

A COMPUTATIONAL TRADE MODEL OF TARIFF WARFARE;
THE EFFECT OF RELATIVE COUNTRY SIZE AND THE STRUCTURE OF
COMPARATIVE ADVANTAGE

by
CHAN WING-YIU
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DEAN

We accept this thesis as conforming
to the required standard

[REDACTED]
Donald G. Ferguson Ph.D.

[REDACTED]
Kenneth G. Stewart Ph.D.

[REDACTED]
Serge Nadeau Ph.D.

[REDACTED]
Denton E. Hewgill Ph.D.

[REDACTED]
Norman J. Ruff Ph.D.

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UNIVERSITY OF VICTORIA

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Supervisor: Professor Donald G. Ferguson

Abstract

In this thesis a small computational general equilibrium trade model is constructed and then used to study the role of relative country size and the international distribution of technology on the outcome of tariff warfare. In contrast to other computational models it is assumed that there is a continuum of goods instead of a finite set of discrete commodities.

To simplify matters and facilitate computation a Ricardian two-country, one-factor model with constant returns to scale in all industries is employed as the modelling framework. As a result, international comparative advantage is dependent upon the trading countries' relative productivity in producing various goods. A continuous density function is then defined to represent the distribution of industries over the countries' relative productivity. Alternate values for the parameters of this density function are used to represent various possible international distributions of technology.

For the purpose of evaluating the welfare impact of changing relative country size and the pattern of comparative advantage special utility functions are defined

for the two countries. The model is completely specified by a system of equations representing international trade balance and the conditions for optimal tariffs for each country. Sensitivity analysis is carried out by varying the appropriate parameters in the system and observing their effects on the outcome of tariff warfare.

Examiners:

[Redacted]

Donald G. Ferguson Ph.D.

[Redacted]

Kenneth G. Stewart Ph.D.

[Redacted]

Serge Nadeau Ph.D.

[Redacted]

Denton E. Hewgill Ph.D.

[Redacted]

Norman J. Ruff Ph.D.

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Chapter 1

Introduction

A major proposition in neoclassical trade theory is that free trade is Pareto superior to autarky. By opening themselves to trade, individual countries can exploit the comparative advantage of each other so that welfare will be increased for all parties. Moreover, the position of free trade represents an efficient point such that it is impossible to increase the welfare of one country without reducing that of at least one other. Modern trade theories, though undoubtedly intellectual advancements over their classical predecessors, are still firmly grounded on the same proposition.

That free trade is optimal for the world as a whole does not preclude the justification of tariffs and other kinds of trade restrictions. From a national point of view, free trade may not be an optimal policy. Given suitable circumstances (e.g. a country which is sizable enough and free of the threat of retaliation) a country may increase its own welfare through imposing unilateral tariffs.¹ In fact, an optimum tariff rate or tariff structure that maximizes the welfare of a single country, given the trading schedules of its trading partners, can be shown to exist under rather general conditions.²

Since it is rarely the case that a country is bestowed

with the favorable conditions for successfully imposing unilateral tariffs, retaliation is most likely to occur following a change in the trade policy of any country. This is because a country levying tariffs on its imports inevitably increases its welfare at the expense of others. If there is an opportunity for the affected countries to regain part or all of their losses by imposing reciprocal tariffs then it is unlikely that the latter will remain idle. In this situation any country attempting to improve its welfare by levying tariffs on imports will incur retaliation from its trading partners, and a tariff war will result.

The nature of the outcome of tariff warfare depends not only on the behavioural assumptions underlying retaliation³ but various objective factors such as the structure of comparative advantage and the relative country sizes. In analysing the outcome of tariff warfare, a particularly important question to be asked is whether one or more parties will emerge as winners. In other words, we would like to know if some countries are better off at the tariff-ridden equilibrium than at free trade. Finally, how significant are the roles of different factors in determining the outcome of tariff retaliation, and what is the tradeoff or interrelationship between them.

It is likely that different models employing different assumptions may give dissimilar answers to these questions. Furthermore, even for a single model definite or unambiguous answers may not exist for some questions. For example, the direction of effects on the equilibrium variables when one or more of the factors changes may not be easily or uniquely determined. Thus despite the enormous volume of literature accumulated on the subject no general agreement has yet been attained on the major issues of tariff warfare.

Until recently most of the studies on tariff warfare were conducted within a discrete commodity framework with one or more factors of production. Moreover, particular emphasis has been placed on the questions of the existence, uniqueness and stability of tariff-ridden equilibrium,⁴ and the conditions for optimal tariff under equilibrium, rather than investigating the roles of specific factors in determining the outcome of tariff warfare. Few attempts have been made to explore the questions stated above.

The objective of this thesis is to construct a specific computational general equilibrium trade model to study the roles of relative country size and the structure of comparative advantage in determining the consequences of tariff warfare. The distinctive feature of this model that

distinguishes it from most conventional trade models is that a continuum of goods is used instead of a finite set of discrete commodities. Theoretical treatments along these lines have been pursued by Dornbush, Fischer and Samuelson (1977,1980) and Wilson (1980). Their studies show that many of the standard issues in international trade can be addressed within a continuum of goods framework.

The assumption of a continuum of goods may be justified by the fact that modern industrial countries produce a wide variety of slightly differentiated products (for example, different models of motor-cycles differing only slightly in design and production methods may be regarded as different goods). In fact we can view the conventional classification of commodities as one which categorizes bundles of close substitutes into discrete groups of commodities. In this sense , therefore, the continuum of goods assumption provides a more flexible representation of the production and consumption process.

The computational model constructed here is based on a Ricardian two-country, one-factor(labour) model with constant returns to scale in all industries. Such a model allows for a simple characterization of comparative advantage in terms of differences in the countries' productivity in various industries. It is therefore possi-

ble to incorporate technology explicitly into the analysis in terms of gaps in factor productivity in different industries without it being obscured by variations in factor intensities.⁵ By assuming a continuum of goods it is possible to represent the distribution of industries over the countries' relative productivity by a continuous density function. Alternative patterns of comparative advantage can be obtained by specifying different forms of density functions or by varying the parameters in a particular density function.

The choice of confining the study to two countries is based on the need to simplify the task of computation and reduce confusion. In principle the model can be extended to a many-country case, but with tremendous complications. Although with appropriate assumptions the latter can be reduced to one with each country trading its own labor against all other countries, the transformation procedure may not be computationally feasible⁶. In addition, it no longer permits a simple specification of comparative advantage by a single density function. The major concern of the present study is the relationship between relative country size and structure of comparative advantage on the one hand, and the welfare consequences of tariff warfare on the other. In particular, attention is drawn to the follow-

ing questions;(i) Are there any combinations of relative country size and pattern of comparative advantage that will give rise to a winner in tariff warfare;(ii) what are these combinations; and (iii) what is the interrelationship between the two factors in determining the final outcome of retaliation.

Modelling and Methodology

In order to focus on the issue of tariff warfare and national welfare it is necessary to make the following fundamental assumptions before proceeding to the task of modelling. First, it is assumed that the objective of each country is to maximize its own welfare. The level of welfare is measured by a homothetic community utility function.⁷ Second, there are no distortions in the domestic economy of each country and therefore tariffs are not imposed for corrective purposes.⁸

Third, the only instrument available to the government of each country for restricting trade is tariffs.⁹ Fourth, the ultimate tariff-ridden equilibrium is either Cournot type or Nash type.¹⁰ The last assumption eliminates the possibility of other types of equilibrium based on different behavioral assumptions(e.g. a Stackleberg or leadership follower equilibrium), thus avoiding a diversity of

results. Each of the other equilibria incontestably has its own value for research, but the limitations of space and time make it desirable to work on the relatively simple Cournot model.

The modelling procedure consists of defining a continuous density function to describe the distribution of industries over the two countries' relative productivity and a special community utility function for each country. Together they are used to derive the total amount of foreign labor imports for each country, as well as their tariff reaction curves.¹¹ The computational model is represented by a system of three equations featuring international trade balance (value of labor imports equal to value of labor exports for each country) and conditions for optimum tariff rates for the two countries. Solving the system gives the equilibrium values of the factorial terms of trade (or the relative wage) and the countries' tariff rates.

The structure of international comparative advantage is characterized by the distribution of industries over the countries' relative productivity which can be changed by either altering the form of the density function or by varying the parameters in a particular function. So long as a specific density function is chosen, the pattern of

comparative advantage is completely determined by the values of the functional parameters.

The relative country size is represented by the ratio of the countries' labour supplies. Both parameters of the density function and the relative country size appear as exogeneous variables in the reduced system of equations. Sensitivity analysis is then carried out by varying these exogeneous variables and observing the changes in the welfare consequences of tariff warfare.

The main reason for addressing the previously stated questions of tariff warfare numerically is that given the complexity of the model, qualitative answers derived from pure theoretical analysis may not be readily obtainable, or even if they can be derived they are so ambiguous that little insight can be gained into the roles of the two factors. To acquire a better understanding of the significance of the two factors in determining the outcome of tariff warfare it is preferable to undertake a simulation analysis to extract qualitative information from quantitative results.

In recent years numerous computational trade models varying both in scale and technical complexity have been constructed to study tariff policies in world trade.¹² However, the majority of them are concerned with the

economic effects of alternative global tariff policies and their implications in solving real world trade problems. The general question of relationship between particular economic factors and the outcome of tariff warfare has not been pursued.

Furthermore, these numerical general equilibrium trade models are usually modelled after the trading relationship between specific countries or groups of countries (such as the trade between Canada and the European Economic Community, and between the United States and Japan). Consequently they are not designed for general simulation analysis of tariff warfare. These considerations make the present computational model stand out among the forest of numerical trade models.

The general structure of the model is explained in the next chapter, where the specific computational model to be used in the simulation analysis is also constructed. The results of computer simulations are presented and discussed in Chapter III. Here the effects of introducing an asymmetry the technology distribution are also discussed. Finally, a summary and conclusions are given in chapter IV.

Chapter 2

The General Model

The numerical general equilibrium trade model is a Ricardian two-country, one-factor(labour) and constant return to scale model with a continuum of goods. To simplify matters a number of working assumptions are introduced. First, all commodities are final goods. There are no intermediate goods produced. Second, perfect competition prevails in all domestic markets so that domestic prices of output are equal to their unit labour costs. Third, the supply of labour is fixed in both countries and wages adjust to ensure that there is full employment. Fourth, no factor flows take place between the two countries. Finally, the two countries use a common currency and hence exchange rates can be ignored.

I. Notations

Before proceeding to the modelling procedure it is necessary to explain the notations used.

$i = \alpha, \beta$ = Indices representing the two countries.

z = Index representing a commodity($z \in [0, 1]$).

$a^i(z)$ = Unit labor requirement for good z . ($i = \alpha, \beta,$
 $z \in [0, 1]$).

$c^i(z)$ = Level of consumption of good z in country i .

L^i = Labor endowment of country i .

M^i = Derived demand for labor imports by country i
from country j . ($i = \alpha, \beta$, $i \neq j$)

$P^i(z)$ = Price of good z in country i .

τ^i = One plus tariff rate imposed by country i on the
imports from country j

U^i = Community utility level in country i .

W^i = Wage rate in country i .

y^i = National income of country i .

Z^i_0 = Boundary of specialization in country i .

II. The Structure of Comparative Advantage

The previous analyses of Ricardian models with a continuum of goods have started by supposing that goods are identified by their position z on the continuum $[0, 1]$. As indicated by the notation introduced above the input requirements are expressed as functions of z and from them the countries' relative productivity

$$\rho = \frac{a^\beta(z)}{a^\alpha(z)} \quad (1)$$

is derived. In turn, this relative productivity function defines the countries' comparative advantage and determines how production in the two countries responds to changing circumstances. Unfortunately, when the model is

presented in this way the form of the relative productivity function appears arbitrary and is not subject to further interpretation.

There is, however, an alternate way of approaching the continuum model, one which derives the continuum and the relative productivity function from a more fundamental construct. Postulate a continuous joint density function

$$f(\theta; a^\alpha, a^\beta) \quad (2)$$

to describe the distribution of goods over the countries' labour requirements (θ is a vector of parameters).

Since it is the countries' relative productivity that matters in determining their production patterns, it is useful to reduce the joint density function to a single variable density function by introducing the following normalization:¹³

$$a^\alpha + a^\beta = 1 \quad (3)$$

Since both input requirement coefficients are non-negative, equation (3) restricts the values of both a^α and a^β to the interval $[0,1]$ and introduces a linear dependency between them. Thus whenever the labor productivity of one country is known that of the other country is simultaneously determined. The normalization (3) transforms (2) into a continuous single variable density function:

$$f(\theta; a^\alpha, 1-a^\alpha) \quad (4)$$

with the usual property of a density function

$$\int_0^1 f(\theta; x, 1-x) dx = 1$$

The integral

$$Z = \int_0^{a^\alpha} f(\theta; x, 1-x) dx \quad (5)$$

gives the proportion of industries with labor requirements less than or equal to a^α in country α . The same proportion of industries, according to (3), have labor requirements greater than or equal to $a^\beta = 1 - a^\alpha$ in country β . Thus the proportion of industries in country β with labour requirements smaller than $1 - a^\alpha$ is equal to $1 - Z$. This is shown in Figure 2.1 where Z is represented by the shaded area.

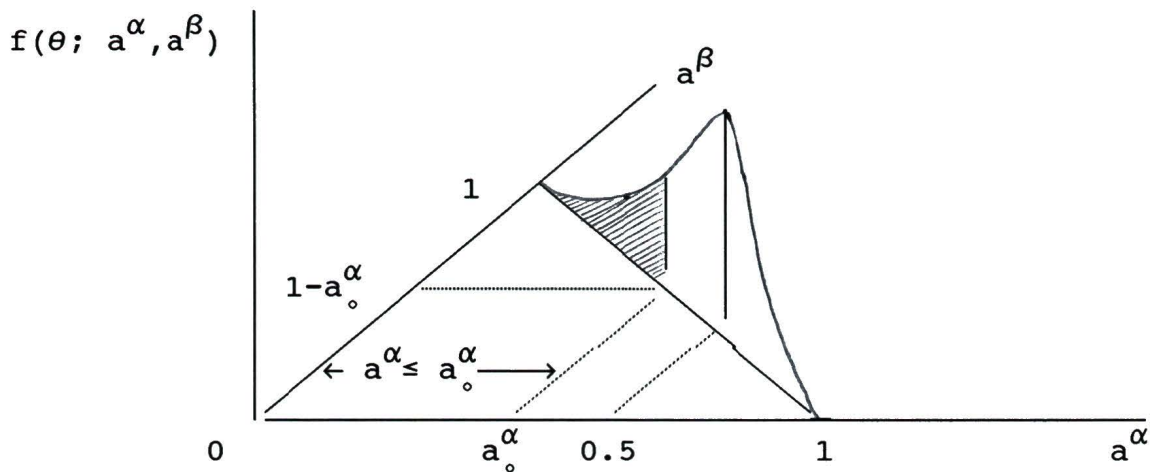


Fig 2.1

The Density Function for the Normalized Labour Requirements of Countries α and β

The density function (4) provides a convenient way of deriving the commodity index z and the continuum on which

it lies. Since (4) is a continuous single-valued function, Z in (5) is a continuous function of a^α . Thus we can index the goods in increasing order of their associated labour requirements in country α . In other words, (5) gives the inverse function of $a^\alpha = a^\alpha(z)$.

We may now define the relative labour productivity

$$\rho = \frac{a^\beta}{a^\alpha} = \frac{1 - a^\alpha}{a^\alpha} \quad (6)$$

from (3). Expressing a^α in terms of ρ and substituting into (5) we get:

$$\begin{aligned} Z &= \int_0^{\frac{1}{1+\rho}} f(\theta; x, 1-x) dx \\ &= Z(\rho) \end{aligned} \quad (7)$$

Since $\frac{dZ}{d\rho} < 0$, Z is a strictly decreasing function of ρ ¹⁴ and can be inverted to define the relative productivity function discussed in the opening paragraph of this section. Moreover, it is possible to relate the properties of $\rho(z)$ to those of the density function, and in what follows it will be used to characterize the structure of comparative advantage.

We turn now to a discussion of how comparative advantage affects production patterns. Let the wage rate of the two countries be W^α and W^β respectively. Then given free trade and perfect competition, country α will produce good Z as long as:

$$W^\alpha \cdot a^\alpha(z) \leq W^\beta \cdot a^\beta(z)$$

$$\text{or } w = \frac{W^\alpha}{W^\beta} \leq \frac{a^\beta(z)}{a^\alpha(z)} = \rho(z) \quad (8)$$

Hence from (7) the proportion of goods produced in country α is given by:

$$Z_0 = \int_0^1 \frac{1}{1+w} f(\theta; x, 1-x) dx \quad (9)$$

and the proportion of goods produced by country β is $1-Z_0$. By virtue of our indexing method, Z_0 also represents the boundary of specialization. Thus we have country α producing goods in the interval $[0, Z_0]$ and country β producing goods in the interval $[Z_0, 1]$.

If each country imposes tariffs on its imports then a wedge is imposed between the prices faced by foreign and domestic consumers. Assuming that a uniform tariff rate is applied to all imports by a country, we have country α producing good z if:

$$W^\alpha a^\alpha(z) \leq \tau^\alpha W^\beta a^\beta(z)$$

$$\text{or } \frac{w}{\tau^\alpha} = \frac{W^\alpha}{\tau^\alpha W^\beta} \leq \rho \quad (10)$$

Similarly country β will produce good z if:

$$W^\alpha \tau^\beta a^\alpha(z) \geq W^\beta a^\beta(z)$$

$$\text{or } \tau^\beta w \geq \rho(z) \quad (11)$$

Substituting (10) into (7) the boundary of specialization for country α is given by;

$$z_{\circ}^{\alpha} = \int_0^1 \frac{1}{1 + \frac{w}{\tau^{\alpha}}} f(\theta; x, 1-x) dx \quad (12)$$

and from (11) the boundary of specialization for country β is given by;

$$z_{\circ}^{\beta} = \int_0^1 \frac{1}{1 + (\tau^{\beta} w)} f(\theta; x, 1-x) dx \quad (13)$$

Since the τ^i ($i = \alpha, \beta$) are greater than or equal to 1 it follows from (12) and (13) that $z_{\circ}^{\beta} \leq z_{\circ}^{\alpha}$ (with equality holding if and only if there is free trade ie. $\tau^{\alpha} = \tau^{\beta} = 1$). The goods lying between the specialization margins will be produced by both countries and, since in both instances the domestic costs will be lower than the tariff-ridden import prices, no trade will take place in those goods (ie. both countries will have fully protected their domestic markets for those goods).

A diagrammatic representation of the tariff effects is provided in Figure 2.2. The shape of the curve shown in Figure 2.2 as well as the specialization margins z_{\circ}^{α} and z_{\circ}^{β} all depend on the form of the density function. Graphically, altering the density function changes the shape of the curve in both Figure 2.1 and Figure 2.2 as well as the

boundaries of specialization in Figure 2.2.

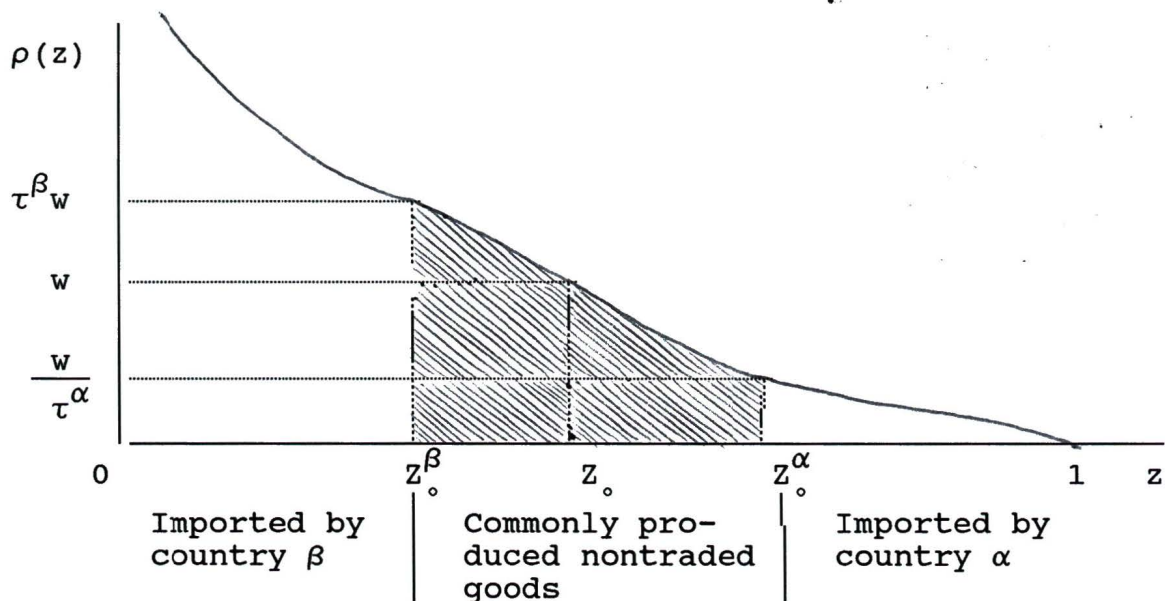


Figure 2.2
Specialization Before and After The Imposition of Tariffs

III. The Derived Demand for Labor Imports

We define a community utility function for each of the two countries α and β :

$$U^i = \int_0^1 g^i [c^i(z), z] dz \quad i = \alpha, \beta \quad (14)$$

Here it is assumed that both countries have identical preferences so that they are identical in all respects except their labour endowments and production technology

Maximization of U^i subject to the national income constraint

$$\int_0^1 p^i(z) c^i(z) dz = y^i$$

yields the Marshallian demand functions for the commodities.

For country α , the national income constraint can be written as:

$$\int_0^{z_\circ^\alpha} w^\alpha a^\alpha(z) c^\alpha(z) dz + \int_{z_\circ^\alpha}^1 \tau^\alpha w^\beta a^\beta(z) c^\alpha(z) dz = y^\alpha \quad (15)$$

where $z_\circ^\alpha = z_\circ^\alpha(w, \tau^\alpha, \theta)$ is the specialization margin as derived in the previous section. National welfare maximization of country α subject to the constraint (15) gives the demand function for good z :

$$c_z^\alpha = c^\alpha(\theta, w^\alpha, w^\beta \tau^\alpha, y^\alpha, z) \quad z \in [0, 1] \quad (16)$$

Similarly for country β we have the income constraint:

$$\int_0^{z_\circ^\beta} \tau^\beta w^\alpha a^\alpha(z) c^\beta(z) dz + \int_{z_\circ^\beta}^1 w^\beta a^\beta(z) c^\beta(z) dz = y^\beta \quad (17)$$

where $z_\circ^\beta = z_\circ^\beta(w, \tau^\beta, \theta)$ is given by (13). Maximizing U^β

subject to (17) gives the demand function for country β :

$$c_z^\beta = c^\beta(\theta, \tau^\beta w^\alpha, w^\beta, y^\beta, z) \quad z \in [0, 1] \quad (18)$$

From (16) and (18) we can deduce the derived demand for foreign labor by each country. Let M_j^i be the derived foreign labor imports by country i . Then we have:

$$M_\beta^\alpha = \int_{z_\circ^\alpha}^1 a^\beta(z) c^\alpha(\theta, w^\alpha, w^\beta \tau^\alpha, y^\alpha, z) dz \quad (19)$$

$$M_\alpha^\beta = \int_0^{z_\circ^\beta} a^\alpha(z) c^\beta(\theta, \tau^\alpha w^\beta, w^\alpha, y^\beta, z) dz \quad (20)$$

The national income of country i consists of its total labour income plus revenue from tariffs, the latter is assumed to be redistributed to the consumers in the economy for their own disposal. Thus we have:

$$y^\alpha = W^\alpha L^\alpha + (\tau^\alpha - 1)W^\beta M_\beta^\alpha \quad (21)$$

$$y^\beta = W^\beta L^\beta + (\tau^\beta - 1)W^\alpha M_\alpha^\beta \quad (22)$$

Both equations (19) and (20) are functions of $W^\alpha, W^\beta, y^\alpha$ or y^β , and τ^α or τ^β . As a result (19) and (21), (20) and (22) together solve for the endogeneous income import demand for foreign labor by each country. Denoting them by m_β^α and m_α^β we have:

$$m_\beta^\alpha = m_\beta^\alpha(\theta, W^\alpha, W^\beta, \tau^\alpha) \quad (23)$$

$$m_\alpha^\beta = m_\alpha^\beta(\theta, W^\alpha, W^\beta, \tau^\beta) \quad (24)$$

Equations (23) and (24) express the derived demand for foreign labour by each country as a function of $\theta, W^\alpha, W^\beta$ and τ^α or τ^β only. They will be used to express one of the international equilibrium conditions in the following section.

Before passing it should be noted that demand functions (23) and (24) can be derived more directly by defining community utility functions with the derived domestic and foreign labor consumption as variables rather than operat-

ing with the conventional utility functions defined on commodity consumption. In fact, for special density functions (4) and utility functions (14) it can be shown that there exist equivalent utility functions defined on labor consumption. Such a relationship is demonstrated in constructing the specific computational model.

IV. Equilibrium Conditions

In the case of free trade, international equilibrium is achieved when the total value of imports by a country equals the total value of its exports, or the value of imports by the foreign country. In a tariff war, the following two conditions are required for an equilibrium: (i) balance of trade for both countries as in free trade. (ii) each country is levying its own optimum tariff rate given the tariff rate of its trading partner.¹⁵

Using the results of the previous section, the first equilibrium condition can be expressed as:

$$W_{\beta}^{\beta\alpha}(\theta, W^{\alpha}, W^{\beta}, \tau^{\alpha}) = W_{\alpha}^{\alpha\beta}(\theta, W^{\alpha}, W^{\beta}, \tau^{\beta}) \quad (25)$$

The second condition is achieved when the tariff reaction curves of the two countries intersect. Here a tariff reaction curve is defined as the locus of tariff rates that maximize a country's equilibrium welfare for given tariff rates of its trading partner.

The tariff reaction curve for each country can be derived by solving a typical optimization problem. We first obtain the indirect utility function for country i by substituting the demand function (16) or (18) into (14). (Note that y^i is further expressed as a function of $W^\alpha, W^\beta, \tau^\alpha$ or τ^β by solving equations (19) and (21), or equations (20) and (22)) Denote the indirect utility function defined on $W^\alpha, W^\beta, \tau^i (i = \alpha, \beta), \theta$ by u^i . Therefore

$$u^i = u^i(\theta, W^\alpha, W^\beta, \tau^i, L^i) \quad i = \alpha, \beta \quad (26)$$

Since only the relative wage matters we can just take the value of W^β as 1 and substitute the solution of W^α (in terms of $\theta, \tau^\alpha, \tau^\beta, L^\alpha$ and L^β) obtained from equation (25) into (26) to get the tariff-ridden welfare level of country i . The reaction curve for country i can then be derived by choosing τ^i to maximize the resulting utility function for i subject to the balance of trade constraint. The first order condition is

$$\frac{\partial u^i}{\partial W^\alpha} \cdot \frac{\partial W^\alpha}{\partial \tau^i} + \frac{\partial u^i}{\partial \tau^i} = 0 \quad (27)$$

where $\partial W^\alpha / \partial \tau^i$ is implicitly defined by

$$\begin{aligned} \frac{\partial m_\beta^\alpha}{\partial \tau^i} + \frac{\partial m_\alpha^\beta}{\partial \tau^i} + \frac{\partial m_\beta^\alpha}{\partial W^\alpha} \cdot \frac{\partial W^\alpha}{\partial \tau^i} \\ + (m_\alpha^\beta + W^\alpha \cdot \frac{\partial m_\alpha^\beta}{\partial W^\alpha}) \cdot \frac{\partial W^\alpha}{\partial \tau^i} = 0 \end{aligned} \quad (28)$$

which was obtained by differentiating (25).¹⁶

The reaction functions

$$R^{\alpha}(\theta, w^{\alpha}, \tau^{\alpha}, \tau^{\beta}) = 0 \quad (29)$$

$$R^{\beta}(\theta, w^{\alpha}, \tau^{\alpha}, \tau^{\beta}) = 0 \quad (30)$$

for the two countries and equation (25) completely determine the equilibrium values of the tariff factors and the relative wage.

When a utility function defined on labor consumption can be defined directly, the model is reduced to a two-country, two-good (labor) model and standard results regarding the conditions for optimum tariff rates apply. In this case offer curves showing the trading schedules of the two countries under different factorial terms of trade can be derived for both countries. The optimum tariff factor (given the tariff rate of the other country) for a country is simply given by:

$$\tau^i - 1 = \frac{1}{\epsilon^j - 1} \quad i, j = \alpha, \beta, \quad i \neq j \quad (31)$$

where $\epsilon^j = \frac{w_j}{m_i^j} \cdot \frac{\partial m_i^j}{\partial w}$ is the elasticity of foreign labor

imports of country j with respect to the relative wage.

Equation (31) gives the same reaction curves as (29) and (30), but in a much more straightforward manner.

We now have a complete system of equations((25), (29), (30) or (25) and two (31)s) representing international

trade balance and optimum tariff conditions. Given θ and L^α, L^β or $L = \frac{L^\alpha}{L^\beta}$ equation (25) alone can be used to solve for the free trade equilibrium relative wage by setting τ^α, τ^β equal to 1. On the other hand, (25), (29), (30) or (31) together yield the tariff-ridden equilibrium values of τ^α, τ^β and $\frac{w^\alpha}{w^\beta}$.¹⁷ By varying the exact values of the exogenous variables θ and $\frac{L^\alpha}{L^\beta}$ a sensitivity analysis can be carried out on the tariff-ridden equilibrium.

V. Welfare Evaluation

The aim of the study is to find out, through simulation analysis, under what circumstances (or what combinations of values of θ and relative country size L) there will be a winner in tariff warfare, and how the welfare consequences of tariff warfare change when the two factors are continuously varied. For this reason it is necessary to find a way to compare the welfare levels of each country at free trade and at the tariff-ridden equilibrium.

The indirect utility function discussed in the previous section provides an explicit measurement of national welfare. With an original utility function defined on commodity consumption the indirect utility function of country

i is given by:

$$u^i(\theta, w^\alpha, w^\beta, \tau^i) = \int_0^1 g^i\{c^i(\theta, w^\alpha, w^\beta, y^i(\theta, w^\alpha, w^\beta, \tau^i, L^i), \tau^i, z)\} dz \quad (32)$$

$$i = \alpha, \beta$$

When a utility function $v^i(M_i^i, M_j^i)$ defined on labor consumption M_i^i, M_j^i is used, the indirect utility function is simply

$$v^i(\theta, w^\alpha, w^\beta, \tau^i) = v^i\left(M_i^i(\theta, w^\alpha, w^\beta, \tau^i, y^i(\theta, w^\alpha, w^\beta, \tau^i)), M_j^i(\theta, w^\alpha, w^\beta, \tau^i, y^i(\theta, w^\alpha, w^\beta, \tau^i))\right) \quad (33)$$

$$i = \alpha, \beta$$

In either case the welfare of a country at free trade can be obtained by setting $\tau^\alpha = \tau^\beta = 1$, $w^\beta = 1$ and substituting the equilibrium value of w^α obtained from equation (25) into the indirect utility function (32) or (33). Denote it by $V_f^i(\theta, L)$, where $L = \frac{L^\alpha}{L^\beta}$. The welfare level of either country at the tariff-ridden equilibrium is obtained by substituting the solutions for τ^α, τ^β and w^α (again setting $w^\beta = 1$) given by the system (25), (29) and (30) (or (25) and the two(31)s) into (32) (or (33)). Let it be denoted by $V_t^i(\theta, L)$.

The relative welfare level at the tariff-ridden equilibrium and at free trade can be evaluated by consider-

ing whether the ratio

$$V_R^i = \frac{V_t^i(\theta, L)}{V_f^i(\theta, L)} \begin{matrix} \geq \\ < \end{matrix} 1 \quad i = \alpha, \beta \quad (34)$$

Since $V_R^i = V_R^i(\theta, L)$ is a function of θ and L only the welfare impact of changing θ and L can be directly observed and followed.

VI. The Computational Model

A specific computational model is constructed out of the general model outlined above by (i) choosing a particular functional form for (4) and (ii) specifying a utility function (14), or defining an equivalent utility function with labor consumption as variables. The procedures will now be outlined in detail;

We choose, for the density function (4), a special (C.E.S) function:

$$f(\theta; a^\alpha, 1-a^\alpha) = \frac{\theta(a^\alpha)^{\theta-1}(1-a^\alpha)^{\theta-1}}{[(a^\alpha)^\theta + (1-a^\alpha)^\theta]^2} \quad (4')$$

It can be verified , though not obvious, that

$$1 = \int_0^1 f(\theta; x, 1-x) dx$$

The density function (4') is symmetric about $a^\alpha = 0.5$ and has only one parameter θ . The symmetric of the distribution implies that both countries have a production advantage

over an equal number of industries. It has a bowl shaped distribution for values of θ smaller than one and a bell-shaped distribution for θ greater than one. The various possible shapes of the density function are shown in Figures 2.3.1, 2.3.2 and 2.3.3.

Qualitatively, raising the value of θ in (4') reduces the variance of the distribution in Figure 2.3.¹⁸ In the case of $\theta > 1$, the distribution becomes more and more concentrated about the mean $a^\alpha = 0.5$. For values of $\theta < 1$, the distribution becomes progressively less bowl-shaped. The structure of comparative advantage, which is characterized by the shape of the distribution curve, is entirely summarized by the single parameter θ .

Although the symmetric density function (4') is the actual form that will be used in the construction of the computational model, an asymmetric distribution can be constructed by slightly modifying (4') and substituting it for the symmetric distribution. The process of introducing asymmetries into the distribution will be discussed in

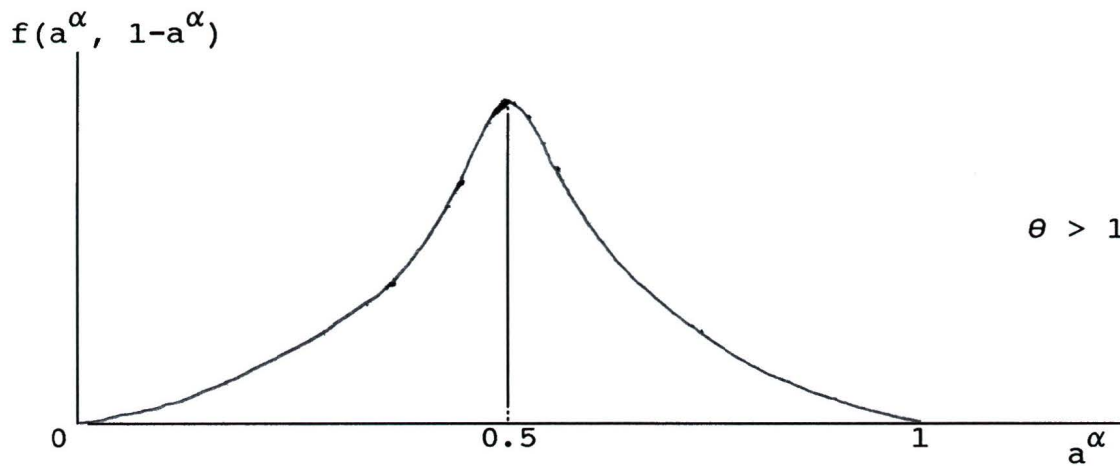


Figure 2.31

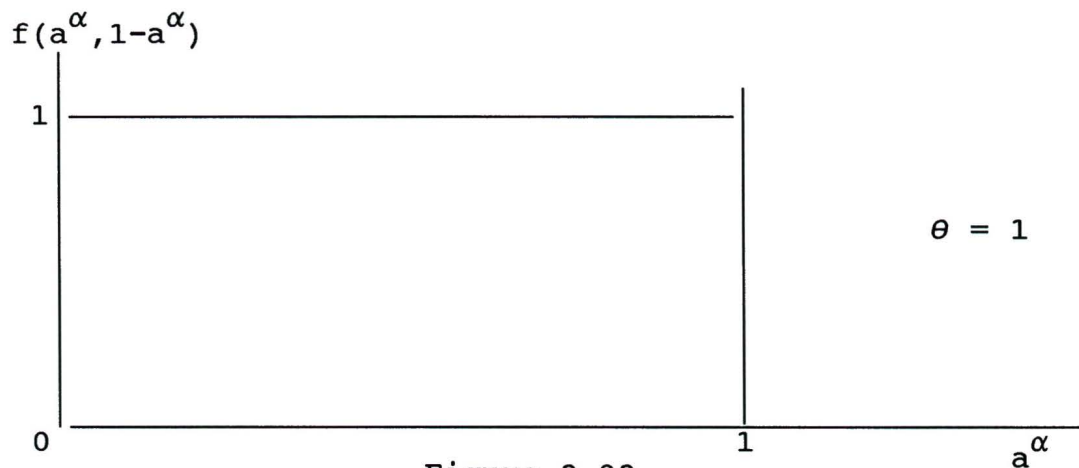
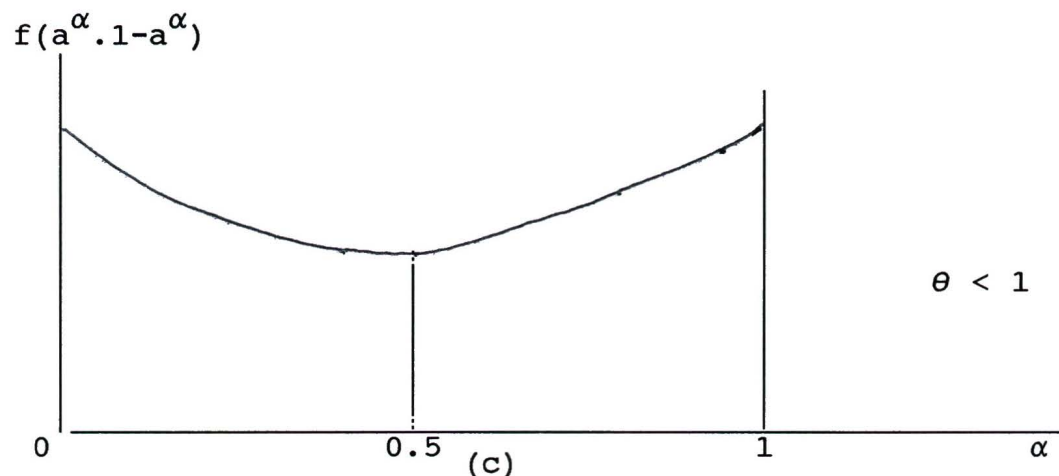


Figure 2.32

Figure 2.33. Shapes of the C.E.S. Distribution for Different Values of the Parameter θ

section VIII below.

Integrating (4'), we get:¹⁹

$$\begin{aligned} Z &= \int_0^{a^\alpha} \frac{\theta x^{\theta-1} (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx \\ &= \frac{(a^\alpha)^\theta}{(a^\alpha)^\theta + (1-a^\alpha)^\theta} \end{aligned} \quad (5')$$

Since $\rho = \frac{1 - a^\alpha}{a^\alpha}$ we have further:

$$Z = \frac{1}{1 + \rho^\theta} \quad (6')$$

The specialization borders for countries α and β are then given by:

$$z_\circ^\alpha = \frac{1}{1 + \left(\frac{w}{\tau^\alpha}\right)^\theta} \quad (11')$$

$$z_\circ^\beta = \frac{1}{1 + (\tau^\beta w)^\theta} \quad (12')$$

such that country α will specialize in the production of goods indexed from $[0, 1/(1 + \left(\frac{w}{\tau^\alpha}\right)^\theta)]$ and country β in goods in the interval $[1/(1 + (\tau^\beta w)^\theta), 1]$.

We specify an identical log-linear utility function

$$U^i = \int_0^1 \ln[c^i(z)] dz \quad i = \alpha, \beta \quad (14')$$

for each country. Optimization with the utility function

(14') subject to the national income constraint (15) and (17) gives the following demand functions for the two countries:

$$c^{\alpha}(z) = \begin{cases} \frac{y^{\alpha}}{w^{\alpha} \cdot a^{\alpha}(z)} & z \in [0, 1/(1 + (\frac{w}{\tau^{\alpha}}))] \\ \frac{y^{\alpha}}{\tau^{\alpha} w^{\beta} a^{\beta}(z)} & z \in [1/(1 + (\frac{w}{\tau^{\alpha}})), 1] \end{cases} \quad (15')$$

$$c^{\beta}(z) = \begin{cases} \frac{y^{\beta}}{\tau^{\beta} w^{\alpha} a^{\alpha}(z)} & z \in [0, 1/(1 + (\tau^{\beta} w))] \\ \frac{y^{\beta}}{w^{\beta} \cdot a^{\beta}(z)} & z \in [1/(1 + (\tau^{\beta} w)), 1] \end{cases} \quad (17')$$

Substituting (15') and (17') into (19) and (20) yields

$$M_{\beta}^{\alpha} = \frac{y^{\alpha}}{\tau^{\alpha} w^{\beta}} (1 - z^{\alpha}) = \frac{y^{\alpha}}{\tau^{\alpha} w^{\beta}} \left(\frac{(w/\tau^{\alpha})^{\theta}}{1 + (w/\tau^{\alpha})^{\theta}} \right) \quad (19')$$

and

$$M_{\alpha}^{\beta} = \frac{y^{\beta}}{\tau^{\beta} w^{\alpha}} \left(\frac{1}{1 + (\tau^{\beta} w)^{\theta}} \right) \quad (20')$$

Equations (19') and (21). (20') and (22) together solve for M_{β}^{α} , M_{α}^{β} , y^{α} and y^{β} in terms of w , L^{α} and τ^{α} or L^{β} and τ^{β} . We have then:

$$m_{\beta}^{\alpha} = \frac{w(w/\tau^{\alpha})^{\theta} L^{\alpha}}{[\tau^{\alpha} + (w/\tau^{\alpha})^{\theta}]} \quad (23')$$

and

$$m_{\alpha}^{\beta} = \frac{L^{\beta}}{w(\tau^{\theta+1} w^{\theta} + 1)} \quad (24')$$

Using (25) and simplifying, the condition for international trade balance is given by:

$$\frac{(\tau^\alpha)^{\theta+1}}{w^\theta} + 1 - L w [\tau^\beta (\tau^\beta w)^\theta + 1] = 0 \quad (25')$$

The tariff reaction curves for the two countries are derived as:²⁰

$$\theta \tau^\alpha \tau^\beta - \tau^\alpha (1 + \theta) - (w/\tau^\alpha)^\theta = 0 \quad (29')$$

$$\theta \tau^\alpha \tau^\beta - \tau^\beta (1 + \theta) - 1/(\tau^\beta w)^\theta = 0 \quad (30')$$

The system of three equations (25'), (29') and (30') completely specifies the model and can be used to solve for equilibrium values of τ^α , τ^β , and w once θ and L are known.

The indirect utility functions for the two countries, in this case, are:

$$u^\alpha(\theta, w, \tau^\alpha, L^\alpha) = \frac{L^\alpha [(w/\tau^\alpha)^\theta \ln(w\tau^\alpha) - \ln(\tau^\alpha + (w/\tau^\alpha)^\theta)]}{1 + (w/\tau^\alpha)^\theta}$$

$$- L^\alpha \int_0^{1/(1+(w/\tau^\alpha)^\theta)} \ln a^\alpha(z) dz$$

$$- L^\alpha \int_{1/(1+(w/\tau^\alpha)^\theta)}^1 \ln a^\beta(z) dz \quad (33'a)$$

$$\begin{aligned}
u^\beta(\theta, w, \tau^\beta, L^\beta) &= \frac{L^\beta [(\tau^\beta w)^\theta \ln(\tau^\beta w) - \ln(\tau^\beta (\tau^\beta w)^\theta + 1)]}{1 + (\tau^\beta w)^\theta} \\
&\quad - L^\beta \int_0^{1/(1+(\tau^\beta w)^\theta)} \ln a^\alpha(z) dz \\
&\quad - L^\beta \int_{1/(1+(\tau^\beta w)^\theta)}^1 \ln a^\beta(z) dz \quad (33'b)
\end{aligned}$$

where $a^\alpha(z) = \frac{(1-z)^\theta}{[z^\theta + (1-z)^\theta]}$ is the inverse function of

$$z = \frac{(a^\alpha)^\theta}{(a^\alpha)^\theta + (1-a^\alpha)^\theta}, \text{ and } a^\beta(z) = 1-a^\alpha(z).$$

With (33'a) and (33'b) the welfare level of each country at free trade and at tariff-ridden equilibrium can be evaluated in principle by substituting the appropriate solutions for τ^α, τ^β and w . The relative welfare level V_R^i can be obtained correspondingly.

Equations (33'a) and (33'b) look messy and are obviously inconvenient to work with. But, as will be shown later, the problem of welfare evaluation is much simplified when a utility function defined on labor consumption is introduced.

VII. Utility Functions Defined on Labour Consumption

As an alternative to using the conventional utility

function(14') we define a community utility function with consumption of foreign and domestic labour as variables. It will be shown that the following (CES) utility function is equivalent to the specification of a symmetric CES density function (4') plus a log-linear utility function defined on commodity consumption.

Postulate the following the CES aggregate labor consumption utility function :

$$V^i(M_i^i, M_j^i) = ((M_i^i)^r + (M_j^i)^r)^{\frac{1}{r}} \quad i, j = \alpha, \beta, \quad i \neq j \quad (35)$$

where M_i^i , M_j^i are the domestic and foreign labour consumption country i respectively.

Maximizing (35) subject to the national income constraint:

$$\tau^i W^j M_j^i + W^i M_i^i = y^i \quad i, j = \alpha, \beta, \quad i \neq j \quad (36)$$

results in the derived labour demand functions:

$$M_i^i = \frac{y^i (W^i)^{1/(r-1)}}{(W^i)^{r/(r-1)} + (\tau^i W^i)^{r/(r-1)}} \quad (37)$$

$$M_j^i = \frac{y^i (\tau^i W^j)^{1/(r-1)}}{(W^i)^{r/(r-1)} + (\tau^i W^i)^{r/(r-1)}} \quad (38)$$

For country α we have therefore:

$$M_\beta^\alpha = \frac{y^\alpha (\tau^\alpha W^\beta)^{1/(r-1)}}{(W^\alpha)^{r/(r-1)} + (\tau^\alpha W^\beta)^{r/(r-1)}} \quad (39)$$

Comparing (39) with (19'), if

$$r = \frac{1}{1 + \theta} \quad (40)$$

then the two are equivalent. Moreover, if the demand functions are of the form in (19') and (20') then the utility function which generated them is unique up to a monotonic transformation and hence, using (40), the C.E.S. utility function (35) will preserve the ordinal ranking of welfare that is given by the model constructed from (4') and (14'). Since we are concerned only with ordinal ranking of the free trade and tariff warfare utility the C.E.S. utility function and the associated labour exchange model can be used.

Equations (38) and (21) or (22) give the partial equilibrium values of labor imports m_j^i and national income. Substituting the latter into (37) we get the partial equilibrium value of domestic labour consumption m_i^i .

Both m_i^i and m_j^i are functions of the relative wage w and the tariff rate τ^i . Thus we can view them as parametric equations of w (given τ^i) and obtain an equation relating $L^i - m_i^i$ (the labor exports of country i) and m_j^i (the labor imports of country i) by eliminating w . This equation corresponds to the offer curve of country i which trades its own technology-embodied labour against country β 's technology embodied labour.²¹ The two offer curves for

country α and β are shown in Figure 2.4.

In Figure 2.4, O^α , O^β , represent the offer curves of country α and β respectively. u^α is the trade indifference curve of country α derived from (35). The slope of OA gives the international factorial terms of trade facing both countries. The positions of the offer curves also depend on the tariff rates imposed.

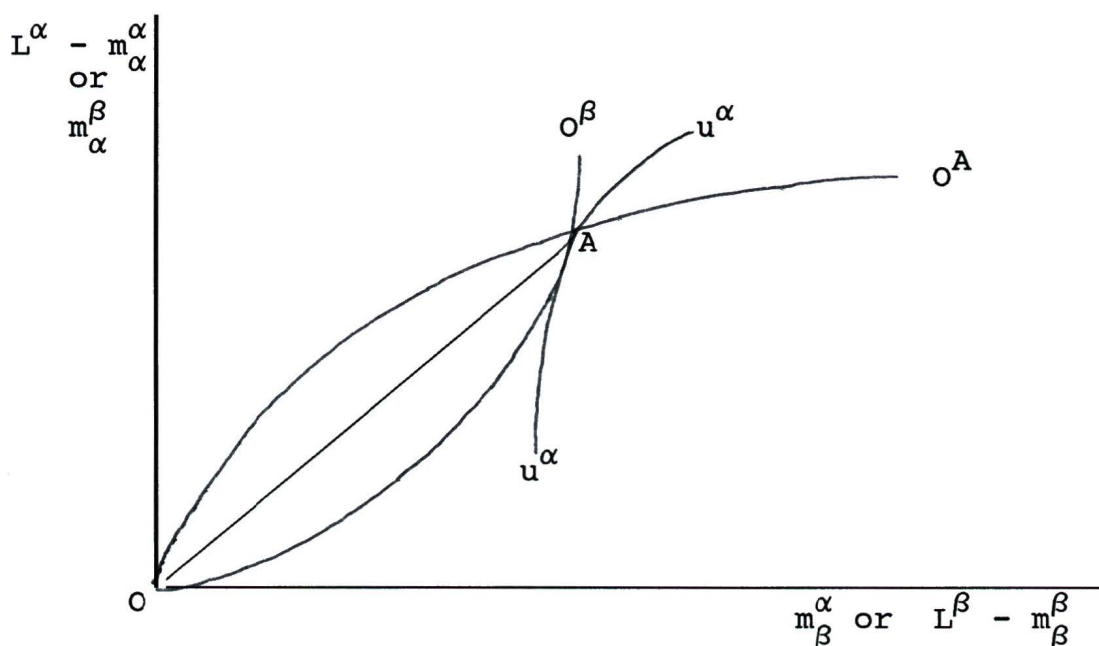


Figure 2.4

Offer curves for countries α and β
and the condition for an optimum tariff for country α

As in the traditional two-country, two-good model, country α imposes its optimum tariff rate, given the tariff rate of country β , when the trade equilibrium point A in Figure 2.4 is such that u^α is tangent to the slope of O^β at A . The condition for an optimum tariff rate for country α is

then given by:

$$\tau^\alpha - 1 = \frac{1}{\epsilon^\beta - 1} \quad (41)$$

and for country β :

$$\tau^\beta - 1 = \frac{1}{\epsilon^\alpha - 1} \quad (42)$$

where $\epsilon^\alpha = (w/m_\beta^\alpha)(\partial m_\beta^\alpha/\partial w)$ and $\epsilon^\beta = (1/w \cdot m_\alpha^\beta)(\partial m_\alpha^\beta/\partial(1/w))$ are the elasticities of foreign labor imports of country α and β respectively, and $m_\beta^\alpha, m_\alpha^\beta$ are given by (23') and (24'). It can be easily verified that (41) and (42) give the reaction curves of (29') and (30'). The same system of equations is thus derived by defining a utility function with labour consumption as variables.

Substituting (40) into (37) and (38) and putting them back into (35) yields the indirect utility function for country α as:

$$v^\alpha(\theta, \tau^\alpha, W^\alpha, W^\beta, Y^\alpha) = Y^\alpha [(W^\alpha)^{-\theta} + (W^\beta/\tau^\alpha)^{-\theta}]^{\frac{1}{\theta}} \quad (43)$$

Since (39) and (21) together solve for y^α in terms of W^α, W^β and τ^α , we have the trade equilibrium indirect utility function:

$$v^\alpha(\theta, w, \tau^\alpha, L^\alpha) = \frac{wL^\alpha [w^{-\theta} + (\tau^\alpha)^{-\theta}]^{\frac{\theta+1}{\theta}}}{(w^{-\theta} + (\tau^\alpha)^{-\theta-1})} \quad (44)$$

Tariff-ridden equilibrium welfare can then be evaluated by inserting the solutions of w , τ^α and τ^β obtained from (25'), (29') and (30') into (44). The free trade welfare is found by setting $\tau^\alpha = \tau^\beta = 1$ and substituting the solution value of w from (25') into (44). For country α the relative welfare level is then

$$V_R^\alpha = \frac{v_t^\alpha}{v_f^\alpha} = \frac{(1 + (w^f)^\theta)}{((w^t)^\theta + (\tau^\alpha)^\theta)} \left(\frac{(w^t)^\theta + (\tau^\alpha)^\theta}{1 + w^f} \right)^{\frac{\theta + 1}{\theta}} \quad (45)$$

where w^f , w^t are the free trade and tariff-ridden equilibrium factoral terms of trade or relative wages. Similarly with country β , we get the following relative welfare function:

$$V_R^\beta = \frac{v_t^\beta}{v_f^\beta} = \frac{(1 + (w^f)^\theta)}{(1 + \tau^\beta (\tau^\beta w^t)^\theta)} \left(\frac{1 + (\tau^\beta w^t)^\theta}{1 + (w^f)^\theta} \right)^{\frac{\theta + 1}{\theta}} \quad (46)$$

The expressions (45) and (46) are much more convenient forms to work with than (33'a) and (33'b). As a result they are the ones that will be used for the purpose of computing the relative welfare levels in the simulation analysis.

VIII. Asymmetric Distribution

Asymmetry in the distribution of industries over the

countries' relative productivity can be introduced by slightly modifying the CES density function (4). In fact, (4) is just a special case of the general CES function:

$$f(\theta; a^\alpha, 1-a^\alpha) = \frac{K_1 K_2 (a^\alpha)^{\theta-1} (1-a^\alpha)^{\theta-1}}{[K_1 (a^\alpha)^\theta + (1-a^\alpha)^\theta]^2} \quad (47)$$

By choosing appropriate values for K_1 and K_2 it is possible to introduce the desired skewness into the distribution. In particular, if $K_1 > K_2$, then the distribution is skewed to the left in fig 2.3, and if $K_1 < K_2$, then it is skewed to the right. Setting $K_1 = K_2$ we get the symmetric density function (4').

Integrating (47) we get:²²

$$\begin{aligned} Z &= \int_0^{a^\alpha} \frac{K_1 K_2 x^{\theta-1} (1-x)^{\theta-1}}{[K_1 x^\theta + K_2 (1-x)^\theta]^2} dx \\ &= \frac{K_1 (a^\alpha)^\theta}{K_1 (a^\alpha)^\theta + K_2 (1-a^\alpha)^\theta} = \frac{1}{1 + K\rho^\theta} \end{aligned} \quad (48)$$

where $K = \frac{K_2}{K_1}$. Repeating the same steps as in the symmetric case we obtain the derived demand for foreign labor by country α :

$$M_\beta^\alpha = \frac{y^\alpha}{\tau^\alpha w^\beta} \left(\frac{K(w/\tau^\alpha)^\theta}{1 + K(w/\tau^\alpha)^\theta} \right) \quad (49)$$

As in the symmetric case, we can introduce a utility function defined on labor consumption, and operate with it.

For the general (CES) density function (47) and log-linear utility functions defined on commodities for the two countries we can define a general CES utility function with labour consumption as variables:²³

$$V^i(M_i^i, M_j^i) = [(a_i M_i^i)^r + (b_i M_j^i)^r]^{\frac{1}{r}} \quad i, j = \alpha, \beta, i \neq j \quad (50)$$

where $a_\alpha = a$, $b_\alpha = b$ and $a_\beta = b$, $b_\beta = a$

Maximization of (50) subject to (36) gives the derived demand function for foreign labour for country α . Comparing it with (49) we have:

$$r = \frac{1}{1 + \theta} \quad (40'a)$$

which is the same as that in the symmetric case, and

$$\frac{a}{b} = \left(\frac{K_1}{K_2} \right)^{\frac{1}{\theta}} \quad (40'b)$$

Thus whenever θ , K_1 and K_2 are chosen, a/b is determined.

and (50) can be used to obtain an ordinal ranking of welfare levels.

Once r is expressed in terms of θ we can rewrite (50) as:

$$V^\alpha(M_\alpha^\alpha, M_\beta^\alpha) = [(M_\alpha^\alpha)^{\frac{1}{\theta+1}} + (M_\beta^\alpha)^{\frac{1}{\theta+1}}]^{\theta+1} \quad (51)$$

for country α , where $M_\alpha^\alpha = aM_\beta^\alpha$ and $M_\beta^\alpha = bM_\beta^\alpha$, and rewrite its national income constraint as:

$$W^\alpha M_\alpha^\alpha + \tau^\alpha W^\beta M_\beta^\alpha = Y^\alpha \quad (52)$$

where $W^\alpha = (W/a)$, $W^\beta = (W/b)$, and $Y^\alpha = W^\alpha (aL^\alpha) + (\tau^\alpha - 1)W^\beta M_\beta^\alpha$.

Equations (51) and (52) are analogous to (35) and (36). Manipulating (51) and (52) and the corresponding equations for country β yield exactly the same system of equations as (25'), (29') and (30') and the same relative welfare functions as (45) and (46), except that $W = W^\alpha/W^\beta$ and $L' = (a/b)L$ have replaced w and L .

Thus, the introduction of an asymmetric distribution affects the relative welfare level (the ratio of welfare at tariff-ridden equilibrium and at free trade) of both countries through changing the effective relative country size from L to L' . In this sense, a value of L_0 in the asymmetric case will give the same relative welfare level as $L_1 = (a/b)L_0$ in the symmetric case.

Consequently, the simulation results for an asymmetric distribution can be deduced from those obtained for the symmetric case by making appropriate modifications. Thus it suffices for us to perform the numerical analysis only for the symmetric case. The numerical results of the computer simulations and their interpretation are presented in the next chapter.

Chapter 3

Simulation Results

I. Results of Computer Simulation

This chapter presents the results obtained from the computer simulation with the numerical trade model constructed in the previous chapter. Specifically, the simulation analysis comprises four major steps:

- (i) specifying numerical values for the exogeneous variables θ and L in the system of equations (25'), (29') and (30').
- (ii) (a) setting $\tau^\alpha = \tau^\beta = 1$ and solving (25') to get the free trade equilibrium factoral terms of trade w .
(b) solving the system (25'), (29') and (30') together to obtain the tariff-ridden equilibrium values of τ^α, τ^β and w .
- (iii) inserting the solutions from (ii) into (45) and (46) to evaluate the relative welfare levels for both countries.
- (iv) repeating the same procedure with a different set of values for θ and L .

The system of equations (25'), (29') and (30') was solved by using the Gauss Non-linear Simultaneous Equation System program. Essentially, the program computes the sol-

ution of the system by means of Broyden's Secant method.²³

Nevertheless, since the system is non-linear, there is a possibility of multiple solutions. In order to determine which of these solutions are true tariff-ridden equilibria it is necessary to gather some qualitative information on the system of equations. Drawing the tariff reaction curves provides an ideal means of locating the possible equilibrium points.

The tariff-reaction curves are represented by the two equations (29') and (30'). However, direct analysis of the reaction curves is complicated by the fact that w , which appears in both equations, is itself a function of τ^α and τ^β from the international trade balance equation (25'). To obtain reaction functions in terms of τ^α and τ^β alone, we have to eliminate w by combining (25') with (29') or (30').

Alternatively, the reaction curves can be drawn by using the same Gauss program to solve two pairs of equations (25') and (29') and (25') and (30') in which one of the variables τ^i is made exogeneous. This was done for $\theta = 0.5$ as dual numerical solutions of the full system were found there.²⁵ For expositional convenience the two equations (29') and (30') representing the reaction curves (in terms of the tariff factors) are rewritten as follows for

$\theta = 0.5$:

$$0.5(\tau^\beta - 1)\tau^\alpha - \tau^\alpha = \frac{w^{0.5}}{\tau^\alpha} \quad (29'')$$

$$0.5(\tau^\alpha - 1)\tau^\beta - \tau^\beta = \frac{1}{(w\tau^\beta)^{0.5}} \quad (30'')$$

The reaction curves and their relationship to relative country size L are shown in Figure 3.1.

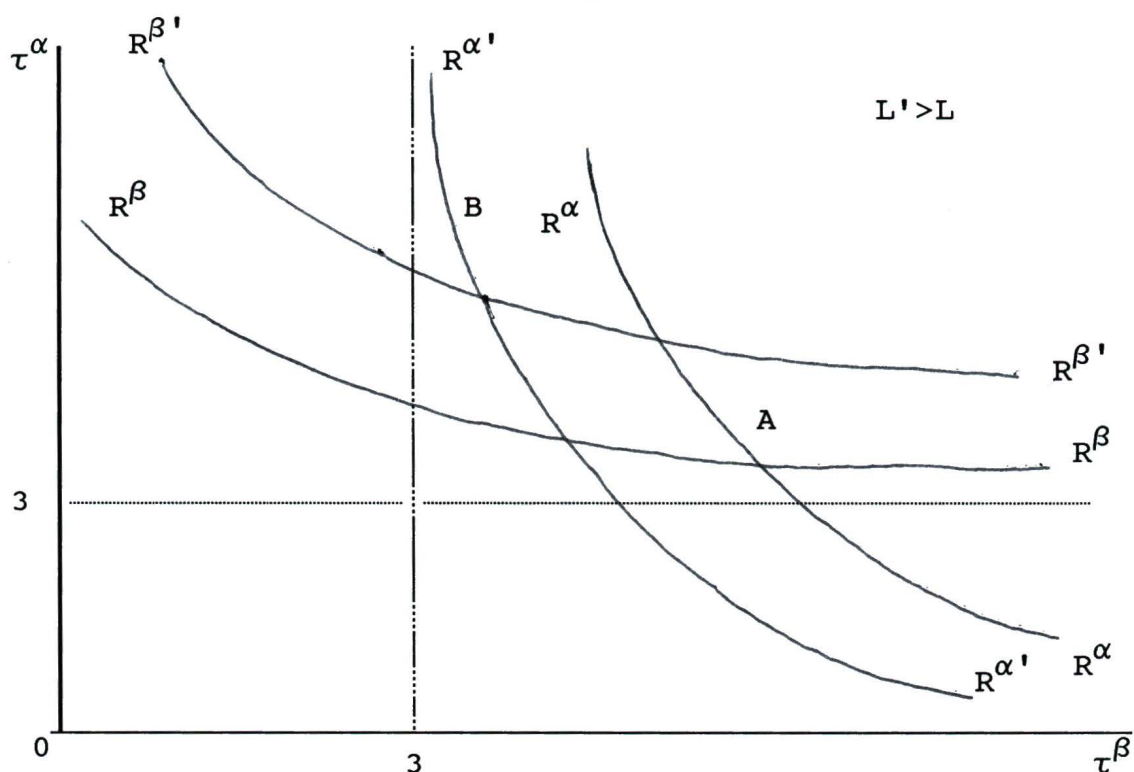


Figure 3.1

Tariff Factor Reaction Curves for Countries α and β

The reaction curve of country α is downward sloping to the right and asymptotic to the vertical line $\tau^\beta = 3$ and that of country β has a similar shape and is asymptotic

to the horizontal line $\tau^\alpha = 3$. The asymptotic behavior of the curves can be derived by considering (29'') and (30''). Since $w, \tau^\alpha, \tau^\beta \geq 0$ the right hand side of both equations is non-negative and therefore we should have:

$$\frac{1}{2}(\tau^i - 1)\tau^j - \tau^j \geq 0$$

or

$$\tau^i \geq 3 \quad i, j = \alpha, \beta, \quad i \neq j \quad (53)$$

where τ^j is the optimum tariff factor of country j given the tariff factor of country i .

Condition (53) implies that the reaction curves of both countries must be asymptotic to the line $\tau^i = 3$. Although (29'') and (30'') may have solutions outside the region defined by (53), such solutions are not sensible and will not be considered here.

The shapes of the reaction curves as shown in Figure 3.1 indicate that there should be at most one observable equilibrium point when $\theta = 0.5$, and at which both τ^α and τ^β are greater than 3. Furthermore, an increase in the relative country size L shifts the reaction curve of country α inward and that of country β upward, thus raising the equilibrium value of τ^α and lowering the equilibrium value of τ^β .

In the case of $\theta = 0.5$, dual numerical solutions have been found for the system (25') (29') and (30') for all

values of L . However, since one of them always lies outside the feasible region as defined by (53) and does not conform to the trend stated above, the relevant solution is determined accordingly. As for all other values of θ unique solutions are obtained and hence there is no need for a graphical analysis of the reaction curves.

The equilibrium values of τ^α, τ^β and w and the corresponding relative welfare levels V_R^α, V_R^β are provided in Tables 3.1, 3.2 and 3.3. Although the results apply only to the symmetric distribution of relative productivities, they can be used to analyse the asymmetric case by simply transforming the variables in the way shown in the previous chapter. The effects of introducing asymmetries in the distribution are explained in detail in Section II below.

Tables 3.1 and 3.2 reveal several interesting and significant trends and features that deserve comment. As can be seen from Table 3.1, at tariff-ridden equilibria the larger country is always imposing a higher tariff rate than the smaller country. At the same time, the terms of trade as shown in Table 3.2 is always favourable to the latter in both the free trade and tariff-ridden equilibria.

These results may be explained in terms of the characteristics of the computational model. Since both countries are identical in all aspects (i.e. they have identical

utility functions, have an equal number of industries with a production advantage and all goods are equally preferable in terms of their unit contribution to national welfare) except their relative size, the larger country must have a greater demand for imported goods and hence a larger derived demand for foreign labour. Consequently, to achieve trade balance the price of imported labour must be higher for the larger country than for the smaller country, thus accounting for the more favourable terms of trade for the latter at free trade.

On the other hand, in order for a country to gain from tariff imposition, it must be able to turn the terms of trade in its own favour through restraining domestic demand for imports so that the benefit derived from the improvement in the terms of trade outweighs the decrease in the gains from trade due to the erection of the trade barrier. As the larger country has a greater demand for imports, higher tariff rates have to be imposed in order to achieve the aim of restricting import demand. This may explain why the larger country has larger equilibrium tariff factors.

Comparison of the free trade and tariff-ridden equilibrium factoral terms of trade also shows that the larger country has been successful in improving its own terms of trade, though the equilibrium terms of trade is still

Table 3.1
Equilibrium Tariff Factors τ^α τ^β
for Countries α and β for Different Relative
Country Sizes L and Patterns of Technology Distribution θ

$\theta \backslash L$	0.5	1	3	8	10	50	100	200
0.5	^a 3.2682	2.1544	1.3897	1.1466	1.1173	1.0235	1.0117	1.0059
	^b 3.4023	2.2706	1.4522	1.1746	1.1402	1.0284	1.0142	1.0071
1	3.3292	2.2056	1.4162	1.1583	1.1269	1.0255	1.0128	1.0064
	3.3292	2.2056	1.4162	1.1583	1.1269	1.0255	1.0128	1.0064
5	3.5208	2.3833	1.5182	1.2046	1.1648	1.0337	1.0169	1.0085
	3.2028	2.1038	1.3658	1.1366	1.1092	1.0218	1.0109	1.0055
10	3.6305	2.4937	1.5851	1.2348	1.1895	1.0389	1.0195	1.0098
	3.1630	2.0756	1.3541	1.1320	1.1055	1.0210	1.0105	1.0052
30	3.8478	2.7256	1.7278	1.2973	1.2401	1.0494	1.0248	1.0124
	3.1138	2.0444	1.3430	1.1279	1.1023	1.0204	1.0102	1.0051
50	3.9705	2.8626	1.8119	1.3328	1.2687	1.0551	1.0276	1.1038
	3.0957	2.0342	1.340	1.1269	1.1015	1.0203	1.0101	1.0051
100	4.1633	3.0847	1.9466	1.3876	1.3125	1.0615	1.0318	1.0159
	3.0751	2.0237	1.3373	1.1261	1.1008	1.0201	1.0101	1.0050
150	4.2919	3.2368	2.0375	1.4233	1.3408	1.0689	1.0345	1.0172
	3.0649	2.0189	1.3362	1.1258	1.1006	1.0201	1.010	1.0050
200	4.3910	3.3558	2.1076	1.4502	1.3621	1.0729	1.0364	1.0180
	3.0584	2.0161	1.3356	1.1256	1.1004	1.020	1.010	1.0050
500	4.7563	3.8054	2.3660	1.5450	1.4364	1.0863	1.0430	1.0215
	3.0414	2.0095	1.3344	1.1253	1.1002	1.020	1.0050	1.0050

* a The upper number in each entry gives the equilibrium value of τ^α , and the lower number gives the value of τ^β .

b All numbers have been rounded to four decimal places.

Table 3.2
Factoral Terms of Trade w^t , w^f
at Tariff-Ridden Equilibria and at Free Trade

θ L	0.5	1	3	8	10	50	100	200
0.5	^a 1.4722	1.2561	1.0998	1.0390	1.0314	1.0063	1.0032	1.0016
	^b 1.5874	1.4142	1.1892	1.0801	1.0650	1.0137	1.0069	1.0035
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.4488	0.5895	0.7953	0.9161	0.9318	0.9857	0.9928	0.9964
	0.3420	0.4472	0.6687	0.6363	0.8639	0.9689	0.9842	0.9920
10	0.3180	0.4702	0.7330	0.8840	0.9056	0.980	0.990	0.9950
	0.2154	0.3162	0.5623	0.7743	0.8111	0.9559	0.9775	0.9886
30	0.1844	0.3298	0.6380	0.8390	0.8686	0.9721	0.9860	0.9930
	0.1036	0.1826	0.4273	0.6853	0.7340	0.9355	0.9669	0.9832
50	0.1432	0.3282	0.600	0.8204	0.8533	0.9688	0.9843	0.9922
	0.0737	0.1414	0.3761	0.6475	0.7007	0.9262	0.9620	0.9807
100	0.1017	0.2251	0.5539	0.7974	0.8344	0.9647	0.9822	0.9911
	0.0464	0.100	0.3162	0.5995	0.6579	0.9137	0.9554	0.9773
150	0.0832	0.1984	0.5297	0.7851	0.8242	0.9625	0.9813	0.9905
	0.0354	0.0817	0.2857	0.5731	0.6341	0.9064	0.9516	0.9754
200	0.0723	0.1815	0.5133	0.7768	0.8174	0.9610	0.9804	0.9902
	0.0292	0.0707	0.2659	0.5550	0.6178	0.9013	0.9489	0.9740
500	0.0461	0.1372	0.4669	0.7526	0.7975	0.9568	0.9782	0.9891
	0.0159	0.0447	0.2115	0.5013	0.5684	0.8853	0.9403	0.9696

* a The upper number in each entry represents the tariff-ridden equilibrium terms of trade w^t and the lower number the free trade equilibrium terms of trade w^f .

b All numbers have been corrected to four decimal places.

Table 3.3
Relative Welfare V_R^α, V_R^β for Countries α and β

$L \backslash \theta$	0.5	1	3	8	10	50	100	200
0.5	^a 0.7421	0.8169	0.9075	0.9576	0.9651	0.9923	0.9961	0.9980
	^b 0.8463	0.9256	0.9811	0.9946	0.9959	0.9993	0.9997	0.9998
1	0.7964	0.8761	0.9504	0.9799	0.9838	0.9966	0.9983	0.9992
	0.7964	0.8761	0.9504	0.9799	0.9838	0.9966	0.9983	0.9992
5	0.9034	0.9740	1.0038	1.0037	1.0032	1.0008	1.0004	1.0002
	0.6665	0.7289	0.8362	0.9178	0.9315	0.9842	0.9920	0.9959
10	0.9388	0.9978	1.0107	1.0055	1.0046	1.0010	1.0005	1.0003
	0.6086	0.6594	0.7760	0.8827	0.9016	0.9768	0.9882	0.9940
30	0.9806	1.0174	1.0118	1.0047	1.0037	1.0007	1.0004	1.0002
	0.5198	0.5518	0.6786	0.8228	0.8502	0.9637	0.9814	0.9906
50	0.9946	1.0211	1.0105	1.0038	1.0030	1.0006	1.0003	1.0001
	0.4807	0.5048	0.6345	0.7943	0.8256	0.9571	0.9780	0.9888
100	1.0085	1.0224	1.0083	1.0027	1.0021	1.0003	1.0002	1.0001
	0.4308	0.4452	0.5768	0.7557	0.7919	0.9480	0.9732	0.9864
150	1.0144	1.0220	1.0070	1.0022	1.0017	1.0003	1.0001	1.0000
	0.4033	0.4127	0.5445	0.7333	0.7723	0.9425	0.9703	0.9849
200	1.0177	1.0212	1.0061	1.0018	1.0014	1.0002	1.0001	1.0000
	0.3847	0.3907	0.5223	0.7175	0.7584	0.9386	0.9683	0.9839
500	1.0242	1.0178	1.0038	1.0010	1.0008	1.0001	1.0000	1.0000
	0.3297	0.3268	0.4556	0.6681	0.7145	0.9258	0.9615	0.9804

* a Upper number represents relative welfare of country α , and lower number represents relative welfare of country β .

b All numbers have been rounded to four decimal places.

favorable to the smaller country. This is probably because the larger country has a more powerful influence over the terms of trade due to the fact that it can bring about more substantial changes in its demand for imports by raising its tariff rates. However, as revealed by Table 3.3, the improvement in the terms of trade may not always be adequate to more than offset the loss incurred by trade restriction.

For a given pattern of international comparative advantage or a given value of θ , it can be seen from Table 3.1 that the equilibrium tariff factor of a country increases with its relative size, α . Again this may be attributed to the fact that the larger the country relative to its trading partner, the higher the tariff rate that is required to curtail domestic demand for imports sufficiently to improve its terms of trade.

On the other hand, keeping the relative country size unchanged, the equilibrium tariff rates of both countries decrease with increasing values of θ . A reason for this is that increasing the value of θ decreases the variance of the distribution curve as shown in Figure 2.1 such that the distribution becomes more concentrated about the vertical line $\alpha = 0.5$. Thus the number of industries with a large international productivity gap (difference in the labour

productivities of the two countries) has decreased. Hence a lower tariff rate is needed by a country to offset the cost advantage possessed by its trading partner.

This implies that a country can achieve the same amount of reduction in domestic demand for imports with a lower tariff rate . Consequently, lower equilibrium tariff rates are obtained for both countries. The effects of reducing the variance of the distribution is also seen in Table 3.2 as both the free trade and the tariff-ridden equilibrium terms of trade improve for the larger country when the value of θ increases.

The relationship between τ^α , τ^β , L and θ are depicted graphically in Figures 3.2 and 3.3. In Figure 3.2, the equilibrium tariff rate of country α is shown as an increasing function of L, its relative country size, and that of country β as a decreasing function of L. The two curves intersect at $L=1$, at which equal tariff rates are imposed by both countries. Increasing the value of the parameter θ shifts both curves downward, reflecting the observed trend.

Figure 3.3 shows the relationship between τ^α , τ^β , and θ and the effects of changing relative country size on the curves. The curves of τ^α , τ^β against θ are downward sloping

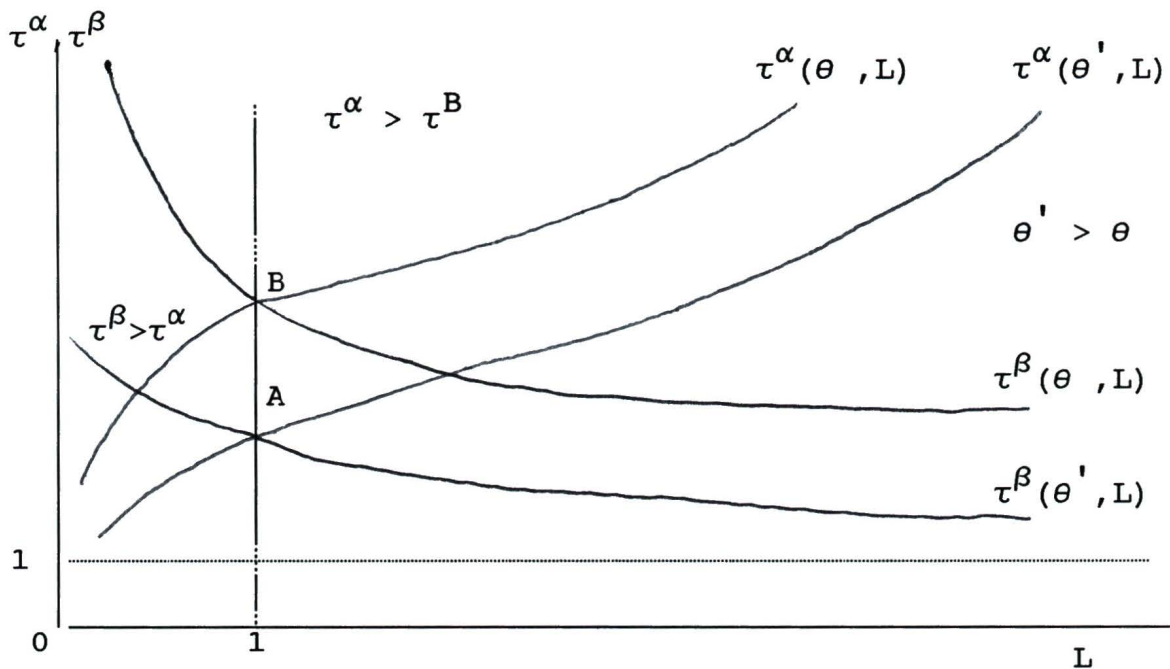


Figure 3.2
Equilibrium Tariff Factors τ^α, τ^β and Relative Country Size

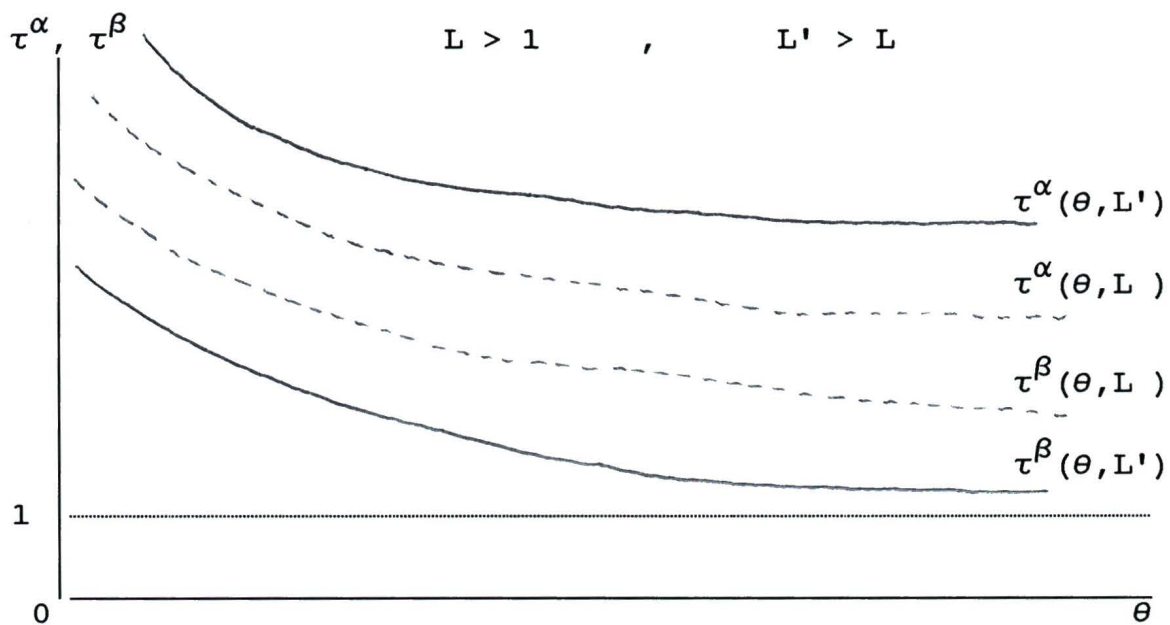


Figure 3.3
Equilibrium Tariff Factors τ^α, τ^β and Parameter θ

with $\tau^\alpha > \tau^\beta$ for $L > 1$. Increasing the relative country size shifts both curves upward. Note that the curves approach the X-axis asymptotically as θ increases.

The relative welfare levels (the ratio of tariff-ridden to free trade welfare levels) for both countries under different combinations of θ and L are provided in Table 3.3. Table 3.3 contains the most important piece of information concerning the welfare consequences of tariff warfare.

As is obvious from Table 3.3, there can be a winner in tariff warfare. In particular, it is the larger country that is likely to emerge as a winner. The smaller country will undoubtedly lose under all circumstances.

The result that a large country is more likely to win a tariff war resembles the standard result obtained with a two-country, two-good model. This is because the larger country is able to produce a more significant change in its terms of trade through tariff imposition than the smaller country. However, there is a tradeoff since the larger country has to endure a higher cost when it erects trade restrictions. But beyond some critical relative country size the improvement in the terms of trade becomes large enough to more than compensate for the loss and the larger country will gain from a tariff war.

The critical relative country size beyond which the larger country will certainly win a tariff war depends on the structure of comparative advantage or the value of θ . From Table 3.3 the critical relative country size decreases continuously from about 100 to less than 10 as θ increases from 0.5 to 50.

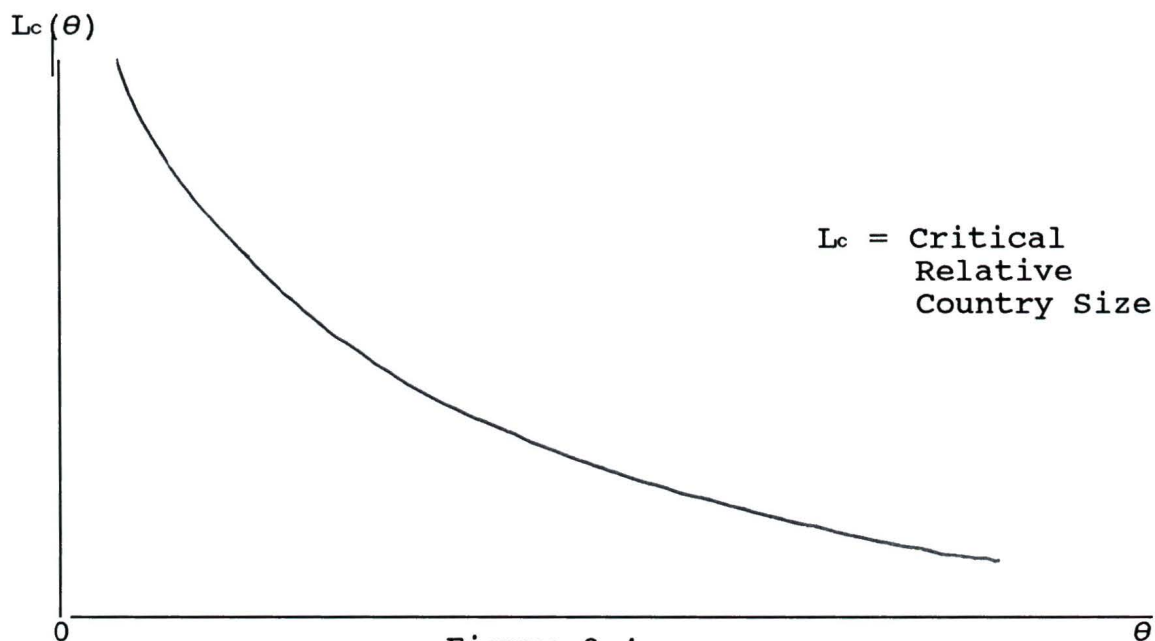
Generally speaking, the higher the value of θ and hence the less dispersed or more concentrated the distribution curve in Figure 2.3, the lower is the critical relative country size. More precisely, when there is a substantial number of industries with a large international productivity gap, (for example, when $\theta = 0.5$) a country can win a tariff war only if it has a population a hundred times larger than its trading partner. This would mean, for instance, a country like the United States confronting a small Pacific island nation.

An explanation for the tradeoff between the pattern of technology distribution and the critical country size may be provided as follows. Intuitively, the greater the number of industries with a wide international productivity gap, the higher the cost of imposing tariffs since the domestic costs of producing the formerly imported goods are much higher than abroad.

Furthermore, when a country is engaged in a tariff war

it suffers an additional loss of welfare resulting from the trade restrictions erected by its rival. Unless it can manage to improve the terms of trade sufficiently to offset these losses its welfare will inevitably deteriorate. Since the ability to shift the terms of trade varies directly with relative country size a larger relative size is therefore required to win a tariff war.

The relationship between the critical relative country size and the value of θ is shown graphically in Figure 3.4.



Despite the existence of a definite relationship between the critical relative country size and the parameter θ , no monotonic trend is observed for the relative welfare of the larger country along the rows or columns of

Table 3.3. Along a given column (i.e.given the value of θ) the relative welfare level for the large country (in this case country α) increases gradually with L until the critical relative country size L_c is reached. After that the relative welfare continues to increase or decrease depending on the value of θ .

Similary the relative welfare along a row increases with θ until the given relative country size becomes the critical relative country size. Thereafter no clear trend is observed for the relative welfare of the larger country.

The relative welfare of the smaller country (in this case country β), on the other hand, does show a continual decline with its decreasing country size. This indicates that the smaller country loses more and more by participating in tariff warfare as the relative size of its rival grows.

Along a given row an opposite trend is observed. Here for a given L the relative welfare of the smaller country increases gradually with the value of θ , indicating that the smaller country suffers less and less loss from the tariff war. This may be explained by the fact that as θ increases, the cost of imposing a tariff decreases since the difference between the domestic and foreign production costs of imported goods has narrowed. Thus the smaller

country, though still a loser in the tariff war, incurs lower costs as a result of its own tariff policy.

The general relationship between the relative welfare levels V_R^α , V_R^β and (θ, L) are shown in Figure 3.5 and Figure 3.6. In Figure 3.5, the curve plotting V_R^α against L increases up to the point $L = L_c(\theta)$ (where $L_c(\theta)$ is the critical relative country size, written as a function of θ). Afterwards the curve does not exhibit a clear trend. At $L = L_c$, the relative welfare of country α , V_R^α is equal to one . The curve of V_R^β against L , in contrast, is shown continuously decreasing as L increases.

In Figure 3.6, the relative welfare of country α and country β , V_R^α , V_R^β when L is greater than one (i.e. when country α has a larger relative size) are drawn against the values of θ . The curve plotting V_R^α against θ is continuously increasing until it reaches the point $\theta = L_c^{-1}(L)$ where L_c^{-1} is the inverse function of $L_c=L_c(\theta)$ (at $\theta=L_c^{-1}(L)$, V_R^α equals to 1 by definition of the critical relative country size). Beyond $\theta = L_c^{-1}(L)$ the curve for V_R^α does not exhibit a clear trend. The curve V_R^β is continuously increasing and asymptotic to the horizontal line $V_R^\beta = 1$.

The welfare consequences of tariff warfare under different circumstances are summarized in Figure 3.7, where the region within which a country will win a tariff war is

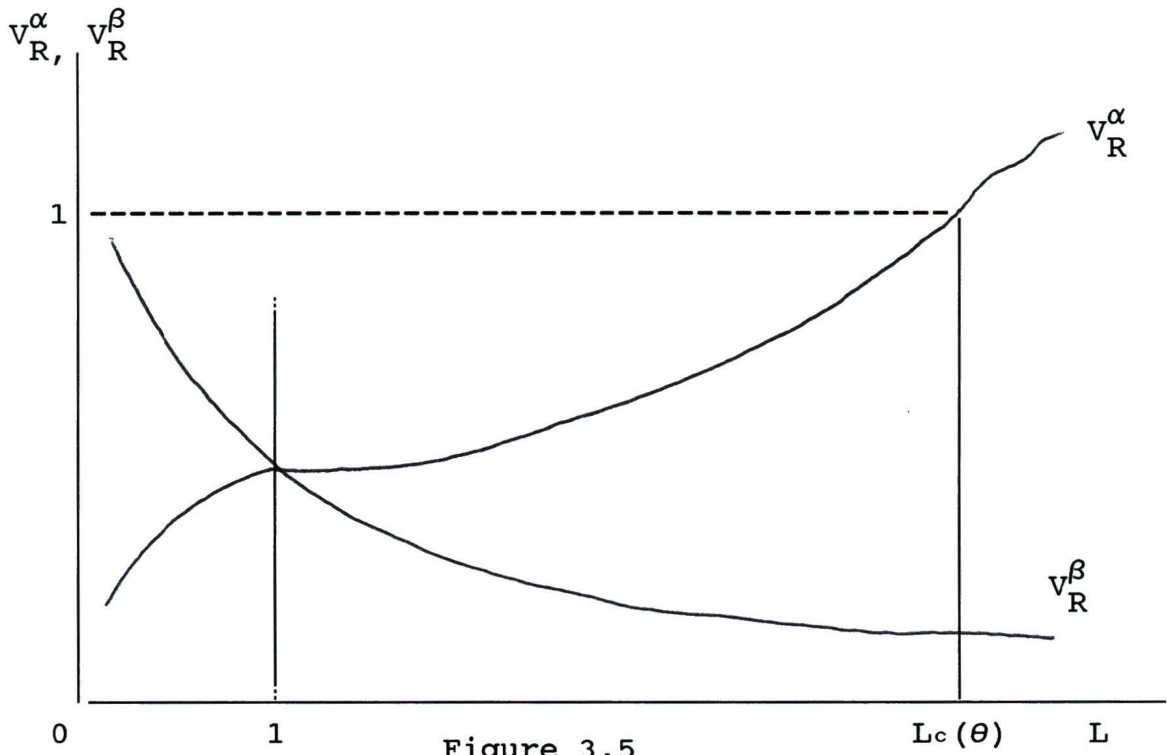


Figure 3.5

Relative Welfare v_R^α, v_R^β at Different Relative Country Sizes

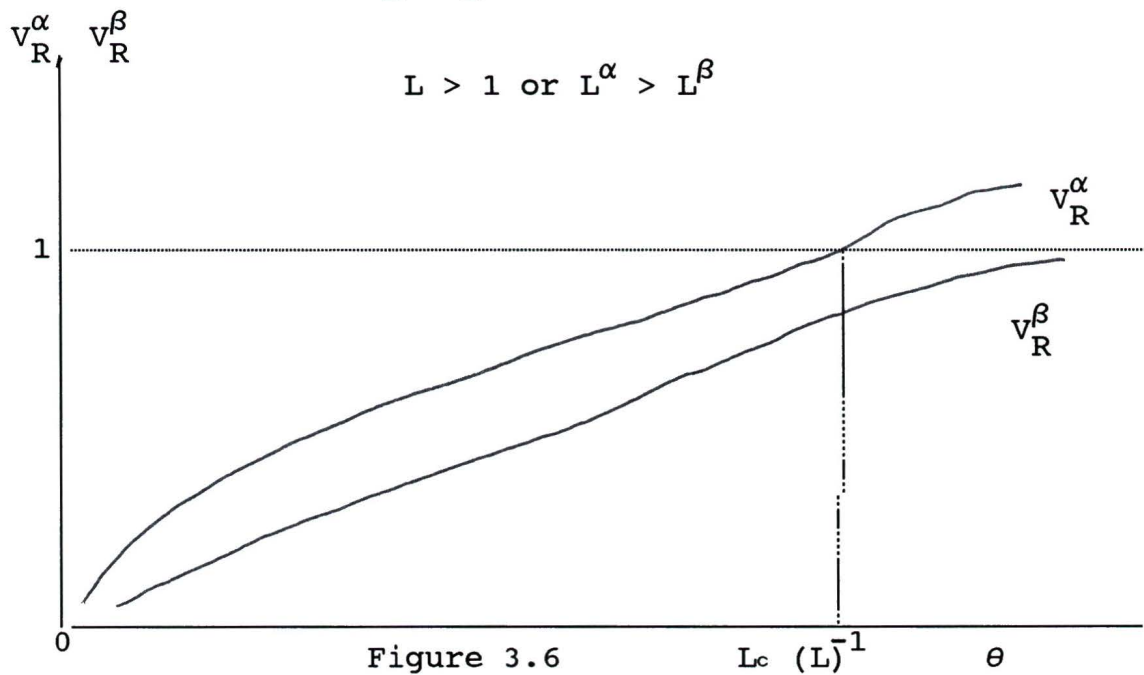


Figure 3.6

Relative Welfare v_R^α, v_R^β and the parameter θ

is shown for each of the two countries α and β .

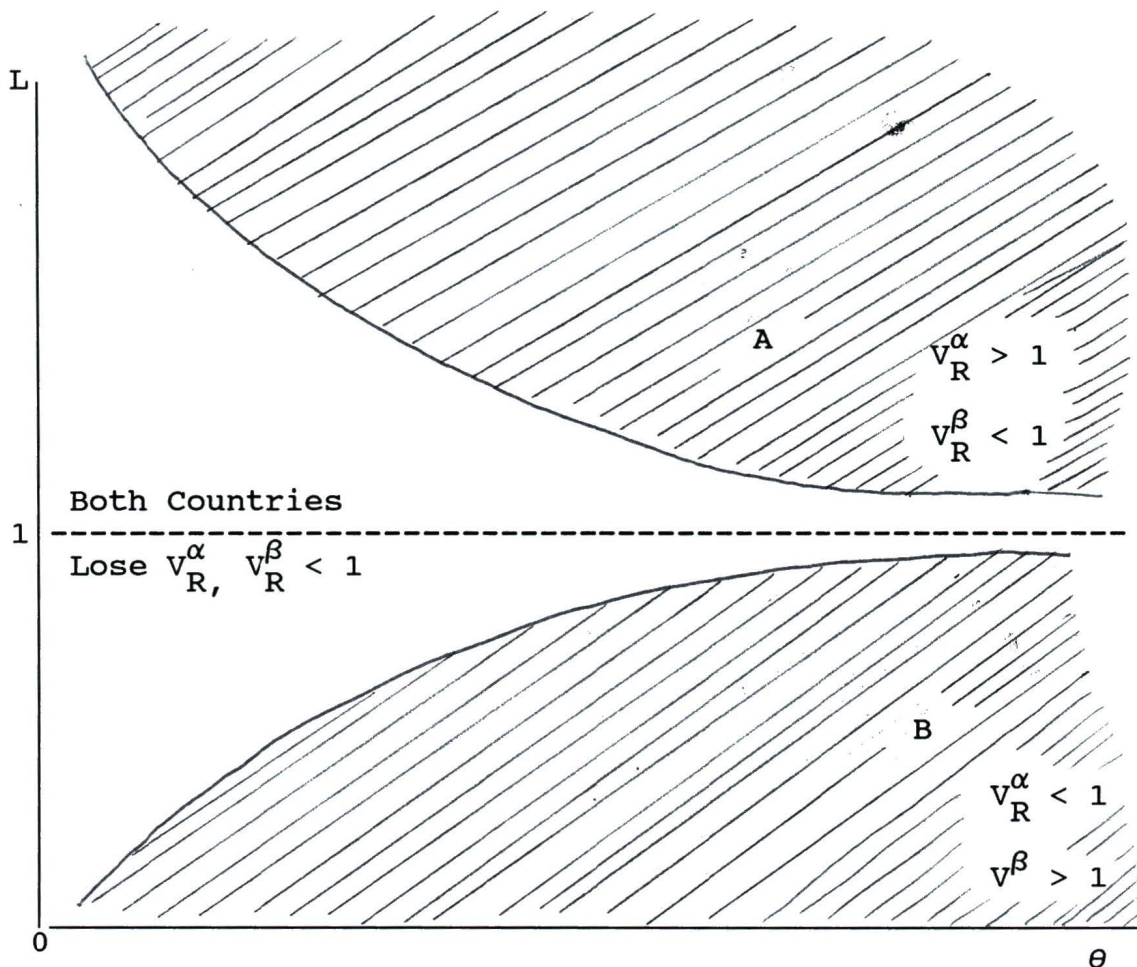


Figure 3.7
Relationship Between Relative Country Size,
Pattern of International Comparative Advantage
and The Welfare Consequences of Tariff Warfare

The sets of all possible pairs of L and θ that will enable country α or country β to win a tariff war are represented by the shaded areas in Figure 3.7. Thus at point A in the upper shaded area the international conditions represented by the vector (θ, L) , are favourable to country α so that it will emerge as a winner. The opposite

is true of point B in the lower shaded area.

Figure 3.7 has particular dynamic implications for tariff warfare. Depending on its future growth path and the relative rate of technological progress or technological diffusion, a country which is initially a loser in tariff warfare may in the near future emerge as a winner. This is so if the international trading environment as described by the vector (θ, L) changes (e.g. from A to B) such that it is no longer favorable to the initial winner. This implies that in the long run, policies concerning population growth and technology also matter.

II. The Effects of Asymmetric Technology Distribution

As shown in the last section of chapter 2, in the case of a CES density function, asymmetries in the technology distribution can be introduced into the computational model by transforming the variables in a way such that $w' = (b/a)w$, and $L' = (a/b)L$, where a/b is a function of the parameters K_1 , K_2 , and θ in the general CES density function (47).

The system of equations (25'), (29') and (30'), and the relative welfare functions (45), (46) can be interpreted in terms of w' and L' and θ . Thus the results in Tables 3.1, 3.2 and 3.3 can be applied to the asymmetric case by simply putting L' in place of L and interpreting the

numbers in Table 3.2 as the equilibrium values of w' (for both free trade and tariff-ridden equilibrium) given L' and θ .

The consequences of introducing asymmetric distribution can be seen more clearly by considering Figure 3.8, which shows the case for $\frac{a}{b} > 1$ or $a > b$.

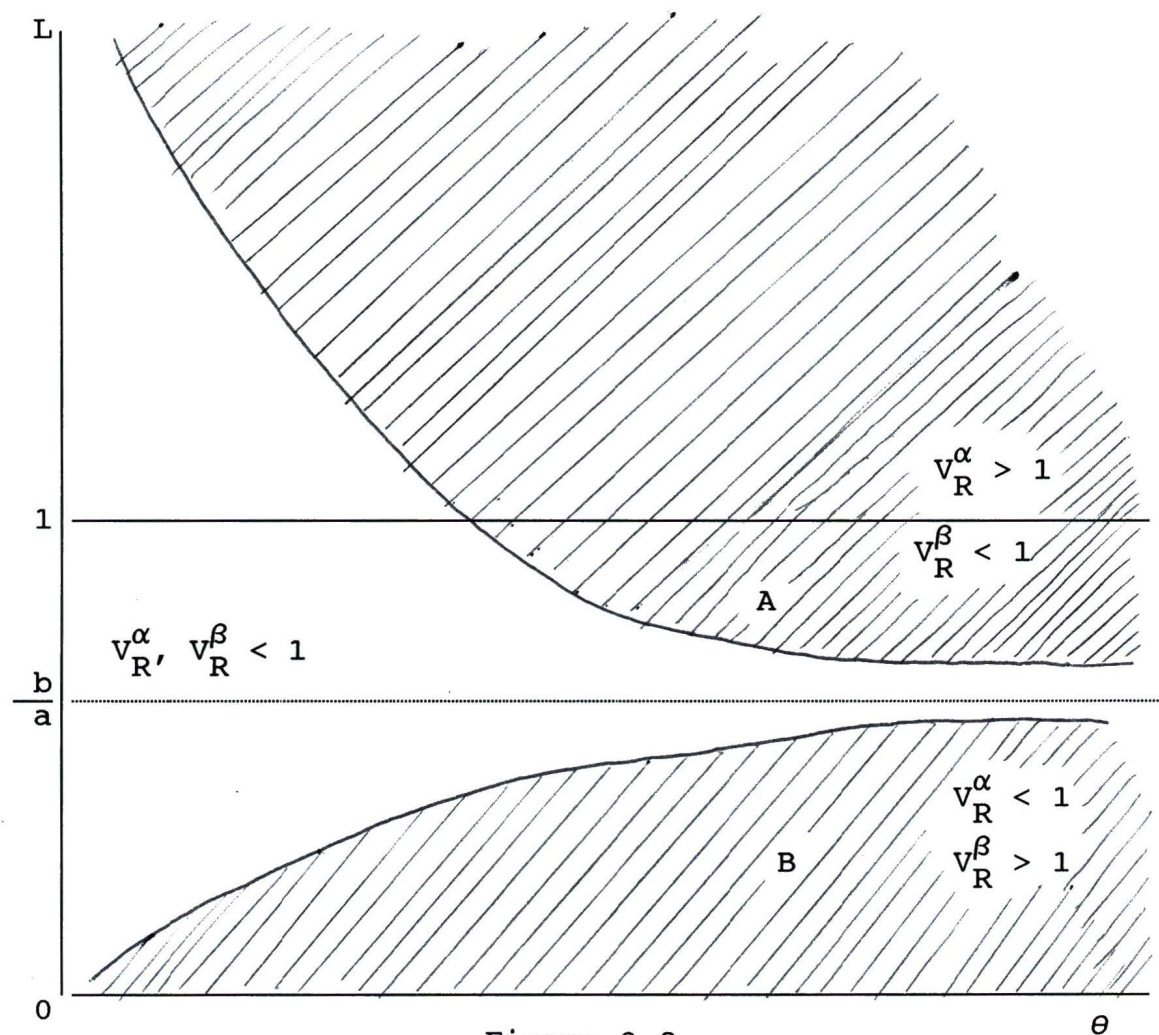


Figure 3.8
Relationship between Relative Country Size, Asymmetric
Technology Distribution and the Welfare Consequences of
Tariff Warfare

Figure 3.8 is obtained by transforming the vertical axis of Figure 3.7, in such a way that the two shaded areas and the asymptotic horizontal line have all moved downward.

Figure 3.8 points to the possibility that the smaller country can win a tariff war. Whenever $\frac{a}{b}$ is sufficiently large (or $\frac{a}{b} \gg 1$) the boundary of the upper shaded area in Figure 3.7 moves down to cut the horizontal line $L = 1$. When this happens country α may be able to emerge as a winner in a tariff war with country β even though the former is smaller. The opposite is true for country β when $\frac{a}{b} \ll 1$.

Since $\frac{a}{b} = \left(\frac{K_1}{K_2}\right)^{1/\theta}$ by construction, $\frac{a}{b} \gg 1$ implies $K_1 \gg K_2$. This means that the density function Figure 2.3 is skewed to the left or, in other words, country α possesses a production advantage over the majority of industries. Thus, despite its small size, α has a much more elastic demand for imports than its larger trading partner, which has a comparative advantage in a small number of industries. Consequently the former has a greater control over the terms of trade and is more likely to win a tariff war.

To conclude, the pattern of comparative advantage does play an important role in determining the destiny of a country participating in tariff warfare. A small country with an overall technological superiority can win .

Conversely, a colossal country with little technological advantage may lose. Moreover, whether there is any winner at all is dependent on the value of θ . The smaller is θ , and hence the greater is the variance in the distribution of technology, the less likely it is that any country will win.

Chapter 4

Summary and Conclusions

This thesis attempts to investigate the welfare implications of tariff warfare by performing a simulation analysis with a computational general equilibrium trade model. More specifically, the roles of relative country size and the structure of comparative advantage in tariff warfare are studied within the context of a relatively simple two-country model with a continuum of goods.

The study aims to provide answers to three fundamental questions concerning tariff warfare. They are restated as follows for reference.

- (i) Can there be a winner in a tariff war, in the sense that a country is better off at the tariff-ridden equilibrium than at free trade?
- (ii) If so, under what circumstances?
- (iii) What is the tradeoff between the factors of relative country size and structure of comparative advantage in determining the welfare consequences of tariff warfare?

The results obtained from the simulation analysis show that there can be a winner in tariff warfare. In the case of the symmetric technology distribution it is the larger country that enjoys the privilege of winning a tariff war.

Furthermore, the more concentrated the distribution, the lower the critical relative country size. Finally, an overwhelming comparative advantage (i.e. a highly asymmetric technology distribution) may compensate for smaller relative size and enable a small country to win a tariff war.

As we can see, notwithstanding the heroic assumption of a continuum of goods, the computational model produces some of the familiar results obtained with a two-good model. The structure of comparative advantage has a highly significant even decisive, role in affecting the consequences of tariff warfare. Relative country size remains a dominant factor in determining the ultimate fate of a country in tariff warfare.

However, these results still differ markedly from those obtained in a general many-country, many-factor and discrete goods model such as that developed by Kuga (1973). In contrast to the latter, only a single tariff-ridden equilibrium is found in any tariff war. At the same time, free trade is obviously not included in the set of final outcomes of tariff warfare under all circumstances , though when θ assumes very high values the tariff- ridden equilibria approach the free trade position asymptotically.

Strictly speaking, these qualitative results for a

continuum of goods are valid only within the context of the present computational model. Just how sensitive the results may be to alternative specifications of the density function and the utility function is yet unknown. Nevertheless, for the general class of symmetric density functions and homothetic utility functions it is still reasonable to believe that similar answers will be obtained for the first two questions.

It should be realized that the present general equilibrium trade model is only one of a broad class of models that can be constructed on a continuum of goods. In order to focus on the factors of technology distribution and relative country size and to facilitate computation a number of restrictive assumptions have been employed. In future research in this area it may be profitable to relax some of these assumptions and construct more general and realistic models.

For instance, different types of tariff-ridden equilibria based on more complex behavioural assumptions can be studied, and the effects of introducing several factors considered.²⁸ The dynamic behaviour of growing countries participating in tariff warfare can also be investigated. Of particular importance is the extension of the study to many-country case, which is more relevant to the under-

standing of real world trade problems.

Results obtained from this research help to provide a more complete picture of the consequences of tariff warfare. It is hoped that the knowledge accumulated on the subject will eventually be useful in guiding the formulation of more sensible and mutually beneficial trade policies.

Footnotes

¹In the simple two-country, two-good case, a country can improve its welfare by levying tariffs on its imports only if by doing so it can shift the terms of trade in its own favour. This condition is satisfied if the tariff-imposing country has a size comparable to its trading partner.

²In the two-country, two-good case with no domestic distortions the optimum tariff rate for a single country is given by:

$$t^i = \frac{1}{\epsilon^j - 1} \quad i \neq j$$

where t^i is the optimum tariff rate of country i and ϵ is the elasticity of country j .

Conditions for optimum tariffs in a world with many countries, goods and factors are discussed by Kuga(1973). Apart from reaping additional gains from trade, tariffs may also be used for correcting domestic distortions. In the latter case a second-best partial equilibrium optimum tariff structure can be shown to exist for the undistorted traded sectors. See Ghosh(1979) for a detailed discussion of optimum tariffs in a distorted multi-commodity economy. The problem of optimum tariffs in a distorted economy was also studied by Bhagwati and Ramaswami(1963, 1969), Bruno (1972) and Kemp and Negishi(1969).

³Like oligopolists, trading countries may follow different strategies in a tariff war and a wide variety of final outcomes are possible. A classic treatment of tariff policy and retaliation in the context of a standard two-country, two-good model is provided by Johnson(1955), who applied the behavioral assumptions underlying a Cournot duopoly to the analysis of tariff warfare. Kuga(1973) extended the analysis to a multi-commodity, country and factor model. An interesting result obtained by the latter is that unlike the simple two-country two-good case free trade may be included in the set of equilibria in the general model.

The welfare implications of cooperative tariff negotiations were explored by Mayer(1977) and El-Agraa(1979).

⁴Depending on the circumstances under which tariff war takes place and the assumptions underlying retaliatory behaviour, a tariff-ridden equilibrium may or may not ex-

ist. However, by making appropriate assumptions Kuga(1973) has been able to prove the existence of such equilibria in a general many-good, many-country and many-factor world.

⁵Here technology refers to the production functions of different industries, though in the multi-factor trade model in which production functions are assumed to be identical across countries technology is usually defined as the factor intensity (ratio of the amount of different factors employed) used in production.

⁶See Wilson(1980) for a detailed discussion of such a transformation and its applications.

⁷In this study the problem of the distribution of gains among members of a country is disregarded by assuming identical homothetic individual preferences. Distributional considerations can be taken into account more explicitly by specifying a social welfare function defined on the utility levels of the individuals in the economy.

⁸See footnote 2.

⁹Other forms of trade restrictions such as export taxes and import quotas can of course be introduced and considered explicitly. However, since there is much equivalence between different forms of trade restrictions it will be desirable to focus exclusively on tariffs in order to avoid unnecessary confusion.

¹⁰A Cournot equilibrium or Nash equilibrium is achieved when each country is imposing its own optimum tariff rate given the tariff rate of the other country rate of the other country. Such equilibria, if they exist, are given by the intersection points of the tariff reaction curves. Beginning from free trade, a Cournot equilibrium is usually achieved after successive iterations along the reaction curves (i.e. only after sequences of retaliation). Therefore some equilibrium points, as determined by certain stability conditions, may never be attained. On the contrary, any Nash equilibrium can be achieved instantly on the grounds that the trading countries recognise their interdependence and hence no time is wasted in intermediate process of adjustment.

¹¹A tariff reaction curve gives the optimum tariff rate

that maximizes a country's welfare given the tariff rate of its rival. It bears a close resemblance to the reaction curve in a Cournot duopoly which gives the profit maximizing output of a firm given the other firm's output.

¹² See, for example, Piggot, J. and Whalley, J. ed. "New Developments in Applied General Equilibrium Analysis"

Cambridge University Press, 1985, for a survey of recent developments in general equilibrium trade models.

¹³ Normalization affects only the absolute magnitude of equilibrium variables but not their qualitative properties, which depend on their relative magnitudes.

¹⁴ Alternatively, we can save many steps by defining directly a density function defined on the countries' relative productivity. That is, define a function $f(\rho)$ such that

$$Z = \int_0^{\rho} f(x) dx$$

is an increasing function of ρ , and index the commodities directly with respect to ρ .

¹⁵ As mentioned earlier, the model deals with only Cournot or Nash equilibria, at which no further welfare improvement can be expected by a country by changing its tariff policy, given the tariff rate of the other country.

¹⁶ Since $\tau^i = 1 + t^i$ where t^i is the tariff rate of country i , we have $d\tau^i = dt^i$ and hence we can operate with τ^i instead of t^i .

¹⁷ Remember that we have set $w^\alpha = 1$; thus the equilibrium relative wage depends entirely on the solution value of w^β .

¹⁸ See appendix.

²⁰ One can verify that this is the proper solution by differentiating (5') with respect to a^α .

²¹ Wilson (1980) has extended the analytical technique to a many-country case and derived the Marshallian demand functions for labour for each country. These functions are found to exhibit the same characteristics as ordinary

demand functions for commodities.

- ²²One can easily verify that this is the proper solution for the general C.E.S. function by differentiating (48) with respect to a^α .
- ²³Here we have non-identical utility functions for the two countries. This is because the asymmetry in distribution has been incorporated in the utility functions defined on labour consumption.
- ²⁴See for example Dennis, J.E. and R.B. Schabel, "Numerical Methods for Unconstrained Optimization and Nonlinear Equations", Prentice Hall, 1983, for a detailed discussion of the algorithm.
- ²⁵Reaction curve analysis is not carried out for values of $\theta > 1$ not only because they give unique numerical solutions, but also because solving the two pairs of equations (25') and (29'), and (25') and (30') is extremely difficult.
- ²⁶See footnote 25.
- ²⁷As shown by equations (23') and (24') the derived demand for foreign labour is an increasing function of country size.
- ²⁸Dornbush, Fischer and Samuelson(1980) have studied a continuum of goods in a two-factor Heckscher-Ohlin model. Their study has shown that most standard results obtained in the two-good model (e.g. Stolper-Samuelson Theorem, and factor price equalization) continue to hold in the context of a continuum of goods.

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APPENDIX I

I. Variance of the C.E.S. Density Function

In this appendix it will be shown that the variance of the symmetric CES density function decreases with the value of θ . By definition the variance of the CES distribution is:

$$\begin{aligned}
 \text{Var}(\theta; x) &= \int_0^1 (x - 0.5)^2 f(\theta; x, 1-x) dx \\
 &= \int_0^1 (x^2 - x + 0.25) \frac{\theta x^{\theta-1} (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx \\
 &= \int_0^1 \frac{\theta x^{\theta+1} (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx - \int_0^1 \frac{\theta x^\theta (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx \\
 &\quad + 0.25 \int_0^1 \frac{\theta x^{\theta-1} (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx \\
 &= \int_0^1 \frac{\theta x^{\theta+1} (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx - 0.25 \quad (1)
 \end{aligned}$$

since $E(x) = \int_0^1 \frac{\theta x^\theta (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx = 0.5$ and

$$\int_0^1 \frac{\theta x^{\theta-1} (1-x)^{\theta-1}}{[x^\theta + (1-x)^\theta]^2} dx = 1$$

Integrating (1) by parts and simplifying, we get:

$$\text{Var}(\theta; x) = 0.75 - 2 \int_0^1 \frac{x^{\theta+1}}{[x^\theta + (1-x)^\theta]} dx \quad (2)$$

To find out how the variance changes with the value of the parameter θ we differentiate (2) with respect to θ .

$$\begin{aligned} \frac{\partial \text{Var}(\theta; x)}{\partial \theta} &= -2 \int_0^1 \frac{x^{\theta+1} (1-x)^\theta [\ln x - \ln(1-x)]}{[x^\theta + (1-x)^\theta]^2} dx \\ &= -2 \int_0^{0.5} \frac{x^{\theta+1} (1-x)^\theta [\ln x - \ln(1-x)]}{[x^\theta + (1-x)^\theta]^2} dx \\ &\quad - 2 \int_{0.5}^1 \frac{x^{\theta+1} (1-x)^\theta [\ln x - \ln(1-x)]}{[x^\theta + (1-x)^\theta]^2} dx \quad (3) \end{aligned}$$

The first term in the (3) is non-negative and the second term is non-positive. However, since the absolute value of the second term is greater than that of the first term we have:

$$\frac{\partial \text{Var}(\theta; \mathbf{x})}{\partial \theta} < 0 \quad \text{for all } \theta > 0$$

Therefore the variance of the CES distribution decreases with increasing value of θ .

APPENDIX II

Solving Non-linear Simultaneous Equation System With Gauss

The non-linear equation system (25'), (29') and (30') were solved by using Gauss Non-linear Simultaneous Equation System (NLSYS) command. The command files are included here for reference.

PART I. Solving for the tariff-ridden equilibrium values of τ^α , τ^β and w with the Gauss NLSYS command.

```
/*=====
NLSYS1.EXP -- Example command file using NLSYS to solve
sets of non-linear simultaneous equations.
Before using this the first time during any
session (or after a NEW command), type:

        RUN NLSYS.SET
        See NLSYS>DOC for additional instructions. */
/*-----*/
/* STEP 1: specify kx1 vector of starting values -- there
MUST be the same number of starting values as
there are equations.
=====*/
let x0 = 1.5 1 1.5 ;
    b1 = 8; b2 = 500;
/* list starting values here -- name should be kept X0 */
/*-----*/
vf = zeros(rows(x0),1); /* size of this vector is determin-
from x0 */
proc f(x);
```

```

/*-----*/
/*STEP 2: specify the equations to be solved, as a function
of the arguments. The objective is to solve for
values such that  $f(x) = 0$ .
=====*/
local x1, x2, x3;
x1 = x[1,1]; x2 = x[2,1]; x3 = x[3,1];
vf[1,1] = (x1^(b1*2 + 2))/(x3^(b1*2)) + 1 - b2*((x2*x3)
          ^ (b1*2) + x3*^2);
vf[2,1] = b1*x1^2*x2^2 - x1^2*(1+b1) - (x3/x1)^(b1*2);
vf[3,1] = b1*x1^2*x2^2 - x2^2*(1+b1) - (1/(x2*x3))^(b1*2));

/*-----*/
retp( vf );
endp;
/*-----*/
/* STEP 3: specify the options you want.
=====*/
output file = n1.out reset: /* specify output file, if
                             desired. */
convtol = 1e-6; /* convergence tolerance.
prntit = 1; /* if 1, will print on every
              iteration -- however, */

pressing ANY key while the
program is running will
toggle the printing on and
off. */
prntout = 1 /* if 1, will print final
              output; NOTE: the solut-
              ion vector is always */
              assigned to x1, and will

```

```

                                be in memory when the
                                program is completed.
fname = &f                      /* change this if the name
                                of the proc containing
                                containing the function
                                is not = f.          */
gradname = &grad;              /* change this to specify
                                another proc to compute
                                the Jacobian matrix, if
                                desired.            */
jco = 0                         /* change this to specify a
                                kxk matrix to be used as
                                the initial value of the
                                Jacobian, if desired. */
/*=====*/
/* The following code actually calls the proc that solves
the equations.                    */
x1 = nlsys(fname, x0, jco, convtol, prntit,prntout);
-----

```

Note: In the above command file, $f[1,1]$, $f[2,1]$ and $f[3,1]$ represent equations (25'), (29') and (30') in the text, and $x1 = (\tau^\alpha)^{1/2}$, $x2 = (\tau^\beta)^{1/2}$ and $x3 = w^{1/2}$. The two parameters $b1$ and $b2$ represent the parameter θ and the relative country size L in the original equations. The free trade equilibrium value of w is solved by using equation (25') alone and setting $\tau^\alpha = \tau^\beta = 1$ (i.e. by deleting $f[2,1]$ and $f[3,1]$ in the above command file and setting $x1 = x2 = 1$ for $f[1,1]$).

PART II. Plotting the tariff factor reaction curves for both countries for $\theta = 0.5$.

The tariff reaction curves for countries α and β are plotted by solving two pairs of equations—(25') and (29') for country α , and (25') and (30') for country β —in which one of the tariff factors (τ^α , τ^β) is made exogeneous.

(i) Command file for plotting the reaction curve of α .

```

/*=====
NLSYS1.EXP -- Example command file using NLSYS to solve
              sets of non-linear simultaneous equations.
              Before using this the first time during any
              session (or after a NEW command), type:

                      RUN NLSYS.SET
              See NLSYS>DOC for additional instructions. */
/*-----*/
/* STEP 1: specify kx1 vector of starting values -- there
              MUST be the same number of starting values as
              there are equations.
              =====*/
let x0 = 2.0  1.3;
      b1 = 0.5; b2 = 50; b3 = 2.5;
/* list starting values here -- name should be kept X0 */
/*-----*/
vf = zeros(rows(x0),1); /* size of this vector is determin-
              from x0 */
proc f(x);
/*-----*/
/*STEP 2: specify the equations to be solved, as a function
              of the arguments. The objective is to solve for
              values such that f(x) = 0.

```

```

=====*/
local x1, x2;
x1 = x[1,1]; x2 = x[2,1];
vf[1,1] = (x1^(b1*2 + 2))/(x2^(b1*2)) + 1 - b2*((b3*x2)
          ^ (b1*2) + x2**2);
vf[2,1] = b1*x1^2*b3^2 - x1^2*(1+b1) - (x2/x1)^(b1*2);

/*-----*/
retp( vf );
endp;
/*-----*/
/* STEP 3: specify the options you want.
=====*/
output file = n1.out reset: /* specify output file, if
                             desired. */
convtol = 1e-6; /* convergence tolerance.
prntit = 1; /* if 1, will print on every
              iteration -- however, */

pressing ANY key while the
program is running will
toggle the printing on and
off. */
prntout = 1 /* if 1, will print final
              output; NOTE: the solut-
              ion vector is always */
              assigned to x1, and will
              be in memory when the
              program is completed.
fname = &f /* change this if the name
              of the proc containing
              containing the function

```

```

                                is not = f.                */
gradname = &grad;                /* change this to specify
                                another proc to compute
                                the Jacobian matrix, if
                                desired.                */
                                jco = 0                    /* change this to specify a
                                kxk matrix to be used as
                                the initial value of the
                                Jacobian, if desired. */
/*=====*/
/* The following code actually calls the proc that solves
the equations.                */
x1 = nlsys(fname, x0, jco, convtol, prntit,prntout);

```

Note: In the above command file, $f[1,1]$, $f[2,1]$ correspond to equations (25') and (29') in the text, and $x1 = (\tau^\alpha)^{1/2}$, $x2 = w^{1/2}$, $b1 = \theta$, $b2 = L$. The parameter $b3$ represents the square root of the exogeneous tariff factor τ^β (or $b3 = (\tau^\beta)^{1/2}$).

(ii) Command file for plotting the reaction curve of β

```

/*=====
NLSYS1.EXP -- Example command file using NLSYS to solve
sets of non-linear simultaneous equations.

Before using this the first time during any
session (or after a NEW command), type:

                                RUN NLSYS.SET

See NLSYS>DOC for additional instructions. */
/*-----*/
/* STEP 1: specify kx1 vector of starting values -- there

```



```

                                pressing ANY key while the
                                program is running will
                                toggle the printing on and
                                off.                                */
prntout = 1                    /* if 1, will print final
                                output; NOTE: the solution
                                vector is always assigned to x1,
                                and will be in memory when the
                                program is completed.
                                */
                                fname = &f                    /* change this if the name
                                of the proc containing the function
                                is not = f.                                */
                                gradname = &grad;                /* change this to specify
                                another proc to compute the Jacobian
                                matrix, if desired.                                */
                                jco = 0                          /* change this to specify a
                                kxk matrix to be used as the initial
                                value of the Jacobian, if desired. */
/*=====*/
/* The following code actually calls the proc that solves
the equations.                                */
    x1 = nlsys(fname, x0, jc0, convtol, prntit,prntout);
-----

```

Note: In the above command file, $f[1,1]$, $f[2,1]$ correspond to equations (25'), (30') in the text, and $x1 = (\tau^\beta)^{1/2}$, $x2 = w^{1/2}$, $b1 = \theta$, $b2 = L$. The parameter $b4$ represents the square root of the exogeneous tariff factor τ^α (or

$$b4 = (\tau^\alpha)^{1/2}.$$

PART III. Computing the relative welfare levels for countries α and β .

The relative welfare levels (ratio of welfare levels at tariff-equilibrium and at free trade) for both countries are computed by Gauss using equations (45) and (46) in the text. The command file are included as follows.

Command file wel.cmd.

```

let b1 = 10;
    x1 = 1.1648;
    x2 = 1.1092;
    x3 = 0.9318;
    x4 = 0.8639;

u1 = ((1 + x4*b1)*(x3^b1 + x1^b1)^(1 + 1/b1))/((x3^b1
    + x1^(b1 + 1))*(1 + x4^b1)^(1 + 1/b1));
u2 = ((1 + x4^b1)*x4*(1 + (x3*x2)^b1)^(1 + 1/b1))/(x3
    *(1 + x3^b1*x3^(b1 + 1))*(1 + x4^b1)^(1 + 1/b1));

```

Note: In the above command file, $x1$, $x2$ correspond to τ^α and τ^β , and $x3$, $x4$ to w^t and w^f . The parameter $b1$ represents the parameter θ in the equations (45) and (46) in the text. Expressions $u1$ and $u2$ correspond to equations (45) and (46) respectively.

VITA

Surname: CHAN Given Names: WING-YIU

Place of Birth: HONG KONG Date of Birth: 26-9-1966

Educational Institutions Attended, with Dates of Entering
and Leaving:

NEW METHOD COLLEGE, HONG KONG	1982 to 1984
LINGNAN COLLEGE, HONG KONG	1984 to 1987
UNIVERSITY OF VICTORIA, B.C., CANADA.	1987 to 1988

Degrees, Diplomas, Etc., Awarded, with Dates and Names of
Institutions:

Higher Diploma.	1986	Lingnan College, Hong Kong
Honours Diploma.	1987	Lingnan College, Hong Kong

Honors and Awards:

Certificate of Merits, Lingnan College, 1987

Canadian Pacific Scholarship, University of Victoria, 1987

Publications:

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Author



CHAN WING-YIU

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