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COUNTING CONVEX SHAPES

Elitza Wise, Department of Mathematics and Statistics

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University of Victoria

INTRODUCTION

The objective of this project was to generate a code that takes a number, n, and outputs the number of convex shapes contaning n points.

DEFINITION

In the non-discrete sense, a set S is called *convex* if the line segment connecting any two points is contained in S.

Figure 1: Convex vs. not convex shape.

ALGORITHM

Abstractly, the code finds every convex shape containg the given number of points by constructing each possible shape row-by-row. The algorithm begins with the first row, then finds the possible coordinates for each subsequent row using recursion.

RESULTS

n	5	6	7	8	9	10	20	30	40	50
C(n)	55	144	338	726	1,445	2,676	228,930	4,409,464	44,312,177	305,101,772

Figure 2: Number of convex shapes C(n) containing n points.

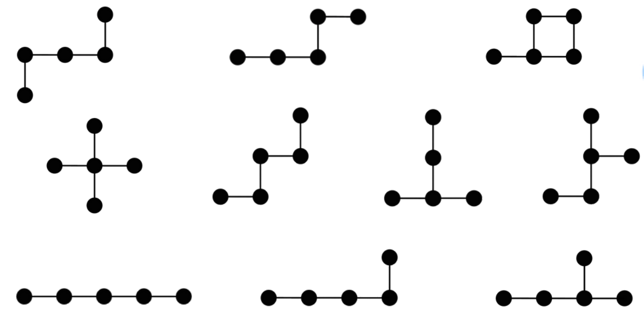

Figure 3: All convex shapes containing five points disregarding rotations and reflections.

It was speculated that $C(n)$ could be approximated with a sub-exponential function of the form $C(n) = c_1 \cdot e^{(c_2 \cdot n^{c_3})}$ based on the integer partition function [1]. The coefficients c_1, c_2, and c_3 were determined using a log-log regression transformation.

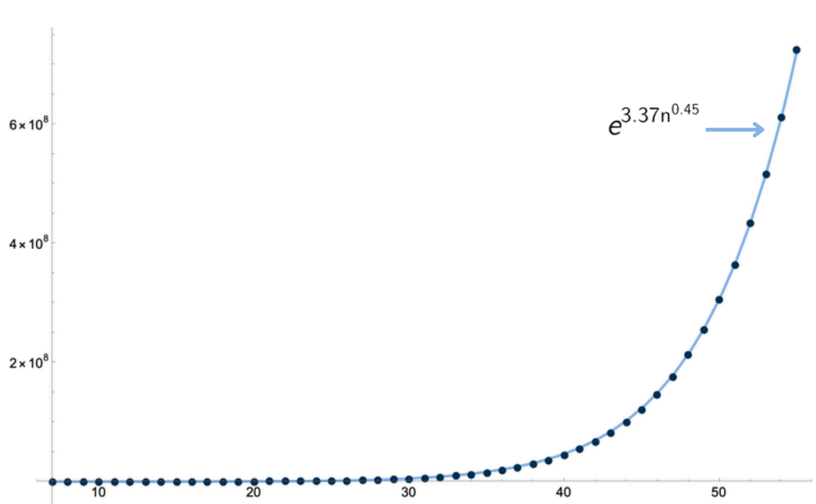

Figure 4: Stretched exponential function fitted to C(n) data.

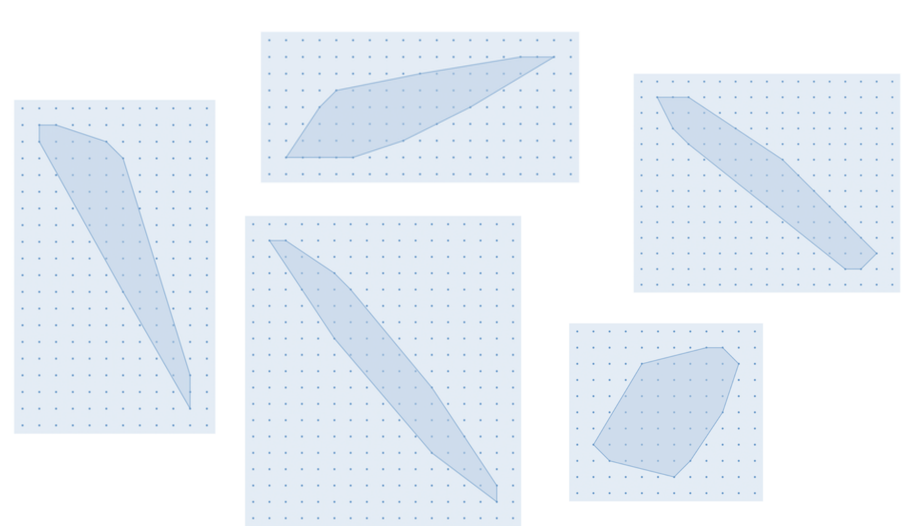

Figure 5: Five randomoly generated convex shapes containing fifty points.

REFERENCES

[1] University of Pennsylvania. “Lectures on Integer Partitions.” 2000, p. 35. Accessed 31 July 2024.