Learning in spaces of tension: Reflecting on my mathematics pedagogy

by

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B.Ed, University of Manitoba, 1994
Diploma in Education (Library Education), University of British Columbia, 2004

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Supervisory Committee

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Abstract

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The purpose of this autoethnographic study was to (re)think my conceptions of teaching and learning mathematics. Using van Manen's (1991) cycle of reflection, Jardine, Friesen, & Clifford's notions of abundance (2006b), and Aoki's (2005 [1993]) notions of "decentering the center", I explored teaching vignettes that pointed to four sites of tension within my lived experiences as a mathematics educator: (a) instrumental vs. relational knowledge; (b) linear vs. recursive relationships between concrete and abstract mathematical experiences; (c) fixing mistakes vs. justification of mathematical thinking; and (d) problem solving vs. problem posing. The three common themes that arose were identified as: (a) my desire to enhance my understanding of mathematics; (b) the importance of occasioning time for students to interact in the mathematics classroom; and (c) my obsession with teaching to the test. I brought each site of tension and theme into conversation, drawing on relevant literature within curriculum studies and mathematics education. During the reflective process, not only did I experience a transformation in my conceptions of teaching and learning mathematics, but also in many fundamental ways my entire being was transformed.

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I wish to acknowledge my parents, Lois and Victor Wiens, who have forever been the wind beneath my wings and the roots beneath my feet. I can only aspire to be the kind of mentor for children that they have been for me. Mom has been with me every step of the way throughout the course of my graduate studies. Her never ending support included listening to me talk about my research with interest, having faith in my abilities, and providing constant encouragement during these last four years.

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I want to acknowledge my supervisor, Dr. Jennifer Thom for her guidance throughout the process of creating this autoethnographic study. Her questions and gentle probing led me to think about mathematics and teaching in ways that I never thought possible. Dr. Thom has told me on many occasions that graduate studies is truly “worth the while” when one’s very being is transformed during the journey. This is exactly what happened to me, and I am truly grateful for this life-long gift.

I also wish to thank Dr. Leslee Francis-Pelton for being part of my committee and for sharing my journey through graduate studies with me. I have greatly appreciated her mentorship in the classroom as well as in the university setting.

Dedication

I dedicate this project to my husband, Don Grubb, whose never-ending patience, support, love, and humour makes both life as a masters student and life in general, an absolute pleasure. Come rain or shine, he has been with me all the way and my heart is filled with gratitude to have such a man in my life.

Chapter 1: Introduction

Participation in school mathematics was always a love of mine. As a grade school student, my teachers taught me to memorize facts, formulas, and algorithms that gave me the right answers, a phenomenon that continues to be a personal strength of mine. The thrill of getting the right answers led me on a journey through the challenge math program in high school and the often-dreaded first year calculus course in University, where I earned a coveted, A grade. I graduated with a Bachelor of Education (K-7) degree with a minor in mathematics and my participation in school mathematics continued as I tutored students through to the tenth grade.

In order to tutor mathematics at this level, my employers required me to take a high school math test. I am not sure why, but I remember cramming for the exam in my favourite study place at my parents' house, math textbook in hand, as I completed question after question and checked my work against the answer key at the back of the book as I went along. I kept many late nights and relished every moment of this interesting read, instantly rewarded by each of my right answers.

This same rush of excitement, of mathematical accomplishment came back to me as I read a chapter by David Jardine (2006) in preparation for a graduate course. The chapter opened with the following problem:

> Eric's family has three members: Eric, his father David, and his mother Gail. David is 2 years older than three times as old as Eric. Gail is 17 years older than the difference between David's age and Eric's age. Altogether they have lived 111 years. How old are Eric and his family members? (p. 61)

Without a moment's hesitation, I picked up my pencil and started to solve the problem in the margin of my textbook. "What a treat, mathematics amidst all this deep philosophy," I said to myself as I pondered a moment about all the abstract theory that I studied during my four years in graduate studies. I sighed with relief as I set up a chart and set variables E, G, and D, to stand in place of the family members and when finished, put down my pencil, feeling proud. "I still have it," I said to myself, feeling a sense of security in the fact that there was still something that I knew for certain.

As I continued on reading the chapter to find out if I got the right answer, this is when my thinking around how well I really understand mathematics changed completely in just one moment. As I considered Jardine's (2006) words about the panic that story problems evoke in children all over North America and Europe, he told me that what I experienced was:

> what many who despise mathematics experience as its horror, and many who love mathematics experience as its relief. What is meant here is this: the answer to what is despised and what is loved is the same: There is already a hidden solution co-present in this question. (p. 63)

He explained, "The example is simply a "site" at which to practice and master a certain mathematical way of thinking" (p. 67). In this light, "we are not doing mathematics, we are in school" (p. 66).

Those who despise mathematics lack recourse for this question, whose answer exists only within it, and they are unable to proceed. Those who love it know that with effort "things are already fine without my own findings" and "my own agency is not experienced as at stake here" (Jardine, 2006, p. 64). The moment these words started to sink in, I became aware that different emotions about math, loving or being phobic of it, can be provoked from the same event, involving a view of mathematics as an object that is independent of the knower. The sudden realization that my view of mathematics might not be so different from those who struggle with mathematics prompted me to consider my mathematical agility in a different manner, as something isolated, disconnected, and irrelevant to my life outside of my role as student and teacher.

My reflection on Jardine's words, written across just seven pages, brought to mind all the nights my mother and I practiced my multiplication tables during my grade four year, as we raced to get them memorized so that my mathematics mark for the term would be the best it could be, as well as a host of other similar experiences with mathematics. I also thought about how difficult it was for me to teach multiplication to my first students during my early years of teaching. The teacher's guide in the textbook that I used with my students told me to teach arrays, and to connect arrays with ideas from student experiences, but the notion of array and its relationship with multiplication was not part of my mathematics vocabulary at the time. As a result, we rarely got beyond references to checkerboards and candy bars, connections to which I could relate, but which felt forced and dictated from the textbook, and not as valued as the timed multiplication drill that I put each of my students through each year. All at once, a notion of there being something more to mathematics than the school math that I grew up with, as well as the desire to understand mathematics differently became clear to me.

Very quickly, a sense of tension and embarrassment came over me where once was incredible pride. Rather than wallowing in my newly discovered mathematical inabilities, I became curious to explore my limited view of mathematics and find out how

I might change my perceptions of mathematics teaching and learning on both conscious and subconscious levels. My experience with Eric and his family provoked me to search the literature and my teaching practice for insight into these queries that I had not thought to question over my fourteen years of teaching until this moment of epiphany.

Three Theoretical Perspectives

After much reading, reflection, and discarding one idea after another, three theoretical perspectives related to the notion of interconnectivity lingered with me, demanding my attention and further exploration. These three perspectives are grounded in the works of: David Jardine, who draws upon the relationship between hermeneutics, ecology, and pedagogy, both within and beyond the domain of mathematics education; Ted Aoki, who explores notions of a living curriculum through the lens of hermeneutics and phenomenology; and Max van Manen, who speaks to the phenomenological qualities of pedagogical reflection and mindfulness, which allows us to truly hear and occasion the type of learning alluded to by Jardine and Aoki.

Mathematics is not an object.

> What if mathematics is much more a world into which we are drawn, a world which we do not and cannot "own", but must rather somehow "inhabit" in order to understand it? What if we cannot own mathematics (either individually or collectively) not because it is some object independent of us and our (individual or collective) ownerships, but because it is not an object at all? (Jardine, Friesen, & Clifford, 2003, p. 90)

This view of mathematics does not portray it as a set of disconnected knowledge and skills that can be produced and consumed, universal to all who encounter it.

Mathematics is something more than those fill in the blank questions (e.g., 5 - 3 = ___) that many of my generation (and plenty after me, I am sorry to say) had to complete in grade school. Jardine's vision of mathematics as a world that we can inhabit evokes a sense of interconnectedness of mathematics to the world and to its own history, as well as a willingness to be drawn beyond the surface of the static, mathematics worksheet where "the site of original difficulty" exists, "demanding our attention" (Jardine et al., 2003).

In his writings, Jardine (1992a, 2006) makes frequent mention of "the site of original difficulty" which he explains "is a return to the essential generativity of human life, a sense of life in which there is something always left to say, with all the difficulty, risk, and ambiguity that such generativity entails" (Jardine, 1992a, p. 120). Involvement with learning procedures, such as that 5 - 3 = ___, is a finished, deadened task when it is memorized along with all the basic addition and subtraction facts, which exist with or without the presence of the knower. When Jardine et al. (2003) asks my students and I to "suffer" with this task and view it as a site of difficulty, he presents an option for us to think about it differently. He invites us to experience and come to an understanding of this "stubborn particular" (Jardine, 2002a), this 5 - 3 = ___, not as a question to answer finitely now and forever, but as a portal, an opening to a world of relations (Jardine, 1998) that I cannot know before the moment that I, as a teacher, begin to contemplate what is lurking beyond the surface of the question with my students. He asks me to ponder how the notion of 5 - 3 and others like it refract my understanding of the whole domain of mathematics, rather than portrayed as lonely, isolated math facts and procedures to be mastered by my students and tested come exam time.

Jardine (2002a) imparts that "understanding the whole involves paying attention to this in its wholeness" (p.143), which involves developing a mindfulness towards, a noticing if you will, of the interconnectivity present both within the domain of mathematics and beyond it, but not necessarily "brought forth" (Jardine, 1988) into either mine or my student's experiences just yet. Jardine's use of the notion of bringing forth should not be taken in the same manner as Piagetian notions of "understanding-as-construction," whereby "our own constructions can be wielded to make the world into an object of our mastery and control" (Jardine, 1988, p. 168). In this latter view, the world asks nothing of us beyond what we ask of it, a unidirectional relationship between each individual and the object of investigation. Instead, Jardine (1988) views understanding as "an engagement with what is other (already present before we arrive), going beyond our own constructions" (p. 149), and thus open to interpretation.

For example, Jardine (2000) speaks of a grade two student's encounter with school mathematics that he observed as a teacher supervisor:

> One student waved me over to them. She said, "I don't understand this one at all." The question she had trouble with was this: Joan went to the post office. She mailed five letters and three packages. How many more letters than packages did she mail?
>
> I squatted down beside the student's desk and put five fingers on one of my hands and three on the other.
>
> "O.K." I said, moving the appropriate hand slightly forward in each case, "she's got five letters and three packages. She's got more letters..."

> The student suddenly grabbed the thumb of my “letters” (five digits extended) hand and bent it down. She then bent down my little “pinky” finger as well, leaving three fingers extended. But then, with a puzzled look, she considered my other (three fingers extended hand). Carefully, she pulled my thumb and little “pinky” finger up, now leaving five fingers extended where there were three, and three fingers extended where there were five.
>
> “Two!” she said, a bit too loud for the enforced quiet that worksheets inevitably demand.
>
> “Yep, two, you’ve got it.”
>
> The student looked back down at the worksheet’s requirements [which included a line upon which to put your answer, and, above this line, a plus and a minus sign, one of which you were instructed to circle, to demonstrate the operation you used in solving the question]. Suddenly, this: “But you know, I’m not sure. Did I add or subtract?” (pp. 108-109)

The child’s query is an engagement with mathematics that lies outside of a bland question on a bland worksheet, a portal into the world of relations of which Jardine speaks. In just one moment, mathematics arose as a living entity that this young girl inhabited rather than possessed. To Jardine et al. (2003), the girl experienced mathematics as a claim made upon her, requiring something of her, pulling her into its question, its repose, its regard (p. 91).

Unfortunately, rather than taking the opportunity to engage with a teachable moment that allows one to initiate conversations about the nature of subtraction and addition and their inextricable bond, the child was silenced by a teacher who told the class that a mistake was made in the writing of the questions, which should be more definitive on which operation to use. Because mathematics, "must be drawn back into its suffering, it's undergoing, it's movement into becoming what it is, its living coming-to-presence, rather than foreclosing being present" (Jardine et al., 2003, p.95), in this moment, what lived, perished, as the child's teacher instructed her to move on to another question.

Jardine's vision of mathematics pedagogy is one of sustainability, in the sense that the historical traditions of mathematics (i.e., "the old") and the mathematics that is deeply experienced by our students in their lived experiences (i.e., "the new") both need each other for survival:

> Only together are the young fecund (and not simply "new") and is the world set right anew once again (and not simply "old"). The teacher stands along this sharp edge which must move like moontides, pulled by this child and this, attentive, wary, interpreting the world. (Jardine, 1992b, p. 142)

Jardine's vision of mathematics opens up a space of sustainable generativity only if teachers are willing to view particularities, such as 5 - 3 = __, as open to interpretation and nested within a network of relations. For example, consideration of nodes that connect to notions of part-part-whole, relating numbers to the benchmarks of 5 and 10 on a ten-frame, visualizing real-life examples and so on, has the potential to alter our conception of this particular number sentence when placed alongside each other, "without which this stubborn particular would not be what it is" (Jardine, 2002a, p. 147).

Jardine recognizes that his vision of mathematics and the suffering it entails is one that may cause tension for educators working within the current curricular climate that values breaking down mathematical ideas into manageable bits that can be reported on and measured (Jardine, Clifford, Friesen, 2002). As I pause to think of how a disconnected vision of mathematics allows me to meet the demands of the school workplace to cover an overburdened curriculum in a relatively short period of time, versus Jardine's vision of mathematics, which takes precious class time to explore with students I feel as though I am caught between two worlds. On one hand, the government of British Columbia requires me not only to teach every learning outcome in our Mathematics Instructional Resource Package (2007) in addition to six other curricular areas, but also to ensure that each of my students meets our school district's standards for achievement in mathematics on a test that students take in February of each school year. Performance scores in mathematics are reported on and shared with the general public, resulting in enormous pressure placed on teachers to occasion learning experiences for our students that might help our students beat the provincial averages. On the other hand, I want to take the time to explore mathematical ideas as they arise, moments experienced like that of Jardine's grade 2 student without the pressure to hurry children along so that we can at least touch upon all elements of the curriculum guide by February. This kind of tension is stressful, but perhaps not such a bad thing if I can view it as a site of inquiry rather than a requirement to make an all or nothing kind of pedagogical decision.

Living in tension is a good thing.

Speaking from a perspective of living in two cultural worlds, Japanese and Canadian, Aoki (2005 [1993]) explores the dualisms that are ripe within Western thought and language such as I/other, right/wrong, mind/body, presence/absence, theory/practice, "this or that" instead of "in a realm of both this and that". These binaries are "frameable in either/or opposition" and are divisive in nature unless we take up Aoki's invitation to embrace living in "a site of tension between this and that, a site of difference that speaks of two or more things at the same time" (Aoki, 2005 [1993]), p. 291). He invites me to both create and linger in a space between subject/object binaries, which he calls "the Zone of Between" (Aoki, 2005 [1986/1991]) or "the third space" (Aoki, 2005 [1996]; 2005 [2003]), which have the potential to become linked by a bridge that is not a bridge:

> any true bridge is more than a merely physical bridge. It is a clearing—a site—into which earth, sky, mortals and divinities are admitted. Indeed it is a dwelling place for humans who, in their longing together, belong together (Aoki, 2005 [1991], p. 438).

More specifically, Aoki (2005 [1986/1991], 2005 [2000]) talks about the relationship between the seeming dichotomous notions of "the curriculum-as-plan" and "the curriculum-as-lived", which he calls "twin moments of the same phenomena, curriculum". He describes the curriculum-as-plan as the mandated school curriculum guides and the curriculum-as-lived as what actually happens in the classroom, those mostly unplanned teachable moments. It "is the curriculum experienced by teachers and students as they move through school and school life" (Aoki, 2005 [2000], p. 232). Between the two there is a tensionality that "emerges, in part, from indwelling in a zone between two curriculum worlds" (Aoki, 2005 [1986/1991], p. 159), "both which require us to give them a hearing simultaneously" (p.162). This space of tension is fraught with ambiguity, and is at times oppressive, while at other times hopeful.

Aoki (2005 [1986/1991]) views this space as the "place of living pedagogy" and invites me not to overcome the tension by choosing to center one or the other as the object of my attention, but to “dwell alright within it” (p. 163). Aoki (2005 [1996]) calls me to consider lingering in the third space between East and West, “wherein the traditions of Western modernist epistemology can meet the Eastern traditions of wisdom” (p. 319). With these notions in mind, the importance of discovering “bridges” on which to dwell within sites of tension that arise from my reflection on my teaching practices to me become paramount.

Decentering the center.

Aoki (2005 [1978/ 1980]) asks me to consider “decentering” the “center” when I talk of these notions of curriculum; relationships between teacher, student, and object of study; theory and practice; multiculturalism; or multiple perspectives towards curriculum inquiry. Each center has certain perspectives to offer the other and it is the conversations that happen between them that both strengthens and decenters them when considered simultaneously. It is their dialectical relationship that becomes the new center of pedagogical thought and inquiry, whereby each is kept alive, distinct yet always considered in relation with the other.

The notion of dwelling between two curriculum worlds, that of the seemingly disconnected and isolated mathematics taught as part of the school curriculum and the interaction between teacher, student, and mathematics which make up the lived mathematics curriculum alluded to by Jardine and Aoki, radically changes my thinking about mathematics pedagogy. Living in this space of tension once caused me enormous stress as I raced to teach all the learning outcomes that the government required me to teach and assess each school year on those aforementioned achievement tests. In my classroom, mathematics became just one more outcome to cover, rather than a world to be lived and experienced. I now feel like I performed a great disservice to my students as I pause to consider the manner in which the curriculum-as-plan was the center of my mathematics program.

I was not so unlike the teacher of Jardine's grade 2 student. Neither of us thought to dwell alright within it, and attempt to decenter the center of our dichotomous mathematical worlds. Perhaps we did not know that we could pay closer attention to something other than the curriculum-as-plan. Or, maybe we did not know how to pay attention and respond to specific lived moments of tension in our lives as mathematics educators.

Learning to pay attention.

van Manen (1991) invites teachers to engage in what he calls pedagogical thoughtfulness, a "multifaceted and complex mindfulness towards children" (p. 8). He bases his notions of pedagogy upon the "intention to strengthen the child's contingent possibility of being and becoming" (pp. 16-17) and "protecting and teaching the young to live in this world and to take responsibility for themselves, for other, and for the continuance and welfare of the world" (p. 7). He reminds me of my "in loco parentis" relation and compassionate, loving responsibility towards my students and asks me to question what motivates my pedagogical interests in children, whether it be in anticipation of understanding what each experience is like for the child himself or herself or in anticipation of realizing my own agenda as an adult, my own fulfilling of dreams and goals for my students, possibly my own unfulfilled aspirations from childhood.

van Manen's notions of pedagogy requires something of us and he calls educators and parents to either act or not act, noticing "what is good for the child" in making that conscious choice. He calls me to notice and to question both my actions and non-actions through critical self-reflection based upon my students' best interests. With Jardine's vision of mathematics in mind, this means keeping the topic at hand alive with my students, a notion that considers both what is best for each individual child as well as what is best for the continuance of mathematics in the future worlds of my students. As a teacher I have the responsibility to act as a bridge between the two:

> In education, we assume responsibility for both, for the life and development of the child and for the continuance of the world. These two responsibilities do not by any means coincide; they may indeed come into conflict with each other. (Arendt, as cited by Jardine, 1992b, pp. 142-143)

Reflection and action, to van Manen, are two distinct kinds of pedagogical practices: "pedagogy first calls us to act, and then to reflect on our actions" (p. 27), which involve a process of deliberation and decision making that necessitates "a temporary stepping back or stepping out of the immediate engagement we have with the world" (p. 101). This distance may pose a conundrum for many a teacher, since teaching that considers the pedagogical good of the child does not allow one to detach oneself in the act of teaching.

van Manen (1991) encourages me to think about four possible different instances during which reflection may occur: reflection before action (anticipatory reflection), reflection-in-action (active reflection), reflection on action (recollective reflection), and "mindful action in pedagogical situations", van Manen describes mindful action to be a qualitatively different type of reflectivity from the other three that should not be confused with reflection in action, which entails those short "stop-and-think" kind of moments where I may have a chance to sort through my repertoire of teaching strategies to choose an appropriate course of action in response to my students. Mindful action can be thought of as a kind of "intuitive thoughtfulness" that usually acts and guides my actions (and non-actions) without my conscious awareness, allowing me to act without consciously knowing that I have acted (van Manen, 1991).

Consideration of what might guide my mindful action and cause me to act in the ways that I do is available to me through reflection on action, which I am particularly interested in as I prepare to leave this place of academic learning and return to the world of classroom teaching full time. It is reflection on action that has the potential to bring about a certain mindfulness to guide anticipatory reflection as well as actions within future pedagogical situations, which allows for a cycle of reflection and pedagogical growth. This cycle of reflection has the potential to provide a bridge between evolving notions of theory and practice, an essential component of my lifelong journey of discovery into how to occasion mathematical learning with children.

It is within these notions of interconnectedness and mindfulness that I set off to explore the mathematics literature in order to articulate and gain insight into underlying tensions that exist within my past practices as a mathematics educator.

Rethinking Notions of Mathematics Teaching and Learning

Throughout my graduate studies, I discovered that old habits do die hard and I still catch myself teaching by telling in the same disconnected and isolated manner in which my teachers taught me over twenty years ago. The purpose of this exploration is to (re)think the way I go about teaching and learning mathematics, insights that will inform my practice of teaching mathematics and perhaps may help other educators do the same. In my view, ongoing transformation of my practice is only possible through a thorough examination and reflection on my underlying assumptions of the nature of mathematical teaching and learning that I am unconsciously conserving. I designed my exploration as an autoethnographic study and will analyze critical teaching vignettes that point to spaces of tension within my past teaching practices in an effort to better articulate and understand my reasoning behind the decisions that I make as I participate in the day to day lived experiences of being a mathematics educator. Since my past interactions with school mathematics seem isolated and disjointed from the real world, it is of great interest to me to reflect upon notions of interconnectivity, as articulated by Jardine and Aoki. I am also curious to discover what this perspective might reveal about my notions of mathematical understanding that shaped my pedagogical decision-making over time.

Questions Under Consideration

My exploration evolved from the following questions that plagued me and demanded my attention: What themes and patterns arise by looking from a perspective of interconnectivity and through examining the sites of tension within particular past experiences amongst myself, mathematics, the BC mathematics curriculum, and students over the course of my life as both student and educator? What will my work in reflection on action within these sites of tension reveal about my underlying assumptions about how children come to learn mathematics and of my conceptions of mathematics itself? And, what insight do these findings provide for my future mathematics teaching?

Self-Reflection is Paramount

Over the past century, educators have had much to say about the need to improve the effectiveness of mathematics education in the U.S. and Canada (Schoenfeld, 2004). In response, the National Council of Teachers of Mathematics (NCTM) called for mathematical reform and created an enriched vision of curricula with grounding in the belief that “all students should learn important mathematical concepts and processes with understanding” (NCTM, 2000, p. ix), which is a shift from more traditional, rote approaches to mathematical learning. While our current BC government mandated curriculum is consistent with the NCTM’s vision, I do not believe that mathematical reform in a global sense is possible unless individual teachers critically examine assumptions about why we choose to teach the way that we do, which is the aim of my research.

Established beliefs are resistant to change (Long & Stuart, 2004) and without reflection, I am in danger of unconsciously conserving traditions from my past, such as promoting a more traditional, rote approach to mathematical learning rather than a more “relational” one (Skemp, 1976). And without critical examination, I may even consciously conserve these traditions out of the belief that my actions might be of pedagogical value. In this day and age as I face what seems to be an information overload, it is so much easier to teach how I was taught, by reverting back to what is familiar. However, what is familiar and more comfortable (for me) is not necessarily applicable to mathematical learning today, nor is it necessarily in keeping with what van Manen (1991) considers to be pedagogically good for our children. As an educator, I want to become aware of the reasons for why I choose to occasion (and not occasion) the learning experiences that I do in the classroom in order to meet the needs of my students.

Chapter 2: Choosing a Research Design

From the moment I began to contemplate the nature of my research questions, I knew that this exploration was to follow principles of a qualitative research design, since my aim is to explain and explore tensions within my teaching practices rather than compare or attribute measurement to a specific phenomenon. The literature points to five common characteristics of qualitative research design: (a) studies take place where the subjects of the study spend most of their time (i.e., naturalistic setting), (b) studies involve descriptive data rather than numbers, (c) studies are concerned with process as opposed to product, with the result being that research questions and design tend to be emergent in nature rather than dictated before one enters into qualitative research, (d) studies are inductive in nature since their questions do not require definitive answers and hypotheses to test, and (e) studies are concerned with making meaning from the research subject's perspective (Bogdan & Biklen, 2007; Creswell, 2007).

As I am not interested in the study of individual student learning, but of my own teaching practices over time, I chose to design my project as a self-study. Within the domain of qualitative research, I considered five different qualitative traditions according to John Creswell (2007), who describes them as (a) case study, (b) phenomenology, (c) grounded theory, (d) ethnography, and (e) biography. Each of these approaches differ in origin and history, emphasis on certain data collection and analysis procedures, narrative form, and most fundamentally in their foci; that is, the purpose behind what each tradition aims to accomplish.

Taking into account that there is much overlap between the five traditions and that each may demonstrate parts of the others to varying degrees, I was careful to keep the different foci in mind while I contemplated my choices. Considered irrelevant in meeting my research goals, I discarded grounded theory, whose purpose is the generation or discovery of a substantive theory, and phenomenology, whose focus is to understand the essence of meaning that an individual or group experience about a phenomenon, rather than the individual him/herself, soon into the process of method selection. I briefly considered case study research, whose focus is a specific case, in this instance, myself as a mathematics teacher, but then moved toward autobiography and autoethnography, since these approaches are recommended when one desires to study a single individual. (Ellis, 2004; Bogdan & Biklen, 2007; Creswell, 2007).

Autoethnography it is

While I made the decision to ground this exploration in the autoethnographic tradition, the choice became blurred along the way, as I pondered whether my work was emerging as more of an autobiography. Carolyn Ellis and Art Bochner (2000) provide insight into the nuances between the two genres and describe autoethnography as a process of doing a study, as well as the written product of research. Autoethnography is an autobiographical genre that connects the personal with the cultural and involves the interaction of three components: the self (auto), culture (ethos), and the research process (graphy) (Ellis & Bochner, 2000). Thus, the emphasis placed on each of these three components varies greatly in the wide variety of approaches to autoethnography that have evolved over the past two decades (Reed-Danahay, 1997; Ellis & Bochner, 2000). What is common to all of the approaches however, is that “autoethnography pursues the ultimate goal of cultural understanding underlying autobiographical experiences.” (Chang, 2007, p. 49). Without consideration of the larger social context within its parameters, autoethnographic work does not transcend the autobiographical (Chang, 2007).

Taking an autoethnographic perspective rather than an autobiographical one opened a space for me to situate sites of tension within my mathematics teaching and learning within the larger social context of school mathematics. The tensions that I experienced did not come from within me, but arose in my interactions with mathematics that contributed to the formation of my ideas and beliefs that guide my students and me to participate in this curriculum-as-lived called mathematics that I formed in my past teaching experiences. It was my intention to do more than merely describe my autobiographical experiences as an educator, but also to notice and reflect upon what my stories tell and do not tell about the nature of mathematics teaching and learning.

Verisimilitude Is the Goal

Another reason for choosing an autoethnographic approach is because of the capacities of stories to “inspire conversation from the point of view of the readers, who enter from the perspective of their own lives” (Ellis & Bochner, 2000, p. 748). The idea is to use my personal experiences with mathematics to generalize to a larger group or school culture, making no claim to an empirical truth, but to a narrative truth which, “seeks verisimilitude; it evokes in readers a feeling that the experience described is lifelike, believable, and possible” (Ellis & Bochner, 2000, p. 751). A well-written autoethnography invites readers into the text and to relate their personal experiences with that of the author, with the goal of bringing the reader and researcher into conversation. In this manner, autoethnographies cannot be read the same way by every person who engages with the text. When viewed from a more traditional view of validity, which seeks empirical truth, the worth of autoethnography as academic text comes into question (Ellis & Bochner, 2000; Ellis, 2004).

A criticism of autoethnography is that this genre of writing and research is an excuse for self/therapeutic storytelling and as such cannot be considered as academic text (Ellis & Bochner, 2000; Ellis, 2004). Those who take this position on autoethnography are asked by Ellis and Bochner (2000) to question, "what's so wrong and threatening about our stories?" which seek to connect the reader with the investigator directly and personally, rather than hiding behind a third person perspective. While some autoethnographies can be considered as sensationalist and voyeuristic, what Ellis and Bochner (2000) call a "reality television" brand of narrative, these authors ask critique to focus on the quality of the conversations inspired by each individual piece of writing, rather than the whole genre itself.

Collecting Personal Memories

As an autoethnographer, my goal was to document the moment-to-moment concrete details of poignant moments in my teaching life, paying attention to physical feelings, thoughts and emotions (Ellis & Bochner, 2000). My main source of memory collection was my filing cabinet. I kept every test and every worksheet that I created for my students over my past 14 years as an educator, even lesson plans that I made as a student teacher, along with planning notes. I also kept sheets of chart paper that documented student thinking, written verbatim, in their own words. I read every scrap of paper in my mathematics file drawer and took notes in a journal as I went along.

Each journal heading included the date and the title of the file I was studying. As I read each file, which documented individual math topics, such as multiplication and measurement, I first made a list of contents, which I entitled “content analysis”, an itemized list of each scrap of paper in the file. Next, I paused to write about teaching anecdotes that came to memory. Some files brought back memories and others did not.

When stories came to me, I used a “process of emotional recall”, whereby I imagined being back in the scene emotionally and physically” (Ellis & Bochner, 2000, p. 752) in order to stimulate my memory. My initial anecdotes and journal entries were very detailed as I wrote down everything that each of my senses allowed me to remember. For example, I closed my eyes and wrote down all that I saw, heard, and touched. Such a focus brought clarity to memories that I did not think possible after so many years gone by.

Making Sense of Personal Memories

After spending weeks combing through the autoethnographic literature looking for definitive advice on how to structure the analysis and interpretation of personal memories, I realized that the process was to be an emergent one (Ellis & Bochner 2000; Ellis, 2004; Chang, 2008). This is not to say that autoethnographic work is neither rigorous nor well thought out, but “gives you freedom to modify your plan as needed so that the most insightful understanding of complex human experiences can be gained with few presumptions and an open mind" (Chang, 2008, p. 67). Heewon Chang (2008) suggests that with an open mind also comes the willingness to let the autoethnographic process be recursive rather than linear and sequential as researchers move back and forth between data management, collection and analysis, each informing the other and dynamically interconnected.

As I collected my stories, I found Chang's (2008) advice to ring true as I abandoned my original plan for data analysis and interpretation and instead began a conversation with my journal entries. Each day that I wrote in my journal, I revisited what I wrote the days before and made note of underlying themes pinpointed by my stories and content analysis using a blue font, which I copied and pasted into a separate document. All the while, I traveled back and forth between my teaching files, my journal entries, and the conversation I had with myself in blue. In the process I used four data collection and analysis strategies suggested by Chang (2008): searching for recurring topics, themes, and patterns; identifying exceptional occurrences (in this case, reoccurring sites of tension); analyzing inclusion and omission; and framing with theories.

All the while, I followed Ellis (2004) and Chang (2008)'s advice to zoom my attention inwards on the details of my experiences as a mathematics educator and zoom out to the broader context in which my experiences are situated. Sometimes I lost sight of the larger context and slipped into thinking that my research might evolve into more of an autobiography than an autoethnography, but I realized what an injustice this would be to not continue to make a conscious effort to zoom out and thus connect themes from my stories to the larger context of mathematics education, and in which my experiences are situated. This movement does not follow a linear path from inside to outside, and instead it is recursive in nature: "Back and forth autoethnographers gaze... As they zoom backward and forward, inward, and outward, distinctions between the personal and the cultural become blurred, sometimes beyond distinct recognition" (Ellis, 2004, p. 38)

I worked with my journal entries and evolving conversation in these described manners and then selected five poignant teaching vignettes that opened a space for me to bring sites of tension within my experiences as a mathematics educator into conversation with the mathematics literature. And, upon response to the emerging dialogue, I continually zoomed into each of my teaching stories, taking the time to deeply reflect upon each lived moment. I next zoomed out to the mathematics literature in an effort to gain insight into each topic at hand as I zoomed back into my stories and continued my reflections and interpretations. This back and forth journey permitted me to consider themes that were woven through all my stories in Chapter 4, which I also took to the literature and then back to my reflections in Chapter 5. Thus, my literature review emerged from this back and forth process of self-reflection and is interspersed with my stories in the next three chapters.

Chapter 3: Zooming In and Out

A Story About Subtraction

Part 1: In the beginning.

Over my fourteen years of teaching I spent much time reviewing the traditional algorithms for the operations with my students. When I taught subtraction, for example, a typical introductory lesson involved me writing a question on the chalkboard as I talked through each of the steps of the traditional procedure for finding the answer. For example, I wrote a question like, 86-9 on the board, careful to line up the ones and tens in their vertical columns. Words like, “six take away nine, can you do it without going into negative numbers which you will learn about later in middle school? No? Knock on the friendly next door neighbour, 80, and rename one group of ten as ten ones. Now you have 16 ones, can you take away 9 now,” came out of my mouth as I explained this step-by-step procedure for subtracting numbers. As I said it out loud, I modeled what to do with the place value blocks and asked student volunteers to count out the blocks as we went along.

When students used the blocks and drew two-dimensional pictures to represent the process of subtraction in their mathematics duo-tangs, I called this the “long way” and the algorithm I wrote on the board was called the “short way”. And, after working through several examples in a like fashion, I asked individual students to model the algorithm as a think-aloud strategy in front of the class. Following much oral and written practice with the short way, we moved on to something else when most were able to find the right answers on a consistent basis.

Part II: A change in teaching.

My teaching of the number operations followed a similar structure until the emergence of the latest BC Mathematics IRP (2007), which does not mention any notion of regrouping and renaming as a means to demonstrate an understanding of addition and subtraction. Instead the document focuses on the development of students' "personal strategies" to demonstrate the processes of addition and subtraction, as well as "creating and solving problems that involve addition and subtraction" (BC Ministry of Education, 2007, p. 52). This change prompted me to try something different the last time I taught subtraction.

This time, I started our study by presenting a word problem situated within the context of a real-life situation, which went something like this: "Ms. Wiens joined a comic book club. The book club has 96 books to share. The first week, She borrowed 18 of them. How many are left?" The children then worked in partner groups to try and find an answer. Some were able to correctly calculate the answer using personal strategies, while others stared at their papers with pencil in hand, not sure of where to start, with words like "I can't do this" and "Miss Wiens how do I do this" heard uttered around the room. So, one by one, I asked the students to share and explain their strategies and how they worked and wrote their responses down on chart paper word for word:

When I subtract numbers:

•I count on a number line

•I count backwards from the starting number

•I count backwards using my fingers

•I count backwards to the nearest ten, stop, and think. (E.g., 56-9...56-6 is 50. Now I take away three more).

•I subtract the tens and then the ones (E.g., 96 - 18...96 -10 is 86. Now I count backwards 8 times)

•I use blocks

•I count by 2s

•I count tens

•I regroup the numbers and take them away

My prompting led to the emergence of more strategies, as the children talked with their desk partners and then together as a whole class, a process that led us into a routine for our math lessons for the remainder of the subtraction unit, which lasted for about a month. First, I selected a student strategy from the chart, whose owner was called the "Math Star of the Day". Next, I made up a worksheet with the math star's ideas, who then modeled their subtractive procedure, which was called "_______'s strategy". Each of my math stars modeled how to find the answers to several subtraction questions in front of the class and I encouraged them to share their thinking at each step of their invented mathematical procedures. As they talked aloud through each step, my stars stopped to accept questions and comments from their classmates to clarify understanding of the new strategy. Each of the worksheets I created emerged from my students' personal strategies and followed the same basic four part structure of (a) introduction of student strategy, (b) prompt to find the answer to three questions using the strategy presented, (c) prompt to answer three questions using any strategy of their choosing, and (d) verbal prompts from me as I circulated around the room and asked students to consider which strategy they liked best and found the most efficient.

One day the math star presented what this individual called “the regrouping strategy” to the class. I followed up the lesson and modeled the procedure with place value blocks, building upon one student’s strategy, “I use blocks”, to show what the steps of regrouping looked like when symbols were used. And this is where my subtraction story goes back to its beginning. I encouraged my students to use the blocks if they needed to, but in the back of my mind I saw them as just a stepping-stone for understanding the regrouping procedure, which I taught step by step. Even though the traditional regrouping algorithm was not part of the curriculum guide at the time, I felt the need to address it with my students due to its presence in the room.

Reflecting on my actions.

Mathematics involves far more than getting the right answer or the ability to follow the steps in an algorithm (the traditional or personal ones). I knew this on some level during the time that this vignette took place, yet I continued to teach the procedures in a teacher directed way, which my students modeled in a student directed way when they had their chances to teach their strategies to the class. While I gave my students the opportunity to demonstrate an understanding of subtraction using personal strategies, occasioned by my curriculum-as-plan, our government mandated curriculum guide, I did not, as Jardine recently helped me to see, go beyond a surface level understanding of the strategies or into the concept of subtraction itself. The student thinking that I captured on my chart paper portrays a notion of subtraction as “counting backwards” or “counting up", a limited and isolated view, that reflects participation in only one kind of subtraction problem, of the type, x-y=___ (Carpenter, Fennema, Franke, Levi, & Empson, 1999).

Missing from this subtraction story are opportunities for student ideas to take shape in interaction with others and to delve deeply into connected concepts within the domain of mathematics, both of which had potential to be prompted with an occasion for my students to participate in a variety of addition and subtraction problem types (see Carpenter et al., 1999) and through insightful questioning on my part. I was not aware of how imperative it is to ask students to justify and explain why their ideas worked and if their ideas would always work, in every type of subtractive situation. The personal strategy became something that each of my students owned, which they doled out to their classmates step by step, only to be memorized, replicated, and judged as worthwhile (i.e., efficient).

Yet, I felt proud of the way that I taught subtraction as well as the entire lived curriculum that I reported on and called mathematics. I thought that I was a great mathematics teacher, perhaps in a similar manner to which I once experienced pride with my mathematical prowess. Now, I question not only my abilities to teach mathematics effectively, but also how I could be so blind. Why did I continue to teach mathematics in such an isolated and disconnected manner, even when I began to consciously try and teach subtraction in a more flexible way with the goal of promoting an understanding of mathematics?

Zooming out towards notions of instrumental vs. relational knowledge.

Upon reading Richard Skemp's (1976) article about mathematical understanding, the incongruence of my behaviour finally began to make sense. Skemp (1976) identifies two distinct types of mathematical knowledge. Relational knowledge, which is "knowing what to do and why" implies mathematics be taught, not as separate, isolated topics, but "as fundamental concepts by which whole areas of mathematics can be inter-related" (Skemp, 1976, p. 93). Relational mathematics has the potential to be adaptable to new tasks and "memorable", since knowledge of how rules are inter-related enables one to remember procedures, rules, and concepts as part of a connected whole and accessed across a wide range of mathematical situations.

Instrumental knowledge, which Skemp (1976) refers to as "rules without reasons" (p. 93), implies mathematics be taught in a manner which entails the memorization of numerous rules for procedures, whereby students can find their way through a mathematical problem or procedure without necessarily understanding why or how the steps work. Rote teaching insinuates that the teacher take the role of "information transmitter", offering direct instruction to students who are seen as passive recipients, or "blank slates" (Baroody & Hume, 1991). While this type of mathematics can help some students get answers quickly and efficiently, it is not necessarily adaptable to new tasks or scenarios: "what has to be done next is determined purely by the local situation...the learner is dependent on outside guidance for learning each new way to 'get there'" (Skemp, 1976, p. 95). This notion of instrumental knowledge is what Jardine (1999) refers to as "memorizable but not memorable", understanding of which does not require understanding ourselves or others any differently for having participated in its formation. Instrumental mathematics proceeds identically in every case, independent of the knower.

Zooming back into my story.

During my early teaching years, even though I laid out the regrouping procedure with place value blocks, I did so instrumentally. I wanted my students to have a true and tried procedure that enabled them to always find the right answer if they followed the steps correctly, and many of my students memorized how to use the blocks in the same manner they memorized the symbolic notation that went with them. Furthermore, I did not explore the possibility that other strategies could exist, nor did I take enough time to develop an understanding of the subtractive action as it emerged from real-life scenarios of relevance to my students outside of the lifeless word problems we studied from the textbook.

When I changed the way that I taught subtraction, my intention was to help my students develop not only a procedure for finding an answer, but also to develop a repertoire of many strategies to find the answers to addition and subtraction problems that connected to a plethora of mathematical concepts my students already had access to. For example, when my students said, "I count backwards to the nearest ten, stop, and think", they deconstructed the number and rounded numbers in ways that suited each subtractive task at hand. Skemp's view of mathematical knowledge enables me to think of my altered perspective as an example of a shift in emphasis from instrumental knowledge to that of relational knowledge as a vital part of student learning.

But if I truly value relational knowledge, why did I encourage my students to share their strategies instrumentally? This was not my intention when I planned instruction. Skemp (1976) helped me to realize that I continually attempted to teach relationally in an instrumental way, an issue that is worthy of my attention and an example of one of three kinds of “mismatch” that can occur between relational and instrumental mathematics. One mismatch is if the goal is relational understanding and the teacher teaches instrumentally, which is what happened to me; the other way around; and lastly when there is a mismatch between the knowledge valued by the textbook/resource that the teacher is using and the teacher’s intentions.

Skemp’s (1976) words help me situate the tension that I experienced between instrumental and relational mathematics in my aforementioned story and help me understand why this mismatch may have occurred. My experience with mathematics in my own elementary schooling involved mostly instrumental mathematics, which is a notion that I did not have the words to express until Skemp shared them with me. Werner Liedtke (2004) suggests that as an educator, I am a product of my own experiences with mathematics. Teachers who miss out on relational understanding in their own schooling become consumers of mathematics and reflect the same practices in their own teaching.

According to Skemp (1976), in order to make a reasoned choice between the two types of knowledge, I must be:

> able to consider the alternative goals of relational and instrumental understanding on their merits and in relation to a particular situation. To make an informed decision of this kind implies an awareness of the distinction, and relational understanding of the mathematics itself. (p. 93)

But must I choose one or the other? Skemp (1976) says that relational thinkers do at times use instrumental thinking because the latter offers the opportunity to get the answer more quickly and reliably. I agree for example, that there is time and place for students to be able to recall basic subtraction facts accurately and efficiently, which may involve memorization. However, one should not confuse fluency with rote understanding. A completely instrumental kind of understanding, for example, about subtraction facts, will not permit a child to bring forth strategies to interact with related mathematical tasks at hand when necessary, which according to Skemp, is what relational thinkers do.

A change in thinking.

I challenge the privileged place of instrumental thinking in my own experiences with school mathematics. And, I wonder how middle school and high school mathematics can be taught effectively and the curricula covered without some kind of conversation between the two types of mathematics. It is between the teacher, student, and mathematical situation at hand to decide. The morals of this story, learning which I will take into future stories as it becomes part of the mindful action that van Manen speaks of, is to notice the type of mathematics I am participating in and why.

A Story about Fractions

One day, 30 copies of a new textbook called Math Makes Sense arrived at the door to my classroom, a gift that purported to be an excellent resource for preparing students to meet the learning outcomes in our curriculum guide. Already feeling confident with my mathematics instruction in the classroom and the worksheets that I developed with student input as a main source of mathematical practice, I was reluctant to pick up the textbook and try something that appeared new and complex. This resource was for teachers that did not know how to teach mathematics conceptually, not for me. My students scored well on their test and assignments.

However, feeling guilty every time I looked over at the shelf of expensive, unused textbooks, I decided that I should at least give the program a try. I was midway through a unit on fractions when I chose a lesson called, “Different names for fractions”. The lesson focused on exploration of equivalent fractions with Cuisenaire rods (see Figure 3-1). My preparation began and ended with a glance through the student textbook, which appeared to make sense to me at that time. I then took the 15 sets of Cuisenaire rods out of the manipulative bin that I shared with another teacher across the hall. Never having laid eyes on these coloured rods of different sizes before, I assumed that each of the pieces was a unit fraction (e.g., one-half, one-third, one-fourth, etc.) as I began to teach the lesson.

The task for my students was:

> Use the orange rod to represent 1 whole. Line up other rods beneath the orange rod to show fractions of one whole. Do this in as many ways as you can. Draw a picture to record your work. Label the orange rod 1 whole. Label each of the other rods with a fraction. Repeat the activity using the brown rod as 1 whole. (Morrow et al., 2004, p. 282)

My introduction to the task included a reading of the question to make sure everyone understood the vocabulary and what the task required of them, as well as instructions to stop with the orange rod, rather than moving on to the brown one. Next, I made sure each student had a partner, a set of rods, and a textbook before I sent them off to explore notions of equivalent fractions on their own with no further teacher direction from me.

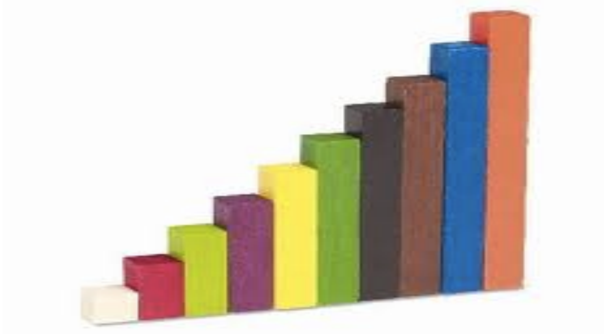

Figure 3-1. **Cuisenaire Rods**

Within moments, it was apparent that my students were completely confused and they soon began to panic. Cries of, “I do not get it,” and, “What am I supposed to do with the rods?” echoed all around me. As I circulated around the room and attempted to help, I too became frustrated. The yellow piece was clearly one-half, the red one-fifth, the tan one-tenth, but none of the other pieces “fit”. I thought that one-quarter, one-eighth, etc., were missing and that we needed much larger blocks than the orange one for the blue piece to be part of something. More than 30 minutes of agony passed before I ended the lesson with a mental note to find the pizza fraction pieces and go back to the way I normally taught equivalence, which only took two math lessons.

As I closed the lesson and pondered upon it later that night, I blamed my frustration not on my own actions and inactions within this situation, but on the textbook itself. I felt as though the textbook authors chose an inappropriate manipulative to model the notion of equivalence and that the activities took too much time, which was so precious in the race to get the curriculum covered each year. Student led lessons, followed by worksheets allowed me to meet this goal. Therefore, those Cuisenaire rods stayed in their box for the remainder of the school year.

Reflecting on my actions.

I consider this to be a terrible lesson due to the chaos that pervaded the room, the lack of progress made towards the development of an image of equivalent fractions, and the frustration felt by both me and my students when we tried to use manipulatives to model mathematical ideas. The lesson that these and other similar lessons held for me is that my thinking was stuck, not willing to move, and I did not know that movement was in itself a possibility. I saw the rods as existing in only one way and I attributed the problem to be the rods, as if they possessed the mathematical concept of equivalence on their own and which my students would somehow discover by following the step-by-step instructions of the textbook.

While I taught my students that fractions represent part/whole relationships, whether that whole is a collection of objects (e.g., the number of students in the classroom), or one object (e.g., a pizza), I did not acknowledge this to be an important concept when I brought out the Cuisenaire rods. Fractions do not represent a particular, static amount that can be considered independently from the notion they embody, a manner of thinking that I knew in theory at the time, but eluded me in practice, and thus allowed me to perceive each rod as only a unit fraction and only as such. However, if one keeps in mind the orange rod as one whole, then the other rods in the set beginning with the white rod become one-tenth, two-tenths, three-tenths, etc. And with this mindset, it is imperative to include that brown rod from the original textbook question in order to occasion students with an opportunity to compare how different rods can show fractions of one whole, depending on which piece is considered to be one whole.

I realize now that *the way* I thought about the rods, or any manipulative model for that matter, has everything to do with my experiences involving the mathematical concepts at hand and the connections with both my physical or mental conceptualizations.

Looking beyond Cuisenaire rods.

I realize that the big idea behind this story is about considering the different, flexible, and connected manners in which Cuisenaire rods and other manipulatives might be used in the classroom. From this story and the many others that appeared in my journal, I learned that I was too quick to move my students on to symbolic notation and procedures (i.e., abstract experiences) deemed the short way, and I thought of continued and prolonged use of manipulatives (i.e., concrete experiences) as cumbersome, time consuming, and a means to an end. I never paused to consider the relationship that exists between the two, the mathematical topic under scrutiny, and the potential that such exploration holds for rich, conceptual learning.

I now wonder if there is a certain point at which symbolic notation should be introduced to students and what relationship manipulatives play towards moving our students from concrete to abstract experiences with mathematics. According to the BC Mathematics IRP (2007), “meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of a variety of manipulatives can...enhance the formation of sound, transferable mathematics concepts” (p. 11). These words are reminiscent of the view of mathematics as an object that resides in the knower, which one can deposit and then transfer from one place to another in a unidirectional fashion. I think that there is much more to this notion of movement between "the concrete to the abstract" than our curriculum-as-plan lets on.

Zooming out towards notions of linear vs. recursive relationships between concrete and abstract mathematical experiences.

Susan Pirie and Thomas Kieren (1994) developed a model for growth in mathematical understanding, which provides a framework to help me situate the tension that arises in consideration of this particular tale, and others, as I seek to bring forth Jardine's vision of mathematics into the domain of my classroom experiences. Pirie and Kieren (1994) challenge the notion of linearity in the development of informal and formal mathematical understanding and view understanding as "a whole dynamic process", that is levelled yet nonlinear, a "transcendently recursive process" (p. 166). They propose a model for the growth in mathematical understanding, which encompasses eight levels that range from "primitive knowing" (i.e., background informal and formal understanding of the learner) to increasing levels of abstraction (i.e., formalisation) with the outermost level of "inventising", which refers to "breaking away from preconceptions that brought about previous understanding and creating new questions that might grow into a completely different concept." (Thom, in press, p. 129)

Presented as an image of eight nested circles (see Figure 3-2). , "each layer contains all previous layers and is embedded in all succeeding layers" (Pirie & Kieren, 1994, p. 172). As one moves outwards through the model, each subsequent layer signifies more structured, not necessarily more sophisticated forms of understandings Moreover, any understanding located in the outer layers has the potential to later become one's

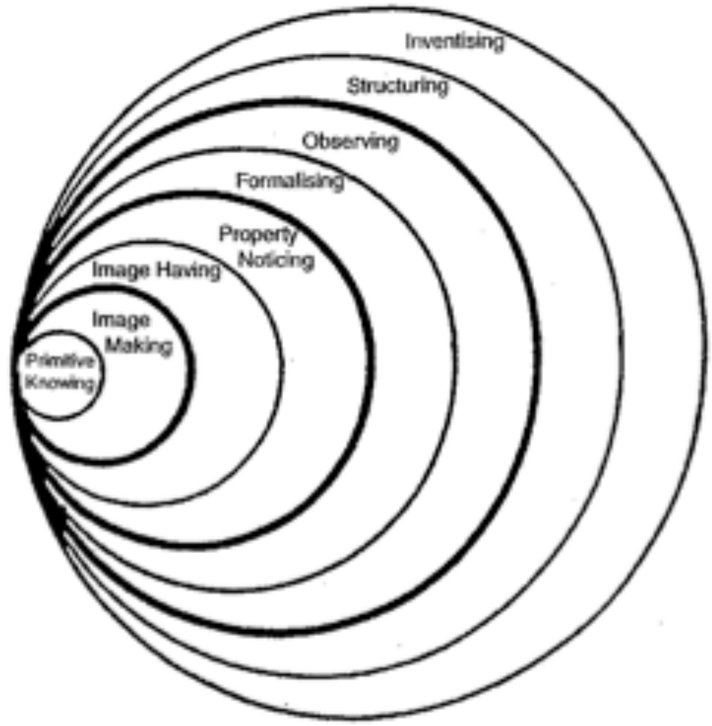

Figure 3-2. **Pirie and Kieren's (1994) model for the growth of mathematical understanding**

primitive knowing as mathematical ideas continue to grow within and beyond one another across one's life span.

When learners engage with mathematical tasks, they move back and forth, inwards and outwards from one layer to another, sometimes jumping ahead more than one layer at a time while in other instances, understanding will "fold back" onto itself. Folding back entails the movement towards more concrete layers in order to develop "thicker understanding" (Pirie & Kieren, 1994; Thom & Pirie, 2006). In this process, inner layer understandings develop "depth and breadth", creating progressively more generalised understandings that continually evolve each time particular mathematical concepts are visited (Pirie & Kieren, 1994; Thom & Pirie, 2006). This back and forth movement contrasts with curricula, including the BC Mathematics IRP (2007), that imply understanding as a progression from the simple (i.e., concrete) to the complex (i.e., abstract). The notion of folding back brings to mind Jardine's vision of mathematics, which entails "knowing something more in knowing the same thing" (Jardine, 2002b, p. 228).

For example, Pirie and Kieren (1994) talk about a student who folded back from outer to inner layers when posed with the question, "What are these things called fractions," while exploring the mathematical task of combining halves and thirds (i.e., adding fractions with different denominators). The student drew two pizzas, one half of it shaded, and the other two thirds of it shaded. She then reformed the halves and thirds into sixths and discovered that 3/6 and 4/6 is 7/6 or 1 and 1/6 and moved towards the generation of an algorithm for the addition of fractions with unlike fractions after practice with similar calculations. In the process, both the student's image of fractions and her formalized understandings were enriched through the experience.

The other key feature of the model is what Pirie and Kieren (1994) call "don't need boundaries", which are marked as three bold circles in Figure 3-2. Once beyond a don't need boundary, a learner operates without conscious reference to the understandings required to work within the inner layers. Thus, a learner devising a theorem for the addition of fractions does not require to mentally or physically work with specific images of a fraction. Instead, images that they are already had (i.e., image having) as already present and can be brought forth when necessary through the process of folding back and collecting. In other words, "what was once formulated (e.g., manipulating fraction pieces) comes to be taken for granted (unformulated), but still remains available for interrogation" (Davis, 1996, p. 204).

In this light, mathematical understanding becomes more than a notion of making connections between mathematical concepts (Skemp, 1976); mathematical understanding focuses on the generativity alluded to by Jardine (1992), where, there is always something more left to say. Thus, each time one revisits a notion such as fractional relationships, old ideas grow with new ones. In the process, both are transformed and mathematics becomes renewed, living on yet always growing with my students. Now, a space opens for me to see mathematics beyond the notion of moving unidirectionally between concrete to abstract mathematical experiences.

Zooming back into my story.

Of particular interest to me as I come back to my fractional tale are the layers of image making and image having, between which is one of the don't need boundaries. Image making involves performing "actions (mental or physical) in order to create some new idea of a topic" (Thom & Pirie, 2006, p. 189). The task of showing fractions of one whole with the Cuisenaire rods had the potential for my students and myself to make meaning of fractional relationships through participation in the activity, rather than replicating a known idea with the rods. My image making is exemplified by my recent conscious awareness that each of the rods can be considered as a different fraction in relation to which rod is considered as one whole.

My non-actions during this time demonstrated that I was not yet working within image having, "a mathematical knowing that has already formed" (Thom & Pirie, 2006, p. 190), regardless of whether or not it is correct. If I perceived my thinking as erroneous during the lesson, the opportunity for me to revisit my image of fractions with the linear model of Cuisenaire rods may have presented itself. Instead, my primitive knowing did not evolve beyond a formal, instrumental understanding of fractional notation, manipulation of which I am proficient. As a result, I attributed the limitations to be that of the rods and the textbook question.

The back and forth movement between informal and formalised mathematical understanding makes irrelevant that “means to an end” notion of concrete experiences alluded to by our curriculum guide (Davis, 1996), opening up a space for me to see how these types of experiences with mathematics are really two side of the same coin, inextricably connected throughout one’s life span. In the past, I was too quick to put away the manipulatives, but now realise how important it is to keep them available throughout the school year.

A change in thinking.

With this framework in mind, I feel as though I traded one source of tension for another. What is most troublesome for me to think about is what happens to those students, such as myself, who have a rule or theorem just given to them (i.e., instrumental knowledge) without experiencing this world of suffering that Jardine speaks of. I want my students to engage in image making and image doing that they can draw from and bring forth into outer levels of understanding. Becoming part of my mindful action as I go forth and begin to consider how I might best go about occasioning interaction with manipulatives with my next classes of students, I question whether the mathematical learning activities in many of my files will be applicable to my future teaching experiences.

A Story in Error about Errors

It is hard for me to let go of the value and power I held for correcting student work during mathematics class. I devoted the first part of every math class to the correction of student work from the previous day's lesson. My students usually sat in groups of four or with desk partners and I always instructed them to either pass their books back, ahead, or to their partner. Sometimes I let them keep their own work to mark, which was a rarity due to the chance happening that my students might copy answers instead of correcting the work on their own. Students who did not finish stepped out into the hallway with their math textbooks or duo-tangs, while the rest of us marked the work together I did not want anyone to copy the right answers!

I wrote the answers to the questions on overhead transparencies ahead of time and put a check beside each one as my students read the answers aloud. When we completed the marking, I invited the banished students from the hallway to return and finish their work as I gave all students a few minutes of negotiation time to discuss any disputes about the marking of their work and to ask questions aloud in front of the class. Next, students moved to a space at the front of the classroom that was of large enough area to seat us all comfortably, the place where I gathered my students to teach our whole group math lessons. Following our lesson time, which I scheduled to occur every day between recess and lunch, my students returned to their desks to complete the day's mathematics assignment.

Before beginning new work, I expected my students to work on their corrections and ask questions of either their classmates or myself if need be. There was a rainbow table at the back of the classroom with four student chairs that I filled with children who required my assistance. Sometimes I sat at the table, but usually I spent the entire math lesson circulating around the room with a red pen and a pair of scissors in hand. During this time, I corrected as many student notebooks as I could and clipped each page that was error free at the top corner. Some students corrected the same work several times before they finally got it right and could move on. I assigned corrections as homework every single night and this became an outcome that I evaluated during report card time. Students with a perfect notebook got a four grade, the top mark on the rating scale I used for every subject area.

The marking and corrections were incredibly time consuming for my students, the educational assistants who worked in the classroom, and me. We spent countless hours clipping pages and making note of the ones who needed extra help. Everyone got tired of correcting the mathematics. The students who had no trouble keeping up with corrections were the ones who got all the right answers in the first place. And, these students with perfect four grades on the correction scale always had the opportunity to use the math centers in my classroom, participate in problem solving activities, or roam around the room with their perfect notebooks that they used to help the others find the right answers as well.

Reflecting on my actions.

When I look back and consider how I let the correction process drive each and every mathematics lesson, it shocks me that I placed so much emphasis on the notion of getting the right answer. The mathematics was right or wrong, an all or nothing phenomenon, and many of the questions that I posed to my students occasioned only these two possibilities. My teaching files reveal worksheets on each strand of the mathematics curriculum guide, some with up to sixty fill in the blank questions, too much mindless practice for anyone, never mind young children!

We circled each mistake and placed a red "x" next to each one, which is why I required all my students to own two red pens as part of their school supply list. If many students had difficulty with a certain kind of question, I planned a lesson to rectify the situation and paced instruction to what I thought at the time met the needs of my students. I tallied the marks, recorded them, gave end of unit tests, and communicated student progress to parents at report card time. This linear process became part of my plan for both formative and summative evaluation.

I gave my students just a few minutes to negotiate whether their partners made a mistake in the marking of mistakes and no structured time to ask questions or clarify insights into their thinking. Instead, I made them go back to the mistakes, again and again, as I subconsciously reinforced the erroneous thinking that I tried to fix!

Zooming out towards notions of fixing mistakes vs. justifying and explaining mathematical thinking.

Brent Davis (1996) invites me to reconsider what he calls a "negative perception of errors," which views errors as indicators of incompetence to be avoided at all cost. Such an attitude towards evaluation focuses only on what one does not know according to the teacher, sending the message to students that evaluation is "something that is done *to* them *by* someone else," an "imposition of authority, a naming, an objectification of one's fluid self" (Davis, 1996, pp. 247-248). Each circled error in my student's mathematics duo-tangs signifies such a perception, with the underlying assumption that "errors can be located, isolated from other understandings, and removed" (Davis, 1996, p. 248), not so unlike a cancer eradicated by chemotherapy.

As an alternative, Davis (1996) suggests thinking about student conceptions and misconceptions as worthy of exploration, rather than evaluation. He thinks that errors should be a focal point for mathematical inquiry rather than something to be avoided, perhaps a portal into Jardine's world of living mathematical relations. Take for example, a student who has an opportunity to justify and explain his/her thinking about an answer of "12 over one" to the question, "What is the reciprocal of 12?" With insightful questioning from a teacher, this student might have the opportunity to fold back to inner layers of mathematical understanding, perhaps calling into question existing images of a fraction as a part of a whole, bringing it forth for consideration, as well as a host of other possibilities.

I have much to learn from the exploration of student thinking, which should be traversed via oral and written communication. Davis (1996) draws attention to the fact that in many mathematics classes, tests and assignments are the only direct communication between teacher and student, which at best offers a "fixed indication of what was known at some point in history" (p. 244). Liedtke (1998), in a different manner, advocates for mathematical intervention based upon diagnostic interviews as opposed to pencil and paper tests and assignments. Interviews with individual students, as well as classroom conversations that emerge from consideration of mathematical inquiries of relevancy to the immediate situation at hand, have the potential to provide information about one's mathematical reasoning, as well as important clues about mathematical understanding and general attitude towards mathematics.

Through conversation, immediate assistance and clarification are possible if students have the opportunity to justify, explain, and re-consider thinking that may be erroneous. To Liedtke (1998), two of the most insightful questions to ask a child are, “How do you know?” and “Why?” At first glance, such an exploration into student thinking may be considered interrogative in nature, which is not the case if questions are posed in a positive, safe, learning atmosphere where students come to understand that their thinking is valued by their classmates and teacher, and that mistakes, if made, are sites of learning. It is insightful questioning and opportunities for conversation that “allows the teacher to view, or the student to confirm, existing understandings” (Pirie & Kieren, 1994, p. 188), an intervention strategy called “validating”. Without such opportunities, the growth of mathematical understanding may be inhibited.

Zooming back into my story.

Pirie and Kieren (1994), Davis (1996), and Liedtke (1998) help me re-assess the emphasis that I put on the correction of mistakes. There is a place for marking and fixing mistakes in the life of the mathematics classroom, but it should not be the focus, nor should mistakes be considered as something so horrible. Time in the mathematics classroom is better spent talking through erroneous thinking with students. A focus on obtaining the right answers has the potential to shut down inquiry and stifle student learning (Jardine, 2002b).

It concerns me greatly that my mathematics lessons followed such onerous, lifeless assessment routines that have little possibility to occasion the kind of mathematical understanding that I am beginning to envision. My assessment practices valued mostly instrumental knowledge and provided routines that made life in the classroom easy for me to manage and report on, but did little to further student learning or confidence with mathematics. I wonder who really benefits from such narrow-minded judgement of a student's work and I feel the desire to reflect back on van Manen's (1991) advice to act in a manner that is mindful of what is pedagogically good for children.

A change in thinking.

Davis (1996) invites me to continually question whether or not my reasons for assessment of student work are closely aligned with the way assessment actually happens in my classroom. A teacher, such as myself, who wants to teach and assess relationally and values a negative perception of errors, reveals a mismatch between intention (i.e., theory) and action (i.e., practice). These words of advice are reminiscent of Skemp (1976) who cautioned me earlier to avoid a mismatch between the desire to teach relationally and the result of instrumental teaching. Thus, the lesson that I take with me into my future teaching is not only the importance of occasioning time for students to talk and justify their mathematical actions, but also the aspiration to consciously consider whether my actions match my intent during my lived experiences as a mathematics educator in all areas, of which assessment is an integral part.

All Roads Lead to School

There is a story that comes to mind when I pause to think of the way that I taught problem solving skills to my students over the years. At some point early in the school year, I always told what I call "the mapping story" as a means to explain how there are many ways to get the right answer. I found that my analogy helped my students, their parents, and fellow educators to deeply understand what I meant by this seemingly simple idea.

The story goes like this: "One of the most important things that you can learn about mathematics is that there is no such thing as one right way to get an answer, but a multitude of ways. And every year, there is always at least one student who teaches me a new way or a method for finding an answer that I have not thought of yet. What is most important to me when I look at your work is that you have a strategy, a tool, or a way of getting to the right answer. Your thinking is more important than the answer itself. This reminds me of how I choose to get to work each morning (a road map is shown on the overhead projector). I start here (circle apartment on map) and end here (circle school). When I am in a hurry and need to be really efficient, I go the shortest possible route. But sometimes, I like to take my time and this way is so much more scenic (point to longer ways on the map). Sometimes, there is construction, and I cannot make my way through, so I change my path part way. I do need to get to school, but there are infinite ways of getting here. We will learn from each other and you will always have the opportunity to choose which route works the best for you."

After I told the story, I provided my students with a variety of open-ended problems, called "math mysteries", in which multiple methods were possible. One of my favourite questions was, "Only 20 animals live in Lisa's barn. All in all, the animals have a total of 62 legs. The only animals that live there are cows and geese. How many cows and how many geese live in Lisa's barn." As students worked through these problems, we discussed their personal strategies in small and large groups and we created a master list of them on chart paper (e.g., draw a picture, act it out, make a table, look for a pattern, talk it out with a partner, etc.) in the same manner as with the addition and subtraction strategies discussed earlier.

The list stayed up for the year, along with any other chart that might be helpful to my students during testing time. If I felt that there was a strategy my students required more practice with, I presented them with a problem that had the potential to elicit the problematic strategy. Each word problem that I chose or occasioned my students to write became sites to practice the use of the strategies. Whenever my students got stuck on their tests, I pointed to that chart and asked them to find a starting place for their work. I permitted them to choose any strategy except their favourite, “talk it out”, because talking was not allowed on the tests.

Reflecting on my actions.

During this time, I felt proud of the way that I taught mathematics through problem solving and I never thought to question the type of mathematics that I valued and inadvertently modeled with my students as I followed this same procedure year after year. It worked for us. My students persevered through difficult tasks and most scored to the best of their ability on their achievement tests.

But, the problems that I presented for my students are just sites to practice their strategies, and not that much different than that problem involving Eric and his family that I spoke of earlier. When I felt that my students needed practice on a certain strategy, I provided questions that required them to use that strategy, as if the strategy was something they owned, a tool they wielded to meet their mathematical needs. Our queries began with the question that I posed to my students, ended with their answers, and then they put away their tools for another occasion.

My actions are both supported and refuted by the BC Mathematics IRP (2007) that offers conflicting messages. On the one hand, it advocates for students to "use mathematics confidently to solve problems" and "be able to use mathematics to make and justify decisions about the world around us" (BC Ministry of Education, 2007, p.14). A true problem solving based activity is defined as one that "asks students to get from what is known to what is sought", implying that "if students have been given ways to solve a problem, it is not a problem, but practice" (BC Ministry of Education, 2007, p. 19). On the other hand, the guide also advocates that students develop "a repertoire of strategies from which to draw upon when mathematical problems are presented to them" (BC Ministry of Education, 2007, p. 33), citing a list of suggested strategies from the 1995 curriculum guide.

I wonder how these strategies can be considered personal strategies when they are listed in the curriculum guide, seconded in our student-recommended textbooks, and displayed on many commercially produced problem solving strategy posters. And, considering Jardine's notion of mathematics as a claim made upon us that pulls us into its question that is posed to us and not by us (Jardine et al., 2003), the question of how the curriculum guide's carefully researched notion of problem solving might somehow fit with notions of problem posing takes root.

Zooming out towards notions of problem solving vs. problem posing.

The mathematics literature (Brown & Walter, 1990; Silver, 1994) challenges the prevalence of problem solving over problem posing in both curriculum guides and actual classroom practices. Problem posing, which refers to the generation of new mathematical problems and the reformulation of old problems, is a role that is usually taken over by teachers and textbook authors, leaving students with the role of problem solvers, potentially stripping them of opportunities to develop, ask, and explore questions that are relevant to their mathematical experiences. In the words of Aoki (2005 [1993]), problem solving has become the center, but it need not be.

Stephen Brown and Marion Walter (1990), who write extensively about problem posing and its relationships to problem solving in the mathematics classroom setting, invite me to consider problem posing with a new perspective, one that includes but goes beyond the activity of asking students to write and test out their own word problems in relation to a mathematical topic under study. Brown and Walter (1990) offer two techniques or what they call, phases of problem posing termed, “accepting the given” and the “what-if-not technique”. The authors suggest that problem posing begin with a simple “object, phenomena, or situation” that they call a “starting point” followed by the prompt, “What are some questions?” The second phase involves the creation of a list of “attributes of the given”. One or more attributes are selected and changed, followed by consideration of both new questions that can be asked and old questions that can be revisited with the added perspective of what the mathematical situation “is not” can offer. As students play with attributes and consider new and old questions, a space is opened to bring forth unanticipated mathematical conversations that may not come up otherwise.

While Brown and Walter (1990) suggest that problem generation is an important pursuit in its own right, as evidenced through the problem posing techniques they developed, they do not negate the inextricable relationship between problem posing and problem solving. Problem solving occurs in the aforementioned phases of problem posing, and in a like manner, problem posing occurs within the process of solving problems posed by teachers and textbooks as well. The latter phenomenon is what Jardine's Grade 2 student experienced, and it is paramount that teachers understand the value of occasioning opportunities for both problem solving and problem posing in the mathematics classroom. It is not an all or nothing phenomenon!

Zooming back into my story.

Going back to my problem solving story, I realize that what is at issue here is not so much whether or not these math mysteries were sites to practice curriculum directed problem solving strategies, but that these lessons were my students primary experiences with math problems. It is what did not happen in my classroom that is of importance to me and ultimately to them. My students had few, if any opportunities to generate and pursue their own inquiries into mathematics.

According to Brown and Walter (1990), "Once we know how to do something in one way, we tend to stop thinking about it further" (p. 83). At the beginning of each school year, the math mysteries that I chose for my students had some potential to occasion generation of new questions while they interacted with these complex problems. However, I never paid attention to student questions as possible sites of inquiry that had the potential to go beyond the original problem. All I heard was whether or not my students were proficient with strategies for finding answers to complex problems, and this is where my thinking stopped!

A change in thinking.

With a new orientation to problem-posing, which decenters the prevalence of problem solving, I am aware of the potential of the math mysteries and strategies from my filing cabinet to become starting points rather than end points of inquiry. Such a shift in my thinking opens a space for me to consider a notion of mathematics that is based on playfulness, inquiry, and convergent thinking as opposed to one based upon divergent thinking, correct answers, and teaching to the test. This shift takes me beyond my notions of school mathematics and towards an evolving notion of what real mathematics may be or perhaps more accurately, what it might mean to be mathematical.

This problem posing story takes me back to the beginning and causes me to re-consider how the “what-if-not strategy” can inform the development of student strategies for subtraction. I feel the inkling to return to all of my stories, but with a different perspective than I had before, this time informed by the collected insight gleaned from consideration of all my stories as one entity.

Chapter 4: Exploring Common Themes

As I continue to mull over each of my stories, I feel called to speak further to my notions about living in two mathematical worlds and to carry on the recursive cycle of reflection that I embarked upon. The conversations between the mathematics literature and sites of tension that arise from my stories transforms my thinking about mathematics learning and teaching in ways that I never thought possible and as such has the potential to deeply impact my future actions as a mathematics educator as mindful action that informs my decision making at each stage of the reflection cycle. However, I have as yet to address the larger tension that I experience from teaching in a manner consistent with my evolving conceptions of mathematics in a school context that puts great emphasis on breaking down mathematics into fragmented, isolated chunks that can be measured, a phenomenon that Jardine calls, “basics as breakdown” (Jardine et al., 2002).

In some ways, my sense of tension is abated as I consider three common threads amongst the fabric of my stories: (a) the desire to enhance my understanding of mathematics in order to make mathematics come alive with my students; (b) the importance of occasioning time for students to interact with one another in order to explain, justify, and develop their mathematical thinking; and (c) the emphasis that I place on getting the right answers and teaching to the test.

The Mathematics that I Want to Know

As I pause to contemplate what it means for my students to inhabit Jardine’s world of mathematics now, and throughout the entire process of reading and reflecting upon my teaching stories, I cannot help but wonder whether or not I have the kind of rich, relational knowledge that I require in order to occasion these kinds of mathematical experiences with my students. I remember most of the basic mathematical procedures that I learned throughout my schooling, procedures that come back to me with ease and precision, but which I do not explain and talk about with my students in a manner that pleases me. For example, when my students generated their list of subtraction strategies, I was not familiar with the expanse of possibilities that exist in response to that original prompt. When I taught about the notion of equivalent fractions, I did not understand how to interpret this mathematical idea with Cuisenaire rods, and had only a circular part/whole representation of fractions upon which to fold back.

And when I consider the notions of opening up errors as sites of inquiry and occasioning opportunities for my students to pose their own mathematical problems, I worry that I may not know the mathematics required to facilitate student learning beyond the static notions of mathematics that I experienced from my elementary schooling. To Jardine (2002a), mathematics teachers must have a solid understanding of mathematics themselves in order to both notice and occasion the types of learning experiences that go beyond a surface level, and I wonder just what this means and whether my mathematical skills fit the bill.

Situating my mathematical knowledge for teaching.

Deborah Ball and her colleagues (Ball & Bass, 2003; Ball, Thames, & Phelps, 2008) are major influences in the development of a research base for understanding teacher’s knowledge of mathematics (Davis & Simmt, 2006). Ball et al. (2003, 2008) developed a practice based, empirical approach to understanding the content knowledge that is needed for teaching mathematics that they call “mathematical knowledge for teaching”. Their theory is based on the premise that the mathematics required for effective teaching is qualitatively different from mathematical knowledge needed by others outside of the profession. Not only do teachers need to know their subject matter beyond that which is taught to students, they must also have a form of “decompressed knowledge”, a knowledge of how to unpack abstract mathematical concepts in order to “make features of particular content visible to and learnable by students” (Ball et al., 2008, p. 400). This striving “to pull concepts apart, make sense of analogies, metaphors, images, and logical constructs” is opposite from mathematical work outside of teaching, whose participants strive to compress mathematical concepts (Davis & Simmt, 2006, p. 300-301).

Ball et al. (2008) identified four distinct, yet overlapping domains of mathematical knowledge for teaching, consideration of which help me situate my current understanding of mathematics. Common content knowledge (CCK) entails the kind of mathematics that is common to other contexts outside of teaching, involving knowing mathematical content and performing procedures, such as the regrouping algorithm for subtraction. As evidenced by my experiences with Eric and his family, this is a type of mathematical knowledge that I feel confident with, although for me it is of a somewhat instrumental nature. Ball et al. (2008) help me to realize that this is not something to be embarrassed about after all, since my common content knowledge (CCK) does encompass part of the mathematics that I should experience fluently in order to be an effective teacher of mathematics. It offers a necessary, albeit a partial picture of how I want to experience mathematics.

A second domain, called specialized content knowledge (SCK) is the type of knowledge and expertise that is unique to teaching mathematics. It involves, for example, selecting representations (e.g., manipulatives, pictures, written symbols, oral language, real-world situations) that reveal key mathematical ideas; making mathematical sense of student errors; posing and selecting mathematical questions, problems, and examples that have the potential to bring forth a given mathematical concept; etc., which are actions that I realize call for more care and attention on my part. According to Brent Davis and Elaine Simmt (2006), who also extensively researched the mathematical knowledge required of teachers, “a key competence of mathematics teachers is the ability to move among underlying images and metaphors - that is to translate notions from one symbolic system to another” (p. 303). This is not an action that is fluent for me, as evidenced by my experiences with the Cuisenaire rods.

Difficult to distinguish from specialized content knowledge (SCK) are the domains of knowledge that blend knowledge of mathematical content and teaching (KCT) and knowledge of content and students (KCS). KCT involves such tasks as sequencing instruction and assessing the instructional advantages and disadvantages of given representations and questions posed to students. KCS involves an “interaction between specific mathematical understanding and familiarity with students and their mathematical thinking” (Ball et al., 2008, p. 401). An example of this kind of insight is anticipation of the types of responses and errors that my students may come up with for the subtraction question that I posed to them about Ms. Wiens and the comic books. It also involves hearing and interpreting my students’ incomplete thinking in the language of “I use blocks” as a strategy for subtraction and asking for further clarification.

Also included in these notions of knowledge of content and students (KCS) is understanding how students move between concrete and abstract experiences with mathematics, which is where familiarity with theories of how students come to know mathematics, such as Pirie and Kieren's (1994) model, fits into this picture of the mathematics that teachers need to know. The mathematical reasoning necessary in order to map learning according to the eight realms of mathematical understanding is where specialized content knowledge (SCK) comes into play, an instance of the overlap that Ball et al. (2008) talk about. For example, in response to a prompt to find a hidden fractional amount that is more than one fourth and less than three fourths, a teacher needs to know that a given set of responses elicited from a student illustrates a good image of fractional quantities (Kieren, 1995).

Figuring out what lies beyond the horizon.

Running across specialized content knowledge (SCK) and common content knowledge (CCK) is what Ball et al. (2008) call "horizon knowledge", which brings to mind Skemp's (1976) notion of relational mathematics. Defined as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p. 403), the notion of horizon knowledge points towards the importance of the ability to place the mathematical topic in the here-and-now, while simultaneously thinking about what may be required of students in their future mathematics experiences in grade school.

However, these aspects of horizon knowledge as addressed by Ball et al. (2008), do not adequately speak to Jardine's vision of bringing forth mathematics in response to interactions with the world around us, or its history, and as such provides an incomplete picture of mathematical knowledge for teaching that I want to know. There is no mention of a teacher's kinship with the generativity with mathematics, or of my "special responsibility to help the students be in touch with the larger world of mathematics and to help students see themselves as part of that larger community as well" (Kieren, 1995, p. 14).

What is also missing from these notions of the mathematical knowledge for teaching is consideration of the skill and know-how involved with managing the complex social dynamics within a classroom of students of which teachers need to be aware. According to Davis and Simmt (2006), "It [mathematics] always occurs in a context that involves others -- and hence, an awareness of how others might be engaged in productive collectivity is an important aspect [of the mathematics that teachers need to know]" (p. 309). Students learn from and with each other as they interact within small and large group settings, whose collective ideas are "greater than the sum of their parts". In other words, the diverse mathematical ideas that students create and ponder together are more robust than each individual idea considered in isolation. Teachers should know how to skillfully create a classroom atmosphere where ideas can "bump together" and "play off one another" without taking a directive role (Davis & Simmt, 2006).

A change in thinking.

The actions and the mathematical reasoning behind these notions of mathematical knowledge for teaching are not necessarily tasks that I do not know how to do, but more accurately, elements of my professional responsibilities that I did not recognize as neglected until these moments of reflection on action. Therefore, the issue at hand is not whether I have the mathematical understanding necessary to be an effective mathematics teacher, but whether I am willing to let my understanding continue to emerge and evolve with that of my students through teaching, self-reflection, teacher workshops, conversations with colleagues, and further reading in the mathematics literature.

In other words, my focus shifted away from considering what mathematical understanding I do not possess and towards noticing that there are many experiences with mathematical thinking that I have yet to encounter. The key word here is "yet" and my mind is open and actively seeking out opportunities to occasion rich, generous inquiry within my daily lived experiences in order to understand the mathematics in the manner I want to know in order to become a more effective mathematics teacher. It is as Jardine et al. (2006a) assure me, "knowledge will come as teachers become more experienced with the ways of this heretofore unexplored territory" (p. 98).

Talking in the Mathematics Classroom

Absent from each of my stories is an emphasis placed on the value of student interactions within the classroom setting. Unless my students played math games, worked on open-ended problem solving tasks, or participated in exploratory activities (e.g., the lesson with the Cuisenaire rods), my classroom was silent for the most part. At least half of my mathematics lessons involved worksheets and required my students to sit at their desks and work silently. I always gave my students the first five to ten minutes of what I called "seat work time" to work together with a partner or small group. When time was up, I permitted my students to participate in what I called "on task talking", but only if absolutely necessary. If individual students did not need to talk to clarify understanding or comment on the mathematics in question, they were to be silent. And, during mathematics time, you could probably have heard a pin drop on most days in my classroom.

My students talked most in our classroom as they participated in opportunities to share ideas during our lesson time. Individual student's modeled their thoughts aloud as each took turns and worked through mathematics questions on the white board. However, I did not give my students much opportunity to justify and explain their thinking, nor did I afford them with the chance to engage each other in purposeful mathematical talk that had the potential to enhance both individual and collective mathematical understanding. I wonder just how I might go about creating a classroom atmosphere whereby Davis and Simmt (2006) suggest that ideas "bump together" and "play off one another" without taking a directive role.

Keeping the purpose in mind.

David Pimm (1987) addresses mathematical talk in the classroom and describes qualities of mathematical talk that make communication meaningful to students and teachers. He calls teachers to consider the purpose for asking questions and for participating in mathematical activities as part of the lesson planning process (i.e., reflection before action). Pimm, like Skemp (1976) and Davis (1996) reminds me of the importance of carefully reflecting upon whether my actions match my intentions. For example, if my purpose for engaging students in mathematical talk is to lead student responses toward a predetermined outcome, and to stay in control of the discourse, then perhaps a style of questioning that Kieren (1995) calls TIRE may be appropriate (for the teacher that is). In this procedure the teacher ***tells*** the students about mathematics; the teacher ***i**nterrogates* or questions students about the mathematics; the student ***r**esponds* to the question; and the teacher ***e**valuates* the student's responses.

According to Pimm (1987), this type of questioning style is an all too common occurrence in the mathematics classroom, whose consequences whether it be intended or unintended, are to ignore or over-rule student responses that do not follow pre-determined criteria (i.e., the curriculum-as-plan) and to pressure students into delivering only those responses that the teacher wants to hear. For example, in my student-led lesson format, my purpose was to get students talking about their mathematical thinking, but my format and manner of interacting with my students led them to parrot back my thinking to the group. This was not my intention.

Now that I am aware that my purpose for engaging in mathematical talk with my students is to occasion opportunities for us to develop, explain and justify our mathematical thinking, rather than focusing on getting the right answer like I did in the past, I realize that my former lesson format and activities are no longer applicable. For example, a problem posing model is much more suitable to this goal than that question about Ms. Wiens and the comic book that I used as a starting point for my subtraction unit. Beginning with a comment such as, "the answer is 78, what are the questions?", sets a much different tone for inquiry, which opens a much wider range of mathematical possibilities to talk about and play with than the original prompt.

My role throughout such an inquiry is to facilitate talk that "deflects responsibility for answering back to the whole group, rather than the teacher" (Pimm, 1987, p. 58). Examples include, echoing back a child's response to the group, asking questions that prompt explanatory reasoning, and drawing attention to a chart, diagram, or some other such feature of a lesson that may have the potential to draw attention away from myself from the centre stage of the lesson. In doing so, I must give up my predetermined conceptions of where the conversation should go, genuinely respond to my students' comments as they arise, help them to see how their actions are related to those of others, and occasion my students to be responsible for their thinking to the classroom community of teacher and students, where all agree on thinking that is "good enough" to fit the mathematical situation at hand (Kieren, 1995).

Talking is not enough.

This classroom interaction style is invaluable to me as I consider relinquishing the silence that pervaded my classroom. However, Kieren (1995) invites me to consider that my view of classroom interaction, which centers on talk *about* mathematics, is based on a limited view of interaction as something that happens between human agents in the room and mathematical reasoning as something that resides in individuals. Kieren asks me to consider mathematical thinking as not solely caused by mathematical artifacts, questions, teachers, the society of the classroom, or actions of the individual. Instead, mathematical cognition is in the interaction between all these components, none of which is the center.

According to Kieren (1995), each individual student responds to the interactive possibilities available within a classroom setting "as determined by his or her lived history -- his or her structure, which includes schemes but also entails affective responses" (p. 13). The mathematical actions of each individual emerge together with the actions of the teacher and classmates, all who respond to the many interactive possibilities according to their unique structures:

> Mathematical cognition allows a person to shape and participate in a world with others. This very participation changes the possibilities for others in the class, but such participation and its expressed results change the possibilities for the person (him)herself. (Kieren, 1995, p. 8)

While it is my role as a teacher to facilitate meaningful mathematical talk as alluded to by David Pimm (1987), this is but one part of a larger responsibility to occasion the possibility of what Kieren (1995) calls, "bringing forth a world of mathematical significance with others" within the classroom setting.

When Kieren talks about bringing forth a world of mathematical significance with others he refers to the work of Humberto Maturana and Francisco Varela (1987) who extensively studied cognition within the domain of biology. This notion goes beyond giving correct answers or finding personal solutions to problems, like what happened in my subtraction story. Instead, it is not so unlike that claim placed upon Jardine's grade 2 student, who posed the question, "Did I add or did I subtract?" The mathematical cognition is located amongst the girl, her conversation with Jardine, and the question. Thus, the mathematical thinking is not located in any one aspect of the mathematical situation. The teacher, who thought the mathematics cognition was in the question, missed out on an opportunity to engage with her student's mathematical reasoning and thus continue the conversation. My primary role as a teacher is to listen for opportunities to occasion my students to expand upon mathematical thinking with each other as well as "being willing to be surprised by that thinking and those explanations." (Kieren, 1995, p. 10).

During my future practice in my mathematics classroom, I want to attend to the collective dynamics that occur between students and our classroom community, which includes me, my students, as well as a host of mathematical possibilities that have the potential to be brought forth by my students.

A change in thinking.

Consideration of this expanded notion of interaction further influences my future mathematics lessons as my focus shifts from the need to provide more time for my students to engage in mathematical talk towards the need to access a host of interactive possibilities through talk. I realize that interactive possibilities within my mathematics classroom may happen between: interconnected nodes of mathematics (such as notions of repeated addition and multiplication; old ideas (i.e., the history of mathematics and how it came to be) and new ones that are grappled with and suffered through by students; students and the mathematical ideas behind a given representation; inner and outer layers of mathematical understanding; participants of mathematical talk, and so on. My awareness of this ever-growing gamut of interactive possibilities opens a space for me to bring forth a world of mathematical significance with my students.

Opportunities for purposeful mathematical talk are crucial for students as they develop in their ability to reason mathematically, but this is only one part of a larger image of interaction that offers my students and I the possibility of inhabiting the world of mathematics that I now envision.

Teaching to the Test

As I read each scrap of paper in my mathematics filing drawer, those looming standardized tests that I spoke of earlier presented themselves and spread like a virus throughout each of my folders. To prepare my students to write the tests, I downloaded the practice tests online and itemized each question by content strands (e.g., multiplication, division, fractions, etc). Next, I cut up the tests and placed the questions into each of my files where they fit best. I pasted the questions into the many worksheets that I designed for my students, and there was always at least one or two on each of my end of the unit tests, which I clipped together neatly at the back of each file.

The tests also invaded my problem solving file, or perhaps more accurately, gave birth to it. As I write these words, I realize that I did not have a problem solving file until I started getting my students ready for the long answer part of the standardized test, which included two open-ended problems! For example,

> Liam bought 2 packages of seeds and 2 posters at the Nature Park. He paid for these with a $20 bill.
>
> | | | |
> |---|---|---|
> | Price List: | Seed Packages | $1.25 each |
> | | Posters | $4.75 each |
> | | Wildflowers | $4.75 each |
>
> Show ALL the different ways his change could be given if he could receive only $1 coins, $2 coins, and $5 bills. Show ALL your work. (Province of British Columbia, 2008, p.20)

I searched the Internet, textbooks, and children's literature for similar kinds of problems and put them in the file, which became a treasure trove of rich problems that now burst through the confines of their file cover.

Before the advent of the tests, the types of problem solving activities that I occasioned for my students were basic word problems on a given topic that were to be studied in class. For example when I taught subtraction, I gave my students a page of subtraction problems of the sort given to Jardine's grade 2 student. Those tests, which became the driving force behind my curriculum-as-plan, became the impetus for me to change the way I occasioned my students to interact with mathematics problems in a similar manner to the way the new curriculum guide conditioned me to change the way I thought about subtraction and regrouping. Considered in this manner, I question whether the social pressures of improving the scores on standardized tests are as much at fault here as the focus that I *chose* to bestow upon them.

Bringing forth notions of abundance.

As I bring Aoki to the forefront of this conversation, I remember that he asks me to decenter my center, which I realize is the curriculum-as-plan. My past actions reveal that I did not give a simultaneous hearing to the lived and planned mathematics curriculum in my classroom. Instead, I privileged one over the other, in a "this" or "that" fashion, rather than in a manner of "this and that"(Aoki, 2005 [1993]). As I let the outcomes and tests drive my instruction and assessment process, I forgot to listen to my students and the questions that they posed as they interacted with mathematics in multifarious ways that I was not able to notice at the time.

As I consider the relationship between the lived and planned curriculum from the perspective of the third space, my previous actions shock me. These pages of reflective writing act as a bridge that opens a space for me to bring my curriculum-as-lived (what happened in my classroom and what I think about it) into conversation with my curriculum-as-plan (what I intended to happen and what I think about it), and create a sense of tension that is indeed for me a space of living pedagogy, as Aoki suggested to me from the very beginning of my research process.

My thinking about mathematics pedagogy is forever changed as I consider tests and curriculum guides as useful and meaningful beacons that have the potential to help me get my bearings and steer me through Jardine's vision of mathematics pedagogy which he calls, a "curriculum in abundance" (Jardine et al., 2006b). As highlighted throughout the course of this written reflection, viewing mathematics as abundant means being able to see that understanding mathematics is about the ability to see "this stubborn particular" in relation to its wholeness or in other words, that "any seemingly isolated thing on earth is in fact the nestling point of vast, living abundance of relations, generations, ancestries, and bloodlines" (Jardine et al., 2006b, p. 7). It means not only experiencing these intergenerational relations, but also understanding that "things are genuinely understood insofar as they are grasped in their abundant interdependence with all things" (Jardine et al., 2006b, p. 8). To experience mathematics in abundance entails leaving behind the all too common notions of the perception of mathematics as an object and involves embracing a view of mathematics as open to interpretation.

Regarded in this light, my curriculum-as-plan has the potential to offer me guidance through a mathematical territory ripe with infinite possibility (Jardine et al., 2006b). Each learning outcome and opportunity for interaction within my mathematics classroom has the potential to become a portal into this abundant world, as something that reaches out and addresses us like that query of Jardine's grade 2 student. However, this can only happen if I experience abundance as a way of thinking not unlike van Manen's notions of mindful action, instead of another tool that I can wield to cover the curriculum.

Abundance thinking has the potential to open up the lived curriculum in a way that I never dreamt possible. It is indeed one of those bridges that Aoki talks about and I am proud to say that my thinking about mathematics inhabits a third space that goes well beyond my notions of my planned and lived curriculum, pointing towards my evolving notions of the nature of mathematics itself.

The focus that I placed on the tests, on my students' corrections, and on the way that I needed to get the right answers throughout my life, are evidence of a view of mathematics as fragmented and measured in isolated bits. An objectified view of mathematics like this makes abundance appear impossible because it sets boundaries on what is imaginable. On the flip side, abundance thinking makes objectified thinking not only undesirable, but also impossible because it enlarges the space of what is imaginable, even within the confines of schooling (Jardine et al., 2006b).

A change in thinking.

I no longer think of mathematics as an object that my students and I can memorize and wield at our will. I am beginning to think abundantly and it is this change of focus that permits me to dwell alright in the space between my seemingly dichotomous curriculum worlds. It is this change in thinking that permits what I once considered to be impossible, Jardine's vision of mathematics within my school culture, to become possible in the domain of mathematics and beyond.

Chapter 5: Taking a Step Back

It is now time to take a step back from this, my autoethnography, and consider the big ideas into mathematics teaching and learning that will work their way into my future teaching as well as to discuss possible contributions to the field of mathematics education. Through van Manen's recursive cycle of reflection, I engaged in a thorough study of my mathematics teaching and now experience Jardine's notions of abundant thinking as a bridge between the seemingly dichotomous worlds of the curriculum-as-lived and the curriculum-as-planned. This is a bridge that I will continue to linger upon and does not merely cross between these worlds, but allows me to dwell within them.

Thinking About the Future

Jardine et al. (2006a, 2006b) say that abundance can only be understood through practicing it in the classroom on a daily, moment-to-moment basis, which for me is only beginning to sink in, with one year's worth of living under my belt since that day I met Eric and his family and started this query into my teaching practice. Understanding abundance requires my lifelong commitment to keep in-sight of the potential of experiencing mathematics abundantly, and my diligent efforts to let go of the confines which my linear, dichotomous thinking unconsciously imposes on my very being and my actions in my classroom. I already discussed many specific instances of my teaching that are likely to change with mindful action, but I have not yet zoomed out towards considering the larger notion of abundance.

There are three goals of my future teaching practice that together have the potential to help me keep abundance thinking at the forefront. I want to make a conscious effort to: (a) provide time for my students to linger in their mathematical inquiries, (b) perceive the constraints that stand in my way of abundant thinking as difficulties instead of impossibilities, and (c) continue my efforts to decenter the center.

Whiling.

In the past, I was guilty of rushing my students through learning outcomes in order to cover all the outcomes in our ten months of time together, which will not be an easy habit to shake. Jardine (2002b) invites me to break away from this notion of clocked time, which "demands the fragmentation of that which it measures" towards "a different sense of time, not slowing exactly, but lingering" (p. 223). He asks me to consider occasioning worthwhile classroom experiences that are "worth the while"; that is, those experiences that need awhile and involve students who want to "while over it".

With this intention in mind, I aspire to occasion certain kinds of activities in my classroom, such as the kinds of problem posing proposed by Brown and Walter (1990). The What-If-Not strategy has the potential to make both my students and myself explicitly aware of the attribute that we choose to vary and opens up the space of mathematical possibilities in which we can "while within". Going back to my subtraction story as an example, if we alter the attribute that all calculations involve positive integers during that scenario that I discussed, the following algorithm, which was invented by a third-grade student becomes not only possible, but likely:

a) 64
  <u>-28</u>
   -4   Four [subtract] eight is negative four...

b) 64
-28
-4
40 and sixty [subtract] twenty is forty...

c) 64
-28
-4
40
36 and forty and negative four are thirty-six.

(Brown & Walter, 1990, pp. 82-83)

Open-ended activities posed as problems that allow for more than one answer have the potential to nudge my students into “whiling” with the mathematics. Another example is an activity like the “broken division key” in which students work in groups to solve division questions without using the division key on a calculator (Van de Walle & Lovin, 2006). These authors provide a host of such activities across each strand of the mathematics curriculum, and I could easily become lost in the quagmire of teaching resources and teachers guides that offer similar advice with explicit lesson plans. However, I must remember to keep in-sight the notion that mathematical cognition does not reside in the activity that I plan for my students, but in the interaction between my students, me, and the mathematics; as well as a host of other possibilities (Kieren, 1995). Thus, the activity that I plan is only one part of the equation, so to speak.

It is in interaction, in whiling, that mathematical experiences ask something of us, like that of Jardine’s grade 2 student. This girl brought forth abundance in the mundane, isolated addition and subtraction problems on her mathematics worksheet. With this address came the possibility for her understanding to grow in the manner envisioned by the Pirie-Kieren theory with a teacher versed in the ways of abundant thinking and eager to take the time to get lost with the girl in her query. What really happened here? The girl did not understand the question asked of her and Jardine helped her to fold back to layers of image making and image having as she used his fingers to compare the letters and packages. This was all she needed as she put down two fingers on one of his hands and then comparing both hands, put two fingers up on the other, and in doing so engaged in property noticing behaviour.

In order for mathematical growth to continue, questions such as, "What do you think, did you add or subtract?" "How do you know?" and "Does this work with other questions?" could help the girl more firmly establish the connections that she makes between addition and subtraction and help her build rich, in other words "thick" understandings of mathematics as she folds backwards and forwards between layers of understanding. And, the opening up this conversation towards the classroom community where: "human beings think for themselves but this thinking is done, at least in anticipation of communicating with others and acting in a community of others interested in mathematics" (Kieren, 1995, p. 7), is just the kind of whiling that Jardine was hoping for when he asked the girl to share her findings with her teacher and classmates.

As I while with my students in the mathematics classroom, I want to remember to deliberately choose questions and prompts to occasion my students to fold backwards and forwards between layers of understanding. Sometimes my prompts may be saying nothing at all and sometimes I may want to invoke students to justify and relate their thinking to the classroom community in order to occasion mathematical growth in understanding. Time will tell. The key is to keep abundance thinking at the forefront, as well as Kieren's notions of interaction and the Pirie-Kieren theory.

Understanding the constraints before me.

My new way of experiencing mathematics can only be sustained if I keep in mind the all too real constraints that may make abundance difficult in my classroom. My classes of 30 students come to me with a variety of special needs, both academic and social that may require a kind of structure that takes us back to worksheets and quiet times in the classroom as we practice the conversation skills necessary to engage in productive mathematical talk (Pimm, 1987). Some classroom communities take a long time to build and gain the kind of trust and feeling of safety that is required for this kind of open-ended learning, and some do not develop the kind of classroom dynamics that Kieren (1995), Davis & Simmt (2006), and myself envision.

I could ramble on with my list and cite all the reasons why teaching abundantly might not work, but what is of importance here is not these constraints in themselves, but my perception of them. They make abundant work challenging, but not impossible and are in essence, what Aoki describes as a site of tension, and therefore a place of living pedagogy:

> living school life means living simultaneously with limitations and with openness, but also that this openness harbours within it risks and possibilities as we quest for a change from the is to the not yet. This tensionality calls us as pedagogues to make time for meaning striving and struggling, time for letting things be, time for questioning, time for singing, time for crying, time for anger, time for praying and hoping" (Aoki, 2005 [1986/1991], p. 164).

As I head back to the classroom setting, my purpose is to occasion mathematical experiences that are "worth whiling" and have the potential to bring forth the kind of abundance that Jardine and I talk about. However, there are times when I might revert to teacher-controlled conversation through TIRE questioning strategies (Kieren, 1995) or a fill in the blank, "guess what I am thinking" type of questioning out of habit (Pimm, 1987). There are times, when I might forget to dwell in the third space and go back to thinking in a dichotomous manner, not so unlike the actions of many educators, since most have us have been schooled this way and taught to school this way (Aoki, 2005 [1993]). This might happen, which is neither right nor wrong, but a fact of life.

The key is to continue with van Manen's cycle of reflection, which allows me to access what was once only available to my subconscious thought and to avoid defaulting back to old habits, both those that I want to change and those I do not. Not all habits are bad, but all should remain open for reflective inquiry. I want to get into the habit of asking myself whether my intent matches my actions within the perceived limitations of each teaching situation.

Decentering the center.

It shocked me to discover how single minded my focus was in each of my teaching stories. Instrumental thinking, linear thinking (e.g., my perception of unidirectional movement from concrete to abstract layers of mathematical ideas), correction of mistakes, and problem solving were privileged to the extent that I lost sight of all other possibilities. Now, this is not to say that these notions are undesirable in and of themselves. The trouble here, as Aoki (2005 [1993]) explains it to me, is that I became "polarized".

Other than the fiasco described with the Cuisenaire rods, I feel proud of the way that I experienced mathematics with my students during the time these lived experiences occurred. In particular, the problem solving sequence is one that I thought was in too high esteem to be questioned. Other educators asked me to travel to their classrooms and present my lesson ideas about problem solving and one of my administrators, who watched me teach this lesson sequence during a routine teacher evaluation, implored me to consider a Masters degree in mathematics education. The point I am trying to make is that I stopped thinking, as Brown and Walter (1990) reminded me earlier and when I did, I let the habit of what I knew in the past to be a good lesson to take over.

The examination of my stories as sites of tension allows me to see my views of mathematics teaching and learning in their wholeness. It allows me to understand what my views were and continue to be. It helps me to understand my views about mathematics through articulation of what they are not, and could therefore be. For example, I did not realize the importance of occasioning opportunities for students to problem pose until I began to consider what I could be doing if I did not ask my students to problem-solve. Whenever I find myself in a place where I think that I have the answer, I want to take a step back and continue to reflect upon alternatives to these moments of singular focus.

The notion of decentering the center, allows me to consider multiple perspectives beyond “a realm of this and that”, as I already described, to what Aoki (2005 [1993]) calls “this and that and more”. It is in this space that newness emerges (Aoki, 2005 [1996]), where abundant thinking permits me to explore the relationships beyond just two seemingly dichotomous ideas, like I did with my stories. It taps open an infinite realm of relations between the world and its history, of which mathematics is just one part.

Contributing to the Field of Mathematics Education

My mathematics teaching practice is forever changed. If my story inspires just one person to go through a similar process of reflection, then I will feel as though I contributed to the field of mathematics education outside of my classroom. And, if even one person begins to heed the NCTM's (2000) call to reform the way they teach mathematics by examining their practice like I did, then I will know that my work has made a difference. This reflective process acts as a bridge between practice and theory, gives insight into deeply rooted views of mathematics of which one may not be consciously aware, and has the potential to transform one's very being.

Bridging practice and theory.

When I began the proposal for this project the idea of letting the literature emerge with my writing overwhelmed me. I thought that I should read everything first and then go about creating a project based on what I read. It was terrifying, but once I let go, I ended up bringing theoretical ideas, such as Pirie and Kieren's (1994) model of the growth of understanding into conversation with my teaching practice. Exploration of the model, which I read about several times during graduate studies and discarded as too theoretical and beyond my understanding, suddenly became relevant and required my further attention when I contemplated my story about fractions.

The literature that I chose to bring into conversation with my stories addressed me, in the same manner as Jardine's grade 2 student. It drew me in and my mathematics pedagogy came out transformed. The back and forth movement between the two, the zooming in and out, bridged my evolving notions of theory and practice in ways I never thought possible. I now view reflection as a precondition to learning and an important method for bridging the gap between theory and practice, as do Long and Stuart (2004); Dolk and Hertog (2008).

Bringing forth a philosophy of mathematics.

"Mathematics is not an object" is a phrase that I repeat many times throughout my writing. Delving into my teaching practices allows me to glimpse the extent to which this philosophy of mathematics permeates every aspect of my teaching without my conscious knowing. Since, "mathematics pedagogy rests on a philosophy of mathematics" (Pimm, 1987, p. 70), it is not a stretch to consider how my beliefs about mathematics and how children learn become part of the mindful action that van Manen talks about. As a teacher, I have the potential to carry the general representation of mathematics within the school culture, which for me, like many other educators, I based upon the practice of breaking down mathematics into fragments to be tested on and reported on standardized tests (Jardine et al., 2002).

The commitment to carry on with van Manen's cycle of reflection outside of graduate studies, allows me to continually consider and re-consider my belief system about what it means to teach and learn mathematics. And, instead of falling back into routine, I want to keep my new routines open for reflective inquiry. For, Pimm (1987) shares with me that, "Many teachers do not see the value or even the possibility of discussion in mathematics as a consequence of the view of mathematics which they hold" (p. 47). It is my shift in thinking with theory that in turn impacts how I go about in practice, occasioning mathematics learning with my students and vice versa.

A change in thinking: How children come to know mathematics

This exercise in critical reflection reveals my underlying assumptions of how children come to learn mathematics, which permeates every aspect of my teaching practice in a manner similar to those standardized tests that invaded my mathematics files. I assumed that children come to learn mathematics by progressing from concrete to abstract notions of mathematics and that mathematics is something that resides outside of the knower, an object at one's disposal that can be deemed as right or wrong, an all or nothing phenomenon.

I no longer see the relationship between simple (concrete) to complex (abstract) mathematical experiences as a unidirectional movement. Instead, I embrace the Pirie-Kieren model, with the notions of recursive layers of understanding that become progressively more structured, yet not necessarily more sophisticated. Sophistication in understanding, rather, is demonstrated by the flexibility of the learner's or learners' movement through the different realms of the model and knowing when such movement is necessary. The notions of folding back and don't need boundaries provide for me a context in which to situate the complex, interconnected relationship between the concrete and abstract that our BC Mathematics IRP (2007) does not.

My past interactions with my students point towards a notion of mathematical cognition that lies in either the individual, for example, my math star's owned strategies, or in the mathematics itself, for example, the Cuisenaire rods fiasco. Mathematical cognition, viewed through the lens of Kieren's (1995) notions of inter-action, also forever changes how I think children come to know mathematics. Mathematics is not an independent endeavour, requiring silence and mindless practice of basic math facts on worksheets! I am excited to practice abundant thinking in my future mathematics teaching and to break free from the confines of linear, "this or that" kind of thinking that will require much conscious reflection to keep at bay.

A change in being.

It is not only my views of mathematics pedagogy that have been transformed, but truly, my very being. What I have discovered or perhaps more accurately, what has addressed me, is a new way of thinking that is definitely about mathematics, but also much more. My new understanding, opens up a whole new world for me that I did not know existed. I can best describe it as a new perspective on life, which prompts me to pause and consider how each lived moment is nested within a larger, interconnected context that deserves my attention and consideration. For example, small conflicts of understanding with friends, family, coworkers, or students, no longer create the same sting or sense of indifference from me when I stop to think about the larger context within which these events are situated. This change in me is a certain mindfulness, a disposition towards noticing the big picture without necessarily the conscious awareness that I am doing so.

Aoki (2005 [1986/1991] reminds me that "teaching is fundamentally a mode of being" (p. 160), which is a notion that I live every day. It is now impossible for me to consider just my teaching practices about mathematics outside of who I am as a person:

> "you *become someone* (not just anyone) as a consequence of how you carry yourself in the world. With practice, you can become more experienced in experiencing things in their abundance...you get better at allowing and taking pedagogical and spiritual pleasure in such happenstance, and at taking care of what is being asked of you in such moments" (Jardine et al., 2006c, p. xxv).

And, this is what is in the pedagogical good for my students.

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