=0.
      DO 9070 IWXY=1,IPLNO
      CRUNCH=ABS(PLMEAN(IWXY)-POPME)
      FINK=FINK+CRUNCH**2.
 9070 CONTINUE
      M=12
      SUCCES=(FINK*M)/(IPLNO-1)
      ATLAST=BANG/(WATE-IPLNO)
      WRITE(6,9080)SUCCES,ATLAST
 9080 FORMAT(1H0,'ESTIMATE OF VARIANCE BETWEEN PLATES = ',F8.4//' ESTIMA
     1TE OF VARIANCE WITHIN A PLATE = ',F8.4)
```



```
      VARRAT=SUCCES/ATLAST
      WRITE(6,9090)VARRAT
 9090 FORMAT(1H0,'VARIANCE RATIO = ',F8.4)
      IF(CHECK)7000,9999,9999
 9999 CALL EXIT
      END
```

APPENDIX B

NOTES ON THE BROWER, GRANT AND ZEISS OSCILLOSCOPE MEASURING MACHINE

This instrument displays, on an oscilloscope screen, the profiles of the line being measured and its mirror image. To set on a particular line the two profiles have to be made to coincide as nearly as possible. This is done by moving the spectrogram in the direction of the dispersion. Descriptions of the instrument have already been given by Laskarides (1970), and by Niehaus (1970), and as a result will not be repeated here. A brief discussion of the advantages and disadvantages of this instrument as applied to this research project follows.

Advantages

(i) Because of the large number of spectra to be measured it was necessary to find a fast, efficient way of measuring all the spectra. The B. G. & Z. machine provides just this.

(ii) This device enables the profiles of lines to be seen. This was a great help in deciding upon the acceptability of lines during the initial stage of choosing a set of lines.

(iii) Because of the nature of the machine it is necessary to measure the spectrogram in one direction only, whereas visual methods require a measurement in both directions along the spectrum. Use of the B. G. & Z. shortens both the measuring and calculating times since no average readings have to be calculated.

(iv) This method is also much easier on the measurer's eyes than visual techniques, and this results in a further speeding up in the measuring process. As many as twenty spectra can be measured in one day without difficulty.

Disadvantages

(i) One problem is to decide how to match the lines. The aim should be to obtain settings which are as close as possible to the 'visual' settings, i.e. the settings obtained if the spectra are measured visually. This generally meant that the core of the lines was being used. In some cases, the stronger lines (eg. $H\delta$) had no well-defined core, and the wings had to be matched. Examples of some settings are given in Figure B1.

(ii) Grain effects in the photographic emulsion can have some effect on the exact position and shape of the minima of the stellar absorption lines. Although this effect is very small, it can be avoided by not using the extreme minima of lines in matching their profiles.

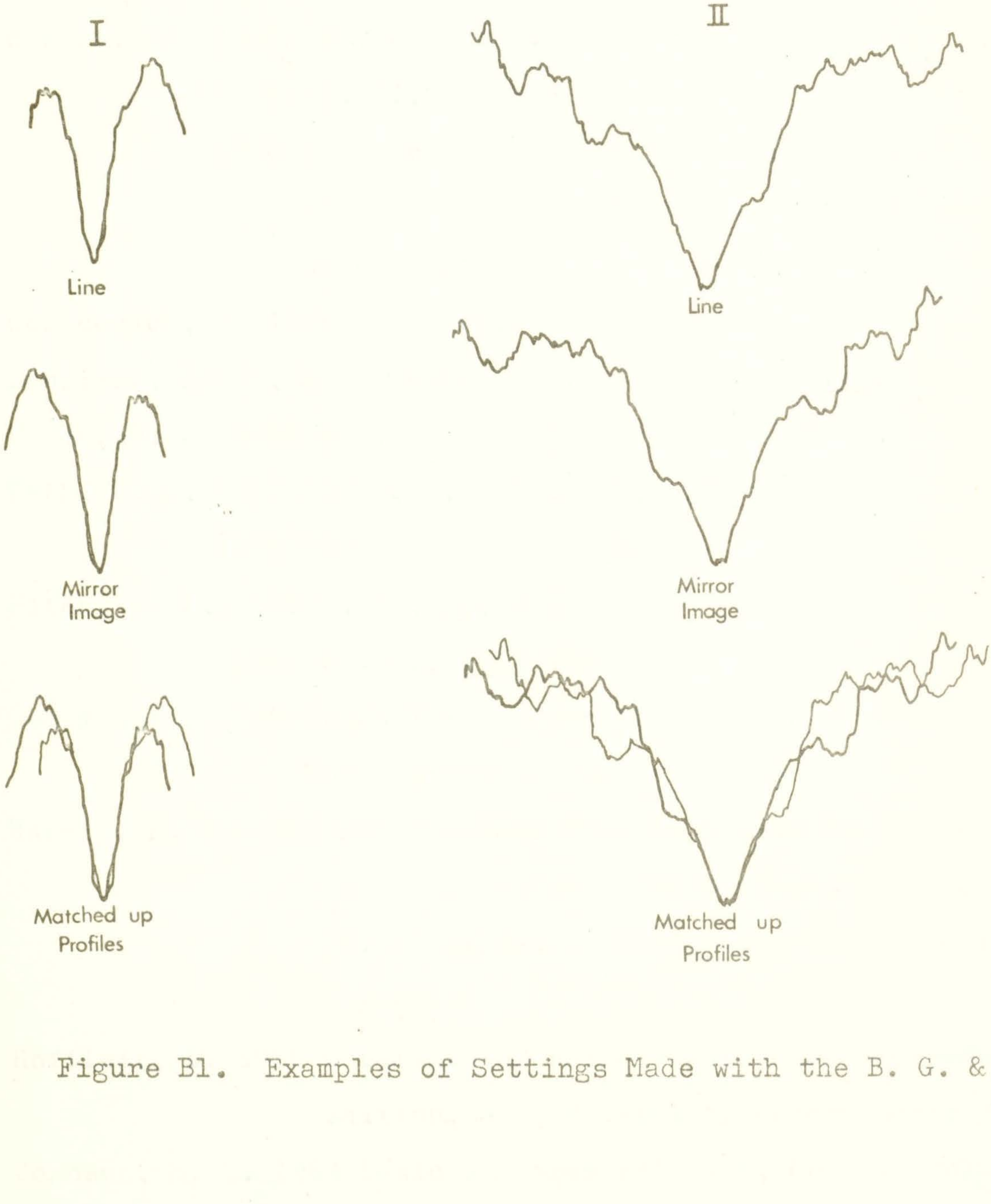

Figure B1. Examples of Settings Made with the B. G. & Z.

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