ACCEPTED
LTY OF GRADUATE STUDIES
[redacted]
DEAN
92/06/27

On Modelling Technological Progress

by

Mark Lynch
B.A., University of Victoria, 1987

A Thesis Submitted in Partial Fulfilment of the
Requirements for the Degree of

MASTER OF ARTS

in the Department of Economics

We accept this thesis as conforming
to the required standard

[redacted]

Dr. S. Nadeau, Supervisor (Department of Economics)

[redacted]

Dr. D. Ferguson, Departmental Member (Department of Economics)

[redacted]

Dr. K. Stewart, Departmental Member (Department of Economics)

[redacted]

Dr. R. Davidson, Outside Member (Department of Mathematics and Statistics)

[redacted]

DR. O. NIEMANN, External Examiner (DEPARTMENT OF GEOGRAPHY)

University of Victoria

Supervisor: Dr. Serge Nadeau

ABSTRACT

Traditional models of output growth have accounted for the impact of technological change under the assumption that technological change is "neutral" to the production process. Technological progress is considered neutral when the relationship between the marginal product of some input and some other economic variable remains unchanged. Three types of technological neutrality have been employed in past studies: Hicks neutral, Harrod neutral and Solow neutral. However, there has been no convincing argument why a tendency for each individual type of neutrality should exist. This thesis develops a direct demand function associated to a Diewert cost function to test for the presence of Hicks, Harrod or Solow technological change using U.S. business investment data for the period 1934 through 1980. The model employs technological paramaters which exclude the a priori assumption that technological progress is Hicks, Harrod or Solow neutral. Maximum likelihood functions are generated in order to conduct a log likelihood ratio test to determine which type of technological neutrality is present over the series tested. The empirical model fails to identify the type of technological neutrality present. The thesis concludes that future research pertaining to technological progress should incorporate parameters which model technological progress as a stochastic event over time.

Examiners: [redacted]

Dr. S. Nadeau, Supervisor (Department of Economics)

[redacted]

Dr. D. Ferguson, Departmental Member (Department of Economics)

[redacted]

Dr. K. Stewart, Departmental Member (Department of Economics)

[redacted]

Dr. R. Davidson, Outside Member (Department of Mathematics and Statistics)

[redacted]

External Examiner

TABLE OF CONTENTS

TITLE PAGE i

ABSTRACT ii

TABLE OF CONTENTS iii

LIST OF TABLES v

1. INTRODUCTION 1

2. LITERATURE REVIEW 7

3. TECHNOLOGICAL CHANGE 16
    3.1 Embodied and Disembodied Technological Change 16
    3.2 Disembodied Technological Change and Neutrality 18
    3.3 Incorporating Technological Change and Duality 22
    3.4 Embodied Technological Change: Putty-Clay Technology 25

4. THE STATISTICAL MODEL 28
    4.1 Functional Form 28
    4.2 The Diewert Cost Function and Technological Change 29
    4.3. Gross Additions to Output 31
    4.4. The Time Structure of Investment Response 33
        4.4.1. Anticipated Investment Need 34
        4.4.2. Patterns of Orders 35
    4.5 Hypothesis Tests 40

5. EMPIRICAL RESULTS 42
    5.1 Data Sources 42
    5.2. The Log Likelihood Ratio Test 43
    5.3 Starting Values 44
    5.4 Lag Identification 44
    5.5 Expected Signs 45
    5.6 Estimation 46
        5.6.1. Model 1 47
            5.6.1.1. Autocorrelation 47
            5.6.1.2. Coefficients 47
            5.6.1.3. Log Likelihood Functions 48
            5.6.1.4. Model Respecification 51

- 5.6.2. Model 2 51
  - 5.6.2.1. Autocorrelation 51
  - 5.6.2.2. Coefficients 52
  - 5.6.2.3. Log Likelihood Functions 53
  - 5.6.2.4. Model Respecification 56
- 5.6.3. Model 3 56
  - 5.6.3.1. Autocorrelation 56
  - 5.6.3.2. Coefficients 57
  - 5.6.3.3. Log Likelihood Functions 58
  - 5.6.3.4. Model Respecification 61
- 5.6.4. Model 4 61
  - 5.6.4.1. Autocorrelation 61
  - 5.6.4.2. Coefficients 62
  - 5.6.4.3. Log Likelihood Functions 62
  - 5.6.4.4. Model Respecification 65
- 5.6.5. Model 5 65
  - 5.6.5.1. Autocorrelation 65
  - 5.6.5.2. Coefficients 67
  - 5.6.5.3. Log Likelihood Functions 67
- 5.7. Summary of Estimation Results 68

6. CONCLUSION 70

APPENDIX 1 74

REFERENCES 76

List of Tables

Model 1 Estimation Results............................49.

Model 2 Estimation Results............................54.

Model 3 Estimation Results............................59.

Model 4 Estimation Results............................63.

Model 5 Estimation Results............................66.

1. INTRODUCTION

As economic activity fluctuates over the business cycle, government fiscal and monetary policies interact with economic growth in an effort to influence the flow of business fixed investment. Short run policies act to counteract the business cycle. Long run policies affect the rate of growth of potential output. Combined fiscal and monetary policies affect investment decisions through changes to the rent on capital goods. In an effort to evaluate the effectiveness of future policies, the rate of economic growth is analyzed using economic theory to provide estimates of the impact of past policies on investment. Such analysis provides policy makers with knowledge regarding the effect which past policies imposed on the rate of investment. This knowledge is then incorporated in the planning process for future policies.

To aid in the formulation of future policies, economic theory is applied to construct investment models. The neo-classical theory of the firm is employed in model development. Each firm attempts to maximize profits using the least cost combination of factor inputs. We assume the firm is a cost minimizer and that we are dealing with a one good (Q), two input case. The firm will employ inputs capital (K) and labour (L) so that the marginal revenue (MR) associated to each factor input equals the marginal cost (MC) of employing the input,

$$MR_i = MC_i \tag{1}$$

where "i" represents the stream of inputs. Capital inputs are a proxy for investment. In other words, the firm will purchase capital so that the price of output (P) multiplied by the marginal product of capital (f_K) equals the user cost of capital (W_K).

$$Pf_K = W_K \tag{2}$$

Assuming the price of output remains constant, and that output increases with capital, the second derivative property states that capital will fall if the user cost of capital rises, and increase if the rate of change of output with respect to capital increases. Policies exert influence on the user cost of capital through interest rates, taxes, and the price of capital goods. Monetary policies affect investment through the interest rate. Fiscal policies affect investment through corporate profit taxes, depreciation allowance, and investment tax credits.

For policy makers to evaluate the effect of fiscal and monetary policies on investment, proper modelling of investment behaviour is required.

At any given time capital and labour are the only visible inputs to the production process. However, there exists a third factor, technological progress, which exerts an influence in the production process. Technological progress can be defined in its broadest terms as useful knowledge pertaining to the art of production. Over time, technological progress acts to obscure the contribution of capital and labour in the production process. For policy makers to disentangle the impact of policies from the impact of technological progress and for industry to determine growth in output due to increases in factor inputs from growth in output due to technological progress, modelling techniques of the production process should be developed which incorporate minimal amount of restrictive assumptions concerning the contribution of technological progress to the production process. However, this has not been the case.

Since we can only observe output and inputs in the firm's production process, we must impose restrictions on the impact of technological change on the marginal products of inputs. Marginal product (MP) is the change in total product resulting from the use of an additional unit of variable factor input. These restrictions usually involve some kind of "neutrality" (see Hahn and Mathews (1964), Solow (1957, Bischoff (1971), and Jorgenson (1966)). Technological progress is considered neutral when the relationship between the marginal product of some input and some other economic variable remains unchanged.

Three classifications of technological neutrality have been employed in past studies: Hicks neutrality, Solow neutrality and Harrod neutrality.

If the marginal products of the inputs increase in precisely the same proportions due to technological change alone, then we say that technological progress is Hicks neutral. An implication of this is that the marginal rate of substitution between inputs in the production process is not affected by technological progress. The marginal rate of substitution measures the rate at which one factor is substituted for another with output held constant.

A second approach suggests that technological change makes capital more effective. An implication of this is that the marginal product of labour is unchanged at a constant ratio of output to labour. This type of technological progress, actually known as capital-augmenting technological progress, is said to be Solow Neutral.

The third approach argues that technological progress augments the effectiveness of the labour stock in a manner analogous to the capital-augmenting Solow-neutral change. This is the so-called labour augmenting case, and technological change is then said to be Harrod neutral.

Although the assumption of neutral technological change has been made as either Hicks, Harrod, or Solow, there is no convincing argument why a tendency for each individual type of neutrality should exist. As technology changes, it influences the profitability of existing methods of producing goods and the pace in which the firm will seek to replace their existing plant and machinery. If technological progress is Hicks neutral, the firm will continue to invest in the same ratio of capital and labour inputs. If technological progress is Harrod neutral, the firm will substitute labour for capital. If Solow neutral, the firm will substitute capital inputs for labour. Since the type of neutrality assumed has an impact on investment and the demand for inputs into the production process, it is imperative that the analyst's assumption of neutrality be valid. Large erroneous conclusions could be reached if the wrong type of neutrality is assumed. Policy makers who incorporate the wrong assumptions regarding the type of technological neutrality may recommend policy tools inappropriate to meet desired needs with unexpected and unwanted results.

The objective of this thesis is to develop an econometric model to test for the presence of Hicks, Harrod or Solow neutral technological change without the a priori restriction of a specific type of technological change. The approach applies microeconomic theory, exploiting the properties of the indirect factor demand functions associated with the Diewert (generalized Leontief) cost function. The perceived advantages of this approach are that it allows the distinction between Hicks-neutral, Harrod-neutral, and Solow-neutral technological progress. Furthermore, the function carries no restrictive qualities which pre-determine the type of technological progress present. Ultimately, this approach will allow the identification of the type of technological progress that best describes reality, providing policy makers with modelling techniques which will refine policy development for the future.

This chapter has outlined the problem of modelling technological change. Chapter two presents a summary of past studies of technological change and their implications on modelling techniques and investment behaviour. The theoretical background underlying the modelling of technological change is presented in Chapter three. Chapter four develops the empirical model to be used in the estimation of the type of technological change. Empirical results are presented in Chapter five along with a description of the testing processes used. Chapter six presents the conclusions derived from the modelling process and suggests areas for further study.

2. LITERATURE REVIEW

In order to develop our model of capital investment allowing for the type of technological progress which best represents reality, a brief summary of past studies is required.

Historically, the problem of estimating the rate and type of technological progress has paralleled the evolution of models of economic growth. In general, economists tend to agree that capital investment is a primary requisite for economic growth. On the other hand, technological progress, although regarded as one of the important causes of economic growth, drew little attention, being regarded as an exogenous variable beyond the scope of economics. However, three distinct approaches to modelling technological change have been utilized by economists.

The first approach utilizes models where technological change is considered a residual factor. For example, early economic models (such as the Harrod-Domar model) were concerned with modelling economic growth in a steady state, or long-period equilibrium context. In steady state growth the rate of growth of all the relevant variables remains constant over time (see Hahn and Mathews (1964)). Technological growth was considered a residual factor, after growth due to capital and labour inputs were accounted for in the modelling process.

The pioneering work of Solow in measuring neutral technological progress drastically changes the situation (see Solow (1957)). Setting the coefficient of neutral technical change in 1909 equal to one, Solow finds that the level of technology in 1949 has advanced more than 80 percent over the series tested. Solow concludes that seven eighths of output increase was attributable to technological change. The residual is due to capital investment.

For economists who had been long acquainted with the theory that investment is one of the major factors of economic development, Solow's results are unacceptable. Several attempts were tried to restore investment as the key element to economic growth. However, as similar results were attained by Abramovitz (1956), Aukrust (1959), Fabricant (1956) and Kendrick and Massel (1960), the focus shifts from the simple concept of investment to the argument that all technological progress takes place through improvements that are embodied in new capital equipment. Embodied technological progress can be defined as capital investment in new equipment. Disembodied technological progress is defined as capital investment undertaken to upgrade used equipment in order to make it more productive. It is impossible to separate the contribution of capital from that of technical change to output growth, for the two may be interacting.

The thesis of embodied technological progress emerged as the product of this argument. New models of capital accumulation are now required to account for the contribution of technological change over time.

In response to the theory of embodied technological change, Solow incorporates the concept that investment (capital) of different vintages has attached a unique embodied technology. He incorporates this concept into an aggregate production function. Solow assumes that embodied technological progress takes place at an exponential rate (see Solow (1959)), holding the labour-output ratio constant. Solow observes that if the rate of embodied technological change is known, then the remaining parameters can be estimated through regression analysis.

Solow tests the validity of the assumption of neutral technological progress, attempting to distinguish a change in output due to technological change from a change in output due to an increase in the factor inputs. Postulating that output is related to inputs, capital, labour, and time, Solow incorporates a Cobb-Douglas production function with constant returns to scale to test for increases in output due to technological change. The production function tested was of the following form:

$$Q = e^{at} K^{1/2} L^{1/2} \tag{3}$$

where the variable "a" represented technological growth, encompassing any kind of shift in the production function. Assuming that capital and labour are paid the value of their marginal products, Solow attributes the residual of total output not paid to the factor inputs, capital and labour, to the growth of technology over time. Solow concludes that technological change, on average, was neutral.

Thirwall (1978) makes the following critique about Solow's methodology. First, since only one combination of factor inputs can be observed at any one time, there is a problem in identifying shifts in the production function from movements along the function unless the assumption of neutral technological progress is made prior to estimation. Since technological change may not be neutral, Solow's results may be biased. Second, there is an implicit assumption that technological progress may be independent of increases in factor inputs. This implicit assumption is questionable. Finally, the Cobb-Douglas function contains the restrictive property of a constant elasticity of substitution between factor inputs (i.e. by assuming a Cobb-Douglas production function, there is an implicit assumption that technical progress is both Hicks and Harrod neutral).

Consequently, growth accredited to technical progress using the Cobb-Douglas production function may not be technical progress at all (i.e. a shift of the production function) but the result of any one or more of a number of factors (substitution of capital for labour, economies of scale, organization improvements and resource shifts from one production process to another). Thirwall concludes that empirical tests of neoclassical growth theory using a Cobb-Douglas production function with its subsequent restrictions are suspect in their representation of reality.

In order to establish how much investment is needed to support the rate of output growth, the rates of both embodied and neutral technological progress needs to be evaluated. Solow's models are incapable of considering such a case, since one type of change is assumed away in each of his models. Jorgenson (1966) constructs a model which encompassed both embodied and disembodied technological change. After estimation, Jorgenson argues that it is statistically impossible to distinguish between embodied and disembodied technological change.

The second approach to the problem of identifying the type of technological progress is to postulate a distinct type of technical progress and to test the relationship between certain variables that this type of technical progress implies (see Jorgenson's (1966)).

For example, technical progress is said to be Hicks-neutral when the relationship between the marginal rate of substitution and the factor proportion remains unchanged. Thus, one way of testing if technical progress is Hicks-neutral is to test the stability of the relationship: marginal rate of substitution - factor proportion over time.

Using this methodology, Beckmann and Sato (1969) develop several models of neutral technological progress and test relationships between economic variables implied by each type of technical progress specified;

i) Product augmenting technological progress; where the product is increased in proportion to the output that would be obtained in the absence of technical change (Hicks neutrality).

ii) Labour augmenting technological progress; where the product is increased in proportion to the augmentation of labour measured in efficiency units (Harrod neutrality).

iii) Capital augmenting technological progress; where the product is increased in proportion to the augmentation of capital as measured in efficiency units (Solow neutral).

iv) Input decreasing technological progress; where the product decreases as a function of time.

Beckmann and Sato also produced a form of technical progress which is not invariant with respect to time;

v) Factor augmenting technological change; where the product increases as a result of increased efficiency in the factors of production.

These various definitions of technical neutrality are tested by Beckmann and Sato using data on the United States, Japan and Germany. They conclude that technical progress was Solow neutral for the sample period tested. However, by specifying the equations as linear or log linear, Beckmann and Sato in the same instance specify the form of the production function. In every case tested, the implied production function is either a modification of the Cobb-Douglas or Constant Elasticity of Substitution production function, with their inherent restrictive qualities (noted previously in the discussion related to Solow's study).

The major problem with postulating a type of technical progress and testing the relationship between economic variables inherently implied is that it does not allow for an easy identification of the type of technical progress that better describes reality. The arguments of relationships among economic variables are different depending on the type of technical progress tested.

The third approach to modelling technological change employs the concept of nonneutral technological change. Technological progress is extensively analyzed in the highly capital intensive energy industry. The role of technological progress as a means of extending energy resources led to several studies of the estimation of nonneutral technological change. Studies by Binswanger (1964), Bernt and Khaled (1987), and Bernt and Woods (1987) provide estimates of biased technological change. All these studies have two features in common. First, to represent the rate at which new technology is introduced, a time trend is employed. Secondly, in classifying the direction of technological change, these studies employ the definitions of technological change as followed by Hicks and Solow. In other words, an a priori assumption regarding the nature of technological change is made which predetermines the nature of technological change present in these studies. Another drawback of the Hicks and Solow definitions of technological change is that estimation reveals nothing about the effects of technological change on the demand for any given factor. Results from studies utilizing Hicks or Solow definitions of technological change could be interpreted in several different fashions, often providing misleading results. For example, results from the aforementioned studies indicate that technological change increases the use of energy, technological change has no impact on the use of energy, or that technological change reduces the use of energy.

In response to the drawbacks stated above concerning a priori assumptions regarding technological change in energy intensive industries, Nelson (1987) incorporated a translog functional form to classify the direction of technological change in relation to inputs into the energy production industry. Incorporating a technological index into the cost-share equations, Nelson uses the resulting coefficients from estimation to classify the direction of technical change on each input. The methodology employed by Nelson has no a-priori assumptions regarding the type of technological change present.

In any case, there is no clear argument why technical progress should be modelled as Hick's, Harrod, or Solow neutral. The assumption of neutral technological change is attractive because it is consistent with steady-state economic growth.

The purpose of this study is to examine a fourth approach to the problem of estimating the rate and type of technical progress. The methodology employed assumes technological progress occurs in a form which is exogenous to the production process. The approach applies microeconomic theory, exploiting the properties of the direct factor demand functions associated with the Diewert (generalized Leontief) cost function (see Diewert (1971)). The perceived advantages of this approach are that it allows the distinction between Hicks-neutral, Harrod-neutral, and Solow neutral technical progress. Furthermore, the function carries no restrictive qualities which pre-determine the type of technological change present.

3. TECHNOLOGICAL CHANGE

3.1 Embodied and Disembodied Technological Change

A distinction is made conventionally between technological change that is embodied (in new equipment) and that which is disembodied (in new and old equipment).

Technological change is considered disembodied if, independent of any changes in the factor inputs, the isoquant contours of the production function shift inward towards the origin as time passes. Even if the inputs of capital and labour are fixed, the maximum output that can be produced from the unchanging factor inputs increases over time as a result of disembodied technological change. This increase in output can be attributed to improvements in technique or organization that enhance the productivity of old equipment along with new.

Disembodied technological change implies a production function of the form;

$$Q(t) = F(K(t), L(t), t) \tag{4}$$

Many inventions or improvements in technique can be introduced only by building new capital equipment of improved design, or by employing new labour of enhanced skill. Investment in new capital equipment or new skills is the essential carrier of improvements in technique; old equipment is left unaltered. This is generally referred to as embodied technological progress.

Embodied technological progress implies a production function of the form;

$$Q(t) = \int_{-\alpha}^{t} Q(v,t)dv \tag{5}$$

where

$$Q(v,t) = F(K_v(t),\ L_v(t),\ v) \tag{6}$$

and where Q(v,t) denotes the amount of output produced on machines of vintage v at time t; $K_v(t)$ is the amount of capital of vintage v still in operation at time t; and $L_v(t)$ is the amount of labour assigned to $K_v(t)$. In other words, embodied technological progress implies that to machines of different vintage is attached a different production function.

3.2 Disembodied Technological Change and Neutrality

This section examines the various definitions of neutrality in the context of disembodied technological progress.

The factor augmenting representation of disembodied technological change is;

$$Q(t) = F[K(t), L(t), t] = G[\alpha_K(t)K(t), \alpha_L(t)L(t)] \tag{7}$$

where α_K and α_L are functions of time alone, and where G is a production function homogeneous of degree one [1]. We may identify $\alpha_K(t)K(t)$ and $\alpha_L(t)L(t)$ as "effective capital" and "effective labour," respectively. Technological change is said to be purely labour augmenting if $\alpha_K(t) = 1$ and $\alpha_L(t) > 0$, whereas it is purely capital augmenting if $\alpha_K(t) > 0$ and $\alpha_L(t) = 1$. It is equally capital and labour augmenting if $\alpha_K(t) = \alpha_L(t)$.

Using the price of output as numeraire and assuming that the holders of capital are profit maximizers, we have that the real wage rate is

---

[1] This section is very much based on Section 2.1 of Burmeister and Dobell (1970).

$$W(t) = G_L[\alpha_K(t)K(t),\ \alpha_L(t)L(t)]\ \alpha_L(t) \tag{8}$$

and that the real rental rate of capital is

$$R(t) = G_K[\alpha_K(t)K(t),\ \alpha_L(t)L(t)]\ \alpha_K(t). \tag{9}$$

Now, consider the index N defined by

$$N \equiv \left(\frac{RK}{WL}\right) * \frac{\partial\left(\frac{WL}{RK}\right)}{\partial t}\Big|_P \tag{10}$$

where RK is the return to capital owners and WL is the return to labour, and where this notation indicates that the time derivative is taken along paths in some specified class P. For a given class P, N measures the rate of change of the percentage of relative shares and thus gives the following definitions:

(i) Technological change is neutral if and only if relative shares remain constant over time and N=0 for movements along paths of the specified type.

(ii) Technological change is labour saving (labour using) if the relative share of labour falls (rises) along the specified paths. Since there are only two production factors, K and L, a labour saving technological change is equivalent to a capital using technological change. Thus technological change is labour saving (and capital using) if and only if $N < 0$.

(iii) Analogously, technological change is capital saving (and labour using) if and only if $N > 0$.

Unfortunately, these definitions depend crucially upon which path is designated. In fact, a particular technological change may be labour saving in the context of one class of paths and capital saving in the context of another class of paths.

The class of paths most often studied are the paths along which there is (1) a constant capital/output ratio, (2) a constant capital/labour ratio, and (3) a constant labour/output ratio. In particular, technological change is said to be Harrod-neutral if and only if $N=0$ for paths of the first type; it is said to be Hicks-neutral if and only if $N=0$ for paths of the second type; and it is said to be Solow-neutral if and only if $N=0$ for paths of the third type.

In terms of the factor augmenting representation of disembodied technological change, it can be shown (see Burmeister and Dobell (1970)) that Harrod neutrality corresponds to pure labour augmentation; Hicks neutrality corresponds to equal labour and capital augmentation; and Solow neutrality corresponds to pure capital augmenting technical change. Specifically, in the context of disembodied technical change:

1) Technological change is Harrod neutral if and only if the production function takes the form:

$$Q(t) = G[K(t),\ \alpha_L(t)L(t)]; \tag{11}$$

2) Technological change is Hicks neutral if and only if the production function takes the form:

$$Q(t) = G[\alpha_K(t)K(t),\ \alpha_L(t)L(t)]; \tag{12}$$

where

$$\alpha_K(t) = \alpha_L(t); \tag{13}$$

3) Technological change is Solow neutral if and only if the production function takes the form:

$$Q(t) = G[\alpha_K(t)K(t),\ L(t)]. \tag{14}$$

3.3 Incorporating Technological Change and Duality

A well known fact in the theory of the firm is that to any neoclassical production function corresponds a dual function. This dual function is usually referred to as the "cost function" and is the solution, in terms of R, W, and Q, to the problem

$$\begin{aligned} C(R,W,Q) = \ & Min\ RK+WL \\ & s.t. G(K,L) = Q \end{aligned} \tag{15}$$

when there is no technological change. Since technological change affects the production function, it also affects the cost function. The objective of this section is to incorporate factor augmenting technological change into the cost function and the associated direct factor demand function.

In the case of disembodied and factor augmenting technological change and using equation (7), the cost function $\chi(R,W,Q,t)$, becomes the solution to the problem

$$\begin{aligned} &Min\ R_t K(t) + W_t L(t) \\ &\quad s.t.\ G[\alpha_K(t)K(t),\ \alpha_L(t)L(t)] = Q(t) \end{aligned} \tag{16}$$

If we let $Z_1(t) = \alpha_K(t)K(t)$, $Z_2(t) = \alpha_L(t)L(t)$, $W_{1t} = R_t/\alpha_K(t)$ and $W_{2t} = W_t/\ \alpha_L(t)$, then equation (16) can be restated as

$$\begin{aligned} &Min\ W_{1t}\, Z_1(t) + W_{2t} Z_2(t) \\ &\quad s.t.\ G[Z_1(t),\ Z_2(t)] = Q(t) \end{aligned} \tag{17}$$

Equation (16) formulates the minimization problem in "natural" units whereas equation (17) formulates the minimization problem in "efficiency" units. From equation (15), the cost function associated to problem (17) takes the form $C[\ W_{1t}, W_{2t}, Q(t)]$. Converting in natural units yields that for the case of disembodied and factor augmenting technological change, the cost function takes the form

$$\chi[R,W,Q,t] = C[\ \frac{R_t}{\alpha_K(t)}, \frac{W_t}{\alpha_L(t)}, Q(t)\ ]. \tag{18}$$

In addition, if we assume that the production function G is homogeneous of degree 1 in Q, then

$$C[R,W,Q] = C[R,W,1]Q \tag{19}$$

and equation (18) becomes

$$\chi[R,W,Q,t] = C[\frac{R_t}{\alpha_k(t)}, \frac{W_t}{\alpha_L(t)}, 1]Q(t). \tag{20}$$

As in the case when there is no technological change, we can also associate a direct factor demand function to the cost function when there is technological change. By definition, the direct demand function for the input K is the function that expresses the amount of K in terms of the factor prices and output level which minimizes cost. From Shepherd's lemma, the direct demand function for capital is

$$\kappa(R,W,Q) = \frac{\partial C(R,W,Q)}{\partial R}. \tag{21}$$

By analogy, when there is factor augmenting technological change the direct demand for capital is

$$K(R,W,Q,t) = \frac{\partial\chi[R,W,Q,t]}{\partial R}. \tag{22}$$

Hence, given equation (20), the direct demand function for capital is

$$K(R,W,Q,t) = \kappa\left[\frac{R_t}{\alpha_K(t)},\frac{W_t}{\alpha_L(t)},1\right]\frac{Q(t)}{\alpha_K(t)} \tag{23}$$

in the case of factor augmenting and disembodied technological change and if the direct demand for capital is κ[R,W,1]Q when there is no technological change.

3.4 Embodied Technological Change: Putty-Clay Technology

In our discussion about neutrality and in our development of the indirect demand function for capital, technological change was of the disembodied type. However, embodied technological change is probably a better description of observed technological change because new machines are clearly more affected by technical progress than old machines, and also because disembodied technological progress implicitly assumes (unrealistically) that capital is as malleable as "jelly." In this section, the formulation of the demand for capital for the case of disembodied technical progress is adapted to the case of embodied technical progress.

To formulate the demand for capital in the case of embodied technical progress, we need to make additional assumptions. The first assumption is that invested capital of a particular vintage (I_t) has a permanently fixed capital-labour ratio. This is commonly referred to as the "putty-clay" hypothesis. The second assumption is that the rate of capital decay (δ) is constant. The third assumption is that the decision makers are risk neutral. Given these assumptions, as well as the assumption of constant returns to scale, it can be shown (see Nadeau (1986)) that at every time period the firm faces the problem

$$\begin{aligned} &Min\ R_t I(t) + W_t L(t) \\ &\quad s.t.\ F[I(t),\ L(t),\ t] = \Delta Q_t \end{aligned} \tag{24}$$

where R_t and W_t respectively denote the cost of capital and the wage rate prevalent at time t; L(t) denotes the amount of labour assigned to I(t); and ΔQ_t denotes gross investment in output, i.e.

$$\Delta Q_t = Q_t - (1-\delta)Q_{t-1} \tag{25}$$

Now, if we specify that technological change is factor augmenting then the production function F[I(t), L(t),t] in (24) becomes

$$F[I(t),\ L(t),t] \ = \ G[\alpha_K(t)I(t),\alpha_L(t)L(t)] \tag{26}$$

and the problem stated in (24) is conceptually identical to the problem stated in (16). Hence, by analogy, we have that the demand for capital investment associated to (24) in the case of embodied and factor augmenting technological change, and constant returns to scale, takes the general form

$$I[R,W,Q,t] \ = \ \kappa[\frac{R_t}{\alpha_K(t)},\frac{W_t}{\alpha_L(t)},\ 1\ ]\frac{\Delta Q(t)}{\alpha_K(t)} \tag{27}$$

where the function $\kappa[R_t/\ \alpha_K(t)\ ,\ W_t/\ \alpha_L(t)\ ,1]$ is defined by equation (23).

4. THE STATISTICAL MODEL

This chapter develops the statistical model used to identify the type of technical progress.

4.1 Functional Form

The model is based on the direct factor demand function associated with the Diewert (generalized Leontief) cost function (developed by Diewert (1976)). The Diewert cost function is the preferred functional form as it meets several requirements for estimation. First, the Diewert cost function exhibits parsimony in parameters, that is the functional form contains no more parameters than are necessary for consistency with the maintained hypothesis. Too many parameters exacerbate problems of multi-collinearity, which tend to be severe in many applications due to market substitution which causes prices, and hence quantities, to be highly correlated.

Secondly, the Diewert cost function allows ease in interpretation. Excessively complex functional forms may contain implausible implications hidden from easy detection making it cumbersome to compute and assess economic benefits of interest. Thus, ceteris paribus, it is better to choose a functional form in which the parameters have an intrinsic and intuitive economic interpretation and in which functional structure is clear.

Thirdly, the Diewert cost function allows interpolative robustness. Within the range of observed data, the chosen functional form is well behaved, displaying consistency with maintained hypotheses such as positive marginal products or convexity.

Finally, as we will see, the Diewert cost function allows, potentially, for the identification of the type of technological progress which best describes reality. For these reasons, we have selected the Diewert cost function to model technological progress over the period observed.

4.2 The Diewert Cost Function and Technological Change

In the two input case, in the absence of technological change and under constant returns to scale, the Diewert cost function takes the form

$$C(R,W,Q) = (a_K R + 2bR^{1/2}W^{1/2} + a_L W)Q \tag{28}$$

where a_K, b, and a_L are unknown coefficients. The direct demand for capital is then

$$K(R,W,Q) = \left(a_K + b\left(\frac{W}{R}\right)^{1/2}\right)Q \tag{29}$$

Hence, according to equation (27), the direct demand for capital investment in the case of embodied and factor augmenting technological progress, and constant returns to scale, is

$$I[R,W,Q,t] = \left[\frac{a_K}{\alpha_K(t)} + b\left[\frac{1}{[\alpha_K(t)\alpha_L(t)]}\right]^{1/2}\left(\frac{W_t}{R_t}\right)^{1/2}\right]\Delta Q_t \tag{30}$$

in the context of a Diewert cost function.

For the statistical analysis, we need to be explicit about the factor augmenting functions $\alpha_K(t)$ and $\alpha_L(t)$. We assume that the efficiency of capital and labour grows exponentially at a constant rate. More specifically, we postulate that

$$\alpha_K(t) = \alpha_1 e^{\mu t} \tag{31}$$

and that

$$\alpha_L(t) = \alpha_2 e^{\lambda t} \tag{32}$$

The coefficients α_1 and α_2 are scale factors, and μ and λ are the rate of growth of the efficiency of capital and labour respectively. Hence, after substitution, equation (30) becomes

$$I[R,W,Q,t] = \left[Ae^{-\mu t} + Be^{-(\mu+\lambda)t/2} \left(\frac{W_t}{R_t} \right)^{1/2} \right] \Delta Q_t \qquad (33)$$

where $A = a_K/\alpha_1$ and $B = b(\alpha_1\ \alpha_2)^{-1/2}$.

4.3. Gross Additions to Output

The investment equation denoted above, although being definitionally valid, is non causal. Obviously, investment precedes production. Hence we must look at how firms formulate expectations about future levels of output.

The process by which expectations are formed about future outputs has not usually been explicitly specified in investment models. Simple moving averages of past outputs have been the most common representation of this process. Furthermore, in that class of models the length of the lags has been based on data considerations rather than on theoretical or predictive grounds (see for example, Cohen (1971) and Bischoff (1971)).

There is an extensive literature dealing with the manner in which expectations are formed in a macro-economy (see Muth (1961)). In these general models it is assumed that economic agents form their expectations as the solution to an econometric model. While it may seem reasonable to have expectations being essentially predictions of the relevant economic theory, Summers (1980) has shown the difficulty of developing even a quite simple model of corporate investment in such a framework.

The approach we have taken here follows Feige and Pierce (1976). Judging, in the context of expected inflation, that information requirements of full rational expectations are too strong, they suggest an alternative which they term as "economically rational expectations". Their basis is that a utility maximizing individual should, in general, add to his information set until the expected marginal costs and benefits of information are equal. From this, they deduce that when information costs are nontrivial, the economic agents may opt for a less expensive forecasting tool than a complete econometric model even if it entails a loss in accuracy. Then, on the grounds that it offers a low application cost compared to that of its predictive power, they suggest the use of the Box-Jenkins univariate (or bivariate) forecasting methodology as a form to derive expectations (see Box and Jenkins (1976)).

Based on this argument we use, as a proxy for the firm's expectations about future outputs, the forecast generated by the univariate Box-Jenkins forecasting procedure. Since, in that framework, an explicit formulation for expected outputs requires a thorough examination of the observed time series, for the time being we simply represent the expectations about future outputs with the general form:

$$Q_{t \mid t-1} = E(Q_t \mid Q_{t-1}, Q_{t-2}, Q_{t-3}, \ldots). \tag{34}$$

To take into account the above considerations, we write the expected gross increment in output as

$$\Delta Q_{t \mid t-1} = Q_{t \mid t-1} - (1-\delta) Q_{t-1}, \tag{35}$$

and we rewrite equation (33) as:

$$I_t = \left[A e^{-\mu t} + B e^{-(\mu+\lambda)t/2} \left(\frac{W_t}{R_t} \right)^{1/2} \right] \Delta Q_{t \mid t-1} \tag{36}$$

4.4. The Time Structure of Investment Response

Until now we have assumed that there was no time lag between the making of plans and the delivery of equipment. However, it is a well known fact that completing a planned investment will require expenditures over a certain number of periods. To relate planned investment to actual outlays, most econometric models make current investment expenditures depend directly on present and past capital shortages and surpluses, and thus assume firms react to capital shortages rather than anticipate them. Such models are in effect "backward looking" and would appear to be somewhat unrealistic. We shall assume, more realistically, that firms are aware that there are necessary lags between the making of plans and the time of delivery. In our view, investment planners order equipment in advance to avoid shortages in the future. To extend our model of investment process to reflect this assumption requires us to look at how firms anticipate future investment needs and that we specify a pattern of orders.

4.4.1. Anticipated Investment Need

To incorporate the above considerations into our statistical model requires that we make additional assumptions about future output demands and future optimal capital-output ratios. We assume that firms act as if the labour wage will increase at a rate $\overline{\upsilon}_t$, calculated as a function of inflation relative to labour costs in the future and act as if tax variables will remain constant. We make the additional assumptions that the price of capital goods will increase on the average at a rate of $\overline{\xi}_t$ in the future and that the nominal cost of funds will stay constant. Incorporating these considerations into the anticipated ratio of factor prices at time t+i then yields:

$$\frac{W_{t+i\,|\,t}}{R_{t+1\,|\,t}} = \frac{w_t(1+v_t)^i(1-r_{u,t})}{\pi_t(1+\overline{\xi}_t)^i(1-ITC_t-r_{u,t}PVDA_t)\left(1+r_t-(1+\overline{\upsilon}_t)(1-\delta)\right)} \qquad (37)$$

where ITC$_t$ represents the rate of investment tax credits on new capital goods purchases, $PVDA_t$ represents the present value of the depreciation allowance accruing to a dollar of new capital, r_{ut} represents the tax rate on undistributed earnings, and r_t represents the discount rate associated to the real (after tax) cost of funds between investment and financing decisions (refer to Appendix).

Finally, we predict output demand one period ahead. We have that investment needs at time t for time t+i is:

$$I_{t+i \mid t} = \left[Ae^{-\mu t} + Be^{-(\mu+\lambda)t/2} \left(\frac{W_{t+i \mid t}}{R_{t+i \mid t}} \right)^{1/2} \right] \Delta Q_{t+i \mid t-1} \tag{38}$$

where

$$\Delta Q_{t+i \mid t-1} = Q_{t+i \mid t-1} - (1-\delta) Q_{t+i-1 \mid t-1} \tag{39}$$

4.4.2. Patterns of Orders

Let us first specify a general form for investment expenditures. If we denote by $I^{o}_{t \mid t-i}$ the equipment ordered at time t-i to be delivered at time t and by I_t the investment expenditures at time t, then by definition

$$I_t=\sum_{i=0}^{h} I^0_{t\,|\,t-i} \tag{40}$$

where h denotes the planning horizon or, if one prefers, the length of the longest lag. Suppose that for each period t-i in the time interval (t-h,t), the firm orders a portion ω(t-i) of the discrepancy between the investment need that the firm anticipates for time period t and the previous orders it made to be delivered at time t. That is

$$I^0_{t\,|\,t-i}=\omega(t-i)\left(I_{t\,|\,t-i}-\sum_{j=i+1}^{h} I^0_{t\,|\,t-j}\right) \tag{41}$$

Then, replacing $I^o_{t|t-i}$ in equation (40) by this expression, we find that the result is a special case of the general form

$$I_t=\sum_{i=0}^{h} \omega(t-i)I_{t\,|\,t-i}. \tag{42}$$

Accordingly, we will refer to the set of parameters {ω(t-i);i=0,...,h} as the speed of completion parameters.

Economic theory provides little guidance either with respect to the length of the planning horizon or with respect to the specification of forms for the speed of completion parameters. In backward looking investment models, one usually assumes that the parameters are constant, and implements such models by choosing the length of the lag according to a best fit criterion. However, recent studies assume that the speed of adjustment is a function of cash flow (see, for example, Nadeau (1986) and Fazzari and Hubbard (1987)). Specifically, it is assumed that the level of internal funds available for investment - a measure of it being cash flow - is a constraint on the volume of investment expenditures. The rationale behind this assumption that more liquidity should result in larger investment expenditures is intuitively appealing.

Although our model is forward looking, we follow Cohen (1971) and use for the speed of completion parameters the specification:

$$\phi(t-i)=\phi_{0,i}+\phi_{1,i}\left(\frac{I_{t\,|\,t-i}}{F_{t-i}}\right) \tag{43}$$

where $\phi_{0,i}$ and $\phi_{1,i}$ are constants and F_{t-i} is the effective cash flow at time t-i. That is:

$$F_{t-i} \equiv RE_{t-i-1} + DEP_{t-i-1} \tag{44}$$

where RE_{t-i-1} is the level of earnings retained at time t-i-1 and DEP_{t-i-i} is the amount of depreciation deduction for tax purposes.

The set $\{\phi_{0,i};i=1,\ldots,h\}$ is supposed to represent the effects of "physical limitations" in the speed of investment completion. It can thus be inferred that $\phi_{0,i}>0$ for $i=0,\ldots,h$. The set $\{\phi_{1,i};i=1,\ldots h\}$ establishes the relationship between liquidity and the investment time structure. Since it can be safely assumed that firms can support, from a liquidity point of view, more investment expenditures at time t the higher the effective cash flow is, we should expect $\phi_{1,i}$ to be greater than zero. However, for i greater than 1, the sign of $\phi_{1,i}$ cannot be inferred a priori because it depends on the magnitude of $\{\phi_{0,i}, \phi_{1,i};i=1,\ldots,h\}$. The larger the values of $\{\phi_{0,i}, \phi_{1,i}\}$ the smaller $\phi_{1,i}$ should be. This simply reflects the fact that, given a desired level of investment at time t, higher levels of realized expenditures from t to t+i-1 means smaller realized expenditures from t+i to t+h.

This last assumption about the time structure of investment completes the specification of this model. Combining equations (33), (43), and (44) yields

$$I_t=\sum_{i=0}^{h} \omega_{0,i}\left[Ae^{-\mu t}+Be^{-(\mu+\lambda)t/2}\left(\frac{W_{t\,|\,t-i}}{R_{t\,|\,t-i}}\right)^{1/2}\right]\Delta Q_{t\,|\,t-i}+\sum_{j=0}^{h}\phi_{1,j}F_{t-j}. \tag{45}$$

which incorporates lagged adjustments on current investment based on the impact of past investment decisions and available cash flow.

Using prior analysis undertaken by Nadeau (1986), we assume that lagged influences of past activity extend no longer than two periods. Hence, equation (45) is respecified as

$$\begin{aligned} I_t= &\left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)^{t/2}}\left(\frac{W_{t\,|\,t}}{R_{t\,|\,t}}\right)^{1/2}\right]\Delta Q_{t\,|\,t} \\ &+\left[\alpha_2 e^{\mu t} + \beta_2 e^{-(\mu+\lambda)^{t/2}}\left(\frac{W_{t\,|\,t-1}}{R_{t\,|\,t-1}}\right)^{1/2}\right]\Delta Q_{t\,|\,t-1} \\ &+\left[\alpha_3 e^{\mu t} + \beta_3 e^{-(\mu+\lambda)^{t/2}}\left(\frac{W_{t\,|\,t-2}}{R_{t\,|\,t-2}}\right)^{1/2}\right]\Delta Q_{t\,|\,t-2} \\ &+\phi_1 F_t + \phi_2 F_{t-1} + \phi_3 F_{t-2} + \epsilon \end{aligned} \tag{46}$$

where α_1, α_2, α_3, β_1, β_2, β_3, ϕ_1, ϕ_2, and ϕ_3 are coefficients and where ε represents the error term.

The identification of the length of the lag for each variable will be performed by trial and error. Variables will be dropped on the basis of the statistical significance of estimated coefficients.

4.5 Hypothesis Tests

From the discussion in chapter 3 about neutrality, we see that equation (46) can help us determine the type of technical progress that best describes reality. In fact, given some data, if the estimation of equation (46) yields one of the following:

1) $\mu = 0$ and $\lambda > 0$;

2) $\mu = \lambda > 0$;[2]

3) $\mu > 0$ and $\lambda = 0$; or

4) $\mu = \lambda = 0$

then we can conclude that technical progress has been

1) Harrod neutral;

2) Hicks neutral;

3) Solow neutral; or

4) There was no technological progress.

---

[2] While the parameters are restricted to be equal, coefficients are expected to be greater than 0.

If, on the other hand, the hypotheses are rejected, we must conclude that technological progress exists and is non-neutral as defined by Hicks, Harrod, and Solow neutrality.

5. EMPIRICAL RESULTS

Since the equation to be estimated is a non-linear function of the parameters and variables, the non-linear regression option of SHAZAM is utilized. The use of maximum likelihood estimation requires that we make some a priori assumption about the distribution of the error term ε. We assume that ε is normally distributed with mean 0 and some variance σ^2_ε.

5.1 Data Sources

Data is sourced from Nadeau (1986). Appendix 1 describes in detail the source of this data.

Investment data represents U.S. investment in producer's nonresidential durable equipment (in billions of 1972 dollars) 1934 through 1980. Output data represents U.S. Business Gross Product (in billions of 1972 dollars). The measure of the return to capital which is used is a variant of Jorgenson's (1967) so called "user cost of capital" where the financial cost of capital is set as a weighted average of the cost of debt and the cost of equity. The return to labour is the price deflator (1972=100) of labour cost.

To account for the possible influence of the Second World War on investment activity, we incorporate a dummy variable, denoted WW*DUM, respecifying equation (46) to its final functional form

$$\begin{aligned} I_t = {} & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\,|\,t}}{R_{t\,|\,t}}\right)^{1/2}\right]\Delta Q_{t\,|\,t} \\ & + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\,|\,t-1}}{R_{t\,|\,t-1}}\right)^{1/2}\right]\Delta Q_{t\,|\,t-1} \\ & + \left[\alpha_3 e^{-\mu t} + \beta_3 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\,|\,t-2}}{R_{t\,|\,t-2}}\right)^{1/2}\right]\Delta Q_{t\,|\,t-2} \\ & + \phi_1 F_t + \phi_2 F_{t-1} + \phi_3 F_{t-2} + \psi WW*DUM + \epsilon \end{aligned} \tag{47}$$

where WW*DUM takes on the value of 1 for the period 1941 through 1945 and 0 for all other periods. The coefficient ψ will relate the influence of the Second World War on investment activity and is expected to have a negative sign.

5.2. The Log Likelihood Ratio Test

The test for the presence of Hicks, Harrod, or Solow neutral technological change and whether no technological change was present will be performed using the log likelihood ratio test.

Let L_{UR} represent the maximum value of the log-likelihood function when the restrictions do not apply, and let L_R represent the maximum value when the restrictions do apply. Then it can be shown that for large samples

$$2(L_{UR} - L_R) \sim \chi_q^2 \tag{48}$$

where q is the number of restrictions (i.e. the number of restricted coefficients under H_o). To do the test we simply compare the value of χ^2_q above with the chi square critical value $\chi^2_{\alpha,q}$. If χ^2_q is greater than $\chi^2_{\alpha,q}$ we reject the associated hypothesis at the α level of signifcance; if χ^2_q is smaller than $\chi^2_{\alpha,q}$, we accept the hypothesis.

5.3 Starting Values

Good starting values are a pre-requirement for the non-linear estimation routine to converge. To obtain good starting values for our coefficients, a grid search procedure utilizing OLS and a linear form of equation (47) is employed. The coefficients μ and λ are assigned values varying between -2 and 2. Values are iterated through four decimal places in order to find coefficients generating the minimum sum of squared errors over the range of initial values. Estimated coefficients are then incorporated into equation (47) as starting values for non-linear estimation.

5.4 Lag Identification

Utilizing starting values generated using OLS, we estimate equation (47) using maximum likelihood estimation to identify the length of the lag and hence the functional form appropriate for our statistical test.

The length of the lag is determined using the statistical significance of the estimated coefficients. The least significant variable is dropped, the equation respecified, new starting values generated, and the modified equation re-estimated.

5.5 Expected Signs

Estimated signs on coefficients should conform with economic theory. A positive sign is expected on the coefficient attached to factor augmenting growth due to capital (μ). A positive sign is an indication that technological change has increased the efficiency of capital intensive factor inputs over time. The coefficient attached to factor augmenting growth due to labour (λ) is also expected to display a positive sign reflecting the positive influence of technological change on the efficiency of labour over time. Since μ and λ are instantaneous growth rates, we should expect the coefficients to be in the neighbourhood of 0.01333.

Scale coefficients, $\alpha_1..\alpha_n$ and $\beta_1..\beta_n$ are expected to display positive signs. The significance of each scale coefficient will vary according to the structure of the lag. Coefficients attached to the cash flow variable ($\phi_1..\phi_3$) are expected to display positive signs. The degree of significance of each coefficient will vary according to the appropriate lag. A positive sign reflects the positive relationship between investment activity and cash flow.

---

[3] Bischoff (1971) estimates a Hicks neutral rate of technological progress of 0.007.

The coefficient attached to the World War 2 dummy is expected to display a negative sign attached to the coefficient. A negative sign will reflect downward pressure on investment activity resulting from the Second World War.

5.6 Estimation

The direct demand function is initially estimated unrestricted to determine the appropriate length of the lag. Results from the estimation are analyzed according to both the sign and statistical significance of each estimated coefficients. Where required, the model is respecified and re-estimated. This process is iterative, reducing the functional form in accordance to both the sign and significance of estimated coefficients.

The log-likelihood functions required for the log likelihood test are generated during estimation of the unrestricted functional form and from restricting the factor augmenting coefficients to zero (e.g. μ, λ) or to be equal (e.g. $\mu = \lambda$).

5.6.1. Model 1

$$\begin{aligned} I_t = & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\mid t}}{R_{t\mid t}}\right)^{1/2}\right]\Delta Q_{t\mid t} \\ & + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\mid t-1}}{R_{t\mid t-1}}\right)^{1/2}\right]\Delta Q_{t\mid t-1} \\ & + \left[\alpha_3 e^{-\mu t} + \beta_3 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\mid t-2}}{R_{t\mid t-2}}\right)^{1/2}\right]\Delta Q_{t\mid t-2} \\ & + \phi_1 F_t + \phi_2 F_{t-1} + \phi_3 F_{t-2} + \psi WW * DUM + \epsilon \end{aligned} \qquad (49)$$

The estimation results are presented in Table 1.

5.6.1.1. Autocorrelation

Estimation of the unrestricted equation (49) reveals error terms are serially correlated over time. Accordingly, the Durbin-Watson Test is employed to evaluate the degree of autocorrelation, the standard correction feature of SHAZAM enabled, and Model 1 re-estimated. An identical procedure is followed when various restrictions are applied to equation (49). Serial correlation is found to be present in all specifications of Model 1.

5.6.1.2. Coefficients

Estimated coefficients display signs which conflict with our a priori expectations. The coefficient attached to capital factor augmenting technological change is negative and statistically significant, implying that a negative relationship exists between investment and capital augmenting technological change. The coefficient attached to the efficiency of labour is also negative, but is statistically insignificant. The magnitude of the estimated coefficients is also much too large. Combined, these coefficients suggest investment is a function of negative factor augmenting technological change over time. These results are clearly inconsistent with economic theory and reality.

Coefficients attached to the cash flow variables are positive and significant for the current period and first period lag. However, the coefficient attached to the second period cash flow variable is negative and significant, conflicting with our expectation of a positive relationship for all cash flow variables. Once again, these estimation results conflict with our theory of a positive relationship between cash flow and investment.

5.6.1.3. Log Likelihood Functions

The log likelihood ratio test is applied at a 5 percent level of significance using the generated log likelihood functions. The hypotheses that technological change has been Harrod, Solow, or that there has been no technological progress are rejected. We accept the hypothesis that technological progress has been Hicks neutral over the series observed. However, caution regarding these results is warranted due to the poor performance of the model with respect to economic theory. Therefore alternative model specifications have been formulated and these results are presented below.

Table 1

$$\begin{aligned} I_t = & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu + \lambda)t/2}\left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right]\Delta Q_{t|t} \\ & + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu + \lambda)t/2}\left(\frac{W_{t|t-1}}{R_{t|t-1}}\right)^{1/2}\right]\Delta Q_{t|t-1} \\ & + \left[\alpha_3 e^{-\mu t} + \beta_3 e^{-(\mu + \lambda)t/2}\left(\frac{W_{t|t-2}}{R_{t|t-2}}\right)^{1/2}\right]\Delta Q_{t|t-2} \\ & + \phi_1 F_t + \phi_2 F_{t-1} + \phi_3 F_{t-2} + \psi WW*DUM + \epsilon \end{aligned}$$

Restriction	No Restriction		$\mu=0$		$\lambda=0$	
Parameter	Est.	S.E	Est.	S.E	Est.	S.E
Likelihood Function	-118.102	na	-123.408	na	-123.680	na
Likelihood Ratio	1	na	10.6116 (p=.00496)	na	11.1566 (p=.00378)	na
μ	-0.1185	.051267	na	na	-0.01765	.000807
λ	-15.402	28.682	0.29386	.19105	na	na
α_1	.001950	.004366	-0.10057	.082429	0.28933	.03545
β_1	-.01680	.050656	0.13717	.08076	0.032137	.047086
α_2	-.02598	.07214	4.063	4.8932	1.565	3.0958
β_2	1.3224	.1104	-0.06778	.13212	-0.04992	.08581
α_3	.02493	.085231	-3.9545	4.868	-1.2964	3.088
β_3	-0.86885	.088718	0.0245	.11229	-0.00012	.047602
ϕ_1	1.1331	.38042	0.64083	.3503	1.1305	.38242
ϕ_2	1.2654	. 9279	0.17627	.53805	1.2698	.52337
ϕ_3	-2.6065	.61842	-2.4527	.54365	-2.6986	.66151
ψ	-4.1626	2.6326	-6.2575	2.75	-5.0803	2.9799
RHO	0.65076	.12577	1.0668	.014366	.78455	.12587

Table 1
(continued)

$$
\begin{aligned}
I_t = & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right]\Delta Q_{t|t} \\
& + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t-1}}{R_{t|t-1}}\right)^{1/2}\right]\Delta Q_{t|t-1} \\
& + \left[\alpha_3 e^{-\mu t} + \beta_3 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t-2}}{R_{t|t-2}}\right)^{1/2}\right]\Delta Q_{t|t-2} \\
& + \phi_1 F_t + \phi_2 F_{t-1} + \phi_3 F_{t-2} + \psi WW{*}DUM + \epsilon
\end{aligned}
$$

Restriction	$\mu=\lambda$		$\mu=0$ and $\lambda=0$	
Parameter	Est.	S.E	Est.	S.E.
Likelihood Function	-118.1808	na	-124.280	na
Likelihood Ratio	.1576 (p=.92422)	na	12.355 (p=.00626)	na
μ	-.11795	.051652	na	na
λ	-.11795	.051652	na	na
α_1	.0019831	.004488	.07366	.064189
β_1	-.015644	.049731	-.03637	.042314
α_2	-.024219	.087272	3.5144	4.9763
β_2	1.0231	.19735	.043381	.057951
α_3	.023169	.085436	-3.5144	4.9763
β_3	-.57174	.18700	-.00447	.033068
ϕ_1	1.294	.37149	.67798	0.37231
ϕ_2	1.2605	.46832	.22969	0.55225
ϕ_3	-2.6034	.63295	-2.5275	0.59456
ψ	-4.1286	2.4436	-4.4536	11.552
RHO	.64783	.12522	1.0612	0.0117

5.6.1.4. Model Respecification

Questionable signs and significance levels of coefficients estimated by Model 1 indicate model respecification is required in order to improve results. We respecify equation (49), and drop the second period cash flow variable due to its statistical insignificance and the negative sign attached to the estimated coefficient. New starting values are generated in accordance with the methodology described using ordinary least squares. Equation (50) is the respecified functional form.

5.6.2. Model 2

$$
\begin{aligned}
I_t = & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\,|\,t}}{R_{t\,|\,t}}\right)^{1/2}\right]\Delta Q_{t\,|\,t} \\
& + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\,|\,t-1}}{R_{t\,|\,t-1}}\right)^{1/2}\right]\Delta Q_{t\,|\,t-1} \\
& + \left[\alpha_3 e^{-\mu t} + \beta_3 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t\,|\,t-2}}{R_{t\,|\,t-2}}\right)^{1/2}\right]\Delta Q_{t\,|\,t-2} \\
& + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW*DUM + \epsilon
\end{aligned}
\qquad (50)
$$

The estimation results are presented in Table 2.

5.6.2.1. Autocorrelation

Estimation of the unrestricted version of equation (50) reveals once again that error terms are serially correlated over time. Estimation of parameter restrictions reveal serial correlation present only when the efficiency of capital is restricted. To correct for serial correlation in both the unrestricted and restricted equation, the standard correction feature of SHAZAM is enabled and Model 2 re-estimated.

5.6.2.2. Coefficients

Unlike estimation results from Model 1, estimation of Model 2 reveals coefficients attached to both factor augmenting technological growth due to capital and labour display expected positive signs. However, the magnitude of the coefficient assigned to labour augmenting technological change is larger than our a priori expectations, while the coefficient attached to capital augmenting technological change is statistically insignificant.

A problem also is apparent with the coefficients attached to the remaining two cash flow variables. Both coefficients attached to the cash flow variables are positive, yet statistically insignificant. Once again, these results conflict with our a priori expectations.

Coefficients attached to our lagged output variables exhibit extremely large standard errors, suggesting the functional form does not fit the time series data effectively.

Combined, these result suggest inherent problems remain in our model specification.

5.6.2.3. Log Likelihood Functions

As in Model 1, the alternative technologies are tested for. However, given the previous discussion, this has only been done for completeness.

As expected, once we correct for autocorrelation in both unrestricted and restricted ($\lambda=0$) equations, log likelihood functions increase in absolute value as the number of restrictions decrease.

We apply the log likelihood ratio test at a 5 percent level of significance using the generated log likelihood functions. Once again, we reject the hypotheses that technological change has been Harrod neutral, Solow neutral, or that no technological progress occurred. We accept the hypothesis that technological progress has been Hicks neutral over the series tested.

Table 2

$$
\begin{aligned}
I_t = & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right]\Delta Q_{t|t} \\
& + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t-1}}{R_{t|t-1}}\right)^{1/2}\right]\Delta Q_{t|t-1} \\
& + \left[\alpha_3 e^{-\mu t} + \beta_3 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t-2}}{R_{t|t-2}}\right)^{1/2}\right]\Delta Q_{t|t-2} \\
& + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW{*}DUM + \epsilon
\end{aligned}
$$

Restriction	**No Restriction**		$\mu=0$		$\lambda=0$	
Parameter	**Est.**	**S.E**	**Est.**	**S.E**	**Est**	**S.E**
Likelihood Function	−127.288	na	−142.656	na	−132.753	na
Likelihood Ratio	1	na	30.736 (p=.0000)	na	10.93 (p=.00423)	na
μ	0.00002	0.000075	na	na	0.26621	0.91382
λ	6.2154	0.11615	101.37	0.53368	na	na
α_1	−394.32	1268.7	.55435	0.2438	−1.2569	0.11456
β_1	394.32	1268.6	−.29806	0.25158	.11017	0.04411
α_2	5929.5	18695	−59.184	28.66	164.73	154.55
β_2	−5930.1	18693	49.358	28.013	.32083	0.54813
α_3	−5715.1	17979	58.743	28.566	−161.32	156.63
β_3	5716.3	17977	−48.721	27.933	.07468	0.528
ϕ_1	.23205	.26405	.80897	.57643	.29572	.37181
ϕ_2	0.6628	.37558	1.3132	.57878	.92317	.56769
ψ_3	−2.8419	3.3198	−11.233	3.1337	−5.8336	7.7616
RHO	0.51162	.16757	.11071	na	.80343	.15575

Table 2
(continued)

$$I_t = \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right]\Delta Q_{t|t} + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t-1}}{R_{t|t-1}}\right)^{1/2}\right]\Delta Q_{t|t-1} + \left[\alpha_3 e^{-\mu t} + \beta_3 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t-2}}{R_{t|t-2}}\right)^{1/2}\right]\Delta Q_{t|t-2} + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW{*}DUM + \epsilon$$

Restriction	$\mu=\lambda$		$\mu=0$ and $\lambda=0$	
Parameter	Est.	S.E	Est.	S.E.
Log Likelihood Function	−128.2747	na	−151.135	na
Likelihood Ratio	1.973 (p=.37288)	na	7.69 (p=.02139)	na
μ	.000028259	.00010 03	na	na
λ	.000028259	.00010 03	na	na
α_1	−316.63	1136.4	0.063185	0.20113
β_1	316.51	1136.3	0.18151	0.11722
α_2	−65.358	237.00	−3.3657	12.445
β_2	60.930	237.10	0.026003	0.19797
α_3	239.46	829.60	3.4075	12.3&6
β_3	−234.56	829.58	0.1425	0.1139
ϕ_1	.27269	.28300	0.81023	0.60685
ϕ_2	.72512	.39062	1.4237	0.61758
ψ_3	−2.9006	3.2384	−89.586	13.444
RHO	.50265	.13455	.14024	na

5.6.2.4. Model Respecification

Estimated coefficients display conflicting signs and large standard errors indicate that Model 2 should be respecified to improve results. Accordingly, we respecify equation (50), dropping the second lag attached to our investment equation, and generate new starting values using ordinary least squares. Equation (51) is the resulting functional form.

5.6.3. Model 3

$$\begin{aligned} I_t = & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)^{t/2}} \left(\frac{W_{t\,|\,t}}{R_{t\,|\,t}}\right)^{1/2}\right] \Delta Q_{t\,|\,t} \\ & + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)^{t/2}} \left(\frac{W_{t\,|\,t-1}}{R_{t\,|\,t-1}}\right)^{1/2}\right] \Delta Q_{t\,|\,t-1} \\ & + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW * DUM + \epsilon \end{aligned} \tag{51}$$

The estimation results are presented on Table 3.

5.6.3.1. Autocorrelation

Estimation of equation (51) reveals the error terms are once again serially correlated in all but one specification of the model (μ and $\lambda = 0$). Autocorrelation increases significantly when restrictions are introduced to the equation.

5.6.3.2. Coefficients

Coefficients generated by the unrestricted estimation of Model 3 are once again inconsistent with expectations. The coefficient attached to factor augmenting technological growth due to capital is negative, suggesting negative augmenting technological growth. The coefficient attached to labour augmenting technological growth displays the expected sign (positive) yet it is statistically insignificant. The coefficient attached to the cash flow variable at time "t" is negative, conflicting with our prior expectations.

These results suggest inherent problems remain in the model specification. Consequently, the Diewert model, which attempts to model technological change as a deterministic event, may have been an inappropriate choice for this experiment. For example, when capital and labour augmenting technological parameters are both restricted to equal zero, estimation results suggest Model Three fits the time series rather well. In other words, the model performs efficiently when the technological change parameters are excluded from the model. Since we know that technological progress takes place, this suggests that technological progress is not modelled appropriately. In particular, this suggests that technological change should be modelled as a stochastic event, rather than a deterministic one.

5.6.3.3. Log Likelihood Functions

We once again apply the log likelihood ratio test at a 5 percent level of significance using log likelihood functions generated from estimation. Unlike Models 1 and 2, the log likelihood ratio test leads to acceptance of the hypotheses that technological change has been Solow neutral and Hicks neutral over time. However, once again, estimated coefficients displaying negative signs and the presence of autocorrelation of error terms suggest model mispecification remains. Therefore, acceptance of the hypothesis that technological change has been either Solow or Hicks neutral is suspect.

Table 3

$$
\begin{aligned}
I_t = & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu + \lambda) t/2} \left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right] \Delta Q_{t|t} \\
& + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu + \lambda) t/2} \left(\frac{W_{t|t-1}}{R_{t|t-1}}\right)^{1/2}\right] \Delta Q_{t|t-1} \\
& + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW*DUM + \epsilon
\end{aligned}
$$

Restriction	No Restriction		$\mu=0$		$\lambda=0$	
Parameter	Est.	S.E	Est.	S.E	Est	S.E
Likelihood Function	-127.219	na	-131.660	na	-130.208	na
Likelihood Ratio	1	na	8.882 (p=.01178)	na	5.978 (p=.05034)	na
μ	-0.08806	0.042603	na	na	0.058126	.0266
λ	2.6671	3.2996	-0.0702	0.2836	na	na
α_1	.006565	.013434	0.008492	0.063958	-0.26548	.18659
β_1	-0.01707	.081291	0.042358	0.049868	0.11202	.044161
α_2	-0.00000	.068494	-0.005 6	0.069527	0.5957	.37756
β_2	0.43553	.076527	-0.08775	0.10079	-0.11017	.07735
ϕ_1	-0.03344	.35704	-0.40999	0.32349	-0.31698	.31254
ϕ_2	0.12802	.52446	-0.82037	0.60272	-0.64556	.55779
ψ_3	-4.7469	3.3479	-6.2897	3.21014	-3.6472	3.4076
RHO	0.44775	.15175	1.0558	0.012361	1.0636	.016019

Table 3
(continued)

$$I_t=\left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right]\Delta Q_{t|t} + \left[\alpha_2 e^{-\mu t} + \beta_2 e^{-(\mu+\lambda)t/2}\left(\frac{W_{t|t-1}}{R_{t|t-1}}\right)^{1/2}\right]\Delta Q_{t|t-1} + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW*DUM + \epsilon$$

Restriction	$\mu=\lambda$		$\mu=0$ and $\lambda=0$	
Parameter	**Est.**	**S.E**	**Est.**	**S.E.**
Likelihood Function	-128.2815	na	-152.822	na
Likelihood Ratio	2.125 (p=.34559)	na	25.603 (p=.0000)	na
μ	-.009967	-.005267	na	na
λ	-.009967	-.005267	na	na
α_1	.70901	.5115	0.1707	0.17183
β_1	-.83286	.55528	0.060969	0.10349
α_2	-.37792	.44103	-0.04860	0.18871
β_2	.86659	.53101	0.27637	0.11523
ϕ_1	.23959	.30298	0.80733	0.79583
ϕ_2	.63550	.42333	1.4795	0.80971
ψ_3	-3.2253	3.3046	-81.462	12.116
RHO	.53490	.12990	0.14	na

5.6.3.4. Model Respecification

Results from estimation of Model 3 indicate respecification of the functional form is once again required to improve results. Accordingly, we drop the second lag attached to our investment equation, generate new starting values for coefficients using ordinary least squares and respecify the functional form. Equation (52) is the revised functional form.

5.6.4. Model 4

$$
\begin{aligned}
I_t \; += & \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu+\lambda)^{t/2}} \left(\frac{W_{t\,|\,t}}{R_{t\,|\,t}} \right)^{1/2} \right] \Delta Q_{t\,|\,t} \\
& + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW * DUM + \epsilon
\end{aligned}
\tag{52}
$$

The estimation results are presented on Table 4.

5.6.4.1. Autocorrelation

Estimation of the unrestricted equation (52) reveals error terms are again serially correlated over time. The level of autocorrelation appears to increase as restrictions are introduced to the equation.

5.6.4.2. Coefficients

Coefficients generated by the unrestricted estimation of Model 4 differ substantially from a priori expectations. The coefficient attached to capital factor augmenting technological change is negative, suggesting negative technological growth over time. Coefficients attached to all the cash flow variables are negative and insignificant. These results continue to imply model mispecification remains inherent in the model. However, as with Model 3, when both capital and labour factor augmenting technological parameters are restricted to equal zero, Model 4 performs relatively well, with estimated coefficients displaying the expected signs. Once again, these results reinforce the case for modelling technological progress as a stochastic event over time.

5.6.4.3. Log Likelihood Functions

As for prior models, we apply the log likelihood ratio test at a 5 percent significance level using the log likelihood functions generated from estimation of unrestricted and restricted versions of equation (52). The degree of error in the model is such that we accept both the hypothesis that technological change has been Harrod neutral and the hypothesis that technological change has been Hicks neutral. However, once again, the presence of serial correlation and signs on coefficients which are inconsistent with expectations suggests that further respecification is required.

Table 4

$$I_t = \left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu + \lambda)t/2}\left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right]\Delta Q_{t|t}$$
$$+\phi_1 F_t + \phi_2 F_{t-1} + \psi WW*DUM + \epsilon$$

Restriction	**No Restriction**		**μ=0**		**λ=0**	
Parameter	**Est.**	**S.E**	**Est.**	**S.E**	**Est**	**S.E**
Likelihood Function	**−130.676**	**na**	**−132.45**	**na**	**−141.689**	**na**
Likelihood Ratio	**1**	**na**	**3.556 (p=.16898)**	**na**	**22.026 (p=.0000)**	**na**
μ	**−0.047928**	**.053065**	**na**	**na**	**−0.00760**	**.0032875**
λ	**0.55067**	**1.9508**	**0.42542**	**0.42017**	**na**	**na**
α_1	**0.036593**	**0.10272**	**0.000552**	**0.019435**	**0.36427**	**0.05471**
β_1	**−0.1022**	**0.17162**	**0.05801**	**0.042639**	**−0.01440**	**0.033021**
ϕ_1	**−0.14625**	**0.32646**	**−0.36401**	**0.0279**	**0.58131**	**0.48357**
ϕ_2	**−0.27778**	**0.5827**	**−0.6897**	**0.59427**	**0.61474**	**0.50588**
ψ_3	**−5.1594**	**3.1012**	**−5.9768**	**3.1636**	**−9.3008**	**3.229**
RHO	**1.0317**	**0.02832**	**1.0587**	**.014835**	**−0.04**	**na**

Table 4
(continued)

$$I_t=\left[\alpha_1 e^{-\mu t} + \beta_1 e^{-(\mu + \lambda)t/2}\left(\frac{W_{t|t}}{R_{t|t}}\right)^{1/2}\right]\Delta Q_{t|t} + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW*DUM + \epsilon$$

Restriction	$\mu=\lambda$		$\mu=0$ and $\lambda=0$	
Parameter	Est	S.E	Est	S.E.
Likelihood Function	-130.7127	na	-158.923	na
Likelihood Ratio	.0734 (p=.96397)	na	56. 9 (p=.0000)	na
μ	-.057173	.040655	na	na
λ	-.057173	.040655	na	na
α_1	.021944	.044514	0.15358	0.05042
β_1	-.072457	.092031	0.27731	0.05107
ϕ_1	-.15501	.33523	0.9489	0.70654
ϕ_2	-.29659	.59347	1.4803	0.71469
ψ_3	-5.2603	3.1655	-82.898	13.676
RHO	1.0315	.021501	-0.005	na

5.6.4.4. Model Respecification

Results from estimating Model 4 indicate respecification of the functional form is once again required to improve results. We drop the constant term attached to our investment equation and generate new starting values for coefficients. Model 5 is the respecified functional form, representing the Diewert model in its simplest form. This model is estimated simply for the sake of completeness.

5.6.5. Model 5

$$\begin{aligned} I_t = & \left[\beta_1 e^{-(\lambda)^{t/2}} \left(\frac{W_{t\,|\,t}}{R_{t\,|\,t}}\right)^{1/2}\right] \Delta Q_{t\,|\,t} \\ & + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW*DUM + \epsilon \end{aligned} \tag{53}$$

The estimation results are presented on Table 5.

5.6.5.1. Autocorrelation

Estimation of both unrestricted and restricted forms of equation (53) reveal error terms are not serially correlated over time. The Durbin Watson statistic falls within the undetermined range when tested for serial correlation.

Table 5

$$I_t = \left[\beta_1 e^{-(\lambda)t/2} \left(\frac{W_{t|t}}{R_{t|t}} \right)^{1/2} \right] \Delta Q_{t|t} + \phi_1 F_t + \phi_2 F_{t-1} + \psi WW*DUM + \epsilon$$

Restriction	No Restrictions		$\lambda=0$	
Parameter	Est.	S.E	Est.	S.E.
Likelihood Function	−136.5405	na	−138.1744	na
Likelihood Ratio	1	na	3.2678 (p=.19517)	na
λ	0.63034	1.2910	na	na
β_1	0.14033	0.32005	0.077422	0.018813
ϕ_1	0.49037	0.11878	0.590117	0.11591
ϕ_2	0.14205	0.11274	0.14889	0.11626
ψ_3	−11.943	0.11274	−11.117	2.5313

5.6.5.2. Coefficients

Coefficients generated by the unrestricted estimation of equation (53) concur with our a priori expectations of positive signs of coefficients attached to technological growth and cash flow variables. However, while the coefficient attached to technological growth displays a positive sign, it is very large and insignificant, suggesting our model remains mispecified, possibly due in part to the underlying assumption of constant technological growth or to an oversimplification of U.S. domestic investment activity over the time period tested. Coefficients attached to the cash flow variables are as expected, displaying positive signs and statistically significant. The World War 2 dummy variable also displays the expected negative sign and is significant.

With the exception of the technological growth coefficient, estimation results of Model 5 are consistent with expectations discussed in Section 5.4.

5.6.5.3. Log Likelihood Functions

The log likelihood test is once again applied at a 5 percent significance level. Since the only restriction remaining in our functional form represents some form of technological progress, the hypothesis that technological change has been neutral is accepted at a 5 percent significance level. Unfortunately, we cannot determine the specific type of neutrality present due to model specification prohibiting distinction between Hicks, Harrod, or Solow neutral technological change. However, due to the statistical insignificance of the coefficient attached to technological progress, accepting the hypothesis that technological progress has been neutral of any type becomes suspect.

5.7. Summary of Estimation Results

Estimation of all equations suggest there are inherent mispecification problems in our functional form.

A number of major assumptions inherent in our model may be invalid. For instance, it has been assumed throughout our analysis that technological progress grows at a constant rate over time. This assumption may be invalid given the erratic nature of technological advances in a modern productive economy. Historically, new technologies occur sporadically over time, increasing the efficiency of factor inputs in a manner which does not conform to a constant growth theory. Certainly, the ramifications of our estimation results when both capital and labour factor augmenting technological parameters are restricted to equal zero in Models Three and Four suggest that technological change should not be modelled as a deterministic event over time. By excluding the technological parameters, both models performed relatively well. Results such as these suggests that technological progress should in fact be modelled as a stochastic event over time.

Secondly, the assumption that invested capital of a particular vintage resides permanently in a fixed capital-labour ratio may also be invalid. It is a well known fact that many companies invest heavily in upgrading productive machinery of several vintages.

Finally, the assumption of constant returns to scale may be invalid, as many U.S. companies faced economies of scale over the time period tested.

Overall, the presence of serial correlation in all unrestricted models where both factor augmenting technology parameters were estimated, the generation of coefficient signs which conflict with expectations, and the failure of the log likelihood test to accept a plausible hypothesis suggest that our model is not appropriate to measure the type of technological progress which best represents reality.

6. CONCLUSION

The objective of this thesis has been to develop an econometric model to test for the presence of Hicks, Harrod, or Solow neutral technological progress without the a priori restriction of a specific type of technological change imposed. Technological progress influences the profitability of existing methods of producing goods and the pace in which a firm will seek to replace their existing plant and machinery. Consequently, the identification of the type of technological growth is critical in determining if a plant should invest in the same ratio of capital and labour inputs, or if the firm will substitute from one input to another. The identification of the type of technological change would also aid policy makers to formulate policies which positively influence the user cost of capital and labour available to the firm using fiscal policy tools.

The empirical model developed is based upon the direct factor demand function associated to the Diewert (generalized Leontief) cost function. Major underlying assumptions include the presence of embodied factor augmenting technological change, constant returns to scale and deterministic technological progress. The model accounts for the influence on investment activity of technological progress, past production levels, past investment activity, and cash flow availability to the firm. In order to test for the presence of Hicks, Harrod, or Solow neutral technological growth, the empirical model is restricted using five independent restrictions in order to test hypotheses which will help us to identify the type of technological progress which best describes reality.

For estimation purposes, the direct factor demand equation is fitted to U.S. data on fixed business investment (1935-1981). A system of lags is employed in estimation to account for the influence of past investment activity. However, when the model is tested to determine the proper specification for the lag structure, the presence of autocorrelation and estimated coefficients which conflict with economic theory render the results suspect. Consequently, lagged variables are dropped based on the statistical significance and resulting sign of the estimated coefficients in order to improve results. A total of five various specifications are estimated, each variation differing from its predecessor by the omission of a lagged variable. Concurrent with estimation, hypothesis tests designed to identify the presence of Hicks, Harrod, or Solow neutral technological growth are performed using the Log Likelihood Ratio test.

Unfortunately, poor estimation results are produced from all five models. The only acceptable model was a simple indirect Diewert demand function accounting only for the lagged influence of cash flow activity in the prior period. The model was tested and found to exhibit an undetermined type of technological progress, due to model specification prohibiting distinction between Hicks, Harrod and Solow neutral technological progress.

Given these results, the development of a model which correctly measures the type and rate of technological change remains a topic where further research is required. The indirect factor demand equation associated to the Diewert (generalized Leontief) cost function presents an ideal starting point from which to develop a model incorporating technological parameters which do not grow at a constant rate over time, but fluctuate according to the rate and direction of technological progress. Our analysis has shown technological progress cannot be modelled as a deterministic event. Consequently, the problem economists face in the development of models with which to measure the rate and type of technological progress lies in the construction of models which allow for stochastic technological progress. Research into the development of such models is currently underway (see Aghion and Howitt).

The requirement for a correct measure of technological change remains critical in today's marketplace, where modern industries engage in world wide factor substitution between capital and labour in an attempt to achieve the structural changes required to remain competitive in a global marketplace. Firms require a sound understanding of technological change in order to invest scarce resources; policy makers require decision tools with which to effectively and efficiently adjust parameters which ultimately affect the firms investment decision. The requirement for further research into modelling technological progress has never been greater. Only through further research will the modelling of technological progress be possible.

APPENDIX 1

DATA SOURCES

This appendix describes the sources for the data used in the analysis.

Data has been collected for the years 1934 to 1980.

Measure of Gross Additions to Output:

To build a causal model, we assumed that firms need to formulate expectations about future levels of output. Then we mentioned that we would follow Feige and Pierce (1976) in using as a proxy for the firms' expectations about future outputs the forecasts generated by the univariate Box-Jenkins procedure. Using this methodology, we found that output Q_t follows an autoregressive process in the first differences whose form is:

$$(1-B)(1-0.3648B + 0.3562B^2)Q_t = e_t,$$

where B is a backshift operator. This form enables us to obtain measures of the variable "expected gross additions to output.

Q_t: Business Gross Product (in billions of 1972 dollars). Source: Observations up to 1971 come from the National Income and Product Accounts of the United States (1929-1974). Data for 1972, 1973, 1974, 1975, and 1976-1980 come from Survey of Current Business July 1976, July 1977, July 1979, and July 1982 respectively.

I_t: Producer's investment in nonresidential durable equipment (in billions of 1972 dollars). Source: Observations up to 1971 come from the National Income and Product Accounts of the United States (1929-1974). Data for 1972, 1973, 1974, 1975, and 1976-1980 come from Survey of Current Business July 1976, July 1977, July 1979, and July 1982 respectively.

r_d: tax rate on corporate distributed earnings (highest marginal statutory rate). Source: Statistics of Income - Corporate Income Tax Returns.

r_u: tax rate on undistributed earnings. For all years, except for 1936 and 1937, it equals r_d. For 1936 and 1937, it was found in Lent (1948).

ITC_t: effective rate of investment tax credit against equipment purchase. A 7 percent investment tax credit was introduced in 1962 for qualified investment in durable goods. The ITC was suspended in 1966, restored a few months later in 1967, and repealed in 1969 in response to inflationary pressures. In 1971, hoping to increase investment, Congress reinstated the 7 percent ITC. In 1975, the ITC was temporarily increased to 10 percent for qualified investment, an increase made permanent in 1980.

π_t: price deflator for producer's durable equipment (1972=100). Source: Observations up to 1971 come from the National Income and Product Accounts of the United States (1929-1974). Data for 1972-1980 come from the Survey of Current Business, July 1976, 1977, 1978, 1979, and 1982.

$\bar{\xi}_t$: inflation rate in the price of capital goods. Calculated from the time series $\{\pi_t\}$.

ξ_t: average expected inflation rate in the price of capital goods. The average was calculated using forecasts of the variable ξ_t over the next five years. The forecast was generated by the autoregressive model:

$$\xi_t = 0.022586 + 0.3155\xi_{t-1} + e_t$$

w_t: price deflator of labour cost (1972=100). Calculated from the average hourly earnings of production workers in all manufacturing industries. The source of this time series is Historical Statistics, and various issues of Statistical Abstracts of the United States.

v_t: inflation rate relative to labour cost. Calculated from the time series $\{w_t\}$.

$\bar{v}_t$: average expected inflation rate in labour cost. The average was calculated using forecasts of the variable v_t over the next five years. The forecast was generated by the moving average model:

$$v_t = 0.04903 - 0.7924e_{t-1} + e_t$$

F_t: Corporate profits after tax plus depreciation allowance less dividends. Source: Nadeau (1986).

Return to Capital: a variant of Jorgensons (1967) so called "user cost of capital" where the financial cost of capital is set as a weighted average of the cost of debt and the cost of equity. Source: Nadeau (1986).

δ_t: the rate of depreciation of capital stock. It was specified as .16, the rate used by Bischoff (1971).

REFERENCES

Aghion, A. and Howitt,P.,"A Model of Growth Through Creative Destruction", Working Paper, University of Western Ontario, London, Ontario, School of Economics, May 1989.

Abramovitz, Moses, "Resource and Output Trends in the United States Since 1870," The American Economic Review, Volume 46, May 1956, pages 5-23.

Aukrust, O., "Investment and Economic Growth" Productivity Measurement Review, Volume 34, 1959, pages 35-53.

Baltagi, B. H. and Griffin, J. M., "A General Index of Technical Change", The Journal of Political Economy, Volume 96, 1988, pages 20-41.

Bechmann, M.J. and Sato, R., "Aggregate Production Functions and Types of Technical Progress: A Statistical Analysis", The American Economic Review, Volume 59A, March 1969, pages 88-101.

Bernt, E. R. and Khaled, M.S., "Parametric Productivity Measurement and Choice Among Flexible Functional Forms", Journal of Political Economy, Volume 96, 1987, pages 1220-1245.

Bernt, E. R. and Wood, D.O., "The Specification and Change of Technical Change in U.S. Manufacturing", Advances in The Economics of Energy and Resources, Volume 4, 1987, pages 199-121.

Binswanger, H.P., "The Measurement of Technical Change Biases With Many Factors of Production", The American Economic Review, Volume 54A, 1964, pages 964-976.

Bischoff, C. W., "The Effect of Alternative Lag Distributions", Tax Incentives and Capital Spending, ed.: G. Forman, Brookings Institute, 1971, pages 131-196.

Box, G.E.P. and G.M. Jenkins, Time Series Analysis: Forecasting and Control, San Francisco: Holden-Day, 1976, 2nd ed.

Bursmeister E. and R. Dobell, "Technological Change in the One Sector Model", Mathematical Theories of Economic Growth, New York: MacMillan, 1970, pages 65-98.

Coen, Robert M., "The Effect of Cash Flows on the Speed of Adjustment", Tax Incentives and Capital Spending", ed. G. Fromm, Washington, Brookings Institute, 1971, pages 131-196.

Diewert, W.E., "Functional Forms for Profit and Transformation Functions", The Journal of Economic Theory, Volume 6, 1973, pages 284-316.

Diewert, W.E., "An Application of the Shepherd Duality Theorem: A Generalized Leontief Production Function", Journal of Political Economy, Volume 79, 1971, pages 481-507.

Dobbs, I.M., Hill, M.B., and Waterson, M., "Industrial Structure and the Employment Consequences of Technical Change" The Oxford Economic Papers, Volume 39, September 1987, pages 552-567.

Fazzari, Steven, Hubbard, R.G., and Peters, B.C., "Financing Constraints and Corporate Investment", NBER Working Paper No. 2387, 1987.

Feller W., "Technological Progress and Recent Growth Theories", The American Economic Review, Volume 57, Number 5, December 1967, pages 1075-1098.

Feige, E.L. and D. K. Pearce, "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy?," Journal of Political Economy, Volume 84, 1976, pages 499-522.

Ferguson, D.and J.P. Gould, Microeconomic Theory, Illinois: R. D. Irwin, 1987.

Forsund, R.F. and Hjalmarsson, L., "Choice of Technology and Long-Run Technical Change in Energy-Intensive Industries", The Energy Journal, Volume 9, No. 3, 1988, pages 79-97.

Galor, O., "The Long Run Implications of a Hicks-Neutral Technical Progress", The International Economic Review, Volume 29, No. 1, February 1988, pages 177-183.

Hahn, F.H. and Mathews, R.C.O., "The Theory of Economic Growth: A Survey", The Economic Journal, December 1964, pages 781-902.

Hall, R.E., "Technical Change and Capital from the Point of View of the Dual", The Review of Economics Studies, Volume 35, 1968, pages 35-36.

Jorgenson, D.W., "The Embodiement Hypothesis", The Journal of Political Economy, Volume 74, February 1966, pages 1-17.

Kennedy, C. and Thirwall, A.P.T., "Surveys in Applied Economics", The Economic Journal, Volume 82, 1972, pages 11-72.

Kendrick, J.W., "Production Trend: Capital and Labour", The Review of Economics and Statistics, Volume 38, August 1956, pages 248-257.

Layard, P.R.G. and Walters, A.P.T., Micro-Economic Theory, New York, New York: McGraw-Hill Book Company, 1978.

Lent, G.E., The Impact of the Undistributed Profit Tax 1936-1937, New York: Columbia University Press, 1948.

Massel, B.F., "Capital Formation and Technical Change in U.S. Manufacturing", The Review of Economics and Statistics, Volume 42, May 1960, pages 182-189.

Muth, J. F., "Rational Expectations and the Theory of Price Movements", JASA, 55 (1960), pages 299-306.

Nadeau, S., "A Theoretical and Empirical Model of the Real and Financial Decisions of the Firms Effects of Tax Incentives and Other Tax Policies" Unpublished Ph.D. Thesis, School of Urban and Public Affairs, Carnegie-Mellon University, 1986.

Nadiri, M.I., "Some Approaches to the Theory and Measurement of Total Factor Productivity: A Survey", The Journal of Economic Literature, Volume 8, 1970, pages 1137-1177.

Nelson, R., "Alternative Technological Indicies and Factor Demand", The Energy Journal, Volume 6, 1987, pages 135-147.

Pindyck, R.S. and Rubinfeld, D.L.,"Econometric Models and Economic Forecasts", 2nd edition, New York, USA: McGraw-Hill Inc., 1981.

Rosenberg, N., "Inside the Black Box: Technology and Economics", London, England: Cambridge University Press, 1982.

Sato, R. and Suzawa, G.S., "Research and Productivity -Endogenous Technical Change, Boston, Mass.: Auburn House Publishing, 1983.

Solow, R., "Investment and Technical Progress", Mathematical Methods in the Social Sciences, Cambridge, Mass.: Arrow, K. J., et al., Standford University Press, 1960.

Solow, R., "Technical Change and Aggregate Production Functions" The Review Economics and Statistics, Volume 39, August 1957, pages 312-20.

Summers, Lawrence H., "Inflation, Taxation and Corporate Investment," National Bureau of Economic Research Working Paper No. 604, December 1980.

Varian, H.R., Microeconomic Analysis, 2nd ed, New York, USA: W.W. Norton & Company Inc., 1984.

Walters, A.A.,"Production and Cost Functions: an Econometric Survey", Econometrica, Volume 31, Jan-April 1963, pages 1-66.

Wills, J., "Technical Change in the U.S. Primary Metals Industry", The Journal of Econometrics, Volume 10, 1979, pages 85-98.

Wolkowitz, B., "Estimation of a Set of Homothetic Production Functions: A Time Econometrics, Volume 10, Series Analysis of American Postwar Manufacturing", Southern Economic Journal, Volume 39,
July 1972 - April 1973, pages 626-637.

VITA

Surname: Lynch

Given Names: Mark

Place of Birth: Ulveston, England

Date of Birth: 10/08/52

Educational Institutions Attended:

University of Victoria	1984 to 1987

Degrees Awarded:

B.A.	University of Victoria 1987

Honours and Awards:

Canadian Pacific Scholarship	1987-88

Publications

PARTIAL COPYRIGHT LICENSE

Title of Thesis or Dissertation: On Modelling Technological Progress

Author [redacted]
(Signature)

MARK LYNCH
(Name in Block Letters)

April 16, 1992.
(Date)