

Scalar-tensor gravity with pseudoscalar couplings

by

Simon Lambert

BSc, University of Alberta, 2003

A Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of

Master of Science

in Physics

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Supervisory Committee

Dr. M. Pospelov, Supervisor (Physics and Astronomy)

Dr. W. Israel, Member (Physics and Astronomy)

Dr. C. Picciotto, Member (Physics and Astronomy)

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Abstract

I examine the observational effects of a light scalar field with a scalar coupling to masses and a pseudoscalar coupling to light and particle spins. The pseudoscalar coupling to light induces a coupling to atomic spins both by inducing a coupling to particle spins directly, and by interactions with electromagnetic fields in the atom. Experiments measuring the interaction of spins to the gradient of the field are the only known way to measure the strength of the interaction with spins. However, limits on the interaction with light derived from these experiments are barely competitive with the separate astronomical limits on the scalar interaction and the interaction with light. Assuming a low mass of the field, as would be the case if the field acts as quintessence, the polarization rotation of the CMB provides a much tighter limit on the product of the pseudoscalar and scalar interaction strengths.

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Chapter 1

Introduction

1.1 Quintessence

The simplest explanation for the acceleration of the expansion of the universe is that there is a cosmological constant. However, the laws of physics as we know them do not provide a good reason for the cosmological constant to take the value implied by current observations, except possibly anthropic arguments, in which one supposes that there are many universes with a wide range of values for the cosmological constant and we of course find ourselves in one with a low enough value to allow galaxies[1]. One alternative is that there could be a non-constant scalar field providing negative pressure. In scenarios in which the scalar field is slowly rolling down a potential and the negative pressure comes from its potential energy it is known as quintessence[2, 3]. An advantage of some quintessence models is that they can alleviate the fine tuning difficulties of the cosmological constant via tracker behaviour; that is, during the radiation dominated era the field tracks the radiation density and after matter-radiation equality switches to having a negative pressure[4]. There are many different models involving scalar fields causing accelerated expansion of the universe, but in this thesis I am concerned with a model that is at least quintessence-like, in the sense of having a standard kinetic energy, low mass, and weak (scalar) coupling to matter, so I will focus on ideas similar enough to seem relevant. The scalar field modelled need not

necessarily be the source of the accelerated expansion of the universe, but it is nice to try to explain as many observations as possible.

Although quintessence obviously interacts with matter via gravitational coupling to its stress-energy, it might or might not have additional couplings to matter. Theoretically, there do not seem to be strong reasons not to expect such couplings, but such couplings would cause the scalar field to mediate additional forces which have not been observed. It has been suggested by Carroll[5] that one would generally expect couplings with matter with strengths suppressed by a high mass scale at which the actual fundamental physics occurs, and that since this would suggest unrealistically large effects on, e.g., Eötvös-type experiments and evolution of the fine structure constant, long range interactions involving a quintessence field might be suppressed by an approximate symmetry of the scalar field ϕ under $\phi \rightarrow \phi + \text{const}$, which would not suppress a pseudoscalar interaction with light. The fact that the pseudoscalar interaction with light would not be so suppressed, and that it could plausibly be relatively large, provides one possible motivation for the model considered in this thesis.

It has also been suggested by Doran and Jäckel [6] that a coupling to fermions would cause difficulties due to quantum corrections changing the quintessence potential. On the assumption that, without some kind of fine tuning of the effective potential, loop corrections should be smaller than or comparable to the classic potential, they find a limit on the change to the fermion mass induced by the coupling. If correct, this would impose an extremely strict limit on the fermion couplings in the model (1.5) assumed in this thesis. However Wetterich[7] has challenged this result on the grounds that in the event that the scalar field is a pseudo Goldstone boson generated by an almost exact fundamental symmetry, the fluctuations generated by QCD will necessarily be cancelled by other fluctuations without the need for fine-tuning.

While in this thesis I will focus on a simple model and find what values of the parameters are compatible, it may be worth pointing out that more complex models may evade the limits, e.g. "chameleon" models in which a strong interaction with matter causes the field to have a high effective mass, and thus a short range, in regions of space with high matter density (such as on Earth)[8].

1.2 Brans-Dicke type scalar interaction

Often quintessence models are proposed with no couplings to matter except the necessary coupling to gravity due to the stress-energy of the field. However, quintessence models in which the scalar field interacts with masses have also been considered[9, 10].

This thesis assumes a coupling between a scalar field and matter fields proportional to the density of matter times the value of the field. A coupling of this sort might occur for many reasons; for example, a coupling to $F_{\mu\nu}F^{\mu\nu}$ would result in such a coupling between the field and baryons due to the nonzero average value of $E^2 - B^2$, and possibly a stronger coupling to dark matter[11]. Such a coupling would produce additional phenomena such as a changing fine structure constant and apparent violation of the equivalence principle since the coupling is not exactly proportional to the masses of objects.

The results here could be easily extended to scalar interactions between the scalar field and ordinary matter not proportional to mass, but the limits on equivalence principle violations are pretty small, so any low mass scalar field that exists must either interact very similarly to different objects with the same mass or interact very weakly. See Schlamminger et al.[12] for a recent test. Instead, I will focus on a scalar field that interacts the same with all mass sources, as would happen in Brans-Dicke theory. Also, even if the field interacted very similarly with different objects of the same mass, if the interaction were not weak it could be detected by observations of the solar system[13].

Brans-Dicke theory is usually thought of as a complete alternate theory of gravity, with the gravitational constant itself essentially being a scalar field, but you can also treat small perturbations of this scalar field as a separate field with a gravitation-like effect in ordinary general relativity.

Brans-Dicke theory follows from the minimization of the Jordan-Brans-Dicke action[14, 15]:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{pl}^2}{2} \phi R - \frac{\omega}{\phi} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} + \mathcal{L}_M \right). \quad (1.1)$$

I use a (+ - - -) metric signature here and throughout the thesis. Through a conformal transformation[14, 16, 17] this can be converted into the ordinary Hilbert action for general relativity plus a scalar field (see Appendix A).

$$S = \int d^4x \sqrt{-\bar{g}} \left(\frac{M_{pl}^2}{2} \bar{R} - \left(\omega + \frac{3}{2} \frac{M_{pl}^2}{2} \right) \bar{\partial}_\mu \alpha \bar{\partial}^\mu \alpha + e^{-2\alpha} \mathcal{L}_M \right) \quad (1.2)$$

where $\alpha = \ln \phi$. If the original ϕ was close to 1, which it is unless there is a very strong interaction (which would be observable in the absence of a "chameleon" effect), α will be small.

For $\omega < -\frac{3}{2} \frac{M_{pl}^2}{2}$ we can rescale α (and rename it ϕ) and use $\alpha \ll 1$ to get

$$S = \int d^4x \sqrt{-\bar{g}} \left(\frac{M_{pl}^2}{2} \bar{R} + \frac{1}{2} \bar{\partial}_\mu \phi \bar{\partial}^\mu \phi + \left(1 - \frac{2}{\sqrt{-2\omega - \frac{3}{2} M_{pl}^2}} \phi \right) \mathcal{L}_M \right) \quad (1.3)$$

Dropping the terms not involving ϕ and renaming $\frac{1}{2} \sqrt{-2\omega - \frac{3}{2} M_{pl}^2}$ as $-M_s$ (the negative sign on M_s being chosen for compatibility with what I have already written in the rest of the paper, but we can make it positive simply by replacing ϕ with $-\phi$), this becomes

$$S = M_{pl}^2 \int d^4x \sqrt{-\bar{g}} \left(\frac{1}{2} \bar{\partial}_\mu \phi \bar{\partial}^\mu \phi + \frac{\phi}{M_s} \mathcal{L}_M \right) \quad (1.4)$$

The dominant effect in the matter dominated era will be the interaction with the rest mass of ordinary particles. Note that the term in the Lagrangian for the rest mass has a negative sign.

1.3 Outline of thesis research

The idea of this thesis is to look at the effects of pseudoscalar couplings of a scalar field with gravitation-like interactions to light and to particle spins. A pseudoscalar coupling with light will induce pseudoscalar couplings with particles: in the case of a (charged, with spin) point particle a divergent pseudoscalar coupling is introduced due to the interaction of the scalar field with the particle's \mathbf{E} and \mathbf{B} fields; while for atoms finite pseudoscalar couplings are produced due to the interaction of the scalar field with a particle's \mathbf{B} field and the nuclear \mathbf{E} field.

This thesis explores the properties of a scalar field ϕ whose behaviour and interactions are described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) + \frac{\phi}{M_p}F_{\mu\nu}\tilde{F}^{\mu\nu} - \sum_j \phi m_j \bar{\psi}_j \left(\frac{1}{M_s} - \frac{i\gamma^5}{M_{p_j}} \right) \psi_j \quad (1.5)$$

where j sums over protons, neutrons, electrons etc. $F^{\mu\nu}$ is the tensor describing electromagnetic fields. M_s and M_p and M_{p_j} are constants with dimensions of mass. It will be discussed later how the pseudoscalar interaction with particle spins relates to the pseudoscalar coupling with light. The M_s term represents the gravitation-like interaction, and the M_p and M_{p_j} terms represent pseudoscalar interactions with light and fermions respectively.

The full Lagrangian describing the physics should be considered to be this Lagrangian plus the standard terms for GR, electromagnetism, etc.

It is interesting to note that the transformation $\phi \rightarrow \phi + \text{const}$ leaves this Lagrangian unchanged except for $V(\phi)$ and the $1/M_s$ term. I will show later that the terms with pseudoscalar couplings depend only on $\partial_\mu\phi$, not on ϕ itself. the effect

on the M_s term is to introduce an additional mass term that can be eliminated by rescaling m_j ; this requires M_s and M_{p_j} to also be rescaled. Noting that the full Lagrangian includes a mass term $-m_j\bar{\psi}_j\psi_j$, the transformation

$$\phi \rightarrow \phi' = \phi + c \quad (1.6)$$

$$m_j \rightarrow m'_j = m_j \left(1 - \frac{c}{M_s}\right) \quad (1.7)$$

$$M_s \rightarrow M'_s = M_s \left(1 - \frac{c}{M_s}\right) \quad (1.8)$$

$$M_{p_j} \rightarrow M'_{p_j} = M_{p_j} \left(1 - \frac{c}{M_s}\right) \quad (1.9)$$

$$V(\phi) \rightarrow V'(\phi') = V(\phi) = V(\phi' - c) \quad (1.10)$$

leaves the Lagrangian unchanged.

If M_s is very large and $V(\phi)$ is close to flat, this corresponds to an approximate symmetry under $\phi \rightarrow \phi + c$ alone.

In Chapter 2 I will describe the effects of the scalar interactions with matter, and in Chapter 3 I will describe the effects of the pseudoscalar interactions. In Chapter 4 I will describe the cosmological evolution of the field and its effects.

1.4 Axions

The axion is a well known hypothetical particle that interacts with light in much the same way as the one described here: in particular, the axion-photon interaction strength $g_{a\gamma\gamma}$ is equivalent to $4/M_p$ in my terminology[18]. Thus limits on M_p can be derived from limits on $g_{a\gamma\gamma}$. Observations of the proportion of helium-burning stars in globular clusters provide a limit of $g_{a\gamma\gamma} < 10^{-10}GeV^{-1}$; if the axions have a mass of less than $0.02eV$ the failure to detect axions from the sun implies that $g_{a\gamma\gamma} < 8.8 \times 10^{-11}GeV^{-1}$ [19] and if the mass is less than 10^{-9} eV, the absence of gamma rays detected from SN 1987A implies a limit of $g_{a\gamma\gamma} \lesssim 10^{-11}GeV^{-1}$ [18]. Since

a mass of 10^{-9} eV corresponds to a Compton wavelength of about 200 metres, which is very small on an astronomical scale, the limits on $M_s M_p$ that I will derive later based on astronomical and cosmological variation of the field only apply to much smaller masses. Thus I will use the tightest limit here. This limit on $g_{a\gamma\gamma}$ implies a limit of

$$M_p > 4 \times 10^{11} GeV \tag{1.11}$$

$$\gtrsim 1.6 \times 10^{-7} M_{pl} \tag{1.12}$$

Note that the recent PVLAS experiment claiming to find a much stronger interaction has not been confirmed [20].

Chapter 2

Effects of scalar couplings

2.1 Scalar field around a spherical mass distribution

Let us look at the effects masses have on ϕ (and vice versa). Ignoring the pseudoscalar couplings, which one would not expect to generate effects on ϕ from astronomical objects due to opposite spins and polarizations canceling each other out, the Lagrangian reduces to

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{\phi}{M_s}m\bar{\Psi}\Psi - V(\phi) \quad (2.1)$$

where $m\bar{\Psi}\Psi$ is really a sum over contributing massive particles.

The Euler-Lagrange equation for ϕ for this Lagrangian is

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)} = \frac{\partial \mathcal{L}}{\partial\phi} \quad (2.2)$$

$$\square\phi = -\frac{m}{M_s}\bar{\Psi}\Psi - \frac{dV}{d\phi} \quad (2.3)$$

For a spherically symmetric, static mass distribution much smaller and denser than the universe, we can presumably assume that $\ddot{\phi}$, which would be driven by changes of the size and composition of the universe, is negligible. Similarly we can presume that $\frac{dV}{d\phi}$ is small if the potential is nearly flat. If the potential is approximately linear, on the other hand, $\frac{dV}{d\phi}$ will be constant, and thus it will drive the

evolution of ϕ throughout the universe, but would not produce local gradients. Only quadratic or higher terms in the potential will affect the local gradients. Ignoring the quadratic or higher terms, we thus have

$$\nabla^2\phi(r) = \frac{m}{M_s}\bar{\Psi}\Psi \quad (2.4)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\phi(r)\right) = \frac{m}{M_s}\bar{\Psi}\Psi \quad (2.5)$$

$$r^2\frac{\partial}{\partial r}\phi(r) = \int_0^r dr' r'^2 \frac{\rho(r')}{M_s} = \frac{m(r)}{4\pi M_s} \quad (2.6)$$

$$\phi(r) = -\int_r^\infty dr' \frac{m(r')}{4\pi M_s r'^2} + \phi(\infty) \quad (2.7)$$

where $m(r)$ is the mass within a radius r . And for a point mass m ,

$$\phi(r) = -\frac{m}{4\pi M_s r} + \phi(\infty) \quad (2.8)$$

If one considers a quadratic term $V_{quad}(\phi) = \frac{1}{2}m_\phi^2\phi^2$, then for a point particle $\phi(r)$ must satisfy

$$\nabla^2\phi(r) - m_\phi^2\phi = -\frac{m}{M_s}\delta^{(3)}(\mathbf{r}) \quad (2.9)$$

Plugging in the ansatz that it will be a Yukawa potential $\frac{c}{r}e^{-\lambda r}$, we get

$$-4\pi c\delta^{(3)}(\mathbf{r}) + \frac{c\lambda^2}{r}e^{-\lambda r} - \frac{cm_\phi^2}{r}e^{-\lambda r} = \frac{m}{M_s}\delta^{(3)}(\mathbf{r}) \quad (2.10)$$

which is satisfied by $\lambda = m_\phi$, $c = -\frac{m}{4\pi M_s}$

Thus, as expected, a point mass will generate a $\phi(r)$ of

$$\phi(r) = -\frac{m}{4\pi M_s r}e^{-m_\phi r} \quad (2.11)$$

If the value of ϕ at infinity is not zero, then $\phi(r)$ will still be given by (2.11) plus a constant, since $\frac{1}{2}m_\phi^2(\phi - \text{const.})^2$ differs from $\frac{1}{2}m_\phi^2\phi^2$ only by an amount linear in ϕ .

If we consider the pseudoscalar couplings in (1.5), we get additional sources for $\square\phi$ of $-\frac{4}{M_p}\mathbf{E}\cdot\mathbf{B}$ and $i\frac{m_j}{M_{p_j}}\bar{\psi}\gamma^5\psi$. The former term may be significant at a single location, but it is difficult to make it nonzero in a large volume: for a situation in which the electric field derives from a potential V_e , the integral of the term over a volume amounts to the integral of $(1/M_p)V_e\mathbf{B}$ over the boundary of the volume, so if V_e diminishes as $1/r$ and \mathbf{B} diminishes as $1/r^3$, the integral will diminish as the inverse square of the radius it is taken over.

The latter term can be evaluated by applying the Dirac equation and its adjoint:

$$i\frac{m_j}{M_{p_j}}\bar{\psi}\gamma^5\psi = \frac{i}{2M_{p_j}} [(m_j\bar{\psi})\gamma^5\psi + \bar{\psi}\gamma^5(m_j\psi)] \quad (2.12)$$

$$= \frac{i}{2M_{p_j}} [((-i\partial_\mu - eA_\mu)\bar{\psi}\gamma^\mu)\gamma^5\psi + \bar{\psi}\gamma^5(\gamma^\mu(i\partial_\mu - eA_\mu)\psi)] \quad (2.13)$$

$$= \frac{1}{2M_{p_j}} [(\partial_\mu\bar{\psi})\gamma^\mu\gamma^5 + \bar{\psi}\gamma^\mu\gamma^5(\partial_\mu\psi)] \quad (2.14)$$

$$= \frac{1}{2M_{p_j}}\partial_\mu(\bar{\psi}\gamma^\mu\gamma^5\psi) \quad (2.15)$$

This is proportional to the divergence of the spin density.

2.2 Gravitation-like effect

The easiest way to find the gravitation-like effect of the scalar field is probably to compare it with Newtonian gravity. The effect of masses on the scalar field is similar to the effect on the Newtonian potential, so it becomes natural to ask if the field has a similar effect on masses. Indeed $\frac{\phi}{M_s}m\bar{\Psi}\Psi$ in (2.1) does look like a gravitation-like potential energy, suggesting a potential energy of a test particle m in the field from a mass M of $-\frac{mM}{4\pi M_s^2 r}$, assuming ϕ is massless or nearly massless.

To compare to Newtonian gravity, if $M_{pl} = (8\pi G_N^0)^{-1/2}$ where G_N^0 is the gravitational constant for gravity alone, the potential energy for two point particles from

gravity alone is

$$-\frac{m_1 m_2}{8\pi M_{pl}^2 r} \quad (2.16)$$

whereas there the additional contribution from ϕ is

$$-\frac{m_1 m_2}{4\pi M_s^2 r} \quad (2.17)$$

Thus the attractive effects of ϕ lead to an effective value of the gravitational constant of

$$G_N = \frac{1}{8\pi M_{pl}^2} + \frac{1}{4\pi M_s^2} \quad (2.18)$$

$$G_N = G_N^0 \left(1 + \frac{2M_{pl}^2}{M_s^2} \right). \quad (2.19)$$

Note that the standard value of M_{pl}^2 is based on the experimental value of G_N , not the bare G_N^0 , so it is different from the true M_{pl} (used in the above equations).

If ϕ has a mass larger than $1/H_0$, then the effective value of G_N will be given by (2.19) for distances from a mass source $r \ll \frac{1}{m_\phi}$ and will be G_N^0 for distances $r \gg \frac{1}{m_\phi}$.

Since the gravitational effects of ϕ are not necessarily equal between different masses, were M_s not very large it would likely already have been detected in Eötvös-type experiments. Since in particular no deflection of light is expected at all due to ϕ , the effect of gravity on light is a promising area to look for deviations from general relativity due to ϕ .

An analysis of the effects of the gravity of the sun on radio transmissions from the sun[21] found $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$. If the apparent value of G_N included a significant contribution from ϕ we would expect the measured value of $\gamma + 1$ to be lower, not higher, than 2. Indeed we would expect the measured value of γ to be

found from

$$(\gamma + 1)G_N = 2G_N^0 \quad (2.20)$$

$$\frac{\gamma + 1}{2} \cong 1 - 2\frac{M_{pl}^2}{M_s^2} \quad (2.21)$$

$$\gamma - 1 \cong -4\frac{M_{pl}^2}{M_s^2} \quad (2.22)$$

Still, conservatively assuming that $\gamma - 1 > -2.5 \times 10^{-5}$ leads to a limit on M_s of

$$-4\frac{M_{pl}^2}{M_s^2} > -2.5 \times 10^{-5} \quad (2.23)$$

$$M_s^2 > 1.6 \times 10^5 M_{pl}^2 \quad (2.24)$$

$$|M_s| > 400M_{pl} \quad (2.25)$$

This limit justifies the assumption that the true value of M_{pl} is approximately equal to the experimental value, and the spin 0 component of the gravitational force is small.

Chapter 3

Effects of pseudoscalar couplings

3.1 Effects of pseudoscalar coupling on light

With the interaction with ϕ from (1.5), the Lagrangian for electromagnetism is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\phi}{M_p}F_{\mu\nu}\tilde{F}^{\mu\nu} \quad (3.1)$$

The interaction with electric charges and currents is omitted as here I am only concerned with the behaviour of light in empty (except for ϕ) space.

The resulting Euler-Lagrange equations for A^ν are

$$0 = \frac{\partial \mathcal{L}}{\partial A_\nu} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} \quad (3.2)$$

$$= \partial_\mu \left(\frac{1}{4} \frac{\partial(F_{\rho\sigma}F^{\rho\sigma})}{\partial(\partial_\mu A_\nu)} - \frac{\phi}{M_p} \frac{\partial(F_{\rho\sigma}\tilde{F}^{\rho\sigma})}{\partial(\partial_\mu A_\nu)} \right) \quad (3.3)$$

$$= \partial_\mu \left(\frac{1}{2} \frac{\partial((\partial^\rho A^\sigma)(\partial_\rho A_\sigma) - (\partial^\rho A^\sigma)(\partial_\sigma A_\rho))}{\partial(\partial_\mu A_\nu)} - \frac{2\phi}{M_p} \frac{\partial(\epsilon^{\pi\rho\sigma\tau}\partial_\pi A_\rho\partial_\sigma A_\tau)}{\partial(\partial_\mu A_\nu)} \right) \quad (3.4)$$

$$= \partial_\mu \left(\partial^\mu A^\nu - \partial^\nu A^\mu - \frac{4\phi}{M_p} \epsilon^{\mu\nu\rho\sigma} \partial_\rho A_\sigma \right) \quad (3.5)$$

$$= \partial_\mu \left(F^{\mu\nu} - \frac{4\phi}{M_p} \tilde{F}^{\mu\nu} \right) \quad (3.6)$$

so, since

$$\partial_\mu \tilde{F}^{\mu\nu} = 0, \quad (3.7)$$

we have

$$\partial_\mu F^{\mu\nu} = \frac{4}{M_p} (\partial_\mu \phi) \tilde{F}^{\mu\nu} \quad (3.8)$$

From (3.7) and (3.8) we can derive a modified wave equation analogously to how we would derive a wave equation for the standard Maxwell's equations.

Applying $\epsilon_{\kappa\rho\lambda\nu} \partial^\lambda$ to both sides of (3.8),

$$\epsilon_{\kappa\rho\lambda\nu} \partial^\lambda \partial_\mu F^{\mu\nu} = \frac{4}{M_p} \epsilon_{\kappa\rho\lambda\nu} \partial^\lambda ((\partial_\mu \phi) \tilde{F}^{\mu\nu}) \quad (3.9)$$

so since

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\sigma\tau} F_{\sigma\tau} \quad (3.10)$$

and

$$F^{\mu\nu} = -\frac{1}{2} \epsilon^{\mu\nu\sigma\tau} \tilde{F}_{\sigma\tau}, \quad (3.11)$$

we have

$$-\epsilon_{\kappa\rho\lambda\nu} \partial^\lambda \partial_\mu \left(\frac{1}{2} \epsilon^{\mu\nu\sigma\tau} \tilde{F}_{\sigma\tau} \right) = \frac{4}{M_p} \epsilon_{\kappa\rho\lambda\nu} \partial^\lambda \left((\partial_\mu \phi) \left(\frac{1}{2} \epsilon^{\mu\nu\sigma\tau} F_{\sigma\tau} \right) \right). \quad (3.12)$$

Now since, (up to an irrelevant constant factor),

$$\epsilon_{\kappa\rho\lambda\nu} \epsilon^{\mu\nu\sigma\tau} = \epsilon_{\nu\kappa\rho\lambda} \epsilon^{\nu\mu\sigma\tau} = \delta_\kappa^\mu \delta_\rho^\sigma \delta_\lambda^\tau - \delta_\kappa^\mu \delta_\lambda^\sigma \delta_\rho^\tau + \delta_\lambda^\mu \delta_\kappa^\sigma \delta_\rho^\tau - \delta_\lambda^\mu \delta_\rho^\sigma \delta_\kappa^\tau + \delta_\rho^\mu \delta_\lambda^\sigma \delta_\kappa^\tau - \delta_\rho^\mu \delta_\kappa^\sigma \delta_\lambda^\tau \quad (3.13)$$

(3.12) becomes

$$\begin{aligned}
& -\frac{1}{2} \left(\partial^\lambda \partial_\kappa \tilde{F}_{\rho\lambda} - \partial^\lambda \partial_\kappa \tilde{F}_{\lambda\rho} + \partial^\mu \partial_\mu \tilde{F}_{\kappa\rho} - \partial^\mu \partial_\mu \tilde{F}_{\rho\kappa} + \partial^\lambda \partial_\rho \tilde{F}_{\lambda\kappa} - \partial^\lambda \partial_\rho \tilde{F}_{\kappa\lambda} \right) \\
& = \frac{2}{M_p} \left(\partial^\lambda ((\partial_\kappa \phi)(F_{\rho\lambda})) - \partial^\lambda ((\partial_\kappa \phi)(F_{\lambda\rho})) + \partial^\mu ((\partial_\mu \phi)(F_{\kappa\rho})) \right. \\
& \quad \left. - \partial^\mu ((\partial_\mu \phi)(F_{\rho\kappa})) + \partial^\lambda ((\partial_\rho \phi)(F_{\lambda\kappa})) - \partial^\lambda ((\partial_\rho \phi)(F_{\kappa\lambda})) \right) \tag{3.14}
\end{aligned}$$

or

$$\partial^\mu \partial_\mu \tilde{F}_{\kappa\rho} = -\frac{4}{M_p} \left(\partial^\mu ((\partial_\mu \phi)F_{\kappa\rho}) + \partial^\mu ((\partial_\rho \phi)F_{\mu\kappa}) - \partial^\mu ((\partial_\kappa \phi)F_{\mu\rho}) \right) \tag{3.15}$$

If we make the approximation that the length scale of changes in $\partial_\mu \phi$ is much larger than the wavelengths of light of interest (which should be good as long as the wavelength is small relative to both the Hubble scale and the distances to the centers of mass sources generating gradients of phi) and also that the components of $\frac{\partial_\mu \phi}{M_p}$ are much smaller than the wavelength (to be justified later once a limit is obtained on it), then

$$\partial^\mu \partial_\mu \tilde{F}_{\kappa\rho} \cong -\frac{4}{M_p} (\partial_\mu \phi) \partial^\mu F_{\kappa\rho} \tag{3.16}$$

Applying $\frac{1}{2}\epsilon^{\sigma\tau\kappa\rho}$ to both sides, we also get

$$\partial^\mu \partial_\mu F^{\sigma\tau} \cong \frac{4}{M_p} (\partial_\mu \phi) \partial^\mu \tilde{F}^{\sigma\tau} \tag{3.17}$$

Define $F_R = F + i\tilde{F}$ and $F_L = F - i\tilde{F}$. With the convention that sinusoidal time dependence is $e^{-i\omega t}$ (as opposed to $e^{i\omega t}$), F_R corresponds to right circularly polarized light and F_L to left circularly polarized light (i.e. $F_L = 0$ for right polarization and $F_L = 0$ for left polarization).

From (3.16) and (3.17),

$$\partial^\mu \partial_\mu F_R \cong -i \frac{4}{M_p} (\partial_\mu \phi) \partial^\mu F_R \quad (3.18)$$

$$\partial^\mu \partial_\mu F_L \cong i \frac{4}{M_p} (\partial_\mu \phi) \partial^\mu F_L. \quad (3.19)$$

Using the already mentioned approximations,

$$(\partial^\mu + i \frac{2}{M_p} (\partial^\mu \phi)) (\partial_\mu + i \frac{2}{M_p} (\partial_\mu \phi)) F_R \cong 0 \quad (3.20)$$

$$(\partial^\mu - i \frac{2}{M_p} (\partial^\mu \phi)) (\partial_\mu - i \frac{2}{M_p} (\partial_\mu \phi)) F_L \cong 0. \quad (3.21)$$

Thus, we have approximate wave equations for $F_R e^{i \frac{2}{M_p} \phi}$ and $F_L e^{-i \frac{2}{M_p} \phi}$:

$$\partial^\mu \partial_\mu (F_R e^{i \frac{2}{M_p} \phi}) \cong 0 \quad (3.22)$$

$$\partial^\mu \partial_\mu (F_L e^{-i \frac{2}{M_p} \phi}) \cong 0. \quad (3.23)$$

These have "plane wave" solutions (Note that I use a (+ - - -) sign convention as throughout this thesis; $\omega t - \mathbf{k} \cdot \mathbf{x} = k_\lambda x^\lambda$. Also, note that $k_{R\lambda}$ and $k_{L\lambda}$ are null vectors.)

$$F_R e^{i \frac{2}{M_p} \phi} \cong e^{-i k_{R\lambda} x^\lambda} \quad (3.24)$$

$$F_L e^{-i \frac{2}{M_p} \phi} \cong e^{-i k_{L\lambda} x^\lambda} \quad (3.25)$$

The corresponding solutions

$$F_R \cong e^{-i k_{R\lambda} x^\lambda - i \frac{2}{M_p} \phi} \quad (3.26)$$

$$F_L \cong e^{-i k_{L\lambda} x^\lambda + i \frac{2}{M_p} \phi} \quad (3.27)$$

are plane waves for constant $\partial\phi$ and given the assumptions used, there should be a

distance scale much larger than the wavelength where they are still close to plane waves. Defining

$$\omega'_R = \omega_R + \frac{2}{M_p} \dot{\phi} \quad (3.28)$$

$$\mathbf{k}'_R = \mathbf{k}_R - \frac{2}{M_p} \nabla \phi \quad (3.29)$$

$$\omega'_L = \omega_L - \frac{2}{M_p} \dot{\phi} \quad (3.30)$$

$$\mathbf{k}'_L = \mathbf{k}_L + \frac{2}{M_p} \nabla \phi, \quad (3.31)$$

(i.e., $k'_{R\lambda} = k_{R\lambda} + \frac{2}{M_p} \partial_\lambda \phi$, $k'_{L\lambda} = k_{L\lambda} - \frac{2}{M_p} \partial_\lambda \phi$) we have

$$F_R \cong e^{-i(k_{R\lambda} + \frac{2}{M_p} \partial_\lambda \phi)x^\lambda} = e^{-i(\omega'_R t - \mathbf{k}'_R \cdot \mathbf{x})} \quad (3.32)$$

$$F_L \cong e^{-i(k_{L\lambda} - \frac{2}{M_p} \partial_\lambda \phi)x^\lambda} = e^{-i(\omega'_L t - \mathbf{k}'_L \cdot \mathbf{x})}. \quad (3.33)$$

Of course, it is the primed quantities which are observed and which are produced at a source.

In order to calculate the group velocity we can use the dispersion relations:

$$0 = \omega_R^2 - k_R^2 = (\omega'_R - \frac{2}{M_p} \dot{\phi})^2 - (\mathbf{k}'_R + \frac{2}{M_p} \nabla \phi)^2 \cong \omega_R'^2 - \mathbf{k}'_R{}^2 - \frac{4}{M_p} \omega'_R \dot{\phi} - \frac{4}{M_p} \mathbf{k}'_R \cdot \nabla \phi, \quad (3.34)$$

so for right-polarized light

$$0 \cong 2\omega'_R \frac{\partial \omega'_R}{\partial \mathbf{k}'_R} - 2\mathbf{k}'_R - \frac{4}{M_p} \frac{\partial \omega'_R}{\partial \mathbf{k}'_R} \dot{\phi} - \frac{4}{M_p} \nabla \phi \quad (3.35)$$

$$\mathbf{v}_g = \frac{\partial \omega'_R}{\partial \mathbf{k}'_R} \cong \frac{\mathbf{k}'_R + \frac{2}{M_p} \nabla \phi}{\omega'_R - \frac{2}{M_p} \dot{\phi}} = \frac{\mathbf{k}_R}{\omega_R} \quad (3.36)$$

where $\partial/\partial\mathbf{k} = \hat{x}\partial/\partial\mathbf{k}_x + \hat{y}\partial/\partial\mathbf{k}_y + \hat{z}\partial/\partial\mathbf{k}_z$. Similarly for left polarized light

$$\mathbf{v}_g \cong \frac{\mathbf{k}_L}{\omega_L} \quad (3.37)$$

Note two things about the group velocity: its magnitude is equal to 1 in $c = 1$ units, so photons still travel at the usual speed; also since its direction is that of the underlying plane wave, not the non-plane ordinary EM waves, there is no deflection of photons due to the scalar field. Of course, one could also guess that there is no bending of light due to the fact that the phase shift of both left and right polarized light is independent of the path taken between two space-time points.

However, the group velocity is not exactly perpendicular to the planes of constant (ordinary EM) phase: since $\mathbf{k}_R = \mathbf{k}'_R + \frac{2}{M_p}\nabla\phi$, for right polarized light it is away from the perpendicular by an angle of about

$$\frac{2(\nabla\phi)_{\perp\mathbf{k}}}{M_p|\mathbf{k}|} \quad (3.38)$$

in the direction of $\nabla\phi$, and of course the opposite direction for left polarized light.

Consider a light source with frequency ω . We have $\omega'_R = \omega'_L \equiv \omega$, so $\omega_R = \omega - \frac{2}{M_p}\dot{\phi}_s$, and $\omega_L = \omega + \frac{2}{M_p}\dot{\phi}_s$, where $\dot{\phi}_s$ is the value of $\dot{\phi}$ at the source. For now, ignore the expansion of the universe. Then the phase change from the source to an observer for right polarized light is $k_{R\lambda}\Delta x^\lambda + \frac{2}{M_p}\Delta\phi = (\omega - \frac{2}{M_p}\dot{\phi}_s)\Delta t - (\omega - \frac{2}{M_p}\dot{\phi}_s)\hat{k}\cdot\Delta x + \frac{2}{M_p}\Delta\phi = \frac{2}{M_p}\Delta\phi$ and similarly for left polarized light the phase change is $-\frac{2}{M_p}\Delta\phi$. We thus have a change in relative phase between left and right polarized light of $\frac{4}{M_p}\Delta\phi$, which will cause a rotation of linearly polarized light of

$$\Delta(\text{polarization angle}) = \frac{2}{M_p}\Delta\phi. \quad (3.39)$$

If the observer has a value of $\dot{\phi}$ of $\dot{\phi}_o \neq \dot{\phi}_s$ then at the observer $\omega'_R = \omega + \frac{2}{M_p}\dot{\phi}_o - \frac{2}{M_p}\dot{\phi}_s$

and $\omega'_L = \omega - \frac{2}{M_p}\dot{\phi}_o + \frac{2}{M_p}\dot{\phi}_s$. This difference can be interpreted simply as the result of the change of phase shift as the value of $\Delta\phi$ between the source and the observer changes. In order to generalize to an expanding universe, note that an expanding universe is equivalent to a universe in which everything is moving away. One can then imagine replacing a system moving away with a stationary system in which time moves much slower, which would look the same from Earth. Thus in an expanding universe one should scale the value of $\dot{\phi}_s$ by $\frac{1}{1+z}$ when calculating the frequency shift.

Of the phenomena listed here, only the polarization rotation effect is expected to be significant (i.e. if the other effects were large enough to be observable any time soon, we would already have observed a polarization rotation).

Carroll reported[5] a rotation of polarization of (in my notation) $\frac{1}{M_p}\Delta\phi$ using the difference of group [sic] velocities between the different circular polarizations. He derived a dispersion relation for ω and k :

$$\omega^2 = k^2 \pm \frac{1}{M_p}\dot{\phi}k \quad (3.40)$$

where (+) is for right-handed and (−) for left handed polarization. Since k and ω are close to proportional, this is similar to my result (3.34) except in the coefficient and the fact that he only considered time dependence. This dispersion relation appears not to lead to any group velocity difference to lowest order either. In any case, it seems that phase velocity is more appropriate here. Certainly if the light source were a steady coherent beam, the polarization rotation would clearly depend on the difference of phase velocities, not group velocity. In the case of a pulsed beam a difference in group velocities would lead the pulses to separate into pulses of each phase, but the linear polarization where they overlap would still be rotated according to the phase velocity difference. In fact, I assume that Carroll simply made a typo in referring to group instead of phase velocity. Nonetheless, there is still a discrepancy

in the dispersion relation and polarization rotation.

Although the dispersion relation looks different from mine, if Carroll used $F_{\mu\nu}F^{\mu\nu}$ instead of $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ as the first term in the Lagrangian (which means that his term $\beta_{F\tilde{F}}/M$ is equivalent to my $-4/M_p$), then it would be almost equivalent to mine considering only $\dot{\phi}$ and not $\nabla\phi$ (differing only in the substitution of k for ω). In that case, however, there would still be a factor of two discrepancy in the polarization rotation (mine is smaller in that case). One possible error he could have made is not taking into account the fact that a relative phase shift of left and right polarized light of some angle leads to a rotation of linearly polarized light of half that angle. Still, without knowing the details of his calculation this is only speculation.

3.2 Coupling with spin induced by interaction with light

The field ϕ , if it has a pseudoscalar interaction with light, will also have an interaction with the spins of charged particles. Even if there is no direct interaction, the interaction with light will induce an interaction of ϕ with the electric and magnetic fields of the particle.

Consider an interaction between a ϕ particle and an ordinary fermion mediated by two photons. I will assume it is an electron for this calculation. The Feynman diagram for this process is a loop diagram with two photons meeting at a single vertex with the ϕ field, each meeting the electron separately. The momenta are labeled as follows: the initial and final momenta of the electron are p_i and p_f respectively and the intermediate momentum of the electron is k . By momentum conservation the incoming ϕ momentum is $p_f - p_i$, which I will call q , and the photon momenta are $p_i - k$ and $p_f - k$.

To calculate the vertex factor for the ϕ - two photon vertex, note that

$$\frac{1}{M_p}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{2}{M_p}\phi\epsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu\partial^\rho A^\sigma \quad (3.41)$$

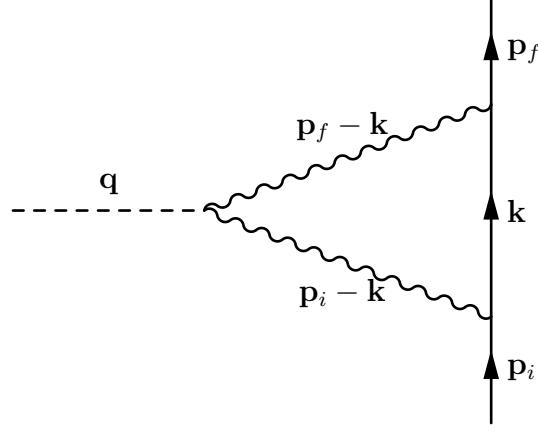


Figure 3.1: Interaction between a charged fermion and a ϕ particle mediated by photons

So the vertex factor consists of $\frac{2}{M_p} \epsilon_{\mu\nu\rho\sigma}$ times the two photon momenta, with the extra two indices contracting with the indices of the photons from the rest of the Feynman diagram.

The other vertex factors, propagators, etc. are the standard ones for Quantum Electrodynamics.

So the amplitude is

$$\begin{aligned}
\mathcal{M} &= \int \frac{d^4k}{(2\pi)^4} \frac{2}{M_p} \epsilon_{\mu\nu\rho\sigma} (p_f^\mu - k^\mu)(p_i^\rho - k^\rho) \bar{u}(p_f, s) (-ie\gamma_\zeta) \left(\frac{-ig^{\nu\zeta}}{(p_f - k)^2 + i\epsilon} \right) \\
&\quad \times \left(\frac{i(k \cdot \gamma + m)}{k^2 - m^2 + i\epsilon} \right) \left(\frac{-ig^{\sigma\xi}}{(p_i - k)^2 + i\epsilon} \right) (-ie\gamma_\xi) u(p_i, s) \quad (3.42) \\
&= \frac{2ie^2}{M_p} \epsilon_{\mu\nu\rho\sigma} \bar{u}(p_f, s) \gamma^\nu \gamma^\tau \gamma^\sigma u(p_i, s) \int \frac{d^4k}{(2\pi)^4} \frac{k_\tau (p_f^\mu - k^\mu)(p_i^\rho - k^\rho)}{(p_f - k)^2 (k^2 - m^2) (p_i - k)^2} \\
&\quad + \frac{2ime^2}{M_p} \epsilon_{\mu\nu\rho\sigma} \bar{u}(p_f, s) \gamma^\nu \gamma^\sigma u(p_i, s) \int \frac{d^4k}{(2\pi)^4} \frac{(p_f^\mu - k^\mu)(p_i^\rho - k^\rho)}{(p_f - k)^2 (k^2 - m^2) (p_i - k)^2}
\end{aligned}$$

Letting $p = (p_i + p_f)/2$, we have $\epsilon_{\mu\nu\rho\sigma} (p_f^\mu - k^\mu)(p_i^\rho - k^\rho) = \epsilon_{\mu\nu\rho\sigma} q^\mu (p^\rho - k^\rho)$. Also, using the Dirac equation ($m\bar{u}(p_f) = \bar{u}(p_f)p_f$ and $mu(p_i) = p_i u(p_i)$) we can convert

the m in the second term into a p and merge that term with the first term:

$$m\epsilon_{\mu\nu\rho\sigma}\bar{u}(p_f)\gamma^\nu\gamma^\sigma u(p_i) = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\bar{u}(p_f)(\not{p}_f\gamma^\nu\gamma^\sigma + \gamma^\nu\gamma^\sigma\not{p}_i)u(p_i) \quad (3.43)$$

$$= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\bar{u}(p_f)(2p_f^\nu\gamma^\sigma - \gamma^\nu\gamma^\tau\gamma^\sigma(p_{f\tau} + p_{i\tau}) + 2\gamma^\nu p_i^\sigma)u(p_i) \quad (3.44)$$

$$= \epsilon_{\mu\nu\rho\sigma}\bar{u}(p_f)(-\gamma^\nu\gamma^\tau\gamma^\sigma p_\tau + q^\nu\gamma^\sigma)u(p_i) \quad (3.45)$$

The q^ν term cancels due to anti-symmetrization with q^μ . That makes the amplitude:

$$\mathcal{M} = \frac{2ie^2}{M_p}\epsilon_{\mu\nu\rho\sigma}\bar{u}(p_f, s)\gamma^\nu\gamma^\tau\gamma^\sigma u(p_i, s)q^\mu \int \frac{d^4k}{(2\pi)^4} \frac{(k_\tau - p_\tau)(p^\rho - k^\rho)}{(p_f - k)^2(k^2 - m^2)(p_i - k)^2} \quad (3.46)$$

In Appendix B I calculate that this amplitude works out to

$$\mathcal{M} = \frac{3ie^2}{8\pi^2 M_p}\bar{u}(p_f, s)\gamma_\mu\gamma^5 u(p_i, s)q^\mu \left(\ln\left(\frac{\Lambda}{m}\right) - \frac{1}{12} - O\left(\sqrt{\frac{-q^2}{m^2}}\right) \right). \quad (3.47)$$

where Λ is a UV cutoff.

If we abstract out the loop and treat this amplitude as arising from a direct tree-level coupling of the scalar field to the fermion, the implied vertex factor is obtained by simply eliminating the \bar{u} and u in (3.47):

$$\frac{3ie^2}{8\pi^2 M_p} \left(\ln\left(\frac{\Lambda}{m}\right) - \frac{1}{12} - O\left(\sqrt{\frac{-q^2}{m^2}}\right) \right) \not{q}\gamma^5 \quad (3.48)$$

For the nonrelativistic limit of $-q^2 \ll m^2$ the amplitude (3.47) becomes

$$\mathcal{M} = \frac{3ie^2}{8\pi^2 M_p} \left(\ln\left(\frac{\Lambda}{m}\right) - \frac{1}{12} \right) \mathbf{q} \cdot \boldsymbol{\sigma} 2m\delta^{ss'} \quad (3.49)$$

For the induced interaction of the spin to the scalar field from the mass M of another particle, we need to add another vertex factor of iM/M_s and a scalar prop-

agator

$$\mathcal{M} = \frac{-3Me^2}{8\pi^2 M_s M_p} \frac{i}{q^2 - m_\phi^2 + i\epsilon} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \mathbf{q} \cdot \boldsymbol{\sigma} 2m\delta^{ss'} \quad (3.50)$$

We can compare this to the scattering amplitude derived from the Born approximation[22] to derive:

$$\tilde{V}(\mathbf{q}, \boldsymbol{\sigma}) = \frac{3iMe^2}{8\pi^2 M_s M_p} \frac{1}{q^2 - m_\phi^2 + i\epsilon} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \mathbf{q} \cdot \boldsymbol{\sigma} \quad (3.51)$$

$$V(\mathbf{x}, \boldsymbol{\sigma}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{3iMe^2}{8\pi^2 M_s M_p} \frac{1}{q^2 - m_\phi^2 + i\epsilon} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \mathbf{q} \cdot \boldsymbol{\sigma} e^{i\mathbf{q}\cdot\mathbf{x}} \quad (3.52)$$

$$= \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{3Me^2}{8\pi^2 M_s M_p} \frac{1}{|\mathbf{q}|^2 + m_\phi^2} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \mathbf{q} \cdot \mathbf{x} \frac{\boldsymbol{\sigma} \cdot \mathbf{x}}{x^2} \sin(\mathbf{q} \cdot \mathbf{x}) \quad (3.53)$$

$$V(r, \sigma_z) = \frac{-3iMe^2\sigma_z}{16\pi^2 M_s M_p} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} \int \frac{dq_z}{2\pi} \frac{q_z}{q_z^2 + |\mathbf{q}_\perp|^2 + m_\phi^2} (e^{iq_z r} - e^{-iq_z r}) \quad (3.54)$$

$$= \frac{3Me^2\sigma_z}{16\pi^2 M_s M_p} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-\sqrt{|\mathbf{q}_\perp|^2 + m_\phi^2} r} \quad (3.55)$$

$$= \frac{3Me^2\sigma_z}{32\pi^3 M_s M_p} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \frac{1 + m_\phi r}{r^2} e^{-m_\phi r} \quad (3.56)$$

You can easily verify my result for the \mathbf{q}_\perp integral by going to polar coordinates, then making a change of variables to $u = \sqrt{|\mathbf{q}_\perp|^2 + m_\phi^2}$ and then integrating by parts.

Using the expression for ϕ from a point source (2.11), we can express this in terms of $\nabla\phi$:

$$V(\boldsymbol{\sigma} \cdot \nabla\phi) = \frac{3e^2}{8\pi^2 M_p} \left(\ln \left(\frac{\Lambda}{m} \right) - \frac{1}{12} \right) \boldsymbol{\sigma} \cdot \nabla\phi \quad (3.57)$$

Note that despite the logarithmic divergence, for $\Lambda \leq M_{pl}$ we still have $\ln \left(\frac{\Lambda}{m} \right) < 50$, so due to the order- α suppression the coefficient here is still smaller than $1/M_p$.

3.3 Experiments measuring spin-gravity interactions

In the following sections I will refer to several experiments measuring the interaction of particle spins with a direction in space, which I will briefly summarize here.

Berglund et. al.[23] measured the ratio of the precession frequencies of Hg^{199} and Cs atoms in a magnetic field. By measuring the change of this ratio as the apparatus rotated with the Earth, they were able to limit the interaction of the spin of a valence neutron in the Hg nucleus to a fixed direction in space to a change in frequency for aligned vs anti-aligned spins of 110 nHz or an energy change of 4.55×10^{-22} eV. They also put a limit on the interaction with electron spins in Cs of 200 μHz or an energy change of 8.27×10^{-19} eV. These limits assume that the electron and nuclear effects do not cancel (it would be an improbable coincidence if they did). Although the limits are for a fixed direction in space, the duration of the experiment (4 runs of 48 hours each with the magnetic field in a different orientation in each run) is probably small enough for the limits to apply to the direction to the sun as well.

Venema et. al.[24] measured the ratio of spin precession frequencies for Hg^{199} and Hg^{201} atoms in a magnetic field, looking for a difference in the ratio for two different orientations of the magnetic field with respect to the Earth's gravitational field. From this, they find a limit on the energy difference for neutrons in mercury aligned and anti-aligned to Earth's gravity of $\Delta E \lesssim 2.1 \times 10^{-20}$ eV.

Heckel et. al [25] measured the torque on a torsion pendulum containing polarized electrons in a magnetic material. By measuring the changes in the torsion as Earth rotated relative to the sun they determined a limit for scalar and pseudoscalar interaction strengths g_P^e and g_S^N of $|g_P^e g_S^N|/(\hbar c) < 1.7 \times 10^{-36}$. This corresponds to $< 5.5 \times 10^{-22} \text{eV}$ for the difference in energy between electrons with spins aligned or anti-aligned with the direction to the sun. In my terminology, the limit corresponds to $|M_s M_{pe}| > 4.8 \times 10^{-5} M_{pl}^2$.

3.4 Electromagnetically mediated interaction with atom or nucleus

In addition to the coupling to spins introduced by the coupling to the electric and magnetic fields of a single particle, there will also be a coupling introduced to the interaction with the electric field of one particle and the magnetic field of another.

In this section I will calculate the effect on the energy of the interaction of a magnetic dipole μ , centred at \mathbf{r}_N , and an electric field with the ϕ field. The dipole will be the spin magnetic field of an electron or a valence nucleon and the electric field that of the nucleus.

The relevant portion of the Lagrangian is $\frac{\phi}{M_p} F \tilde{F}$; assuming spherical symmetry for the probability distribution of \mathbf{r}_N , and an electric field from a spherically symmetric potential $A_0(r)$, and $\mathbf{r}_- = \mathbf{r} - \mathbf{r}_N$, and assuming that $\phi = \phi_0 + \mathbf{r} \cdot \nabla \phi$:

$$\left\langle \int \frac{\phi}{M_p} F \tilde{F} \right\rangle = -4 \left\langle \int d^3\mathbf{r} \frac{\phi}{M_p} \mathbf{E} \cdot \mathbf{B} \right\rangle \quad (3.58)$$

$$= 4 \left\langle \int d^3\mathbf{r} \frac{\phi}{M_p} \nabla A_0(r) \cdot \mathbf{B}(\mathbf{r}_-) \right\rangle \quad (3.59)$$

where

$$\mathbf{B}(\mathbf{r}_-) = \frac{3(\mu \cdot \hat{r}_-) \hat{r}_- - \mu}{|\mathbf{r}_-|^3} + \frac{8\pi}{3} \mu \delta^3(\mathbf{r}_-) \quad (3.60)$$

Using integration by parts, and noting that $\nabla \cdot \mathbf{B} = 0$, we get

$$\left\langle \int \frac{\phi}{M_p} F \tilde{F} \right\rangle = 4 \left\langle \oint_{r_- \rightarrow \infty} \frac{\mathbf{r}_- \cdot \nabla \phi}{M_p} A_0(\infty) \mathbf{B} \cdot \hat{r}_- \right\rangle - 4 \left\langle \int d^3\mathbf{r} \frac{A_0(r)}{M_p} \nabla \phi \cdot \mathbf{B} \right\rangle \quad (3.61)$$

For the boundary term I take for convenience a sphere oriented around \mathbf{r}_N . I assume that $\phi \simeq \phi(\mathbf{r}_N) + \mathbf{r}_- \cdot \nabla \phi$ (the $\phi(\mathbf{r}_N)$ term will not contribute in the limit as $r_- \rightarrow \infty$ as \mathbf{B} drops off as r_-^3 ; also I assume that A_0 approaches a constant value as r (and r_-) $\rightarrow \infty$, ie the charge is finite and no external fields).

Note that the integrals depend linearly on $\nabla\phi$ and μ so we can express them in the form

$$I_{ij}m_j\partial_i\phi \quad (3.62)$$

but these are the only directions in the problem not spherically symmetrically integrated over. Thus by symmetry

$$I_{ij} \equiv I\delta_{ij}. \quad (3.63)$$

We can find I by tracing over I_{ij} :

$$I\delta_{ij}\delta_{ij} = \delta_{ij}I_{ij} \quad (3.64)$$

$$I = \frac{1}{3}\delta_{ij}I_{ij} \quad (3.65)$$

$$\begin{aligned} I_{ij} &= 4 \left\langle \oint_{r_- \rightarrow \infty} \frac{A_0(\infty)}{M_p} (r_-)_i (\hat{r}_-)_k \left(\frac{3(\hat{r}_-)_j (\hat{r}_-)_k - \delta_{jk}}{|\mathbf{r}_-|^3} \right) \right\rangle \\ &\quad - 4 \left\langle \int d^3\mathbf{r} \frac{A_0(r)}{M_p} \left(\frac{3(\hat{r}_-)_j (\hat{r}_-)_i - \delta_{ij}}{|\mathbf{r}_-|^3} + \frac{8\pi}{3} \delta_{ij} \delta^3(\mathbf{r}_-) \right) \right\rangle \end{aligned} \quad (3.66)$$

$$I = \frac{4}{3} \frac{A_0(\infty)}{M_p} \left\langle \oint_{r_- \rightarrow \infty} \frac{2}{|r_-|^2} \right\rangle - \frac{4}{3M_p} \left\langle \int d^3\mathbf{r} A_0(r) 8\pi \delta^3\mathbf{r}_- \right\rangle \quad (3.67)$$

$$= \frac{32\pi}{3M_p} A_0(\infty) - \frac{32\pi}{3M_p} \int d^3\mathbf{r}_N A_0(r_N) |\psi(r_N)|^2 \quad (3.68)$$

In the case of a hydrogen atom in the ground state, $A_0(r) = \frac{e}{4\pi r}$ and $\psi^2(r) = \frac{1}{\pi a_0^3} e^{-\frac{2r}{a_0}}$, leading to

$$\left\langle \int \frac{\phi}{M_p} F \tilde{F} \right\rangle = \frac{8e}{3M_p a_0} \mu \cdot \nabla\phi \quad (3.69)$$

$$= -\frac{4e^2 g_e}{3m_e M_p a_0} \mathbf{s} \cdot \nabla\phi \quad (3.70)$$

$$= -\frac{4\alpha^2}{3M_p} \hat{S} \cdot \nabla\phi \quad (3.71)$$

where I used $g_e \cong 2$, $\alpha = e^2$ and $a_0 = 1/m_e\alpha$.

For the nucleus, for simplicity I will assume that $\psi^2(\mathbf{r}_n)$ is uniform within the nucleus. I will also assume a bare nucleus; the effect of electron clouds should be small.

For the bare nucleus, the potential takes the form:

$$A_0(r) = \begin{cases} \frac{eZ}{4\pi r} & r > R \\ -\frac{eZ}{8\pi R^3}r^2 + \frac{3eZ}{8\pi R} & r < R \end{cases} \quad (3.72)$$

where R is the nuclear radius.

For the nucleus with these approximations,

$$\left\langle \int \frac{\phi}{M_p} F \tilde{F} \right\rangle \cong \frac{16eZ}{5M_p R} \mu \cdot \nabla \phi \quad (3.73)$$

$$\cong \frac{4e^2 g_n Z}{5m_p M_p R} \hat{S} \cdot \nabla \phi \quad (3.74)$$

$$\cong \frac{4\alpha^2 g_n}{5M_p} \frac{Z m_e a_0}{m_p R} \hat{S} \cdot \nabla \phi \quad (3.75)$$

where I use $|\mu_n| = \frac{g_n e}{4m_p}$.

3.5 Local effects from electromagnetically mediated spin interaction

Using (3.75), the energy difference between a nucleus with valence neutron spin aligned with $\nabla\phi$, and one anti-aligned is

$$\Delta E = \frac{8\alpha^2 g_n}{5M_p} \frac{Z m_e a_0}{m_p R} |\nabla\phi|. \quad (3.76)$$

From (2.8), $\nabla\phi = \frac{m}{4\pi M_s r^2} \hat{r}$, whereas for gravity $\mathbf{g} = -\frac{m}{8\pi M_{pl}^2 r^2} \hat{r}$ so

$$\nabla\phi = -\mathbf{g} \frac{2M_{pl}^2}{M_s} \quad (3.77)$$

thus

$$\Delta E = -\frac{16\alpha^2 g_n}{5} \frac{Z m_e a_0}{m_p R} g \frac{M_{pl}^2}{M_s M_p} \quad (3.78)$$

This calculation was done with $\hbar = c = 1$; in order to have the units work out for numerical calculations, the RHS will have to be multiplied by \hbar/c .

Given an observational limit of $\Delta E \lesssim 4.55 \times 10^{-31} GeV$ for ^{199}Hg for the gradient of ϕ from the sun from Berglund et. al.[23], we get

$$\frac{M_s M_p}{M_{Pl}^2} \gtrsim 6 \times 10^{-6} \quad (3.79)$$

while from Venema et. al.[24], we have a limit of $\Delta E \lesssim 2.1 \times 10^{-20} eV$ for neutrons in mercury due to the gradient of ϕ from the Earth, resulting in a limit of

$$\frac{M_s M_p}{M_{Pl}^2} \gtrsim 2 \times 10^{-4}, \quad (3.80)$$

which is much better.

3.6 Local effects from direct interaction with spin

There will also be an interaction with the spins of particles. This includes both any interaction with spin completely unmediated by electromagnetism, represented by the $i\gamma_5$ term in $\sum_j \phi m_j \bar{\psi}_j \left(-\frac{1}{M_s} + \frac{i\gamma_5}{M_{pj}} \right) \psi_j$, and the interaction with the electric and magnetic fields of a single particle calculated in §3.4. For simplicity I will combine these effects into a single term.

Using the Dirac equation

$$\gamma^\mu (i\partial_\mu - eA_\mu) \psi = m\psi \quad (3.81)$$

and its adjoint version

$$(i\partial_\mu + eA_\mu)\bar{\psi}\gamma^\mu = -m\bar{\psi} \quad (3.82)$$

we can re-express the average value of this term for a single particle as

$$\begin{aligned} \left\langle \phi m_j \bar{\psi} \left(\frac{i\gamma^5}{M_{p_j}} \right) \psi \right\rangle &= \frac{i}{2M_{p_j}} \left(-\langle \phi ((i\partial_\mu + eA_\mu)\bar{\psi})\gamma^\mu\gamma^5\psi \rangle \right. \\ &\quad \left. + \langle \phi \bar{\psi}\gamma^5(i\not{\partial} - e\not{A})\psi \rangle \right) \end{aligned} \quad (3.83)$$

$$= \frac{1}{2M_{p_j}} \langle \phi \partial_\mu (\bar{\psi}\gamma^\mu\gamma^5\psi) \rangle \quad (3.84)$$

$$= -\frac{1}{2M_{p_j}} \langle \partial_\mu \phi (\bar{\psi}\gamma^\mu\gamma^5\psi) \rangle \quad (3.85)$$

So the change in energy between a spin aligned with $\nabla\phi$ and one anti-aligned is

$$\begin{aligned} \Delta E &= \left\langle \frac{\partial_\mu \phi}{M_{p_j}} \bar{\psi}\gamma^\mu\gamma^5\psi \right\rangle \\ &\simeq \frac{m}{4\pi M_s M_{p_j} r^2} \end{aligned} \quad (3.86)$$

$$\simeq \frac{2M_{pl}^2}{M_s M_{p_j}} g \quad (3.87)$$

$$\simeq (\text{in S.I. units}) \frac{2M_{pl}^2}{M_s M_{p_j}} g \frac{\hbar}{c} \quad (3.88)$$

where g is the local gravitational acceleration due to the same mass concentration causing the gradient of interest in ϕ . It is assumed that the particle is spin- $\frac{1}{2}$. The limits on these direct interactions from the experiments mentioned in §3.3 are summarized in Table 3.1

3.7 Local effects summary

Of the experimental limits derived here, only (3.80) provides a limit on $M_p M_s$ that appears to be competitive to the limits on M_p and M_s individually. Local experiments

Table 3.1: Experimental limits

experiment	mass source	limit on ΔE	limit on $M_S M_{p_j}$ obtained
Berglund et. al. [23]	Sun	$\lesssim 4.55 \times 10^{-22}$ eV	$ M_s M_{p_n} \gtrsim 5.7 \times 10^{-5} M_{pl}^2$
Venema et. al. [24]	Earth	$\lesssim 2.1 \times 10^{-20}$ eV	$ M_s M_{p_n} \gtrsim 2.0 \times 10^{-3} M_{pl}^2$
Heckel et. al. [25]	Sun	$\lesssim 5.5 \times 10^{-22}$ eV	$ M_s M_{p_e} \gtrsim 4.8 \times 10^{-5} M_{pl}^2$

thus do not seem to be a very promising approach unless one expects a higher coupling to spins than the coupling induced by the electromagnetic interaction with the scalar field.

Chapter 4

Cosmological evolution of ϕ and its effects

4.1 Cosmological variation of the scalar field

The dominant cosmological effects on ϕ come from the interaction with matter and (in principle) the cosmological constant, and the effects of the expansion of the universe. However, if the cosmological constant (apart from the contribution of ϕ itself) is truly a constant, it and any interaction between it and ϕ can be included in $V(\phi)$. Indeed, one can define

$$V'(\phi) \equiv V(\phi) + \rho_\Lambda + V_{int}(\phi, \Lambda) \quad (4.1)$$

Thus even though (since I am mainly considering the scalar field in the context of a quintessence model) I am assuming that there is no separate cosmological constant and that the apparent cosmological constant is generated by $V(\phi)$, the calculation easily generalizes to the addition of a genuine cosmological constant to the model.

If the apparent cosmological constant is caused by a nonzero potential energy of the scalar field ϕ (assume that the kinetic energy from changes of ϕ are small due to ϕ changing only slowly), then $V(\phi) = \rho_\Lambda$ for the present value of ϕ .

If we consider small changes of ϕ , $V(\phi)$ can be approximated as linear, and we

can define M_Λ such that $V(\phi)$ is

$$V(\phi) \simeq \rho_\Lambda \left(1 + \frac{\phi}{M_\Lambda} \right) \quad (4.2)$$

near present values of ϕ , assuming we set the present value of ϕ to zero. We can do this using the transformation (1.6-1.10).

If we assume that ϕ is varying slowly enough that (4.2) is valid over cosmological timescales, then the evolution of ϕ resulting from (2.3) is

$$\square\phi = -\frac{\rho_m}{M_s} - \frac{\rho_\Lambda}{M_\Lambda} \quad (4.3)$$

Here since we are dealing with an expanding universe we must use the covariant derivative to construct the \square operator. $\square\phi$ is

$$\square\phi = D_\mu D^\mu \phi = g^{\mu\nu} D_\mu D_\nu \phi = \partial_\mu \partial^\mu \phi - g^{\mu\nu} \Gamma^\lambda_{\mu\nu} \partial_\lambda \phi \quad (4.4)$$

where D_μ is the covariant derivative. For the Robertson-Walker metric, $g^{\mu\nu} \Gamma^\lambda_{\mu\nu} = 3\frac{\dot{a}}{a}\delta_0^\lambda$, so $\square\phi = \ddot{\phi} - (1/a^2)\nabla^2\phi + 3H\dot{\phi}$ where $H = \frac{\dot{a}}{a}$. On a cosmological scale $\nabla^2\phi$ must average to 0 because by homogeneity we expect no net flow of $\nabla\phi$ across a distant bounding surface. Thus the evolution of ϕ is described by[11]

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\rho_m}{M_s} - \frac{\rho_\Lambda}{M_\Lambda} = -\frac{\rho_c}{M_s}\Omega_m \left(\frac{a_0}{a}\right)^3 - \frac{\rho_c}{M_\Lambda}\Omega_\Lambda. \quad (4.5)$$

Assuming that $V(\phi)$ changes little over cosmological timescales, and that the kinetic energy of ϕ is small relative to the potential, Λ CDM holds at least approximately. Also assuming that $\Omega_\Lambda + \Omega_M = 1$, we have the following expression for the evolution of the scale factor, with $b = \frac{3}{2}\Omega_\Lambda^{1/2}H_0$, H_0 and a_0 being the present values

of H and a respectively,

$$a(t)^3 = a_0^3 \frac{\Omega_M}{\Omega_\Lambda} [\sinh(bt)]^2. \quad (4.6)$$

This can easily be verified by plugging it in to the Friedman equations (for Λ CDM)

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_M \left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda \right) \quad (4.7)$$

$$\frac{\ddot{a}}{a} = H_0^2 \left(-\frac{\Omega_M}{2} \left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda \right). \quad (4.8)$$

Noting that

$$\dot{a} = \frac{2}{3} a_0 b \left(\frac{\Omega_M}{\Omega_\Lambda} \right)^{\frac{1}{3}} [\sinh(bt)]^{-\frac{1}{3}} \cosh(bt) \quad (4.9)$$

so

$$H = \frac{\dot{a}}{a} = \frac{2b \cosh(bt)}{3 \sinh(bt)} \quad (4.10)$$

we can integrate (4.5) by multiplying both sides by $\sinh(bt)^2$ to get

$$\sinh(bt)^2 \ddot{\phi} + 2b \sinh(bt) \cosh(bt) \dot{\phi} = -\frac{\rho_c}{M_s} \Omega_\Lambda - \frac{\rho_c}{M_\Lambda} \Omega_\Lambda \sinh(bt)^2 \quad (4.11)$$

$$\frac{d}{dt} \left(\sinh(bt)^2 \dot{\phi} \right) = -\frac{\rho_c}{M_s} \Omega_\Lambda - \frac{\rho_c}{M_\Lambda} \Omega_\Lambda \sinh(bt)^2 \quad (4.12)$$

$$\sinh(bt)^2 \dot{\phi} - t_c = -\frac{\rho_c}{M_s} \Omega_\Lambda t - \frac{\rho_c}{M_\Lambda} \Omega_\Lambda \left(-\frac{t}{2} + \frac{\sinh(2bt)}{4b} \right) \quad (4.13)$$

where t_c is a constant of integration.

So we have

$$\dot{\phi} = - \left(\frac{1}{M_s} - \frac{1}{2M_\Lambda} \right) \frac{\rho_c \Omega_\Lambda t}{\sinh(bt)^2} - \frac{\rho_c \Omega_\Lambda \cosh(bt)}{2b M_\Lambda \sinh(bt)} + \frac{t_c}{\sinh(bt)^2}. \quad (4.14)$$

In Appendix C I calculate that the t_c term is about twice the size of the other terms at matter-radiation equality and thus will be smaller by the times we are interested in (after decoupling).

Before integrating this to get the formula for ϕ itself, we can check the kinetic

energy. Observations of the accelerated expansion of the universe require that at the present time the kinetic energy is smaller than the cosmological constant. Setting $t_c = 0$ and approximating t_0 as $1/H_0$:

$$\dot{\phi} = - \left(\frac{1}{M_s} - \frac{1}{2M_\Lambda} \right) \frac{\rho_c \Omega_M}{H_0} - \frac{\rho_c}{3H_0 M_\Lambda} \quad (4.15)$$

$$= - \frac{\rho_c}{H_0} \left(\frac{\Omega_M}{M_s} + \frac{1}{M_\Lambda} \left(\frac{1}{3} - \frac{\Omega_M}{2} \right) \right) \quad (4.16)$$

The kinetic energy is $\frac{1}{2}\dot{\phi}^2$; requiring that it be smaller than ρ_Λ , and using $\rho_c = 3H_0^2 M_{pl}^2$ gives:

$$\Omega_\Lambda > \frac{3}{2} M_{pl}^2 \left(\frac{\Omega_M^2}{M_s^2} + 2 \frac{\Omega_M}{M_s} \left(\frac{1}{3M_\Lambda} - \frac{\Omega_M}{2M_\Lambda} \right) + \left(\frac{1}{3M_\Lambda} - \frac{\Omega_M}{2M_\Lambda} \right)^2 \right). \quad (4.17)$$

Since we already have a much stricter limit (2.25) on M_s , we can approximate M_s as infinite to get a limit on M_Λ of

$$|M_\Lambda| > \sqrt{\frac{3}{2\Omega_\Lambda}} \left(\frac{1}{3} - \frac{\Omega_M}{2} \right) M_{pl} = 0.3 M_{pl} \quad (4.18)$$

Now to derive the formula for ϕ : setting $t_c = 0$ for simplicity, integrating (4.14) results in

$$\begin{aligned} \phi - \phi_c &= \left(\frac{1}{M_s} - \frac{1}{2M_\Lambda} \right) \frac{\rho_c \Omega_\Lambda}{b^2} \left(\frac{bt \cosh(bt)}{\sinh(bt)} - \ln(\sinh(bt)) \right) \\ &\quad - \frac{\rho_c \Omega_\Lambda}{2b^2 M_\Lambda} \ln(\sinh(bt)) - \frac{t_c \cosh(bt)}{b \sinh(bt)} \end{aligned} \quad (4.19)$$

$$\begin{aligned} &= \left(\frac{1}{M_s} - \frac{1}{2M_\Lambda} \right) \frac{\rho_c \Omega_\Lambda t}{b} \coth(bt) \\ &\quad - \frac{1}{M_s} \frac{\rho_c \Omega_\Lambda}{b^2} \ln(\sinh(bt)) - \frac{t_c}{b} \coth(bt). \end{aligned} \quad (4.20)$$

Letting t_0 be the present time and recalling that we have chosen $\phi(t_0) = 0$,

$$\begin{aligned} \phi(t) = & \left(\frac{1}{M_s} - \frac{1}{2M_\Lambda} \right) \frac{\rho_c \Omega_\Lambda}{b} (t \coth(bt) - t_0 \coth(bt_0)) \\ & - \frac{1}{M_s} \frac{\rho_c \Omega_\Lambda}{b^2} \ln \frac{\sinh(bt)}{\sinh(bt_0)} - \frac{t_c}{b} (\coth(bt) - \coth(bt_0)). \end{aligned} \quad (4.21)$$

using $\rho_c = 3H_0^2 M_{pl}^2$,

$$\begin{aligned} \phi(t) = & \frac{4}{3} M_{pl}^2 \left(\frac{1}{M_s} - \frac{1}{2M_\Lambda} \right) (bt \coth(bt) - bt_0 \coth(bt_0)) \\ & - \frac{4}{3} \frac{M_{pl}^2}{M_s} \ln \frac{\sinh(bt)}{\sinh(bt_0)} - \frac{t_c}{b} (\coth(bt) - \coth(bt_0)) \end{aligned} \quad (4.22)$$

Now, given a z value, we can approximate $bt(z)$ from

$$z(t) + 1 = \frac{a(t_0)}{a(t)} = \left(\frac{\sinh(bt_0)}{\sinh(bt)} \right)^{\frac{2}{3}} \simeq \left(\frac{1.7}{\sinh(bt)} \right)^{\frac{2}{3}} \quad (4.23)$$

$$bt(z) \simeq \sinh^{-1} \frac{1.7}{(z+1)^{\frac{3}{2}}} \quad (4.24)$$

For $t \ll 1/b$, t_0 if $t_c = 0$,

$$\begin{aligned} \phi(t) & \simeq -\frac{4}{3} \frac{M_{pl}^2}{M_s} \ln \frac{\sinh(bt)}{\sinh(bt_0)} + \text{const} = -\frac{4}{3} \frac{M_{pl}^2}{M_s} \ln (1+z)^{-\frac{3}{2}} + \text{const} \\ & = 2 \frac{M_{pl}^2}{M_s} \ln(1+z) + \text{const} \end{aligned} \quad (4.25)$$

Carroll's paper[5] mentions a source at $z=2.012$ and $\Delta\chi = 2^\circ \pm 3^\circ$, originally analyzed in [26]

$$\Delta angle(z = 2.012) \leq 0.087 \quad (4.26)$$

Table 4.1: Observational limits from a $z=2$ source

Assumptions on M_s and M_Λ	limit
$M_s = M_\Lambda$	$ M_p M_s \gtrsim 43 M_{pl}^2$
$ M_\Lambda \rightarrow \infty$	$ M_p M_s \gtrsim 36 M_{pl}^2$
$ M_s \rightarrow \infty$	$ M_p M_\Lambda \gtrsim 7 M_{pl}^2$

Table 4.2: Observational limits from the CMB

Assumptions on M_s and M_Λ	limit
$M_s = M_\Lambda$	$ M_p M_s \gtrsim 261 M_{pl}^2$
$ M_\Lambda \rightarrow \infty$	$ M_p M_s \gtrsim 255 M_{pl}^2$

putting (4.22) (with $\Delta angle = (2/M_p)\phi$) into (4.26) with $t_c = 0$,

$$0.087 \geq \left| \frac{8}{3} \left(\frac{M_{pl}^2}{M_p M_s} - \frac{M_{pl}^2}{2M_p M_\Lambda} \right) (bt \coth(bt) - bt_0 \coth(bt_0)) - \frac{8}{3} \frac{M_{pl}^2}{M_p M_s} \ln \frac{\sinh(bt)}{\sinh(bt_0)} \right| \quad (4.27)$$

$$\gtrsim \left| \frac{8}{3} \left(\frac{M_{pl}^2}{M_p M_s} - \frac{M_{pl}^2}{2M_p M_\Lambda} \right) (1.03 - 1.51) + 4 \frac{M_{pl}^2}{M_p M_s} \ln(3.012) \right| \quad (4.28)$$

$$\gtrsim \left| 3.1 \frac{M_{pl}^2}{M_p M_s} + 0.6 \frac{M_{pl}^2}{M_p M_\Lambda} \right| \quad (4.29)$$

From this we can derive limits on M_p and M_s shown in Table 4.1.

Using a limit of $\Delta angle < 6^\circ$ at $z \simeq 1100$ from CMB polarization [27, 28], see also [29] for a weaker limit,

$$0.2 \gtrsim \left| \frac{8}{3} \left(\frac{M_{pl}^2}{M_p M_s} - \frac{M_{pl}^2}{2M_p M_\Lambda} \right) (1 - 1.51) + 4 \frac{M_{pl}^2}{M_p M_s} \ln(1100) \right| \quad (4.30)$$

$$\gtrsim \left| 26.7 \frac{M_{pl}^2}{M_p M_s} + 0.7 \frac{M_{pl}^2}{M_p M_\Lambda} \right|, \quad (4.31)$$

which results in the limits shown in Table 4.2.

These limits are far tighter than the limits obtained from experimental measurements of spins on Earth. They are also far tighter than the combined individual limits on M_p (1.12) and M_s (2.25).

Note that if we apply (4.25) to the expression (4.2) to derive the expected change of the dark energy density over time, we get

$$\frac{\Delta\rho_\Lambda}{\rho_\Lambda} = \frac{\Delta\phi}{M_\Lambda} = 2\frac{M_{pl}^2}{M_s M_\Lambda} \ln(1+z) \quad (4.32)$$

This could in principle be used to derive limits on $M_s M_\Lambda$ from limits on the change of the dark energy density, but there is already a tight limit on M_s .

If $V(\phi)$ is not approximately linear for $\phi \cong \phi_0$, but instead approaches a local minimum so that

$$V(\phi) \cong \rho_\Lambda + \frac{1}{2}m_\phi^2(\phi - \phi_0)^2, \quad (4.33)$$

where ϕ_0 is the local minimum and not necessarily exactly the present value of ϕ , then the equation for ϕ

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2(\phi - \phi_0) = -\frac{\rho_\Lambda}{M_s} \quad (4.34)$$

is basically a damped harmonic oscillator. If the frequency is large compared to H ($m_\phi \gg H$) then we can treat the RHS as approximately constant and define

$$\phi'_0 = \phi_0 - \frac{\rho_m}{m_\phi^2 M_s} \quad (4.35)$$

in order to absorb it. Hence ϕ will oscillate around the slowly moving ϕ'_0 rather than the constant ϕ_0 that represents the bottom of the potential.

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2(\phi - \phi'_0) \cong 0 \quad (4.36)$$

If we also assume that H is approximately constant, then this is a standard damped harmonic oscillator with (approximately) constant coefficients with solution

$$\phi = \phi'_0 + ce^{-\frac{3}{2} \int H dt} \cos\left(\sqrt{m_\phi^2 - \frac{9}{4}H^2}t + \alpha\right) \quad (4.37)$$

where c and α are arbitrary constants. Noting the assumption that $m_\phi \gg H$ this becomes

$$\phi = \phi'_0 + (\phi_{in} - \phi'_0) \left(\frac{a_{in}}{a(t)}\right)^{\frac{3}{2}} \cos(m_\phi(t - t_{in})) \quad (4.38)$$

where t_{in} is chosen to be at a point at which the cosine is at a maximum, and ϕ_{in} and a_{in} are the values of ϕ and a at that point.

The amplitude of the fluctuations will be determined by matching the solution to the overdamped region in which $H_0 > m_\phi$ through a critically damped region in which ϕ decreases as $a^{-3/2}$ with no oscillation. It seems likely that they will tend to be small.

The displacement of ϕ'_0 from ϕ_0 effectively causes an increase in the potential of $\frac{\rho_m^2}{2m_\phi^2 M_s^2}$. In order for this to be significant compared to ρ_Λ we would need m_ϕ to be almost as small as $\frac{\sqrt{\rho_m}}{2M_s}$. Given that $\rho_\Lambda \sim 10^{-120} M_{pl}^4$ and from (2.25) $M_s > 400 M_{pl}$, we would need m_ϕ almost as small as $10^{-63} M_{pl}$. Since the Hubble constant is around $6 \times 10^{-61} M_{pl}$, this clearly requires $m_\phi < H_0$. So given that the oscillating solution requires $m_\phi > H$, the displacement of the equilibrium point only leads to a small change of ρ_Λ .

Chapter 5

Conclusions

In this thesis I looked at the implications of a light scalar field interacting linearly with mass and with a pseudoscalar interaction with light and with fermion spins. A limit on the gravitational parameter γ from Cassini implies that M_s is conservatively more than 400 times the Planck mass, a tight limit on the interaction with mass. I derived to the order of a single loop the coupling of the scalar field to particle spins induced by the interaction with light, and also estimated the additional coupling to atoms from the interaction of the field with electromagnetic fields within the atom. I compared the calculated interactions of atom or electron spins with the gradient of the scalar field to experimental limits to constrain the strengths of the interaction of the field. This is the only way I know of to constrain the interaction with spin, but this does not provide good constraints on the interaction with light.

I derived the initial state of the field at the beginning of the matter dominated era by finding its behaviour in the radiation dominated era, and found a limit on the interaction strength with the cosmological constant (or in the case that the field is a quintessence field and there is no cosmological constant, a limit on the slope of the potential). Using the observational limits on polarization rotation I found a limit on the product of the strengths of interaction with light and with mass. I found this limit to be much tighter than the limits derived from local measurements.

If there is no significant evolution of the field over time, then the limits derived from polarization rotation would not apply, but the separate limits on the scalar and pseudoscalar interaction strengths still would apply as long as the mass is sufficiently small that the interaction range is at least \gtrsim AU (for the scalar) or \gtrsim 200 m (for the pseudoscalar).

The limits on the interaction with mass are sufficiently tight to make it seem somewhat unnatural, but a near-symmetry under shifts in the value of the field (as would be expected if it were a pseudo-Goldstone boson) would keep the interaction weak. For a quintessence field such an approximate symmetry would also help to explain the flatness of the potential. The mass interaction could also potentially serve as a calculational proxy for other interactions roughly proportional to mass. Any such interaction would also have to be very small.

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Appendix A

Jordan to Einstein frame using a conformal transformation

The metric used in the Jordan-Brans-Dicke action is known as the Jordan frame:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{pl}^2}{2} \phi R - \frac{\omega}{\phi} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} + \mathcal{L}_M \right) \quad (\text{A.1})$$

In order to get ordinary general relativity (the Einstein frame) with a separate scalar field, we can scale the metric by $g_{ab} \rightarrow \bar{g}_{ab} = \phi g_{ab}$ in order to eliminate the ϕ from ϕR [14]. It should be noted that in order for the two frames to be physically equivalent, it is necessary for the units used in the Einstein frame to also be scaled[16,

17, 30]. Under this transformation:

$$g_{ab} \rightarrow \bar{g}_{ab} = \phi g_{ab} \quad (\text{A.2})$$

$$g^{ab} \rightarrow \bar{g}^{ab} = \frac{g^{ab}}{\phi} \quad (\text{A.3})$$

$$\sqrt{-g} \rightarrow \sqrt{-\bar{g}} = \phi^2 \sqrt{-g} \quad (\text{A.4})$$

$$\partial_\mu \rightarrow \bar{\partial}_\mu = \partial_\mu \quad (\text{A.5})$$

$$\partial^\mu \rightarrow \bar{\partial}^\mu = \bar{g}^{\mu\nu} \bar{\partial}_\nu = \frac{\partial^\mu}{\phi} \quad (\text{A.6})$$

$$\partial^\mu \partial_\mu \rightarrow \bar{\partial}^\mu \bar{\partial}_\mu = \frac{\partial^\mu \partial_\mu}{\phi} \quad (\text{A.7})$$

$$\partial_\mu \partial^\mu \rightarrow \bar{\partial}_\mu \bar{\partial}^\mu = \partial_\mu \frac{\partial^\mu}{\phi} = \frac{\partial_\mu \partial^\mu}{\phi} - \frac{(\partial_\mu \phi)}{\phi} \frac{\partial^\mu}{\phi} \quad (\text{A.8})$$

$$\begin{aligned} \Gamma^\mu{}_{\nu\rho} \rightarrow \bar{\Gamma}^\mu{}_{\nu\rho} &= \frac{1}{2} \bar{g}^{\mu\lambda} (\bar{g}_{\lambda\nu,\rho} + \bar{g}_{\lambda\rho,\nu} - \bar{g}_{\nu\rho,\lambda}) \\ &= \Gamma^\mu{}_{\nu\rho} + \frac{1}{2} \left(\delta_\nu^\mu \frac{\phi_{,\rho}}{\phi} + \delta_\rho^\mu \frac{\phi_{,\nu}}{\phi} - g_{\nu\rho} \frac{\phi^{,\mu}}{\phi} \right) \end{aligned} \quad (\text{A.9})$$

As the difference

$$\bar{\Gamma}^\mu{}_{\nu\rho} - \Gamma^\mu{}_{\nu\rho} = \frac{1}{2} (\delta_\nu^\mu \nabla_\rho + \delta_\rho^\mu \nabla_\nu - g_{\nu\rho} \nabla^\mu) \ln \phi \quad (\text{A.10})$$

is a tensor, the ordinary rules of covariant differentiation will apply to it. Note that in this appendix $\bar{\nabla}$ refers to the covariant derivative whereas elsewhere I use ∇ for the spatial derivative and D for the covariant derivative. Comparing the difference between the new and old Ricci tensor

$$\bar{R}_{ab} - R_{ab} = \partial_c (\bar{\Gamma}^c{}_{ab} - \Gamma^c{}_{ab}) - \partial_a (\bar{\Gamma}^c{}_{bc} - \Gamma^c{}_{bc}) + \bar{\Gamma}^c{}_{cd} \bar{\Gamma}^d{}_{ab} - \Gamma^c{}_{cd} \Gamma^d{}_{ab} - \bar{\Gamma}^d{}_{bc} \bar{\Gamma}^c{}_{ad} + \Gamma^d{}_{bc} \Gamma^c{}_{ad} \quad (\text{A.11})$$

to $\nabla_c (\bar{\Gamma}^c{}_{ab} - \Gamma^c{}_{ab})$, $\bar{\nabla}_c (\bar{\Gamma}^c{}_{ab} - \Gamma^c{}_{ab})$, $\nabla_a (\bar{\Gamma}^c{}_{bc} - \Gamma^c{}_{bc})$, and $\bar{\nabla}_a (\bar{\Gamma}^c{}_{bc} - \Gamma^c{}_{bc})$, it turns

out that it is equal to

$$\bar{R}_{ab} - R_{ab} = \frac{1}{2} [(\bar{\nabla}_c + \nabla_c) (\bar{\Gamma}^c_{ab} - \Gamma^c_{ab}) - (\bar{\nabla}_a + \nabla_a) (\bar{\Gamma}^c_{bc} - \Gamma^c_{bc})] \quad (\text{A.12})$$

$$= \frac{1}{4} [(\bar{\nabla}_c + \nabla_c) (\delta_a^c \partial_b + \delta_b^c \partial_a - g_{ab} \partial^c) \ln \phi - (\bar{\nabla}_a + \nabla_a) (4 \partial_b \ln \phi)] \quad (\text{A.13})$$

$$= \frac{1}{4} [(\bar{\nabla}_b + \nabla_b) \partial_a \ln \phi - (\bar{\nabla}_a + \nabla_a) (3 \partial_b \ln \phi) - \bar{g}_{ab} \bar{\square} \ln \phi - g_{ab} \square \ln \phi]. \quad (\text{A.14})$$

The Ricci scalar transforms as

$$R \rightarrow \bar{R} = \bar{g}^{ab} \bar{R}_{ab} \quad (\text{A.15})$$

$$= \frac{g^{ab}}{\phi} R_{ab} + \frac{g^{ab}}{\phi} (\bar{R}_{ab} - R_{ab}) \quad (\text{A.16})$$

$$= \frac{R}{\phi} - \frac{3}{2} \bar{\square} \ln \phi - \frac{3}{2\phi} \square \ln \phi. \quad (\text{A.17})$$

To relate $\bar{\square} \ln \phi$ and $\square \ln \phi$,

$$\bar{\square} \ln \phi - \frac{\square}{\phi} \ln \phi = \bar{\nabla}_\mu \bar{\partial}^\mu \ln \phi - \frac{1}{\phi} \nabla_\mu \partial^\mu \ln \phi \quad (\text{A.18})$$

$$= \bar{\partial}_\mu \bar{\partial}^\mu \ln \phi - \frac{1}{\phi} \partial_\mu \partial^\mu \ln \phi + \bar{\Gamma}^\mu_{\mu\lambda} \bar{\partial}^\lambda \ln \phi - \frac{1}{\phi} \Gamma^\mu_{\mu\lambda} \partial^\lambda \ln \phi \quad (\text{A.19})$$

$$= -\frac{(\partial \ln \phi)^2}{\phi} + \frac{1}{\phi} (\bar{\Gamma}^\mu_{\mu\lambda} - \Gamma^\mu_{\mu\lambda}) \partial^\lambda \ln \phi \quad (\text{A.20})$$

$$= -(\bar{\partial} \ln \phi)^2 + \frac{2}{\phi} (\partial \ln \phi)^2 \quad (\text{A.21})$$

$$= (\bar{\partial} \ln \phi)^2. \quad (\text{A.22})$$

The action thus becomes

$$S = \int d^4x \sqrt{-\bar{g}} \left(\frac{M_{pl}^2}{2} \bar{R} + \frac{3}{2} \frac{M_{pl}^2}{2} \bar{\square} \ln \phi + \frac{3}{2} \frac{M_{pl}^2}{2} \frac{\square \ln \phi}{\phi} - \omega \frac{\bar{\partial}_\mu \phi \bar{\partial}^\mu \phi}{\phi^2} + \frac{\mathcal{L}_M}{\phi^2} \right) \quad (\text{A.23})$$

$$= \int d^4x \sqrt{-\bar{g}} \left(\frac{M_{pl}^2}{2} \bar{R} + 3 \frac{M_{pl}^2}{2} \bar{\square} \ln \phi - \frac{3}{2} \frac{M_{pl}^2}{2} (\bar{\partial} \ln \phi)^2 - \omega (\bar{\partial} \ln \phi)^2 + \frac{\mathcal{L}_M}{\phi^2} \right). \quad (\text{A.24})$$

Substituting $\phi = e^\alpha$, and noting that the $\bar{\square}$ term when integrated amounts to an integral of $\bar{\partial} \ln(\phi)$ over a boundary surface, and hence can be ignored, the action can now be expressed as

$$S = \int d^4x \sqrt{-\bar{g}} \left(\frac{M_{pl}^2}{2} \bar{R} - \left(\omega + \frac{3}{2} \frac{M_{pl}^2}{2} \right) (\bar{\partial} \alpha)^2 + e^{-2\alpha} \mathcal{L}_M \right), \quad (\text{A.25})$$

confirming the result stated by Brans[14].

Appendix B

Single loop calculation of induced interaction with spin

In this appendix I will evaluate the amplitude

$$\mathcal{M} = \frac{2ie^2}{M_p} \epsilon_{\mu\nu\rho\sigma} \bar{u}(p_f, s) \gamma^\nu \gamma^\tau \gamma^\sigma u(p_i, s) q^\mu \int \frac{d^4k}{(2\pi)^4} \frac{(k_\tau - p_\tau)(p^\rho - k^\rho)}{(p_f - k)^2 (k^2 - m^2) (p_i - k)^2} \quad (\text{B.1})$$

The integral is logarithmically UV-divergent, as the integrand is $\sim 1/k^4$ at large k . To evaluate the integral, I will use Feynman parametrization[22].

$$\frac{1}{((p_f - k)^2 + i\epsilon)(k^2 - m^2 + i\epsilon)((p_i - k)^2 + i\epsilon)} = \int_0^1 dx dy dz \delta(x + y + z - 1) \frac{2}{D^3} \quad (\text{B.2})$$

where

$$D = x(k - p_f)^2 + y(k^2 - m^2) + z(k - p_i)^2 + (x + y + z)i\epsilon \quad (\text{B.3})$$

let

$$l = k - xp_f - zp_i \quad (\text{B.4})$$

then

$$D = l^2 - \Delta + i\epsilon \quad (\text{B.5})$$

where

$$\Delta = -xz(p_f - p_i)^2 + y^2m^2 = -xzq^2 + y^2m^2 \quad (\text{B.6})$$

If this interaction is with a virtual ϕ particle, such that there exists a frame in which there is no energy transferred by the ϕ particle, then q^2 will be negative. Thus $\Delta > 0$. I will make the assumption that $q^2 < 0$ from now on since it is valid for the interactions I am interested in.

The numerator can be re-expressed as follows, dropping terms that will cancel due to symmetry in l :

$$\begin{aligned} (k_\tau - p_\tau)(p^\rho - k^\rho) &= -(k_\tau - \frac{1}{2}(p_{f\tau} + p_{i\tau}))(k^\rho - \frac{1}{2}(p_f^\rho + p_i^\rho)) \\ &= -(l_\tau + (x - \frac{1}{2})p_{f\tau} + (z - \frac{1}{2})p_{i\tau})(l^\rho + (x - \frac{1}{2})p_f^\rho + (z - \frac{1}{2})p_i^\rho) \\ &= -\frac{1}{4}\delta_\tau^\rho l^2 - ((x - \frac{1}{2})(x - \frac{1}{2})p_{f\tau}p_f^\rho + (x - \frac{1}{2})(z - \frac{1}{2})p_{f\tau}p_i^\rho \\ &\quad + (z - \frac{1}{2})(x - \frac{1}{2})p_{i\tau}p_f^\rho + (z - \frac{1}{2})(z - \frac{1}{2})p_{i\tau}p_i^\rho) \end{aligned} \quad (\text{B.7})$$

I will first evaluate the l^2 term, which gives the logarithmically divergent part of the integral. I will use a fictitious heavy photon with mass Λ as a Pauli-Villars regulator.[22]

Instead of

$$\int \frac{d^4l}{(2\pi)^4} \frac{l^2}{(l^2 - \Delta)^3} \quad (\text{B.8})$$

,

we now have

$$\int \frac{d^4 l}{(2\pi)^4} \left(\frac{l^2}{(l^2 - \Delta)^3} - \frac{l^2}{(l^2 - \Delta_1)^3} - \frac{l^2}{(l^2 - \Delta_2)^3} + \frac{l^2}{(l^2 - \Delta_3)^3} \right) \quad (\text{B.9})$$

where $\Delta_1 = \Delta + x\Lambda^2$, $\Delta_2 = \Delta + z\Lambda^2$ and $\Delta_3 = \Delta + (x+z)\Lambda^2$

By Wick-rotating[22] this we get

$$\begin{aligned} \frac{i}{(4\pi)^2} \int_0^\infty dl_E^2 \left(\frac{l_E^4}{(l_E^2 + \Delta)^3} - \frac{l_E^4}{(l_E^2 + \Delta_1)^3} - \frac{l_E^4}{(l_E^2 + \Delta_2)^3} + \frac{l_E^4}{(l_E^2 + \Delta_3)^3} \right) \\ = \frac{i}{(4\pi)^2} \ln \left(\frac{\Delta_1 \Delta_2}{\Delta \Delta_3} \right) \end{aligned} \quad (\text{B.10})$$

plugging this in to (B.1) (and using (B.2) and (B.7)) we get for the logarithmically divergent contribution to the amplitude:

$$\begin{aligned} \mathcal{M}_{logdiv} &= \frac{e^2}{M_p} \epsilon_{\mu\nu\rho\sigma} \bar{u}(p_f, s) \gamma^\nu \gamma^\tau \gamma^\sigma u(p_i, s) q^\mu \delta_\tau^\rho \frac{1}{(4\pi)^2} \int_0^1 dx dy dz \delta(x+y+z-1) \ln \left(\frac{\Delta_1 \Delta_2}{\Delta \Delta_3} \right) \\ &= \frac{3ie^2}{8\pi^2 M_p} q_\mu \bar{u}(p_f, s) \gamma^\mu \gamma^5 u(p_i, s) \int_0^1 dx dy dz \delta(x+y+z-1) \ln \left(\frac{\Delta_1 \Delta_2}{\Delta \Delta_3} \right) \end{aligned} \quad (\text{B.11})$$

The xyz integral looks difficult to integrate exactly but the logarithm takes values of the order $\ln(\Lambda^2/m^2) = 2\ln(\Lambda/m)$, for typical values of x,y and z. For specially chosen values it can be as low as 0 or as high as $\ln(\Lambda^2/-q^2)$. However the area in which the $-q^2$ term in Δ is larger than the m^2 term is $\sim \sqrt{-q^2/m^2}$, so the contribution of $-q^2$ is small for $-q^2 \ll m^2$.

For generic values of x, y and z, assuming $\Lambda^2 \gg m^2 \gg -q^2$, we can approximate

the logarithm as

$$\ln\left(\frac{\Delta_1\Delta_2}{\Delta\Delta_3}\right) = \ln\left(1 + \frac{xz\Lambda^4}{\Delta^2 + (x+z)\Lambda^2\Delta}\right) \quad (\text{B.12})$$

$$\cong \ln\left(\frac{xz\Lambda^4}{(x+z)\Lambda^2y^2m^2\left(1 + \frac{xz-q^2}{y^2m^2}\right)}\right) \quad (\text{B.13})$$

$$\cong 2\ln\left(\frac{\Lambda}{m}\right) + \ln\left(\frac{xz}{y^2}\right) - \ln(x+z) - \ln\left(1 + \frac{xz-q^2}{y^2m^2}\right) \quad (\text{B.14})$$

The integral of the terms not including the last is $\cong \frac{1}{4} + \ln\left(\frac{\Lambda}{m}\right)$. The integral of $\ln\left(1 + \frac{xz-q^2}{y^2m^2}\right)$ is bounded above by

$$\int_0^1 dy \ln\left(1 + \frac{1}{4y^2} \frac{-q^2}{m^2}\right) = \ln\left(1 + \frac{-q^2}{4m^2}\right) + \sqrt{\frac{-q^2}{m^2}} \arctan\left(\frac{1}{2}\sqrt{\frac{m^2}{-q^2}}\right) \quad (\text{B.15})$$

which approaches $\frac{\pi}{2}\sqrt{\frac{-q^2}{m^2}}$ for small $-q^2/m^2$.

So we have

$$\mathcal{M}_{\log div} \cong \frac{3ie^2}{8\pi^2 M_p} q_\mu \bar{u}(p_f, s) \gamma^\mu \gamma^5 u(p_i, s) \left(\frac{1}{4} + \ln\left(\frac{\Lambda}{m}\right) - O\left(\sqrt{-q^2/m^2}\right)\right) \quad (\text{B.16})$$

which of course includes a non-divergent component.

For the non-divergent terms, taking into account the x-z symmetry of the integral,

$$\begin{aligned} & \int_0^1 dx dy dz \delta(x+y+z-1) \left((x-\frac{1}{2})(x-\frac{1}{2}) p_{f\tau} p_f^\rho + (x-\frac{1}{2})(z-\frac{1}{2}) p_{f\tau} p_i^\rho \right. \\ & \quad \left. + (z-\frac{1}{2})(x-\frac{1}{2}) p_{i\tau} p_f^\rho + (z-\frac{1}{2})(z-\frac{1}{2}) p_{i\tau} p_i^\rho \right) \\ &= \int_0^1 dx dy dz \delta(x+y+z-1) \left(x-\frac{1}{2} \right) \left((x-\frac{1}{2}) (p_{f\tau} p_f^\rho + p_{i\tau} p_i^\rho) \right. \\ & \quad \left. + (z-\frac{1}{2}) (p_{f\tau} p_i^\rho + p_{i\tau} p_f^\rho) \right) \quad (\text{B.17}) \end{aligned}$$

Next, let us multiply this by some of the factors outside the integral.

$$\epsilon_{\mu\nu\rho\sigma}\gamma^\nu\gamma^\tau\gamma^\sigma = -2i(\delta_\mu^\tau\gamma_\rho - \delta_\rho^\tau\gamma_\mu)\gamma^5 \quad (\text{B.18})$$

so, using the momentum space Dirac equation and adjoint equation $\not{p}_i u(p_i) = m u(p_i)$ and $\bar{u}(p_f)\not{p}_f = m\bar{u}(p_f)$,

$$\begin{aligned} & \epsilon_{\mu\nu\rho\sigma}\bar{u}(p_f, s)\gamma^\nu\gamma^\tau\gamma^\sigma u(p_i, s)(p_{f\tau}p_f^\rho + p_{i\tau}p_i^\rho) \\ &= 4im^2\bar{u}(p_f, s)\gamma_\mu\gamma^5 u(p_i, s) - 2i\bar{u}(p_f, s)(\not{p}_f p_{f\mu}\gamma^5 - \gamma^5\not{p}_i p_{i\mu})u(p_i, s) \\ &= 4im^2\bar{u}(p_f, s)\gamma_\mu\gamma^5 u(p_i, s) - 2imq_\mu\bar{u}(p_f, s)\gamma^5 u(p_i, s) \end{aligned} \quad (\text{B.19})$$

and

$$\begin{aligned} & \epsilon_{\mu\nu\rho\sigma}\bar{u}(p_f, s)\gamma^\nu\gamma^\tau\gamma^\sigma u(p_i, s)(p_{f\tau}p_i^\rho + p_{i\tau}p_f^\rho) \\ &= 4ip_f \cdot p_i\bar{u}(p_f, s)\gamma_\mu\gamma^5 u(p_i, s) - 2i\bar{u}(p_f, s)(\not{p}_f p_{i\mu}\gamma^5 - \gamma^5\not{p}_i p_{f\mu})u(p_i, s) \\ &= 4i(m^2 - \frac{q^2}{2})\bar{u}(p_f, s)\gamma_\mu\gamma^5 u(p_i, s) + 2imq_\mu\bar{u}(p_f, s)\gamma^5 u(p_i, s) \end{aligned} \quad (\text{B.20})$$

To get the γ^5 terms in the same form ($\gamma_\mu\gamma^5$) as the other terms, multiply by the q^μ factor to eliminate μ and then reintroduce it.

$$\begin{aligned} 2imq_\mu\bar{u}(p_f, s)\gamma^5 u(p_i, s)q^\mu &= 2imq^2\bar{u}(p_f, s)\gamma^5 u(p_i, s) \\ &= iq^2\bar{u}(p_f, s)(\gamma^5\not{p}_i + \not{p}_f\gamma^5)u(p_i, s) \\ &= iq^2\bar{u}(p_f, s)\gamma_\mu\gamma^5 u(p_i, s)q^\mu \end{aligned} \quad (\text{B.21})$$

This makes (B.19) and (B.20) each become

$$4i(m^2 - \frac{q^2}{4})\bar{u}(p_f, s)\gamma_\mu\gamma^5 u(p_i, s) \quad (\text{B.22})$$

The non-(UV)divergent contribution to the amplitude will be

$$\begin{aligned}
\mathcal{M}_{nondiv} &= \frac{16e^2}{M_p} \left(m^2 - \frac{q^2}{4}\right) \bar{u}(p_f, s) \gamma_\mu \gamma^5 u(p_i, s) q^\mu \int \frac{d^4 l}{(2\pi)^4} \times \\
&\quad \times \int_0^1 dx dy dz \delta(x + y + z - 1) \frac{(x - \frac{1}{2})(x + z - 1)}{(l^2 - \Delta + i\epsilon)^3} \\
&= \frac{-ie^2}{2\pi^2 M_p} \left(1 + \frac{-q^2}{4m^2}\right) \bar{u}(p_f, s) \gamma_\mu \gamma^5 u(p_i, s) q^\mu I\left(\frac{-q^2}{m^2}\right) \quad (\text{B.23})
\end{aligned}$$

where I did the l integral by looking up the formula in Peskin and Schroeder[22], and

$$I\left(\frac{-q^2}{m^2}\right) = \int_0^1 dx dy dz \delta(x + y + z - 1) \frac{(x - \frac{1}{2})(x + z - 1)}{xz \frac{-q^2}{m^2} + y^2} \quad (\text{B.24})$$

Earlier I sacrificed the explicit symmetry under the exchange of x with z in order to reduce the number of terms. Restoring it, this integral becomes

$$I\left(\frac{-q^2}{m^2}\right) = \int_0^1 dx dy dz \delta(x + y + z - 1) \frac{\frac{1}{2}(x + z - 1)^2}{xz \frac{-q^2}{m^2} + y^2} \quad (\text{B.25})$$

$$= \int_0^1 dy \int_0^{1-y} dx \frac{\frac{1}{2}y^2}{x(1-x-y) \frac{-q^2}{m^2} + y^2} \quad (\text{B.26})$$

which clearly integrates to $\frac{1}{4}$ for $q^2 = 0$. To estimate how much a nonzero q^2 value reduces this I will rescale x :

$$= \int_0^1 dy \int_0^1 dx \frac{\frac{1}{2}y^2(1-y)}{x(1-x)(1-y)^2 \frac{-q^2}{m^2} + y^2}. \quad (\text{B.27})$$

Since $x(1-x) \leq \frac{1}{4}$, for small $\frac{-q^2}{m^2}$ the first term in the denominator is larger than or equal to the other only for $y < \frac{1}{2} \sqrt{\frac{-q^2}{m^2}}$. For larger values of y the second term will grow quadratically and the first term will shrink. Thus the effect of nonzero q^2 is $O(\sqrt{-q^2/m^2})$.

The final non-divergent component of the amplitude (not including the non-divergent part of the "divergent" part) is

$$\mathcal{M}_{nondiv} = \frac{-ie^2}{2\pi^2 M_p} \left(1 + \frac{-q^2}{4m^2}\right) \bar{u}(p_f, s) \gamma_\mu \gamma^5 u(p_i, s) q^\mu \left(\frac{1}{4} - O\left(\sqrt{\frac{-q^2}{m^2}}\right)\right) \quad (\text{B.28})$$

$$= \frac{-ie^2}{8\pi^2 M_p} \bar{u}(p_f, s) \gamma_\mu \gamma^5 u(p_i, s) q^\mu \left(1 - O\left(\sqrt{\frac{-q^2}{m^2}}\right)\right) \quad (\text{B.29})$$

putting together (B.16) and (B.29), we get a final amplitude of

$$\mathcal{M} = \frac{3ie^2}{8\pi^2 M_p} \bar{u}(p_f, s) \gamma_\mu \gamma^5 u(p_i, s) q^\mu \left(\ln\left(\frac{\Lambda}{m}\right) - \frac{1}{12} - O\left(\sqrt{\frac{-q^2}{m^2}}\right)\right) \quad (\text{B.30})$$

where the O notation is intended to indicate the dependence on q only. I am less concerned about inaccuracies due to the assumption that Λ/m is large, since Λ/m is constant.

Appendix C

Calculation of the initial conditions of the ϕ field

Olive and Pospelov[11] set the constant of integration t_c to be zero on the grounds that during the radiation dominated period the right side of (4.5) is small and that therefore $\dot{\phi}$ scales as a^{-3} during that period and thus should be very small by the end of that period (and that therefore we should chose the constant so as to make $\dot{\phi} \rightarrow 0$ as $t \rightarrow 0$).

According to Olive and Pospelov, the mass term $m\bar{\psi}\psi$ coincides with the ψ contribution to the trace of the stress-energy tensor, $\rho - 3p$. They go on to claim that since during the radiation dominated era $\rho = 3p$, only Λ will contribute during that time. However, for massive particles, ρ is only approximately equal to $3p$, even during the radiation era. The easiest way to calculate the real contribution is to note that $m\bar{\psi}\psi$ is Lorentz invariant, so the contribution is simply proportional to the number of particles[31]. Thus once a kind of matter particle has lost its corresponding antimatter, (or if it does not decay or annihilate much with its antimatter) its contribution will scale as a^{-3} ; and while it is still highly relativistic (with the antimatter still around) the contribution of the matter and antimatter together will scale as the component of energy density corresponding to the particle type divided by the en-

ergy per particle. Since one would expect an asymptotically constant fraction of the energy density for each particle type and a per particle energy scaling as a^{-1} , this corresponds to a contribution scaling as $a^{-4}/a^{-1} = a^{-3}$ in the ultra-relativistic regime as well. However in the interim between the ultra-relativistic and sub-relativistic regimes there will be an abrupt drop as matter-antimatter particle pairs annihilate and their energy is released to form lighter particles and radiation.

In the radiation era $a \sim t^{-1/2}$, so a contribution that scales as a^{-3} scales as $t^{-3/2}$.

$$\ddot{\phi} + 3H\dot{\phi} = \frac{C}{t^{\frac{3}{2}}} \quad (\text{C.1})$$

$$\ddot{\phi}t^{\frac{3}{2}} + \frac{3t^{\frac{1}{2}}}{2}\dot{\phi} = C \quad (\text{C.2})$$

$$\frac{d}{dt} \left(\dot{\phi}t^{\frac{3}{2}} \right) = C \quad (\text{C.3})$$

$$\dot{\phi}t^{\frac{3}{2}} = Ct + C_1 \quad (\text{C.4})$$

$$\dot{\phi} = Ct^{-\frac{1}{2}} + C_1t^{-\frac{3}{2}} \quad (\text{C.5})$$

Each time the energy drops low enough for a particle type to annihilate with its antiparticles, if they do so quickly then this corresponds to a drop in the value of C ; the solutions can be matched by the choice of C_1 . The kinetic energy density of the ϕ field is $\frac{1}{2}\dot{\phi}^2$ which scales as

$$\frac{1}{2}C^2t^{-1} + CC_1t^{-2} + \frac{1}{2}C_1^2t^{-3} \quad (\text{C.6})$$

Note that C is fixed by the interaction strength and the history of the number of particles and antiparticles of each type. Since the radiation density scales as $a^{-4} = t^{-2}$ an initial value (at a small nonzero t) of the kinetic energy comparable to the radiation density will lead to a small value of C_1 , and this term will quickly become negligible. The C^2t^{-1} term represents a kind of attractor which is moving over time due to the drops in the value of C ; the value of $\dot{\phi}$ at the end of the radiation era is

determined by the history of the number of particles and antiparticles over time. The most important consideration however, is whether there was time for $\dot{\phi}$ to converge to the most recent attractor after the preceding shifts. A simple argument suggests that this will always be the case by the end of the radiation era as long as the rest mass density of the universe is comparable to the mass density at matter-radiation equality. Indeed, each time the value of C drops, the value of the integration constant C_1 rises in order to keep $\dot{\phi}$ continuous, and at the same time, the rest mass density drops by the same factor that C does, the excess energy going to radiation. But between shifts the ratio between the C_1 term in (C.5) and the C term scales as a^{-2} , whereas the ratio between the radiation and matter densities scales as a^{-1} . Thus by the time matter-radiation equality can be reached the C_1 term will have become negligible relative to the C term.

Thus, the value of $\dot{\phi}$ at the end of the radiation era is determined by the value of the C term at the end of the radiation era, and hence is determined by the interaction strength and matter density towards the end of the radiation era:

$$\frac{C}{t^{\frac{3}{2}}} = -\frac{\rho_m}{M_s} \quad (\text{C.7})$$

so if $a = ct^{1/2}$ during the radiation era, then $C = -c^{-3}\rho_m a^3/M_s$. We can find c by matching the expression for a before and after matter-radiation equality. For simplicity at the cost of accuracy I will assume each is valid up to the point of equality. Since the current matter density is currently around 5360 times the CMB radiation density, matter radiation equality must have happened when a was 5360 times smaller. So,

$$\frac{1}{5360^3} = \frac{\Omega_M}{\Omega_\Lambda} [\sinh(bt_{eq})]^2 \cong \frac{\Omega_M}{\Omega_\Lambda} (bt_{eq})^2 = \frac{9}{4} \Omega_M (H_0 t_{eq})^2 \quad (\text{C.8})$$

$$t_{eq} = \frac{2}{3H_0} \frac{1}{5360^{\frac{3}{2}}} \frac{1}{\sqrt{\Omega_M}} = 3.3 \times 10^{-6} H_0^{-1} \quad (\text{C.9})$$

so,

$$\frac{a_0}{5360} = ct_{eq}^{\frac{1}{2}} = c \left(\frac{3.3 \times 10^{-6}}{H_0} \right)^{\frac{1}{2}} \quad (\text{C.10})$$

$$c = 0.1 a_0 H_0^{1/2}. \quad (\text{C.11})$$

Again assuming the validity of $a = ct^{1/2}$ right up to t_{eq} , at t_{eq} we have

$$\dot{\phi}(t_{eq}) = Ct_{eq}^{-1/2} \quad (\text{C.12})$$

$$= -(0.1 a_0 H_0^{1/2})^{-3} \frac{\rho_m a^3}{M_s \sqrt{3.3 \times 10^{-6} H_0^{-1}}} \quad (\text{C.13})$$

$$= 5 \times 10^5 \frac{\rho_m a^3}{M_s a_0^3} H_0^{-1}. \quad (\text{C.14})$$

Since $\rho_m a^3$ doesn't change over time, we can put in the present day values:

$$\dot{\phi}(t_{eq}) = 5 \times 10^5 \frac{\Omega_M \rho_c}{M_s H_0} \quad (\text{C.15})$$

Now, putting this into (4.14) for $t = t_{eq}$, using the approximations that (4.14) is valid all the way to t_{eq} , and that $\sinh(bt) = bt$ and $\cosh(bt) = 1$ for these small values of t ,

$$5 \times 10^5 \frac{\Omega_M \rho_c}{M_s H_0} = - \left(\frac{1}{M_s} - \frac{1}{2M_\Lambda} \right) \frac{\rho_c \Omega_\Lambda}{b^2 t_{eq}} - \frac{\rho_c \Omega_\Lambda}{2b^2 M_\Lambda t_{eq}} + \frac{t_c}{b^2 t_{eq}^2} \quad (\text{C.16})$$

$$= - \frac{\rho_c \Omega_\Lambda}{M_s b^2 t_{eq}} + \frac{t_c}{b^2 t_{eq}^2} \quad (\text{C.17})$$

It turns out that the non- t_c term on the right hand side equals exactly (given these approximations) minus one times the left hand side, so the t_c term is twice as large. However, by the time of CMB last scattering (which is the first time $\dot{\phi}$ begins affecting the change in ϕ between space-time points we can observe photons from) it will

already have dropped to less than the non- t_c term. So it seems that the assumption that $t_c = 0$ can be numerically justified.