

# Race Car Suspension Modelling

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## Abstract

This study develops and validates a suspension model for a Formula SAE race car, aiming to enhance performance optimization and system integration. The vehicle's dynamic response was modeled using a seven-degree-of-freedom (7DoF) state-space vibration framework coupled with a two-dimensional, double-track yaw model. Experimental modal analysis was employed to validate the computational models, involving bump and hammer tests to identify dynamic parameters such as natural frequencies and damping ratios. The circle fit method was used to estimate damping ratios and natural frequencies from Nyquist plots and was critical for identifying closely spaced modes.

## Background

Tires are complicated, non-linear mechanical elements. A common practice for modelling tire behaviour is to curve fit an easy-to-use mathematical function. One of the most common functions to use is the 'magic formula' by Hans Pacejka [6]. It uses the two dominant first-order inputs: normal force and tire slip. Experimental data was used to create surface-fitted model, shown in Figure 1.

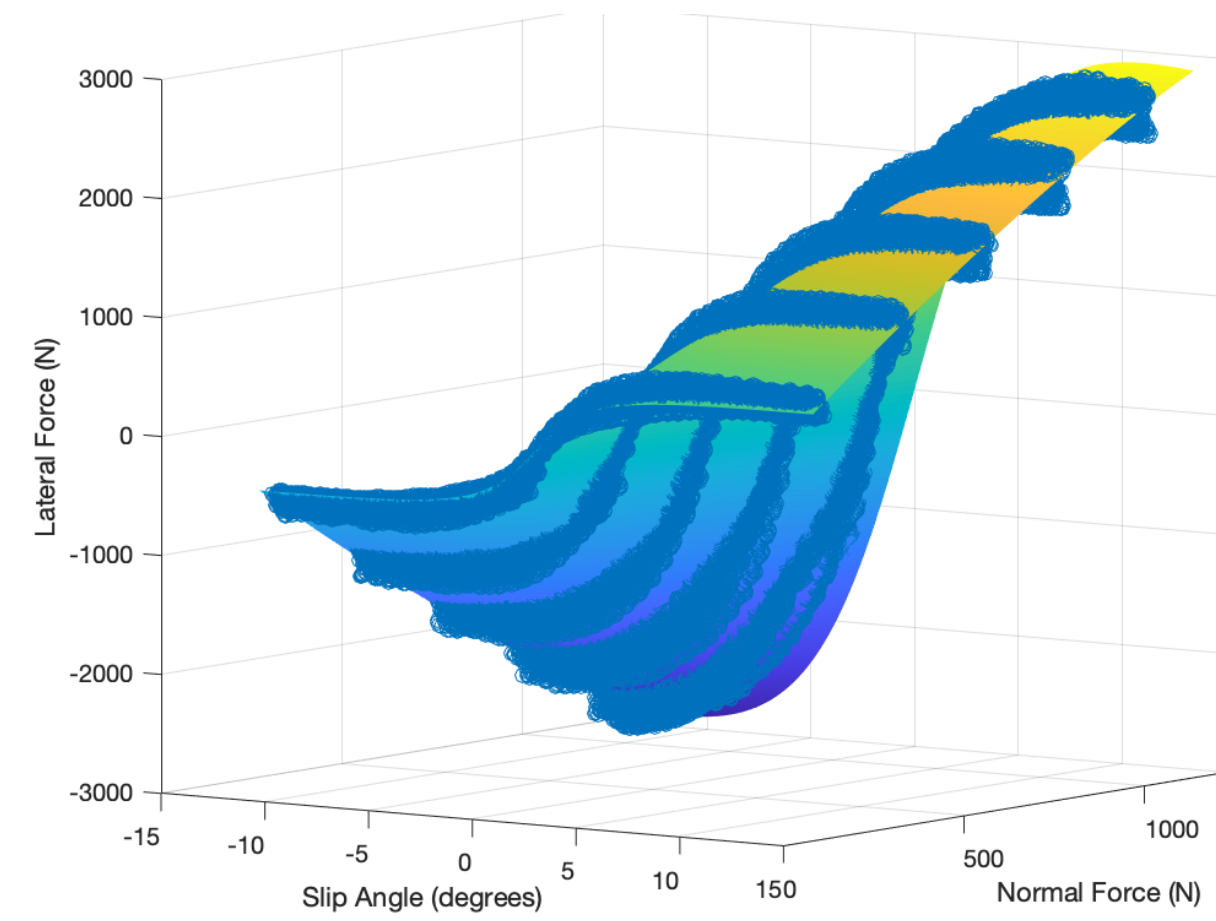


Figure 1: Lateral tire force model [5]

The differential has a large impact on the vehicle handling, so it is hard to ignore when building a vehicle simulation. An overview of the working principles and governing equations was given. The equations were then integrated into the full vehicle model. In summary, the UVic FSAE vehicle uses a limited slip differential (LSD). The LSD keeps the left a right wheels locked at the same speed until the ratio of external differential torque to driven (engine) torque reaches a threshold – at this point, the clutch disks unlock, and the two wheels turn at different speeds [8]. This gives a tunable operating envelope, like the one shown in Figure 2.

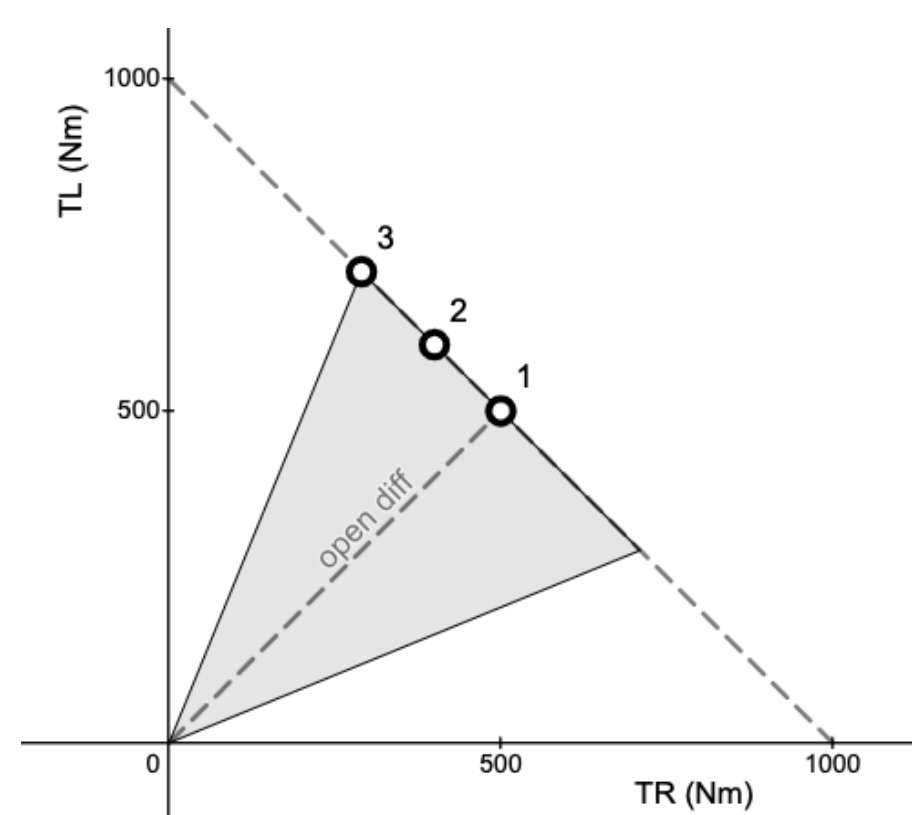


Figure 2: Limited Slip Differential Bias Diagram

## Suspension Model

A 7-degree-of-freedom (7 DoF) model was created to simulate the vehicle's suspension. The chassis has three degrees of freedom, and each wheel assembly has one. Each degree of freedom has an inertia, and they are all connected via springs, dampers, and linkages. A schematic is shown in Figure 3. Inertia, spring stiffness, and viscous damping is captured in the equations of motion.

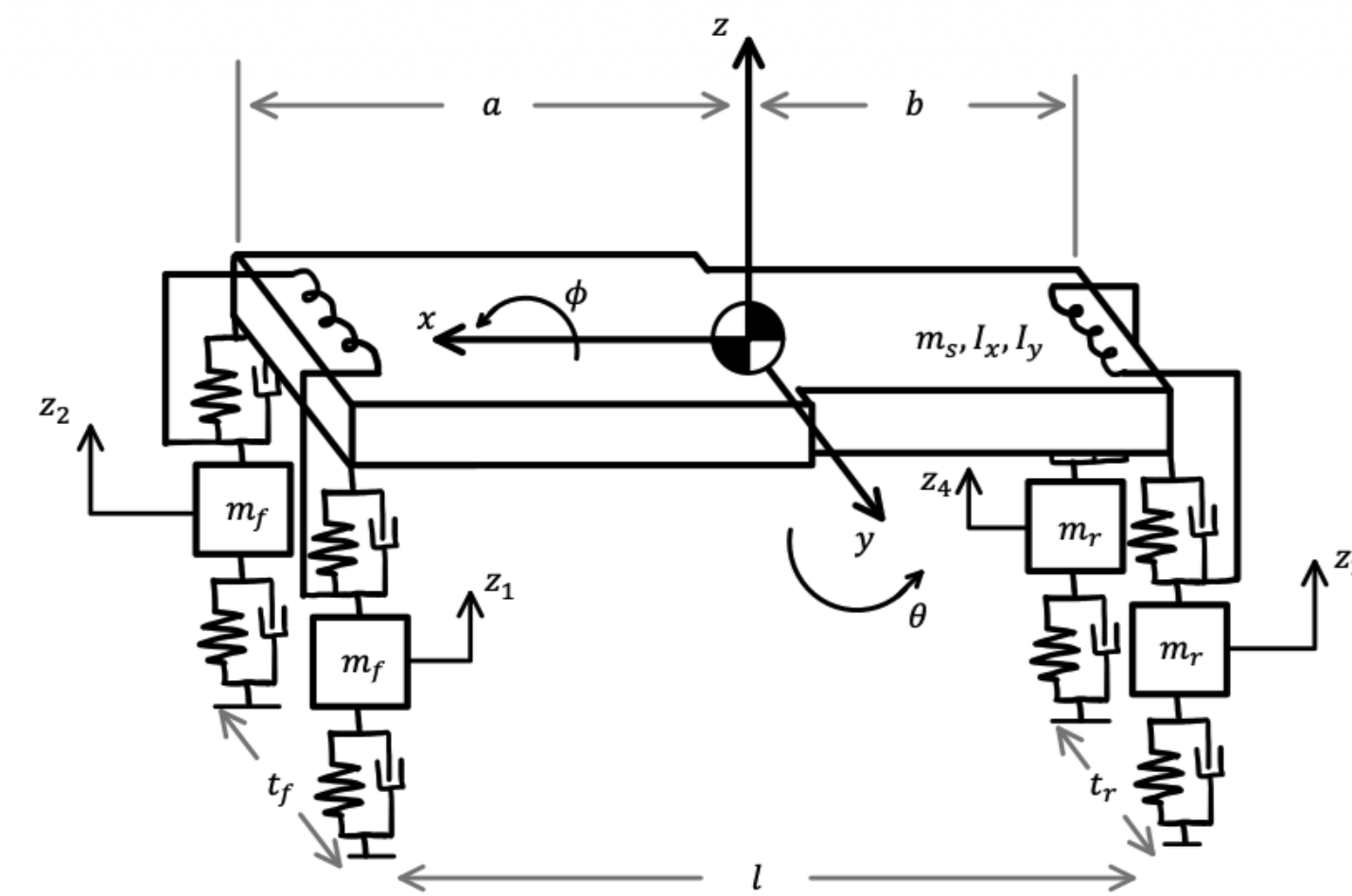


Figure 3: 7 DoF Model Schematic

The equations of motion were derived in standard form:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = F(t)$$

For time domain modelling, these equations are transformed to state space form [12]:

$$\dot{x}(t) = \begin{bmatrix} \dot{q}(t) \\ \ddot{q}(t) \end{bmatrix} = \begin{bmatrix} 0_{7 \times 7} & I_{7 \times 7} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} + \begin{bmatrix} 0_{7 \times 7} \\ M^{-1} \end{bmatrix} F(t)$$

$$\dot{x}(t) = A \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} + BF(t)$$

As modelled, this system's motion is defined by 7 mode shape vectors, each with a natural frequency and phase lag. These can be extracted by eigenvalue decomposition of the system matrices. They can also be identified by frequency response, otherwise known as modal analysis [12].

## Test Methods

Accelerometers and gyroscopes were attached at each degree of freedom and integrated into the vehicle's CAN bus data acquisition system (DAQ). The layout is shown in Figure 4. The vehicle was subject to a series of disturbances. First, a simple hammer blow to each wheel. Second, a series of carefully arranged bumps on the racetrack were driven over. The system's response to these stimuli was recorded on the DAQ for post-processing and analysis.

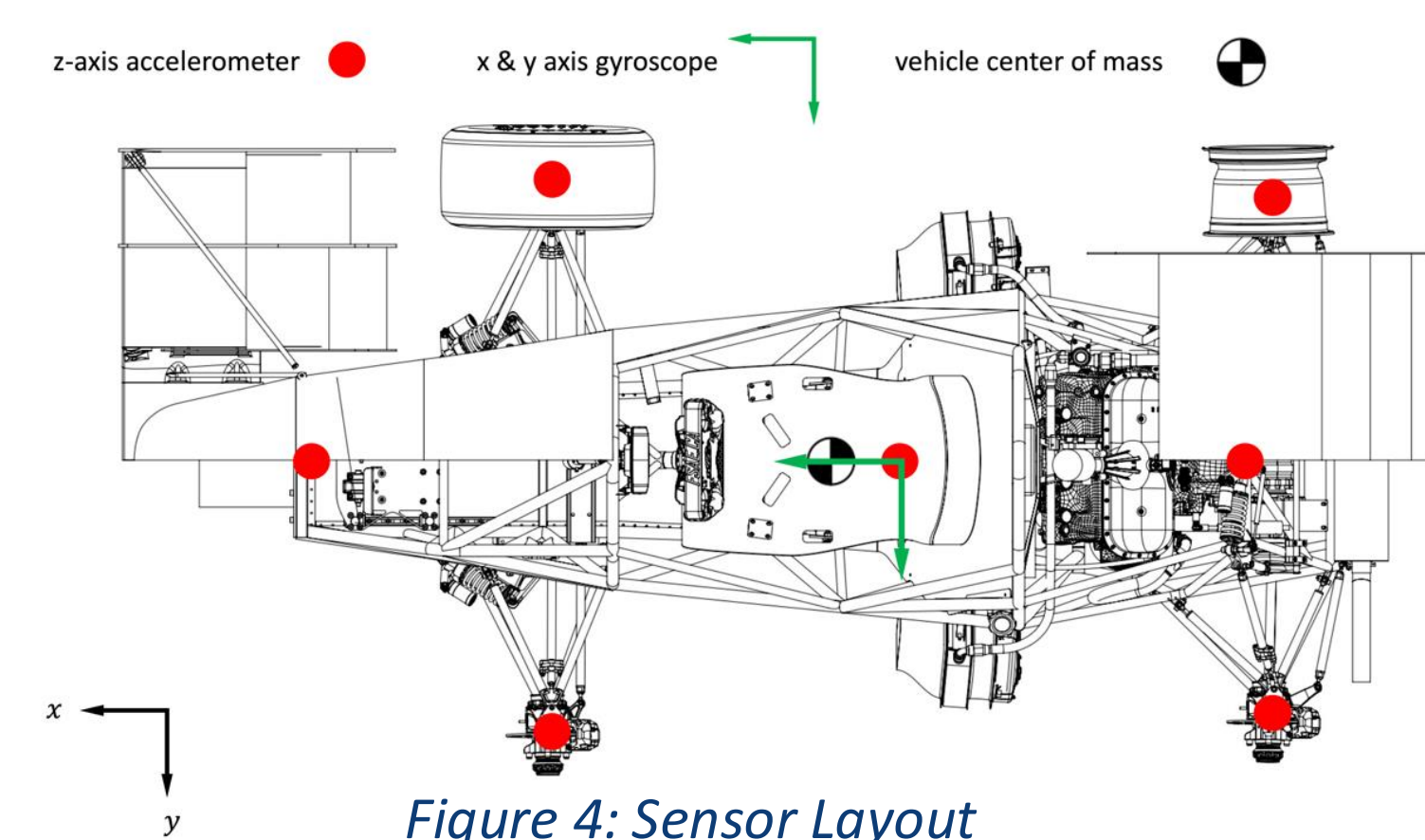


Figure 4: Sensor Layout

## Results

The first analysis was done on the free response of each wheel, where the theoretical single-degree-of-freedom (SDoF) free response equation was fit to find the natural frequency and damping ratio. A pair of time histories is shown in Figure 5, and the aggregated results are shown in Figure 6.

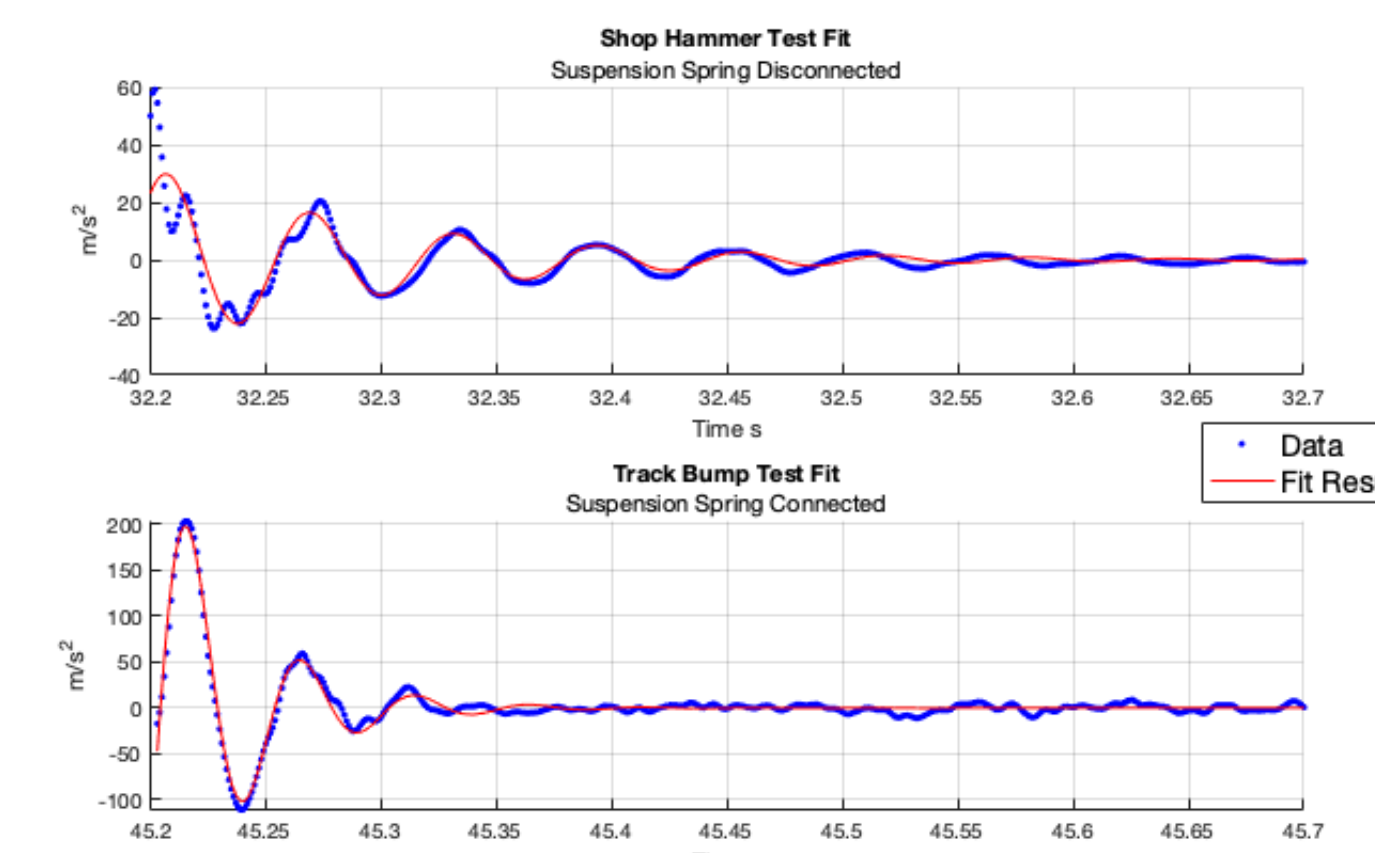


Figure 5: Single wheel time history fit example

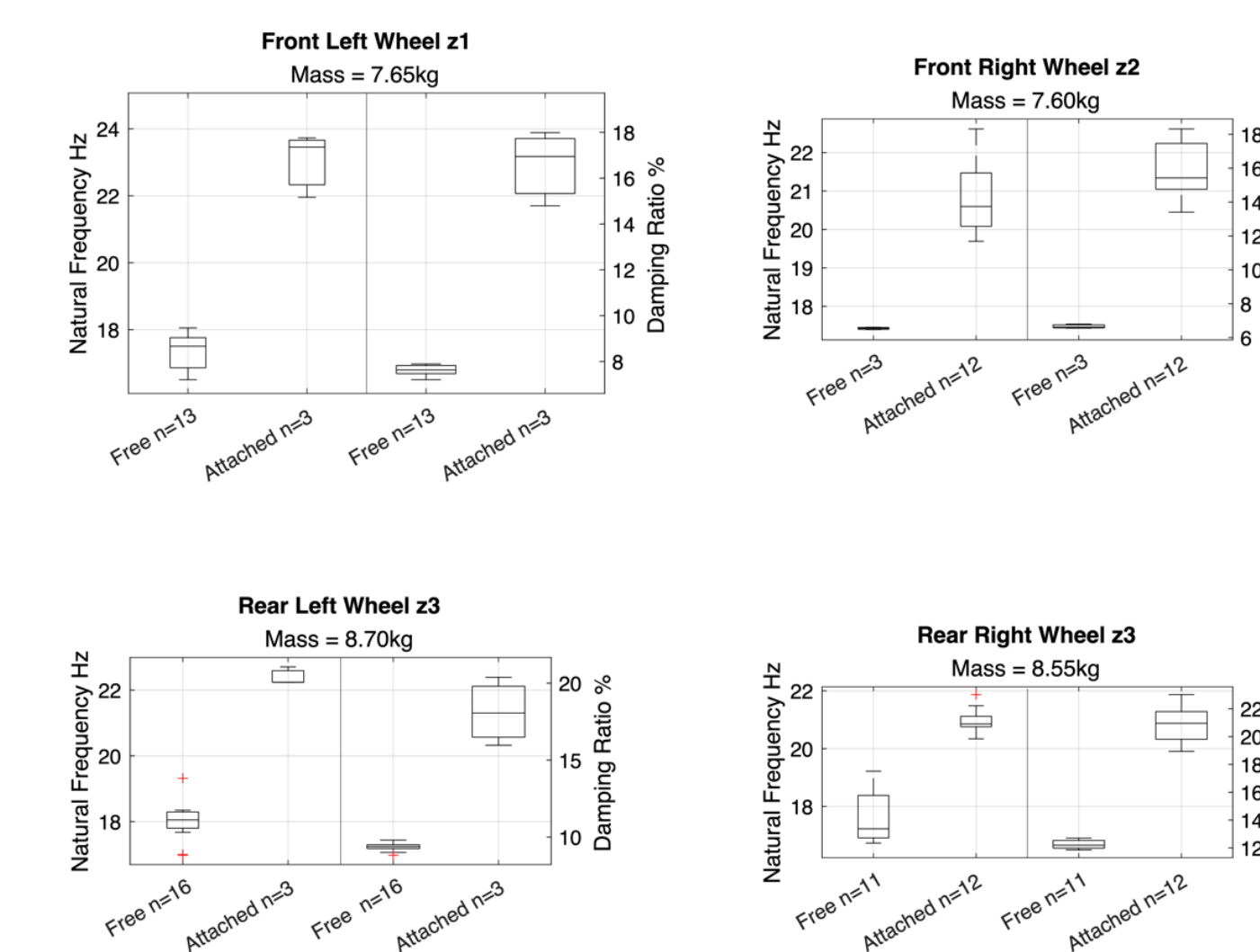


Figure 6: Aggregated single wheel data.

The chassis response was difficult to isolate in the time domain, as multi-mode excitation was always present. The circle fit frequency domain technique was used to identify each the mode shapes, natural frequencies, and damping ratios. This is shown in the bottom left of Figure 7, where a clean circle can be fit in the Nyquist plane. The natural frequency for this mode can be found where the discrete spacing between points is largest. The damping ratio can then be calculated based on the relative spacing of points on either side of the natural frequency [18].

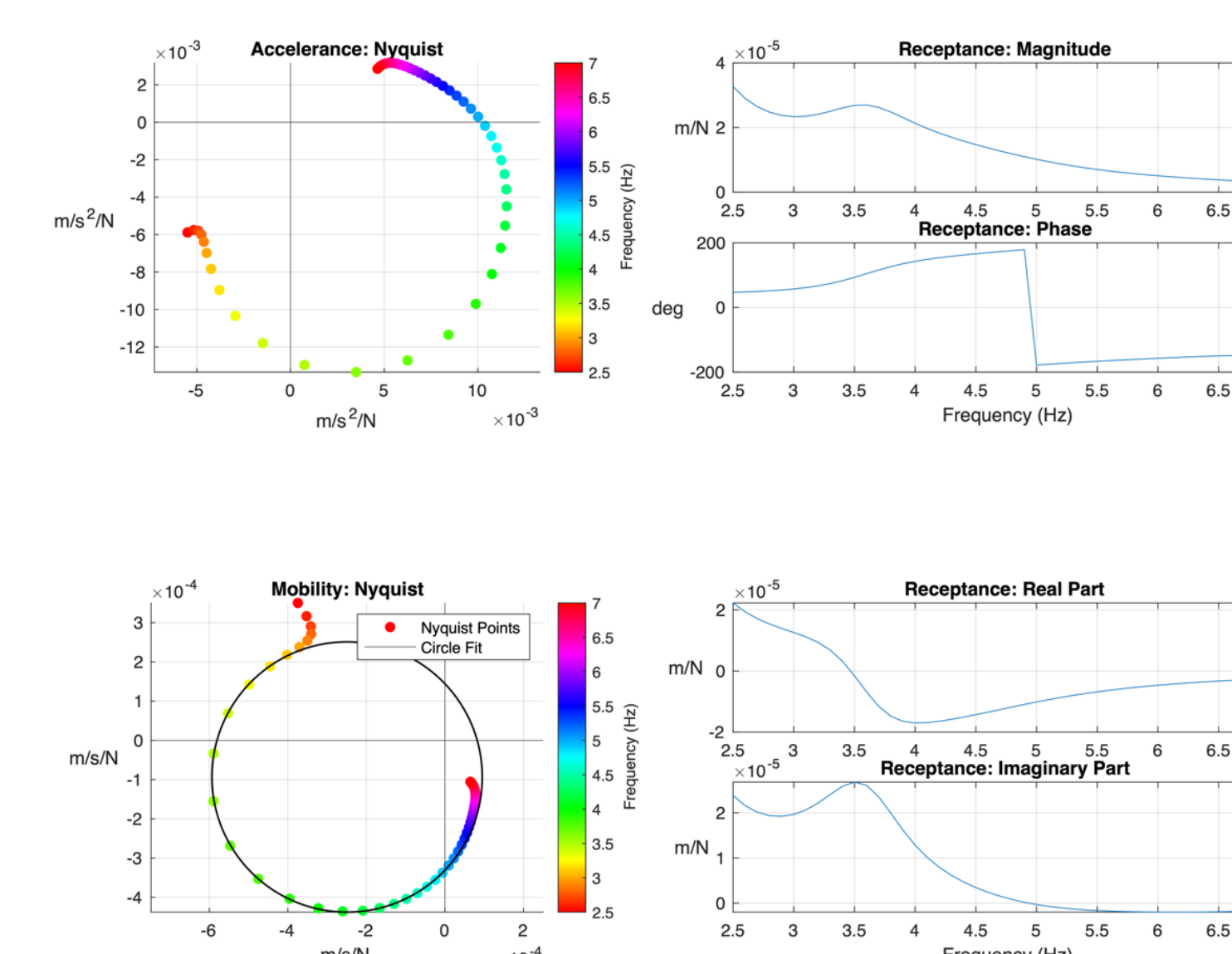


Figure 7: Chassis vertical frequency response function & circle fit. Several of these plots/analyses were done.

## Conclusion

The research presented a framework for modeling and analyzing the suspension dynamics of a Formula SAE race car. The 7DoF suspension model offered a holistic representation of vehicle behavior under dynamic conditions.

Experimental validation through bump and hammer tests successfully identified dynamic parameters, including natural frequencies and damping ratios. The circle fit method was critical for identifying natural frequencies and damping ratios, enabling pole identification even for closely spaced modes. By visualizing damping and natural frequency characteristics in Nyquist plots, this method provided precise estimations that were otherwise challenging. Overall agreement between the theoretical and measured natural frequencies indicated, at minimum, the model captured the essential dynamic characteristics of the system.

	Heave $f_n$	Roll $f_n$	Pitch $f_n$
	Hz	Hz	Hz
Theoretical	3.31	10.88	5.24
Measured	3.55	10.48/11.90	6.24

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