ESTIMATING THE NAIRU:
An Analysis of the Bank of Canada Model

by

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ABSTRACT

The Bank of Canada proposes a model consisting of four equations to estimate simultaneously the NAIRU, the non-accelerating-inflation rate of unemployment. The model consists of an aggregate production function of the kind used in neo-classical growth models, an Okun's Law equation, a Phillips curve and a fourth equation that defines the NAIRU to be determined by Unemployment Insurance payments and demographic factors. This last equation is substituted into the system wherever the NAIRU appears. The Phillips curve as the centrepiece of the model is of the accelerationist type. The theoretical concepts underlying the Phillips curve and the production function lead to certain restrictions being imposed on the parameters of the model.

The importance of having robust and reliable estimates of the NAIRU and the controversial theoretical basis of the model motivate the analysis in this thesis of the restrictions and the estimates of the model. Less restricted model versions are specified and likelihood-ratio tests are used in order to test the restrictions. For the analysis of the estimates of the Bank of Canada model and of the less restricted versions of the Bank of Canada model estimated here, expected values are

determined on the basis of theoretical considerations and previous empirical studies. In addition, NAIRUs are generated with the estimates of the different models to see whether they are meaningful. The restrictions are mainly rejected but the less restricted models are not satisfactory in comparison to the Bank of Canada model because their parameter estimates often have the wrong sign or are out of the expected range. Furthermore, the NAIRUs generated have unrealistic values in many cases. On balance, the results fail to support the theory underlying the model. This does not necessarily mean that the theory is wrong. It can very well be the case that the model does not appropriately reflect the theory. Nevertheless, the NAIRU estimates of the Bank of Canada model are not reliable and should be regarded with great care.

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I Introduction

After the second world war there was a consensus in most of the capitalist countries that unemployment is a hardship and can be fought by governments through engaging in aggregate demand management. The developments of the 1970's exhibiting unemployment that could not be reduced by the known tools of economic policy and that was accompanied by high inflation rates, destroyed this consensus. Nevertheless, the success of governments with respect to economic policy is still measured to a large extent by the unemployment rate and people expect governments to react to it.

While it is generally accepted that a zero rate of unemployment is not possible because a part of the labour force is engaged in job search at every point in time, the question of exactly what level of the unemployment rate should be regarded as being the "full employment rate of unemployment" remains to be answered. This rate is not only important for macroeconomic policy makers to base their decisions on according to their economic belief be it Keynesian, Monetarist or New Classical, but also crucial for all other individuals because they have to make their own economic decisions on this unemployment rate and the probable actions of the government and the central bank with respect to it.

According to a majority of economists this "full employment rate of unemployment" rate should occur when the economy is in general equilibrium. This is the case when all markets are cleared, economic agents correctly perceive the distribution of prices and wages throughout the economy and inflation is neither accelerating nor decelerating. "The full employment rate of unemployment" is, therefore, referred to as the NAIRU : the non-accelerating-inflation-rate-of-unemployment or simply the natural rate of unemployment.[1] Unfortunately, the NAIRU is unobservable and has to be estimated.

In a recent paper, the Bank of Canada compares different approaches to the estimation of the NAIRU and proposes one of their own.[2] This procedure is based on a Phillips curve, relating wage inflation to the difference of the actual and natural rates of unemployment, price expectations and some other factors; an Okun's Law equation showing the link between actual and potential output using the difference between the actual and natural rates of unemployment; a production function relating capital and labour input to output and a behaviourial relationship between the NAIRU and demographical

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[1]Note that the equilibrium approach and the concept of a natural rate is open to critique as may be seen below.

[2]D.Rose, The NAIRU in Canada:Concepts, determinants and estimates, Bank of Canada, Technical Report No. 50, Ottawa 1988

factors and unemployment insurance benefits. This behaviourial relationship is substituted into the other equations for the NAIRU and then the system is estimated simultaneously. Using two different sample periods, the results yield NAIRU estimates of 8% or 9.3% for the fourth quarter of 1987.

If the Bank of Canada model were able to yield robust and reliable estimates of the NAIRU it would be an important tool to analyze the state and the functioning of the economy and to design economic policy as well. In addition, the theoretical concepts underlying the model are controversial so that any analysis of the model can also lead to conclusions about the theory the model is based on. The importance of having a good estimate of the equilibrium unemployment rate or NAIRU and the theoretical dispute about it are motivating this thesis. Therefore, the aim of this thesis is to analyze the Bank of Canada model with respect to the underlying theory in order to find out whether it generates reliable estimates or not and to draw conclusions about the theory, if possible.

In the first place, theoretical concepts that are related to the model will be discussed in order to describe the theory underlying the model as well as to describe alternative approaches to give appropriate background information. A main focus will be on the Phillips curve which is regarded as the theoretical centrepiece of the model. Then, it will be shown how theory leads to certain parameter restrictions of the model. These restrictions are imposed on the parameters related to the NAIRU and on some other parameters. After reproducing the Bank of Canada results[3], the restrictions will be tested by likelihood-ratio tests. In order to do this less restricted versions of the Bank of Canada model will be used. In another step, expected values of the parameter estimates in the different models are analyzed. The estimates are compared with the expected values in order to decide whether they are reasonable or out of range. This is done in order to obtain information about the various models additional to the likelihood ratio tests.

The paper is organized in three parts. At the very beginning (Chapter II) the Bank of Canada Model is outlined and its variables are explained. The second part of the analysis (Chapter III) discusses theoretical approaches related to the NAIRU to give the reader a background as well as to determine which part of the theory the model is based on, since the restrictions imposed upon the model are developed from theory. The presentation of the theory is

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[3]The estimation has been realized with the same software package the Bank of Canada used, TSP. The programm has been slightly changed in order to make estimation possible with the TSP version available on the University of Victoria mainframe. See B.H.Hall, R.Schnake, C.Cummins, Time Series Processor Version 4.1, User's Manual, Palo Alto 1987 and B.H.Hall, R.Schnake, C.Cummins, Time Series Processor Version 4.1., Reference Manual, Palo Alto 1986 for a reference.

centred on the Phillips Curve as a key relationship in understanding and modelling the supply side of the macro economy. The development of the concept and alternative approaches are described.

The literature used with respect to the Phillips Curve contains among others an article by Lipsey[4], that summarizes the Phillips curve discussion from a Keynesian point of view and also an article by Santomero and Seater[5] that surveys the main features of the research up to 1978. Friedman's article about the role of monetary policy is to be mentioned here as well.[6]This part will also describe the discussion about search and contract theory and present an approach by Okun[7] based on the so called price tag economy. It will also contain a post-Keynesian model proposed by Tuscherer[8].

Chapter III also presents theoretical background about the

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[4]R.G.Lipsey, "The Place of the Phillips Curve in Macroeconomic Models", Stability and Inflation, A.R.Bergstrom, et al. (eds), Chichester, 1978

[5]A.J.Santomero and J.J.Seater, "The Inflation-Unemployment Trade-Off: A Critique of the Literature", Journal of Economic Literature, Vol.16 (1978), p. 499-544.

[6]M.Friedman, "The Role of Monetary Policy", The American Economic Review, Vol.58 (1968), p.4-17

[7]A.M.Okun, Prices and Quantities, The Brookings Institution, Washington, 1981

[8]T.Tuscherer, "The Unnatural 'Natural' Rate of Unemployment", Journal of Post-Keynesian Economics, Vol.4 (1981), p.25-31

other theoretical or technical concepts lying behind the model. The first concept discussed is Okun's Law which is mainly based on Okun's article[9] and the critiques on it. After that, the idea of an aggregate production function and its properties is surveyed. Following that the possible determinants of the NAIRU are presented. Important sources for this are an article by Cousineau[10] about unemployment insurance and labour market adjustments and the Bank of Canada paper by Rose. Chapter III concludes with an analysis of the theory underlying the Bank of Canada model and the restrictions required by the theory.

The third part of the thesis (Chapters IV and V) presents the estimation and test procedures in a detailed form and summarizes the conclusions of the study. The estimation results will be reported as well as the statistical properties of the estimates. The results will be compared with those of the Bank of Canada and interpreted on the basis of the theory outlined in Chapter III.

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[9] A.M.Okun, "Potential GNP: Its Measurement and Significance", 1962 Business Proceedings, Business and Economics Statistics Section, American Statistical Association.

[10] J.M.Cousineau, "Unemployment Insurance and Labour Market Adjustments", Income Distribution and Economic Security in Canada, F.Vaillancour(ed.), Toronto 1985.

II The Bank of Canada Model

The Bank of Canada model to estimate the NAIRU consists of four equations. The first equation is a production function, the second an Okun's Law equation and the third a Phillips curve. Equation four presents the determinants of the NAIRU and is substituted into the first three equations wherever the NAIRU appears.[11]

$$(1)\quad \log Y = \alpha_0 + (1 - \alpha_2)K + \alpha_2 LS + ETFP + \epsilon_1$$

$$(2)\quad RNU - NAIRU = \beta_1(RNU_{t-1} - NAIRU_{t-1}) + \beta_2(\log Y - \log\hat{Y}) + \epsilon_2$$ [12]

$$(3)\quad W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3(RNU - NAIRU) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$$

$$(4)\quad NAIRU = \delta_{0i} + \delta_{1i}DEM + \delta_{2i}UI$$

Note that the δ_{0i}, δ_{1i} and δ_{2i} may differ among equations according to the restriction imposed upon the system.

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[11] For a definition of the data and its sources see Appendix III.

[12] $\log\hat{Y} = \hat{\alpha}_0 + (1 - \hat{\alpha}_2)K + \hat{\alpha}_2 LS + ETFP$; note that in the equation (2) reported in the Rose and Ford paper there obviously is a typing error; the intercept coefficient β_0 does not appear in the computer programm of the Bank of Canada and should, in addition, not be there.See, D.Rose and R.Ford, "Estimates of the NAIRU Using an Extended Okun's Law", Bank of Canada Working Paper 89-3, Ottawa 1989., p.4.

The production function is of the Cobb-Douglas type. K denotes the input of capital in its logarithmic form, measured by capital stock data. LS is the log of the labour input when the economy is at its equilibrium. Labour input is defined as the product of the labour force population, labor force participation rate , trend average hours worked and the yearly number of weeks (i.e. 52) multiplied with (1- NAIRU)[13]. In other words, the labour input used in the Bank of Canada model consists of the total potential labour supply reduced by the amount of labour unemployed in equilibrium as given by the natural rate of unemployment. Thus, the right hand side of the production function represents an expression of potential output.[14] Since the dependent variable of the production function is the log of actual output the residual of the equation (1) can be used to define the output market gap of

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[13] LS = log(LF(1 - NAIRU)), where LF is a measure of the total labour force composed of a trend participation rate obtained from the Bank of Canada's RDXF econometric model, an estimate of the labour force population, provided by Statistics Canada, and trend average hours worked.

[14] Note that actual output is regressed on potential input here. This may cause the estimates of α_2 to be too low. Potential output can be probably better estimated individually than through using a production function in a simultaneous equation system (See for example J. Brox and M. Cluff, Potential Output and the Real GNP Gap, Canadian Statistical Review, January 1979). To do so one needs, however, an appropriate estimate of the natural rate of unemployment. Nevertheless, since potential output and the NAIRU are parallel phenomena it is an appealing feature of the Bank of Canada model that both are estimated simultaneously.

the Okun's Law equation.[15]

ETFP is a productivity term measured as the Solow residual of the described production function using actual capital and labour use, i.e. the labour supply is multiplied by (1 -RNU). ETFP is smoothed assuming that its growth is on the average equal to growth in productivity. It is determined prior to the estimation of the simultaneous model.[16]

The Okun's Law equation relates the labour market gap (RNU - NAIRU), actual minus natural rate of unemployment, to its past value and to the output gap which is the difference between actual and potential output. Potential output is the output that is produced when the unemployment rate is at its natural level.[17] The output gap simply is the residual of equation (1)[18]

The Phillips curve equation relates the rate of wage increase (W), expressed as the first difference of the log of

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[15]D.Rose and R.Ford (1989), op.cit., p.5.

[16]D.Rose and R.Ford (1989), op.cit., pp 4-5.

[17]Note that the whole concept of the natural rate of unemployment is contentious, see below chapter II

[18]This gives rise to an identification problem which is solved by imposing a diagonal variance-covariance matrix. See D.Rose and R.Ford (1989), op.cit. , pp. 5-6, especially note 7.

wages, to the following variables: 1) expected rate of inflation (P), estimated as a second order Almon-lag of the first difference of the log of the consumer price index; 2) the trend growth rate of average hours worked (H); 3) the labour market gap (RNU-NAIRU); 4) the growth rate of productivity measured as the first difference of the productivity term ETFP (G); 5) the dummy variable (AIB) that accounts for the Anti-Inflation-Board period; 6) the variable PREL is defined as a seven quarter moving average of the first difference of the log of the ratio of the GDP deflator divided by the CPI. It measures the expected difference between the growth of producer prices[19] and consumer prices and is "meant to capture the willingness-to-pay of firms, and might represent changes in Canada's international terms of trade or cyclical productivity effects"[20]. PREL together with the expected rate of inflation P makes this Phillips curve

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[19] Rose and Ford call it producer prices; input prices seems clearer here because it is meant to measure the prices faced by producers for the input they need for manufacturing;see D.Rose and R.Ford, op.cit., p.6

[20]ibid., PREL rises (declines) if the prices firms pay for their inputs are expected to increase (decrease) relatively to consumer prices; other things being equal, higher (lower) wages will be asked for and paid so that the rate of wage increae goes up (down) as a consequence. The terms of trade affect PREL through the change of the level of import prices together with the composition of the imports, i.e. the share of consumer goods. Cyclical productivity changes may have different impacts on input and consumer goods; rising productivity leads to wage increases and consequently to rises of the index that measures input prices whereas consumer prices may increase at a slower pace or decrease through rising supply brought about by higher productivity.

accelerationist.[21]

The last equation is not stochastic and relates the NAIRU to its explaining variables. These are, in this model, DEM, a demographic variable, defined as one minus the adult male proportion of the labour force and UI defined as the maximum Unemployment Insurance wage-replacement rate multiplied with the proportion of the labour forced covered by the Unemployment Insurance programme.

Equation (4) is substituted for the NAIRU in equations (1), (2) and (3) restricting the δ_{0i}, δ_{1i} and δ_{2i} to be the same in every equation of the model. Then, the model is simultaneously estimated and the estimates $\hat{\delta}_{0i}$, $\hat{\delta}_{1i}$ and $\hat{\delta}_{2i}$ are used to compute the NAIRUs for the sample period and to predict the NAIRUs for subsequent time periods.

The data used to estimate the model cover the period from the first quarter of 1967 until the fourth quarter of 1985.

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[21]D.Rose and R.Ford, op.cit.,p.6; see also p.48 and 77 of this thesis.

III The Theoretical Basis of the Bank of Canada Model

III.1 The Phillips Curve

> "Since 1970 the Phillips Curve has been an unidentified flying object and has eluded all econometric effort to nail it down." (A.M.Okun)[22]

III.1.1 The Original Phillips Curve

In 1958 A.W.Phillips published a study in which he showed that the rate of change of nominal wages and the unemployment rate are inversely related.[23] He based his results on UK data for 1861 to 1957. However, Phillips was not the first who analyzed the relation between unemployment and inflation. In 1926, for example, Irving Fisher studied a similar problem[24]. Nevertheless, Phillips became famous with the relationship he discovered. This is partly because the Phillips curve allows for a closure of a Keynesian macromodel of the economy.[25]

In a simple Keynesian model where aggregate demand is

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[22]A.M.Okun, Postwar Macroeconomic Performance, in: M.Feldstein (ed.), The American Economy in Transition, Chicago, 1982, p. 166

[23]A.W.Phillips, "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957", Economica, Vol.25 (1958)

[24]A.J.Santomero and J.J.Seater, op.cit., p.500

[25]R.G.Lipsey, op.cit., p.51

determined by the IS/LM model, prices are either fixed when there is unemployment, or totally income elastic in the case of full or equilibrium employment. This means that a rising aggregate demand increases output without affecting prices in the case of unemployment and generates inflation with no effect on output when there is full employment. The model is not very realistic and does not provide a satisfactory explanation for the behaviour in the disequilibrium case when aggregate demand exceeds the full employment output. In this case the IS curve intersects the LM curve at a level of output greater than the full employment output. Inserting this full employment output into both the IS and LM equations and solving both equation for the rate of interest yields two different results.[26] In the simple Keynesian model this problem is solved by an immediate leftward shift of the LM curve such that the IS and LM curves meet at the full employment output at a higher interest rate. If a Phillips curve is used to close the system[27] we have a satisfactory explanation of disequilibrium behaviour. In the case of unemployment, the model implies that price changes are too slow to bring about a fast adjustment back to equilibrium so that there is room for fiscal or monetary policy to increase

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[26]R.G.Lipsey, op.cit., p.53

[27]The curve is modified here to be a positive non-linear relationship between output and inflation which is the same as being a negative non-linear relationship between inflation and unemployment.

aggregate demand and thus lead the economy into equilibrium. In the opposite case, if output is above equilibrium output prices start to rise. Thus, the real amount of money declines and so does aggregate demand because of rising interest rates. The model returns to equilibrium. In conclusion, this model allows for both Keynesian adjustment policy and the classical adjustment mechanism. It represents a form of the Neo-Classical-Keynesian synthesis. However, this model does not explain why wages have risen since World War II during all periods even through recessions.[28] Furthermore, it cannot explain why there is no deflation when the output is below potential or why we observe both unemployment and inflation.

So far, no explanation has been given to provide a theoretical rational for the Phillips curve. Lipsey offers a rather simple explanation by using micro labour markets.[29] In a situation of excess demand there is a positive rate of wage increase, whereas with excess supply this rate is negative. In the first case wages are bid up until equilibrium is reached. In the latter case the opposite occurs. If wages rise beyond the market clearing level the number of employed individuals starts to decline as supply exceeds demand in this situation and only the amount demanded is contracted. Furthermore, the number of unemployed individuals can be related to the number

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[28]A.M.Okun 1981, op.cit., p.234

[29]R.G.Lipsey, op.cit., p.58-60

of vacancies. Assuming search unemployment, there is always a certain number of unemployed workers for any number of vacancies such that there is negative non-linear relationship between vacancies and the number of unemployed. As vacancies reflect excess demand, unemployment can now be linked with the rate of wage increase and one obtains the usual Phillips curve if it is additionally assumed that the wage rate is sticky downwards.

The Keynesian Phillips curve as described by Lipsey leads to the conclusion that governments and/or monetary authorities can choose any point on the Phillips curve according to their preferences by engaging in aggregate demand management. In other words, this form of the Phillips curve offers a trade-off between unemployment and inflation in the long run as well as in the short run.

III.1.2 The Natural Rate Hypothesis

This trade-off was criticized by Friedman[30] who argues that the rate of unemployment at zero inflation (actual and expected value) is governed by real forces. He also says that this rate occurs when the economy is in equilibrium. It is therefore called the natural rate of unemployment. If the government wishes to lower unemployment it raises the money

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[30] M.Friedman, op.cit.

supply and thus decreases interest rates and stimulates spending due to a rise of real cash balances. As people have been expecting prices to be stable, output and employment will rise as a consequence of higher investments and spending. Employees accept new jobs because they face higher nominal wage offers and evaluate the price level according to their expectations, formed on the basis of the constant price level they observed before the money supply increased. Thus, they think that their real wage has risen. However, the employers perceive that real wages are declining as a consequence of the rise in prices. This is equivalent to a move along the Phillips curve to a point where there is lower unemployment and higher inflation. Once the employees realise the decline in real wages they commence to bid up nominal wages in order to return to their initial real wage. Therefore, the unemployment rate returns to the natural rate after being lower than the natural rate for a short period. In the meantime though, the expected inflation rate has risen. This shifts the Phillips curve upwards so that the new rate of inflation (actual and expected) is compatible with the natural rate of unemployment. The Phillips curve is no longer stable but shifts according to monetary policy and subsequent changes in the price level. If the government still wants to maintain the lower unemployment rate it has to increase the rate of monetary growth even further. This will again have only a short run effect on unemployment but will again raise inflation. Therefore, if governments want to have an unemployment rate that is continuously lower than the natural rate they will have to accept accelerating inflation. The Phillips curve of this theoretical framework is consequently called an accelerationist Phillips curve. Following this branch of economic theory there is no long run Phillips curve trade-off. Only in the short run real quantities can be affected by nominal quantities through misperceptions of prices and wages.

It should be noted that the underlying theory of Friedman is an auction economy where the general equilibrium is optimal with respect to social welfare. Here, the natural rate of unemployment reflects the equilibrium so that any deviation from it, even if it is only for a short period of time, represents a loss of social welfare. Any aggregate demand policy that leads to a rate of unemployment lower than the natural rate of unemployment generates more output than optimal.[31]

This is opposed to the Keynesian view, as pointed out above, where the labour market is not perfectly competitive through wage stickiness. In this case, social welfare can be increased by engaging in aggregate demand policy. Now the rise in employment brings about a rise in social welfare, because

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[31]A.M.Okun 1981, op.cit., p.263-269

workers who are employed were involuntarily unemployed before. The aggregate demand policy leads to the social optimum finding an optimal trade-off between unemployment and inflation.

III.1.3 Search Theories

The auction market approach to the labour market underlying Friedman's model suggests that in a recession where wage cuts occur, there will be high quit rates, whereas in reality the opposite is the case.[32] The model also fails to explain lay-offs. Furthermore, there is no explanation for job search because markets clear instantaneously. As an alternative with similar results as in the Friedman model, there are search models. In these models, workers are exposed to imperfect information about jobs and wages, so that they have to engage in a search process.[33]

In one category of these models, for example, individuals face a distribution of wages over vacancies. They set an acceptance wage according to the distribution of wages known to them and the cost of search. If again, the employees

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[32] D.F.Gordon, "A Neo-Classical Theory of Keynesian Unemployment", K.Brunner, A.H.Meltzer, (eds), The Phillips Curve and Labour Markets, Amsterdam, 1976, p.65-97

[33] Santomero and Seater, op.cit., p.518-525

misperceive inflation and receive high wage offers that were not expected they accept more job offers because their acceptance wage is still based on their past experience. After realising the error they raise the acceptance wage so that the unemployment rate returns to its old state. This analysis yields a Phillips curve with a short run trade-off.

Unfortunately, the search model also has no room for lay-offs. Again, the quit rate is supposed to rise during a recession as workers face lower wages and would quit to start a search for better jobs. Furthermore, search theory cannot explain the unemployment during recessions where wages have risen in spite of massive unemployment.

III.1.4 Contract Theory

A third theoretical approach to the Phillips curve, described by D.F.Gordon, is represented by contract-theory.[34] According to this theory employees are facing risks of losing their jobs and of changing wages. It is assumed that these employees are also risk averse whereas employers are risk neutral. Now, employers offer relatively low wages in exchange for a long term implicit contract that guarantees job security and no wage fluctuations. Thus, in a recession employees with an implicit contract earn above market real

[34] D.F.Gordon, op.cit., p.76-92

wages which in turn leads to Keynesian unemployment, since there are workers who are willing to work at that real wage but are not allowed to do so. Here, the rigidity of wages does not come from the employees, as suggested by Keynes, but from the employers.

In the framework of this model, it can also be shown that there is a possibility for lay-offs. Companies have a certain number of employees with tenure whose wage lies below the wage non-tenure workers could be hired for. In the case of sufficient demand the firms hire non-tenure workers in addition to their tenure workers up to the point where the marginal factor cost of these non-tenure workers is equal to the companies demand. If now a sharp decline in demand occurs such that the number of employees needed is equal to or less than the number of tenure workers, the non-tenure workers have to be laid off. This is because the wage of the tenure workers is lower than the lowest wage a non-tenure worker can be employed for. Hence, there is no room for a mutually beneficial wage decrease so that some of the non-tenured workers could keep their jobs. The lay-offs are interpreted as a substitute for wage cuts and subsequent quits.[35]

Gordon argues that his analysis allows for a Phillips

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[35]With such an interpretation lay-offs could also be incorporated into search and auction market models. A specific contract model does not seem to be necessary.

curve because there are always non-tenured workers. The shifts of the Phillips curve are explained by changes of the money terms of the quasi-contracts following altered considerations about future inflation. The Phillips curve of the contract model moves at a slower rate than the Phillips curve of the search model. In the contract model, the employers change their considerations with respect to the influences current changes might have on their long term predictions of future wages and prices. In the search model, the employees change their considerations with respect to the influences current changes might have on the current distribution of wages.

Furthermore, Gordon points out that there might be involuntary unemployment at the long run full employment equilibrium if aggregate demand fluctuates over separate markets and supply cannot adjust instantaneously. In this case, there will simultaneously be excess supply and excess demand in markets of the economy. Higher nominal demand will not be able to reduce this unemployment due to shifts of the Phillips curve.

III.1.5 Price Tag Economy

Another approach that combines the existence of a trade-off, the contract theory and the Friedman model, is presented by Okun.[36] He bases his analysis on a model of the so called price tag economy where product markets exhibit stable seller-customer relations and shopping costs, created by the cost of search being a reflection of the labour markets with stable employer-employee relations based on explicit or implicit contracts. Prices are set by sellers as a mark-up over marginal costs so that prices are no longer equal to marginal costs as they are in the auction market model that underlies the Friedman analysis. Thus, there are elements of monopoly in the price tag economy which lead to a restriction of output. Therefore, the price tag economy is most probably below a social optimum and additional output generated by higher aggregate demand may increase social welfare.[37] Okun argues that the optimal properties of the equilibrium in the pure auction world is based on the implicit assumption that all products and services are homogeneous and simple, i.e. can be easily traded in auction markets. On the contrary, goods are, according to Okun, complex and heterogeneous which is more efficiently reflected by the price tag economy model than by the auction market model.[38]

Okun then argues that the social losses brought about by inflation may not be greater than the welfare gain due to a

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[36]A.Okun 1981, op.cit., chapters 2-4, 6-8

[37]A.Okun 1981, op.cit. pp. 266-267

[38]A.Okun 1981, op.cit., p.268

limited period of higher output and less unemployment. He presents a model[39] based on a social loss function. This function depends on the difference between the actual unemployment rate and the unemployment rate associated with critical shortages. It is also contingent on the inflation rate to which the economy is adapted, its variance and the difference between actual and adapted price level. To obtain a Phillips curve he presents the price level to be a function of the adapted inflation rate[40] and of unemployment. Its partial derivative with respect to the adapted price level is positive and with respect to unemployment negative. The rate of unemployment for which this function becomes zero, so that the actual inflation rate depends only on the adapted inflation rate, is comparable to, but still different from the natural rate of unemployment of the Friedman model. This function being equal to zero shows the long run level of unemployment with a rate of inflation always equal to the adapted one. This is consistent with Friedman's considerations. The Phillips curve shifts according to

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[39]A.Okun 1981,op.cit., p.297

[40]"Adapted rate of inflation" means the rate of inflation the economic behaviour of the economic agents is used to and adjusted to; it changes if the development of the inflation rate over the following business cycles is likely to be considerably different from the rate that governs present economic decisions. Thus, it is different from the "expected rate of inflation" which reflects expectations about the inflation rate of the near future. A change of the expected rate of inflation does not necessarily lead to a change of the adapted rate of inflation. See A.Okun 1981, op. cit., p. 301.

different levels of the adapted rate of inflation. In addition, Okun introduces an average inflation rate perceived to be the long run rate of inflation. The adapted rate of inflation changes when it differs from the average rate of inflation. Hence, the Phillips curve shifts up when the average rate of inflation is greater than the adapted rate of inflation. It does not necessarily shift when the actual rate of inflation differs from the expected rate of inflation as is the case in the Friedman model.

Okun's point is that any change in the rate of inflation will generate a long chain of real consequences. A rise of the rate of inflation will raise the expectations about future price levels and thus affect the average price level that will in turn change the adapted price level. But the adjustment to a new adapted rate of inflation is time consuming due to costs and may last very long.[41] During this adjustment period the nominal disturbance has real effects. It is also possible that the short run curve will be left intact. Although this model offers a Phillips curve trade-off that might be used if the social gain is positive Okun remains careful about the possibilities of demand management especially if the decision maker faces uncertainty or imperfect control of his tools.

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[41]A.Okun 1981,op.cit., p.302

Comparable results are obtained by R.E.Hall[42] who uses a simulation model with adaptive expectations and a social welfare function to show that there is a usable Phillips curve trade-off that may increase social welfare.

III.1.6 Rational Expectations

To incorporate the change of price perceptions into the Phillips curve normally a price expectations term is added. The expectations have often been modelled to be adaptive which implies that the individuals make systematic forecasting errors when they adjust their price considerations to shocks. This has been criticized. Lucas and Sargent and Wallace provide models based on rational expectations which lead to the conclusion that there is no feasible trade-off between unemployment and inflation even in the short run.[43]

Their models are generally based on equilibrium and market clearing considerations. However, the assumption of complete information is removed. In other words, economic

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[42]R.E.Hall,"The Phillips Curve and Macroeconomic Policy", in K.Brunner, A.Meltzer,(eds), The Phillips Curve and Labour Markets, Amsterdam, 1976, p.127-141

[43]R.E.Lucas and L.A.Rapping, "Price Expectations and the Phillips Curve", American Economic Review, Vol.59 (1969), p.342-350 and R.E.Lucas, "Some International Evidence on Output-Inflation Tradeoffs", American Economic Review, Vol.63 (1973), p.326-334

agents can misperceive nominal price changes as relative price changes in their particular markets. They adjust their behaviour according to their misperceptions. Therefore, nominal changes can have real effects.

Lucas presents a model[44] where economic agents make inferences about the general price level they can expect. They do so on the basis of the observed price in their particular markets and the mean value of the general price level which is based on information from prior periods. The difference between the observed price and the mean value of the general price level determines the quantitiy of goods the economic agents are supplying on their particular markets. This supply varies according to alterations of the perceived prices. It is, therefore, called cyclical supply. It has to be distinguished from the normal supply that follows a time trend. Cyclical and normal supply together yield total supply. Closing the model by an aggregate demand function, Lucas shows that the cyclical component of aggregate supply as well as the change of the price level, depend positively on aggregate demand through a coefficient π. This π is positively contingent on the variance of the particular market prices and negatively contingent on the variance of the price level. In another expression it depends inversely on the variance of the aggregate demand.

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[44]R.E.Lucas 1973, op.cit.

Through testing the model for several countries it is shown that the coefficient π is relatively high for countries with stable prices and low for countries with inflation experience. Thus, there is a short term trade-off between output and inflation in countries with stable prices. But, once this trade-off has been used it no longer is available because the variances change and so does π so that there is no stable Phillips curve, not even in the short run.

Lucas and Sargent criticize models that lead to a Phillips curve trade-off because these ignore the fact that structural parameters are not stable over time but rather depend on policy and other variables.[45] Furthermore, they criticize Keynesian macroeconometric models because important parameters, for example, are restricted to zero and there are no behaviourial foundations that are based on theory.[46] According to Lucas and Sargent these models must lead to wrong results.

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[45] R.E.Lucas, "Econometric Policy Evaluation: A Critique", K.Brunner, A.H.Meltzer, (eds), op.cit., p.19-46

R.E.Lucas and T.Sargent, "After Keynesian Macroeconomics", R.E.Lucas and T.Sargent, (eds), Rational Expectations and Econometric Practice, Minneapolis, 1981, p.295-320

[46] E.g. the liquidity preference relation expresses the demand for money as a function of only income and interest rate and no other variables. R.E. Lucas and T. Sargent 1981, op. cit., p. 299.

III.1.7 A Post-Keynesian Model

A post-Keynesian point of view is emphasised by Tuscherer[47]. He starts with a macroeconomic market model where aggregate demand is determined by a fraction of an exogenously given level of wealth, by investment (governed by the rate of interest) and by consumption. Consumption is defined as the marginal propensity to consume multiplied with disposable income, which itself is defined as employment multiplied by money wages.

Aggregate demand is determined through firms that set their level of output and the amount of labour to produce it on the basis of short run expectations. Then, through an aggregate supply function the price level is determined and hence the real wage. When this real wage is greater than the marginal disutility of employment, then there is involuntary unemployment. More workers than actually hired are willing to work at this real wage. Tuscherer then derives a Phillips curve that is partly vertical for a certain very low unemployment rate. As for higher unemployment rates the curve exhibits the usual functional form. The vertical part represents the case where there is no involuntary unemployment. At any other point of the non-vertical part of this Phillips curve, involuntary unemployment exists.

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[47]T.Tuscherer,op.cit.

Therefore, there is also involuntary unemployment at a zero rate of inflation. Thus, this model allows for an unemployment equilibrium of the economy.

If the monetary authorities face this equilibrium and increase the quantity of money they raise the aggregate demand through a rise of wealth. This in turn will increase employment and the price level and lead to a decline of the real wage. This is a movement along the Phillips curve. Now the workers start bargaining for higher money wages which increases aggregate demand further but decreases aggregate supply. Prices rise again. This leads to a rightward movement away from the original Phillips curve. Tuscherer now detects a negative real balance effect as a consequence of the second increase of the price level. He believes that creditors decrease buying by more than debtors raise their demand. Unfortunately, he provides no explanation for this assumption.[48] This leads the economy towards the initial unemployment rate and inflation is declining but not to the zero level of the above mentioned equilibrium. Furthermore, the price increase has brought about a decline of the real money supply and thus a rise in the interest rates. This generates a decline in investment which in turn decreases aggregate demand and employment. Finally, unemployment is back

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[48]Normally, creditors and debtors are assumed to be equally affected and no real balance affect should occur.

at the old level but inflation is higher than before. Therefore, the Phillips curve seems to be vertical in the long run.

Tuscherer argues that the initial unemployment rate to which the model returns, is not a natural rate, because it contains involuntary unemployment. According to his line of argument, there is a vast variety of vertical Phillips curves depending on the initial unemployment rate the analysis is based on. Only one of these unemployment rates represents the natural rate. It is the one rate compatible with the vertical part of the initial curve (see above). For Tuscherer, the vertical Phillips curve in Friedman's sense is just another expression for the classical equilibrium rejected by Keynes. In Tuscherer's analysis the position of the vertical Phillips curve in the unemployment-inflation space depends on the unemployment rate the analysis begins with. Since there are many possible unemployment rates that could be used as a starting point there are as many vertical Phillips curves. Any of these curves would be misleading in determining the natural rate of unemployment.Therefore, for Tuscherer,there is no use in dealing with a natural rate of unemployment as determined by a vertical Phillips curve.

III.1.8 The Hysteresis Approach

In connection with the Tuscherer model it is worthwhile to mention an approach proposed by Summers.[49] According to Summers, conventional Keynesian models contain a Phillips curve to describe the movement of prices and wages and to determine a natural rate of unemployment or rather the NAIRU. He argues that this is misleading with respect to Keynes' own ideas about economic reality and rejects the rigidity of prices and wages pointed out by Keynesians as being illogical. In addition, Summers does not find any empirical support for the notion that economic fluctuations are transitory movements around an unique equilibrium. Unemployment, for example, rather follows a random walk. Summers states that there are many possible equilibria the economy can reach. The switch between different equilibria is determined by history and external factors. He clarifies this point with a labour market example where firms try to pay wages that are close to the relative wage paid by all other employers. He argues, that in such an analytic framework multiple equilibria are possible and then extends this conclusion to the whole economy. According to Summers, economic policy is able to impose equilibria upon the economy in certain limits. This means,

[49] Lawrence H. Summers, "Should Keynesian Economics Dispense with the Phillips Curve?", in: R.Cross(ed), Unemployment, Hysteresis and the Natural Rate Hypothesis , Oxford 1988, pp 11 - 25

economic policy can have effects as opposed to the results of the Monetarists and New Classical School where this is not the case. In this framework of multiple equilibria the Phillips curve that reflects an unique equilibrium is senseless and should be banned from analytical models.

III.2 Okun's Law

A.Okun developed a concept of measuring potential productive capacity or rather potential GNP.[50] This is a short run concept and therefore assumes capital stock, technological knowledge, natural resources, skill and education of the labour force as given. Potential output differs from actual if aggregate demand is lower than needed to produce the potential output. The idea behind Okun's concept is therefore basically Keynesian, i.e. the economy is at an unemployment equilibrium due to lack of aggregate demand. Okun argues that today's failure to produce potential GNP can endanger the future potential GNP through low investment in plant, equipment, research and education which is caused by low profits and incomes. Thus, engaging in aggregate demand management to reach potential GNP has also an effect on future production capacity and is therefore beneficial (see above discussion of Okun's price tag economy,too).

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[50] A.Okun 1962,op.cit.

Okun's Law states that potential GNP is equal to actual GNP plus actual GNP multiplied by a link factor times the difference between actual and natural rate of unemployment:

$$Y^* = Y^a(1 + c(U^a - U^*))$$ [51]

The unemployment rate is used as a proxy for the influence of idle resources on output. It stands for a variety of factors that affect output. Simply, output can be considered as consisting of the multiplication of population size, labour force participation rate, employment rate, productivity per man hour and hours worked.[52] Starting from a recessionary situation all these factors will change when the economy moves towards full employment. Okun states that the labour force participation depends on the demand for labour so that the labour force changes pro-cyclically. Average hours worked also rise during recovery because employers try to raise output by using overtime hours instead of hiring new workers; whereas, during a recession, they decline more than the GNP because entrepreneurs resist lay-offs due to hiring and firing costs. Okun provides a rationale for this, saying that existing contracts guarantee employment for certain periods, that

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[51] The * denotes potential or equilibrium values whereas the a denotes actual or observed values.

[52] R.J.Gordon, "Unemployment and Potential Output in the 1980s", in: <u>Brookings Papers on Economic Activity</u> 2:1984, p.537

workers have acquired skills which the employer does not want to loose and/or that technological factors require a fixed staff of specialized employees. In other words, the labour market is not a continuous auction market but rather a market with rigidities. Therefore, during a recession, there are fewer lay-offs than one would expect for an auction market, so that productivity rises in times of expansion and falls in recession periods.

Okun's Law has been criticized for several reasons. It does not for example allow influences of capital on output.[53] The omission of capital reflects the assumption that capital is abundant for any output up to full employment output. Furthermore, the simple version of Okun's Law does not take into account the lagged reaction of labour input on output so that only the impact of actual unemployment on potential GNP is computed.[54] In addition, the link factor c is computed on the basis of past values of the GNP and unemployment so that current influences do not affect it. Okun's Law also generates fluctuations in potential GNP as a consequence of changes in the actual rate of unemployment. This is a contradiction to theory which assumes potential GNP to follow a time trend

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[53] O. Eckstein, "Potential Output and Productivity:Comment",in: Brookings Papers on Economic Activity, 1973, p.11-60.

[54] J.Schaafsma, "Okun's Law and the Interaction between Actual and Potential GNP: The Canadian Experience", unpublished paper, University of Victoria, 1975.

without fluctuations. Therefore, potential GNP series generated by Okun's Law need to be smoothed.

On the other hand, it is considered to yield fairly stable and robust results and offers a short and simple relationship of employment and output instead of the complex identity that accounts for every single factor influencing output.[55]

III.3 The Aggregate Production Function

Neo-Classical growth models have introduced an aggregate production function into macroeconomic theory. This function is of the Cobb-Douglas form which was developed for microeconomic purposes:

$$Y = A\ K^{1-\alpha}\ L^{\alpha}\ e^{gt}\ ,\ 0 < \alpha < 1$$

where K denotes capital input raised to the power of $1-\alpha$; L denotes labour input raised to the power of α; and g is the growth rate of the output that can be attributed to technical progress.

Cobb-Douglas production functions exhibit constant

[55] R.J.Gordon, op.cit., p.539

elasticity of substitution that is equal to one.[56] Furthermore, as the coefficient α is smaller than one, the marginal productivity of either input factor is declining, the more the particular factor is used. In addition, the coefficient $1-\alpha$ denotes the share of output that can be attributed to capital and α denotes the share that can be attributed to labour. Any change of factor prices leads to a change of the quantity of each input. Therefore, it is assumed that factors are homogeneous and malleable. If prices are presumed to be flexible, in equilibrium the real rate of interest will equal the marginal product of capital and the real wage will be equal to the marginal product of labour.

The Cobb-Douglas form imposes a serious restriction on the production process. The constant elasticity of substitution is forced to be equal to one. As there is no particular reason for this, a production function is preferable that does not force the elasticity of substitution to have certain values. Such a production function is the CES or constant elasticity of substitution production function.[57] There are also production functions that allow for variable elasticity of substitution, the so called translog production

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[56]A.C.Chiang, Fundamental Methods of Mathematical Economics, New York, 1984 (3rd ed), p. 425-430

[57]H.L.Varian, Microeconomic Analysis, New York, 1984 (2nd ed.), p.179

functions[58].

Nevertheless, there is a problem that affects every specification of a production function. This problem appears when input factors are omitted in the function, that are taken into account by the firms. Therefore, the error term of the function to be estimated may be correlated with the other input factors and thus generate biased estimates of the coefficients.[59]

The aggregate Cobb-Douglas production function was subject to a theoretical critique by Neo-Keynesians which is summarized in Hacche.[60] The assumptions about perfect competition, on which the aggregate production function is based, are criticized as being unrealistic and hence invalid. According to the Neo-Keynesians, monopoly theory is more appropriate when the economy is analyzed as a whole. They argue that prices are set by firms on the basis of costs and that profit maximisation over the long run is a "meaningless phrase". Neo-Keynesians also reject the smooth substitutability of capital. Substitution possibilities are rather limited and costly because capital goods are

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[58]R.L.Thomas, Introductory Econometrics, Theory and Application, New York 1985, pp. 241-243

[59]H.L.Varian, op.cit., p.171-188

[60]G.Hacche, The Theory of Economic Growth: An Introduction, London Basingstoke, 1979, chapter 10

specifically designed for particular uses where they are utilised together with specific material and labour inputs. However, as Hacche points out, factor substitution in a whole economy can be regarded as altering the composition of output among goods that are produced with different fixed factor proportions and through technological change in the production process of individual goods.

Neo-Keynesians also doubt the homogeneity assumption. For them, capital goods are rather heterogeneous. This destroys, in their view, the usefulness of aggregate production functions. If capital is heterogenous the measurement of the aggregate capital stock requires the knowledge of the price vector of the economy and hence, the profit rate. Therefore, the production function, the marginal products of the factors and the aggregate capital stock are meaningless unless the rate of profit has already been observed.

The counter argument to this critique states that the aggregate production function is a "parable" still useful for analysis and for empirical studies. Furthermore, it has been argued that every capital good may be considered individually and "the rates of return on capital goods become a question of intertemporal equilibrium".[61] In turn, considering capital goods individually has been criticized because it does not

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[61]G.Hacche, op.cit., p.180

lead to a uniform rate of profit. This is countered by the observation that there is nothing like a uniform rate of profit and that assumptions, as special as those underlying the production function, can generate such a uniform rate.

Despite all the criticisms, the aggregate production function is an essential part of many macroeconomic models, especially if the model is intended to compute potential GNP.[62]

III.4. Determinants of the NAIRU

There are several factors that are presumed to influence the NAIRU. Riddell and Smith, on the basis of empirical studies, reached the conclusion that the Unemployment Insurance (UI) Act and changes in the demographic composition of the labour force (youth and women) are the most important contributors. [63]

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[62]For Canada see for example J.Brox and M.Cluff, "Potential Output and Real GNP Gap", Canadian Statistical Review, Jan.1979.

See also R.L.Thomas, op.cit., pp 217 - 219, for a description of aggregational problems.

[63]W.C.Riddell and M.P.Smith, "Expected Inflation and Wage Changes in Canada 1967-1981", Canadian Journal of Economics, Vol.15 (1982), p.385

In Canada[64], an employee is entitled to UI benefits after he or she has worked at least ten to fourteen weeks. Then, the worker has to wait two weeks after being laid off until he or she obtains the first payment of UI benefits. These benefits cover 60 % of the previous earnings up to a maximum that changes from year to year, according to the formula for the computation of the average wage. The benefits are paid as long as the previous working period lasted plus two additional weeks. For every half point the regional unemployment rate exceeds the lowest level observed throughout the country the payment of benefits is extended by two weeks up to a maximum of thirty-two weeks. If the employment period is more than twenty-five weeks every two additional weeks worked, will entitle the employee for an extra week of benefits. The maximum period of benefit payments is fifty weeks.

UI benefits will lead entitled workers to attach a higher value to leisure than in the non-benefit case, as long as leisure is regarded as a commodity. Their net gain of returning to work is reduced by the amount of benefits. Therefore, entitled workers tend to remain unemployed for longer periods than those who are not entitled to UI benefits. On the other hand, an early acceptance of a new job would not

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[64]For the following see J.M.Cousineau, op.cit.. Note that the Unemployment Insurance Act has recently been changed, but the description by Cousineau is still valid because the sample period in the empirical analysis of this thesis ends in 1985.

be efficient if the worker had to return to an occupation that exhibits lower skill requirements and a salary that do not correspond to the worker's abilities. This can lead to a decline of productivity. In other words, UI benefits may encourage search for better jobs.

The above analysis does not incorporate the impact of UI benefits on the search process itself. In search theory models (see above) the UI benefits lower the cost of searching and, therefore, raise the acceptance wage so that the number of jobs the worker would return to declines. Hence, the duration of unemployment is prolonged through UI benefits.

UI benefits may also have effects on the mobility of the unemployed workers. The mobility may be either regional or occupational. If the unemployed worker changes occupations in order to find work, he will face costs in the form of forgone earnings as the result of training. UI benefits reduce the possible gain from occupational change even further. Thus, UI benefits again may extend the unemployment period. In the case of regional mobility, the worker willing to move is subject to monetary and psychological costs. UI benefits raise these costs because they represent forgone earnings after the unemployed worker has moved and found a new job. Therefore, UI benefits reduce the net gain of moving. In particular, the Canadian system of UI benefits does not support mobility because the benefits are high in regions with high unemployment.

Finally, UI benefits may raise the labour force participation rate because twenty weeks of work yield twenty-two to fifty weeks of UI benefits. This encourages people to look for temporary jobs that are quickly available. It is also an incentive for industries to create jobs with low payment and low productivity. Industries that face seasonal, cyclical or otherwise unstable demand benefit from UI benefits because it is easier for them to find workers and to lay them off later on, once the worker is entitled to UI benefits. Here, the UI benefits are like a lay-off subsidy.

Cousineau cites his own and other empirical results to support his theoretical deductions that UI benefits create unemployment. But, empirical studies that use UI benefits to determine the NAIRU show that the coefficient that measures the influence of the UI benefits on the NAIRU is not always significantly different from zero.[65]

During the 1960s and 1970s the labour force participation of several sub-groups of the population rose. One such sub-group were young workers, the so called baby-boomers. Their

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[65]W.C.Riddell and P.M.Smith, op.cit., p.389 and D.Rose, op.cit. p.30.

share of the labour force rose between 1962 and the mid 1970s and then declined sharply after 1980. Young workers have higher than average unemployment rates so that this together with the higher participation rate might have influenced the aggregate unemployment rate and the NAIRU.[66] The other important sub-group is female workers who have a steadily increasing share of the labour force since the early 1950s. The rising participation rate of female workers has driven up the unemployment rate because the rise of female participation was not fully anticipated and the additional workers were difficult to absorb.[67] This is reflected by the fact that women have had higher unemployment rates than men since the mid 1960s. However, the developments of the 1980s lead to the conclusion that some effects of the rise in the female share have been transitory, e.g. the results of lack of job experience. Since the early 1980s both male and female unemployment rates are fairly equal. Women now behave more like men with respect to work. Therefore, the changes in the female participation rate might have raised the NAIRU especially durimg the 1970s but are less likely to have had an impact on it during the 1980s. [68]

Another factor with impact on the NAIRU is the

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[66]D.Rose, op.cit., p.10-12

[67]ibid.,p.12

[68]ibid, pp. 13-15

consequence of economic shocks that lead to temporary rises in structural unemployment. Lilien [69] points out that "a significant fraction of cyclical unemployment can be explained by the slow adjustment of labour to exogenous shifts of sectoral employment demand".[70] This is due to sectoral attachments because of specific skills and wage premiums with seniority. Lilien criticizes the notion that the natural rate of unemployment is relatively constant over time and evolves rather slow by depending on the long run changes on the supply side of the economy. On the basis of his model that includes a variable that measures the sectoral shifts of demand for labour, Lilien concludes that the natural rate of unemployment changed with real factors in the 1970s. The sectoral shifts turn out to be significant and exogoneous to the process. They seem to be a variable that influences the cyclical behaviour of unemployment and also the path of the natural rate of unemployment over time. Lilien uses US data for his study. A similar study is realized by Samson[71]. She obtains results similar to those of Lilien. In addition, she finds that the transmission of the business cycle from the U.S. to Canada is to a high degree influenced by real variables.

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[69]D.M.Lilien, "Sectoral Shifts and Cyclical Unemployment", Journal of Political Economy, Vol.90 (1982), p.777-793

[70]ibid., p.779

[71]Lucie Samson, A study of the impact of sectoral shifts on aggregate unemployment in Canada, Canadian Journal of Economics, Vol XVIII, No.3, August 1985

McCallum presents comparable results which emphasize, however, the role of aggregate demand variables. Aggregate demand variables are found to be more important in explaining the causes of unemployment in both the US and Canada. Sectoral shifts play a secondary role. [72]

The next factor assumed to have influence on the NAIRU are trade unions. Summers[73] finds higher unemployment in states of the US where unionization rates are high. The same kind of argument could be made for Canada. Unions use temporary high prices to bargain for higher wages and to realize real wage gains after prices have declined. Unions are regarded as being able to push wages above their competitive equilibrium value and thus to increase unemployment. However, unionization rates are far below 50% of the labour force so that this line of argument should be doubted.

Following Friedman, aggregate demand does not affect the NAIRU. It only generates the fluctuations around it. Opposed to this view, Jenkinson[74] points out that with low aggregate

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[72]J.McCallum, "Unemployment in Canada and the United States", Canadian Journal of Economics, Vol 20 (1987), p.802-822

[73]L.H.Summers, "Why is the Unemployment Rate So Very High Near Full Employment?", Brookings Papers on Economic Activity, Washington 1986.

[74]T.J.Jenkinson, The NAIRU: Statistical Fact or Theoretical Straitjacket, R.Cross (ed), Unemployment, Hysteresis and the Natural Rate Hypothesis, Oxford 1988, pp

demand the productive capacity of an economy will be declining. As this process is not easily reversible and reconstructing the capacity is time consuming the level of demand has an influence on the NAIRU. Jenkinson uses a reduced form equation where both supply side and demand variables affect the NAIRU. He runs two simple OLS regressions on the variables measured in levels with the unemployment rate being the dependent variable to estimate the long run parameters of the relationship. Then, the residuals contain the information of the data generating process. It can now be tested whether they move systematically in the direction of an equilibrium which means that there is a unique NAIRU . The first regression excludes the demand variables, in the second they are included. To test whether the residuals move in the direction of an equilibrium, Jenkinson tests whether the series of the residuals is stationary. If it is, there is an equilibrium, if it is a random walk there is none. He finds the residuals to be a random walk when the demand variables are excluded and stationary when the demand is considered. Hence, Jenkinson questions the NAIRU to be a structural parameter of the economy and states that demand policies may have some influence on it.

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Jenkinson's results are criticized by Nickell[75] for regarding the relationship of the NAIRU and its explanatory variables to be linear, whereas in reality it is non-linear. He estimates a similar model and accounts for the non-linearity. The results show that the demand variables can be excluded without creating any problems. Nevertheless, Nickell states that the government can influence unemployment through a change in demand accompanied by a change in competitiveness without affecting the rate of inflation[76].

III.5 Influence of the Theory on the Model Restrictions

The Bank of Canada model is estimated under the assumption that the NAIRUs appearing in the different equations of the system are identical, i.e. there is a unique NAIRU for the whole economy. This basically goes back to Friedman's considerations about the natural rate of unemployment:

> "The 'Natural rate of unemployment', in other words, is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labour

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[75] S.Nickell, The NAIRU: Some Theory and Statistical Facts, R.Cross (ed), op.cit., pp 379 - 385

[76] Stephen Nickell, op.cit., p 381

> and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labour availabilities, the costs of mobility, and so on.[77]

Thus, the natural rate of unemployment reflects the Walrasian equilibrium which is unique, so that there must be a unique natural rate of unemployment. Therefore, the restrictions imposed upon the δ's (in equation (4)) have to be such that the NAIRU is the same in every single equation of the model. This is also obvious if we analyze how the model equations are related to the labour market since the NAIRU is observed when the labour market is in equilibrium. The Phillips curve connects the labour market via the unemployment rate with wage inflation so that any equilibrium on the labour market appears in the Phillips curve through the equilibrium or natural rate of unemployment. The same is effective for an Okun's Law equation that relates the difference of actual and potential output to the difference of actual and natural rates of unemployment. If the labour market exhibits a disequilibrium there will be an output gap according to Okun's law; in the equilibrium case actual output will be equal to potential since the actual unemployment rate is the natural. The production function also mirrors the labour market through the labour input variable. Equilibrium on the labour market means that there is a specific amount of labour input

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[77]M.Friedman, op.cit., p.8

available to the economy and that this specific amount coincides with a certain rate of unemployment, which is the natural rate. Assuming that capital formation is on its long run trend, the economy produces potential output when the labour market is at its equilibrium. Any deviations from the equilibrium labour input imply deviations from potential output, a result that also follows from Okun's law. Therefore, the labour market influences all of three equations and the NAIRU, as a result of the labour market forces, must be the same in every equation.

Friedman's considerations are also reflected in the Phillips curve. The Phillips curve used in the model contains two price expectations variables. The first one, P, expresses consumer price expectations and reflects the theory of rational expectations because it is directly linked to the rate of wage increase, i.e. the parameter σ_0 in equation (3) is bound to be one. The other variable is PREL, the expected difference of producer and consumer price expectations. In the long run consumer and producer prices are growing at the same constant rate so that PREL disappears. In the short run, however, PREL may obtain values different from zero as a consequence of different expectations of consumers and producers due to nominal shocks.[78] Incorporating these two

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[78]The variable PREL seems to reflect Friedman's ideas about the effects of an increase of the money supply. The employers facing higher demand are willing to pay higher

price expectation variables ensures that the Phillips curve is accelerationist or follows Friedman's ideas.[79] In other words, in the long run when unemployment is at its natural rate and consumer and producer prices grow at the same constant rate, changes of the price expectations directly alter the growth rate of nominal wages without affecting the unemployment rate. Thus, there is a vertical Phillips curve in the long run.

In the short run, however, the Phillips curve of the Bank of Canada model allows for an unemployment rate different from the natural rate and for the expected growth rate of producer prices to be different from that of the consumer prices. Hence, there is an unemployment (wage)-inflation trade-off in the short run.

As well as the consumer price expectations the rate of change of the hours worked is also directly related to the rate of wage increase. Other things being equal, a change of the growth rate of the hours worked leads to a change of the

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producer prices (e.g. nominal wages), while the consumer still expects the consumer prices to remain on its long run trend. Thus, the workers believe that the real wage has increased whereas the producers perceive it to have declined or remained unchanged because of the higher demand that brings about increased prices. Therefore, additional workers are hired and unemployment declines. After the price expectations have been corrected the unemployment will return to its old level.See Friedman, op.cit., p.10.

[79] D.Rose, R.Ford, op.cit., p.6

growth rate of output per capita, i.e. if the average employee is for example working more hours per year his output will increase. This in turn brings about a variation of the rate of change of real wages which results in a similar variation of the growth rate of nominal wages. Consequently, σ_1 (in equation (3)) is also restricted to one.

Moreover, the rate of wage increase additionally depends on the rate of change of productivity. The coefficient on productivity growth (σ_2 in equation (3)) is set to be equal to $1/\alpha_2$. This is derived from growth theory where, on the basis of a Cobb-Douglas production function, in the equilibrium steady state the income per capita is growing at a rate that is equal to $1/\alpha_2$ multiplied with the rate of productivity growth.[80] This restriction implies that the production

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[80]The aggregate production function of the neoclassical growth model is as follows, Y denoting output, K capital and L labour input, g the growth rate of technical progress and t time:

$$Y = K^{1-\alpha} L^{\alpha} e^{gt}, \quad 0<\alpha<1$$

$$\Leftrightarrow \quad Y/L = (K/L)^{(1-\alpha)} e^{gt}$$

$$\Leftrightarrow \quad y = k^{(1-\alpha)} e^{gt}$$

$$\Leftrightarrow \quad \log y = (1-\alpha) \log k + gt$$

$$\Leftrightarrow \quad (\delta/\delta t) \log y = (\delta/\delta t)((1-\alpha) \log k) + g$$

$$\Leftrightarrow \quad \dot{y} = (1-\alpha)\, \dot{k} + g$$

$$\Leftrightarrow \quad \dot{k} = (1-\alpha)\, \dot{k} + g, \text{ since } \dot{y}=\dot{k} \text{ in equilibrium}$$

$$\Leftrightarrow \quad \dot{k} = g/\alpha$$

$\Leftrightarrow \quad \dot{y} = g/\alpha$; The growth rate of income per capita is equal to the growth rate of productivity divided by the elasticity of output with respect to labour input, α. Note that g is equal to the growth rate of ETFP in the Bank of Canada function of the model is of the Cobb-Douglas type so that α_1 is equal to $1-\alpha_2$ and consequently restricted accordingly. Otherwise the restriction that $\sigma_2=1/\alpha_2$ cannot be derived.[81]

It should be noted that the rate of change of hours worked and the growth rate of productivity are independent from each other because productivity is not bound to specific factors of production in growth models using a Cobb-Douglas production function.

As described above, the NAIRU as a concept and its theoretical background is open to discussion. If it is possible to show that the real world data supports the above model then the underlying theory of the model would be supported. A rejection of this model, however, does not necessarily imply that the theory has flaws; it may as well be that the model does not reflect the theory appropriately.

Therefore, this thesis intends to analyze the Bank of Canada model in order to find out whether it is supported by

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model.Set $\alpha = \alpha_2$.

[81]Nevertheless, the two restrictions are also tested separately because there will be the possibility that nominal wages do not rise, other things being equal, at rate equal to $1/\alpha_2$ times the rate of productivity growth due to market imperfections even if the Cobb-Douglas production function is an appropriate description of the production process of an economy. On the other hand, if α_1 and α_2 do not add up to one $1/\alpha_2$ times the rate of productivity growth may still be a fair approximation.

the data. This is done in two steps. First, the restrictions, derived from theory, that make the coefficients of the NAIRU equation identical for every equation of the model are tested. To perform the tests less restricted versions of the model are defined and compared with the original Bank of Canada model. It is also analyzed whether the aggregate production function used in the model is an appropriate way to describe the production technology of the economy. Second, the coefficient estimates of the Bank of Canada model and of the less restricted model versions as well as the NAIRU's that can be generated with them are analyzed on the basis of theoretical considerations about their values. This yields information about the validity of the different models additional to the information obtained through testing the restrictions. The results of the two steps of the analysis are intended to show whether the theory behind the Bank of Canada model is favoured by the data and whether less restricted model versions lead to reasonable results.

IV Empirical Results

IV.1 The Testing Procedure

For convenience it is appropriate to present the model at this point again:

$$(1)\ \log Y = \alpha_0 + (1 - \alpha_2)K + \alpha_2 LS + ETFP + \epsilon_1$$ [82]

$$(2)\ RNU - NAIRU = \beta_1(RNU_{t-1} - NAIRU_{t-1}) + \beta_2(\log Y - \log\hat{Y}) + \epsilon_2$$

$$(3)\ W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3(RNU - NAIRU) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$$

$$(4)\ NAIRU = \delta_{0i} + \delta_{1i} DEM + \delta_{2i} UI$$

In the Bank of Canada model the δ are restricted such that there is a unique δ_{0i} as well as unique δ_{1i} and δ_{2i} wherever the NAIRU appears in the system.[83]

At the beginning of the testing procedure the Bank of Canada model has to be re-estimated in order to obtain a basis and a reference for the analysis. Without this reproduction an analysis of the kind presented below is impossible.[84]

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[82] See footnote number 13.

[83] See Appendix I for the exact versions of the models.

[84] For the reproduction results see Table 12 on p. 81

In general, three testing procedures are available to test restrictions of the kind the Bank of Canada model exhibits. These are the Likelihood Ratio test (LR), the Wald test (W) and the Lagrange multiplier test (LM). To perform the Wald test only unrestricted versions of the model are needed whereas for the Lagrange multiplier test restricted versions are necessary. The three tests are asymptotically equivalent because they all have the same asymptotic distribution.[85] However, for finite samples the results of the three tests may differ. Since the application of LR tests is much more convenient than using LM tests and W tests for this specific model in TSP and since it can be shown that the values of the three tests are such that

$$LM<LR<W$$

if the restrictions are linear[86] LR tests are chosen to do the testing. The above inequality shows that the acceptance of restrictions can be favoured with LM tests, while its

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[85]G.Judge and W.E.Griffith et al., The Theory and Practice of Econometrics, New York, 1985 (2^{nd} edition), p. 184

[86]J.Kmenta, Elements of Econometrics, New York McMillan, 1986(2^{nd} edition), pp. 491-495. See also G.E.Mizon, Inferential Procedures in Nonlinear Models: An Application in a UK Industrial Cross Section Study of Factor Substitution and Returns to Scale, Econometrica, Vol.45, No.5 (July 1977), pp. 1228-1229. Note further that a diagonal variance-covariance matrix to solve the identification problem of the system is imposed (See page 9). This makes the log-likelihood value reported by the computer meaningless and it has to be computed otherwise, which may generate problems for the computation of the LM test and , besides the problems of designing an LM test for this particular model in TSP, it is questionable if meaningful results could be obtained.

rejection can be favoured by W tests. In addition, since the restricted and unrestricted versions are estimated LR tests allow for analyzing other features of the models, for example, as to whether the parameter estimates have meaningful values and are in accord with economic theory .

Likelihood-ratio tests are used in order to analyze the restrictions on the coefficients. The idea of this test is to compare the maximized values of the likelihood-functions of a restricted and an unrestricted version of the same model in order to decide whether the restrictions are valid or not. It can be shown that:

$$2(LogLike_1 - LogLike_0) \rightarrow X^2(q) \text{ [87]},$$

i.e. is asymptotically X^2 distributed. $LogLike_1$ is the maximized value of the logarithm of the likelihood-function under the hypothesis that the model is unrestricted. $LogLike_0$ denotes this value under the null hypothesis that the restrictions are valid. q denotes the degrees of freedom of the X^2 distribution. The restrictions are valid if the two LogLike values are close to each other such that the X^2 value is not significantly different from zero. On the other hand, if the restrictions do not hold the values will be different and the X^2 value will be significantly greater than zero. In

[87]G.Judge and W.E.Griffiths et.al., op.cit.,p.184

other words, the restrictions are rejected if the X^2 value of the test is greater than the critical value given by the table of the X^2-distribution at an appropriate level of significance.

As mentioned, to realize LR tests one needs restricted and unrestricted versions of the model in question. An ideal way to proceed in the direction of the optimal model version is to start with the least restricted version and to impose restrictions step by step in order to choose the most restricted version that is still acceptable by the testing procedure.[88]

In the case of the Bank of Canada model this procedure is not feasible. Many of the less restricted versions of the model cannot be estimated.[89] As a consequence, a less desirable method has to be used. Beginning with the most restricted version, the restrictions are stepwisely relaxed in order to proceed in the direction of the optimal model. This procedure leads to a sequential testing procedure and a hierarchy of models that are nested into each other or in

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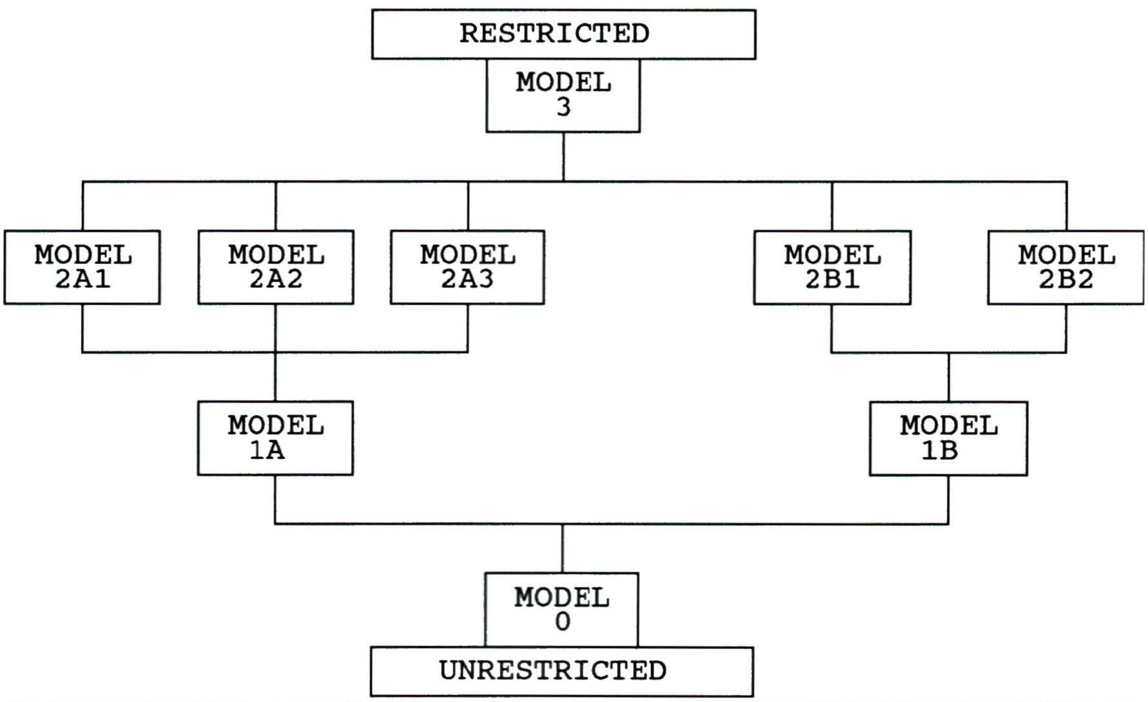
[88]G.E.Mizon, op. cit., p.1225

[89]This is mainly due to two reasons: 1) some models do not converge to the maximum of the log-likelihood function; 2) the relaxing of some restrictions makes estimation impossible because arithmetic errors occur such that the error limit is exceeded and the estimation procedure is terminated by the program.

other words the more restricted versions are special cases of the less restricted ones. Unfortunately, this procedure is not optimal because estimation difficulties prevent the development of a complete hypothesis testing framework. This means that any model found to be optimal by this procedure is not necessarily optimal in an absolute sense. Nevertheless, in the absence of a better way to proceed this method can still lead to a meaningful analysis of some properties of the Bank of Canada model. The following figure shows how the models tested are related to each other with respect to restrictions on the δ's. The models are named and will be explained below.

Figure 1; The Relation of the Models to each other according to their Level of Restriction



The restrictions on the δ's represent the most important influence of the theory in the model and determine, therefore, the way to proceed. In the first place, the models are estimated and tests on the δ's performed leaving the restrictions on α_1 and σ_2 untouched. One way to move away from the restricted Bank of Canada model (Model 3) is to assume the δ's in one of the three equations are different from those in the remaining two. This leads to three models: 2A1, 2A2 and 2A3. For model 2A1 the δ's of the first equation are different from those of the second and third equation whereas for model 2A2 those of the second equation are different from the other ones. The δ's of model 2A3 behave in an analogous way. Model 2A3 is the most appealing because for models 2A1 and 2A2 different δ's are assumed for the production function which enters the system at two places: in equation (1) and in the Okun's Law equation (equation (2)). Since it does not make much sense to presume different NAIRUs for the same function, model 2A3 is of more interest than the other two. Therefore, models 2A1 and 2A2 have only been estimated for the case where the restrictions on α_2 and σ_2 are not relaxed.

The models 2A1, 2A2 and 2A3 are themselves restricted versions of model 1A where the NAIRUs, the δ's, are different among the three equations. Unfortunately, this model is not estimable.

Another way to proceed from the Bank of Canada model

(model 3) is to assume that the NAIRU appearing in the production function is different from that of the labour market gap (RNU-NAIRU). Now, assuming that the NAIRU of the Okun's Law[90] equation (equation (2)), that is seemingly neither related to a labour gap nor to the production function, is equal to the NAIRU of the production function we reach model 2B1; assuming it to be equal to the NAIRU of the labour market gap we have model 2B2. Model 2B2 seems to be more appealing because the NAIRU of the Okun's Law equation that is neither related to the production function nor to the labour market gap in fact comes from a labour market gap term and becomes isolated just by rearranging the Okun's Law equation.[91] Both models , 2B1 and 2B2, are restricted versions of model 1B where the above mentioned NAIRU of the Okun's Law equation is not equal to either of the production function or the labour gap NAIRU.

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[90]Considering the Okun's Law equation (equation (2)) we see three NAIRUs:

$$RNU = NAIRU + \beta_1(RNU_{t-1} - NAIRU_{t-1}) + \beta_2(\log Y - \log\hat{Y}) + \epsilon_2$$

The first Nairu term on the right hand side of the above equation does not seem to be related to the production function or to the labour market gap, the second NAIRU clearly is part of a labour market gap term and the third one hides behind $\hat{Y}$ which is the estimate of the production function.

[91]The Okun's Law equation can be rewritten as follows:

$$RNU - NAIRU = \beta_1(RNU_{t-1} - NAIRU_{t-1}) + \beta_2(\log Y - \log\hat{Y}) + \epsilon_2$$

This equation relates the actual labour market gap to its previous value and the actual output gap, so that the NAIRU term that appears on the left hand side of the equation clearly is related to the labour market gap.

The models 1A and 1B are special cases of model 0, the totally unrestricted version. At this point, it should be noted that there are other versions of the model not mentioned here that could be incorporated in Figure 2. They do not appear because they are not estimable, i.e. they are less restricted than the models 1A and 1B, or they exhibit different NAIRUs for the production function which is not very realistic[92].

In order to perform the Likelihood-Ratio tests the different models are estimated to obtain the value of the maximized log-likelihood function. Then, these values are used to compute the X^2 value and to compare it with the X^2 value of the table at a level of significance of 5% in order to decide whether the restrictions are rejected or not.

After having analyzed solely the restrictions on the δ's the same estimation and testing procedure is repeated for three different cases: 1) the restriction that $\sigma_2=1/\alpha_2$ is relaxed; 2) the restriction that $\alpha_2=1-\alpha_1$ is relaxed, i.e. the production function is no longer necessarily of the Cobb-Douglas type; 3) both the restrictions on α_2 and σ_2 are

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[92]There are models that are different from model 1A where the production functions of the model have different NAIRUs, e.g. a model whose labour market gap NAIRUs are identical. Models less restricted than 1A are for example ones that have the same properties as 1A but have different NAIRUs in equation (2); a model less restricted than 1B is the one described at the beginning of this footnote.

relaxed.

The results of the estimation procedure are used to:

a) test specific restrictions on the δ's given the described restrictions on α_2 and σ_2; and

b) test the restrictions on α_2 and σ_2 given specific restrictions on the δ's.

Some of the models are also estimated in order to test for the validity of the restrictions on σ_0, the coefficient of the price expectations, and σ_1, the coefficient of the growth rate of hours worked. These restrictions are considered to be less important with respect to theory. The growth rate of hours worked is directly linked to the rate of wage increase because a change of this growth rate leads, ceteris paribus, to a change of the growth rate of per capita output and hence to a change of the real wage growth rate. This together with a constant rate of inflation brings about a change of the nominal wage growth rate. The unit coefficient on the price expectations variable reflects the rational expectations approach. A change of expectations is supposed to have a similar effect on nominal wages, ceteris paribus. Any other expectations formation with systematic errors involved in the process is not compatible with a rationally acting economic subject, generally assumed in economic theory.

The estimation and testing is performed for two sample periods:1)1967:1 - 1985:4 and 2)1967:1 - 1981:4. The different periods are motivated by the severe 1981-1982 recession in Canada to see how this event affects the NAIRU estimates.[93]

IV.2 Test Results

In this section of the thesis the results of the likelihood ratio tests are presented. Subsection IV.2.1 contains the results for the restrictions of the δ's, i.e. the Bank of Canada model (model 3) is tested against less restricted models. Subsection IV.2.2 presents the results for the restrictions on α_1 and σ_2 for each estimated model, i.e each model is tested against itself with respect to the restrictions on α_1 and σ_2. For model 3 the results for σ_0 and σ_1 are given as well.

[93]D.Rose, R.Ford, op.cit., p.11

IV.2.1 Results for the restrictions on the δ's

The following figure explains the organisation of this subsection:

Figure 2; The organisation of Subsection IV.2.1

- IV.2.1.1 Results for the restrictions on the δ's when $\alpha_1=1-\alpha_2$ and $\sigma_2=1/\alpha_2$
  - Model 3 vs. all other Models; Table 1
  - Models 2B1 and 2B2 vs. Model 1B; Table 2
- IV.2.1.2 Results for the Restrictions on the δ's when $\sigma_2\neq1/\alpha_2$
  - Model 3 vs. all other Models; Table 3
- IV.2.1.3 Results for the Restrictions on the δ's when $\alpha_1\neq1-\alpha_2$
  - Model 3 vs. all other Models; Table 4
- IV.2.1.4 Results for the Restrictions on the δ's when $\sigma_2\neq1/\alpha_2$ and $\alpha_1\neq1-\alpha_2$
  - Model 3 vs. all other Models; Table 5

IV.2.1.1 Results for the restriction on the δ's when $\alpha_1=1-\alpha_2$ and $\sigma_2=1/\alpha_2$[94]

The results reported in this chapter come out of the estimation process leaving the restrictions on α_1 and σ_2 untouched. In the first place, δ_{2i}, the coefficient of the Unemployment Insurance variable, is insignificant on the basis of the LR test. A result that is identical with those of the Bank of Canada. The LR tests for the significance of δ_{2i} have been performed because the Bank of Canada finds it to be insignificant, and its t-ratio is always too low for significance.

Model 3, the most restricted version, is accepted when tested against model 2A3 for both $\delta_{2i}=0$ and $\delta_{2i}\neq0$.[95], the same result the Bank of Canada presents.[96] It is rejected, however, when tested against models 2B1, 2B2 and 1B.[97] It is

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[94] For the exact version of the models tested see Appendix I.

[95]Note that every model is estimated two times:1) $\delta_{2i}=0$ and 2) $\delta_{2i}\neq0$. This is done in order to test for the significance of δ_{2i} with every model. The values of the Likelihood ratio tests of the $\delta_{2i}\neq0$ case do not lead to results different from the $\delta_{2i}=0$ case. Therefore, the test results of the $\delta_{2i}=0$ case will not be reported.

[96]D.Rose, R.Ford, op.cit., p.10

[97]It is a property of a sequential testing procedure that the rejection of restrictions on a more restricted level of models (2B1 and 2B2) should lead to a rejection of the same restrictions on a less restricted level (model 1B).

also rejected when tested against the , less important, models 2A1 and 2A2. In addition, models 2B1 and 2B2 can be tested against model 1B and it turns out that both are rejected in comparison to model 1B. Thus, the results for this round of testing are ambiguous because the restrictions imposed on model 3 hold in one case, model 2A3, and are rejected for all others.

For the shorter sample period the results are somewhat different. Model 2B1 is not estimable due to convergence problems as well as model 1B. Therefore, the restrictions are only tested with respect to model 2A3 and model 2B2 and it follows from the tests that the restrictions are accepted.

See the following table for the X^2 values that result from testing the restrictions of model 3 as compared to the less restricted models.

Table 1[98]; Testing Model 3 against Models 2A3, 2B1, 2B2 and 1B

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 2A3	1.89	1.78	5.99	2
Model 2B1	36.05	--	5.99	2
Model 2B2	34.40	5.28	5.99	2
Model 1B	42.87	--	9.49	4

The next table shows the results for the tests that compare model 1B with models 2B1 and 2B2.

Table 2; Testing Models 2B1 and 2B2 against Model 1B

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 2B1	6.88	1.78	5.99	2
Model 2B2	8.46	--	5.99	2

[98]For the maximized values of the log-likelihood functions see Appendix II.

IV.2.1.2 Results for the restrictions on the δ's when $\sigma_2 \neq 1/\alpha_2$

Now, the restriction that $\sigma_2=1/\alpha_2$ is relaxed. In other words, we drop the assumption that in the long run equilibrium income per capita grows at a rate equal to $1/\alpha_2$ times the growth rate of productivity. The restrictions on the δ's are tested again. As before δ_{2i} is insignificant for the estimated models, given that $\sigma_2 \neq 1/\alpha_2$, with the exception of model 2A3 for the short sample period. However, the restrictions of model 3 are now rejected when tested against model 2A3. They are also rejected, as before, when compared with models 2B1 and 2B2. Model 1B is not estimable for this particular case, neither is model 2B2 for the short sample period. See the following table for the X^2-values of the tests for both sample periods.

Table 3; Model 3 tested against Models 2A3, 2B1,2B2 with $\sigma_2 \neq 1/\alpha_2$

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 2A3	16.79	8.51	5.99	2
Model 2B1	38.33	12.98	5.99	2
Model 2B2	38.33	--	5.99	2
Model 1B	--	--	9.49	4

In this case the result is unambiguous; the restrictions of model 3 are rejected in all cases.

IV.2.1.3 Results for Restrictions on the δ's when $\alpha_1 \neq 1-\alpha_2$

Here, the restrictions on the δ's are analyzed for the case where the assumtion that the production function is of the Cobb-Douglas type is relaxed; namely, the assumption that $\alpha_1=1-\alpha_2$ is no longer valid.

As for the case where α_1 and σ_2 are restricted the restrictions are accepted when model 2A3 is compared with model 3 through a LR test. On the other hand, models 2B1 and 2B2 are superior to model 3 on the basis of the LR test. Model 1B is not estimable for this case, as well as model 2B2 for the shorter sample period. Table 4 gives the results.

Table 4; Model 3 tested against Models 2A3, 2B1, 2B2, 1B with $\alpha_1 \neq 1-\alpha_2$

	X^2-value of LR test, sample period: 67:1-85:4	67:1-81:4	Critical values 5%	Degrees of freedom
Model 2A3	4.59	5.22	5.99	2
Model 2B1	46.84	7.56	5.99	2
Model 2B2	42.64	--	5.99	2
Model 1B	--	--	9.49	4

The results are ambiguous because model 3 is not rejected in all cases.

IV.2.1.4 Results for Restrictions on the δ's when $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$

In the following the results of the tests are presented where both restrictions that $\sigma_2=1/\alpha_2$ and $\alpha_1=1-\alpha_2$ are lifted. As for the $\sigma_2 \neq 1/\alpha_2$ case models 2A3, 2B1 and 2B2 are preferred to model 3 on the basis of the LR test. See Table 5 for the results.

Table 5; Model 3 tested against Models 2A3, 2B1, 2B2 and 1B with $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 2A3	15.67	7.27	5.99	2
Model 2B1	47.23	11.37	5.99	2
Model 2B2	41.46	--	5.99	2
Model 1B	--	--	9.49	4

IV.2.2. Results for the Restrictions on α_1 and σ_2

The following figure gives the organisation of this subsection:

Figure 3; The organisation of Subsection IV.2.2

- IV.2.2.1 Results for $\sigma_2 \neq 1/\alpha_2$,
  - it is tested whether $\sigma_2=1/\alpha_2$ is a valid restriction using the $\sigma_2 \neq 1/\alpha_2$ and the $\sigma_2=1/\alpha_2$ version of each model.
- IV.2.2.2 Results for $\alpha_1 \neq 1-\alpha_2$
  - each model is tested for the validity of the restriction that $\alpha_1=1-\alpha_2$ using the $\alpha_1 \neq 1-\alpha_2$ and the $\alpha_1=1-\alpha_2$ version of the models.
- IV.2.1.3 Results for $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$
  - each model is tested for the validity of the restriction that $\sigma_2=1/\alpha_2$ and $\alpha_1=1-\alpha_2$ using the $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$ and the $\sigma_2=1/\alpha_2$ and $\alpha_1=1-\alpha_2$ versions of the models.
- IV.2.2.3 Results for $\sigma_0=1$ and $\sigma_1=1$
  - for model 3, it is tested whether the restrictions that $\sigma_0=1$ and $\sigma_1=1$ are valid.

IV.2.2.1 Results for $\sigma_2 \neq 1/\alpha_2$

This section of the thesis presents the results of the tests that analyze the validity of the assumtion that $\sigma_2=1/\alpha_2$ or in other words the assumtion that income per capita grows at a rate $1/\alpha_2$ times the rate of productivity growth. These tests are performed for every estimable set of restrictions on the δ's, i.e. for all estimable models. The results show that this restriction is rejected for every model that is estimable. Table 6 gives the X^2 values of the LR tests for both sample periods.

Table 6; Testing for the validity of $\sigma_2=1/\alpha_2$, assuming $\alpha_1=1-\alpha_2$

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 3	9.94	21.49	3.84	1
Model 2A3	24.85	28.23	3.84	1
Model 2B1	12.22	--	3.84	1
Model 2B2	13.88	--	3.84	1
Model 1B	--	--	3.84	1

It is also tested whether the restriction on σ_2 is valid given that the restriction that $\alpha_1=1-\alpha_2$ does not hold. Table 7 gives the respective results.

Table 7; Testing for the validity of $\sigma_2=1/\alpha_2$, assuming $\alpha_1\neq1-\alpha_2$

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 3	9.26	8.11	3.84	1
Model 2A3	20.34	10.16	3.84	1
Model 2B1	9.66	11.93	3.84	1
Model 2B2	8.08	--	3.84	1
Model 1B	--	--	3.84	1

As well as in the former case the restriction is rejected for all estimable models.

IV.2.2.2 Results for $\alpha_1\neq1-\alpha_2$

Now, the assumption is tested whether the production function is of the Cobb-Douglas form or in other words whether the restriction that $\alpha_1=1-\alpha_2$ holds. This restriction is accepted for model 3 on the basis of the long sample period, but surprisingly rejected on the basis of the short one. For all other estimable models the restriction is rejected. Table 8 presents the results.

Table 8; Testing for the validity of $\alpha_1=1-\alpha_2$, assuming $\sigma_2=1/\alpha_2$

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 3	3.51	16.01	3.84	1
Model 2A3	6.21	19.45	3.84	1
Model 2B1	14.30	--	3.84	1
Model 2B2	11.76	--	3.84	1
Model 1B	--	--	3.84	1

It is also tested whether the restriction that $\alpha_1=1-\alpha_2$ is valid when the restriction that $\sigma_2=1/\alpha_2$ is relaxed. See Table 9 for the results of the respective LR tests.

Table 9; Testing for the validity of $\alpha_1=1-\alpha_2$, assuming $\sigma_2\neq 1/\alpha_2$

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
Model 3	2.83	2.63	3.84	1
Model 2A3	1.71	1.38	3.84	1
Model 2B1	11.74	1.01	3.84	1
Model 2B2	5.95	--	3.84	1
Model 1B	--	--	3.84	1

For the long sample period, the restriction on α_1 is accepted for model 3 and 2A3 and rejected for the models 2B1 and 2B2 (Table 9). It is accepted for all estimable models for the short sample period.

IV.2.2.3 Results for $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$

The joint hypothesis that both restriction, on α_1 and σ_2 are valid, is rejected for every estimable model in both sample periods. Table 10 gives the X^2 values of the respective tests.

Table 10; Testing for the validity of $\sigma_2=1/\alpha_2$ and $\alpha_1=1-\alpha_2$

	X^2-value of LR test, sample period: 67:1- 85:4	81:4	Critical values 5%	Degrees of freedom
Model 3	12.78	24.11	5.99	2
Model 2A3	26.56	29.61	5.99	2
Model 2B1	23.96	--	5.99	2
Model 2B2	19.84	--	5.99	2
Model 1B	--	--	5.99	2

IV.2.3 Results for the Restrictions on σ_0 and σ_1

The restrictions that σ_0 and σ_1 are equal to one are only tested for model 3 because they are of minor interest with respect to the validity of the whole model.[99] For model 3 the results indicate that both restrictions are accepted separately and jointly for both sample periods. Table 11 presents the results of the LR-tests.

Table 11; Testing for the validity of $\sigma_0=1$ and $\sigma_1=1$, assuming all other restrictions to be valid

	X^2-value of LR test, sample period: 67:1-		Critical values 5%	Degrees of freedom
	85:4	81:4		
$\sigma_0=1$	0.003	1.516	3.84	1
$\sigma_1=1$	0.434	0.679	3.84	1
$\sigma_0=1$ and $\sigma_1=1$	1.162	1.625	5.99	2

[99]See Section IV.1 above.

IV.3 Analysis of the Parameter Estimates

As shown above, less restricted models are, with few exceptions preferable to model 3 on the basis of a likelihood-ratio test. It is, therefore, of interest to analyze whether these models generate parameter estimates that are meaningful in terms of economic theory and other empirical studies. Figure 4 gives the organisation of this section:

Figure 4; The organisation of Section IV.3

- IV.3.1 Expectations with Respect to Parameter Estimates
  Based on theoretical considerations, expectations are be presented about the range of values and the sign the parameter estimates for every equation of the model.

- IV.3.2 Parameter Estimates of Model 3
  The parameter estimates of Model 3 are analyzed on the basis of the expectations described in section IV.3.1.

- IV.3.3 Parameter Estimates of Model 2A3
  The parameter estimates of model 2A3 are analyzed on the basis of the considerations of section IV.3.1

- IV.3.4 Parameter Estimates of Model 2B1
  The parameter estimates of model 2B1 are analyzed on the basis of the considerations of section IV.3.1

- IV.3.5 Parameter Estimates of Models 2B2 and 1B
  The parameter estimates of models 2B2 and 1B are analyzed on the basis of the considerations of section IV.3.1

IV.3.1 Expectations with Respect to Parameter Estimates

$$(1)\ \log Y = \alpha_0 + (1 - \alpha_2)K + \alpha_2 LS + ETFP + \epsilon_1$$

The parameter estimates for α_2 are expected to be positive and to roughly range between 0.6 and 0.75.[100] These figures are derived from an empirical study by Brox and Cluff[101] who estimate this parameter to be equal to 0.69 for a sample period from 1960 to 1976. Accordingly, $\hat{\alpha}_1$ is supposed to obtain values between 0.25 and 0.4 in the cases where it is estimated separately.

$$(2)\ RNU - NAIRU = \beta_1(RNU_{t-1} - NAIRU_{t-1}) + \beta_2(\log Y - \log\hat{Y}) + \epsilon_2$$

The estimate $\hat{\beta}_1$ is expected to be greater than zero because a positive labour market gap in the previous period is supposed to have a positive effect on the gap in the current period due to unemployment that is persistent and cannot disappear in one period.

On the other hand, $\hat{\beta}_2$ should be negative because it links the output gap with the labour market gap. In other words, a rise (decline) of output means a decline (rise) of

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[100] The specification of the model may cause a downward bias of the estimates of α_2; see note 14.

[101] J.Brox and M.Cluff, Potential Output and the Real GNP Gap, Canadian Statistical Review, January 1979, p. x

unemployment so that a factor linking the two has to be less than zero.[102]

$$(3)\ W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3 (RNU - NAIRU) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$$

σ_0 and σ_1 are assumed to be equal to one for reasons explained above. σ_2 is supposed to be positive because a growing productivity leads to higher real wages through rising marginal products of the employees. The rise of real wages is realized through an increase of nominal wages, other things being equal.

σ_3 should be negative since a positive unemployment gap or in other words an actual unemployment rate higher than the natural is a sign of excess supply on the labour market. An excess supply on the labour market is supposed to have an adverse effect on the rate of wage increase.

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[102]Based on the original Okun's Law equation, it can be shown that $\hat{\beta}_2$ should be negative. See the following derivation with Y denoting output, U unemployment rate and the (*) the potential values.

$$Y^* = Y(1 + c\,(U-U^*)),\ 0<c<1$$

$$\Leftrightarrow \log Y^* = \log Y + \log(1 + c(U-U^*))$$

$$\Leftrightarrow \log Y^* = \log Y + c(U-U^*),\ \text{since } c(U-U^*) \approx \log(1 + c\,(U-U^*))$$

$$\Leftrightarrow (-c)\,(U-U^*) = (\log Y - \log Y^*)$$

$$\Leftrightarrow U = U^* + (-c^{-1})\,(\log Y - \log Y^*)$$

Set $(-c^{-1}) = \beta_2$.

σ_4 can be expected to be negative as well. The variable AIB is a dummy obtaining the value 1 for the Anti-Inflation-Board period and 0 otherwise. Assuming that the measures of this board had at least some effects, the increase of wages should be slowed down.

σ_5 is supposed to be non-negative. It is the coefficient of the variable PREL, the expected difference of the growth of producer (see footnote 20) and consumer prices. If producer prices are expected to rise faster than consumer prices employers are expected to pay higher input prices than in equilibrium so that there is a positive effect on wages.

(4) NAIRU $= \delta_{0i} + \delta_{1i}DEM + \delta_{2i}UI$

δ_{0i} is expected to be positive. Even if there were no Unemployment Insurance benefits and no demographic factors influencing the NAIRU one would still expect some positive rate of unemployment in the equilibrium due to search unemployment. δ_{1i} and δ_{2i} are supposed to be positive because the influence of demographic pressure and Unemployment Insurance benefits is positively correlated with the NAIRU.[103]

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[103]See chapter III.4 for the theory.

IV.3.2 Parameter Estimates of Model 3

Reproducing the estimation of the Bank of Canada model yields the coefficient estimates shown in Table 12.

Table 12; Estimates of Model 3, σ_2 and α_2 restricted

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R2=0.99		R^2=0.97	
Intercept ($\hat{\alpha}_0$)	-0.60	(0.037)	-0.73	(0.036)
Labour Input ($\hat{\alpha}_2$)	0.45	(0.011)	0.41	(0.011)
Capital Input ($\hat{\alpha}_1$)	0.55	(=1-$\hat{\alpha}_2$)	0.59	(=1-$\hat{\alpha}_2$)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.71	(0.039)	0.77	(0.050)
Output gap ($\hat{\beta}_2$)	-22.39	(2.63)	-16.97	(2.72)
Phillips Curve	R^2=0.21		R^2=0.10	
Technical Change ($\hat{\sigma}_2$)	2.2	(=1/$\hat{\alpha}_2$)	2.43	(=1/$\hat{\alpha}_2$)
Labour Gap ($\hat{\sigma}_3$)	-0.42	(0.10)	-0.69	(0.26)
AIB ($\hat{\sigma}_4$)	-0.07*	(0.39)	0.49*	(0.46)
PREL ($\hat{\sigma}_5$)	0.57*	(0.49)	0.09*	(0.60)
NAIRU				
Intercept ($\hat{\delta}_0$)	-22.3	(5.2)	-13.58	(2.62)
Demographics ($\hat{\delta}_1$)	57.7	(10.1)	39.63	(5.12)
Unemployment Insur. ($\hat{\delta}_2$)	--	--	--	--
NAIRU (87:4) computed with the estimates		9.36%		8.09%

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

The reproduction of the Bank of Canada model yields several estimates that are not in accord with the expectations outlined in the previous section or have statistical properties that are not satisfactory. For both sample periods, δ_0 is smaller than zero and σ_4 is positive for the short sample period. α_2 is too small in both sample periods. In addition, σ_4 and σ_5 are not significant on the basis of the t-ratio. From theory one would expect the variable PREL to have a significant influence on the rate of wage increase. The Anti-Inflation -Board, on the other hand, possibly had no influence on nominal wages[104]. The NAIRU is estimated for the fourth quarter of 1987 in order to make it comparable to the Bank of Canada results that are reported for the same period.

In the $\sigma_2 \neq 1/\alpha_2$ case (see Table 13) $\hat{\delta}_0$ has the wrong sign for the long sample period again. $\hat{\delta}_0$ and $\hat{\delta}1$ are insignificant in the short period. $\hat{\sigma}_4$ is insignificant for both sample periods, whereas $\hat{\sigma}_5$ is insignificant solely for the long sample. $\hat{\alpha}_2$ is smaller than one would expect. $\hat{\sigma}_2$ is now estimated. For both samples it is insignificant on the basis of the t-ratio and has the wrong sign for the short period.

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[104]This result is at variance with the result of Ridell and Smith who find the Anti-Inflation-Board to have had a significant impact on wage settlements. W.Ridell and P. Smith op. cit., p.390. See also J.Schaafsma and W.D. Walsh, An Empirical Analysis of the Impact of Canadian Wage Controls on Negotiated Wage Settlements, 1967-87, Discussion Paper 89-10, Department of Economics, University of Victoria, Victoria, 1989, pp. 20-22.

Table 13; Estimates of Model 3, assuming $\sigma_2 \neq 1/\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	$R^2=0.99$		$R^2=0.99$	
Intercept ($\hat{\alpha}_0$)	-0.63	(0.039)	-0.58	(0.086)
Labour Input ($\hat{\alpha}_2$)	0.44	(0.012)	0.45	(0.025)
Capital Input ($\hat{\alpha}_1$)	0.56	($=1-\hat{\alpha}_2$)	0.55	($=1-\hat{\alpha}_2$)
Okun's Law	$R^2=0.98$		$R^2=0.97$	
Lagged Labour gap ($\hat{\beta}_1$)	0.72	(0.039)	0.95	(0.019)
Output gap ($\hat{\beta}_2$)	-21.93	(2.63)	-13.24	(2.78)
Phillips Curve	$R^2=0.33$		$R^2=0.35$	
Technical Change ($\hat{\sigma}_2$)	0.78*	(0.43)	-1.09	(0.551)
Labour Gap ($\hat{\sigma}_3$)	-0.37	(0.10)	-0.38	(0.16)
AIB ($\hat{\sigma}_4$)	0.24*	(0.37)	0.45*	(0.39)
PREL ($\hat{\sigma}_5$)	0.94*	(0.49)	1.06	(0.49)
NAIRU				
Intercept ($\hat{\delta}_0$)	-20.0	(3.45)	7.74*	(8.59)
Demographics ($\hat{\delta}_1$)	53.8	(6.63)	1.37*	(15.5)
Unemployment Insur. ($\hat{\delta}_2$)	--	--	--	--
NAIRU (87:4) computed with the estimates	9.43%		8.21%	

The * denotes an insignificant t-ratio. Standard errors in parenthesis.

Table 14 gives the estimation results for model 3 assuming $\alpha_1 \neq 1-\alpha_2$. Now, α_1 is no longer computed but estimated. For the long sample period $\hat{\alpha}_1$ and $\hat{\alpha}_2$ do not exactly add up to one, as they should if the production function of the macroeconomy is of the Cobb-Douglas type. In addition, their

Table 14; Estimates of Model 3, assuming $\alpha_1 \neq 1-\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	-1.29	(0.51)	-4.13	(1.45)
Labour Input ($\hat{\alpha}_2$)	0.48	(0.059)	1.37	(0.41)
Capital Input ($\hat{\alpha}_1$)	0.62	(0.13)	0.13	(0.20)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.69	(0.042)	0.86	(0.044)
Output gap ($\hat{\beta}_2$)	-22.64	(2.63)	-15.60	(2.80)
Phillips Curve	R^2=0.26		R^2=0.25	
Technical Change ($\hat{\sigma}_2$)	2.08	(=1/$\hat{\alpha}_2$)	0.73	(=1/$\hat{\alpha}_2$)
Labour Gap ($\hat{\sigma}_3$)	-0.39	(0.10)	-0.67	(0.21)
AIB ($\hat{\sigma}_4$)	0.15*	(0.38)	0.59*	(0.51)
PREL ($\hat{\sigma}_5$)	0.75*	(0.48)	0.51*	(0.53)
NAIRU				
Intercept ($\hat{\delta}_0$)	-22.1	(3.20)	-8.24	(3.49)
Demographics ($\hat{\delta}_1$)	57.6	(6.19)	29.9	(6.68)
Unemployment Insur.($\hat{\delta}_2$)	--	--	--	--
NAIRU (87:4) computed with the estimates		9.38%		8.12%

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

values for the long sample period are somewhat out of range because $\hat{\alpha}_1$ exceeds $\hat{\alpha}_2$ with $\hat{\alpha}_2$ lying below 0.6. For the short period they are totally out of range, $\hat{\alpha}_1$ is far too low and insignificant , $\hat{\alpha}_2$ too high. $\hat{\sigma}_4$ and $\hat{\sigma}_5$ are insignificant for both sample periods.

If the restrictions that $\sigma_2=1/\alpha 2$ and $\alpha 1=1-\alpha_2$ are jointly relaxed (see Table 15 on the following page) $\hat{\delta}_0$ has the wrong sign for the long sample period and is, together with $\hat{\delta}1$, insignificant for the short sample period. In addition, $\hat{\alpha}_0$, $\hat{\alpha}_2$, $\hat{\sigma}_2$, $\hat{\sigma}_4$ and $\hat{\sigma}_5$ are insignificant for the long sample and $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\sigma}_2$ and $\hat{\sigma}_4$ for the short. $\hat{\alpha}_1$ is too small for the long sample period and $\hat{\alpha}_2$ too high for the short period.

Table 15; Estimates of Model 3, assuming $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	0.97*	(0.88)	-2.82	(1.42)
Labour Input ($\hat{\alpha}_2$)	0.75*	(0.23)	1.20	(0.46)
Capital Input (α_1)	0.025	(0.11)	0.16	(0.24)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.78	(0.042)	0.95	(0.019)
Output gap ($\hat{\beta}2$)	-19.86	(2.66)	-13.41	(2.85)
Phillips Curve	R^2=0.34		R^2=0.35	
Technical Change ($\hat{\sigma}_2$)	0.67*	(0.44)	-1.09*	(0.55)
Labour Gap ($\hat{\sigma}_3$)	-0.38	(0.09)	-0.38	(0.16)
AIB ($\hat{\sigma}_4$)	0.23*	(0.36)	0.44*	(0.39)
PREL (σ_5)	0.88*	(0.47)	1.06	(0.49)
NAIRU				
Intercept ($\hat{\delta}_0$)	-18.1	(3.86)	7.83*	(8.46)
Demographics ($\hat{\delta}_1$)	50.2	(7.38)	1.25*	(15.3)
Unemployment Insur. ($\hat{\delta}_2$)	--	--	--	--
NAIRU (87:4) computed with estimates		9.42%		8.51%

The * denotes an insignificant t-ratio. Standard erors are in parenthesis.

IV.3.3 Parameter Estimates of Model 2A3

Model 2A3 is the model where it is assumed that the NAIRUs of the production function (equation 1) and the Okun's Law equation (equation 2) are identical but different from the NAIRU of the Phillips curve equation (equation 3). Therefore, there are two sets of NAIRUs reported in the respective tables. Set 1 gives the NAIRU of equations 1 and 2 [105], set 2 the NAIRU of equation 3 [106].

Table 16 gives the estimates of model 2A3 leaving the restrictions on α_1 and σ_2 untouched. $\hat{\delta}_{01}$ and $\hat{\delta}_{02}$ have the wrong sign for both sample periods[107] as well as $\hat{\sigma}_4$. $\hat{\alpha}_2$ is too small again. As before, some estimates are not significant on the basis of the t-ratio.

Relaxing the restriction that $\sigma_2=1/\alpha_2$ (see Table 17) leads to results that are contradictory with respect to the estimates for the δ's. While $\hat{\delta}_{01}$ is negative for both sample periods, $\hat{\delta}_{02}$ is positive for both periods. The $\hat{\delta}_{11}$ and $\hat{\delta}_{12}$ behave just the opposite way. $\hat{\alpha}_2$ is too small again and $\hat{\sigma}_2$ has

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[105] The respective $\hat{\delta}$s are $\hat{\delta}_{01}$ (intercept), $\hat{\delta}_{11}$ (demographics) and $\hat{\delta}_{21}$ (unemployment insurance).

[106] The respective $\hat{\delta}$s are $\hat{\delta}_{02}$ (intercept), $\hat{\delta}_{12}$ (demographics) and $\hat{\delta}_{22}$ (unemployment insurance).

[107] Note that there are now two sets of estimates for the δ's. The first one contains δ-estimates from equations (1) and (2) and the second one from equation (3).

the wrong sign.

The estimates of the δ's behave well for both sets and periods when the restriction that $\alpha_1=1-\alpha_2$ is relaxed (see Table 18), although $\hat{\delta}_{01}$ and $\hat{\delta}_{02}$ have the wrong sign for both samples. $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are out of range for the 1967-1981 period.

Table 19 gives the results for α_1 and σ_2 being jointly unrestricted. The same contradictory behaviour of the δ estimates as in the $\sigma_2\neq1/\alpha_2$ case occurs here. The coefficients of the production function are also out range, especially for the long sample period.

Tables 16, 17, 18 and 19 are presented on the following pages.

Table 16; Parameter Estimates of Model 2A3, σ_2 and α_2 restricted

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.98		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	-0.59	(0.037)	-0.73	(0.037)
Labour Input ($\hat{\alpha}_2$)	0.46	(0.01)	0.41	(0.01)
Capital Input ($\hat{\alpha}_1$)	0.54	(=1-$\hat{\alpha}_2$)	0.59	(=1-$\hat{\alpha}_2$)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.70	(0.039)	0.77	(0.051)
Output gap ($\hat{\beta}2$)	-22.70	(2.64)	-16.96	(2.72)
Phillips Curve	R^2=0.21		R^2=0.11	
Technical Change ($\hat{\sigma}_2$)	2.17	(=1/$\hat{\alpha}_2$)	2.44	(=1/$\hat{\alpha}_2$)
Labour Gap ($\hat{\sigma}_3$)	-0.42	(0.10)	-0.60	(0.27)
AIB ($\hat{\sigma}_4$)	0.09*	(0.41)	0.21*	(0.50)
PREL ($\hat{\sigma}_5$)	0.32*	(0.53)	0.15*	(0.61)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	-26.25	(3.75)	-12.84	(3.29)
Demographics ($\hat{\delta}_{11}$)	65.32	(7.23)	37.95	(6.41)
Unemployment Insur. ($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)			8.51%	
Intercept ($\hat{\delta}_{02}$)	-16.77	(5.99)	-16.33	(5.23)
Demographics ($\hat{\delta}_{12}$)	47.17	(11.7)	45.63	(10.4)
Unemployment Insur. ($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	9.48%		7.90%	
Set 2	9.04%		8.64%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

Table 17; Parameter Estimates of Model 2A3, assuming $\sigma_2 \neq 1/\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	-0.60	(0.038)	-0.74	(0.037)
Labour Input ($\hat{\alpha}_2$)	0.45	(0.01)	0.41	(0.01)
Capital Input (α_1)	0.55	(=1-$\hat{\alpha}_2$)	0.59	(=1-$\hat{\alpha}_2$)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.70	(0.039)	0.77	(0.051)
Output gap ($\hat{\beta}2$)	-22.70	(2.62)	-17.08	(2.72)
Phillips Curve	R^2=0.39		R^2=0.35	
Technical Change ($\hat{\sigma}_2$)	-0.64*	(0.53)	-1.11*	(0.59)
Labour Gap ($\hat{\sigma}_3$)	-0.21	(0.09)	-0.32*	(0.22)
AIB ($\hat{\sigma}_4$)	0.16*	(0.35)	0.34*	(0.39)
PREL (σ_5)	1.19*	(0.48)	1.13	(0.51)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	-25.45	(3.73)	-12.71	(3.25)
Demographics ($\hat{\delta}_{11}$)	63.77	(7.20)	37.69	(6.34)
Unemployment Insur. ($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	11.01*	(19.9)	9.03	(18.4)
Demographics ($\hat{\delta}_{12}$)	-1.61*	(36.1)	-0.17*	(32.9)
Unemployment Insur. ($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	9.43%		7.91%	
Set 2	10.13%		8.94%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

Table 18; Parameter Estimates of Model 2A3, assuming $\alpha_1 \neq 1-\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	$R^2=0.99$		$R^2=0.99$	
Intercept ($\hat{\alpha}_0$)	-1.64	(0.64)	-4.23	(1.48)
Labour Input ($\hat{\alpha}_2$)	0.71	(0.16)	1.35	(0.39)
Capital Input ($\hat{\alpha}_1$)	0.43	(0.072)	0.15*	(0.19)
Okun's Law	$R^2=0.99$		$R^2=0.97$	
Lagged Labour gap ($\hat{\beta}_1$)	0.68	(0.043)	0.79	(0.051)
Output gap ($\hat{\beta}2$)	-23.14	(2.66)	-16.63	(2.73)
Phillips Curve	$R^2=0.28$		$R^2=0.25$	
Technical Change ($\hat{\sigma}_2$)	1.41	$(=1/\hat{\alpha}_2)$	0.74	$(=1/\hat{\alpha}_2)$
Labour Gap ($\hat{\sigma}_3$)	-0.36	(0.10)	-0.47	(0.23)
AIB ($\hat{\sigma}_4$)	0.11*	(0.38)	0.28*	(0.43)
PREL ($\hat{\sigma}_5$)	0.56*	(0.50)	0.62*	(0.53)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	-25.78	(3.61)	-12.28	(3.57)
Demographics ($\hat{\delta}_{11}$)	64.39	(6.97)	36.87	(6.95)
Unemployment Insur. ($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	-12.29	(7.12)	-7.97*	(6.31)
Demographics ($\hat{\delta}_{12}$)	39.37	(13.6)	30.53	(11.9)
Unemployment Insur. ($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	9.45%		7.89%	
Set 2	9.22%		8.74%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

Table 19; Parameter Estimates of Model 2A3, assuming $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	0.53*	(0.81)	-2.58*	(1.54)
Labour Input ($\hat{\alpha}_2$)	0.17*	(0.21)	0.90	(0.42)
Capital Input ($\hat{\alpha}_1$)	0.68	(0.094)	0.36*	(0.19)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.73	(0.043)	0.78	(0.051)
Output gap ($\hat{\beta}2$)	-21.52	(2.67)	-17.08	(2.74)
Phillips Curve	R^2=0.39		R^2=0.35	
Technical Change ($\hat{\sigma}_2$)	-0.64	(0.52)	-1.11*	(0.592)
Labour Gap ($\hat{\sigma}_3$)	-0.21	(0.10)	-0.32*	(0.22)
AIB ($\hat{\sigma}_4$)	0.16*	(0.35)	0.34*	(0.40)
PREL (σ_5)	1.19	(0.48)	1.13	(0.51)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	-25.04	(4.01)	-12.59	(3.39)
Demographics ($\hat{\delta}_{11}$)	62.99	(7.73)	37.48	(6.61)
Unemployment Insur.($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	11.00*	(19.9)	9.03*	(18.4)
Demographics ($\hat{\delta}_{12}$)	-1.61*	(36.1)	-0.17*	(32.9)
Unemployment Insur.($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	9.42%		7.91%	
Set 2	10.13%		8.94%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

IV.3.4 Parameter Estimates of Model 2B1

For model 2B1 it is assumed that the NAIRUs of the production function and the Okun's Law equation (equations 1 and 2) are identical [108] except for the NAIRU in equation 2 that is related to the labour gap. This NAIRU is assumed to be equal to the NAIRU of the Phillips curve equation (equation 3).[109] Therefore,as in the previous section, there are two sets of estimates of NAIRU coefficients.

Model 2B1 is not estimable for the short sample period for α_1 and σ_2 being restricted. For the long sample period the estimate of α_2 is out of range and , again, the estimates of the δ_{1i} are negative and therefore in contradiction to the expectations about this parameter. The NAIRU estimates generated by the estimated coefficients of both sets are beyond every reasonable value. The results are reported in Table 20.

[108] This is NAIRU (Set 1) in the tables and the respective $\hat{\delta}$s are $\hat{\delta}_{01}$ (intercept), $\hat{\delta}_{11}$ (demographics) and $\hat{\delta}_{21}$ (unemployment insurance).

[109] This is NAIRU (Set 2) in the tables and the respective $\hat{\delta}$s are $\hat{\delta}_{02}$ (intercept), $\hat{\delta}_{12}$ (demographics) and $\hat{\delta}_{22}$ (unemployment insurance).

Table 20; Parameter Estimates of Model 2B1, σ_2 and α_2 restricted

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2= --	
Intercept ($\hat{\alpha}_0$)	2.07	(0.53)	--	(--)
Labour Input ($\hat{\alpha}_2$)	0.93	(0.07)	--	(--)
Capital Input ($\hat{\alpha}_1$)	0.07	(=1-$\hat{\alpha}_2$)	--	(--)
Okun's Law	R^2=0.98		R^2= --	
Lagged Labour gap ($\hat{\beta}_1$)	0.86	(0.035)	--	(--)
Output gap ($\hat{\beta}$2)	-16.35	(2.85)	--	(--)
Phillips Curve	R^2=0.19		R^2= --	
Technical Change ($\hat{\sigma}_2$)	1.08	(=1/$\hat{\alpha}_2$)	--	(--)
Labour Gap ($\hat{\sigma}_3$)	-0.004	(0.002)	--	(--)
AIB ($\hat{\sigma}_4$)	0.11*	(0.40)	--	(--)
PREL ($\hat{\sigma}_5$)	1.75	(0.41)	--	(--)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	95.03	(3.45)	--	(--)
Demographics ($\hat{\delta}_{11}$)	-46.76	(23.1)	--	(--)
Unemployment Insur.($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	113.53	(5.87)	--	(--)
Demographics ($\hat{\delta}_{12}$)	-63.29	(26.7)	--	(--)
Unemployment Insur.($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	69.5%		--	
Set 2	78.9%		--	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

Table 21 gives the results when the restriction on σ_2 is relaxed. Here, the long sample period yields the $\hat{\delta}_{1i}$ with the wrong sign for both sets, the short period only for one. All estimates of the δ's are insignificant for the short period. For both periods the estimates of σ_2 are negative which should not be the case.

In Table 22 the results for the $\alpha_1 \neq 1-\alpha_2$ case are presented. The estimates of δ_{1i} have the wrong sign except for the first set of δ estimates of the short sample period. The long sample period yields a negative $\hat{\alpha}_1$ and the short one a negative $\hat{\alpha}_2$, both of which are unacceptable. In addition, the $\hat{\delta}$s of Set 2 (labour gap) are not significant on the basis of the t-ratio for the short sample period.

Relaxing the restrictions on α_1 and σ_2 yield the results presented in Table 23. The estimate of α_1 is negative for the long and insignificant for the short sample period. As before, the estimates of the δ's are all insignificant in the short sample period. Furthermore, the $\hat{\delta}_{0i}$ are less than zero except for one set of δ estimates in the short sample period. $\hat{\sigma}_2$ has the wrong sign in both periods.

Table 21; Parameter Estimates of Model 2B1, assuming $\sigma_2 \neq 1/\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	0.11*	(0.17)	-0.57	(0.09)
Labour Input ($\hat{\alpha}_2$)	0.65	(0.05)	0.46	(0.027)
Capital Input (α_1)	0.35	(=1-$\hat{\alpha}_2$)	0.54	(=1-$\hat{\alpha}_2$)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.78	(0.037)	0.78	(0.049)
Output gap ($\hat{\beta}_2$)	-19.60	(2.70)	-16.55	(2.66)
Phillips Curve	R^2=0.37		R^2=0.35	
Technical Change ($\hat{\sigma}_2$)	-0.95	(0.47)	-1.15	(0.55)
Labour Gap ($\hat{\sigma}_3$)	-0.098	(0.03)	-0.29	(0.14)
AIB ($\hat{\sigma}_4$)	0.19*	(0.35)	0.33*	(0.39)
PREL (σ_5)	1.45	(0.44)	1.16	(0.49)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	40.09	(11.4)	6.87*	(8.86)
Demographics ($\hat{\delta}_{11}$)	-53.28	(20.2)	3.59*	(15.7)
Unemployment Insur.($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	58.72	(13.5)	12.73*	(11.1)
Demographics ($\hat{\delta}_{12}$)	-86.85	(23.9)	-6.79*	(19.7)
Unemployment Insur.($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	10.95%		8.84%	
Set 2	11.21%		9.01%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

Table 22; Parameter Estimates of Model 2B1, assuming $\alpha_1 \neq 1-\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	-1.48	(0.72)	2.48	(0.55)
Labour Input ($\hat{\alpha}_2$)	1.99	(0.30)	-0.56	(0.16)
Capital Input ($\hat{\alpha}_1$)	-0.52	(0.18)	1.06	(0.08)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.86	(0.037)	0.76	(0.05)
Output gap ($\hat{\beta}2$)	-15.88	(3.28)	-15.51	(2.53)
Phillips Curve	R^2=0.29		R^2=0.34	
Technical Change ($\hat{\sigma}_2$)	0.50	(=1/$\hat{\alpha}_2$)	-1.79	(=1/$\hat{\alpha}_2$)
Labour Gap ($\hat{\sigma}_3$)	-0.07	(0.03)	-0.31	(0.13)
AIB ($\hat{\sigma}_4$)	0.22*	(0.37)	0.38*	(0.39)
PREL ($\hat{\sigma}_5$)	1.24	(0.46)	1.29	(0.49)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	53.26	(6.14)	7.94	(7.98)
Demographics ($\hat{\delta}_{11}$)	-79.89	(11.2)	1.53	(14.2)
Unemployment Insur. ($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	64.40	(7.06)	14.59*	(10.2)
Demographics ($\hat{\delta}_{12}$)	-100.5	(12.9)	-10.30*	(18.2)
Unemployment Insur. ($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	9.56%		8.78%	
Set 2	9.43%		8.95%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

Table 23; Parameter Estimates of Model 2B1, assuming $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	-1.12*	(0.71)	-1.91*	(1.32)
Labour Input ($\hat{\alpha}_2$)	1.82	(0.30)	0.89	(0.42)
Capital Input ($\hat{\alpha}_1$)	-0.42	(0.18)	0.32*	(0.22)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.86	(0.037)	0.78	(0.05)
Output gap ($\hat{\beta}_2$)	-16.59	(3.25)	-16.47	(2.71)
Phillips Curve	R^2=0.37		R^2=0.35	
Technical Change ($\hat{\sigma}_2$)	-0.97	(0.47)	-1.14	(0.55)
Labour Gap ($\hat{\sigma}_3$)	-0.08	(0.003)	-0.29	(0.14)
AIB ($\hat{\sigma}_4$)	0.19*	(0.35)	0.33*	(0.39)
PREL ($\hat{\sigma}_5$)	1.48	(0.44)	1.16	(0.49)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	55.53	(6.69)	6.40*	(8.83)
Demographics ($\hat{\delta}_{11}$)	-80.67	(11.8)	4.44*	(15.7)
Unemployment Insur. ($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	67.89	(7.67)	11.81*	(10.9)
Demographics ($\hat{\delta}_{12}$)	-102.9	(13.6)	-5.15*	(19.5)
Unemployment Insur. ($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	11.40%		8.83%	
Set 2	11.60%		8.99%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

IV.3.5 Parameter Estimates of Model 2B2 and Model 1B

In Model 2B2 the NAIRUs of equation 2 (Okun's Law) are assumed to be equal to the NAIRU of equation 3 (Phillips Curve) except for the NAIRU related to the output gap which is set equal to the NAIRU of equation 1 (Production Function). Again, there are two different sets of parameter estimates of the NAIRU equation: Set 1 contains the estimates for the coefficients of the NAIRU related to the output gap[110] and Set 2 those of the NAIRU related to the labour gap.[111]

Model 2B2 is not estimable for the short sample period except for the case where α_1 and σ_2 are restricted (see Table 24). In addition, the otherwise estimable versions of model 2B2 do not yield estimates for δ_{11} due to singularity of the data. δ_{11} is the coefficient of the demographic variable in the equation of the NAIRU that is related to the output gap, i.e. it appears in the production function (equation 1).

If α_1 and σ_2 are restricted, the estimation results of Model 2B2 for the long sample are not convincing because $\hat{\alpha}_2$ and all estimates for the δ's as well as $\hat{\sigma}_5$ are insignificant.

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[110] The respective $\hat{\delta}$s are $\hat{\delta}_{01}$ (intercept), $\hat{\delta}_{11}$ (demographics) and $\hat{\delta}_{21}$ (unemployment insurance).

[111] The respective $\hat{\delta}$s are $\hat{\delta}_{02}$ (intercept), $\hat{\delta}_{12}$ (demographics) and $\hat{\delta}_{22}$ (unemployment insurance).

In addition, $\hat{\alpha}_2$ is out of range. In both periods the $\hat{\delta}_{0i}$ have the wrong sign. Due to the singularity problem for the estimate of δ_{11} the Nairu generated by Set 1 does not make sense and is not reported in Table 24.

Table 25 presents the long sample period results of the versions of model 2B2 where either α_1 or σ_2 are unrestricted. In the latter case $\hat{\alpha}_2$ is out of range and $\hat{\delta}_{12}$ [112] has the wrong sign as well as $\hat{\sigma}_2$ which is additionally insignificant. For α_1 being unrestricted the estimate of α_2 is too high and that of α_1 bears the wrong sign. All estimates of the δ's are insignificant and $\hat{\delta}_{01}$ [113] and $\hat{\delta}_{12}$ [114] are, in contradiction to the expectations, negative. Again, the NAIRU based on Set 1 estimates is not reported.

Having no restrictions on α_1 and σ_2 (See Table 26) leads to results that are not very promising. $\hat{\alpha}_1$ and the insignificant $\hat{\sigma}_2$ have the wrong signs. Except for $\hat{\delta}_{02}$, the intercept of the equation of the labour gap NAIRU, all other estimates of the δ's are insignificant or could not be

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[112] $\hat{\delta}_{12}$ is the coefficient of the demographics variable in the equation of the NAIRU that is related to the labour gap; that is, the NAIRU of the Phillips curve equation (equation (3)) and the second NAIRU of the Okun's Law equation.

[113] $\hat{\delta}_{01}$ is the intercept of the equation of the NAIRU related to the output gap.

[114] $\hat{\delta}_{12}$ is the coefficient of the demographics variable of the output gap NAIRU equation for the $\alpha_1 \neq 1-\alpha_2$ case.

estimated due to singularity of the data.

The same table also presents the results for model 1B. Model 1B is a less restricted version of models 2B1 and 2B2.It contains three different NAIRU's: 1. The output gap NAIRU appearing in the production function (equation (1)) and in the Okun's Law equation (equation (2), third NAIRU term); 2. The NAIRU related to the labour market gap being part of the Phillips curve equation (equation (3)) and of the Okun's Law equation (second NAIRU term); 3. The NAIRU of the Okun's Law equation seemingly neither related to the output nor to the labour gap (first NAIRU term of this equation). This model is only estimable for the long sample period, α_1 and σ_2 being restricted. It generates three sets of estimates of the δ's. Except for $\hat{\delta}_{01}$ and $\hat{\delta}_{02}$ all the $\hat{\delta}_{ji}$ are insignificant or not estimable due to singularity ($\hat{\delta}_{11}$). Every $\hat{\delta}_{1i}$ has the wrong sign. In addition, $\hat{\alpha}_2$ is out of range .

Table 24; Parameter Estimates of Model 2B2 , σ_2 and α_1 restricted

Sample Period	1967:1-1985:4		1967:1-1981:4	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	-1.54	(0.11)	-1.44	(0.47)
Labour Input ($\hat{\alpha}_2$)	0.99	(0.07)	0.59	(0.10)
Capital Input (α_1)	0.01	(1-$\hat{\alpha}_2$)	0.41	(1-$\hat{\alpha}_2$)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.95	(0.019)	0.81	(0.05)
Output gap ($\hat{\beta}2$)	-12.66	(2.95)	-15.79	(2.89)
Phillips Curve	R^2=0.29		R^2=0.16	
Technical Change ($\hat{\sigma}_2$)	1.01	(=1/$\hat{\alpha}_2$)	1.69	(=1/$\hat{\alpha}_2$)
Labour Gap ($\hat{\sigma}_3$)	-0.23	(0.08)	-0.69	(0.24)
AIB ($\hat{\sigma}_4$)	0.18*	(0.36)	0.56*	(0.43)
PREL ($\hat{\sigma}_5$)	0.73*	(0.46)	0.29*	(0.57)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	-12.54	(115.3)	-409 *	(513.6)
Demographics ($\hat{\delta}_{11}$)	(-2702)	(0.00)	(-653)*	(0.00)
Unemployment Insur.($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	4.45*	(9.44)	-11.69	(2.95)
Demographics ($\hat{\delta}_{12}$)	8.11*	(17.7)	36.17	(5.74)
Unemployment Insur.($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	--		--	
Set 2	8.88%		8.09%	

The * denotes an insignificant t-ratio. Standard errors in parenthesis.

Table 25; Parameter Estimates of Model 2B2, assuming $\sigma_2 \neq 1/\alpha_2$ or $\alpha_1 \neq 1-\alpha_2$

1967:1-1985:4, Version	$\sigma_2 \neq 1/\alpha_2$		$\alpha_1 = 1-\alpha_2$	
Production Function	R^2=0.99		R^2=0.99	
Intercept ($\hat{\alpha}_0$)	-1.26	(0.13)	-4.06	(1.57)
Labour Input ($\hat{\alpha}_2$)	0.99	(0.07)	2.04	(0.35)
Capital Input ($\hat{\alpha}_1$)	0.01	($1-\hat{\alpha}_2$)	-0.58	(0.20)
Okun's Law	R^2=0.98		R^2=0.97	
Lagged Labour gap ($\hat{\beta}_1$)	0.97	(0.01)	0.96	(0.01)
Output gap ($\hat{\beta}2$)	-12.24	(2.84)	-10.99	(3.24)
Phillips Curve	R^2=0.38		R^2=0.32	
Technical Change ($\hat{\sigma}_2$)	-0.81*	(0.46)	0.49	($=1/\hat{\alpha}_2$)
Labour Gap ($\hat{\sigma}_3$)	-0.15	(0.05)	-0.17	(0.06)
AIB ($\hat{\sigma}_4$)	0.18*	(0.34)	0.19*	(0.37)
PREL ($\hat{\sigma}_5$)	1.30	(0.44)	1.00	(0.46)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	-15.07*	(85.8)	-71.6*	(55.0)
Demographics ($\hat{\delta}_{11}$)	(-2019)	(0.00)	(-399)*	(0.00)
Unemployment Insur. ($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	28.76	(13.0)	17.29*	(12.1)
Demographics ($\hat{\delta}_{12}$)	-33.60*	(23.3)	-14.66*	(22.3)
Unemployment Insur. ($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	--		--	
Set 2	10.38%		9.23%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

Table 26; Parameter Estimates of Model 2B2, assuming $\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$ and Model 1B, σ_2 and α_2 restricted

1967:1-1985:4, Model	Model 2B2 $\sigma_2 \neq 1/\alpha_2$; $\alpha_1 \neq 1-\alpha_2$		Model 1B $\sigma_2 = 1/\alpha_2$; $\alpha_1 = 1-\alpha_2$	
Production Function	$R^2=0.99$		$R^2=0.99$	
Intercept ($\hat{\alpha}_0$)	-4.99	(1.86)	0.49*	(0.27)
Labour Input ($\hat{\alpha}_2$)	1.88	(0.34)	0.93	(0.07)
Capital Input ($\hat{\alpha}_1$)	-0.50	(0.20)	0.07	($1-\hat{\alpha}_2$)
Okun's Law	$R^2=0.98$		$R^2=0.98$	
Lagged Labour gap ($\hat{\beta}_1$)	0.97	(0.01)	0.87	(0.04)
Output gap ($\hat{\beta}2$)	-11.38	(3.16)	-15.96	(2.88)
Phillips Curve	$R^2=0.38$		$R^2=0.24$	
Technical Change ($\hat{\sigma}_2$)	-0.84*	(0.46)	1.08	($=1/\hat{\alpha}_2$)
Labour Gap ($\hat{\sigma}_3$)	-0.15	(0.047)	-0.12	(0.05)
AIB ($\hat{\sigma}_4$)	0.18*	(0.34)	0.24*	(0.38)
PREL ($\hat{\sigma}_5$)	1.33	(0.44)	0.93*	(0.49)
NAIRU (Set 1)				
Intercept ($\hat{\delta}_{01}$)	-265.4*	(85.8)	73.0	(16.5)
Demographics ($\hat{\delta}_{11}$)	(-990)*	(0.00)	(-265)*	(0.00)
Unemployment Insur.($\hat{\delta}_{21}$)	--	--	--	--
NAIRU (Set 2)				
Intercept ($\hat{\delta}_{02}$)	30.97	(13.6)	30.41	(13.7)
Demographics ($\hat{\delta}_{12}$)	-37.40*	(24.3)	-40.58*	(25.5)
Unemployment Insur.($\hat{\delta}_{22}$)	--	--	--	--
NAIRU (Set 3)				
Intercept ($\hat{\delta}_{03}$)	--	--	23.55*	(12.1)
Demographics ($\hat{\delta}_{13}$)	--	--	-27.59*	(22.5)
Unemployment Insur.($\hat{\delta}_{23}$)	--	--	--	--
NAIRU (87:4) computed with estimates				
Set 1	--		--	
Set 2	10.51%		8.21%	
Set 3	--		8.46%	

The * denotes an insignificant t-ratio. Standard errors are in parenthesis.

IV.3.6 Concluding Remarks about the Analysis of the Estimates

None of the models yields estimates that are acceptable for each and every parameter. The parameter estimates of the production function, $\hat{\alpha}_0$, $\hat{\alpha}_1$ and $\hat{\alpha}_2$, are out of the expected range in most of the cases. Every set of δ estimates contains at least one estimate that has either the wrong sign or is insignificant or both. $\hat{\sigma}_2$ is, if estimated, almost always negative in contradiction to the expectation about this parameter. The parameter $\hat{\sigma}_4$ is insignificant with a very few exceptions which means that according to this model the Anti-Inflation-Board had no influence on nominal wage increases.

The NAIRUs for the fourth quarter of 1987 generated with the estimates range from 8.88% to 11.6% in the long sample, except for the two NAIRUs of the restricted version of model 2B1, 69.5% and 78.9%, and from 7.9% to 9.01% in the short sample. These results do not seem to be too far from those generated by the Bank of Canada model, 9.36% for the long and 8.09 for the short period. Nevertheless, it must be noted that many of the less restricted models generate NAIRUs for the first quarter of 1967 that are higher than or equal to those for the fourth quarter of 1987. This is an unrealistic result because the NAIRU is assumed to have risen over the last two decades. Declining or constant NAIRU estimates are observed for model 2B1, 2B2 , 1B and partly for model 2A3.

V Conclusions

The likelihood ratio tests performed in order to test the validity of the restrictions of the Bank of Canada model do not lead to an unambiguous solution in the sense that an optimal model is found. This is due to the fact that the restrictions are not rejected in all cases. Although most of the less restricted models are superior to the Bank of Canada model there are still a few versions of less restricted models that are not accepted when tested against the Bank of Canada model. In addition, the difficulties of estimating certain less restricted models do not allow the determination of the optimal model. This would only be possible if a sufficient number of models could be estimated so that a decision based on the testing procedure could be realized.

This situation is even more complicated by an analysis of the parameter estimates that come out of the different models. The models that are superior to the Bank of Canada model on the basis of the likelihood ratio test yield parameter estimates that are in contradiction to the values one would expect from economic theory or previous empirical studies. It should be noted, however, that the Bank of Canada model itself does not generate estimates that meet such criteria in each case. Furthermore, the NAIRU's that are generated with the estimates of the less restricted models are in most cases not realistic for the beginning of the sample period although the values for the fourth quarter of 1987 seem to be acceptable in comparison to the values the Bank of Canada model generates.

Regarding both the results of the likelihood ratio tests and the analysis of the parameter estimates and the NAIRUs generated with them, it is impossible to present a model that is capable of yielding reliable estimates of the natural rate of unemployment.

At this point the question has to be raised why the results of testing the models and analyzing the estimates are the way they are. Here, two possible explanations can be offered. First, the theory underlying the model may be wrong or, second, the theory is right but the model misspecified.

As derived above, the Bank of Canada model is mainly based on Friedman's considerations about the Phillips curve and on the concept of an aggregate Cobb-Douglas production function. It is shown that the restrictions of the models are a consequence of this theoretical approach. However, the results of the likelihood ratio tests show that these restrictions are, with few exceptions, not valid. In other words, less restricted versions of the model are in most cases superior to the Bank of Canada model. These less restricted models are not in accordance with the theory underlying the Bank of Canada model. Since a unique NAIRU is a centrepiece of this theory the rejection of the restrictions that are necessary to the derivation of this unique NAIRU can lead to the conclusion that this theoretical approach does not properly reflect the real world. Here, it is to be emphasized that from the perspective of economic theory the Phillips curve is the most important behaviourial part of the model while Okun's Law and the aggregate production function are more properly viewed as technical relationships. Rejecting the restrictions, then, means rejecting the Phillips curve as a way of modelling the supply side of an economy.

At this point it is worthwhile to recall several theoretical approaches (from Chapter III) that are alternative to the theory underlying the Bank of Canada model. Thus, Lucas' approach, Okun's price tag economy, Tuscherer's Neo-Keynesian model and the hysteresis approach of Summer will be briefly reviewed in the content of the findings of this thesis.

First, Lucas[115], for example rejects the concept of any usable trade-off between unemployment and inflation even in the short run. His supply side of the economy is composed of a normal supply that follows a time trend and a cyclical supply that depends on relative prices as perceived by economic agents and on its lagged value. The cyclical

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[115] R.E.Lucas and L.A.Rapping, op.cit.

R.E.Lucas 1973, op.cit.

component does not allow for a usable unemployment-inflation trade off. Therefore, the Phillips curve equation of the Bank of Canada model which allows for a short run trade off is contradictory to Lucas' theory. If Lucas is right, the Phillips curve equation in the Bank of Canada model must lead to incorrect results; the restrictions imposed on the equations of the Bank of Canada model are unlikely to be valid and the results of the likelihood ratio tests are consistent with this view. In other words, the results of the tests may reject Friedman's theory and support different approaches to the supply side of an economy, for example, that of Lucas'.

Second, the test results may also support other theoretical approaches such as that of A.Okun.[116] In Okun's Phillips relation the actual rate of price increase does not depend on the expected rate of inflation but on a rate of inflation to which the economy is adjusted to, i.e. the adapted rate of inflation. The Phillips curve does not necessarily change its position in the inflation unemployment space if the adapted rate of inflation is different from the actual rate. This is different to Friedman's Phillips curve where the Phillips curve shifts after alterations of the expected rate of inflation. Consequently, the Bank of Canada model with a Phillips relation based on the expected rate of inflation may lead to incorrect results, provided Okun's

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[116] A.M.Okun 1981, op.cit.

approach is correct. The results of the likelihood ratio tests possibly indicate that Okun's approach is a more appropriate way of presenting the Phillips relation. Therefore, it should be noted that the Bank of Canada model could yield reasonable results if it were possible to model the Phillips curve according to Okun's approach.

Third, the results of the likelihood ratio tests may as well give support to the neo-Keynesian model of Tuscherer.[117] His model can lead to a variety of natural rates of unemployment which is contradictory to the theory behind the Bank of Canada model that allows for only one NAIRU at one point in time. Following Tuscherer it would be senseless to search for a unique NAIRU the way it is done with the Bank of Canada model because the resulting natural rate is just one out of the variety of possible rates. Any natural rate estimates with the model would most probably reflect an equilibrium with involuntary unemployment. From such an equilibrium the economy should be moved to an equilibrium with an unemployment rate that reflects voluntary unemployment only. This is different from Friedman's theory where a NAIRU estimated with the Bank of Canada model, provided the theory is correctly modelled, would be an unemployment rate that mirrors an equilibrium the economy should be at, i.e. at which social welfare is maximized.

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[117] T.Tuscherer, op.cit.

Finally, L.Summer's approach should be mentioned.[118] Summers states that an economy can reach many possible equilibria and that economic policy is able to impose such equilibria within certain limits. The way from one equilibrium to another is determined by history and external factors. Hence, Summer's ideas are opposed to Friedman's view that is based on a unique equilibrium and does not allow for economic policy to have long run effects. If Summers is right, any econometric model based on Friedman's ideas cannot lead to reasonable results.

As noted above, these lines of argument represent just one of the possible interpretations of the findings of this thesis. The second possibility is that the theory is basically right but the model does not reflect it properly.

In the first place, the modelling of the NAIRU, equation (4), may be incomplete. Demographic pressure and unemployment insurance may not be the only factors that influence the NAIRU. Changes of aggregate demand, union power and alterations of the attitude towards work may be mentioned as possible additional influences on the NAIRU. It is also worthwhile to note here that the Unemployment Insurance variable is not significant for almost all models which is in contradiction to the expectations one would have on the basis

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[118] L.H.Summers 1987, op.cit.

of the theory. Furthermore, the chosen production function is simplistic and does not necessarily match reality in the way it should. Even the specification of the Okun's Law equation and the Phillips curve could be wrong, although they are in accordance with the literature and other studies and there do not seem to be many obvious possibilities to model things differently.

Unfortunately, it is impossible to come to a final decision on the basis of the above analysis as to whether the theory is false or the model is misspecified. The only conclusion is that the data fails to support the theory underlying the Bank of Canada model. In other words, Friedman's approach to the Phillips curve with its unique NAIRU and the possibility of a short run trade off between unemployment and inflation is not corroborated by the results of the likelihood ratio tests.

It follows, on the basis of the test and analysis results of this thesis, that the NAIRU's generated by the Bank of Canada model have to be considered with great care and are not beyond some appreciable doubt. In addition, the analysis of several variants of the Phillips curve model of the Bank of Canada performed in this thesis shows that none of these variants yields parameter estimates that behave consistently well. Therefore it may be concluded that it is more promising to search for an alternative to the Phillips curve instead of attempting to refine still further the Phillips curve model.

Bibliography

Chiang, A.C., Fundamental Methods of Mathematical Economics, New York, 1984 (2nd Ed.)

Cousineau, J.M., "Unemployment Insurance and Labour Market Adjustments", Income Distribution and Economic Security in Canada, F.Vaillancour (ed.), Toronto, 1985

Eckstein, O., "Potential Output and Productivity: Comment", Brookings Papers of Economic Activity 1973, Washington, 1973, p. 11-60

Feldstein, M.(ed.), The American Economy in Transition, Chicago, 1982

Ford, R. and Rose, D., "Estimates of the NAIRU Using an Extended Okun's Law", Bank of Canada Working Paper 89-3, Ottawa, 1989

Friedman, M., "The Role of Monetary Policy", The American Economic Review, Vol.58 (1968), p. 4-17

Gordon, D.F., "A Neo-Classical Theory of Keynesian Unemployment", The Phillips Curve and Labour Markets, K.Brunner, A.H.Meltzer (eds), Amsterdam, 1976, p. 65-97

Gordon, R.J., "Unemployment and Potential Output in the 1980s", Brookings Papers of Economic Activity 1984, Washington, 1984

Hacche, G., The Theory of Economic Growth: An Introduction, London Basingstoke, 1979

Hall, R.E., "The Phillips Curve and Macroeconomic Policy", The Phillips Curve and Labour Markets, K.Brunner, A.H.Meltzer, (eds), Amsterdam, 1976, p. 127-141

Hall, B.H., Schnake, R., Cummins, C., Time Series Processor Version 4.1, User's Manual, Palo Alto 1987

Hall, B.H., Schnake, R., Cummins, C., Time Series Processor Version 4.1., Reference Manual, Palo Alto 1986 for a reference.

Jenkinson, T.J., The NAIRU: Statistical Fact or Theoretical Straitjacket, R.Cross (ed), Unemployment, Hysteresis and the Natural Rate Hypothesis, Oxford 1988, pp 365 - 377

Judge, G.G., et.al., The Theory and Practice of Econometrics, New York, 1985 (2nd Ed.)

Kmenta, J., Elements of Econometrics, New York McMillan, [2]1986

Lilien, D.M., "Sectoral Shifts and Cyclical Unemployment", Journal of Political Economy, Vol.90 (1982)

Lipsey, R.G., "The Place of the Phillips Curve in Macroeconomic Models", Stability and Inflation, A.R.Bergstrom, et.al. (eds), Chichester, 1978

Lucas, R.E., "Some International Evidence on Output-Inflation Tradeoffs", The American Economic Review, Vol.63 (1973), p. 326-334

Lucas, R.E., "Econometric Policy Evaluation: A Critique", The Phillips Curve and Labour Markets, K.Brunner, A.H.Meltzer, (eds), Amsterdam, 1976

Lucas, R.E. and Rapping, L.A., "Price Expectations and the Phillips Curve", The American Economic Review, Vol. 59 (1969), p. 342-350

Lucas, R.E. and Sargent, T., "After Keynesian Macroeconomics", Rational Expectations and Econometric Practice, R.E. Lucas, T.Sargent, (eds), Minneapolis, 1981, pp.295-320

McCallum, J.M., "Unemployment in Canada and the United States", Canadian Journal of Economics, Vol 20 (1987), p. 802-822

Mizon, G.E., Inferential Procedures in Nonlinear Models: An Application in a UK Industrial Cross Section Study of Factor Substitution and Returns to Scale, Econometrica, Vol.45, No.5 (July 1977), pp. 1228-1229

Nickell, S., The NAIRU: Some Theory and Statistical Facts, R.Cross (ed), Unemployment, Hysteresis and the Natural Rate Hypothesis, Oxford 1988, pp 379 - 385

Okun, A.M., "Potential GNP: Its Measurement and Significance", 1962 Business Proceedings, Business and Economics Statistics Section, American Statistical Association

Okun, A.M., Prices and Quantities, The Brookings Institution, Washington, 1981

Phillips, A.W., "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom 1861-1957", Economica, Vol 25 (1958)

Pindyk, R.S. and Rubinfeld, D.L., Econometric Models and Economic Forecasts, New York, 1981 (2nd Ed.)

Reid, F. and Smith, D.A., "The Impact of Demographic Changes on Unemployment", Canadian Public Policy, Vol 7 (1981), p. 348-351

Riddell, W.C. and Smith, M.P., "Expected Inflation and Wage Changes in Canada 1967-1981", Canadian Journal of Economics, Vol 15 (1982)

Rose, D., "The NAIRU in Canada: Concepts, Determinants and Estimates", Bank of Canada, Technical Report No. 50, Ottawa, 1988

Samson, L., "A Study of the Impact of Sectoral Shifts on Aggregate Unemployment in Canada", Canadian Journal of Economics, Vol. 18 (1985), p. 518-530

Santomero, A.J. and Seater, J.J., "The Inflation-Unemployment Trade-Off: A Critique of the Literature", Journal of Economic Literature, Vol. 16 (1978), p. 499-544

Schaafsma, J., "Okun's Law and the Interaction between Actual and Potential GNP: The Canadian Experience", unpublished paper, University of Victoria, 1975

Schaafsma, J. and Walsh, W.D., "An Empirical Analysis of the Impact of Canadian Wage Controls on Negotiated Wage Settlements, 1967-87", Discussion Paper 89-10, Department of Economics, University of Victoria, 1989

Summers, L.H., "Why is the Unemployment Rate So Very High Near Full Employment?", Brookings Papers of Economic Activity 1986, Washington, 1986

Summers, L.H., "Should Keynesian Economics dispense with the Phillips Curve?", Unemployment, Hysteresis and the Natural Rate Hypothesis, R.Cross, (ed), New York, 1987

Thomas, R.L., Introductory Econometrics, Theory and Application, New York 1985

Tuscherer, T., "The Unnatural 'Natural' Rate of Unemployment", Journal of Post-Keynesian Economics, Vol 4 (1981), p. 25-31

Varian, H.L., Microeconomic Analysis, New York, 1984 (2nd Ed.)

APPENDIX I The Complete Estimated Models

Model 3

(1) $\log Y = \alpha_0 + (1 - \alpha_2)K + \alpha_2 \log(LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP + \epsilon_1$

(2) $RNU = \delta_{01} + \delta_{11}DEM + \delta_{21}UI + \beta_1(RNU_{t-1} - (\delta_{01} + \delta_{11}DEM_{t-1} + \delta_{21}UI_{t-1})) + \beta_2(\log Y - (\alpha_0 + (1-\alpha_2)K + \alpha_2 \log (LF(1- 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP)) + \epsilon_2$

(3) $W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3 (RNU - (\delta_{01} + \delta_{11}DEM + \delta_{21}UI)) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$

(4) $NAIRU = \delta_{0i} + \delta_{1i}DEM + \delta_{2i}UI$

Model 2A3

(1) $\log Y = \alpha_0 + (1-\alpha_2)K + \alpha_2 \log(LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP + \epsilon_1$

(2) $RNU = \delta_{01} + \delta_{11}DEM + \delta_{21}UI + \beta_1(RNU_{t-1} - (\delta_{01} + \delta_{11}DEM_{t-1} + \delta_{21}UI_{t-1})) + \beta_2(\log Y - (\alpha_0 + (1-\alpha_2)K + \alpha_2 \log (LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP)) + \epsilon_2$

(3) $W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3(RNU - (\delta_{02} + \delta_{12}DEM + \delta_{22} UI)) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$

(4) $NAIRU = \delta_{0i} + \delta_{1i}DEM + \delta_{2i}UI$

Model 2B1

$$(1)\ \log Y = \alpha_0 + (1 - \alpha_2)K + \alpha_2 \log(LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP + \epsilon_1$$

$$(2)\ RNU = \delta_{01} + \delta_{11}DEM + \delta_{21}UI + \beta_1(RNU_{t-1} - (\delta_{02} + \delta_{12}DEM_{t-1} + \delta_{22}UI_{t-1})) + \beta_2(\log Y - (\alpha_0 + (1-\alpha_2)K + \alpha_2 \log(LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP)) + \epsilon_2$$

$$(3)\ W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3(RNU - (\delta_{02} + \delta_{12}DEM + \delta_{22}UI)) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$$

$$(4)\ NAIRU = \delta_{0i} + \delta_{1i}DEM + \delta_{2i}UI$$

Model 2B2

(1) $\log Y = \alpha_0 + (1-\alpha_2)K + \alpha_2 \log(LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP + \epsilon_1$

(2) $RNU = \delta_{02} + \delta_{12}DEM + \delta_{22}UI + \beta_1(RNU_{t-1} - (\delta_{02} + \delta_{12}DEM_{t-1} + \delta_{22}UI_{t-1})) + \beta_2(\log Y - (\alpha_0 + (1-\alpha_2)K + \alpha_2 \log (LF(1- 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP)) + \epsilon_2$

(3) $W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3 (RNU - (\delta_{02} + \delta_{12}DEM + \delta_{22}UI)) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$

(4) $NAIRU = \delta_{0i} + \delta_{1i}DEM + \delta_{2i}UI$

Model 1B

(1) $\log Y = \alpha_0 + (1 - \alpha_2)K + \alpha_2 \log(LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP + \epsilon_1$

(2) $RNU = \delta_{03} + \delta_{13}DEM + \delta_{23}UI + \beta_1(RNU_{t-1} - (\delta_{02} + \delta_{12}DEM_{t-1} + \delta_{22}UI_{t-1})) + \beta_2(\log Y - (\alpha_0 + (1-\alpha_2)K + \alpha_2 \log (LF(1 - 0.01(\delta_{01} + \delta_{11}DEM + \delta_{21}UI))) + ETFP)) + \epsilon_2$

(3) $W = \sigma_0 P + \sigma_1 H + \sigma_2 G + \sigma_3 (RNU - (\delta_{02} + \delta_{12}DEM + \delta_{22}UI)) + \sigma_4 AIB + \sigma_5 PREL + \epsilon_3$

(4) $NAIRU = \delta_{0i} + \delta_{1i}DEM + \delta_{2i}UI$

APPENDIX II The Values of the Log-Likelihood Functions of the Estimated Models

Table 1AII shows the maximized values of the log-likelihood functions of the estimated models for the 1967:1 - 1985:4 sample period including the values for the $\delta_{2i} \neq 0$ case.

Table 1AII; Values of the Log-Likelihood Functions, long sample period

Re-strictions	MODEL 3 $\delta_{2i}=0$ $(\delta_{2i} \neq 0)$	MODEL 2A3 $\delta_{2i}=0$ $(\delta_{2i} \neq 0)$	MODEL 2B1 $\delta_{2i}=0$ $(\delta_{2i} \neq 0)$	MODEL 2B2 $\delta_{2i}=0$	MODEL 1B $\delta_{2i}=0$
$\sigma_2=1/\alpha_2$ and $\alpha_1=1-\alpha_2$	406.1 (406.9)	407.0 (407.7)	424.1 (426.8)	423.3	427.5
$\sigma_2 \neq 1/\alpha_2$	411.0 (411.6)	419.4 (422.3)	430.2 (431.5)	430.2	--
$\alpha_1 \neq 1-\alpha_2$	(407.8) (408.5)	410.1 (410.8)	431.2 (432.9)	429.1	--
$\sigma_2 \neq 1/\alpha_2$ and $\alpha_1 \neq 1-\alpha_2$	(412.5) (413.0)	420.3 (423.2)	436.1 (437.8)	433.2	--

Table 2AII shows the maximized values of the log-likelihood functions of the different models for the 1967:1 - 1981:4 sample period. The values for the $\delta_{2i}\neq0$ case are reported as well.

Table 2AII; Values of the Log-Likelihood Functions, short sample period

Re-strictions	MODEL 3 $\delta_{2i}=0$ $(\delta_{2i}\neq0)$	MODEL 2A3 $\delta_{2i}=0$ $(\delta_{2i}\neq0)$	MODEL 2B1 $\delta_{2i}=0$ $(\delta_{2i}\neq0)$	MODEL 2B2 1B $\delta_{2i}=0$
$\sigma_2=1/\alpha_2$ and $\alpha_1=1-\alpha_2$	359.6 (360.0)	360.5 (361.0)	-- (--)	362.3 --
$\sigma_2\neq1/\alpha_2$	370.4 (370.6)	374.6 (378.3)	376.9 (377.7)	-- --
$\alpha_1\neq1-\alpha_2$	(367.7) (368.0)	370.3 (371.4)	371.4 (373.2)	-- --
$\sigma_2\neq1/\alpha_2$ and $\alpha_1\neq1-\alpha_2$	(371.7) (371.8)	375.3 (379.0)	377.4 (378.0)	-- --

Appendix III The Variables and their Data

logY	:	log of the real Gross Domestic Product
K	:	log of the real capital stock
LS	:	log of the full employment labour force defined as the product of (1 - NAIRU) and trend total hours worked. Trend total hours worked is the product of the following factors: Trend participation rate from the Bank of Canadas quarterly RDXF econometric model; Estimate of the Labour Force Population (Statistics Canada); Trend Average Hours Worked (Labour Force Survey), detrended.
ETFP	:	log of the total trend factor productivity measured as the smoothed Solow residual from the production function of the model using actual capital and labour use instead of potential. Determined prior to model estimation.
RNU	:	National Unemployment Rate
W	:	First difference if the log of wages, measured as National Accounts Labour Income divided by Paid Employment from the Labour Force Survey.
P	:	Inflation expectations, estimated as a second order Almon-lag of the first difference of the log of the consumer price index.
PREL	:	Seven-quarter moving average of the first difference of the log of the GDP deflator divided by the CPI.
G	:	Productivity growth, first difference of of ETFP
H	:	Growth rate of trend average hours
AIB	:	Dummy variable for the Anti-Inflation-Board wage and price control period.

DEM : 1 - Adult Male proportion of the Labour Force.

UI : Maximum Unemployment Insurance wage-replacement rate times the proportion of the labour force covered.

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