Precoding for MIMO Full-Duplex Relay Communication Systems

by

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ABSTRACT

Multiple antennas combined with cooperative relaying, called multiple-input multiple-output (MIMO) relay communications, can be used to improve the reliability and capacity of wireless communications systems. The precoding design is crucial to realize the full potential of MIMO relay systems. Full-duplex (FD) relay communications has become realistic with the development of effective loop interference (LI) cancellation techniques. The focus of this dissertation is on the precoding design for MIMO FD amplify-and-forward (AF) relay communication systems. First, the transceiver design for MIMO FD AF relay communication systems is considered with residual LI, which will exist in any FD system. Then the precoding design is extended to two-way MIMO FD relay communication systems. Iterative algorithms are presented for both systems based on minimizing the mean squared error (MSE) to obtain the source and relay precoders and destination combiner. Finally, the precoding design for MIMO FD relay communication systems with multiple users is investigated. Two systems are examined, namely a multiuser uplink system and a multiuser paired downlink system. By converting the original problems into convex subproblems, locally optimal solutions are found for these systems considering the existence of residual LI. The performance improvement for the proposed FD systems over the corresponding half-duplex (HD) systems is evaluated via simulation.

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Abbreviations

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
CF	Compress-and-Forward
CSI	Channel State Information
DF	Decode-and-Forward
DPC	Dirty Paper Coding
e2e	End-to-End
EVD	Eigenvalue Decomposition
FD	Full-Duplex
GSM	Global System for Mobile Communications
HD	Half-Duplex
HSPA	High Speed Packet Access
i.i.d.	Independent and Identically Distributed
KKT	Karush Kuhn Tucker
LI	Loop Interference
LTE	Long-Term Evolution
LTE-A	Long-Term Evolution Advanced
MIMO	Multiple Input Multiple Output
MMSE	Minimum Mean Squared Error
MRT	Maximum Ratio Transmission
MSE	Mean Squared Error
OFDM	Orthogonal Frequency Division Multiplexing
PSD	Positive Semidefinite
QCQP	Quadratically Constrained Quadratic Problem
QoS	Quality of Service
SDP	Semidefinite Programming
SINR	Signal to Interference and Noise Ratio
SMSE	Sum Mean Squared Error
SNR	Signal to Noise Ratio

SVD	Singular Value Decomposition
TDMA	Time-Division Multiple Access
THP	Tomlinson-Harashima Precoding
UMTS	Universal Mobile Telecommunications System
WCDMA	Wideband Code Division Multiple Access
ZF	Zero-Forcing
1G	First Generation
2G	Second Generation
3G	Third Generation
4G	Fourth Generation

Notations

Unless stated otherwise, in this dissertation boldface upper-case and lower-case letters denote matrices and vectors, respectively, and normal letters denote scalars.

x^*	the conjugate of a complex scalar x
$\mathbf{X}^{\mathrm{T}}$	the transpose of matrix $\mathbf{X}$
$\mathbf{X}^{\mathrm{H}}$	the conjugate transpose (Hermitian) of matrix $\mathbf{X}$
$\mathbf{X}_{i,j}$	the (i,j)th element of $\mathbf{X}$
$\mathrm{tr}\,(\mathbf{X})$	the trace of $\mathbf{X}$
$\mathbf{I}_M$	the identity matrix of size $M \times M$
$\mathbf{0}$	a zero vector or matrix
$\|x\|$	the absolute value of a real scalar x or magnitude of a complex scalar x
$\\|\mathbf{x}\\|$	the Euclidean norm of a vector $\mathbf{x}$
$\\|\mathbf{X}\\|_{\mathrm{F}}$	the Frobenius norm of a matrix $\mathbf{X}$
$\mathbb{E}\{\mathbf{X}\}$	the expectation of a random matrix $\mathbf{X}$
$\mathbb{V}\mathrm{ar}\{\mathbf{X}\}$	the variance of a random matrix $\mathbf{X}$
$\mathbb{R}$	the set of real numbers
$\mathbb{C}$	the set of complex numbers
$\mathbb{C}^m$	the set of complex column vectors with size m
$\mathbb{C}^{m\times n}$	the set of complex matrices with size $m \times n$
$\mathcal{CN}(\mathbf{m}, \mathbf{\Sigma})$	complex Gaussian distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{\Sigma}$
$\max$	maximize
$\min$	minimize
$\max\{x, y\}$	the maximum of x and y
$\min\{x, y\}$	the minimum of x and y
$\mathrm{diag}\{\mathbf{x}\}$	matrix with diagonal entries as the elements of $\mathbf{x}$
$\mathrm{blkdiag}\{\mathbf{A}_1, \ldots, \mathbf{A}_L\}$	matrix with diagonal entries as $\mathbf{A}_1, \ldots, \mathbf{A}_L$ in sequence
$\Re\{\mathbf{X}\}$	real part of matrix $\mathbf{X}$
$\Im\{\mathbf{X}\}$	imaginary part of matrix $\mathbf{X}$
$\det\{\mathbf{X}\}$	determinant of matrix $\mathbf{X}$
$\mathrm{vec}\{\mathbf{X}\}$	vectorization of matrix $\mathbf{X}$

$tr\{\mathbf{X}\}$	trace of matrix $\mathbf{X}$
$\mathbf{a} \succ \mathbf{b}$	$\mathbf{a}$ dominates $\mathbf{b}$
$\mathbf{a} \prec \mathbf{b}$	$\mathbf{b}$ dominates $\mathbf{a}$
$\otimes$	matrix Kronecker product

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DEDICATION

To

My parents

My wife

My son

Chapter 1

Introduction

COMMUNICATION system using electrical technology have had a significant impact on modern society. The first cellular phone was developed in 1947 by Bell Labs. In the 1980s, first generation (1G) cellular systems [1] were launched using analog technology to support simultaneous calls. The second generation (2G) cellular systems were digital [2]. The Global System for Mobile Communications (GSM) standard was launched in Finland in July 1991. It is the default standard for mobile communications in over 219 countries and territories with over 90% market occupancy. In 2001, third generation (3G) cellular systems were launched in Japan using the Wideband Code Division Multiple Access (WCDMA) standard. Subsequently, High-Speed Packet Access (HSPA) was developed, allowing Universal Mobile Telecommunications System (UMTS) networks to attain higher data rates and capacity [3].

The demand for digital technology has exploded in the 21st century with the development of electronic devices such as smart phones, smart watches, and tablets. This has resulted in a dramatic increase in data requirements and bandwidth consumption. Thus, 3G networks will be overwhelmed by the growth of bandwidth-intensive applications such as streaming media. As a consequence, the Long Term Evolution (LTE) standard was released in December 2008 and later the LTE-Advanced (LTE-A) standard was introduced, which is known as Fourth Generation (4G) cellular [4].

Mobile phone applications are increasing dramatically, resulting in demands for more bandwidth and more reliable communication systems. Thus, there is an urgent need to develop the next generation communication systems. However, different from wired media such as coaxial cables, twisted pairs and optical fibers, the open transmission environment for wireless communications has significantly more severe attenuation and signal fluctuations, as well as noise and interference. Therefore, sophisticated techniques must be used to achieve satisfactory wireless system performance. This along with the need for more flexible and reliable wireless communications has led to the development of technologies such as multiple-input multiple-output (MIMO) communications, cooperation and full-duplex (FD) communications.

1.1 Cooperative Wireless Communications

The concept of cooperative communications was first introduced in 1998 [5]. An approach was proposed to utilize the signals overheard at a third party other than the source and destination nodes to aid communications. The concept of cooperative communications with the work of Cover and El Gamal which provided an information theoretic perspective [6].

Relays are employed to extend cell coverage in a multiuser environment and improve performance. The simplest relay system model has one source node, one destination node and one relay node. The source transmits the signal which is received by the relay. Then the relay forwards the received signal to the destination. Several cooperative protocols were discussed in [7]. The most popular cooperation strategies are amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF).

With the AF strategy, the relay simply amplifies and forwards the received signal to the destination. The signal received at the relay is noisy, and the noise is also amplified. Nevertheless, AF is a simple method that has been used in many cooperative communication systems. AF is also referred to as non-regenerative relaying. A key advantage of non-regenerative relaying is that the relay is transparent to the modulation and coding employed by the source and destination and thus is simple to implement. Furthermore, the signal processing delay is negligible with this strategy [8]. The AF protocol is adopted as the cooperative model in this dissertation.

The DF strategy was proposed because the AF strategy is susceptible to noise amplification. With the DF approach, the relay decodes the received signal and then modulates the resulting message and forwards it to the destination. A good source-to-relay channel is required to guarantee that the message encoded by the relay is correct. Otherwise an incorrect message will be forwarded to the destination, which may result in error propagation. The relay can use a different channel code than the one used by the source to reduce the end to end (e2e) error rate.

The CF strategy uses a quantization codebook at the relay to encode the quantized samples of the received signal. The destination performs combining and decoding on the received signal. Therefore the CF strategy is suitable for cooperative wireless systems with good relay-to-destination channels.

Most of the cooperative communication systems considered in the literature assume that the relay receives and forwards the signal separately. In time-division multiple access (TDMA) systems, the uplink and downlink transmissions are on separate frequency bands and the relay works in half-duplex (HD) mode within the same frequency band, so source-to-destination communications requires two time slots. This extra time slot reduces the spectral efficiency, thus full-duplex (FD) mode has been proposed as it has the potential to double the spectral efficiency of HD systems.

1.2 Full-Duplex Wireless Communications

In the past, it was generally not possible for radios to receive and transmit on the same frequency band simultaneously because of the resulting interference. Thus, bidirectional systems must separate the uplink and downlink channels into orthogonal signaling dimensions typically using time or frequency dimensions [9], so radios operate in HD mode. This changed with the design and implementation of an in-band full-duplex WiFi radio that can transmit and receive simultaneously on the same frequency channel [10]. Since then, FD radios have been investigated as a promising technique for next generation wireless communication systems because of the potential to double the spectral efficiency of HD systems. However, the performance of FD systems is degraded by the loop interference (LI) introduced by signal leakage from the transmitter to the receiver. The LI can be significantly higher ($60 - 90$ dB) than the received source signal [11], thus LI cancellation in FD systems is a critical issue. As a result, numerous LI mitigation schemes have been proposed which can be classified as natural isolation, time domain cancellation, and spatial suppression [12].

Loop interference can be suppressed in the spatial domain by employing multiple antennas. For instance, linear receive and transmit filters can be used to reduce the effects of LI. Note that time domain cancellation and spatial suppression are not mutually exclusive schemes. The residual LI after using one technique can be further reduced by applying the other scheme. In [13], null space projection and Minimum Mean Squared Error (MMSE) filters were used for spatial and time domain LI cancellation, respectively. In [14], a spatial loop interference nullification method employing additional transmit antennas was employed for FD AF MIMO relay systems. The additional antennas are used to null the interference using the increased number of degrees of freedom for the precoding matrix at the relay.

In [15], joint analog and digital loop interference cancellation was considered for a FD AF relay with a single receive antenna but multiple transmit antennas. It was assumed that the residual LI after analog suppression in the relay receive RF chain is perfectly canceled, which is not realistic. In [11], a FD radio design using signal inversion and adaptive cancellation was proposed. This method supports wideband and high power systems and there is no limitation on the bandwidth or power. It was shown that signal inversion can reduce the LI by at least 45 dB over a 40 MHz bandwidth. Further, adding adaptive cancellation can reduce the LI by up to 73 dB for a 10 MHz OFDM signal. In [16], three LI cancellation mechanisms were considered for a FD off-the-shelf MIMO radio. These mechanisms are antenna separation with digital cancellation, antenna separation with analog cancellation, and antenna separation with analog and digital cancellation. The LI signal was suppressed by more than 70 dB and the power of the interfering signal after cancellation is well approximated as a linear function of the transmit power of the interfering antenna. Results were obtained which show that if the LI is canceled before the interfering signal reaches the receiver front end, then the achievable rate of the FD system can be higher than the achievable rate of a half-duplex system with the same resources. Active cancellation mechanisms exploit the fact that a node has knowledge of its transmitted signal to cancel the interference. Spatial cancellation is also an active cancellation mechanism. Passive cancellation techniques isolates the transmit and receive antennas using techniques such as directional antennas, shielding, and cross-polarization. Moreover, LI can also be eliminated using physical isolation of the antennas. This can be achieved by the introduction of physical obstacles between transmit and receive antennas. As a practical consideration, imperfect loop interference cancellation is assumed and so that the residual LI is considered throughout this dissertation.

1.3 Precoding Design in MIMO Relay Systems

The advantages of MIMO systems have been widely acknowledged and thus MIMO technology has been incorporated into many wireless standards such as IEEE 802.11ac/n (WiFi), IEEE 802.16e (WiMAX), and LTE/LTE-A (4G) [17]. MIMO techniques deliver significant performance enhancements in terms of data rate and interference reduction. Precoding is a efficient way to fully realize the benefits of MIMO systems and can be classified into two groups, those designed to increase the transmission rate and those designed to improve the reliability. Precoding requires knowledge of the channel state information (CSI), so it is very important to understand the nature of wireless channels.

In conventional single stream beamforming, the same signal is transmitted from each transmit antenna with appropriate weights so that the signal power is maximized at the receiver. However, this beamforming cannot maximize the signal power at all receive antennas when the receiver is equipped with multiple antennas. MIMO precoding is a generalization of beamforming which supports multi-stream transmission to improve the system performance. This precoding can be either linear or nonlinear. Linear precoding includes maximum-ratio transmission (MRT), zero-forcing (ZF) precoding and minimum mean square error (MMSE) precoding [18]. ZF is a suboptimal approach that is popular because it has low computational complexity [19]. MMSE precoding can be interpreted as a Wiener transmit filter which is obtained by minimizing the sum mean square error (SMSE). Nonlinear precoding is based on the concept of dirty paper coding (DPC). Tomlinson-Harashima precoding (THP) is a well-known nonlinear precoding technique which employs modulo operations and successive interference cancellation [20]. DPC can be used to remove interference if the channel state information (CSI) is known at the transmitter. However, in practice, this CSI is limited due to estimation errors and quantization. In a communication system, the channel is estimated from the received signal, and pilot signals are typically inserted in the transmitted signal to facilitate channel estimation. Then the transmitter acquires the CSI from the receiver using feedback [21].

Joint precoding design at both the source and relay for MIMO relaying with AF has been investigated [22–24]. In [22], joint source, relay, and destination precoder design for a multiuser MIMO relay communication system with all nodes equipped with multiple antennas was considered using the MMSE criterion. The system has multiple sources but only one destination. Conversely, the precoder optimization problem was investigated in [22] for a relay system with multiple source nodes and multiple destination nodes. A joint source and relay precoding design for MIMO two-way relaying was considered in [24] with multiple antennas at both the source and relay.

The systems considered in [22–24] have a HD relay node. Thus, precoding with full-duplex relaying can be employed to improve the spectral efficiency [25–29]. In [25], a joint precoding/decoding design that maximizes the end-to-end (e2e) performance was investigated for a MIMO FD relay system. Spatial mitigation of the LI was employed at the relay and the LI was assumed to be canceled completely. Joint source and relay precoding for a one-way full-duplex MIMO system was investigated in [26]. The sum rate was maximized with a threshold for the loop interference at the relay. In [27], several precoder and weight vector designs were developed considering the signal to leakage plus noise ratio, minimum mean square error (MMSE) and zero forcing for a FD MIMO relay system. In [28], full-duplex radios was used to improve the spectrum efficiency in a two-way relay system where two sources exchange information through a multi-antenna FD relay. Instead of just suppressing the loop interference, the goal is to maximize the e2e performance by jointly optimizing the beamforming matrix at the AF relay and the power at the sources. A joint multi-filter design scheme was considered in [29] for inter-antenna/multi-stream interference and LI suppression to minimize the MSE at the destinations. Different from the approaches in the literature, FD AF MIMO relay systems are considered in this dissertation where the source, relay and destination nodes have multiple antennas. The precoding matrices at the nodes are designed considering the residual LI.

1.4 Summary of Contributions

This dissertation considers the precoding design for a MIMO full-duplex relay communication system from a practical perspective. The main results are presented in Chapters 2, 3 and 4 and are summarized below.

Chapter 2 investigates the linear source and relay precoder and destination combiner design for multiple-input multiple-output (MIMO) full-duplex relay communication systems. The design criterion is minimizing the MSE under transmit power constraints at the source and relay. This problem is non-convex and a closed-form solution is intractable, so the original problem is translated into three subproblems which are solved iteratively. The convergence of the proposed algorithm is analyzed. In order to reduce the computational complexity, a simpler bi-step iterative solution is given. Simulation results are presented which show that the proposed FD system has almost doubled the achievable rate of the corresponding HD system. The effect of the residual loop interference is examined with respect to the achievable rate. The bi-step solution provides performance comparable to the tri-step iterative solution with lower complexity. Thus, this approach provides a good tradeoff between performance and complexity.

Chapter 3 presents the precoding design for two-way MIMO full-duplex amplify-and-forward relay communication systems. The joint precoding design for both the source and relay nodes for MIMO two-way relaying with the AF strategy is investigated to minimize the SMSE under transmit power constraints at the source and relay nodes. The system has multiple antennas at the source and relay nodes. Two iterative algorithms are introduced to solve this non convex optimization problem by translating the original problem into three/two convex subproblems which are solved alternately. It is shown that both iterative algorithms converge. Results are given which show that the sum achievable rate of the FD system is greater than that of the corresponding HD system, but the residual LI degrades this rate. In addition, the bi-step algorithm has lower computational complexity and comparable performance to the tri-step algorithm.

Chapter 4 considers the precoding design for MIMO full-duplex AF relaying with multiple users. Two systems are investigated. The first is a multiuser uplink MIMO FD AF relaying system where multiple sources have multiple antennas. The second is a multiuser paired system where multiple sources and destinations which have multiple antennas. Iterative algorithms are introduced for both systems to minimize the MSE with transmit power constraints at the sources and relay. The effects of the residual LI on the achievable rate of the system are considered.

1.5 Dissertation Organization

The remainder of this dissertation is organized as follows. Chapter 2 considers the precoding design for MIMO one-way FD relay communication systems. To further improve the spectral performance of MIMO relaying systems, Chapter 3 proposes precoding for MIMO two-way FD relaying systems and the performance of the proposed schemes are examined. Chapter 4 considers precoding for multiple users in MIMO FD relay communication systems. Finally, Chapter 5 provides the conclusions of this dissertation and some suggestions for future work.

1.6 Publications

Submitted papers

Y. Shao, Y. Dai, X. Dong, and T. A. Gulliver, "Transceiver design for MIMO full-duplex amplify-and-forward relay communication systems".

Y. Shao, Y. Dai, X. Dong, and T. A. Gulliver, "Precoding for MIMO full-duplex amplify-and-forward relay communication systems".

Y. Shao, and T. A. Gulliver, "Precoding design for two-way MIMO full-duplex amplify-and-forward relay communication systems".

Y. Shao, and T. A. Gulliver, "Precoding for multiuser MIMO full-duplex amplify-and-forward relay uplink communication systems".

Y. Shao, and T. A. Gulliver, "Precoding design for multiuser MIMO full-duplex amplify-and-forward relay downlink communication systems".

Chapter 2

Precoding Design for MIMO Full-Duplex Amplify-and-Forward Relay Communication Systems

In order to increase the spectral efficiency of communication systems, Full-duplex (FD) multiple-input multiple-output (MIMO) relay has been considered as an effective scheme and have attracted considerable research from both academia and industry. Further, the loop interference (LI) is one of the key channellings in FD systems. In this chapter the linear source and relay precoder and destination combiner design for multiple-input multiple-output (MIMO) full-duplex (FD) relay communication systems is examined. The effect of the residual interference due to imperfect LI cancellation is considered in the design. Two design algorithms are proposed to minimize the mean squared error (MSE) of the received signal at the destination. The first is a tri-step alternating iterative algorithm while the second is a bi-step iterative algorithm which has lower complexity and performance comparable to that of the first algorithm. The convergence of these iterative algorithms is analyzed. Results are presented which show that the proposed FD relay system can provide approximately double the achievable rate of the corresponding half-duplex (HD) system if the residual interference is not high.

2.1 Introduction

MULTIPLE transmit and receive antennas in wireless systems, known as MIMO (multiple-input multiple-output) systems, were first devised in the 1970s [30]. Since then, they have been extensively investigated due to the advantages of improved spectral efficiency and higher reliability. The concept of relaying was first introduced in [31] and has been extended to MIMO communication systems. Relay systems have been shown to increase achievable rate and extend the coverage of wireless communication systems.

There are two main techniques for relay communication systems [32]. With the regenerative approach, the relay node first decodes the received data, then re-encodes the data and sends it to the destination. However, with the non-regenerative approach, the relay only amplifies the received signal and forwards it. The regenerative technique is also called decode-and-forward (DF), and the non-regenerative technique is known as amplify-and-forward (AF). The AF approach has been the subject of significant research [33–35] due to the improved system coverage and low implementation complexity.

The focus in the literature has been on half-duplex relaying [22, 24, 32–39]. However, with the development of new signal processing techniques and antenna designs, full-duplex relaying in MIMO systems has become practical. The most critical issue with full-duplex relaying is the loop interference (LI) at the relay due to the simultaneous signal transmission and reception. Loop interference cancellation techniques can be categorized as passive or active. Passive methods isolate the transmit and receive antennas using techniques such as directional antennas, shielding, and cross-polarization. Active methods exploit the fact that a node has knowledge of its transmitted signal to suppress the interference using techniques such as temporal or spatial cancellation [40].

The development of loop interference cancellation techniques has led to a significant increase in interest in full-duplex (FD) relaying because of its potential to provide increased capacity compared to half-duplex (HD) relaying [41–44]. The design and implementation of a full-duplex WiFi-PHY based MIMO system that practically achieves the theoretical doubling of throughput was presented in [10]. The key challenge is the residual loop interference left after cancellation which translates into a decrease in the signal-to-noise ratio (SNR). In [44], the achievable rate of a FD MIMO system was analyzed considering the effect of the residual LI.

Precoding can be employed in MIMO relay systems to enhance the system performance [22, 25, 36, 42, 43, 45]. The optimal source and relay precoder and destination combiner matrix designs for a linear AF uplink MIMO relay system were considered in [22]. A robust design for AF MIMO relay systems with a direct link and imperfect channel state information (CSI) was proposed in [45]. In [36], the optimal source and relay precoder designs were considered for a multiuser MIMO AF relaying system with direct links and imperfect CSI. Solutions were obtained for a HD system based on the minimum mean squared error (MMSE) using an alternating optimization strategy. In [37], joint precoding optimization for multiuser multi-antenna relay down links using quadratic programming was studied in terms of maximizing the capacity.

The recent introduction of MIMO full-duplex systems [10] has led to research on precoding for MIMO FD relay systems. A FD relay precoder design was presented in [42], and a comparison given of the capacity with HD and FD relaying. It was shown that a FD system can achieve almost double the capacity of a HD system if there is no residual loop interference. A low complexity FD MIMO relay system with joint ZF-based precoding was developed in [25], but it was assumed that the relay employs multiple antennas while the source and destination nodes have only single antennas. The ergodic capacity with imperfect CSI was derived and zero-forcing (ZF) precoding was used to mitigate the residual loop interference. A more practical FD AF relay network was investigated in [46] considering the impact of loop interference. It was found that the capacity and outage performance are improved as the transmit power at the source is increased. However, an increase in the relay transmit power results in an increase in the loop interference which can degrade the capacity and outage performance. To the best of our knowledge, the joint design of the source and relay precoders and the destination combiner matrices has not been considered with FD relaying. Thus, this problem is considered here including the effects of the residual loop interference (LI) caused by imperfect LI cancellation.

In this chapter, precoding design for a MIMO full-duplex (FD) relay communication system is presented based on optimization of the source precoder, relay precoder and destination combiner using the MMSE criterion. Different from the approaches in [25, 42], the mean squared error is minimized under transmit power constraints at the source and relay nodes. Since the original optimization problem is non-convex and a closed-form solution is intractable, the original problem is translated into subproblems which can be solved iteratively. It is shown that this algorithm converges to a locally optimal solution. Since the computational complexity of the proposed tri-step iterative algorithm is high, a low complexity bi-step solution is given. Results are presented which show that this bi-step solution provides performance comparable to that with tri-step iterative algorithm. Such a complexity performance trade off is desirable for practical FD MIMO relay communication systems. The achievable rate improvement with a FD relay over that with a HD relay is also illustrated, along with the impact of residual loop interference.

The remainder of this chapter is organized as follows. In Section 2.2, the system model of the precoding FD MIMO AF relay communication system is presented. The optimization problem based on the MMSE criterion is also given. Two iterative algorithms are introduced in Section 2.3, and the corresponding achievable rate is given. Section 2.4 provides numerical results which demonstrate the performance improvements. Finally, some conclusions are given in Section 2.5.

2.2 System Model

A three node MIMO full-duplex (FD) relay system is considered where the direct link between the source and destination is negligible due to large-scale fading and the long distance between the two nodes. As shown in Fig. 2.1, the source and destination are equipped with N_s and N_d antennas, respectively. A non-regenerative relay is employed to amplify the signal from the source and forward it to the destination. The relay operates in FD mode and employs N_r and N_t antennas to receive and transmit signals simultaneously. Thus, source to destination communications is accomplished in one time slot compared to a HD system that requires two time slots.

Let $\mathbf{s}[n] \in \mathbb{C}^{L\times 1}$ be the length L signal vector transmitted by the source at time n. Without loss of generality, it is assumed that $L \leq \min\{N_s, N_d, N_t, N_r\}$. In addition, we assume $\mathbb{E}[\mathbf{s}[n]\mathbf{s}[n]^H] = \mathbf{I}_L$, where $(\cdot)^H$ denotes conjugate transpose (Hermitian) and $\mathbb{E}(\cdot)$ represents expectation. Let $\mathbf{H}_{sr} \in \mathbb{C}^{N_r\times N_s}$ and $\mathbf{H}_{rd} \in \mathbb{C}^{N_d\times N_t}$ denote the source-to-relay and relay-to-destination channel matrices, respectively.

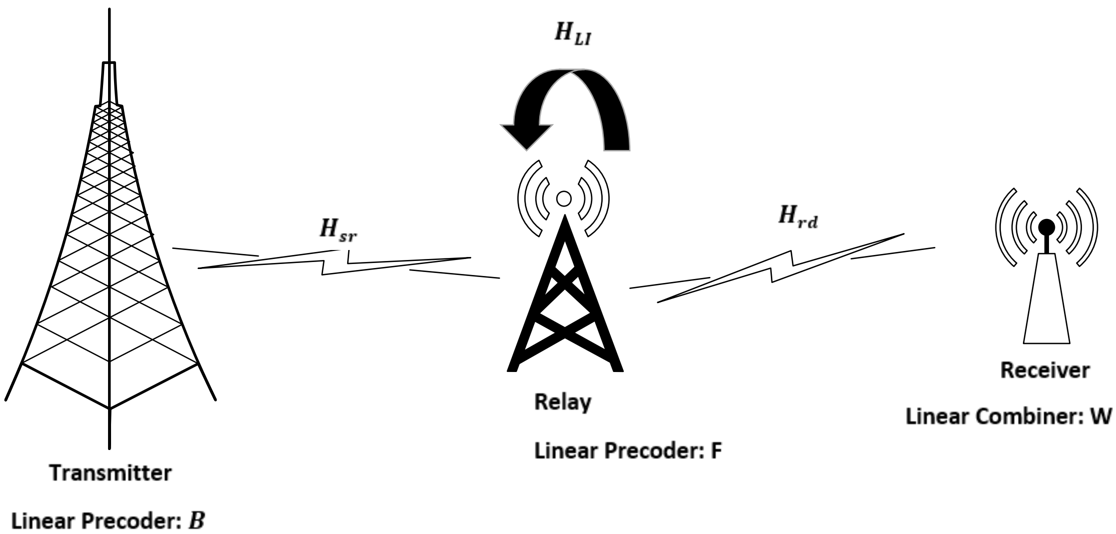

Figure 2.1: The MIMO full-duplex (FD) relay system model.

As shown in Fig. 2.1, the source precoding matrix $\mathbf{B}[n] \in \mathbb{C}^{L\times N_s}$ is applied to $\mathbf{s}[n]$ and the resulting signal is sent to the relay. The received signal at the relay can be expressed as

$$\mathbf{y}_r[n] = \mathbf{H}_{sr}[n]\mathbf{B}[n]\mathbf{s}[n] + \mathbf{H}_{LI}[n]\mathbf{t}[n] + \mathbf{n}_r[n], \tag{2.1}$$

where $\mathbf{H}_{LI} \in \mathbb{C}^{N_r\times N_t}$ denotes the loop interference (LI) channel matrix and $\mathbf{n}_r \in \mathbb{C}^{N_r\times 1}$ is the noise vector at the relay. After employing a LI cancellation technique, (2.1) can be written as

$$\mathbf{y}_r[n] = \mathbf{H}_{sr}[n]\mathbf{B}[n]\mathbf{s}[n] + \mathbf{H}_{LI}[n]\mathbf{t}[n] + \mathbf{T}[n] + \mathbf{n}_r[n], \tag{2.2}$$

where $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\mathbf{t}[n]$ when perfect LI cancellation is applied. However, in an actual system $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\tilde{\mathbf{t}}[n]$ where $\tilde{\mathbf{t}}[n]$ is a noisy version of $\mathbf{t}[n]$ due to imperfect LI cancellation. Therefore, the received signal at the relay can be expressed as

$$\mathbf{y}_r[n] = \mathbf{H}_{sr}[n]\mathbf{B}[n]\mathbf{s}[n] + \mathbf{H}_{LI}[n]\Delta\mathrm{t}[n] + \mathbf{n}_r[n], \tag{2.3}$$

where $\Delta\mathrm{t}[n] = \mathrm{t}[n] - \tilde{\mathrm{t}}[n]$ and $\mathbf{H}_{LI}[n]\Delta\mathrm{t}[n]$ is the residual LI after imperfect LI cancellation.

At time $n+1$, the FD relay amplifies the received signal with a relay precoder $\mathbf{F}[n+1] \in \mathbb{C}^{N_r \times N_t}$ and then forwards the amplified signal to the destination immediately. The resulting signal at the destination is

$$\begin{aligned} \mathbf{y}_d[n+1] = \; & \mathbf{H}_{rd}[n+1]\mathbf{F}[n+1]\mathbf{H}_{sr}[n]\mathbf{B}[n]\mathbf{s}[n] + \mathbf{H}_{rd}[n+1]\mathbf{F}[n+1] \\ & \times \mathbf{H}_{LI}[n]\Delta\mathrm{t}[n] + \mathbf{H}_{rd}[n+1]\mathbf{F}[n+1]\mathbf{n}_r[n] + \mathbf{n}_d[n+1], \end{aligned} \tag{2.4}$$

where $\mathbf{F}[n+1] \in \mathbb{C}^{N_r \times N_t}$ is the relay precoder and $\mathbf{n}_d \in \mathbb{C}^{N_r \times 1}$ is an independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) vector with zero mean and unit variance. It is assumed that the channel variations during the precoder update interval are relatively small and so can be ignored. Thus, the time index has no influence on the precoder design. For simplicity, this index is omitted to obtain a more concise expression for the received signal which can be written as [47]

$$\begin{aligned} \mathbf{y}_d \; & = \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B}\mathbf{s} + \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{LI}\Delta\mathrm{t} + \mathbf{H}_{rd}\mathbf{F}\mathbf{n}_r + \mathbf{n}_d \\ & = \bar{\mathbf{H}}\mathbf{s} + \mathbf{n}, \end{aligned} \tag{2.5}$$

where $\bar{\mathbf{H}} = \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B}$ and $\mathbf{n} = \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{LI}\Delta\mathrm{t} + \mathbf{H}_{rd}\mathbf{F}\mathbf{n}_r + \mathbf{n}_d$ are the equivalent channel and noise matrices, respectively. Δt can be modeled as white Gaussian noise [48] which is independent of n_r and n_d. Consequently, the covariance matrix of $\mathbf{n}$ can be expressed as

$$\mathbf{C}_n = \mathbb{E}[\mathbf{n}\mathbf{n}^H] = \; \sigma_t^2\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{I}_{N_d}, \tag{2.6}$$

where σ_t^2 is the variance of Δt.

A combiner $\mathbf{W} \in \mathbb{C}^{N_r \times L}$ is employed at the destination and the resulting estimate of the transmitted signal can be written as

$$\hat{\mathrm{s}} = \mathbf{W}^H\mathbf{y}_d. \tag{2.7}$$

Since i.i.d. AWGN with zero mean and unit variance is assumed, we have $\mathbb{E}[\mathbf{n}_r\mathbf{n}_r^H] =$

$\sigma_{n,r}^2\mathbf{I}_{N_r}$ and $\mathbb{E}[\mathbf{n}_d\mathbf{n}_d^H] = \sigma_{n,d}^2\mathbf{I}_{N_r}$, where $\sigma_{n,r}^2 = 1$ and $\sigma_{n,d}^2 = 1$ are the variances of $\mathbf{n}_r$ and $\mathbf{n}_d$, respectively. The problem now is how to design the linear precoders $\mathbf{B}$ and $\mathbf{F}$ at the source and relay, and the linear combiner $\mathbf{W}$ at the destination, to minimize the mean squared error (MSE) of the received signal at the destination which can be expressed as

$$\begin{aligned}\mathrm{MSE}(\mathbf{B},\mathbf{F},\mathbf{W}) &= \mathbb{E}[(\hat{\mathbf{s}}-\mathbf{s})(\hat{\mathbf{s}}-\mathbf{s})^H] \\ &= (\mathbf{W}^H\bar{\mathbf{H}}-\mathbf{I}_L)(\mathbf{W}^H\bar{\mathbf{H}}-\mathbf{I}_L)^H + \mathbf{W}^H\mathbf{C}_n\mathbf{W}.\end{aligned} \tag{2.8}$$

The corresponding optimization problem can be formulated as

$$\min_{\mathbf{B},\mathbf{F},\mathbf{W}} \ \mathrm{MSE} \tag{2.9a}$$

$$s.t.\ tr(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \le P_r \tag{2.9b}$$

$$tr(\mathbf{B}\mathbf{B}^H) \le P_s \tag{2.9c}$$

where $P_s > 0$ and $P_r > 0$ are the power budget constraints at the source and relay nodes, respectively. Unfortunately, the problem in (2.9) is non-convex, which makes determining an optimal solution difficult. Thus in the following section two iterative algorithms are developed to solve this optimization problem.

2.3 Solution of the Optimization Problem

2.3.1 Tri-step iterative algorithm

In this section, a tri-step algorithm [36, 45] is presented which is based on alternating optimization that updates $\mathbf{B}$, $\mathbf{F}$ and $\mathbf{W}$ individually while the others are fixed to solve the convex subproblems. First, given $\mathbf{B}$ and $\mathbf{F}$, the optimal combiner $\mathbf{W}$ can be obtained by solving the unconstrained convex problem since $\mathbf{W}$ is independent of the constraints in (2.9a). The optimal solution can then be obtained by taking the derivative of (2.9a) with respect to $\mathbf{W}$ and setting it to zero. Solving $\frac{\partial}{\partial \mathbf{W}}\mathrm{MSE} = 0$ gives

$$\mathbf{W} = (\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \mathbf{C}_n)^{-1}\mathbf{H}. \tag{2.10}$$

This solution is known as the Wiener filter [39].

Second, with $\mathbf{W}$ from (2.10) and given $\mathbf{B}$, $\mathbf{F}$ can be obtained by solving the following problem. Since $\mathbf{B}$ is known, the constraint in (2.9c) is eliminated so the optimization problem can be formulated as

$$\min_{\mathbf{F}} \; tr((\bar{\mathbf{H}}_{rd}\mathbf{F}\bar{\mathbf{H}}_{sr} - \mathbf{I}_L)(\bar{\mathbf{H}}_{rd}\mathbf{F}\bar{\mathbf{H}}_{sr} - \mathbf{I}_L)^H + \mathbf{W}^H\mathbf{C}_n\mathbf{W}) \tag{2.11a}$$

$$s.t. \; tr(\mathbf{F}(\bar{\mathbf{H}}_{sr}\bar{\mathbf{H}}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r, \tag{2.11b}$$

where $\bar{\mathbf{H}}_{rd} = \mathbf{W}^H\mathbf{H}_{rd}$ and $\bar{\mathbf{H}}_{sr} = \mathbf{H}_{sr}\mathbf{B}$ are the equivalent relay-to-destination and source-to-relay MIMO channels, respectively. As the problem in (2.11) is convex [24], the optimal relay precoder can be obtained by employing the KKT conditions [49]. The Lagrangian of the problem can be expressed as

$$\begin{aligned} L = \; & tr((\bar{\mathbf{H}}_{rd}\mathbf{F}\bar{\mathbf{H}}_{sr} - \mathbf{I}_L)(\bar{\mathbf{H}}_{rd}\mathbf{F}\bar{\mathbf{H}}_{sr} - \mathbf{I}_L)^H + \mathbf{W}^H\mathbf{C}_n\mathbf{W}) \\ & + \mu(tr(\mathbf{F}(\bar{\mathbf{H}}_{sr}\bar{\mathbf{H}}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) - P_r). \end{aligned} \tag{2.12}$$

By differentiating L in (2.12) with respect to $\mathbf{B}$ and $\mathbf{W}$ and equating the result to zero, the optimal $\mathbf{F}$ from (2.12) can be expressed as

$$\mathbf{F} = \; \bar{\mathbf{H}}_{rd}^H(\bar{\mathbf{H}}_{rd}\bar{\mathbf{H}}_{rd}^H + \mu\mathbf{I}_L)^{-1}\bar{\mathbf{H}}_{sr}^H(\bar{\mathbf{H}}_{sr}\bar{\mathbf{H}}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1}, \tag{2.13}$$

where $\mu \geq 0$ is the Lagrange multiplier which can be found from the complementary slackness condition given by

$$\mu(tr(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) - P_r) = 0. \tag{2.14}$$

If $\mu = 0$, we have from (2.13) that

$$\mathbf{F} = \; \bar{\mathbf{H}}_{rd}^H(\bar{\mathbf{H}}_{rd}\bar{\mathbf{H}}_{rd}^H)^{-1}\bar{\mathbf{H}}_{sr}^H(\bar{\mathbf{H}}_{sr}\bar{\mathbf{H}}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1}. \tag{2.15}$$

Since in this case $\mu = 0$ already satisfies $\mu \geq 0$, if $\mathbf{F}$ in (2.15) satisfies the constraint in (2.11b), then (2.15) is a solution to the problem in (2.11).[] If $\mu > 0$, then

$$tr(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r. \tag{2.16}$$

To find μ, substitute (2.13) into (2.16) and solve the following nonlinear equation

$$\begin{aligned} & tr(\bar{\mathbf{H}}_{rd}^H(\bar{\mathbf{H}}_{rd}\bar{\mathbf{H}}_{rd}^H + \mu\mathbf{I}_L)^{-1}\bar{\mathbf{H}}_{sr}^H(\bar{\mathbf{H}}_{sr}\bar{\mathbf{H}}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H \\ & + \mathbf{I}_{N_r})^{-1} \times \bar{\mathbf{H}}_{sr}(\bar{\mathbf{H}}_{rd}\bar{\mathbf{H}}_{rd}^H + \mu\mathbf{I}_L)^{-1}\bar{\mathbf{H}}_{rd}) = P_r. \end{aligned} \tag{2.17}$$

Using the singular value decomposition (SVD) of $\bar{\mathbf{H}}_{rd}$ given by $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ where $\mathbf{U} \in$

$\mathbb{C}^{L\times L}$ and $\mathbf{V} \in \mathbb{C}^{N_t\times N_t}$ are unitary matrices, $\mathbf{\Sigma} = \begin{bmatrix} \mathbf{S} & 0 \\ 0 & 0 \end{bmatrix}_{L\times N_t}$, and $\mathbf{S} = diag\{\sigma_1, \sigma_2, \ldots, \sigma_L\}$, (2.17) can be expressed as

$$tr(\mathbf{\Sigma}(\mathbf{\Sigma}^2 + \mu\mathbf{I}_L)^{-1}\mathbf{U}^H\bar{\mathbf{H}}_{sr}^H(\bar{\mathbf{H}}_{sr}\bar{\mathbf{H}}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H \\ + \mathbf{I}_{N_r})^{-1} \times \bar{\mathbf{H}}_{sr}\mathbf{U}(\mathbf{\Sigma}^2 + \mu\mathbf{I}_L)^{-1}\mathbf{\Sigma}) = P_r. \tag{2.18}$$

Equation (2.18) can be shown to be equivalent to [36]

$$\sum_{i=1}^{L} \frac{\sigma_i^2\gamma_i}{(\sigma_i^2 + \mu)^2} = P_r, \tag{2.19}$$

where σ_i and γ_i are the main diagonal elements of $\mathbf{\Sigma}$ and $\mathbf{\Gamma}$, respectively. We have that $\mathbf{\Gamma} = \mathbf{U}^H\bar{\mathbf{H}}_{sr}^H(\bar{\mathbf{H}}_{sr}\bar{\mathbf{H}}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1}\bar{\mathbf{H}}_{sr}\mathbf{U}$. A technique such as the bisection method can be used to find μ since the left hand side of (2.19) is monotonically decreasing with respect to μ [49].

The third subproblem is to optimize the source precoder $\mathbf{B}$ given the previously obtained $\mathbf{W}$ and $\mathbf{F}$. It is obvious that updating the source precoder can affect the power constraint at the relay. Thus, the relay power constraint (2.11b) should be included, so (2.9a) is written as

$$\begin{aligned} \text{MSE} = \; & tr((\mathbf{W}^H\mathbf{H} - \mathbf{I}_L)(\mathbf{W}^H\mathbf{H} - \mathbf{I}_L)^H + \mathbf{W}^H\mathbf{C}_n\mathbf{W}) \\ = \; & tr((\mathbf{Q}_1\mathbf{B} - \mathbf{I}_L)(\mathbf{Q}_1\mathbf{B} - \mathbf{I}_L)^H + \mathbf{\Psi}_1) \\ = \; & tr(\mathbf{Q}_1\mathbf{B}\mathbf{B}^H\mathbf{Q}_1^H) - tr(\mathbf{Q}_1\mathbf{B}) - tr(\mathbf{B}^H\mathbf{Q}_1^H) + \Psi_2, \end{aligned} \tag{2.20}$$

where $\mathbf{Q}_1 = \mathbf{W}^H\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}$, $\mathbf{\Psi}_1 = \mathbf{W}^H(\mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{W}$ and $\Psi_2 = tr(\mathbf{\Psi}_1) + tr(\mathbf{I}_L)$. Using the matrix identities $tr(\mathbf{C}^T\mathbf{D}) = (\text{vec}(\mathbf{C}))^T\text{vec}(\mathbf{D})$, $tr(\mathbf{A}\mathbf{B}) = tr(\mathbf{B}\mathbf{A})$, $tr(\mathbf{A}^H\mathbf{B}\mathbf{A}\mathbf{C}) = (\text{vec}(\mathbf{A}))^H(\mathbf{C}^T \otimes \mathbf{B})\text{vec}(\mathbf{A})$ and $\text{vec}(\mathbf{C}\mathbf{D}) = (\mathbf{I} \otimes \mathbf{C})\text{vec}(\mathbf{D})$ gives

$$\begin{aligned} tr(\mathbf{Q}_1\mathbf{B}\mathbf{B}^H\mathbf{Q}_1^H) & = (\text{vec}(\mathbf{B}))^H\text{vec}(\mathbf{Q}_1^H\mathbf{Q}_1\mathbf{B}) \\ & = (\text{vec}(\mathbf{B}))^H(\mathbf{I}_{N_t} \otimes (\mathbf{Q}_1^H\mathbf{Q}_1))(\text{vec}(\mathbf{B})) \\ & = \mathbf{b}^H(\mathbf{I}_{N_t} \otimes (\mathbf{Q}_1^H\mathbf{Q}_1))\mathbf{b}, \end{aligned} \tag{2.21}$$

and

$$tr(\mathbf{Q}_1\mathbf{B}) = (\text{vec}(\mathbf{Q}_1^T))^T\mathbf{b}, \tag{2.22}$$

where $\mathbf{b} = \text{vec}(\mathbf{B})$. Equation (2.20) can then be written as

$$\begin{aligned}\text{MSE} = & \ \mathbf{b}^H(\mathbf{I}_{N_t} \otimes (\mathbf{Q}_1^H\mathbf{Q}_1))\mathbf{b} - (\text{vec}(\mathbf{Q}_1^H))^H\mathbf{b} - \mathbf{b}^H(\text{vec}(\mathbf{Q}_1^H)) + \Psi_2 \\ = & \ \mathbf{b}^H\mathbf{\Omega}_1\mathbf{b} - \mathbf{c}_1^H\mathbf{b} - \mathbf{b}^H\mathbf{c}_1 + \Psi_2 \\ = & \ (\mathbf{b}^H\mathbf{\Omega}_1^{\frac{1}{2}} - \mathbf{c}_1^H\mathbf{\Omega}_1^{-\frac{1}{2}})(\mathbf{\Omega}_1^{\frac{1}{2}}\mathbf{b} - \mathbf{\Omega}_1^{-\frac{1}{2}}\mathbf{c}_1) + \Psi_3,\end{aligned} \tag{2.23}$$

where $\mathbf{\Omega}_1 = bd(\mathbf{I}_{N_t} \otimes (\mathbf{Q}_1^H\mathbf{Q}_1))$, $\mathbf{c}_1 = (\text{vec}(\mathbf{Q}_1^H))$, $\Psi_3 = \Psi_2 - \mathbf{c}_1^H\Omega_1^{-1}\mathbf{c}_1$, $bd(\cdot)$ denotes a block-diagonal matrix, $\mathbf{\Omega}_1^{\frac{1}{2}}\mathbf{\Omega}_1^{\frac{1}{2}} = \mathbf{\Omega}_1$ and $\mathbf{\Omega}_1^{\frac{1}{2}} = \mathbf{\Omega}_1^{\frac{H}{2}}$. The power constraint in (2.9b) can be formulated as

$$\begin{aligned} & tr(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r \\ = & \ tr(\mathbf{Q}_2\mathbf{B}\mathbf{B}^H\mathbf{Q}_2^H) + tr(\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \\ = & \ \mathbf{b}^H(\mathbf{I}_{N_t} \otimes (\mathbf{Q}_2^H\mathbf{Q}_2))\mathbf{b} + tr(\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \\ = & \ \mathbf{b}^H\mathbf{\Omega}_2\mathbf{b} + tr(\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r,\end{aligned} \tag{2.24}$$

where $\mathbf{Q}_2 = \mathbf{F}\mathbf{H}_{SR}$ and $\mathbf{\Omega}_2 = bd(\mathbf{I}_{N_t} \otimes (\mathbf{Q}_2^H\mathbf{Q}_2))$.

The original problem in (2.9) is equivalent to the following convex quadratically constrained quadratic program (QCQP) problem

$$\min_{\mathbf{b}} \ (\mathbf{\Omega_1}^{\frac{1}{2}}\mathbf{b} - \mathbf{\Omega}_1^{-\frac{1}{2}}\mathbf{c}_1)^H(\mathbf{\Omega}_1^{\frac{1}{2}}\mathbf{b} - \mathbf{\Omega}_1^{-\frac{1}{2}}\mathbf{c}_1) + \Psi_3 \tag{2.25a}$$

$$s.t. \ \mathbf{b}^H\mathbf{\Omega}_2\mathbf{b} \leq \bar{P}_r \tag{2.25b}$$

$$\mathbf{b}^H\mathbf{D}\mathbf{b} \leq P_s \tag{2.25c}$$

where $\bar{P}_r = P_r - tr(\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H)$ and $\mathbf{D} = bd(\mathbf{I}_L)$. A QCQP problem can be solved efficiently using the disciplined convex programming toolbox CVX [50]. A proof of the convexity of a problem similar to (2.25a) was given in [24]. Since Ψ_3 in (2.25a) is a constant, it does not affect the optimization, so the QCQP problem can be rewritten as

$$\min_{\mathbf{b}} \ (\mathbf{\Omega}_1^{\frac{1}{2}}\mathbf{b} - \mathbf{\Omega}_1^{-\frac{1}{2}}\mathbf{c}_1)^H(\mathbf{\Omega}_1^{\frac{1}{2}}\mathbf{b} - \mathbf{\Omega}_1^{-\frac{1}{2}}\mathbf{c}_1) \tag{2.26a}$$

$$s.t. \ \mathbf{b}^H\mathbf{\Omega}_2\mathbf{b} \leq \bar{P}_r \tag{2.26b}$$

$$\mathbf{b}^H\mathbf{D}\mathbf{b} \leq P_s \tag{2.26c}$$

a) Iterative algorithm and convergence

The proposed tri-step iterative algorithm is summarized in Algorithm 1 below. This algorithm can be shown to converge as follows. It is obvious that the three subproblems are convex. It then follows that each update of $\mathbf{B}$, $\mathbf{F}$ and $\mathbf{W}$ will decrease or at least not increase the value of the objective function, and thus the iterative algorithm converges to a locally optimum solution.

Algorithm 1 Iterative Algorithm to Design $\mathbf{B}$, $\mathbf{F}$ and $\mathbf{W}$

1: Initialize the algorithm with $\mathbf{B}^{(0)} = \sqrt{\frac{P_s}{L}}\mathbf{I}_L$ and $\mathbf{F}^{(0)} = \sqrt{\frac{P_r}{tr(\mathbf{H}_{sr}\mathbf{B}^{(0)}(\mathbf{H}_{sr}\mathbf{B}^{(0)})^H+\mathbf{I}_{N_r})}}\mathbf{I}_{N_r}$ and set $i = 0$.
2: Update $\mathbf{W}^{(i)}$ using $\mathbf{F}^{(i)}$ and $\mathbf{B}^{(i)}$ using (2.10).
3: Update $\mathbf{F}^{(i+1)}$ using $\mathbf{W}^{(i)}$ and $\mathbf{B}^{(i)}$ using (2.13) and (2.19).
4: Update $\mathbf{B}^{(i+1)}$ using $\mathbf{W}^{(i)}$ and $\mathbf{F}^{(i+1)}$ by solving the problem (2.26).
5: If $(\text{MSE}^{(i)} - \text{MSE}^{(i+1)})/\text{MSE}^{(i)} > \epsilon$, go to step 2.
6: End

2.3.2 Bi-step iterative algorithm

The tri-step iterative algorithm presented in Section 2.3.1 provides good performance as verified in Section 2.4, but has high computational complexity. In this section, an iterative algorithm is developed which has a lower computational complexity than the tri-step algorithm. It was proven in [22] and [39] that the optimal precoding solution for one-way relaying is to first parallelize the channels between the source and the relay and between the relay and the destination using singular value decomposition (SVD), and then match the eigenchannels in the two hops. Substituting (2.10) into (2.8), the MSE in (2.8) becomes a function of $\mathbf{B}$ and $\mathbf{F}$ given by

$$\text{MSE} = tr\{[\mathbf{I}_{N_d} + \bar{\mathbf{H}}\mathbf{C}_n^{-1}\bar{\mathbf{H}}^H]^{-1}\} \tag{2.27}$$

Therefore, the optimization problem can be formulated as

$$\min_{\mathbf{F},\mathbf{B}} \; tr\{[\mathbf{I}_{N_d} + \bar{\mathbf{H}}\mathbf{C}_n^{-1}\bar{\mathbf{H}}^H]^{-1}\} \tag{2.28a}$$

$$s.t. \; tr(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r \tag{2.28b}$$

$$tr\{\mathbf{B}\mathbf{B}^H\} \leq P_s. \tag{2.28c}$$

In the bi-step algorithm, the source and relay matrices are updated alternately. First, for a given source matrix $\mathbf{B}$ satisfying (2.28c), the optimal relay matrix $\mathbf{F}$ can be found by solving the following problem

$$\min_{\mathbf{F}} \; tr\{[\mathbf{I}_{N_d} + \bar{\mathbf{H}}\mathbf{C}_n^{-1}\bar{\mathbf{H}}^H]^{-1}\} \tag{2.29a}$$

$$s.t. \; tr(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r \tag{2.29b}$$

Define the following SVDs

$$\bar{\mathbf{H}}_{sr} = \mathbf{H}_{sr}\mathbf{B} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{V}_s^H, \tag{2.30}$$

$$\mathbf{H}_{rd} = \mathbf{U}_r\mathbf{\Lambda}_r\mathbf{V}_r^H. \tag{2.31}$$

Similar to the approach in [24], the relay precoder can be expressed as

$$\mathbf{F} = \mathbf{V}_r\mathbf{\Lambda}_f\mathbf{U}_r^H, \tag{2.32}$$

where $\mathbf{\Lambda}_f$ is an $L \times L$ diagonal matrix. Substituting (2.30), (2.31) and (2.32) into (2.29a), the MSE can be written as

$$\begin{aligned} \text{MSE} &= tr\{[\mathbf{I}_{N_d} + (\mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_s)(\mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_{H_{LI}}\mathbf{\Lambda}_f\mathbf{\Lambda}_r \\ &\quad + \mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_f\mathbf{\Lambda}_r + \mathbf{I}_{N_D})^{-1}(\mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_s)]^{-1}\}, \end{aligned} \tag{2.33}$$

where $\mathbf{\Lambda}_{H_{LI}} = \mathbf{U}_s^H(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H)\mathbf{U}_s$. Clearly $\mathbf{\Lambda}_{H_{LI}}$ is not diagonal, so solving the optimization problem directly is difficult. However, a tractable upper bound on the MSE can be considered to simplify the problem. Defining $\mathbf{C} = \mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_{H_{LI}}\mathbf{\Lambda}_f\mathbf{\Lambda}_r + \mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_f\mathbf{\Lambda}_r + \mathbf{I}_{N_D}$ and $\mathbf{D} = \mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_s$, the MSE in (2.33) becomes

$$\begin{aligned} \text{MSE} = &\; tr\{[\mathbf{I}_L + \mathbf{D}\mathbf{C}^{-1}\mathbf{D}]^{-1}\} \\ = &\; tr\{\mathbf{I}_L - (\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{C}\mathbf{D}^{-1})^{-1}\}, \end{aligned} \tag{2.34}$$

where the matrix inversion lemma $(\mathbf{I} + \mathbf{A}^{-1})^{-1} = \mathbf{I} - (\mathbf{I} + \mathbf{A})^{-1}$ has been used. Since for any positive definite square matrix $\mathbf{A}$, it has $tr\{\mathbf{A}^{-1}\} \geq \sum_i [\mathbf{A}(i,i)]^{-1}$ [39], we have

$$\begin{aligned} \text{MSE} &\leq \mathbf{I}_{N_D} - \sum_i^L [(\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{C}\mathbf{D}^{-1})(i,i)] \\ &= tr\{\mathbf{I}_L - (\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{\Lambda}_C\mathbf{D}^{-1})\} \\ &= tr\{[\mathbf{I}_L + \mathbf{D}\mathbf{\Lambda}_C^{-1}\mathbf{D}]^{-1}\}. \end{aligned} \tag{2.35}$$

Therefore, the upper bound on the MSE is

$$\begin{aligned} \text{MSE}^{\mu} &= tr\{[\mathbf{I}_{N_d} + (\mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_s)(\mathbf{\Lambda}_r\mathbf{\Lambda}_f\tilde{\mathbf{\Lambda}}_{H_{LI}}\mathbf{\Lambda}_f\mathbf{\Lambda}_r \\ &\quad + \mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_f\mathbf{\Lambda}_r + \mathbf{I}_{N_d})^{-1}(\mathbf{\Lambda}_r\mathbf{\Lambda}_f\mathbf{\Lambda}_s)]^{-1}\}, \end{aligned} \tag{2.36}$$

where $\tilde{\mathbf{\Lambda}}_{H_{LI}}$ is a diagonal matrix that contains the diagonal entries of $\mathbf{\Lambda}_{H_{LI}}$. A similar approach can be applied to the constraint in (2.29b). Then the original optimization problem can be written as

$$\min_{\lambda_{f_i}} \sum_{i=1}^{L} \left(1 + \frac{\lambda_{s_i}^2 \lambda_{r_i}^2 \lambda_{f_i}^2}{1 + \lambda_{r_i}^2 \lambda_{f_i}^2 (\lambda_{H_{LI_i}}^2 + 1)}\right) \tag{2.37a}$$

$$s.t. \sum_{i=1}^{L} \lambda_{f_i}^2 (\lambda_{s_i}^2 + \lambda_{H_{LI_i}}^2 + 1) \leq P_r \tag{2.37b}$$

where λ_{s_i}, λ_{r_i}, λ_{f_i}, and $\lambda_{H_{LI_i}}$ are the ith main diagonal elements of $\mathbf{\Lambda}_s$, $\mathbf{\Lambda}_r$, $\mathbf{\Lambda}_f$, and $\mathbf{\Lambda}_{H_{LI}}$, respectively. This optimization problem can be solved by employing the KKT conditions. The problem in (2.37) has a water-filling solution which is given by

$$\lambda_{f_i} = \frac{1}{\lambda_{r_i}} \left[\frac{1}{\lambda_{s_i}^2 + 1 + \lambda_{H_{LI_i}}^2} \left(\frac{\lambda_{s_i}\lambda_{r_i}}{[\lambda_{s_i}^2 + (1 + \lambda_{H_{LI_i}}^2)\mu]^{\frac{1}{2}}} - 1\right)^{+}\right]^{\frac{1}{2}}, \tag{2.38}$$

where for real valued number x, $(x)^+ = \max(x, 0)$ and $\mu \geq 0$ is the solution of the nonlinear problem

$$\sum_{i=1}^{L} \frac{1}{\lambda_{r_i}^2} \left[\left(\frac{\lambda_{s_i}\lambda_{r_i}}{[\lambda_{s_i}^2 + (1 + \lambda_{H_{LI_i}}^2)\mu]^{\frac{1}{2}}} - 1\right)^{+}\right]^{\frac{1}{2}} = P_r. \tag{2.39}$$

Since (2.39) is a monotonically decreasing function of μ, it can be efficiently solved using the bisection method [49].

Next, fixing the relay precoder from (2.32) and applying the matrix identity $tr\{[\mathbf{I}_m + \mathbf{A}_{m\times n}\mathbf{B}_{n\times m}]^{-1}\} = tr\{[\mathbf{I}_n + \mathbf{B}_{n\times m}\mathbf{A}_{m\times n}]^{-1}\} + m - n$, the objective function (2.29a) can be rewritten as

$$\begin{aligned} \text{MSE} = \; & tr\{[\mathbf{I}_L + \bar{\mathbf{H}}\mathbf{C}_n^{-1}\bar{\mathbf{H}}^H]^{-1}\} \\ = \; & tr\{[\mathbf{I}_L + \bar{\mathbf{H}}^H\bar{\mathbf{H}}\mathbf{C}_n^{-1}]^{-1}\} + N_s - N_d \\ = \; & tr\{[\mathbf{I}_L + \mathbf{C}_n^{-\frac{1}{2}}\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{H}_{rd}^H \times \mathbf{C}_n^{-\frac{1}{2}}]^{-1}\} + N_s - N_d \\ = \; & tr\{[\mathbf{I}_L + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^H]^{-1}\} + N_s - N_d. \end{aligned} \tag{2.40}$$

where $\tilde{\mathbf{H}} = \mathbf{C}_n^{-\frac{1}{2}}\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}$ and $\mathbf{Q} = \mathbf{B}\mathbf{B}^H$. The optimal $\mathbf{B}$ is $\mathbf{B} = \mathbf{\Theta}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{\Phi}$ where $\mathbf{\Theta}\mathbf{\Lambda}\mathbf{\Theta}^H$ is the eigenvalue decomposition (EVD) of $\mathbf{Q}$, and $\mathbf{\Phi}$ is an arbitrary $L \times L$ unitary matrix. The optimization problem in (2.28) can now be formulated as

$$\begin{aligned} & \min_{\mathbf{Q}} \; tr\{[\mathbf{I}_{N_d} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^H]^{-1}\} \\ & s.t. \; tr\{\mathbf{Q}\} \leq P_s \\ & \quad\;\; tr\{\mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{sr}\mathbf{Q}\} \leq \bar{P}_r. \end{aligned} \tag{2.41}$$

where $\bar{P}_r = P_r - tr\{\sigma_t^2 \mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H + \mathbf{F}\mathbf{F}^H\}$. Introducing a positive semidefinite (PSD) matrix $\mathbf{X}$ that satisfies

$$[\mathbf{I}_{N_d} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^H]^{-1} \preceq \mathbf{X}, \tag{2.42}$$

and using the Schur complement [49], the problem in (2.41) can be converted to the following equivalent SDP problem

$$\begin{aligned} \min_{\mathbf{Q}} \; & tr\{\mathbf{X}\} \\ s.t. \; & \begin{bmatrix} \mathbf{X} & \mathbf{I}_{N_d} \\ \mathbf{I}_{N_d} & \mathbf{I}_{N_d} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^H \end{bmatrix} \succeq 0 \\ & tr\{\mathbf{Q}\} \leq P_s \\ & tr\{\mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{sr}\mathbf{Q}\} \leq \bar{P}_r \\ & tr\{\mathbf{Q}\} \succeq 0 \end{aligned} \tag{2.43}$$

An SDP problem can be solved efficiently using the disciplined convex programming toolbox CVX [50].

a) Iterative algorithm and convergence

The proposed bi-step iterative algorithm is summarized in Algorithm 2 below. This algorithm can be shown to converge as follows. It is obvious that the two subproblems are convex. It then follows that each update of $\mathbf{B}$, $\mathbf{F}$ will decrease or at least not increase the value of the objective function, and thus the iterative algorithm converges to a locally optimum solution.

Algorithm 2 Iterative Algorithm to Design $\mathbf{B}$, $\mathbf{F}$ and $\mathbf{W}$

1: Initialize the algorithm with $\mathbf{B}^{(0)} = \sqrt{\frac{P_s}{L}}\mathbf{I}_L$ and set $i = 0$.
2: Solve (2.37) to obtain $\mathbf{\Lambda}_f$ and substitute into (2.32) to obtain the relay precoder $\mathbf{F}^{(i)}$.
3: Solve the problem (2.43) to obtain $\mathbf{B}^{(i)}$.
4: If $(\text{MSE}^{(i)} - \text{MSE}^{(i+1)})/\text{MSE}^{(i)} > \epsilon$, go to step 2.
5: End

b) Achievable rate

The achievable rate for the model (2.5) can be obtained using an approach similar to that in [42] and is written as

$$\begin{aligned} R = \log_2 \det \Big[& \mathbf{I}_{N_r} + \frac{P_s}{N_t}(\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B})(\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B})^H \\ & \times(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{I}_{N_r})^{-1} \Big], \end{aligned} \tag{2.44}$$

where $\det(\cdot)$ denotes determinant.

2.4 Numerical Results

In this section, the performance of the proposed precoding algorithm for a full-duplex (FD) MIMO relay system is examined and compared with that of a half-duplex (HD) relay system. The HD algorithm is the same as the proposed FD algorithm but with no LI. The achievable rate of the HD system is half that of the FD system without residual LI since it requires two time slots for source to destination data transmission. Similar to the related literature, a flat fading environment is considered where the estimated channel matrices $\mathbf{H}_{sr}$, $\mathbf{H}_{LI}$, and $\mathbf{H}_{rd}$ are composed of i.i.d. complex Gaussian random variables with zero mean and unit variance. The signal-to-noise ratios (SNRs) of the source-to-relay and relay-to-destination channels are $\text{SNR}_{s-r} = \frac{P_s}{N_s}$ and $\text{SNR}_{r-d} = \frac{P_r}{N_t}$, respectively. For simplicity, it is assumed that perfect channel state information (CSI) is available for all channels. As discussed in [10], the residual LI can vary from 0 dB to 15 dB larger than the channel noise. Therefore, the residual LI levels considered here are 0 dB, 5 dB and 10 dB. All results given are averaged over 1000 trials with independent channel realizations. In all cases, results are given for an average of 1000 independent channel realizations. Note that the optimization procedure for the HD system mentioned in this chapter is as the same as the proposed FD system except residual LI term and the achievable rate for the HD system is dropped by half than FD system since two time slots are required for the transmission between the source and destination.

Fig. 2.2 presents the MSE of the proposed tri-step iterative method versus SNR_{s-r} with $\text{SNR}_{r-d} = 30$ dB and $N_s = N_r = N_t = N_r = L = 2$. The convergence tolerance is set to $\epsilon = 0.00001$ and the maximum number of iterations is 30. It is clear that the FD system has a higher MSE than the HD system due to the existence of residual LI. Further, the MSE increases as the residual LI level increases.

Fig. 2.3 presents the achievable rate of the HD and FD systems. The FD achievable rate is twice the HD if the LI is canceled completely. The FD system outperform the HD system in the high SNR_{s-r} region for all levels of residual LI. Further, when the residual LI level is 10 dB, the HD system outperforms the FD system only when $\text{SNR}_{s-r} < 10$ dB.

Figs. 2.4 and 2.5 present the MSE and achievable rate with a fixed SNR of 30 dB between the source and relay and an SNR between the relay and destination from 0 dB to 30 dB. The MSE in Fig. 2.4 is better than that in Fig. 2.2 in the low SNR_{r-d} region because a higher transmit power results in greater residual LI. Fig. 2.5 shows that the achievable rate of the FD system is always higher than that of the HD system for the

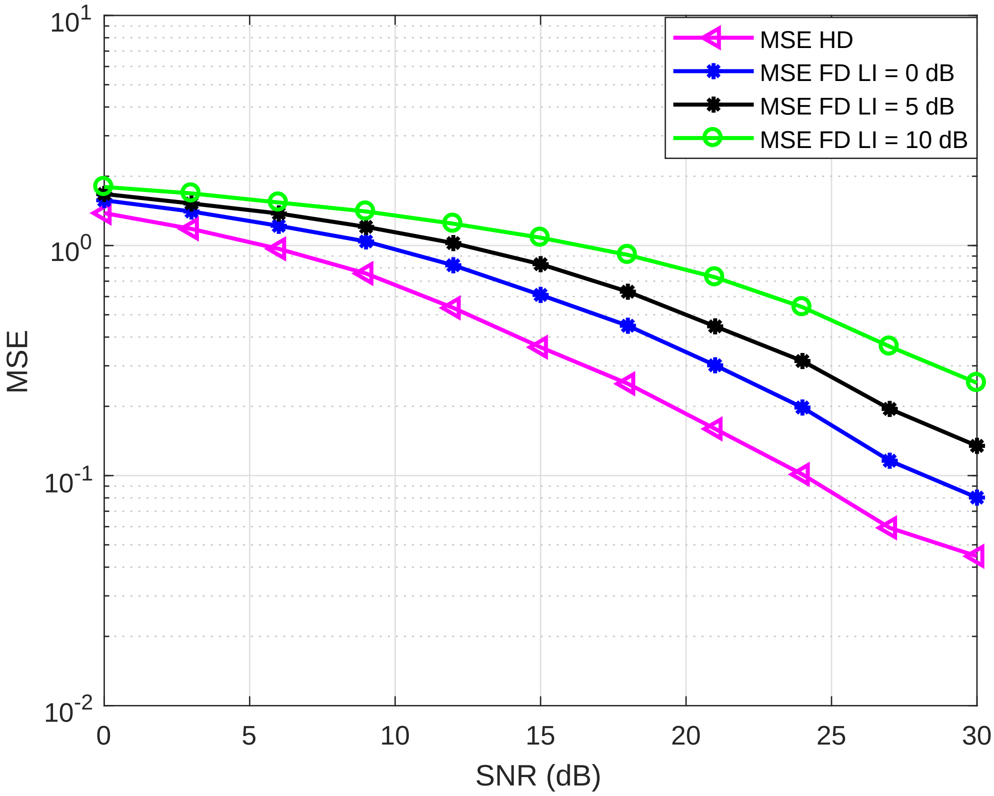

Figure 2.2: Tri-step algorithm MSE versus SNR_{s-r} with $\text{SNR}_{r-d} = 30$ dB.

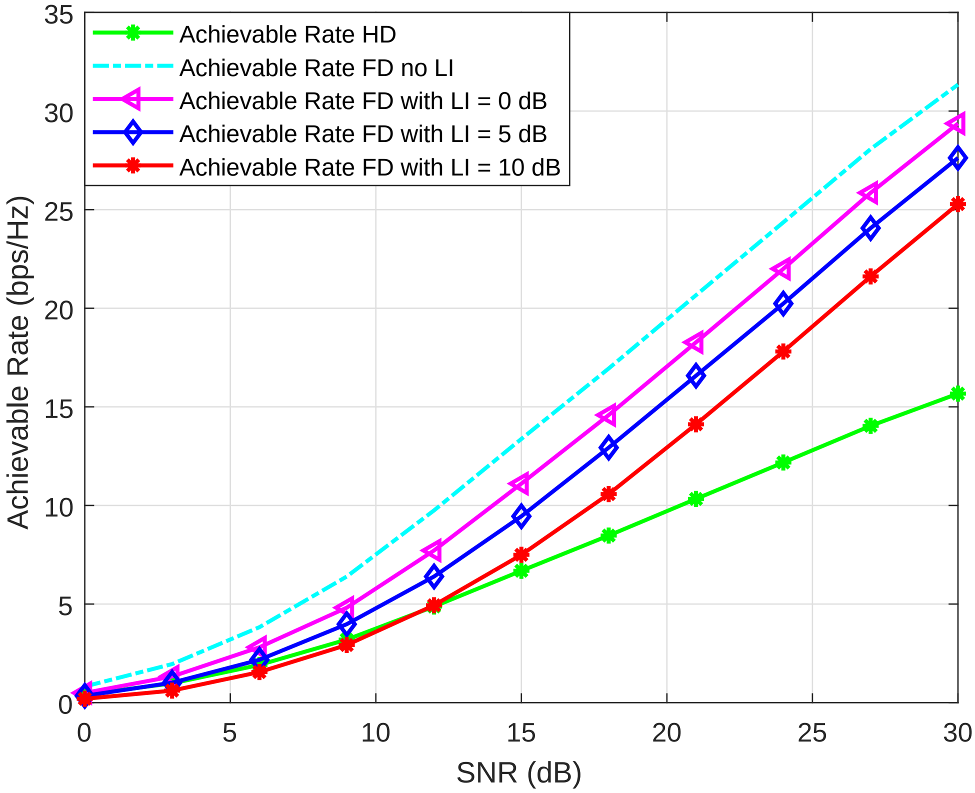

Figure 2.3: Tri-step algorithm achievable rate versus SNR_{s-r} with $\text{SNR}_{r-d} = 30$ dB.

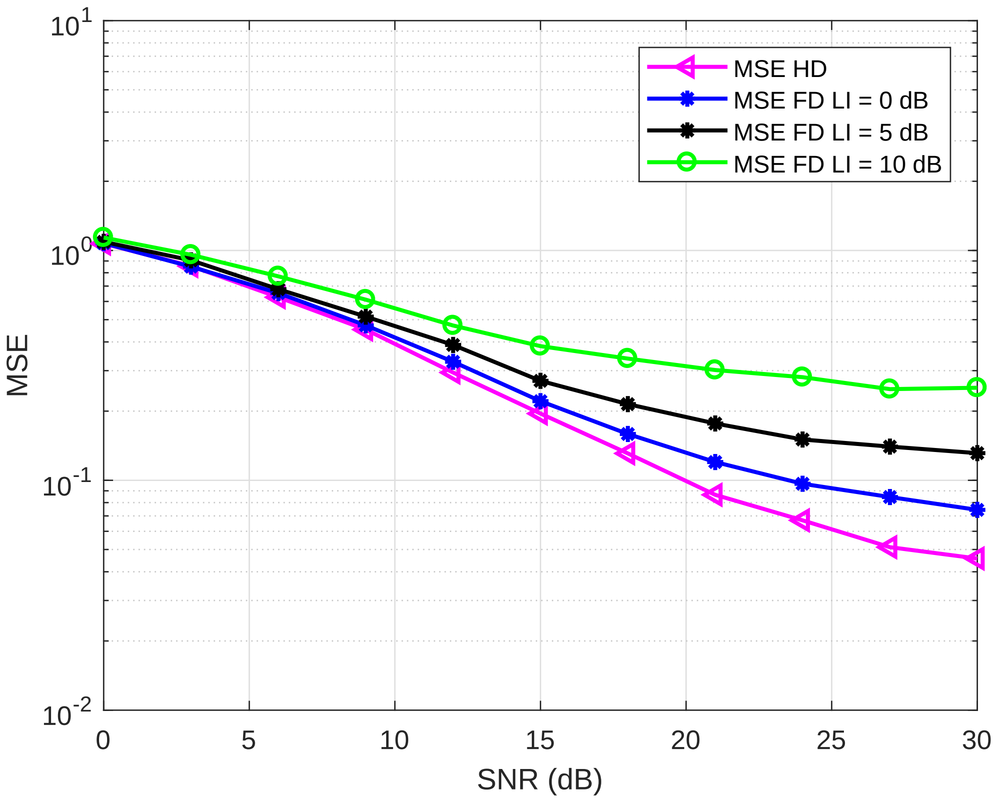

Figure 2.4: Tri-step algorithm MSE versus SNR_{r-d} with $\text{SNR}_{s-r} = 30$ dB.

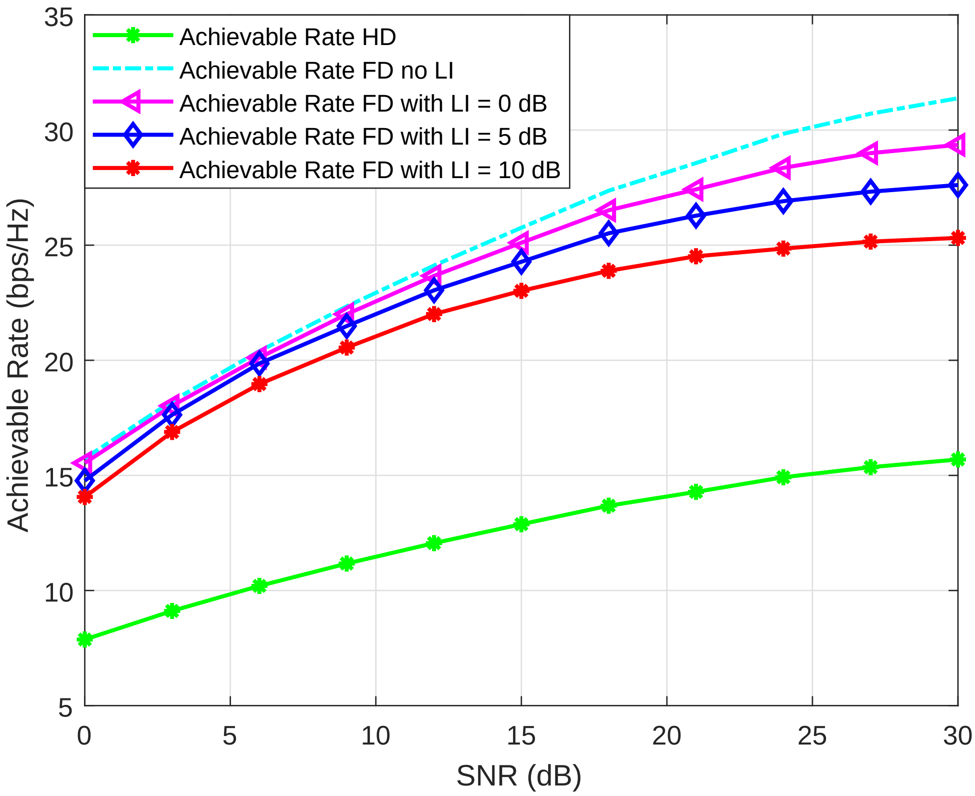

Figure 2.5: Tri-step algorithm achievable rate versus SNR_{r-d} with $\text{SNR}_{s-r} = 30$ dB.

residual LI levels considered.

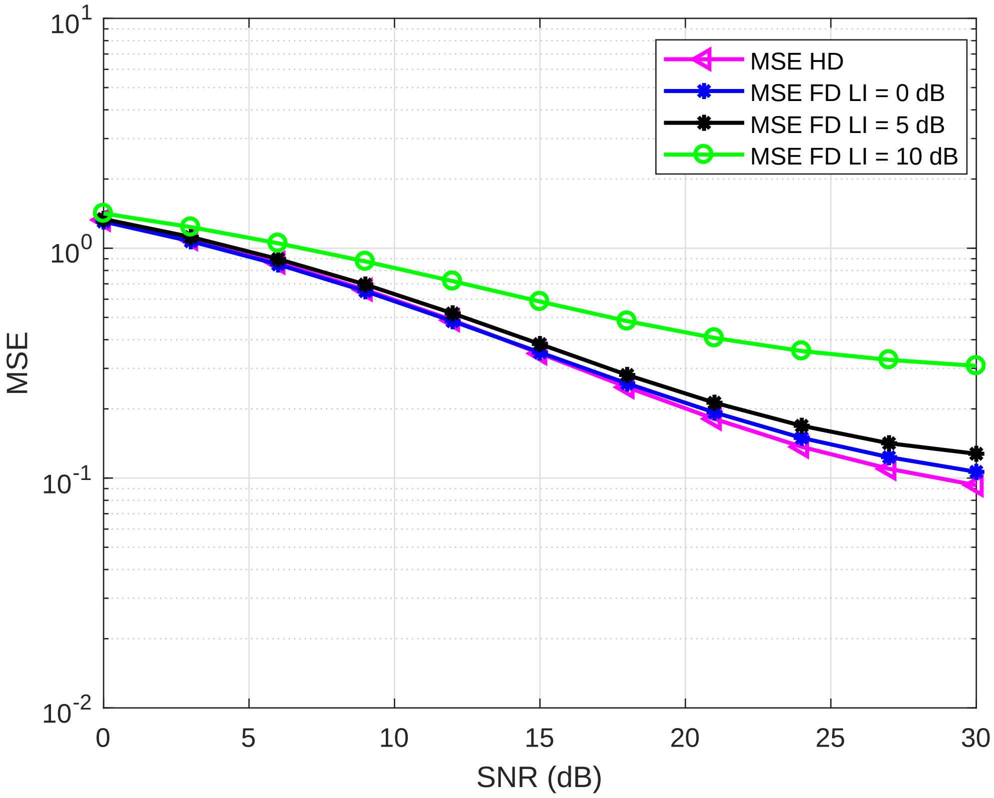

Figure 2.6: Bi-step algorithm MSE versus SNR_{r-d} with $\text{SNR}_{s-r} = 30$ dB.

Figs. 2.6 and 2.7 present the MSE and achievable rate for the proposed bi-step algorithm with a fixed SNR of 30 dB between the source and relay and an SNR between the relay and destination from 0 dB to 30 dB. In Fig. 2.6, the HD system has better MSE performance than the FD system through all the residual SNR region. Fig. 2.7 shows the achievable rate of the proposed bi-step algorithm. The achievable rate of the FD system is greater than that of the HD system for all values of residual LI.

Fig. 2.8 presents the achievable rate for the proposed tri-step and bi-step algorithms. This shows that achievable rate of the bi-step algorithm is comparable to that of the tri-step algorithm. Table 2.1 compares the number of iterations required for convergence with a tolerance $\epsilon = 10^{-3}$ for both algorithms. The number of antennas is $N_s = N_r = N_t = N_d = 2$. The source transmit power is fixed at 30 dB and the SNR of the relay to destination link varies from 0 dB to 30 dB. These results show that the bi-step algorithm requires fewer iterations and so has lower computational complexity. This performance-complexity trade off is an important consideration in the design of practical MIMO FD relay communication systems.

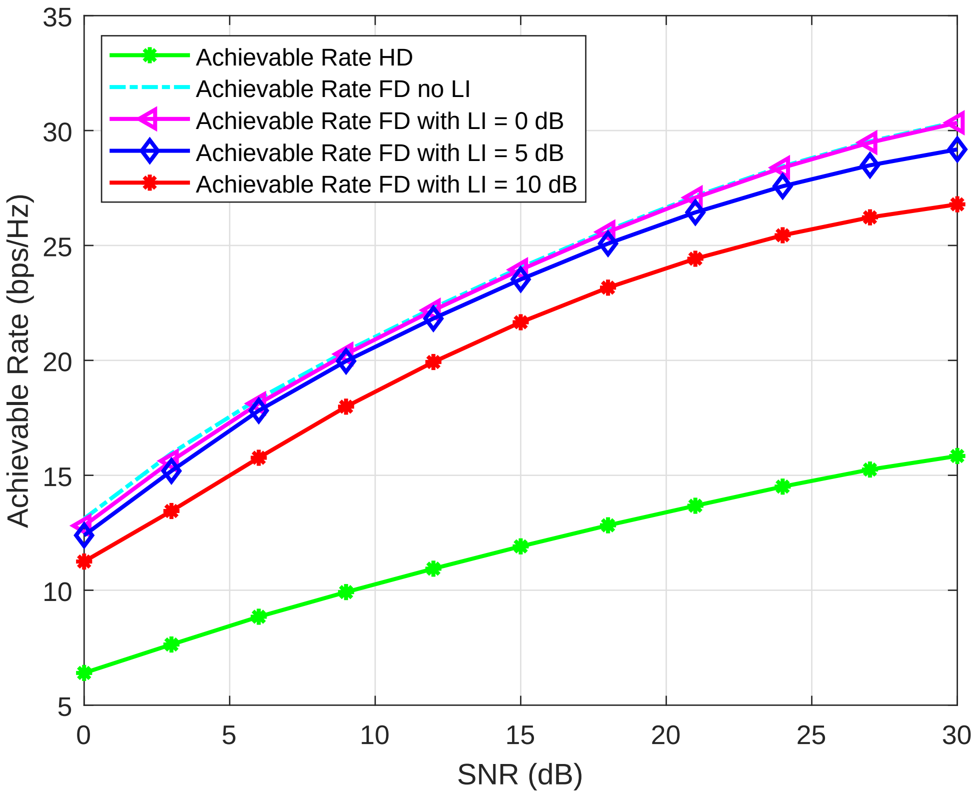

Figure 2.7: Bi-step algorithm achievable rate versus SNR_{r-d} with $\text{SNR}_{s-r} = 30$ dB.

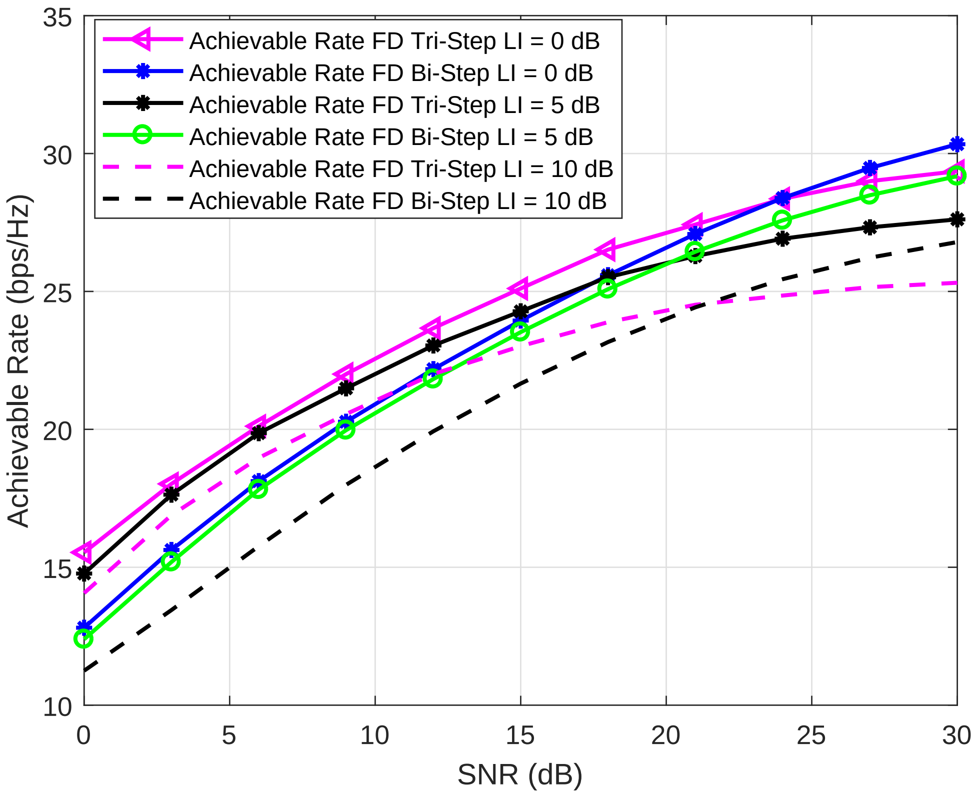

Figure 2.8: Achievable rate comparison between the tri-step and bi-step algorithms.

Table 2.1: Average Number of Iterations Required for Convergence for the One-Way System

SNR_{rd_i} (dB)	0	5	10	15	20
Tri-Step Algorithm	4	6	10	11	15
Bi-Step Algorithm	2	4	4	6	6

2.5 Conclusions

In this chapter, a locally optimal source and relay precoding and destination combiner design problem was proposed for MIMO FD AF relay communication systems. To make the optimization problem tractable, two efficient MMSE algorithms were developed to obtain the source and relay precoding, and destination combining, matrices. The tri-step iterative algorithm gives optimal solutions to the three corresponding subproblems, while the bi-step iterative algorithm provides optimal solutions to the two corresponding subproblems. The convergence of the algorithms was examined, and the effect of the residual loop interference at the relay on the achievable rate was evaluated. Simulation results were presented which demonstrate that both algorithms outperform the corresponding HD relay system in terms of achievable rate and MSE.

Chapter 3

Precoding Design for Two-Way MIMO Full-Duplex Amplify-and-Forward Relay Communication Systems

In the previous chapter, a one-way MIMO full-duplex amplify-and-forward relay communication system has been proposed and analyzed. In contrast to the one-way relaying, two-way relaying is a highly efficient scheme to realize the information exchange between two node. Different from the one-way relaying, the two-way relay node receives signals from all source nodes simultaneously and then broadcasts the forwarding signals to all destination nodes. To further improve the spectral efficiency, this chapter considers a two-way multiple-input multiple-output (MIMO) full-duplex (FD) amplify-and-forward (AF) relay by exploiting the physical layer network coding (PNC) technique.

3.1 Introduction

MULTIPLE-input multiple-output (MIMO) relay communication systems have been extensively investigated in recent years because they can enhance capacity by increasing coverage and reliability [34]. In an amplify-and-forward (AF) relay system, the relay node amplifies the received signal and then forwards the amplified signal to the destination node. Since the relay only performs amplification, the complexity of this strategy is much lower than decode and forward (DF), which is a regenerative relaying scheme. In half-duplex (HD) relay systems [22, 43, 51], communications from the source to destination requires two time slots so the source node transmits only half of the time, which limits the spectral efficiency.

In contrast to one-way relaying which needs four time slots to exchange information between two nodes, two-way relaying only needs two time slots to complete a round of information exchange. Therefore, two-way relaying has a higher spectral efficiency than one-way relaying. Physical-layer network coding (PNC) which exploit the self-information at the nodes has been used with two-way relaying [52–56]. There are two steps in HD two-way relaying communications. First, the nodes transmit their signals to the relay node during the multiplexing access (MAC) phase. Then the relay node broadcasts (BC) the received signal to the two nodes. Each node can cancel the interference they generate from the signal received from the relay to recover the signal transmitted by the other node.

In [52], a novel two-way relaying scheme which approaches the sum capacity of the MIMO cellular two-way relay channel was investigated. In order to achieve efficient interference-free decoding at the relay, a new non-linear lattice-based precoding technique was used to compensate for the inter stream interference. The sum capacity of the proposed system was asymptotically achieved in the high signal-to-noise ratio (SNR) region. The trade off between the capacity and diversity-multiplexing of the two-way relay channel was examined in [53]. An iterative algorithm was proposed to maximize the achievable rate with AF relaying subject to minimum signal-to-interference-and-noise ratio (SINR) constraints. An energy efficient two-way AF relaying system with multiple antennas at both the sources and relay was presented in [54]. The transmit power was minimized while satisfying the quality of service (QoS) requirements of both sources. Transmit beamformers and receive combiners were designed with a zero forcing (ZF) based relay precoding matrix. In [55], it was shown that the optimal diversity-multiplexing gain trade off can be achieved by a compress-and-forward (CF) strategy in which the relay quantizes its received signal and transmits the corresponding codeword.

Multiple-input multiple-output (MIMO) can be employed to improve the transmission reliability and enhance the channel capacity of a wireless communication system. Employing MIMO in a two-way relaying system is an efficient way to increase the performance over single antenna systems. In order to fully realize the benefits of MIMO two-way relaying, precoding should be employed at both the source nodes and relay node by making use of channel state information (CSI) [24, 39, 57–60]. In [57], a nonlinear precoder design was presented for a MIMO two-way relay system using minimum mean squared error (MMSE) decision feedback equalizers. The design first considers the nonlinear source precoding at the two sources with a fixed relay precoder, and then considers the joint precoder design to incorporate the relay precoder. In [58], a constrained optimization problem with respect to the relay precoder was formulated for the general case of multiple relays each with multiple antennas. Under the assumption that complete CSI is available at the relays, the problem was converted to a convex optimization problem with respect to only the non-zero entries of the relay precoder matrix, which leads to a closed-form relay precoding solution. In [59], a low-complexity joint beamforming and power management scheme was proposed. The beamformer first aligns the channel matrices of the node pairs and then decomposes the aligned channel into parallel subchannels. It was shown that the proposed joint scheme gives improved sum capacity performance and can be used to lower the required transmit power. Two iterative algorithms were proposed in [24] for joint source and relay precoder design based on the MSE criterion in a MIMO two-way relay system. In this system, two multiple antenna source nodes exchange information with the help of a multiple antenna amplify-and-forward relay node. In [60], the problem of precoder design to suppress co-channel interference in a multiuser two-way relay system was considered. The uplink performance including the overall MSE and sum rate was optimized while maintaining individual downlink SINR requirements.

While most of the results in the literature focus on half-duplex relay systems [24, 39, 55–60], the development of new signal processing techniques and antenna designs has made FD relaying in MIMO systems a reality [10, 40]. A full-duplex AF relaying system under Nakagami-m fading was considered in [46] and closed-form expressions for the outage probability and ergodic capacity were derived. In [61], an interference suppression scheme was investigated to mitigate the residual LI and interference in a multi-user FD relaying system. Rather than applying HD in two-way relaying as in [24, 39, 55–60], a two-way FD relay design was presented in [62]. It was shown that FD relaying can achieve almost double the capacity of HD relaying if there is no residual LI. In [63], distributed space-time coding was investigated for a two-way FD relaying network which allows relay communications in both directions simultaneously. The direct source to destination link was also considered. A two-way FD relaying system with residual LI was presented in [64]. Exact and approximate closed-form expressions were given for the outage probability with both perfect and imperfect channel state information (CSI). A joint precoder/combiner design that maximizes the end-to-end (e2e) performance was investigated in [25]. ZF LI suppression at the relay was considered and a closed-form solution was obtained. In [65], rate and outage probability trade offs were examined for full-duplex one-way and two-way relaying systems considering the residual LI.

An algorithm was presented in [28] to maximize the e2e performance by jointly optimizing the beamforming matrix at an AF relay and the transmit power at the source. If multiple antennas are employed at both the source and destination sides, it has been shown that the channel sum rate increases linearly with the minimum number of antennas [24]. In contrast to [28] which employs only a single antenna at the source and destination, this chapter considers a MIMO FD two-way relaying system where the source, relay and destination have multiple antennas. Further, the AF protocol with physical layer network coding is employed. As this is a FD system, the residual loop interference at the relay is considered. Since the transmission power at the source nodes are low, the residual LI at the source nodes is assumed very small and can be ignored. The source precoders, relay precoder and destination combiners are optimized using the MSE criterion. Since the original optimization problem is non-convex and a closed-form solution is intractable, it is translated into three subproblems which can be solved iteratively. It is shown that this algorithm converges to an optimal solution. Since the computational complexity of the proposed tri-step iterative algorithm is high, a low complexity bi-step iterative approach is obtained. Results are presented which show that this bi-step iterative algorithm provides performance comparable to that with the tri-step iterative algorithm, so the complexity-performance trade off is favorable. The sum achievable rate improvement with FD relaying over HD relaying is illustrated, and the effects of the residual LI are examined.

The remainder of this chapter is organized as follows. In Section 3.2, the system model of the MIMO two-way full-duplex relay system is introduced, and the problem formulation is presented in Section 3.3. Two iterative algorithms for solving the proposed optimization problem are developed in Section 3.4. The sum mean squared error (MSE) performance, sum achievable rate and complexity of the proposed algorithms are analyzed in Section 3.5. Numerical results are presented to demonstrate the performance improvement with FD relaying and precoding. Finally, some conclusion are given in Section 3.6.

3.2 System Model

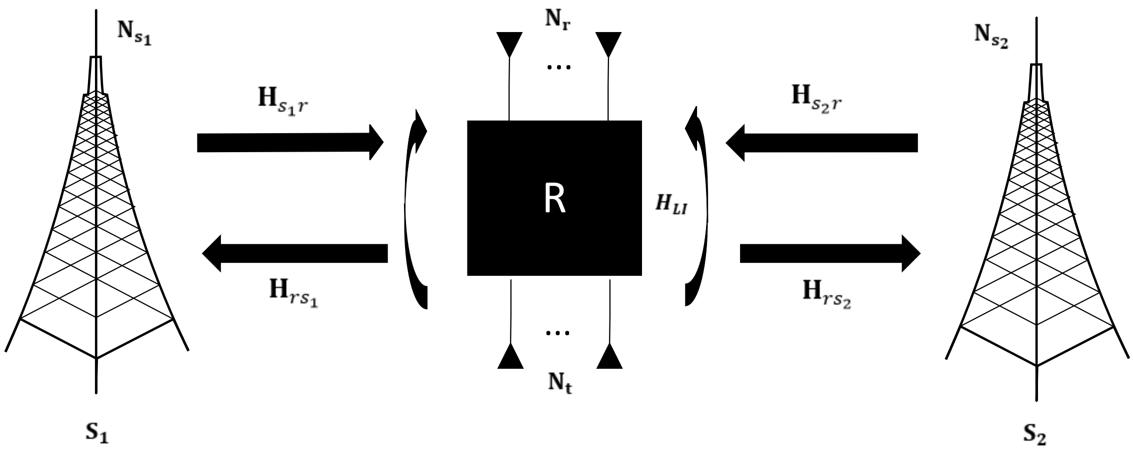

Figure 3.1: The MIMO two-way FD AF relay system model.

We consider a three node, two-way MIMO full-duplex (FD) relay system. As shown in Fig. 3.1, two source nodes want to exchange messages via a relay R. Source S_1 is equipped with N_{s_1} antennas for transmitting and another group with the same number of antennas for receiving. Source S_2 is equipped with N_{s_2} antennas for transmitting and another group with the same number for receiving. The relay node works in full-duplex (FD) mode with physical layer network coding [28], which has N_r and N_t antennas to receive and transmit, respectively. This non-regenerative relay amplifies the received signals from both source nodes and then broadcasts the resulting signal to the destinations simultaneously. Therefore, communications between the two source is accomplished in one time slot compared to a half-duplex (HD) system that requires two time slots. Note that in the two-way relay system, the source nodes are the destination nodes during the relay broadcast phase.

Let $\mathbf{s}_i[n] \in \mathbb{C}^{L\times 1}$ represents the $L \times 1$ signal vector transmitted at time n for node i, $i = 1, 2$. Without loss of generality, we assume that $L \leq \min\{N_{S_i}, N_t, N_r\}$, $i = 1, 2$. In addition, it is assumed that $\mathbb{E}[\mathbf{s}_i[n]\mathbf{s}_i[n]^H] = \mathbf{I}_L$, where $(\cdot)^H$ represents conjugate transpose (Hermitian) and $\mathbb{E}$ denotes expectation. A linear precoding matrix $\mathbf{B}_i[n]$ is applied to the signal vector $\mathbf{s}_i[n]$ before transmission. The received signal at the relay can be expressed as

$$\mathbf{y}_R[n] = \mathbf{H}_{S_1R}[n]\mathbf{B}_1[n]\mathbf{s}_1[n] + \mathbf{H}_{S_2R}[n]\mathbf{B}_2[n]\mathbf{s}_2[n] + \mathbf{H}_{LI}[n]\mathbf{t}[n] + \mathbf{n}_R[n], \qquad (3.1)$$

where $\mathbf{H}_{S_iR}[n] \in \mathbb{C}^{N_r\times N_{s_i}}$ is the ith source to relay channel matrix, $\mathbf{H}_{RS_i} \in \mathbb{C}^{N_{s_i}\times N_t}$ is the relay to ith destination channel matrix, $\mathbf{H}_{LI}[n] \in \mathbb{C}^{N_r\times N_t}$ is the loop interference (LI) channel matrix, and $\mathbf{n}_r[n] \in \mathbb{C}^{N_r\times 1}$ is an independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) matrix. After employing a LI cancellation technique, (3.1) can be written as

$$\mathbf{y}_R[n] = \mathbf{H}_{S_1R}[n]\mathbf{B}_1[n]\mathbf{s}_1[n] + \mathbf{H}_{S_2R}[n]\mathbf{B}_2[n]\mathbf{s}_2[n] + \mathbf{H}_{LI}[n]\mathbf{t}[n] + \mathbf{T}[n] + \mathbf{n}_R[n], \qquad (3.2)$$

where $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\mathbf{t}[n]$ when perfect LI cancellation is applied. However, in an actual system $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\tilde{\mathbf{t}}[n]$ where $\tilde{\mathbf{t}}[n]$ is a noisy version of $\mathbf{t}[n]$ due to imperfect LI cancellation. As discussed in [66], $\mathbf{y}_R[n]$ can be rewritten as

$$\mathbf{y}_R[n] = \mathbf{H}_{S_1R}[n]\mathbf{B}_1[n]\mathbf{s}_1[n] + \mathbf{H}_{S_2R}[n]\mathbf{B}_2[n]\mathbf{s}_2[n] + \mathbf{H}_{LI}[n]\Delta\mathbf{t}[n] + \mathbf{n}_R[n], \qquad (3.3)$$

where $\Delta\mathrm{t}[n] = \mathrm{t}[n] - \tilde{\mathrm{t}}[n]$ and $\mathbf{H}_{LI}[n]\Delta\mathrm{t}[n]$ is the residual LI after imperfect LI cancellation.

At time $n + 1$, the full-duplex relay applies a precoding matrix $\mathbf{F}[n + 1] \in \mathbb{C}^{N_t\times N_t}$ to the received signal and then broadcasts the result to the nodes. The received signal at node i can be expressed as

$$\begin{aligned}\mathbf{y}_i[n+1] =&\mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{y}_R[n] + \mathbf{n}_{D_i}[n+1] \\ =&\mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{H}_{S_iR}[n]\mathbf{B}_i[n]\mathbf{s}_i[n] + \mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1] \\ &\times \mathbf{H}_{S_{\bar{i}}R}[n]\mathbf{B}_{\bar{i}}[n]\mathbf{s}_{\bar{i}}[n] + \mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{H}_{LI}[n]\Delta\mathbf{t}[n] \\ &+ \mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{n}_R[n+1] + \mathbf{n}_{D_i}[n],\end{aligned} \tag{3.4}$$

where $\bar{i} = 2$ if $i = 1$ and $\bar{i} = 1$ if $i = 2$.

Similar to [60], we assume that the channel characteristics of each link change very slowly so they can be perfectly estimated using pilot symbols or training sequences. The channel $\mathbf{H}_{S_iR}$ at the relay can be estimated by S_i sending a training sequence. The LI channel $\mathbf{H}_{LI}$ can be estimated at the relay by sending an N_t-symbol pilot sequence. Although channel reciprocity does not hold exactly, as discussed in [22, 24, 28, 58] for the purposes of analysis it can be assumed to hold during the MAC and BC phases so that $\mathbf{H}_{S_iR} = \mathbf{H}_{RS_i}^T$. Thus, the back propagated self-interference term $\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{s}_i$ from (3.4) can be canceled. Further, we assume that the channel variations during the precoder update interval are relatively small so the time index has no influence on the precoder design and can be omitted. Therefore, a more concise expression for (3.4) is

$$\mathbf{y}_i = \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{s}_{\bar{i}} + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_iR}\mathbf{H}_{LI}\Delta\mathbf{t} + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{n}_R + \mathbf{n}_{D_i} \tag{3.5}$$

A combiner $\mathbf{W}_i \in \mathbb{C}^{N_{s_{\bar{i}}} \times L}$ is employed on the received signal at node i, so the estimated signal from node $\bar{i}$ received by node i can be written as

$$\hat{\mathbf{s}}_{\bar{i}} = \mathbf{W}_i^H \mathbf{y}_i. \tag{3.6}$$

Since i.i.d. AWGN with zero mean and unit variance is assumed, $\mathbb{E}[\mathbf{n}_R\mathbf{n}_R^H] = \sigma_{n,r}^2\mathbf{I}_{N_r}$ and $\mathbb{E}[\mathbf{n}_{D_i}\mathbf{n}_{D_i}^H] = \sigma_{n,d}^2\mathbf{I}_{N_{S_i}}$, where $\sigma_{n,r}^2 = 1$ and $\sigma_{n,d}^2 = 1$ are the variances of $\mathbf{n}_R$ and $\mathbf{n}_{D_i}$, respectively. The problem now is how to design the linear precoders $\mathbf{B}_i$ and $\mathbf{F}$ and the linear combiners $\mathbf{W}_i$ to minimize the sum mean squared error (SMSE) of the received signals at the destinations.

3.3 Problem Formulation

In this section, we first formulate the joint source and relay precoding optimization problem to minimize the SMSE in the MIMO two-way relay system. Considering the received signal (3.5) and the estimated signal after applying the linear combiner (3.6), the MSE at node i can be expressed as

$$\begin{aligned} J_i &= \mathbb{E}[(\hat{\mathbf{s}}_{\bar{i}} - \mathbf{s}_{\bar{i}})(\hat{\mathbf{s}}_{\bar{i}} - \mathbf{s}_{\bar{i}})^H] \\ &= tr\{(\mathbf{W}_i^H \mathbf{H}_{RS_i} \mathbf{F} \mathbf{H}_{S_{\bar{i}}R} \mathbf{B}_{\bar{i}} - \mathbf{I}_{N_{S_i}})(\mathbf{W}_i^H \mathbf{H}_{RS_i} \mathbf{F} \mathbf{H}_{S_{\bar{i}}R} \mathbf{B}_{\bar{i}} - \mathbf{I}_{N_{S_i}})^H \\ &\quad + \mathbf{W}_i^H \mathbf{C}_{n_i} \mathbf{W}_i\}, \end{aligned} \tag{3.7}$$

where $\mathbf{C}_{n_i} = \sigma_t^2 \mathbf{H}_{RS_i} \mathbf{F} \mathbf{H}_{LI} \mathbf{H}_{LI}^H \mathbf{F}^H \mathbf{H}_{RS_i}^H + \mathbf{H}_{RS_i} \mathbf{F} \mathbf{F}^H \mathbf{H}_{RS_i}^H + \mathbf{I}_{N_{D_i}}$, and σ_t^2 is the variance of $\Delta \mathbf{t}$.

The problem is to find the matrices $\mathbf{F}, \mathbf{B}_i, \mathbf{W}_i$ such that the SMSE at the two destinations is minimized. The optimization problem can be formulated as

$$\min_{\mathbf{B}_i, \mathbf{F}, \mathbf{W}_i, i=1,2} \quad J_1 + J_2 \tag{3.8a}$$

$$s.t.\ tr\left(\mathbf{F}\left(\sum_{i=1}^{2} \mathbf{H}_{S_iR} \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_{S_iR}^H + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}\right) \mathbf{F}^H\right) \leq P_r \tag{3.8b}$$

$$tr\left(\mathbf{B}_i \mathbf{B}_i^H\right) \leq P_i \tag{3.8c}$$

where $P_i > 0$ and $P_r > 0$ are the power constraints at source node i and the relay node, respectively.

3.4 The Proposed Iterative Algorithms

The original optimization problem in (3.8) is non-convex and a closed-form solution is intractable. Thus, in this section two algorithms are proposed to solve this problem. One is a tri-step iterative algorithm and the other is a bi-step iterative approach with lower computational complexity.

3.4.1 Tri-step Algorithm

In this subsection, a tri-step algorithm [22, 24], is presented which is based on alternating optimization that updates one group of precoders at a time while fixing the others to solve the corresponding convex subproblems to obtain $\mathbf{B}_i$, $\mathbf{F}$ and $\mathbf{W}_i$. First, given $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{F}$, we find the optimal combining matrices $\mathbf{W}_1$ and $\mathbf{W}_2$. Since the power constraints in (3.8b) and (3.8c) are not related to the destination combiners $\mathbf{W}_1$ and $\mathbf{W}_2$, the optimization problem is unconstrained and so is given by

$$\min_{\mathbf{W}_i, i=1,2} J_{W_1} + J_{W_2} \tag{3.9}$$

where

$$\begin{aligned} J_{W_i} = tr\{&\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H\mathbf{W}_i + \mathbf{I}_{N_{S_i}} \\ &- \mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}} - \mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H\mathbf{W}_i + \mathbf{W}_i^H\mathbf{C}_{n_i}\mathbf{W}_i\}. \end{aligned} \tag{3.10}$$

Differentiating J_{W_i} with respect to $\mathbf{W}_i$ and setting the result to zero, the optimal combining matrix can be expressed as

$$\mathbf{W}_i = (\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{C}_{n_i})^{-1}\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}, i = 1, 2. \tag{3.11}$$

This solution is also known as a Wiener filter [22].

Second, the optimal relay precoding matrix $\mathbf{F}$ is obtained by assuming $\mathbf{W}_i$ and $\mathbf{B}_i$, $i = 1, 2$, are fixed and solving the optimization problem

$$\begin{aligned} &\min_{\mathbf{F}} \; J_1 + J_2 \\ &s.t.\; tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) \leq P_r \end{aligned} \tag{3.12}$$

where $\mathbf{K}_x = \left(\sum_{i=1}^{2}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{S_iR}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}\right)$, $i = 1, 2$. The MSE at S_i is

$$\begin{aligned} J_i = tr(&\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H - \mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F} \\ &- \mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H + \mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H \\ &+ \mathbf{W}_i\mathbf{W}_i^H + \sigma_t^2\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H + \mathbf{I}_L) \\ = tr(&\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{K}_{x_{\bar{i}}}\mathbf{F}^H - \mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F} \\ &- \mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H + \sigma_{n_r}^2\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H \\ &+ \mathbf{W}_i\mathbf{W}_i^H + \sigma_t^2\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H + \mathbf{I}_L) \end{aligned} \tag{3.13}$$

where $\mathbf{K}_{x_{\bar{i}}} = \mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H$. As with similar problems [24], (3.13) is convex so the optimal relay precoder can be obtained by employing the KKT conditions. The Lagrangian function of (3.13) is

$$\mathcal{L} = J_1 + J_2 + \lambda(tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) - P_r), \tag{3.14}$$

where $\lambda \geq 0$ is the Lagrange multiplier. Differentiating $\mathcal{L}$ respect to $\mathbf{F}$ with $\mathbf{B}_i$ and $\mathbf{W}_i$

fixed and equating the result to zero gives

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \mathbf{F}} &= \mathbf{H}_{RS_1}^H \mathbf{W}_1 \mathbf{W}_1^H \mathbf{H}_{RS_1} \mathbf{F} \mathbf{K}_{x_2} - \mathbf{H}_{RS_1}^H \mathbf{W}_1 \mathbf{B}_2^H \mathbf{H}_{S_2R}^H \\ &\quad + \mathbf{H}_{RS_2}^H \mathbf{W}_2 \mathbf{W}_2^H \mathbf{H}_{RS_2} \mathbf{F} \mathbf{K}_{x_1} - \mathbf{H}_{RS_2}^H \mathbf{W}_2 \mathbf{B}_1^H \mathbf{H}_{S_1R}^H + \lambda \mathbf{F} \mathbf{K}_x \\ &\quad + \mathbf{H}_{RS_1}^H \mathbf{W}_1 \mathbf{W}_1^H \mathbf{H}_{RS_1} \mathbf{F} (\sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &\quad + \mathbf{H}_{RS_2}^H \mathbf{W}_2 \mathbf{W}_2^H \mathbf{H}_{RS_2} \mathbf{F} (\sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &= \mathbf{K}_{r_1} \mathbf{F} (\mathbf{K}_{x_2} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) + \mathbf{K}_{r_2} \mathbf{F} (\mathbf{K}_{x_1} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &\quad - \mathbf{K}_r + \lambda \mathbf{F} \mathbf{K}_x = 0, \end{aligned} \tag{3.15}$$

where $\mathbf{K}_r = \mathbf{H}_{RS_1}^H \mathbf{W}_1 \mathbf{B}_2^H \mathbf{H}_{S_2R}^H + \mathbf{H}_{RS_2}^H \mathbf{W}_2 \mathbf{B}_1^H \mathbf{H}_{S_1R}^H$, $\mathbf{K}_{r_1} = \mathbf{H}_{RS_1}^H \mathbf{W}_1 \mathbf{W}_1^H \mathbf{H}_{RS_1}$ and $\mathbf{K}_{r_2} = \mathbf{H}_{RS_2}^H \mathbf{W}_2 \mathbf{W}_2^H \mathbf{H}_{RS_2}$. From the properties of the vector operators $\text{vec}(\mathbf{AXB})) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$ and $\text{vec}(\mathbf{A} + \mathbf{B}) = \text{vec}(\mathbf{A}) + \text{vec}(\mathbf{B})$, so we have

$$\begin{aligned}&((\mathbf{K}_{x_2} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_1})\text{vec}(\mathbf{F}) \\ &+ ((\mathbf{K}_{x_1} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_2})\text{vec}(\mathbf{F}) = \text{vec}(\mathbf{K}_r).\end{aligned} \tag{3.16}$$

The optimal solution is then

$$\begin{aligned}\mathbf{F} = \text{mat}\{&[(\mathbf{K}_{x_2} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_1} \\ &+ (\mathbf{K}_{x_1} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_2} + (\lambda \mathbf{K}_x^H)^T \otimes \mathbf{I}_{N_r}]^{-1} \text{vec}(\mathbf{K}_r)\},\end{aligned} \tag{3.17}$$

where $\text{mat}\{\cdot\}$ is the inverse operation of $\text{vec}(\cdot)$. In the case $\lambda = 0$, we have

$$\mathbf{F} = \text{mat}\{[(\mathbf{K}_{x_2} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_1} + (\mathbf{K}_{x_1} + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_2}]^{-1} \text{vec}(\mathbf{K}_r)\}, \tag{3.18}$$

and

$$\lambda(tr(\mathbf{F} \mathbf{K}_x \mathbf{F}^H) - P_r) = 0. \tag{3.19}$$

If $\mathbf{F}$ in (3.18) satisfies the condition in (3.19), then (3.18) is the optimal relay precoder. Otherwise, let $\lambda > 0$ so that

$$tr(\mathbf{F} \mathbf{K}_x \mathbf{F}^H) \leq P_r, \tag{3.20}$$

and substituting (3.17) into (3.20) and solving the resulting nonlinear equation gives

$$tr(\mathbf{F} \mathbf{K}_x \mathbf{F}^H) = P_r. \tag{3.21}$$

In this case, $\mathbf{F}$ decreases with λ because of the inverse in (3.17). The optimal λ that satisfies $tr(\mathbf{F} \mathbf{K}_x \mathbf{F}^H) = P_r$ can then be readily obtained using numerical methods such as bisection search.

An upper bound on λ can be found by following an approach similar to that in [24]. Let $\mathbf{K}_r = \mathbf{E}_1 + \mathbf{E}_2$ where $\mathbf{E}_1 = \mathbf{K}_{r_1}\mathbf{F}(\mathbf{K}_{x_2} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) + \mathbf{K}_{r_2}\mathbf{F}(\mathbf{K}_{x_1} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})$ and $\mathbf{E}_2 = \lambda\mathbf{F}\mathbf{K}_x = 0$. If $\mathbf{F}$ and λ are the optimal primal and dual solutions of (3.8), respectively, then

$$\mathbf{F} = \frac{1}{\lambda}\mathbf{E}_2\mathbf{K}_x^{-1}, \tag{3.22}$$

and

$$tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) = tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{K}_x\mathbf{K}_x^{-1}\mathbf{E}_2^H) = tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_2^H) = P_r. \tag{3.23}$$

On the other hand, if $\lambda > 0$ we have

$$\begin{aligned} tr\left(\frac{1}{\lambda^2}\mathbf{K}_r\mathbf{K}_x^{-1}\mathbf{K}_r^H\right) &= tr(\frac{1}{\lambda^2}(\mathbf{E}_1 + \mathbf{E}_2)\mathbf{K}_x^{-1}(\mathbf{E}_1 + \mathbf{E}_2)^H) \\ &= tr(\frac{1}{\lambda^2}\mathbf{E}_1\mathbf{K}_x^{-1}\mathbf{E}_1^H) + tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_2^H) \\ &\quad + tr(\frac{1}{\lambda^2}\mathbf{E}_1\mathbf{K}_x^{-1}\mathbf{E}_2^H) + tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_1^H) \end{aligned} \tag{3.24}$$

Applying the matrix property that if $\mathbf{Z_1} \geq 0$ and $\mathbf{Z_2} \geq 0$ then $tr(\mathbf{Z_1}\mathbf{Z_2}) \geq 0$ gives $tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_2^H)$
≥ 0, so that

$$tr(\mathbf{E}_1\mathbf{F}^H) = tr(\mathbf{K}_{r_1}\mathbf{F}\mathbf{K}_{x_2}\mathbf{F}^H + \mathbf{K}_{r_2}\mathbf{F}\mathbf{K}_{x_1}\mathbf{F}^H) \geq 0 \tag{3.25}$$

$$tr(\mathbf{E}_1\mathbf{F}^H) = tr(\frac{1}{\lambda^2}\mathbf{E}_1\mathbf{K}_x^{-1}\mathbf{E}_2^H) \geq 0 \tag{3.26}$$

$$tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_1^H) \geq 0 \tag{3.27}$$

Since all the terms in (3.24) are greater than or equal zero, it can be concluded that

$$tr(\frac{1}{\lambda^2}\mathbf{K}_r\mathbf{K}_x^{-1}\mathbf{K}_r^H) \geq P_r. \tag{3.28}$$

Now, we have $\lambda \leq \sqrt{\frac{\mathbf{K}_r\mathbf{K}_x^{-1}\mathbf{K}_r^H}{P_r}}$ which is an upper bound on λ.

Third, the optimal source precoders $\mathbf{B}_i$, $i = 1, 2$, are derived using the previously obtained $\mathbf{F}$ and $\mathbf{W}_i$. From (3.8b), updating the source precoder can affect the power constraint at the relay. Thus the relay power constraint in (3.8b) should be included, so

(3.8) is rewritten as

$$\begin{aligned} \min_{\mathbf{B}_i, i=1,2} \quad & J_{B_1} + J_{B_2} \\ s.t.\ & tr(\mathbf{B}_i\mathbf{B}_i^H) \leq P_i \\ & tr(\sum_{i=1}^{2} \mathbf{H}_{S_iR}^H \mathbf{F}^H \mathbf{F} \mathbf{H}_{S_iR} \mathbf{B}_i \mathbf{B}_i^H) \leq \bar{P}_r \end{aligned} \tag{3.29}$$

where $\bar{P}_r = P_r - tr(\mathbf{F}(\sigma_t^2 \mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H)$. Now let

$$\mathbf{K}_{O_i} = \mathbf{H}_{S_iR}^H \mathbf{F}^H \mathbf{F} \mathbf{H}_{S_iR}, \tag{3.30}$$

$$J_{B_i} = tr\{\mathbf{K}_{S_i1}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H - 2\Re(\mathbf{K}_{S_i2}\mathbf{B}_{\bar{i}}) + \mathbf{K}_{S_i3}\}, \tag{3.31}$$

$$\mathbf{K}_{S_i1} = \mathbf{H}_{S_{\bar{i}}R}^H \mathbf{F}^H \mathbf{H}_{RS_i}^H \mathbf{W}_i \mathbf{W}_i^H \mathbf{H}_{RS_i} \mathbf{F} \mathbf{H}_{S_{\bar{i}}R}, \tag{3.32}$$

$$\mathbf{K}_{S_i2} = \mathbf{W}_i^H \mathbf{H}_{RS_i} \mathbf{F} \mathbf{H}_{S_{\bar{i}}R}, \tag{3.33}$$

$$\mathbf{K}_{S_i3} = \mathbf{W}_i^H \mathbf{C}_{n_i} \mathbf{W}_i + \mathbf{I}_{N_{S_i}}. \tag{3.34}$$

Applying the trace operator identity $tr\{\mathbf{ABCD}\} = (\text{vec}(\mathbf{D})^T)^T(\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B})$ gives

$$J_{B_i} = \hat{\mathbf{b}}_{\bar{i}}^H \hat{\mathbf{O}}_i \hat{\mathbf{b}}_{\bar{i}} - 2\Re\{\hat{\mathbf{a}}_i^T \hat{\mathbf{b}}_{\bar{i}}\} + tr\{\mathbf{K}_{S_i3}\}, i = 1, 2, \tag{3.35}$$

where $\hat{\mathbf{O}}_i = \mathbf{I}_N \otimes \mathbf{K}_{S_i1}$, $\hat{\mathbf{a}}_i = \text{vec}(\mathbf{K}_{S_i2}^T)$ and $\hat{\mathbf{b}}_i = \text{vec}(\mathbf{B}_i)$. Because $\hat{\mathbf{O}}_i$ is positive semidefinite (PSD), (34) can be transformed into

$$J_{B_i} = \| \hat{\mathbf{O}}_i^{\frac{1}{2}} \hat{\mathbf{b}}_{\bar{i}} \|_2^2 - 2\Re\{\hat{\mathbf{a}}_i^T \hat{\mathbf{b}}_{\bar{i}}\} + tr\{\mathbf{K}_{S_i3}\}, i = 1, 2 \tag{3.36}$$

To eliminate the $\Re\{\cdot\}$ operation, let $\mathbf{b}_i = [\Re\{\hat{\mathbf{b}}_i^T\}, \Im\{\hat{\mathbf{b}}_i^T\}]^T, i = 1, 2$. This gives

$$J_{B_i} = \mathbf{b}_{\bar{i}}^T \mathbf{O}_i \mathbf{b}_{\bar{i}} - 2\mathbf{a}_i^T \mathbf{b}_{\bar{i}} + tr\{\mathbf{K}_{S_i3}\}, i = 1, 2 \tag{3.37}$$

where $\mathbf{O}_i = \tilde{\mathbf{O}}_i^T \tilde{\mathbf{O}}_i$ with $\tilde{\mathbf{O}}_i = \begin{bmatrix} \Re\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} & -\Im\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} \\ \Im\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} & \Re\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} \end{bmatrix}$, $\mathbf{a}_i = \begin{bmatrix} \Re\{\hat{\mathbf{a}}_i^T\} & -\Im\{\hat{\mathbf{a}}_i^T\} \end{bmatrix}^T$, $i = 1, 2$. In addition, for the power constraints in (3.29) we have $tr\{\mathbf{B}_i\mathbf{B}_i^H\} = \mathbf{b}_i^T \hat{\mathbf{E}}_i \mathbf{b}_i$ with $\hat{\mathbf{E}}_i = \mathbf{I}_{2L^2 \times 2L^2}, i = 1, 2$, and $tr\{\mathbf{K}_{o_1}\mathbf{B}_1\mathbf{B}_1^H + \mathbf{K}_{o_2}\mathbf{B}_2\mathbf{B}_2^H\} = \mathbf{b}_1^H \hat{\mathbf{E}}_3 \mathbf{b}_1 + \mathbf{b}_2^H \hat{\mathbf{E}}_4 \mathbf{b}_2$, where $\hat{\mathbf{E}}_3 = \tilde{\mathbf{E}}_3^T \tilde{\mathbf{E}}_3$ and $\hat{\mathbf{E}}_4 = \tilde{\mathbf{E}}_4^T \tilde{\mathbf{E}}_4$ are positive semidefinite matrices with

$$\tilde{\mathbf{E}}_3 = \begin{bmatrix} \Re\{(\mathbf{I}_N \otimes \mathbf{K}_{O_1})^{\frac{1}{2}}\} & -\Im\{(\mathbf{I}_N \otimes \mathbf{K}_{O_1})^{\frac{1}{2}}\} \\ \Im\{(\mathbf{I}_N \otimes \mathbf{K}_{O_1})^{\frac{1}{2}}\} & \Re\{(\mathbf{I}_N \otimes \mathbf{K}_{O_1})^{\frac{1}{2}}\} \end{bmatrix}, \tag{3.38}$$

and

$$\tilde{\mathbf{E}}_4 = \begin{bmatrix} \Re\{(\mathbf{I}_N \otimes \mathbf{K}_{O_2})^{\frac{1}{2}}\} & -\Im\{(\mathbf{I}_N \otimes \mathbf{K}_{O_2})^{\frac{1}{2}}\} \\ \Im\{(\mathbf{I}_N \otimes \mathbf{K}_{O_2})^{\frac{1}{2}}\} & \Re\{(\mathbf{I}_N \otimes \mathbf{K}_{O_2})^{\frac{1}{2}}\} \end{bmatrix}. \tag{3.39}$$

The resulting optimization problem has the form

$$\begin{aligned} &\min_{\mathbf{b}} \ \mathbf{b}^T\mathbf{O}\mathbf{b} - \mathbf{a}^T\mathbf{b} + tr\{\mathbf{K}_{S_13} + \mathbf{K}_{S_23}\} \\ &s.t. \ \mathbf{a}^T\mathbf{E}_1\mathbf{b} \le P_1, \mathbf{b}^T\mathbf{E}_2\mathbf{b} \le P_2 \\ &\quad\ \ \mathbf{a}^T\mathbf{E}_3\mathbf{b} \le \bar{P}_r \end{aligned} \tag{3.40}$$

where $\mathbf{O} = \begin{bmatrix} \mathbf{O}_2 & 0 \\ 0 & \mathbf{O}_1 \end{bmatrix}$, $\mathbf{a} = [2\mathbf{a}_2^T, 2\mathbf{a}_1^T]^T$, $\mathbf{b} = [\mathbf{b}_1^T, \mathbf{b}_2^T]^T$, $\mathbf{E}_1 = \begin{bmatrix} \hat{\mathbf{E}}_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{E}_2 = \begin{bmatrix} 0 & 0 \\ 0 & \hat{\mathbf{E}}_2 \end{bmatrix}$, and $\mathbf{E}_3 = \begin{bmatrix} \hat{\mathbf{E}}_3 & 0 \\ 0 & \hat{\mathbf{E}}_4 \end{bmatrix}$. Note that the term $tr\{\mathbf{K}_{S_13} + \mathbf{K}_{S_23}\}$ does not affect the result of the optimization problem and so can be ignored. Since $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ and $\mathbf{O}$ are positive semidefinite, the problem can be transformed into a convex QCQP problem and efficiently solved using CVX [50]. The algorithm to solve the original optimization problem (3.8) is summarized in Algorithm 3.

Algorithm 3 Tri-Step Iterative Algorithm to Design $\mathbf{B}_i$, $\mathbf{F}$ and $\mathbf{W}_i$

1: Initialize the algorithm with $\mathbf{B}_i^{(n)} = \sqrt{\frac{P_{si}}{L}}\mathbf{I}_L$, $\mathbf{F}^{(n)} = \sqrt{\frac{P_r}{tr(\sum_{i=1}^2 \mathbf{H}_{S_iR}\mathbf{B}_i^{(n)}(\mathbf{H}_{S_iR}\mathbf{B}_i^{(n)})^H + \mathbf{I}_{N_r})}}\mathbf{I}_{N_r}$, $i = 1, 2$, and set $n = 0$.
2: Update $\mathbf{W}_i^{(n)}$ using (3.11) with $\mathbf{F}^{(n)}$ and $\mathbf{B}_i^{(n)}$.
3: Update $\mathbf{F}^{(n+1)}$ using (3.17) and (3.28) with $\mathbf{W}_i^{(n)}$ and $\mathbf{B}_i^{(n)}$.
4: Update $\mathbf{B}_i^{(n+1)}$ by solving the problem (3.40) using $\mathbf{W}_i^{(n)}$ and $\mathbf{F}^{(n+1)}$.
5: If $(\text{SMSE}^{(n)} - \text{SMSE}^{(n+1)})/\text{SMSE}^{(n)} > \epsilon$, go to step 2.
6: End

3.4.2 Bi-step Algorithm

The tri-step iterative algorithm presented above provides good performance according to the results presented in Section 3.5, but the computational complexity is high due to the number of iterations required for convergence. In this section, a bi-step iterative approach for source and relay precoding matrices design is presented which has a smaller computational complexity than the tri-step algorithm. Applying the combiner (3.11) at the destinations, the MSE of the signal estimate at node i in (3.7) is a function of $\mathbf{B}_i$ and $\mathbf{F}$ given by

$$J_i = tr\{[\mathbf{I}_{N_{S_i}} + \mathbf{H}_i\mathbf{C}_{n_i}^{-1}\mathbf{H}_i^H]^{-1}\}, \tag{3.41}$$

where $\mathbf{H}_i = \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}$. Thus, the joint source and relay precoder optimization problem for the proposed two-way full-duplex relaying system is

$$\min_{\mathbf{B}_i,\mathbf{F},i=1,2} \quad J_1 + J_2 \tag{3.42a}$$

$$s.t.\ tr(\mathbf{F}(\sum_{i=1}^{2}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{S_iR}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_{D_i}})\mathbf{F}^H) \leq P_r \tag{3.42b}$$

$$tr(\mathbf{B}_i\mathbf{B}_i^H) \leq P_i \tag{3.42c}$$

In this iterative algorithm, the source and relay precoders are found by solving two convex subproblems.

Assuming source matrices $\mathbf{B}_i$ satisfying (3.42c) are given, and eliminating the constraint in (3.42c), the relay matrix $\mathbf{F}$ is optimized by solving the following problem

$$\min_{\mathbf{F},i=1,2} \quad J_1 + J_2 \tag{3.43a}$$

$$s.t.\ tr(\mathbf{F}(\sum_{i=1}^{2}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{S_iR}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_{D_i}})\mathbf{F}^H) \leq P_r \tag{3.43b}$$

It was proven in [39] that the optimal precoding in one-way relaying has the channels between the source and relay parallel. Then, singular value decomposition (SVD) can be used between the relay and destination to match the eigenchannels in the two communication hops. Similar to the approach in [24], a heuristic channel parallelization method for bidirectional communications can be employed which uses generalized singular value decomposition (GSVD) for the MAC phase and SVD for the BC phase. Applying GSVD for the MAC phase channel pair $(\mathbf{H}_{S_1R})^H$, $(\mathbf{H}_{S_2R})^H$ gives

$$\mathbf{H}_{S_1R} = \mathbf{V}_h\mathbf{\Gamma}_{h_1}\mathbf{U}_{h_1}^H, \tag{3.44}$$

$$\mathbf{H}_{S_2R} = \mathbf{V}_h\mathbf{\Gamma}_{h_2}\mathbf{U}_{h_2}^H, \tag{3.45}$$

where $\mathbf{V}_h$ is a nonsingular $N_r \times N_r$ complex matrix, $\mathbf{U}_{h_1}^H$ and $\mathbf{U}_{h_2}^H$ are $L \times L$ unitary matrices, and $\mathbf{\Gamma}_{h_1} = [\mathbf{0}_{(N_r-L)\times L}^T, \mathbf{\Lambda}_{h_1}^T]^T$ and $\mathbf{\Gamma}_{h_2} = [\mathbf{\Lambda}_{h_2}^T, \mathbf{0}_{(N_r-L)\times L}^T]^T$ where $\mathbf{\Lambda}_{h_1}$ and $\mathbf{\Lambda}_{h_2}$ are $L \times L$ nonnegative diagonal matrices.

For the BC phase, since the superimposed signal is simultaneously transmitted to the two nodes, a virtual point-to-point MIMO channel $\mathbf{H}_{rs} = [(\mathbf{H}_{RS_1})^T, (\mathbf{H}_{RS_2})^T]^T$ can be established. Employing SVD on $\mathbf{H}_{rs}$ gives

$$\mathbf{H}_{rs} = \mathbf{V}_g\mathbf{\Gamma}_g\mathbf{U}_g^H, \tag{3.46}$$

where $\mathbf{V}_g$ and $\mathbf{U}_g$ are $2N_r \times 2N_r$ and $N_t \times N_t$ unitary matrices, respectively, $\mathbf{\Gamma}_g = [\mathbf{\Lambda}_g^T, \mathbf{0}_{(2N_r-N_t)\times N_t}^T]^T$, and $\mathbf{\Lambda}_g$ is an $N_t \times N_t$ nonnegative diagonal matrix. Employing SVD on $\mathbf{H}_{RS_1}$ and $\mathbf{H}_{RS_2}$ gives

$$\mathbf{H}_{RS_1} = \mathbf{V}_{g_1}\mathbf{\Gamma}_g\mathbf{U}_g^H \tag{3.47}$$

$$\mathbf{H}_{RS_2} = \mathbf{V}_{g_2}\mathbf{\Gamma}_g\mathbf{U}_g^H \tag{3.48}$$

where $\mathbf{V}_{g_1} = \mathbf{V}_g(1 : N_r, 1 : 2N_r)$ and $\mathbf{V}_{g_2} = \mathbf{V}_g(N_r + 1 : 2N_r, 1 : 2N_r)$. Note that $\mathbf{V}_{g_1}$ and $\mathbf{V}_{g_2}$ are not unitary matrices. Based on the solution of a similar problem in [24, (29)], the optimal relay precoding matrix obtained by solving problem (3.43) is

$$\mathbf{F} = \mathbf{U}_g\mathbf{\Lambda}_F\mathbf{V}_h^{-1}, \tag{3.49}$$

and the ith source precoder is

$$\mathbf{B}_i = \mathbf{U}_{h_i}\mathbf{\Lambda}_{B_i}\mathbf{V}_{B_i}, i = 1, 2. \tag{3.50}$$

Substituting (3.47) and (3.48) in (3.41) gives

$$\begin{aligned} J_i = \ & tr\{[\mathbf{I}_L + (\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}})(\sigma_t^2\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H \\ & + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{I}_{D_i})^{-1}(\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}})^H]^{-1}\}. \end{aligned} \tag{3.51}$$

Due to the similarity between J_1 and J_2, we focus on the derivation of J_1 and the results for J_2 can be obtained using the same approach. Substituting (3.44)-(3.50) in (3.51) gives

$$\begin{aligned} J_1 = \ & tr\{[\mathbf{I}_{N_{s_1}} + (\mathbf{V}_{g_1}\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{\Gamma}_{h_2}\mathbf{\Lambda}_{B_2}\mathbf{V}_{B_2}^H)^H(\mathbf{V}_{g_1}\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{V}_h^{-1}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ & \times \mathbf{V}_h^{-H}\mathbf{\Lambda}_F^H\mathbf{\Gamma}_g^H\mathbf{V}_{g_1}^H + \mathbf{I}_{N_{D_1}})^{-1}(\mathbf{V}_{g_1}\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{\Gamma}_{h_2}\mathbf{\Lambda}_{B_2}\mathbf{V}_{B_2}^H)]^{-1}\} \end{aligned} \tag{3.52}$$

Denoting $\mathbf{B}_{h_{LI}} = \mathbf{V}_h^{-1}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{V}_h^{-H}$ and $\mathbf{B}_{g_i} = (\mathbf{V}_{g_i}\mathbf{V}_{g_i}^H)^{-1}$ gives

$$J_1 = \ tr\{[\mathbf{I}_L + (\mathbf{\Lambda}_{B_2}\mathbf{\Gamma}_{h_2}\mathbf{\Lambda}_F\mathbf{\Gamma}_g)(\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{B}_{h_{LI}}\mathbf{\Lambda}_F\mathbf{\Gamma}_g + \mathbf{B}_{g_1})^{-1}(\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{\Gamma}_{h_2}\mathbf{\Lambda}_{B_2})]^{-1}\} \tag{3.53}$$

It is obvious that the MSE covariance matrix in (3.53) is not diagonal since $\mathbf{B}_{h_{LI}}$ is an non-diagonal matrix. To solve this issue, let $\mathbf{C} = \mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{B}_{h_{LI}}\mathbf{\Lambda}_F\mathbf{\Gamma}_g + \mathbf{B}_{g_1}$, and $\mathbf{D} =$

$\mathbf{\Lambda}_{B_2}\mathbf{\Gamma}_{h_2}\mathbf{\Lambda}_F\mathbf{\Gamma}_g$ gives

$$\begin{aligned} J_1 =& \; tr\{[\mathbf{I}_L + \mathbf{D}\mathbf{C}^{-1}\mathbf{D}]^{-1}\} \\ =& \; tr\{\mathbf{I}_L - (\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{C}\mathbf{D}^{-1})^{-1}\}, \end{aligned} \tag{3.54}$$

where the matrix inversion lemma $(\mathbf{I}+\mathbf{A}^{-1})^{-1} = \mathbf{I}-(\mathbf{I}+\mathbf{A})^{-1}$ has been applied. Since $tr\{\mathbf{A}^{-1}\} \geq \sum_i [\mathbf{A}(i,i)]^{-1}$ for any positive definite square matrix $\mathbf{A}$, we have

$$\begin{aligned} J_1 &\leq \mathbf{I}_L - \sum_{i=1}^{L}[(\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{C}\mathbf{D}^{-1})(i,i)] \\ &= tr\{\mathbf{I}_L - (\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{\Lambda}_c\mathbf{D}^{-1})^{-1}\} \\ &= tr\{[\mathbf{I}_L + \mathbf{D}\mathbf{\Lambda}_c^{-1}\mathbf{D}]^{-1}\}, \end{aligned} \tag{3.55}$$

so the upper bound on J_1 is

$$\begin{aligned} J_1 \leq J_1^u = tr\{[\mathbf{I}_L + (\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{\Gamma}_{h_2}\mathbf{\Lambda}_{B_2})(\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{\Lambda}_{B_{h_{LI}}}\mathbf{\Lambda}_F\mathbf{\Gamma}_g \\ + \mathbf{\Lambda}_{B_{g_1}})^{-1}(\mathbf{\Gamma}_g\mathbf{\Lambda}_F\mathbf{\Gamma}_{h_2}\mathbf{\Lambda}_{B_2})]^{-1}\} \end{aligned} \tag{3.56}$$

where $\mathbf{\Lambda}_{B_{h_{LI}}}$ and $\mathbf{\Lambda}_{B_{g_1}}$ are diagonal matrices containing the diagonal entries of $\mathbf{B}_{h_{LI}}$ and $\mathbf{B}_{g_1}$, respectively. Now the upper bound in (3.56) has a diagonal structure, so the precoders can be obtained by minimizing this bound. Assuming $\mathbf{P}_k = \mathbf{\Lambda}_k^2$ for $k \in \{h_1, h_2, F, g, B_2, B_2\}$, the upper bound in (3.56) can be reformulated as

$$J_1^u = \sum_{n=1}^{L}\left(1 + \frac{p_g^n p_{h_2}^n p_F^n p_{B_2}^n}{\lambda_{B_{g_1}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n}\right)^{-1}, \tag{3.57}$$

where the p_k^n are the diagonal entries of $\mathbf{P}_k$ and λ_k^n f $k \in \{B_{g_1}, B_{g_2}, B_1, B_2, B_{h_{LI}}\}$ are the diagonal entries of $\mathbf{\Lambda}_k$. The precoder design can then be simplified to the following optimization problem

$$\min_{p_F} \; J_1^u + J_2^u \tag{3.58a}$$

$$s.t. \; \sum_{n=1}^{L} p_F^n (p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) \leq P_r \tag{3.58b}$$

This problem is convex and thus can be solved using the KKT conditions. The Lagrangian function of (3.58) is

$$\mathcal{L} = \sum_{n=1}^{L} \left[\frac{\lambda_{B_{g_1}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n}{\lambda_{B_{g_1}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_2}^n p_F^n p_{B_2}^n} + \frac{\lambda_{B_{g_2}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n}{\lambda_{B_{g_2}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_1}^n p_F^n p_{B_1}^n} \right] + \mu \left[\sum_{n=1}^{L} p_F^n (p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) - P_r \right], \tag{3.59}$$

where $\mu \geq 0$ is the Lagrange multiplier. Taking the derivative with respect to p_F^n gives

$$\frac{\partial \mathcal{L}}{\partial p_F^n} = \sum_{i=1}^{2} \frac{-(\lambda_{B_{g_i}}^n p_{h_{\bar{i}}}^n p_g^n p_{B_{\bar{i}}}^n)}{(\lambda_{B_{g_i}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_{\bar{i}}}^n p_F^n p_{B_{\bar{i}}}^n)^2} + \mu (p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) = 0 \tag{3.60}$$

and the complementarity condition can be expressed as

$$\mu \left[\sum_{n=1}^{L} p_F^n (p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) - P_r \right] = 0. \tag{3.61}$$

From (3.60) and (3.61)

$$p_F^n = \max[0, \text{Root}(f^n)], \forall n, \tag{3.62}$$

where $\text{Root}(f^n)$ denotes the maximum real root of

$$f^n = \mu (p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) = \sum_{i=1}^{2} \frac{\lambda_{B_{g_i}}^n p_{h_{\bar{i}}}^n p_g^n p_{B_{\bar{i}}}^n}{(\lambda_{B_{g_i}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_{\bar{i}}}^n p_F^n p_{B_{\bar{i}}}^n)^2}, \tag{3.63}$$

and μ should be chosen to satisfy

$$\sum_{n=1}^{L} p_F^n (p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) = P_r. \tag{3.64}$$

Equation (3.64) can be efficiently solved numerically as follows.

In order to find the optimal solution for the relay precoder $\mathbf{F}$ in (3.49), $\mathbf{\Lambda}_F$ must be found. The diagonal entries of $\mathbf{\Lambda}_F$ are given by p_F^n from (3.62), and (3.63) can be expressed as

$$
\begin{aligned}
f^n = \; & \mu(p^n_{h_1}p^n_{B_1} + p^n_{B_2}p^n_{h_2} + \lambda^n_{B_{h_{LI}}}) - \frac{\lambda^n_{B_{g_1}}p^n_{h_2}p^n_g p^n_{B_2}}{[\lambda^n_{B_{g_1}} + p^n_F(p^n_g\lambda^n_{B_{h_{LI}}} + p^n_{h_2}p^n_g p^n_{B_2})]^2} \\
& - \frac{\lambda^n_{B_{g_2}}p^n_{h_1}p^n_g p^n_{B_1}}{[\lambda^n_{B_{g_2}} + p^n_F(p^n_g\lambda^n_{B_{h_{LI}}} + p^n_{h_1}p^n_g p^n_{B_1})]^2} = 0.
\end{aligned}
\tag{3.65}
$$

This can be rewritten as

$$
\begin{aligned}
f^n = \; & (p^n_F)^4[T^n(S^n_1)^2(S^n_2)^2)] + (p^n_F)^3[T^n(2R^n_2)(S^n_1)^2S^n_2 + (2R^n_1)(S^n_2)^2S^n_1] \\
& + (p^n_F)^2[T^n((R^n_1)^2(S^n_2)^2 + (S^n_1)^2(R^n_2)^2 + 4R^n_1R^n_2S^n_1S^n_2) - Q^n_1(S^n_2)^2 - Q^n_2(S^n_1)^2] \\
& + (p^n_F)[T^n(2(R^n_1)^2R^n_2s^n_2 + 2R^n_1(R^n_2)^2s^n_1) - (2Q^n_1R^n_2s^n_2 + 2Q^n_2R^n_1s^n_1)] \\
& + T^n(R^n_1)^2(R^n_2)^2 - Q^n_1(R^n_2)^2 - Q^n_2(R^n_1)^2 = 0
\end{aligned}
\tag{3.66}
$$

where $T^n = \mu(p^n_{h_1}p^n_{B_1} + p^n_{B_2}p^n_{h_2} + \lambda^n_{B_{h_{LI}}})$, $Q^n_1 = \lambda^n_{B_{g_1}}p^n_{h_2}p^n_g p^n_{B_2}$, $Q^n_2 = \lambda^n_{B_{g_2}}p^n_{h_1}p^n_g p^n_{B_1}$, $R^n_1 = \lambda^n_{B_{g_1}}$, $R^n_2 = \lambda^n_{B_{g_2}}$, $S^n_1 = p^n_g\lambda^n_{B_{h_{LI}}} + p^n_{h_2}p^n_g p^n_{B_2}$ and $S^n_2 = p^n_g\lambda^n_{B_{h_{LI}}} + p^n_{h_1}p^n_g p^n_{B_1}$. p^n_F is the maximum real root of the polynomial in (3.66). The precoder $\mathbf{F}$ in (3.49) is then obtained using $\mathbf{\Lambda}_F$.

The next task is to obtain $\mathbf{B}_i$ using the optimal relay precoder $\mathbf{F}$ from (3.49). Using the identity

$$
tr\{[\mathbf{I}_m + \mathbf{A}_{m\times n}\mathbf{B}_{n\times m}]^{-1}\} = tr\{[\mathbf{I}_n + \mathbf{B}_{n\times m}\mathbf{A}_{m\times n}]^{-1}\} + m - n,
$$

the objective function (3.41) can be expressed as

$$
\begin{aligned}
J_i &= tr\{[\mathbf{I}_L + \mathbf{H}^H_i\mathbf{C}^{-1}_{n_i}\mathbf{H}_i]^{-1}\} + N_t - L \\
&= tr\{[\mathbf{I}_L + \mathbf{C}^{-\frac{1}{2}}_{n_i}\mathbf{H}_i\mathbf{H}^H_i\mathbf{C}^{-\frac{1}{2}}_{n_i}]^{-1}\} + N_t - L \\
&= tr\{[\mathbf{I}_L + \mathbf{C}^{-\frac{1}{2}}_{n_i}\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}^H_{\bar{i}}\mathbf{H}^H_{S_{\bar{i}}R}\mathbf{F}^H\mathbf{H}^H_{RS_i}\mathbf{C}^{-\frac{1}{2}}_{n_i}]^{-1}\} \\
&= tr\{[\mathbf{I}_L + \mathbf{F}_i\mathbf{Q}_{\bar{i}}\mathbf{F}^H_i]^{-1}\},
\end{aligned}
\tag{3.67}
$$

where $\mathbf{C}_{n_i} = \sigma^2_t\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}^H_{LI}\mathbf{F}^H\mathbf{H}^H_{RS_i} + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H\mathbf{H}^H_{RS_i} + \mathbf{I}_{N_{D_i}}$, $\mathbf{Q}_i = \mathbf{B}_{\bar{i}}\mathbf{B}^H_{\bar{i}}$ and $\mathbf{F}_i = \mathbf{C}^{-\frac{1}{2}}_{n_i}\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}$. Using the above results, the original optimization problem can be transformed into

$$
\min_{\mathbf{Q}_i} \; tr\{[\mathbf{I}_L + \mathbf{F}_1\mathbf{Q}_2\mathbf{F}^H_1]^{-1}\} + tr\{[\mathbf{I}_L + \mathbf{F}_2\mathbf{Q}_1\mathbf{F}^H_2]^{-1}\} \tag{3.68a}
$$

$$
s.t. \; tr\{(\mathbf{Q}_1(\mathbf{H}^H_{S_1R}\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_1R}) + (\mathbf{Q}_2(\mathbf{H}^H_{S_1R}\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_1R})\} \le \bar{P}_r \tag{3.68b}
$$

$$
tr\{\mathbf{B}_i\mathbf{B}^H_i\} \le P_i \tag{3.68c}
$$

where $\bar{P}_r = P_r - tr(\mathbf{F}(\sigma_t^2 \mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H)$.

We now introduce positive semidefinite (PSD) matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ that satisfy

$$[\mathbf{I}_L + \mathbf{F}_1\mathbf{Q}_2\mathbf{F}_1^H]^{-1} \leq \mathbf{X}_1, \tag{3.69a}$$

$$[\mathbf{I}_L + \mathbf{F}_2\mathbf{Q}_1\mathbf{F}_2^H]^{-1} \leq \mathbf{X}_2, \tag{3.69b}$$

and using the Schur complement gives

$$\begin{bmatrix} \mathbf{X}_1 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_1\mathbf{Q}_2\mathbf{F}_1^H \end{bmatrix} \succeq 0, \tag{3.70a}$$

$$\begin{bmatrix} \mathbf{X}_2 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_2\mathbf{Q}_1\mathbf{F}_2^H \end{bmatrix} \succeq 0, \tag{3.70b}$$

where $\mathbf{X} \succeq 0$ means that $\mathbf{X}$ is PSD. Since the sum of two PSD matrices with the same dimensions is still PSD, let $\mathbf{X} = \mathbf{X}_1 + \mathbf{X}_2$ so that problem (3.68) can be converted to the following positive semidefinite programming optimization problem

$$\min_{\mathbf{Q}_i, \mathbf{X}} \; tr\{\mathbf{X}\} \tag{3.71a}$$

$$s.t. \; \begin{bmatrix} \mathbf{X}_1 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_1\mathbf{Q}_2\mathbf{F}_1^H \end{bmatrix} \succeq 0 \tag{3.71b}$$

$$\begin{bmatrix} \mathbf{X}_2 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_2\mathbf{Q}_1\mathbf{F}_2^H \end{bmatrix} \succeq 0 \tag{3.71c}$$

$$tr\{(\mathbf{Q}_1(\mathbf{H}_{S_1R}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_1R}) + (\mathbf{Q}_2(\mathbf{H}_{S_2R}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_2R})\} \leq \bar{P}_r \tag{3.71d}$$

$$tr\{\mathbf{Q}_i\} \leq P_i \tag{3.71e}$$

$$\mathbf{Q}_i \geq 0, i = 1, 2 \tag{3.71f}$$

The CVX software package [50] can be used to solve problem (3.71). Then the original source and relay precoding optimization problem given in (3.42) can be solved with the iterative algorithm given in Algorithm 4.

Algorithm 4 Bi-Step Iterative Algorithm to Design $\mathbf{B}_i$ and $\mathbf{F}$

1: Initialize the algorithm with $\mathbf{B}_i^{(n)} = \sqrt{\frac{P_{si}}{L}}\mathbf{I}_L$, $i = 1, 2$, and set $n = 0$.
2: $\mathbf{F}^{(n)}$ is first solved with (3.49) and (3.62) using $\mathbf{B}_i^{(n)}$.
3: Update the subproblem (3.71) using $\mathbf{F}^{(n)}$ to obtain $\mathbf{B}_i^{(n+1)}$.
4: If $(\text{SMSE}^{(n)} - \text{SMSE}^{(n+1)})/\text{SMSE}^{(n)} > \epsilon$, go to step 2.
5: End

a) Convergence Analysis

The tri-step algorithm can be shown to converge as follows. It is obvious that the subproblems are convex. It then follows that each update of $\mathbf{B}_i$, $\mathbf{F}$ and $\mathbf{W}_i$ will decrease or at least not increase the value of the objective function, and thus the iterative algorithm converges to at least a local optimum solution. Similarity, the two subproblems in the bi-step algorithm are convex. It then follows that each update of $\mathbf{B}_i$ and $\mathbf{F}$ will decrease or at least not increase the value of the objective function, and thus the bi-step iterative algorithm also converges to at least a local optimum solution.

b) Complexity Comparison

The number of iterations required for convergence for the two algorithms is given in Table 3.1 for the same tolerance $\epsilon = 0.001$. The parameters used are $N_{s_1} = N_{s_2} = N_t = N_r = 2$ and $\text{SNR}_{s-r} = 30$ dB with SNR_{r-d} set to $0, 5, 10, 15, 20, 25$ and 30 dB. The residual loop interference level is set to 10 dB. These results shows that the proposed tri-step algorithm requires a larger number of iterations when the SNR is high. When the SNR is 10 dB or less, the algorithms require a similar number of iterations, but the results for this region are not important as the performance is poor. Comparing the performance and complexity of the two algorithms, the bi-step algorithm provides a good trade off between performance and computational complexity.

Table 3.1: Average Number of Iterations Required for Convergence for the Two-Way System

SNR_{rd_i} (dB)	0	5	10	15	20	25	30
Tri-Step Algorithm	4	8	8	12	16	21	24
Bi-Step Algorithm	4	8	8	8	11	11	12

3.5 Simulation Results

In this section, the performance of the proposed optimization algorithms is studied through numerical simulation. A frequency-flat block-fading channel is considered as in the related literature and the system employs Orthogonal Frequency Division Multiplexing (OFDM) for broadband transmission over multi path channels, so the transmitted signal represent narrowband sub-carriers. Therefore, flat-fading MIMO channels are considered. The entries of $\mathbf{H}_{S_iR}$ and $\mathbf{H}_{RS_i}$ are identically and independent distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, and the entries of $\mathbf{H}_{LI}$ are i.i.d. complex Gaussian random variables with zero mean and variance σ_{LI}^2.

The received signal-to-interference-plus-noise ratio (SINR) at node S_1 is

$$\mathbf{\Theta}_1 = \frac{\frac{P_{s_1}}{N_{t_1}} \left\|\mathbf{H}_{RS_1}\mathbf{F}\mathbf{H}_{S_2R}\right\|^2}{\sigma_t^2 \left\|\mathbf{H}_{RS_1}\mathbf{F}\mathbf{H}_{LI}\right\|^2 + \left\|\mathbf{H}_{RS_1}\mathbf{F}\right\|^2 + \mathbf{I}_{N_{D_1}}}, \tag{3.72}$$

and the received SINR at node S_2 is

$$\mathbf{\Theta}_2 = \frac{\frac{P_{s_2}}{N_{t_2}} \left\|\mathbf{H}_{RS_2}\mathbf{F}\mathbf{H}_{S_1R}\right\|^2}{\sigma_t^2 \left\|\mathbf{H}_{RS_2}\mathbf{F}\mathbf{H}_{LI}\right\|^2 + \left\|\mathbf{H}_{RS_2}\mathbf{F}\right\|^2 + \mathbf{I}_{N_{D_2}}}. \tag{3.73}$$

The achievable rates are given by $R_1 = \log_2 \det[\mathbf{I}_{N_r} + \mathbf{\Theta}_1]$ and $R_2 = \log_2 \det[\mathbf{I}_{N_r} + \mathbf{\Theta}_2]$, respectively, where $\det(\mathbf{A})$ denotes the determinant of $\mathbf{A}$. Therefore, the sum achievable rate of the proposed two-way FD relay system can be written as $R_{sum} = R_1 + R_2$.

The performance of the proposed precoding algorithms for a two-way MIMO full-duplex relaying system is examined in terms of the sum mean squared error (SMSE) and the sum achievable rate. The results are compared with those of the corresponding half-duplex (HD) relay system. Note that the precoding algorithms for the HD system are the same as for the FD system except that the residual LI term is zero. Further, the achievable rate for the HD system is reduced by half because two time slots are required for information exchange between the two nodes. The signal-to-noise ratios (SNRs) of the source-to-relay and relay-to-destination channels are $\text{SNR}_{s_i-r} = \frac{P_{s_i}}{N_t}$ and $\text{SNR}_{r-d_i} = \frac{P_r}{M_r}$, respectively. For simplicity, it is assumed that perfect channel state information (CSI) is available for all channels. Further, $N_{s_1} = N_{s_2} = N_t = N_r = 2$ is assumed in all simulations. The extension to the case with more than 2 antennas is straightforward. All the results are averaged over 1000 trials with independent channel realizations. As discussed in [10], the residual LI can vary from 0 dB to 15 dB larger than the channel noise. Therefore, the residual LI levels considered here are 0 dB, 5 dB and 10 dB. The convergence tolerance for the tri-step iterative algorithm is set to $\epsilon = 1e^{-6}$ and the maximum number of iterations is 30.

Fig. 3.2 presents the SMSE of the proposed tri-step iterative method versus SNR_{s-r} with $\text{SNR}_{r-d} = 30$ dB. It is clear that the FD system has a higher SMSE than the HD system due to the existence of residual LI. Further, the SMSE increases as the residual LI level increases. Fig. 3.3 presents the achievable rate of the HD and FD systems. The FD system sum achievable rate is twice that of the HD system when the LI is canceled completely. The FD system outperform the HD system in the region when $\text{SNR}_{s-r} \geq 17$ dB for all levels of residual LI. Further, when the residual LI level is great than 5 dB, the HD system outperforms the FD system only when $\text{SNR}_{s-r} < 10$ dB. The HD system outperform the FD system when the residual LI is great than 10 dB and $\text{SNR}_{s-r} < 17$ dB.

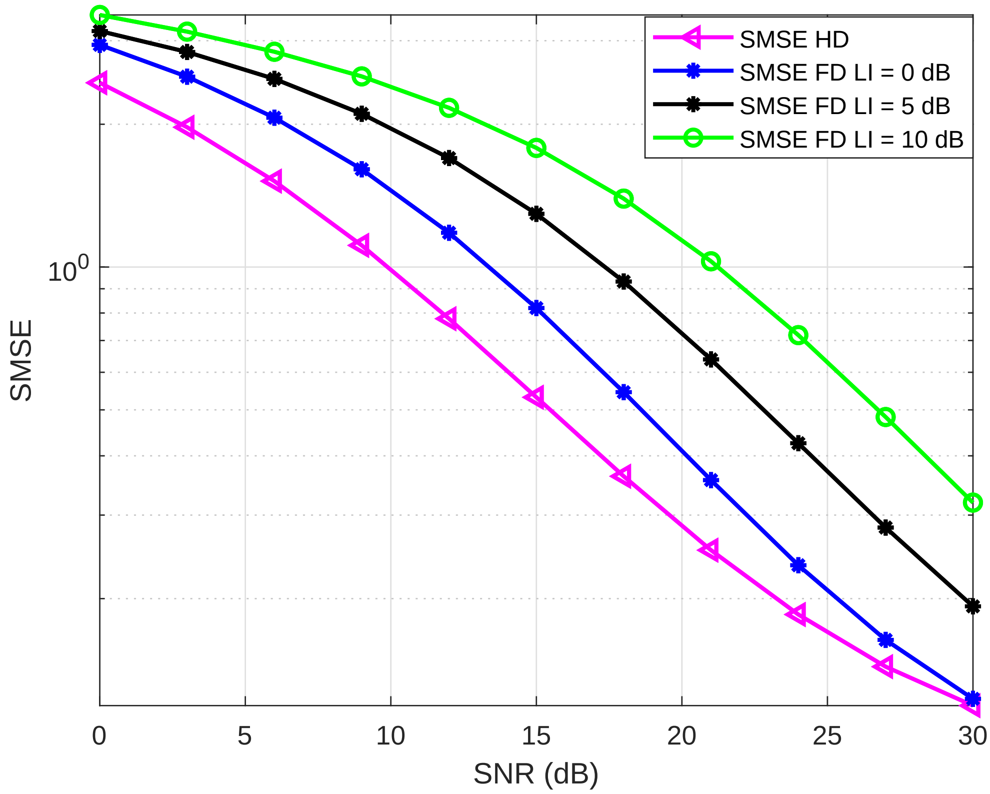

Figure 3.2: Tri-step algorithm SMSE versus SNR_{s_i-r} with $\text{SNR}_{r-d_i} = 30$ dB.

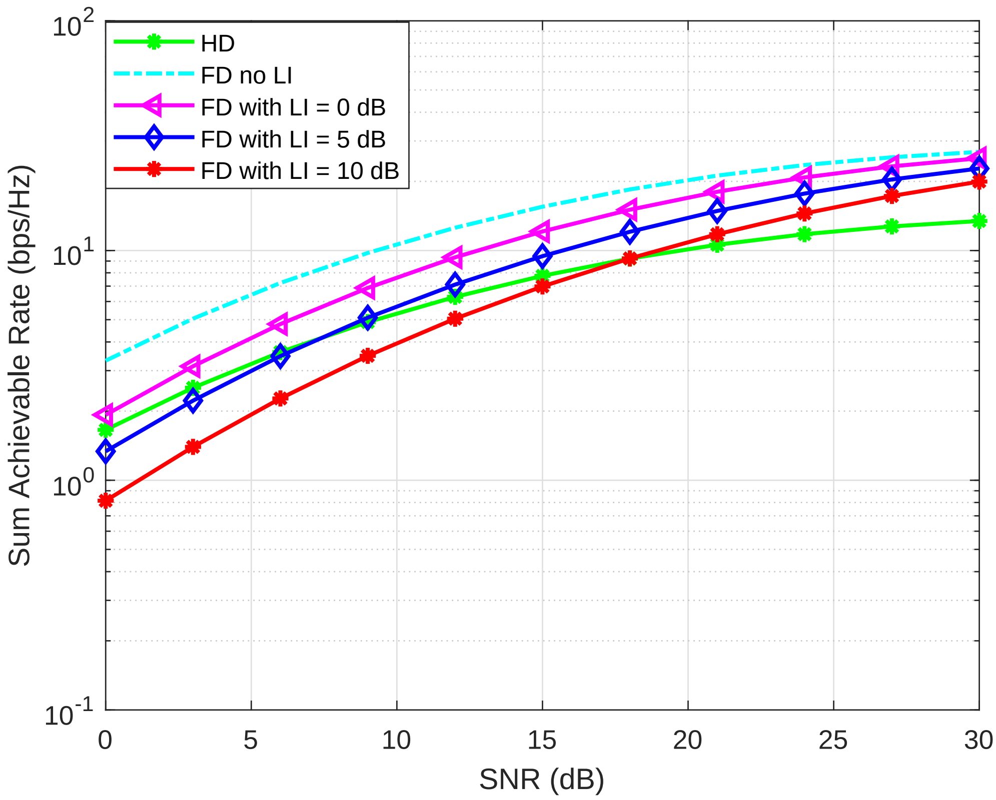

Figure 3.3: Tri-step algorithm sum achievable rate versus $\text{SNR}_{s_i - r}$ with $\text{SNR}_{r-d_i} = 30$ dB.

Figs. 3.4 and 3.5 present the SMSE and sum achievable rate with a fixed SNR of 30 dB between the source and relay and an SNR between the relay and destination from 0 dB to 30 dB. The SMSE in Fig. 3.4 is better than that in Fig. 3.2 in the low SNR_{r-d} region because a higher transmit power at the relay results in greater residual LI. Fig. 3.5 shows that the sum achievable rate of the FD system is always higher than that of the HD system for the residual LI levels considered.

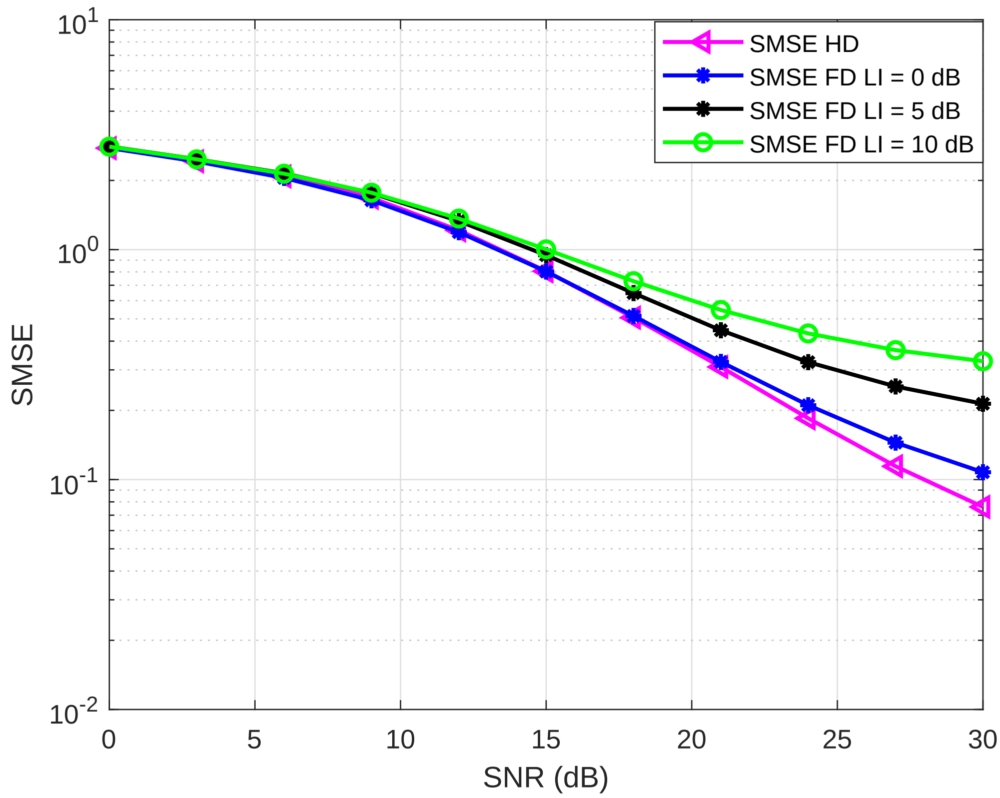

Figure 3.4: Tri-step algorithm SMSE versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

Figs. 3.6 and 3.7 present the SMSE and achievable rate for the proposed bi-step algorithm with a fixed SNR of 30 dB between the source and relay and an SNR between the relay and destination from 0 dB to 30 dB. In Fig. 3.6, the HD system has better SMSE than the FD system for all residual LI levels. The SMSE of the FD system is degraded as the residual LI level increases. Fig. 3.7 shows that the sum achievable rate of the FD system is greater than that of the HD system for all values of residual LI.

Fig. 3.8 presents the sum achievable rate for the proposed tri-step and bi-step iterative algorithms. This shows that sum achievable rates of the two algorithms are comparable. This performance-complexity trade off is an important consideration in the design of practical MIMO FD relay communication systems.

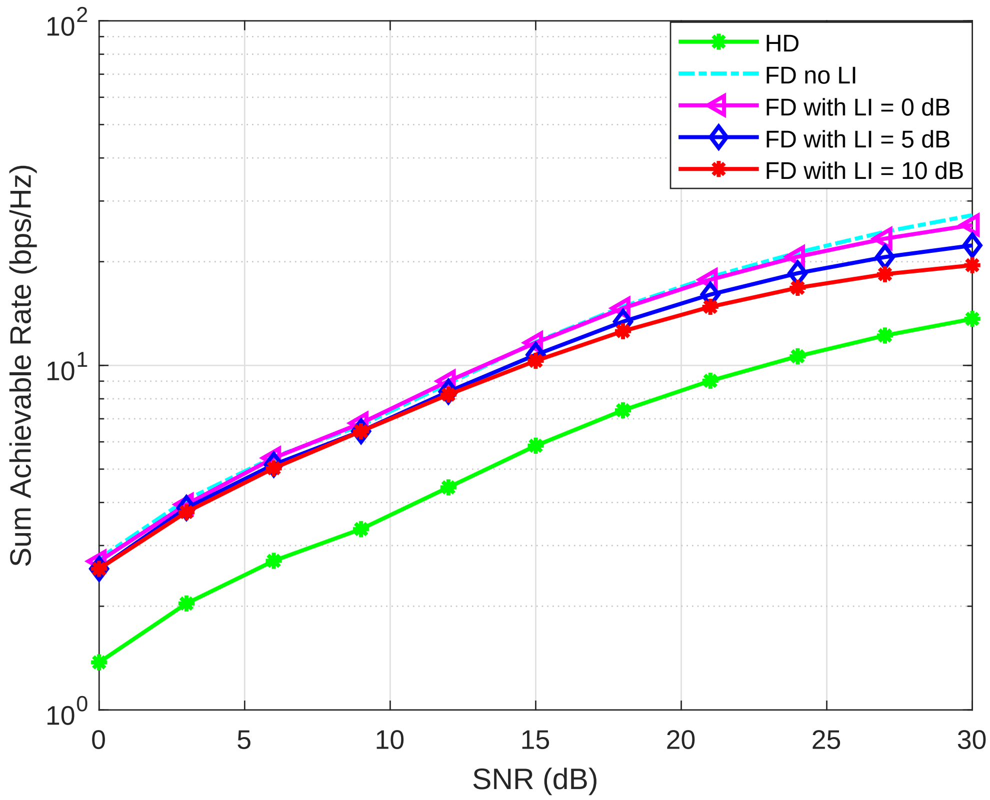

Figure 3.5: Tri-step algorithm sum achievable rate versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

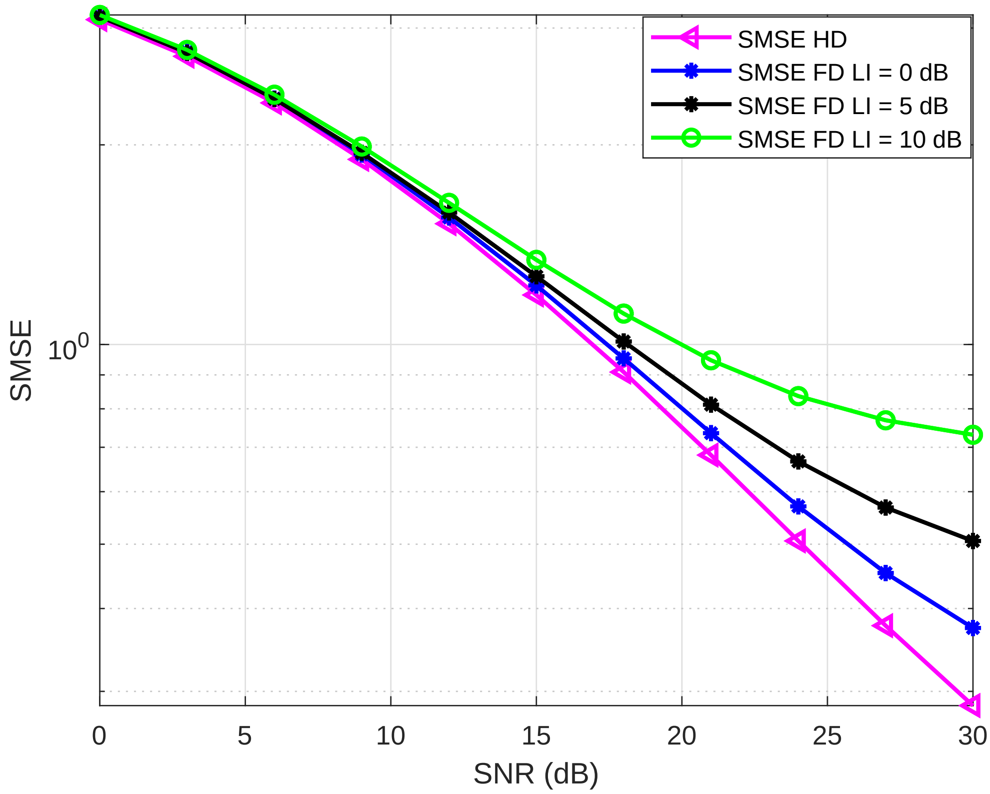

Figure 3.6: Bi-step algorithm SMSE versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

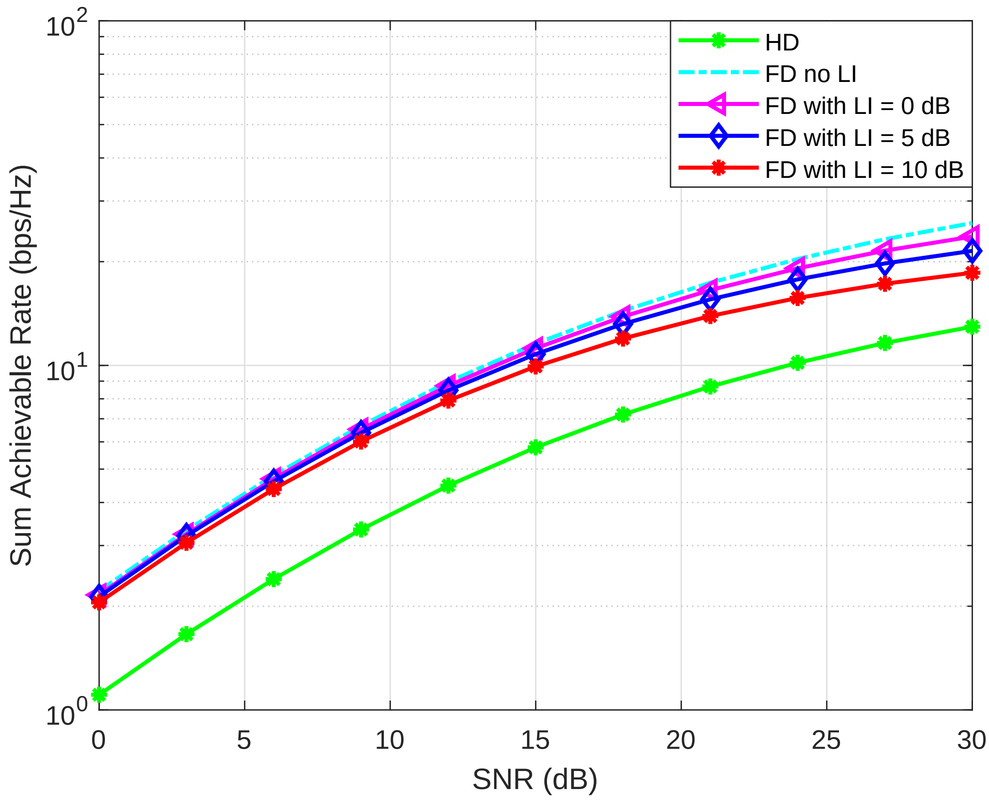

Figure 3.7: Bi-step algorithm sum achievable rate versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

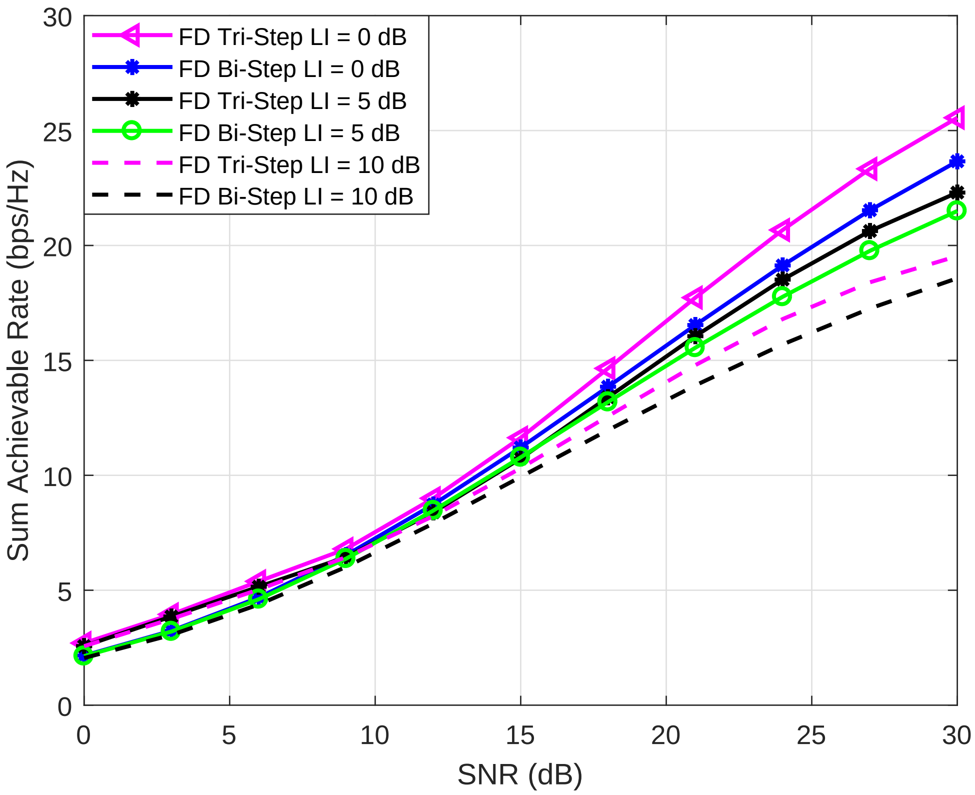

Figure 3.8: Sum achievable rate for the tri-step and bi-step algorithms.

3.6 Conclusion

In this chapter, a locally optimal source and relay precoding and destination combiner design was considered for MIMO two-way FD relay communication systems. To make the optimization problem tractable, two efficient MSE based algorithms were developed to obtain the source and relay precoding and destination combining matrices. The tri-step iterative algorithm provides optimal solutions to the three corresponding subproblems, while the bi-step iterative algorithm provides optimal solutions to the two corresponding subproblems. The convergence of the algorithms was examined, and the effect of the residual loop interference at the relay on the sum achievable rate was evaluated. Simulation results were presented which demonstrate that both algorithms outperform the corresponding HD relay system in terms of sum achievable rate and SMSE.

Chapter 4

Precoding Design for Multiuser MIMO Full-Duplex Amplify-and-Forward Relay Communication Systems

In the previous chapter, a precoding in two-way full-duplex relaying systems has been proposed and analyzed. In contrast to the one-way relaying, the two-way relaying is a highly efficient scheme to realize the bi-directional information exchange. In this chapter, precoding designs are investigated in two multiuser MIMO FD relay communication systems. The first is a uplink system where multiple users are at the transmit side. The other one is a multiuser paired downlink FD relay system where multiple nodes are employed at both transmit and receiver sides.

4.1 Uplink Communication Systems

This section considers the precoding matrix design for a multiuser multiple-input multiple-output (MIMO) full-duplex (FD) uplink communication system with an amplify-and-forward (AF) relay. The main problem in FD systems is the loop interference (LI), and perfect LI cancellation is intractable so residual LI exists. Linear precoders are used at the sources and relay, and a minimum mean-squared-error (MMSE) receiver is employed at the destination to reduce the influence of the residual LI. An iterative method is developed to solve the non-convex joint source, relay, and receiver optimization problem. Simulation results are presented which show that the proposed iterative source and relay optimization algorithm provides better performance than existing half-duplex systems in terms of the capacity under residual LI.

4.1.1 Introduction

Multiple transmit and receive antenna wireless communication systems, known as MIMO (multiple-input multiple-output) systems, were first devised in the 1970s [30]. Subsequently, MIMO technology has been shown to significantly improve the spectral and energy efficiency of wireless systems. Complex wireless propagation environments such as multi-path fading, shadowing and interference cause errors at the destination receiver. To reduce these errors and improve performance, relay assisted cooperative communication systems were investigated [60]. There has been significant research on one way relay communication systems. In [67], an iterative algorithm was developed to design the source and relay precoders and destination combiner for a single-user MIMO relay system. The joint design of source and relay precoders in a two-way relay system where both the source and relay nodes are equipped with multiple antennas was presented in [24]. In [22], joint transceiver optimization for a multiuser MIMO relay uplink communication system was studied.

In [22, 24, 67], the signal is transmitted via the source to relay and then the relay to destination links. Since source to destination transmission takes two time slots, the capacity is reduced by half. Full-duplex (FD) MIMO relaying has been proposed to increase the capacity compared to conventional half-duplex (HD) systems [68]. The loop interference (LI) is a critical issue in FD systems because the relay transmits and receives simultaneously. In general, the LI is much larger than the channel noise and so can degrade the system performance significantly. Therefore, LI cancellation techniques have been developed [69]. Temporal cancellation methods such as antenna isolation and analog/digital precancellation have been shown to be effective [10, 12]. However, it is impossible to cancel the LI completely, and the residual LI can still be larger than the noise level.

This section presents a MIMO full-duplex relaying uplink communication system with multiple users. Precoder design for the users and relay, and the linear combiner design at the destination are investigated considering the residual LI. This is based on minimizing the MSE of the received signal at the destination. An iterative method is developed to update the source and relay precoding matrices and the linear combining matrix at the destination.

The remainder of this section is organized as follows. Section 4.1.2 presents the system model and the solution of the optimization problem is given in Section 4.1.3. Simulation results are presented in Section 4.1.4 to demonstrate the effectiveness of this solution, and finally some concluding remarks are given in Section 4.1.5.

4.1.2 System Model

We consider a MIMO full-duplex (FD) relay system where the direct link between the source and destination is negligible due to large-scale fading and the large distance between the sources and destination nodes. As shown in Fig. 4.1, K users transmit to

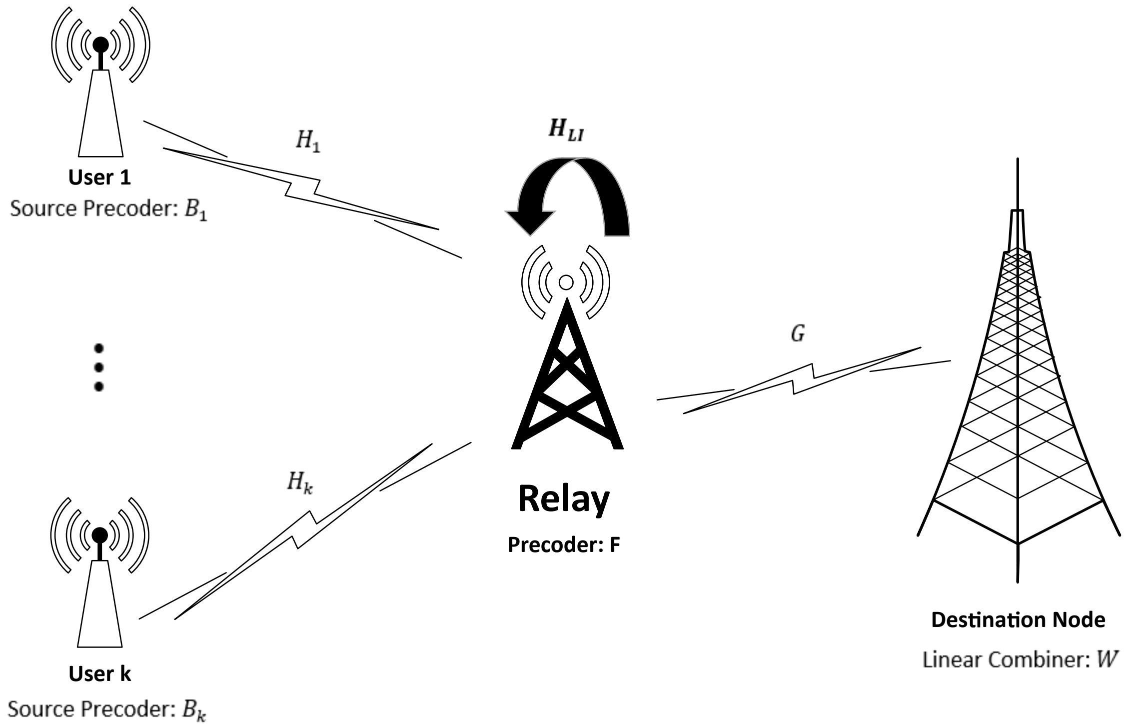

Figure 4.1: The multiuser MIMO full-duplex (FD) relay system model.

the destination with the help of a FD relay. The ith user and the destination are equipped with N_i and N_d antennas, respectively. The FD relay node is equipped with N_r and N_t antennas for receiving and transmitting simultaneously. Let $\mathbf{s}_i[n]$ denote the length N_i signal vector at time n for source node i. The number of independent data streams from all users is $N_b = \sum_{i=1}^{K} N_i$. Without loss of generality, we assume $N_b \leq \min(N_r, N_t, N_d)$. A linear precoding matrix $\mathbf{B}_i$ is applied to the signal vector s_i before transmission. The received signal at the relay can be expressed as

$$\mathrm{y}_r[n] = \sum_{i=1}^{K} \mathbf{H}_i[n]\mathbf{B}_i[n]\mathrm{s}_i[n] + \mathbf{H}_{LI}[n]\mathrm{t}[n] + \mathbf{n}_r[n], \tag{4.1}$$

where $\mathbf{H}_i[n] \in \mathbb{C}^{N_r \times N_i}$ is the channel matrix between the ith user and relay, $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$ is the noise vector at the relay and $\mathbf{H}_{LI} \in \mathbb{C}^{N_r \times N_t}$ is the loop interference (LI) channel matrix. Equation (4.1) shows that the signal received at the relay is corrupted by LI, but loop interference cancellation techniques can be employed. Then from [69], (4.1) can be rewritten as

$$\mathrm{y}_r[n] = \sum_{i=1}^{K} \mathbf{H}_i[n]\mathbf{B}_i[n]\mathrm{s}_i[n] + \mathbf{H}_{LI}[n]\mathrm{t}[n] + \mathbf{T}[n] + \mathbf{n}_r[n], \tag{4.2}$$

where $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\mathrm{t}[n]$ when perfect LI cancellation is applied. However, in practical communication systems $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\tilde{\mathrm{t}}[n]$, where $\tilde{\mathrm{t}}[n]$ is a noisy version of $\mathrm{t}[n]$, so that

$$\mathrm{y}_r[n] = \sum_{i=1}^{K} \mathbf{H}_i[n]\mathbf{B}_i[n]\mathrm{s}_i[n] + \mathbf{H}_{LI}[n]\Delta\mathrm{t}[n] + \mathbf{n}_r[n], \tag{4.3}$$

where $\Delta\mathrm{t}[n] = \mathrm{t}[n] - \tilde{\mathrm{t}}[n]$ and $\mathbf{H}_{LI}[n]\Delta\mathrm{t}[n]$ is the residual LI after the imperfect loop interference cancellation.

At time $n+1$, the full-duplex relay multiplies the received signal with a relay precoder $\mathbf{F}[n+1] \in \mathbb{C}^{N_r \times N_t}$ and immediately transmits this signal to the destination. The signal at the destination can be expressed as

$$\begin{aligned} \mathrm{y}_d[n+1] = \; & \mathbf{G}[n+1]\mathbf{F}[n+1]\bar{\mathbf{H}}[n]\mathbf{s}[n] + \mathbf{G}[n+1]\mathbf{F}[n+1]\mathbf{H}_{LI}[n]\Delta\mathrm{t}[n] \\ & + \mathbf{G}[n+1]\mathbf{F}[n+1]\mathbf{n}_r[n] + \mathbf{n}_d[n+1]. \end{aligned} \tag{4.4}$$

where $\bar{\mathbf{H}}[n] = [\mathbf{H}_1[n]\mathbf{B}_1[n], \ldots, \mathbf{H}_K[n]\mathbf{B}_K[n]]$ is the equivalent multiple access MIMO channel matrix of the source to relay link, and $\mathbf{s}[n] = [\mathrm{s}_1[n]^T, \ldots, \mathrm{s}_k[n]^T]^T$ is the equivalent transmitted signal vector where $(\cdot)^T$ denotes vector transpose. Let $\mathbb{E}[\mathbf{ss}^H] = \mathbf{I}_{N_b}$ where, $(\cdot)^H$ denotes matrix Hermitian transpose and $\mathbb{E}[\cdot]$ denotes expectation. We assume that the channel variation during the precoder update interval is relatively small and so can be ignored. Thus, the time index has no influence on the precoder design. For simplicity, this index is omitted to obtain a more concise expression for the received signal in (4.4) which is

$$\begin{aligned} \mathbf{y}_d = \; & \mathbf{GF}\bar{\mathbf{H}}\mathbf{s} + \mathbf{n} \\ = \; & \mathbf{Hs} + \mathbf{n}, \end{aligned} \tag{4.5}$$

where $\mathbf{H} = [\mathbf{GFH}_1\mathbf{B}_1, \ldots, \mathbf{GFH}_k\mathbf{B}_K]$, $\mathbf{n} = \mathbf{GFH}_{LI}\Delta\mathrm{t} + \mathbf{GFn}_r + \mathbf{n}_d$ is the equivalent noise vector, $\mathbf{G}$ is the $N_d \times N_t$ MIMO channel matrix between the relay and destination, and $\mathbf{n}_d \in \mathbb{C}^{N_d \times N_b}$ is the noise vector at the destination. $\Delta\mathrm{t}$ can be modeled as white Gaussian noise [48] which is independent of $\mathbf{n}_r$ and $\mathbf{n}_d$. The corresponding covariance matrix of $\mathbf{n}$ can be expressed as

$$\mathbf{C}_n = \mathbb{E}[nn^H] = \sigma_t^2 \mathbf{GFH}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{G}^H + \mathbf{GFF}^H\mathbf{G}^H + \mathbf{I}_{N_d}, \tag{4.6}$$

where σ_t^2 is the variance of Δt.

A linear combiner matrix $\mathbf{W} \in \mathbb{C}^{N_d \times N_b}$ is applied on the received signal at the destination and the resulting estimated signal is

$$\hat{\text{s}} = \mathbf{W}^H \text{y}_d. \tag{4.7}$$

The mean squared error can be expressed as

$$\mathbf{MSE}(\mathbf{B}, \mathbf{F}, \mathbf{W}) = \mathbb{E}[(\hat{\mathbf{s}} - \mathbf{s})(\hat{\mathbf{s}} - \mathbf{s})^H] \tag{4.8}$$

The optimization problem is to minimize the mean square error of the received signal at the destination under the constraints of individual user power and relay power budget, which can be expressed as

$$\min_{\mathbf{B}_i, \mathbf{F}, \mathbf{W}} \ \mathbf{MSE} \tag{4.9a}$$

$$s.t. \ tr(\mathbf{F}(\sum_{i=1}^{K} \mathbf{H}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_i^H + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \mathbf{F}^H) \leq P_r \tag{4.9b}$$

$$tr(\mathbf{B}_i \mathbf{B}_i^H) \leq P_{s_i}, i = 1, \dots, K \tag{4.9c}$$

4.1.3 Solution of the Optimization Problem

The problem in (4.9) is non-convex, which makes determining an optimal solution difficult. Thus in this section, an iterative algorithm is developed to solve this optimization problem. The source precoding matrix $\mathbf{B}_i$, relay precoding matrix $\mathbf{F}$ and destination combiner matrix $\mathbf{W}$ are determined to minimize the mean-squared-error (MSE) of the signal estimate

$$\mathbf{MSE} = tr((\mathbf{W}^H \mathbf{H} - \mathbf{I}_{N_b})(\mathbf{W}^H \mathbf{H} - \mathbf{I}_{N_b})^H + \mathbf{W}^H \mathbf{C}_n \mathbf{W}), \tag{4.10}$$

A closed form solution for the optimization problem in (4.10) is intractable. Therefore, an iterative algorithm is used to solve this problem by introducing three convex subproblems.

Given the relay precoder $\mathbf{F}$ and ith source precoder $\mathbf{B}_i$, the original optimization problem becomes an unconstrained problem. The destination combiner can then be obtained by taking the derivative of the MSE with respect to $\mathbf{W}$, and the solution can be expressed as

$$\mathbf{W} = (\mathbf{H}\mathbf{H}^H + \mathbf{C}_n)^{-1} \mathbf{H}. \tag{4.11}$$

Next, given the linear combiner $\mathbf{W}$ and $\mathbf{B}_i$, $\mathbf{F}$ can be obtained by solving the following problem

$$\min_{\mathbf{F}} \; tr((\bar{\mathbf{G}}\mathbf{F}\bar{\mathbf{H}} - \mathbf{I}_{N_b})(\bar{\mathbf{G}}\mathbf{F}\bar{\mathbf{H}} - \mathbf{I}_{N_b})^H + \bar{\mathbf{G}}\mathbf{F}\mathbf{F}^H\bar{\mathbf{G}}^H) \tag{4.12a}$$

$$s.t. \; tr(\mathbf{F}(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r, \tag{4.12b}$$

where $\bar{\mathbf{G}} = \mathbf{W}^H\mathbf{G}$ and $\bar{\mathbf{H}} = [\mathbf{H}_1\mathbf{B}_1, \ldots, \mathbf{H}_k\mathbf{B}_k]$. The convexity of the problem in (4.12) was proven in [22].

As in [22], the KKT conditions can be applied to the convex subproblem in (4.12). Then the optimal relay precoder can be expressed as

$$\mathbf{F} = \bar{\mathbf{G}}^H(\bar{\mathbf{G}}\bar{\mathbf{G}}^H + \mu\mathbf{I}_{N_b})^{-1}\bar{\mathbf{H}}^H(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1}, \tag{4.13}$$

where $\mu \geq 0$ is the Lagrange multiplier which can be found from the complementary slackness condition given by

$$\mu(tr(\mathbf{F}(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) - P_r) = 0. \tag{4.14}$$

If $\mu = 0$, we have from (4.13) that

$$\mathbf{F} = \bar{\mathbf{G}}^H(\bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{-1}\bar{\mathbf{H}}^H(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1}, \tag{4.15}$$

Since in this case $\mu = 0$ already satisfies $\mu \geq 0$, if $\mathbf{F}$ in (4.15) satisfies the constraint in (4.12b), then (4.15) is a solution to the problem in (4.12). Otherwise, if $\mu > 0$, then

$$tr(\mathbf{F}(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H) \leq P_r. \tag{4.16}$$

To find μ, substitute (4.13) into (4.16) and solve the following nonlinear equation

$$\begin{aligned} tr(\bar{\mathbf{G}}^H(\bar{\mathbf{G}}\bar{\mathbf{G}}^H \; + \mu\mathbf{I}_{N_b})^{-1}\bar{\mathbf{H}}^H(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1} \\ \times \bar{\mathbf{H}}(\bar{\mathbf{G}}\bar{\mathbf{G}}^H + \mu\mathbf{I}_{N_b})^{-1}\bar{\mathbf{G}}) = P_r. \end{aligned} \tag{4.17}$$

Using the singular value decomposition (SVD) of $\bar{\mathbf{G}}$ given by $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ where $\mathbf{U} \in \mathbb{C}^{N_b \times N_b}$ and $\mathbf{V} \in \mathbb{C}^{N_t \times N_t}$ are unitary matrices, $\mathbf{\Sigma} = \begin{bmatrix} \mathbf{O} & 0 \\ 0 & 0 \end{bmatrix}_{N_b \times N_t}$, and $\mathbf{O} = diag\{\sigma_1, \sigma_2, \ldots, \sigma_L\}$, (4.17) can be expressed as

$$\begin{aligned} tr(\mathbf{\Sigma}(\mathbf{\Sigma}^2 + \mu\mathbf{I}_{N_b})^{-1}\mathbf{U}^H \; \bar{\mathbf{H}}^H(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1} \\ \times \bar{\mathbf{H}}\mathbf{U}(\mathbf{\Sigma}^2 + \mu\mathbf{I}_{N_b})^{-1}\mathbf{\Sigma}) = P_r. \end{aligned} \tag{4.18}$$

Equation (4.18) can be shown to be equivalent to [22]

$$\sum_{i=1}^{N_b} \frac{\sigma_i^2 \gamma_i}{(\sigma_i^2 + \mu)^2} = P_r, \tag{4.19}$$

where σ_i and γ_i are the main diagonal elements of $\mathbf{\Sigma}$ and $\mathbf{\Gamma}$, respectively. We have that $\mathbf{\Gamma} = \mathbf{U}^H \bar{\mathbf{H}}^H (\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \sigma_t^2 \mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^{-1} \bar{\mathbf{H}}\mathbf{U}$. A technique such as the bisection method can be used to find μ since the left hand side of (4.19) is monotonically decreasing with respect to μ [49].

In the third subproblem, the source precoder $\mathbf{B}_i$ can be determined using $\mathbf{W}$ and $\mathbf{F}$ obtained above. The optimization problem can be reformulated as a convex quadratically constrained quadratic program (QCQP) problem using the following steps. Using the matrix identities

$$\begin{aligned} tr(\mathbf{C}^T\mathbf{D}) &= (\text{vec}(\mathbf{C}))^T \text{vec}(\mathbf{D}), \\ tr(\mathbf{AB}) &= tr(\mathbf{BA}), \\ \text{vec}(\mathbf{CD}) &= (\mathbf{I} \otimes \mathbf{C})\text{vec}(\mathbf{D}), \end{aligned}$$

in (4.12) and $\mathbf{A}_i = \mathbf{W}^H \mathbf{G}\mathbf{F}\mathbf{H}_i$, we have

$$\begin{aligned} tr(\mathbf{A}_i\mathbf{B}_i\mathbf{B}_i^H\mathbf{A}_i^H) &= (\text{vec}(\mathbf{B}_i))^H \text{vec}(\mathbf{A}_i^H\mathbf{A}_i\mathbf{B}_i) \\ &= (\text{vec}(\mathbf{B}_i))^H (\mathbf{I}_{N_b} \otimes (\mathbf{A}_i^H\mathbf{A}_i))(\text{vec}(\mathbf{B}_i)) \\ &= \mathbf{b}_i^H (\mathbf{I}_{N_b} \otimes (\mathbf{A}_i^H\mathbf{A}_i))\mathbf{b}_i, \end{aligned} \tag{4.20}$$

and

$$\begin{aligned} & tr(\mathbf{W}^H\mathbf{G}\mathbf{F}[\mathbf{H}_1\mathbf{B}_1, \ldots, \mathbf{H}_K\mathbf{B}_K]) \\ & = tr([\mathbf{A}_1\mathbf{B}_1, ..., \mathbf{A}_K\mathbf{B}_K]) \\ & = \sum_{i=1}^{K} tr(\mathbf{A}_{ii}\mathbf{B}_i) = \sum_{i=1}^{K} (\text{vec}(\mathbf{A}_{ii}^T))^T \mathbf{b}_i \end{aligned} \tag{4.21}$$

where $\mathbf{b}_i = \text{vec}(\mathbf{B}_i)$, $\mathbf{A}_{ii}$ is a matrix containing rows $(\sum_{j=1}^{i-1} N_j + 1)$ to $(\sum_{j=1}^{i-1} N_j)$ of

$\mathbf{A}_i$. The MSE can be written as

$$\begin{aligned}
\text{MSE} = &\; tr((\mathbf{W}^H\mathbf{G}\mathbf{F}(\sum_{i=1}^{K}\mathbf{H}_i\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_i^H)\mathbf{F}^H\mathbf{G}^H\mathbf{W} - \mathbf{W}^H\mathbf{G}\mathbf{F}[\mathbf{H}_1\mathbf{B}_1,\ldots,\mathbf{H}_K\mathbf{B}_K] \\
&- (\mathbf{W}^H\mathbf{G}\mathbf{F}[\mathbf{H}_1\mathbf{B}_1,\ldots,\mathbf{H}_K\mathbf{B}_K])^H + \mathbf{I}_{N_b} + \mathbf{W}^H(\sigma_t^2\mathbf{G}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{G}^H \\
&+ \mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H + \mathbf{I}_{N_b})\mathbf{W} \\
= &\; \sum_{i=1}^{K}\mathbf{b}_i^H(\mathbf{I}_{N_i}\otimes(\mathbf{A}_i^H\mathbf{A}_i))\mathbf{b}_i - \sum_{i=1}^{K}(\text{vec}(\mathbf{A}_{ii}^T))^T\mathbf{b}_i - \sum_{i=1}^{K}\mathbf{b}_i^H\text{vec}(\mathbf{A}_{ii}^H) + \mathbf{t} \\
= &\; \mathbf{b}^H\mathbf{A}\mathbf{b} - \mathbf{c}^H\mathbf{b} - \mathbf{b}^H\mathbf{c} + \mathbf{t},
\end{aligned} \tag{4.22}$$

where

$$\begin{aligned}
\mathbf{t} &= tr(\mathbf{I}_{N_b} + \mathbf{W}^H(\sigma_t^2\mathbf{G}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{G}^H + \mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H + \mathbf{I}_{N_d})\mathbf{W}), \\
\mathbf{A} &= bd(\mathbf{I}_{N_1}\otimes(\mathbf{A}_1^H\mathbf{A}_1),\ldots,\mathbf{I}_{N_k}\otimes(\mathbf{A}_K^H\mathbf{A}_K)), \\
\mathbf{b} &= [\mathbf{b}_1^T,\ldots,\mathbf{b}_K^T]^T, \\
\mathbf{c} &= [(\text{vec}(\mathbf{A}_{11}^H))^T,\ldots,(\text{vec}(\mathbf{A}_{KK}^H))^T]^T.
\end{aligned}$$

The MSE can be equivalently expressed as

$$\begin{aligned}
\text{MSE} = &\; \mathbf{b}^H\mathbf{A}^{\frac{1}{2}}\mathbf{A}^{\frac{1}{2}}\mathbf{b} - \mathbf{c}^H\mathbf{A}^{-\frac{1}{2}}\mathbf{A}^{\frac{1}{2}}\mathbf{b} - \mathbf{b}^H\mathbf{A}^{\frac{1}{2}}\mathbf{A}^{-\frac{1}{2}}\mathbf{c} \\
&+ \mathbf{c}^H\mathbf{A}^{-\frac{1}{2}}\mathbf{A}^{-\frac{1}{2}}\mathbf{c} - \mathbf{c}^H\mathbf{A}^{-1}\mathbf{c} + t \\
= &\; (\mathbf{b}^H\mathbf{A}^{\frac{1}{2}} - \mathbf{c}^H\mathbf{A}^{-\frac{1}{2}})(\mathbf{A}^{\frac{1}{2}}\mathbf{b} - \mathbf{A}^{-\frac{1}{2}}\mathbf{c}) - \mathbf{c}^H\mathbf{A}^{-1}\mathbf{c} + \mathbf{t},
\end{aligned} \tag{4.23}$$

where $\mathbf{A}^{\frac{1}{2}}\mathbf{A}^{\frac{1}{2}} = \mathbf{A}$ and $\mathbf{A}^{\frac{1}{2}} = \mathbf{A}^{\frac{H}{2}}$. Note that the term $-\mathbf{c}^H\mathbf{A}^{-1}\mathbf{c} + \mathbf{t}$ in (4.23) can be eliminated since it is not related to $\mathbf{b}$.

The original problem in (4.10) is equivalent to the following QCQP problem

$$\min_{b}\; (\mathbf{A}^{\frac{1}{2}}\mathbf{b} - \mathbf{A}^{-\frac{1}{2}}\mathbf{c})^H(\mathbf{A}^{\frac{1}{2}}\mathbf{b} - \mathbf{A}^{-\frac{1}{2}}\mathbf{c}) \tag{4.24a}$$

$$s.t.\; \mathbf{b}^H\mathbf{C}\mathbf{b} \le \bar{P}_r \tag{4.24b}$$

$$\mathbf{b}^H\mathbf{D}\mathbf{b} \le P_{s_i}, i = 1,\ldots,K \tag{4.24c}$$

where $\bar{P}_r = P_r - tr(\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H)$, $\mathbf{C}_i = \mathbf{F}\mathbf{H}_i$, $\mathbf{C} = bd(\mathbf{I}_{N_K}\otimes(\mathbf{C}_1^H\mathbf{C}_1),\ldots,\mathbf{I}_{N_1}\otimes(\mathbf{C}_K^H\mathbf{C}_K))$ and $\mathbf{D}_i = bd(\mathbf{D}_{i1},\mathbf{D}_{i2},\ldots,\mathbf{D}_{iK})$ with $\mathbf{D}_{ii} = \mathbf{I}_{N_i}$ and $\mathbf{D}_{ij} = 0, j = 1,\ldots,K, j \ne i$. A QCQP problem can be solved efficiently using the disciplined convex programming toolbox CVX[50]. A proof of the convexity of a problem similar to (4.24a) was given in [22].

The proposed iterative algorithm is summarized in Algorithm 5. This algorithm can be shown to converge as follows. It is obvious that the three subproblems are convex. It then follows that each update of $\mathbf{B}_k$, $\mathbf{F}$ and $\mathbf{W}$ will decrease or at least not increase the value of the objective function, and thus the iterative algorithm converges to a locally optimum solution.

Algorithm 5 Iterative Algorithm to Design $\mathbf{B}_k$, $\mathbf{F}$ and $\mathbf{W}$ for the uplink model

1: Initialize the algorithm with $\mathbf{B}_k^{(0)} = \sqrt{\frac{P_{s_k}}{L}}\mathbf{I}_L$ and $\mathbf{F}^{(0)} = \sqrt{\frac{P_r}{tr(\sum_{i=1}^{K}\mathbf{H}_i\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_i^H+\sigma_i^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H+\mathbf{I}_{N_r})}}\mathbf{I}_{N_r}$, and set $n = 0$.
2: Update $\mathbf{W}^{(n)}$ using $\mathbf{F}^{(n)}$ and $\mathbf{B}^{(n)}$ using (4.11).
3: Update $\mathbf{F}^{(n+1)}$ using $\mathbf{W}^{(n)}$ and $\mathbf{B}^{(n)}$ using (4.13) and (4.19).
4: Update $\mathbf{B}^{(n+1)}$ using $\mathbf{W}^{(n)}$ and $\mathbf{F}^{(n+1)}$ by solving the problem in (4.24).
5: If $(\mathbf{MSE}^{(n)} - \mathbf{MSE}^{(n+1)})/\mathbf{MSE}^{(n)} > \epsilon$, go to step 2.
6: End

4.1.4 Numerical Results

In this section, the performance of the proposed multiuser MIMO FD relay algorithm is evaluated using numerical simulation. For simplicity, we consider a system with two source users. The extension to more than two users is straightforward. We assume the source nodes are equipped with two antennas and the destination node has four antennas. The relay node is equipped with four receive antennas and four transmit antennas.

As in the related literature, a flat-fading MIMO channel model are considered. Thus, the entries of $\mathbf{H}_i$, $\mathbf{G}$ and $\mathbf{H}_{LI}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Further, all noise terms are i.i.d. complex circularly symmetric Gaussian random variables with zero mean and unit variance. The signal to noise ratio (SNR) of the relay to destination link is fixed at 30 dB while the SNR of the source to relay link varies from 0 dB to 30 dB. As discussed in [10], the minimum residual LI magnitude is 0 dB. Therefore, the residual LI is set to 0 dB, 5 dB and 10 dB here. In all cases, the average results for 1000 independent channel realizations are given.

Fig. 4.2 presents the MSE for a conventional half-duplex system and a full-duplex system with different levels of residual LI. It is clear that the HD system provides the best MSE performance. Further, the FD MSE performance is degraded as the residual LI level increases.

The achievable rate for the system in (5) can be obtained using an approach similar

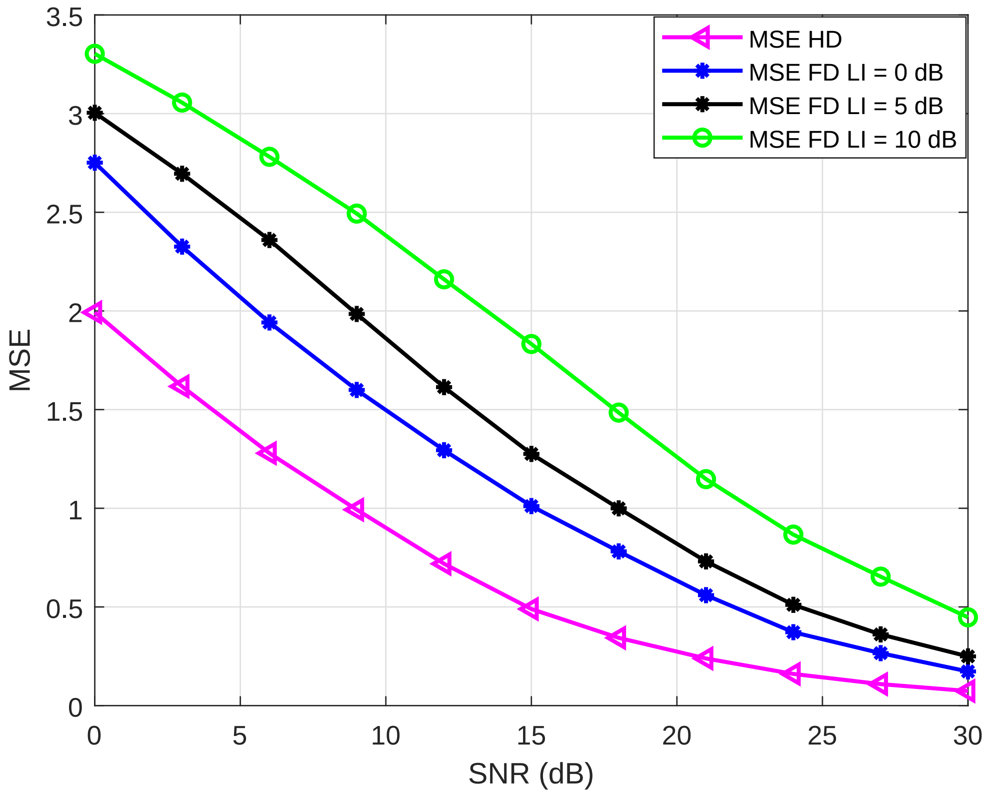

Figure 4.2: MSE for the HD and FD systems with different levels of residual LI.

to that in [42] which gives

$$R = \log_2 \det \big[\mathbf{I}_{N_d} + (\mathbf{G}\mathbf{F}\bar{\mathbf{H}})(\mathbf{G}\mathbf{F}\bar{\mathbf{H}})^H \\ \times(\sigma_t^2 \mathbf{G}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{G}^H + \mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H + \mathbf{I}_{N_d})^{-1}\big] . \tag{4.25}$$

Fig. 4.3 shows the achievable rate of the half- duplex and full-duplex systems. The FD achievable rate is twice the HD achievable rate if the LI is canceled completely. Further, the FD system achievable rate is higher than that of the HD system when the residual LI magnitude is 0 dB and 5 dB. However, when the residual LI magnitude is 10 dB, the achievable rate of the FD system is better only when the source SNR is greater than 20 dB.

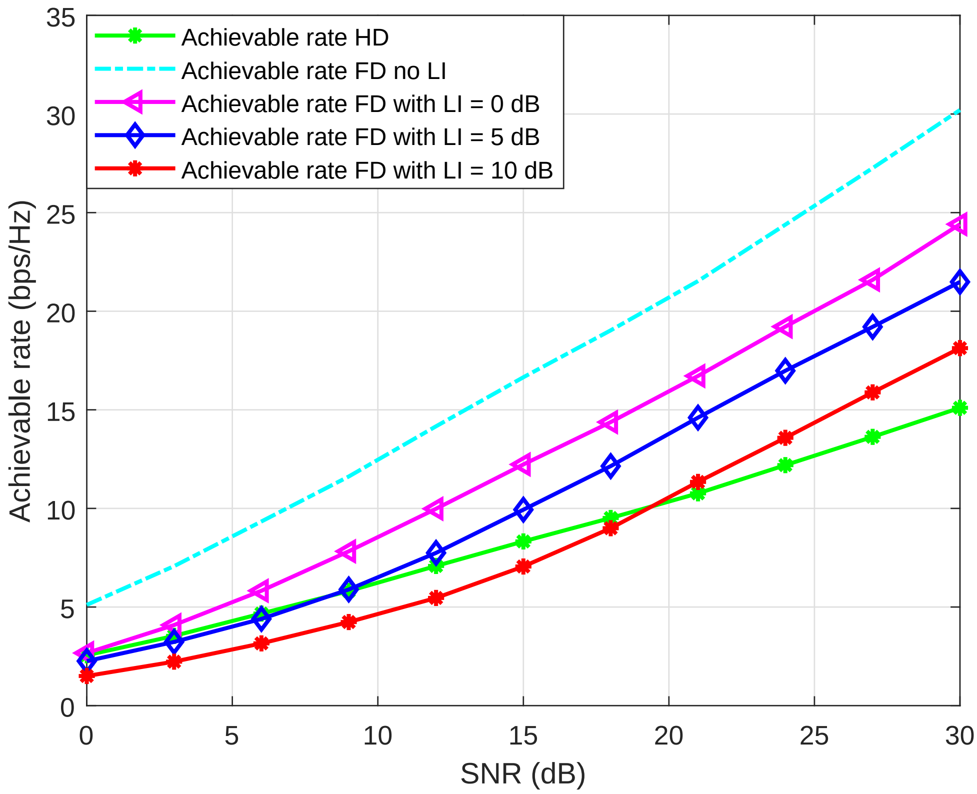

Figure 4.3: Achievable rate of the HD and FD relay systems with different levels of residual LI.

The convergence of the proposed iterative algorithm is now considered. Since the sub-problems are convex, it then follows that each update of $\mathbf{B}_i$, $\mathbf{F}$ and $\mathbf{W}$ will decrease or at least not increase the value of the objective function. Thus, the iterative algorithm converges to a local optimum solution. Fig. 4.4 presents the normalized MSE of the proposed algorithm versus the number of iterations at different levels of residual LI. The transmit power is fixed at 30 dB for all transmitters and the SNR at the relay is 20 dB.

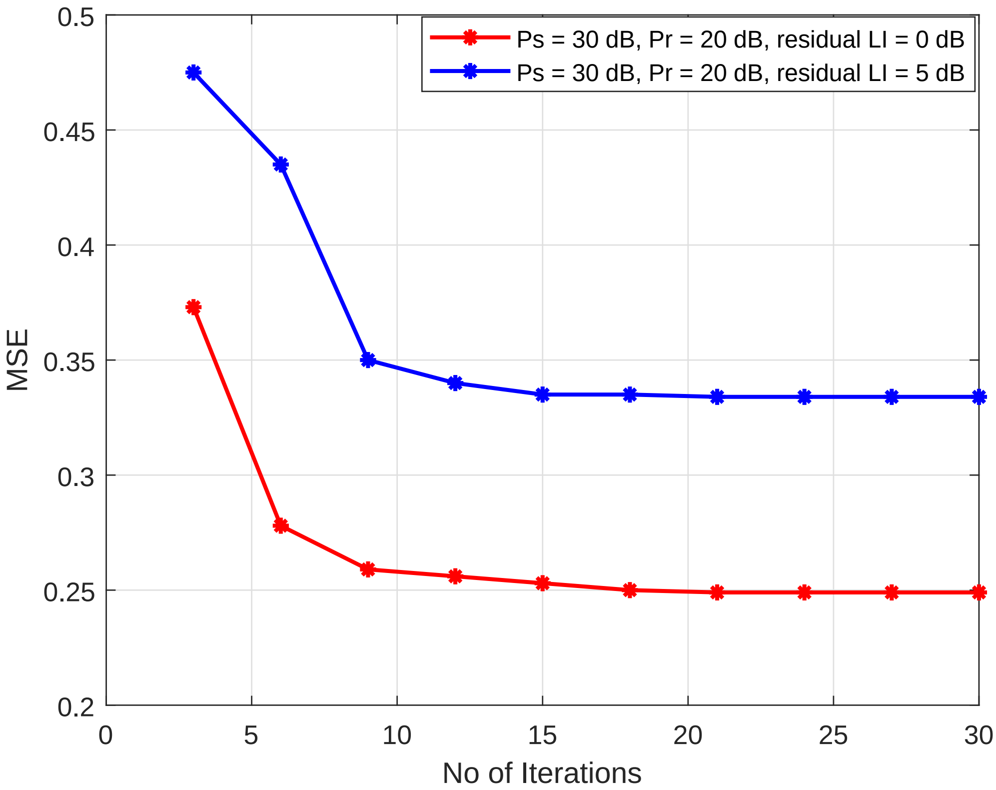

Figure 4.4: MSE versus the number of iterations.

This shows that good performance is achieved after 10 iterations, so it is suggested that 12 iterations be used with the proposed algorithm.

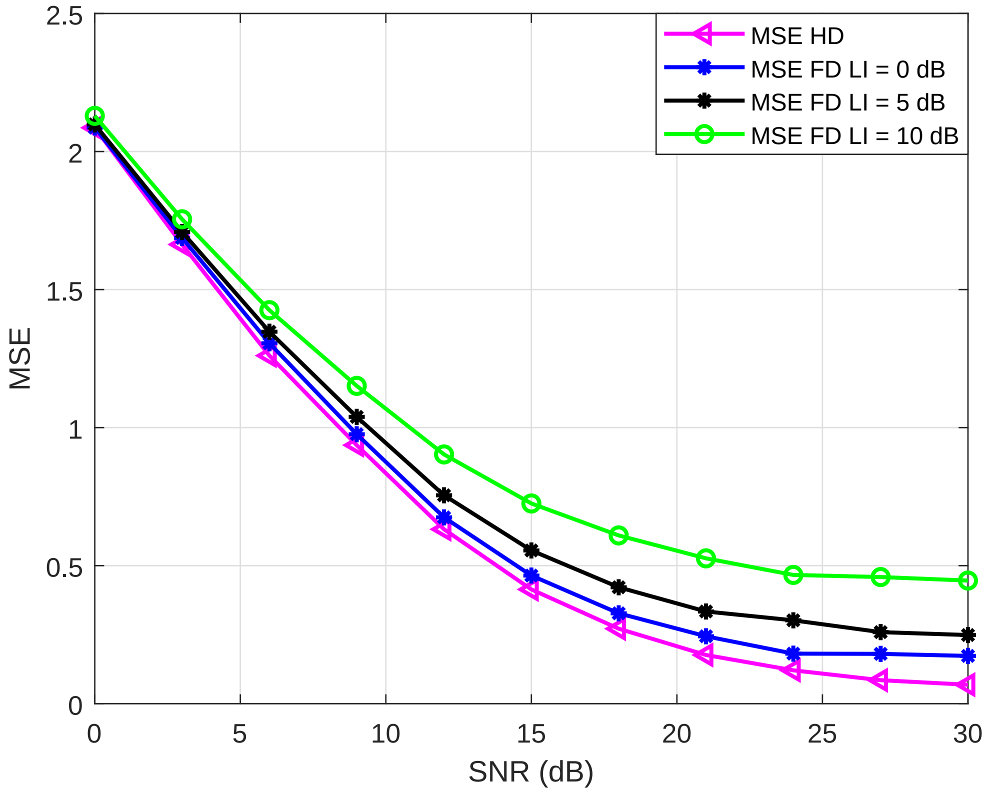

Figure 4.5: MSE for the HD and FD relay systems with different levels of residual LI.

Figs. 4.5 and 4.6 present the normalized MSE and achievable rate with a fixed SNR of 30 dB between the transmitters and relay and an SNR between the relay and destination from 0 dB to 30 dB. The MSE in Fig. 4.5 is better than in Fig. 4.2 in the low SNR region because a higher transmit power results in more residual LI. Fig. 4.6 shows that the achievable rate of the FD system is always higher than that of the HD system for residual LI levels up to 10 dB.

4.1.5 Conclusions

In this section, the design of uplink precoders for a MIMO full-duplex (FD) relay multiuser system with residual loop interference (LI) was investigated. The linear source and relay precoding matrices and the destination combining matrix were optimized to minimize the mean squared error (MSE). The original non convex problem was converted into three convex subproblems and an iterative algorithm was used to optimize the source, relay and destination matrices. Simulation results were presented which confirm that the

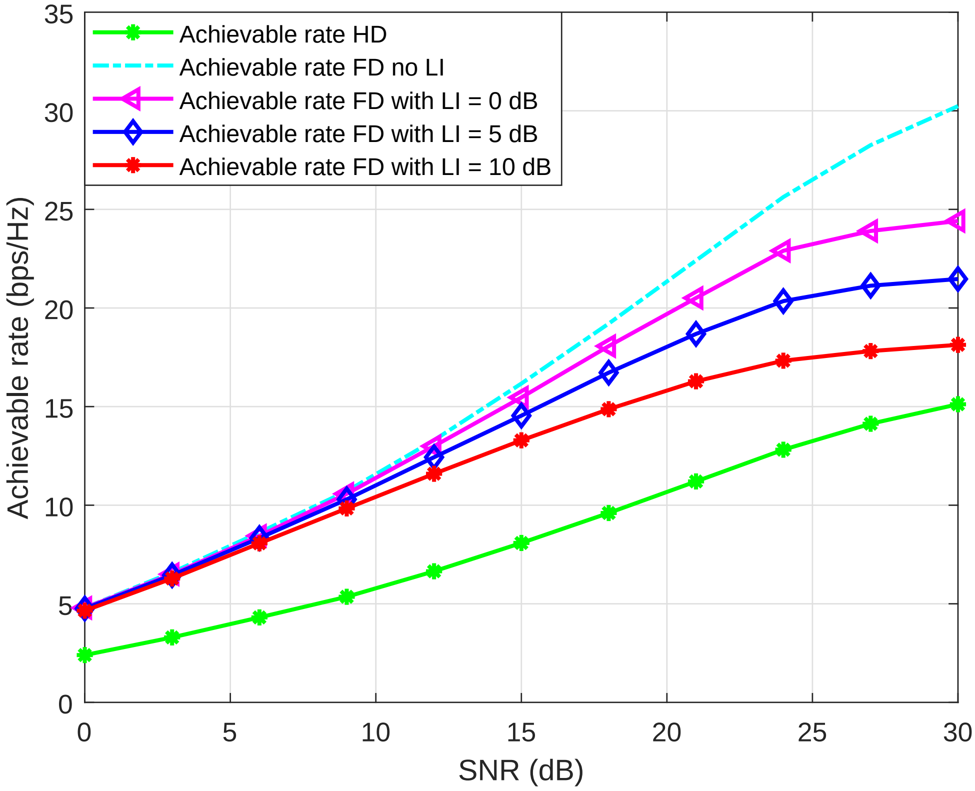

Figure 4.6: Achievable rate of the HD and FD relay systems with different levels of residual LI.

proposed iterative method outperforms existing half-duplex (HD) relaying schemes in terms of achievable rate and MSE.

4.2 Multiuser paired Downlink Communication Systems

This section investigates the precoding matrix design for a multiuser multiple-input multiple-output (MIMO) full-duplex (FD) amplify-and-forward (AF) relay downlink communication system where users simultaneously transmit data streams to other users via a relay node. In a practical FD system, the loop interference (LI) cancellation at the relay is imperfect so that residual LI exists. Linear precoders are used at the sources and relay, and minimum mean-squared-error (MMSE) combiners are employed at the destinations to reduce the effect of the residual LI. As the corresponding design problem is non convex, a closed-form solution is intractable. Therefore, an iterative method is developed to solve the optimization problem. Simulation results are presented which show that the proposed iterative algorithm provides better performance than the corresponding half-duplex (HD) solution in terms of the achievable rate under residual LI.

4.2.1 Introduction

Multiple-input multiple-output (MIMO) relaying communication systems have been the subject of considerable research due to their ability to improve the achievable rate and coverage [37, 70–74]. A relay is employed between the source and destination so the signal is transmitted from the source to the relay and then from the relay to the destination. The relay node can employ either the decode-and-forward (DF) or amplify-and-forward (AF) protocols. AF simply amplifies the received signal and then forwards it, so it has lower complexity than the DF protocol [23].

Precoding is a well known technique for interference mitigation. Joint precoding optimization for a multiuser relay downlink system was investigated in [37]. The sum achievable rate was maximized by using quadratic programming, but multiple antennas are employed only at the relay. The performance can be improved by using multiple antennas at the source and destination. Transceiver design for a multiuser non-regenerative MIMO relay system with multiple antennas at both the source and destination was investigated in [70–74]. However, a half-duplex (HD) relay is employed, so the transmission from source to destination requires two time slots, which limits the potential achievable rate.

Recently, full-duplex (FD) relay systems have attracted attention because data transmission can be completed in one time slot [10, 12, 23, 42, 75]. As a consequence, FD MIMO relaying can increase the achievable rate compared to HD systems [10]. The loop interference (LI) is a critical issue because the relay transmits and receives simultaneously. In general, the LI is much larger than the channel noise and so can significantly degrade performance. Therefore, LI cancellation techniques have been developed [12]. Temporal cancellation methods such as antenna isolation and analog/digital precancellation have been shown to be effective. However, it is impossible to cancel the LI completely, and the residual LI can still be larger than the noise level [66].

This section presents a MIMO FD relaying downlink communication system with multiple source-destination pairs. Precoder design for the sources and relay, and the linear combiner design at the destinations are investigated considering the residual LI with the goal of minimizing the sum MSE of the received signals at the destinations. An iterative algorithm is developed to update the source and relay precoding matrices and the linear combining matrix at the destinations.

The remainder of this section is organized as follows. Section 4.2.2 presents the system model and the solution of the optimization problem is given in Section 4.2.3. Simulation results are presented in Section 4.2.4 to demonstrate the effectiveness of this solution, and finally some concluding remarks are given in Section 4.2.5.

4.2.2 System Model

Consider a MIMO full-duplex (FD) relay system with K source-destination pairs communicating simultaneously with the aid of a relay. The direct links between the sources and destinations are assumed to be negligible due to large-scale fading and the long distances between them. As shown in Fig. 4.7, the kth source-destination pair is equipped with N_{s_k} and N_{d_k} antennas, respectively. The relay operates in FD mode and employs N_r and N_t antennas to receive and transmit signals simultaneously. Thus, communication between source-destination pairs is accomplished in one time slot compared to a HD system that requires two time slots.

Let $\mathbf{s}_k[n]$ represent the length d signal vector at time n for the kth source. A linear precoding matrix $\mathbf{B}_k[n]$ is applied to $\mathbf{s}_k[n]$ before transmission. The received signal at the relay can be expressed as

$$\mathbf{y}_r[n] = \sum_{k=1}^{K} \mathbf{H}_k[n]\mathbf{B}_k[n]\mathbf{s}_k[n] + \mathbf{H}_{LI}[n]\mathbf{t}[n] + \mathbf{n}_r[n], \tag{4.26}$$

where $\mathbf{H}_k[n] \in \mathbb{C}^{N_r \times N_{s_k}}$ denotes the channel between source k and the relay node, $\mathbf{H}_{LI}[n] \in \mathbb{C}^{N_r \times N_t}$ denotes the loop interference (LI) channels, and $\mathbf{n}_r[n] \in \mathbb{C}^{N_r \times 1}$ is an independent and identically distributed (i.i.d.) additive white Gaussian noise matrix. As the received signal $\mathbf{y}_r[n]$ is corrupted by residual LI, according to [66] it can be rewritten

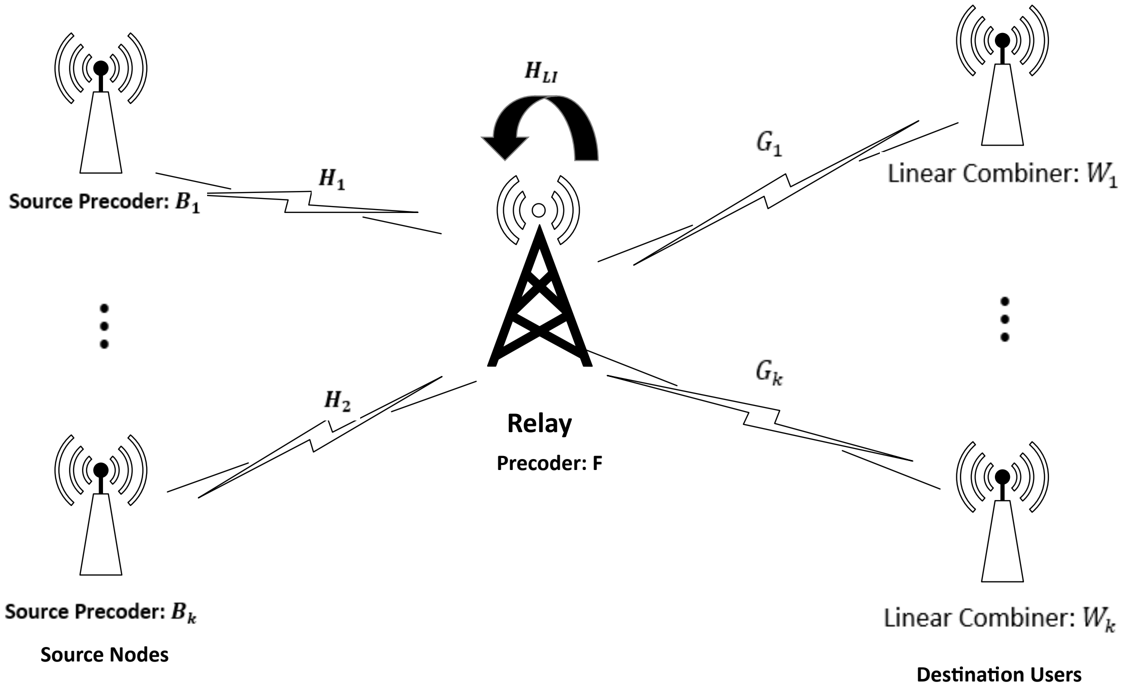

Figure 4.7: The multiuser MIMO full-duplex (FD) relay system.

as

$$\mathbf{y}_r[n] = \sum_{k=1}^{K} \mathbf{H}_k[n]\mathbf{B}_k[n]\mathbf{s}_k[n] + \mathbf{H}_{LI}[n]\Delta\mathbf{t}[n] + \mathbf{n}_r[n], \tag{4.27}$$

where $\Delta\mathbf{t}[n] = \mathrm{t}[n] - \tilde{\mathbf{t}}[n]$ and $\mathbf{H}_{LI}[n]\Delta\mathbf{t}[n]$ is the residual LI.

At time $n+1$, the relay amplifies the received signal with a relay precoding matrix $\mathbf{F}[n+1] \in \mathbb{C}^{N_t \times N_r}$ and then immediately forwards the resulting signal to the destinations. The received signal at the kth destination can be written as

$$\mathbf{y}_k[n+1] = \mathbf{G}_k[n+1]\mathbf{F}[n+1]\mathbf{y}_r[n] + \mathbf{n}_{d_k}[n+1], \tag{4.28}$$

where $\mathbf{G}_k[n+1] \in \mathbb{C}^{N_i \times N_t}$ is the channel matrix between the relay node and destination k, and $\mathbf{n}_{d_k}[n+1] \in \mathbb{C}^{N_k \times 1}$ is an i.i.d. AWGN vector with zero mean and unit variance.

Similar to [23, 66, 74], the channel state information (CSI) is assumed to be available at all nodes. In addition, channel variations during the precoder update interval are assumed to be relatively small. Under this assumption, the time index has no influence on the precoder design [66]. Thus, the received signal at the kth destination can be expressed more concisely as

$$\mathbf{y}_k = \mathbf{G}_k\mathbf{F}\sum_{k=1}^{K} \mathbf{H}_k\mathbf{B}_k\mathbf{s}_k + \mathbf{n}_k, \tag{4.29}$$

where

$$\mathbf{n}_k = \mathbf{G}_k\mathbf{F}\mathbf{H}_{LI}\Delta\mathbf{t} + \mathbf{G}_k\mathbf{F}\mathbf{n}_r + \mathbf{n}_{d_k}. \tag{4.30}$$

As shown in [48], $\Delta\mathbf{t}$ can be modeled as a white Gaussian noise which is independent of $\mathbf{n}_r$ and $\mathbf{n}_{d_k}$ and has variance σ_t^2. Consequently, the covariance matrix of $\mathbf{n}_k$ can be expressed as

$$\mathbf{C}_{n_k} = \mathbb{E}[\mathbf{n}_k\mathbf{n}_k^H]. \tag{4.31}$$

A linear combiner $\mathbf{W}_k \in \mathbb{C}^{N_k \times N_b}$ is applied at the kth destination to obtain the signal estimate

$$\hat{\mathbf{s}}_k = \mathbf{W}_k^H \mathbf{y}_k. \tag{4.32}$$

The mean square error (MSE) at the kth user is defined as

$$\begin{aligned} \mathrm{MSE}_k &= \mathbb{E}[(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H] \\ &= (\mathbf{W}_k^H\mathbf{L}_k - \mathbf{I}_d)(\mathbf{W}_k^H\mathbf{L}_k - \mathbf{I}_d)^H + \mathbf{W}_k^H\mathbf{C}_{n_k}\mathbf{W}_k + \mathbf{W}_k^H\mathbf{E}_{n_k}\mathbf{W}_k, \end{aligned} \tag{4.33}$$

where $\mathbf{L}_k = \mathbf{G}_k\mathbf{F}\bar{\mathbf{H}}_k$, and $\mathbf{E}_{n_k} = \mathbf{G}_k\mathbf{F}\sum_{m=1,m\neq k}^{K}\bar{\mathbf{H}}_m\bar{\mathbf{H}}_m^H\mathbf{F}^H$ $\mathbf{G}_k^H, k = 1, \ldots, K$. The optimization problem is to obtain the linear precoder matrices $\mathbf{B}_k$ and $\mathbf{F}$ at the source and relay, and the linear combiners $\mathbf{W}_k$ at the destinations, which minimize the sum mean squared error (SMSE) of the received signals at the destinations which can be expressed as

$$\min_{\mathbf{B}_k,\mathbf{F},\mathbf{W}_k} \quad \mathrm{SMSE} \tag{4.34a}$$

$$s.t. \; tr(\mathbf{F}(\mathbf{y}_r\mathbf{y}_r^H)\mathbf{F}^H) \leq P_r \tag{4.34b}$$

$$tr(\mathbf{B}_k\mathbf{B}_k^H) \leq P_{s_k}, k = 1, \ldots, K, \tag{4.34c}$$

where $\mathrm{SMSE} = \sum_{k=1}^{K} tr(\mathrm{MSE}_k)$, and $P_{s_k} > 0$ and $P_r > 0$ are the power constraints at the kth source and relay, respectively.

4.2.3 Solution of the Optimization Problem

The problem in (4.34) is non-convex which makes a globally optimal solution intractable. Thus in this section, an iterative algorithm is presented which is based on alternating optimization that updates $\mathbf{B}_k$, $\mathbf{F}$ and $\mathbf{W}_k$ individually while the others are fixed to solve the three convex subproblems.

First, given $\mathbf{B}_k$ and $\mathbf{F}$, the optimal combiner at the kth destination $\mathbf{W}_k$ can be obtained by solving the unconstrained convex problem since $\mathbf{W}_k$ is independent of the constraints in (4.34b) and (4.34c). The optimal solution can then be obtained by taking the derivative of (4.33) with respect to $\mathbf{W}_k$ and setting it to zero. Solving $\frac{\partial}{\partial \mathbf{W}_k} tr(\text{MSE}_k) = 0$ gives

$$\mathbf{W}_k = \mathbf{C}_{w_k}^{-1} \mathbf{G}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k, \tag{4.35}$$

where $\mathbf{C}_{w_k} = (\mathbf{G}_k \mathbf{F} \mathbf{H} \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k^H \mathbf{F}^H \mathbf{G}_k^H + \mathbf{C}_{n_k} + \mathbf{E}_{n_k})^{-1} \mathbf{G}_k$
$\mathbf{F} \mathbf{H}_k \mathbf{B}_k$. This solution is known as the Wiener filter.

Second, with $\mathbf{W}_k$ from (4.35) and given $\mathbf{B}_k$, $\mathbf{F}$ can be obtained by solving the following problem. The objective function in (4.34) can be expressed as

$$\begin{aligned} \text{SMSE} = & \sum_{k=1}^{K} tr((\mathbf{W}_k^H \mathbf{G}_k \mathbf{F} \mathbf{H} \mathbf{B}_k - \mathbf{I}_{N_k})(\mathbf{W}_k^H \mathbf{G}_k \mathbf{F} \mathbf{H} \mathbf{B}_k - \mathbf{I}_{N_k})^H \\ & + \mathbf{W}_k^H (\sigma_t^2 \mathbf{G}_k \mathbf{F} \mathbf{H}_{LI} \mathbf{H}_{LI}^H \mathbf{F}^H \mathbf{G}_k^H + \mathbf{G}_k \mathbf{F} \mathbf{F}^H \mathbf{G}_k^H + \mathbf{I}_{N_k}) \mathbf{W}_k \\ & + \mathbf{W}_k^H \mathbf{G}_k \mathbf{F} \sum_{m=1, m\neq k}^{K} \mathbf{H} \mathbf{B}_m \mathbf{B}_m^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}_k^H \mathbf{W}_k). \end{aligned} \tag{4.36}$$

Note that since $\mathbf{B}_k$ is known, the constraint in (4.34c) is eliminated. The original optimization problem then becomes

$$\begin{aligned} & \min_{\mathbf{F}} \ \text{SMSE} \\ & s.t. \ tr(\mathbf{F}(\sum_{k=1}^{K} \mathbf{H}_k \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k^H + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \mathbf{F}^H) \leq P_r. \end{aligned} \tag{4.37}$$

Consider the singular-value decomposition (SVD) of the equivalent source-to-relay and relay-to-destination channels

$$\mathbf{H} = [\mathbf{H}_1 \mathbf{B}_1, \ldots, \mathbf{H}_K \mathbf{B}_K] = \mathbf{U}_h \mathbf{\Lambda}_h \mathbf{V}_h^H, \tag{4.38}$$

and

$$\mathbf{G} = [\mathbf{G}_1^T, \ldots, \mathbf{W}_K^T]^T = \mathbf{U}_g \mathbf{\Lambda}_g \mathbf{V}_g^H. \tag{4.39}$$

The optimal structure of the relay precoding matrix is similar to that in [23]

$$\mathbf{F} = \mathbf{V}_g \mathbf{A} \mathbf{U}_h^H, \tag{4.40}$$

and from (4.38) and (4.39) we have

$$\mathbf{H}_k \mathbf{B}_k = \mathbf{U}_h \mathbf{\Lambda}_h \mathbf{V}_{h_k}^H, \tag{4.41}$$

and

$$\mathbf{G}_k = \mathbf{U}_{g_k} \mathbf{\Lambda}_g \mathbf{V}_g^H. \tag{4.42}$$

Note that $\mathbf{V}_h = [\mathbf{V}_{h1}^T, \ldots, \mathbf{V}_{h_K}^T]^T$ and $\mathbf{U}_g = [\mathbf{U}_{g1}^T, \ldots, \mathbf{U}_{g_K}^T]^T$ which have dimensions $d \times L_1$ and $N_i \times L_2$, respectively. Substituting (4.38), (4.39) and (4.40) into (4.36) gives

$$\begin{aligned}\text{SMSE} = & \sum_{k=1}^{K} tr((\mathbf{W}_k^H \mathbf{U}_{g_k} \mathbf{\Lambda}_g \mathbf{A} \mathbf{\Lambda}_h \mathbf{V}_{h_k}^H - \mathbf{I}_{N_k}) \times (\mathbf{W}_k^H \mathbf{U}_{g_k} \mathbf{\Lambda}_g \mathbf{A} \mathbf{\Lambda}_h \mathbf{V}_{h_k}^H - \mathbf{I}_{N_k})^H \\ & + \mathbf{W}_k^H (\sigma_t^2 \mathbf{U}_{g_k} \mathbf{\Lambda}_g \mathbf{A} \mathbf{U}_h^H \mathbf{H}_{LI} \mathbf{H}_{LI}^H \mathbf{U}_h \mathbf{A}^H \mathbf{\Lambda}_g \mathbf{U}_{g_k}^H + \mathbf{U}_{g_k} \mathbf{\Lambda}_g \mathbf{A} \mathbf{A}^H \mathbf{\Lambda}_g \mathbf{U}_{g_k}^H \\ & + \mathbf{I}_{N_k} + \mathbf{U}_{g_k} \mathbf{\Lambda}_g \mathbf{A} \sum_{m=1, m \neq k}^{K} \mathbf{\Lambda}_h \mathbf{V}_{h_m}^H \mathbf{V}_{h_m} \mathbf{\Lambda}_h \mathbf{A}^H \mathbf{\Lambda}_g \mathbf{U}_{g_k}^H) \mathbf{W}_k).\end{aligned} \tag{4.43}$$

Using the matrix identities

$$\begin{aligned} tr(\mathbf{C}^T \mathbf{D}) &= (\text{vec}(\mathbf{C}))^H \text{vec}(\mathbf{D}), \\ tr(\mathbf{A}^H \mathbf{B} \mathbf{A} \mathbf{C}) &= tr(\text{vec}(\mathbf{A}))^H (\mathbf{C}^T \otimes \mathbf{B}) \text{vec}(\mathbf{A}) \\ \text{vec} \mathbf{A} \mathbf{B} \mathbf{C} &= (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B}), \end{aligned}$$

where $\text{vec}(\cdot)$ concatenates the columns of a matrix into a single vector. The SMSE in (4.36) can be represented as a function of $\mathbf{a} = \text{vec}(\mathbf{A})$ as

$$\begin{aligned}\text{SMSE} = & \sum_{k=1}^{K} (\mathbf{P}_k \mathbf{a} - \text{vec}(\mathbf{I}_{N_k}))^H (\mathbf{P}_k \mathbf{a} - \text{vec}(\mathbf{I}_{N_k})) \\ & + \mathbf{a}^H \mathbf{Q}_k \mathbf{a} + \mathbf{a}^H \mathbf{S}_k \mathbf{a} + \mathbf{a}^H \mathbf{R}_k \mathbf{a} + \mathbf{t}_1 \end{aligned} \tag{4.44}$$

where $\mathbf{t}_1 = \sum_{k=1}^{K} tr(\mathbf{W}_k^H \mathbf{W}_k)$ does not depend on $\mathbf{a}$ and

$$\mathbf{P}_k = (\mathbf{\Lambda}_h \mathbf{V}_{h_k}^H)^T \otimes (\mathbf{W}_k^H \mathbf{U}_{g_k} \mathbf{\Lambda}_g) \tag{4.45}$$

$$\mathbf{Q}_k = \sigma_t^2 ((\mathbf{U}_h^H \mathbf{H}_{LI} \mathbf{H}_{LI}^H \mathbf{U}_h)^T \otimes \mathbf{\Lambda}_g \mathbf{U}_{g_k}^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{U}_{g_k} \mathbf{\Lambda}_g) \tag{4.46}$$

$$\mathbf{S}_k = \mathbf{I}_L \otimes (\mathbf{\Lambda}_g \mathbf{U}_{\mathbf{g}_k}^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{U}_{g_k} \mathbf{\Lambda}_g) \tag{4.47}$$

$$\mathbf{R}_k = (\sum_{m=1, m \neq k}^{K} \mathbf{\Lambda}_h \mathbf{V}_{h_m}^H \mathbf{V}_{h_m} \mathbf{\Lambda}_h)^T \otimes (\mathbf{\Lambda}_g \mathbf{U}_{g_k}^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{U}_{g_k} \mathbf{\Lambda}_g) \tag{4.48}$$

Further, the relay transmit power constraint in (4.37) can be written as

$$\begin{aligned} & tr(\mathbf{F} (\sum_{k=1}^{K} \mathbf{H}_k \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k^H + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \mathbf{F}^H) \\ & = tr(\mathbf{A} \times (\mathbf{\Lambda}_h^2 + \sigma_t^2 \mathbf{U}_h^H \mathbf{H}_{LI} \mathbf{H}_{LI}^H \mathbf{U}_h + \mathbf{I}_{L_1}) \mathbf{A}^H). \end{aligned} \tag{4.49}$$

Introducing $\mathbf{D} = (\mathbf{\Lambda}_h^2 + \sigma_t^2 \mathbf{U}_h^H \mathbf{H}_{LI} \mathbf{H}_{LI}^H \mathbf{U}_h + \mathbf{I}_{L_1}) \otimes \mathbf{I}_{L_1}$, (24) can be rewritten as

$$\mathbf{a}^H \mathbf{D} \mathbf{a} \leq P_r. \tag{4.50}$$

The original relay optimization problem is then given by

$$\begin{aligned} &\min_{\mathbf{A}} \ \text{SMSE} \\ &s.t. \ \mathbf{a}^H \mathbf{D} \mathbf{a} \leq P_r. \end{aligned} \tag{4.51}$$

This is a quadratically constrained quadratic programming (QCQP) problem which is convex and so can be efficiently solved using the interior point method. For example, the CVX toolbox for disciplined convex programming [50] can be employed.

Third, the kth source precoding matrix $\mathbf{B}_k$ can be obtained using $\mathbf{W}_k$ and $\mathbf{F}$ given above. The corresponding optimization problem can be formulated as the following QCQP problem

$$\begin{aligned} \text{SMSE} = & \sum_{k=1}^{K} tr((\bar{\mathbf{G}}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k - \mathbf{I}_{N_k})(\bar{\mathbf{G}}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k - \mathbf{I}_{N_k})^H \\ & + \bar{\mathbf{G}}_k \mathbf{F} \sum_{m=1, m \neq k}^{K} \mathbf{H}_k \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k^H \mathbf{F}^H \bar{\mathbf{G}}_k^H) + \mathbf{t}_2, \end{aligned} \tag{4.52}$$

where $\bar{\mathbf{G}}_k = \mathbf{W}_k^H \mathbf{G}_k$ and $\mathbf{t}_2 = \sum_{k=1}^{K} tr(\mathbf{W}_k^H \mathbf{C}_k \mathbf{W}_k)$ can be ignored in the optimization as it does not depend on $\mathbf{B}_k$. Using the matrix identities, (4.52) can be written as a function of $\mathbf{b}_k = \text{vec}(\mathbf{B}_k)$ which gives

$$\begin{aligned} \text{SMSE} = & \sum_{k=1}^{K} [(\mathbf{S}_k \mathbf{b}_k - \text{vec}(\mathbf{I}_{N_k}))^H (\mathbf{S}_k \mathbf{b}_k - \text{vec}(\mathbf{I}_{N_k})) \\ & + \sum_{m=1, m \neq k}^{K} \mathbf{b}_m^H (\mathbf{I}_d \otimes \mathbf{H}_m^H \mathbf{F}^H \bar{\mathbf{G}}_k^H \bar{\mathbf{G}}_k \mathbf{F} \mathbf{H}_m) \mathbf{b}_m] + \mathbf{t}_2 \\ = & \sum_{k=1}^{K} [(\mathbf{S}_k \mathbf{b}_k - \text{vec}(\mathbf{I}_{N_k}))^H (\mathbf{S}_k \mathbf{b}_k - \text{vec}(\mathbf{I}_{N_k})) + \mathbf{b}_k^H \mathbf{T}_k \mathbf{b}_k] + \mathbf{t}_2, \end{aligned} \tag{4.53}$$

where $\mathbf{S}_k = \mathbf{I}_{N_k} \otimes (\bar{\mathbf{G}}_k \mathbf{F} \mathbf{H})$, $\mathbf{T}_k = \mathbf{I}_{N_k} \otimes \sum_{m=1, m \neq k}^{K} (\mathbf{H}^H \mathbf{F}^H \bar{\mathbf{G}}_k^H \bar{\mathbf{G}}_k \mathbf{F} \mathbf{H})$, and $\mathbf{t}_2 = \sum_{k=1}^{K} tr(\mathbf{W}_k^H \mathbf{C}_{n_k} \mathbf{W}_k)$ which is independent of $\mathbf{B}_k$ and so can be ignored. Introducing $\mathbf{T} = bd(\mathbf{T}_1, \ldots, \mathbf{T}_K)$ where $bd(\cdot)$ denotes a block-diagonal matrix, and $\bar{\mathbf{S}}_k = [\mathbf{S}_{k1}, \ldots, \mathbf{S}_{kK}]$ where $\mathbf{S}_{kk} = \mathbf{S}_k$ and $\mathbf{S}_{jk} = 0, j \neq k$, (4.52) can be written as a function of $\mathbf{b} = [\mathbf{b}_1^T, \mathbf{b}_1^T, \ldots, \mathbf{b}_K^T]^T$ which gives

$$\Phi_1(\mathbf{b}) = \sum_{k=1}^{K} (\bar{\mathbf{S}}_k \mathbf{b} - \text{vec}(\mathbf{I}_{N_k}))^H (\bar{\mathbf{S}}_k \mathbf{b} - \text{vec}(\mathbf{I}_{N_k})) + \mathbf{b}^H \mathbf{T} \mathbf{b} \tag{4.54}$$

Now introducing $\mathbf{E}_j = \mathbf{I}_{N_k} \otimes (\mathbf{H}_j^H \mathbf{F}^H \mathbf{F} \mathbf{H}_j)$, $\mathbf{E} = bd(\mathbf{E}_1, \mathbf{E}_2, \ldots, \mathbf{E}_K)$ and $\bar{\mathbf{E}} = bd(\bar{\mathbf{E}}_{i1}, \bar{\mathbf{E}}_{i2}, \ldots, \bar{\mathbf{E}}_{iK})$ where $\bar{\mathbf{E}}_{iK} = \mathbf{I}_{d_{N_s}}$ and $\bar{\mathbf{E}}_{ij} = 0, i \neq j$, the optimal $\mathbf{b}$ can be obtained by solving the following problem

$$\begin{aligned} \min_{\mathbf{b}} \ & \Phi_1(\mathbf{b}) \\ s.t. \ & \sum_{i=1}^{K} \mathbf{b}^H \bar{\mathbf{E}}_i \mathbf{b} \leq P_s \\ & \mathbf{b}^H \mathbf{E} \mathbf{b} \leq P_r - \sigma_r^2 tr(\mathbf{F}(\sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \mathbf{F}^H). \end{aligned} \tag{4.55}$$

This is a QCQP problem and can be solved using the CVX MATLAB toolbox for disciplined convex programming.

The proposed iterative algorithm is summarized in Algorithm 6. This algorithm can be shown to converge as follows. It is obvious that the three subproblems are convex. It then follows that each update of $\mathbf{B}_k$, $\mathbf{F}$ and $\mathbf{W}_k$ will decrease or at least not increase the value of the objective function, and thus the iterative algorithm converges to a locally optimum solution.

Algorithm 6 Iterative design of $\mathbf{B}_k$, $\mathbf{F}$ and $\mathbf{W}_k$ for the uplink model

1: Initialize the algorithm with $\mathbf{B}^{(0)} = \sqrt{\frac{P_s}{L}} \mathbf{I}_L$ and $\mathbf{F}^{(0)} = \sqrt{\frac{P_r}{tr(\mathbf{H}_{SR} \mathbf{B}_k^{(0)} (\mathbf{H}_{SR} \mathbf{B}_k^{(0)})^H + \sigma_t^2 \mathbf{H}_{LI} \mathbf{H}_{LI})^H + \mathbf{I}_{N_r})}} \mathbf{I}_{N_r}$, and set $n = 0$.
2: Update $\mathbf{W}_k^{(n)}$ using $\mathbf{F}^{(n)}$ and $\mathbf{B}_k^{(n)}$ using (4.35).
3: Update $\mathbf{F}^{(n+1)}$ using $\mathbf{W}_k^{(n)}$ and $\mathbf{B}_k^{(n)}$, $k = 1, \ldots, K$ in (4.40).
4: Update $\mathbf{B}_k^{(n+1)}$ using $\mathbf{W}_k^{(n)}$ and $\mathbf{F}^{(n+1)}$ by solving the problem in (4.55).
5: If $(\text{SMSE}^{(n)} - \text{SMSE}^{(n+1)})/\text{SMSE}^{(n)} > \epsilon$, go to step 2.
6: End

4.2.4 Numerical Results

In this section, the performance of the proposed multiuser MIMO FD relay algorithm is evaluated using numerical simulation. For simplicity, we consider a system with two sources and two destinations. The extension to more than two source-destination pairs is straightforward. We assume that the sources and destinations are equipped with two antennas, and the relay is equipped with four receive antennas and four transmit antennas.

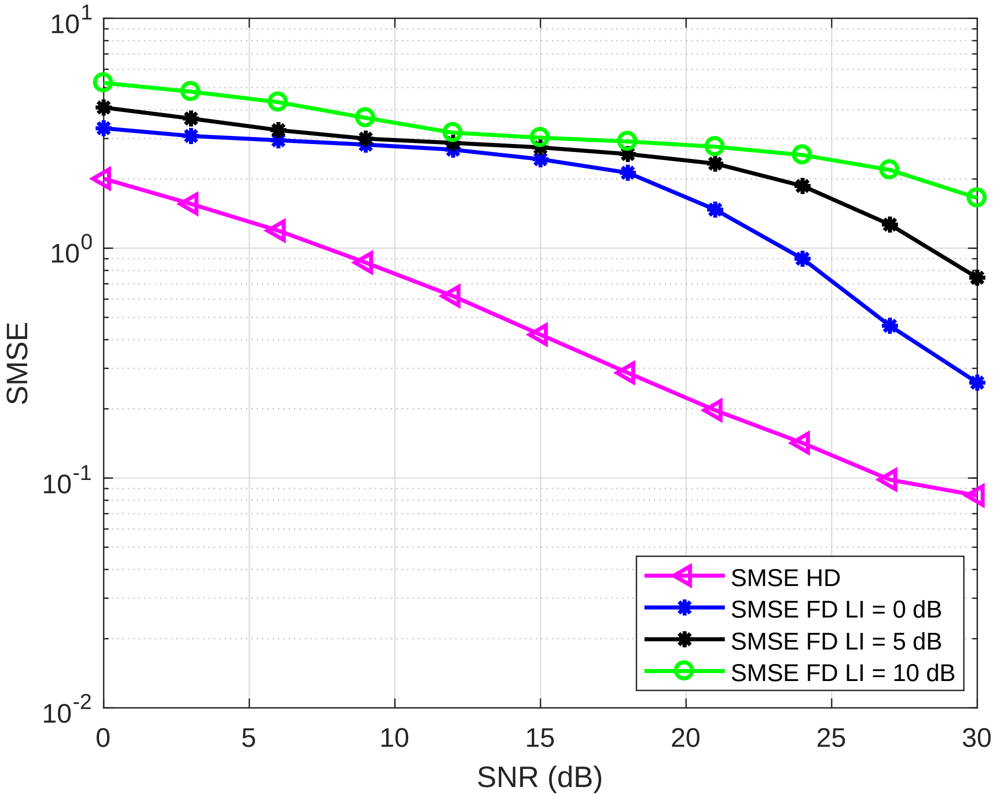

Figure 4.8: SMSE versus SNR_{s-r} with $\text{SNR}_{r-d} = 30$ dB.

As in the related literature, flat-fading MIMO channels are considered. It is assumed that the entries of $\mathbf{H}_k$, $\mathbf{G}_k$ and $\mathbf{H}_{LI}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Further, all noise terms are i.i.d. complex circularly symmetric Gaussian random variables with zero mean and unit variance. As discussed in [10], the residual LI has magnitude 0 dB to 30 dB. Therefore, LI levels of 0 dB, 5 dB and 10 dB are considered. In all cases, results are given for an average of 1000 independent channel realizations. Note that the optimization procedure for the HD system mentioned in this section is as the same as the proposed FD system except residual LI term and the achievable rate for the HD system is dropped by half than FD system since two time slots are required for the transmission between the source and destination.

Fig. 4.8 presents the SMSE for the half-duplex (HD) system and the full-duplex (FD) system with different levels of residual LI. The signal-to-noise-ratio between the sources and relay varies from 0 dB to 30 dB and the signal-to-noise-ratio between the relay and destinations is fixed at 30 dB. It is clear that the HD system provides the best SMSE as the FD performance is degraded as the LI increases.

The achievable rate for the system in (4.29) can be obtained using an approach similar

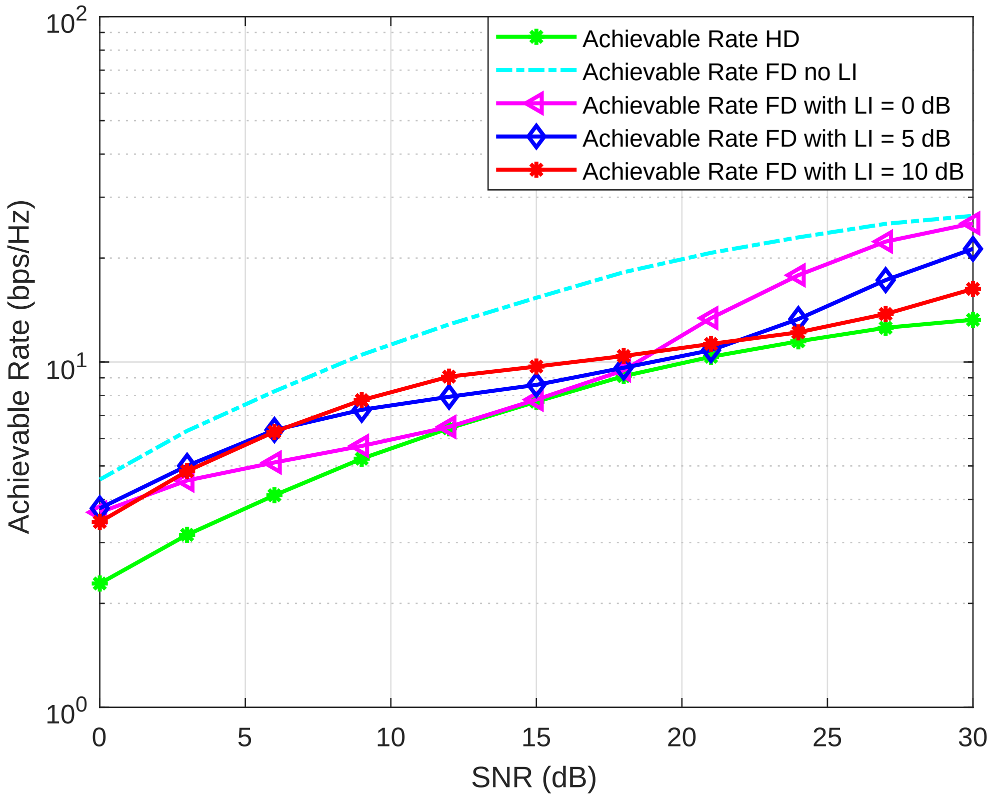

Figure 4.9: Achievable rate versus SNR_{s-r} with $\text{SNR}_{r-d} = 30$ dB.

to that in [42], and can be written as

$$R = \sum_{k=1}^{K} R_k, \tag{4.56}$$

where

$$\begin{aligned} R_k &= \log_2 \det \left[\mathbf{I}_{N_d} + (\mathbf{G}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k)(\mathbf{G}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k)^H \right. \\ &\quad \left. \times (\sigma_t^2 \mathbf{G}_k \mathbf{F} \mathbf{H}_{LI} \mathbf{H}_{LI}^H \mathbf{F}^H \mathbf{G}_k^H + \mathbf{G}_k \mathbf{F} \mathbf{F}^H \mathbf{G}_k^H + \mathbf{I}_{N_d})^{-1} \right]. \end{aligned} \tag{4.57}$$

Fig. 4.9 shows the achievable rate for the HD and FD systems. In this figure, the SNR between the sources and relay varies from 0 dB to 30 dB while the SNR between the relay and destinations is fixed at 30 dB. The HD system corresponds to the case when the residual LI is zero and two time slots are required for transmission from the sources to the destinations. Thus, the FD achievable rate is twice the HD achievable rate if the LI is canceled completely. Further, the achievable rate with the FD system is higher when the residual LI is 0 dB to 10 dB. The achievable rate of the FD system is degraded as the residual LI increases in the high SNR region since the residual LI is greater than the multiuser interference. In the low SNR region, the multiuser interference dominates the residual LI, so the effect of the residual LI is minimal.

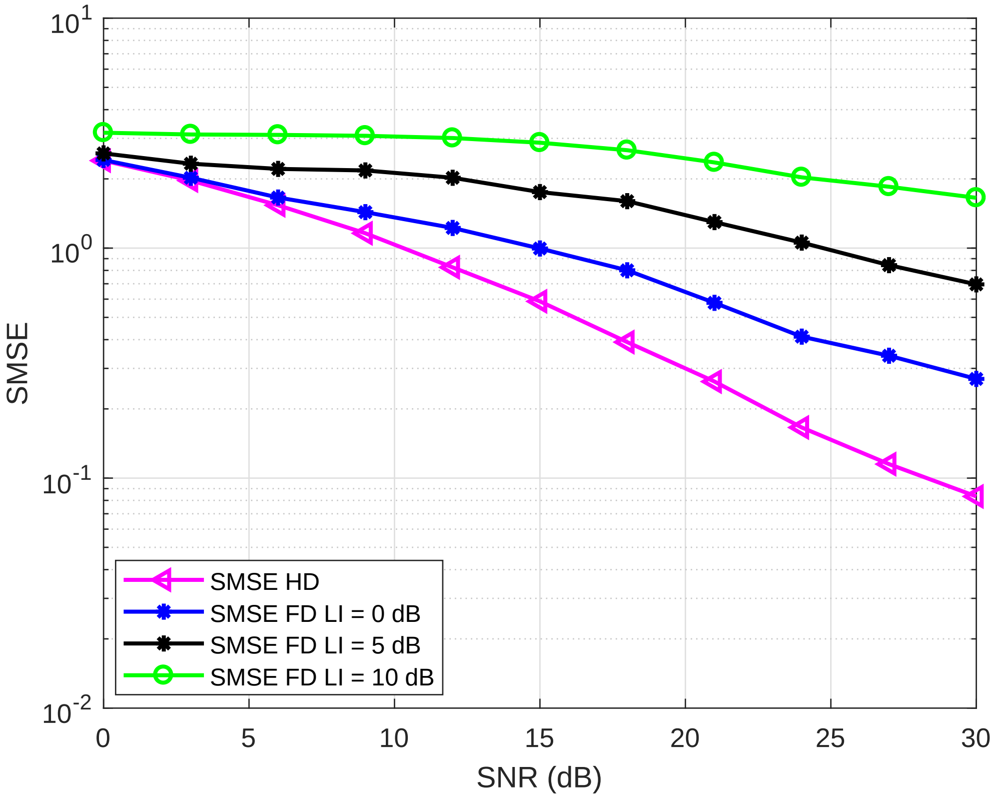

Figure 4.10: SMSE versus SNR_{r-d} with $\text{SNR}_{s-r} = 30$ dB.

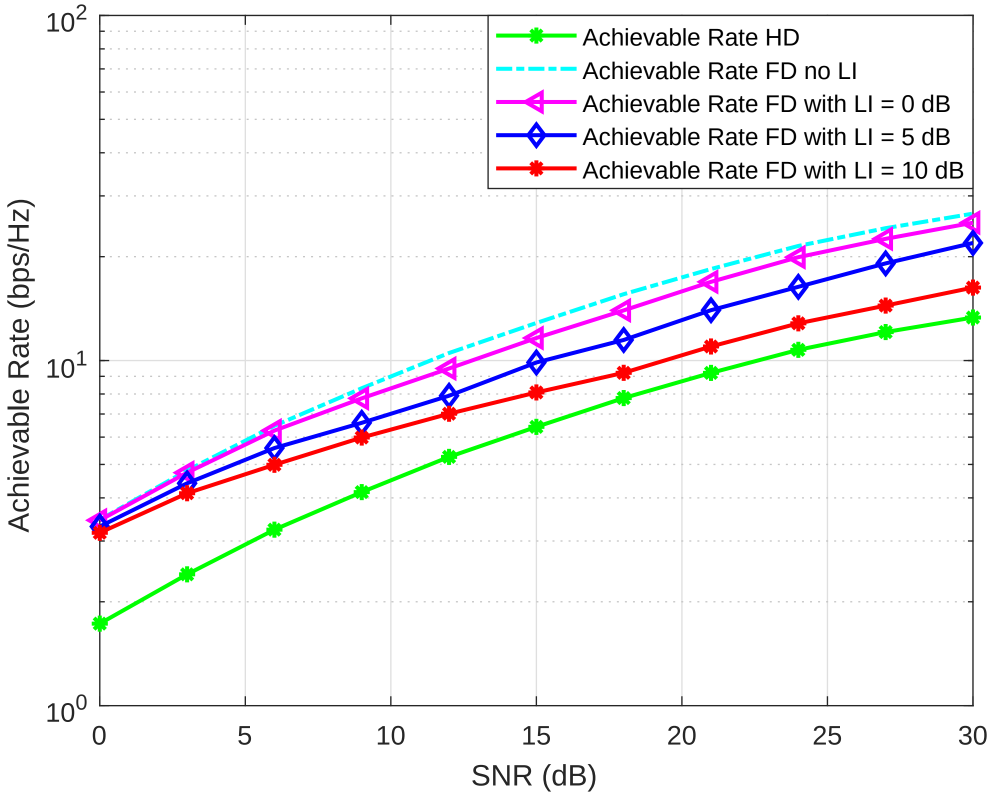

Figure 4.11: Achievable rate versus SNR_{r-d} with $\text{SNR}_{s-r} = 30$ dB.

Figs. 4.10 and 4.11 present the SMSE and achievable rate with a fixed SNR of 30 dB between the sources and relay and an SNR between the relay and destinations from 0 dB to 30 dB. The SMSE in Fig. 4.10 is better than in Fig. 4.8 in the low SNR region because a higher transmit power results in more residual LI. Fig. 4.11 shows that the achievable rate of the FD system is always higher than that of the HD system for residual LI levels up to 10 dB.

4.2.5 Conclusions

In this section, the precoder design for the downlink of a multiuser non-regenerative MIMO full-duplex (FD) relay system with residual loop interference (LI) was investigated. The source and relay precoding matrices and the destination combining matrix were optimized to minimize the sum mean squared error (SMSE). The original non convex problem was converted into three convex subproblems and an iterative algorithm was used to optimize the source, relay and destination matrices. Simulation results were presented which confirm that the proposed iterative method outperforms the corresponding half-duplex (HD) relay system in terms of the SMSE and achievable rate.

Chapter 5

Conclusions and Future Work

The previous three chapters investigated the precoding design for multiple-input multiple-output (MIMO) full-duplex (FD) one-way and two-way relaying communication systems and multiuser uplink and multiuser paired downlink MIMO systems. This chapter summarizes the research contributions and presents some issues and expectations as guidelines for future research.

5.1 Precoding Design for MIMO Full-Duplex Amplify-and-Forward Relay Communication Systems

Precoding for a full-duplex (FD) one-way MIMO amplify-and-forward (AF) relay communication system was investigated. The effect of residual loop interference (LI) was considered in the design as a practical consideration. Two iterative algorithms were presented to solve the non convex optimization problem. These are guaranteed to converge to locally optimum solutions. Numerical results were presented which illustrate the achievable rate improvement of the proposed scheme over the corresponding half-duplex (HD) relaying system.

5.2 Precoding Design for Two-Way MIMO Full-Duplex Amplify-and-Forward Relay Communication Systems

Locally optimal source and relay precoding and destination combiner design was considered for MIMO two-way FD relay communication systems. The residual LI introduced by the FD relay was considered in the design. To solve the corresponding non convex problem, two efficient MSE based algorithms were developed. Simulation results were presented which demonstrate that both algorithms outperform the corresponding HD relay system in terms of the sum achievable rate and sum MSE. The computational complexity of the two algorithms was compared to examine the tradeoff between performance and complexity which is important when considering implementation.

5.3 Precoding Design for Multiuser MIMO Full-Duplex Amplify-and-Forward Relay Communication Systems

The precoding design for MIMO FD AF relay communication systems with multiple users was examined. Two models were considered, one for the multiuser uplink system and another for the multiuser paired downlink system. Both design problems are non convex which makes the globally optimum solution intractable. Iterative algorithms were proposed for these problems to obtain locally optimum solutions. The improvement of the FD system over the corresponding HD system was shown via simulation. With good LI cancellation, the FD system is an efficient means of improving system performance and thus should be considered for future generation communication systems.

5.4 Future Research

There are many directions for further research related to the topics in this dissertation, some of which are summarized below.

- In this dissertation, the precoding designs are based on the assumption of perfect CSI, and this can only be acquired by channel estimation. The effects of imperfect channel state information (CSI) is a practical consideration which degrades system performance and thus should be examined. It is also worthwhile to consider channel estimation errors in the channel model.
- The two-way MIMO FD system in Chapter 3 only considered two nodes exchanging information. It would be interesting to extend this to a two-way MIMO FD system with more than two nodes. Since inter user interference is introduced, the objective function will change.
- The mean squared error (MSE) of the received signal at the destination was considered in this dissertation. Another approach is to maximize the achievable rate as the

design criteria. This is important as the achievable rate of MIMO FD relay systems has not been well investigated.

Bibliography

[1] E. B. Carne, *Connections for the Digital Age:Multimedia Communications for Mobile, Nomadic and Fixed Devices*. Hoboken, NJ : Wiley, 2011.

[2] V. Hachenburg, B. D. Holm, and S. J. I., "Data signaling functions for a cellular mobile telephone system," *IEEE Transactions on Vehicular Technology*, vol. VT-26, no. 1, pp. 82–88, Feb. 1977.

[3] R. Patachaianand and K. Sandrasegaran, "User speed estimation techniques for UMTS," *Electronics Letters*, vol. 43, no. 19, pp. 1036–1037, Sep. 2007.

[4] M. Akselrod, N. Becker, M. Fidler, and R. Lunnen, "4G LTE on the road - What impacts download speeds most?" in *Proc. IEEE Vehicular Technology Conference*, Sep. 2017, pp. 1–6.

[5] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Proc. IEEE International Symposium on Information Theory*, Aug. 1988, p. 156.

[6] T. M. Cover and A. A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.

[7] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 74–80, Oct. 2004.

[8] J. Zhang, Z. K. M. Ho, E. A. Jorswieck, and M. Haardt, "Interference neutralization for non-regenerative two-way relaying networks - Part I: Feasibility," *IEEE Transactions on Signal Processing*, vol. 63, no. 17, pp. 4544–4556, Sep. 2015.

[9] A. Goldsmith, *Wireless Communications*. Cambridge University Press, New York, NY, USA, 2005.

[10] D. Bharadia, E. McMilin, and S. Katti, "Full duplex radios," in *Proc. ACM SIGCOMM*, New York, NY, USA, 2013, pp. 375–386.

[11] M. Jain *et al.*, "Practical, real-time, full duplex wireless," in *Proc. Annual International Conference on Mobile Computing and Networking*, New York, NY, USA, 2011, pp. 301–312.

[12] T. Riihonen, S. Werner, and R. Wichman, "Mitigation of loopback self-interference in full-duplex MIMO relays," *IEEE Transactions on Signal Processing*, vol. 59, no. 12, pp. 5983–5993, Dec. 2011.

[13] ——, "Spatial loop interference suppression in full-duplex MIMO relays," in *Proc. Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, USA, Nov. 2009, pp. 1508–1512.

[14] B. Chun and Y. H. Lee, "A spatial self-interference nullification method for full duplex amplify-and-forward MIMO relays," in *Proc. IEEE Wireless Communication and Networking*, Apr. 2010, pp. 1–6.

[15] Y. Y. Kang, B.-J. Kwak, and J. H. Cho, "An optimal full-duplex AF relay for joint analog and digital domain self-interference cancellation," *IEEE Transactions on Wireless Communications*, vol. 62, no. 8, pp. 2758–2772, Aug. 2014.

[16] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," in *Proc. Asilomar Conference on Signals, Systems and Computers*, Nov. 2010, pp. 1558–1562.

[17] N. Fatema, G. Hua, Y. Xiang, D. Peng, and I. Natgunanathan, "Massive MIMO linear precoding: A survey," *IEEE Systems Journal*, no. 99, pp. 1–12, 2017.

[18] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[19] A. Wiesel, Y. C. Eldar, and S. Shamai, "Zero-forcing precoding and generalized inverses," *IEEE Transactions on Signal Processing*, vol. 56, no. 9, pp. 4409–4418, Sep. 2008.

[20] F. Liu, L. Jiang, and C. He, "Low complexity lattice reduction aided MMSE precoding design for MIMO systems," in *Proc. International Conference on Communications and Networking*, China, Aug. 2007, pp. 498–502.

[21] M. Vu and A. Paulraj, "MIMO wireless linear precoding," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 86–105, Sep. 2007.

[22] M. R. A. Khandaker and Y. Rong, "Joint transceiver optimization for multiuser MIMO relay communication systems," *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 5977–5986, Nov. 2012.

[23] K. X. Nguyen, Y. Rong, and S. Nordholm, "Transceiver optimization for interference MIMO relay systems using the structure of relay matrix," in *Proc. Wireless and Optical Communication Conference*, Oct. 2015, pp. 29–33.

[24] R. Wang and M. Tao, "Joint source and relay precoding designs for MIMO two-way relaying based on MSE criterion," *IEEE Transactions on Signal Processing*, vol. 60, no. 3, pp. 1352–1365, Mar. 2012.

[25] H. A. Suraweera, I. Krikidis, G. Zheng, C. Yuen, and P. J. Smith, "Low-complexity end-to-end performance optimization in MIMO full-duplex relay systems," *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 913–927, Feb. 2014.

[26] J. Zhang, O. Taghizadeh, and M. Haardt, "Joint source and relay precoding design for one-way full-duplex MIMO relaying systems," in *Proc. International Symposium on Wireless Communication Systems*, Aug. 2013, pp. 1–5.

[27] C. Y. A. Shang, P. J. Smith, G. K. Woodward, and H. A. Suraweera, "Linear transceivers for full duplex MIMO relays," in *Proc. Australian Communication Theory Workshop*, Feb. 2014, pp. 11–16.

[28] G. Zheng, "Joint beamforming optimization and power control for full-duplex MIMO two-way relay channel," *IEEE Transactions on Signal Processing*, vol. 63, no. 3, pp. 555–566, Feb. 2015.

[29] J. Sangiamwong, T. Asai, J. Hagiwara, Y. Okumura, and T. Ohya, "Joint multi-filter design for full-duplex MU-MIMO relaying," in *Proc. IEEE Vehicular Technology Conference*, Apr. 2009, pp. 1–5.

[30] A. Kaye and D. George, "Transmission of multiplexed PAM signals over multiple channel and diversity systems," *IEEE Transactions on Communication Technology*, vol. 18, pp. 520–526, Oct. 1970.

[31] E. C. Van Der Meulen, "Three-terminal communication channels," *Advances in Applied Probability*, vol. 3, no. 1, pp. 120–154, 1971.

[32] Y. Rong, "MMSE-based non-regenerative multicarrier MIMO wireless relay communications with direct source-destination link," in *Proc. IEEE International Conference on Communications*, Dresden, Germany, Jun. 2009, pp. 1–5.

[33] Y. Dai and X. Dong, "Power allocation for multi-pair massive MIMO two-way AF relaying with linear processing," *IEEE Transactions on Wireless Communications*, vol. 15, no. 9, pp. 5932–5946, Sep. 2016.

[34] C. Xing, S. Ma, and Y. Wu, "Robust joint design of linear relay precoder and destination equalizer for dual-hop amplify-and-forward MIMO relay systems," *IEEE Transactions on Signal Processing*, vol. 58, no. 4, pp. 2273–2283, Apr. 2010.

[35] O. Munoz-Medina, J. Vidal, and A. Agustin, "Linear transceiver design in non-regenerative relays with channel state information," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2593–2604, Jun. 2007.

[36] Z. He, X. Zhang, Y. Bi, and Y. Rong, "Optimal source and relay design for multiuser MIMO af relay communication systems with direct links and imperfect channel information," *IEEE Transactions on Wireless Communications*, vol. 15, no. 3, pp. 2025–2038, Mar. 2016.

[37] W. Xu, X. Dong, and W. Lu, "Joint precoding optimization for multiuser multi-antenna relaying downlinks using quadratic programming," *IEEE Transactions on Communications*, vol. 59, no. 5, pp. 1228–1235, May 2011.

[38] C. Song, K. Lee, and I. Lee, "MMSE-based MIMO cooperative relaying systems: Closed-form designs and outage behavior," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 8, pp. 1390–1401, Sep. 2012.

[39] R. Mo and Y. Chew, "MMSE-based joint source and relay precoding design for amplify-and-forward MIMO relay networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 9, pp. 4668–4676, Sep. 2009.

[40] A. S. et al., "In-band full-duplex wireless: Challenges and opportunities," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 9, pp. 1637–1652, Sep. 2014.

[41] A. Almradi and K. A. Hamdi, "Ergodic capacity analysis of MIMO full duplex relaying with imperfect CSI," in *Proc. IEEE Global Communications Conference*, San Diego, CA, Dec. 2015, pp. 1–6.

[42] Y. Y. Kang and J. H. Cho, "Capacity of MIMO wireless channel with full-duplex amplify-and-forward relay," in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, Tokyo, Japan, Sep. 2009, pp. 117–121.

[43] K. C. Dheeraj, A. Thangaraj, and R. Ganti, "Equalization in amplify-forward full-duplex relay with direct link," in *Proc. National Conference on Communications*, Mumbai, India, Feb. 2015, pp. 1–6.

[44] A. C. Cirik, Y. Rong, and Y. Hua, "Achievable rates of full-duplex MIMO radios in fast fading channels with imperfect channel estimation," *IEEE Transactions on Signal Processing*, vol. 62, no. 15, pp. 3874–3886, Aug. 2014.

[45] Z. He, W. Jiang, and Y. Rong, "Robust design for amplify-and-forward MIMO relay systems with direct link and imperfect channel information," *IEEE Transactions on Wireless Communications*, vol. 14, no. 1, pp. 353–363, Jan. 2015.

[46] Z. Shi, S. Ma, F. Hou, and K. Tam, "Analysis on full duplex amplify-and-forward relay networks under Nakagami fading channels," in *Proc. IEEE Vehicular Technology Conference*, Quebec City, QC, Sep. 2012, pp. 1–5.

[47] C. T. Lin and W. R. Wu, "Linear transceiver design for full-duplex MIMO relay systems: A non-iterative approach," *IEEE Wireless Communications Letters*, vol. 6, no. 4, pp. 518–521, Aug. 2017.

[48] L. J. Rodriguez, N. H. Tran, and T. Le-Ngoc, "Performance of full-duplex AF relaying in the presence of residual self-interference," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 9, pp. 1752–1764, Sep. 2014.

[49] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, Cambridge, UK, 2004.

[50] M. C. Grant and S. P. Boyd, "CVX: Matlab software for disciplined convex programming."

[51] Z. He, J. Zhang, W. Liu, and Y. Rong, "New results on transceiver design for two-hop amplify-and-forward MIMO relay systems with direct link," *IEEE Transactions on Signal Processing*, vol. 64, no. 20, pp. 5232–5241, Oct. 2016.

[52] Z. Fang, X. Yuan, and X. Wang, "Towards the asymptotic sum capacity of the MIMO cellular two-way relay channel," *IEEE Transactions on Signal Processing*, vol. 62, no. 16, pp. 4039–4051, Aug. 2014.

[53] R. Vaze and R. W. Heath, "On the capacity and diversity-multiplexing tradeoff of the two-way relay channel," *IEEE Transactions on Information Theory*, vol. 57, no. 7, pp. 4219–4234, Jul. 2011.

[54] H. Kim, N. Lee, and J. Kang, “Energy efficient two-way AF relay system with multiple-antennas,” in *Proc. IEEE Vehicular Technology Conference*, Budapest, Hungary, Sep. 2011, pp. 1–5.

[55] D. Gunduz, A. Goldsmith, and H. V. Poor, “MIMO two-way relay channel: Diversity-multiplexing tradeoff analysis,” in *Proc. Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, Oct. 2008, pp. 1474–1478.

[56] T. Unger and A. Klein, “On the performance of two-way relaying with multiple-antenna relay stations,” in *Proc. IST Mobile and Wireless Communications Summit*, Jul. 2007, pp. 1–5.

[57] R. Wang, M. Tao, and Z. Xiang, “Nonlinear precoding design for MIMO amplify-and-forward two-way relay systems,” *IEEE Transactions on Vehicular Technology*, vol. 61, no. 9, pp. 3984–3995, Nov. 2012.

[58] C. Li, L. Yang, and W. P. Zhu, “Two-way MIMO relay precoder design with channel state information,” *IEEE Transactions on Communications*, vol. 58, no. 12, pp. 3358–3363, Dec. 2010.

[59] C. Y. Leow, Z. Ding, and K. K. Leung, “Joint beamforming and power management for nonregenerative MIMO two-way relaying channels,” *IEEE Transactions on Vehicular Technology*, vol. 60, no. 9, pp. 4374–4383, Dec. 2011.

[60] R. Wang, M. Tao, and Y. Huang, “Linear precoding designs for amplify-and-forward multiuser two-way relay systems,” *IEEE Transactions on Communications*, vol. 11, no. 12, pp. 4457–4469, Dec. 2012.

[61] X. X. et al., “Power-efficient distributed beamforming for multiple full-duplex relays aided multiuser networks,” in *Proc. IEEE Global Communications Conference*, San Diego, CA, Dec. 2015, pp. 1–7.

[62] H. Alves, D. B. da Costa, R. D. Souza, and M. Latva-aho, “On the performance of two-way half-duplex and one-way full-duplex relaying,” in *Proc. IEEE International Workshop on Signal Processing Advances in Wireless Communications*, Darmstadt, Germany, Jun. 2013, pp. 56–60.

[63] Y. Liu, X. G. Xia, Z. Zhang, and H. Zhang, “Self-coded distributed space-time coding for two-way full-duplex relay networks,” in *Proc. IEEE Global Communications Conference*, Dec. 2015, pp. 1–6.

[64] D. Choi and J. H. Lee, "Outage probability of two-way full-duplex relaying with imperfect channel state information," *IEEE Communications Letters*, vol. 18, no. 6, pp. 933–936, Jun. 2014.

[65] Z. Zhang, Z. Ma, Z. Ding, M. Xiao, and G. K. Karagiannidis, "Full-duplex two-way and one-way relaying: Average rate, outage probability, and tradeoffs," *IEEE Transactions on Wireless Communications*, vol. 15, no. 6, pp. 3920–3933, Jun. 2016.

[66] C. T. Lin, F. S. T. W. R. Wu, and F. J. Jheng, "Joint precoders design for full-duplex MIMO relay systems with QR-SIC detector," in *Proc. IEEE Global Communications Conference*, San Diego, CA, 2015, pp. 1–6.

[67] C. Xing, S. Ma, Y. C. Wu, and T. S. Ng, "Transceiver design for dual-hop non-regenerative MIMO-OFDM relay systems under channel uncertainties," *IEEE Transactions on Signal Processing*, vol. 58, no. 12, pp. 6325–6339, Dec. 2010.

[68] B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-duplex MIMO relaying: Achievable rates under limited dynamic range," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 8, pp. 1541–1553, Sep. 2012.

[69] C. Lin, F. Tseng, and W. Wu, "MMSE transceiver design for full-duplex MIMO relay systems," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 8, pp. 6849–6861, Aug. 2017.

[70] J. Liu, Z. Liu, and Z. Qiu, "Joint MMSE transceiver design for multiuser non-regenerative MIMO relay downlink systems," in *Proc. Internation Conference on Wireless Communications and Mobile Computing*, Istanbul, Turkey, Jul. 2011, pp. 877–882.

[71] J. Liu, F. Gao, and Z. Qiu, "Robust transceiver design for downlink multiuser MIMO af relay systems," *IEEE Transactions on Wireless Communications*, vol. 14, no. 4, pp. 1228–1235, Apr. 2015.

[72] D. Xu, Y. Huang, L. Yang, and B. Li, "Linear transceiver design for multiuser MIMO downlink," in *Proc. IEEE Internation Conference on Communications*, Beijing, China, May 2008, pp. 1–5.

[73] C. Zhai, X. Li, and Y. Hei, "A novel decomposed transceiver design for multiuser MIMO relay downlink systems," in *Proc. IEEE Wireless Communications and Networking Conference*, Paris, France, Apr. 2012, pp. 1921–1924.

[74] L. Gopal, Y. Rong, and Z. Zang, "Robust MMSE transceiver design for nonregenerative multicasting MIMO relay systems," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 10, pp. 8979–8989, Oct. 2017.

[75] A. C. Cirik, S. Biswas, S. Vuppala, and T. Ratnarajah, "Robust transceiver design for full duplex multiuser MIMO systems," *IEEE Wireless Communications Letters*, vol. 5, no. 3, pp. 260–263, Jun. 2016.