Performance Analysis of Cognitive Transmission in Multiple Cell Environment

by

Dong Zhang
B.Eng., University of Victoria, 2010

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Supervisory Committee

Dr. Hong-Chuan Yang, Supervisor
(Department of Electrical and Computer Engineering)

Dr. Pan Agathoklis, Departmental Member
(Department of Electrical and Computer Engineering)

Dr. Kui Wu, Outside Member
(Department of Computer Science)

Supervisory Committee

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(Department of Electrical and Computer Engineering)

Dr. Kui Wu, Outside Member
(Department of Computer Science)

ABSTRACT

This thesis conducts a performance analysis of cognitive transmission in a multiple-cell environment. Most of the cognitive radio (CR)-related research has focused on scenarios where a secondary system operates in the presence of a single primary communication system. In this work, we extend the study of a single-cell scenario to a more practical scenario where the secondary system is subjected to two independent primary users (PU). In particular, we investigate the performance of a secondary system operating in an interweaving fashion to explore the spectrum of opportunities. Under the assumption of the Poison traffic model for PU activities, we apply the Markov chain model to first determine the system parameters for combined PUs activities, and then characterize the dynamics of the spectrum opportunities for a secondary user (SU) for both single- and multiple-channel access. To fully investigate the proposed system, we also consider some possible drawbacks and provide corresponding solutions in the extension section.

We derive the exact mathematical expressions for the performance metrics, including average waiting time and average service time of the SU transmission. To enrich the performance analysis, other performance metrics, such as average throughput and collision frequency/ratio are also presented. Through selected numerical examples, we examine the effect of different operation parameters on the SU system

performance. We believe that those analytical results can help predict which types of SU applications can be supported under certain practical PU activities.

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DEDICATION

To my family

Chapter 1

Introduction

Cognitive Radio (CR), characterized by its reconfigurability and intelligent adaptive behavior, has found various applications in the radio and wireless network industry. With its potential to revolutionize our future communication interface management, CR has drawn much attention from different research groups. For example, the IEEE 802.22 research group has already made standards for using spectrum holes for secondary users (SU) in the TV bands. The objective of this thesis is to conduct an extensive performance analysis of the potential uses of CR.

In this chapter, we briefly introduce the background and motivation for CR, and the existing literature on the performance analysis of CR. In addition, we present the contributions and outline of the thesis.

1.1 Background and Motivation

The concept of CR can be traced to software-defined radio (SDR). In contrast to conventional hardware-implemented radio systems, SDR is implemented by using digital signal processors and other general purpose processors for radio communications. Due to its programmable configuration, SDR has an advantage in being able to ideally cover all of the communication frequency band, and supporting various communication standards (e.g., GSM, CDMA, and Wifi) in a single end device. SDR has disadvantages because of the high power consumption and large physical size. For example, SDR technology is currently used mainly in base stations.

SDR also opens new possibilities for future portable hand-held devices. Because SDR can support multiple standards, portable devices can have more service options.

Nevertheless, each change of functions requires a specific software reconfiguration, and more intelligent technology that can automatically adapt itself to the current surroundings, with the ability to switch functions accordingly. As a result, CR must have reconfigurability and adaptivity.

CR was first termed by Joseph Mitola III, in his doctoral dissertation [25]. According to the definition, CR is a self-reconfigurable software radio environment with high computational intelligence, and it is further defined by its cognition cycle . As of today, CR is defined as a paradigm for wireless communication, in which either the network or the wireless node can change its transmitting/receiving parameters to communicate efficiently, while avoiding interference from licensed or unlicensed users. The two primary objectives of CR are to have:

- highly reliable communication whenever and wherever needed;
- efficient utilization of the radio spectrum [11];

In the US, a majority of research groups have narrowed the study of CR to dynamic spectrum access (DSA) and unlicensed spectrum usage, due to a report from the Federal Communication Commission (FCC) in 2002 [16]. According to the report, some frequency bands are largely unoccupied most of time, some are partially occupied, and the rest are heavily used . To better utilize the spectrum, CR is widely recognized as a very promising solution, which serves as the motivation for this thesis.

1.2 Existing Works on CR

This section summarizes the literature on the design and performance analyses of CR. Some basic explanations of CR-related terms are also given. Readers, who are familiar with CR or more interested in the core material, may skip this section.

1.2.1 Overview of CR

Fig 1.1 [11] illustrates a basic cognitive cycle for most CR designs. In the figure, the main tasks for CR, from the transmitter/receiver point of view, are presented. The receiver plays various roles for detections, such as spectrum holes, noise level of the radio environment, channel state information (CSI), etc. Once the detections are finished, the information is sent back to the transmitter, so that the transmitter can fulfill its tasks for transmitting power control and dynamic spectrum management.

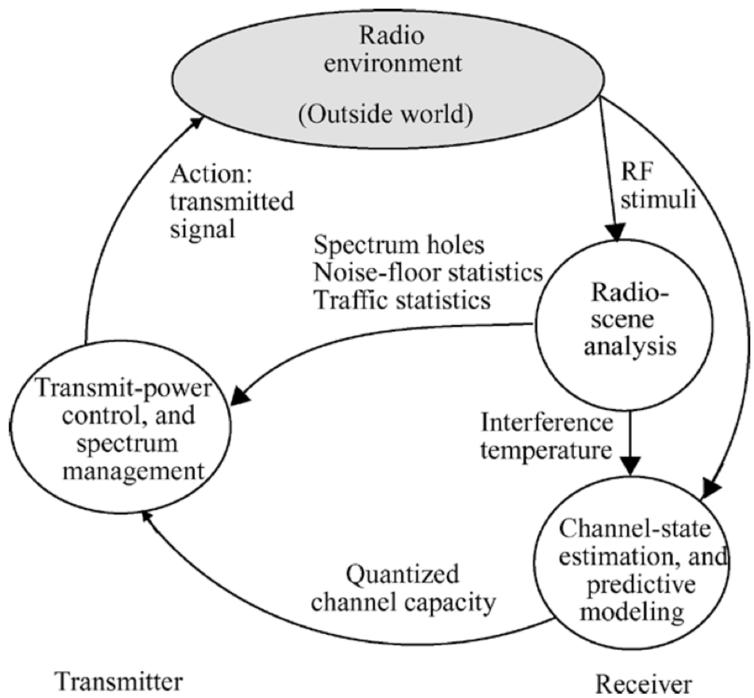

Figure 1.1: Cognitive Cycle

The cognitive cycle gives a general idea of how to build a cognitive model around the spectrum holes. The model can vary based on different degrees of cognition, requirement, or constraints. Since CR is concerned with utilizing the spectrum holes, they need to be categorized. Spectrum holes can be classified into three types based on the FCC report:

- black spaces, represent the heavily-used spectrum band by PU, and are often detected with high power.
- grey spaces, represent the partially-occupied spectrum band by PU, using relatively low power.
- white spaces, represent the unoccupied spectrum band, where only ambient noise exists.

Clearly, grey spaces and white spaces are the natural candidates for CR. After we have a clear idea of the different types of spectrum holes, we want to know how the spectrum holes are being accessed and detected.

The three main spectrum access techniques are: interweave, underlay and overlay [4] . Interweave means that a CR user will constantly exploit spectrum holes for communication. Once the spectrum holes disappear, the CR user will stop transmitting to avoid interference for the PU. Underlay is based on interweave, and conditionally allows for continued transmitting, based on the presence of the PU. The condition is that interferences to the PU caused by the CR user must be under a certain threshold. Unlike interweave and underlay, the overlay technique operates in a way that the CR user split the power into two parts. One part maintains or improves the PUs performance, while the other part supports its own transmission. Specifically, these spectrum access techniques can be explained with the system model in Figure 1.2 [10]. In the system model, two users $i = 1, 2$ are considered, where 1 represents the PU and 2 represents the SU. Each user is assumed to have their own operating band, and the SU is assumed to have perfect sensing detection for the PU. The above three spectrum access techniques can be further explained as follows:

- Interweave: As long as the PU is sensed to be active, the SU accesses its own band B_2. Otherwise, the SU can access both bands B_1 and B_2.
- Underlay: While the SU accesses its own band B_2, it also senses B_1. Regardless of the PU's presence, the SU also accesses B_1 under the condition that interference caused by SU will not affect the PU's performance. In this situation, the power allocation of the SU follows the water-filling technique.
- Overlay: While the PU is active, the SU accesses both bands in a different manner from underlay. The SU splits its power into these two bands. One is for its own transmission in B_2, and the other is used to enhance the PU's transmission to achieve the goal of coexisting PU and SU.

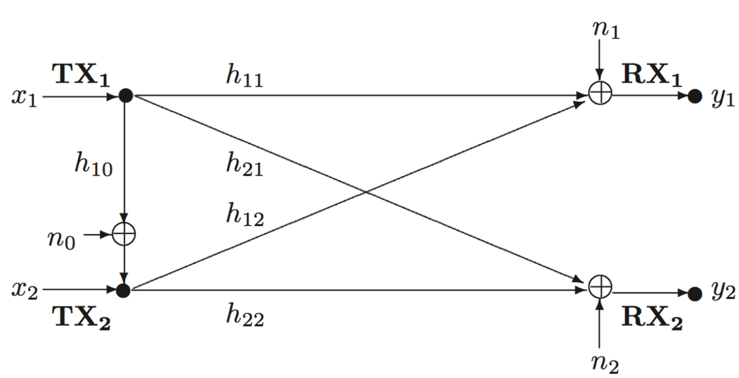

Figure 1.2: System Model for Spectrum Access

The objective of CR is to efficiently use the spectrum. To achieve this, an instantaneous exploration of the spectrum holes must be done very rapidly; therefore, spectrum-sensing is a critical element of CR. In most examples of CR, three well-known detection methods are used:

- energy detection,
- matched filtering detection,
- cyclostationary detection [30].

Energy detection is the most commonly used method in signal detection for CR because only the signal power is required for the detection decision. Also, the signal power is available for most of the primary signals. The matched filter detection and cyclostationary detection methods are used when more information about the primary signal is known (i.e., data rates, carrier frequency, preambles, pilots, synchronization symbols, and modulation schemes). Nevertheless, most of the information is not available due to security or privacy reasons. The above three sensing techniques are fundamental methods, and each method can be applied to any single detector. Moreover, the performance of a signal detector can be degraded because of such factors as fading, shadowing, or a broken sensor. The degradation has led to the development of cooperative spectrum sensing (CCS). CCS takes advantage of having multiple CRs operating in the same band to reduce the detection time, and thus, increasing their agility, to greatly improve the performance. Representative papers, such as a survey of CSS [17] demonstrate the improved performance over realistic fading channels. In [30], CSS is used to study maximum channel throughput in CR networks. Also, CSS in a two-user network [5] whowed a 35% increased agility. As a result of many related works [30, 34, 27], CCS has been shown to be the optimal sensing setting for a CR network under interference constraints.

1.2.2 Common Performance Metrics of CR

Since the design of a CR network may vary, due to different degrees of cognition, system requirements, and constraints, the diversity of CR designs makes their performance evaluation challenging. Performance metrics can be defined at node, network, and application levels [44].

From the node level perspective, radio environment awareness provides for some metric candidates to evaluate CR performance. The performance of CR depends highly on how accurate each node can grasp the information from the radio environment or from the PU. Consequently, the detection probability, P_D, and the false alarm probability, P_{FA}, can provide sufficient information about the environment awareness. A larger value of P_D indicate less interference for the PU, so the PU can be better protected; whereas, a smaller value of P_{FA} indicates more chances that the channel can be reused by the SU, leading to higher spectrum efficiency. In a cognitive network, a trade-off always exists between P_D and P_{FA}. Numerous authors have used probability of detection and false alarm as the performance metrics, and usually evaluate one, given constraints against the other one [20, 28, 30, 23]. For example, in [28], the author maximizes the probability of detection, given constraints for the false alarm probability. In [30], constraints on both P_D and P_{FA} are used to solve the optimal system throughput. In most papers, P_D and P_{FA} are usually defined as follows:

$$P_D = \Pr(Y > \Lambda | H_1), \tag{1.1}$$

$$P_{FA} = \Pr(Y > \Lambda | H_0), \tag{1.2}$$

where H_0 and H_1 represent the hypothesis of no PU signal transmitting and the hypothesis of with PU signal transmitting, respectively. Y describes the decision statistic, and Λ is the decision threshold. Once the decision statistic Y is given, P_D and P_{FA} can be easily calculated. If fading channels are introduced, the detection probability can be derived by averaging P_D over the fading statistic.

On a network level, several interesting results have emerged with regards to throughput, capacity, and outage probability for CR systems. (See for example [42, 6, 13, 31] and the references therein). Capacity is a useful performance metric for understanding the performance limits, and is usually studied in different fading environments, and access or diversity schemes. Related works that use capacity as a performance metric include [4, 32, 15]. In [32], the authors studied cognitive system performance with imperfect channel knowledge in terms of ergodic capacity. In [4], the throughput is represented by the channel capacity,

$$C = B \log_2(1 + \frac{S}{N}). \tag{1.3}$$

where B is the channel bandwidth, and $\frac{S}{N}$ is the signal-to-nosie ratio. In [15], use of the OFDMA technique showed a increasing capacity over CDMA. In [18], the author investigated the ergodic capacity under the effect of maximum ratio combining (MRC) diversity.

Delay is another very useful QoS metric for CR, because of its importance in real-time applications. Unfortunately, delay analysis is under-explored due to the analyzing difficulties. In most existing works, (for example in [36, 1, 22]), delay is approached by queueing analysis. In [36], the arriving traffic for the SU is assumed to follow a Poisson distribution, and transmission delay is obtained by studying the steady state queue lengths of the SUs. In [1], the average end-to-end delay is considered in multi-hop wireless networks, and the close form expression for end-to-end delay is approximately derived as in an open G/G/1 queuing networks. In [22] the priority fluid queueing model is used to study the performance of routers with priority service. Many other delay analysis have used queueing analysis, (such as [24, 35, 3, 26, 33] and references therein). Besides the queueing analysis approach, other methods for studying delay are also available. For example in [21], delay is studied in a CR sensor network (CRSN). Two types of channel switching schemes were investigated for the derivation of average packet transmission delay.

1.2.3 Statistical Models

Most scientific analysis of CR are built on mathematical models. A stochastic model enables the prediction of white space and direct CR, to access idle primary channel efficiently. Therefore, choosing a reasonable statistical model is essential for CR analysis.

In most of the studies of CR, the traffic of the PU is modelled by continuous-time Markov-Chain (CTMC) with ON-OFF states. Although this model is not always realistic, it reasonably approximates the PU behavior in IEEE 802.11 Wireless LAN for various traffic models, and has been verified in [7, 8]. The main benefit of using CTMC is its simplified mathematical derivation. By using CTMC, we can examine the performance of a proposed system by assuming Poisson traffic arrival processes and exponential service time distribution for the PUs and SUs. For example, in [45], the authors use a Markov chain analysis to show that the throughput of CR users increases significantly. In [39], using the CTMC model, the authors studied the impact of primary channel loads and spectrum band allocation on delay performance of CR users. Many other interesting results from CR performance analysis have used CTMC, (such as [14, 37, 41] and references therein).

1.3 Contribution and Thesis Outline

Most of the studies related to CR have focused on modelling operations of CR in a single cell, primary system, where the transmitter and receiver of the CR are located in the coverage of a single PU transmitter. Nevertheless, this model is not always realistic, especially for future generations of cellular systems. In this work, we investigate the situation where the end nodes of CR are located in separate PUs coverage. The main contributions of this study are as follows:

- derivation of mathematical formulations for key metrics: average waiting/service time for the single channel access case
- extended analysis of the multi-channel access case
- study of throughput of such CR system
- consideration of two extensions to improve the proposed system model

All the work described above is in the category of interweave study. The limitation of interweave is in the requirement for perfect sensing, which means that interference is not tolerated. To make this study more practical, two extensions are also considered. In this case, collisions/interference evaluations are involved. Some useful mathematical formulations are also derived.

The rest of this thesis is organized as follows: Chapter 2 summarizes works on single cell primary system. In Chapter 3, we present the system model for two adjacent PU networks and the corresponding analysis, together with some numerical examples. Chapter 4 introduces two extensions based on our proposed system model. Finally, conclusions and future works are discussed in Chapter 5.

Chapter 2

Single Cell Primary System Scenario

In this chapter, CR in a single cell primary system is reviewed to provide the background of the problems that are studied in the next chapter. We begin with a brief introduction of a single cell CR network. Then, the system model and performance evaluation are described in detail. Numerical examples are also given. The material serves as the foundation for Chapter 3.

2.1 Introduction

Consider a single cell primary system,where L parallel communications channels are used by the PU and SU [4]. Transmission of the PU is assumed to be unslotted so that the PU can access channels at any time. The SU follows a interweave mode of operation, and accesses only one channel at a time, opportunistically.

The behavior of the PU can be captured as a continuous-time ON-OFF Markov process. Although this model is not always realistic, it has been used as an approximation of existing wireless access applications by many researchers [4, 19, 40]. The model also makes the future derivations of the performance metrics more traceable. The objective of the SU is to access primary channels opportunistically. When the primary channel is busy, It sense the channel periodically. Once the primary channel is sensed to be idle, the SU will access the channel and start its transmission. During its transmission, the SU will continuously sense the channel so that it can evacuate the channel immediately when the PU is present. Thus, interference caused by the

SU is avoided.

Two important performance metrics are proposed, which evaluate the system Performance (i.e., the average waiting time and the average service time). These two metrics differ from the common performance metrics, and can be viewed as application level metrics. Defining the application metrics can be challenging since each application has its own features and objectives. In this work, since we adopt IEEE 802.11 Wireless LAN to model the PU activity, these two metrics provide some insight into the lower layer system design.Ultimately, by evaluating the metrics, the types of applications supported by the SU can be predicted.

2.2 System Model

As illustrated in Fig. 2.1, a single cell network with L parallel channels is available for transmission. The occupancy of each channel is modeled according to continuous-time Markov chains with ON-OFF state. In particular, the ON state indicates that the channel is busy, and its duration is denoted by T^P_{on}; the OFF state indicates that the channel is idle, and its duration is denoted by T^P_{off}. The holding times are exponentially distributed with parameter λ for the busy state and parameter μ for the idle state, respectively. In this work, parameters λ and μ are the average period instead of the rate.

In this single cell network, the secondary system is assumed to be synchronized. In other words, the sensing result from the transmitter and receiver are the same. This is not always true in a practical systems, since the two end nodes are in different locations. Their sensing results may differ due to such factors as imperfect detecting technique, shadowing, and fading. While this issue is not the main concern in this work, we believe that techniques are available to solve this problem. A potential solution would be to use a feedback link to maintain the synchronization. Also, perfect sensing is assumed for the secondary system. By perfect sensing, no sensing errors are present and the sensing time is negligible[12]. Even with perfect sensing, collisions still occur when the SU is not finishing its transmission before the PU returns to the channel. To avoid interferences caused by collisions, the SU continuously senses the channel during the transmission time. Under this condition, the SU can evacuate the channel immediately when the PU is returning. While the PU is transmitting, the SU senses the channel periodically with a time duration, T_s. Another assumption for this model is that the SU always has packets to transmit. Since the SU aims to

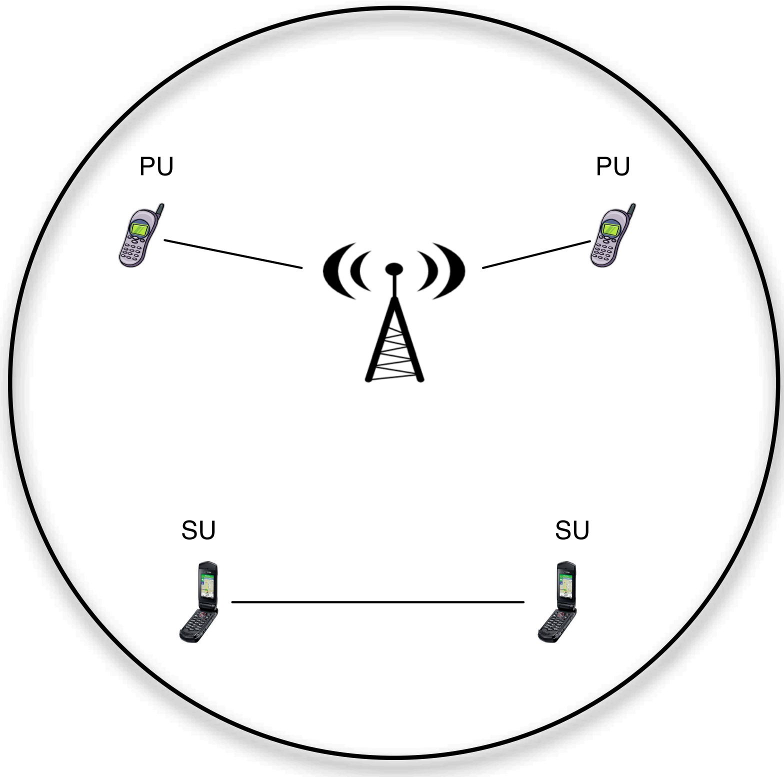

Figure 2.1: System Model

improve the spectrum usage, this is consistent with most CR related works [4, 12].

2.3 Performance Analysis

In this section, we study the performance metrics of the single cell primary system. The two metrics are: average waiting time and average service time. For a secondary system, they measure (on average) how long the CR would wait to transmit and the length of the transmission duration. With these metrics, we can predict types of applications that the SU can support under the PU's activity.

Consider the scenario where the SU can learn the transmission opportunities by sensing only one channel at a time. The PU's activity follows a continuous-time Markov chain model with ON-OFF state. Transmission of the PU is unslotted. The PU is able to access the channel at any time, and the PU's channel access is not affected by the SU. The holding time of the ON and OFF states are exponentially distributed; therefore, the PDF of ON and OFF are given as:

$$f_{T_{ON}^P}(t) = \frac{1}{\lambda} e^{-\frac{t}{\lambda}}, t \geq 0, \tag{2.1}$$

$$f_{T_{OFF}^P}(t) = \frac{1}{\mu} e^{-\frac{t}{\mu}}, t \geq 0. \tag{2.2}$$

Fig. 2.2 shows the block diagram of the SU's transmission opportunity. When the PU is sensed to be busy, the SU senses the channel in every T_s period. In other words, the SU follows a slotted structure. The total number of time slots (N) that the SU has to wait is a random variable, which depends on the duration of T_{ON}^P. Also, from the block diagram, we have the following expression for one ON-OFF cycle,

$$T_{OFF}^S = NT_s \geq T_{ON}^P. \tag{2.3}$$

Therefore, periodic sensing may cause a certain waste of the potential transmission opportunity for the SU. We use τ to represent the lost opportunity time, and clearly τ falls in between 0 and T_S. When the channel is sensed to be idle, the SU starts transmission, and during the transmission, the SU continuously senses the channel until the PU is returning. Hence, the SU finishes a complete wait-and-transmit cycle.

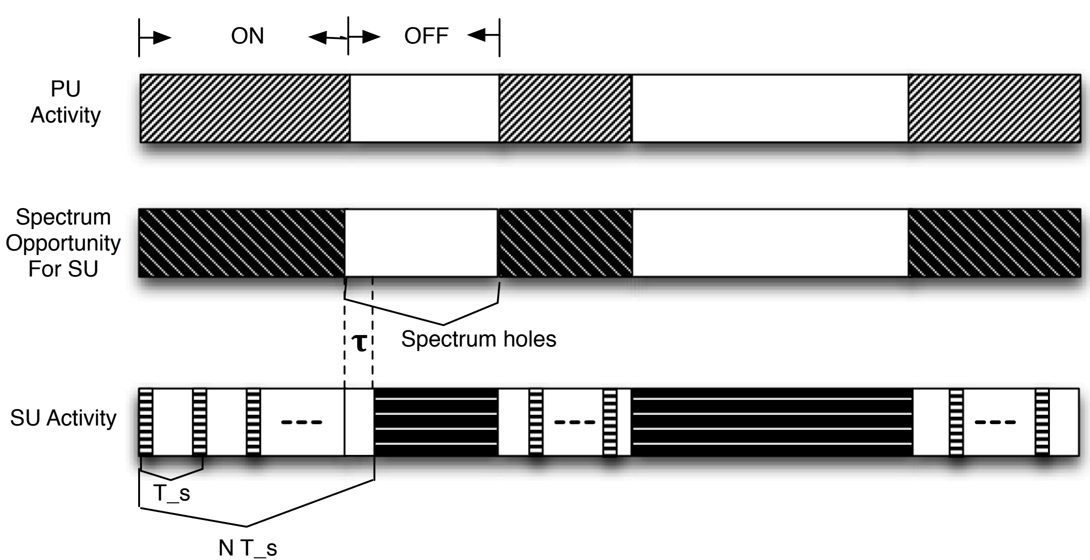

Figure 2.2: Sample of SU Transmission Opportunities

To mathematically derive the average waiting time and average service time, denoted by $E[T^S_{OFF}]$ and $E[T^S_{ON}]$ respectively, we begin with the equation

$$T^P_{ON} + T^P_{OFF} = T^S_{OFF} + T^S_{ON}. \tag{2.4}$$

Note that the above equality are only true within one complete ON-OFF cycle, since each SU's ON-OFF cycle are highly dependant on the corresponding PU's ON-OFF cycle. In other words, the duration of the SU's OFF-ON cycle should match the PU's ON-OFF cycle for one complete wait-and-transmit operation of SU. Similarly, from Fig 2.2, τ can be written as

$$\tau = NT_s - T^P_{ON}. \tag{2.5}$$

τ is the time period when the channel is idle and the SU is not transmitting. With the help of equation (2.4) and (2.5), the service time of the SU can be expressed as

$$\begin{aligned} T^S_{ON} &= T^P_{ON} + T^P_{OFF} - T^S_{OFF} \\ &= T^P_{ON} + T^P_{OFF} - NT_S \\ &= T^P_{OFF} - \tau \end{aligned} \tag{2.6}$$

To get the average service time, now we only need to find out the average of τ and T^P_{OFF}. Since T^P_{OFF} is exponentially distributed, the average of T^P_{OFF} is its parameter μ. Once we find out $E[\tau]$, we can get $E[T^S_{ON}]$ eventually. Note that to simplify the mathematical derivation, we assume that T^P_{OFF} is always larger than τ. The case where T^P_{OFF} is smaller than τ will be discussed in details in Chapter 4. When N is given, the range of T^P_{ON} is determined to be between $(N-1)T_s$ and NT_s. Then, we have the conditional PDF of T^P_{ON} for a given N as a truncated exponential random variable,

$$f_{T^P_{ON}|N}(t) = \frac{1}{\lambda}\frac{e^{-\frac{t}{\lambda}}}{e^{-\frac{(N-1)T_s}{\lambda}} - e^{-\frac{NT_s}{\lambda}}},$$

$$(N-1)T_s \le t \le NT_s. \tag{2.7}$$

Furthermore, the PDF of τ conditional on N can be determined as

$$\begin{aligned} f_{\tau|N}(t) &= f_{T_{ON}^P|N}(NT_s - t) \\ &= \frac{1}{\lambda}\frac{e^{\frac{t-T_s}{\lambda}}}{1-e^{-\frac{T_s}{\lambda}}} \\ &= 0 \le t \le T_s. \end{aligned} \tag{2.8}$$

From equation (2.8), we see that τ and N are independent; therefore, $f_\tau(t) = f_{\tau|N}(t)$. This can be also explained by the fact that the PU's activity is assumed to be memoryless. As a result, we get the average service time for the SU as

$$\begin{aligned} E[T_{ON}^S] &= E[T_{OFF}^P] - E[\tau] \\ &= \mu + \lambda - \frac{T_s}{1-e^{-\frac{T_s}{\lambda}}} \end{aligned} \tag{2.9}$$

The average waiting time $E[T_{OFF}^S]$ can be solved by finding the statistic of N, since we have $T_{OFF}^S = NT_s$. If we denote the probability that N is equal to n as $P_n = Prob\{N = n\}$, then the PMF of N can be expressed as a function of CDF of T_{ON}^P as

$$\begin{aligned} P_n &= Prob\{N = n\} \\ &= Prob\{(n-1)T_s \le T_{ON}^P \le nT_s\} \\ &= F_{T_{ON}^P}(nT_s) - F_{T_{ON}^P}((n-1)T_s) \end{aligned} \tag{2.10}$$

Because T_{ON}^P is exponentially distributed, we have the average waiting time as

$$\begin{aligned} E[T_{OFF}^S] &= E[N]T_s \\ &= T_s \sum_{n=1}^{+\infty} nP_n \\ &= T_s \sum_{n=1}^{+\infty} ne^{-(n-1)\frac{T_s}{\lambda}}(1-e^{-\frac{T_s}{\lambda}}) \\ &= \frac{T_s}{1-e^{-\frac{T_s}{\lambda}}} \end{aligned} \tag{2.11}$$

2.4 Numerical Example

In this section, we present and discuss some numerical examples based on the above results that we derived. From equation (2.9) and (2.11), we see that the average waiting time and average service time are functions of sensing time T_s. To examine the relation between sensing period and performance metrics, we have the following two figures. From Fig 2.3, we can see that under two different PU system parameters ($\lambda_1 = 6\ ms$ $\mu_1 = 3\ ms$, and $\lambda_2 = 4\ ms$, $\mu_2 = 7\ ms$), when the sensing period is increasing, the average service time is decreased. This makes sensing because when the sensing period is very small, the SU is approximately continuously sensing the primary channel. Take PU2 as an example, when the sensing period is very small, the average service time of the SU is almost equal to the OFF time of the PU, which is 7 ms. Similarly, we observe that average waiting time is increasing as the sensing period increases. This can be explained by the average waiting time as the product of N and sensing period.

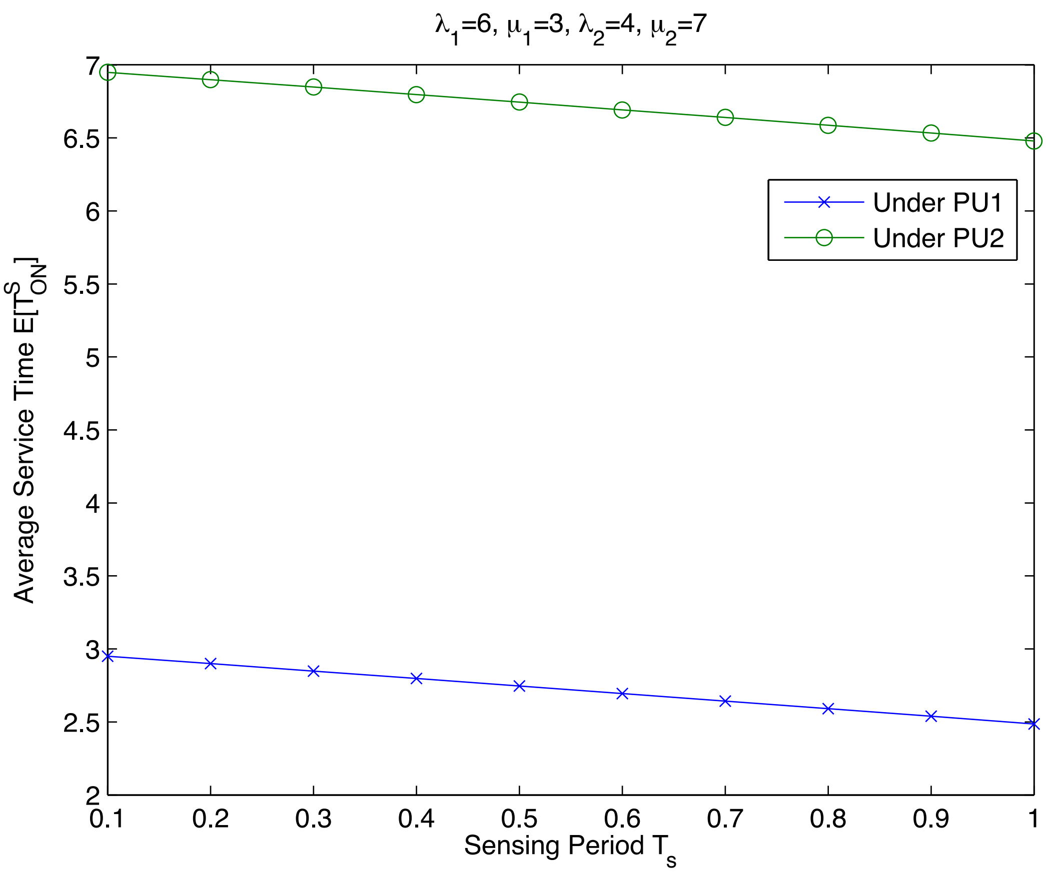

Figure 2.3: Average Service Time Over Sensing Period

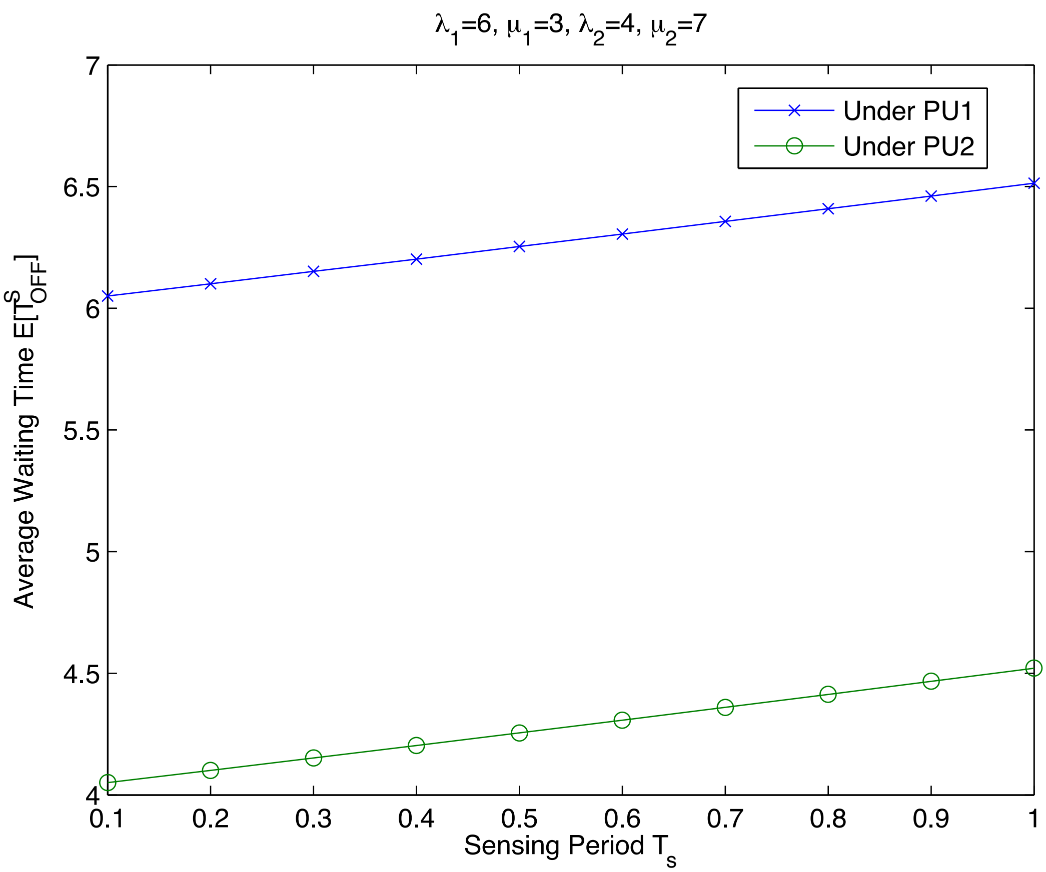

Figure 2.4: Average Service Time Over Sensing Period

Chapter 3

Dual-cell Primary System Scenario

Most studies focus on single cell primary systems, where only a single primary system affects the secondary transmission. If multiple primary systems are involved, one of the advantages of the proposed model is in providing a larger radio coverage over a wide geographic area for the secondary system. Another advantage is that system capacity of the SU is greatly increased by means of frequency reuse in the different area. The above advantages are important for practical examples, such as cellular networks.

In this chapter, we extend our study from a single cell primary system scenario to a scenario where SU transmissions are affected by two different primary systems. In this dual-cell primary system, the transmitter and receiver of CR are separately located in those two primary systems, as illustrated in Fig 3.1. We evaluate the performance of the dual-cell system through statistical analysis. In particular, we derive the exact expressions of the average waiting time and average service time of the SU. More specifically, with the Markov model for the transmission activities of the primary systems, we developed a model for the spectrum opportunities for the secondary system, with single and multiple channel access. For the primary system parameter calculation, we developed a new method based on the Markov chain super state. Furthermore, other performance metrics such as average throughput are presented to enrich our performance evaluation. Finally, the mathematical formulations are illustrated with selected numerical examples.

The rest of the chapter is organized as follows: In Section 3.1, we introduce the system model. Performance analysis for the average waiting/service time and average throughput are given in Section 3.2, which is followed by the numerical results in Section 3.3. Finally, the conclusions and discussions are given.

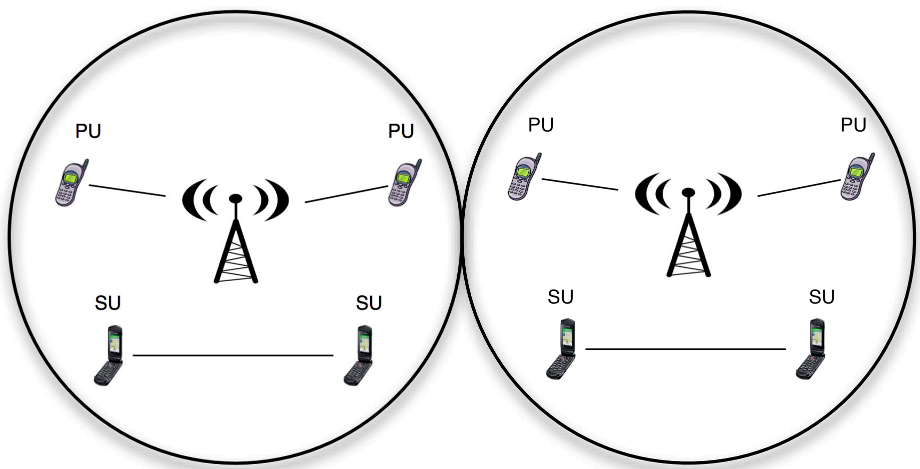

Figure 3.1: Dual-cell Primary System Model

3.1 System Model and Notations

For consistency, and to avoid confusion, all of the system parameter notations are the same as in the single cell primary system analysis in Chapter 2. New notations are explained in the following analysis.

The system model under consideration consists of two primary systems and a single secondary system, where the transmitter and receiver of the SU are located in the coverage area of different PUs as illustrated in Fig 3.1. To avoid the interference introduced by simultaneous transmission, we assume that the SU operates in an interweave fashion. Specifically, the SU transmitter and receiver can jointly sense primary channels periodically. When either the PUs is in the ON state, the SU will continue sensing the channel for spectrum opportunities in every period of T_S. Only when both PUs are in the OFF state will the SU transmit. When the SU is transmitting, it will sense the channel continuously and evacuate the channel when either PU becomes active. We assume that the spectrum sensing of SU is perfect in this work and consider the case of imperfect sensing in our future studies. The sample operation of the SU under two PUs, with the proposed sensing and transmission strategy, is illustrated in Fig 3.2.

The transmission of PU is unslotted and the PU activities are modelled as independent continuous-time Markov chain (CTMC) with ON and OFF state. In particular, the duration of the ON and OFF period for each channel for both PUs follows an independent exponential distribution. The ON and OFF duration of PU i for channel j has an average λ_{ij} and μ_{ij}, respectively. Following the same approximation of PU behavior for the single primary system analysis, the PDF of the ON state duration and the OFF state duration for PU i on channel j are given by

$$f_{T_{ON}^P}(t) = \frac{1}{\lambda_{ij}} e^{-\frac{t}{\lambda_{ij}}}, t \geq 0, \tag{3.1}$$

and

$$f_{T_{OFF}^P}(t) = \frac{1}{\mu_{ij}} e^{-\frac{t}{\mu_{ij}}}, t \geq 0. \tag{3.2}$$

respectively.

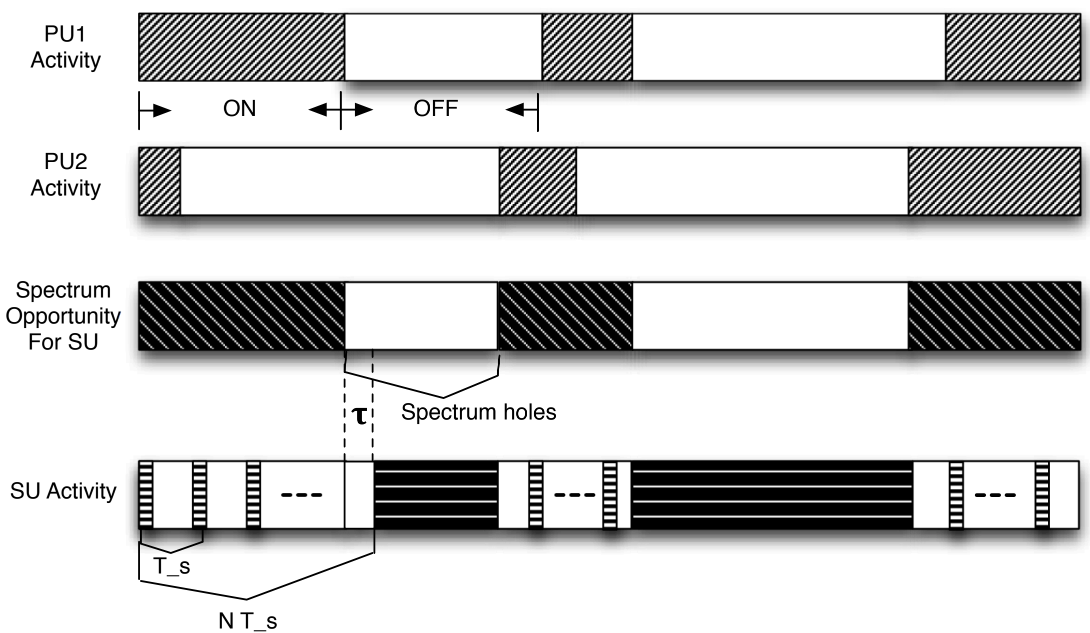

Figure 3.2: Spectrum Opportunity for SU in Presence of Two Primary System

3.2 Performance Evaluation

The objective of this section is to derive the mathematical formulation for the performance metrics, average waiting time ($E[T^S_{OFF}]$) and average service time($E[T^S_{ON}]$). We first consider the case that SU can sense a single channel that is used by both PUs and access it in an interweave fashion. Then, we investigate the multiple channel access case. Note that a successful communication for the SU only happens when both primary channels are in the OFF state. The spectrum opportunities depend on the activities of both PUs. In particular, in order to calculate the performance metrics for the SU, following the approach of single cell primary system, we first need to derive the statistics for the combined activities; i.e., the time duration that both PUs are in the OFF state simultaneously, and the time duration that at least one PU is in the ON state, denoted by μ_c and λ_c, respectively. For PU i on the single channel of interest, we denote the duration of the ON and OFF states by λ_i and μ_i, respectively. After the derivation of average waiting time and average service time in both single and multiple channel access, we then present the analysis of average throughput for the SU. Later, some selected numerical examples are discussed.

3.2.1 Single Channel Access Case

We first construct a four-state continuouse-time Markov chain with state space defined as: state A: both PUs in ON state; state B: PU 1 in ON state and PU 2 in OFF state; state C: PU 1 in OFF state and PU 2 in ON state; and state D: both PUs in OFF state. Based on the Markov modeling of individual PU activities, we can obtain the rate matrix of the new Markov chain, denoted by Q, as

$$Q = \begin{bmatrix} -\frac{1}{\lambda_1} - \frac{1}{\lambda_2} & \frac{1}{\lambda_2} & \frac{1}{\lambda_1} & 0 \\ \frac{1}{\mu_2} & -\frac{1}{\mu_2} - \frac{1}{\lambda_1} & 0 & \frac{1}{\lambda_1} \\ \frac{1}{\mu_1} & 0 & -\frac{1}{\mu_1} - \frac{1}{\lambda_2} & \frac{1}{\lambda_2} \\ 0 & \frac{1}{\mu_1} & \frac{1}{\mu_2} & -\frac{1}{\mu_1} - \frac{1}{\mu_2} \end{bmatrix} \tag{3.3}$$

The state transition diagram for the four-state Markov chain is shown in Fig 3.3.

Note that the steady state probabilities satisfy the following system of linear equations [38]

$$\sum_i \pi_i Q_{ij} = 0, for\ all\ j \tag{3.4}$$

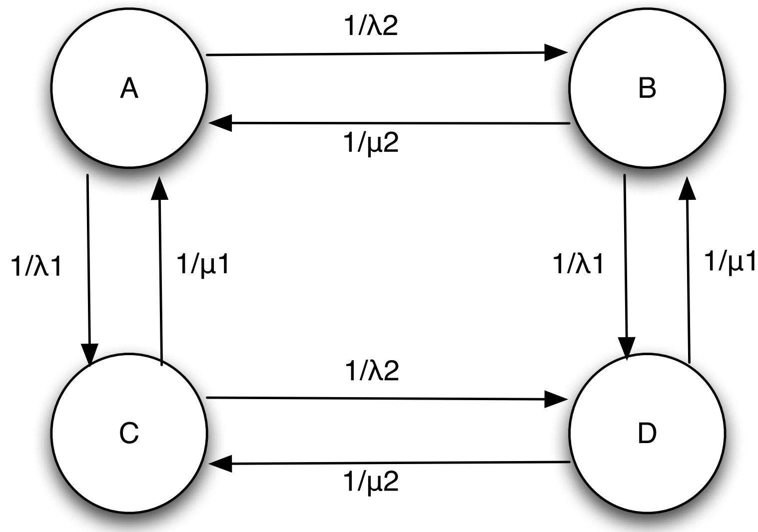

Figure 3.3: Rate Diagram

$$\sum_i \pi_i = 1 \tag{3.5}$$

we can calculate the steady state probability vector, denoted by $\vec{\pi} = [\pi_A, \pi_B, \pi_C, \pi_D]$ as

$$\pi_A = \frac{\lambda_1\lambda_2}{\lambda_1\lambda_2 + \lambda_1\mu_2 + \mu_1\lambda_2 + \mu_1\mu_2} \tag{3.6}$$

$$\pi_B = \frac{\lambda_1\mu_2}{\lambda_1\lambda_2 + \lambda_1\mu_2 + \mu_1\lambda_2 + \mu_1\mu_2} \tag{3.7}$$

$$\pi_C = \frac{\lambda_2\mu_1}{\lambda_1\lambda_2 + \lambda_1\mu_2 + \mu_1\lambda_2 + \mu_1\mu_2} \tag{3.8}$$

$$\pi_D = \frac{\mu_1\mu_2}{\lambda_1\lambda_2 + \lambda_1\mu_2 + \mu_1\lambda_2 + \mu_1\mu_2} \tag{3.9}$$

Based on the mode of operation, λ_c can be calculated as the average length of time that the Markov chain stays continuously in states A, B, and C and μ_c the average length of time in state D. It immediately follows that:

$$\mu_c = \frac{1}{\frac{1}{\mu_1} + \frac{1}{\mu_2}} = \frac{\mu_1\mu_2}{\mu_1 + \mu_2}. \tag{3.10}$$

To determine λ_c, we first aggregate states A, B, and C together to form a "super state". The state aggregation is also indicated in the rate matrix as:

$$\left[\begin{array}{ccc|c} -\frac{1}{\lambda_1}-\frac{1}{\lambda_2} & \frac{1}{\lambda_2} & \frac{1}{\lambda_1} & 0 \\ \frac{1}{\mu_2} & -\frac{1}{\mu_2}-\frac{1}{\lambda_1} & 0 & \frac{1}{\lambda_1} \\ \frac{1}{\mu_1} & 0 & -\frac{1}{\mu_1}-\frac{1}{\lambda_2} & \frac{1}{\lambda_2} \\ \hline 0 & \frac{1}{\mu_1} & \frac{1}{\mu_2} & -\frac{1}{\mu_1}-\frac{1}{\mu_2} \end{array}\right] \tag{3.11}$$

Therefore, the rate, at which the four-state Markov chain leaves the super state, can then be calculated as

$$\frac{1}{\lambda_c} = \frac{\frac{1}{\lambda_1}\pi_B + \frac{1}{\lambda_2}\pi_C + 0 * \pi_A}{\pi_A + \pi_B + \pi_C}. \tag{3.12}$$

After proper substitution and manipulation, we derive the analytical expression of λ_c as

$$\lambda_c = \frac{\lambda_1\lambda_2 + \lambda_1\mu_2 + \lambda_2\mu_1}{\mu_1 + \mu_2}. \tag{3.13}$$

Alternatively, we determine λ_c by generalizing the balance equation of continuous time Markov chain [29, eq. 6.18]

$$v_j\pi_j = \sum_{k \neq j} v_{kj}\pi_k, \text{ all states } j, \tag{3.14}$$

where v_{kj} denotes the transition rate from state k to state j and v_j the rate leaving state j. Specifically, applying to the super state, we have

$$\frac{1}{\lambda_c}(\pi_A + \pi_B + \pi_C) = \frac{1}{\mu_c}\pi_D, \tag{3.15}$$

which will lead to the same expression for λ_c as in equation (3.13) With the above state aggregation process, we essentially arrive at a new two-state Markov chain with state U, corresponding the super state, and state D: both PUs in the OFF state and spectrum opportunities exist. The rate matrix of the new chain is

$$Q_c = \begin{bmatrix} -1/\lambda_c & 1/\lambda_c \\ 1/\mu_c & -1/\mu_c \end{bmatrix}, \tag{3.16}$$

where λ_c and μ_c were given above.

With the above parameters results, we are now ready to develop closed form expressions for performance metrics: average waiting time ($E[T^S_{OFF}]$) and average service time ($E[T^S_{ON}]$) for the SU. We assume that the SU will periodically sense the

PU's activity in every T_s time. The average waiting time T_{OFF}^S will be an integer multiple of T_s, i.e. $T_{OFF}^S = NT_s$, where N represent the number of sensing periods before both PUs are ON. Since T_s is not negligible, worst and best case scenarios exist for our sensing schemes. The best case occurs as soon as the PU is idle, the SU starts transmitting so that the SU will not waste any time to transmit. The worst case occurs at the time the PU is OFF, the SU just finishes sensing, and will waste one T_s period to transmit. We use τ to represent this wasted time, and it can also be expressed as $\tau = NT_s - T_{ON}^C$, and clearly τ falls in the range from 0 to T_s. From Fig 3.2, it is reasonable to assume that every time period of an SU operation cycle should be equal to the time period of the PU operation cycle , which is

$$T_{ON}^C + T_{OFF}^C = T_{OFF}^S + T_{ON}^S \tag{3.17}$$

where T_{ON}^C denotes the ON duration of the combined PU activities, and T_{OFF}^C denotes the OFF duration of the combined PU activities. After substituting τ and N into equation (3.17), T_{ON}^S can now be expressed as

$$\begin{aligned} T_{ON}^S &= T_{ON}^C + T_{OFF}^C - T_{OFF}^S \\ &= T_{ON}^C + T_{OFF}^C - NT_S \\ &= T_{OFF}^C - \tau \end{aligned} \tag{3.18}$$

To get the average of T_{ON}^S, we only need to know the average of τ since T_{OFF}^C is exponentially distributed with parameter μ_c. Knowing that T_{ON}^C is exponentially distributed and $T_{ON}^C|N$ is between $(N-1)T_s$ and NT_s, we have PDF of $T_{ON}^C|N$ as a truncated exponential random variable,

$$f_{T_{ON}^C|N}(t) = \frac{1}{\lambda_c} \frac{e^{-\frac{t}{\lambda_c}}}{e^{-\frac{(N-1)T_s}{\lambda_c}} - e^{-\frac{NT_s}{\lambda_c}}},$$

$$(N-1)T_s \le t \le NT_s. \tag{3.19}$$

The PDF of τ conditional on N can be determined as

$$\begin{aligned} f_{\tau|N}(t) &= f_{T_{ON}^{C}|N}(NT_s - t) \\ &= \frac{1}{\lambda_c}\frac{e^{\frac{t-T_s}{\lambda_c}}}{1-e^{-\frac{T_s}{\lambda_c}}} \\ &, \quad 0 \le t \le T_s. \end{aligned} \tag{3.20}$$

From equation (3.27), we see that τ and N are independent; therefore, $f_{\tau}(t) = f_{\tau|N}(t)$. This can be also explained by the fact that considered primary user activity which is exponential is memoryless. As a result, we get the average service time for secondary user as

$$\begin{aligned} E[T_{ON}^{S}] &= E[T_{OFF}^{C}] - E[\tau] \\ &= \mu_c + \lambda_c - \frac{T_s}{1-e^{-\frac{T_s}{\lambda_c}}} \end{aligned} \tag{3.21}$$

The average waiting time $E[T_{OFF}^{S}]$ can be achieved by finding the statistic of N, since we have $T_{OFF}^{S} = NT_s$. If we denote $P_n = Prob\{N = n\}$, then the PMF of N can be expressed as a function of CDF of T_{ON}^{C} as

$$\begin{aligned} P_n &= \Pr\{N = n\} \\ &= \Pr\{(n-1)T_s \le T_{ON}^{C} \le nT_s\} \\ &= F_{T_{ON}^{C}}(nT_s) - F_{T_{ON}^{C}}((n-1)T_s) \end{aligned} \tag{3.22}$$

Because T_{ON}^{C} is exponentially distributed, we have the average waiting time as

$$\begin{aligned} E[T_{OFF}^{S}] &= E[N]T_s \\ &= T_s\sum_{n=1}^{+\infty} nP_n \\ &= T_s\sum_{n=1}^{+\infty} ne^{-(n-1)\frac{T_s}{\lambda_c}}(1-e^{-\frac{T_s}{\lambda_c}}) \\ &= \frac{T_s}{1-e^{-\frac{T_s}{\lambda_c}}} \end{aligned} \tag{3.23}$$

3.2.2 Multi-Channel Access Case

In this section, we consider the case where the SU can sense all L available channels at the same time and can access any one of them should it become idle. In particular, the SU will sense all available L channels every T_s time period and start transmission on the first available idle channel. During its transmission on the idle channel, the SU will continue sensing all the L channels. When PUs appears on the currently used channel, the SU will switch to one of the other idle channels, if available. Otherwise, the SU will return to the periodic sensing state.

Following the same approach as in previous section, we can model the combined PU activities on the jth channel using a two-state Markov chain with rate matrix given by

$$Q_{cj} = \begin{bmatrix} -1/\lambda_{cj} & 1/\lambda_{cj} \\ 1/\mu_{cj} & -1/\mu_{cj} \end{bmatrix}, \tag{3.24}$$

where λ_{cj} and μ_{cj} are the average duration of busy and idle state, given by

$$\lambda_{cj} = \frac{\lambda_{1j}\lambda_{2j} + \lambda_{1j}\mu_{2j} + \lambda_{2j}\mu_{1j}}{\mu_{1j} + \mu_{2j}}, \tag{3.25}$$

and

$$\mu_{cj} = \frac{\mu_{1j}\mu_{2j}}{\mu_{1j} + \mu_{2j}}, \tag{3.26}$$

respectively. λ_{ij} and μ_{ij} are the parameters for PU i's behaviour on channel j, as defined in section 3.1. We can then build a 2^L state Markov chain with states denoted by a sequence of L 1's or 0's. In particular, the first state is denoted by $(11\cdots 11)$, corresponding to the case that all channels are idle. The second state, denoted by $(11\cdots 10)$, represents the case that all channels except for channel L are idle. The last state $(00\cdots 00)$ denotes the case that all L channels are busy and, as such, spectrum opportunity is not available. The rate matrix of the Markov chain is given by

$$Q_L = \left[\begin{array}{cccccc|c} -q_1 & \frac{1}{\mu_{cL}} & \frac{1}{\mu_{c(L-1)}} & \cdots & 0 & & 0 \\ \frac{1}{\lambda_{cL}} & -q_2 & 0 & \cdots & 0 & & 0 \\ \frac{1}{\lambda_{c(L-1)}} & 0 & -q_3 & \cdots & 0 & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & -q_{L-1} & & \frac{1}{\mu_{cL}} \\ \hline 0 & 0 & 0 & \cdots & \frac{1}{\lambda_{cL}} & & -q_L \end{array}\right] \tag{3.27}$$

where the diagonal entry q_i is given in terms of off-diagonal entries q_{ij} as

$$q_i = \sum_{j,j\neq i}^{L} q_{ij}, i = 1, 2, \cdots L. \tag{3.28}$$

For example, we have

$$q_L = \frac{1}{\lambda_{c1}} + \frac{1}{\lambda_{c2}} + \cdots + \frac{1}{\lambda_{cL}}. \tag{3.29}$$

Based on the mode of operation of multiple channel access, the spectrum opportunities exist when at least one channel is idle. We can apply the state aggregation again and use a two-state Markov chain to characterize the spectrum opportunities. In this case, state T is the aggregation of states $(11\cdots 11)$, $(11\cdots 10)$, $(11\cdots 01)$, $\cdots$, and $(00\cdots 01)$, where spectrum opportunity exists, whereas state U corresponds state $(00\cdots 00)$. Following a similar approach as in previous section, we can calculate the parameter for the new Markov chain as

$$\lambda_U = \frac{1}{q_m} = \frac{1}{\frac{1}{\lambda_{c1}} + \frac{1}{\lambda_{c2}} + \cdots + \frac{1}{\lambda_{cm}}}, \tag{3.30}$$

and

$$\mu_T = \frac{1 - \pi_U}{q_m \pi_U} \tag{3.31}$$

where π_U is the stationary probability of state $(00\cdots 00)$ that can be calculated from the rate matrix Q_L. As an example, for the special case of two available channels, we can obtain λ_U and μ_T as

$$\lambda_U^{L=2} = \frac{\lambda_{c1}\lambda_{c2}}{\lambda_{c1} + \lambda_{c2}}, \tag{3.32}$$

$$\mu_T^{L=2} = \frac{\lambda_{c1}\mu_{c2} + \lambda_{c2}\mu_{c1} + \mu_{c1}\mu_{c2}}{\lambda_{c1} + \lambda_{c2}}. \tag{3.33}$$

After substituting λ_U and μ_T into proper equation in previous section, we calculate the performance metrics of average service time and average wait time for multiple channel access case.

3.3 Average Throughput

To enrich our performance analysis in this work, we also consider average throughput based on previous metrics, average waiting and service time. The average throughput can be expressed as

$$\Gamma = \frac{E[T_{ON}^S]}{E[T_{ON}^S] + E[T_{OFF}^S]} \log_2(1+\gamma), \tag{3.34}$$

where γ is the signal-to-noise ratio of the SU channel.

3.4 Numerical Results

In this section, we present selected numerical examples investigating the performance of cognitive transmission systems in a dual cell environment. Specifically, we examine the effect of the sensing period T_s and the number of available channels L on the performance metrics of interest. Without loss of generality, we assume that the activities of the PUs in cell 1, denoted by PU1, have parameter $\lambda_1 = 6\ ms$, $\mu_1 = 3\ ms$ and those in cell 2, denoted by PU2, have common parameters $\lambda_2 = 4\ ms$, and $\mu_2 = 7\ ms$. It follows that $\lambda_c = 7.80\ ms$ and $\mu_c = 2.10\ ms$.

Fig.3.4 and 3.5 show the performance of the single channel access case. In particular, we plot the average service time and the average waiting time as a function of the sensing period for the cases where the SU is operating under a single PU and under both PUs. We can clearly see that the service time of the SU is much smaller when operating under both PUs and that the wait time becomes longer on average. We also observe from Fig 3.4 that the average service time increases as the sensing period decreases. This is due to the SU being able to continuously monitor the PU's activity, so that it will detect the PU's transition state more accurately. Therefore, the average service time is improved. In the case of PU2 for example, we see that when the sensing period is 0.1 ms, the average service time of SU approaches to the average OFF time of PU2. Thus, the SU starts transmission exactly when PU2 is idle. Fig.3.5 shows that the waiting time increases as the sensing period increases. This make sense as the SU is more actively exploring the spectrum opportunities with smaller T_s.

For multiple channel results, we continue to plot system performance metrics versus sensing period. We compared four sets of performance metrics based on different number of channels, which are L=1, 2, 3, and 4. In Figs. 3.6 and 3.7, we let all channels have the same parameters. That is $\lambda_c = 7.80\ ms$ and $\mu_c = 2.10\ ms$.

Fig. 3.6 shows the effect of the number of available channels L on the average service time of the SU. Similar to the single channel case, the average service time decreases as the sensing period Ts increases. More importantly, when the number of channels increases, the average service time is greatly improved with growing gain. This is because the SU can monitor all available channels and switch to an idle channel when the currently used channel becomes busy. Fig. 3.7 shows the effects of number of channels on the average waiting time. As the number of channels increases, the average waiting time decreases, but with diminishing gain. Increasing the number of available channels can considerably improve the performance of the SU.

3.5 Conclusion

In this chapter, we presented the main contributions of this work. We proposed a system model which is more related to practical applications, and evaluated its performance. In particular, we first derived the system parameters from two different methods for one channel access. Then we extended it to multiple channel access. The numerical results successfully identity and validate our proposed system model, the model parameters, and performance metrics. Moreover, we analyzed the average throughput for the secondary system to enrich our system evaluation. Some possible drawback exists. One drawback is that the PU's traffic is assumed to follow an exponential distribution, which is not always realistic. Nevertheless, it makes our mathematical formulation more tractable, and can give us approximate results on the performance evaluation. The second drawback is that in our spectrum opportunity diagram (Fig 3.3), where a negligible probability exists that τ is larger than the OFF duration of the combined PU's. In this case, the SU has to wait for the next ON-OFF cycle. Another drawback is that the SU processes have two different sensing algorithms. When the PU is present, the SU periodically sense the channel; and when the PU is absent, the SU is continuously sensing the channel. The key to the continuous sensing is to avoid interference to the PU. In reality, continuous sensing is difficult to achieve. Most studies have suggested a slotted structure for the SU [7, 8]. In the following chapter, we extend our study and solve these drawbacks.

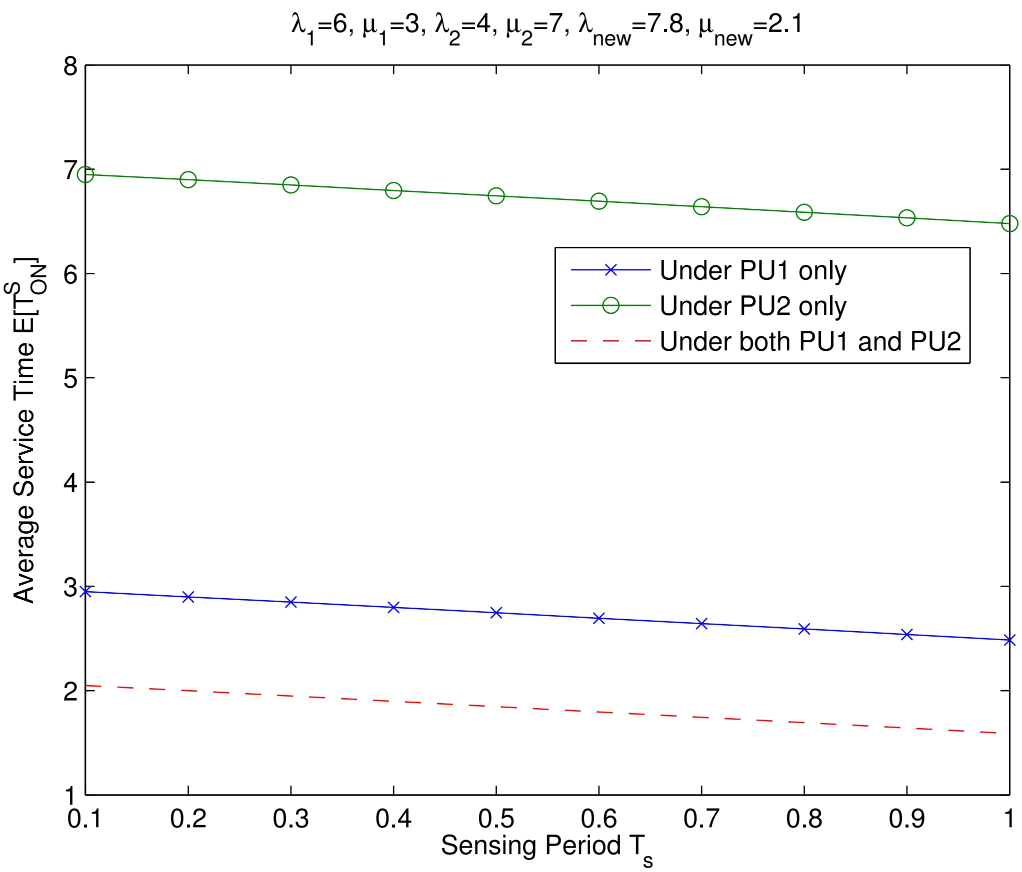

Figure 3.4: Average Service Time vs. T_s

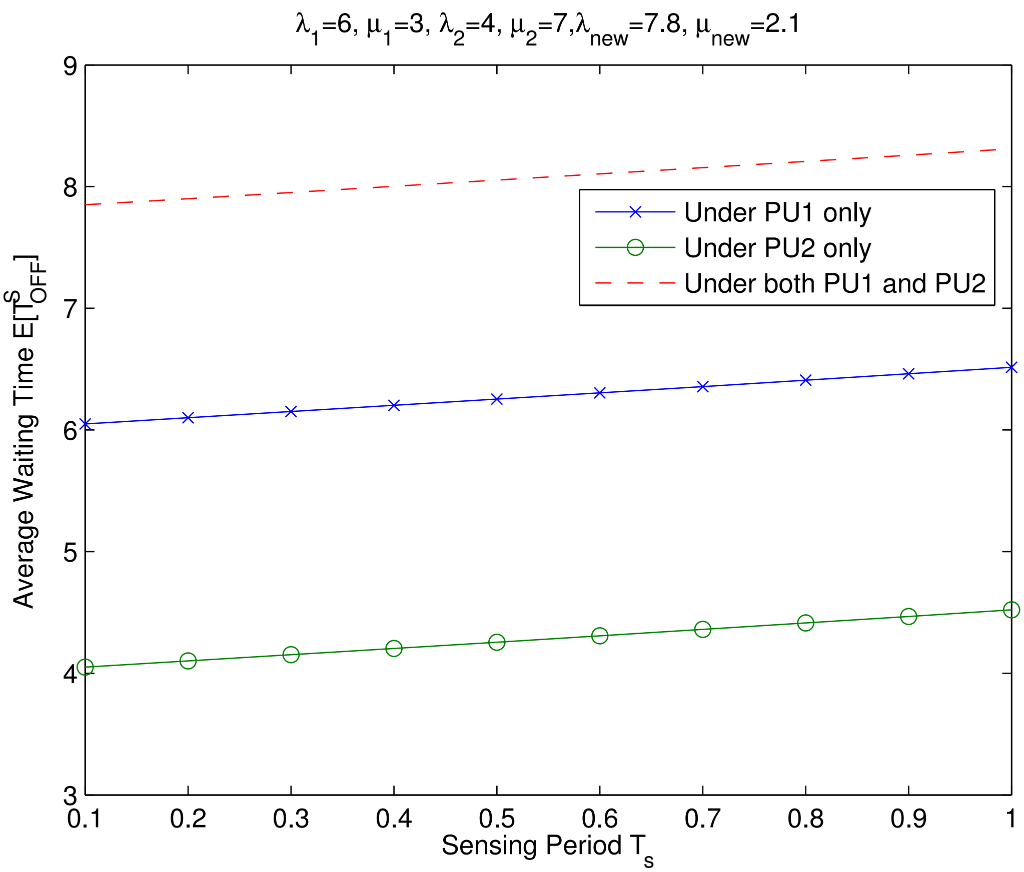

Figure 3.5: Average Waiting Time vs. T_s

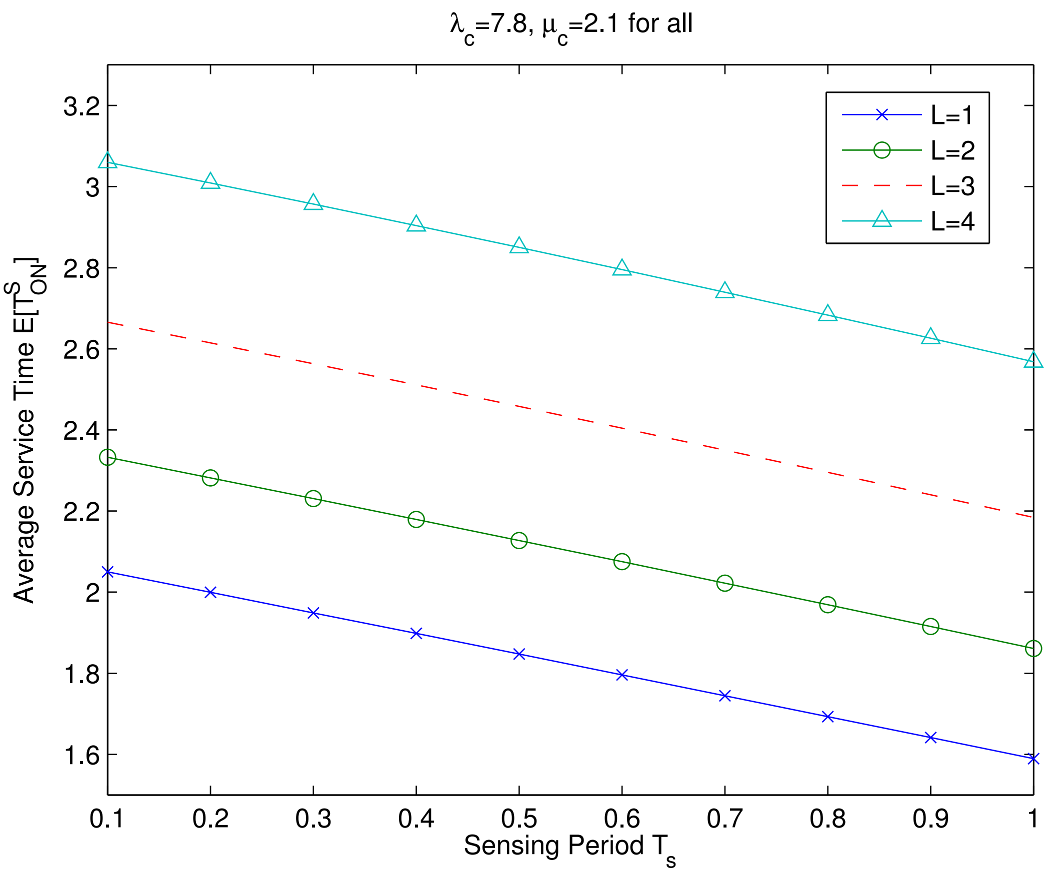

Figure 3.6: Average Service Time vs. T_s for Multi-channel

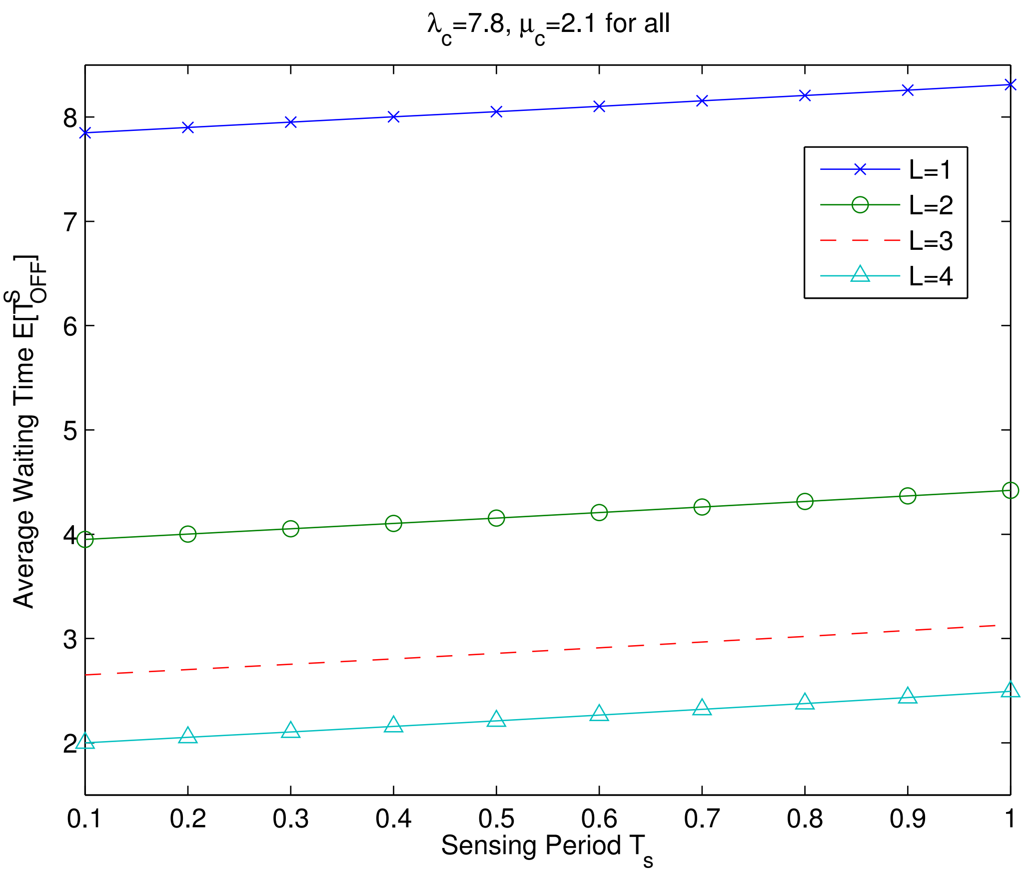

Figure 3.7: Average Waiting Time vs. T_s

Chapter 4

Extension Study for a Dual-cell Primary System

In the dual-cell network analysis in Chapter 3, we have discussed some possible drawbacks, which leads to some further analysis and extensions, and eventually to some solutions and improvement to the system model. In this chapter, we focus on two extensions. The first extension one considers the case where τ is larger than the OFF duration of the combined PU's. The other extension adopts a slotted structure for a secondary system consistently instead of using the two separate sensing method. For the latter extension, a collision happens in a scenario where the SU senses the channel idle and starts transmission, and the PU returns to the channel before the SU finishes its transmission. Therefore, to capture the impact of collision on PU thus to protect PU, we introduce new performance metrics, collision frequency and collision ratio, from both the PU and SU points of view, respectively.

4.1 Analysis for Extremely Small PU OFF Time

Although τ, described as a missing opportunity duration, is a very small value (falling in the range between 0 to T_s), we can not neglect the possibility that it may be larger than the OFF duration of combined PU's. In this case illustrated in Fig. 4.1, the SU completely misses a transmission opportunity and gets into the next ON-OFF cycle; therefore, the average waiting time is increased, and average service time is decreased.

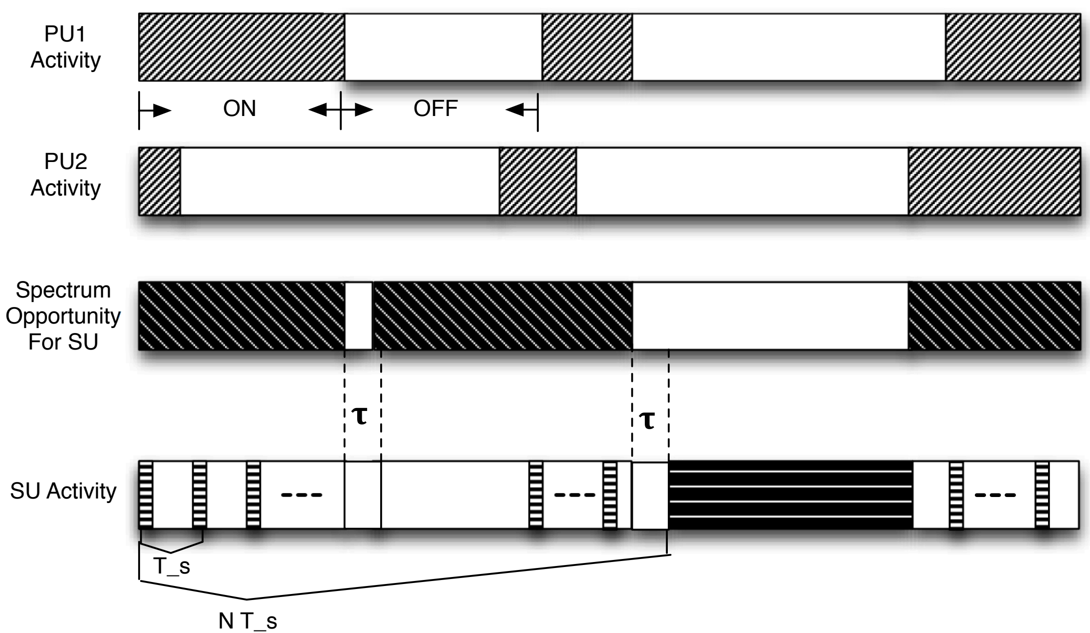

Figure 4.1: Spectrum Opportunity Diagram under Different τ

4.1.1 Performance Analysis

Considering the case that T_{OFF}^C is less than τ, our mathematical derivations of average waiting time and service time are altered accordingly. The system model and its operation remains the same except that now we need to add the case where $T_{OFF}^C < \tau$ into our formulation. Start with the equality between the ON-OFF cycle of the combined PUs and the OFF-ON cycle of the SU, i.e.

$$T_{ON}^C + T_{OFF}^C = T_{OFF}^S + T_{ON}^S, \tag{4.1}$$

Then T_{ON}^S can be expressed as:

$$\begin{aligned} T_{ON}^S &= T_{ON}^C + T_{OFF}^C - T_{OFF}^S \\ &= T_{ON}^C + T_{OFF}^C - NT_S \\ &= T_{OFF}^C - \tau. \end{aligned} \tag{4.2}$$

To calculate the average service time $E[T_{ON}^S]$, we need to obtain the distribution of T_{ON}^S, which is the difference of two random variables. Note that T_{OFF}^C may be less than τ with non-zero probability. In this case, we set T_{ON}^S in equation (4.2) to zero and start the new ON-OFF cycle after T_{OFF}^C. It follows that the service time in each ON-OFF cycle T_{ON}^S has a mixed distribution, with PDF given by

$$f_{T_{ON}^S}(t) = \Pr[T_{OFF}^C < \tau]\delta(t) + f_{T_{ON}^S, T_{OFF}^C > \tau}(t), \tag{4.3}$$

where $f_{T_{ON}^S, T_{OFF}^C > \tau}(t)$ denotes the PDF of T_{ON}^S for the case that T_{OFF}^C is greater than τ. The probability $\Pr[T_{OFF}^C < \tau]$ can be calculated using the PDFs of T_{OFF}^C and τ as

$$\Pr[T_{OFF}^C < \tau] = \int_0^{T_s} \int_0^y f_{T_{OFF}^C}(x) f_\tau(y) dx dy. \tag{4.4}$$

Similarly, the PDF $f_{T_{ON}^S, T_{OFF}^C > \tau}(t)$ can be calculated as

$$f_{T_{ON}^S, T_{OFF}^C > \tau}(t) = \int_0^{T_s} f_{T_{OFF}^C}(t + y) f_\tau(y) dy. \tag{4.5}$$

Based on the Markov chain formulation in previous subsection, both T_{ON}^C and T_{OFF}^C are exponentially distributed with parameter λ_c and μ_c, respectively. Conditioning on the number of sensing period N, the PDF of T_{ON}^{C} can be obtained as

$$f_{T_{ON}^{C}|N}(t) = \frac{1}{\lambda_c} \frac{e^{-\frac{t}{\lambda_c}}}{e^{-\frac{(N-1)T_s}{\lambda_c}} - e^{-\frac{NT_s}{\lambda_c}}},$$

$$(N-1)T_s \leq t \leq NT_s. \tag{4.6}$$

It follows that the PDF of τ conditioned on N can be determined as

$$\begin{aligned} f_{\tau|N}(t) &= f_{T_{ON}^{C}|N}(NT_s - t) \\ &= \frac{1}{\lambda_c} \frac{e^{\frac{t}{\lambda_c}}}{e^{\frac{T_s}{\lambda_c}} - 1}, 0 \leq t \leq T_s. \end{aligned} \tag{4.7}$$

From equation (4.7), we can see that τ is actually independent of N. Therefore, $f_{\tau}(t) = f_{\tau|N}(t)$. This can be also explained with the memoryless property of exponential random variables.

Finally, the average service time can be calculated using the PDF of T_{ON}^{S} as

$$E[T_{ON}^{S}] = \int_0^{\infty} t f_{T_{ON}^{S}}(t) dt. \tag{4.8}$$

Finally, after proper substitution and carrying out integration, we arrive at the following closed form expression of the average service time.

$$\begin{aligned} E[T_{ON}^{S}] &= \int_0^{\infty} t \int_0^{T_s} \frac{1}{\mu_c} e^{-\frac{t+y}{\mu_c}} \cdot \frac{1}{\lambda_c} \frac{e^{\frac{y-T_s}{\lambda_c}}}{1 - e^{-\frac{T_s}{\lambda_c}}} dy dt \\ &= \frac{\mu_c^2}{\mu_c - \lambda_c} \frac{e^{-\frac{T_s}{\mu_c}} - e^{-\frac{T_s}{\lambda_c}}}{1 - e^{-\frac{T_s}{\lambda_c}}}. \end{aligned} \tag{4.9}$$

We now consider the average waiting time $E[T_{OFF}^{S}]$. We first note that if T_{OFF}^{C} is greater than τ in a particular ON-OFF cycle of combined PU activities, SU needs to wait for $T_{OFF}^{S} = NT_s$, where N is the number of sensing periods, before transmission. The probability mass function (PMF) of N can be calculated, using the CDF of T_{ON}^{C}, as

$$\begin{aligned} \Pr[N = n] &= \Pr[(n-1)T_s \leq T_{ON}^{c} \leq nT_s] \\ &= F_{T_{ON}^{C}}(nT_s) - F_{T_{ON}^{C}}((n-1)T_s). \end{aligned} \tag{4.10}$$

When T_{OFF}^C is less than τ in a particular ON-OFF cycle, SU needs to continue to wait for the next ON-OFF cycle. The average number of sensing period that SU needs to wait in each ON-OFF cycle is always equal to

$$E[N] = \sum_{n=1}^{+\infty} n \Pr[N = n]. \tag{4.11}$$

Based on these observations, we can calculate the average waiting time of SU before accessing the channel, by averaging the number ON-OFF cycle and sensing period that it needs to wait, as

$$\begin{aligned} E[T_{OFF}^S] &= \sum_{i=1}^{+\infty} \Pr[T_{OFF}^C < \tau]^{i-1}(1 - \Pr[T_{OFF}^C < \tau]) i T_s E[N] \\ &= \frac{1}{1 - \Pr[T_{OFF}^C < \tau]} \frac{T_s}{1 - e^{-\frac{T_s}{\lambda_c}}}. \end{aligned} \tag{4.12}$$

Since $\Pr[T_{OFF}^C < \tau]$ can be calculated from equation (4.4) as

$$\Pr[T_{OFF}^C < \tau] = 1 - \frac{\mu_c}{\mu_c - \lambda_c} \frac{e^{-\frac{T_s}{\mu_c}} - e^{-\frac{T_s}{\lambda_c}}}{1 - e^{-\frac{T_s}{\lambda_c}}}, \tag{4.13}$$

we can finally obtain the analytical expression of the average waiting time as

$$E[T_{OFF}^S] = \frac{T_s(\mu_c - \lambda_c)}{\mu_c(e^{-\frac{T_s}{\mu_c}} - e^{-\frac{T_s}{\lambda_c}})}. \tag{4.14}$$

4.1.2 Numerical Examples

In this section, we present the selected numerical examples to investigate the effect of extremely small OFF time of the combined PUs on the performance of cognitive transmission systems in the dual cell environment. Similar to the examples in Chapter 3, we again plot the average service time and average waiting time as a function of sensing period according to the proposed extension, and compare the result with the ones in Chapter 3. All the system parameters keep the same. We assume the activities of PUs in cell 1, denoted by PU1, have common parameter $\lambda_{1j} = 6$ ms, $\mu_{1j} = 3$ ms and those in cell 2, denoted by PU2, have common parameter $\lambda_{2j} = 4$ ms, and $\mu_{2j} = 7$ ms, where $j = 1, 2, \cdots, L$. It follows that $\lambda_{cj} = 7.80$ ms and $\mu_{cj} = 2.10$ ms.

Fig. 4.2 and Fig. 4.3 show the performance of the SU in the single channel access before extension and after extension. In general, the average service time under both situations are decreasing as the sensing period increases, and the average waiting time are increasing as the sensing period increases. These results are consistent with the results in Chapter 3. The differences are that in Fig 4.2, the average service time after extension is smaller than the service time before extension. This is due to that the consideration of case $T^{C}_{OFF} < \tau$ make the SU completely miss one transmission opportunity thus decrease the average service time. In Fig 4.3, we observe that the average waiting time after the extension is larger than the one before extension. This is because when $T^{C}_{OFF} < \tau$, SU has to continue to wait for the next ON-OFF, hence the average waiting time is increased.

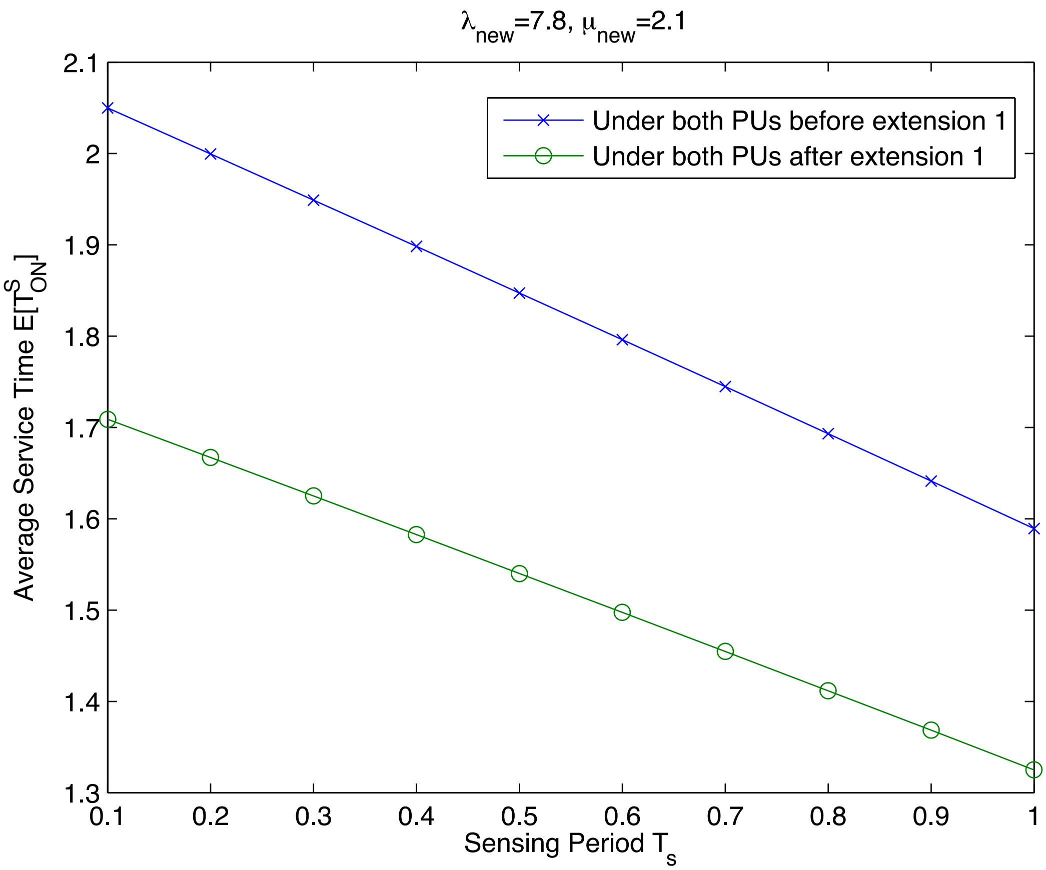

Figure 4.2: Average Service Time vs. T_s

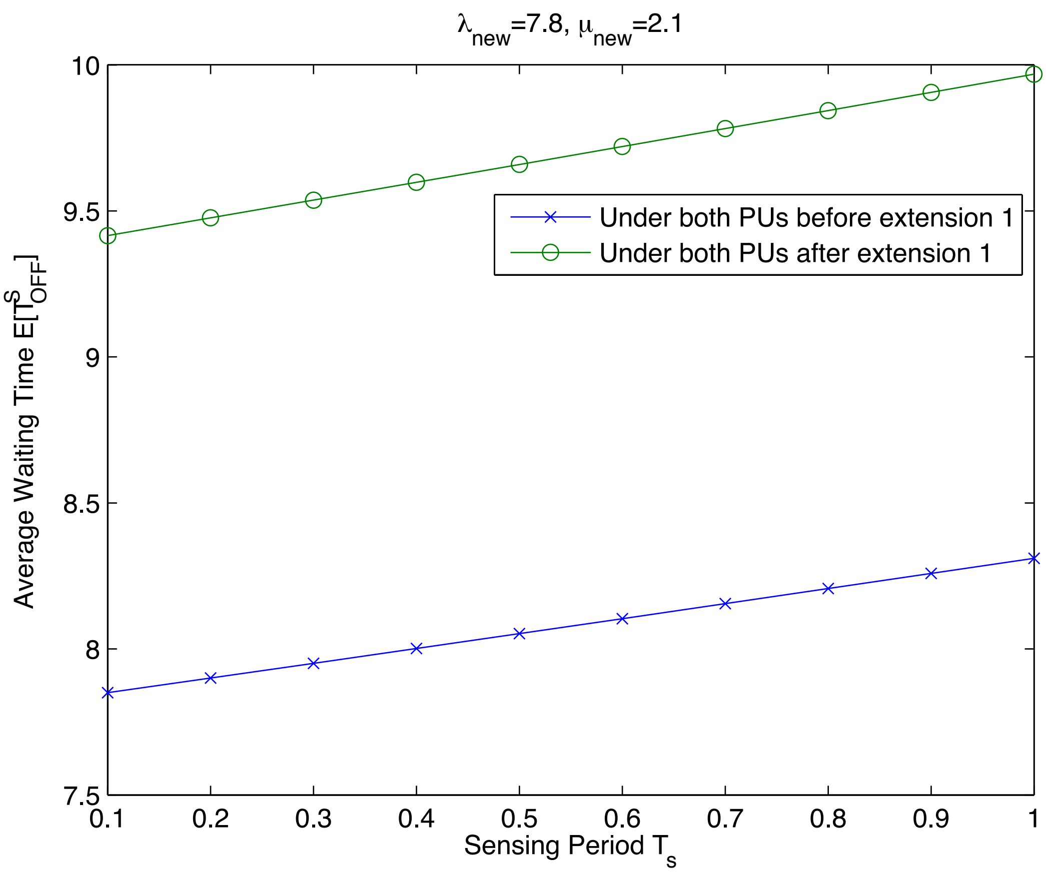

Figure 4.3: Average Waiting Time vs. T_s

4.2 Slotted SU Transmission

Due to hardware limitations and the energy cost of spectrum sensing [2], the SU may not be able to sense the channel continuously. If this is in a multiple-channel system, it is even more difficult for the SU to continuously and simultaneously sense the spectrum holes. In this section, we investigate the case where the SU adopts slotted structure. The slotted structure has been applied in many studies of CR [43, 9, 12]. Besides considering the cost-effectiveness, the slotted structure simplifies the physical layer design, and makes the theoretical analysis more mathematically tractable. Also, by finding the optimal slot duration, the performance of the SU can be greatly improved; hence, limited bandwidth resources can be used more efficiently. In the following, we investigate the system performance by calculating collision frequency and collision ratio.

4.2.1 Model of Slotted SU

In previous chapters, we assumed that when the PU is active, the SU senses the channel periodically. While the SU is transmitting, it continuously monitors the channel. In this operation mode, once the PU returns, the SU immediately evacuates the channel so that interference is avoided. When the SU completely adopts a slotted structure, the collision between the PU and SU can not be avoided. The collision happens in a situation where the SU senses the channel idle and starts transmission, and the PU returns to the channel before SU finishes its transmission,as illustrated in Fig 4.4. In this section, all the assumptions remain the same as in the dual cell scenario except for the SU. We recall the assumption that the PU follows continuous-time Markov Chain so that it can access the channel at any time. The SU adopts a slotted structure, and it senses the channel every period of T_s. Perfect sensing is applied to the SU, i.e., the false alarm and missing probability of the sensing is zero. In addition, the sensing occurs at the beginning of each time slot, and the sensing time is negligible.

4.2.2 Collision Frequency and Collision Ratio

We use collision frequency (C_P) as one performance metric, which is from the PU's point of view. C_P is defined as a fraction of collision probability over probability of

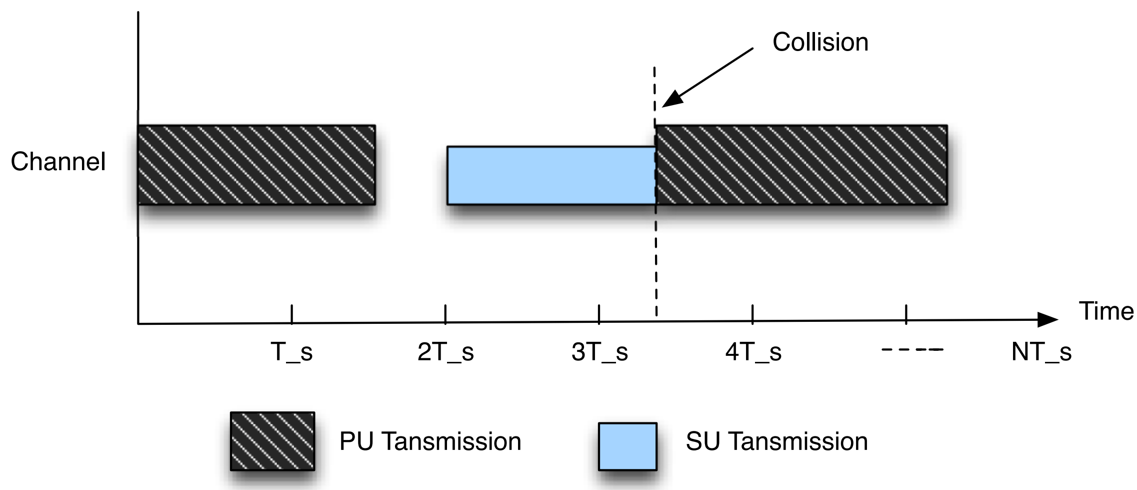

Figure 4.4: Illustration of a Collision

PU transmission in one slot duration, T_s. In mathematics, it can be expressed as

$$C_P = \frac{\Pr[collsion]}{\Pr[PU\ transmits]} \tag{4.15}$$

According to the stationarity, we start with the PU statistics. We continue using the model of CTMC with ON-OFF states for the PUs in a dual cell environment. Each state of the combined PUs follows an exponential distribution with parameter λ_c for ON and μ_c for OFF. The state transition matrix Q is then given by

$$Q = \begin{bmatrix} -\frac{1}{\mu_c} & \frac{1}{\mu_c} \\ \frac{1}{\lambda_c} & -\frac{1}{\lambda_c} \end{bmatrix} \tag{4.16}$$

The stationary probabilities of the two states can be solved as

$$\pi_{ON} = \frac{\mu_c}{\lambda_c + \mu_c} \tag{4.17}$$

$$\pi_{OFF} = \frac{\lambda_c}{\lambda_c + \mu_c} \tag{4.18}$$

Since a collision only occurs when both the PU and SU transmit when channel is sensed to be idle, $\Pr[collision]$ is given as the product of $\Pr[PU\ transmits]$ and $\Pr[SU\ transmits]$ provide that channel is idle.

$$\Pr[collision] = (1 - e^{-\frac{T_s}{\mu_c}})\pi_{OFF} \tag{4.19}$$

The unconditional probability of PU transmitting can be expressed as

$$\Pr[PU\ transmits] = 1 - \pi_{OFF}e^{-\frac{T_s}{\mu_c}} \tag{4.20}$$

As a result, collision frequency is expressed as

$$C_P = \frac{(1 - e^{-\frac{T_s}{\mu_c}})\pi_{OFF}}{1 - \pi_{OFF}e^{-\frac{T_s}{\mu_c}}} \tag{4.21}$$

We use collision ratio (C_S) as the other performance metric, which is from the SU's point of view. C_S is defined as a fraction of number of collision slots over the number of SU transmissions slots. It can be expressed as

$$C_S = \frac{\#\ of\ collision\ slots}{\#\ of\ SU\ transmission\ slots}. \tag{4.22}$$

When the SU transmits its packets in a slotted structure, it will always produce a collision when the PU returns. Therefore, the number of collision slots is equal to one. We denote the number of SU transmission slots to be M, and M is a random variable. Collision ratio is then expressed as

$$C_S = \frac{1}{E[M]}. \tag{4.23}$$

If we denote $P_m = \Pr\{M = m\}$, the PMF of M can be expressed as a function of CDF of T_{ON}^S as

$$\begin{aligned} P_m &= \Pr\{M = m\} \\ &= \Pr\{(m)T_s \leq T_{ON}^S \leq (m+1)T_s\} \\ &= F_{T_{ON}^S}((m+1)T_s) - F_{T_{ON}^S}(mT_s) \end{aligned} \tag{4.24}$$

The average number of transmission slots is equal to

$$E[M] = \sum_{m=1}^{+\infty} m \Pr[M = m]. \tag{4.25}$$

We can finally obtain the collision ratio C_S as

$$C_S = \frac{1}{\sum_{m=1}^{+\infty} m \Pr[M = m]}. \tag{4.26}$$

4.2.3 Numerical Results

To evaluate the performance of collision, we selected the collision frequency as an example. Because in our future study of the underlay scenario, collision frequency can be used in an optimization problem as a constraint to minimize the interference to the PU.

Fig. 4.5 shows the performance of slotted structure SU transmission in multiple cell environment. We assume combined PUs have parameters $\lambda_c = 7.80\ ms$ and $\mu_c = 2.10$ ms. We plot the collision frequency as function of sensing period T_s. We can observe that the collision frequency increases as the sensing period T_s increases. This is due to that when SU transmits, the probability of returned PU increases as the sensing window becomes larger.

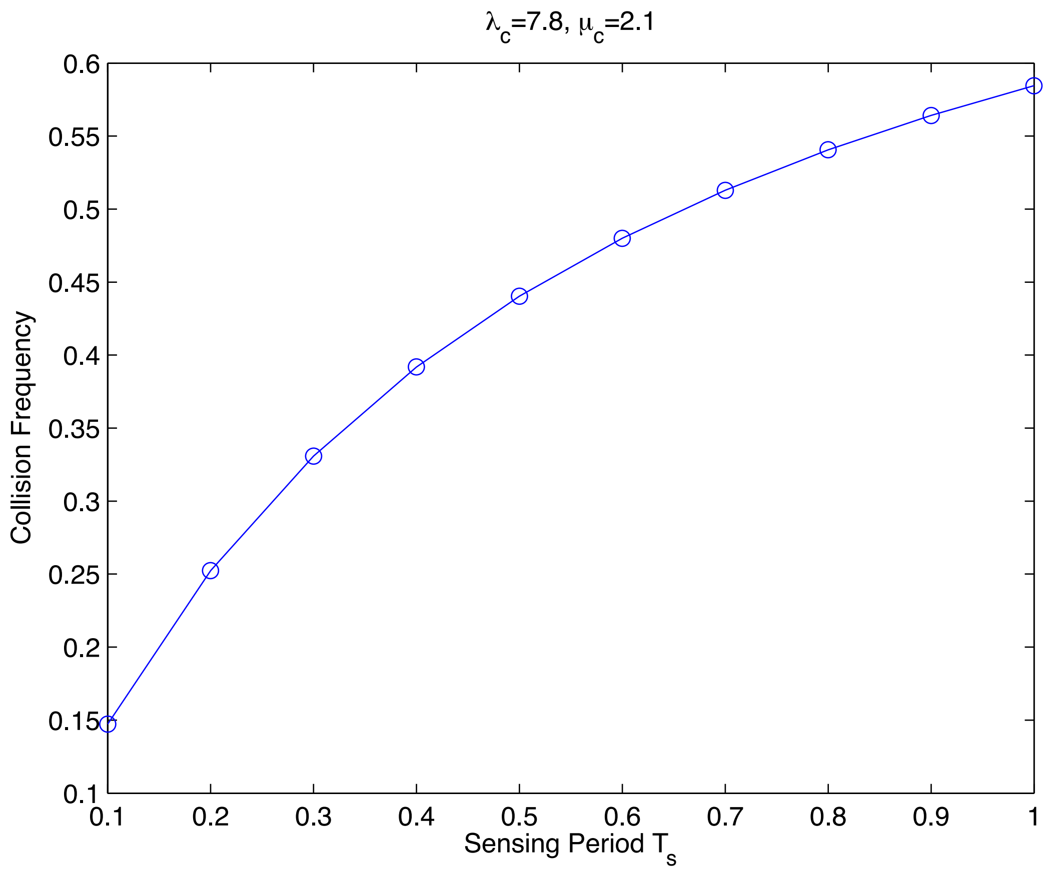

Figure 4.5: Collision Frequency vs. T_s

Chapter 5

Conclusions

In this thesis, we focus on the design and performance evaluation of cognitive transmission in a dual-cell environment. The proposed system has been built from some recent studies of a single cell environment, in conjunction with several critical observations. We started with a study and reviews of a single cell environment, and looked into the exact mathematic expressions for its performance metrics, such as average waiting time and average service time. The review provided us with sufficient background information for our proposed system model and for its performance evaluation. Also, we discovered several drawbacks and the realization of a multiple cell environment.

Based on the drawbacks and the realization, we proposed a new system model, which involves a dual-cell environment, and examined its performance. In particular, we introduced the super state method into our derivation of combined PU parameters. Then, based on these parameters, we found the exact expressions for the average waiting time and the service time for the proposed model, for both the single channel and the multiple channel cases. To enrich our performance evaluation, the average throughput was discussed. Through our selected numerical examples, our proposed system was successfully verified and validated. In addition, we made two improvement for our system model. First, we considered the case where τ is bigger than the OFF time of the combined PUs, which made our system model more practical. Then, by changing the SU to a slotted structure, we evaluated our proposed model by investigating the collision probability. Numerical examples verified our improved results by comparing the extension with the originally proposed model.

For future work, several scenarios would be worthwhile to explore. First, a multiple SU-access scheme will be investigated. Second, we will study the average delay in the secondary system with the help of queueing analysis. Third, since we considered the collision probability, we will extend our interweave study to an underly scenario. By studying the maximum throughput subject to the collision threshold for the PU, we can formulate our system model into an optimization problem. Moreover, since our proposed system model is built on Poisson traffic of the PU, the Poisson traffic and exponential distribution makes our analysis more tractable in mathematics. In the next step, we intend to investigate cases with more general distributions of the PU, and adopt the results into our system model to determine the outcome. These considerations present interesting challenges for future research of cognitive transmission in multiple cell environments.

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