A Program Evaluation of Applications of Mathematics 10
in British Columbia Public Schools

by

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A Dissertation Submitted in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR of PHILOSOPHY

in the Department of Interdisciplinary Studies

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ABSTRACT

This is an outcomes based program evaluation which used a nonequivalent control-group design to determine the effectiveness of the implementation of Applications of Mathematics 10 in British Columbia public schools. The Experimental Group (10 classes, N = 154) and Control Group (13 classes, N = 232) were selected as intact classes from a population of schools offering Applications of Mathematics 10 (Experimental Group) and Principles of Mathematics 10 and Mathematics 10A (Control Group) in the 1998/99 school year. The criteria used to evaluate this program consisted of:

1) a comparison of teaching methodology used in the 23 classes participating in the study based upon teacher surveys (pre-test and post-test) and logbooks kept by the teachers reporting the methodology used in each class;
2) a comparison of student achievement in the three courses based upon student achievement scores (pre-test and post-test) on multiple choice mathematics assessments; and,
3) a comparison of student attitudes towards mathematics in the three courses based upon student attitude scores (pre-test and post-test) on two surveys.

Teachers in the Experimental and Control groups reported using different teaching strategies (but similar assessment strategies) in their respective classes. The Experimental Group teachers reported using teaching methodologies more consistent with the desired constructivist treatment than did the Control Group teachers.

Using analysis of variance and subsequent post-hoc multiple comparisons of pre- and post-test means it was determined that student achievement scores in the Control Sub-Group, Principles of Mathematics 10 (pre-test $\underline{M}$ = 17.5, $\underline{SD}$ = 5.1; post-test $\underline{M}$ = 23.0, $\underline{SD}$ = 6.8), were significantly higher (pre- and post-test) than the Experimental Group scores ($\underline{M}$ = 12.6, $\underline{SD}$ = 4.2; post-test $\underline{M}$ = 14.8, $\underline{SD}$ = 4.8) and the Control Sub-Group scores, Mathematics 10A, (pre-test $\underline{M}$ = 10.6, $\underline{SD}$ = 3.9; post-test $\underline{M}$ = 12.8, SD = 4.6). The Experimental Group scored significantly higher than the Mathematics 10A sub-group on the pre-test assessment, but not the post-test assessment.

It was also determined that student attitude toward mathematics scores in the Control Sub-Group, PM 10 (pre-test M = 13.2, SD = 3.0; post-test M = 12.8, SD = 3.2), were significantly higher (pre- and post-test) than the Experimental Group scores (M = 10.1, SD = 2.8; post-test M = 10.0, SD = 3.0) and the Control Sub-Group scores, 10A, (pre-test M = 9.1, SD = 2.6; post-test M = 8.4, SD = 2.9). The Experimental Group scored significantly higher than the 10A sub-group on the post-test attitude toward mathematics assessment, but not the pre-test assessment.

It was concluded that the Applications of Mathematics 10 implementation is a qualified success and that this model of delivering mathematics instruction should be pursued.

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CHAPTER 1

INTRODUCTION

Background

The province of British Columbia has undergone a major educational reform over the past decade, initiated by a Royal Commission on Education led by Barry M. Sullivan (1988). This commission resulted in the Ministry of Education, Skills and Training developing and implementing a program entitled Year 2000: A Framework for Learning (Province of British Columbia, 1989). One of the more significant policies that evolved out of this program was a determination that the school system should ensure "that school activities and the procedures used to assess student learning are meaningful to students" (p.10). It was interpreted that "the focus at the provincial level should be on identifying the intended general learning outcomes for educational programs which will prepare students to take their place in society after they leave school" (p.10).

The Year 2000: A Framework for Learning initiative subsequently evolved into The Kindergarten to Grade 12 Education Plan (Province of British Columbia, 1994). This plan noted that schools have been highly effective in preparing young people who have academic interests, but have been less successful in providing a high-quality education for those who enter the workplace or vocational or technical institutions directly from school. As a result of the Kindergarten to Grade 12 Education Plan, there were wholesale revisions to British Columbia's school curriculum. This revision process included changes to the structure and content of the Mathematics K to 12 curriculum. In particular, the secondary course structure was re-designed to provide students with a viable alternative to the pre-calculus program of study, Principles of Mathematics 9 to 12 (PM), and the "modified" program of study (Mathematics 9A, 10A, and 11A). This new program of study, Applications of Mathematics (AM) 9 to 12, was developed and distributed to schools in 1996.

It is the new Applications of Mathematics program that is the focus of this program evaluation. This program evaluation centers on three specific questions outlined below:

1. Do teachers of Applications of Mathematics 10 classes (Experimental Group) use significantly different teaching methodologies compared to teachers of Principles of Mathematics 10 and Mathematics 10A classes (Control Group)?

2. Are students' scores on mathematics achievement assessments (pre-test and post-test) significantly different for students who have taken Applications of Mathematics 10 (Experimental Group) compared to those who have taken Principles of Mathematics 10 or Mathematics 10A (Control Group)?

3. Are students' scores on attitude towards mathematics assessments (pre-test and post-test) significantly different for students who have taken Applications of Mathematics 10 (Experimental Group) compared to those who have taken Principles of Mathematics 10 or Mathematics 10A (Control Group)?

Statement of the Problem

Development of the secondary mathematics curriculum was completed and delivered to B.C. schools in May 1996 in the form of three Integrated Resource Packages (IRP):

1. Mathematics 11 and 12: Introductory Mathematics 11, Principles of Mathematics 11 and 12 (IRP 026), (Province of British Columbia, 1996a);
2. Mathematics 8 to 10 (IRP 031), (Province of British Columbia, 1996b); and,
3. Mathematics 11 and 12: Applications of Mathematics 11 and 12 (IRP 044), (Province of British Columbia, 1996c).

Implementation of the AM 10 curriculum began in September 1996 on a small scale in the form of 7 pilot schools involving approximately 210 students. Implementation continued the following year (1997/98) with 20 schools involving approximately 800 students enrolled in Applications of Mathematics 10. As of June, 1999 there were over 30 schools in British Columbia with approximately 1200 students enrolled in AM 10.

There are two primary goals of the Applications of Mathematics curriculum as stated in the Mathematics 8 to 10 (Province of British Columbia, 1996b) and Mathematics 11 and 12: Applications of Mathematics 11 and 12 (Province of British Columbia, 1996c) IRPs:

1. The Applications of Mathematics 9 to 12 program is intended to provide an applied, hands-on approach to learning mathematics that will appeal to a broad range of

students and give them the skills and knowledge they need to use mathematics at home, in the marketplace, and in their careers;

2. The kindergarten to Grade 12 mathematics curriculum, including the Applications of Mathematics 9 to 12 program, is intended to focus on a number of key principles and processes: positive attitudes, problem solving, communicating mathematically, connecting and applying mathematical ideas, contextual mathematics, reasoning mathematically, using technology, and estimation and mental mathematics.

The Applications of Mathematics 9 to 12 curriculum uses, as a developmental basis, the National Council of Teachers of Mathematics (1989) Curriculum and Evaluation Standards for School Mathematics. Since the NCTM's work advocates a constructivist approach to the learning and teaching of mathematics (Cobb, Yackel & Wood, 1992; Orton R., 1995; Wheatley, Blumsack & Jakubowski, 1995), it follows that a constructivist approach to teaching and learning in the Applications of Mathematics curriculum is desired and expected.

The prescribed learning outcomes in the AM courses are based upon a form of constructivist learning, described by Lerman (1989) and A. Orton (1994) as the active construction of knowledge by the learner rather than passive reception from the environment. At the grade 11 and 12 levels the outcomes of the two mathematics curricula (AM and PM) differ in learning and teaching approaches as well as mathematics topics. The PM curriculum retains the mathematics content (e.g., algebra, polynomial functions, trigonometry, geometry) which identifies it as a traditional pre-calculus program of studies while the AM curriculum, following the NCTM (1989) Curriculum and Evaluation Standards for School Mathematics, contains discrete mathematics topics such as combinatorics, probability (discrete and continuous), and linear programming. As well, the AM curriculum contains other non-traditional topics such as fractal geometry, matrices, and financial decision making. As a result of these topic differences there is no way of comparing student achievement for students in PM 11 and 12 and AM 11 and 12 except on a very limited range of skills.

At the grade 10 level 39% of the learning outcomes in the Applications and Principles curricula are identical; 23% of the learning outcomes address the same topics, but with differing pedagogical emphases; and the remainder (38%) of the learning outcomes are unique to each course. Appendix B provides a detailed breakdown and comparison of the learning outcomes for each course.

There are several methodological implications for teaching as a result of the Applications of Mathematics curriculum. As the AM curriculum design was based on constructivist learning theory, there are resulting implicit and explicit pedagogical differences in the way these courses are intended to be taught. The Mathematics 8 to 10 IRP (Province of British Columbia, 1996b) states that:

> ...The difference between the courses is in the instructional approach.
> The Applications of Mathematics courses emphasize developmental methods for building concepts using a variety of contexts that are based on real-world situations. The students' learning centres on concrete activities and models, with less emphasis on formalism, computation, and symbol manipulation. There is also a greater emphasis on the use of technology as a tool to enable students to conceptualize mathematical ideas and solve real problems.

The Principles of Mathematics courses share some similar methods but also include a more formal structure with a greater emphasis on theory, symbolic manipulations, and analysis of the interconnections within mathematics (p. 5)

The Mathematics 8 to 10 IRP also lists prescribed learning outcomes for Applications of Mathematics 10 and Principles of Mathematics 10 on the topic of trigonometry that make definite statements concerning teacher methodology.

Applications of Mathematics 10:

It is expected that students will apply trigonometry to solve problems using appropriate technology. (p. A-22)

Principles of Mathematics 10:

It is expected that students will solve applied trigonometry problems using exact values. (p. A-29)

The Principles of Mathematics 10 teachers are restricted to instruction that promotes the use of exact values. Although calculators can be used to assist with instruction on this unit, the wording of the outcome minimizes student use of technology by requiring them to answer problems in exact values. The Applications of Mathematics 10 teachers are clearly required to provide their students with experiences using a calculator or other appropriate technology.

In addition to the information concerning the intended methodological changes needed to teach Applications of Mathematics which are included in the IRP, the B.C. Ministry of Education also made in-service support available to interested teachers through the establishment (in 1996) of the Centre for Applied Academics (CFAA). The following is an excerpt from a report commissioned by the Ministry of Education (Applied Academics Final Evaluation Report: Haslin, 2000):

> The mandate of the Centre is to work in partnership with the Ministry to promote, support, and assist in the development of Applied Academics programs within the Province. According to Ministry records, this mandate includes the following outcomes:
>
> - to create awareness of Applied Academics among students, teachers, counselors, parents, etc.;
> - to perform an advocacy role for the promotion and development of Applied Academics;
> - to articulate Applied Academics courses for entry into the post-secondary system and to career pathways (i.e. facilitate student transition either to further study or to employment); and
> - to develop and provide support material and services for implementation (e.g. learning resources and in-service training).
>
> The initial set-up costs and operating funding in 1997/98 for CFAA was $470,064. Operating funding for 1998/99 was $335,150, and for 1999 to 2000 is $305,000, annually. In addition to the operating funds, there are funds for learning resources, $195,000 in 1997/98, and roughly $159,000 in 1999 and 2000, as well as special initiative funding of $25,000 in 1999 and 2000. The CFAA has also obtained federal government funding of approximately $600,000 for the Applications of Working and Learning (AWAL) Project, a professional

> development initiative to assist teachers to create lessons based on authentic workplace applications. (p. 2)

The CFAA was thus given a three year mandate and approximately $2,000,000 in provincial funding and $600,000 in federal funding to support teachers in the implementation of Applied Academic courses (which included Applications of Mathematics 9 to 12.) Haslin (2000) reports that the CFAA was involved in a number of activities that supported teachers in implementing Applications of Mathematics:

- Teacher in-service (in-service sessions were offered in the Lower Mainland, Fraser Valley, Vancouver Island, the Interior, Kootenays and Northern BC);
- Developing and distributing implementation resources;
- Providing information for teachers, administrators, students, and parents through the publication of a CFAA newsletter;
- Organizing a listserv for interested teachers to share implementation concerns and ideas;
- Organizing three annual Applied Academics Conferences that focused on providing in-service for teachers and administrators on the instructional practices espoused by the Applied Academics philosophy;
- Providing additional teacher in-service through the Applications of Working and Learning (AWAL) project, funded by Human Resources Canada and the Ministry of Education. This included thirty AWAL workshops (10 in the interior, 3 on Vancouver Island and 17 in the Lower Mainland) that were provided between May 1998 and August 1999;

- Developing and maintaining a website that provided teachers and students with information, news and student activities; and,
- Development of web-based learning resources for Applications of Mathematics that include lessons, employability skills, classroom activities and other relevant website addresses.

Additional information concerning in-service opportunities was made known to teachers across British Columbia through a variety of Ministry of Education publications including: BC Education News (December 1995, February/March 1997, and April/May/June 1997 issues) and Update on Implementation (February 1996 and Fall 1997 issues). The British Columbia Association of Mathematics Teachers (BCAMT) also provided its membership with information concerning the Applications of Mathematics courses (curriculum changes and in-service opportunities) through the BCAMT Newsletter (April 1996, September 1996, June 1997, September 1997, December 1997, and September 1998 issues).

It is important to determine if these (and other) specific teaching methodologies stated explicitly in the introduction to the Mathematics 8 to 10 IRP, reflected in the learning outcomes, and shared with teachers through extensive in-service opportunities province-wide are being practiced in the AM classroom. These approaches hold the key for the successful implementation of the Applications curriculum. Any observed student achievement or attitudinal differences (between AM 10 and PM 10 students) may be attributable, in part, to the different teacher methodologies the students experience.

The first assessment criterion used in this program evaluation was the teacher methodology used in the two groups. This criterion was assessed by determining if there were identifiable differences in teaching methodology used in the Applications of Mathematics and the Principles of Mathematics and Mathematics 10A classrooms and if student achievement and attitude differences correlate to the identified differences in teaching methodology. The second assessment criterion used was AM 10 students' achievement. AM 10 student (experimental group) achievement was compared to that of PM 10 and 10A students (control group) to determine whether there were significant differences between the two groups and from the pre-test to the post-test assessment. The third and final assessment criterion used was AM 10 students' attitudes toward mathematics. A comparison of students' attitude towards mathematics was needed to determine if it changed in one of the groups more or less significantly over the same time frame.

To effectively assess the goals of the Applications of Mathematics program the following three forms of assessment were used:

1. A comparison of the teaching methods that teachers of the Experimental Group (Applications of Mathematics 10 teachers) and the Control Group (Principles of Mathematics 10 and Mathematics 10A teachers) have reported using in their classrooms;
2. A comparison of student achievement in the Experimental Group to student achievement in the Control Group (pre-test and post-test); and,
3. A comparison of students' attitudes toward mathematics in the Experimental Group to student attitude toward mathematics in the Control Group (pre-test and post-test).

CHAPTER 2

REVIEW OF RELATED LITERATURE

Introduction

The review of related literature identifies three areas which suggest further study is needed. First, although there is a substantial body of research which indicates that using a constructivist approach to teaching mathematics (or teaching in a constructivist learning environment) results in increased student understanding of the concepts, few studies have been conducted which look at the effectiveness of a curriculum designed on this basis.

Second, the research on student streaming appears to be ambiguous as to its effects on students. Consequently it is clear that there is a need for quantifiable (and qualitative) research which relates different teaching methodologies in the different streams, tracks, or pathways to student achievement and attitudes towards mathematics.

Third, although there have been a number of program evaluations on applied academics conducted since 1990, they have tended to focus on student achievement while ignoring the issues of students' attitudes towards mathematics and of teacher methodology

This literature review is organized into three sections that are relevant to this program evaluation. The three sections:

1. provide an overview of constructivism as it relates to mathematics education including:
   a) Piaget's cognitive development theory;

b) constructivism as a learning theory for mathematics education;

c) constructivism in mathematics curriculum development; and,

d) teaching models organized from the perspective of constructivist learning;

2. review the research on the effects of ability grouping on student academic achievement (AA) and self-concept (SC); and,

3. identify and critique previous evaluations of mathematics programs that were based upon constructivism or contextual learning.

Constructivism in Mathematics

Cognitive Developmental Basis for Constructivism

Jean Piaget's cognitive development theory focuses on the stage-like developmental sequence of rational thinking of the developing individual (Santrock, 1988). The stages of cognitive development consist of the following:

1. Sensorimotor stage - lasts from birth to about two years of age during which time the infant develops the ability to organize and coordinate her sensations and perceptions with her physical movements and actions;
2. Properational stage - lasts from two to about seven years of age during which time the use of language and perceptual images increases. The child has difficulty manipulating images and representations and easily becomes centered and unable to reverse situations mentally.

3. Concrete operational stage - lasts from seven to approximately eleven years of age during which time the child's thinking crystallizes into more of a system due, in part, to a shift from egocentrism to relativism.
4. Formal operational stage - lasts from approximately eleven to fifteen years of age during which time the child learns hypothetical reasoning and is able to move beyond concrete experiences to a symbolic, abstract level.

One of the principal deficiencies of Piaget's work was his focus on children under the age of twelve (Santrock, 1988). Kohlberg, as noted by Santrock, compensated for this deficiency by using adolescents in a series of later studies which confirmed Piaget's conclusions and showed their validity when applied to adolescents. Other weaknesses of Piaget's work are that:

- the Piagetian stages are not exactly pure;
- his concepts are somewhat loosely defined;
- not all adolescents and adults appear to reach the formal level;
- investigation suggests that progressive changes in thought structures may extend beyond the level of formal operations;
- maturation of the nervous system and level of intelligence appear to influence cognitive development;
- Piaget acknowledges that social environment can accelerate or delay the onset of formal operations;
- it has been found that different people have different aptitudes for solving different types of problems; and,

- Piaget's models are qualitative, not quantitative measurements and do not necessarily predict in depth the performance of adolescents (Rice, 1992; Santrock, 1988).

Piaget addresses these issues by admitting that they play a role in the cognitive development of adolescents and that the underlying stages hold true even if their onset is shifted and the duration is not always consistent (Rice, 1992).

Jean Piaget's work on cognitive development plays an important role in the development of constructivism (Cobb, 1994; Cobb & Yackel, 1995; von Glasersfeld, 1995; Orton, A., 1994; Orton. R., 1988; Steffe & Kieren 1994). Ernst von Glasersfeld (1995) describes Piaget's early work as an attempt to show that learners can construct for themselves the reality they experience. Von Glasersfeld goes on to describe Piaget's stage theory as a way of organizing an observer's view of developing children because:

> ...whatever theory a psychological investigator builds up, it will not be a description of the observed subjects' *objective* mental reality but rather a conceptual tool for systematizing the investigator's experiences with the subjects. (p. 71)

Stephen Lerman (1989) interprets Piaget's work as an attempt to provide an alternative to empiricism or platonism and as such, "places the roots of knowledge in the individual, and thus borders on private thought and language" (p. 213). Lerman defends the Piagetian stages of development, describing them as Piaget's attempt to establish objectivity.

The most fundamental idea that is borrowed from Piaget, and subsequently forms the basis of constructivism, is that knowledge is connected with action. Knowledge "arises from interactions that take place mid-way between [the child and her environment] and thus involves both at the same time..." (Piaget, 1972, p. 19). As a result of Piaget's genetic epistemology, it was believed that concrete operational children could learn fundamental structures of mathematics (Steffe & Kieren, 1994). Steffe and Kieren go on to describe "the Piagetian studies" of which several were devoted to investigating the readiness of young children to learn mathematics. Although Steffe and Kieren identify two types of readiness studies (correlational and training), they do not give any indication of the findings.

Constructivism as a Learning Theory for Mathematics Education

Two opposing views on mathematical learning are best described by Cobb, Yackel and Wood (1992) in the following manner:

> The first difficulty concerns a tension in eclectic characterizations of mathematical learning. On the one hand, learning is described as a process in which students actively construct mathematical knowledge as they strive to make sense of their worlds. On the other hand, learning can, in practice, be treated as a process of apprehending or recognizing mathematical relationships presented in instructional representations. These two characterizations of mathematical learning reflect differences in the emphasis given to the students and to the teacher's interpretations of instructional representations. (p.6)

The first characterization views learning as an active construction based on the premise that the student builds on and modifies their current mathematical thinking. The second characterization views learning as the correct recognition of mathematical relationships which places the emphasis on the teacher and their expert interpretation of the instructional material.

The characterization that "knowledge cannot simply be transferred ready-made from parent to child or from teacher to student but has to be actively built up by each learner in his or her own mind" (von Glasersfeld, 1991) provides a clear starting point for defining constructivism. From this initial description two hypotheses have emerged which attempt to more clearly define constructivism (Cobb, et al., 1992; Orton, A., 1992; Orton, R., 1988; Steffe & Kieren, 1994; Wheatley, et al., 1995). The two hypotheses are summarized by Lerman (1989):

1) Knowledge is actively constructed by the cognizing subject, not passively received from the environment.
2) Coming to know is an adaptive process that organizes one's experiential world; it does not discover an independent, pre-existing world outside the mind of the knower. (p. 210)

Constructivists who adhere only to hypothesis (1) are referred to as 'weak' constructivists, whereas, those who accept both hypothesis (1) and (2) are termed 'radical' constructivists. The weak constructivist hypothesis is uncontroversial in comparison to the radical version, but there are still some aspects of the weak hypotheses that need to be discussed further.

Weak constructivism implies that the emphasis in the classroom should be on mathematical activity not passive reception (Orton, A., 1992). The issue is what is meant by mathematical activity? Mathematical activity can range from physical manipulation of concrete objects to mental activity resulting from listening to a lecture. Within this range of mathematical activities, it is the job of the constructivist teacher to look for a balance. This attempt to balance mathematical activities produces a tension between constructivism and representationalism (Cobb, et al., 1992; Orton, R., 1995; Steffe & Kieren, 1994).

The representational perspective is that learning is a process of acquiring fixed mathematical structures which are independent of the learner (Orton, R. 1988 & 1995; Steffe & Kieren, 1994). The tension arises when the constructivist educator attempts to help the student construct a meaning for a mathematical concept which is an exact representation of the educator's construct. The educator often has difficulty holding back from direct interference and 'telling' the student and therefore appears to fall back to the representational model. Cobb, Yackel and Wood (1992) attempt to address this tension by arguing that constructivist theory should not be interpreted as implying that students' learning must be natural and that teachers should not tell them anything as they attempt to create meaning out of the world. Rather, the teacher should be free to specify mathematical relationships for the student. Cobb et al. as well as Steffe and Kieren and R. Orton suggest that students must construct their mathematical knowledge in any setting whatsoever, including the traditional lecture format.

Radical constructivism has been criticized as a solipsistic position. If radical constructivism does imply that there is no world outside the mind of the knower, then this may be true. Stephen Lerman (1989) and Steffe and Kieren (1994) argue that radical constructivism does not attempt to link 'understanding' to certain and absolute mathematical concepts, but rather it recognizes that we interact with human beings and it is through this interaction that mathematical realities are constructed by the individual.

Although there is no certain absolute knowledge, there may be, as von Glasersfeld (1991) explains it in Orton, A. (1992), a consensual domain:

> If...people look through distorting lenses and agree on what they see, this does not make what they see any more *real* - it merely means that on the basis of such agreements they can build up a consensus in certain areas of their subjective experiential worlds...one of the oldest [such area] is the consensual domain of numbers. (p. 124)

Curriculum Development Based on Constructivism

Although there is a large body of research associated with constructivism as a learning theory and its relationship to Piaget's stage theory of cognitive development (Steffe & Kieren, 1994), there appears to have been little work done on constructivism and curriculum development. Most studies, for example Roper and Carter's (1992) discussion of Great Britain's National Curriculum, deal with constructivism and curriculum development by presenting research on the learning theory and its associated teaching practices and then discussing curriculum development independent of the teaching practices.

At present one research paper has been uncovered which specifically deals with curriculum development in the context of constructivism. Wheatley, Blumsack and Jakubowski (1995), in a paper presented to the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, describe the use of radical constructivism and the NCTM Professional Standards as the basis for curriculum reform in two university mathematics courses (geometry and problem solving). Wheatley et al. report four significant findings based upon their qualitative research:

1) students were challenged to rethink mathematics concepts previously studied but not understood;
2) students developed confidence in their mathematics knowledge;
3) students became more positive as mathematics learners; and,
4) students increased their competence and made connections among algebra, geometry, and calculus concepts. (p. 7)

The results reported in this study are intriguing as they relate significantly to the subject of this study. Unfortunately, the authors' qualitative research procedures are not clearly indicated and there is an absence of quantifiable data to triangulate their findings.

Lochhead (1992) details the use of constructivism as the primary instructional philosophy in a number of American high schools in an article in Educational Studies in Mathematics. Lochhead reports on the Ventures program which is described as an ambitious project involving roughly 40 high schools. The project is designed to prepare students for admission to selective four-year colleges and is based on four key elements:

1. Students are expected to master the program of study. This is expected of all students including those that were previously assumed to be incapable of benefiting from such rigor;
2. Teachers design instructional sequences that provide students the opportunity to build their competence and master academic material;

3. Students are made responsible for their own learning; and,
4. The role of teachers is to nurture and lead students to take control of their own education.

Lochhead reports that students exposed to this environment increased their achievement in mathematics and that college enrollment increased.

Constructivist Oriented Teaching Models

Although constructivism provides a useful framework for viewing mathematics learning, many believe that it does not explicitly tell teachers how to teach mathematics (Simon, 1995). The following is a brief review of some of the teaching models that have been developed using constructivism as its basis. The review includes a summary of the common methodological elements of the various models along with their effectiveness based upon the research evidence provided.

Pirie and Kieren (1992) identify the following four beliefs about teaching, the classroom, and students which provide the basis for creating a constructivist environment where effective learning of mathematics can take place:

1. Students' progress towards particular mathematics learning goals may not be achieved by some and may not be achieved as expected by others. This suggests that teachers must continually re-create the environment in order to accommodate students' changing understandings.
2. It is necessary for the teacher to act on the belief that there are different pathways to similar mathematics understanding. The result of this is that there is no specific form

or sequence of instruction which guarantees student understanding in a constructivist environment.

3. Different people hold different mathematical understanding. This implies that students' understandings of particular mathematical concepts are not the same, nor are they the same as the teacher's, mathematician's, or textbook writers'. An understanding of a topic is not an acquisition, but is an ongoing process unique to each student.
4. There are different levels of understanding for each topic and these levels are never achieved 'once and for all'. This tenet, like the previous two, is concerned with the growth of students' mathematical understanding. Pirie and Kiren identify eight levels of mathematical understanding: primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventing. The various levels of understanding have embedded in them previous layers which can be accessed to permit a back-and-forth movement between activities at the different levels. The implication of this for teachers is that although two students may appear to exhibit the same understanding of a mathematical topic, this may not be the case. To compensate for this the teacher must prompt students to justify what they say in order to reveal their thinking and logic.

Pirie and Kieren argue that the above four tenets are intended to guide teachers in developing a constructivist classroom environment through a set of constructivist beliefs in action. They also argue that since "a teacher is consciously responding to the diversity of student constructions, any of a variety of instructional acts might be appropriate" (p. 509).

Pirie and Kieren collected data during detailed observations taken in classes of 8 year olds and 12 year olds working on the topic of fractions. Seven specific teaching episodes were analyzed and although Pirie and Kieren did not themselves summarize the teacher activities as a whole, the following constructivist methodologies can be observed:

1. Teachers ask their students to articulate their understanding so that future interactions may be directed and modified to further guide the students' learning;
2. Teachers use activities which extend their students' understanding of previous concepts;
3. Teachers use provocative teaching acts (pushing students to outer levels of understanding) or invocative teaching acts (encouraging students to use prior, simpler experiences to understand) in order to extend student knowledge; and,
4. Teachers allow students to select activities meaningful to them when learning mathematical concepts.

Another teaching model based upon constructivism is offered by Simon (1995) in Reconstructing Mathematics Pedagogy from a Constructivist Perspective. Simon conducted a teaching experiment as part of the Construction of Elementary Mathematics (CEM) Project, a 3 year study of the mathematical and pedagogical development of prospective elementary teachers. The project studied an experimental teacher preparation program designed to increase pre-service teachers' mathematical knowledge and encourage the development of positive views of mathematics. The data were collected on 26 prospective teachers over a 5-week pre-student-teaching practicum and a 15-week student-teaching practicum.

One of the results of the CEM Project was the development of the Mathematics Teaching Cycle as a schematic model of the cyclical interrelationship of aspects of teacher knowledge, thinking, decision making, and activity. Simon describes a *hypothetical learning trajectory* which "assumes that an individual's learning has some regularity to it and that the classroom community constrains mathematical activity often in predictable ways, and that many of the students in the same class can benefit from the same mathematical task" (p. 135).

The hypothetical learning trajectory provides the teacher with a rationale for choosing a specific instructional methodology, but is termed hypothetical because the true learning trajectory can not be known in advance and is based, in part, on the teacher's knowledge as well as an assessment of students' prior knowledge. There are three components to the hypothetical learning trajectory: the learning goal that defines the direction; the learning activities; and, the hypothetical learning process. As with the Pirie and Kieren model, the Mathematics Teaching Cycle is an iterative process whereby the teacher is constantly using student responses to modify his or her teaching on an ongoing basis. The Mathematics Teaching Cycle (Figure 1) is shown on the next page.

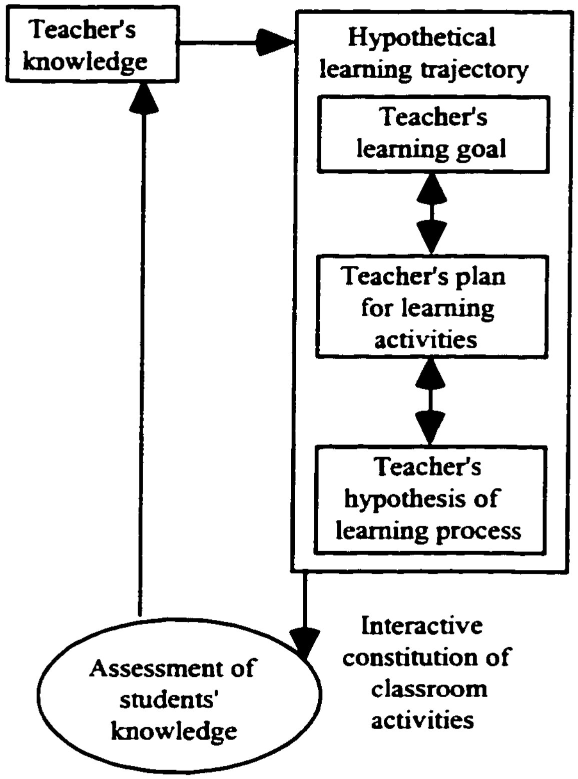

Figure 1. Simon's Mathematics Teaching Cycle

Simon concludes by identifying several themes related to the decision making process used by teachers who use constructivism to meet the challenges of the classroom:

1. Students' thinking and understanding is taken seriously and given a central place in the design and implementation of instruction. Understanding students' thinking is a continual process of data collection and hypothesis generation.
2. The teacher's knowledge evolves simultaneously with the growth in the students' knowledge. As the students are learning mathematics, the teacher is learning about mathematics, learning, teaching, and about the mathematical thinking of his students.

3. Planning for instruction is seen as including the generation of a hypothetical learning trajectory. This view acknowledges and values the goals of the teacher for instruction and the importance of hypotheses about students' learning processes.

4. The continually changing knowledge of the teacher creates continual change in the teacher's hypothetical learning trajectory. (p.141)

In "Toward a Working Model of Constructivist Teaching: A Reaction to Simon", Steffe and D'Ambrosio (1995) compare their model of constructivist teaching to that proposed by Simon. Steffe and D'Ambrosio maintain that there is such a thing as 'constructivist teaching' which is more than using different instructional designs within a constructivist framework as Simon (1995) asserts. Although Steffe and D'Ambrosio agree with Simon that posing problems or tasks is a principal strategy of a mathematics teacher, they prefer to replace 'problems or tasks' with 'situations' with the understanding that situations also include genuine problems. They also include situations which lead to generalizing assimilation (or transfer of learning) and functional accommodations.

Simon's hypothetical learning trajectory is viewed by Steffe and D'Ambrosio as compatible with their model, especially as it includes a basic tenet of constructivism that knowledge is not passively received but is actively built up by the cognizing subject. Steffe and D'Ambrosio believe that, "Simon's emphasis on the social processes involved in teaching mathematics makes it quite difficult to focus on the mathematics of his students" (p. 153).

In their model, Steffe and D'Ambrosio use the phrase "zone of potential construction" to refer to a teacher's working hypotheses of what the student can learn.

This again is consistent with Simon's concept of a hypothetical learning trajectory where Steffe's and D'Ambrosio's zone of potential construction (shown in Figure 2 below) is an implicit part of the hypothetical learning trajectory.

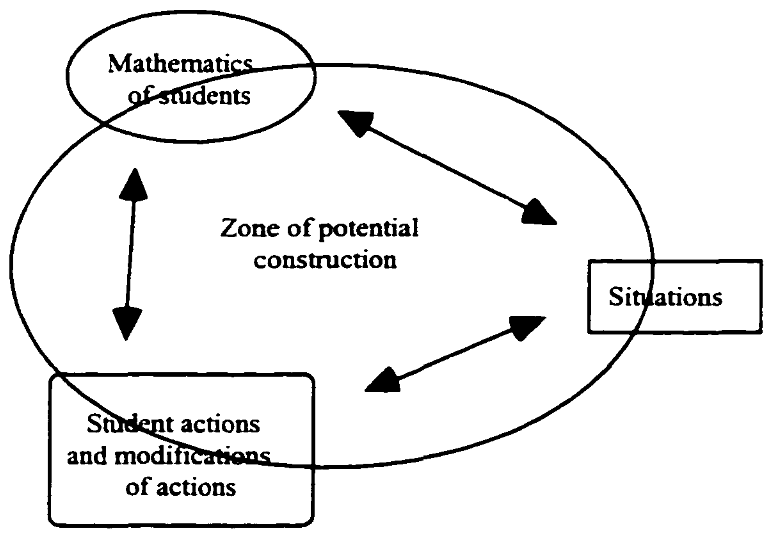

Figure 2. Steffe's and D'Ambrosio's Zone of Potential Construction as Part of Simon's Hypothetical Learning Trajectory.

The principal issue appears to be that Simon's model maintains that the purposes and the means of posing situations and encouraging reflection would be modified by teachers as their knowledge changes as the result of being involved in the culture of the mathematics classroom. Steffe and D'Ambrosio advocate that situations be posed by teachers to bring forth, sustain and encourage, and modify the mathematics of students.

The book In Search of Understanding: The Case for Constructivist Classrooms by Brooks and Brooks (1993) is of particular interest as it provides explicit strategies for educators wishing to connect theory to practice. Brooks and Brooks provide detailed descriptions of both classroom practice and its underlying theoretical connections. They provide five overarching principles of constructivist pedagogy:

Principle 1: Teachers should pose problems of emerging relevance to students. Brooks and Brooks note that relevance of a problem does not have to be pre-existing for the student, but can emerge through teacher mediation. Of critical importance is that when posing problems for students to study, teachers should avoid:

- isolating the variable for the students;
- providing the student with more information than they need or want; and,
- simplifying the complexity of the problem too early.

Constructivist teachers should ask one big question and give the students time to think about it and lead them to the resources to answer it. This is in opposition to the usually prescribed scope, sequence, and timeline of most state or provincial curricula, which Brooks and Brooks state often interfere with teachers' ability to help students understand complex concepts.

Principle 2: Students' learning should be structured around primary concepts. Teachers need to organize information around conceptual clusters of problems and questions. Brooks and Brooks maintain that a holistic presentation of problems helps students become more engaged than if the same problem were presented in isolated parts. Two different examples of conceptual clusterings are offered:

- conceptual themes - centered around the big ideas of a course; and,
- polar conflicts - which can be applied in all subject areas to serve as the big ideas around which investigations can take place (e.g., independence / interdependence / dependence, impulsivity / reflection, individual / group, etc.).

Brooks and Brooks explain their reasoning as follows:

> Problems structured around "big ideas" are intended to provide a context in which students learn component skills, gather information, and build knowledge.
>
> Attempts to linearize concept formation quickly stifle the learning process. (p. 49)

Principle 3: Seeking to understand students' points of view is essential to constructivist teaching. Brooks and Brooks take the position that the existence of other perspectives must be acknowledged and that many "truths", which are commonly accepted, should be reflected upon. They maintain that the teacher's role is one of talking *and* listening and that listening is at least as important a teaching skill as talking. To accomplish this the teacher must be prepared to ask questions which reveal the students conceptions and encourages their students to reflect on their conceptions. One method of accomplishing this is for the teacher to ask students to elaborate on their responses, but to do so in a non-challenging manner (i.e., make the request for elaboration a regular part of the classroom and show students that their responses are desired and respected.)

Principle 4: Learning is enhanced when the teacher adapts the curriculum to address students' suppositions. This includes the curriculum's cognitive, social and emotional demands on students. Brooks and Brooks embrace Piaget's stages of cognitive development but take them a step further because, " categorizing students' general abilities does not help teachers in developing appropriate instructional strategies for particular topics and concepts because at any one point in time, people use several different cognitive structures" (p. 71). This results in curricular adaptations which address student suppositions based, not upon a single cognitive categorization, but upon an understanding of the cognitive demands of the curricular tasks (or questions being asked) and an understanding of the student's present understanding of the concept (as gained through the practice of principles 1, 2, & 3).

Adapting the curriculum does not necessarily take the form of reducing the content or changing the order of presentation. It can also consist of changing the activities the student is involved in to make them more relevant. Brooks and Brooks point out, that only by listening to the student, can teachers effectively collect the needed information regarding cognitive and affective functioning and subsequently adapt their teaching methodologies as necessary.

Principle 5: Assessment of student learning should be made in the context of teaching. Rather than simply indicating that a student's response is 'right' or 'wrong' Brooks and Brooks believe that teachers should listen to the student and use the response to assess their present level of understanding and adapt the curriculum (and instructional methodology) as needed. The use of nonjudgmental feedback is stressed by Brooks and Brooks. They are quick to admit that it is difficult to structure assessment in this manner as schools are accustomed to the use of tests and grades. Since constructivist teachers provide meaningful contexts in which learning is encouraged, it is important that they use authentic tasks to assess their students. As tests are structured to determine what information a student knows in relation to a body of knowledge, the focus is on the material and not on the student's personal constructions. Brooks and Brooks therefore suggest that meaningful tasks be devised to assess a student's understanding. This has the advantages that:

1. Learning continues while assessment occurs;

2. The teacher can differentiate between what the students have memorized and what they have internalized; and,
3. It models that there can be multiple paths to the same end which are equally valid.

Brooks and Brooks (1993) offer the following set of twelve descriptors of what would characterize a successful constructivist teacher:

1. Constructivist teachers encourage and accept student autonomy and initiative;
2. Constructivist teachers use raw data and primary sources along with manipulative, interactive, and physical materials;
3. When framing tasks, constructivist teachers use cognitive terminology such as "classify", "analyze", "predict", and "create";
4. Constructivist teachers allow student responses to drive lessons, shift instructional strategies, and alter content;
5. Constructivist teachers inquire about students' understandings of concepts before sharing their own understandings of those concepts;
6. Constructivist teachers encourage students to engage in dialogue, both with the teacher and with one another;
7. Constructivist teachers encourage student inquiry by asking thoughtful, open-ended questions and encouraging students to ask questions of each other;
8. Constructivist teachers seek elaboration of students' initial responses;
9. Constructivist teachers engage students in experiences that might engender contradictions to their initial hypotheses and then encourage discussion;
10. Constructivist teachers allow wait time after posing questions;

11. Constructivist teachers provide time for students to construct relationships and create metaphors; and,
12. Constructivist teachers nurture students' natural curiosity through frequent use of the learning cycle model.

Brooks and Brooks conclude by suggesting that the following six changes must be made by educational institutions (including schools, school boards, provincial/state authorities and national authorities) before constructivist classrooms can be successful:

1. Structure preservice and inservice teacher education around constructivist principles and practices;
2. Jettison most standardized testing and make assessment meaningful for students;
3. Focus more on teachers' professional development than on textbooks and workbooks;
4. Eliminate letter and number grades;
5. Form school-based study groups focused on human developmental principles; and,
6. Require annual seminars on teaching and learning for administration and school board members.

The Effects of Ability Grouping on Student Academic Achievement and Self-concept

A review of relevant research dealing with student streaming in secondary schools is necessary as the three stream course structure for which the Applications of Mathematics 10 curriculum was developed attempts to address several issues related to student achievement, learning and teaching styles, and placement of students into alternate 'pathways' of mathematics education. The Mathematics 8 to 10 IRP (1996) states that, "Each student's choice will be guided by personal needs, interests, and career aspirations" (p. 2). The following review is intended to show that this approach to streaming is unique and therefore requires further study.

The review identifies issues concerning student streaming in secondary mathematics classes including: alternatives to streaming; the effects of ability grouping on student academic achievement (AA) and self-concept (SC); the effects (other than those related to student achievement or student self-concept) that student streaming has on the education system; and, a survey of current streaming practices within Canada and other countries.

Streaming (also termed ability-grouping or tracking) is the grouping of students, either by school or classroom organization in order to reduce the heterogeneity of a given group (see for example: Kulik & Kulik, 1982; Wolff, 1989; Reuman, 1989; Slavin, 1993). The assignment of a student to a specific class or group within a class is usually based upon some combination of composite achievement, IQ, and teacher judgments (Slavin, 1993). Streaming can be categorized into two main types:

1. Between-class grouping occurs when students at a given grade are assigned to a certain class on the basis of their academic performance. Examples of between-class grouping include:
    - ability-grouped class assignment - students are assigned on the basis of ability or achievement to one or more self-contained classes;
    - block scheduling - students spend all or most of the day with one homogeneous group of students. This is reportedly more common in middle schools than in senior secondary schools (Slavin, 1993);

- assigning students to "academic", "general" or "vocational" tracks or to "advanced", "basic" or "remedial" tracks. This type of streaming often involves different courses or course requirements and is most common at the senior secondary level;
- special classes for low or high achievers;
- Joplin Plan - this is an American program designed to group elementary students heterogeneously except for reading where they are regrouped across grade lines;
- nongraded plans - grade-level designations are removed entirely and students are placed in flexible groups according to their performance level, not their age; and,
- assigning students to ability-grouped classes for all academic subjects, but allowing for some variation in student placement (high or low) within those subjects.

2. Within-class grouping occurs when students within a given class are assigned to certain groups based upon their past performance. Mason and Good (1993) and Slavin (1987a, 1987b) describe the following examples of within-class grouping:
   - ability grouping for selected subjects (e.g., reading or mathematics) - students in heterogeneous classes are regrouped according to achievement for specific subjects. These groups work on different materials at rates specific to each group;
   - a variation of the above occurs when the teacher presents a lesson to the class as a whole and then forms groups to provide enrichment or extension to high-achievers or remediation to low achievers;
   - group-paced mastery learning is a form of flexible within-class grouping where students are placed into "masters" and "non-masters" groups after each lesson.

Placement is based upon a formative test and remediation or enrichment takes place subsequent to that; and,

- an extreme form of within-class grouping is continuous progress where each student essentially becomes an ability group of one.

Brewer, Rees, and Argys (1995), Slavin (1990a), and Kulik and Kulik (1982) all report that at the secondary level the overwhelming method of student streaming is between-class grouping.

For the past 80 years the arguments for and against streaming have remained consistent. Turney (1931) as reported in Slavin's "Achievement Effects of Ability Grouping in Secondary Schools: A Best-Evidence Synthesis" (1990a) summarizes writings of the 1920's listing the main reasons for ability grouping as follows:

1. It permits pupils to make progress commensurate with their abilities.
2. It makes possible an adaptation of the technique of instruction to the needs of the group.
3. It reduces failures.
4. It helps to maintain interest and incentive, because bright students are not bored by the participation of the dull.
5. Slower pupil's participate more when not eclipsed by those much brighter.
6. It makes teaching easier.
7. It makes possible individual instruction to small slow groups. (p. 473)

Turney also reported that the main arguments against ability grouping were as follows:

1. Slow pupils need the presence of the able students to stimulate them and encourage them.
2. A stigma is attached to low sections, operating to discourage the pupils in these sections.
3. Teachers are unable, or do not have time, to differentiate the work for different levels of ability.
4. Teachers object to the slower groups. (p. 473)

Interestingly, in 1997 the Province of British Columbia released a working paper (Addressing Student Differences: Next Steps) which looked, in part, at the issue of student streaming in British Columbia's secondary schools. Part of the Ministry's paper includes the results of field responses concerning the possible elimination of British Columbia's low-ability mathematics stream (Mathematics 9A to 11A). The comments submitted by teachers responding to Addressing Student Differences: Next Steps essentially mirror those reported by educators of Turney's generation (about 70 years ago).

Recently most discussions for and against streaming have centered on two principle issues:

1. Student achievement in streamed courses. Proponents for streaming (e.g., Allan, 1991; Feldhusen, 1991; Hallinan, 1990; Kulik & Kulik, 1982) maintain that although student streaming does not necessarily improve lower ability student achievement, it does not diminish it and the positive effect on higher-ability student achievement is

desirable. Opponents (e.g., Gamoran, et al., 1995; Slavin, 1990a; Slavin & Karweit, 1985) point out that the quality of instruction in lower-ability classes is poorer than in higher-ability classes and as a result (combined with resulting lower expectations) may have an impact on lower-ability student achievement which was not previously accounted for.

2. Student self-concept in streamed courses. Critics of ability grouping (Barquet, 1992; Byrne, 1990; French & Rothman, 1990; Gamoran & Berends, 1987; Oakes, 1987) maintain that the demoralizing effects of being placed in a lower stream in addition to lower expectations, and poor behavioral models contribute to a higher rate of delinquency, absenteeism, and dropout for those students. Barquet (1992) takes this a step further and maintains that streaming is often based on biased placement practices which denies equal access to education for minorities.

Slavin (1990a) insists that the burden of proof is on those that view streaming as effective even if it is considered distasteful. If streaming by ability is to be accepted it must be proven that there is a definite positive increase in student achievement (for specific groups) without a corresponding decrease (for other groups) in student achievement, equity, or self concept. On the other hand, anti-streaming groups can provide arguments to discontinue streaming even if it was proven not to have adverse effects on student achievement.

Other researchers who have attempted to determine if student self-concept is adversely affected by ability grouping include Feldhusen (1989) and Kolloff and Feldhusen (1984). Their studies have noted that gifted students' self concept is not diminished when placed in high-ability groups. In fact, Feldhusen (1989) concluded that through interaction with other gifted students in ability-grouped classes, talented youth have a better chance of developing a positive self-image than in a more heterogeneous environment.

Alternatives to Streaming

The following is a brief description of three alternatives to streaming that have been put forward by various critics of this practice.

1. Heterogeneous Grouping - Midkiff, Towery and Roark (1991) and Manning and Lucking (1990) advocate heterogeneous grouping as an alternative to streaming. Classes and groups of students are organized with a mixture of learners at all ability levels. The onus is placed on the teacher to adapt the learning environment to meet the needs of the students.
2. Cooperative Learning - A number of researchers have suggested the use of cooperative learning as an alternative to streaming. Some of the supporters of cooperative learning include Legters and McDill (1995), Slavin (1993, 1990a, 1990b, 1987a, 1987b), Lebow (1992), Mills and Durden (1992), Manning and Lucking (1990), and Moore and Davenport (1988). Cooperative learning involves students working in small, heterogeneous groups. Students generally work together to help each other master material introduced by the teacher. Cooperative learning may also include groups working on projects or other activities where the students are finding or discovering information. Slavin (1993) reports that research on cooperative learning in the middle grades consistently shows positive

effects as long as two elements are maintained: group goals and individual accountability. Slavin also indicates that the use of cooperative learning has had "positive effects on such outcomes as self-esteem, race relations, acceptance of main-streamed students, and ability to work cooperatively" (p. 546). A study conducted by Slavin and Karweit (1985) comparing the effects of whole class, ability grouped and individualized instruction (modified to use a cooperative learning model) on student achievement found that individualized instruction resulted in significantly higher student achievement than whole group instruction, but not significantly different than within-class grouping instruction.

3. Mastery Learning - Slavin has also suggested that streaming could be replaced with mastery learning. Mastery learning has three variations:

   a) Group-based mastery learning - this involves testing students at the end of a series of lessons, and providing those students who do not achieve a pre-set mastery criterion with additional assistance while others do enrichment activities. This form of mastery learning is most commonly used in elementary schools (Slavin, 1987b).

   b) Individualized or continuous progress - Slavin (1987b) notes that this form of mastery learning, in which students are constantly re-grouped as they proceed at their own rates, has been found to be more effective than group-based mastery learning.

   c) Keller Plan - this is a form of mastery learning where students take as much time as they need to pass a series of tests covering the content of a course using self-study materials, peer tutoring, and/or lectures. This form of mastery learning is not commonly used in elementary or secondary schools, but is used extensively at the college level.

Student Streaming Effectiveness

As noted previously, there are two primary measures which have been commonly used to assess the effectiveness of streaming students in mathematics: student academic achievement (AA) and student self-concept (SC). The following is a sampling of the research based upon these two measures.

Assessing Streaming Based Upon Student Achievement

A total of fifteen papers that deal with student ability streaming in the context of student academic achievement were reviewed. Of the fifteen, eight focused on secondary students while the remainder focused on elementary students.

Secondary school studies.

Two papers that are most noteworthy are "Achievement Effect of Ability Grouping in Secondary Schools: A Best-Evidence Synthesis" by Slavin (1990a) and "Effects of Ability Grouping on Secondary School Students: A Meta-analysis of Evaluation Findings" by Kulik and Kulik (1982). Although both of these studies used similar meta-analysis techniques to look at a large number of research studies (Kulik & Kulik - 36 studies; Slavin - 29 studies), they came to opposing conclusions.

Kulik and Kulik concluded that grouping had positive effects on student achievement with the greatest Effect Size (ES) occurring in classes for talented or gifted students. They reported an average ES of .10 which they interpret as statistically significant but caution that this relatively low value also suggests that "factors other than grouping played a role in determining experimental outcomes" (p. 422).

Slavin, on the other hand, found that the median ES for the 29 studies included in his meta-analysis was effectively .00. This would indicate that grouping had no effect on student achievement.

Allan (1991) is critical of Slavin's findings pointing out that the criteria he used to select studies for inclusion in his best-evidence synthesis unnecessarily eliminated many of the types of studies included in Kulik and Kulik's meta-analysis. In addition, both Slavin's and Kuliks' work have a significant flaw which has been pointed out by Allan and Feldhusen (1991). They maintain that the use of standardized test scores as criterion measures may not provide the best measures of achievement for a given school system (curriculum). Kulik (1991) admits that both he and Slavin have shown that local tests provide a better measure than do standardized tests.

Other studies which use student achievement as a measure of ability grouping effectiveness include: Bode, (1996); Brewer, et al., (1995); Byrne (1990); Gamoran, (1992); Hallinan, (1994); and, Hoffer, (1992). Once again the results of these studies provide somewhat conflicting information.

Bode (1996) compared student achievement in eighth grade math classes between those that received within-class grouped instruction and those that received instructional tailoring. Bode concluded that within-class grouping and instructional tailoring had no effect on average eighth grade mathematics achievement but high achievers performed to their maximum without a negative effect on low-achievers.

Brewer, Rees and Argys (1995) studied the effect of detracking tenth grade mathematics students and concluded that "tracking *was* an important determinant of student achievement" (p. 212). They concluded that placement in a below-average math class, as compared to a heterogeneous class, resulted in a decrease in achievement of approximately five percentage points. Students placed in an above-average class gained roughly the same in achievement as the below-average students lost. The average student gained approximately two percentage points. In other words, by placing below-average students into a heterogeneous class their achievement would increase, but at the expense of the other students. These findings only reinforce the dichotomy between equity and excellence which student streaming raises. As Gallagher (1995) asks in a review of Brewer, Rees and Argys' work "Should students, regardless of past performance or current aptitude or vocational interests, be receiving identical curricular experiences in secondary education?" (p. 216).

Hoffer (1992) in a study comparing average student achievement growth from seventh to ninth grades in grouped and ungrouped schools found that, after controlling for student social background and initial levels of achievement, high-group placement had a weak positive effect on student achievement while low-group placement had a strong negative effect. Hoffer concludes that ability grouping does not benefit students as a whole and that alternatives, such as detracking or improving the opportunities for average- and lower-group students within an ability grouped system, be implemented and monitored. In related studies by Hallinan (1994) and Gamoran (1992) similar observations were made with regard to the positive effect of grouping on high-ability students. In addition Hallinan's and Gamoran's studies also looked at school differences in tracking effects and found that they were not consistent from school to school. The resulting inequity in student achievement, as it relates to ability-grouping, suggests that factors in addition to student achievement need to be considered when ability grouping/tracking practices and policies are being developed.

Finally, Byrne's (1990) study intended to assess the impact of general SC, academic SC, and AA in discriminating between low- and high-track high school students found that both AA and academic SC differentiated between low- and high-track students. Surprisingly, academic SC, rather than AA, was found to be the primary variable discriminating between low- and high-track students. Byrne interpreted this to mean that "low-track students, cognizant of the discrepancy between their school marks and those of their high-track peers, perceive themselves as academically less capable" (p. 178). Byrne is quick to point out that more importantly, students could not be distinguished on the basis of general SC. She suggests that this finding could be interpreted that although low-track students may know of their inferior academic ability, they place little value on academic achievement and it subsequently does not play a significant role in their overall concept of self.

The preceding three studies highlight what appears to be a central issue emerging around the debate on streaming. As mentioned previously, the recurring theme is the tension between equity and excellence. Many educators, politicians, parents and public in general want all of our students to have the exact same educational opportunities. There are also a great number of people in the same areas who want to make sure that students get the best education possible. The emerging question appears to be: Is this possible?

Elementary school studies.

Of the seven studies reviewed that look at ability streaming in the context of elementary student academic achievement three were authored or co-authored by Slavin (Slavin 1987a, 1987b; Slavin & Karweit, 1985). These three studies will be discussed at the same time as they all take the same position on the use of student streaming.

Slavin (1987a) conducted a best-evidence synthesis similar to that conducted at the secondary level. Four grouping plans were examined: ability-grouped class assignment, regrouping for reading or mathematics, the Joplin Plan, and within-class ability grouping. Slavin's conclusions include the following:

1. Ability-grouped class assignment does not enhance student achievement in the elementary school;
2. There is some evidence to suggest that regrouping for reading or mathematics by ability within grade levels can be instructionally effective if the level and pace of instruction is adapted to the achievement level of the regrouped class and if students are not regrouped for more than one or two different subjects;
3. The Joplin Plan and similar forms of non-graded grouping have had consistent positive effects on reading achievement; and,

4. Within-class ability grouping results in positive effects on mathematics achievement if the number of groups is kept small.

Slavin (1987a, 1987b) suggests that if ability grouping is decided upon in an elementary school, the following features should be incorporated:

> 1. Students remain in heterogeneous classes most of the day and are regrouped by performance level only in such subjects as reading and mathematics in which reducing heterogeneity is particularly important.
> 2. The grouping plan reduces heterogeneity in the specific skill being taught.
> 3. Group assignments are flexible and are frequently reassessed.
> 4. Teachers adapt their level and pace of instruction in regrouped classes to accommodate students' levels of readiness and learning rates. (p. 116)

Slavin's and Karweit's (1985) work reinforces the use of within-class ability grouping and cooperative learning strategies as compared to whole-class instruction to increase student achievement. As with the secondary student achievement findings, these results include some surprises which lead to possible contradictory interpretation. Slavin and Karweit report that the positive effects of individual instruction and within-class ability grouping were the greatest for average students rather than those farthest from the mean and in settings with the greatest degree of student heterogeneity (as the programs were designed for).

Hallinan and Soerensen (1985) looked at student achievement in the context of class size and ability group size. They conclude that in ungrouped classes the size of the class has no significant effect on student achievement while in ability-grouped classes the class size has an inverse effect on student achievement. This supports the belief that smaller class sizes are more conducive to learning than large ones.

In contrast to Hallinan and Soerensen's findings, Mason and Good (1993) studied the effects on fifth and sixth grade student mathematics achievement focusing on whole class instruction versus within-class ability grouped instruction (two groups). Mason and Good chose nine schools from among 16 in a mid-western district and randomly assigned students to one of three groups (two-group model, whole class ad hoc model, or two-group control model). Three independent variables were assessed including: treatment (two-group, whole class, two-group control); aptitude/achievement (high, average, low); and, grade (fourth, fifth, sixth). It was found that students in whole-class settings were provided more developmental time than those in two-group classes. In all areas of achievement assessed (computation, concepts, problem solving, and mental math/estimation) students experiencing whole-class instruction outperformed both those in the control group classes and the two-group classes with effect sizes ranging from +.13 to +.32.

Reuman (1989) found that when comparing between-class ability grouped students and within-class ability grouped students, students in low-ability groups received lower grades than students in low-ability classrooms. The same held true for students in high-ability groups who received higher grades than students in high-ability classrooms.

Wolff (1989) authored a literature review which focused on the effectiveness of ability grouping in grades kindergarten to six. She reviewed 28 studies in terms of types of grouping, and effects of ability grouping on behavior, self concept and academic achievement. Wolff's findings concerning the effectiveness of ability-grouping were inconclusive in that, as with the case of the literature dealing with ability grouping in secondary schools, different studies yielded contradictory results. Wolff concludes with eight recommendations concerning the use of ability grouping which are very similar to Slavin's noted previously.

Assessing Streaming Based Upon Student Self-concept

The literature reviewed on elementary or secondary school ability grouping is consistently ambiguous. No trends observed at this point should be considered without first discussing a second important factor, namely the impact of student streaming on student self-concept. Three of the studies reviewed previously contain information related to student self-concept in the context of ability grouping (Byrne, 1990; Kulik & Kulik, 1982; Wolff, 1989). In addition, two studies have been found that focus exclusively on this topic (Byrne, 1988; Kolloff & Feldhusen, 1984).

In Kulik's and Kulik's (1982) meta-analysis 15 studies that reported results on student self-concept were analyzed. The average effect size was found to be .01 with a standard deviation of .40. The Kuliks conclude that ability grouping has no effect on student self-concept. It should be noted that Kulik and Kulik also included attitudes toward subject matter and attitudes toward school in their meta-analysis and found that although there was no significant effect on attitudes toward school, there was a statistically significant average ES (.37, SD = .32) with regard to attitudes toward subject matter. It should also be noted that the data does not distinguish by subject and there is no way to be sure that the ES holds for mathematics classes.

Wolff's (1989) annotated bibliography cites three studies that looked at ability grouping and student self-concept (Abadzi, 1984; Eder, 1983; Van Fossen, 1987). All three studies concluded that self-concept for high-ability students increases while it decreases for low-ability students. Abadzi, as noted in Wolff (1989), states that students who felt they had been misplaced (i.e., had realistic expectations of being placed in a higher ability group, but were placed in an average-ability group) had a lower self-concept than their peers of average ability. Thus it would appear that it is not the placement of students in ability groups that may cause a student to develop a low self-concept, but rather the mis-placement of students into lower ability groups which results in low student self-concept.

Byrne (1988, 1990) conducted a study of four suburban, upper-middle class secondary schools in Ottawa to determine the impact of general self-concept, academic self-concept and academic achievement in discriminating between low- and high-track high school students. Byrne (1988, 1990) found that both student AA and academic SC were able to differentiate between low- and high-track students. An unexpected finding was that academic SC was more effective than AA in this differentiation. Confounding this was the finding that general self-concept appeared to be unaffected by track placement. Byrne (1988) suggests that "low-ability students place more importance on their social and/or physical, rather then their academic competencies. Accordingly, their SCs in these areas may be higher, or at least equivalent to those of their high track peers" (p. 23). Byrne (1990) concludes that although low-track students are aware of discrepancy between their school marks and those of their high-track peers and subsequently perceive themselves as academically less capable, the low-track students place little value on AA and it therefore has little effect on their general SC.

Studies cited have concentrated on the differences between low- and high-track students and the resulting focus has been on the effect of tracking on the low-ability students. It is also important to consider the impact of ability grouping on gifted students. Kolloff and Feldhusen (1984) attempted to determine the effects of an enrichment program on self-concept and creative thinking abilities of gifted elementary students (grades 3, 4, 5, & 6). The students (N = 420) were in the top seven percent based on achievement tests and teacher ratings and were randomly assigned to either an experimental or control group. Students in the experimental group participated in a pull-out program (Program for Academic and Creative Enrichment [PACE]) while students in the control group participated in their regular class at all times. Kolloff and Feldhusen found that students in the PACE program achieved significantly higher scores on two creative thinking variables (Verbal Originality and Figural Originality). There were no significant differences between the groups on two measures of self concept (*Piers-Harris Children's Self-Concept Scale* and the *ME Scale*).

It appears that ability grouping or tracking produces no pattern of improvement or decline on student self-concept (Rogers, 1993). It is likely that there are many personal, environmental, family, and other extraneous variables which affect self-concept more directly than tracking or ability grouping (Byrne, 1988; Fiedler, et al., 1993; Gamoran, 1992; Rogers, 1993.)

Additional Issues Related to Student Streaming

In addition to the impact of student streaming on student academic achievement and self-concept, it is important to review other issues that are potentially affected by a policy of ability grouping. The following is a summary of the recurring issues that appear in the reviewed literature.

Instructional Differences in Streamed Classes.

The issue of equity of access versus excellence, which is pervasive in the literature on student streaming, includes an alternative definition of equity. Do students receive the same type and quality of instruction in different streamed classes? A number of studies and surveys have been undertaken to study this question. The results have been very consistent on this question. In studies conducted by Gamoran, Nystrand, Berends, and LePore (1995), Oakes and Guiton (1995), Smith-Maddox and Wheelock (1995), Hallinan (1994), Gamoran (1993), Oakes (1992, 1987), and Moore and Davenport (1988), it has been consistently found that there are differences in curriculum content, instructional activities, and classroom climate. Oakes (1987) argues that these differences prevent low-ability streamed students from gaining access to high-status knowledge.

In a report to the National Center on Effective Secondary Schools, Moore and Davenport (1988) analyzed the placement policies of schools in New York, Chicago, Philadelphia, and Boston. Moore and Davenport concluded that in high-track classes, teachers:

> ...have more positive expectations for students, offer more challenging work at a faster pace, expect regular homework, expect more creative work, and expect more critical writing and discussion. Students in low-track classes, in contrast, are more likely to spend time completing worksheets and other exercises that teach discrete skills, listening to lectures, and doing individual seatwork. (p. 153)

Oakes (1992) notes that in elementary schools with ability grouping students in lower-ability groups fall further behind even when the grouping is for the purpose of "catching up." The resulting gap in prerequisites makes it very difficult for students to move from the lower to a higher track and only manages to widen the gap. In addition to the curriculum gap, Hallinan (1994) and Oakes and Guiton (1995) point out that not only are lower-ability students not exposed to the same quality of instruction as their higher-ability peers, they are more often taught by inexperienced teachers. The inexperience comes in the form of either subject inexperience or overall teaching inexperience.

The extent of teacher inexperience in British Columbia has been clearly identified in The 1995 British Columbia Assessment of Mathematics and Science (Marshall, et. al., 1996) which notes that seven percent of Mathematics 10A teachers are first year teachers compared to five percent of Mathematics 10 teachers. In addition 11% of Math 10A teachers reported having taken no post-secondary mathematics content courses compared to just two percent of Math 10 teachers. It was also noted by Marshall, et al., that:

> Teachers of Math 10 tend to specialize more in the teaching of mathematics as a subject than those of Math 10A. For example, only 4% of Math 10 teachers spend twenty percent or less of their time teaching the subject, compared to 15% of

> Math 10A teachers. Further, 63% of Math 10 teachers spend more than 80% of their time teaching mathematics, compared to 41% of Math 10A teachers. (p. 212)

In summary, the evidence suggests that there are inequities associated with streaming as it relates to the overall lower quality of instruction that low-ability students receive compared to that experienced by high-ability students. There does not appear to be any evidence which indicates that low-ability students in streamed classes experience similar mathematics curriculum and instruction, and general benefits of experienced teachers that high-ability students do. Even if there is student academic achievement to be gained with streaming, it appears to be gained at the cost of providing low-ability students with a less than suitable educational experience.

Student Placement Practices.

A third dimension of the equity of student streaming involves the placement process. The use of ability grouping within individual schools is largely dependent upon the preferences, priorities, and competence of the school's principal, department heads, counselors and other staff involved (Moore & Davenport, 1988). Oakes and Guiton (1995) conclude "that high school tracking decisions result from the synergy of three powerful factors: differentiated, hierarchical curriculum structures; school cultures alternately committed to common schooling and accommodating differences; and political actions by individuals within those structures and cultures aimed at influencing the distribution of advantage" (p. 30).

What then is the effect of these forces involved in student placement practices? The first effect is that the placement criteria are likely to be different from school to school (Gamoran et. al., 1995; Oakes, 1987). This could mean that the same student could be placed into completely different classes depending upon the school. Even though students' prior achievement affects track placement most strongly (Oakes, 1992), the cumulative effect of school characteristics (e.g., school size, entry criteria for particular tracks, scheduling practices) combine to increase the likelihood that a student with particular characteristics will be placed in particular classes.

Useem (1990) assessed parental involvement in student placement decisions for grade 7 mathematics students. She concluded that highly educated parents were more likely than less-educated parents to be involved in the placement of their children, and that parents with college degrees "succeeded much more often than non-college graduates in ensuring that their children were placed in a track of sequential courses that would lead to better preparation through the high school (and college) years" (p. 17). Useem interpreted this to mean that a student's placement also depended on his or her parents' willingness to ensure that their children had the opportunity to learn more advanced material even when this was counter to recommendations made by school personnel.

The student-assignment factors are ultimately so inconsistent that it is questionable whether the process is fair (Oakes, 1992). Oakes also questions claims about the neutrality of placements, arguing that race and social-class bias is so much a part of the school environment that course offerings and tracking policies have resulted in a curriculum which is designed to be appropriate for stereotypical racial and social-class backgrounds.

Monetary Cost of Streaming.

The most obvious direct monetary cost associated with the practice of ability grouping in schools is the process of placement. Moore and Davenport (1988) point out that, "In many schools logistical considerations were preeminent in assigning students to courses. Staying within budget; maintaining required class sizes; and juggling available classrooms and available teachers became the overriding administrative preoccupation." (p. 9) The direct salary costs of having administrators spending so much of their time on the placement process instead of on higher priority tasks (such as teaching) is significant.

There are also indirect consequences of streaming students which are ultimately born by the taxpayer. Ability grouping of students can predetermine their future opportunities and aspirations (French & Rothman, 1990). The result of this may be that, for some students, acceptance to post-secondary institutions is not a viable option until such time as they are able to complete the prerequisites for entrance. The cost of this (even if the student is paying tuition) is largely subsidized by the taxpayer.

Research (e.g., Barquet, 1992; Byrne, 1988; Oakes, 1987) has also shown that students in low-ability groupings have a higher incidence of dropout. The economic impact of having youth attempting to find meaningful employment and being able to contribute to society is very difficult to assess directly. No research has been uncovered that looks specifically at the monetary cost of streaming students, and as a result, this side effect seems to be overlooked when decisions with regards to ability grouping of students are made.

Status of Student Streaming in Canada and Other Jurisdictions

The following is a brief survey of student streaming practices in British Columbia and other provinces in Canada as well as in the countries of the United States, Japan, Great Britain, and Germany.

British Columbia

From 1989 to 1996 British Columbia had two streams of mathematics courses at the secondary level that consisted of:

1. Mathematics 8, 9, 10, 11, 12, often referred to as the 'regular' stream. This was a pre-calculus curriculum designed to meet graduation and post-secondary entrance requirements upon completion of Mathematics 11.
2. Mathematics 8, Mathematics 9A, 10A, 11A or Mathematics 8, Mathematics 9A, 10A, Introductory Mathematics 11, Mathematics 11, 12. The 'A' stream led to four possible routes to graduation (Math 11A, Intro Math 11, Math 11, or Accounting 11). Of the four possible courses, only Mathematics 11 met the majority of post-secondary entrance requirements. Approximately 25% of students in grade 9 province wide were (and still are) enrolled in Mathematics 9A while enrollment in Mathematics 10A is approximately 29% (Province of British Columbia, 1997).

The Curriculum and Resources Branch of the Ministry of Education, Skills and Training (Province of British Columbia, 1996a, 1996b) revised the "regular" stream into Mathematics 8, Principles of Mathematics 9, 10, 11, 12. The content and intent of these courses was largely unchanged. The Ministry also developed a new program of studies, Applications of Mathematics 9, 10, 11, 12 (Province of British Columbia, 1996b, 1996c). These courses, which are the focus of this program evaluation, were designed to offer a constructivist approach to learning mathematics. The following (Figure 3) is a schematic of the British Columbia secondary mathematics course structure for the period September 1996 to August 2001:

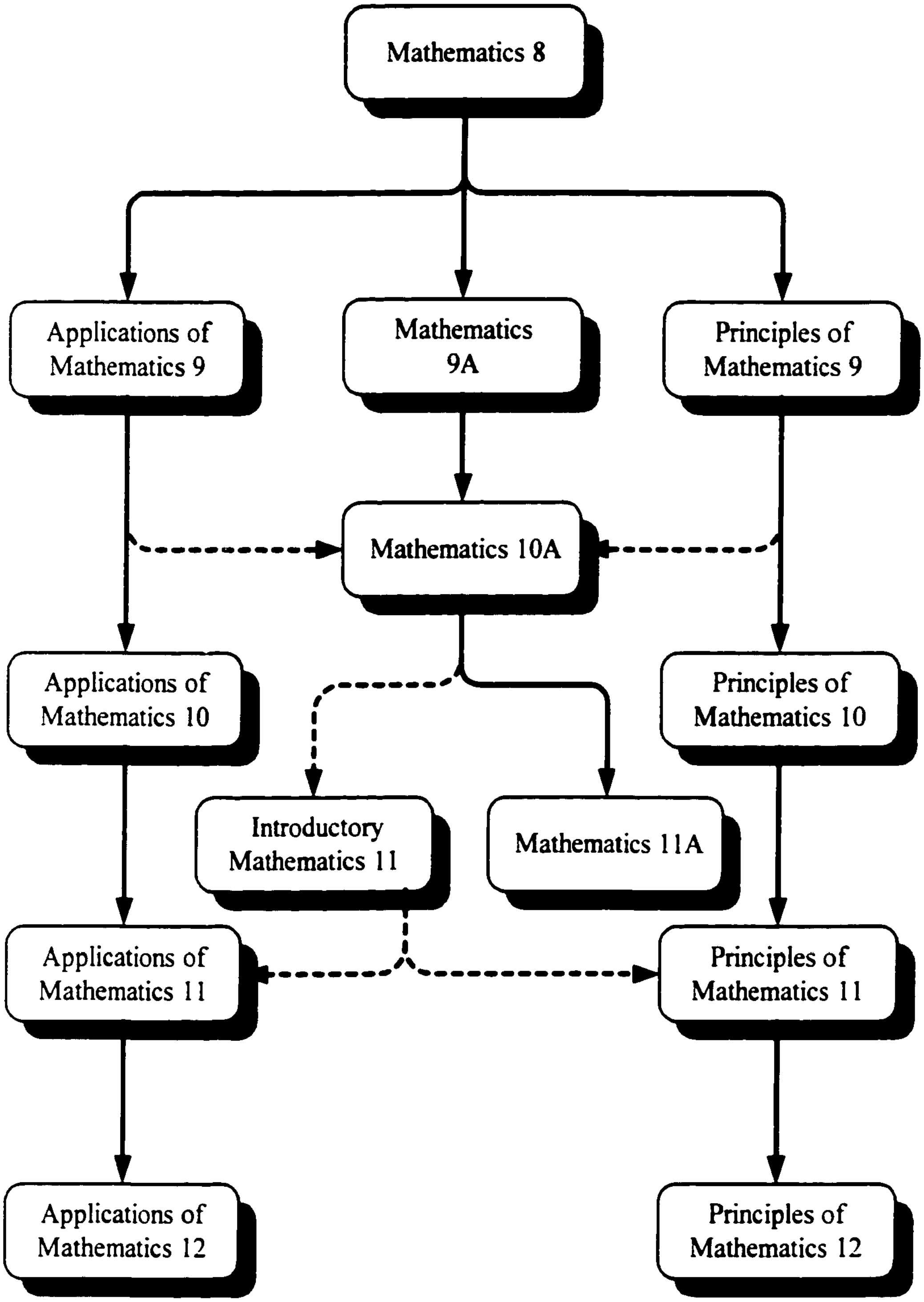

Figure 3. British Columbia's Secondary Mathematics Course Structure (September 1996 to August 2001)

Beginning in July, 1999 the Curriculum Branch began revising the curricula for both Principles of Mathematics 10 to 12 and Applications of Mathematics 10 - 12 to completely align with The Common Curriculum Framework for K-12 Mathematics (Western Canadian Protocol for Collaboration in Basic Education, 1995, 1996). The resulting curricula at the grade 10 level have greater content differences between the Application of Mathematics and Principles of Mathematics courses, but the AM courses maintain the emphasis on the use of constructivist teaching methodologies.

A Mathematics Task Force, appointed in January 1999 by the British Columbia Minister of Education, submitted a paper (The Report of the Mathematics Task Force, Province of British Columbia, 1999) that recommended that the present A-stream courses be replaced with a more relevant workplace oriented curriculum. The report recommended that new courses (Essentials of Mathematics 10 to 12) be developed with content and philosophy similar to those recently developed by Manitoba Education and Training (Consumer Mathematics Senior 2 to 4). The B.C. Ministry of Education accepted this recommendation and the curriculum for this third pathway was subsequently developed and incorporated into the Mathematics 10 to 12 Integrated Resource Package (Province of British Columbia, 2000b) and distributed to schools for implementation beginning September 2001. The resulting three stream secondary mathematics course structure is shown below in Figure 4:

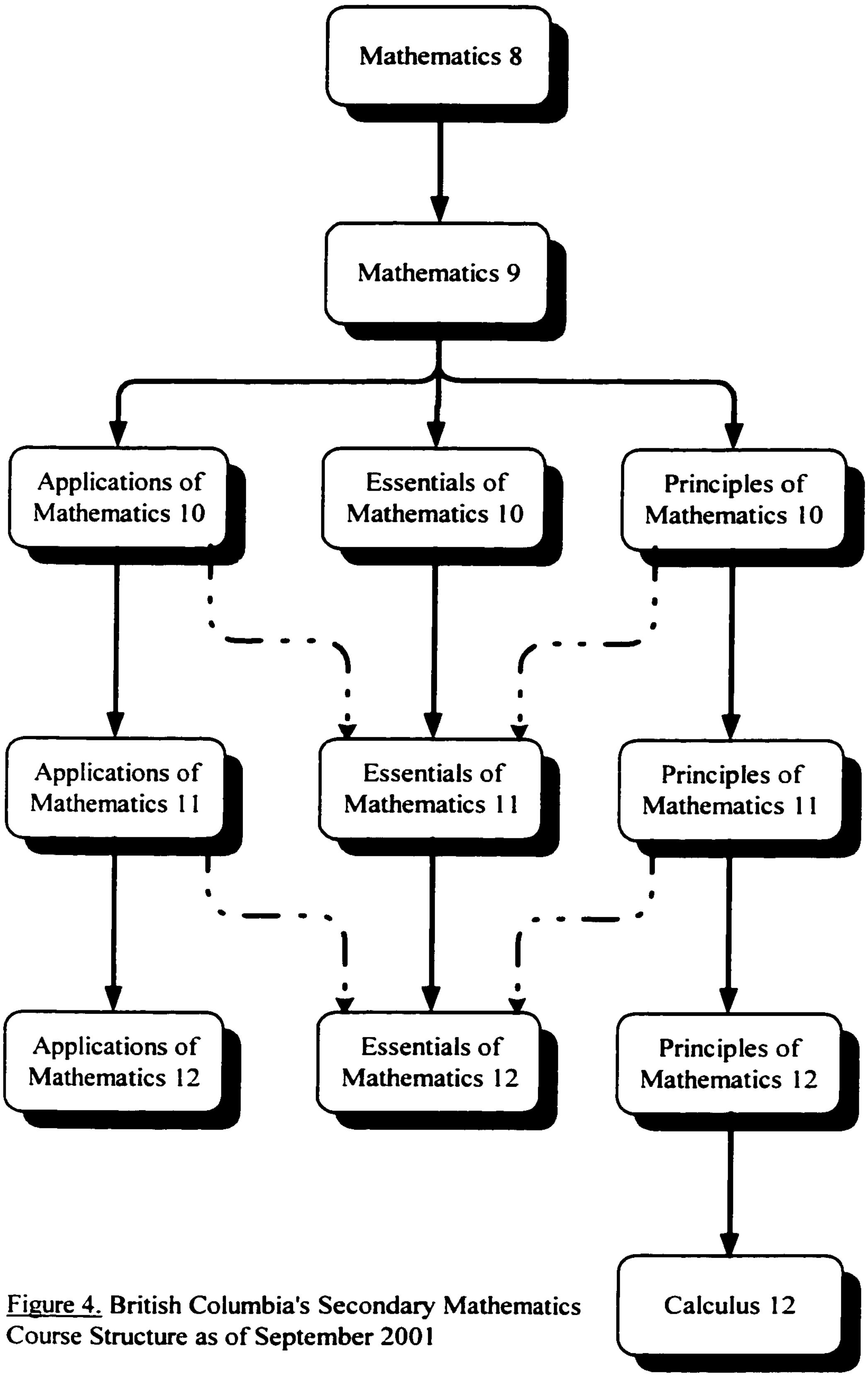

Figure 4. British Columbia's Secondary Mathematics Course Structure as of September 2001

Alberta

Presently Alberta has three streams in place at the secondary level (Grades 10 to 12) (Alberta Education, 1991):

1. Mathematics 14/24 - designed for that group of students whose needs, interests and abilities focus on basic mathematical understanding. Mathematics 24 meets the requirements for the General High School Diploma.
2. Applied Mathematics 10/20/30 - designed to provide students with a clearer picture of why they are learning the mathematics and motivates them in learning. The approach used in applied mathematics is "primarily data driven, using numerical and geometrical problem solving techniques" (Alberta Education, 1998a, p.1).
3. Pure Mathematics 10/20/30 - designed to emphasize theory and the testing of mathematical hypotheses. The pure mathematics courses "endeavor to show that concepts are valid all the time, or valid within a well-defined set of restrictions" (Alberta Education, 1998b, p. 1).

These courses (with the exception of Mathematics 14/24) were developed using The Common Curriculum Framework for K-12 Mathematics (Western Canadian Protocol for Collaboration in Basic Education; 1995, 1996). In personal communications with Mr. Hugh Sanders, Assistant Director of the Curriculum Standards Branch, Alberta Learning, Mr. Sanders has stated that some form of the stream consisting of Mathematics 14/24 will be retained and that it will be consistent with those developed in British Columbia and Manitoba.

Saskatchewan

In Charting the Course: A Guide for Revising the Mathematics Program in the Province of Saskatchewan Hope (1990) looked at three possible alternatives to the then current secondary program. The three structures reviewed were:

1. A modification of the current dual university entrance program structure.
2. A structure similar to British Columbia's and Alberta's two stream structure in use at that time.
3. A single program structure with multiple exits.

Hope recommended to Saskatchewan Education that a single program with multiple exits be adopted. This recommendation was accepted and as a result Saskatchewan has implemented a mathematics curriculum at the secondary level (Grades 10 to 12) which consists of Mathematics 10, 20, A30, B30, and C30 (Saskatchewan Education, 1995a, 1995b). Mathematics 20 is suitable for high school graduation while Mathematics A30 meets general university requirements and various combinations of Mathematics A30, B30 and C30 meet the requirements for post-secondary mathematics courses and entry to mathematically-dependent post-secondary programs.

Manitoba

Wayne Watt, Mathematics Coordinator, School Programs Division, Manitoba Education and Training (personal communication) has indicated that Manitoba will complete implementation of the Applied/Pure Mathematics course structure set out in

The Common Curriculum Framework for K-12 Mathematics (1995, 1996) by the year 2001. This will result in Manitoba offering the same secondary mathematics courses, Mathematics Senior 2, 3, and 4 (grades 10, 11, 12) and Applied Mathematics Senior 2, 3, and 4, as Alberta's Pure Mathematics 10, 11, 12 and Applied Mathematics 10, 11, 12 and British Columbia's Principles of Mathematics 10, 11, 12 and Applications of Mathematics 10, 11, 12. Manitoba has developed a third stream, Consumer Mathematics Senior 2, 3, and 4, for students that are unable to meet the expectations of either of the other two streams. Manitoba differs from the rest of the provinces and territories in Canada in that its graduation requirement is for students to successfully complete a grade 12 mathematics course. The rest of the Canadian jurisdictions require a grade 11 mathematics course for graduation.

Ontario

The province of Ontario is presently undergoing a large-scale educational reform. In a background research report commissioned by the Ontario Ministry of Education and Training, Mathematics: Secondary School Curriculum (Roulet, 1997) specific recommendations concerning streaming in mathematics are not made. The report looks at several different models including The Common Curriculum Framework for K-12 Mathematics and the Harvard Group's, Assessing Mathematical Understanding and Skills Effectively.

Ontario later released a curriculum document, The Ontario School, Grades 9 and 10: Mathematics (Province of Ontario, 1999), which indicated that Ontario has developed a grades nine and ten curriculum similar to British Columbia's. Starting in grade nine and continuing through to grade 10 there are two types of courses:

1. Applied courses - focus on the essential concepts of a subject, and develop students' knowledge and skills through practical applications and concrete examples. Familiar situations are used to illustrate ideas, and students are given more opportunities to experience hands-on applications of the concepts and theories they study.
2. Academic courses - develop students' knowledge and skills through the study of theory and abstract problems. These courses focus on the essential concepts of a subject and explore related concepts as well. They incorporate practical applications as appropriate.

The grade nine and ten mathematics curriculum has accordingly been broken into the following two streams:

1. Principles of Mathematics 9 and 10 - these Academic courses, as described in The Ontario Curriculum: Grades 9 and 10: Mathematics (1999), are designed to enable students to "engage in abstract extensions of core learning that will deepen their mathematical knowledge and enrich their understanding" (p. 9).
2. Foundations of Mathematics 9 and 10 - these Applied courses are designed to provide students with "opportunities to consolidate core skills and deepen their understanding of key mathematical concepts" (p. 18).

An unedited draft of The Ontario Curriculum, Grades 11 and 12: Mathematics, 1999 (Educational Computing Organization of Ontario, January 1999) describes five types of courses designed for various disciplines: university preparation, university/college preparation, college preparation, workplace preparation, and open. The university preparation, university/college preparation, college preparation, and workplace preparation courses are being designed in collaboration with the respective universities, colleges, or business community to meet entrance requirements for the respective post-secondary institutions, apprenticeship or training programs, or the needs of employers in the workplace.

Open courses are appropriate for all students and are not linked to any specific post-secondary institution. These courses are designed to allow students to "broaden their knowledge and skills in a particular subject that reflects their interests and that may or may not relate to their post-secondary goals" (p. 11).

A later draft of that document (Educational Computing Organization of Ontario, July, 1999) entitled Mathematics 2000: Grades 11 - 12: Curriculum policy document reports that the number of Grade 12 courses will be 7, including: Mathematics for Everyday Life; College and Apprenticeship Mathematics; Technology Mathematics; Calculus with Applications to Business and Social Sciences; Mathematics of Information Management; Mathematics, Science, Engineering Differential Calculus; and, Geometry and Discrete Mathematics. The most recent official publications concerning the Ontario Grade 11 to 12 mathematics curriculum, The Ontario Curriculum: Grades 11 and 12: Course descriptions and prerequisites, (Province of Ontario, 2000a) and The Ontario Curriculum: Grades 11 and 12: Mathematics, (Province of Ontario, 2000b) describe a secondary mathematics course structure (see Figure 5) which includes 4 grade 11 and 6 grade 12 mathematics courses identified to be implemented by September 2001.

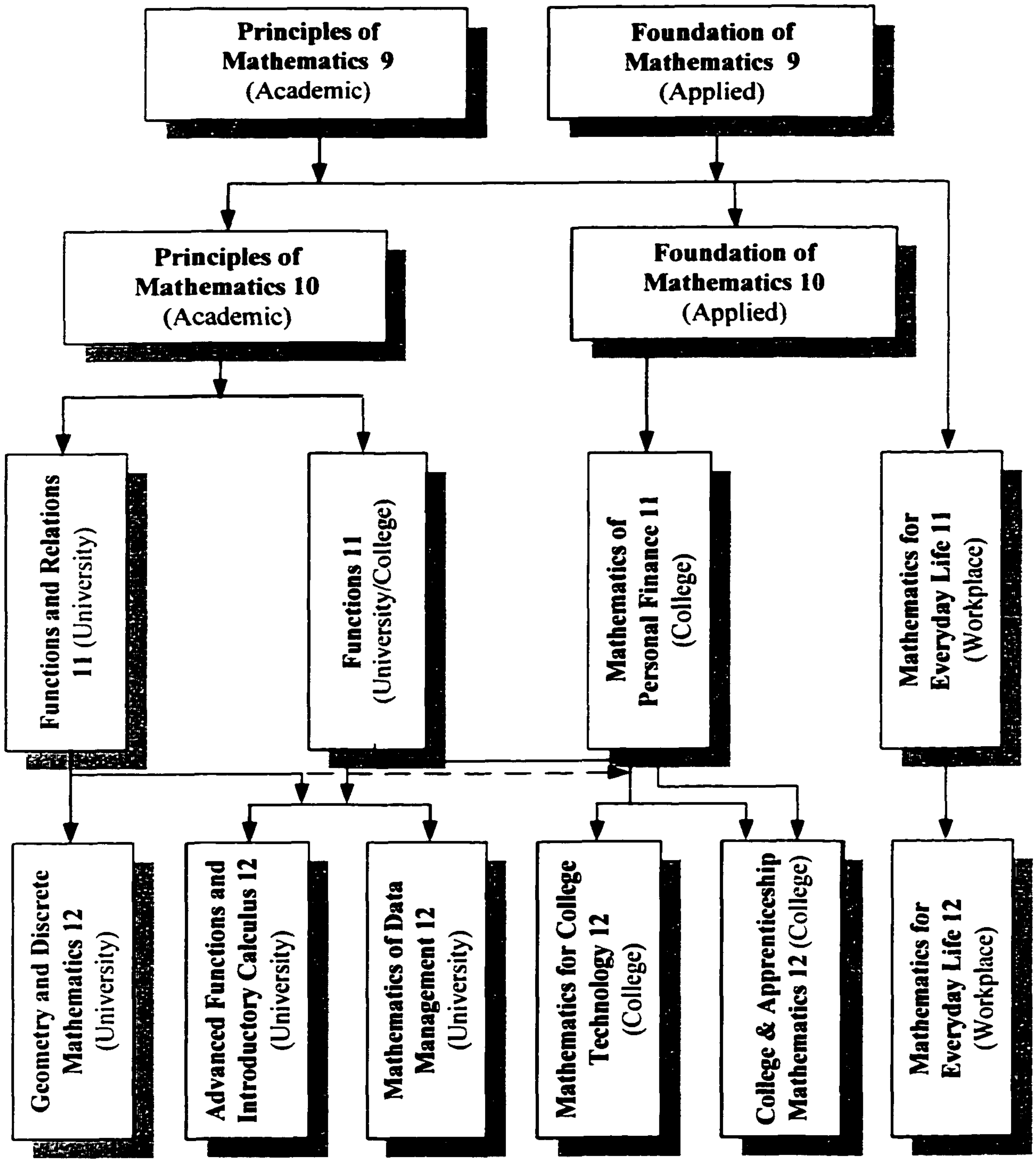

Figure 5. Ontario's Secondary Mathematics Courses Structure as of September 2001

Atlantic Provinces

The Departments of Education of New Brunswick, Nova Scotia, Newfoundland and Labrador and Prince Edward Island have produced the Foundation for the Atlantic Canada Mathematics Curriculum, (Atlantic Provinces Education Foundation, 1996) which is a framework document for mathematics curriculum development. The only statement found in this document that deals with the issue of ability grouping states that:

> ...a teacher is responsible for tailoring instruction to reach as many students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters but should be designed to help all students, whether strong, weak or average, to reach their highest potential. (p. 32)

Within Canada it appears that the various Ministries of Education are attempting to implement change with respect to the use of ability grouping.

Jurisdictions Outside Canada

Lofthouse (1996) conducted a selected review of the literature and current practice for the British Columbia Ministry of Education, Skills and Training and reported on the following countries:

United States.

Detracking and destreaming efforts are continuing in most American states but this appears to be an idealized goal that is being resisted more so in secondary schools than elementary schools, especially in mathematics.

Japan.

Nearly all students complete a rigorous core curriculum during nine years of schooling. Transition to Upper Secondary is achieved by passing an entrance examination. The highest achievers go to the most prestigious academic high schools while lower achievers go to vocational schools and failures go to night school or into the labor market.

Great Britain.

Throughout the eighties, there was a growing concern for the quality of education which resulted in the creation of a National Curriculum with extensive syllabi in each subject. The National Curriculum was introduced into schools in September 1995 and consists of four Key Stages. Each "stage" encompasses more time than the typical "grade". National assessments occur at the end of each stage. Foundation subjects (including mathematics) are mandated but the method of instruction is not and it is further recognized that a complete curriculum will go beyond the designated Foundation subjects to include other areas.

Germany.

There is no central government dictating specific educational policies. The 16 separate school systems issue broad guidelines, allowing local school districts to determine course content. A leaving examination, the Abitur, is required only if a student wished to enter university. There are three types of schools: The Gymnasium - university preparation; the Realschule - a practical alternative stressing math, science, modern languages, and occupational opportunities; the Hauptschule - a "second-class" technical school. Students are streamed at grade 5 into one of these schools and transfers can occur from school type to school type at the end of grades six and ten. Students in all three schools take essentially the same subjects, but with differing emphasis (Gymnasium - "pure" subjects; Realschule - "practical" subjects; and, Hauptschule - "applied" subjects).

As within Canada, the streaming practices of the various countries reviewed, with the exception of Great Britain, all use ability streaming to separate academic- and vocational-track students. Even in Great Britain where a single National Curriculum is presently being implemented, there is considerable resistance by teachers on the basis that many feel that the low-ability students will be unable to succeed in the relatively rigorous courses (Orton, 1994).

Applied Academics Program Evaluations

As the primary focus of this study is to conduct a program evaluation of the Applications of Mathematics 10 curriculum, it important that a thorough review of studies of similar programs be part of this literature review. There appear to be very few mathematics programs explicitly based on constructivism. There are, however, a number of program evaluations of a mathematics program developed by the Center for Occupational Research Development (CORD). CORD's program, Applied Mathematics, bears a striking similarity to British Columbia's Applications of Mathematics curriculum in that it uses constructivism as the basis for instruction and assessment. The Applied Mathematics programs are a subset of an educational reform commonly called applied academics.

The term 'applied academics' refers to courses which, as Hull (1993) describes them, "use teaching approaches such as hands-on, real world, apprenticeship, and work-based." Hull describes the learning theory behind these teaching approaches as contextual learning:

> ...learning occurs only when students (learners) process new information or knowledge in such a way that it makes sense to them in their frame of reference (their own inner world of memory, experience, and response). (p. 41)

As this definition closely matches that of previously stated definitions of constructivism (weak and radical), it is appropriate to consider the results of applied academics studies within the scope of this review.

Applied Mathematics curriculum, as described by Leon Pedrotti (1995) of the Center for Occupational Research and Development, is an applied academics course which covers topics in arithmetic, algebra, geometry, trigonometry, statistics, computer spreadsheets, computer graphics, and quality assurance and quality control at the grade 9 and 10 level. Problems are presented in contexts such as agriculture, agribusiness, health occupations, home economics, business and marketing, and industrial technology. The curriculum provides teachers with a suggested four point teaching strategy: getting started; learning the skills; applying the skills; and looking back. The purpose of the two courses is to provide students with the equivalent of Algebra I over two years using the teaching approaches described previously by Hull.

There are a number of program evaluations directly associated with the CORD version of Applied Mathematics (Center for Occupational Research and Development, 1995; McKillip, Davis, Koballa, & Oliver, 1993; Pepple & O'Connor, 1992; Wang & Owens, 1992, 1994, 1995).

Wang and Owens (1992, 1994, 1995) were involved in an ongoing program evaluation of Applied Mathematics for the Boeing Company (1990-1994). Over the course of four years curriculum specific student achievement assessments were administered in 63 schools throughout the state of Washington. The results of the student achievement assessments are consistent from year to year and indicate that Applied Mathematics provides students with a challenging alternative to traditional academic classes. The achievement scores of students in Applied Mathematics were consistently higher than those of the comparison students (taking Algebra I).

Students' confidence in their ability to do mathematics increased and perhaps more importantly, students at the lower end of academic achievement tended to gain the most in applied academic courses. Wang and Owens conclude their evaluation with the recommendation that Applied Mathematics be made available to all students, not just those in vocational programs or with low 'traditional' academic ability.

There appear to be two significant problems with Wang and Owens' statistical reporting. The first difficulty is that the Applied Mathematics (experimental) and Applied Mathematics Comparison (control) groups were of significantly different sizes (N=309 for Applied Mathematics and N=75 for Applied Mathematics Comparison). This raises some questions about the generalizability of the achievement comparison.

A second problem arises from the student achievement instrument. Wang and Owens provide no evidence for the validity of the test scores. Specifically, construct validity, content validity, predictive validity and concurrent validity are not substantiated in any way throughout the four years of evaluation. In addition to this omission, Wang and Owens fail to provide any measure of test-retest reliability, although they do indicate that an ANOVA shows significant differences in student achievement at $p < .05$.

It must be noted that Wang and Owens' evaluations were based on the assumption that the teachers were using the Applied Mathematics curriculum developed by CORD. Although the CORD curriculum materials appear to have been designed with a constructivist teaching methodology in mind, at no time in the evaluation is there any indication that the evaluators confirmed that they were being used in the classroom. In the second year evaluation report Wang and Owens (1992) had the opportunity to do so as they indicate that their evaluation was designed to be multi-dimensional. They report using case studies, teacher surveys and interviews in an attempt to triangulate their findings. Upon closer scrutiny it can be seen that:

1. the case studies are self-reported profiles of the applied academics sites;
2. the teacher surveys do not address the issue of teacher methodology; and,
3. the interviews deal with teacher training for potential applied academics teachers and did not provide for any follow-up once the teachers were back in the classroom.

It is ironic that Wang and Owens (1994) presented a paper at the Annual Meeting of the American Educational Research Association which reports that:

> A multiple approach must be used to measure the different indicators of success of applied academic courses. These different approaches are complementary so that data collected through one procedure can be validated by other procedures. (p. 16)

This is a sound premise upon which to conduct a program evaluation, but unfortunately it appears to have been distorted to create a one-sided report on the success of applied academics rather than truly evaluate it.

The Center for Occupational Research and Development (1995) conducted an evaluation of the effectiveness of Applied Mathematics 1 and 2 as compared to Algebra 1. The authors of this report (no names specified) concluded that students completing both Applied Mathematics 1 and 2 courses (N=326) did not score significantly different (95% confidence level) than did students who completed Algebra 1 (N=766).

This study suffers from the same problems that were observed in Wang and Owens' studies. Although the CORD authors report a 95% confidence level, they do not indicate what statistic this reflects. Once again test validity and reliability is in question as no evidence of either is presented. As well, the emphasis is once again on student achievement and completely ignores the issue of teacher methodology. It can not be assumed that if the CORD materials for Applied Mathematics 1 and 2 are being used, that it follows the teachers are using them as desired.

At roughly the same time that Wang and Owens submitted their second year evaluation report to The Boeing Company, Pepple and O'Connor (1992) were completing a detailed evaluation of the Applied Mathematics demonstration sites in Indiana. Their evaluation focused on a comparison of student achievement between Applied Mathematics students and Algebra 1 students. Their findings substantiate those of Wang and Owens (1992, 1994). It should be noted that the instrument used in the Indiana study was created by the Illinois State Board of Education and the Mathematics Assessment Advisory Committee to assess grade 10 student achievement in the state goals for learning: computation, ratios and percentages, measurement, algebra, geometry, data collection and analysis, and estimation. These goals were matched to the outcomes of the Applied Mathematics materials. As a result, Pepple and O'Connor's findings have a higher construct and content validity than do Wang and Owens. At the same time these findings reinforce Wang and Owens' somewhat unsubstantiated results.

Pepple and O'Connor conclude that the Applied Mathematics materials:

1. enable students to perform at higher academic levels across a broader range of proficiency areas in mathematics than traditional materials used in comparison classes;
2. created an opportunity to use a greater variety of teaching methods as compared to other courses they taught; and,

3. need to be evaluated further to more accurately define the program and student characteristics.

A study conducted by the University of Georgia (McKillip, et al., 1993) focuses on several aspects of Applied Mathematics: acceptance of Applied Mathematics at the post-secondary level; certification of Applied Mathematics teachers; performance of Applied Mathematics students compared to Algebra 1 students; problems students subsequently encounter at the post-secondary level; and, which students should be taking Applied Mathematics.

McKillip et al. (1993) present an evaluation which attempts to provide a true multiple approach that could provide the validity absent from previous studies noted in this review. Of particular note is the use of teacher interviews and classroom observations in addition to student performance comparisons.

The results of the evaluation indicate that the tests used for student assessment are "somewhat uneven in coverage of topics in algebra, but only a few items were slanted toward applications" (p. 10). This statement supports concerns expressed previously with regards to Wang and Owens' evaluation reports. The test referred to by McKillip et al. is the same one used by CORD and Wang and Owens in their evaluations.

McKillip's teacher observations suggest that teachers make considerable use of cooperative learning groups, experimentation, and solving detailed mathematics applications in their classes. It was also noted that calculators were made available to students at all times. Teacher interviews indicate that the majority of teachers were concerned that the materials may be inappropriate for the lower achieving students and the courses are better suited for grade 10 students than grade 9.

The main conclusions that McKillip made in this program evaluation are that:

1. Applied Mathematics 1 and 2 are accepted by most post-secondary institutions and are approved as electives in the College Preparatory Curriculum;
2. Applied Mathematics 1 and 2 together are a reasonable alternative to Algebra 1;
3. Mathematics teachers can, and usually do, add supplementary material to the course to cover the few gaps in the curriculum (when compared to Algebra 1);
4. The Applied Mathematics courses are suitable for the middle 50% of students and that students in the third quartile need help with the reading and problem solving; and,
5. The Applied Mathematics courses are more suitable for grade 10, 11, and 12 students than grade 9.

Summary

The review of related literature identifies three areas which suggest further study is needed. First, there is evidence which indicates that using a constructivist approach to teaching mathematics results in increased student understanding of the concepts. As well, the student's ability to apply these concepts in problem solving situations is improved. To date very few studies have been conducted which look at the effectiveness of a curriculum designed on this basis. Previous research has been focused on evaluating the effectiveness of using a constructivist approach when offering parts of an existing traditional curriculum as opposed to evaluating the effectiveness of a constructivist oriented curriculum.

Second, it appears that ability grouping or streaming (such as is found in the Applications of Mathematics / Principles of Mathematics course structure) may benefit high ability students with regards to academic achievement. High ability student self concept appears to be unaffected by streaming, but it is not clear what happens to the balance of the student population. Average- and low-ability students appear to be more likely to experience an educational climate which is not as good as it would be in high ability classes. Although average- and low-ability students are aware of the discrepancy between their academic achievement and that of their high-ability peers, it does not seem to play a significant role in their self-concept. There are just too many other things going on their lives which have a greater impact than a mathematics class. Even though the research on student streaming appears to be ambiguous as to its effects, it is very clear that there is a need for quantifiable (and qualitative) research which relates different teaching methodologies in the different streams, tracks, or pathways to student achievement and attitudes towards mathematics.

Third, although there have been a number of program evaluations on applied academics conducted since 1990, they have tended to focus on student achievement while ignoring the issues of students' attitudes towards mathematics and of teacher methodology. Although one evaluation, which uses teacher methodology as part of the evaluation criteria, is in evidence, duplication under similar conditions and using related instruments should take place in order to confirm or refute the findings. This is relevant to British Columbia's Applications of Mathematics 10 curriculum as it is based upon teachers using constructivist methodologies similar to those described in CORD's Applied Mathematics curriculum (Pedrotti & Chamberlain, 1995).

CHAPER 3

EXPERIMENTAL DESIGN AND RESEARCH PROCEDURES

Introduction

This program evaluation attempts to determine whether British Columbia's Applications of Mathematics 10 (AM 10) curriculum is being taught as intended. The primary goal of the Applications of Mathematics 10 course is to provide students (using constructivist teaching methodologies) with a mathematics education which is equivalent to that provided by the Principles of Mathematics 10 (PM 10) curriculum. At the same time, the course is intended to promote in students a positive attitude towards mathematics.

This study attempts to determine whether teachers of AM 10 courses are using the intended methodologies more or less than teachers of 10A or PM 10 courses. It also compares AM 10 student achievement in mathematics (pre-test and post-test) and attitude towards mathematics to that of PM 10 and 10A students. The purpose of the student score comparisons is to determine if there are significant differences in the pre-test and/or post-test mean scores between the three groups and if there are significant changes in the group mean scores from pre-test to post-test.

It is important to note that this study does not attempt to evaluate the Applications of Mathematics 10 program with respect to implementation issues such as: teacher inservice; post-secondary acceptance; and, availability or appropriateness of learning resources. Accordingly, the study focuses solely on the three questions related to teacher practice, student achievement, and student attitude towards mathematics.

Experimental Design

A quasi-experimental nonequivalent control-group design forms the basis of this program evaluation. The experimental group was made up of students taking Applications of Mathematics 10 during the 1998/99 school year, while the control group was made up of students taking Principles of Mathematics or Mathematics 10A during the same time period. A quasi-experimental design was necessitated as the research subjects could not be randomly assigned to either control (PM 10 or 10A) or experimental (AM 10) groups. The control and experimental groups consisted of intact AM 10, PM 10, and 10A classes. With the exception of the non-random subject assignment, the essential feature of this design is the administration of pre-test and post-test student achievement and attitude (towards mathematics) assessments.

This study design originally attempted to provide a degree of assurance that teachers in the experimental and control groups were as similar as possible with respect to: educational backgrounds; teaching experience; age; and, gender. Due to the smaller than anticipated sample size, these factors could not be controlled for by matching teachers in each group with similar characteristics. Comparison of the teacher characteristics a priori was conducted to identify if there were noticeable differences between the groups (only small differences were noted).

The design also attempted to provide for effective control of threats to internal validity such as: history, maturation, testing, instrumentation, statistical regression, differential selection, experimental mortality, and selection-maturation interaction (Gall et al., 1996). Similar designs had been used in many of the Applied Academics program evaluations previously reviewed (e.g., Center for Occupational Research and Development, 1995; McKillip, Davis, Koballa, & Oliver, 1993; Pepple & O'Connor, 1992; Wang & Owens, 1992, 1994, 1995). The notable difference between this study and previous Applied Academic studies is that an additional assessment criterion for the program evaluation was included (i.e., identification of teachers' self reported instructional and assessment strategies used in the control and experimental groups).

The program evaluation consisted of the following steps:

1. Identification of schools offering Applications of Mathematics 10 in the 1998/99 school year;
2. Identification of teachers (and their classes) of AM 10, PM 10, 10A willing to participate in the study;
3. Administration of a Teacher Identification Survey to identify the characteristics of the teachers in each of the control and experimental groups;
4. Administration of student achievement tests (pre-test and post-test) to ten intact Applications of Mathematics 10 classes (N = 154 subjects), ten intact Principles of Mathematics 10 classes (N = 195 subjects), and 3 intact Mathematics 10A classes (N = 39 subjects);
5. Administration of student attitude surveys (pre-test and post-test) using the same intact classes as identified in (4);
6. Administration of teacher surveys (pre-test and post-test) to all participating teachers to identify self-reported teaching practices used over the duration of the study; and,

7. Collection of monthly logbooks (completed by a subset of the participating teachers) which ask them to identify the instructional and assessment strategies used during each teaching period.

Subject Selection

The following procedures were used to select the subjects for this study:

1. In February, 1998 the British Columbia Ministry of Education distributed a survey to the mathematics department head or school administrator of every public secondary school in British Columbia asking respondents to indicate if they would be offering at least one Applications of Mathematics 10 course during the 1998/99 school year. Thirty-one schools (out of approximately 600 potential schools) indicated that they planned to offer Applications of Mathematics 10 in the 1998/99 school year.
2. Ethics approval for the study was obtained from the University of Victoria's Ethics Committee on April 24, 1998 (see Appendix A).
3. In May 1998 a follow-up letter was sent to the 31 schools identified in the Ministry of Education survey asking if they were interested in participating in a program evaluation of the Applications of Mathematics curriculum. Eleven of the 31 schools indicated they were interested in participating in the study.
4. During July and August 1998, letters were sent to the school district superintendent of each potential school asking that permission be granted for the teacher(s) to be involved in the study. All superintendents granted permission.

5. During the first teaching week in September a Teacher Identification Questionnaire was sent to the participating schools with the request that the interested grade 10 mathematics teachers (preferably one teacher for each of AM 10, PM 10, and 10A) fill out the questionnaire and return it by September 15, 1998;

   Upon receipt of the completed Teacher Identification Questionnaires it became apparent that insufficient numbers of teachers had agreed to participate in the study to allow for a random sample of suitable size to be selected. The decision was then made to include all interested teachers and their classes in the study. This resulted in thirteen Applications of Mathematics 10 teachers, fourteen Principles of Mathematics 10 teachers, and eight Mathematics 10A teachers initially agreeing to participate in the program evaluation. From this initial group of 35 classes in 10 schools, ten AM 10 classes, ten PM 10 classes, and three 10A classes (involving 19 teachers) eventually completed the study. The subject selection section of the next chapter describes the characteristics of the schools involved in the study.
6. The 19 teachers involved in the study were asked to maintain a logbook of the instructional and assessment strategies they used in each class. This logbook was kept for the duration of the teaching of the course (AM 10, PM 10 or 10A).
7. Initially all students enrolled in the twenty-three classes were included in the study. Those classes or students who did not write both the pre-test and post-test student achievement and attitude assessments were excluded from the data set.

Treatment

One of the main goals of the Applications of Mathematics 10 curriculum, as stated previously, is to provide an applied, hands-on approach to learning mathematics. Accordingly, it was expected that the treatment used in this study would consist of teaching students in the AM 10 courses (experimental group) using constructivist oriented instructional strategies. The first part of the study was designed to determine if the 'treatment' took place in the Experimental Group classes.

Although the Experimental Group teachers were not provided with any specific inservice related to constructivist teaching methodology specifically related to this study, they did have the opportunity to attend a number of in-service sessions offered by the Centre for Applied Academics. Additionally, the integrated resource package that included the Applications of Mathematics 10 and Principles of Mathematics 10 curriculum provided direction concerning the expected differences in instruction. All but one of the Experimental Group teachers had access to learning resources being developed by Pearson Publishing (at that time called Addison Wesley Longman Publishing) in cooperation with the Western Canadian Protocol jurisdictions. These resources were developed specifically for the WCP version of Applied Mathematics 10 and were based upon a constructivist learning philosophy. The match between the learning outcomes addressed by these learning resources and the Applications of Mathematics curriculum was approximately 60%, therefore the teachers could not use them for the entire course.

It was expected (and later confirmed) that the students in PM 10 and 10A (control group) would be taught using what are often referred to as traditional strategies. The learning resources for these courses had been in use in British Columbia schools since 1988 and consisted primarily of explanatory notes, exercises and 'word problems' that were conducive to a direct instruction teaching style.

Experimental Group Instructional and Assessment Strategies

Using Brooks and Brooks' (1993) five principles of constructivist pedagogy and corresponding set of twelve descriptors of what would characterize a constructivist teacher, the following instructional and assessment strategies (categorized into a number of groups), were selected from a list developed by Herrington, Sparrow, Herrington, and Oliver (1997a, 1997b). The following are the groupings of instructional and assessment strategies expected to be used by the Experimental Group teachers and therefore make up the 'treatment' for this group:

Experimental Group Instructional Strategies

Discussion.

- Group work - includes grouping by students' ability, for convenience, to teach specific skills, by self-selection, by mixed ability, etc.;
- Role play - conceptual learning through drama and movement;
- Explaining - an extended use of talk (in contrast to short answers to teacher questions). Emphasis is on the role of students using talk to develop their ideas and understandings.

Practical work.

- Manipulatives - classified into structured (e.g., Dienes' Multibase Arithmetic Blocks, Cuisenaire rods) and unstructured (e.g., multilink cubes, shells, matchsticks) materials;
- Game playing - games used to practice facts and skills, development of concepts, strategy building and problem solving;
- Outdoors - requires the students being involved in developing mathematical ideas in kinesthetic ways;
- Mathematics centres - also called task centres. Generally based around puzzles, problems and investigations. Necessary materials are prepared by the teachers and placed in a container for student use;

Investigating.

- Guided discovery - the teacher plans and sets up a situation and task that has embedded within it the desired learning outcomes. Usually followed by class discussion;
- Projects - involves students investigating, solving problems, researching and applying mathematics. Projects are usually completed by a group and involve group work skills, identification and collection of resources, and presentation skills;
- Open-ended tasks - tasks which allow for a variety of solution approaches and can be answered at several levels. The tasks should challenge all students, can be worked on in groups or by individuals, can be easily extended, and require minimal teacher direction;

Problem solving.

- Puzzles - puzzles such as number puzzles and the Tower of Hanoi require students to think through the situation and employ pattern searching to arrive at generalization;
- Modeling - a term used to describe the processes involved in using mathematics to solve real-world problems;
- Applications - tasks (including word problems) in which students will apply previously learnt procedures. Students not only demonstrate their knowledge of procedures, but also their ability to choose the most appropriate ones;
- Themes - using a theme or context to investigate related mathematical ideas;
- Problem solving - involves a situation that does not have an immediate solution or method;
- Problem posing - initially may come directly from the instigation of the teacher, but eventually the intention is that students will pose their own problems and therefore develop an inquiry approach to mathematics;

Using technology.

- Calculators and graphing calculators - as a teaching and learning aid (e.g., calculating large numbers, operations involving algebra, statistics, programming and graphing). This does not include trivial activities such as checking calculation done by another method; and,
- Computers - Used as electronic blackboards for displaying graphs or introducing a problem or investigation. Includes student use of spreadsheets, mathematical adventure games and simulations.

Experimental Group Assessment Strategies

Observing.

- Anecdotal - an informal type of assessment. Records (of achievements, beliefs and attitudes) are made as the teacher observes students engaged in some mathematics;

Questioning.

- Higher order questioning - can be either written or oral. Attempts to find out whether a student has made important connections that underpin certain facts or procedures. Question stems include: What if...? How Does...? Why does...? How could...?;
- Open-ended questioning - questions that allow for more than one answer. They involve mathematical thinking that goes beyond recalling facts or demonstrating skills;

Interviewing.

- Structured interviews - contain the same set of questions given in the same sequence. Often used to diagnose difficulties that students may be encountering;
- Open interviews - does not follow a set sequence of questions. Interview relies on discussion between the teacher and the student with questions arising incidentally;

Testing.

- Diagnosis - can be done through testing or interviews. Once problem areas are identified, the teacher can take steps to initiate the appropriate teaching strategy to address the misconception;

- Performance-based - involves making a formal assessment while the student is involved in performing a mathematical activity. Activities often include the use of materials;
- Problem solving - a mathematical problem requires students to reach a solution that does not involve the simple application of a standard procedure or algorithm. The choice of strategies provide an indication of how good the students are at solving problems;
- Attitude - students' attitudes may be considered as feeling or beliefs about mathematics. Information about student attitudes can be obtained through teacher observation and attitudinal tests;

Reporting.

- Oral report - can involve students individually or in groups communicating the results of a mathematical activity;
- Written report - can be a short summary describing what the student learnt in a particular lesson or it can be a longer report describing an extended mathematical idea;
- Portfolio - a collection of student work generally compiled over a long period of time. Pieces of work may be chosen by the teacher, student, or both;
- Investigation - an investigation results from the presentation of an open situation that can be extended and explored with the use of mathematics. Students will use mathematical processes such as drawing diagrams, tabulating, making conjectures, looking for and finding patterns, generalizing results, explaining and proving;

- Modeling - involves students using mathematics to solve problems that occur in the real world. The assessment task is embedded in a context that students would normally experience in real life;

Self-Assessment.

- Journals - journals can provide insight into student attitudes towards mathematics as well as student achievements;
- Reflective prompts - involves using a structured set of questions that prompt students to reflect on their own learning;
- Self-questioning - a student self assessment which can be prompted by the teacher with such questions as: How shall we go about solving this problem? What factors do I need to consider before I start? Is this making sense? How is this method helping? Can I find a better way to do this? Does the answer make sense?; and,
- Peer assessment - students develop their own assessment tasks for their peers to complete.

Control Group Instructional and Assessment Strategies

The control group (students in Principles of Mathematics 10 and Mathematics 10A) was expected to be exposed to a more traditional instructional and assessment style. This traditional style can be characterized by the following list of instructional and assessment strategies:

Control Group Instructional Strategies

Consolidation and practice.

- Drill and practice - Students are shown a method often broken into smaller steps. They then practice the method by completing a series of examples of the same kind of calculation;
- Homework - assigned to students to complete unfinished work; consolidate and practice skills; develop independent study habits;
- Textbooks - students working from page to page for all or part of the class time;
- Worksheets - may be photocopied from commercially produced blackline masters, or may be teacher generated, either handwritten or computer generated;

Teacher centered.

- Exposition - traditional teaching approach where the main purpose is to transmit information from teacher to student as quickly as possible. May incorporate questions and demonstrations that build upon prior knowledge of students; and,
- Beginning a lesson - intended to set the scene for learning, link ideas with previous and future learning and motivate the students to learn.

Control Group Assessment Strategies

Questioning.

- Factual questioning - usually requires students to respond automatically with the answer to a question (e.g., basic number facts);

Testing.

- Pencil and paper test - traditional form of assessment which often assess skills, facts but can also assess conceptual understanding, applications and problem solving abilities; and,
- Multiple choice test - included in many formal tests (particularly provincial or national tests). Easy to score and provide statistical data. Questions tend to focus on the proauct (answer) rather than the process.

It was not expected that the above instructional and assessment strategies be exclusive to either the control or experimental groups. Rather the classification represents an expectation that the strategies would be used predominantly in one group or another. Part of the study design is intended to confirm whether the teaching and instructional strategies were, in fact, administered to the Experimental and Control Groups as expected.

Development of Student and Teacher Assessment Instruments

Teacher Identification Survey

Once the schools for the study had been identified, and it had been determined that the number of schools participating would not permit for random matching of teachers in each group, it was decided that it would be necessary to administer a Teacher Identification Survey, based upon The 1995 British Columbia Assessment of Mathematics and Science (Marshall, et al., 1996). The purpose of the Teacher

Identification Survey was to determine if the teachers representing each group (AM 10 , PM 10, and 10A) had roughly the same characteristics including gender, age, teaching assignment, educational background, teaching experience, general teaching practices. Appendix C includes the specific survey questions and percentage responses for each group of teachers.

Teacher Pre- and Post-test Surveys

In addition to the initial Teacher Identification Survey, teachers were asked to complete an in-depth survey to identify their general mathematics teaching and assessment practices. This survey was also adapted from The 1995 British Columbia Assessment of Mathematics and Science (Marshall, et al., 1996).

The Teacher Pre- and Post-test Surveys were comprised of three parts, the first of which asked teachers questions related to topics including school timetable organization, teaching and assessment strategies commonly used, use of technology in the classroom, amount of homework assigned, and types and frequency of other student work assigned. The second part of the survey asked teachers questions about their views on the importance and difficulty of teaching different mathematics topics in the curriculum. The third and final section of this survey focused on teacher assessment practices including the type of assessment and evaluation used as well as the frequency of evaluation.

The results of these surveys were used to determine if there were any changes in teachers' instructional and assessment practices during the period of this study as well as to triangulate another self reported measure of teaching practices, the Teacher Logbook.

The complete Teacher Pre- and Post-test Survey and a summary of results can be found in Appendix D.

Teacher Logbook

As previously stated, it was assumed that the treatment the experimental group experienced would be the teaching of the Applications of Mathematics 10 curriculum using constructivist oriented instructional and assessment strategies. When determining if there were any differences in student achievement or attitude towards mathematics between the control and the experimental group, it was necessary to determine whether the treatment actually took place as intended. To that end, all teachers participating in the study were asked to maintain a logbook of the teaching and assessment strategies used during each teaching period of their class for the duration of the course.

The logbook is based upon a list of twenty-eight instructional and twenty-three assessment strategies identified by Herrington, Sparrow, Herrington, and Oliver (1997a, 1997b). The strategies were grouped under the headings outlined in Table 1 on the next page:

Table 1

Categories of Teaching and Assessment Strategies Used in the Teacher Logbook

Teaching Strategies	Assessment Strategies
Consolidation and Practice	Observing
Discussion	Questioning
Practical Work	Interviewing
Investigating	Testing
Teacher Centered	Reporting
Problem Solving	Self-assessment
Using technology	

A glossary describing the various strategies was included with each logbook to ensure that there was consistency in how the terms were applied by the various teachers. Teachers were asked to indicate, for each class, the specific teaching and assessment strategies used and to indicate the approximate percentage of class time spent using the various strategies. The logbook was to be completed daily and submitted monthly to help ensure that the data were not lost. Appendix E contains an example of the Teacher Logbook.

Student Achievement and Attitude Assessments

Both experimental and control group students wrote two parallel mathematics achievement and two identical attitude assessments (pre-test and post-test). The same pre-test assessment was written by all students as early in the course as possible (teachers were asked to ensure that their class wrote the assessment within the first 10 hours of classroom instruction). The post-test assessment (different from the pre-test assessment) was written by all students as close as possible to the end of the course. Teachers were allowed to use their discretion in both cases so as not to interfere unduly with class routines.

The pre- and post-test assessments were developed with permission from a bank of test items that the British Columbia Ministry of Education used to create the multiple choice portion of The 1995 British Columbia Assessment of Mathematics and Science. This bank of questions was also used to develop the 1999 Provincial Assessment of Reading Comprehension, First-Draft Writing and Numeracy: Grade 10 that all grade 10 students in British Columbia were required to write in May 1999. The use of this standardized achievement test format had the following advantages:

- the items are well written;
- internal reliability is usually high (Cronbach's alpha for the pre-test student mathematics assessment developed for this study was calculated to be .75 and for the post-test assessment it was calculated to be .86); and,
- standard conditions of administration and scoring have been established (Gall, Borg, & Gall, 1996; Kubiszyn & Borich, 1984).

Gall, et al. also identify the drawbacks of using standardized achievement tests, including:

- that guessing, response set, or random or careless answers can distort the scores;
- the use of a restricted time limit may not accurately reflect the characteristics of individuals who are much slower, more deliberate or more thoughtful in responding than their peers;
- the scores do not reflect the unique experiences of different types of individuals.

The student achievement items were selected using the following procedures:

1. Each curriculum organizer for the Applications of Math 10, Principles of Math 10 and Mathematics 10A curricula was assigned a percent weighting to reflect the amount of time typically spent teaching each (see Table 2 below).

Table 2

Pre- and Post-test Student Achievement Assessments Specifications: Percent Weighting of Curriculum Organizers Based on the Teaching Emphasis in Each Course

	Percent Weight by Course			
Curriculum Organizer	10A	AM 10	PM 10	Average
Number	30%	10%	10%	17%
Patterns & Relations	15%	25%	40%	27%
Shape & Space	35%	40%	40%	38%
Statistics & Probability	20%	25%	10%	18%
Total	100%	100%	100%	100%

2. This information was then used to determine the number of questions of each type that were to be selected for the assessment instruments. Table 34 indicates the corresponding number of questions (based on a 60 item instrument) that were selected for the two pilot assessments. Grade 9 mathematics test items were selected using the "Average" weightings and were included to further broaden the range of student abilities the achievement tests would cover.

Table 3

Pre- and Post-test Pilot Student Achievement Assessment Specifications: Number of Test Items per Curriculum Organizer per Course

	Number of Questions by Course				
Curriculum Organizer	9	10A	AM 10	PM 10	Total
Number	3	5	2	2	12
Patterns & Relations	4	2	4	6	16
Shape & Space	6	5	6	6	23
Statistics & Probability	2	3	3	1	9
Total Questions	15	15	15	15	60

3. In June 1998, the pilot pre- and post-test pilot assessments were distributed to five schools previously identified as offering Applications of Mathematics 10 and Principles of Mathematics 10 at that time. Due to printing and distribution delays

there was not sufficient time for the schools to field test the instruments as planned. As an alternative, participating teachers were asked to critique the instruments. The feedback provided by these teachers was used to select the final questions for the pre- and post-test student achievement assessments. Table 4 indicates the corresponding number of questions (based on a 40 item instrument) that were selected for the two instruments.

Table 4

Pre- and Post-test Student Achievement Assessment Specifications: Number of Test Items per Curriculum Organizer per Course

	Number of Questions by Course				
Curriculum Organizer	9	10A	AM 10	PM 10	Total
Number	2	3	1	1	7
Patterns & Relations	3	2	3	4	12
Shape & Space	3	3	4	4	14
Statistics & Probability	2	2	2	1	7
Total Questions	10	10	10	10	40

4. For each Student Achievement & Attitude Assessment (Forms A & B), the final distribution of the forty test questions corresponding to each of the grades, courses and curriculum organizers is indicated in Table 5.

Table 5

Student Achievement Assessment (Forms A & B): Question Organization

	Corresponding Question Numbers by Course			
Curriculum Organizer	9	10A	AM 10	PM 10
Number	14, 15	16 - 18	19	20
Patterns & Relations	21 - 23	24, 25	26 - 28	29 - 32
Shape & Space	33 - 35	36 - 38	39 - 42	43 - 46
Statistics & Probability	47, 48	49, 50	51, 52	53

The pre- and post-test assessments used in this study are not true standardized achievement tests as the items were selected from a bank of questions, rather than from a complete standardized test. The advantage to this adaptation is that the tests were constructed to assess students with respect to the specific curriculum learning outcomes in Applications of Mathematics 10, Principles of Mathematics 10, and Mathematics 10A. It should be noted that one half of the achievement assessment items were identified for those learning outcomes that were common to both the Applications of Mathematics 10 and Principles of Mathematics 10 curricula. The other half of the achievement assessment items were identified for those learning outcomes that were common to Mathematics 10A and Mathematics 9. As a consequence, the resulting test has greater content validity then if an unadapted standardized test had been used.

Although piloting of the pre- and post-test instruments was preferred so that the internal reliability of the two instruments could be determined, the internal reliability of the instruments was calculated a priori. Cronbach's alpha were calculated for each set of questions corresponding to a curriculum organizer as well as for the complete achievement instruments. Table 6 summarizes the reliability coefficients for each part of the achievement instruments:

Table 6

Reliability Coefficients (Cronbach's Alpha) for Each Curriculum Organizer for Both Pre- and Post-Test Student Mathematics Achievement Assessment Instruments

Curriculum Organizer	Number of Items	Pre-Test Reliability Coefficients (Cronbach's Alpha)	Post-test Reliability Coefficients (Cronbach's Alpha)
Number	7	.50	.60
Patterns & Relations	12	.56	.62
Shape & Space	14	.50	.72
Statistics & Probability	7	.52	.66
Complete Instrument	40	.75	.86

The attitudinal section of the pre- and post-test assessments was designed to assess three components of students' attitudes towards mathematics:

1. affective - which consists of the individual's feeling about mathematics;
2. cognitive - which is the individual's beliefs or knowledge about mathematics; and,
3. behavioral - which is the individual's predisposition to act toward mathematics in a particular way. (Gall, et al., 1996)

As with the student mathematics achievement portion of the assessment instruments, reliability coefficients (Cronbach's alpha) for the pre- and post-test student attitudes towards mathematics assessments were calculated a priori. For the six-item pre-test attitude assessment the reliability coefficient is $\alpha = .62$ and for the post-test attitude assessment reliability coefficient is $\alpha = .66$.

The first 7 items (questions 1 through 7) of each achievement and attitudinal assessment include questions related to student demographics (i.e., age, gender, previous mathematics course taken, grade in previous mathematics course, presence of calculators or computers in the home). The next 6 items (questions 8 through 13) relate to students' attitudes towards mathematics. The intent was to determine whether students' attitudes towards mathematics were different for each group and to determine if they changed (pre-test to post-test) based upon the treatment (i.e., how the mathematics was taught). The remaining assessment items (questions 14 through 53) deal with student achievement in mathematics. Once again, the intent was to determine whether students' mathematics achievement was significantly different between the Experimental and Control Groups and to determine if it changed (pre-test to post-test) based upon the treatment. Appendix F contains the parental and student permission to participate in the study letters and Appendix G contains the Pre- and Post-test Student Achievement and Attitude Assessments (Forms A [pre-test] & B [post-test] respectively).

CHAPTER 4

DATA ANALYSIS AND RESULTS

Each section of this chapter begins with a description of the procedures used to process and analyse the pertinent research data. The results are then presented with specific points being noted for later discussion. Included are the results for the following sets of research data:

- School Characteristics
- Teacher Identification Surveys;
- Teacher Pre- and Post-test Surveys;
- Teacher Logbooks; and,
- Student Achievement and Attitudes Towards Mathematics Assessments (Pre- and Post-test).

Participating School Characteristics

Ten schools agreed to participate in the study. The identified characteristics of the schools include: the total student population of each school; the geographic location of the school (Lower Mainland, Vancouver Island, Thompson-Nicola, Kootenay, Cariboo Chilcotin, Skeena, Peace River and Okanagan); the teaching timetable used by the school (linear [full year], 2 semesters, 4 quarters); the number of teachers teaching each course and their class sizes; and, the school type (grades 8 to 10, grades 8 to 12, and grades 10 to 12). These characteristics are outlined in Table 7:

Table 7

Characteristics of Participating Schools and Grade 10 Mathematics Classes

School	Total School Population	School Type	Geographic Location of School	School Teaching Timetable Type	Number of Application of Mathematics 10 Classes (class size)	Number of Principles of Mathematics 10 Classes (class size)	Number of Mathematics 10A Classes (class size)
1	753	8 - 12	Lower Mainland	linear	1 (20)	0	0
2	900	8 - 12	Cariboo Chilcotin	2 semesters	1[a] (11)	1[a] (21)	1 (21)
3	295	8 - 10	Vancouver Island	2 semesters	2[b] (21 & 21)	1[b] (33)	0
4	1257	10 - 12	Vancouver Island	linear	1[a] (19)	1[a] (28)	0
5	908	9 - 12	Okanagan	4 quarters	1[b] (18)	2[b] (16 & 22)	1 (22)
6	516	8 - 12	Skeena	2 semesters	1 (16)	1 (24)	0
7	1055	8 - 12	Okanagan	2 semesters	1 (19)	2[b] (18 & 23)	1[b] (12)
8	752	8 - 12	Okanagan	2 semesters	1 (39)	0	0
9	634	10 - 12	Okanagan	2 semesters	1 (24)	1 (23)	0
10	885	9 - 12	Vancouver Island	2 semesters	0	1 (29)	0
Total	7955				10 (208)	10 (237)	3 (55)
Ave.	796				21	24	18
				Participating Students	154	193	39
				Participation Rate (%)	74	81	71

[a] indicates that the same teacher taught both classes.
[b] indicates that one teacher taught one of each class (AM 10 and PM 10 or PM 10 and 10A) and a second teacher taught the remaining class.

From this it can be seen that seven of the schools are full secondary schools (grades 8 to 12 or grades 9 to 12), two are senior secondary schools (grades 10 to 12) and one is a junior secondary (grades 8 to 10). The schools are primarily from outside the Lower Mainland of British Columbia with four from the Okanagan region, three from Vancouver Island, and one each from the Lower Mainland, Skeena, and Cariboo

Chilcotin regions. The average student population of the schools participating in the study is 796. Of the original population of students agreeing to participate in the study 74% of the Applications of Mathematics 10 students completed both the pre-test and the post-test assessments, while 81 % and 71% of the Principles of Mathematics 10 and Mathematics 10A students respectively completed both assessments (combined full participation rate for the Control Group is 79%).

Teacher Identification Surveys

The results of the Teacher Identification Survey were tabulated to determine if there were any noticeable differences in the identified teacher characteristics. The data were initially grouped by course (AM 10, PM 10, 10A) and once it was determined that the number of Mathematics 10A teachers was very limited (three teachers), the data for these teachers were added to that of the Principles of Mathematics 10 teachers to provide an overall profile for the Control Group. As the number of participating teachers and their corresponding classes was lower then expected, there was no attempt to match the Experimental and Control Group teachers as originally planned. The resulting teacher profiles for the two groups were reviewed to determine if there were any significant anomalies before the next phase of the program evaluation was undertaken.

The results of the questionnaire are reported as the percentage of each group (Experimental and Control) responding in the affirmative to each question. A complete breakdown of the teacher responses is included in Appendix C. Table 8 below summarizes the Teacher Identification Survey results.

Table 8

Experimental and Control Group Teacher Characteristic Profiles

Teacher Characteristic	Experimental Group (n=10) (% of Group)	Control Group (n=13) (% of Group)
Male	80	85
Female	20	15
Under 40 years old	30	23
40 to 59 years old	70	69
Teaching full time	80	100
Mathematics specialist	60	69
Specialist in another subject area	30	23
Bachelor's degree (Education, Arts, or other)	90	100
Master's degree (Education, Science, or other)	30	31
Ten years or less teaching experience	30	31
More than 10 years teaching experience	70	69
Ten years or less mathematics teaching experience	50	53
More than 10 years mathematics teaching experience	50	46
No post-secondary mathematics content courses completed	0	15
1 or 2 post-secondary mathematics content courses completed	20	15
3 or more post-secondary mathematics content courses completed	80	69
No post-secondary mathematics methods courses completed	10	31
1 or 2 post-secondary mathematics methods courses completed	50	31
3 or more post-secondary mathematics methods courses completed	40	38
60% or less of teaching load is mathematics	30	16
61% to 100% of teaching load is mathematics	70	85
2 hours or less per week spent grading student tests or exams	50	23
More than 2 hours per week spent grading student tests or exams	50	77
2 hours or less per week spent reading or grading other student work	40	77
More than 2 hours per week spent reading or grading other student work	60	23
2 hours or less per week planning lessons by yourself	30	39
More than 2 hours per week spent planning lessons by yourself	70	61
2 hours or less per week spent meeting with students outside of classroom time	60	61
More than 2 hours per week spent meeting with students outside of classroom time	40	38
Less than 1 hour per week spent meeting with parents	90	85
1 hour or more per week spent meeting with parents	10	15
Less than 1 hour per week spent on profession reading and development activities	80	69
1 hour or more per week spent on profession reading and development activities	20	30
Less than 1 hour per week spent keeping students' records up to date	10	23
1 hour or more per week spent keeping students' records up to date	90	77
2 hours or less per week spent on administrative tasks	50	61
More than 2 hours per week spent on administrative tasks	50	39

Figures 6, 7 & 8 starting below show the results of the Teacher Identification Survey and illustrate that the Experimental and Control Group teachers have essentially the same profiles. The differences and similarities are analyzed more easily when the responses are presented graphically.

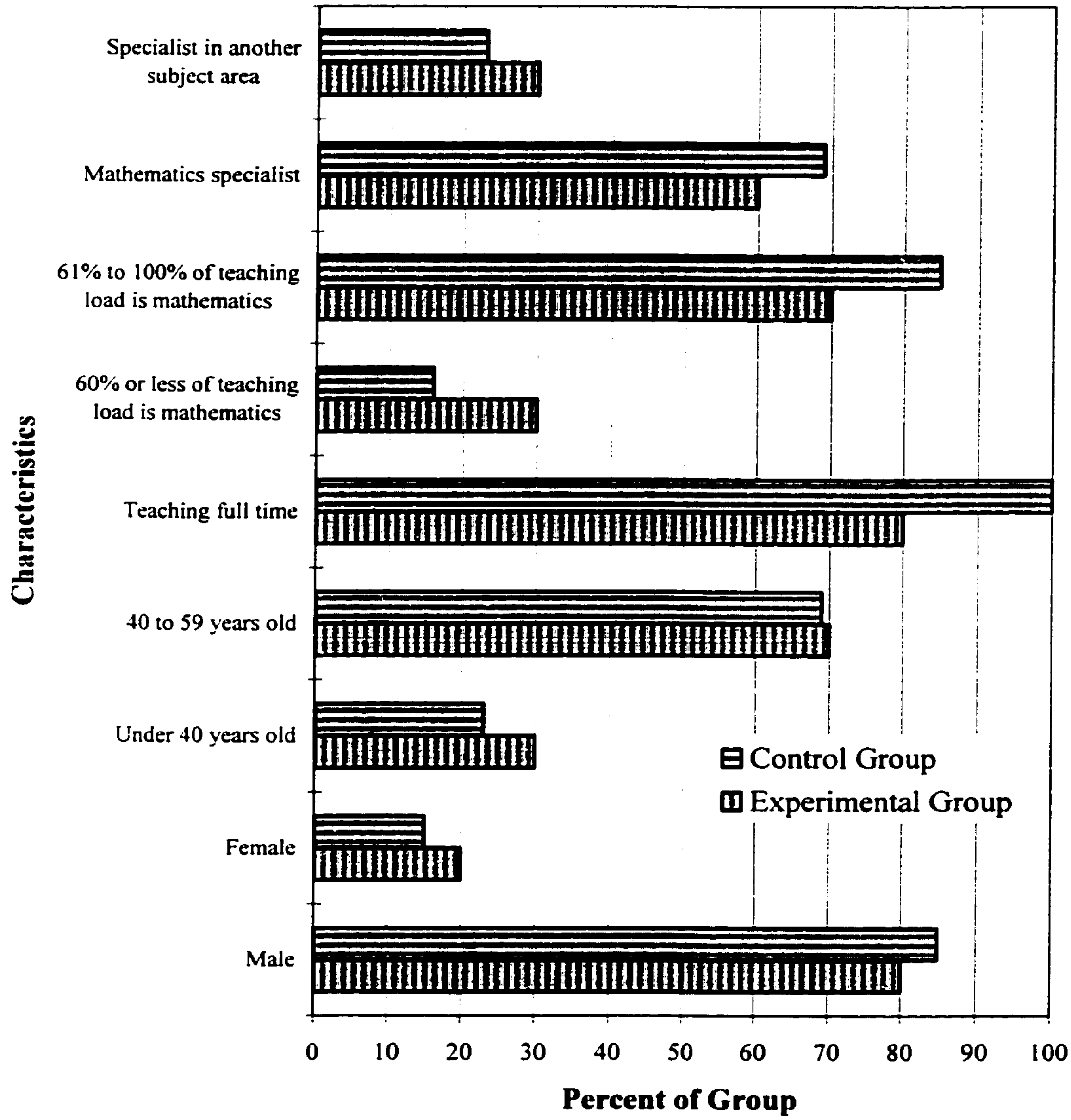

Figure 6. Teacher Profile Characteristics: Gender, Age, Teaching Specialization.

The two groups have similar gender and age distributions. Although the Control Group teachers tend to teach mathematics as a greater percentage of their teaching load (85% of Control Group Teachers versus 70% of Experimental Group Teachers), it should be noted that all of the Control Group teachers were full-time while 80% of the Experimental Group teachers taught part-time. This may account for the difference on average in mathematics teaching load.

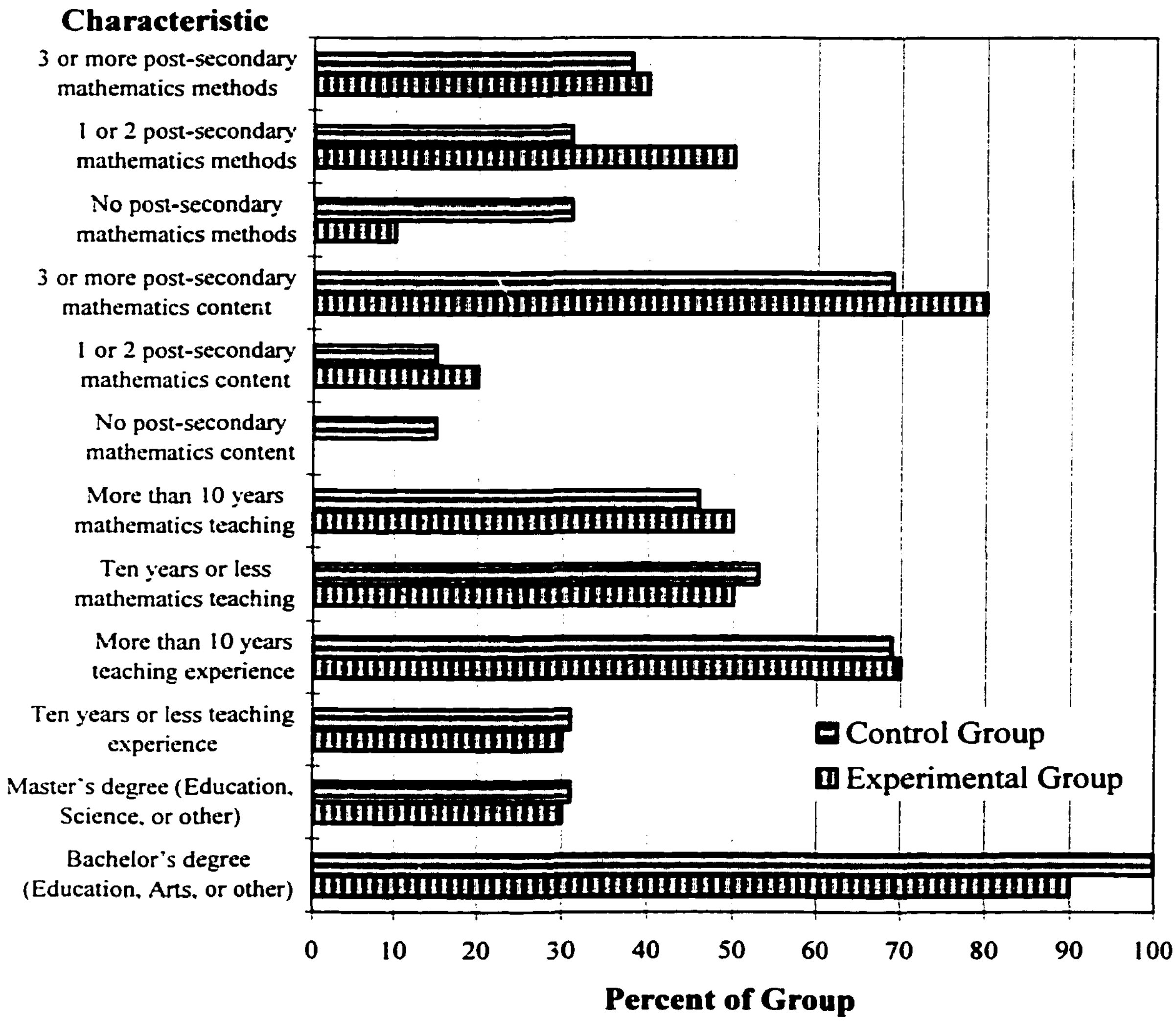

Figure 7. Teacher Profile Characteristics: Education Background.

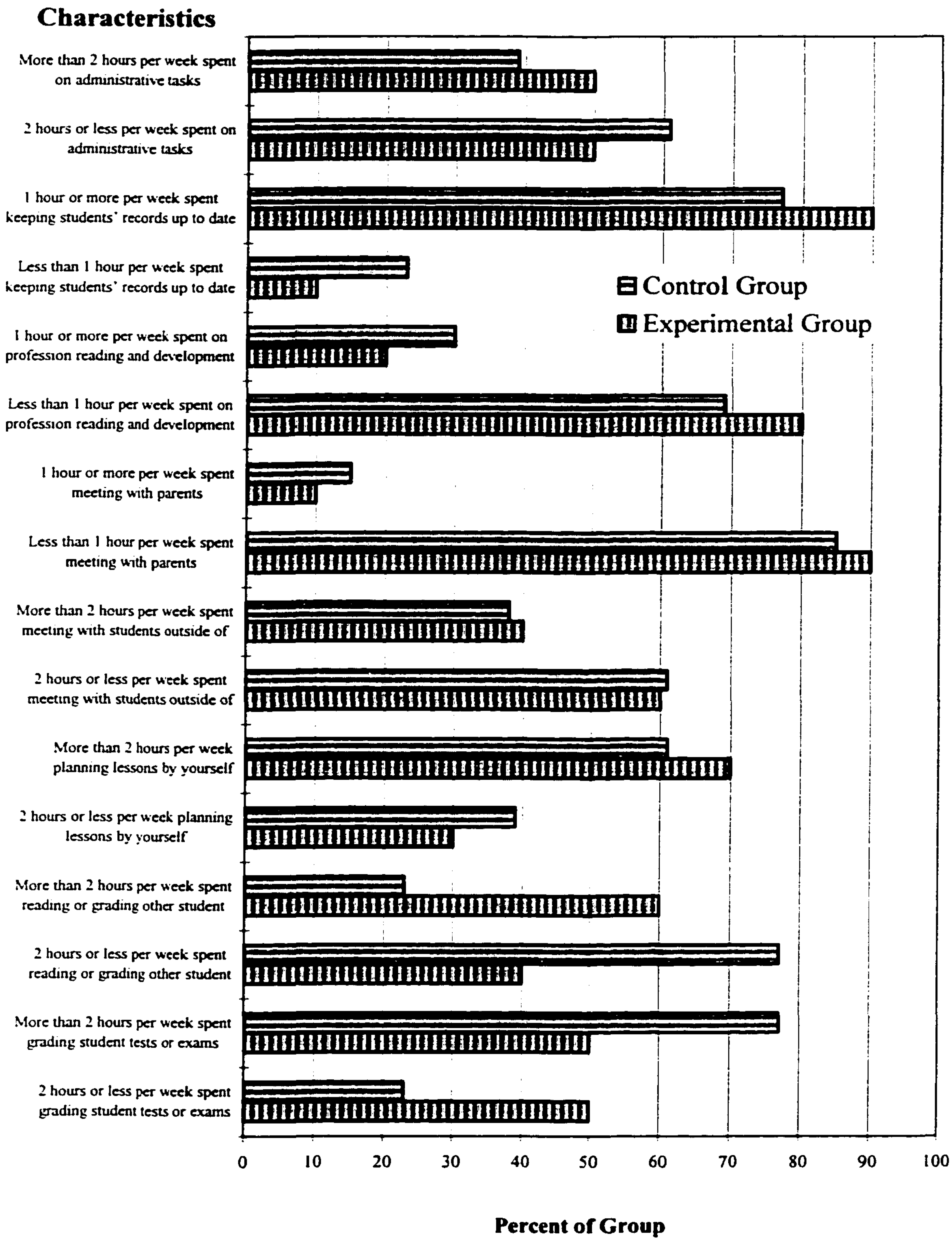

Figure 8. Teacher Profile Characteristics: Teaching Practices.

The two groups can be seen to possess similar educational backgrounds. It should be noted that the Experimental Group teachers reported having taken more mathematics content and methodology courses than the Control Group teachers. Although more of the Control Group teachers teach mathematics full time than their Experimental Group counterparts, it appears that the Control Group teachers have taken fewer mathematics courses and have less pedagogical training than the Experimental Group teachers.

A number of differences between the Experimental and Control Group teachers can be identified with regard to teaching practices. Most notable is that the Experimental Group teachers report spending more time than the Control Group teachers outside the classroom in teaching related activities such as grading student work (not including marking exams), planning lessons, meeting with parents, keeping student records up to date, and other administrative tasks.

Teacher Pre- and Post-test Surveys

The results of the Teacher Pre- and Post-test Surveys were compiled in a manner similar to those of the Teacher Identification Surveys. The survey findings were compared to the teaching and instructional strategies recorded in the Teacher Logbooks by participating teachers as they taught the course. This comparison was made to substantiate the self-reported Teacher Logbooks and to determine if the subjects in each group received the treatment as expected.

Depending upon the question, the results are reported as either the percentage of each group (Experimental and Control) responding in the affirmative to each question, or as an average score on a 4 or 5 point Likert scale. Detailed responses to the pre- and post-test surveys are presented in Appendix D. Table 9 summarizes the average of each group's responses.

Table 9

Summary of Pre-test and Post-test Teacher Surveys on Mathematics Teaching Practices

Pre-test and post-test questions	Experimental Group (Pre-test N = 10, Post-test N = 9) (% Yes) or Likert Scale Score Pre-test/Post-test	Control Group (Pre-test N = 13, Post-test N = 12) (% Yes) or Likert Scale Score Pre-test/Post-test
Has your approach to teaching mathematics changed in the past 4 years?	100/78	77/67
Likely to use cooperative learning groups?	90/89	54/50
Likely to encourage individuals to progress at their own rate?	56/33	46/33
Likely to have students use concrete materials?	70/78	38/50
Likely to encourage students to use calculators?	100/100	100/100
Likely to have students use computers?	60/56	0/17
Likely to focus on problem solving activities?	60/89	62/58
Likely to use an activities approach to teaching mathematics?	90/89	46/67
Assign mathematics homework some lessons?	50/56	8/17
Assign mathematics homework most lessons?	40/44	46/42
Assign mathematics homework every lesson?	10/0	46/42
Assign 30 minutes or less of homework?	80/89	62/58
Assign 31 minutes or more of homework?	20/11	38/42
0 = Don't use one, 1 = Dislike a lot, 2=Dislike, 3 = Like, 4 = Like a lot		
Like using a computer in math class?	2.1/2.5	0.8/1.2
Like using a computer at home?	3.1/3.6	3.2/3.0
Like using a calculator in math class?	2.9/3.4	3.2/3.0
Like using a calculator at home?	3.2/3.1	3.2/2.8

table continues

Table 9 (Cont'd)

Summary of Pre-test and Post-test Teacher Surveys on Mathematics Teaching Practices

Pre-test and post-test questions	Experimental Group (Pre-test N = 10, Post-test N = 9) (% Yes) or Likert Scale Score Pre-test/Post-test	Control Group (Pre-test N = 13, Post-test N = 12) (% Yes) or Likert Scale Score Pre-test/Post-test
0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson		
Show students what to do on the blackboard or the overhead projector?	1.5/1.8	2.6/2.2
Students use object like blocks and counters?	0.7/0.7	0.4/0.5
Students work individually on problems from textbooks or other exercises as assigned?	1.5/1.3	1.9/2.1
0 = None, 1 = About 1/4, 2 = About 1/2, 3 = About 3/4, 4 = Almost all		
How many of your students have access to scientific calculators during most mathematics lessons?	3.7/3.9	3.6/3.6
How many of your students have access to graphing calculators during most mathematics lessons?	2.3/2.2	1.7/1.7
How many of your students have access to computers during most mathematics lessons?	0.7/1.6	0/0.4
0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson		
How often do students in your class use calculators to check answers?	2.0/1.9	2.1/2.4
How often do students in your class use calculators on tests and exams?	2.4/2.1	2.3/2.6
How often do students in your class use calculators for routine computation?	2.2/2.2	2.3/2.5
How often do students in your class use calculators for solving complex problems?	2.1/1.9	2.1/2.2
How often do students in your class use calculators for exploring number concepts?	1.8/1.6	1.9/2.1

table continues

Table 9 (Cont'd)

Summary of Pre-test and Post-test Teacher Surveys on Mathematics Teaching Practices

Pre-test and post-test questions	Experimental Group (Pre-test N = 10, Post-test N = 9) (% Yes) or Likert Scale Score Pre-test/Post-test	Control Group (Pre-test N = 13, Post-test N = 12) (% Yes) or Likert Scale Score Pre-test/Post-test
0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson		
How often do you usually ask students to explain the reasoning behind the idea?	1.8/1.7	1.8/2.0
How often do you usually ask students to represent and analyze relationships using tables, charts, or graphs?	1.0/1.1	1.2/0.9
How often do you usually ask students to work on problems for which there is no immediately obvious method of solution?	0.9/0.9	0.8/0.7
How often do you usually ask students to use computers to solve exercises or problems?	0.3/0.6	0/0.2
0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson		
How often do you usually ask students to write equations to represent relationships?	1.1/1.0	1.2/1.3
How often do you usually ask students to practice computational skills	1.3/1.6	1.3/1.3
How frequently do you correct the student's error in front of the class when a student gives an incorrect response during a class discussion?	0.8/0.7	0.5/0.5
How frequently do you ask the student another question to help arrive at the correct response when a student gives an incorrect response during a class discussion?	2.2/1.8	1.8/1.8
How frequently do you call on another student who is likely to give the correct response when a student gives an incorrect response during a class discussion?	0.6/0.9	0.6/1.0
How frequently do you call on other students to get their responses and then discuss what is correct when a student gives an incorrect response during a class discussion?	2.0/1.7	1.6/1.6

table continues

Table 9 (Cont'd)

Summary of Pre-test and Post-test Teacher Surveys on Mathematics Teaching Practices

Pre-test and post-test questions	Experimental Group (Pre-test N = 10, Post-test N = 9) (% Yes) or Likert Scale Score Pre-test/Post-test	Control Group (Pre-test N = 13, Post-test N = 12) (% Yes) or Likert Scale Score Pre-test/Post-test
0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson		
How often do students work individually without assistance from the teacher?	1.0/1.1	1.1/0.8
How often do students work individually with assistance from the teacher?	1.3/1.5	1.8/1.9
How often do students work together as a class with the teacher teaching the whole class?	1.7/1.2	1.7/1.4
How often do students work together as a class with students responding to one another?	1.3/1.0	1.2/1.1
How often do students work in pairs or small groups without assistance from the teacher?	1.5/1.0	1.4/0.7
How often do students work in pairs or small groups with assistance from the teacher?	1.5/1.2	1.4/1.2
0 = Never, 1 = Rarely, 2 = Sometimes, 3 = Always		
When you assign homework how often do you assign worksheets or workbooks?	2.0/2.2	1.8/1.8
When you assign homework how often do you assign problem/question sets in a textbook?	2.0/2.0	2.5/2.4
When you assign homework how often do you assign reading in a textbook or supplementary materials?	1.2/1.3	1.3/1.2
When you assign homework how often do you assign writing definitions or other short writing assignments?	1.0/1.2	0.8/1.1
When you assign homework how often do you assign small investigation(s) or gathering data?	1.8/1.8	1.2/1.4
When you assign homework how often do you assign working individually on long term projects or experiments?	1.6/1.9	0.6/1.2

table continues

Table 9 (Cont'd)

Summary of Pre-test and Post-test Teacher Surveys on Mathematics Teaching Practices

Pre-test and post-test questions	Experimental Group (Pre-test N = 10, Post-test N = 9) (% Yes) or Likert Scale Score Pre-test/Post-test	Control Group (Pre-test N = 13, Post-test N = 12) (% Yes) or Likert Scale Score Pre-test/Post-test
0 = Never, 1 = Rarely, 2 = Sometimes, 3 = Always		
When you assign homework how often do you assign working as a small group on long term projects or experiments?	1.5/1.9	0.5/0.8
When you assign homework how often do you assign finding one or more uses of the content covered?	1.6/1.5	0.8/0.9
When you assign homework how often do you assign preparing oral reports either individually or as a small group?	0.6/0.8	0.4/0.7
When you assign written homework, how often do you record whether or not the homework was completed?	2.6/3.1	2.5/2.6
When you assign written homework, how often do you collect, correct and keep assignments?	2.1/2.1	1.5/1.1
When you assign written homework, how often do you give feedback on homework and then return to students?	2.3/2.6	1.8/1.8
When you assign written homework, how often do you give feedback on homework to the whole class?	2.3/2.7	2.2/2.1
When you assign written homework, how often do you have students correct their own assignments in class?	1.9/2.2	1.9/1.9
When you assign written homework, how often do you use it as a basis for class discussion?	2.0/2.3	1.2/1.6
When you assign written homework, how often do you use it to contribute towards grades or marks?	2.3/3.1	2.2/2.7

When the pre-test and post-test responses are compared for each group of teachers there appear to be few changes in their responses during the term of the study. One notable change is that the Experimental Group teachers indicated that after teaching the

Applications of Mathematics 10 course, they were more likely to focus on problem solving activities (60% pre-test increased to 89% post-test).

When the two groups (Control and Experimental) are compared to each other, there are a number of differences that appear to be consistent with the intended treatments. Seventy percent (pre-test) to 78% (post-test) of the Experimental Group teachers reported that they were likely to have students use concrete materials, while only 38% (pre-test) to 50% (post-test) of the Control Group teachers reported the same. Far more of the Experimental Group teachers (60% pre-test & 56% post-test) were likely to have students use computers as compared to Control Group teachers (0% pre-test & 17% post-test). In addition, the surveys indicate that Control Group teachers assign more homework more often than their Experimental Group teachers.

Other differences between the two groups relate to the types of assignments given to students. Experimental Group results indicate that these teachers "rarely" or "sometimes" assign small investigations or long term projects. The Control Group teachers' scores indicate that they "never" or "rarely" do the same thing.

With respect to grading homework, the Experimental Group teachers indicate that they "sometimes" or "always":

- record completion of homework;
- collect, correct and keep assignments;
- provide feedback to individual students; and,
- use the homework results for class discussion.

Control Group teachers answered the same questions with "rarely" or "sometimes".

Teacher Logbooks

The data for each participating teacher's monthly Teacher Logbook were compiled on a monthly basis using the following procedures:

1. For each teacher's monthly logbook the total number of occurrences of teaching or assessment strategies used was determined by summing the number of times each strategy was reported as being used.
2. The average amount of class time spent each month on the different categories of teaching or assessment strategies was then calculated by summing the time reported as spent daily using the strategies and then dividing it by the number of classes available that month for the teaching of the course.

At the completion of each course the monthly data from the teacher's logbook were compiled into monthly average and course totals for each of the Experimental and Control Group teachers as follows:

3. The Average Monthly Total (Number of Occurrences of Each Strategy) was calculated by summing the separate monthly totals and dividing this sum by the number of months the course was taught (i.e., 10, 5, or 2.5 depending upon whether the course was taught using a linear, semester or quarter system timetable).
4. The Average Monthly Time (% of Class Time) was calculated in a similar manner by adding up the separate monthly times and dividing this total by the number of months the course was taught (i.e., 10, 5, or 2.5 depending upon whether the course was taught using a linear, semester or quarter system timetable).

5. The Group Average Course Totals and Group Average Course Time were calculated by summing the Average Course Totals or the Average Course Times for all of the teachers in each group and dividing these totals by the number of teachers in each group.

All calculations used daily or monthly averages as not all participating teachers completed all of the monthly logbooks for the duration of the teaching of their course. In addition, the different timetables in which the courses were taught (linear, semester quarter) made it impossible to combine the results of the time spent using each group of teaching or assessment strategies reported in the monthly logbooks except as a percentage of class time.

Tables 10 and 11 summarize, for the Experimental Group, the individual teacher and Group Average data for the number of occurrences of each instructional and assessment strategy as reported by the teachers in their logbooks. Of the ten Experimental Group teachers participating in the study, seven regularly maintained a logbook.

Table 10

Monthly Average Number of Occurrences of Different Teaching Strategies Used by the Experimental Group Teachers

Strategy Category / Specific Strategy	Average Monthly Totals (Number of Occurrences of Each Strategy - by Teacher)							Experimental Group Average Course Total (Occurrences)
	A	B	C	D	E	F	G	
Consolidation & Practice								86
Drill and Practice	0.8	4.3	10.8	1.4	10	1.3	1.3	21
Homework	1	5.7	10	4.8	8	1	2.8	26
Textbooks	0	3.2	9.6	6	3	1	4	20
Worksheets	2.3	4.7	3.8	1.4	7.7	2.3	1.5	19
Discussion								29
Group Work	4.3	0.4	3.4	1.8	5	2.7	5.5	18
Role Play	0	0.1	0	0	0	0.3	0	0
Explaining	1	0.6	0.2	0.6	5.3	3	2.3	9
Peer Tutoring	0	0.1	0.2	0.2	2.3	0	0.3	2
Practical Work								8
Manipulatives	0.5	0	0.2	1.6	1	1	1.5	4
Game Playing	0	0	0.4	0.2	0.7	0.7	0	1
Outdoors	0.3	0	0	0.4	0.3	1.7	0	2
Mathematics Centres	0	0	0	0.8	1	0	0	1
Investigating								41
Guided Discovery	0.8	1.1	0	6	6.7	2.7	8	17
Projects	4	1.6	3.4	7.8	0	1	2.5	19
Open-ended Tasks	0	0.9	0.4	2.2	0.3	0	2	5
Teacher Centered								71
Exposition	0.3	8.7	10.8	4	6	1.3	3.5	29
Beginning a Lesson	2.8	8.6	12	3.6	8	5	6	38
Team Teaching	0	0.1	0	0	0.3	0	0	0
Involving Others	0.3	0	0	0.2	9.7	0	0	4
Problem Solving								48
Puzzles	0.5	2.1	0.8	0.4	0.7	0.7	0	5
Modeling	0	2.9	0.8	1.4	6.3	1.3	0.8	10
Applications	1.5	4.8	1	1.6	10.7	2	2.3	18
Themes	0	0	0	0.2	10	0.3	0	4
Problem Solving	1.5	3.7	0.4	0.6	2.3	1	0	10
Problem Posing	0	0	0.4	0.2	0.3	0.7	0.5	1
Using Technology								28
Calculators	3	4.7	0.2	1	9.7	1.3	3.5	19
Graphing Calculators	1	1.3	0.8	0	0.3	0.3	1	5
Computers	0	0	0.4	0.8	0.3	1	2.8	4
Other	0	0	0	1.8	8	0	0	4
							Total	315

Table 11

Monthly Average Number of Occurrences of Different Assessment Strategies Used by the Experimental Group Teachers

Strategy Category / Specific Strategy	Average Monthly Totals (Number of Occurrences of Each Strategy - by Teacher)							Experimental Group Average Course Total (Occurrences)
	A	B	C	D	E	F	G	
Observing								14
Checklists	0	0	0.4	0	0	0.8	9.5	8
Anecdotal	3.3	0	0	0	1.3	0.4	0	6
Questioning								33
Higher order	3	4.2	0.4	0	4.7	0.8	0.3	13
Factual	3.3	4.3	0.6	0	9.3	1.2	1	16
Open-ended	0.3	1.6	0.2	0	2	0.2	0.8	4
Interviewing								3
Structured	0	0	0	0.2	0.3	0	0	0
Open	0	0	0	0	5.7	0.6	0.8	3
Parent	0	0	0	0	0.7	0	0	0
Testing								27
Diagnosis	0	0.1	0	0.8	2	0	0	1
Performance-based	1.8	0.2	0.8	1.6	1.7	0	0	5
Pencil and Paper	1	2.4	1.4	2.6	10.7	1.2	0.5	13
Multiple Choice	0	0.2	0	0.6	1.7	0.4	0	2
Problem Solving	0.3	0.7	0.4	1.6	4.3	1	0	5
Attitude	0	0	0	0.2	0.7	0.4	0	1
Reporting								12
Oral	0.8	0.3	0	0	0.7	0.8	0.5	3
Written	2.8	0.3	0	0	3.3	1	0.8	7
Portfolio	0	0	0	0	0	0.2	0	0
Investigation	0.5	0	0	0	0	0	0.3	1
Modeling	0.3	0	0	0	1	0	0.3	1
Self - Assessment								9
Journals	0	0	0	0	0	0	0	0
Reflective Prompts	0	0	0	1.2	4	0	0.3	3
Self-questioning	0.8	0	0	0.6	6.3	0	0	4
Peer Assessment	0.3	0	0	0.6	0.7	0	1.3	2
Other	0	0	0	0.4	2	0	0	1
							Total	99

Tables 12 and 13 summarize, for the Experimental Group, the individual teacher and Group Average data for the monthly average time spent using each instructional and assessment strategy as reported by the teachers in their logbooks.

Table 12

Monthly Average Time Spent by Experimental Group Teachers Using Different Teaching Strategies

Teaching Strategy Category	Average Monthly Time (% of Class Time - by Teacher)							Experimental Group Average Course Time (% of Class Time)
	A*	B	C	D	E	F	G	
Consolidation & Practice		25	39	22	14	16	18	22
Discussion		2	6	5	5	10	15	7
Practical Work		0	2	5	3	12	2	4
Investigating		8	13	31	4	12	21	15
Teacher Centered		27	15	5	8	12	7	12
Problem Solving		18	6	6	13	15	6	11
Using Technology		2	7	7	8	6	17	8
Other		0	0	2	8	0	0	2
Total		82	88	83	63	83	86	81

Table 13

Monthly Average Time Spent by Experimental Group Teachers Using Different Assessment Strategies

Assessment Strategy Category	Average Monthly Time (% of Class Time- by Teacher)							Experimental Group Average Course Time (% of Class Time)
	A*	B	C	D	E	F	G	
Observing		0	0	0	0	5	3	1
Questioning		7	1	0	5	6	1	3
Interviewing		0	0	1	2	1	0	1
Testing		10	6	10	24	9	3	10
Reporting		1	0	0	1	3	1	1
Self - Assessment		0	0	2	3	0	1	1
Total		18	7	13	35	24	9	18

* Teacher A did not report the time spent on each teaching or assessment strategy.

Tables 14 and 15 summarize, for the Control Group, the individual teacher and Group Average data for the number of occurrences of each instructional and assessment strategy as reported by the teachers in their logbooks. Of the thirteen Control Group teachers participating in the study (three 10A teachers and seven PM 10 teachers), four PM 10 teachers and none of the 10A teachers regularly maintained a logbook.

Table 14

Monthly Average Number of Occurrences of Different Teaching Strategies Used by the Control Group Teachers

Strategy Category / Specific Strategy	Average Monthly Totals by Teacher (Number of Occurrences of Each Strategy - by Teacher)				Control Group Average Course Total (Occurrences)
	H	I	J	K	
Consolidation & Practice					134
Drill and Practice	8	5.1	4.6	3	32
Homework	11.3	5.4	10.6	5.5	48
Textbooks	9	4.8	10.6	5.8	44
Worksheets	3.8	2.1	0	0.3	10
Discussion					13
Group Work	3	0.1	0	1.8	6
Role Play	0	0	0	0	0
Explaining	0.8	0.3	0	1.8	4
Peer Tutoring	1.3	0	0	1	3
Practical Work					1
Manipulatives	0	0	0	0.8	1
Game Playing	0	0	0	0	0
Outdoors	0	0	0	0.3	0
Mathematics Centres	0	0	0	0	0
Investigating					5
Guided Discovery	1.3	0.3	0	0.8	3
Projects	0	0.1	0	0.3	1
Open-ended Tasks	0	0.2	0	0.5	1
Teacher Centered					104
Exposition	10	8	9.4	1.8	47
Beginning a Lesson	11.5	8	8.8	8.8	56
Team Teaching	0	0	0	0	0
Involving Others	0	0.2	0.2	0	1
Problem Solving					36
Puzzles	0	1.6	0	1	5
Modeling	0	2.7	0	0.3	7
Applications	0.8	3.6	0	0	10
Themes	0	0	0	0	0
Problem Solving	1.8	2.8	0.2	1.3	11
Problem Posing	0.8	0.3	0	1	3
Using Technology					14
Calculators	1.3	3.9	1.4	0.3	14
Graphing Calculators	0	0	0	0.3	0
Computers	0	0	0	0	0
Other	0	0	5.6	0	7
				Total	314

Table 15

Monthly Average Number of Occurrences of Different Assessment Strategies Used by the Control Group Teachers

Strategy Category / Specific Strategy	Average Monthly Totals by Teacher (Number of Occurrences of Each Strategy - by Teacher)				Control Group Average Course Total (Occurrences)
	H	I	J	K	
Observing					11
Checklists	8.3	0	0	0	10
Anecdotal	0.5	0	0	0.5	1
Questioning					28
Higher order	0.8	3.9	0	0.3	11
Factual	1.5	4.8	0	0	14
Open-ended	0.3	1.1	0	0	3
Interviewing					0
Structured	0	0	0	0	0
Open	0.3	0	0	0	0
Parent	0	0	0	0	0
Testing					28
Diagnosis	0	0	0.2	0.3	1
Performance-based	0	0.6	0	0.8	3
Pencil and Paper	5.5	2.1	1.6	1.8	16
Multiple Choice	0	0.1	2	0.3	3
Problem Solving	0.3	0.3	1.2	1.8	5
Attitude	0	0	0.2	0	0
Reporting					0
Oral	0	0	0	0	0
Written	0	0	0	0	0
Portfolio	0	0	0	0	0
Investigation	0	0	0	0	0
Modeling	0	0	0	0	0
Self - Assessment					2
Journals	0	0	0	0	0
Reflective Prompts	0	0	1.2	0	2
Self-questioning	0	0	0.2	0	0
Peer Assessment	0	0	0	0	0
Other	0.3	0	0.2	0	1
				Total	70

Tables 16 and 17 summarize, for the Control Group, the individual teacher and Group Average data for the monthly average time spent using each instructional and assessment strategy as reported by the teachers in their logbooks.

Table 16

Monthly Average Time Spent by Control Group Teachers Using Different Teaching Strategies

Teaching Strategy Category	Average Monthly Time (% of Class Time- by Teacher)				Control Group Average Course Time (% of Class Time)
	H	I	J	K	
Consolidation & Practice	59	31	46	44	45
Discussion	6	1	0	7	4
Practical Work	0	0	0	1	0
Investigating	2	1	0	5	2
Teacher Centered	17	34	21	19	23
Problem Solving	2	13	0	5	5
Using Technology	1	2	3	3	2
Other	0	0	6	0	1
Total	87	82	76	84	82

Table 17

Monthly Average Time Spent by Control Group Teachers Using Different Assessment Strategies

Assessment Strategy Category	Average Monthly Time (% of Class Time - by Teacher)				Control Group Average Course Time (% of Class Time)
	H	I	J	K	
Observing	3	0	0	1	1
Questioning	1	8	0	1	3
Interviewing	0	0	0	0	0
Testing	12	10	14	19	14
Reporting	0	0	0	0	0
Self - Assessment	0	0	2	0	0
Total	16	18	16	21	18

Tables 18 and 19 and Figures 9 and 10 compare the Control and Experimental Groups with respect to the course average time spent using different teaching and assessment strategies as reported in the teacher logbooks

Table 18

Comparison Between Experimental and Control Group Teachers of the Course Average Time Spent Using Different Teaching Strategies

Teaching Strategy Category	Experimental Group Average Course Time (% of Class Time)	Control Group Average Course Time (% of Class Time)
Consolidation & Practice	22	45
Discussion	7	4
Practical Work	4	0
Investigating	15	2
Teacher Centered	12	23
Problem Solving	11	5
Using Technology	8	2
Other	2	1
Total	81	82

Table 19

Comparison Between Experimental and Control Group Teachers of the Course Average Time Spent Using Different Assessment Strategies

Assessment Strategy Category	Experimental Group Average Course Time (% of Class Time)	Control Group Average Course Time (% of Class Time)
Observing	1	1
Questioning	3	3
Interviewing	1	0
Testing	10	14
Reporting	1	0
Self - Assessment	1	0
Total	18	18

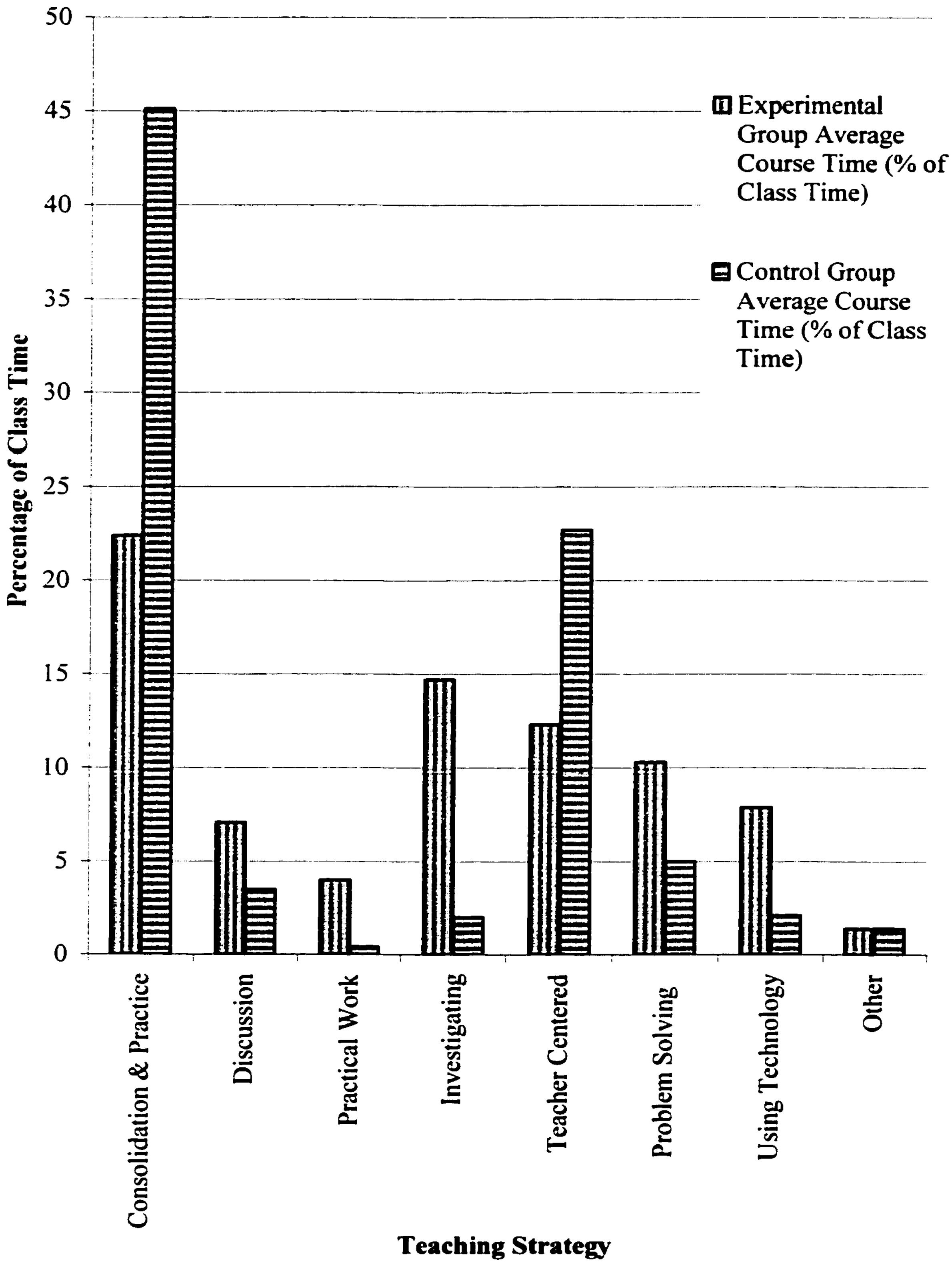

Figure 9. Comparison Between Experimental and Control Group Teachers of the Course Average Time Spent Using Different Teaching Strategies.

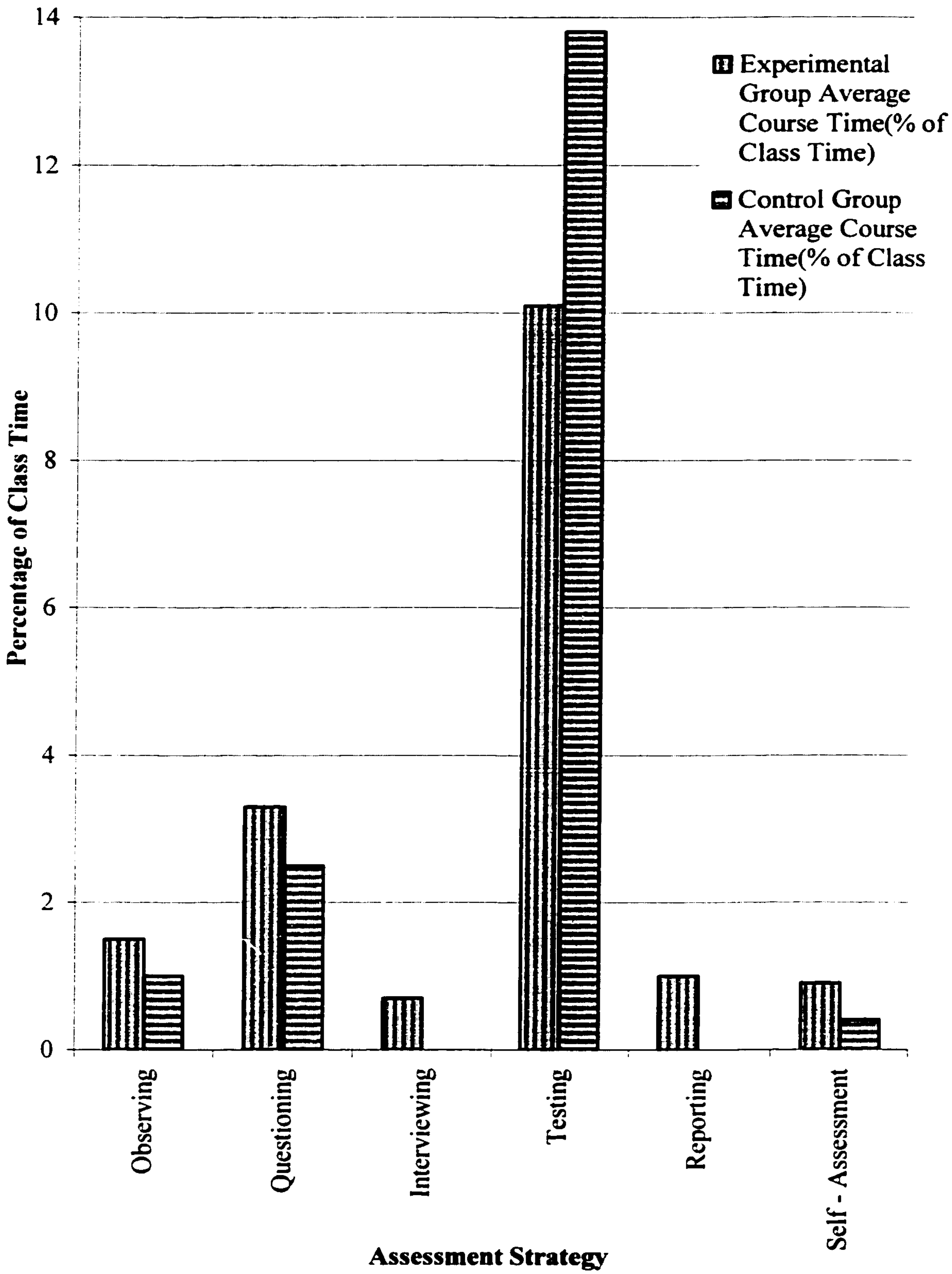

Figure 10. Comparison Between Experimental and Control Group Teachers of the Course Average Time Spent Using Different Assessment Strategies.

The comparison between the amount of time that the Experimental and Control Group teachers spent using different teaching and assessment strategies indicates that both groups spend the same amount of class time involved in the acts of teaching (81% to 82%) and assessment (18%). [Note that the total time does not equal 100% due to minor calculation errors made by the teachers in their Logbooks, administrative management, etc.]

There is considerable difference in the amount of time spent on specific categories of teaching or assessment strategies. For example, the Control Group (N = 4) teachers reported spending more than twice as much time on average using Consolidation and Practice as did the Experimental Group (N = 7) teachers (45% compared to 22%). In addition, the amount of time spent using Teacher Centered teaching strategies is also considerably more for the Control Group teachers than for the Experimental Group teachers (23% compared to 12%). While the Experimental Group teachers spent a considerable amount of class time using Investigative teaching strategies (15%), the Control Group Teachers used only 2% of class time for this. In general, the Control Group teachers used two primary sets of teaching strategies (Consolidation and Practice, and Teacher Centered) that amount to 84% of the actual teaching time, whereas the Experimental Group teachers reported using these strategies for 47% of the actual teaching time. They also reported using a variety of additional teaching strategies such as Discussion, Practical Work, Problem Solving, and Using Technology.

Similar differences are found in the amount of time spent using different assessment strategies. Although both groups primarily used Testing, the Control Group teachers reported testing for 40% more time than did Experimental Group teachers.

Experimental Group Teachers reported using other assessment strategies including Observing, Questioning, Interviewing, Reporting, and Self-Assessment more of the actual assessment time (57%) than did the Control Group teachers (25% of assessment time).

Tables 20 and 21provide a comparison between the Control and Experimental Groups with respect to the average number of different teaching and assessment strategies used during the teaching of each course. The comparisons are also shown graphically in Figures 11 and 12.

Table 20

Comparison Between Experimental and Control Group Teachers of the Number of Different Teaching Strategies Used During the Teaching of Each Course

Teaching Strategy Category / Specific Strategy	Experimental Group Average Monthly Total (Number of Occurrences)	Control Group Average Monthly Total (Number of Occurrences)
Consolidation & Practice	86	134
Drill and Practice	21	32
Homework	26	48
Textbooks	20	44
Worksheets	19	10
Discussion	29	13
Group Work	18	6
Role Play	0	0
Explaining	9	4
Peer Tutoring	2	3
Practical Work	8	1
Manipulatives	4	1
Game Playing	1	0
Outdoors	2	0
Mathematics Centres	1	0
Investigating	41	5
Guided Discovery	17	3
Projects	19	1
Open-ended Tasks	5	1
Teacher Centered	71	104
Exposition	29	47
Beginning a Lesson	38	56
Team Teaching	0	0
Involving Others	4	1
Problem Solving	48	36
Puzzles	5	5
Modeling	10	7
Applications	18	10
Themes	4	0
Problem Solving	10	11
Problem Posing	1	3
Using Technology	28	14
Calculators	19	14
Graphing Calculators	5	0
Computers	4	0
Other	4	7
Total	314	315

Table 21

Comparison Between Experimental and Control Group Teachers of the Number of Different Assessment Strategies Used During the Teaching of Each Course

Assessment Strategy Category	Specific Strategy	Experimental Group Average Course Total (Number of Occurrences)	Control Group Average Course Total (Number of Occurrences)
Observing		14	11
	Checklists	8	10
	Anecdotal	6	1
Questioning		33	28
	Higher order	13	11
	Factual	16	14
	Open-ended	4	3
Interviewing		3	0
	Structured	0	0
	Open	3	0
	Parent	0	0
Testing		27	28
	Diagnosis	1	1
	Performance-based	5	3
	Pencil and Paper	13	16
	Multiple Choice	2	3
	Problem Solving	5	5
	Attitude	1	0
Reporting		12	0
	Oral	3	0
	Written	7	0
	Portfolio	0	0
	Investigation	1	0
	Modeling	1	0
Self - Assessment		9	2
	Journals	0	0
	Reflective Prompts	3	2
	Self-questioning	4	0
	Peer Assessment	2	0
Other		1	1
	Total	99	70

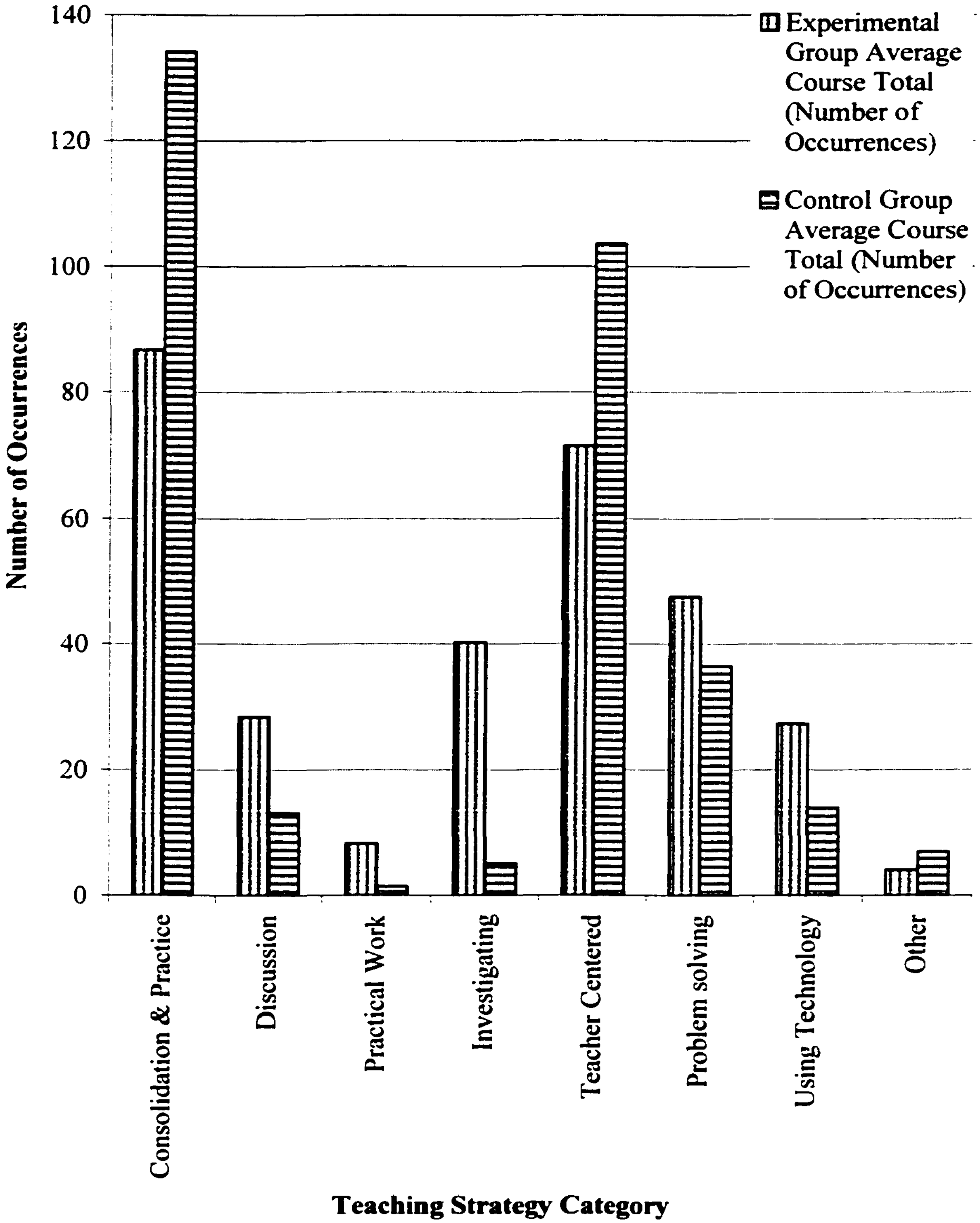

Figure 11. Comparison Between Experimental and Control Group Teachers of the Average Number of Different Teaching Strategies Used During the Teaching of Each Course

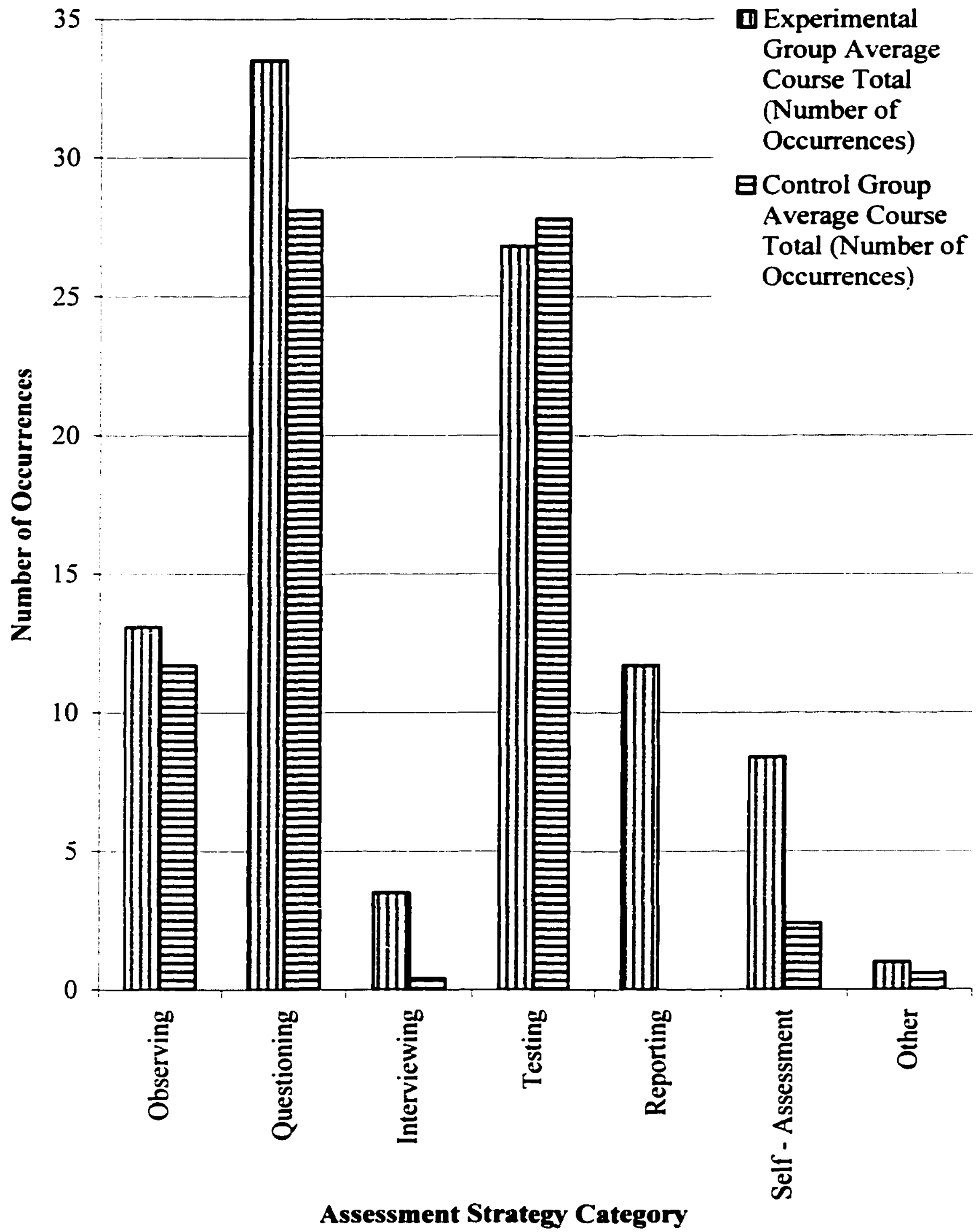

Figure 12. Comparison Between Experimental and Control Group Teachers of the Average Number of Different Assessment Strategies Used During the Teaching of Each Course

A general observation from these data is that the Experimental Group and Control Group teachers reported using the same number of teaching strategies (in the same amount of class time). The relative differences between the two groups occur in the breakdown of the different teaching strategies. Control Group teachers reported using Consolidation and Practice strategies 55% more often than did the Experimental Group teachers. Similarly, Control Group teachers reported using Teacher Centered teaching strategies 44% more often than did Experimental Group teachers. At the same time, the Experimental Group teachers reported using the following teaching strategies more often than did the Control Group teachers: Discussion (more than twice as often); Practical Work (almost fives times more often); Investigating (almost 7 times more often); Problem Solving (30% more often); and, Using Technology (almost twice as often).

The most obvious difference between the Control and Experimental Group teachers with respect to assessment practices is that the Experimental Group teachers reported assessing their students 41% more often than did the Control Group teachers. The second obvious difference is that the Experimental Group teachers reported using a wider variety of assessment strategies than did the Control Group teachers. These strategies include: Interviewing (almost eight times as often); Reporting (12 times vs. none for Control Group teachers); and, Self-Assessment (2.5 times more often).

Both groups reported using Testing equally to assess their students. The Experimental Group teachers used Testing strategies 27 times (on average) while teaching the course compared to 28 times for the Control Group teachers. The differences in Testing frequency relate to the types of testing used by the two groups. Experimental Group teachers reported using a wider range of testing techniques than did the Control

Group teachers. Of particular note is that Experimental Group teachers reported using diagnostic and performance-based testing more than twice as often as did Control Group teachers while at the same time, using pencil and paper testing 28% less often.

Student Achievement and Attitude Toward Mathematics Assessments (Pre- and Post-Test)

The student achievement and attitude assessment results (pre- and post-test) were scanned electronically and converted into text files. The text files were then imported and converted into SPSS data documents (using SPSS Base 9.0). The pre-test and post-test data were matched by pre-assigned student identification numbers and any students who did not complete both assessments were not included in subsequent analyses.

Experimental and Control Group Student Population Characteristics

Tables 22 through 24 and Figures 13 through 17 provide a profile of the students (by Control and Experimental Group and course) using the characteristics of gender, age, previous courses taken, and grades in previous courses.

Table 22

Experimental and Control Group Student Populations by Gender

Group (Course)	Male Population (% of Pop.)	Female Population (% of Pop.)	Student Population
Experimental Group (Applications of Mathematics 10)	78 (50.7)	76 (49.3)	154
Control Group (PM 10 & 10A Combined)	112 (48.3)	120 (51.7)	232
Control Group (Mathematics 10A)	25 (64.1)	14 (35.9)	39
Control Group (Principles of Mathematics 10)	87 (45.1)	106 (54.9)	193
Total Population	190 (49.3)	196 (50.7)	386

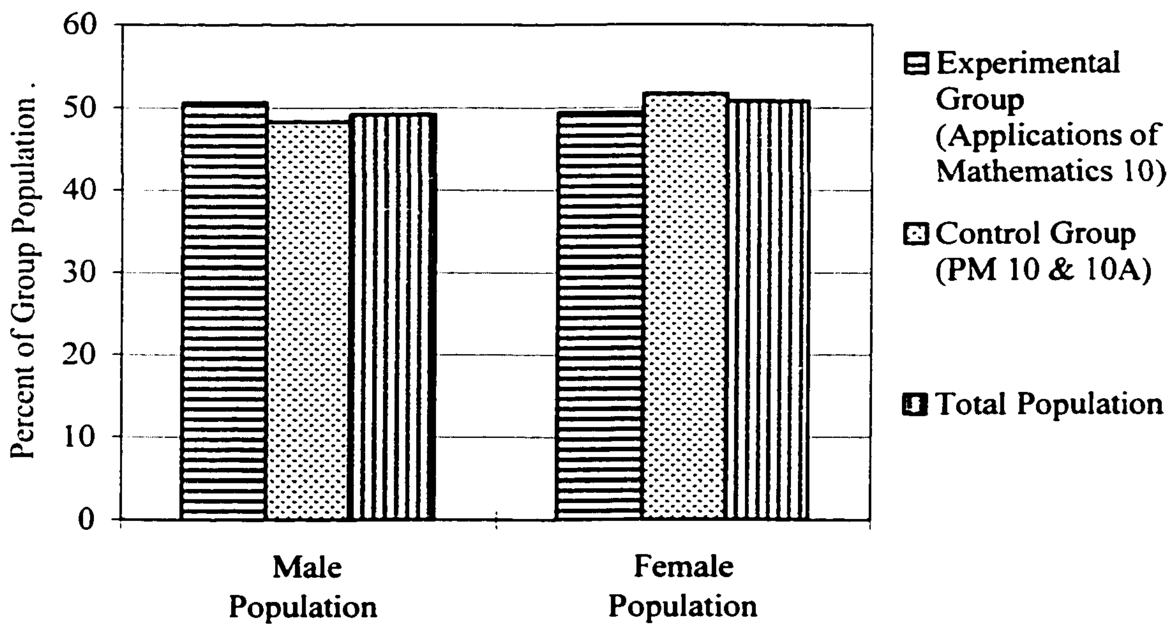

Figure 13. Comparison of Experimental and Control Group Student Populations by Gender

Both the Experimental and Control Group student populations are balanced with respect to gender. It should be noted that the Mathematics 10A sub-population within the Control Group is predominantly male (64.1%) while the Experimental and Control Groups have virtually the same gender balance (50.7% and 48.3 % male vs. 49.3% and 51.7% female respectively).

The age distributions within the two groups, broken down by the previous course taken, are shown below in Table 23 and Figure 14.

Table 23

Experimental and Control Group Student Populations by Previous Course and Age at Time of Writing Pre-Test Assessment

Group (Course(s))	Age (yrs)	Previous Course (% of Group Population) 9A	AM 9	PM 9	Gr. 10	Total (%)
Experimental Group (AM 10)						
	14	1.1	5.4	4.9		11.4
	15	7.0	33.0	28.1		68.1
	16	0.5	7.6	3.8	3.8	15.7
	17	0.5	0.5	0.5	2.2	3.7
	18				1.1	1.1
	Total	9.1	46.5	37.3	8.1	100
Control Group (PM 10 & 10A Combined)						
	14	0.6		18.1		18.7
	15	4.5	5.9	56.8	0.9	68.1
	16	3.9	0.6	3.3	1.8	9.6
	17	1.2		0.9	0.6	2.7
	18			0.6		0.6
	Total	10.2	6.5	79.7	3.3	99.7

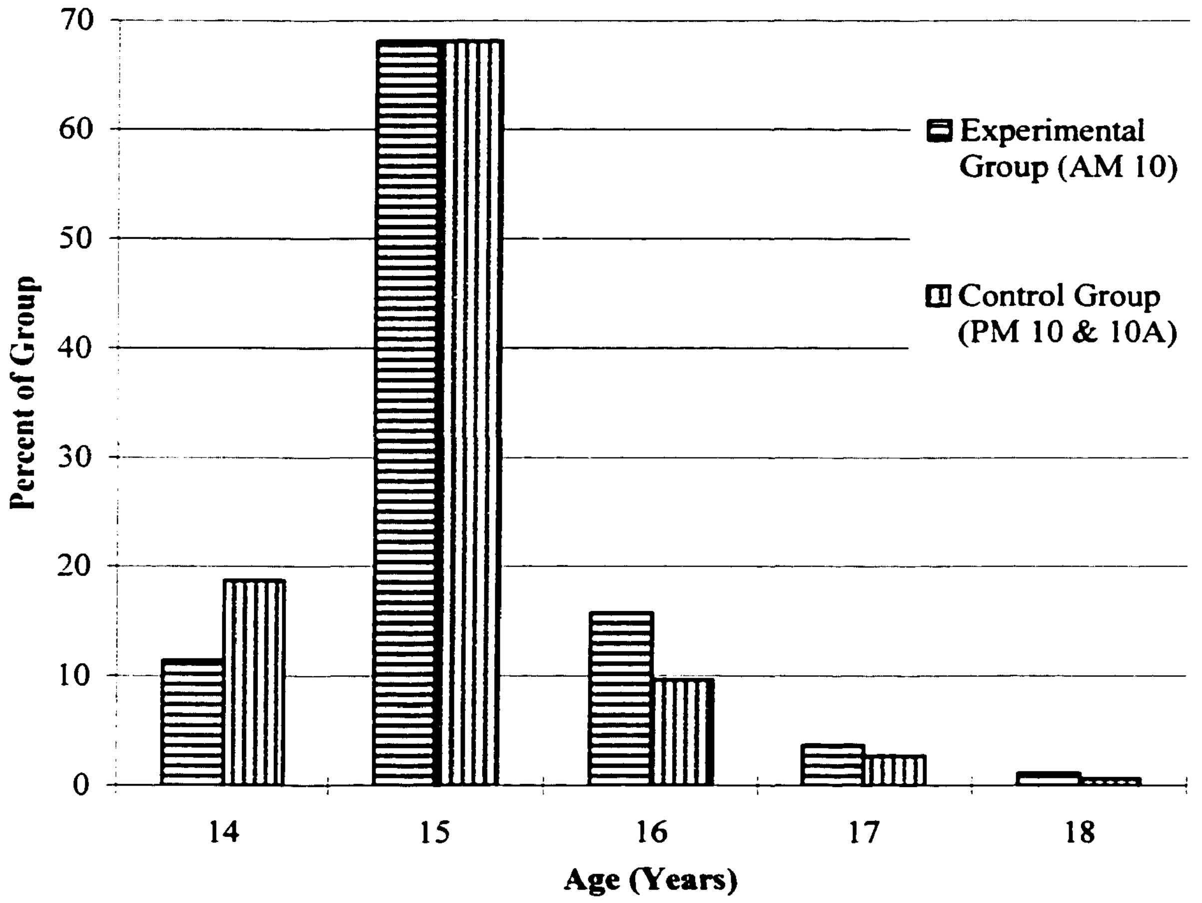

Figure 14. Comparison of Experimental and Control Group Student Populations by Age.

The mode for both groups is 15 years of age (68.1% of students in both groups). This is expected as the majority of the students wrote the pre-test assessment at the beginning of the school year (September 1998). The Experimental Group has a higher percentage of students who are 16 to 18 years old (20.5%) than does the Control Group (12.9%), while the Control Group has a corresponding higher percentage of 14 year olds (18.7%) than does the Experimental Group (11.4%). These differences may best be accounted for when the students' grades in their previous mathematics course are examined (see Table 24 and Figure 15).

Table 24

Experimental and Control Group Student Populations by Previous Course Taken and Grade Reported in Previous Course

Group (Course(s))	Grade	Previous Course (% of Group Population) 9A	AM 9	PM 9	Gr. 10	Total (%)
Experimental Group (AM 10)						
	A	1.0	4.2	0.5		5.7
	B	4.2	12.0	6.3	0.5	23.0
	C or C+	3.7	28.3	26.7	1.0	59.7
	P		2.6	2.6	0.5	5.7
	F			1.0	4.7	5.7
	Total	8.9	47.1	37.1	6.7	99.8
Control Group (PM 10 & 10A)						
	A	0.3	1.2	21.4		22.9
	B	2.7	1.2	32.7	0.6	37.2
	C or C+	6.5	3.9	21.7	0.6	32.7
	P	0.3	0.6	2.7	0.3	3.9
	F		0.3	0.9	2.1	3.3
	Total	9.8	7.2	79.4	3.6	100

The Experimental Group sample includes a higher percentage of students who had failed their previous mathematics course (5.7%) than did the Control Group sample (3.3%). The Experimental Group students repeating the course due to past failure result in a corresponding increase in the percentage of the student population in the 17 to 18 year age range (4.8%).

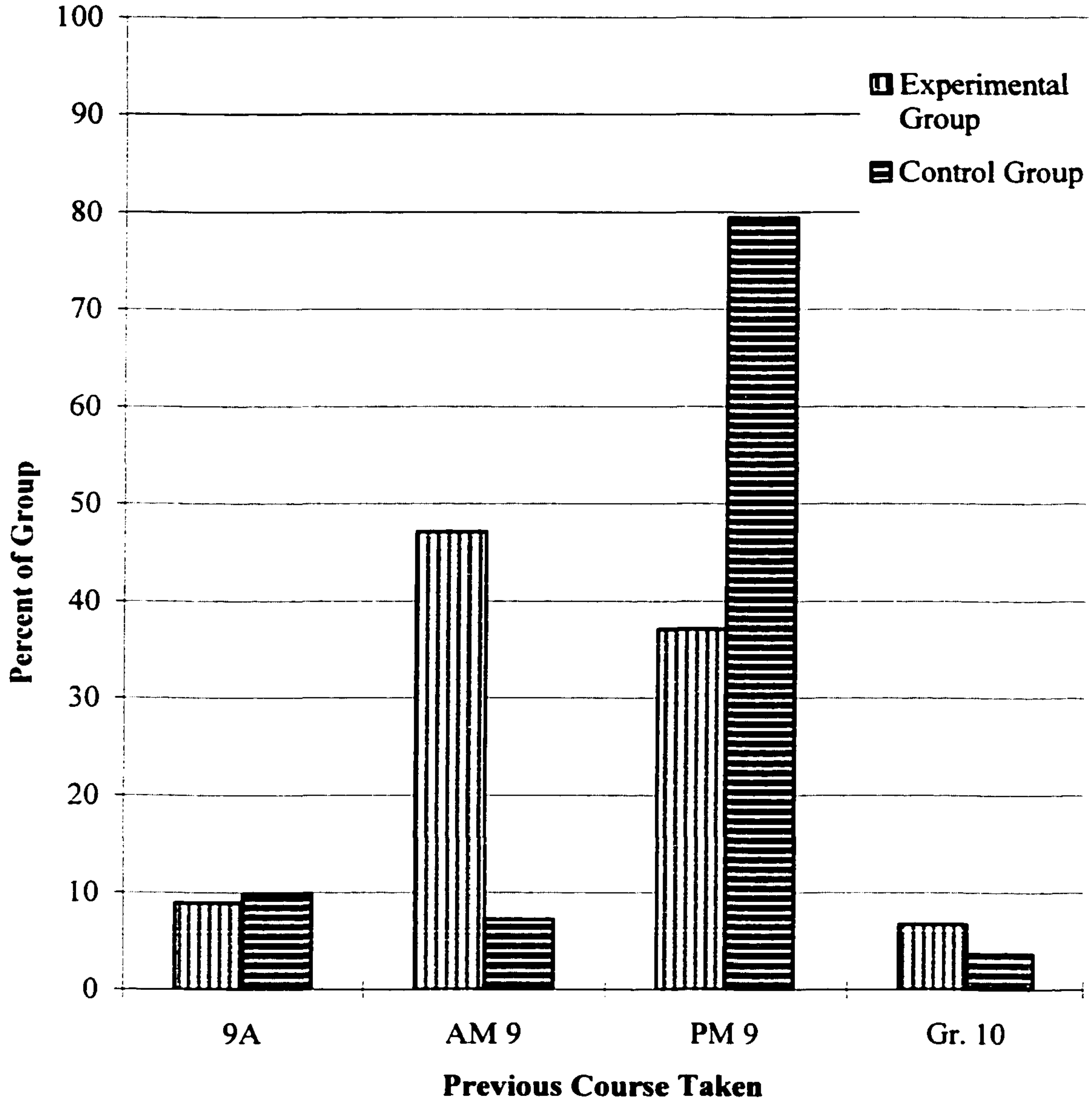

Figure 15. Comparison of Experimental and Control Group Student Populations by Previous Course Taken

Figure 15 illustrates that the majority of students in the Experimental Group previously took either Applications of Mathematics 9 (47.1%) or Mathematics 9A (8.9%) prior to taking Applications of Mathematics 10. The data also confirm that the majority of students in the Control Group previously took Principles of Mathematics 9 (79.4%) before taking Principles of Mathematics 10. It should be noted that, of those students in the Control Group identified as previously taking Mathematics 9A (9.8%), almost all are taking Mathematics 10A. The Experimental Group population includes students who have taken courses other than Principles of Mathematics while the Control Group population consists primarily of students who have previously taken Principles of Mathematics.

Further analysis of the Control and Experimental Group students' grade distribution in their previous courses provides additional information concerning the characteristics of the two populations. Figures 16 and 17 provide a different perspective on the profiles of the Experimental and Control Group students.

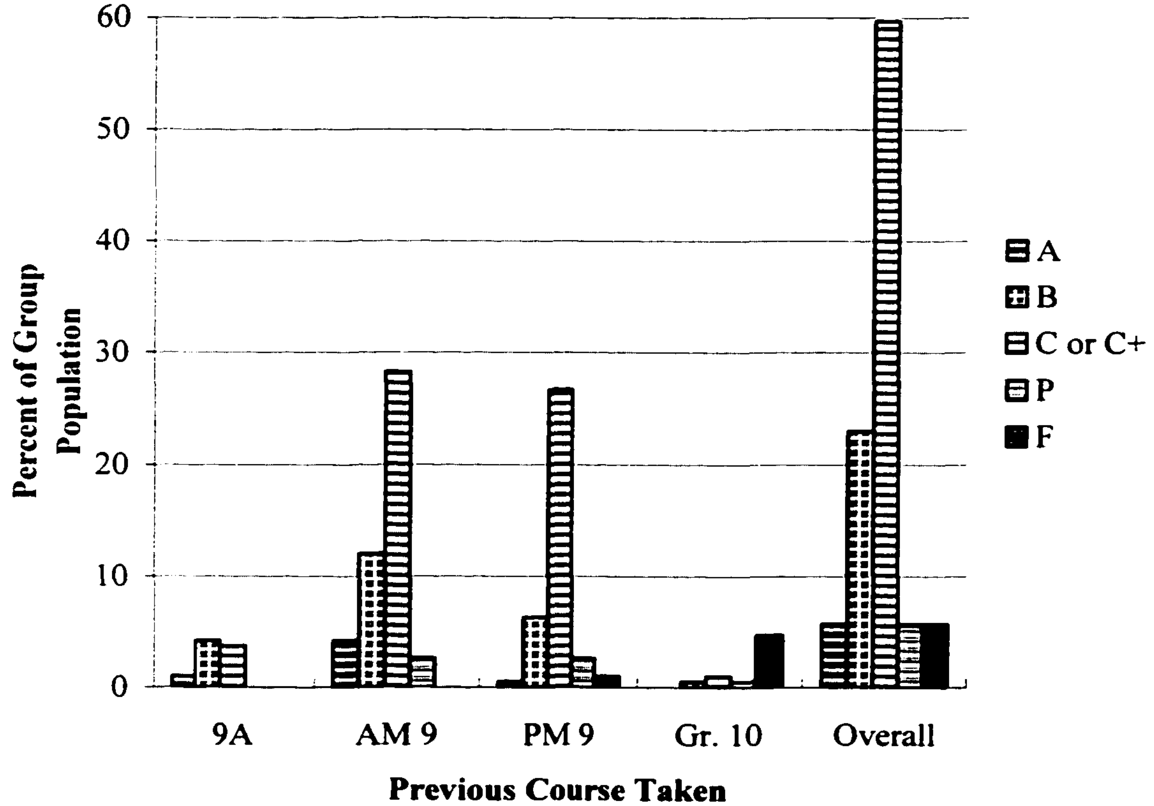

Figure 16. Experimental Group Population Profile: Grouped by Previous Course Taken and Grade in Previous Course.

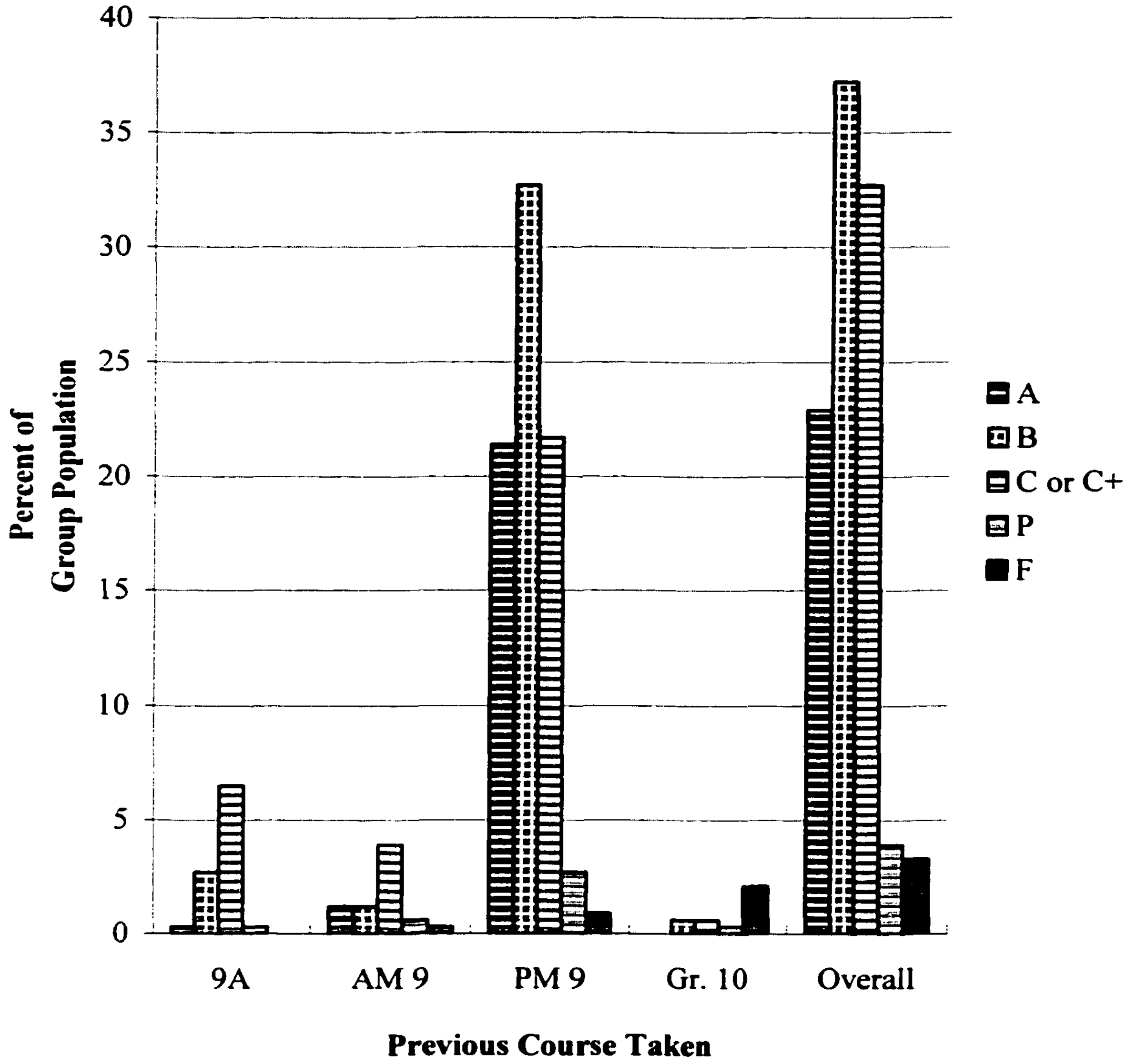

Figure 17. Control Group Population Profile: Grouped by Previous Course Taken and Grade in Previous Course.

When the Control and Experimental Group students' prior courses grades are analyzed, it becomes apparent that the Experimental Group students have not had the same success at mathematics as the Control Group students. Seventy-one percent of the Experimental Group students received a letter grade of "C+" or lower in their previous course (including 5.7% with a failing grade). The Control Group students' letter grade distribution is different in that only 40% of these student received a "C+" or lower in their previous course (3.3% with a failing grade). This difference could indicate that the Control Group students are starting with a history of success in mathematics that may have a greater positive impact on their attitude towards mathematics and/or greater self-confidence.

Experimental and Control Group Student Results

The student achievement total scores (pre- and post-test) were divided into four sub-scores (Number, Patterns and Relations, Shape and Space, and Statistics and Probability) corresponding to the curriculum organizers. In addition, an Attitude Toward Mathematics score was also calculated for both the pre-test and post- test assessments. Tables 25 (Pre-Test) and 26 (Post-Test) include the student scores for both the Experimental Group (AM 10), and the Control Group (PM 10 & 10A); and the separate scores for the PM 10 and 10A Control Sub-groups.

Table 25

Experimental and Control Group Pre-Test Assessment Results

Group (course)	N	Pre-Test Assessment Results					
		Curriculum Organizer Scores					
		Number	Patterns & Relations	Shape & Space	Statistics & Probability	Total Score	Attitude Score
Max. Score		7	12	14	7	40	20
Experimental (AM 10)	154						
M		3.5	2.7	3.5	2.8	12.6	10.1
SD		1.3	1.7	1.9	1.6	4.2	2.8
Control (Combined)	232						
M		4.3	4.1	4.7	3.2	16.3	12.6
SD		1.4	2.2	2.4	1.5	5.5	3.3
Control (10A)	39						
M		3.4	2.2	2.9	2.2	10.6	9.1
SD		1.2	1.5	1.8	1.3	3.9	2.6
Control (PM 10)	193						
M		4.5	4.5	5.1	3.5	17.5	13.2
SD		1.3	2.1	2.3	1.5	5.1	3.0
Total	386						
M		4.0	3.6	4.2	3.1	14.9	11.6
SD		1.4	2.1	2.3	1.6	5.4	3.3

Table 26

Experimental and Control Group Post-Test Assessment Results

Group (course)	N	Number	Patterns & Relations	Shape & Space	Statistics & Probability	Total Score	Attitude Score
		Post-Test Assessment Results					
		Curriculum Organizer Scores					
Max. Score		7	12	14	7	40	20
Experimental (AM 10)	154						
M		3.4	3.5	5.0	2.8	14.8	10.0
SD		1.3	1.8	2.4	1.8	4.8	3.0
Control (Combined)	232						
M		4.7	5.5	7.0	4.1	21.3	12.1
SD		1.7	2.4	3.2	1.9	7.5	3.6
Control (10A)	39						
M		3.1	2.9	4.0	2.8	12.8	8.4
SD		1.2	1.5	2.4	1.3	4.6	2.9
Skewness		-.336	.411	.693	-.034	.918	.345
Kurtosis		-1.073	-.467	.591	.637	1.676	-.135
Control (PM 10)	193						
M		5.0	6.0	7.6	4.4	23.0	12.8
SD		1.6	2.2	3.0	1.9	6.8	3.2
Total	386						
M		4.2	4.7	6.2	3.6	18.7	11.2
SD		1.7	2.4	3.1	2.0	7.3	3.5

Figures 18 and 19 provide a visual comparison of the groups' Attitude Toward Mathematics and Total scores as well as the sub-scores for the curriculum organizers Number, Patterns & Relations, Shape & Space, and Statistics & Probability.

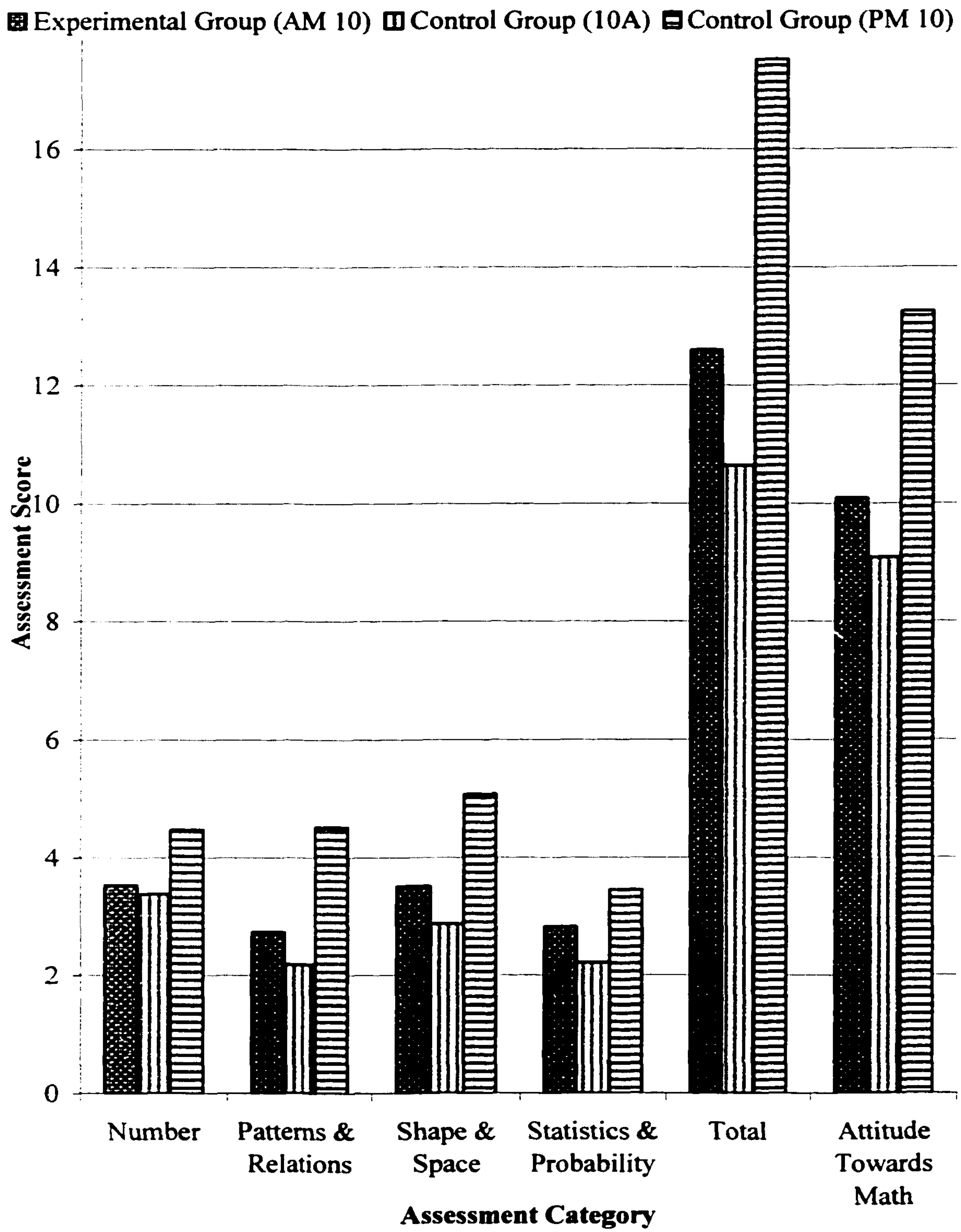

Figure 18. Pre-test Assessment Results for Experimental and Control Groups

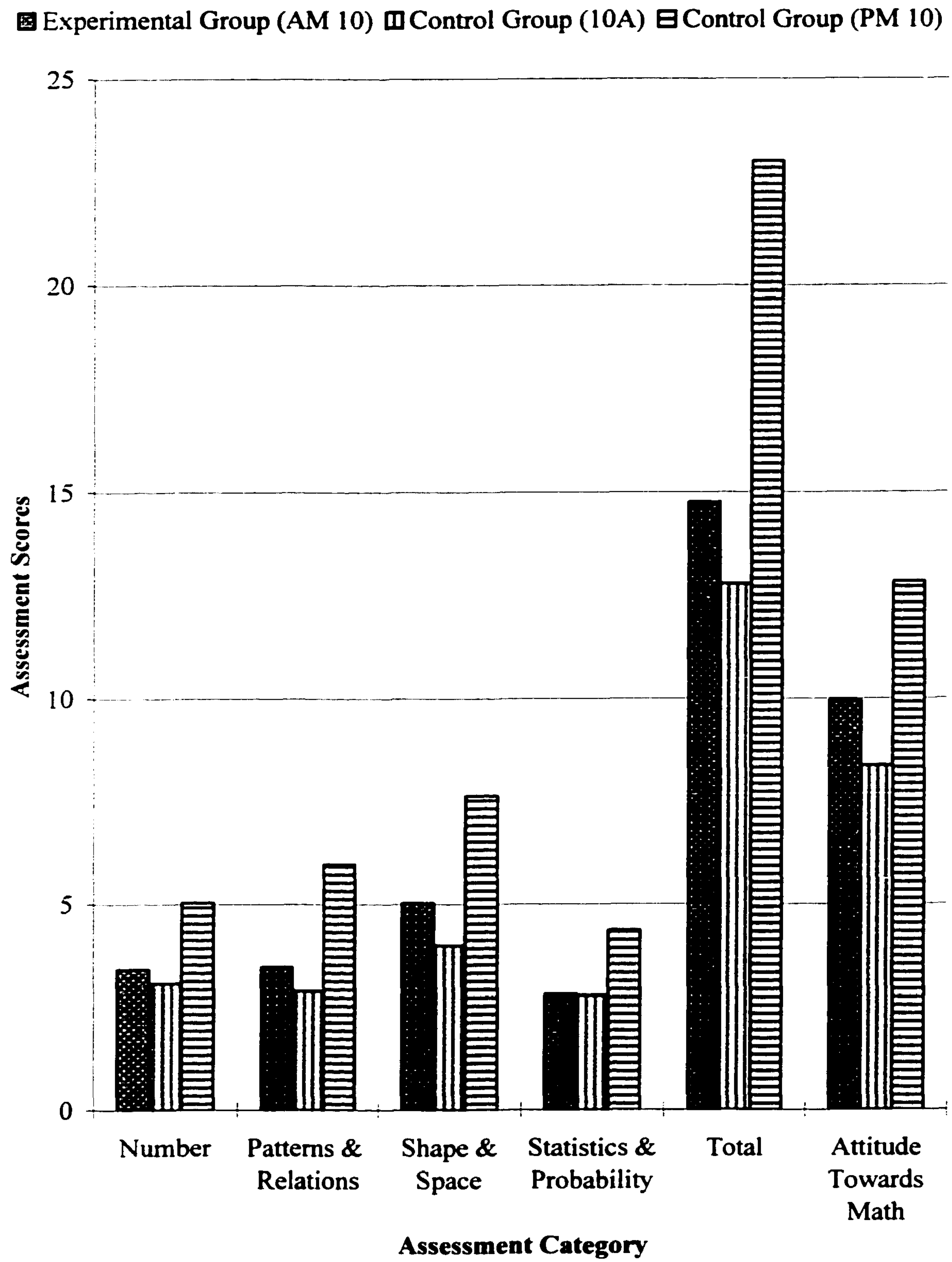

Figure 19. Post-test Assessment Results for Experimental and Control Groups

Assessing the Normality, Homogeneity of Variance and Independence of Observations of the Experimental and Control Group Student Data

It is important to test a number of assumptions concerning the student data before an analysis of variance (ANOVA) can be undertaken (Gall, Borg, and Gall, 1996). The three assumptions that were tested include normality (determining if the within-group data are normally distributed), homogeneity of variance (determining if the data come from populations with the same variance) and independence of observations (determining if the observations are all independent of one another).

Complete details of the statistical analyses used to test these assumptions are provided in Appendix H. The analysis indicates that the student results are not distributed normally about the mean scores and that the variances between Experimental and Control Groups are not equal. The analysis also indicates that the departure from normality of the data set is consistent with the distribution of the population from which the sample was drawn (based upon an analysis of census data obtained from the 1999 Foundation Skills Assessment for Numeracy - see Appendix H). As the ratio of the highest to lowest variance was within the acceptable range of no more than five-to-one (Howell, 1995), it was determined that the lack of normality and homogeneity of variance would not adversely affect the results of the ANOVA.

In both data sets, the lack of normality of the distribution of the sample and population means may be attributable to a number of factors that can not be controlled through the assessment administration procedures. These factors include: administering the assessments in different environmental settings (e.g., classroom, gymnasium);

students not writing the assessment while in their mathematics classroom (e.g., all Grade 10 students writing the assessment at the same time - regardless of whether they were taking a mathematics course at that time); students not treating the assessment in a serious manner and as a consequence, not providing answers that reflect their understanding of the mathematics in question, and students misidentifying which course they were taking.

The third assumption that was considered was whether the observations were independent of one another. This assumption would be violated if, for example, any of the students in the study cheated and copied off their neighbors during the writing of either of the assessments. One possible way of addressing this assumption is to randomly assign the students to groups. As this was not feasible with intact classes, the study design relies on the classroom teachers to monitor this issue during the assessment writing periods. Each teacher was provided with clear instructions for administering the pre- and post-test assessments and as there were no instances of cheating reported by any of the teachers, it is assumed that the individual student results are truly independent. Even if the independence of observations assumption were violated by a few individuals, the sample sizes for each group (Control = 232, Experimental = 154) minimizes the impact this would have on the overall results.

Although the student results may be viewed as independent by themselves, they are not independent when teacher methodology is considered. As the students' results are presumed to be dependent upon the teaching methodology used during the period of the study, it follows that student results from any particular class may be higher or lower than the rest of the sample due to the actions of the teacher. As the teacher methodologies could not be controlled (and in fact are one of the factors being studied), it is important to keep this violation of independence in mind as the results are being interpreted.

Assessing the Homogeneity of Regression of the Experimental and Control Group Student Data

The initial study design called for an analysis of covariance (ANCOVA) of the paired student data. The intent was to use the ANCOVA process to adjust the means of the dependent variable (post-test scores) to what they would be if all subjects scored equally on the covariate (pre-test scores) (Tabachnick & Fidell, 1996). Before this analysis was attempted, the homogeneity of regression slopes assumption was tested. This assumption was that the regression of post-test scores on pre-test scores was the same for both treatments. This analysis was accomplished using the General Linear Model (Univariate) analysis mode of SPSS Base 9.0.0. The negative results of this test (see Table 27) indicates that an ANCOVA could not be conducted with this data but that a one-way analysis of variance (ANOVA) was appropriate.

Table 27

Test of Homogeneity of Regression Slopes for the Pre-test and Post-test Experimental and Control Group Student Populations.

Dependent Variable: Post-test Total Score

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Observed Power [a]
Corrected Model	10101 [b]	5	2020	74	.00	1.00
Intercept	3094	1	3094	113	.00	1.00
Course * Pre-test Total Score	446	2	223	8	.00	.96
Course	6	2	3	.1	.89	.07
Pre-test Total Score	537	1	537	19	.00	.99
Error	10443	380	27			
Total	155255	386				
Corrected Total	20544	385				

[a] Computed using alpha = .05
[b] R Squared = .49 (Adjusted R Squared = .49)

The test of between subjects effects shown in Table 35 indicates that the interaction term, Course * Pre-test Total Score, violates the equal slopes assumption ($F = 8, p < .005$). The homogeneity of regression assumption is therefore rejected and as a result an ANCOVA can not be used to analyze the data.

Further analysis is indicated to identify trends that could provide insight as to possible causative factors. Accordingly, scatterplots of pre-test vs. post-test scores (and corresponding lines of best fit and regression equations) for each of the three groups were analyzed. Figures 20 to 23 show the scatterplots and lines of best fit for the data for each of the groups:

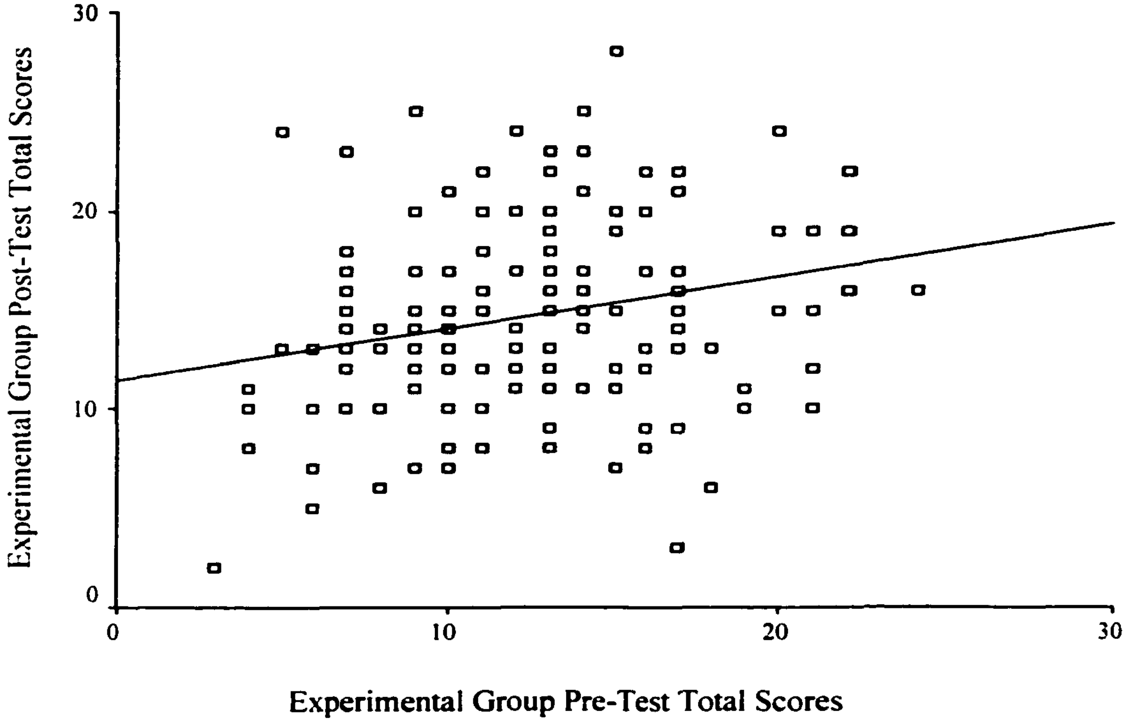

Figure 20. Line of Best Fit for Plot of Experimental Group Student Pre-Test Total Scores vs. Post-Test Total Scores

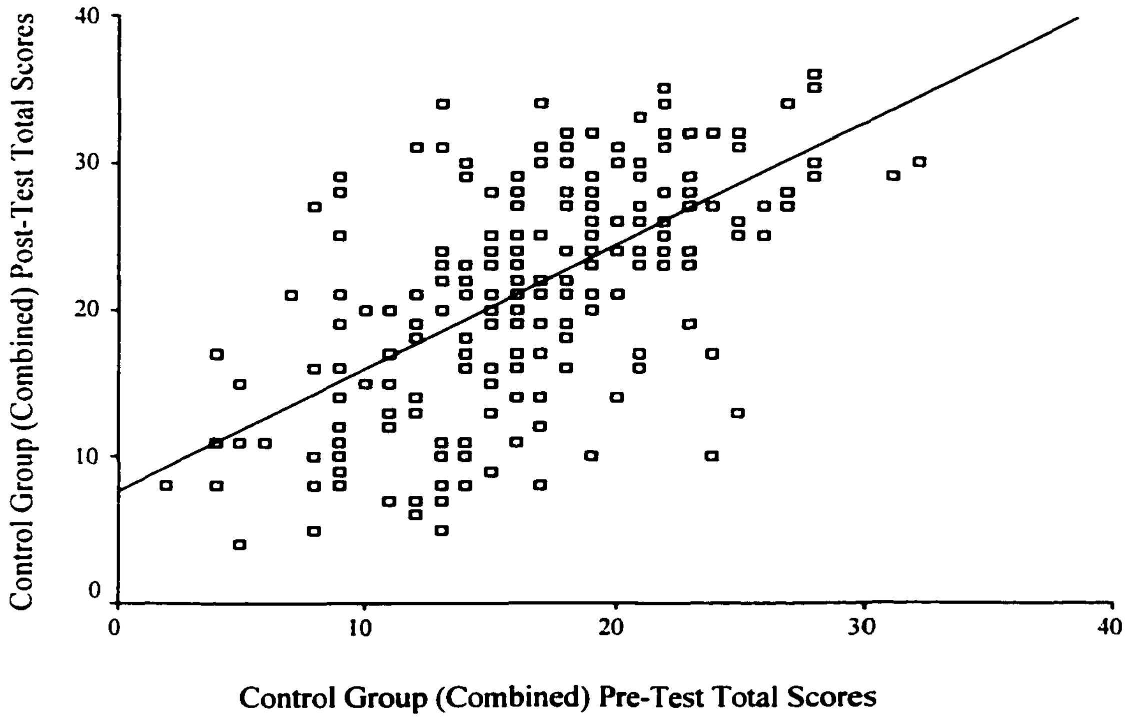

Figure 21. Line of Best Fit for Plot of Control Group (Combined) Student Pre-Test Total Scores vs. Post-Test Total Scores

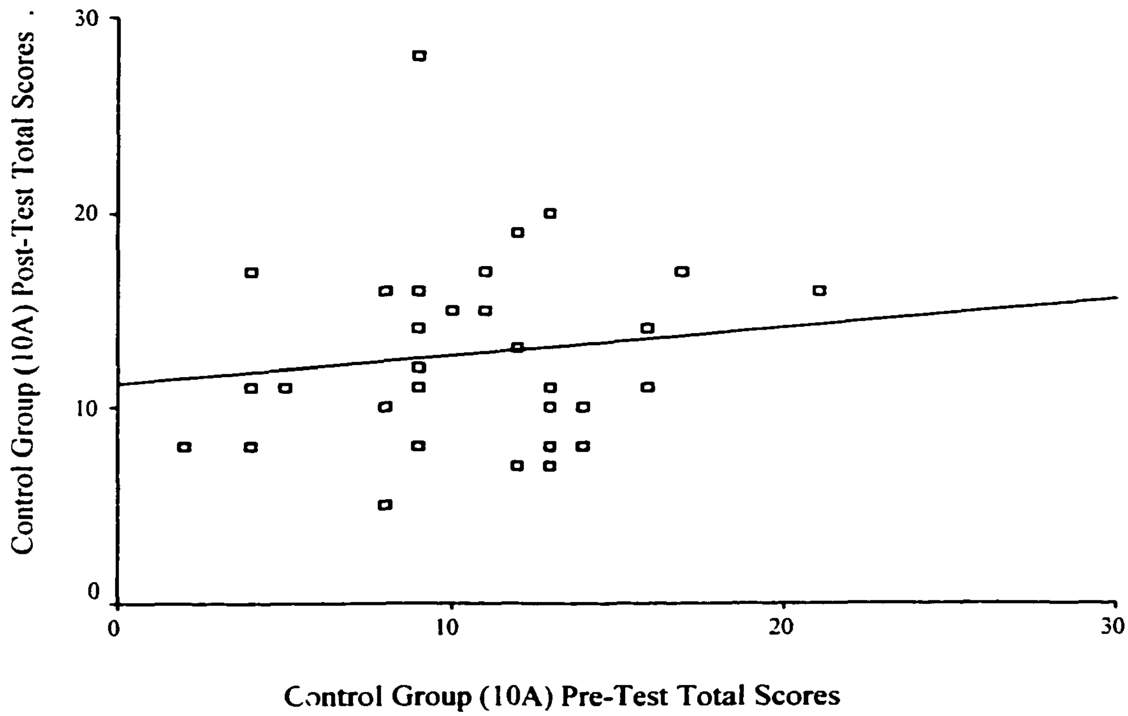

Figure 22. Line of Best Fit for Plot of Control Group (10A) Student Pre-Test Total Scores vs. Post-Test Total Scores

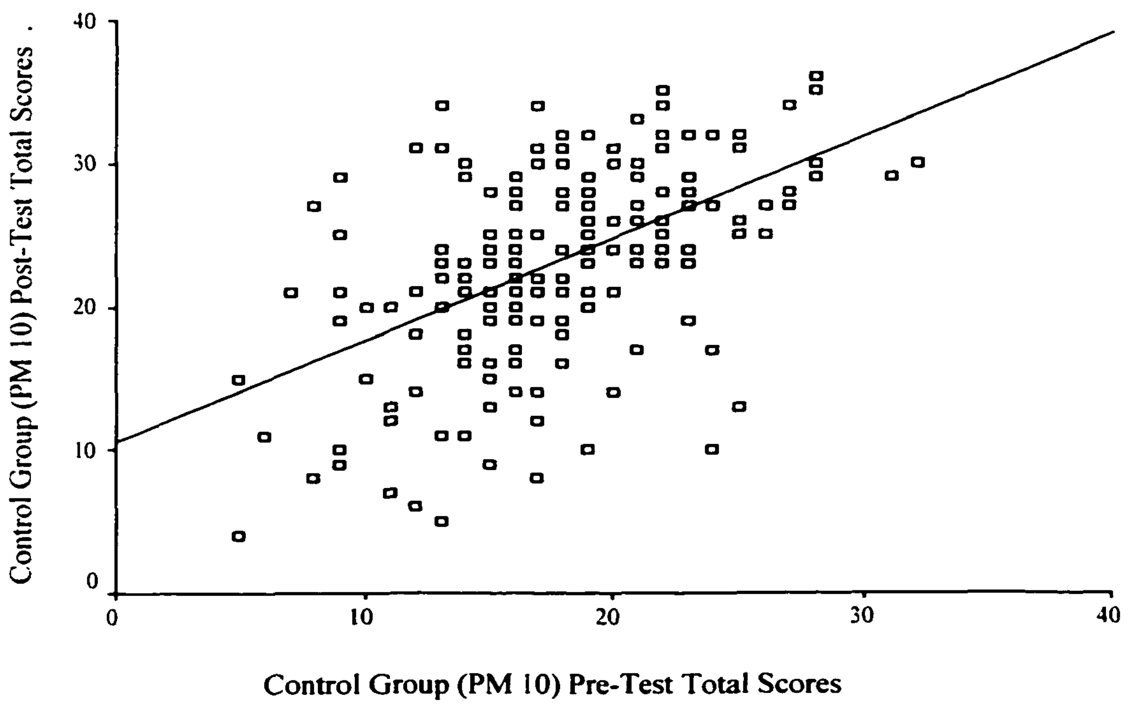

Figure 23. Line of Best Fit for Plot of Control Group (PM 10) Student Pre-Test Total Scores vs. Post-Test Total Scores

Table 28 summarizes, by group, the model coefficients, correlations, and correlation significance of the regression equation.

Table 28

Summary of Regression Models for Student Pre-Test Total Scores vs. Post-Test Total Scores by Group

Statistic	Experimental Group (AM 10)	Control Group (Combined)	Control Group (10A)	Control Group (PM 10)
Pearson Correlation	.23	.62	.12	.54
Sig. (1-tailed)	.00	.00	.23	.00
r^2	.06	.38	.01	.29
Intercept (β_o)	11.42	7.64	11.25	10.54
Std. Error	1.18	1.22	2.19	1.49
t	9.66	6.27	5.14	7.10
Significance	.00	.00	.00	.00
Slope (β_1)	0.26	0.84	0.14	0.71
Std. Error	0.09	0.07	0.19	0.08
t	2.97	11.83	0.74	8.75
Significance	.00	.00	.46	.00

Dependent Variable: Post-Test Total Score

The resulting regression equations are as follows:

Experimental

Group: $\text{Post - Test Total Scores} = 0.26 \times \text{Pre - Test Total Score} + 11.43$

Control Group (Combined):

$\text{Post - Test Total Scores} = 0.84 \times \text{Pre - Test Total Score} + 7.64$

Control Group (10A): $\text{Post - Test Total Scores} = 0.14 \times \text{Pre - Test Total Score} + 11.25$

Control Group (PM 10):

$$\text{Post - Test Total Scores} = 0.71 \times \text{Pre - Test Total Score} + 10.54$$

The slopes of the regression equations vary considerably from 0.14 for the 10A Control Sub-Group (which is not significant at $t = .74$ with $p = .45$) to 0.26 for the Experimental Group (which is significant at $t = 2.97$ with $p < .005$) to 0.71 for the PM 10 Control sub-Group (which is also significant at $t = 8.75$ with $p < .005$). The cause for this difference in regression slopes may be due to the large increase in achievement scores that the PM 10 Control Sub-Group demonstrates compared to the other two groups. It can also be seen that Pre-Test scores account for less than 6% of the variability of the Post-Test scores in the Experimental Group whereas it accounts for almost 38% of the variability in the Combined Control Group Post-Test. These differences can be examined further by analyzing the differences in scores (from pre-test to post-test) of the various groups.

Table 29 summarizes the Pre-Test and Post-Test Total Scores for each of the groups. The difference between the two scores is included along with the percent change in score relative to the pre-test score. The two values were calculated as follows:

- $\text{Difference Between Mean Scores} = (\text{Post - Test Score}) - (\text{Pre - Test Score})$; and,
- $\text{Percent Difference Between Mean Scores} = \dfrac{\text{Difference Between Mean Scores}}{\text{Pre - Test Score}} \times 100\%$.

Table 29

Summary of Differences Between Student Pre-Test and Post-Test Total Scores by Assessment Category and Group

Assessment Category (Score) Group	Pre-Test Mean Score	Post-Test Mean Score	Difference Between Mean Scores	Percent Difference Between Mean Scores
Total Score (20)				
Experimental	12.6	14.8	2.2	17%
Control (10A)	10.6	12.8	2.2	20%
Control (PM 10)	17.5	23.0	5.5	32%
Number (7)				
Experimental	3.5	3.4	-0.1	-3%
Control (10A)	3.4	3.1	-0.3	-9%
Control (PM 10)	4.5	5.0	0.5	13%
Patterns and Relations (12)				
Experimental	2.7	3.5	0.8	28%
Control (10A)	2.2	2.9	0.7	33%
Control (PM 10)	4.5	6.0	1.5	32%
Shape and Space (14)				
Experimental	3.5	5.0	1.5	43%
Control (10A)	2.9	4.0	1.1	39%
Control (PM 10)	5.1	7.6	2.5	50%
Statistics and Probability (7)				
Experimental	2.8	2.8	0	0%
Control (10A)	2.2	2.8	0.6	26%
Control (PM 10)	3.5	4.4	0.9	26%
Attitude Toward Mathematics (20)				
Experimental	10.1	10.0	-0.1	-1%
Control (10A)	9.1	8.3	-0.8	-9%
Control (PM 10)	13.2	12.8	-0.4	-3%

The relative percent increase in post-test student scores is substantially larger for the PM 10 students than it is for the AM 10 or 10A students (1.6 to 1.8 times larger). The closeness of the AM 10 and 10A students' increases in post-test total scores and the larger increase in PM 10 student post-test total scores is consistent with the differences in the slopes of the regression lines. Figure 24 illustrates the different changes from pre-test to post-test scores for the three groups.

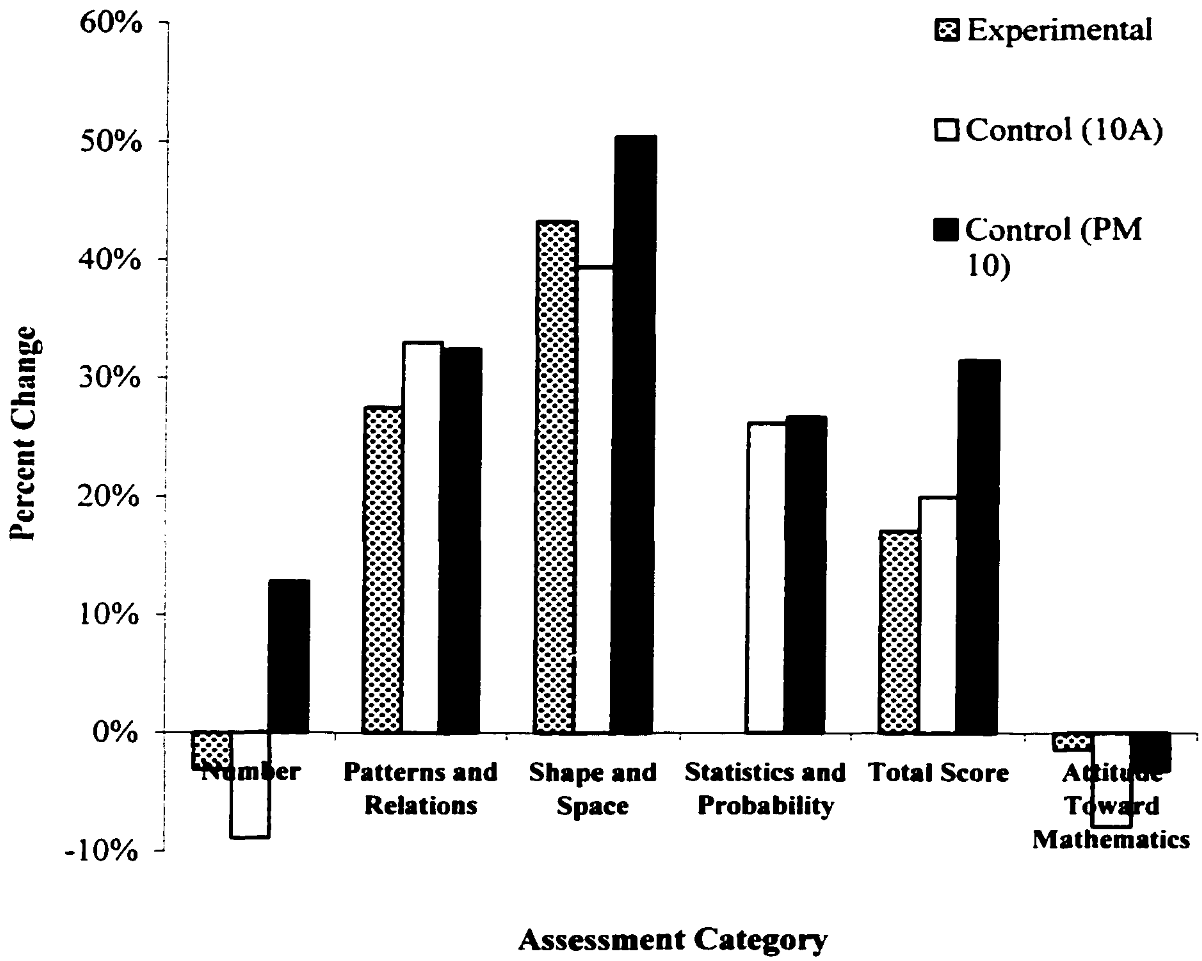

Figure 24. Comparison of Percent Changes Between Pre-Test and Post-Test Scores by Group

Both the AM 10 and 10A groups scored lower on the Number post-test questions (-.1 and -.3 points respectively). This may indicate that number operations are not stressed to any great extent in these classes. All groups improved their scores with respect to the Shape and Space organizer while the AM 10 group did not improve at all with respect to Statistics and Probability. It is possible that the teachers did not cover this material; they had previously indicated that the curriculum was too full and that something had to be removed to permit other parts of the curriculum being covered adequately. Although all groups scored a lower post-test assessment with respect Attitude Toward Mathematics than on the pre-test assessment, the Experimental Group mean score dropped the least (relative to the original pre-test score). For all of the above observations, it is important that it be determined whether the changes are statistically significant or are to be considered normal variance.

Analysis of Variance of the Experimental and Control Group Student Data

A one-way analysis of variance (ANOVA) was performed on the pre-test and post-test student group mean scores as well as on the change in group mean scores from pre-test to post-test to determine whether there were any significant differences between the sub-total means and between the overall mean scores (achievement and attitude towards mathematics). As was previously noted, the Levene test was used to test the hypothesis that the variances across the three sub-groups (AM 10, PM 10, & 10A) were the same. In addition, a post hoc pairwise multiple comparison (using the Tamahane method as variances were previously shown not to be equal) was conducted to identify any significant differences between group means (see Table 30).

Table 30

Analysis of Variance of Student Pre-Test and Post-Test Scores for Curriculum Organizers, Total Scores and Attitude Toward Mathematics

Score Category		Sum of Squares	df	Mean Square	F	Sig.	η^2
Pre-Test Scores							
Number	Between Groups	91	2	45.6	26.0	.00	.12
	Within Groups	672	383	1.8			
	Total	763	385				
Patterns and Relations	Between Groups	354	2	177.4	49.5	.00	.21
	Within Groups	1372	383	3.6			
	Total	1727	385				
Shape and Space	Between Groups	286	2	143.2	32.3	.00	.14
	Within Groups	1699	383	4.4			
	Total	1985	385				
Statistics and Probability	Between Groups	66	2	33.1	14.4	.00	.07
	Within Groups	877	383	2.3			
	Total	943	385				
Total Score	Between Groups	2825	2	1412.5	65.0	.00	.25
	Within Groups	8320	383	21.7			
	Total	11145	385				
Attitude Toward Mathematics	Between Groups	1127	2	563.3	68.0	.00	.26
	Within Groups	3170	383	8.3			
	Total	4297	385				
Post-Test Scores							
Number	Between Groups	279	2	139.5	68.5	.00	.26
	Within Groups	780	383	2.0			
	Total	1059	385				
Patterns and Relations	Between Groups	669	2	334.3	82.1	.00	.30
	Within Groups	1559	383	4.1			
	Total	2228	385				
Shape and Space	Between Groups	794	2	396.9	53.9	.00	.22
	Within Groups	2819	383	7.4			
	Total	3613	385				
Statistics and Probability	Between Groups	232	2	115.8	35.8	.00	.16
	Within Groups	1240	383	3.2			
	Total	1471	385				
Total Score	Between Groups	7356	2	3678.1	106.8	.00	.36
	Within Groups	13188	383	34.4			
	Total	20544	385				
Attitude Toward Mathematics	Between Groups	1062	2	531.1	55.1	.00	.22
	Within Groups	3694	383	9.6			
	Total	4756	385				

Results of the ANOVA clearly indicate that there are statistical differences ($p < .05$ in all cases) between groups for all of the curriculum organizer sub-scores, total scores, and Attitude Toward Mathematics scores (pre- and post-test). The measure of the magnitude of effect (η^2) suggests that up to 36% (Post-Test Total score) of the variability of the results can be attributed to group effects. Although this measure is biased (Howell, 1995), it provides a useful first approximation of the true significance of the differences between the group means. The subsequent post hoc analysis results, summarized in Table 31 below, provides additional information concerning the direction of the effects.

Table 31

Post Hoc Analysis of Student Pre-Test and Post-Test Scores for Curriculum Organizers, Total Scores and Attitude Toward Mathematics

Dependent Variable	(I) course	(J) course	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Pre-Test Scores							
Number	AM 10	10A	.15	.24	.89	-.40	.70
		PM 10	-.94*	.14	.00	-1.29	-.59
	10A	PM 10	-1.09*	.23	.00	-1.62	-.55
Patterns and Relations	AM 10	10A	.55	.34	.15	-.13	1.24
		PM 10	-1.78*	.20	.00	-2.27	-1.29
	10A	PM 10	-2.33*	.33	.00	-3.04	-1.63
Shape and Space	AM 10	10A	.64	.38	.14	-.14	1.42
		PM 10	-1.55*	.23	.00	-2.10	-1.01
	10A	PM 10	-2.20*	.37	.00	-2.99	-1.40
Statistics and Probability	AM 10	10A	.61*	.27	.04	.01	1.21
		PM 10	-.63*	.16	.00	-1.03	-.23
	10A	PM 10	-1.24*	.27	.00	-1.82	-.67
Total Score	AM 10	10A	1.96*	.84	.02	.23	3.69
		PM 10	-4.90*	.50	.00	-6.10	-3.70
	10A	PM 10	-6.86*	.82	.00	-8.62	-5.10
Attitude Toward Math	AM 10	10A	1.01	.52	.10	-.13	2.16
		PM 10	-3.16*	.31	.00	-3.91	-2.41
	10A	PM 10	-4.18*	.51	.00	-5.31	-3.04

table continues

Table 31 (Cont'd)

Post Hoc Analysis of Student Pre-Test and Post-Test Scores for Curriculum Organizers, Total Scores and Attitude Toward Mathematics

Dependent Variable	(I) course	(J) course	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Post-Test Scores							
Number	AM 10	10A	.35	.26	.33	-.20	.89
		PM 10	-1.62*	.15	.00	-1.99	-1.25
	10A	PM 10	-1.96*	.25	.00	-2.52	-1.41
Patterns and Relations	AM 10	10A	.58	.36	.13	-.12	1.28
		PM 10	-2.49*	.22	.00	-3.02	-1.97
	10A	PM 10	-3.08*	.35	.00	-3.79	-2.36
Shape and Space	AM 10	10A	1.03	.49	.06	-.03	2.09
		PM 10	-2.60*	.29	.00	-3.30	-1.90
	10A	PM 10	-3.63*	.48	.00	-4.71	-2.55
Statistics and Probability	AM 10	10A	.03	.32	1.00	-.60	.65
		PM 10	-1.54*	.19	.00	-2.02	-1.07
	10A	PM 10	-1.57*	.32	.00	-2.19	-.96
Total Score	AM 10	10A	1.98	1.05	.06	-.05	4.02
		PM 10	-8.26*	.63	.00	-9.75	-6.76
	10A	PM 10	-10.24*	1.03	.00	-12.39	-8.09
Attitude Toward Math	AM 10	10A	1.60*	.56	.01	.30	2.90
		PM 10	-2.87*	.34	.00	-3.67	-2.06
	10A	PM 10	-4.46*	.55	.00	-5.75	-3.18

* The mean difference is significant at the .05 level.

The results of the ANOVA and subsequent post hoc analysis are summarized in the following line plots and provide the basis for the following observations about the groups with respect to each of the assessment categories:

Number

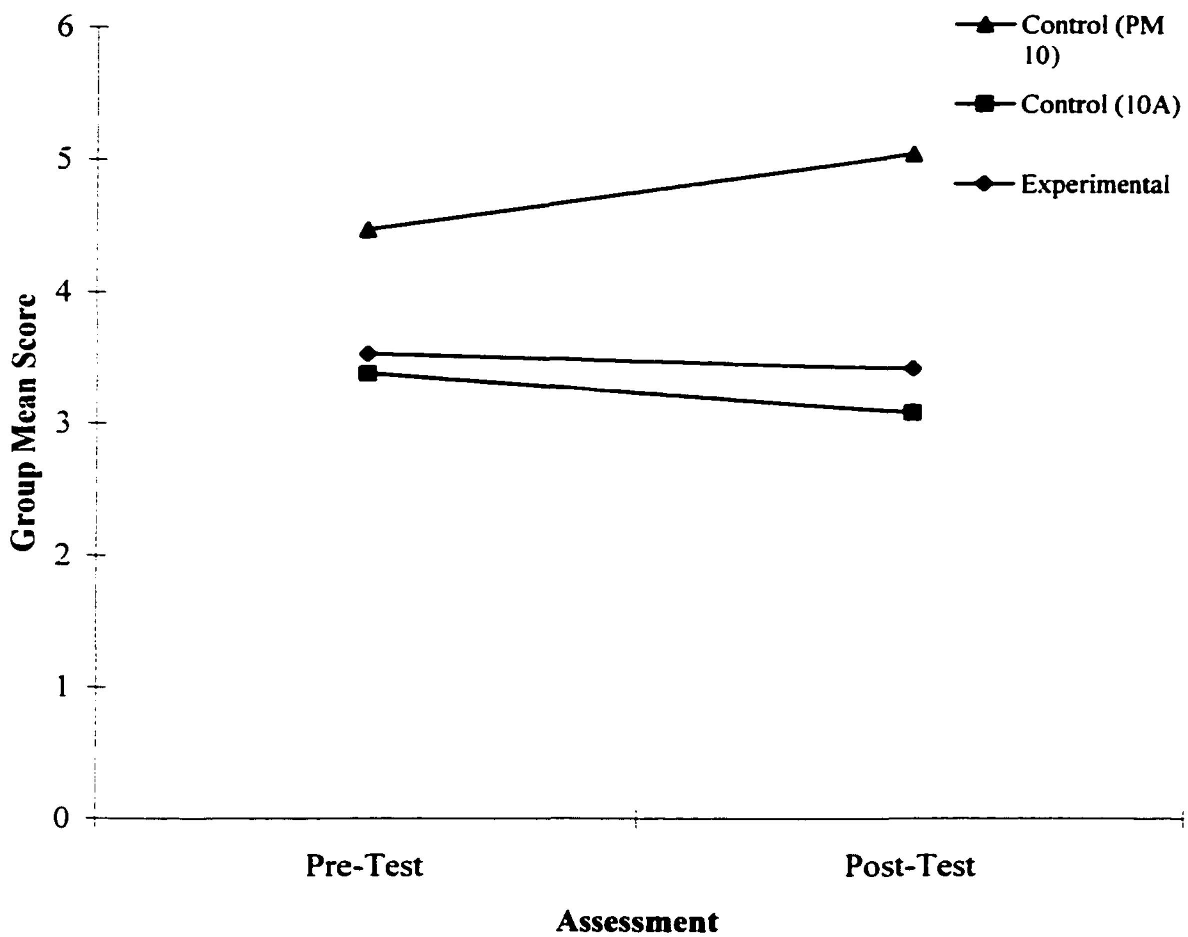

Figure 25. Comparison of Group Pre-Test and Post-Test Scores for the *Number* Curriculum Organizer

- in both pre- and post-test assessments AM 10 and 10A students did not have significantly different scores
- in both pre- and post-test assessments PM 10 students scored significantly higher than did both AM 10 and 10A students
- 12% (pre-test) to 26% (post-test) of the variability in Number scores can be attributed to group membership

Patterns and Relations

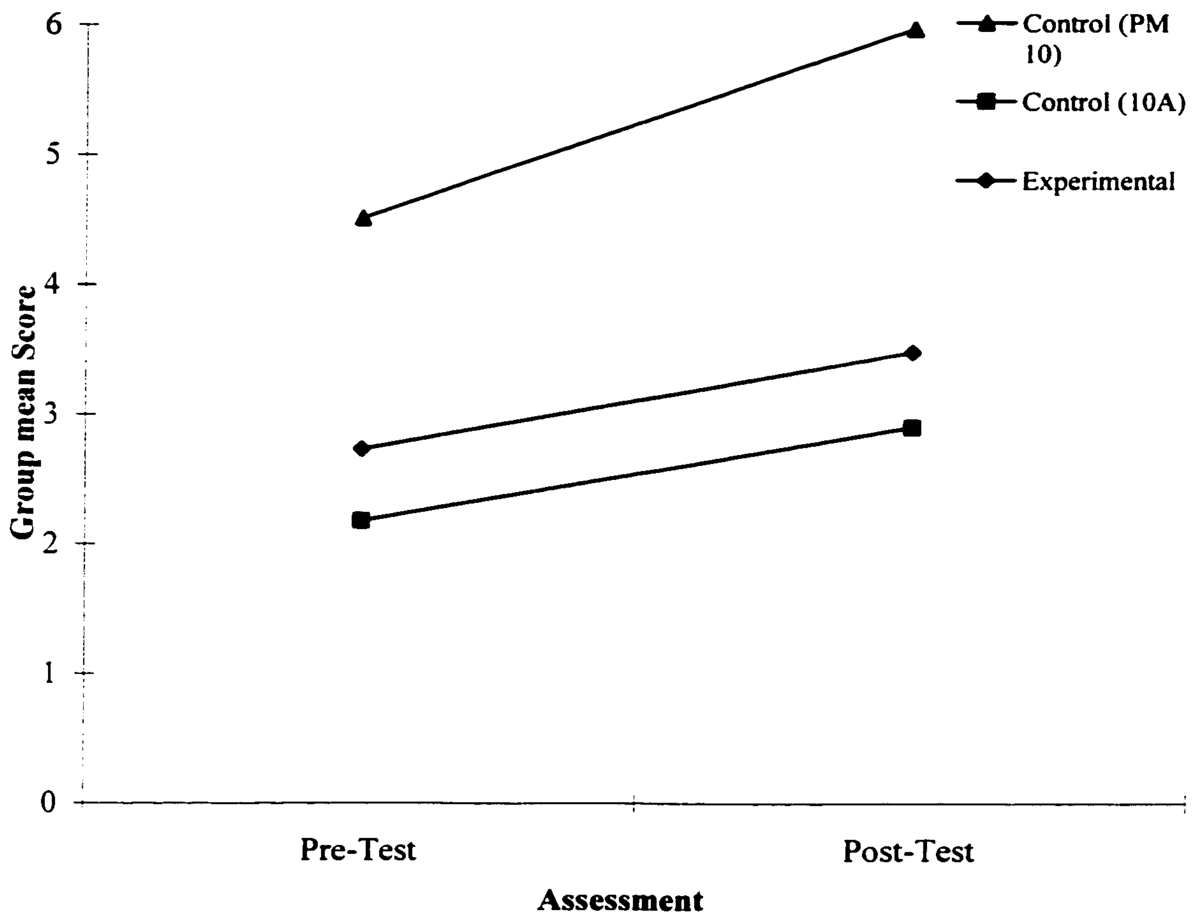

Figure 26. Comparison of Group Pre-Test and Post-Test Scores for the *Patterns & Relations* Curriculum Organizer

- in both pre- and post-test assessments AM 10 and 10A students did not have significantly different scores
- in both pre- and post-test assessments PM 10 students scored significantly higher than did both AM 10 and 10A students
- 20% (pre-test) to 30% (post-test) of the variability in Patterns and Relations scores can be attributed to group membership

Shape and Space

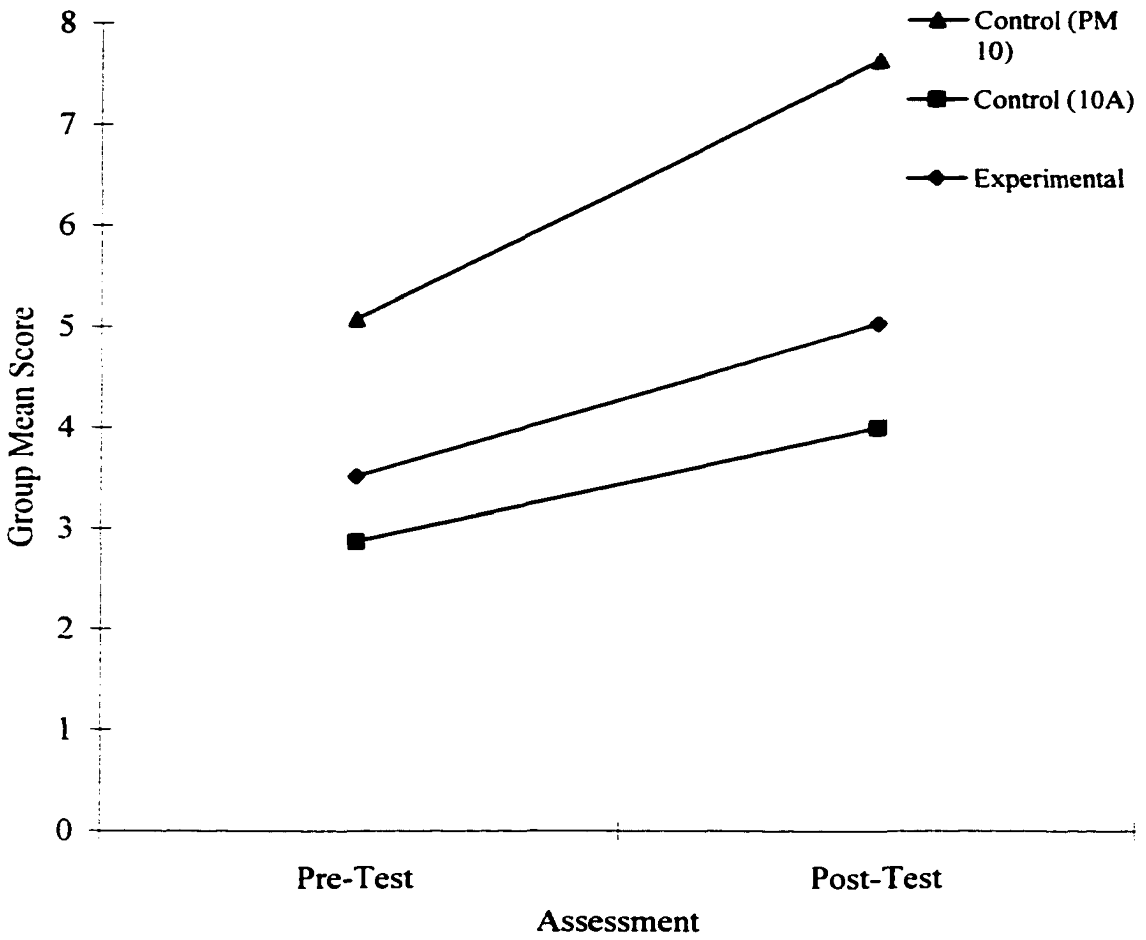

Figure 27. Comparison of Group Pre-Test and Post-Test Scores for the *Shape & Shape* Curriculum Organizer

- in both pre- and post-test assessments AM 10 and 10A students did not have a significantly different score
- in both pre- and post-test assessments PM 10 students scored significantly higher than did both AM 10 and 10A students
- 14% (pre-test) to 22% (post-test) of the variability in Shape and Space scores can be attributed to group membership

Statistics and Probability

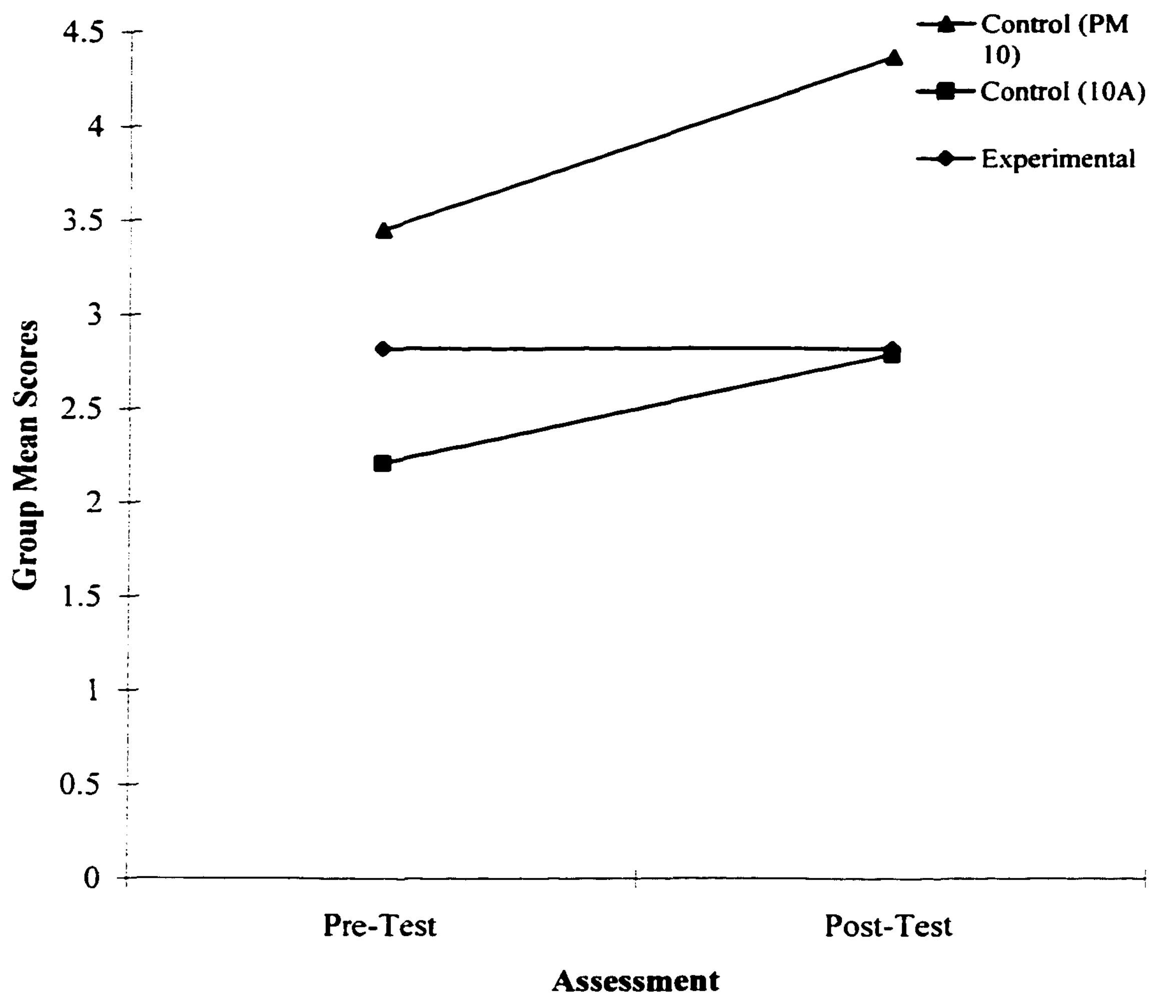

Figure 28. Comparison of Group Pre-Test and Post-Test Scores for the *Statistics & Probability* Curriculum Organizer

- AM 10 students scored significantly better than did 10A students in the pre-test assessment, but statistically the same in the post-test assessment
- in both pre- and post-test assessments PM 10 students scored significantly higher than did both AM 10 and 10A students
- 7% (pre-test) to 15% (post-test) of the variability in Statistics and Probability scores can be attributed to group membership

Total Score

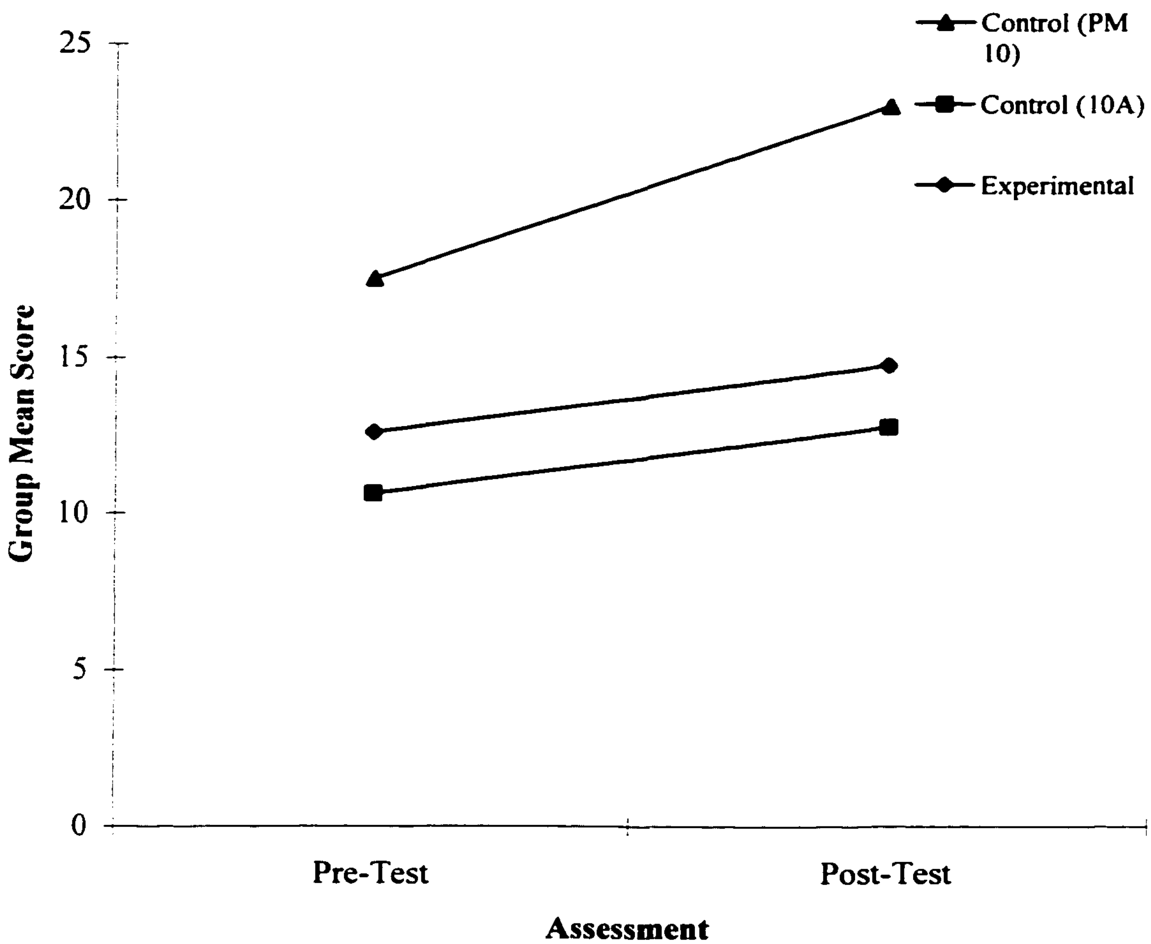

Figure 29. Comparison of Group Pre-Test and Post-Test Scores for the *Total Score*

- AM 10 students scored significantly better than did 10A students in the pre-test assessment, but statistically the same in the post-test assessment
- in both pre- and post-test assessments PM 10 students scored significantly higher than did both AM 10 and 10A students
- 25% (pre-test) to 36% (post-test) of the variability in Total scores can be attributed to group membership

Attitude Toward Mathematics

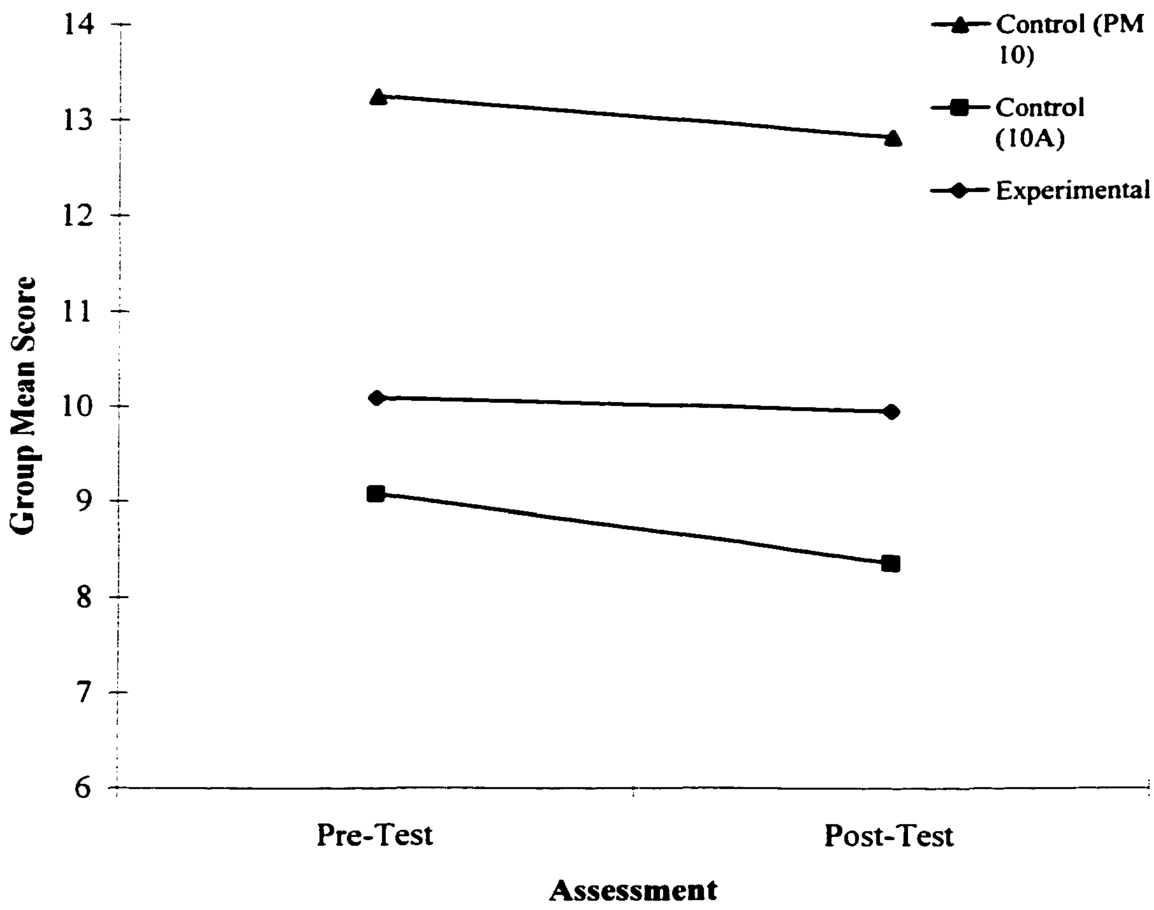

Figure 30. Comparison of Group Pre-Test and Post-Test Scores for *Attitude Towards Mathematics*

- AM 10 students did not score significantly better than did 10A students in the pre-test assessment, but did score statistically higher in the post-test assessment
- in both pre- and post-test assessments PM 10 students scored significantly higher than did both AM 10 and 10A students
- 26% (pre-test) to 22% (post-test) of the variability in Attitude Toward Mathematics scores can be attributed to group membership

The group results were also analyzed with respect to changes in scores from pre-test to post-test. The same ANOVA and post hoc analysis procedures were used as with the previous data, the single difference being that the test of homogeneity of variances indicates that the hypothesis of equal variances across the groups (for the pre-test to post-test difference of means) is accepted. Table 32 summarizes the results of the Levene test for the mean differences data.

Table 32

Test of Homogeneity of Variance for Differences Between Student Pre-Test and Post-Test Scores by Curriculum Organizer, Total Score and Attitude Toward Mathematics

Difference in Means for	Levene Statistic	df1	df2	Sig.
Number	.046	2	383	.96
Patterns and Relations	1.81	2	383	.17
Shape and Space	.05	2	383	.96
Statistics and Probability	2.97	2	383	.05
Total Score	.28	2	383	.76
Attitude Toward Mathematics	4.01	2	383	.02

In all categories but one (Attitude Toward Mathematics: Levene statistic of 4.01 with $p = .02$), the Levene test results indicate that the hypothesis of equal variances across the three groups is not rejected.

Results of the ANOVA for the differences between the mean scores (pre-test to post-test) are summarized in Table 33.

Table 33

Analysis of Variance of Differences Between Means (Student Pre-Test to Post-Test Scores) for Curriculum Organizers, Total Scores and Attitude Toward Mathematics

Difference Between Means for:		Sum of Squares	df	Mean Square	F	Sig.	η^2
Number	Between Groups	51	2	25.6	9.2	.00	.05
	Within Groups	1059	383	2.8			
	Total	1110	385				
Patterns and Relations	Between Groups	50	2	25.0	4.3	.01	.02
	Within Groups	2207	383	5.8			
	Total	2257	385				
Shape and Space	Between Groups	127	2	63.3	7.2	.00	.04
	Within Groups	3364	383	8.8			
	Total	3491	385				
Statistics and Probability	Between Groups	72	2	36.0	8.5	.00	.04
	Within Groups	1630	383	4.3			
	Total	1702	385				
Total Score	Between Groups	1091	2	545.6	16.4	.00	.08
	Within Groups	12761	383	33.3			
	Total	13852	385				
Attitude Toward Mathematics	Between Groups	14	2	6.8	.8	.47	.00
	Within Groups	3389	383	8.8			
	Total	3403	385				

The ANOVA results indicate that a post-hoc analysis is needed for all curriculum organizer sub-scores and the Total Score assessment categories. In each case the value of F and its associated level of significance, p, is less than .05 the exception being the Attitude Toward Mathematics results ($F = 0.8$, with $p = .47$). The conclusion is that the difference between the means for the Attitude Toward Mathematics scores are statistically the same.

Table 34 summarizes the results of the multiple comparison post-hoc analysis using the Bonferroni test (assumes equal variances).

Table 34

Post Hoc Analysis of Differences Between Means (Student Pre-Test to Post-Test Scores) for Curriculum Organizers, Total Scores and Attitude Toward Mathematics

Difference Between Means of:	(I) course	(J) course	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Number	AM 10	10A	.2	.30	1.00	-.5	.9
		PM 10	-.7*	.18	.00	-1.1	-.3
	10A	PM 10	-.9*	.29	.01	-1.6	-.2
Patterns and Relations	AM 10	10A	.0	.43	1.00	-1.0	1.1
		PM 10	-.7*	.26	.02	-1.3	-.1
	10A	PM 10	-.7	.42	.24	-1.8	.3
Shape and Space	AM 10	10A	.4	.53	1.00	-.9	1.7
		PM 10	-1.1*	.32	.00	-1.8	-.3
	10A	PM 10	-1.4*	.52	.02	-2.7	-.2
Statistics and Probability	AM 10	10A	-.6	.37	.35	-1.5	.3
		PM 10	-.9*	.22	.00	-1.5	-.4
	10A	PM 10	-.3	.36	1.00	-1.2	.5
Total Score	AM 10	10A	.03	1.03	1.00	-2.5	2.5
		PM 10	-3.4*	.62	.00	-4.9	-1.9
	10A	PM 10	-3.4*	1.01	.00	-5.8	-1.0

* The mean difference is significant at the .05 level.

The results of the ANOVA and subsequent post hoc analysis provide the basis for the following observations about the groups with respect to each of the changes in assessment categories:

Number

- AM 10 students did not increase their scores significantly more or less than did 10A students
- PM 10 students increased their scores significantly more than did both AM 10 and 10A students
- less than 5% of the variability in the change in Number scores can be attributed to group membership

Patterns and Relations

- AM 10 students did not increase their scores significantly more or less than did 10A students
- PM 10 students did not increase their scores significantly more or less than did 10A students
- PM 10 students increased their scores significantly more than did AM 10 students
- 2% of the variability in the change in Patterns and Relations scores can be attributed to group membership

Shape and Space

- AM 10 students did not increase their scores significantly more or less than did 10A students
- PM 10 students increased their scores significantly more than did both AM 10 and 10A students
- less than 4% of the variability in the change in Shape and Space scores can be attributed to group membership

Statistics and Probability

- AM 10 students did not increase their scores significantly more or less than did 10A students
- PM 10 students did not increase their scores significantly more or less than did 10A students
- PM 10 students increased their scores significantly more than did AM 10 students
- 4% of the variability in the change in Number scores can be attributed to group membership

<u>Total Score</u>

- AM 10 students did not increase their scores significantly more or less than did 10A students
- PM 10 students increased their scores significantly more than did both AM 10 and 10A students
- less than 8% of the variability in the change in Total scores can be attributed to group membership

As the students involved in this study have had a wide range of mathematical experience, depending upon the previous mathematics course taken, it is important to determine if any of the differences in the group means can be attributed to the previous courses (9A, AM 9, PM 9, Grade 10 course). The Experimental and Control Groups do not contain the same distribution of students with respect to the previous mathematics courses taken. Accordingly, the first step in this analysis is to identify the student mean scores by previous course for Total Achievement and Attitude Toward Mathematics scores (pre-test and post-test). These means are shown in Table 35.

Table 35

Comparison of Experimental and Control Group Student Achievement and Attitude Toward Mathematics Means (Pre-Test and Post-Test) Grouped by Previous Course Taken

Group (Course)	Previous Course	Statistic	Pre-Test Total Score	Post-Test Total Score	Pre-Test Attitude Toward Mathematics	Post-Test Attitude Toward Mathematics
Experimental	AM 9	M	12.2	15.2	10.4	10.2
(AM 10)		N	72	72	72	72
		S.D.	3.7	4.6	2.5	3.0
	9A	M	11.1	12.8	10.7	10.9
		N	10	10	10	10
		S.D.	6.4	6.0	3.0	3.1
	PM 9	M	13.2	14.4	10.0	9.8
		N	63	63	63	63
		S.D.	4.4	4.6	3.0	2.8
	Gr 10	M	13.8	16.0	7.9	8.3
		N	9	9	9	9
		S.D.	3.9	6.2	3.3	4.3
Control	AM 9	M	12.0	13.4	8.6	8.4
(10A)		N	9	9	9	9
		S.D.	4.3	3.1	1.9	3.0
	9A	M	10.0	11.8	9.1	7.8
		N	22	22	22	22
		S.D.	4.1	4.0	2.5	2.5
	PM 9	M	11.4	15.4	10.0	9.9
		N	7	7	7	7
		S.D.	2.0	7.1	3.6	4.2
	Gr 10	M	8.0	10.0	7.0	9.0
		N	1	1	1	1
		S.D.	.	.	.	.

table continues

Table 35 (Cont'd)

Comparison of Experimental and Control Group Student Achievement and Attitude Toward Mathematics Means (Pre-Test and Post-Test) Grouped by Previous Course Taken

Group (Course)	Previous Course	Statistic	Pre-Test Total Score	Post-Test Total Score	Pre-Test Attitude Toward Mathematics	Post-Test Attitude Toward Mathematics
Control (PM 10)	AM 9	M	17.7	20.7	11.3	10.7
		N	3	3	3	3
		S.D.	4.9	8.3	6.1	6.4
	9A	M	6.0	11.0	6.0	6.0
		N	1	1	1	1
		S.D.	.	.	.	.
	PM 9	M	17.6	23.2	13.4	12.9
		N	184	184	184	184
		S.D.	5.1	6.7	2.8	3.1
	Gr 10	M	16.4	19.0	9.8	11.6
		N	5	5	5	5
		S.D.	5.0	8.3	3.3	2.7
Total Population	AM 9	M	12.4	15.2	10.2	10.0
		N	84	84	84	84
		S.D.	3.9	4.7	2.6	3.2
	9A	M	10.2	12.1	9.5	8.7
		N	33	33	33	33
		S.D.	4.9	4.6	2.8	3.0
	PM 9	M	16.3	20.8	12.5	12.1
		N	254	254	254	254
		S.D.	5.3	7.4	3.3	3.4
	Gr 10	M	14.3	16.6	8.5	9.5
		N	15	15	15	15
		S.D.	4.5	6.9	3.2	3.9

These Student Achievement and Attitude Toward Mathematics mean scores were subjected to an analysis of variance using the students previous course (9A, AM9, PM 9, or Grade 10 course) as a second layered independent variable. The results of this analysis are shown in Table 36.

Table 36

Analysis of Variance of Student Achievement and Attitude Toward Mathematics Mean Scores (Pre-Test and Post-Test) Grouped by Previous Mathematics Course Taken

Difference Between Means for:		Sum of Squares	df	Mean Square	F	Sig.	η^2
Post-Test Total	Between Groups	3662	3	1221	27.6	.00	.18
* Previous course	Within Groups	16882	382	44			
	Total	20544	385				
Post-Test Attitude	Between Groups	568	3	189	17.3	.00	.12
* Previous course	Within Groups	4188	382	11			
	Total	4756	385				
Pre-Test Total	Between Groups	1799	3	600	24.5	.00	.16
* Previous course	Within Groups	9346	383	24			
	Total	11145	385				
Pre-Test Attitude	Between Groups	649	3	216	22.7	.00	.15
* Previous course	Within Groups	3647	383	10			
	Total	4297	385				

As the ANOVA results confirm that there are significant differences between means (in all cases $p < .005$), a post hoc multiple comparison analysis is needed to identify the specific group means that are significantly different. Table 37 summarizes the results of the post-hoc analysis.

Table 37

Post Hoc Analysis of Differences Between Means (Pre-Test and Post-Test): Grouped by Previous Mathematics Course Taken

Difference Between Means of:	(I) Previous Course	(J) Previous Course	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Post-Test Total Score	AM 9	9A	3.2*	1.4	.01	.6	5.7
		PM 9	-5.6 *	.8	.00	-7.4	-3.8
		Gr 10	-1.4	1.9	.98	-6.9	4.1
	9A	PM 9	-8.8 *	1.2	.00	-11.3	-6.2
		Gr 10	-4.5	2.1	.17	-10.2	1.1
	PM 9	Gr 10	4.2	1.8	.20	-1.3	9.7
Post-Test Attitude Toward Mathematics	AM 9	9A	1.3	.7	.24	-.4	3.0
		PM 9	-2.1 *	.4	.00	-3.2	-1.0
		Gr 10	.5	.9	1.00	-2.6	3.7
	9A	PM 9	-3.4 *	.6	.00	-4.9	-1.8
		Gr 10	-.8	1.0	.99	-4.1	2.5
	PM 9	Gr 10	2.6	.9	.13	-.5	5.7

table continues

Table 37 (Cont'd)

Post Hoc Analysis of Differences Between Means (Pre-Test and Post-Test): Grouped by Previous Mathematics Course Taken

Difference Between Means of:	(I) Previous Course	(J) Previous Course	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Pre-Test Total Score	AM 9	9A	2.2	1.0	.15	-.4	4.8
		PM 9	-4.0 *	.6	.00	-5.4	-2.5
		Gr 10	-1.9	1.4	.60	-5.6	1.8
	9A	PM 9	-6.1 *	.9	.00	-8.6	-3.6
		Gr 10	-4.1 *	1.5	.05	-8.2	-.0
	PM 9	Gr 10	2.1	1.3	.50	-1.6	5.7
Pre-Test Attitude Toward Mathematics	AM 9	9A	.7	.6	.73	-.8	2.3
		PM 9	-2.3 *	.4	.00	-3.2	-1.3
		Gr 10	1.8	.9	.32	-.9	4.34
	9A	PM 9	-3.0 *	.6	.00	-4.4	-1.6
		Gr 10	1.0	1.0	.88	-1.7	3.8
	PM 9	Gr 10	4.0 *	.8	.00	1.4	6.6

* The mean difference is significant at the .05 level.

The results of the ANOVA and subsequent post hoc analysis provide the basis for the following observations about the groups with respect to each of the changes in assessment categories:

Pre-Test Achievement Score

- Grade 10 students scored significantly higher than did 9A students
- PM 9 students scored significantly higher than did both AM 9 and 9A students
- approximately 16% of the variability in the Pre-Test Achievement scores can be attributed to group membership

Post-Test Achievement Score

- AM 9 students scored significantly higher than did 9A students
- PM 9 students scored significantly higher than did both AM 9 and 9A students
- approximately 18% of the variability in the Post-Test Achievement scores can be attributed to group membership

Pre-Test Attitude Toward Mathematics Score

- AM 9 students scored significantly higher than did the 9A students
- PM 9 students scored significantly higher than did AM 9, 9A, and Grade 10 students
- approximately 15% of the variability in the Pre-Test Attitude Toward Mathematics scores can be attributed to group membership

Post-Test Attitude Toward Mathematics Score

- PM 9 students scored significantly higher than did both AM 9 and 9A students
- approximately 12% of the variability in the Pre-Test Attitude Toward Mathematics scores can be attributed to group membership

Figures 43 through 46 provide a detailed graphical presentation of this data.

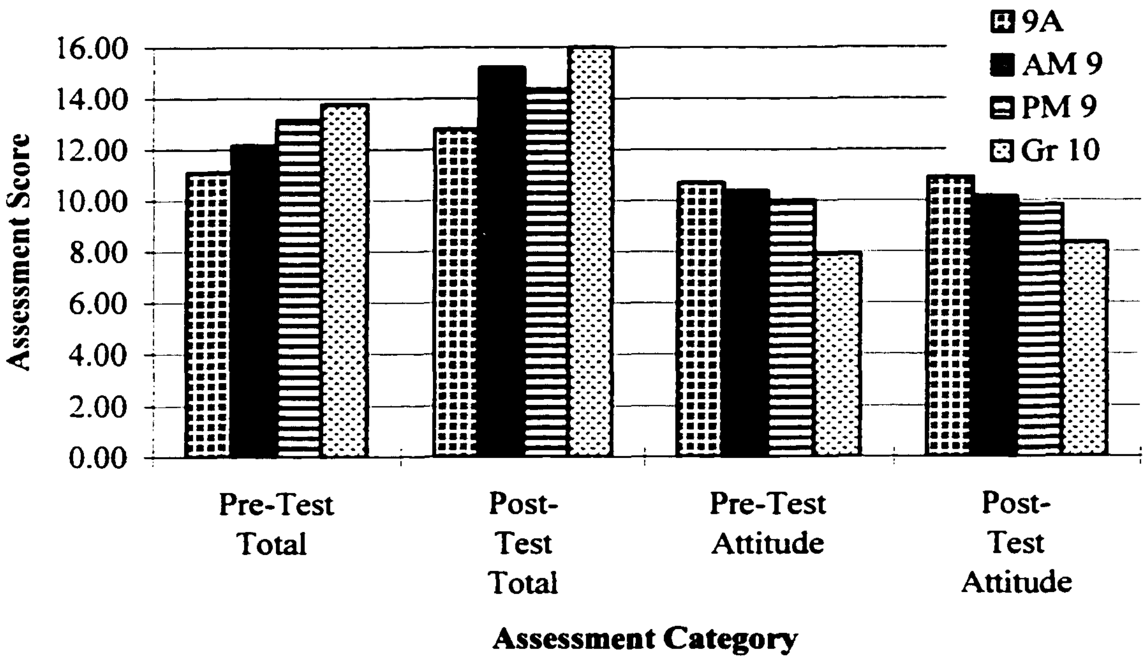

Figure 31. Comparison of Experimental Group Student Total Achievement and Attitude Toward Mathematics Scores: Grouped by Previous Course Taken

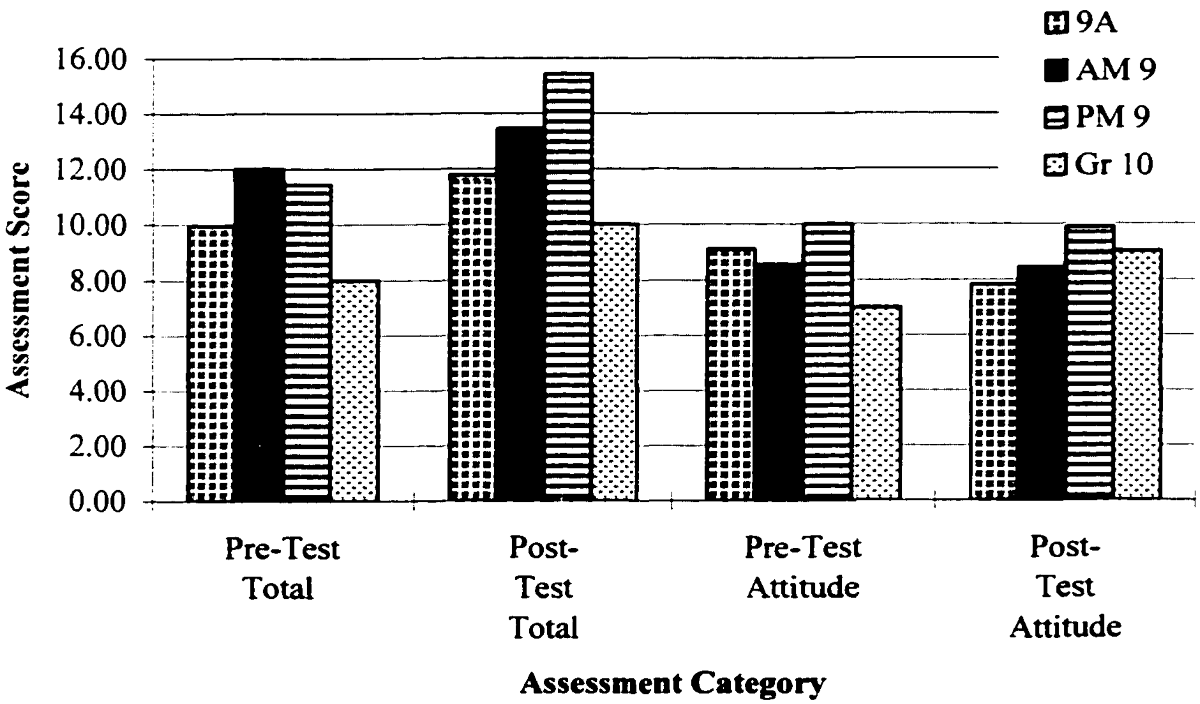

Figure 32. Comparison of Control Group (10A) Student Total Achievement and Attitude Toward Mathematics Scores: Grouped by Previous Course Taken

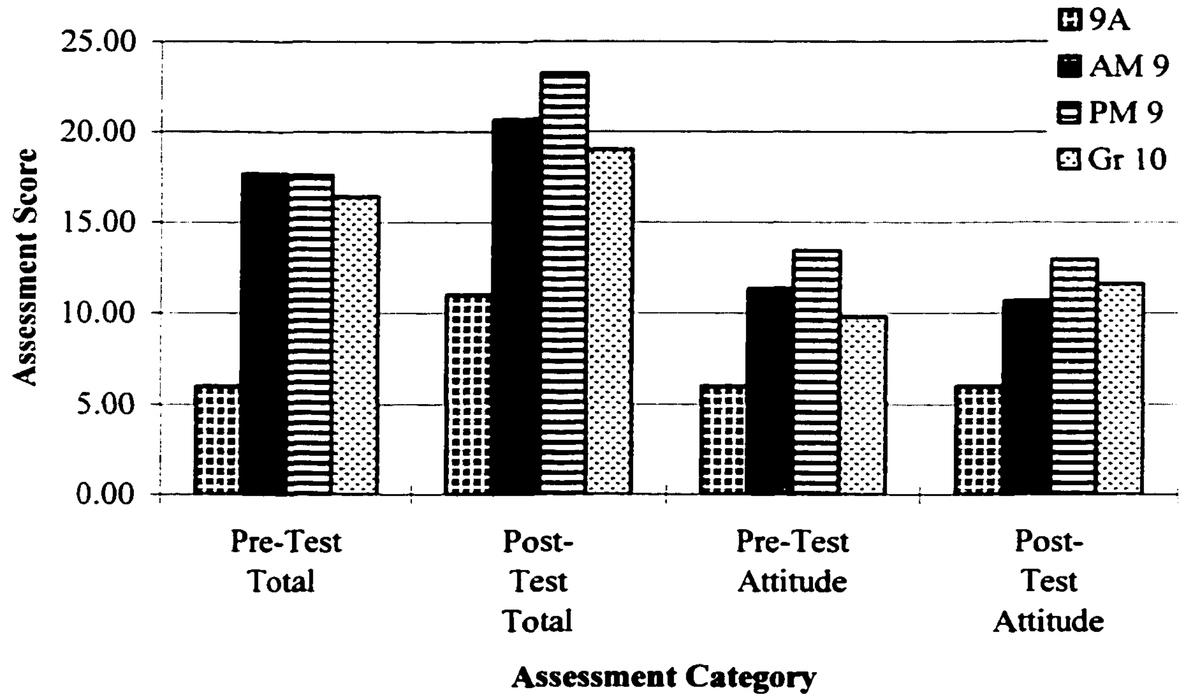

Figure 33. Comparison of Control Group (PM 10) Student Total Achievement and Attitude Toward Mathematics Scores: Grouped by Previous Course Taken

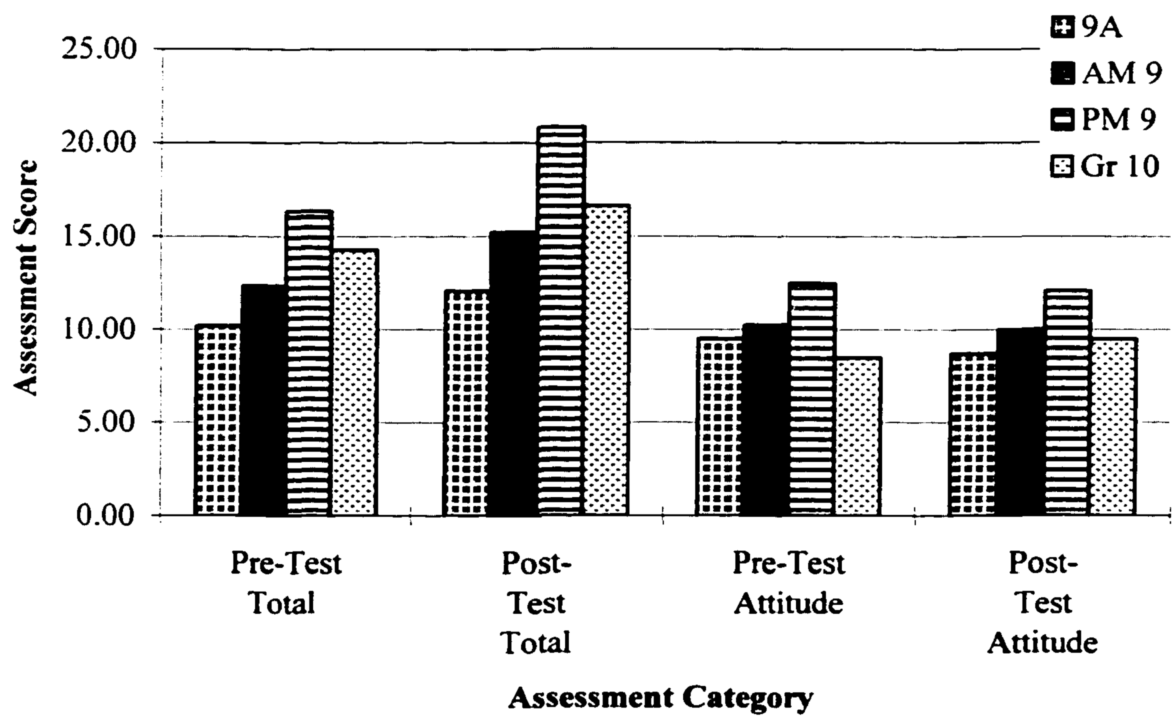

Figure 34. Comparison of Combined Student Population Total Achievement and Attitude Toward Mathematics Scores: Grouped by Previous Course Taken

A number of observations can be made with respect to these results. Although the 9A students (N = 33) scored statistically the same as the AM 9 students (N = 84) on the pre-test assessment (for both Total Score and Attitude Toward Mathematics score), the gap widened between the two groups and the 9A students subsequently scored significantly lower on the post-test assessment (Total Score).

Another interesting point is that PM 9 students (N = 63), who took AM 10, scored higher than did the AM 9 students (N = 72) on the Pre-Test achievement assessment (PM 9: M = 13.2, SD = 4.4 vs. AM 9: M = 12.2, SD = 3.7), but scored noticeably lower than did the AM 9 students on the subsequent Post-Test achievement assessment (PM 9: M = 14.4, SD = 4.6 vs. AM 9: M = 15.2, SD = 4.6). While the AM 9 sub-group gained 3.0 points on the assessment, the PM 9 sub-group only gained 1.2 points.

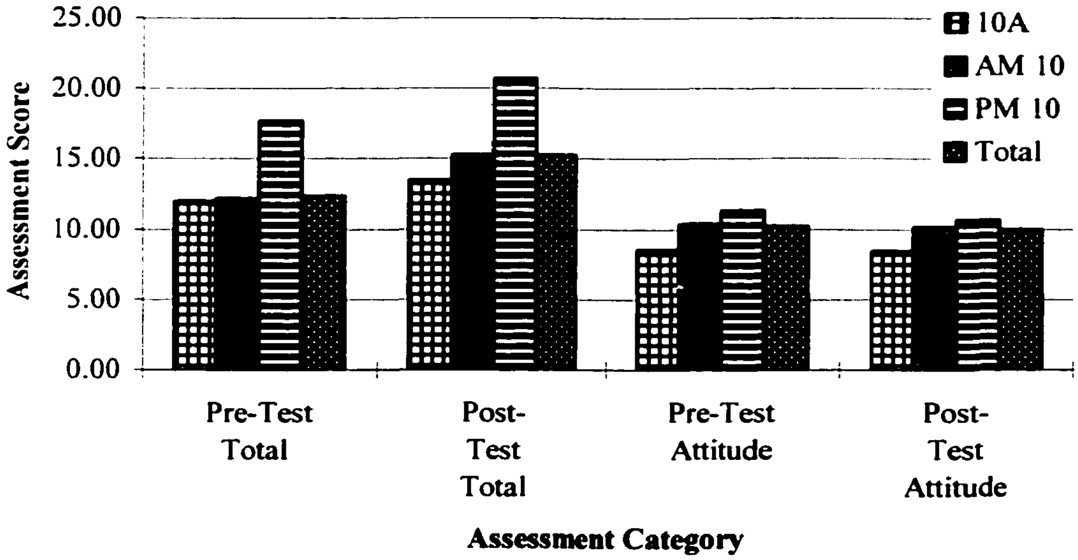

Figure 35. Comparison of AM 9 Student Scores for Total Achievement and Attitude Toward Mathematics Scores: Grouped by Course Taken the Following Year

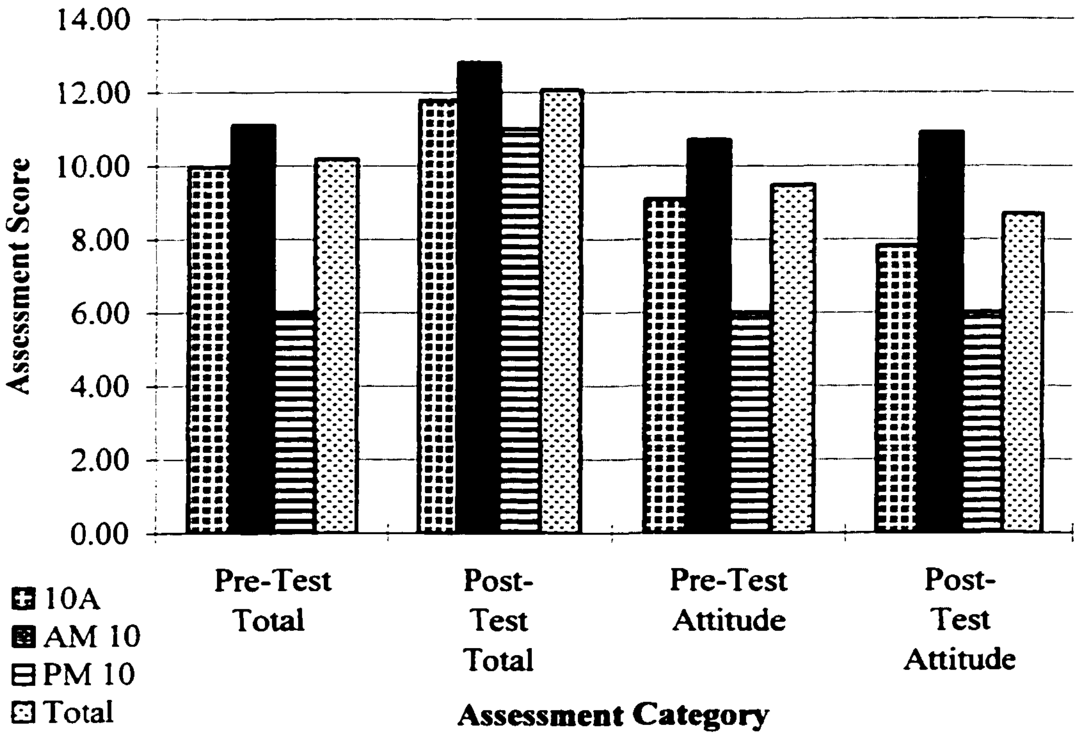

Figure 36. Comparison of 9A Student Scores for Total Achievement and Attitude Toward Mathematics Scores: Grouped by Course Taken the Following Year

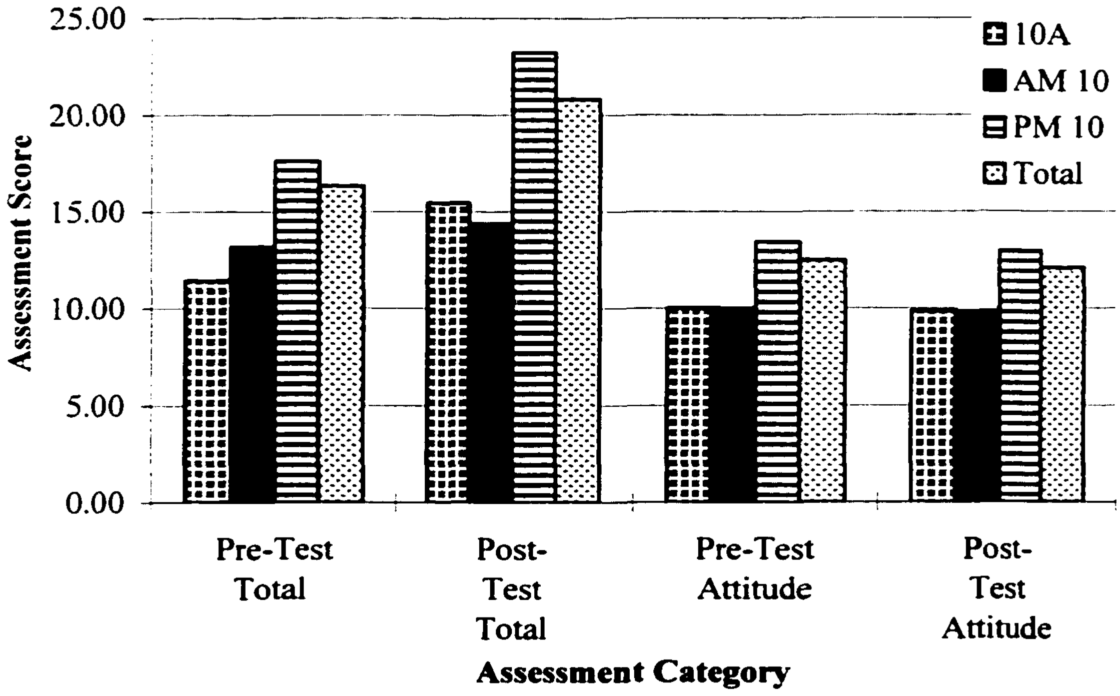

Figure 37. Comparison of PM 9 Student Scores for Total Achievement and Attitude Toward Mathematics Scores: Grouped by Course Taken the Following Year

Figures 38 through 41 portray the same results as the previous figures (42 through 46), but with the grouping altered to focus on the assessment results for each of the previous course sub-groups (9A, AM 9, and PM 9) as they pertain to the Experimental and Control Groups. Of particular interest is the fact that PM 9 students who took 10A (N = 7) rather than AM 10 (N = 63) increased their Post-Test achievement scores more in the 10A course (10A increase was from M = 11.4, SD = 2.0 to M = 15.4, SD = 7.1 compared to the AM 10 increase from M = 13.2, SD = 4.4 to M = 14.4, SD = 4.6).

The previous results indicate that there are significant differences, in terms of achievement and attitude towards mathematics scores, between the various groups as well as from pre-test to post-test assessment within each group. A logical question arising from these differences is whether there is a correlation between mathematics achievement and attitude toward mathematics for each of the groups. The following figures plot the achievement scores (Total) against the attitude toward mathematics scores (Attitude) for each of the Experimental and Control Groups (10A and PM 10). The regression models for each of the lines of best fit for the Total vs. Attitude plots are summarized in Table 47. The resulting regression equations are then identified for those lines of best fit with a significant Pearson correlation coefficient ($\alpha = .05$).

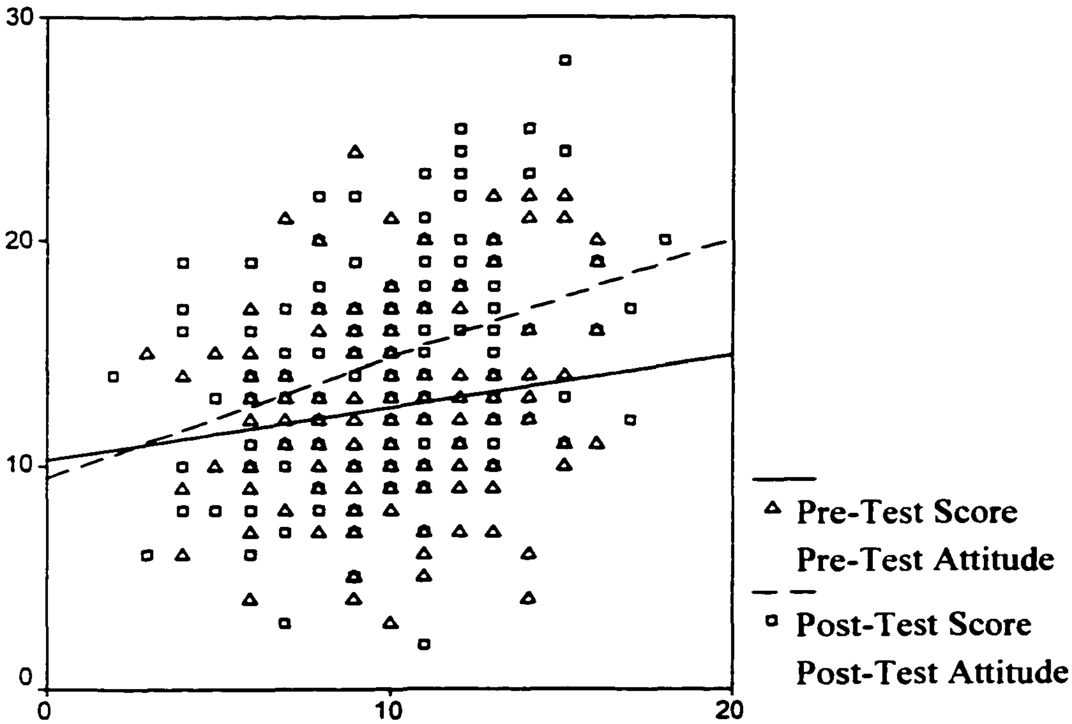

Figure 38. Experimental Group (AM 10) Plot of Achievement Score vs. Attitude Toward Mathematics Score for Pre-Test and Post-Test Assessments

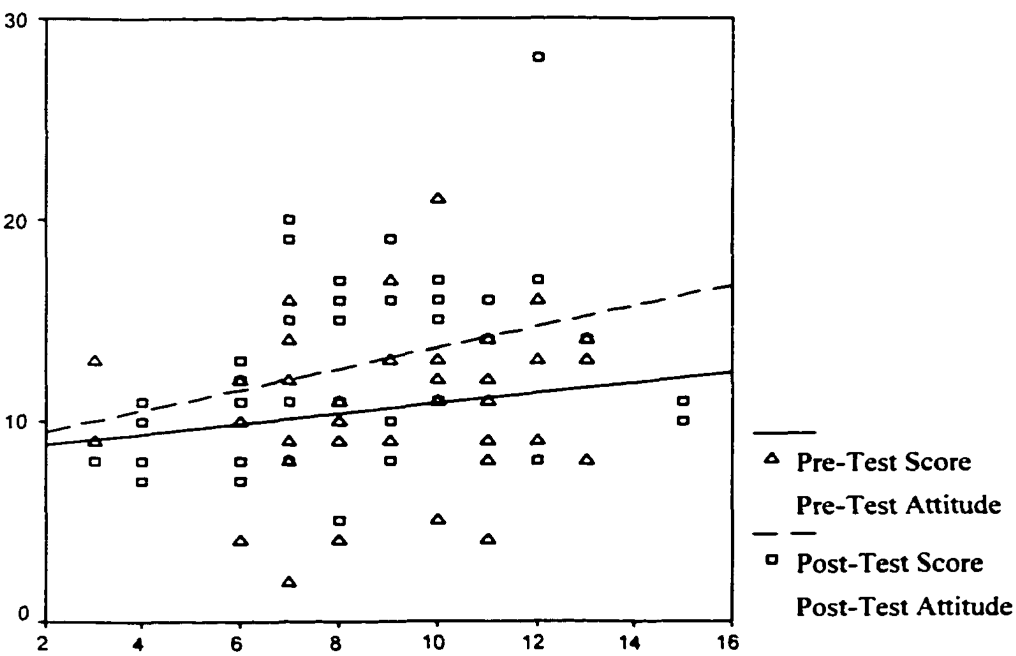

Figure 39. Control Group (10A) Plot of Achievement Score vs. Attitude Toward Mathematics Score for Pre-Test and Post-Test Assessments

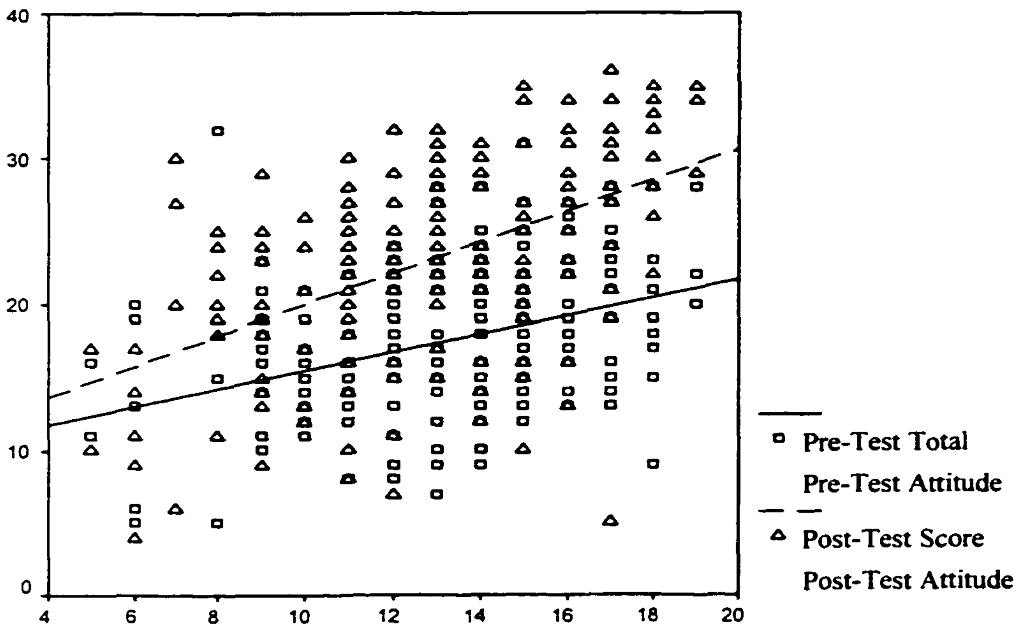

Figure 40. Control Group (PM 10) Plot of Achievement Score vs. Attitude Toward Mathematics Score for Pre-Test and Post-Test Assessments

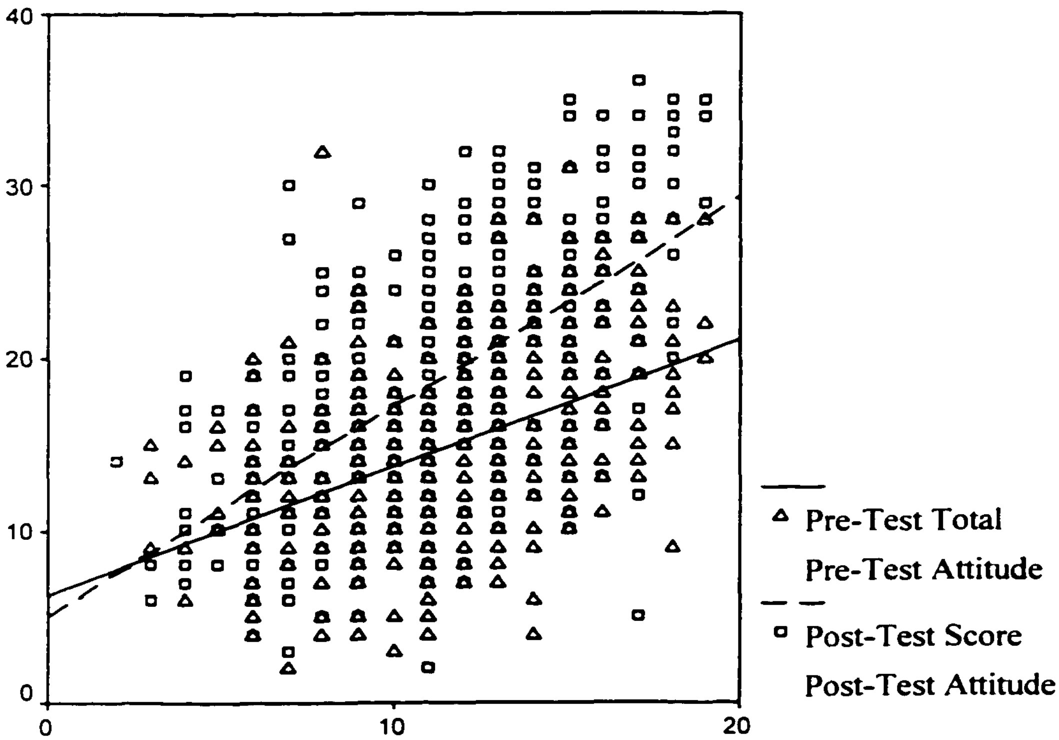

Figure 41. Combined Control Group Plots of Achievement Score vs. Attitude Toward Mathematics Score for Pre-Test and Post-Test Assessments

Table 38

Summary of Regression Models for Student Achievement Total Score vs. Attitude Toward Mathematics Scores (Pre- and Post-Test) for Experimental and Control Groups

Statistic	Exper. Group (Pre-Test)	Exper. Group (Post-Test)	Control Group (10A) (Pre-Test)	Control Group (10A) (Post-Test)	Control Group (PM 10) (Pre-Test)	Control Group (PM 10) (Post-Test)	All Groups Combined (Pre- Test)	All Groups Combined (Post-Test)
Pearson Corr.	.15	.34	.17	.33	.36	.49	.46	.58
Sig. (1-tailed)	.06	.00	.30	.04	.00	.00	.00	.00
r^2	.02	.11	.03	.11	.13	.24	.21	.33
Intercept (β_o)	8.8	6.8	7.9	5.6	9.6	7.4	7.3	6.0
Std. Error	.71	.75	1.21	1.34	.72	.71	.44	.40
t	12.4	9.1	6.5	4.2	13.3	10.5	16.5	15.0
Sig.	.00	.00	.00	.00	.00	.00	.00	.00
Slope (β_1)	.10	.22	.11	.21	.21	.23	.29	.28
Std. Error	.05	.05	.11	.10	.04	.03	.03	.02
t	1.9	4.4	1.1	2.2	5.4	7.9	10.2	14.1
Sig.	.06	.00	.30	.04	.00	.00	.00	.00

The resulting regression equations (where the Pearson correlation coefficient is significant at the .05 level) are as follows:

Experimental Group (Post-Test):

$$\text{Post - Test Score (AM 10)} = 0.22 \times \text{Post - Test Attitude (AM 10)} + 6.8$$

Control Group (Pre-Test):

$$\text{Pre - Test Score (PM 10)} = 0.21 \times \text{Pre - Test Attitude (PM 10)} + 9.6$$

Control Group (Post-Test):

$$\text{Post - Test Score (10A)} = 0.21 \times \text{Post - Test Attitude (10A)} + 5.6$$

$$\text{Post - Test Score (PM 10)} = 0.23 \times \text{Post - Test Attitude (PM 10)} + 7.4$$

All Groups Combined:

$$\text{Post - Test Score (Combined)} = 0.28 \times \text{Post - Test Attitude (Combined)} + 6.0$$

$$\text{Pre - Test Score (Combined)} = 0.29 \times \text{Pre - Test Attitude (Combined)} + 7.3$$

In all cases the values of r^2 indicate that the Post-Test assessment of Attitude Toward Mathematics is associated with the Post-Test achievement score. The Post-Test achievement score accounts for 11% of the variability in the Experimental Group Attitude score (as does the Control (10A) Group achievement score), but this value increases to 24% for the Control (PM 10) Group. When all three groups are combined for the Post-Test assessment, the achievement score accounts for 33% of the variability of the Attitude Toward Mathematics score. Placed into perspective, although all groups increased their achievement scores from the pre-test to the post-test assessment, the Attitude Toward Mathematics scores decreased at the same time that the variability in the Attitude scores that could be accounted for by the achievement scores increased.

CHAPTER 5

DISCUSSION

The first section of this chapter presents and discusses the conclusions arrived at for the three study questions, upon which this program evaluation is based. The second section identifies and discusses the limitations inherent in this study. The last section of this chapter describes how the study results relate to previous research and provides a number of suggestions for additional research.

Conclusions

This study was carried out to ascertain answers to three primary questions with respect to the implementation of Applications of Mathematics 10 in British Columbia public schools. These questions and the conclusions arrived at are as follows:

1. Do teachers of Applications of Mathematics 10 classes (Experimental Group) use significantly different teaching methodologies compared to teachers of Principles of Mathematics 10 and Mathematics 10A classes (Control Group)?

Conclusion: The Applications of Mathematics 10 teachers in this study use noticeably different teaching methodologies compared to teachers of Principles of Mathematics 10 and Mathematics 10A. Statistical significance could not be assigned to these results as the sample of teachers completing the self-reported teacher logbooks was smaller than anticipated (7 of the AM 10 teachers (70%) but only 4 of the Control Group teachers (31%) participated in this part of the assessment).

A number of instructional and assessment strategies were identified from the literature as being associated with constructivist teaching (the Experimental Group treatment). This pragmatic constructivism toolbox (Perkins, 1999) includes the use of instructional strategies that were anticipated to be used as the treatment on the Control Group. This included consolidation and practice strategies (drill and practice, homework and textbooks) and teacher centered strategies (exposition). The toolbox also includes those strategies more commonly associated with constructivist teaching, including: discussion (group work, role playing, and explaining); practical work (manipulatives, game playing, outdoor activities, and mathematics centres); investigating (guided discovery, projects and open-ended tasks); problem solving (puzzles, applications, themes and problem posing); and, using technology teaching (calculators, graphing calculators and computers).

The assessment strategies expected of both groups (Control Group treatment) included factual questioning and pencil and paper/multiple choice testing. The (assessment) practices expected of the Experimental Group teachers also included: anecdotal observations; questioning (higher order and open-ended); interviewing (structured and open); testing (diagnosis, performance-based, problem solving and attitude); reporting (oral, written, portfolio investigation and modeling); and, self-assessment (journals, reflective prompts, self-questioning and peer-assessment).

Results of the self-reported teacher logbooks and teacher surveys indicate that there were identifiable trends in the treatments the Control and Experimental Groups received. Both groups of teachers spent the same amount of time instructing (approximately 80% of class time) and assessing (approximately 20% of class time) their students. There were differences between the two groups with respect to the proportion of time spent using different strategies and the number of times different strategies were used in the classroom.

The Experimental Group teachers reported spending approximately half the time on consolidation and practice and teacher centered instructional strategies that the Control Group teachers did. Additionally, Experimental Group teachers reported spending approximately twice as much time as the Control Group teachers using strategies such as discussion, practical work, investigating, problem solving, and using technology.

These results are corroborated by the number of times the different instructional strategies were reported being used in the classroom. Although the average total number of instructional strategies used by each group is identical (315 occurrences during the teaching of the courses), Control Group teachers ($N = 9$) used strategies such as exposition, drill and practice, homework, textbooks and worksheets for 75% of those instances while Experimental Group teachers ($N = 12$) used the same strategies for 50% of the 315 occurrences. Experimental Group teachers used other instructional strategies (e.g., group work, manipulatives, guided discovery, applications) twice as often as the Control Group teachers.

The Experimental Group teachers reported spending 27% less time on pencil and paper and multiple choice testing than the Control Group teachers did. In place of this time, the Experimental Group teachers used other assessment strategies including: observation; questioning; interviewing; reporting; and, self-assessment. Of particular note is that the Experimental Group teachers assessed their students 41% more often than did the Control Group teachers in the same amount of time.

These results would suggest that the Experimental Group teachers used a wider range of instructional and assessment strategies than their Control Group counterparts. This information is further corroborated by the results of the Pre- and Post-test Teacher Surveys on Mathematics Teaching Practices. The survey results indicate that the Experimental Group teachers favor using concrete materials, technology, and problem-solving activities consistent with constructivist teaching methodologies (for example see: Brooks & Brooks, 1993; Crawford & Witte, 1999) in the classroom more than the Control Group teachers. At the same time, the survey results also corroborate that the Control Group teachers relied more heavily on traditional summative forms of assessment.

The results of the Teacher Identification Surveys indicate that a number of possible confounding factors including teachers' age, gender, and overall education and teaching experience can be discounted with respect to the Experimental and Control Group. There are a number of factors (e.g., teaching specialization, mathematics content and pedagogical training, school involvement, etc.) that will be discussed further in the Limitations section of this chapter.

Although further qualitative research is needed to confirm or clarify these findings, it appears that students in the two groups experienced different treatments as anticipated. With the instructional and assessment differences identified (at least tentatively), the next question was to determine if there were any achievement or attitudinal differences between the Experimental and Control Group students.

2. Are students' achievement on mathematics assessments scores (pre-test and post-test) significantly different for students who have taken Applications of Mathematics 10 (experimental group) compared to those who have taken Principles of Mathematics 10 or Mathematics 10A (control group)?

3. Are students' attitude towards mathematics scores (pre-test and post-test) significantly different for students who have taken Applications of Mathematics 10 (experimental group) compared to those who have taken Principles of Mathematics 10 or Mathematics 10A(control group)?

Conclusion: There are significant differences between Applications of Mathematics 10, Mathematics 10A and Principles of Mathematics 10 subjects with respect to both mathematics achievement and attitude towards mathematics. These include significant differences between the group mean scores as well as significant difference between the pre- and post-test scores within the same group.

Analysis of variance and subsequent post-hoc analysis of differences between mean scores of the pre-test and post-test assessments indicate that Principles of Mathematics 10 students scored significantly higher ($\alpha = .05$ for analyses) than both the Applications of Mathematics 10 and Mathematics 10A students. The result holds true for the sub-scores for each of the four curriculum organizers (Number, Patterns and Relations, Shape and Space, and Statistics and Probability), the total achievement score and the attitude toward mathematics score.

Although the AM 10 students achieved significantly lower scores than the PM 10 students, they scored significantly higher than the 10A students on: the pre-test Statistics and Probability sub-score; the pre-test total achievement score; and, the post-test attitude toward mathematics score. In all other sub-scores, there was no statistical difference. Calculations of η^2 for the significant differences indicate that the variability in the various total achievement and attitude toward mathematics scores is not trivial (η^2 values indicate that 22% to 36% of the variability in the different scores could be attributed to group membership).

When a similar analysis of variance and post hoc analysis of differences between means was conducted on the change in group mean scores (from pre-test to post-test) it was found that PM 10 students increased their scores significantly more than both AM 10 and 10A students for the Number, Shape and Space, and total achievement categories. The PM 10 students also increased their mean scores significantly more than the AM 10 students for Patterns and Relations and Statistics and Probability. The AM 10 students did not increase their scores significantly more than the 10A students.

The variability in the change in the mean scores (η^2) ranged from 2% to 8%. This indicates that group membership attributed for a small part of the variability of the change in scores.

When the students were grouped by the previous mathematics course taken (Mathematics 9A, Applications of Mathematics 9, Principles of Mathematics 9, and Grade 10) and an analysis of variance and post hoc analysis of mean differences (for pre- and post-test achievement and attitude toward mathematics scores only) was conducted, a number of significant differences were identified.

Those students who were previously in Principles of Mathematics 9 scored significantly higher than students from both Applications of Mathematics 9 and Principles of Mathematics 9 for all scores.

Although the AM 9 students had statistically the same pre-test achievement mean score as the 9A students, their pre-test attitude toward mathematics mean score and subsequent post-test mean scores (achievement and attitude toward mathematics) were significantly higher than the 9A students. This is corroborated when the mean scores of the AM 9 students are compared. It can be seen that those AM 9 students (N = 72), who went on to take AM 10, obtained higher post achievement (M = 15.2, SD = 4.6) and attitude toward mathematics mean scores (M = 10.2, SD = 3.0) than did those who went on to take 10A (N = 9) the following year (Achievement: M = 13.4, SD = 3.1; and, Attitude Toward Mathematics: M = 8.4, SD = 3.0).

Although the AM 10 sample population was largely made up of students who had previously taken PM 9 (37% of the AM 10 population) or AM 9 (47% of the AM 10 population), it is important to note that there are noticeable differences in the distribution of letter grades for these students in their grade 9 courses. Of the AM 9 students who subsequently took AM 10, 34 % had received a letter grade of at least "B", 60% had received a grade of "C" or "C+", 5% had received a pass, and none of them had received a failing grade. Contrasting this, of the PM 9 students who subsequently took AM 10, only 18% had a received a grade of "B" or better, 72% had received a "C" or "C+" grade, 7% had received a pass and 3% had failed the grade 9 course. Thus, it appears that those PM 9 students who either entered AM 10 on their own, or were counseled to take the course, did not experience the same success in AM 10 as did their AM 9 peers. This is born out when the post-test mean scores for both achievement and attitude toward mathematics for the two sub-groups are compared. The PM 9 students taking AM 10 averaged lower achievement (M = 14.4, SD = 4.6) and attitude toward mathematics (M = 9.8, SD = 2.8) scores than did the AM 9 students who then took AM 10. These results suggest that students who take two courses based upon a constructivist philosophy (i.e., Applications of Mathematics), develop better understanding of the mathematical concepts involved and have better attitudes toward mathematics (as measured by the student assessment instruments used in this study) than those who are exposed to just one course.

The final point for consideration in this program evaluation of Applications of Mathematics 10 is the connection between student mathematics achievement and attitude towards mathematics and the teacher methodology used in the classroom. This study confirms the previous result that there is a significant correlation between students' mathematics achievement and their attitudes towards mathematics. The post-test correlation between achievement and attitude (for all three groups) indicates that the achievement score accounts for 33% of the variability of the attitude toward mathematics score. The observation that those students who previously would have taken the 10A course, but instead took the AM 10 course and subsequently scored significantly higher on both mathematics achievement and attitude towards mathematics scores, suggests that the treatment (i.e., constructivist methodology) may have played a role. This association appears to be stronger when the student has experienced the constructivist methodology (intended in the Applications of Mathematics course) for two courses compared to just one.

Limitations

For the student achievement and attitude toward mathematics results of this study to be considered conclusive:

- the Experimental Group subjects would have needed to score statistically different from the Control Group subjects on all post-test assessments (including the total achievement score, the attitude toward mathematics score, and all 4 curriculum sub-scores); and,
- the relative change in scores from pre-test to post-test would also have needed to be statistically significant for the total achievement score, the attitude toward mathematics score, and all 4 curriculum sub-scores.

As this did not occur, it is important to consider what were some of the possible limitations of this study that may have contributed to the less than conclusive results. These include limitations associated with:

- researcher bias;
- sampling techniques used;
- intended treatments;
- instrumentation used to collect teacher and student data;
- statistical techniques used to analyze the teacher and student data; and,
- generalizability of conclusions.

Limitations Due to Researcher Bias

The researcher in this study is employed by the British Columbia Ministry of Education as the Mathematics Coordinator in the Curriculum Branch. As such, the researcher was partly responsible for the development of the secondary course structure in use for the period of the study. The researcher was also partially responsible for the development of the Applications of Mathematics curriculum. This involvement would suggest that the researcher had a vested interest in demonstrating that the Applications of Mathematics curriculum was succeeding as originally planned. This potential for bias can best be addressed by ensuring that the study design and its execution are well documented and open to scrutiny and possible replication.

Limitations in Sampling Techniques

The teachers who decided to offer the Applications of Mathematics 10 curriculum in the 1998/99 school year appear more willing to partake in an innovative project than their colleagues. It follows that there is a degree of teacher self-selection both in respect to teaching the course in the first place, and then in volunteering to participate in the study itself. The results of the Teacher Identification Survey indicate that the Experimental Group teachers have taken more mathematics content and mathematics methods courses than their Control Group counterparts. Additionally, the survey results show that the Experimental Group teachers were predisposed to spend more time on teaching activities such as: professional reading and development; keeping student records updated; meeting with parents; planning lessons; and, grading student work (other than exams) than the Control Group teachers. These activities indicate that the Experimental Group teachers were previously inclined to embrace the philosophy and intent of the Applications of Mathematics curriculum.

This self-selection cannot be avoided until all secondary schools in the province are offering the curriculum. At that point a random sample would have a reduced possibility of sample bias. Another possible method to address this limitation is to randomly assign teachers to teach the different courses and then require them to use the different teaching methodologies. This is impractical as there is no way of guaranteeing that the intended treatments would be used in the sample classrooms.

Compounding the self-selection limitation is the fact that the population of schools offering Applications of Mathematics during the period of the study was small (31 schools province wide out of approximately 700 secondary schools). Out of this small population only ten schools volunteered to participate in the study, thus further reducing the possibility of the sample being considered random. A possible solution to this limitation is to conduct the program evaluation when there is a larger population of schools offering the course. This would then ensure that a true random sample of schools would provide a large enough sample population to generate statistically reliable results. As the intent of conducting the program evaluation during the 1998/99 school year was to get information early in the implementation of the program, such a delay was ruled out.

In addition to teacher self-selection and the resulting chance of sample bias, the Hawthorne Effect should also affect how the results of this study are to be interpreted. The fact that the students and teachers were in a new learning environment and aware that they are being studied may have had unpredictable effects on the results. Once again this can be minimized if the population from which the sample is drawn consists of all secondary schools in the province and the Applications of Mathematics courses have been in place long enough to diminish the 'newness' of them. A more viable alternative is to ensure that the students and teachers had been exposed to the constructivist learning environment in previous courses. Thus, students entering the grade 10 year would be acclimatized to the inherent methodologies and would not perceive of them as 'new'.

Limitations in Intended Treatment

The teachers involved in this study (Experimental or Control Group) were teaching one of three specific curricula. Although there were implicit and explicit pedagogical expectations with respect to instructional practices to be used in the classroom, there is a degree of uncertainty about what is or is not 'constructivist teaching'. For some it is a process and a teaching model, while for others it is a definite set of instructional and assessment strategies. In between these two definitions is an unlimited repertoire of models and practices that teachers could be using and modifying as they teach. To partially compensate for this, the teachers involved in the study were provided with a glossary of instructional and assessment strategies (see Appendix E). The glossary made no mention of which of the strategies had been identified by the researcher as being constructivist oriented. Although the very presence of the list of strategies may have influenced the teachers' choices of how to teach, this could not be avoided as a common set of operational definitions was required to ensure accurate results being reported by the teachers in their logbooks. It was anticipated that any initial influence on the teachers' choice of instructional or assessment strategies would dissipate as the term of the course continued. This possible influence could be reduced by using in-class observations of the teachers for a prolonged period in place of (or in addition to) the teacher logbooks. This was not possible for the purposes of this study due to financial and geographic constraints associated with this type of data gathering. As this type of data would be invaluable in validating the teacher reported logbook data, this could be a logical next step.

Limitations in Data Collection Instruments

As was noted in the previous section, the teacher logbooks had inherent limitations as they relied on the teachers' self-reporting of what they did in their classrooms. The most effective way of addressing this issue is to conduct direct observations of the teachers in their classrooms to confirm their self-reported data.

The student assessments consisted of 40 multiple choice achievement items and 13 demographic and attitudinal items. This multiple choice assessment format is consistent with the style that the Control Group teachers were anticipated to use in their classes. This was confirmed through the longitudinal data obtained from the teacher logbooks. Although many of the questions were contextual in nature, the lack of open-ended questions may put the Experimental Group students at a disadvantage. It was expected that the treatment that these students would be receiving would include a strong focus on 'big idea' problems that would lend themselves to open-ended assessment techniques. It would have been preferable for the student assessments to include an open-ended problem solving section, but once again financial constraints dictated that the student instruments be restricted to multiple choice questions.

A second limitation of the student assessment instruments is that they were not piloted prior to formal use in the Experimental and Control Group classrooms. An attempt was made to pilot the instruments in several Principles of Mathematics 10 and Applications of Mathematics 10 classrooms in June 1998. Unfortunately, printing and distribution delays resulted in the pilot assessment instruments not reaching the piloting teachers until late in June. By that time the teachers were preparing their students for final examinations and did not have time to have their students write the assessments. To compensate for this, the pilot teachers were asked to critique the pilot assessment instruments with respect to: curriculum fit; age/course appropriateness; and, clarity of language and illustrations. This teacher feedback was incorporated into the final instruments prior to them being administered to the Experimental and Control Group students.

Limitations in Statistical Analysis Procedures

The original research design called for an analysis of covariance (ANCOVA) of the pre- and post-test student data. When the data were tested for homogeneity of regression it was determined that the slopes of the regression plots of pre-test scores onto post-scores were significantly different. Because of this difference, an ANCOVA was not possible and as a result each set of pre- and post-test data were subjected to an ANOVA instead. This had the effect of not allowing for any adjustment of the post-test results (dependent variable) to what they would be if all students scored equally on the pre-test assessment (covariate). Although ANOVA is useful in identifying significant differences between group mean scores, the results can not compensate for the students' initial mathematical understanding and attitudes towards mathematics. This places a limit on how the data can be interpreted, but ANOVA is still able to identify if there are significant differences between the three group mean scores.

The grouped student data were further broken down by previous course taken. The purpose for this was to see if there were any identifiable trends stemming from the grade 9 mathematics experiences. Although trends could be observed (and were reported and discussed in Chapters 4 and 5) and the use of ANOVA could determine that significant differences exist between the grade 9 sub-group mean scores for achievement and attitude toward mathematics, the analysis procedures could not determine if the differences observed between the sub-groups of the Experimental Group and the two Control Groups were significant. This analysis might have been possible if the original sample size were larger to accommodate the fact that there are potentially 12 subgroups involved. As the sample population was limited due to circumstances beyond the researcher's control, this limitation could be addressed only if done on a provincial scale (possibly as part of a Provincial Learning Assessment Program that the British Columbia Ministry of Education is considering conducting in the next few years).

Limitations in Generalizability of the Results

As the teachers involved in this study are primarily self-selected, as the students may be as well, the teacher and student results may not be generalizable to the population of grade 10 students and mathematics teachers in British Columbia. The teachers involved in teaching the Applications of Mathematics 10 classes early in the implementation of the program are likely innovative teachers more willing to try new things than are the rest of the teaching population. This limitation can be addressed once the Applications of Mathematics 10 courses have been implemented on a wider scale. At that time, random selection of Experimental and Control Group classes will ensure that the teacher and student results are more generalizable to the population as a whole.

Relationship to Previous Research

The trends observed in the teacher logbooks support much of the research on constructivist oriented teaching. Although the logbook data can not be directly related to the teaching models proposed by: Pirie and Kieren (1992); Simon's (1995) "hypothetical learning trajectory"; Steffe's and D'Ambrosio's (1995) "zone of potential construction"; or Brooks' and Brooks' (1993) five principles of constructivist pedagogy, the data are consistent with many of Brooks' and Brooks' twelve descriptors of what would characterize a constructivist teacher. The Experimental Group teacher logbook data suggest that these teachers: use manipulative, interactive, and physical materials; use discussion and group work to determine their students' understanding of concepts; use open-ended questions, projects, and investigations; and, seek elaboration of students' initial responses.

The logbook data are also consistent with the contextual teaching strategies described by Crawford and Witte (1999) as: relating (providing context for the student); experiencing (using exploration, discovery, and invention as well as laboratory and problem-solving activities); applying (putting the concepts to use); cooperating (learning in the context of sharing, responding, and communicating with other learners); and, transferring (using knowledge in a new context or situation).

The newly released Principles and Standards for School Mathematics (NCTM, 2000) includes a number of principles that the Experimental Group teachers appear to be following more closely than their Control Group colleagues. These include:

- Equity (high expectations and strong support for all students) - AM 10 teachers are offering their students a challenging curriculum that most of the students would not experience if they were to take Mathematics 10A;
- Teaching (understanding what students know and need to learn and then challenging and supporting them to learn it well) - the wide range of instructional strategies used by the Experimental Group teachers suggests that they attempting to do this;
- Learning (actively building knowledge from experience and prior knowledge) - the Experimental Group teachers' use of questioning and diagnostic assessment strategies suggests that they are attempting to build on their students' prior knowledge. The use of practical work and investigating instructional strategies also suggests that these teachers are providing their student's with opportunities to actively build their knowledge from experience;

- Assessment (furnish useful information to both teachers and students) - the Experimental Group teachers use twice as many types of assessment strategies than do the Control Group teachers. This range of assessment activities provides the teacher with ongoing formative assessment information (as compared to pencil and paper tests which tend to be more summative in nature) that also provides their students with opportunities to gain useful information; and,
- Technology (influences the mathematics that is taught and enhances students' learning) - the Experimental Group teachers make use of technology, including graphing calculators and computers, considerably more than the Control Group teachers.

The study data and subsequent statistical analyses are consistent with the census data provided by the 1999 Provincial Assessment of Reading Comprehension, First-Draft Writing, and Numeracy (Province of British Columbia, 2000a). Using the same ANOVA and post-hoc analysis of mean differences used on the sample data (see Appendix H), it was determined that the group mean scores for both student achievement and attitude towards mathematics were significantly different with the PM students scoring highest, followed by the AM 10 students and then the 10A students. The numeracy assessment also included four open-ended written response questions that provided a more contextual setting for students to demonstrate their mathematical understanding. Analysis of the student scores on these questions is also consistent with the FSA multiple choice results and the pre-test and post-test student results in this study.

The most relevant research previously reviewed with respect to student achievement in constructivist oriented settings deals with the Applied Academics (AA) program evaluations (including: Center for Occupational Research and Development, 1993; McKillip, Davis, Koballa, & Oliver, 1993; Pepple & O'Connor, 1992; Wang & Owens, 1992, 1994, 1995). The curriculum model used in the Applied Academics program evaluations is based upon teaching the equivalent of Algebra I over a two year period "using teaching approaches such as hands-on, real world, apprenticeship, and work-based" (Hull, 1993). This differs from the Applications of Mathematics/Principles of Mathematics model, which is based upon the teaching of 60% of the same mathematical content, using constructivist teaching methodologies in the Applications of Mathematics 10 class more so than in the Principles of Mathematics 10 class.

The Applied Academics program evaluations concluded that student achievement in the AA classes was either significantly higher or at least the same as that of students in the regular Algebra I class. This is not consistent with the results obtained in this study, but as the two models differ in design with respect to the amount of time (1 year vs. 2 years spent on similar content), this is not unexpected. The AA program evaluations support the results of the teacher logbooks in that: 1) Pepple and O'Connor (1992) also concluded that the AA teachers used a greater variety of teaching methods compared to other courses they taught; and 2) McKillip et al. (1993) suggest that the AA teachers make considerable use of cooperative learning groups, experimentation, and solving applications type problems in their classes.

Applications of Mathematics courses take a different approach to the concept of streaming students compared to the more traditional low ability/high ability model of the previous secondary course structure (Mathematics 10A/Principles of Mathematics 10). With the advent of the Applications of Mathematics pathway, students now have a mathematics course alternative where placement is not based solely upon their success in previous mathematics courses. Although the above may not be generalizable to the entire mathematics teaching population in the long run, it should be noted that, unlike previous streaming programs, the teachers of the AM courses in this study have the same, if not more, mathematics content and methods education. Therefore, even if students are placed in the AM course solely because they are not succeeding in the "regular" class, they are still gaining the benefit of experienced, trained teachers that is generally not available to students in the 10A classes (Marshall, et al., 1996). It may be that the Applications of Mathematics pathway provides an academic alternative to the traditional high-ability course without the negative side effects of being taught by inexperienced teachers, who tend to use the same instructional strategies that didn't work in the first place.

Suggestions for Further Research

The results of this study suggest a number of areas that require further study including:

1. Validation of the teacher logbook data through classroom observations of teachers use of different instructional and assessment strategies. This would also provide qualitative research that could provide information as to what works and possibly why it works.
2. Validation of the student assessment results. This validation should be done using an assessment instrument that is more consistent with the constructivist teaching expected in the Applications of Mathematics classrooms. This instrument should

include open-ended problems and possibly a performance-based component where students are expected to solve problems that require: physical manipulation of objects; data collection and analysis; and/or, computers (spreadsheets).

3. A longitudinal study of students in the Applications of Mathematics and Principles of Mathematics pathways is needed to determine the long-term effects that constructivist teaching has on student learning and how it affects post-secondary and/or workplace choices. This type of long-term data may provide the "big picture" view of mathematics education that most mathematics educators (and the public) may not be able to perceive.
4. The above longitudinal study should also include a more detailed examination of students' attitudes toward mathematics and how they change over time (depending upon their later educational and workplace experiences).
5. A study on the effect that the new Applied Mathematics learning resources, developed by Pearson Publishing in cooperation with the Western Canadian Protocol, have on both the teaching and the learning of mathematics is needed. These resources are based upon the Western Canadian Protocol Mathematics Framework for Kindergarten to Grade 12 Mathematics (1996). This Framework has been adopted by British Columbia for both the Applications of Mathematics and Principles of Mathematics pathways. With an approximate 60% overlap between the two pathways at the grade 10 level, the impact that these learning resources have on the teaching and learning of the common content must be assessed.

Recommendations

The Applications of Mathematics 10 curriculum in British Columbia appears to be a qualified success within the constraints of this study. It appears that this program may be affecting Applications of Mathematics 10 teachers' instructional practices as intended. Having said that, there are a number of recommendations that can be made which may further improve the effectiveness of the Applications of Mathematics implementation. Many of these recommendations are consistent with those already identified by Brooks and Brooks (1993) and also found in The Report of the Mathematics Task Force (Province of British Columbia, 1999).

It is recommended that:

1. The B.C. Ministry of Education and school districts ensure that secondary mathematics teachers have regular access to in-service that assists them in becoming familiar with constructivist principles and practices.
2. The B.C. College of Teachers and the Faculties of education structure pre-service programs around constructivist principles and practices.
3. B.C. school district and school level administration staff provide active support to encourage more secondary schools to offer and teach Applications of Mathematics courses.
4. The B.C. Ministry of Education develop assessment models (including Provincial Examinations and Provincial Learning Assessments) that are consistent with constructivist principles and practices.

5. The B.C. Ministry of Education conduct an ongoing review of the Applications of Mathematics curriculum for the purpose of reducing content where possible. A reduction in specific content, while maintaining broad mathematical goals, frees teachers to more easily use appropriate constructivist principles and practices in their teaching.
6. B.C. school districts and schools provide students and parents with information concerning the different mathematics pathways. This should include information concerning post-secondary and workplace options resulting from each pathway AND the educational consequences of each of the pathways.

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Appendix A

Ethics Approval

University of Victoria

Human Research Ethics Committee

CERTIFICATE OF APPROVAL

Principal Investigators	Department/School	Supervisor
Bruce McAskill	**Social & Natural Sciences**	**Dr. L. Pelton**
Graduate Student		

Title: A Program Evaluation of Applications of Mathematics 10

Project No.	Start Date	End Date	Approval Date
119-98	**1 May 98**	**30 Jun 99**	**24 Apr 98**

Certification

This is to certify that the University of Victoria Ethics Review Committee on Research and Other Activities Involving Human Subjects has examined the research proposal and concludes that, in all respects, the proposed research meets appropriate standards of ethics as outlined by the University of Victoria Research Regulations Involving Human Subjects.

J. Howard Brunt,
Acting Associate Vice-President, Research

This Certificate of Approval is valid for the above term provided there is no change in the procedures. Extensions/minor amendments may be granted upon receipt of "Request for Continuing Review or Amendment of an Approved Project" form.

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Appendix R

Prescribed Learning Outcomes Comparison for Applications of Mathematics 10 and Principles of Mathematics 10

Number (Number Concepts)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will explain and illustrate the structure and interrelationships of the sets of numbers within a number system.	It is expected that students will explain and illustrate the structure and interrelationships of the sets of numbers within a number system.
Identical Outcomes	
• identify exceptions to the real number system	• identify exceptions to the real number system
Similar Outcomes with a Different Emphasis	
• demonstrate an understanding of radical notation of index 2 • describe the difference between exact and approximate values for irrational numbers	• demonstrate appropriate use of radical notation • distinguish between exact and approximate values for irrational numbers
Unique Outcomes	
• illustrate the similarities and differences between rational and irrational numbers • identify situations where answers involve only the positive (principal) square root, or both positive and negative square roots of a number	• demonstrate an understanding of rational exponents as an equivalent form of radical notation

Number (Number Operations)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will demonstrate understanding of and proficiency with the basic operations on real numbers, and apply these ideas and skills in solving applied problems.	It is expected that students will demonstrate understanding and proficiency as they perform the basic operations on real numbers, and apply these skills in solving practical problems.
Identical Outcomes	
• apply the laws of exponents to variable expressions with integral exponents • evaluate numerical expressions involving absolute value notation • use appropriate technology to solve problems involving simple and compound interest from annual to daily compounding	• apply the laws of exponents to variable expressions with integral exponents • evaluate numerical expressions involving absolute value notation • use appropriate technology to solve problems involving simple and compound interest from annual to daily compounding
Similar Outcomes with a Different Emphasis	
• evaluate numerical expressions with rational exponents and positive integral bases using a calculator • extend the operations of addition and multiplication to include radicals • use a calculator to evaluate radical expressions and solve problems involving radicals	• evaluate numerical expressions with rational exponents and positive integral bases using a calculator or other technology • add, subtract, multiply, and divide radicals of index 2 having a numerical radicand • apply addition, subtraction, and multiplication of radicals to solve problems
Unique Outcomes	
• solve problems with formulae from industry and commerce involving radicals or exponents	• convert repeating decimals to a ratio of integers and vice versa • convert between expressions with rational exponents and their radical equivalents • simplify radicals of index 2 having numerical radicands • convert an entire radical to a mixed radical and vice versa • determine the effective annual rate of interest, given the nominal rate and the number of compounding periods per year

Patterns and Relations (Patterns)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will use patterns to describe the world and solve problems.	It is expected that students will use patterns to describe certain aspects of the world mathematically and to solve problems.
Identical Outcomes	
• use properly annotated graphical representation to model data from physical situations • interpolate, extrapolate, and draw conclusions from graphs representing naturally occurring data • use data from physical or experimental situations to determine the equations of a line, including definitions of the variables and units involved	• use properly annotated graphical representation to model data from physical situations • interpolate, extrapolate, and draw conclusions from graphs representing naturally occurring data • use data from physical or experimental situations to represent the equations of a line, including definitions of the variables and units involved
Similar Outcomes with a Different Emphasis	
• translate a given linear equation to slope-intercept form ($y = mx + b$) or general form ($ax + by + c = 0$)	• represent a given linear equation in slope-intercept form ($y = mx + b$) and general form ($ax + by + c = 0$)

Patterns and Relations (Variables and Equations)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will extend and consolidate skills in simplifying and manipulating algebraic expressions to expand their use of the language of mathematics in problem solving.	It is expected that students will represent algebraic expressions in different ways, and extend and consolidate skills in simplifying and manipulating algebraic expressions. students will:
Identical Outcomes	
• multiply a polynomial by a polynomial • multiply and divide rational expressions • add and subtract rational expressions with different monomial denominators • solve linear equations involving rational expressions with monomial denominators	• multiply a polynomial by a polynomial • multiply and divide rational expressions • add and subtract rational expressions with different monomial denominators • solve linear equations involving rational expressions with monomial denominators
Similar Outcomes with a Different Emphasis	
• factor polynomials of the following types: – difference of squares – perfect square trinomials – combinations of the difference of square and perfect square trinomials, including common factor and $x^2 = bx + c$ • list any excluded values for the variables in a rational expression • use factoring to simplify rational expressions • solve a formula or multi-variable equation for a specific variable in terms of the other variables, involving no more than two steps • solve applied problems using algebra skills and appropriate technology with attention to units	• completely factor polynomials of the following types: – trinomials of degree 2 with leading coefficient not equal to 1 – difference of squares – perfect square trinomials – combinations of the previous types • identify any excluded values for the variables in a rational expression • apply factoring to simplify rational expressions • solve a formula or multi-variable equation for a specific variable in terms of the other variables • solve applied problems using algebra skills and appropriate technology

Patterns and Relations (Relations and Functions)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will develop an understanding of the linear relations and will use technology to make connections between algebraic and graphical representations, as well as between mathematics and the real world.	It is expected that students will develop an understanding of the linear model and will use technology to make connections between algebraic and graphical representations, as well as between mathematics and the real world.
Identical Outcomes	
• demonstrate an understanding of the one-to-one correspondence of an ordered pair and points on the coordinate plane • determine coordinate pairs from a linear equation and draw the graph of the line • demonstrate an understanding of the relationship between points on the graph and the linear equations • graph a linear relationship given various conditions: – the equation in $y = mx + b$ form – slope and y-intercept – slope and any point • graph a linear inequality in two variables determine, from a given equation, the slope and y-intercept of the graph • derive the equation of a line in $y = mx + b$ form, given various conditions: – the slope and y-intercept – the graph of the line – the slope and any point on the line – any two points on the line – an equation of a parallel line and a point on the line – an equation of a perpendicular line and a point on the line • use slope and y-intercept to identify from the equation pairs of lines that are: – parallel – coincident – intersecting – perpendicular	• demonstrate an understanding of the one-to-one correspondence of an ordered pair and points on the coordinate plane • determine coordinate pairs from a linear equation and draw the graph of the line • demonstrate an understanding of the relationship between points on the graph and the linear equations • graph a linear relationship given various conditions: – the equation in $y = mx + b$ form – slope and y-intercept – slope and any point • graph a linear inequality in two variables determine, from a given equation, the slope and y-intercept of the graph • derive the equation of a line in $y = mx + b$ form, given various conditions: – the slope and y-intercept – the graph of the line – the slope and any point on the line – any two points on the line – an equation of a parallel line and a point on the line – an equation of a perpendicular line and a point on the line • use slope and y-intercept to identify from the equation pairs of lines that are: – parallel – coincident – intersecting – perpendicular
Unique Outcomes	
• use data from real-world sources to develop linear models • apply the linear relations to solve real-world problems	• solve systems of linear equations by graphing, using appropriate technology

Shape and Space (Measurement)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will describe and analyse real-life situations using trigonometry.	It is expected that students will describe and analyse real-life situations using vector representations and trigonometry.
Identical Outcomes	
• apply the Pythagorean relationship to determine the length of the internal diagonal of a box	• apply the Pythagorean relationship to determine the length of the internal diagonal of a box
Similar Outcomes with a Different Emphasis	
• apply trigonometry to solve problems using appropriate technology	• solve applied trigonometry problems using exact values
Unique Outcomes	
• select and apply appropriate instruments and units of measurement for determining length, area, volume, time, mass, and rates • explain the limitations of a measuring instrument and the measurement strategy used, using the concepts of precision and accuracy • enlarge or reduce a dimensioned object to a specified scale	• calculate exact values of the primary trigonometric ratios for 30°, 45°, and 60° • determine the area of a triangle using Heron's formula • define and use scalar and vector representations using appropriate terminology and notation • perform head-to-tail vector additions on triangles • solve problems involving right triangles using vectors

Shape and Space (3-D Objects and 2-D Shapes)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will use geometry to model the world around them and to analyse the interrelationships among shapes.	It is expected that students will use geometry to model the world around them and to analyse the interrelationships among shapes.
Identical Outcomes	
• demonstrate an understanding of the following terms: – midpoint – median – altitude – bisector of an angle or a segment – perpendicular bisector of a segment • use coordinate geometry to solve problems involving geometric figures	• demonstrate an understanding of the following terms: – midpoint – median – altitude – bisector of an angle or a segment – perpendicular bisector of a segment • use coordinate geometry to solve problems involving geometric figures
Unique Outcomes	
• determine whether given triangles are congruent using the following: – SSS (side-side-side) – ASA (angle-side-angle) – SAS (side-angle-side) – AAS (angle-angle-side) – HL (hypotenuse-leg) • demonstrate an ability to interpret 2-D representations of 3-D space	• solve abstract and concrete problems using geometric properties of angles, lines, triangles, and polygons

Shape and Space (Transformations)

Applications of Mathematics 10	Principles of Mathematics 10
Unique Outcomes	
It is expected that students will create and analyse design patterns. • design interlocking shapes to fill regions and tile 2-D space • identify examples of tiling in the real world • apply multiple transformations to polygons on the coordinate plane	

Statistics and Probability (Data Analysis)

Applications of Mathematics 10	Principles of Mathematics 10
It is expected that students will demonstrate an ability to acquire, organize, process, interpret, and communicate statistical information.	It is expected that students will describe, implement, and analyse sampling procedures and draw appropriate inferences from the data collected.
Identical Outcomes	
• draw conclusions about the population from which a sample was taken	• draw conclusions about the population from which a sample was taken
Similar Outcomes with a Different Emphasis	
• choose and justify sampling techniques that will result in an appropriate sample, including biased and unbiased samples	• choose, justify, and apply sampling techniques to achieve an appropriate sample of a given population
Unique Outcomes	
• describe the difference between census and survey, and between population and sample • define a population from which a sample is to be drawn • determine a sample that will represent the population • establish and support a position on generalizations made about populations based on data from samples • identify and solve problems using data analysis	• analyse a sample for bias and validity • interpret other forms of bias in the use of samples • demonstrate the variability of data by giving examples of different distributions

Statistics and Probability (Chance and Uncertainty)

<table>
<tr><th>Applications of Mathematics 10</th><th>Principles of Mathematics 10</th></tr>
<tr><th colspan="2">Unique Outcomes</th></tr>
<tr><td>It is expected that students will be able to communicate their understanding of probability using appropriate terminology.

• define independent and dependent events and identify them in real-world Applications of Mathematics 10
• explain why the probability of mutually exclusive events is zero
• explain the difference between AND and OR when used to connect statements
• apply the product of appropriate probabilities to calculate the probability of pairs of independent and dependent events</td><td>It is expected that students will determine permutations and combinations of possible events using the fundamental counting principle, organized lists, and formulae.

• solve problems involving the number of ordered arrangements of objects placed in a linear or circular manner by listing the outcomes
• derive, use, and evaluate factorial expressions
• apply permutation and combination formulae to determine the number of possible outcomes
• solve elementary probability situations that involve permutations and combinations</td></tr>
</table>

Summary

The total number of learning outcomes for each of the courses have been counted and classified as **Identical Outcomes**, **Similar Outcomes with a Different Emphasis** or **Unique Outcomes**. The Problem Solving organizer is not included in the calculations as these outcomes are process oriented rather than content oriented

Applications of Mathematics 10 vs. Principles of Mathematics 10

Sub-organizer	Identical Outcomes	Similar Outcomes with a Different Emphasis	Unique Outcomes	Total Outcomes*
Number Concepts	1	2	3	9
Number Operations	3	3	6	18
Patterns	3	1	0	8
Variables & Equations	4	5	0	18
Relations & Functions	7	0	3	17
Measurement	1	1	8	12
3-D Objects/2-D Shapes	2	0	3	7
Transformations	0	0	3	3
Data Analysis	1	1	8	12
Chance & Uncertainty	0	0	8	8
Total	22 X 2 = 44	13 X 2 = 26	42	112
Percent of the Curriculum	**39.3%**	**23.2%**	**37.5%**	**100%**

* **Total Outcomes** were calculated by doubling the number of **Identical Outcomes** and **Similar Outcomes with a Different Emphasis** and adding the result to the number of **Unique Outcomes**.

Appendix C: Teacher Identification Survey With Results

Program Evaluation of

Applications of Mathematics 10

1998/99

Teacher

Identification Survey

Purpose:

The purpose of this questionnaire is to identify teachers, in each of the three groups (Applications of Math10, Principles of Math 10, and Mathematics 10A), with comparable educational and teaching experiences.

Principal Investigator: Bruce McAskill

Phone: (250) 356-7687

Supervisor: Dr. Leslee Francis-Pelton

Social & Natural Sciences

University of Victoria

Phone: (250) 721-7794

NAME:____________________________ **SCHOOL:**________________________

SECTION A: BACKGROUND INFORMATION

The purpose of this questionnaire is to identify teachers, in each of the three groups (Applications of Math10, Principles of Math 10, and Mathematics 10A), with comparable educational and teaching experiences.

To answer the following questions, fill in the appropriate circle(s).

Teacher Identification Questionnaire Results

1. **Which Grade 10 MATHEMATICS course(s) will you be teaching this school year?**

	Applications of Mathematics 10	Mathematics 10A	Principles of Mathematics 10	Total by School Org.
Full year	2	0	1	3
First semester	5	3	7	15
Second semester	2	0	0	2
First quarter	1	0	2	3
Second quarter	0	0	0	0
Third quarter	0	0	0	0
Fourth quarter	0	0	0	0
Total Classes	10	3	10	23

The following results are listed as the percentage responding in each group

2. **Are you male or female?**

	Experimental	10A	PM 10	Control
A) Male	80	100	80	85
B) Female	20	0	20	15

3. **What is your AGE group?**

		Experimental	10A	PM 10	Control
A)	Under 30	10	33	0	8
B)	30 to 39	20	0	10	7
C)	40 to 49	40	0	30	23
D)	50 to 59	30	67	50	54
E)	60 or over	0	0	10	8

4. **What is your current TEACHING ASSIGNMENT?**

		Experimental	10A	PM 10	Control
A)	Full-time	80	100	100	100
B)	Part-time	20	0	0	9
C)	Substitute	0	0	0	0
D)	Continuing	50	0	50	38
E)	Temporary	0	33	0	8
F)	Mathematics specialist	60	33	80	69
G)	Specialist in another subject area	30	33	10	23
H)	Classroom generalist	10	0	0	0
I)	Other	20	0	10	8

5. **What is your EDUCATIONAL BACKGROUND?**

		Experimental	10A	PM 10	Control
A)	No degree	0	0	0	0
B)	Bachelor of Education	50	67	40	54
C)	Bachelor of Science	40	33	50	46
D)	Bachelor of Arts	0	0	10	8
E)	Bachelors degree in another area	20	0	0	8
F)	Master of Education	30	0	30	23
G)	Master of Science	0	33	10	8
H)	Masters degree in another area	0	0	0	0
I)	Other	10	33	0	8

6. **What will be your TEACHING EXPERIENCE as of June 1999?**

6a. Total years in teaching:

		Experimental	10A	PM 10	Control
A)	My first year.	10	0	0	0
B)	2 to 5 years.	10	0	10	8
C)	6 to 10 years.	10	33	10	23
D)	More than 10 years.	70	67	80	69

6b. Years teaching MATHEMATICS:

		Experimental	10A	PM 10	Control
A)	1-2 years.	10	0	0	0
B)	3 to 5 years.	10	33	10	15
C)	6 to 10 years.	30	33	30	38
D)	11 to 15 years.	30	0	20	15
E)	More than 15 years.	20	33	40	31

7. **How many post-secondary courses in MATHEMATICS CONTENT have you successfully completed? (e.g., For U.B.C. 6 credits or 3 units = 2 courses).**

		Experimental	10A	PM 10	Control
A)	0	0	33	10	15
B)	1 or 2	20	33	10	15
C)	3 or more	80	33	80	69

8. **How many post-secondary courses in MATHEMATICS METHODS (pedagogy) have you successfully completed? (e.g., For U.B.C. 6 credits or 3 units = 2 courses).**

		Experimental	10A	PM 10	Control
A)	0	10	67	20	31
B)	1 or 2	50	33	30	31
C)	3 or more	40	0	50	38

9. What percent of your current teaching load is MATHEMATICS?

		Experimental	10A	PM 10	Control
A)	0 to 20%	0	0	0	0
B)	21 to 40%	0	0	0	8
C)	41 to 60%	30	0	10	8
D)	61 to 80%	40	33	20	23
E)	81 to 100%	30	67	70	62

10. Approximately how many hours per week do you normally spend on each of the following activities outside the formal school day?

A) preparing or grading student tests or exams

	Experimental	10A	PM 10	Control
None	0	0	0	0
Less than 1 hour	10	0	0	0
1-2 hours	40	0	30	23
3-4 hours	40	33	50	62
More than 4 hours	10	67	20	15

B) reading or grading other student work

	Experimental	10A	PM 10	Control
None	0	33	0	8
Less than 1 hour	0	0	10	15
1-2 hours	40	67	60	54
3-4 hours	60	0	30	23
More than 4 hours	0	0	0	0

C) planning lessons by yourself

	Experimental	10A	PM 10	Control
None	0	0	0	0
Less than 1 hour	10	0	10	8
1-2 hours	20	67	20	31
3-4 hours	40	33	40	38
More than 4 hours	30	0	30	23

D) meeting with students outside of classroom time (e.g., tutoring, guidance)

	Experimental	10A	PM 10	Control
None	0	0	0	0
Less than 1 hour	0	33	10	15
1-2 hours	60	67	40	46
3-4 hours	30	0	50	38
More than 4 hours	10	0	0	0

E) meeting with parents

	Experimental	10A	PM 10	Control
None	10	33	0	8
Less than 1 hour	80	67	90	77
1-2 hours	10	0	10	15
3-4 hours	0	0	0	0
More than 4 hours	0	0	0	0

F) professional reading and development activity

	Experimental	10A	PM 10	Control
None	10	33	20	23
Less than 1 hour	70	33	50	46
1-2 hours	10	33	20	15
3-4 hours	10	0	10	15
More than 4 hours	0	0	0	0

G) keeping students' records up to date

	Experimental	10A	PM 10	Control
None	0	0	0	0
Less than 1 hour	10	0	30	23
1-2 hours	90	67	70	69
3-4 hours	0	33	0	8
More than 4 hours	0	0	0	0

H) administrative tasks including staff meetings

	Experimental	10A	PM 10	Control
None	0	33	0	8
Less than 1 hour	0	0	20	15
1-2 hours	50	67	30	38
3-4 hours	30	0	40	31
More than 4 hours	20	0	10	8

Appendix D: Teacher Pre-Test and Post-Test Surveys (With Results)

September 7, 1998

Re: Program Evaluation of Applications of Mathematics 10

Dear <Teacher Name>:

I wish to take this opportunity to thank you for your continued support of this study. It is only with your assistance will I be able to complete the program evaluation of Applications of Math 10.

During the teaching of your <course> class I ask that you administer two student achievement and attitude assessments. The first assessment needs to be completed as close as possible to the beginning of the course (before 10 hours of instruction have been completed). The second assessment is to be completed at the end of the course. I ask that, while your students are completing each of the achievement and attitude assessments, you fill out a questionnaire as well. I also ask that you maintain a logbook of your teaching and assessment practices during the term of the class.

Please carefully follow the instructions listed below to ensure the validity and reliability of this study.

Student Achievement & Attitude Assessment Instructions - This assessment is to be written before the first ten hours of instruction have been completed. *Prior to writing the assessment the students must complete and return to you the enclosed Student Consent Form.*

1. Distribute one Student Achievement & Attitude Assessment - Form A and Bubble Response Form to each student in your <course> class.
2. **IMPORTANT - ENSURE THAT EACH STUDENT WRITES THEIR NAME IN FULL ON THE GENERAL PURPOSE ANSWER FORM AND FILLS IN THE CORRESPONDING BUBBLES.**
3. Please ensure that your students use a pencil to fill each bubble completely and darkly.
4. Students are to be permitted a maximum of 60 minutes to complete the assessment.
5. Courier the completed General Purpose Answer Forms, Student Achievement & Attitude Assessments - Form A, and completed Student Consent Forms (using the enclosed courier slip) to me within one week of completion.

Teacher Questionnaire Instructions

1. Please complete the eleven page Teacher Questionnaire at the same time your students write the Student Achievement & Attitude Assessment - Form A.
2. Courier the completed Teacher Questionnaire with the completed student materials (using the enclosed courier slip).

Teacher Logbook Instructions

1. After each <course> class identify each of the teaching and/or assessment methods that you used. Indicate approximately what percent of class time was spent on each. Refer to the Glossary of Terms and the Example for further clarification if needed. If necessary, there is space provided for you to list additional teaching or assessment methods.
2. At the end of each month courier the completed logbook to me. I will provide you with a fresh logbook and courier slips prior to the start of each month.

Before the completion of this class, I will send you the second Student Achievement & Attitude Assessment and a Follow-up Teacher Questionnaire. Please feel free to contact me if you have any questions concerning this study. Once again I wish to thank you for your assistance.

Sincerely,

Bruce McAskill

Program Evaluation of

Applications of Mathematics 10

1998/99

Teacher Pre-test and Post-test Survey

Directions:

Please answer the questions in this booklet by referring to the Grade 10 Mathematics class that is writing the student achievement and attitude towards mathematics assessment.

Principal Investigator: Bruce McAskill

Phone: (250) 356-7687

Supervisor: Dr. Leslee Francis-Pelton

Social & Natural Sciences

University of Victoria

Phone: (250) 721-7794

NAME:__________________________ SCHOOL:__________________

SECTION B: ON THE TEACHING OF MATHEMATICS

I. Mathematics Implementation and Classroom Practices

Your responses to the following items will enable us to generate a composite picture of the mathematics curriculum as it has been implemented, describe the kinds of practices associated with the teaching of the curriculum.

To answer the following questions, fill in the appropriate circle(s).

1. What is the length of each class period in MATHEMATICS?

		Experimental		Control	
		Pre-test	Post-test	Pre-test	Post-test
A)	30 minutes or less	0	0	0	0
B)	31-45 minutes	0	0	0	0
C)	46-60 minutes	0	0	8	8
D)	61-75 minutes	30	30	54	54
E)	More than 75 minutes	70	70	38	38

2. How many periods of MATHEMATICS instruction does your class receive each week?

		Experimental		Control	
		Pre-test	Post-test	Pre-test	Post-test
A)	3 or fewer	30	30	8	8
B)	4	10	10	8	8
C)	5	60	60	85	85
E)	6 or more	0	0	0	0

3. In the past four years, has your approach to teaching mathematics changed?

		Experimental		Control	
		Pre-test	Post-test	Pre-test	Post-test
A)	Yes	100	78	77	67
B)	No	0	22	23	33

4. Select from the following list of teaching strategies those which may reflect your approach to teaching MATHEMATICS. Percent Responses

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) I am likely to use cooperative learning groups.	90	89	54	50
B) I am likely to encourage individuals to progress at their own rate.	56	33	46	33
C) I am likely to have students use concrete materials.	70	78	38	50
D) I am likely to encourage students to use calculators.	100	100	100	100
E) I am likely to have students use computers.	60	56	0	17
F) I am likely to focus on problem solving activities.	60	89	62	58
G) I am likely to use and activities approach to teaching mathematics.	90	89	46	67

5. How much do you like...

(0-4 scale)

0 = Don't use one, 1 = Dislike a lot, 2 = Dislike, 3 = Like, 4 = Like a lot

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) using a computer in math class?	2.1	2.5	0.8	1.2
B) using a computer at home?	3.1	3.6	3.1	3.0
C) using a calculator in math class?	2.9	3.4	3.2	3.0
D) using a calculator at home?	3.2	3.1	3.2	2.8

6. How often do you do each of the following? *(0-3 scale)*

0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) I show students what to do on the black-board or the overhead projector	1.5	1.8	2.6	2.2
B) Students use objects like blocks and counters.	0.7	0.7	0.4	0.5
C) Students work individually on problems from textbooks or other exercises as assigned	1.5	1.3	1.9	2.1

7. How many of your students have access to calculators and/or computers during most mathematics lessons? (*0-4 Scale*)

0 = None, 1 = About 1/4, 2 = About 1/2, 3 = About 3/4, 4 = Almost all

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Scientific Calculators	3.7	3.9	3.6	3.6
B) Graphing Calculators	2.3	2.2	1.7	1.7
C) Computers	0.7	1.6	0.0	0.4

8. How often do students in your class use calculators for the following activities? (*0-3 Scale*)

0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Checking answers	2.0	1.9	2.1	2.4
B) Tests and exams	2.4	2.1	2.3	2.6
C) Routine computation	2.2	2.2	2.3	2.5
D) Solving complex problems	2.1	1.9	2.1	2.2
E) Exploring number concepts	1.8	1.6	1.9	2.1

9. In your MATHEMATICS lessons, how often do you usually ask students to do the following? *(0-3 Scale)*

0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Explain the reasoning behind the idea	1.8	1.7	1.8	2.0
B) Represent and analyze relationships using tables, charts, or graphs	1.0	1.1	1.2	0.9
C) Work on problems for which there is no immediately obvious method of solution	0.9	0.9	0.8	0.7
D) Use computers to solve exercises or problems	0.3	0.6	0.0	0.2
E) Write equations to represent relationships	1.1	1.0	1.2	1.5
F) Practice computational skills	1.3	1.6	1.3	1.3

10. In your MATHEMATICS lessons, how frequently do you do the following when a student gives an incorrect response during a class discussion? *(0-3 Scale)*

0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Correct the student's error in front of the class	0.8	0.7	0.5	0.5
B) Ask the student another question to help arrive at the correct response	2.2	1.8	1.8	1.8
C) Call on another student who is likely to give the correct response	0.6	0.9	0.6	1.0
D) Call on other students to get their responses and then discuss what is correct	2.0	1.7	1.6	1.6

11. In MATHEMATICS lessons, how often do students...
(0-3 Scale)
0 = Never or almost never, 1 = Some lessons, 2 = Most lessons, 3 = Every lesson

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) work individually without assistance from the teacher?	1.0	1.1	1.1	0.8
B) work individually with assistance from the teacher?	1.3	1.5	1.8	1.9
C) work together as a class with the teacher teaching the whole class?	1.7	1.2	1.7	1.4
D) work together as a class with students responding to one another?	1.3	1.0	1.2	1.1
E) work in pairs or small groups without assistance from the teacher?	1.5	1.0	1.4	0.7
F) work in pairs or small groups with assistance from the teacher?	1.5	1.2	1.4	1.2

12. How often do you usually assign MATHEMATICS homework?

		Experimental		Control	
		Pre-test	Post-test	Pre-test	Post-test
A)	Never or almost never	0	0	0	0
B)	Some lessons	50	56	8	17
C)	Most lessons	40	44	46	42
D)	Every lesson	10	0	46	42

13. If you assign MATHEMATICS homework, how many minutes of mathematics homework do you usually assign to your students?

		Experimental		Control	
		Pre-test	Post-test	Pre-test	Post-test
A)	I do not assign homework	0	0	0	0
B)	Less than 15 minutes	10	0	0	0
C)	15 to 30 minutes	70	89	62	58
D)	31 to 60 minutes	20	11	38	42
E)	61 to 90 minutes	0	0	0	0
F)	More than 90 minutes	0	0	0	0

14. If you assign MATHEMATICS homework, how often do you assign each of the following kinds of tasks?(*0-3 Scale*)

0 = Never, 1 = Rarely, 2 = Sometimes, 3 = Always

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Worksheets or workbook	2.0	2.2	1.8	1.8
B) Problem/question sets in textbook	2.0	2.0	2.5	2.4
C) Reading in a textbook or supplementary materials	1.2	1.3	1.3	1.2
D) Writing definitions or other short writing assignments	1.0	1.2	0.8	1.1
E) Small investigation(s) or gathering data	1.8	1.8	1.2	1.4
F) Working individually on long term projects or experiments	1.6	1.9	0.6	1.2
G) Working as a small group on long term projects or experiments	1.5	1.9	0.5	0.8
H) Finding one or more uses of the content covered	1.6	1.5	0.8	0.9
I) Preparing oral reports either individually or as a small group	0.6	0.8	0.4	0.7
J) Keeping a journal	0.9	0.6	0.7	0.7

15. If students are assigned <u>written</u> MATHEMATICS homework, how often do you do each of the following? (*0-3 Scale*)

0 = Never, 1 = Rarely, 2 = Sometimes, 3 = Always

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Record whether or not the homework was completed	2.6	3.1	2.5	2.6
B) Collect, correct and keep assignments	2.1	2.1	1.5	1.1
C) Give feedback on homework and then return to students	2.3	2.6	1.8	1.8
D) Give feedback on homework to whole class	2.3	2.7	2.2	2.1
E) Have students correct their own assignments in class	1.9	2.2	1.9	1.9
F) Working individually on long term projects or experiments	1.0	1.4	0.4	0.5
G) Use it as a basis for class discussion.	2.0	2.3	1.2	1.6
H) Use it to contribute towards grades or marks	2.3	3.1	2.2	2.7

For each of the next 12 items, three answers are needed.

A) Tell how important you think the topic is for the this class
B) Tell how easy it is to teach the topic to this class
C) Tell how much you like teaching the topic in this class
(*0-4 Scale*)

A	B	C
0 not at all important	0 very difficult	0 dislike a lot
1 not important	1 difficult	1 dislike
2 undecided	2 undecided	2 undecided
3 important	3 easy	3 like
4 very important	4 very easy	4 like a lot

1. Finding area, perimeter, and volume

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	2.3	3.2	2.2	2.9
B	2.7	3.0	2.9	2.9
C	2.9	2.9	2.9	2.8

2. Problem solving

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	3.5	3.7	3.6	3.8
B	0.9	0.9	1.4	1.1
C	2.7	3.0	2.9	2.9

3. Solving equations and simplifying expressions

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	2.8	2.9	3.2	3.3
B	2.3	2.1	2.3	2.0
C	2.8	2.8	2.8	3.1

4. Working with exponents

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	2.6	2.6	2.7	2.8
B	2.6	2.6	1.9	2.3
C	2.8	2.6	2.7	2.5

5. Estimating answers

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	3.7	3.3	3.2	3.3
B	1.3	1.8	1.5	1.6
C	2.6	2.5	2.5	2.6

6. Geometry

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	2.9	2.8	3.2	2.9
B	2.3	2.3	2.2	2.2
C	2.8	2.9	2.8	2.9

7. Data analysis

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	3.3	2.9	2.9	2.9
B	1.9	2.6	1.9	2.6
C	2.7	2.6	2.4	2.7

8. Trigonometry

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	2.8	2.7	3.0	2.9
B	2.2	2.9	2.0	2.6
C	3.1	3.2	3.2	3.2

9. Working with decimals, fractions, and percent

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	3.4	3.3	3.4	3.1
B	2.0	2.0	2.1	2.9
C	2.8	2.8	2.4	2.9

10. Estimation

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	3.2	3.2	2.9	3.2
B	1.7	2.1	1.8	2.1
C	2.8	2.6	2.7	2.7

11. Memorizing basic facts

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	2.4	2.3	2.8	2.4
B	1.2	1.9	1.5	1.6
C	2.0	1.9	2.1	1.8

12. Learning to use computers for use in mathematics

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A	3.0	3.0	2.5	3.1
B	1.9	2.1	1.5	2.0
C	2.7	2.6	2.2	2.6

III. Student Evaluation

1. In the past four years has your approach to student evaluation in MATHEMATICS changed?

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Yes	80	67	69	33
B) No	20	33	31	67

2. Select from the following list of evaluation strategies those which may reflect your approach to student evaluation in MATHEMATICS. Percent Responses

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) I am likely to have students involved in self assessment.	50	33	23	17
B) I am likely to assess students through informal observations during class time.	80	89	54	42
C) I am likely to give frequent quizzes and tests.	50	56	92	83
D) I am less likely to give frequent quizzes and tests.	50	44	8	17
E) I am likely to evaluate problem-solving strategies as well as answers.	80	100	85	83

3. How often do you use the assessment information you gather from students to...
(0-3 Scale)

0 = None, 1 = Little, 2 = Quite a lot, 3 = A great deal

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) provide students' grade or marks?	2.6	2.6	2.5	2.5
B) provide feedback to students?	2.3	2.3	2.4	2.4
C) diagnose students' learning problems?	1.6	1.6	1.8	1.7
D) report to parents?	2.3	2.0	2.0	2.1
E) assign students to different programs or tracks?	1.4	1.3	1.5	1.8
F) plan for future lessons?	2.1	1.9	1.7	1.8

4. In assessing the work of the students in your MATHEMATICS class, how much weight do you give each of the following types of assessment?
(0-3 Scale)

0 = None, 1 = Little, 2 = Quite a lot, 3 = A great deal

	Experimental		Control	
	Pre-test	Post-test	Pre-test	Post-test
A) Standardized tests produced outside the school	0.3	0.7	0.4	0.3
B) Teacher-made short answer or essay tests that require students to describe or explain their reasoning.	1.5	1.4	1.5	1.4
C) Teacher-made multiple choice, true-false and matching tests	1.2	0.9	1.0	1.3
D) How well students do on homework assignments	1.6	1.8	1.1	1.3
E) How well students do on projects or practical/laboratory exercises	1.7	2.0	0.6	0.8
F) Observations of students	1.4	1.2	0.9	0.8
G) Responses of students in class	1.0	0.9	0.8	0.8

Appendix E: Teacher Logbook Sample

Program Evaluation of

Applications of Mathematics 10

1998/99

Teacher Logbook for

September

<Teacher>, <Course>, <School>

Directions:

1. **After each class note the approximate percentage of class time that was spent on each group of teaching and/or assessment strategies (shaded area of logbook).**

2. **Check each specific strategy that was used during each class. Refer to the Glossary of Terms and the Example for clarification.**

3. **Space is provided for you to list additional teaching or assessment strategies.**

4. **At the end of each month courier the completed logbook to me. I will provide you with a fresh logbook prior to the start of each month (along with courier slips).**

Principal Investigator: Bruce McAskill

Phone: (250) 356-7687

Supervisor: Dr. Leslee Francis-Pelton

Social & Natural Sciences

University of Victoria

Phone: (250) 721-7794

Glossary of Terms

In order to ensure that we have a common understanding of the teaching and/or assessment strategies listed in this logbook, I have provided a brief description of each based upon the program *Investigating teaching strategies in mathematics classrooms* by Herrington, Sparrow, Herrington, and Oliver (1997). You may use a different term for a particular strategy, but I ask that you consider these descriptions when completing your logbook.

Teaching Strategies	Description
Drill and Practice	Students are shown a method often broken into smaller steps. They then practice the method by completing a series of examples of the same kind of calculation .
Homework	Assigned to students to complete unfinished work; consolidate and practice skills; develop independent study habits.
Textbooks	Students working from page to page for all or part of the class time.
Worksheets	May be photocopied from commercially produced blackline masters, or may be teacher generated, either handwritten or computer generated.
Group Work	Includes grouping: by student ability, for convenience, to teach specific skills, by self-selection, by mixed ability, etc.
Role Play	Conceptual learning through drama and movement (e.g., shopkeeper & customer).
Explaining	An extended use of talk (in contrast to short answers to teacher questions). Emphasis is on the role of students using talk to develop their ideas and understandings.
Peer Tutoring	The general practice of pairing one student with another of greater ability and knowledge in the subject (within or between classes)
Manipulatives	Classified into structured (e.g., Dienes' Multibase Arithmetic Blocks, Cuisenaire rods) and unstructured (e.g., multilink cubes, shells, matchsticks) materials.
Game Playing	Games used to practice facts and skills, development of concepts, strategy building and problem solving.
Outdoors	Requires the students being involved in developing mathematical ideas in kinesthetic ways. Examples include mathematics trails and human graphs.
Mathematics Centres	Also called task centers. Generally based around puzzles, problems and investigations. Necessary materials are prepared by the teacher and placed in a container for student use.
Guided Discovery	The teacher plans and sets up a situation and task that has embedded within it the desired learning outcomes. Usually followed by class discussion. Teacher acts as guide and questioner rather than provider of knowledge

Projects	Involves students investigating, solving problems, researching and applying mathematics. Projects are usually completed by a group and involve group work skills, identification and collection of resources, and presentation skills.
Open-ended Tasks	Tasks which allow for a variety of solution approaches and can be answered at several levels. The tasks should: challenge all students, can be worked on in groups or by individuals, can be easily extended, and require minimal teacher direction.
Exposition	Traditional teaching approach where the main purpose is to transmit information from teacher to student as quickly as possible. May incorporate questions and demonstrations that build upon prior knowledge of students.
Beginning a Lesson	Intended to set the scene for learning, link ideas with previous and future learning and motivate the students to learn.
Team Teaching	Two or more teachers work with a group of students.
Involving Others	Includes parents, speakers, helpers (including volunteers), etc.
Puzzles	Puzzles such as number puzzles and the Tower of Hanoi require students to think through the situation and employ pattern searching to arrive at generalizations.
Modeling	A term used to describe the processes involved in using mathematics to solve real-world problems.
Applications	Tasks (including word problems) in which students will apply previously learnt procedures. Students not only demonstrate their knowledge of procedures, but also their ability to choose the most appropriate ones.
Themes	Using a theme or context to investigate related mathematical ideas. Connections can be made between different mathematical content areas as well as across other disciplines.
Problem Solving	Involves a situation that does not have an immediate solution method.
Problem Posing	Initially may come directly from the instigation of the teacher but eventually the intention is that students will pose their own problems and therefore develop an inquiry approach to mathematics.
Calculators	As a teaching and learning aid (e.g., calculating large numbers, developing the concept of place value). This does not include trivial activities such as checking calculations done by another method.
Graphing Calculators	Same as Calculators above but includes operations that involve algebra, statistics, programming and graphing.
Computers	Used as electronic blackboards for displaying graphs or introducing a problem or investigation. Includes student use of spreadsheets, mathematical adventure games and simulations.

Assessment Strategies	Description
Checklists	Can be used to record information as students work individually or in groups. They can be used to cover content, processes and attitudes.
Anecdotal	An informal type of assessment. Records (of achievements, beliefs and attitudes) are made as the teacher observes students engaged in some mathematics
Higher Order Questioning	Can be either written or oral. Attempts to find out whether a student has made important connections that underpin certain facts or procedures. Question stems include: What if...? How Does...? Why does...? How could...?
Factual Questioning	Usually requires students to respond automatically with the answer to a question (e.g., basic number facts).
Open-ended Questioning	Questions that allow for more than one answer. They involve mathematical thinking that goes beyond recalling facts or demonstrating skills.
Structured Interviews	Contain the same set of questions given in the same sequence. Often used to diagnose difficulties that students may be encountering.
Open Interviews	Does not follow a set sequence of questions. Interview relies on discussion between the teacher and the student with questions arising incidentally.
Parent Interviews	Information concerning a student's motivation or areas of difficulty they are experiencing may be brought to the teacher's attention by the parent as parents see the student doing mathematics in a different context than the teacher.
Diagnosis	Can be done through testing or interviews. Once problem areas are identified, the teacher can take steps to initiate the appropriate teaching strategy to address the misconception.
Performance-based	Involves making a formal assessment while the student is involved in performing a mathematical activity. Activities often include the use of materials.
Pencil and Paper Test	Traditional form of assessment which often assess skills, facts but can also assess conceptual understanding, applications and problem solving abilities.
Multiple Choice Test	Included in many formal tests (particularly provincial or national tests). Easy to score and provide statistical data. Questions tend to focus on the product (answer) rather than the process.
Problem Solving	A mathematical problem requires students to reach a solution that does not involve the simple application of a standard procedure or algorithm. The choice of strategies provide an indication of how good the students are at solving problems.

Attitude	Students' attitudes may be considered as feeling or beliefs about mathematics. Information about student attitudes can be obtained through teacher observation and attitudinal tests.
Oral Report	Can involve students individually or in groups communicating the results of a mathematical activity.
Written Report	Can be a short summary describing what the student learnt in a particular lesson or it can be a longer report describing an extended mathematical idea
Portfolio	A collection of student work generally compiled over a long period of time. Pieces of work may be chosen by the teacher, student, or both.
Investigation	An investigation results from the presentation of an open situation that can be extended and explored with the use of mathematics. Students will use mathematical processes such as drawing diagrams, tabulating, making conjectures, looking for and finding patterns, generalizing results, explaining and proving.
Modeling	Involves students using mathematics to solve problems that occur in the real world. The assessment task is embedded in a context that students would normally experience in real life.
Journals	Journals can provide insight into student attitudes towards mathematics as well as student achievements.
Reflective Prompts	Involves using a structured set of questions that prompt students to reflect on their own learning.
Self-questioning	A student self assessment which can be prompted by the teacher with such questions as: How shall we go about solving this problem? What factors do I need to consider before I start? Is this making sense? How is this method helping? Can I find a better way to do this? Does the answer make sense?
Peer Assessment	Students develop their own assessment tasks for their peers to complete.

Example

Teaching Strategies	Mon. 14th	Tues. 15th	Wed. 16th	Thurs. 17th	Fri. 18th
Consolidation & Practice	**10%**	**15%**			
Drill and Practice					
Homework	√				
Textbooks					
Worksheets					
Discussion		**10%**		**20%**	
Group Work		√			
Role Play				√	
Explaining				√	
Peer Tutoring					
Practical Work		**25%**		**15%**	
Manipulatives				√	
Game Playing				√	
Outdoors		√			
Mathematics Centres					
Investigating	**30%**				
Guided Discovery	√				
Projects					
Open-ended Tasks					
Teacher Centered	**20%**			**10%**	
Exposition	√			√	
Beginning a Lesson	√				
Team Teaching					
Involving Others					
Problem Solving		**10%**			
Puzzles					
Modeling					
Applications					
Themes					
Problem Solving		√			
Problem Posing					
Using Technology	**10%**			**10%**	
Calculators	√				
Graphing Calculators					
Computers				√	
Other					

Example

Assessment Strategies	Mon. 14th	Tues. 15th	Wed. 16th	Thurs. 17th	Fri. 18th
Observing	**5%**				
Checklists					
Anecdotal	√				
Questioning	**10%**				
Higher order	√				
Factual					
Open-ended					
Interviewing				**30%**	
Structured				√	
Open					
Parent					
Testing		**55%**			
Diagnosis					
Performance-based					
Pencil and Paper		√			
Multiple Choice		√			
Problem Solving		√			
Attitude		√			
Reporting				**15%**	
Oral				√	
Written					
Portfolio				√	
Investigation					
Modeling					
Self - Assessment	**15%**				
Journals	√				
Reflective Prompts					
Self-questioning					
Peer Assessment					
Other					

Appendix F: Student/Parent Permission Letters

September 17, 1998

Dear Student:

I would like your help in studying how well you are learning mathematics in this type of course (Principles of Mathematics 10, Applications of Mathematics 10 or Mathematics 10A). I also want to know what you think about the mathematics you are taking in this course.

I am asking you if I can:

1. Give you two tests on mathematics which include questions to help me determine how well you understand the mathematics you are taking in class and also what you think about the mathematics you are taking in class.

2. Look at your school grades.

I will not tell anyone your name or anything about you. Your scores on the two mathematics tests will not affect your grade in this course. You do not have to be in this study. If you want to be in the study, and then change your mind, you can quit any time.

If you ever have any questions you can ask your teacher, myself or Dr Francis Pelton.

Thank you for your help

Sincerely,

Dear Parent/Guardian:

Re: Program Evaluation of Applications of Mathematics 10

I am asking for your cooperation in helping me determine whether the new Applications of Mathematics curriculum is being successfully implemented in schools. I want to determine how well students taking Applications of Mathematics 10, Principles of Mathematics 10 and Mathematics 10A achieve on standardized mathematics tests. I also want to determine if there are difference in students' attitudes towards mathematics in the different courses. In order to study these questions, I will be working with teachers and students in your child's school over the next year.

I am asking your permission to:

1. Administer to your child two mathematics achievement attitude assessments (the first at the beginning of the course and the second near the end of the course.)

2. Access school records to determine your child's mathematics grade prior to taking this mathematics course.

All of the information obtained will be kept confidential and will be stored in a locked office at the University of Victoria. Any reports or publications in which the information is used will not reveal the identity of your child in any way.

Participation in this project is purely voluntary. If, for any reason you do not wish your child to participate in this project, or if he or she does not wish to continue participating, withdrawal will not effect his or her school status or grades in any way. The results of the mathematics achievement and attitude tests will not effect your child's grade in any way. If you have questions, please contact me or my supervisor at one of the phone numbers below. If you agree to your child's participation, please sign the attached permission form and return it to his/her teacher. Thank you.

Thank you for your help

Sincerely,

Dear Parents or Guardian and Student:

Please complete the attached forms and return them within one week. Thank you for your help with this project. If you have any questions or concerns please contact myself (250-356-7687) or Dr. Francis Pelton (250-721-7794).

Sincerely,

Bruce M^{c}Askill

For Parent or Guardians:

I do/do not (circle one) give permission for __________________________ to participate in "The Program Evaluation of Applications of Mathematics 10" as described in the attached letter.

______________________________________ __________________

Parent(s) or Guardian(s) Signature(s) **Date**

For the Student:

I do/do not (circle one) agree to join in "The Program Evaluation of Applications of Mathematics 10" as described in the attached letter.

______________________________________ __________________

Student's Signature **Date**

Appendix G:
Student Achievement & Attitude Assessments (Forms A [Pre-test] & B [Post-test])

FORM **A**

Program Evaluation of Applications of Mathematics 10

1998/99

Student Achievement & Attitude Assessment

Directions:

This booklet contains questions about mathematics. Follow the instructions for each section. For your answers, use the bubbles on the BUBBLE FORM provided. You MUST use a pencil to fill the bubbles, because the machine that will read the booklet can not see ink marks! Fill each bubble completely and darkly. If you make a mistake, erase it completely and then fill in the bubble for your new answer

Principal Investigator: Bruce McAskill

Phone: (250) 356-7687

Supervisor: Dr. Leslee Francis-Pelton

Social & Natural Sciences

University of Victoria

Phone: (250) 721-7794

Form A

SECTION A: BACKGROUND INFORMATION & ATTITUDE TOWARDS MATHEMATICS

Please use the bubbles numbered 1 to 13 to answer **SECTION A** questions 1 to 13.

Completely and darkly fill in the bubble which represents your answer. If you make a mistake, erase it completely and then fill in the bubble for your new answer.

---

1. **How old are you?**

A) 14 or less
B) 15
C) 16
D) 17
E) 18 or older

---

2. **Are you male or female?**

A) Male
B) Female

---

3. **Do you have a calculator in your home?**

A) Yes
B) No

---

4. **Do you have a computer in your home?**

A) Yes
B) No

5. Which MATHEMATICS course are you presently taking?

A) Applications of Mathematics 10
B) Mathematics 10A
C) Principles of Mathematics 10
D) Other

---

6. What MATHEMATICS course did you take previous to this one?

A) Applications of Mathematics 9
B) Mathematics 9A
C) Principles of Mathematics 9 or Mathematics 9
D) A Grade 10 course

---

7. What grade did you receive in your last MATHEMATICS course?

A) A
B) B
C) C or C+
D) P
E) F

---

8. What Grade 11 MATHEMATICS course do you plan to take?

A) Applications of Mathematics 11
B) Introductory Mathematics 11
C) Mathematics 11A
D) Principles of Mathematics 11
E) Accounting 11

9. **What Grade 12 MATHEMATICS course do you plan to take?**

A) None
B) Applications of Mathematics 12
C) Principles of Mathematics 12
D) An enriched mathematics course (e.g., Advanced Placement, International Baccalaureate, Calculus, etc.)

---

10. **Do you plan on taking more MATHEMATICS courses once you have finished secondary school?**

A) No
B) Yes, maybe one or two
C) Yes, I plan on making mathematics a major part of my life
D) I don't know

---

11. **How many years of future education do you plan to take after you leave secondary school?**

A) None
B) 3 years or fewer
C) 4 or 5 years
D) 6 years or more
E) I don't know

---

12. **Do you agree with the statement,** "*I usually do well in mathematics*"?

A) Strongly agree
B) Agree
C) Disagree
D) Strongly disagree

13. How much do you like mathematics?

A) Like a lot
B) Like
C) Dislike
D) Dislike a lot

SECTION B: MATHEMATICS ACHIEVEMENT

Please use the bubbles numbered 14 to 53 to answer **SECTION B** questions 14 to 53.

Completely and darkly fill in the bubble which represents your answer. If you make a mistake, erase it completely and then fill in the bubble for your new answer. Please try as hard as you can and make the best selections you can.

---

14. Rounded to the nearest kilometre, 1958.0989 km =

A) 1958 km
B) 1958.099 km
C) 1959 km
D) 2000 km
E) I don't know.

---

15. If there are 300 calories in 900 g of a certain food, how many calories are there in a 300 g portion of that same food?

A) 27
B) 33
C) 100
D) 270
E) I don't know.

16. The sales tax is 5%. How much would the sales tax be on a new car that costs $6750.00?

A) $675.00
B) $337.50
C) $67.50
D) $33.75
E) I don't know.

---

17. What is the value of $2^4 \times 3^2$?

A) 144
B) 48
C) 96
D) 72
E) I don't know.

---

18. Multiply: $3\frac{1}{2} \times 2\frac{1}{7}$

A) $\frac{6}{14}$
B) $5\frac{9}{14}$
C) $6\frac{1}{14}$
D) $7\frac{1}{2}$
E) I don't know.

19. Write in simplest radical form

$3\sqrt{48}$

A) $12\sqrt{3}$

B) $7\sqrt{3}$

C) $6\sqrt{12}$

D) $5\sqrt{12}$

E) I don't know.

---

20. Find the product in simplest radical form.

$(3\sqrt{6})(4\sqrt{2})$

A) $12\sqrt{12}$

B) $24\sqrt{3}$

C) $7\sqrt{12}$

D) $14\sqrt{3}$

E) I don't know.

---

21. Which expression represents a number that is 9 more than two-thirds of a given number?

A) $\frac{2}{3}z + 9$

B) $\frac{3}{2}z + 9$

C) $\frac{2}{3}(z + 9)$

D) $\frac{3}{2}(z + 9)$

E) I don't know.

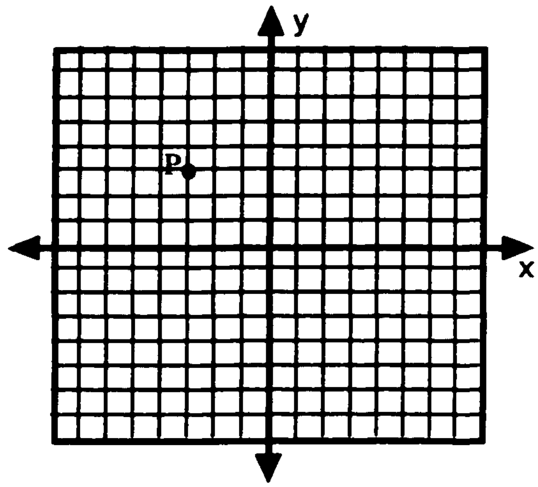
22. Solve: 2 - (2 - y) = 2 - y

A) y = 0
B) y = - 0.5
C) y = 0.5
D) y = 1
E) I don't know.

---

23. The coordinates of point P are:

A) (- 3, - 3)
B) (- 3, 3)
C) (3, - 3)
D) (3, 3)
E) I don't know.



---

24. Solve for x: 3x + 7 = 5x + 4

A) $x = -\frac{11}{2}$
B) $x = -\frac{3}{2}$
C) $x = \frac{3}{2}$
D) $x = \frac{11}{2}$
E) I don't know.

25. Which equation represents the following problem?

How long is a rectangular lawn of width **w** if the length is 25 m longer than the width and the perimeter measures 110 m?

A) $2w + 25 = 100$
B) $2(w + 25) = 110$
C) $w(w + 25) = 110$
D) $2[w + (w + 25)] = 110$
E) I don't know.

---

26. Factor completely: $a^2 - 36b^2$

A) $(a - 12b)(a + 3b)$
B) $(a - 6b)(a - 6b)$
C) $(a - 6b)(a + 6b)$
D) $(a - 4b)(a + 9b)$
E) I don't know.

---

27. Expand: $(3x + 4y^2)^2$

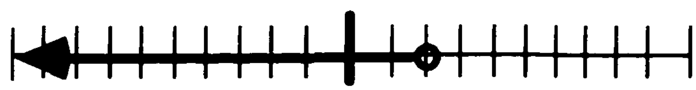
A) $6x^2 + 8y^4$
B) $9x^2 + 16y^4$
C) $9x^2 + 12xy^2 + 16y^4$
D) $9x^2 + 24xy^2 + 16y^4$
E) I don't know.

28. Solve for x: $x + 3 = \frac{1}{2}x - 1$

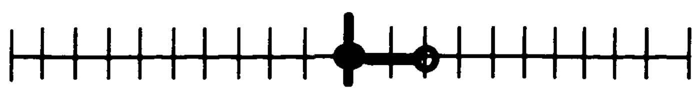
A) x= - 8
B) x= $-\frac{8}{3}$
C) x= 4
D) x= 8
E) I don't know.

---

29. The graph of $x \leq 0$ or $x > 2$ is:

A) 

B) 

C) 

D) 

E) I don't know.

---

30. Which one of the following statements about the equation $2(x - 7) = 2x + 5$ is true?

A) The equation has no solution.
B) The equation has infinitely many solutions.
C) $x = 0$
D) $x = 19$
E) I don't know.

31. At what point does the graph of the line $2y + 7x - 17 = 0$ intersect the y axis?

A) $(\frac{17}{2},0)$
B) $(0,\frac{17}{2})$
C) $(\frac{17}{7},0)$
D) $(0,\frac{17}{7})$
E) I don't know.

---

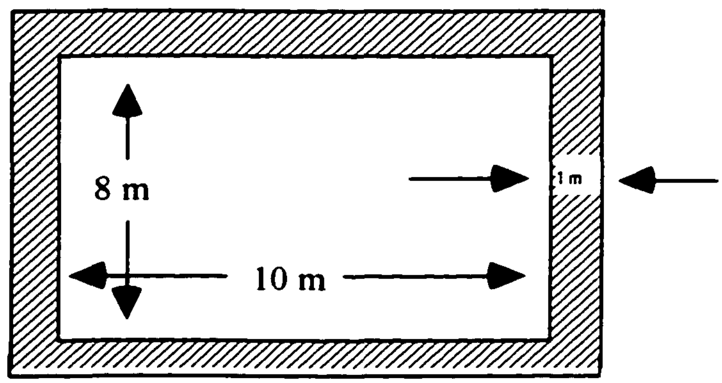
32. Find the missing factor: $42a^2bc^5 = (2bc^2)(\quad)$

A) $21a^2c^3$
B) $21a^2bc^3$
C) $40a^2c^3$
D) $40a^2bc^3$
E) I don't know.

---

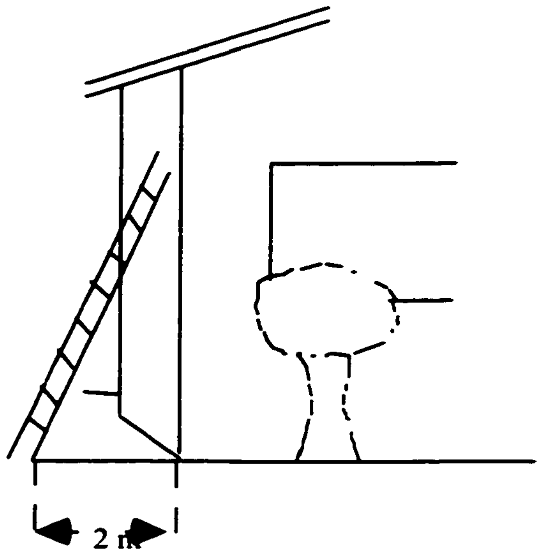
33. A rectangular pool is to be surrounded by a rectangular cement walk 1 m wide. If cement costs \$2.50 per square metre and the dimensions of the pool are 10 m by 8 m, what would the walk cost?

A) \$40
B) \$80
C) \$100
D) \$120
E) I don't know.



34. Joe, the window cleaner, uses a ladder that is 5 m long. When he places the foot of the ladder 2 m from the wall of the house, how high up the wall does the ladder reach?

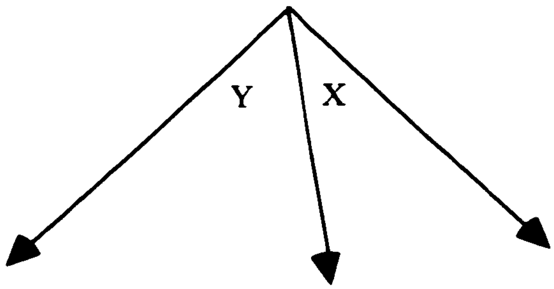
A) $\sqrt{10}$ m
B) $\sqrt{29}$ m
C) $\sqrt{21}$ m
D) 7 m
E) I don't know.



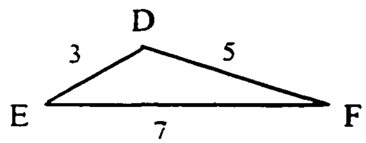
---

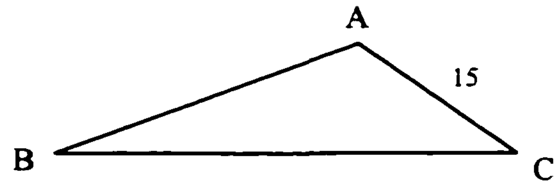
35. Angles X and Y in the figure below are complementary. If the measure of angle X is 24° less than the measure of angle Y, then angle X is:

A) 48°
B) 23°
C) 57°
D) 33°
E) I don't know.



36. Triangle ABC is similar to triangle DFE. Find the length of segment BC.





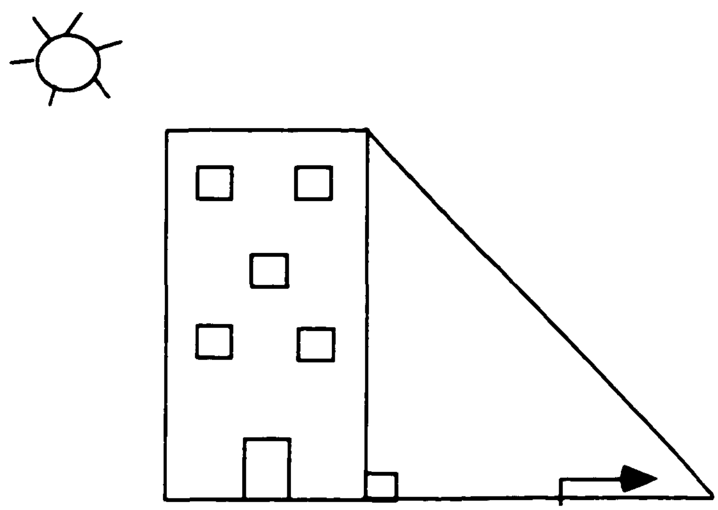
A) 21
B) 15
C) 35
D) $\frac{47}{5}$
E) I don't know.

---

37. A building cast a shadow 100 m long when the angle of elevation of the sun was 42°. What is the approximate height of the building?

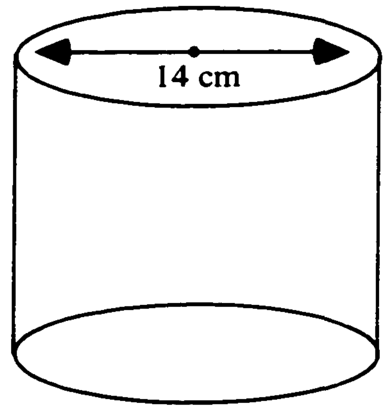
$\sin 42° = 0.6691$
$\cos 42° = 0.7432$
$\tan 42° = 0.9004$

A) 67 m
B) 74 m
C) 90 m
D) 112 m
E) I don't know.



38. What is the approximate total surface area of the solid cylinder below? Use the formula S.A. = $2\pi r^2 + 2\pi rh$.

A) 880 cm^2
B) 1187 cm^2
C) 308 cm^2
D) 934 cm^2
E) I don't know.



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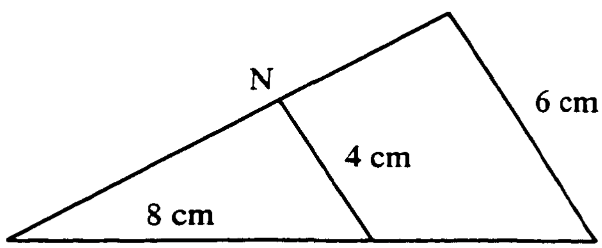
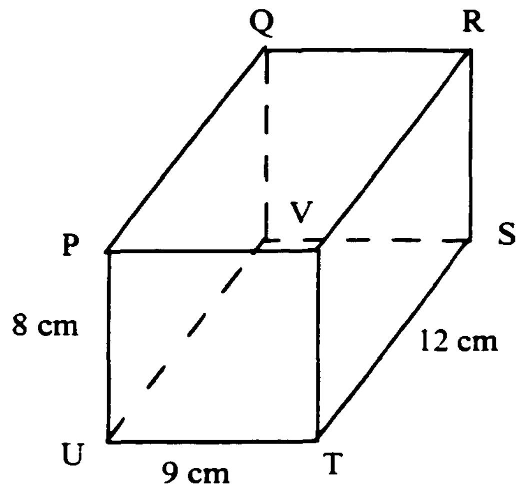
39. In the figure below, line NQ is parallel to line OP, NQ = 4 cm, OP = 6 cm and MQ = 8 cm. Find the length of MP.

A) 10 cm
B) 12 cm
C) 14 cm
D) 16 cm
E) I don't know.



40. **In the rectangular solid below, the length of the diagonal PS is:**

A) 21 cm
B) 17 cm
C) 20 cm
D) $\sqrt{200}$ cm
E) I don't know.



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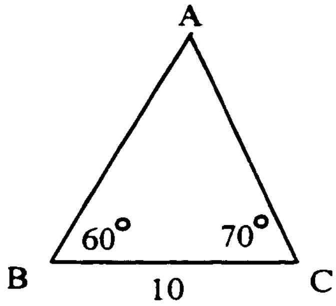
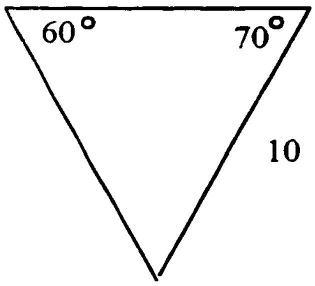
41. ABCD is a trapezoid with AD = AB = BC = $\frac{1}{2}$ DC. What is the measure of $\angle$ D.

A) 30°
B) 45°
C) 60°
D) more than 60°
E) I don't know.

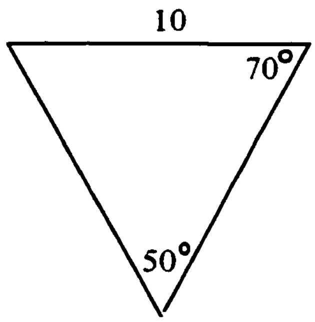
42. Which triangle is congruent to Δ ABC?



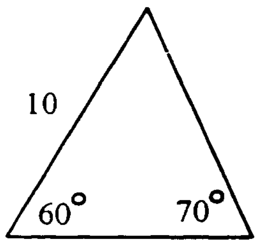
A)



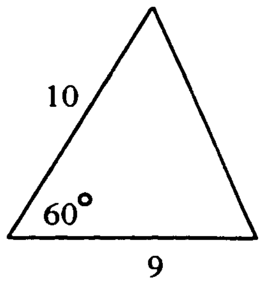
B)



C)



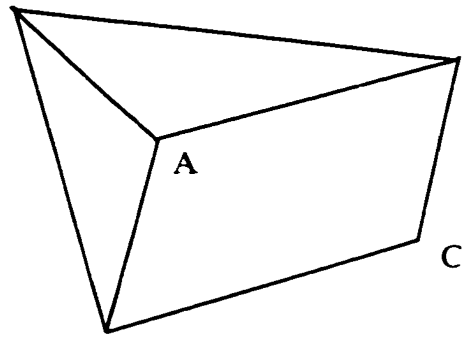
D)



E) I don't know.

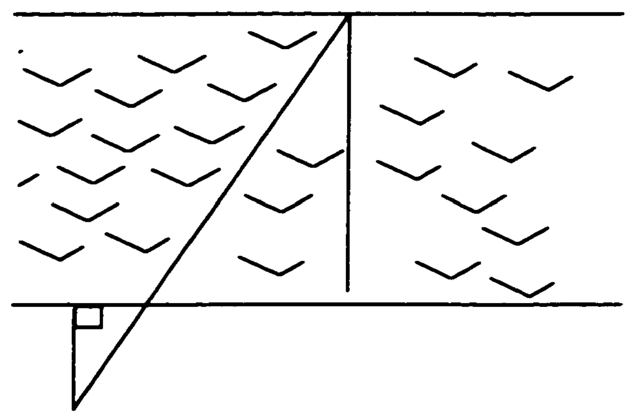
43. ABCD is a parallelogram. Δ ADE is isosceles with EA = AD. If ∠ ADC = 60° and ED ⊥ DC, ∠ EAB = ?

A) 60°
B) 90°
C) 120°
D) 150°
E) I don't know.



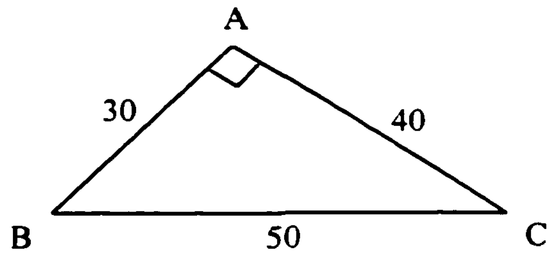
44. The figure below illustrates a water canal and a method of measuring its width. If PS = 24 m, PR = 3 m and RT = 5 m, how wide is the canal?

A) 24 m
B) 32 m
C) 40 m
D) 60 m
E) I don't know.



45. Given Δ ABC as labeled below. Use the following information to determine the measure of ∠ B.

$\tan 31° = 0.6000$
$\tan 37° = 0.7535$
$\tan 39° = 0.8098$
$\tan 53° = 1.3270$



A) 53°
B) 37°
C) 31°
D) 39°
E) I don't know.

---

46. An ocean liner travels 10 km north and then 6 km east. How far is the ship from its starting point?

A) 11.7 km
B) 16 km
C) 8 km
D) 4 km
E) I don't know.

---

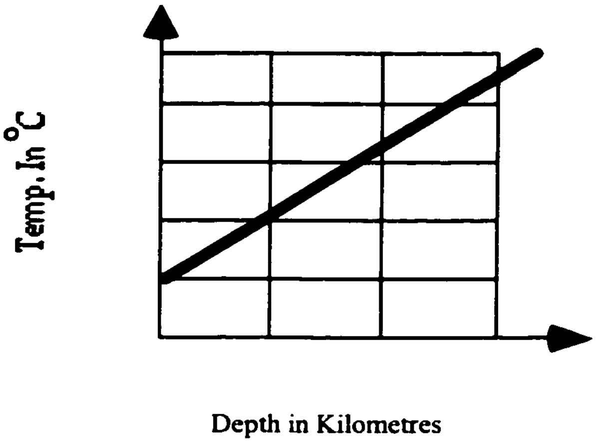
47. What is the mode of the following numbers?

2, 2, 2, 3, 4, 5, 10

A) 2
B) 3
C) 4
D) 10
E) I don't know.

48. From the graph below, the temperature at a depth of 2.5 km is closest to:

A) 30° C
B) 40° C
C) 50° C
D) 60° C
E) I don't know.



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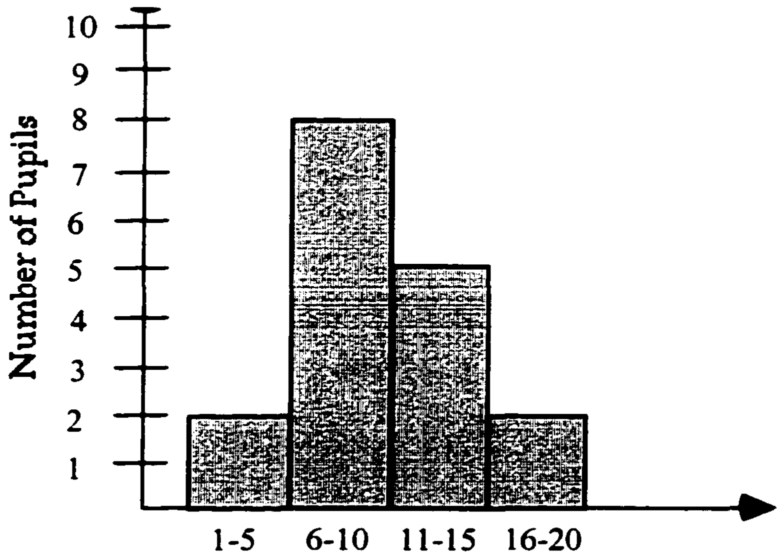
49. Find the arithmetic mean of the following numbers: 1.50, 2.40, 3.75

A) 2.40
B) 2.55
C) 3.75
D) 7.65
E) I don't know.

50. The graph below shows the time taken by pupils to travel from home to school. How many pupils must travel for MORE than 10 minutes?

A) 2
B) 5
C) 7
D) 8
E) I don't know.



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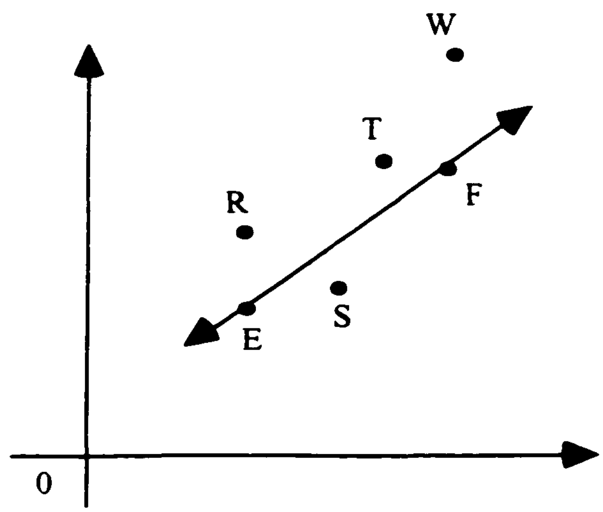
51. A man has 5 quarters, 2 dimes, 6 pennies, 3 nickels and 4 one-dollar coins in his pocket. What is the likelihood that if one is drawn at random, it will be a nickel?

A) 15%
B) 12%
C) 25%
D) 10%
E) I don't know.

52. The points R, S, T, and W were obtained experimentally to describe a theoretical linear relationship. Which point is most likely to have the greatest experimental error?

A) R
B) S
C) T
D) W
E) I don't know.



53. Maria tossed a coin in the air 4 times and it came up heads each time. If she tosses the coin one more time, what will happen?

A) It will come up heads because Maria is lucky.
B) It will come up tails because she can't be lucky five times in a row.
C) It may come up heads or it may come up tails, but it is more likely to come up heads.
D) It may come up heads or it may come up tails, and both are equally likely.
E) I don't know.

END

THANK YOU

FORM **B**

Program Evaluation of Applications of Mathematics 10

1998/99

Student Achievement & Attitude Assessment

Directions:

This booklet contains questions about mathematics. Follow the instructions for each section. For your answers, use the bubbles on the BUBBLE FORM provided.

You MUST use a pencil to fill the bubbles, because the machine that will read the booklet can not see ink marks! Fill each bubble completely and darkly. If you make a mistake, erase it completely and then fill in the bubble for your new answer

Principal Investigator: Bruce McAskill
Phone: (250) 356-7687

Supervisor: Dr. Leslee Francis-Pelton
Social & Natural Sciences
University of Victoria
Phone: (250) 721-7794

Form B

SECTION A: BACKGROUND INFORMATION & ATTITUDE TOWARDS MATHEMATICS

Please use the bubbles numbered 1 to 13 to answer **SECTION A** questions 1 to 13. Completely and darkly fill in the bubble which represents your answer. If you make a mistake, erase it completely and then fill in the bubble for your new answer.

---

1. **How old are you?**

A) 14 or less
B) 15
C) 16
D) 17
E) 18 or older

---

2. **Are you male or female?**

A) Male
B) Female

---

3. **Do you have a calculator in your home?**

A) Yes
B) No

---

4. **Do you have a computer in your home?**

A) Yes
B) No

5. **Which MATHEMATICS course are you presently taking?**

A) Applications of Mathematics 10
B) Mathematics 10A
C) Principles of Mathematics 10
D) Other

---

6. **What MATHEMATICS course did you take previous to this one?**

A) Applications of Mathematics 9
B) Mathematics 9A
C) Principles of Mathematics 9 or Mathematics 9
D) A Grade 10 course

---

7. **What grade did you receive in your last MATHEMATICS course?**

A) A
B) B
C) C or C+
D) P
E) F

---

8. **What Grade 11 MATHEMATICS course do you plan to take?**

A) Applications of Mathematics 11
B) Introductory Mathematics 11
C) Mathematics 11A
D) Principles of Mathematics 11
E) Accounting 11

9. What Grade 12 MATHEMATICS course do you plan to take?

A) None
B) Applications of Mathematics 12
C) Principles of Mathematics 12
D) An enriched mathematics course (e.g., Advanced Placement, International Baccalaureate, Calculus, etc.)

---

10. Do you plan on taking more MATHEMATICS courses once you have finished secondary school?

A) No
B) Yes, maybe one or two
C) Yes, I plan on making mathematics a major part of my life
D) I don't know

---

11. How many years of future education do you plan to take after you leave secondary school?

A) None
B) 3 years or fewer
C) 4 or 5 years
D) 6 years or more
E) I don't know

---

12. Do you agree with the statement, "*I usually do well in mathematics*"?

A) Strongly agree
B) Agree
C) Disagree
D) Strongly disagree

13. How much do you like mathematics?

A) Like a lot
B) Like
C) Dislike
D) Dislike a lot

SECTION B: MATHEMATICS ACHIEVEMENT

Please use the bubbles numbered 14 to 53 to answer **SECTION B** questions 14 to 53.

Completely and darkly fill in the bubble which represents your answer. If you make a mistake, erase it completely and then fill in the bubble for your new answer. Please try as hard as you can and make the best selections you can.

---

14. Rounded to the nearest centimetre, 358.358 cm =

A) 358 cm
B) 358.36 cm
C) 358.4 cm
D) 360 cm
E) I don't know.

---

15. If the city's property tax is $29.87 per $1000 of assessed value, the tax on a property assessed at $14 900 would be closest to which one of the following amounts?

A) $400
B) $420
C) $450
D) $470
E) I don't know.

16. Linda's new bike cost $159.99 and the sales tax was 5%. How much did she pay including tax?

A) $164.99
B) $167.99
C) $172.98
D) $177.99
E) I don't know.

---

17. Evaluate: $\frac{6^2}{3}$

A) 12
B) 4
C) 108
D) 6
E) I don't know.

---

18. Divide: $1\frac{1}{2} \div 2\frac{2}{3}$

A) $\frac{1}{3}$
B) $\frac{1}{2}$
C) $1\frac{1}{3}$
D) $\frac{32}{9}$
E) I don't know.

19. Write in simplest radical form

$3\sqrt{72}$

A) $18\sqrt{2}$

B) $9\sqrt{2}$

C) $6\sqrt{8}$

D) $9\sqrt{8}$

E) I don't know.

---

20. Find the product in simplest radical form.

$(3\sqrt{6})(4\sqrt{2})$

A) $12\sqrt{12}$

B) $24\sqrt{3}$

C) $7\sqrt{12}$

D) $14\sqrt{3}$

E) I don't know.

---

21. Write an expression for the quotient of seven and twice a number.

A) 7(2x)

B) 7 + 2x

C) $\frac{7}{2x}$

D) $\frac{7}{2}x$

E) I don't know.

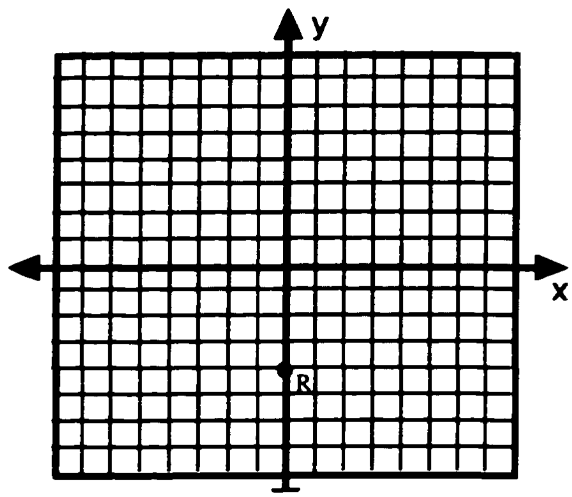
22. Solve: -12w - 12 = 36

A) w = 2
B) w = -2
C) w = -4
D) w = 4
E) I don't know.

---

23. The coordinates of point R are:

A) (- 4, 0)
B) (0, - 4)
C) (0, 4)
D) (4, 0)
E) I don't know.



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24. Solve for n: 4 (n - 3) - 5 = 7n

A) $n = -\frac{17}{3}$
B) $n = \frac{17}{3}$
C) $n = \frac{7}{3}$
D) $n = -\frac{7}{3}$
E) I don't know.

25. The cost of a new car is less than 4 times the cost of a used car. If x represents the cost of a new car, and y represents the cost of a used car, which one of these is true?

A) $x < 4y$
B) $x - y = 4$
C) $y > 4x$
D) $x = y + 4$
E) I don't know.

---

26. Factor completely over the rational numbers: $4a^2 - 8$

A) $(2a + 4)(2a - 2)$
B) $2(2a + 2)(a - 2)$
C) $2^2(a^2 - 2)$
D) $2^2(a + 2)(a - 1)$
E) I don't know.

---

27. Expand and simplify: $(2x - 5)^2 - 3(x - 7)$

A) $4x^2 - 3x - 32$
B) $4x^2 - 23x + 4$
C) $4x^2 - 23x + 46$
D) $4x^2 - 3x + 18$
E) I don't know.

28. Solve for x: $3x + 7 = 5x + 4$

A) $x = -\frac{11}{2}$

B) $x = -\frac{3}{2}$

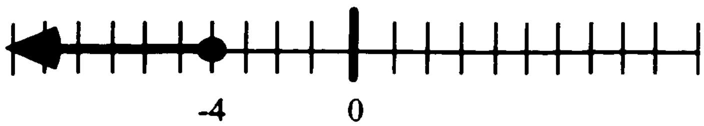
C) $x = \frac{3}{2}$

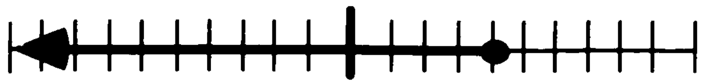
D) $x = \frac{11}{2}$

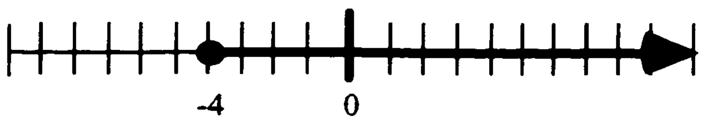
E) I don't know.

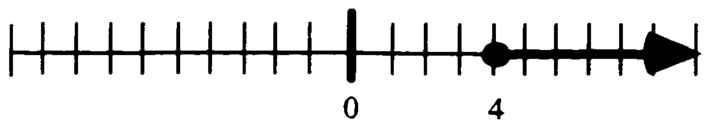
---

29. Which one of the following is the graph of $2x - 3 \geq 5$?

A) 

B) 

C) 

D) 

E) I don't know.

30. How many solutions does the following system of linear equations have?

$3x = y - 4$

$6x - 2y = 8$

A) none
B) One
C) Two
D) More than 2
E) I don't know.

---

31. What are the co-ordinates of the point of intersection of the y-axis and $3x - 4y = -8$?

A) $(-\frac{8}{3}, 0)$
B) $(0, -4)$
C) $(-4, \frac{3}{8})$
D) $(0, 2)$
E) I don't know.

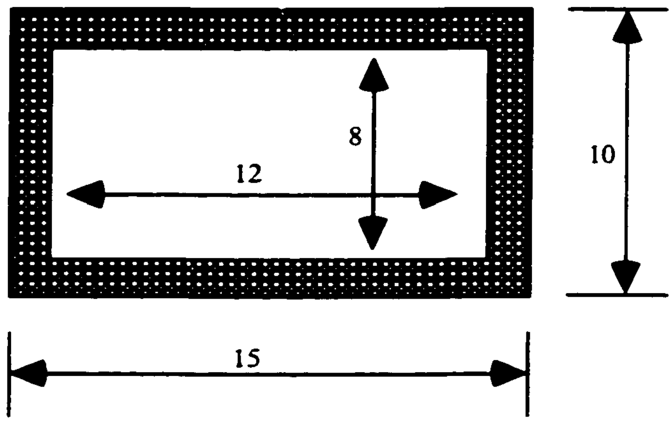
---

32. Find the missing factor: $2a^3 (\quad) = 16a^{18}$

A) $8a^6$
B) $8a^{15}$
C) $14a^{15}$
D) $32a^{21}$
E) I don't know.

33. What is the area of the shaded portion of this figure?

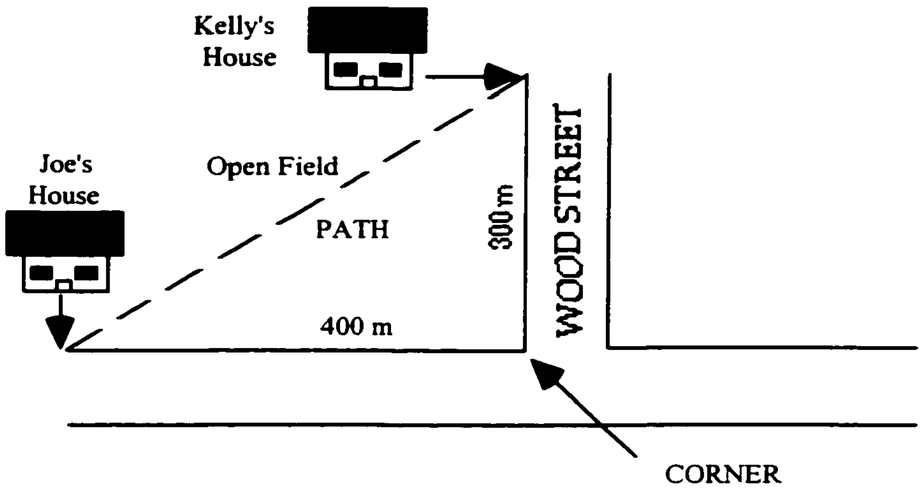
A) 54
B) 96
C) 120
D) 60
E) I don't know.



---

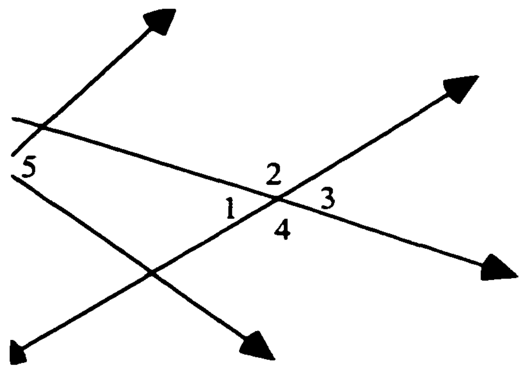
34. When Joe walks from his house to Kelly's house, he follows the path through the open field. How far does he walk?

A) 450 m
B) 500 m
C) 550 m
D) 600 m
E) I don't know.



35. Which two angles are each supplementary to ∠ 4?

A) – 1 and – 2
B) – 2 and – 3
C) – 5 and – 1
D) – 1 and –3
E) I don't know.



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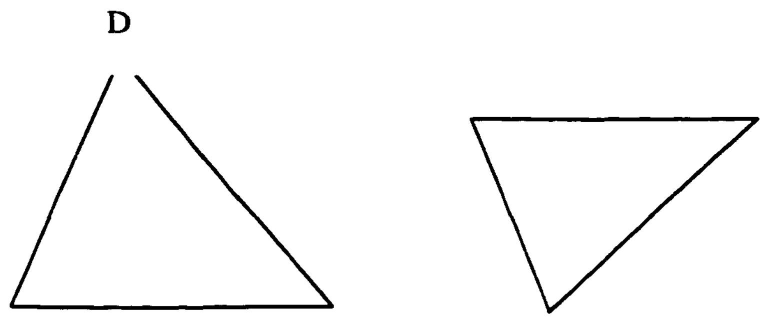
36. Δ DEF is similar to Δ XYZ. Find the length of YZ.

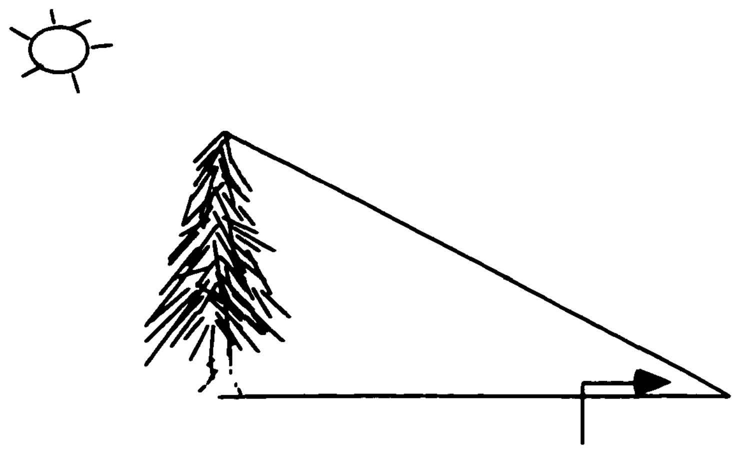
A) 26.25
B) 28
C) $37.\overline{3}$
D) 43.75
E) I don't know.



37. A tree cast a shadow 10 m long when the angle of elevation of the sun was 42°. What is the approximate height of the tree?

sin 42° = 0.6691
cos 42° = 0.7431
tan 42° = 0.9004

A) 6.7 m
B) 7.4 m
C) 9.0 m
D) 11.1 m
E) I don't know.



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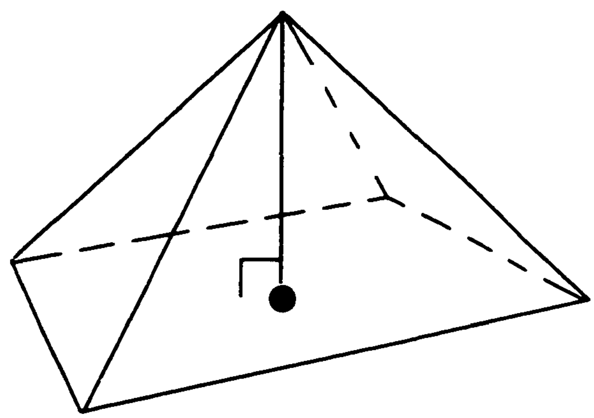
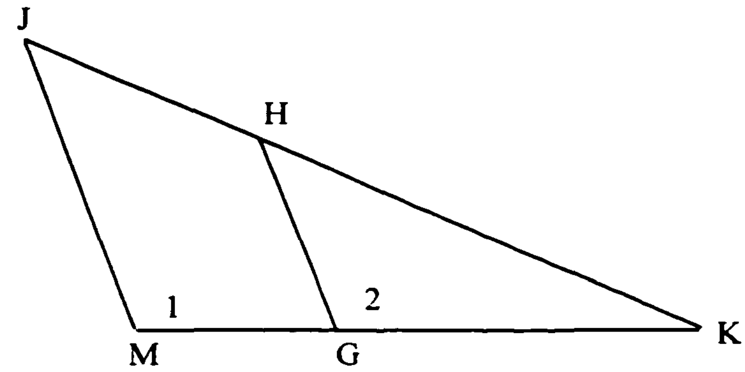
38. The volume of a pyramid is equal to $V=\frac{1}{3}Bh$, where B is the area of the base and h is the vertical height. The volume of the pyramid shown below is:

A) 1200 m^3
B) 1800 m^3
C) 2400 m^3
D) 3600 m^3
E) I don't know.



39. If $\angle 1 = \angle 2$, JH = 7, JK = 21 and GK = 10, what is MG?

A) 5
B) 7
C) 10
D) 15
E) I don't know.



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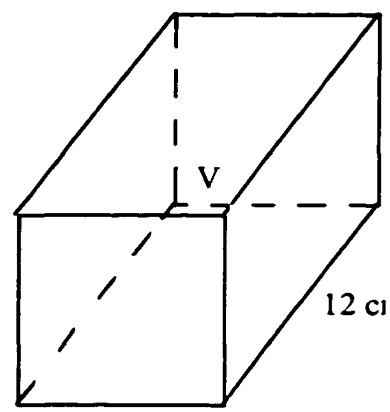
40. In the rectangular solid below, the length of the diagonal PS is:

A) 21 cm
B) 17 cm
C) 20 cm
D) $\sqrt{200}$ cm
E) I don't know.



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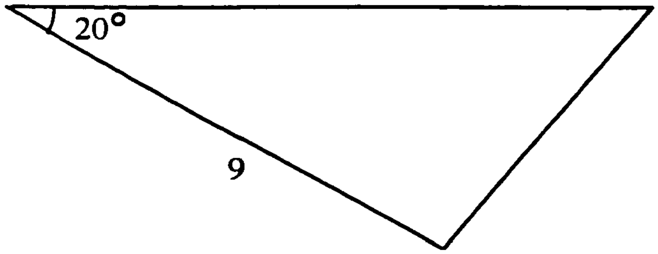
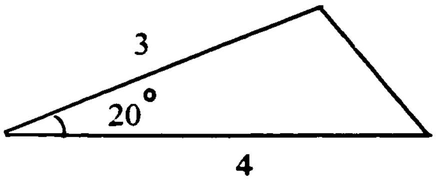
41. ABCD is a trapezoid with AD = AB = BC = $\frac{1}{2}$ DC. What is the measure of $\angle$ D.

A) 30°
B) 45°
C) 60°
D) more than 60°
E) I don't know.

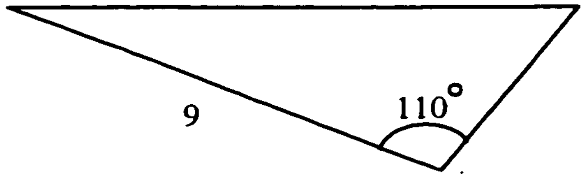
42. Which triangle can you be sure is similar to ∠MNP?



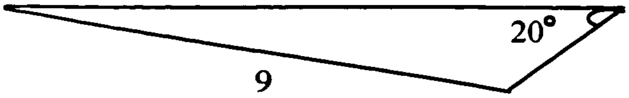
A)



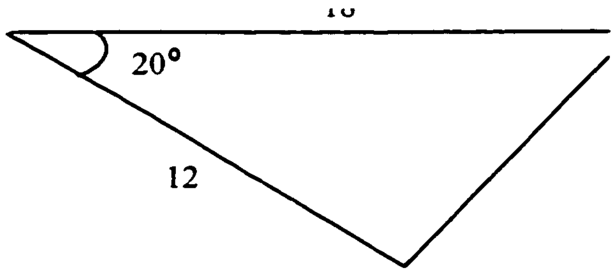
B)



C)



D)



E) I don't know.

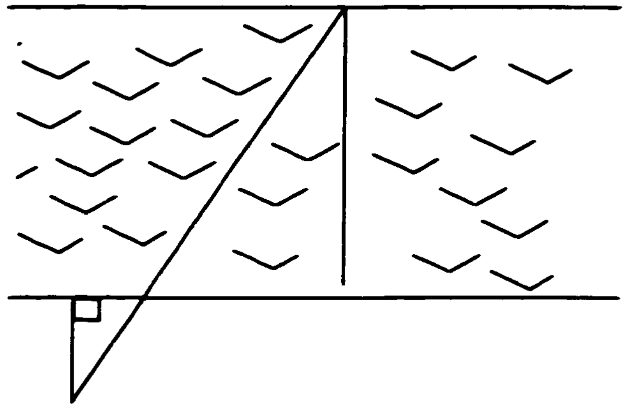
---

43. Quadrilateral ABDC is made up of 2 equilateral triangles ABC and BCD. The measure of ∠ ABD is:

A) 60°
B) 90°
C) 120°
D) 150°
E) I don't know.

44. The figure below illustrates a water canal and a method of measuring its width. If PS = 24 m, PR = 2 m and RT = 5 m, how wide is the canal?

A) 24 m
B) 32 m
C) 40 m
D) 60 m
E) I don't know.



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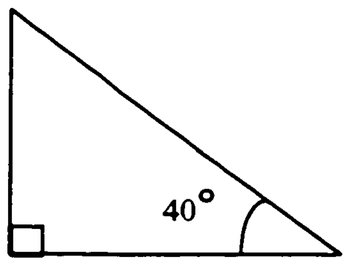
45. If ∠ C = 40° and BC = 20 in the right triangle below, use the following information to find AB.

sin 40° = 0.6428
cos 40° = 0.7660
tan 40° = 0.8391

A) 16.8
B) 15.3
C) 12.9
D) 23.8
E) I don't know.



46. An ocean liner travels 10 km north and then 12.5 km east. How far is the ship from its starting point?

A) 11.7 km
B) 16 km
C) 8 km
D) 4 km
E) I don't know.

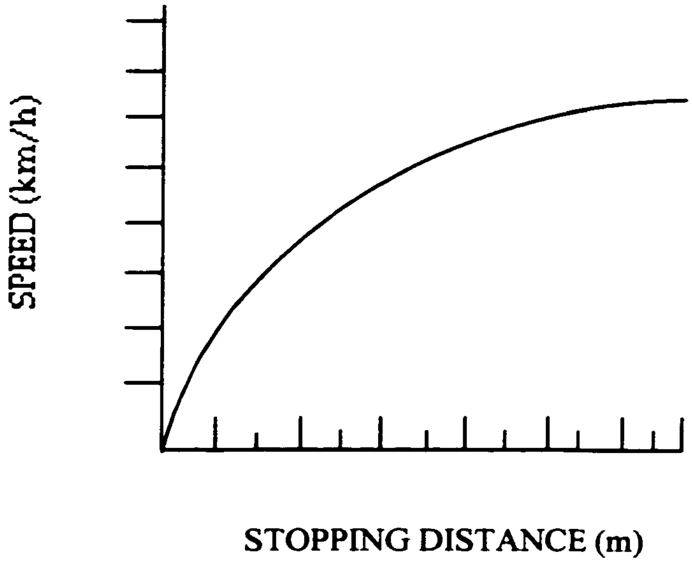
---

47. What is the mean of the following numbers?

2, 2, 2, 3, 4, 5, 10

A) 3
B) 2
C) 10
D) 4
E) I don't know.

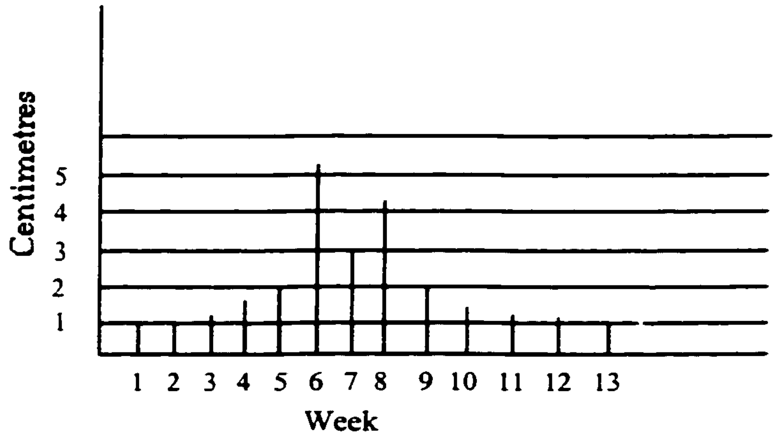
48. This graph represents the relationship between the speed of a car in kilometres per hour (km/h) and the stopping distance in metres (m) after first applying the brakes. If the skid marks were 45 m long, about how fast was the car travelling when the brakes were first applied?



A) 40 km/h
B) 56 km/h
C) 72 km/h
D) 88 km/h
E) I don't know.

49. In the graph below, rainfall in centimetres is plotted for 13 weeks. The average weekly rainfall during the period is approximately:

A) 1 cm
B) 2 cm
C) 3 cm
D) 4 cm
E) I don't know.



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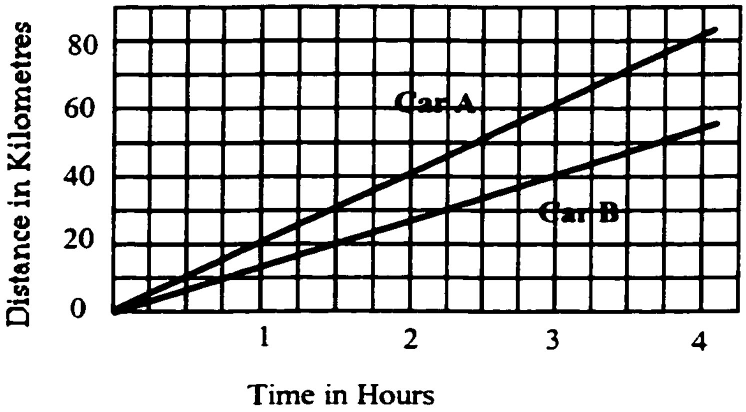
50. The distance travelled by two cars during a period of 4 hours is shown in the graph below. How much longer does it take Car B to go 50 km than it does for Car A to go 50 km?

A) 1 hour 15 minutes
B) 1 hour 30 minutes
C) 2 hours
D) 2 hours 30 minutes
E) I don't know.



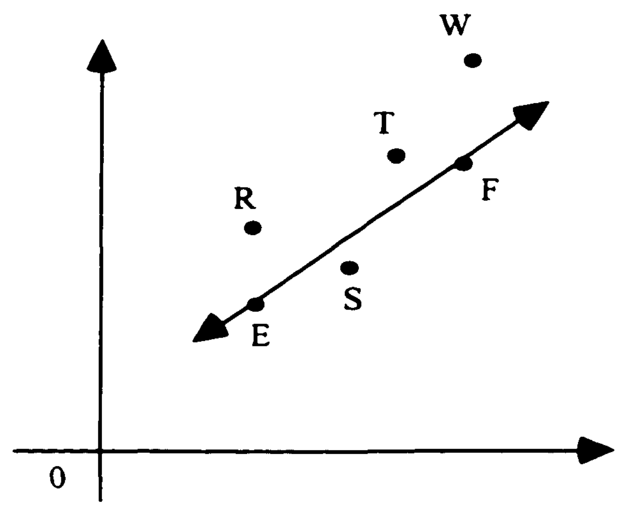
51. A man has 5 quarters, 2 dimes, 6 pennies, 3 nickels and 4 one-dollar coins in his pocket. What is the likelihood that if one is drawn at random, it will be a dime?

A) 15%
B) 12%
C) 25%
D) 10%
E) I don't know.

---

52. The points R, S, T, and W were obtained experimentally to describe a theoretical linear relationship. Which point is most likely to have the least experimental error?

A) R
B) S
C) T
D) W
E) I don't know.



53. Maria tossed a coin in the air 4 times and it came up heads each time. If she tosses the coin one more time, what will happen?

A) It will come up heads because Maria is lucky.

B) It will come up tails because she can't be lucky five times in a row.

C) It may come up heads or it may come up tails, but it is more likely to come up heads.

D) It may come up heads or it may come up tails, and both are equally likely.

E) I don't know.

END

THANK YOU

Appendix H: Testing of Assumptions for Analysis of Variance

Assessing the Normality of the Experimental and Control Group Student Data

The assessment of normality for each sample was done first by analyzing the expected normal probability and the detrended expected normal probability plots. If the actual distributions are normal, then the points for the cases fall along the diagonal line of the normal probability plots running from lower left to upper right, with some minor deviations due to random processes (Tabachnick & Fidell, 1996). The detrended normal probability plots would (for normal distributions) be similar to normal probability plots except that deviations from the diagonal are plotted instead of values along the diagonal. A normal distribution will result in an even distribution of the cases above and below the horizontal line (the line of zero deviation from expected normal values). This process has the advantage of being able to identify univariate outliers as cases that are far from the other cases.

To confirm the results of the graphical displays, both the Kolmogorov-Smirnov and Shapiro-Wilk normality tests ($\alpha = 0.05$) are used on the data. The Shapiro-Wilk test is included in the analysis because the sample size for the 10A group ($N = 39$) is less than fifty.

Figures 42 through 53 show Normal Q-Q Plots and Detrended Normal Q-Q Plots for the pre-test and post-test results by group (Experimental, Control (10A), and Control (PM 10)).

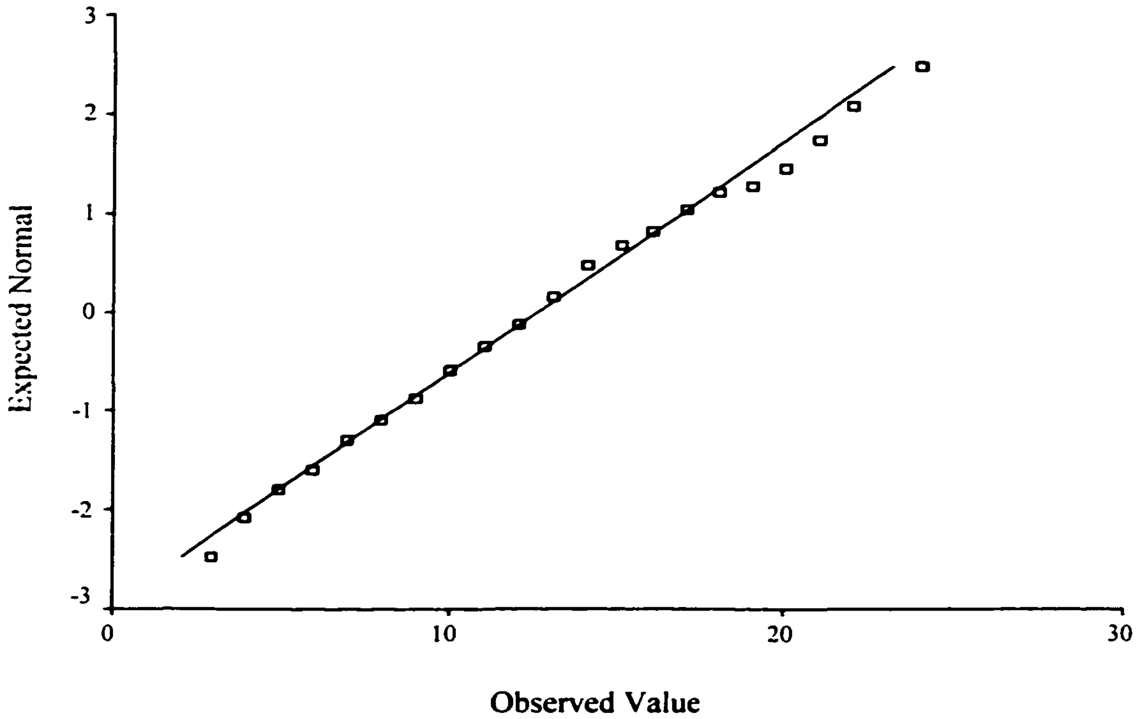


Figure 42. Normal Q-Q Plot of Experimental Group Pre-Test Total Scores

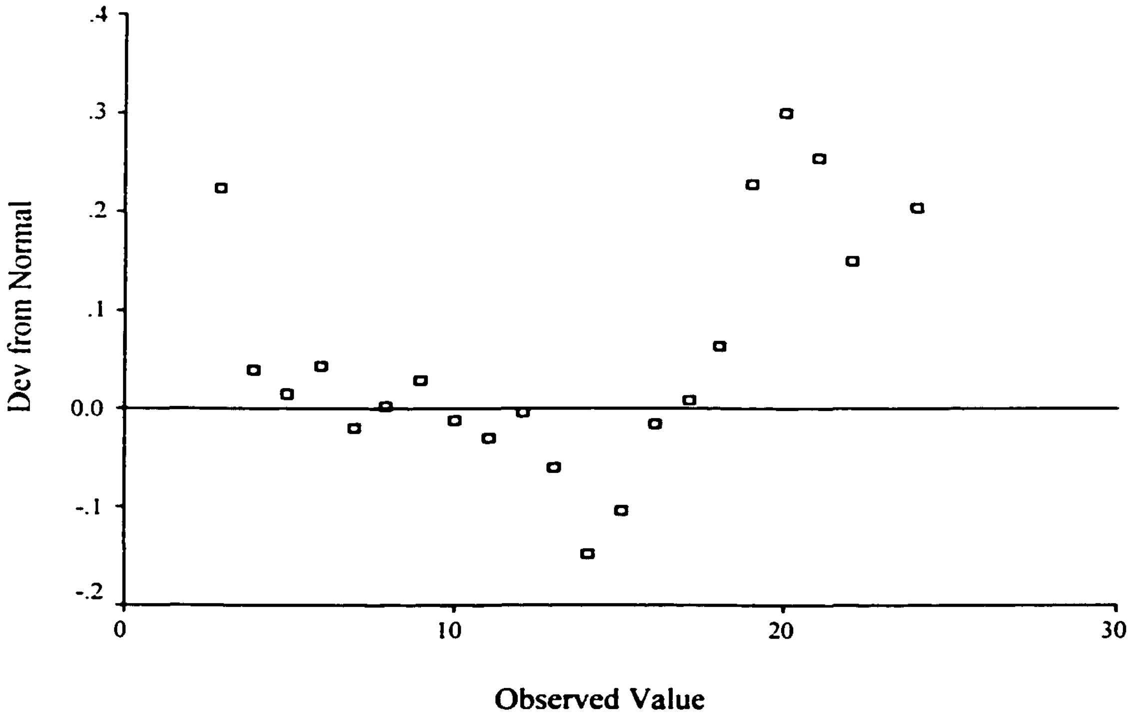


Figure 43. Detrended Normal Q-Q Plot of Experimental Group Pre-Test Total Scores

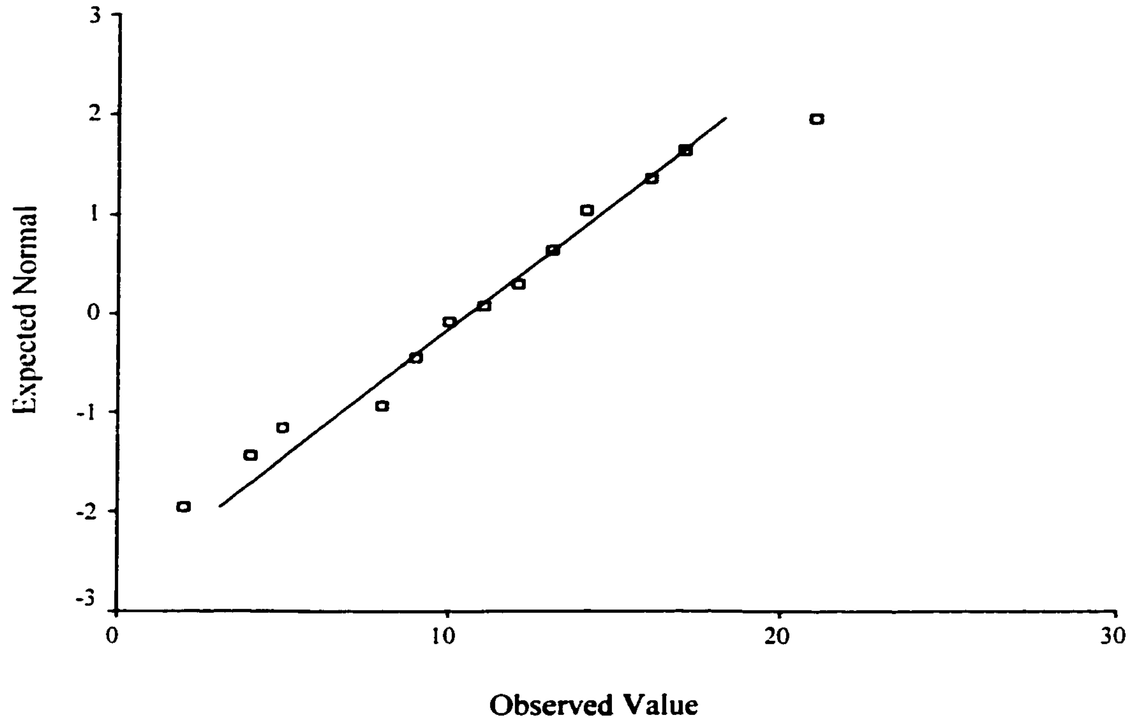


Figure 44. Normal Q-Q Plot of Control Group (10A) Pre-Test Total Scores

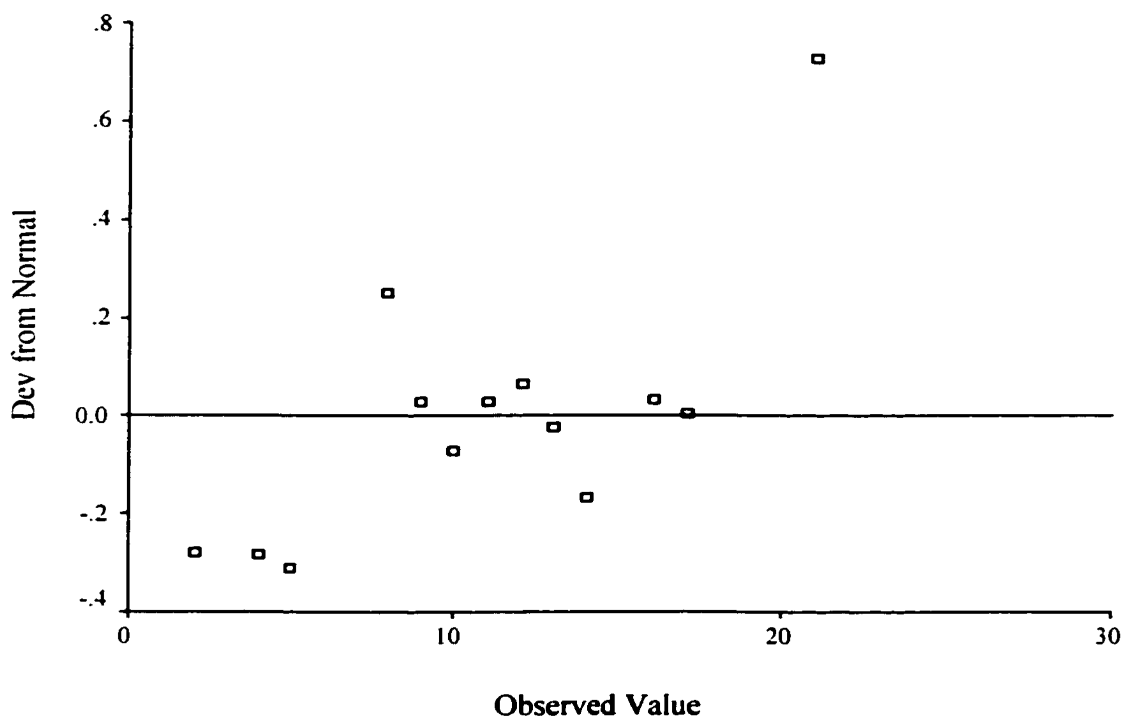


Figure 45. Detrended Normal Q-Q Plot of Control Group (10A) Pre-Test Total Scores

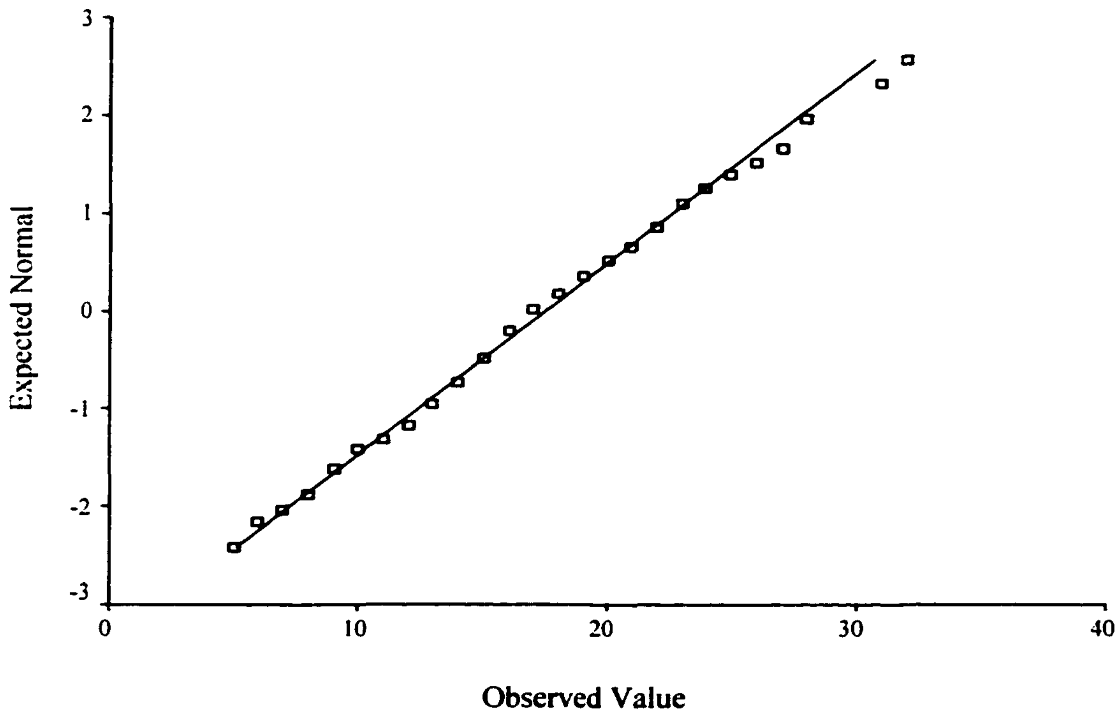


Figure 46. Normal Q-Q Plot of Control Group (PM 10) Pre-Test Total Scores

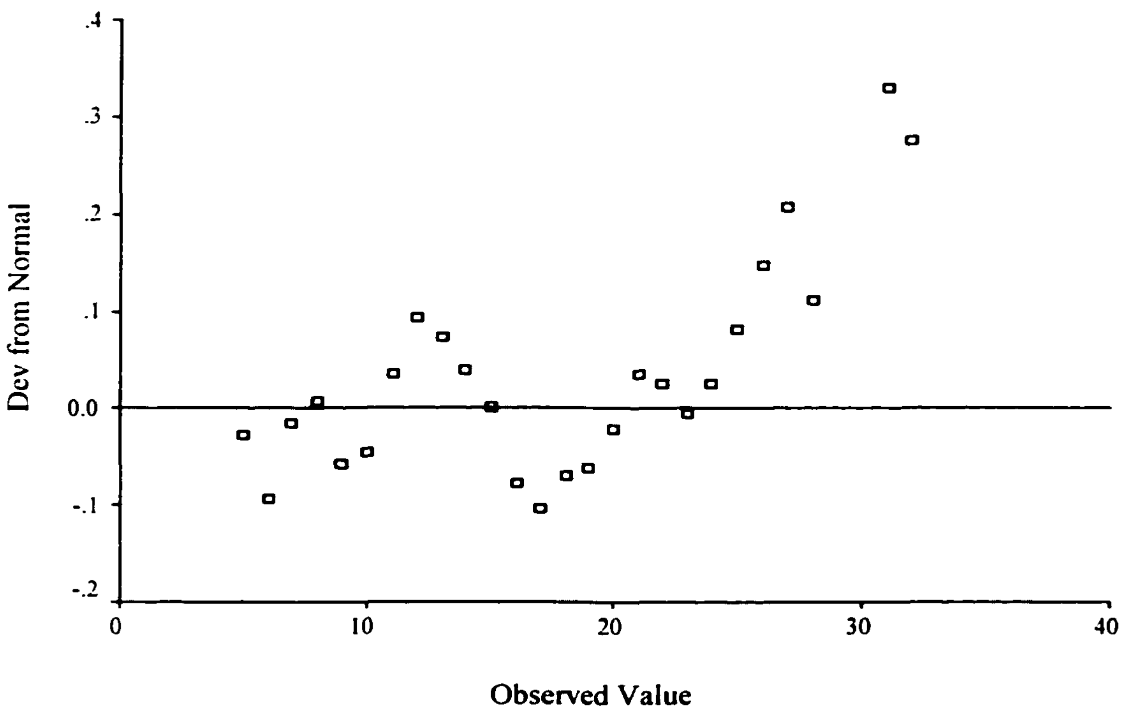


Figure 47. Detrended Normal Q-Q Plot of Control Group (PM 10) Pre-Test Total Scores

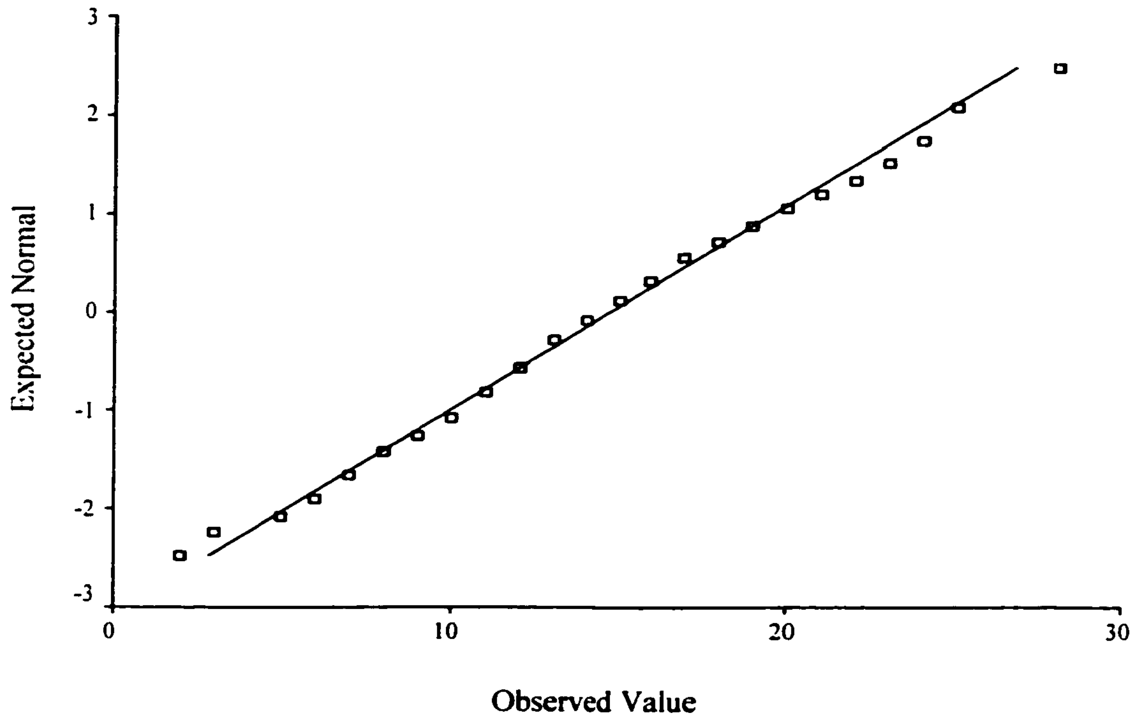


Figure 48. Normal Q-Q Plot of Experimental Group Post-Test Total Scores

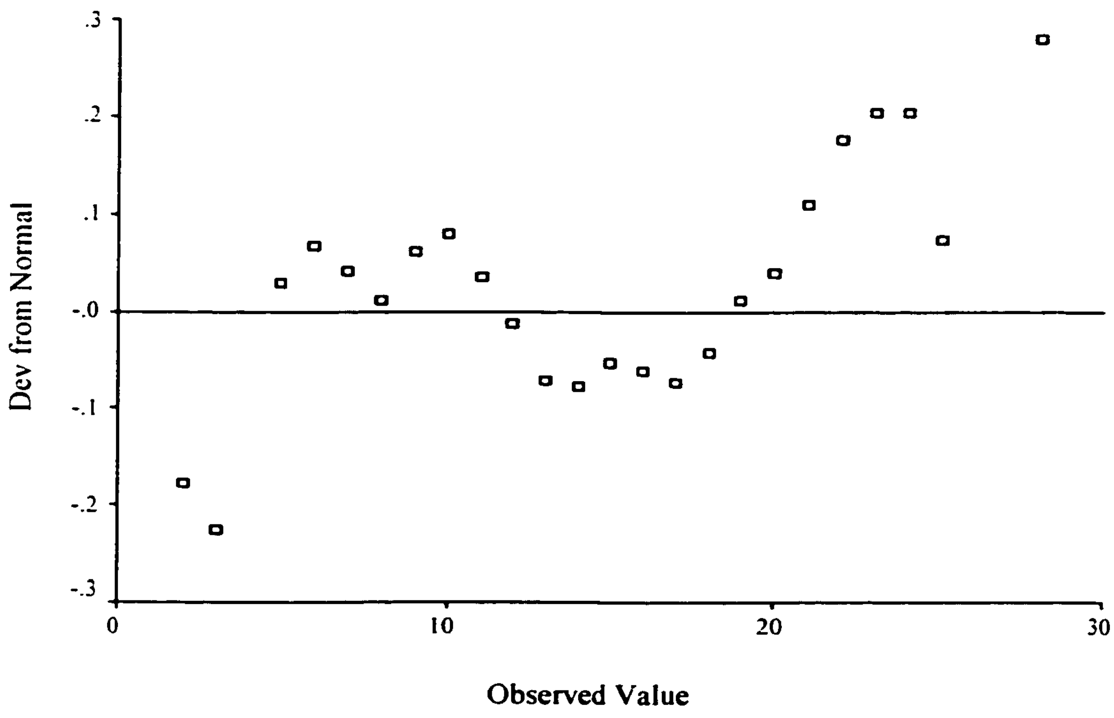


Figure 49. Detrended Normal Q-Q Plot of Experimental Group Post-Test Total Scores

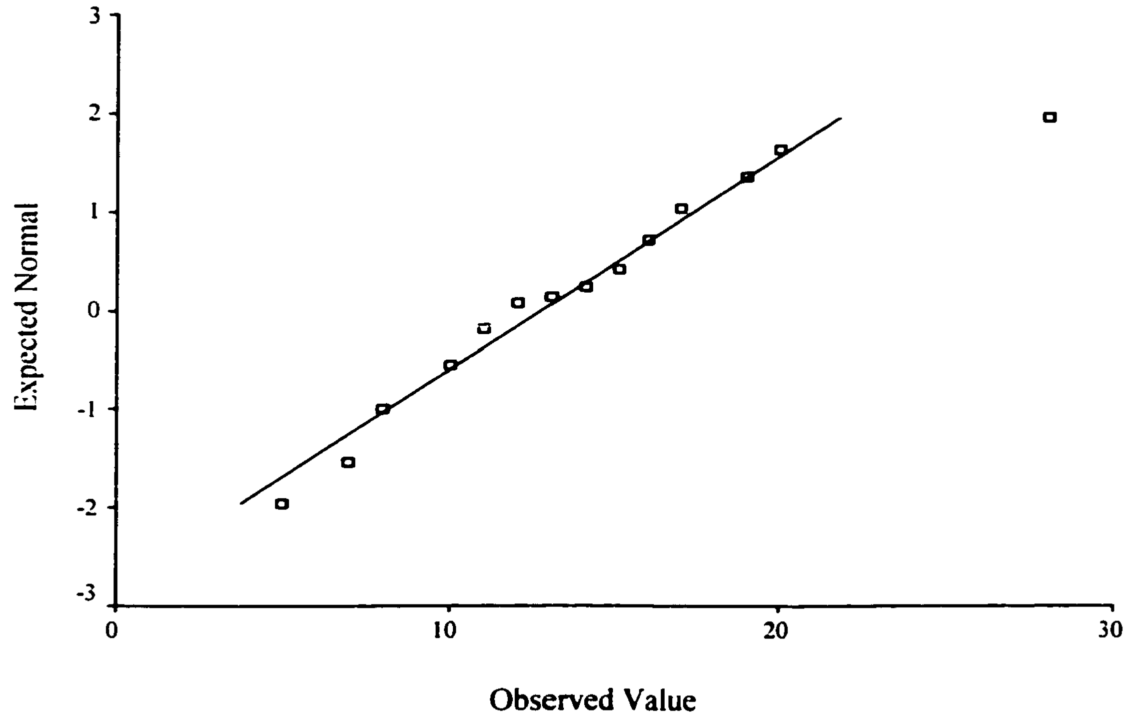


Figure 50. Normal Q-Q Plot of Control Group (10A) Post-Test Total Scores

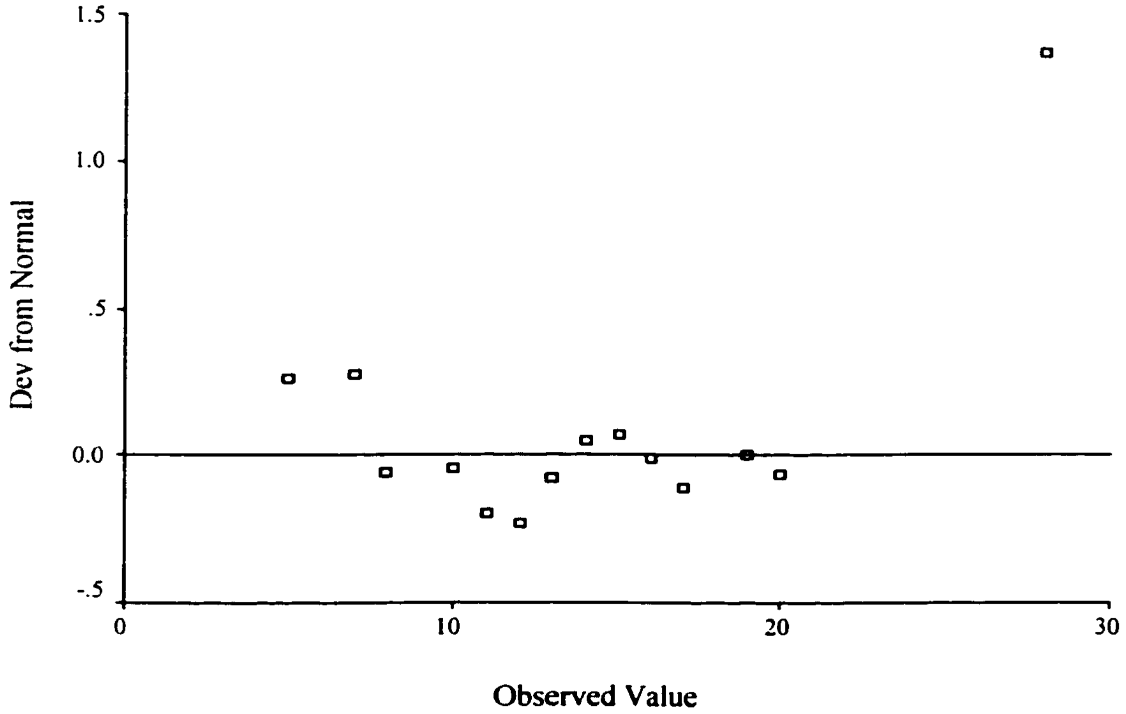


Figure 51. Detrended Normal Q-Q Plot of Control Group (10A) Post-Test Total Scores

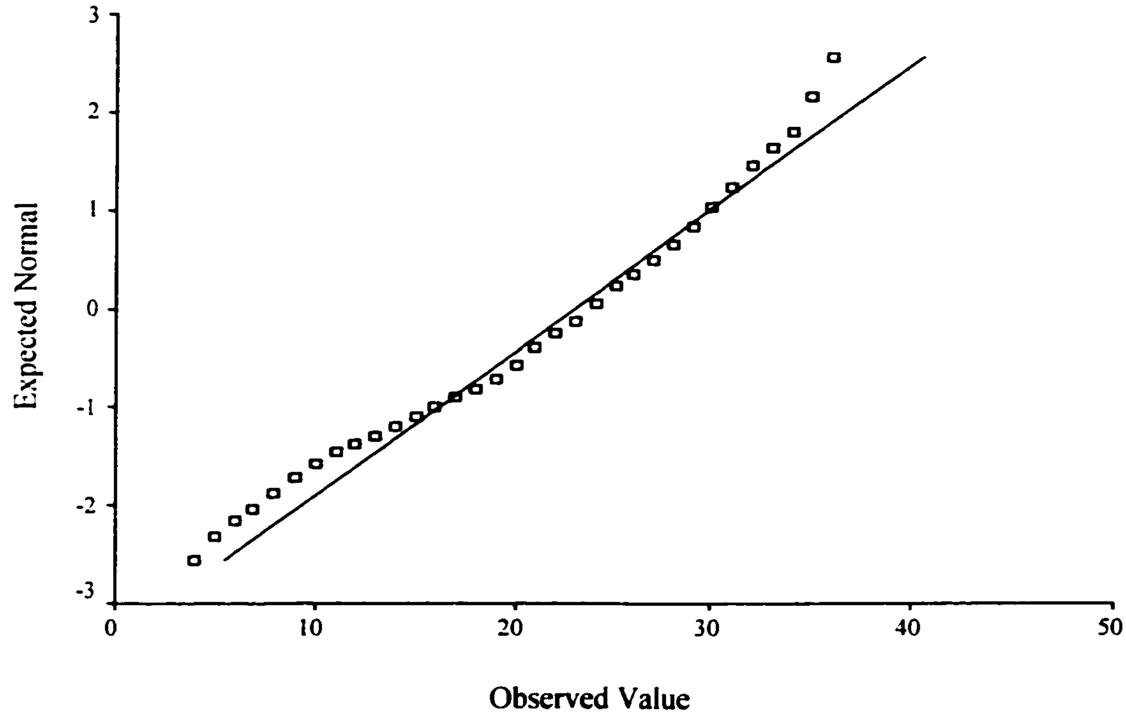


Figure 52. Normal Q-Q Plot of Control Group (PM 10) Post-Test Total Scores

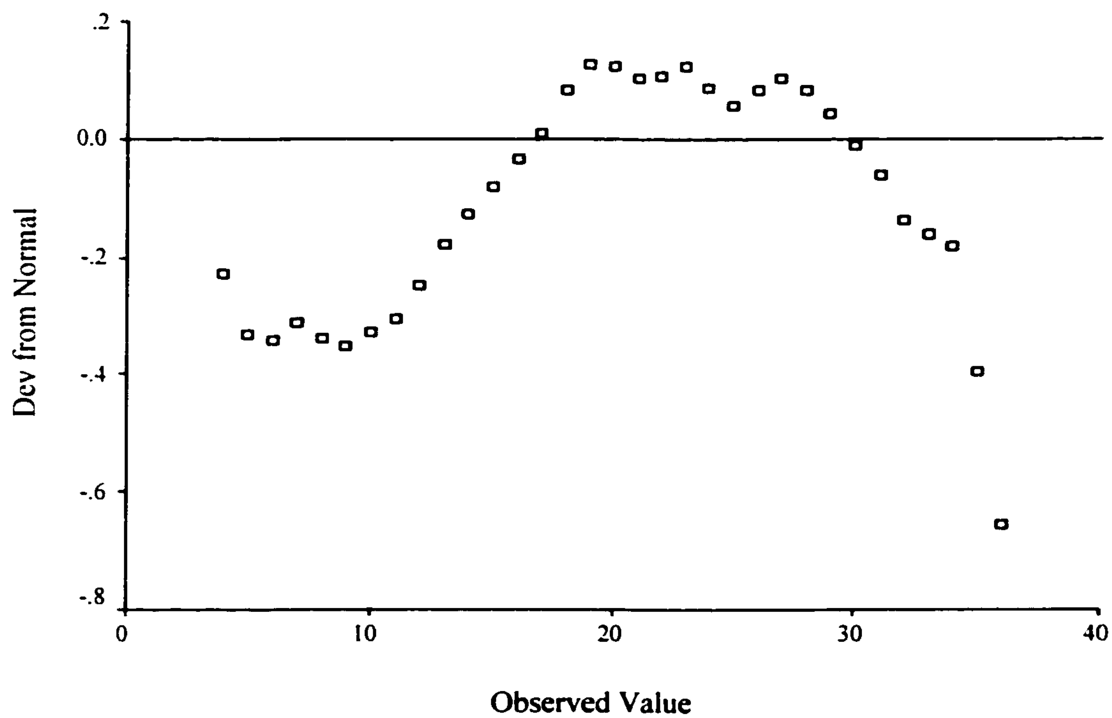


Figure 53. Detrended Normal Q-Q Plot of Control Group (PM 10) Post-Test Total Scores

Table 39

Test of Normality for the Pre-test and Post-test Experimental and Control Group Student Populations.

	Course	Kolmogorov-Smirnov[a] Statistic	df	Sig.	Shapiro-Wilk Statistic	df	Sig.
Pre-Test							
Number	AM 10	.190	154	.00			
	10A	.262	39	.00	.866	39	.01**
	PM 10	.168	193	.00			
Patterns & Relations	AM 10	.130	154	.00			
	10A	.182	39	.00	.932	39	.03
	PM 10	.116	193	.00			
Shape & Space	AM 10	.123	154	.00			
	10A	.174	39	.00	.944	39	.08
	PM 10	.104	193	.00			
Statistics & Probability	AM 10	.145	154	.00			
	10A	.208	39	.00	.920	39	.01
	PM 10	.170	193	.00			
Total	AM 10	.078	154	.02			
	10A	.163	39	.01	.937	39	.05
	PM 10	.080	193	.01			
Attitude Towards Mathematics	AM 10	.115	154	.00			
	10A	.089	39	.20*	.964	39	.37
	PM 10	.092	193	.00			
Post-Test							
Number	AM 10	.180	154	.00			
	10A	.161	39	.01	.932	39	.03
	PM 10	.185	193	.00			
Patterns & Relations	AM 10	.177	154	.00			
	10A	.139	39	.06	.916	39	.01**
	PM 10	.115	193	.00			
Shape & Space	AM 10	.119	154	.00			
	10A	.127	39	.12	.939	39	.05
	PM 10	.123	193	.00			
Statistics & Probability	AM 10	.175	154	.00			
	10A	.181	39	.00	.922	39	.02
	PM 10	.169	193	.00			
Total	AM 10	.105	154	.00			
	10A	.130	39	.09	.967	39	.43
	PM 10	.087	193	.00			
Attitude Towards Mathematics	AM 10	.078	154	.02			
	10A	.133	39	.08	.939	39	.05
	PM 10	.098	193	.00			

** This is an upper bound of the true significance.
* This is a lower bound of the true significance.
[a] Lilliefors Significance Correction

The results of the Kolmogorov-Smirnov and Shapiro-Wilk normality tests displayed in Table 39 confirm that the normal probability and detrended normal probability plots indicate the samples are not generally normally distributed about their respective means. In all cases but one, the normality tests indicate that the normal population hypothesis must be rejected.

Assessing the Homogeneity of Variance of the Student Data

The assumption of homogeneity of variance was tested using the Levene test. This procedure was selected as it is fairly robust to departures from normality. The results of this test are shown in Table 40 below. Included in the analysis are the sub-scores for each of the curriculum organizers as well as the Attitude Toward Mathematics scores.

Table 40

Test of Homogeneity of Variance for the Pre-test and Post-test Experimental and Control Group Student Populations.

	Levene Statistic	df1	df2	Sig.
Post-test				
Number	1.10	2	383	.34
Patterns & Relations	4.39	2	383	.01
Shape & Space	3.52	2	383	.03
Statistics & Probability	5.83	2	383	.00
Total	9.80	2	383	.00
Attitude Towards Mathematics	.60	2	383	.55
Pre-test				
Number	.22	2	383	.81
Patterns & Relations	8.48	2	383	.00
Shape & Space	4.21	2	383	.02
Statistics A Probability	1.46	2	383	.24
Total	4.27	2	383	.02
Attitude Towards Mathematics	.54	2	383	.59

The results of the Levene test for homogeneity of variance of the sample means are mixed as they indicate that the assumption hypothesis of equal variances is rejected for:

- Post-test: Patterns and Relations ($F = 4.39, p < .005$), Shape and Space ($F = 3.52$, p=.03), Statistics and Probability ($F = 5.83, p < .005$), Total Score ($F = 9.80$, p<.005); and,
- Pre-test: Pattern and Relations ($F = 8.48, p = .01$), Shape and Space ($F = 4.21$, $p = .02$), Total Score ($F = 4.27, p = .02$).

Whereas, the hypothesis of equal variances is not rejected for:

- Post-test: Number ($F = 1.10, p = .34$), Attitude Towards Mathematics ($F = 0.60$, p=.55); and,
- Pre-test: Number ($F = 0.22, p = .81$), Statistics and Probability ($F = 1.46$, p=.24), Attitude Towards Mathematics ($F = 0.54, p$=.59).

The lack of normality in the sample populations, combined with the unequal variances between groups, suggests that a possible data transformation is required. Prior to attempting any data transformation, the results of the 1999 Provincial Assessment of Reading Comprehension, First-Draft Writing, and Numeracy (Province of British Columbia, 2000), made available by the British Columbia Ministry of Education, were analyzed for normality and homogeneity of variances. As the Numeracy component of this provincial assessment was developed from the same pool of questions as the pre-test and post-test assessments used in this study (many of the questions are identical) and the provincial assessment was done on a census basis (involving all Grade 10 students in British Columbia public schools, N = 43,637 students), it was believed that an analysis of the normality and homogeneity of variance of the total population would assist in determining whether a data transformation is necessary.

The results of the normality test performed on the Numeracy assessment data are shown below:

Table 41

Test of Normality for the Student Numeracy Results of the "1999 Provincial Assessment of Reading Comprehension, First-Draft Writing, and Numeracy".

	Course	Kolmogorov-Smirnov[a] Statistic	df	Sig.
Total Score	PM 10	.067	25121	.00
	AM 10	.098	1147	.00
	10A	.125	4697	.00
	OTHER	.101	4233	.00
Attitude Towards Mathematics	PM 10	.050	25121	.00
	AM 10	.048	1147	.00
	10A	.057	4697	.00
	OTHER	.064	4233	.00
Written Response	PM 10	.080	25121	.00
	AM 10	.103	1147	.00
	10A	.116	4697	.00
	OTHER	.094	4233	.00

[a] Lilliefors Significance Correction

The results of the Kolmogorov-Smirnov test strongly reject the hypothesis of normality for the population from which the samples were drawn (all p values less than .0005).

In addition to the multiple-choice achievement and attitude toward mathematics assessment items similar to those used in this study, the provincial assessment also included a written thematic-based component. The written component provides an additional dimension to this study as students' written work could not be included in the original study design, but was considered desirable. Table 42 consists of the Numeracy assessment results including the population size, mean, standard deviation, skewness and kurtosis for each of the identified sub-populations (PM 10, AM 10, 10A, Other).

Table 42

1999 Provincial Assessment of Reading Comprehension, First-Draft Writing, and Numeracy: Grade 10 Numeracy Results for Achievement (Multiple Choice and Written Response) and Attitude Toward Mathematics

		N	M	SD	Skewness	Std. Error of Skewness	Kurtosis	Std. Error of Kurtosis
Multiple Choice Score	PM 10	29853	15.0	6.2	-.069	.014	-.315	.028
	AM 10	1471	11.5	5.3	.396	.064	.391	.128
	10A	6688	9.7	4.9	.550	.030	1.098	.060
	OTHER	5625	15.3	8.5	-.092	.033	-1.093	.065
	Total	43637	14.1	6.7	.098	.012	-.481	.023
Attitude Toward Mathematics	PM 10	29851	20.8	5.3	.247	.014	.051	.028
	AM 10	1470	21.8	5.1	.115	.064	.379	.128
	10A	6687	23.0	5.0	.247	.030	.568	.060
	OTHER	5624	20.3	6.1	.525	.033	.559	.065
	Total	43632	21.1	5.4	.257	.012	.200	.023
Written Response Score	PM 10	25123	6.1	3.1	.421	.015	.617	.031
	AM 10	1148	4.8	2.7	.659	.072	.912	.144
	10A	4698	4.2	2.5	1.151	.036	4.652	.071
	OTHER	4233	6.9	3.9	.397	.038	-.187	.075
	Total	35202	5.9	3.2	.553	.013	.682	.026

To test whether these provincial census assessment results were statistically significant an analysis of variance (ANOVA) was conducted using SPSS for Windows (Standard Version 9.0.0) to first test the normality of the populations (see Table 43); then SPSS was used to conduct a one-way ANOVA (see Table 31); and a post-hoc multiple comparison of means was performed (see Table 32) as indicated by the results of the ANOVA.

The results of the test for homogeneity of variance of the student provincial data are shown in the table below:

Table 43

Test of Homogeneity of Variance for the 1999 Provincial Assessment of Reading Comprehension, First-Draft Writing, and Numeracy.

Category	Levene Statistic	df1	df2	Sig.
Total Score	694	3	35194	.00
Attitude Towards Mathematics	50	3	35194	.00
Written Response Score	345	3	35194	.00

The results of the Levene test on the provincial assessment data are consistent with those of this study as they indicate that the hypotheses of equal variances for the provincial population are rejected for all three categories: Total Score ($F = 694$, $p<.005$), Attitude Towards Mathematics ($F = 50$, $p < .005$), and Written Response ($F = 345$, $p<.005$). These results also indicate that any post-hoc multiple comparison should use Tamhane's T2 method that allows for unequal variances.

Table 44

Analysis of Variance of the 1999 Provincial Assessment of Reading Comprehension, First-Draft Writing, and Numeracy: Grade 10 Numeracy Results for Achievement (Multiple Choice and Written Response) and Attitude Toward Mathematics

Score Category		Sum of Squares	df	Mean Square	F	Sig.
Multiple Choice	Between Groups	170840	3	56947	1409	.00
Achievement	Within Groups	1763175	43633	40		
Score	Total	1934015	43636			
Attitude	Between Groups	31444	3	10481	365	.00
Toward	Within Groups	1252714	43628	29		
Mathematics	Total	1284159	43631			
Written	Between Groups	19896	3	6632	678	.00
Response	Within Groups	343852	35198	10		
Score	Total	363748	35201			

The results of the ANOVA (Table 44) indicate that there are significant differences in the mean scores for all three assessment categories (Multiple Choice Achievement Score: $F = 1409$ with $p<.005$; Written Response Achievement Score: $F=6631$ with $p<.005$, and Attitude Toward Mathematics Score: $F = 10481$ with $p<.005$). The subsequent post-hoc multiple comparison (Table 45) indicates that there are significant differences between the group mean scores for each assessment category.

Table 45

Post Hoc Analysis of the 1999 Provincial Assessment of Reading Comprehension, First-Draft Writing, and Numeracy: Grade 10 Numeracy Results for Achievement (Multiple Choice and Written Response) and Attitude Toward Mathematics

Dependent Variable	(I) Course	(J) Course	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Multiple Choice	PM 10	AM 10	3.5*	.17	.00	3.1	3.9
		10A	5.3*	.01	.00	5.1	5.5
		OTHER	-.3*	.01	.04	-.6	0
	AM 10	10A	1.8*	.18	.00	1.4	2.2
		OTHER	-3.8*	.19	.00	-4.3	-3.4
	10A	OTHER	-5.6*	.12	.00	-5.9	-5.3
Attitude Toward Mathematics	PM 10	AM 10	-1.0*	.14	.00	-1.4	-.6
		10A	-2.2*	.07	.00	-2.4	-2.0
		OTHER	.6*	.08	.00	.34	.8
	AM 10	10A	-1.2*	.15	.00	-1.6	-.8
		OTHER	1.6*	.16	.00	1.2	2.0
	10A	OTHER	2.8*	.10	.00	2.5	3.0
Written Response Score	PM 10	AM 10	1.3*	.09	.00	1.1	1.5
		10A	1.9*	.05	.00	1.8	2.0
		OTHER	-.8*	.05	.00	-.9	-.6
	AM 10	10A	.6*	.10	.00	.3	.8
		OTHER	-2.1*	.10	.00	-2.4	-1.8
	10A	OTHER	-2.7*	.06	.00	-2.9	-2.5

* The mean difference is significant at the .05 level.

The results of the post-hoc multiple comparisons clearly indicate that, for all three assessment categories there were statistical differences between the mean scores ($p < .005$ for all cases). The scores can be ranked in the following order: Other Scores > PM 10 Scores > AM 10 Scores > 10A Scores. In the case of the Attitude Toward Mathematics score, a high score indicates a poor attitude toward mathematics (the scores obtained in this study use a high score to indicate a more positive attitude toward mathematics.)

These results also indicate that the original population from which the samples were drawn is not distributed normally about the mean scores and the variances between groups are not equal. A comparison of the distribution characteristics (variance, skewness, and kurtosis) of the sample populations with one another for each assessment category provides sufficient information to determine if a data transformation is necessary. Table 46 compares the skewness and kurtosis of each of the populations (pre-test and post-test) while Table 47 compares the variances of the distribution and identifies the ratio of the highest variance to the lowest variance within each assessment category.

Table 46

Comparison of Skewness and Kurtosis of Pre-Test and Post-Test Sample Populations by Assessment Category

Assessment Category	Skewness				Kurtosis			
	AM 10 (.195) [a]	10 A (.378) [a]	PM 10 (.175) [a]	Control (.124) [a]	AM 10 (.389) [b]	10 A (.741) [b]	PM 10 (.348) [b]	Control (.248) [b]
Pre-Test								
Total Score	.281	.039	.168	.344	-.099	.654	-.009	-.041
Number	-.339	.011	-.229	-.218	-.164	-.668	.123	-.091
Patterns & Relations	.664	.141	.191	.493	.117	-.929	-.626	-.342
Shape & Space	.150	.082	.283	.397	-.298	-.828	-.488	-.131
Statistics & Probability	-.165	.051	-.460	-.264	-.581	-.176	-.252	-.536
Attitude Toward Mathematics	-.039	-.448	-.447	-.104	-.393	-.206	-.085	-.506

Table Continues

Table 46 (Cont'd)

Comparison of Skewness and Kurtosis of Pre-Test and Post-Test Sample Populations by Assessment Category

Assessment Category	Skewness				Kurtosis			
	AM 10 (.195) [a]	10 A (.378) [a]	PM 10 (.175) [a]	Control (.124) [a]	AM 10 (.389) [b]	10 A (.741) [b]	PM 10 (.348) [b]	Control (.248) [b]
Post-Test								
Total Score	.181	.918	-.544	.213	-.010	1.676	-.079	-.758
Number	-.201	-.336	-.569	-.076	-.411	-1.073	-.229	-.640
Patterns & Relations	.186	.411	-.348	.171	-.388	-.467	-.420	-.730
Shape & Space	-.031	.693	-.379	.058	-.590	.591	-.399	-.681
Statistics & Probability	.205	-.034	-.571	-.089	-.843	.637	-.387	-.893
Attitude Toward Mathematics	-.141	.345	-.296	-.083	.084	-.135	-.451	-.452

[a] Standard error of Skewness for both the pre-test and post-test sample populations
[b] Standard error of Kurtosis for both the pre-test and post-test sample populations.

When comparing the skewness and kurtosis of the various distributions, the standard error of each should be taken into account. When this is done the distributions are judged to be relatively similar in shape. Although the population distributions about the mean are not symmetrical, the consistently similar shape indicates that an analysis of variance is most likely to be valid (Howell, 1995). The most noticeable difference across the groups with respect to skewness and kurtosis is seen in the Mathematics 10A sample distribution. When the 10A and PM 10 sub-groups are considered together as a Combined Control Group population, the difference is eliminated (i.e., both the Control and Experimental Group population distributions are skewed in the same direction, or if in opposite directions, the difference is very slight). The small sample size for the 10A sub-group can cause this deviation from normality and as a result, the ANOVA results with respect to 10A should be considered suspect.

As long as the largest variance is no more than five times the smallest, the analysis of variance is most likely to be valid (Howell, 1995). Table 34 compares the variances and shows the largest ratio of variances for given assessment categories.

Table 47

Comparison of Variances of Pre-Test and Post-Test Sample Populations by Assessment Category

	Variance in Pre-Test Scores				Variance in Post-Test Scores			
Assessment Category	AM 10	10 A	PM 10	Largest Ratio	AM 10	10 A	PM 10	Largest Ratio
Total Score	17.98	14.90	26.11	1.75	22.94	20.98	46.24	2.20
Number	1.85	1.51	1.74	1.23	1.72	1.49	2.40	1.61
Patterns & Relations	2.76	2.37	4.45	1.88	3.31	2.37	5.02	2.12
Shape & Space	3.57	3.06	5.38	1.76	5.95	5.81	8.82	1.52
Statistics & Probability	2.56	1.69	2.19	1.51	3.20	1.74	3.57	2.05
Attitude Toward Mathematics	7.95	6.55	8.88	1.36	9.18	8.58	10.18	1.19

As none of the variance ratios comes close to a value of five (highest ratio being 2.20), it is apparent that transformation of the student data is not warranted. Although ANOVA is robust with respect to unequal variances, the small sample size of the 10A group (n = 39), if combined with a large variance, can result in the *F* test being too liberal, leading to increased Type I error and an inflated alpha level (Tabachnick & Fidell, 1996). The unequal variances (within the ratio range noted above), can be compensated for using Tamhane's T2 method during the post hoc multiple comparison calculation.