Efficient Models of Choice for Examining Risk and Ambiguity: A Prospect Theory Approach

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Abstract

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Uncertainty in economic decision-making can present itself in a variety of forms however the most commonly researched are risk and ambiguity. Recently, there has been a resurgence of interest in evaluating the relationship between individuals' preference for risk and ambiguity through the use of cognitive decision models. Accurately characterizing these preferences therefore relies on the use of appropriate models and model fitting techniques. Huettel, Stowe, Gordon, Warner & Platt (2006) used Expected Utility Theory (EUT) and the alpha maxmin model to evaluate individuals risk and ambiguity preference, respectively. Their results suggest that risk and ambiguity evoke disparate cognitive processes at both a behavioural and neural level. However, the use of EUT in characterizing risk preference calls into question the accuracy of their results. The present study attempts to re-evaluate the relationship between risk and ambiguity using a more appropriate and well-established model of risky decision-making, Cumulative Prospect Theory (CPT). Using a similar task design as Huettel et al. (2006), participants (N = 93) were required to make a series of decisions between two options that involved monetary outcomes. Each trial consisted of choices between two of the following options: risky, certain and ambiguous. Parameters for both EUT and CPT were estimated on risky trials and used to inform the estimation of ambiguity parameters using the α-maxmin on ambiguous trials. Moreover, each model was estimated using two methods of model fitting, optimization and hierarchical Bayesian analysis methods. Overall, CPT outperformed EUT on risky trials as well, ambiguity parameters from α-maxmin informed by CPT risk parameters outperformed EUT informed α-maxmin parameters. Finally, CPT estimated alpha and beta values were found to be uncorrelated. However, the present results demonstrate that ambiguity preference parameters correlate with the probability distortion parameters that may be a more

accurate depiction of an individuals' level of risk preference. These results can be used to inform future endeavours uncovering the neural correlates of levels of uncertainty in decision-making.

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Introduction

Our everyday lives are fraught with an incalculable number of decisions. These decisions range from whether or not you bring an umbrella to work or whether to invest money in the stock market. Amongst these decisions, rarely do we encounter one that provides all available consequences with complete certainty. Therefore, we are constantly required to incorporate uncertainty into our choices. Within the context of uncertainty in decision-making, options that offer probabilistic outcomes are referred to as risky. Risk can be associated with a wide variety of behaviours such as skydiving or deciding to drive while intoxicated. From a neuroeconomics standpoint however, risk is assessed using monetary outcomes with associated probabilities.

Successful decision-making requires the ability to accurately assess the available options and incorporate the necessary information to make an informed choice. How the available information guides individuals' choices in an economic context has been of interest to researchers dating back to the late 1700's (Bernoulli, 1738; as cited in Stearns, 2000), the full history of which is beyond the scope of this thesis. Of primary importance however, is the development of normative and descriptive models of decision-making. Normative and descriptive models of decision-making can be distinguished by the underlying goal of the model. That is, normative models of decision-making seek to outline how individuals *should* decide whereas descriptive models attempt to explain the actual decisions that individuals make. It therefore follows that the goal of a research endeavour should be mirrored in the decision-model chosen for analysis. In other words, if the goal of the researcher is to account for actual decision-making behaviour and to understand what factors influence real decisions, appropriate descriptive models should be chosen over normative models of choice.

One of the most prominent normative models of decision-making is Expected Utility Theory (EUT), originally based on a proposition stated in Bernoulli (as cited in Stearns, 2000). Here it is suggested that a decision-maker evaluates the available options in terms of a subjective value, or utility, rather than objective values (see Section 1.1.1. for model definition). In the case of a certain option, one that offers a monetary outcome with complete certainty, the objectively stated value is transformed into subjective value via a diminishing marginal utility function. When presented with a risky alternative, the utility of the overall option is calculated by summing the individual subjective values of the option multiplied by the corresponding probabilities. Moreover, EUT assumes that decision-makers evaluate the outcomes based on final states of wealth. That is, an individuals overall wealth influences the extent to which they discount the available outcomes.

Although EUT remained a prominent theory of decision-making, several shortcomings were brought to light through the famous Allais paradox **(**Allais, 1953**).** Later, Kahneman and Tversky (1979) presented a thorough review of the explanatory limitations of EUT and offered an alternative descriptive model of decision-making, Cumulative Prospect Theory[1] (CPT). Under the rubric of CPT, a decision-makers evaluation of the available options is determined by both value and probability distortions (see Section 1.1.2. for model definition). That is, the utility of an option is the sum of the underlying subjective values multiplied by a decision-weight, rather than an objective probability. Moreover, CPT incorporates the concept of loss aversion into the valuation process, allowing the model to account for the behavioural phenomenon that losses loom larger than gains (Tversky & Kahneman, 1991). An additional feature of CPT is the

[1] The original version of Prospect Theory was presented in Kahneman & Tversky (1979). The present study utilizes the most recent version of this model, Cumulative Prospect Theory, presented in Kahneman & Tversky (1992).

concept of a common reference point. Here, rather than basing the valuation of options on final states of wealth, the decision-maker evaluates the outcomes in relation to a reference frame.

Since its conception, CPT has been validated as an accurate and robust descriptive model (e.g. Mishra, Gregson & Lalumière, 2012; Rottenstreich & Hsee, 2001; Ungemach, Steward, & Reimers, 2011) that is able to account for the commonly observed four-fold pattern of decision-making: risk seeking for low probability gains and high probability losses, and risk aversion for high probability gains and low probability losses. In addition, recent work evaluated the temporal stability of CPT parameters and found that parameter estimates remain generally consistent at the individual level across time (Glockner & Pachur, 2012). Moreover, Bruhin, Fehr-Duda and Epper (2010) demonstrated that the heterogeneity of individual choice preferences exists cross-culturally and can be accounted for within the framework of the CPT model.

Although both EUT and CPT are capable of describing aspects of risky decision-making, they are unable to account for decision-making at all levels of economic uncertainty. The potential for multiple levels of uncertainty can be demonstrated through the well-known Ellsberg paradox (Ellsberg, 1961). Here, an individual is given a choice between two urns of which they can place a bet. In one of the urns, there is a known proportion of marbles while in the other urn, the proportion of marbles is unknown. For example, the decision-maker is told that in one urn there are 50 blue marbles and 50 red marbles while the other urn contains a total of 100 marbles but the proportion of red and blue marbles is unknown. If a red marble is selected, the individual will win $100. The individual is then asked to select the urn that they wish to pick a marble from. In this circumstance it is common that the individual will select the urn with the known proportion of marbles. This indicates that the individual believes that there is a greater number of red marbles, or therefore a greater likelihood of selecting a red marble from the known urn. It

follows then that the decision-maker believes that the proportion of blue marbles in the unknown urn must be greater than 50. After making this choice, the individual is given the same scenario with the same urns and is told to bet on selecting a blue marble that is now worth $100. In this circumstance, the decision-maker commonly selects the known urn again. This implies that they now believe there is greater than 50 red marbles in the unknown urn. This example demonstrates that the decision-makers beliefs about the likelihood of outcomes are non-transitive. Therefore from a purely behavioural perspective, Ellsberg (1961) demonstrated that when confronted with decisions that involve options with unknown information, individuals often violate several of the Savage Axioms[2]. These types of options with unknown information are now commonly referred to as ambiguous. They therefore differ from risk as risky options provide information regarding the likelihood of an outcome in the form of probability while ambiguous options require that an individual infer the likelihood of events. Therefore, ambiguous options offer the risk that the decision-maker is wrong about their beliefs regarding the likelihood of an event (Ellsberg, 1961). It is also possible that individuals are willing to pay a premium for the additional information provided in the risky options. For instance, some research has shown that individuals are generally willing to incur a 10 – 20% loss based on expected value in order to avoid ambiguity (Yates & Zukowski, 1976). Moreover, this and other behavioural research observing decision-making between risk and ambiguity has demonstrated that individuals show high ambiguity aversion in decisions where ambiguity is pitted against a risky option offering a high probability of the larger outcome (Becker & Brownson, 1964; Ellsberg, 1961). These findings led to the assumption that individuals are ambiguity averse in general. Ambiguity aversion does not hold across all circumstances and in fact, in some contexts, ambiguity seeking can be evoked. For

[2] For a description of the Savage Axioms, see Fishburn (1986).

instance, individuals appear to be risk seeking when the probability of winning the larger outcome in risky option is below 40% (Curley & Yates, 1985).

Recent work from Huettel, Stowe, Gordon, Warner & Platt (2006) suggests that risk and ambiguity evoke disparate neural mechanisms and therefore represent distinct forms of uncertainty. In this experiment, subjects performed a two-alternative forced choice task requiring them to choose between a combination of ambiguous, risky and certain options. In other words, each trial provided the participant with one of the following choices: risky versus certain (RC), risky versus risky (RR), ambiguous versus certain (AC) or ambiguous versus risky (AR). Each participant's preference for risk and ambiguity was then estimated using the EUT and α-maxmin model (as described later in section 1.2.3), respectively. Following parameter estimation, the researchers evaluated the relationship between individuals' preference for ambiguity and preference for risk. The results demonstrated that an individuals risk preference is uncorrelated with their ambiguity preference. To investigate the neural correlates of these preferences, the subjects risk and ambiguity preference parameters were analyzed with respect to the differential activation observed in the decision-phase of the trials. Further analysis observing the correlation of subjects' risk and ambiguity parameters showed that the differential activation in the posterior inferior frontal sulcus (pIFS) was positively correlated with subjects' risk preference and uncorrelated with ambiguity preference. In contrast, differential activation in the posterior parietal cortex (pPAR) was correlated with ambiguity preference but uncorrelated with risk preference. Therefore, the authors demonstrate a double dissociation for the neural processes related to risk and ambiguity preference. Given this, the authors conclude that risk and ambiguity represent disparate forms of uncertainty.

At first glance, the authors appear to provide a clear account of the separate processes involved in risky and ambiguous decision-making. However, several issues arise on closer examination of the methodological approach taken by Huettel et al. (2006). The first issue surrounds the decision-model used to estimate and represent each subjects' risk preference. As discussed above, modelling decision-making with EUT is useful when the researcher intends to describe how an individual *should* chose in an economic context. However, it is clearly stated the intention of Huettel et al. (2006) was to describe the underlying components that contribute to each subjects' *actual* decision-making behaviour. Therefore, the model chosen to account for risk preference should be a strong descriptive model rather than a normative model. The use of EUT calls into question the accuracy and efficiency of the estimated risk preference parameters. Moreover, this diminishing marginal utility parameter, which in the case of EUT defines an individuals' preference for risk, is subsequently used to determine the utility of the values that comprise the ambiguous options. These estimated utilities are subsequently used to estimate the subjects' ambiguity preference via the α-maxmin model. Therefore, the accuracy of the estimated ambiguity preference parameter relies on an accurate representation of the underlying utilities of the options. In other words, accurately estimating ambiguity preference relies on an accurate estimate of the utility parameter.

In order to support the efficiency of the model, the authors state that EUT correctly predicted the thirteen subjects' choices on an average of 75% of the trials with a minimum of 68% and a maximum of 86%. Initially, the maximum of 86% correct predictions seems sufficient. However, these values are not surprising given that EUT is embedded in the CPT model. That is, a decision-maker who approaches a decision by maximizing utility and treating

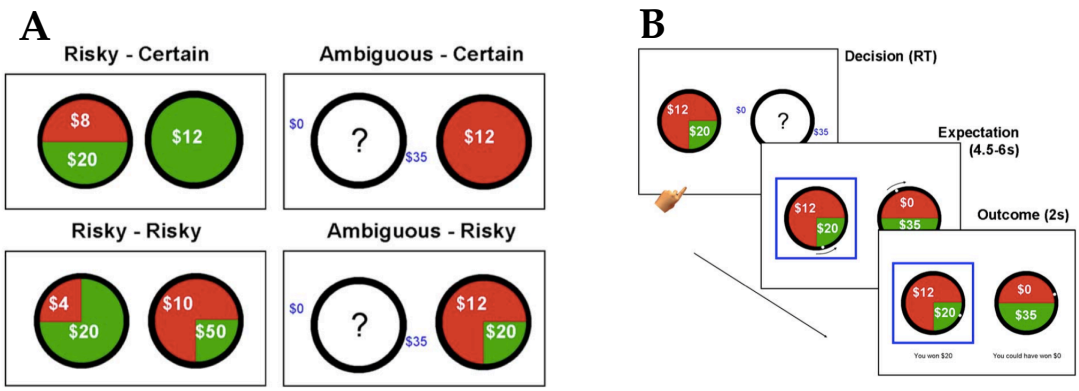

Figure 1. Schematic of trial types (A) and experimental procedure (B) used in Huettel et al. (2006).

probability objectively can be represented under the CPT model by fixing the probability parameter at a value of one. Cross-cultural research suggests that utility maximizers consistently comprise a small proportion of a given population (Bruhin, Fehr-Duda, Epper, 2010). Therefore, it is possible that model was able to accurately predict choices for a small set of subjects in the study.

A second caveat of Huettel et al. (2006) surrounds the task design itself (see Figure 1A). One issue pertains to that values that comprise the ambiguous options. Across all of the option types, ambiguous options were the only ones that presented the possibility of winning nothing if it was chosen. This becomes an issue if one considers the fact that all other options, whether greater or less in expected value, offer the possibility of winning something compared to nothing. A second issue with the ambiguous options is that the format they are presented in is substantially different from the format of all other available options. For instance, the ambiguous trials are presented as a white circle with a question mark appearing in the center and all values appearing outside the circle. In contrast, all other options are presented in colour format with outcome values appearing within the circles. Therefore, overall the ambiguous options presented are fundamentally different than all other options.

One final issue with the trial configuration is the presentation of feedback following each decision. Providing feedback could have some influencing effects on the decisions that subjects make in an experimental context regardless of whether or not this influence is intentional. In other words, subjects' could misinterpret feedback as an indication of the rightness or wrongness of their previous decision and modify their subsequent decision accordingly, regardless of their inherent preference. Some researchers have demonstrated that subjects' tend to evoke a competitive and suspicious strategy in psychological gambling tasks, assuming that the experimenter is more knowledgeable about the outcomes for ambiguous options (Kühberger & Perner, 2003). This competitive strategy presumably arises because the experimenter will inevitably lose money if the subject gains money based on the outcome of their choices. This strategy is in contrast to the cooperative strategy evoked if the individual creating the ambiguous scenario is a friend of the subject. Overall, competitive strategies result in ambiguity aversion while cooperative strategies result in ambiguity seeking. In the case of Huettel et al. (2006), it is possible that subjects' approached the task in a competitive manner, believing that some unstated rule governs the likelihood of the outcomes on ambiguous trials. In this case, subjects' could place more weight on the feedback, believing it provides information about the rule and the likelihood of events on subsequent ambiguous trials. Therefore, providing subjects' with feedback regarding the outcome of their decision could ultimately drive them to choose in a task-specific manner rather than selecting the options they would in a non-experimental situation. It follows that if the estimated parameters are derived from the participants' choices, the values obtained could be confounded by the feedback provided. Moreover, as will be discussed later, the method for fitting EUT to the model was deterministic and therefore does not account for inconsistencies in subjects' choices.

The issues presented above call into question the accuracy of the parameter estimates and the evaluated relationship between risk and ambiguity parameters as well as their neural correlates. The primary goal of the present work is to re-evaluate the relationship between risk and ambiguity in economic decision-making. Based on the extant literature, it is hypothesized that a CPT model of decision-making with a shifted-reference point should provide a more accurate representation of an individuals' preference for risk when compared to the EUT model. As risk preference parameters are used to inform the estimation of ambiguity preference, this increased accuracy should then allow for more precise evaluation of ambiguity preferences.

A secondary goal of the present work is to investigate the appropriate model fitting methods. As will be described below, Huettel et al. (2006) fit the EUT model using an optimization method. Previous work suggests hierarchical Bayesian analysis (HBA) methods may be more optimal when estimating parameters across populations and at the individual level (Ahn, Krawitz, Kim, Busemeyer & Brown, 2011). In order to provide a clean comparison to Huettel et al. (2006) and to evaluate the method of model fitting, all models will be fit using both optimization and HBA. Finally, previous literature suggests that analyzing decision data by averaging across all subjects' has the potential to mask prominent heterogeneity at the individual level (Gonzalez and Wu, 1999). Moreover, Bruhin et al. (2010) demonstrated that subpopulations exhibiting similar choice behaviour exist cross-culturally. These researchers demonstrated that decision-making preferences are accurately represented by three distinct subpopulations. Therefore it was hypothesized that these subpopulations would exist in the present data set.

1.1. Defining the Models

1.1.1. Expect Utility Theory

According to Expected Utility Theory, the decision-maker determines the expected utility (EU) of an option (O_j) by summing across the *n* possible outcomes of that option weighted by the options associated probabilities as follows:

$$EU(O_j) = \sum_{i=1}^{n} p_{i,j} v(x_{i,j}) \qquad [1]$$

Each of the available outcomes is characterized by a corresponding probability, p, and value, x. The objective values are transformed into subjective values by a value function, $v()$, defined as follows:

$$v(x_i) = x_i^{\beta} \qquad [2]$$

The diminishing marginal utility parameter, $0 \leq \beta \leq 1$, determines the extent to which an individual discounts large magnitudes, where $\beta = 1$ would signify that values are represented objectively.

1.1.2. Cumulative Prospect Theory

According to Cumulative Prospect Theory (CPT), the decision-maker determines the subjective expected utility (SEU) of an option (O_j) by summing across the *n* possible outcomes of that option weighted by the subjective representation of the outcome probabilities as follows:

$$SEU(O_j) = \sum_{i=1}^{n} w(p_{i,j}) v(x_{i,j}) \qquad [3]$$

Each of the available outcomes is characterized by a corresponding probability, p, and value, x. The objective probabilities are transformed into subjective decision weights by a probability function, $w()$, defined as follows:

$$w(p_i) = \frac{p_i^{\gamma}}{(p_i^{\gamma} + (1 - p_i)^{\gamma})^{\frac{1}{\gamma}}} \qquad [4]$$

The probability distortion parameter, $0 \leq \gamma \leq 1$, represents an individuals tendency to overweight small probabilities and underweight large probabilities, where a γ value of 1 would signify that probabilities are evaluated objectively. The options objective values, x, are transformed into subjective values by a value function, $v()$, defined as follows:

$$v(x_i) = \begin{cases} x_i^{\beta} & if \;\; x_i \geq 0 \\ -\lambda(x_i)^{\beta} & if \; x_i \; < 0 \end{cases} \qquad [5]$$

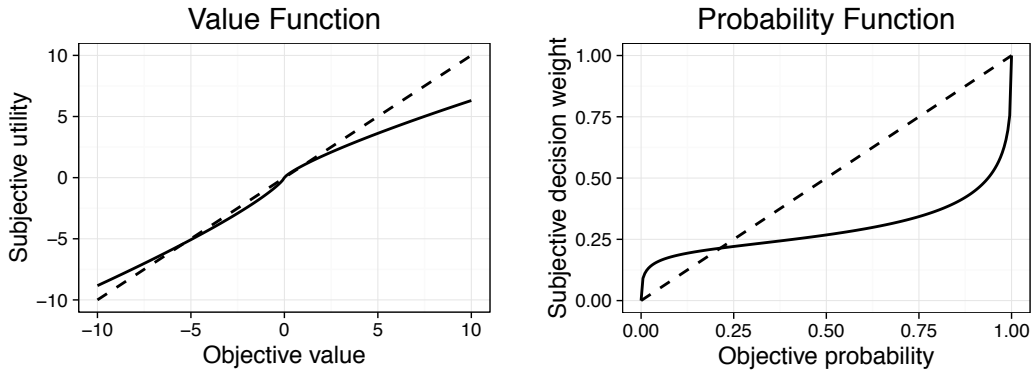

Figure 2. Graphical example of CPT parameter functions.

The diminishing marginal utility parameter, $0 \leq \beta \leq 1$, determines the extent to which an individual discounts large magnitudes (rephrase), where a $\beta = 1$ would signify that values are represented objectively. The loss aversion parameter, $0 \leq \lambda < \infty$, determines the extent to which an individual overweights losses compared to gains; as λ increases beyond a value of 1, an individual is said to subjectively interpret the absolute value of a loss greater than the absolute value of a gain.

1.1.3. *Implementations of CPT*

As previously addressed, CPT asserts that individuals evaluate the available outcomes that comprise each option relative to a common reference point. Past research has supported the notion that reference points are determined in a trial specific context rather than by final states of wealth (Barkan & Busemeyer, 2003; Tversky & Kahneman, 1991). To investigate this further,

CPT was fit to the data using an unshifted (CPTU), shifted (CPTS) and mixed shifted (CPTSm) reference frame.

The CPTU fit the data as though outcomes were evaluated in terms of final states of wealth. That is, each of the outcome values was considered independently of the other outcome values. The CPTS fit the data according to the standard shifted reference frame, whereby the available outcomes are evaluated relative to the most certain outcome. On trials where the options were ambiguous or risky versus a certain option, the certain option value was used as a reference point and thus converted to a subjective value of $0. On trials that presented either two risky or one risky and one ambiguous alternative, the lowest possible outcome across both alternatives was used as the reference point and thus converted to a subjective value of $0. The CPTSm fit the data using a combination of both the CPTU and CPTS approach. On trials where the options were risky or ambiguous versus a certain option, the certain option was again used as a reference point. However, with trials where either two risky or one risky and one ambiguous alternative was presented, the outcomes were evaluated independently of one another, thus relative to final states of wealth.

The purpose of fitting the models in such a way was to evaluate all potential approaches that a decision-maker may take. While the use of the certain outcome as a reference point is a logical assumption, it is less obvious how an individual may cope when presented with more complex alternatives.

All implementations of CPT are based on the four-parameter model discussed in Glöckner and Pachur (2012). Here, using eight separate CPT based models, it was demonstrated that the four-parameter model was sufficiently robust compared to the largest seven-parameter model.

Lastly, it should be noted that all CPT model fitting was done using code taken from the open-sourced method of CPT parameter estimation in Nilsson et al. (2011).

1.1.4. α-Maxmin Model

The α-maxmin model[3] expresses the utilities of options with unstated probabilities by summing across the *n* possible outcomes of that option weighted by a decision weight as follows:

$$U(O_j) = \alpha \cdot v(x_{1,j}) + (1 - \alpha) \cdot v(x_{2,j}) \qquad [6]$$

where $x_{1,j} < x_{2,j}$. The decision weight parameter, $0 \leq \alpha \leq 1$, represents the weight applied to the subjective value of an outcome under the worst probability distribution. That is, the model assumes that the decision-maker applies a subjective probability distribution to the outcomes of an ambiguous option and these distributions represent the decision-makers beliefs about the likelihood of receiving the lowest outcome. For instance, a decision-maker with $\alpha = 0.5$ implies that they believe either outcome is equally likely to occur. Values greater than 0.5 demonstrate that a decision-maker is pessimistic about the option and believes that the lower value is more likely to occur.

2. Methods

2.1. Participants

Participants were recruited using the online University of Victoria SONA research participation system and were compensated with course credit in addition to a small monetary bonus. Participants were informed prior to beginning the task that a bonus of up to $5.00 would be awarded at the end of the experiment and that it depended on their choices made throughout the task. Following completion of the task, participants were asked to report any history of traumatic brain injury, neurological disorders and current medications. Finally, participants were

[3] This version of the α-maxmin model was used in Huettel et al. (2006) to characterize ambiguity preference. For a detailed description of the model see Ghirardato, Maccheroni & Marinacci (2004).

given their monetary bonus and were informed that the amount awarded was calculated based on the ratio of the expected value of the options they chose to the expected value of the optimal choices on all the trials. All participants provided informed consent and the study was approved by the University of Victoria research ethics board.

Ninety-three undergraduate students at the University of Victoria (25 male; mean age = 21.70, *SD* = 5.71) participated in the experiment. Of these 93 subjects, a total of 4 were excluded from the analysis. The exclusion of one participant was based on the reported history of multiple traumatic brain injuries and strokes in addition to a neurological disorder that resulted in five points of central vision. This participant and the three others also performed below a 65% accuracy threshold on catch trials (see task design). Of the 89 participants (12 left handed) included in the analysis, the majority reported having a grade-point average between a B+ and an A.

2.2. Materials

On each trial, participants were required to select between two options that involved monetary outcomes, each with a corresponding probability and value(s). The trials were comprised of options presented in pie-chart format and consisted of four trial types, synonymous with Huettel et al. (2006) (Figure 1).

2.2.1. Stimuli. The decision task was generated using E-Prime software. The options were displayed on a 19-inch computer screen with a white background in a coloured pie chart format (see Figure 3). Options were divided into three categories: risky, certain, and ambiguous. The risky options were comprised of 2 values each with an associated probability of the value occurring. The section of

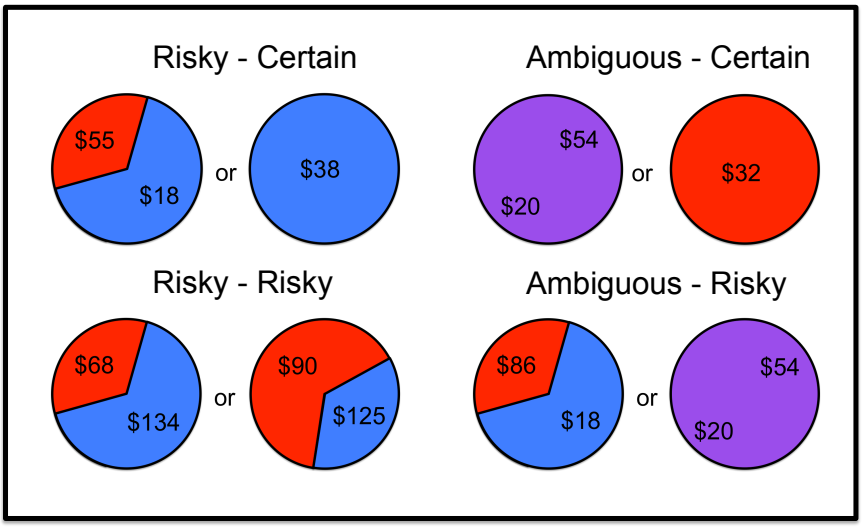

Figure 3. Schematic of trial types use in the present experiment.

the pie representing the probabilities were red and blue and were counterbalanced across the risky options such that the larger probability was never associated with either colour exclusively. The certain options displayed a single value within the center of either a blue or red circle. Lastly, the ambiguous options displayed two values within a purple circle. The values for the ambiguous and risky options were always displayed in opposite sections toward the outer border of the circle subtended at a 45° angle from the midline of the circle.

The values of all the options were classified as either small or large magnitude such that small magnitude values ranged from $2 to $72 and large magnitude values ranged from $80 to $150. Within each of the magnitude domains, ambiguous options were further classified as having either low or high variance such that the values of low variance options were separated by $30 to $40 and the values of high variance options were separated by $60 to $70. On risky options probabilities were classified as low or high such that low probabilities ranged from 10% to 40% whereas high probabilities ranged from 60% to 90%. On each trial, one of the options was classified as a better choice based on the expected value of the option. The expected value of the better choice ranged between 2% to 20% greater than the expected value of the alternative.

In order to minimize the differences across trial types, the values for the options were designed as follows. The values used for certain options in RC trials were used as the expected values for the ambiguous options in AC, AR trials, and one of the risky options in RR trials. Likewise, the values presented for certain options in AC trials were used as the expected values for risky options in RC, AR trials, and one of the risky options in RR trials.

2.2.2. Task. Each trial began with a central fixation cross on a white screen that remained on the screen for 1000ms. The two options then appeared simultaneously and remained on the screen for a total of 3000ms, regardless of whether a response was made. After 3000ms, the central fixation cross re-appeared signifying the beginning of a new trial. After a response was made, a tone was presented through headphones to indicate that the computer had registered the participant's response. If the participant failed to respond within the 3000ms, an alternative tone was presented to indicate that they did not respond in time.

The options were centered along the horizontal plane of the screen with one option on the left and the other on the right hand side of the screen, separated by the word "OR". The side the options were presented on was counterbalanced across options types. The entire task consisted of one round of 8 practice trials and 336 experimental trials. The 8 practice trials consisted of 2 trials for each trial type. Practice trials were used to allow participants to familiarize themselves with the task, the significance of the tones and the speed at which the information was presented. A second purpose of the practice trials was to demonstrate that, while hidden, the ambiguous options did have underlying probabilities and to provide participants with some indication of these underlying probabilities. To do so, practice trials that presented a choice between either an ambiguous and risky option or an ambiguous and certain option, displayed the probability of the ambiguous option after the participant made their choice. To avoid biasing

effects, the underlying probabilities corresponding to the larger outcome were 30%, 40%, 60% and 70%, thus averaging to 50% overall. This aspect of the task was not used in the experimental trials. Lastly, the values presented in practice trials were not used in any of the experimental trials.

The remainder of the task consisted of a total of 336 trials, comprised of 320 experimental trials and 16 catch trials. There were 4 catch trials for each trial type that included an obvious better option such that all values comprising this option were greater than the values comprising the other option. Accuracy on catch trials was used to assess whether participants were actively engaged in the task. The experimental trials were comprised of 80 trials for each trial type. The order of all trials was randomized across all participants.

2.3. Procedure

Upon arrival, participants were greeted by a trained research assistant and taken to one of the laboratory rooms. Participants were seated comfortably in front of a 19-inch computer monitor in a laboratory room at the University of Victoria.

After completing informed consent, participants were provided with both verbal and visual task instructions. The visual instructions included an example of each of the option types, however question marks were displayed instead of values. While the schematic of the options was displayed on the screen, the participants received a verbal description of what each of the option types signified. Participants were also reminded that although the sections of the pies were blue and red, these colours did not provide any additional information regarding the salience of the outcomes (i.e. blue is not better than red and red is not better than blue). This was to avoid a potentially negative association with the colour red. Prior to beginning the task, participants were informed that there is not necessarily a right answer for each decision rather,

they should select the option they would prefer if they were presented with the choice in their everyday lives.

It is important to note that prior to performing the task, it was emphasized that selecting the preferred response was more critical than responding quickly[4]. So, although participants were informed that they would have 3000ms to respond, they should not sacrifice accuracy for speed. However, in order to ensure participants were paying attention to the task and responding to the maximum number of trials, they were also told that a hypothetical $100 would be deducted from their earnings if they failed to respond within the allotted time. They were also told that the hypothetical $100 deduction would be used at the end of the experiment to calculate their overall $5 bonus. Participants made their choices using the "Z" and "/" keys on a keyboard for the left and right option, respectively. Participants were instructed to use their index fingers to make their choices and to keep their index fingers on the keys to ensure the appropriate key was pressed.

Prior to performing the experimental task, participants completed the practice trials (see task description). If participants failed to respond to more than 2 of the practice trials, they were asked to re-do the practice session. During the practice, participants were also instructed to acclimatize to the 3000ms response period. Once participants felt comfortable with the task, they began the experimental trials.

The experimental trials were evenly divided into 4 sections (i.e. 84 trials each), therefore providing participants with 3 untimed break periods. When the break screen appeared, participants were instructed to take some time to look away from the screen or stretch and then proceed when they felt comfortable to do so.

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[4] Dror, Busemeyer, & Basola (1999) demonstrated that placing time pressure on participants in economic decision tasks leads to greater risk taking behaviour.

After completing all of the trials, participants completed a brief demographic form and answered open-ended questions regarding their approach to the task. They were subsequently debriefed and provided with their monetary bonus.

2.4. Parameter Estimation Methods

For both estimation methods, parameters for CPT and EUT models were estimated across all RC and RR trials. For EUT estimation, β parameters were then used to calculate the utility of the outcomes on AC and AR trials. Likewise, CPT β and λ parameter estimates for each individual were used to calculate the utility of the outcomes on AC and AR trials. As well, γ parameters were used to calculate decision weights on for the risky options on AR trials. These utility estimates were then used to estimate α values for each participant using the α-maxmin model. It is important to note that the number of parameters differs across the different model fitting methods (Table 1). All analyses were performed in R 2.14.2.

Table 1. Parameters involved in model fitting techniques for EUT and CPT

Fit Type	**Model**	$\boldsymbol{\beta}$	$\boldsymbol{\lambda}$	$\boldsymbol{\gamma}$	$\boldsymbol{\varphi}$
Optimization	EUT	X	-	-	-
	CPT	X	X	X	-
HBA	EUT	X	-	-	X
	CPT	X	X	X	X

2.3.1. Optimization

In order to replicate the analytical approach of Huettel et al. (2006), each of the models was fit to the data by finding the parameters that maximized the number of correct model predictions. In order to do this, the models were optimized in R using the EasyABC package (Jabot, Faure, Dumoulin & Albert, 2015). Each of the models was estimated using uninformative

priors characterized by uniform distributions bounded by the feasible parameter values as follows. Beta, gamma and alpha parameters were bounded between 0 and 1 whereas lambda values were bounded between 0 and 5.

2.3.2. Hierarchical Bayesian Estimation

While a maximum likelihood estimation (MLE) method is common for parameter estimation, the use of Hierarchical Bayesian analysis (HBA) methods has been shown to provide more accurate parameter estimates at the individual level (Nilsson, Rieskamp & Wagenmakers, 2011). Here, parameters are estimated at the individual level while simultaneously estimating the overall population hyperparameters. Therefore, group-level and individual level distributions are estimated in a mutually constraining and informative manner. To fit the models with HBA, parameters were estimated using the *rjags* package (Plummer, Stukalov, & Denwood, 2016).

For HBA estimation, models were fit in a probabilistic manner as opposed to the deterministic manner used in the optimization estimation. When fitting each model probabilistically, an additional parameter is added to the estimation procedure. In contrast to providing a binary output for each trial for each participant, the model is fit by outputting a probability that the individual chose a particular option on a given trial. The probability, $Pr()$, of choosing an option, A, over an alternative option, B, is determined from their subjective utilities according to a version of the Luce choice rule[5]:

$$Pr(A,B) = \frac{1}{1+ e^{\varphi[SEU(B)-SEU(A)]}} \qquad [7]$$

The sensitivity parameter, $0 < \varphi$, specifies the likelihood that the decision-maker selects the option with the larger subjective expected value. As φ increases, an individual becomes more sensitive to differences in the subjective values of the available options (Figure 4).

[5] This version of the Luce Choice Rule was taken from Nilsson, Rieskamp & Wagenmakers (2012).

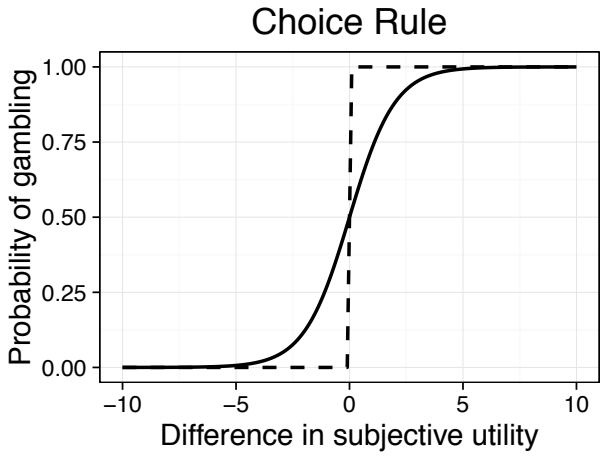

Figure 4. Graphical display of Luce choice parameter

Results

A pairwise t-test was conducted to assess subjects' response times across trial types. Mean response time was fastest on AC trials (M = 1.41s, SD = 0.48) and RC trials (M = 1.47s, SD = 0.51) and slowest for AR (M = 1.70s, SD = 0.51) and RR trials (M = 1.70, SD = 0.51). All reaction time differences across conditions were significant with the exception of AR and RR trials (p = .44).

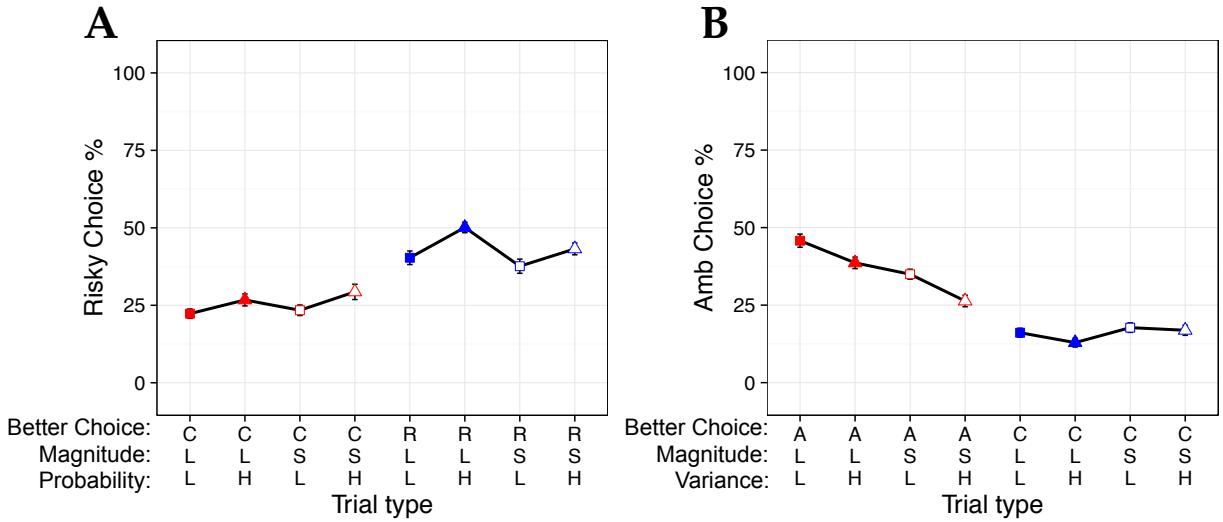

Figure 5. Overall choice percentage for on RC (A) and AC (B) trials plotted as a function of factor types. On the x-axis, the factor levels are indicated for better choice being either certain (C), risky (R), or ambiguous (A), large (L) and small (S) magnitude, low (L) and high (H) probability and low (L) and high (H) variance.

Analyzing across all participants choice data and trial types demonstrates that overall participants chose the risky option more than ambiguous options**,** $t(88) = 7.28$**,** $p < .01$. Within trial type overall participants chose the risky option in RC trials (34.13%) more than they chose the ambiguous option in AC trials (26.17%). In both trial conditions, participants choice percentage reflects a sensitivity to better choice as they selected the risky option in RC trials when it was a better option 42.82% of the time and 25.44% of the time when the certain option was better. Similarly, on AC trials, participants' chose the ambiguous option 36.43% of the time when it was the better choice and 15.91% of the time when the certain option was a better

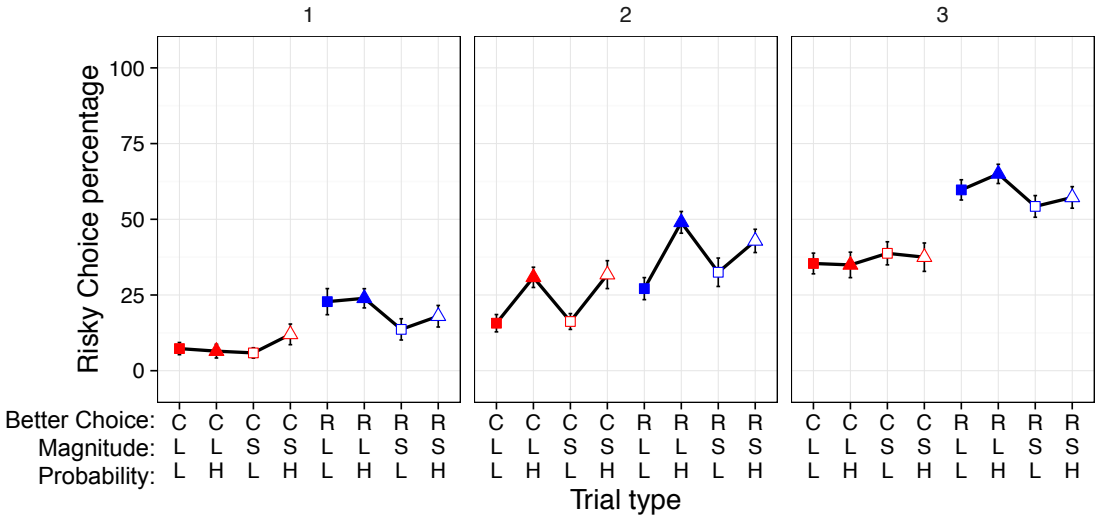

Figure 6. Risky choice percentage on risky-certain trials plotted at cluster level.

choice. Choice data from the RR and AR trials show a similar tendency, indicating that participants were sensitive to better choice overall. Overall risky and ambiguous choice percentage in RC and AC trials are plotted by factor type in Figure 5.

An ANOVA was conducted to assess the effect of probability, magnitude, variance and better choice on choice percentage for each individual trial type [see Appendix B for all ANOVA

summary tables]. On RC trials, there was a significant effect of probability and better choice on participants' risky choice percentage. On AC trials, there was a significant effect of variance, magnitude and better choice on ambiguous choice percentage.

As previously mentioned, past research has shown that distinct sub-populations exist whereby individuals within each sub-population tend to decide in a similar manner. In the present results, visual inspection of the individual level data suggests that overall mean percentage of choice does not provide an adequate summary of the subjects' choice behaviour [see Appendix A]. To assess the existence of distinct sub-populations in the present data, a cluster analysis was performed on risky and ambiguous trials separately, using the method of partitioning around medoids (Kaufman & Rousseew, 1990) as implemented in the *fpc* package in R (Henning, 2015). The Duda-Hart test (Duda & Hart, 1973) indicated significant evidence in favour of more than one cluster on both risky ($p = .0061$) and ambiguous ($p < .001$) trials. Subjects were then partitioned into three clusters in accordance with Bruhin et al. (2010) who found that across cultures, three distinct sub-populations of individuals exist. For risky trials clusters were determined by the percentage of risky or risky option B choices made for all RC and RR trials combined. Likewise, clusters for ambiguous trials were determined by percentage of ambiguous option choices on all AC and AR trials. In comparison to the overall plots depicted in Figure 5, examples of plots for risky and ambiguous choice percentage for individual subjects demonstrates the high degree of variability in responses [See Appendix A]. The number of subjects' within each cluster differed, with 20 subjects appearing in cluster 1, 30 subjects in cluster 2 and 39 subjects in cluster 3 on risky trials. Similarly, on ambiguous trials, 19 subjects comprised cluster 1, 47 subjects comprised cluster 2 and 23 subjects were in cluster 3.

Figure 6 illustrates the risky choice percentage for the three clusters on RC trials. The ANOVA output for RC trials assessing the effect of magnitude, probability and better choice on risky choice percentage for each cluster are provided in Appendix B. Cluster 1 appears highly risk averse as they demonstrate a low risky choice percentage overall whereas Cluster 3 appears

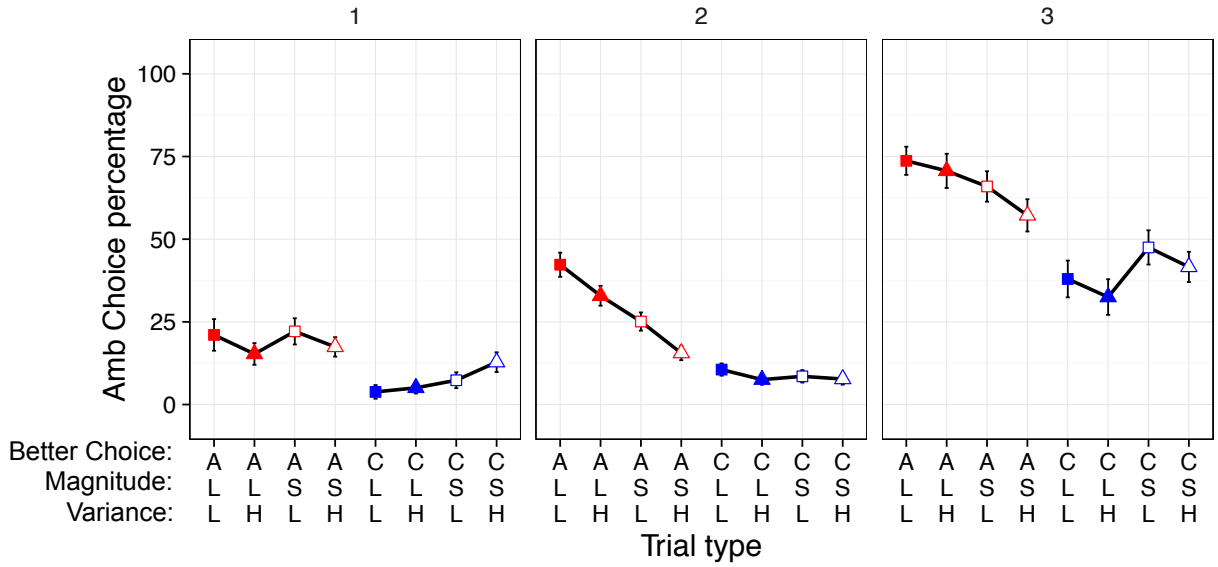

Figure 7. Ambiguous choice percentage plotted at cluster level for ambiguous-certain trials.

to be sensitive to better choice and tends to select the risky option more than the other two clusters. The effect of better choice on risky choice selection is significant for Cluster 1, $F(1, 19) = 16.66$, $p<0.001$, and 3, $F(1, 38) = 83.55$, $p < .001$, meaning that subjects within these clusters chose the risky option more often when it was a better choice. Moreover, Cluster 1 shows a significant interaction between magnitude and better choice, $F(1, 19) = 9.09$, $p = .007$, whereas Cluster 3 shows a significant interaction between probability and better choice, $F(1, 38) = 4.13$, $p = .049$, as well as magnitude and better choice, $F(1, 38) = 11.56$, $p = .002$. Cluster 2 shows a sensitivity to changes in probability, whereby they selected the risky option when the probability of the larger outcome is high, independent of the magnitude of the outcomes. Furthermore, the

ANOVA output indicates that this effect of probability on risky choice percentage is significant, $F(1, 28) = 16.01$, $p < .001$.

Figure 7 illustrates the ambiguous choice percentage for the three clusters on AC trials. The ANOVA output for AC trials assessing the effect of magnitude, variance and better choice on ambiguous choice percentage for each cluster is depicted in Table 2. Here a similar pattern of clusters is shown whereby Cluster 1 appears highly ambiguity averse, rarely selecting the ambiguous option overall. Cluster 3 appears to be less ambiguity averse, selecting the ambiguous option more than any of the other clusters and appears highly sensitive to better choice. The effect of better choice on ambiguous choice selection is significant for Cluster 1, $F(1, 18) = 29.45$, $p<0.001$, and 3, $F(1, 21) = 52.87$, $p < .001$, meaning that subjects within this cluster chose the ambiguous option more often when it was the better choice. Cluster 2 shows a strong sensitivity to the variance and magnitude of the ambiguous option. That is, the perceived riskiness of the variance appears to be modulated by the magnitude of the options whereby changes in the variance within the low magnitude domain leads to a decrease in ambiguous option selection. As indicated in the ANOVA output in Table 2, there is a significant effect of variance, $F(1, 46) = 15.61$, $p < .001$, and magnitude, $F(1,46) = 22.34$, $p <.001$, on ambiguous choice percentage.

It is important to note that the subjects' within each cluster were not identical for risky and ambiguous trials. Of the subjects in cluster 1 on risky trials, 70% of these appeared in cluster 2 on ambiguous trials. Meaning that, on risky trials they generally avoided risk while on ambiguous trials, they were more sensitive to the variance of the ambiguous option. The remaining 30% in cluster 1 for risky trials appeared in cluster 1 on ambiguous trials showing a similar degree of risk and ambiguity aversion overall. For cluster 2 on risky trials, the majority of

subjects (60%) appeared in cluster 2 on ambiguous trials demonstrating a sensitivity to probability on risky trials and sensitivity variance on the ambiguous trials. Of the remaining subjects, 16.7% and 23% of cluster 2 on risky trials appeared in cluster 3 and 1 on ambiguous trials, respectively. Therefore a small portion of the subjects who appeared to be sensitive to the probability of the risky options, were either highly sensitive to better choice or highly ambiguity averse. Lastly, for cluster 3 on risky trials, the majority of the subjects appeared in either cluster 2 (38.5%) or cluster 3 (46.5%) on ambiguous trials. Therefore, most of the subjects who showed a high degree of sensitivity to better choice on risky trials, were either highly sensitive to better choice or variance on ambiguous trials.

3.1. Model Fitting

3.1.1. Optimization. In line with the estimation method used in Huettel et al. (2006), EUT and all reference point implementations of CPT were fit using an optimization function in R. As previously described, the optimization function finds a parameter value that maximizes the number of correct predictions made by the model in comparison to the participants actual choices.

For all models, the first step was estimating the parameters using EUT and CPT for all RC and RR trials. Next, ambiguity preference was estimated by fitting the α-maxmin model to all AC and AR trials using the individual parameter estimates from EUT and each of the implementations of CPT on risky trials. The mean percent correct predictions at the individual level for each of the models demonstrates that overall, all implementations of CPT predicted a greater average of subjects' choices with CPTS and CPTSm performing best overall (Table 2). The EUT estimates of β predicted an average of 64.54% of the subjects' choices, with a minimum of 53.38% and a maximum of 77.93%. In contrast, CPTS parameter estimates

Table 2. Summary of percent correct predictions made on risky versus certain and risky versus risky trials for all models at the individual and cluster level estimated using optimization method.

Model	Level of Fit	Mean % Correct	Min (%)	Max (%)	SD	SE
EUT	Individual	64.64	53.38	77.93	5.54	0.59
	Cluster	65.06	61.94	68.91	3.54	2.04
CPTU	Individual	67.29	54.55	86.93	7.18	0.76
	Cluster	70.52	66.10	78.76	7.14	4.12
CPTS	Individual	70.63	55.84	90.85	7.17	0.76
	Cluster	71.68	67.50	79.17	6.50	3.75
CPTSm	Individual	70.73	57.14	85.71	6.83	0.72
	Cluster	71.79	68.17	79.01	6.25	3.60

Table 3. Summary of percent correct predictions made on ambiguous versus certain and ambiguous versus risky trials using α-maxmin model at the individual and cluster level estimated using optimization method. Model column represents the risky model parameters used to inform the ambiguity preference estimation with α-maxmin.

Model	Level of Fit	Mean % Correct	Min	Max	SD	SE
EUT	Individual	75.82	57.33	97.35	8.38	0.89
	Cluster	75.80	67.58	83.12	7.81	4.51
CPTU	Individual	74.26	56.67	91.39	8.12	0.86
	Cluster	73.65	68.24	76.47	4.60	2.66
CPTS	Individual	74.93	55.70	96.69	8.86	0.94
	Cluster	74.93	67.04	81.87	7.46	4.31
CPTSm	Individual	71.40	45.57	97.35	10.07	1.07
	Cluster	71.37	64.41	77.51	6.59	3.81

correctly predicted an average of 70.63% of the subjects' choices with a minimum of 55.84% and a maximum of 90.85%. Similarly, CPTSm correctly predicted an average of 70.73% of choices with a minimum of 57.14% and maximum of 85.71%. Parameter estimates from each of the models on risky trials were then used to estimate the ambiguity preference using α-maxmin model on all AC and AR trials. Overall, α parameter estimates from all of the models

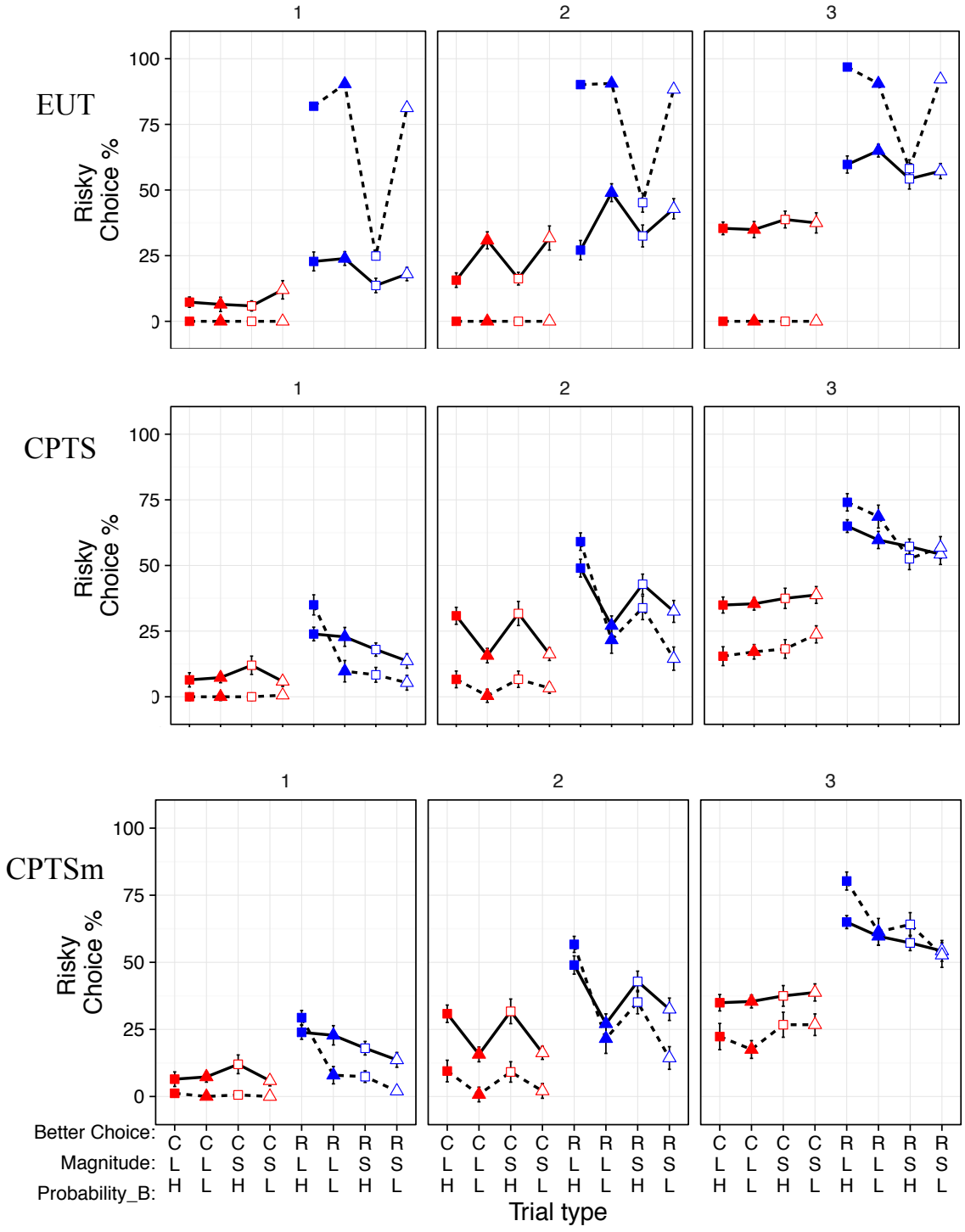

Figure 8. Model prediction for risky choice percentage on RC trials plotted by cluster for EUT, CPTS and CPTSm fit with optimization method.

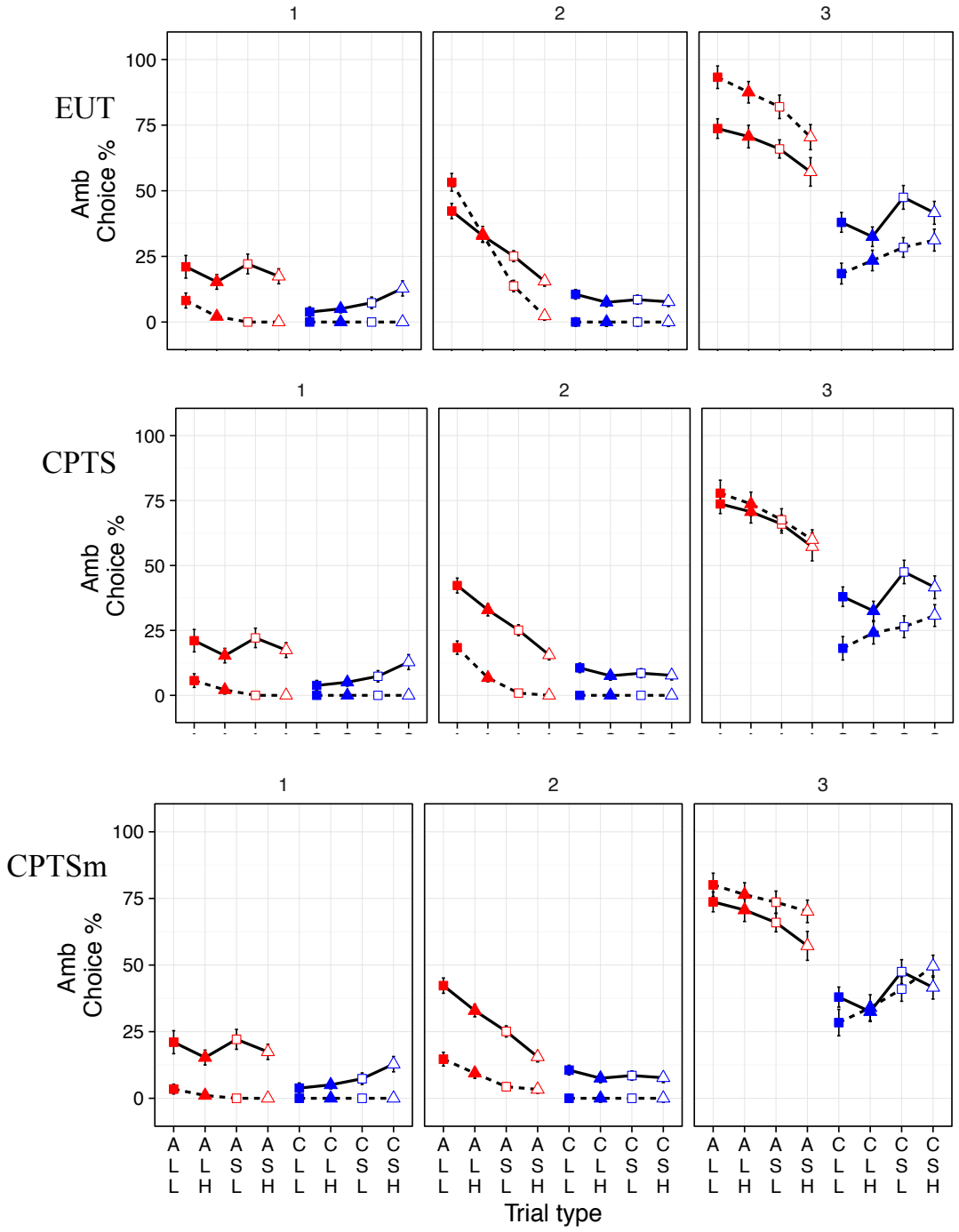

Figure 9. Model prediction for ambiguous choice percentage on AC trials plotted by cluster for EUT, CPTS and CPTSm fit with optimization method.

demonstrated a similar percentage of correct predictions (Table 3). The EUT estimates predicted an average of 75.82% of the subjects' choices, with a minimum of 57.33% and a maximum of 97.32%. Similarly, CPTS estimates predicted an average of 74.93% of the subjects' choices with a minimum of 55.84% and a maximum of 90.85%. Finally, CPTSm again performed similar to CPTS correctly predicting an average of 71.40% of choices with a minimum of 45.57% and maximum of 97.35%. For both risky and ambiguity trial fits, CPTS and CPTSm were able to correctly predict a greater average of subjects' choices overall and will therefore be used for comparison against EUT for the remainder of this section.

The fit at the cluster level on RC trials for EUT, CPTS and CPTSm are shown in Figure 8. The mean correct predictions at the cluster level are shown in Table 4. Overall, the models' fit showed a similar degree of prediction accuracy at the cluster level. Similarly, for AC and AR trials the models showed no improvement in prediction accuracy at the cluster level. Figure 9 shows the fit at the cluster level for AC trials for EUT, CPTS and CPTSm.

3.1.2. Optimization Parameter Evaluation. Mean parameters across all participants for EUT, CPTS and CPTSm are shown in Appendix C. In contrast to Huettel et al. (2006), there was a significant correlation between beta and alpha values obtained using EUT model fitting, $r(87) = -.30$, $p = 0.004$. When fitting the data with CPTS, the correlation between beta and alpha values was not significant, ($p > .10$). However, there was a significant correlation between gamma, the probability distortion parameter, and alpha values, $r(87) = -0.32$, $p < .001$. Similarly, CPTSm parameters demonstrated no correlation between alpha and beta parameters but a significant correlation between gamma and alpha values, $r(87) = -.28$, $p < .001$. Lastly, beta and alpha parameters were significantly correlated across models. Figure 10 displays the scatterplots for beta versus alpha for EUT, CPTS and CPTSm. Gamma versus alpha plots are displayed in

Figure 11 for CPTS and CPTsm. See Appendix D for value and probability functions plotted using cluster level parameters for each model.

Table 4. Correlations for EUT, CPTS and CPTSm parameters using optimization method

Model	**EUT**	**CPTS**				**CPTSm**			
	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\lambda}$	$\boldsymbol{\gamma}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\lambda}$	$\boldsymbol{\gamma}$	$\boldsymbol{\alpha}$
EUT. β	**-.30 ****	**.47 ****	**-.34 ****	.08	-.16	**.46****	**-.42****	**.34****	**-.21***
EUT.α		-.17	**.34 ****	.19	**.56 ****	**-.27***	**.36****	-.08	**.57****
CPTS.β			-.07	**.24 ***	-.05	**.56****	**-.21***	**.23***	-.11
CPTS.λ				.16	-.05	**-.28****	**0.77****	-.13	.04
CPTS.γ					**-.32 ****	**-.22***	.14	**.51****	**-.21***
CPTS.α						.05	.03	**-.24***	**.65****
CPTSm.β							**-.26***	-.04	.02
CPTSm.λ								-.01	.15
CPTSm.γ									**-.28****

Note. Significant value correlations are in bold. *$p < .05$, **$p < .001$.

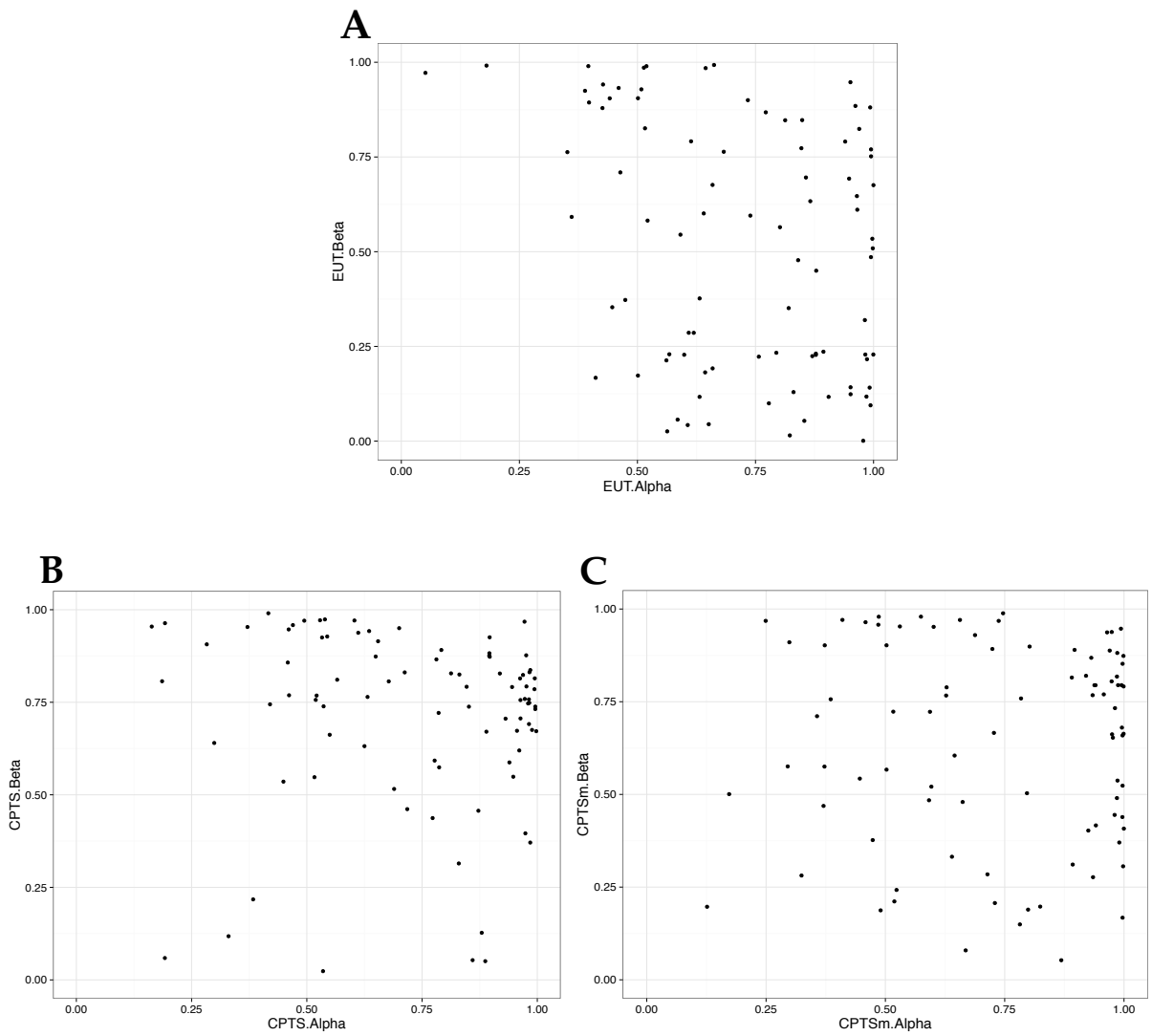

Figure 10. Scatterplots of beta and alpha parameter values from EUT (A), CPTS (B) and CPTSm(C) estimated using optimization method.

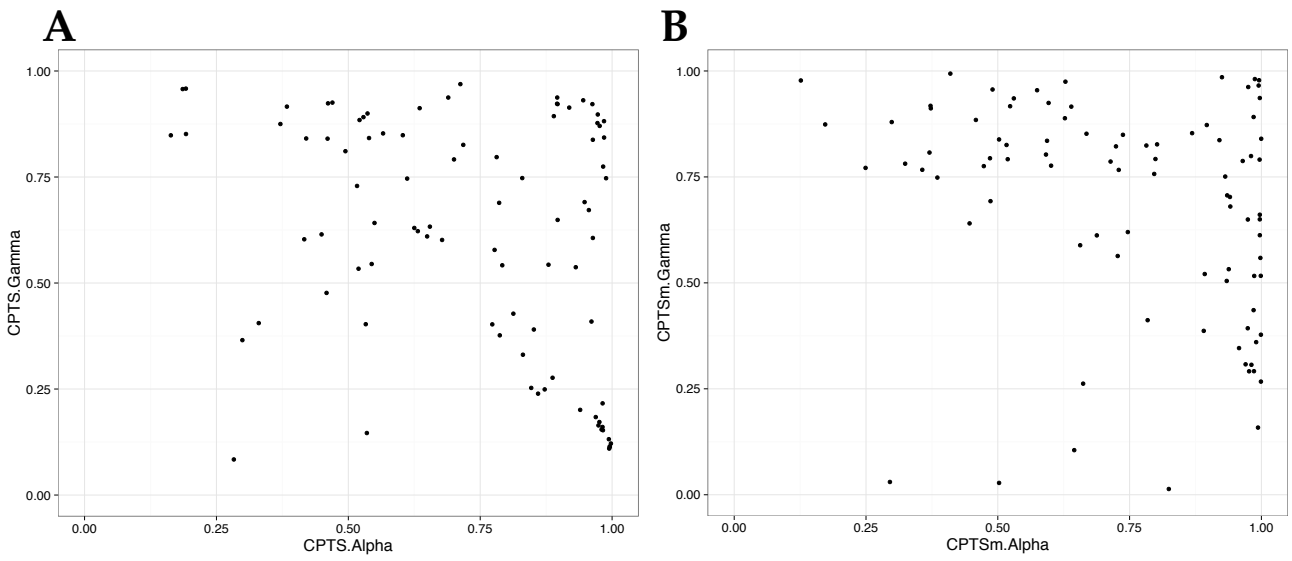

Figure 11. Scatterplots of gamma and alpha parameter values from CPTS (A) and CPTSm(B) estimated using optimization method.

Hierarchical Bayesian Analysis. The mean percent correct predictions at the individual level for each of the models demonstrates that overall, all implementations of CPT predicted a greater average of subjects' choices (Table 5). The EUT estimates of β predicted an average of 80.26% of the subjects' choices, with a minimum of 61.89% and a maximum of 89.16%. In contrast, CPTS parameter estimates predicted an average of 86.66% of the subjects' choices with a minimum of 75.00% and a maximum of 94.39%. CTSm performed similar to CPTS with demonstrating an average correct prediction of 86.21% with a minimum of 77.95% and a maximum of 94.47%. Parameter estimates from each of the models on risky trials were then used to estimate the ambiguity preference using α-maxmin model on all AC and AR trials. Overall, α parameter estimates from all of the models demonstrated a similar percentage of correct predictions (Table 6). The EUT estimates predicted an average of 84.38% of the subjects' choices, with a minimum of 73.45% and a maximum of 94.32%. Similarly, CPTS estimates predicted an average of 86.54% of the subjects' choices with a minimum of 77.54% and a maximum of 95.62%. Estimates from the CPTSm model demonstrated a correct prediction

Table 5. Summary of percent correct predictions made on risky versus certain and risky versus risky trials for all models at the individual and cluster level estimated using HBA estimation method.

Model	Level of Fit	Mean % Correct	Min (%)	Max (%)	SD	SE
EUT	Individual	80.26	61.89	89.16	5.41	0.57
	Cluster	87.89	82.58	92.29	4.92	2.84
CPTU	Individual	84.37	62.18	93.77	5.31	0.56
	Cluster	93.43	92.36	94.81	1.25	0.72
CPTS	Individual	86.66	75.00	94.39	3.66	0.39
	Cluster	94.55	93.55	95.72	1.10	0.63
CPTSm	Individual	86.21	77.95	94.47	3.30	0.35
	Cluster	94.20	92.84	95.64	1.40	0.81

Table 6. Summary of percent correct predictions made on ambiguous versus certain and ambiguous versus risky trials using α-maxmin model at the individual and cluster level estimated using HBA estimation method. Model column represents the risky model parameters used to inform the ambiguity preference estimation with α-maxmin.

Model	Level of Fit	Mean % Correct	Min (%)	Max (%)	SD	SE
EUT	Individual	84.38	73.45	94.32	4.65	0.49
	Cluster	91.18	86.85	94.80	4.02	2.32
CPTU	Individual	68.35	24.39	99.38	14.84	1.57
	Cluster	70.67	52.43	84.60	16.51	9.53
CPTS	Individual	86.54	77.54	95.62	3.49	0.37
	Cluster	94.28	92.31	96.62	2.11	1.22
CPTSm	Individual	85.40	76.51	95.79	3.78	0.40
	Cluster	92.88	89.62	96.47	3.42	1.98

average of 85.50% with a minimum of 76.51% and a maximum of 95.79%. As with the optimization fit, overall the CPTS and CPTSm showed a similar prediction accuracy, correctly prediction more subjects' choices than EUT parameter estimates. Finally, using HBA estimation methods provides the added benefit of model comparison with the use of the Deviance

Information Criterion (see Berg, Meyer & Yu, 2002 for example). When comparing between two models, this approach provides a measure of the goodness-of-fit while penalizing a model for increasing model complexity. When computing the DIC values, a negative value is associated with an overall better fit to the data. On risky trials, CPTS performed better when compared to EUT (DIC = -1558.88, *SD* = 76.77) and CPTU (DIC = -1119.92, *SD* = 64.98). Similarly, CPTSm performed better when compared to EUT (DIC = -438.96, *SD* = 52.76) and CPTU (DIC = -280.56, *SD* = 54.12). Finally, the CPTS fit was marginally better than CPTSm (DIC = -175.39, *SD* = 21.43).

The fit at the cluster level on RC trials for EUT, CPTS and CPTSm are shown in Figure 12. The mean correct predictions at the cluster level are shown in Table 5. Overall, all model fits showed an increase in prediction accuracy at the cluster level where CPTS and CPTSm correctly predicted 94.55% and 94.20% of choices, respectively, on RC and RR trials. EUT showed an increase in prediction accuracy on cluster trials (*M* = 87.89%) but was lower than both CPT models. A similar trend is shown on AC and AR trials whereby CPTS and CPTSm correctly predicted 94.28% and 92.88% of subjects' choices at the cluster level. EUT performed better at the cluster level on AC and AR trials compared to RC and RR trials showing a mean correct prediction of 91.18% at the cluster level. Model predictions are at the cluster level are shown in Figures 11 and 12 for RC and AC trials, respectively. Overall, fitting the models using HBA method appeared to provide more accurate predictions at the individual level and cluster level on both risky (Figure 14) and ambiguous trials (Figure 15).

3.1.4. HBA Parameter Evaluation. Mean parameters across all participants for EUT, CPTS and CPTSm are shown in Appendix C. Alpha and beta values within each of the model fits were not significantly correlated with one another. For the CPTS fit, the correlation between gamma and

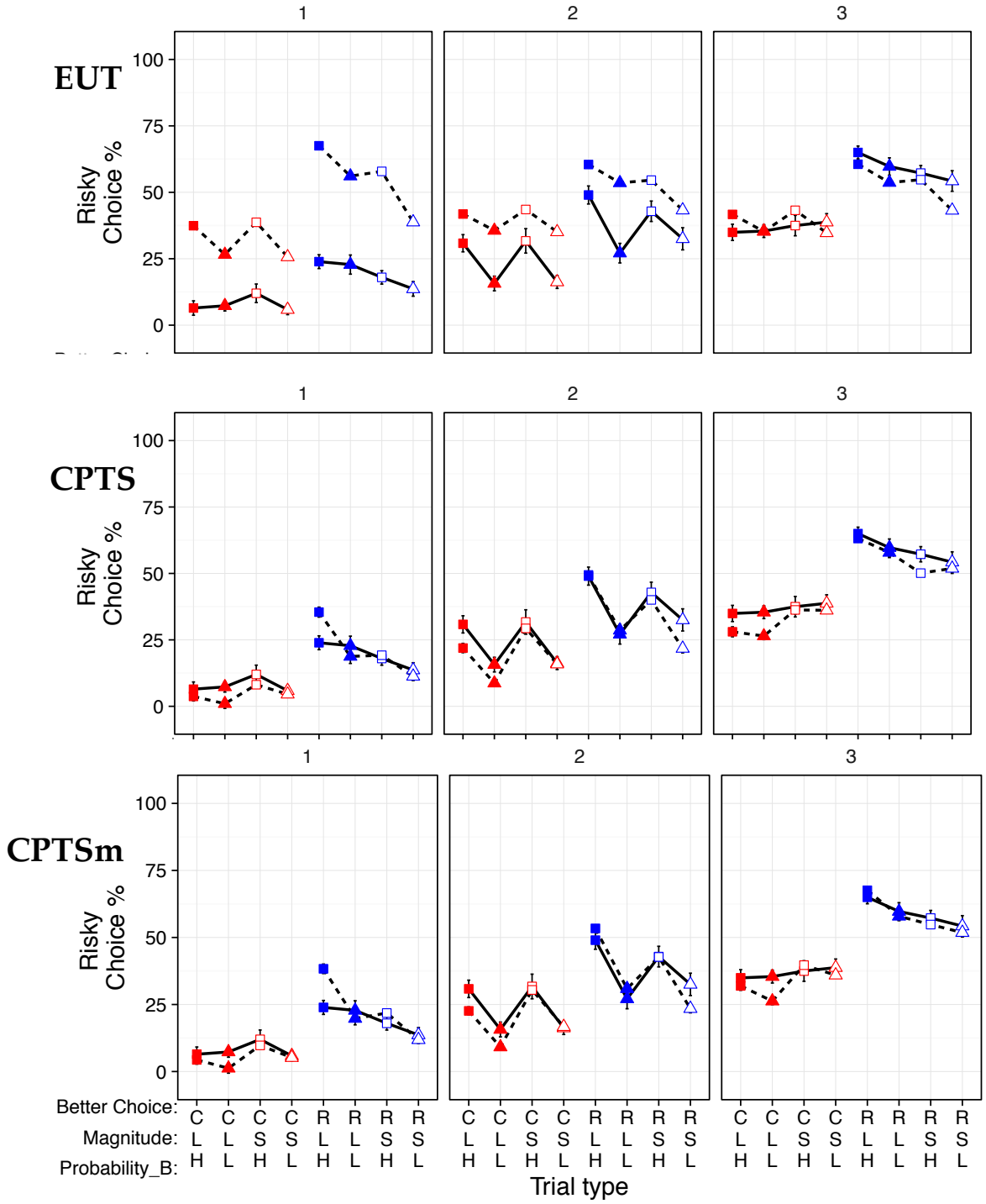

Figure 12. Model predictions from HBA fit for EUT, CPTS and CPTSm for RC trials plotted at the cluster level.

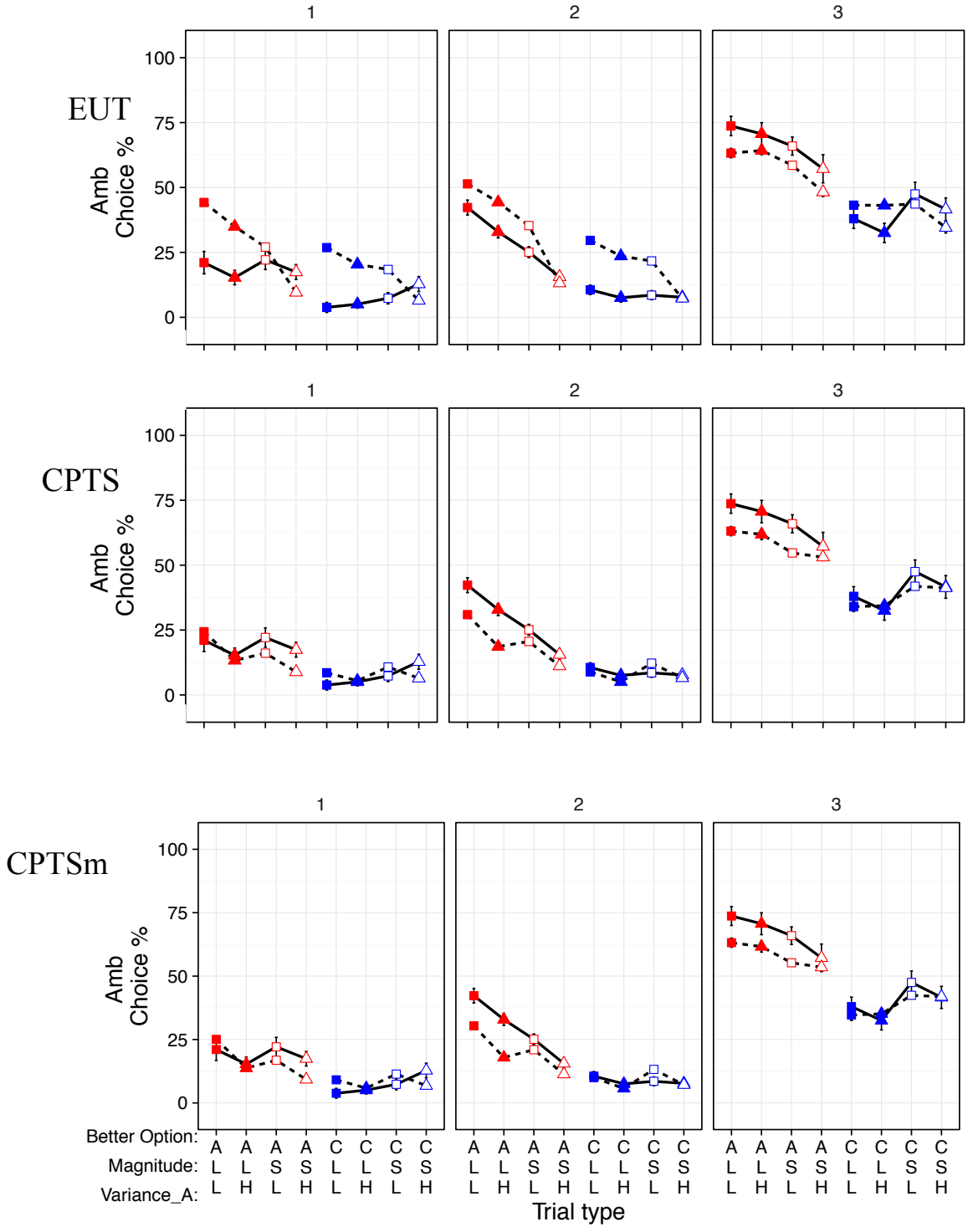

Figure 13. Model predictions from HBA fit for EUT, CPTS and CPTSm for AC trials plotted at the cluster level.

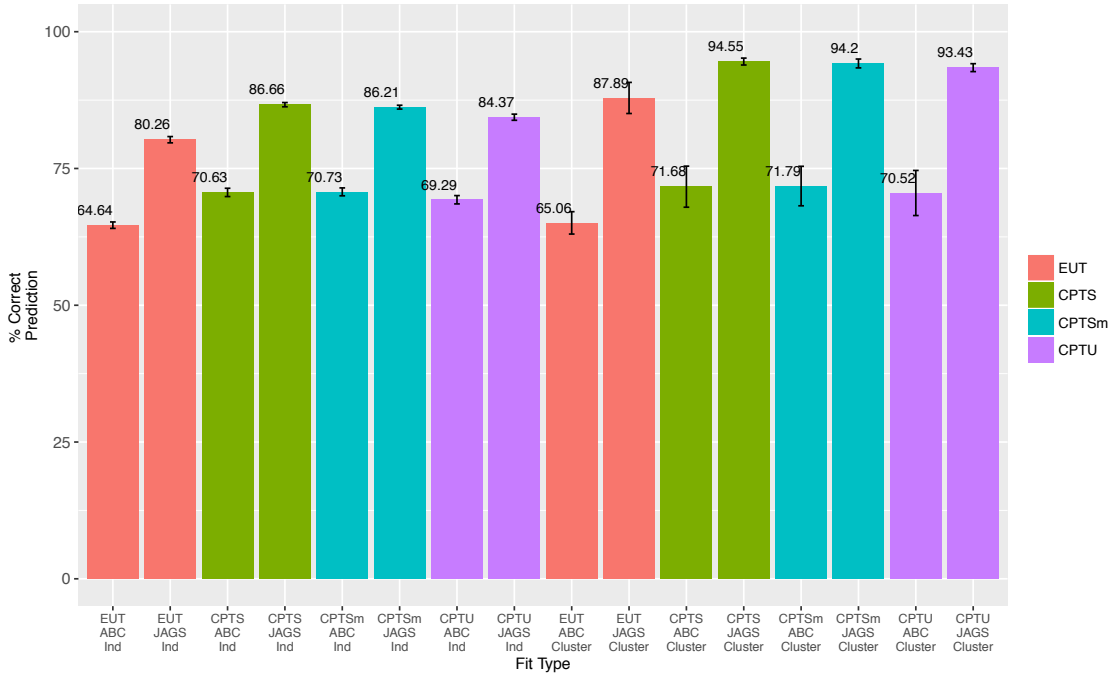

Figure 14. Percent correct predictions for all risky trials across each fit type and model. Error bars represent standard error of the mean. On the x-axis, ABC and JAGS represent the optimization and HBA method, respectively.

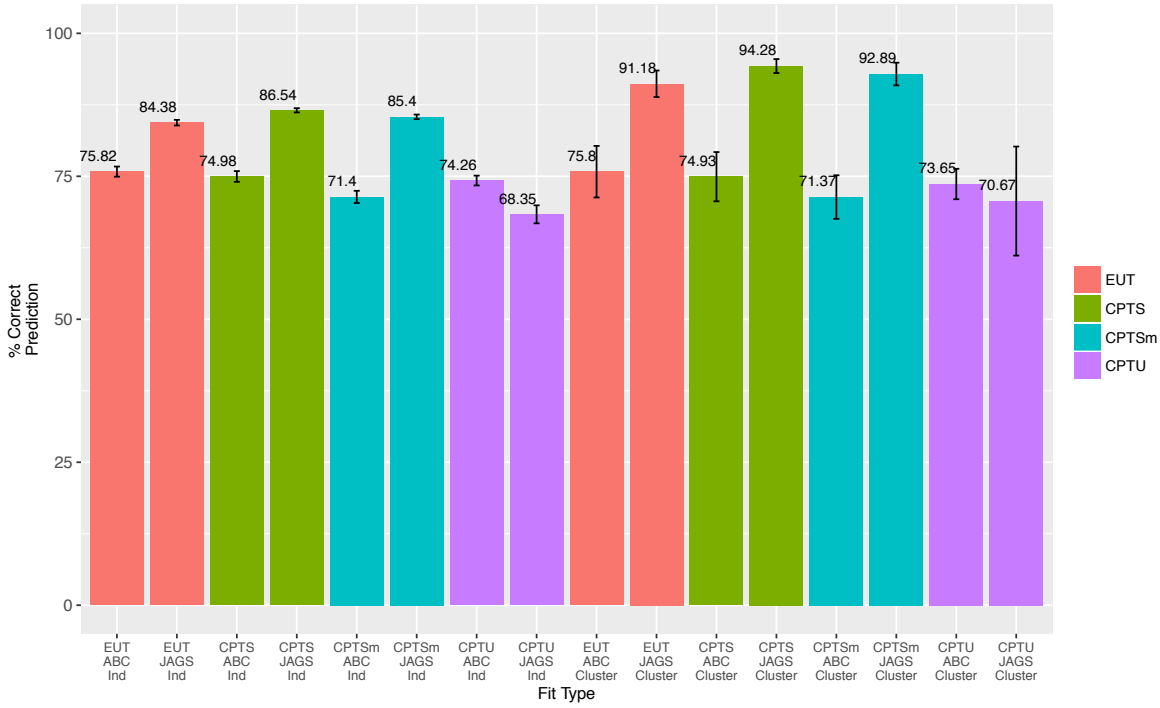

Figure 15. Percent correct predictions for all ambiguous trials across each fit type and model. Error bars represent standard error of the mean.

alpha was not significant, $r(87) = -.16$, $p = .08$. However, for the CPTSm fit, the gamma and alpha parameters were significantly correlated with one another, $r(87) = -.21$, $p < .01$. As with the optimization method, alpha and beta values across each of the models significantly correlated with one another. Figure 16 displays the scatterplots for gamma and alpha values from CPTS and CPTSm. Figure 17 displays the scatterplots for beta and alpha parameters estimated from EUT, CPTS and CPTSm. See Appendix D for value, probability and choice rule functions plotted using cluster level parameter values for each of the models.

Table 7. Correlations for EUT, CPTS, CPTSm and α-maxmin estimates obtained using HBA fit method

Model	**EUT**	**CPTS**				**CPTSm**			
	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\lambda}$	$\boldsymbol{\gamma}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\lambda}$	$\boldsymbol{\gamma}$	$\boldsymbol{\alpha}$
EUT. β	.05	**.49****	**-.36****	.07	.10	**.57****	-.06	**-.34****	-.09
EUT.α		**-.33****	**.34****	.05	**.86****	-.15	-.14	**-.40****	**.88****
CPTS.β			**-.38****	**.39****	-.11	**.90****	**-.27****	**-.43****	-.15
CPTS.λ				.00	.00	**-.43****	-.02	**-.97****	.16
CPTS.γ					-.16	**-.52****	**.96****	-.04	**-.20***
CPTS.α						.07	-.19	.15	**.96****
CPTSm.β							**-.45****	**-.40****	.03
CPTSm.λ								-.07	**-.26***
CPTSm.γ									**-.21***

Note. Significant value correlations are in bold. *$p < .05$, **$p < .001$.

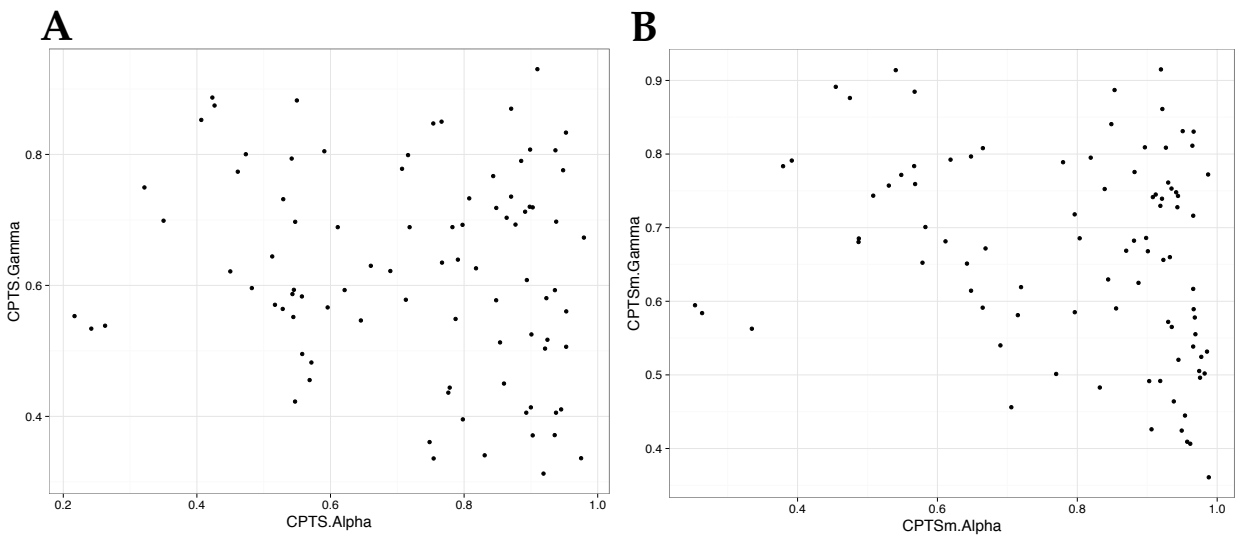

Figure 16. Scatterplots of gamma and alpha parameter values from CPTS (A) and CPTSm(B) estimated using HBA method.

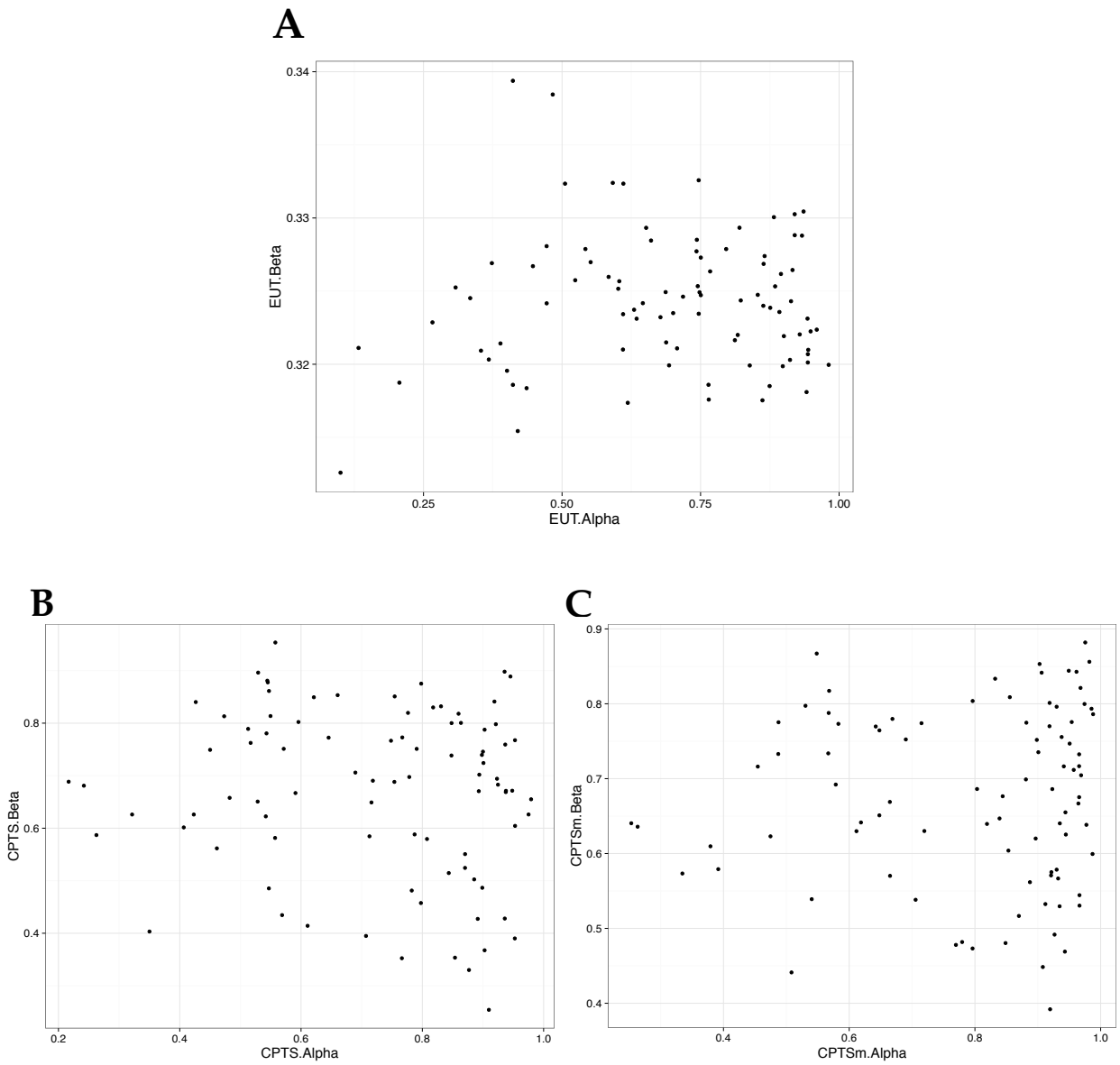

Figure 17. Scatterplots of beta and alpha parameter values from EUT (A), CPTS (B) and CPTSm(C) estimated using HBA method.

4. Discussion

The goal of the present work was to re-evaluate the relationship between risk and ambiguity in the context of economic uncertainty. As previously discussed, past research concluded that risk and ambiguity are independent forms of uncertainty that utilize disparate neural processes (Huettel et al., 2006). Considering the methodological approach taken in

Huettel et al. (2006) raises concerns about the accuracy of the conclusions drawn regarding the processing of risk and ambiguity from both a behavioural and a neural standpoint. Therefore, the present work attempted to improve upon these concerns by evaluating risk and ambiguity preferences using more robust models of choice.

From a purely behavioural standpoint, the results support previous literature that suggests individuals are generally ambiguity averse (Becker and Brownson, 1964; Ellsberg, 1961). When looking at decision-preferences across all trial types, it is evident that overall subjects' tended to exhibit ambiguity aversion to a greater extent than risk aversion. That is, overall subjects' tended to select risky options more so than ambiguous options. However, observing decision-making by collapsing across all participants has the potential to mask heterogeneity in decision preferences (Gonzalez & Wu, 1999). Moreover, past research has also indicated that this heterogeneity can be accounted for at the level of subpopulations or clusters (Bruhin et al., 2010). Therefore, in the present study, decision-making was assessed at both the individual and cluster level.

The findings presented above suggest that important individual differences exist in decision-making styles and that these differences can be accurately represented at the cluster level for options that involve both risk and ambiguity. With respect to trials that involved only risk and certainty (i.e. RC and RR trials), individuals' choices were driven by the characteristics of the options, namely: probability, magnitude and better choice. Similarly, on trials that involved ambiguity (i.e. AC and AR trials), individuals' choices appeared to be influenced by variance of the ambiguous option, magnitude and better choice. In both sets of trials, there appears to be independent subpopulations that tend to be highly ambiguity and risk averse as well as a separate subpopulation that is highly sensitive to better choice. For those subjects who fall within the highly risk averse group, a large portion of them appear to demonstrate high

ambiguity aversion however their approach to ambiguity appears to be modulated by the variance of the outcomes. The risky trials in the present experiment did not increase the variance on the risky options and therefore, it is possible that providing a wider range of variance on risky options would have a similar effect on their choices. Alternatively, the group of individuals who appeared highly sensitive to better choice and demonstrated a greater overall risky choice percentage than other groups, appeared to be split between two of the ambiguous trial groups. A large portion of these individuals remained in a similar group within the ambiguous trials, selecting the ambiguous option more overall compared to the other groups and being highly sensitive to better choice. The second large portion of the subjects appeared in the group highly sensitive to variance on ambiguous trials. Most interesting however, is cluster 2 in both types of trials. On risky trials, this cluster appears to be sensitive to the riskiness of the options in terms of probability. That is, their proportion of risky option selections increases as the probability of winning the larger outcome increases, independent of better choice. Interestingly, the majority of individuals that comprise this risky cluster are present in the cluster of individuals on ambiguous trials that appear to base their decision-making on the variance between the outcomes of an ambiguous option. Within this cluster, the proportion of ambiguous options selected increases as the variance between the options decrease. As discussed in Ellsberg (1961), variance between the outcomes of an ambiguous option may be used as a signal of the riskiness of that option. Therefore, it is possible that this cluster of subjects' use the variance of the outcomes as a signal for the likelihood that they will receive the higher outcome in the option. In the absence of objective probabilities, individuals may use the variance of the outcomes within an option to guide their perception of likelihood.

4.1. Model Fitting

In order to evaluate the choice models for comparison with Huettel et al. (2006), all models were estimated using an optimization method. This method fits each model by finding the parameters that maximize the correct model predictions for each subject individually. Across all the models, CPTS and CPTSm were shown to correctly predict a greater percentage of subjects' choices when compared to EUT and CPTU. In comparison with Huettel et al. (2006), the EUT model here correctly predicted a lower average of subjects' choices on risky trials. This could be explained by the larger variety in the trial types used in the present experiment. As Glöckner and Pachur (2012) discuss, simple models tend to be suboptimal when a given a more complex array of conditions. However, this does not imply that the parameter estimates described in Huettel et al. (2006) were sufficiently accurate for the more simple trials. As previously discussed, the range of correct predictions in the previous work does not indicate that the EUT model accounting for all types of choice preferences. In fact, it is entirely possible that the EUT model used was able to accurately account for the decisions of a small proportion of the subjects that tend to approach decisions by maximizing utility. Moreover, regardless of predicting an average of 64% of the subjects' choices, the fit at the cluster level demonstrates the inability for an EUT model to accurate account for variations within the factor levels (Figure 7). For instance, although subjects' displayed a significant variation in choice percentage when the certain option is classified as the better choice, and EUT model fit in a deterministic manner consistently selects the certain option. Here, the use of graphical displays of model predictions clearly demonstrates that basing model success on prediction average alone does not provide any indication of how the model actually represents individuals' choices.

In comparison with the optimization fit method, HBA fitting was shown to be more optimal as each of the models appear to correctly predict a greater proportion of subjects' choices when compared to their optimization analogue. First, it should be clarified that the increase in prediction accuracy could be due to the added Luce choice parameter. It is well known that increasing the number of parameters in a model leads to increased model flexibility and therefore a greater ability to account for variability in the data (Glöckner & Pachur, 2012). Moreover, the increased flexibility could lead to over fitting the data. Apart from model flexibility, the increase in prediction accuracy could be due to the ability of a probabilistic model to account for the inherent 'noise' in decision-making. It seems reasonable that when confronted with a series of decisions, individuals' decision styles may not be deterministic. That is, although an individual generally prefers a particular option, they may not select that option on every occasion under every individual circumstance.

Within the HBA fitting method, the models were assessed for fit using the DIC that takes into account the number of parameters in a model. On risky trials, all implementations of CPT outperformed EUT. Comparing across CPT implementations, CPTS and CPTSm appeared to fit the data equally well, with CPTS slightly outperforming CPTSm. When fitting the α-maxmin model to the ambiguous trials however, EUT outperformed CPTU however both CPTS and CPTSm outperformed EUT. Together these results support prior research that suggests individuals evaluate risky options (Kahneman & Tversky, 1991) and ambiguous options relative to a common reference point and not final states of wealth.

4.2. Parameter Values

In contrast to the findings presented in Huettel et al. (2006), the alpha and beta parameters resulting from the optimization fit with EUT were significantly correlated with one another.

Again, this discrepancy could be due to the increased complexity in the array of decisions provided to participants. Looking at the CPT parameter estimates however, shows no correlation across alpha and beta values for both HBA and optimization. Given that these models demonstrated a better fit to the actual data at both the individual and cluster level, it seems likely that the parameter estimates provided by these models are more accurate. Therefore it is possible to conclude that the processes used in a decision processes represented by these two parameters are unrelated to one another. While the lack of a relationship between these two parameters is similar to the result found in Huettel et al. (2006), this does not mean that their conclusions about risk and ambiguity with respect to alpha and beta values were accurate. That is, the lack of relationship is not surprising if one considers how the values of these parameters are used to evaluate the utility of risky and ambiguous options. Beta values define the curvature of the diminishing marginal utility function and therefore represent the extent to which an individual discounts the underlying values of an option. Alpha parameters however, represent the weight applied to the already discounted values of an ambiguous option or, an individuals' belief in the likelihood that an outcome will occur. There is no evidence to suggest that value discounting and likelihood evaluation are related processes, nor does the lack of relationship between these two parameters illustrate anything in regards to the relationship between how individuals process risk and ambiguity. Given the increased success of the CPTS and CPTSm models compared to EUT model fits using HBA estimation, it can be assumed that the parameter estimates from these models are sufficient in demonstrating that alpha and beta parameters are unrelated.

An interesting finding in the present work shows a potential relationship between gamma values obtained from CPT estimates and alpha values obtained from the α-maxmin model. When fit with both optimization and HBA, CPTSm showed a significant negative correlation between

gamma and alpha values. This is not surprising given the similarities in the way each parameter is incorporated in the evaluation of an option's utilities. With the CPT model, gamma values define the extent to which an individual distorts objective probabilities. Therefore, an individual with a low gamma value tends to distort the objective probability that they will receive a given outcome. Similarly, with the α-maxmin model, the extent to which an individual discounts the likelihood of receiving the larger of two outcomes is represented by the alpha parameter. For instance, an individual with a high alpha value tends to overweight the likelihood that they will receive the smaller of two outcomes within an option. In the absence of information, it would seem rational for a decision-maker to assume that each outcome in an ambiguous option is equally likely. In this circumstance, the rational decision-maker would be characterized with an alpha value of 0.5. However, as an individuals' alpha value deviates from 0.5, it would suggest that the likelihood of outcomes occurring is being distorted. In the case of a risky option, a rational decision-maker is assumed to objectively evaluate the probabilities that are provided to them, therefore displaying a high gamma value. It is possible then that those who demonstrate a larger deviation from an alpha value of 0.5, distorting rational likelihood, would also tend to distort probability, the symbol of likelihood within a risky option.

4.3. Implications for Risk and Ambiguity

The present results demonstrate that the relationship between risk and ambiguity is complex and may not be accurately described by estimates derived from EUT models of choice. One issue with evaluating the relationship between these two levels of uncertainty with EUT is that EUT associates risk preference with a single parameter that defines the curvature of the utility function or, the extent to which an individual discounts objectively stated values. The inherent riskiness of an option however is not necessarily represented in the values of that option.

Rather, an option becomes risky when its associated outcomes become probabilistic, that is, when the outcomes no longer occur with certainty. Therefore, a more appropriate assessment of risk preference would be to evaluate how an individual copes with probability. In line with this, the present results suggest that risk and ambiguity may be related to one another and that, this relationship can be represented by an individuals' CPT derived probability discounting function and their ambiguity preference parameter derived from the α-maxmin model.

In Huettel et al. (2006), the associated neural processes of risk and ambiguity were evaluated with respect to EUT beta and the EUT informed α-maxmin parameter. The authors demonstrated that these parameters were independently correlated with separate neural components and thus concluded that risk and ambiguity evoke disparate neural processes. The first issue with this conclusion is that beta parameters are not necessarily related to risk preference, as previously discussed. Moreover, as demonstrated in the present work, EUT derived parameters are highly inaccurate when describing actual decision behaviour. Therefore, it is logical to conclude that the true neural signatures of risk and ambiguity preference were inaccurately depicted in Huettel et al. (2006). The results from the present work suggest that CPT derived parameters and a CPT informed α-maxmin parameter more accurately represent the subjects' choices and therefore provide a more efficient representation of risk and ambiguity preference.

4.4. Limitations & Future Directions

As addressed previously, there is a potential for over fitting the data using HBA methods given the added flexibility of the model fitting technique (Glöckner & Pachur, 2012). In order to assess whether the over fitting led to the observed increase in prediction accuracy, future analysis could utilize a cross-validation method. Here, parameters for each of the models and fits could be

estimated on a random subset of trials, excluding a subset for the evaluation procedure. These parameter estimates would then be applied to the excluded trials to assess the prediction accuracy of the models.

In addition, model estimates from CPTS and CPTSm provided slightly different results with respect to the parameter evaluations. It is difficult to assess the differences of the fits for these models as they both appeared to accurately account for subjects' decision-making to a similar extent. The two models differed in the extent to which a shifted-reference frame was applied to the estimation processes. For CPTS, the shifted-reference frame, all values were referenced to the certain outcome on RC and AC trials. On RR and AR trials, the reference point was the lowest possible outcome across both options. For CPTSm however, the shifted-reference point was only applied to AC and RC trials whereas no reference point was associated with RR and AR trials. While past research supports the idea that when available, the certain option acts as a reference point, less is known about decision involving more complex options. The present results fail to adequately distinguish between these two types of CPT implementations. Therefore, future research could use self-report measures to assess how individual approach these complex options in order to determine the more appropriate choice of reference point.

It is important to note that the present analysis is not exhaustive and was not intended to assess the quality of the α-maxmin model. Within the literature, several models for ambiguity preference have been proposed, however the superiority of all of these models remains controversial (Dimmock, Kouwenberg, Mitchell, & Peijnenburg, 2015, Einhorn & Hogarth, 1985). A complete evaluation of all models of ambiguity is beyond the scope of the present thesis. Rather, the purpose here was to demonstrate that CPT models of choice are more optimal for representing actual individual choice preferences in the context of economic risk. Given the

results it is evident that, should a model that estimates an individuals' preference for ambiguity require the use of risk preference parameters, these parameters should be estimated using a CPT model.

5. Conclusions

The present results suggest that, overall subjects appear to select risky options more so than ambiguous options however, this trend does not represent the underlying individual decision-making styles that exist within a population. When individual decision-making styles are taken into consideration, some similarities between ambiguity and risk assessment are observed at the behavioural level. Moreover, if the goal of decision-making research is to represent the underlying facets of these decision-making styles using cognitive models of choice, it is necessary to select the appropriate descriptive models to do so. The present work demonstrates that for risky decision-making behaviour, the CPT model is superior to EUT in its descriptive quality of individual choice preference. Furthermore, estimating these parameters using a probabilistic, rather than deterministic, choice model provides a more accurate representation of the minor inconsistencies in human choice behaviour. Finally, when examining an individual's perception of risk it is important to consider that parameter used to represent this perception.

6. References

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Appendix A
Example of Individual Plots

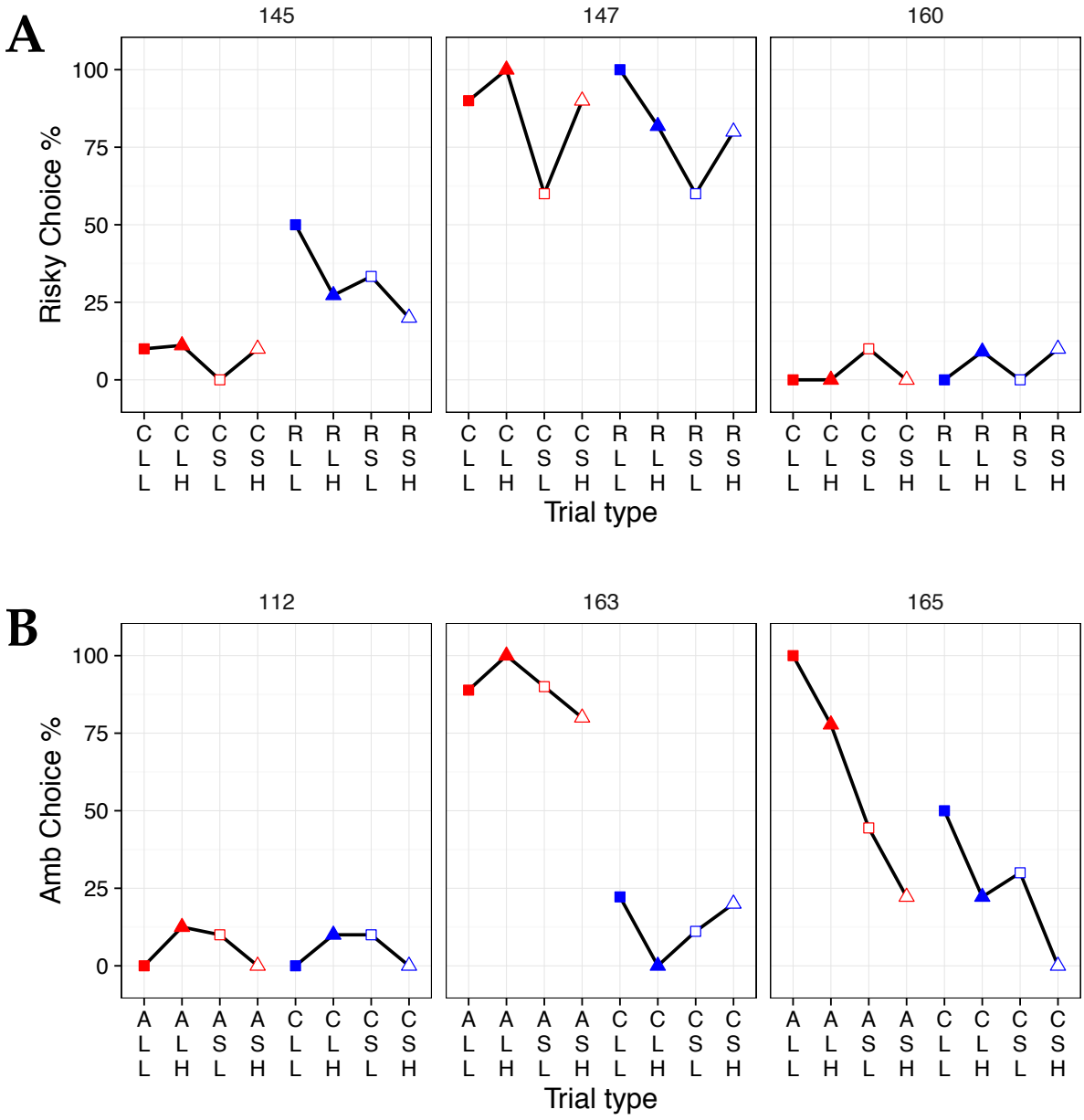

Figure 18. Example plots depicting choice percentage for individual subjects on RC (A) and AC (B) trials.

Appendix B
ANOVA Summary Tables

Table 8. Summary of ANOVA effects for all participants for RC trials.

Effect	**DF**	**SS**	**DFE**	**SSE**	**F**	**p**
Probability	1	7360.07	88	73648.13	8.79	0.004*
Magnitude	1	393.31	88	40925.62	0.85	0.360
Better Choice	1	53792.29	88	37180.78	127.32	<0.001*
Probability X Magnitude	1	80.51	88	20788.05	0.34	0.561
Probability X Better Choice	1	280.28	88	13846.24	1.78	0.185
Magnitude X Better Choice	1	1981.68	88	13750.09	12.68	< 0.001*
P X M X BC	1	360.29	88	15636.77	2.03	0.158

Table 9. Summary of ANOVA effects for all participants for AC trials.

Effect	**DF**	**SS**	**DFE**	**SSE**	**F**	**p**
Variance	1	4366.27	88	17655.52	21.76	< 0.001*
Magnitude	1	3407.20	88	50464.86	5.94	0.017 *
Better Choice	1	74915.50	88	39423.68	167.22	< .001*
Variance X Magnitude	1	8.22	88	13759.04	0.053	0.819
Variance X Better Choice	1	1533.98	88	13462.64	10.03	0.002*
Magnitude X Better Choice	1	9189.09	88	15422.02	52.43	< 0.001*
V X M X BC	1	163.88	88	8686.14	1.66	0.201

Table 10. Summary of ANOVA effects for cluster 1 level for RC trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
1	Probability (P)	1	301.71	19	4234.97	1.42	0.247
1	Magnitude (M)	1	315.26	19	2931.66	2.15	0.158
1	Better Choice (BC)	1	5732.17	19	6881.46	16.66	< 0.001*
1	P X M	1	273.38	19	2271.71	2.41	0.136
1	P X BC	1	0.075	19	1709.54	0.00088	0.977
1	M X BC	1	966.69	19	2129.76	9.08	0.007 *
1	P X M X BC	1	36.73	19	2075.89	0.35	0.559

Table 11. Summary of ANOVA effects for cluster 2 for RC trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
2	Probability	1	14127.26	28	24706.16	16.01	<0.001 *
2	Magnitude	1	0.81	28	14290.98	0.0016	0.968
2	Better Choice	1	12658.87	28	8788.06	40.33	< 0.001 *
2	P X M	1	475.05	28	11209.93	1.19	0.285
2	P X BC	1	12.15	28	7431.02	0.046	0.832
2	M X BC	1	6.44	28	4977.33	0.036	0.850
2	P X M X BC	1	587.80	28	5595.37	2.94	0.097

Table 12. Summary of ANOVA effects for cluster 3 for RC trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
3	Probability	1	206.22	38	37431.88	0.21	0.650
3	Magnitude	1	260.92	38	23519.30	0.42	0.520
3	Better Choice	1	39120.58	38	17791.93	83.55	< 0.001 *
3	P X M	1	47.51	38	6590.98	0.27	0.604
3	P X BC	1	487.71	38	4486.03	4.13	0.049 *
3	M X BC	1	1784.63	38	5866.91	11.56	0.002 *
3	P X M X BC	1	11.03	38	7690.24	0.054	0.817

Table 13. Summary of ANOVA effects for cluster 1 for AC trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
1	Variance (V)	1	33.53	18	1818.35	0.33	0.572
1	Magnitude (M)	1	498.54	18	7934.55	1.13	0.302
1	Better Choice (BC)	1	5220.95	18	3191.16	29.45	< 0.001*
1	V X M	1	66.89	18	860.18	1.40	0.252
1	V X BC	1	698.47	18	1574.28	7.99	0.011
1	M X BC	1	155.24	18	3373.96	0.83	0.375
1	V X M X BC	1	23.90	18	1440.48	0.30	0.591

Table 14. Summary of ANOVA effects for cluster 2 for AC trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
2	Variance	1	3116.81	46	9381.77	15.61	< 0.001 *
2	Magnitude	1	7935.16	46	16695.20	22.34	< 0.001 *
2	Better Choice	1	39842.67	46	16424.19	114.02	< 0.001 *
2	V X M	1	25.81	46	6458.62	0.19	0.667
2	V X BC	1	1363.87	46	5739.50	11.17	0.002 *
2	M X BC	1	6394.70	46	8135.79	36.94	< 0.001 *
2	V X M X BC	1	33.90	47	3584.50	0.44	0.508

Table 15. Summary of ANOVA effects for cluster 3 for AC trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
3	Variance	1	2030.33	21	5641.00	7.56	0.012 *
3	Magnitude	1	43.23	21	20765.38	0.044	0.836
3	Better Choice	1	35542.94	21	14117.29	52.87	< 0.001*
3	V X M	1	87.21	21	6268.54	0.29	0.595
3	V X BC	1	0.11	21	5620.37	0.00042	0.984
3	M X BC	1	3982.62	21	2568.81	32.56	< 0.001*
3	V X M X BC	1	158.88	21	3608.36	0.92	0.347

Table 16. Summary of ANOVA effects for cluster 1 on RR trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
1	Probability (P)	1	3879.17	19	28128.29	2.76	0.112
1	Magnitude (M)	1	32116.74	19	8749.06	73.42	< 0.001*
1	Better Choice (BC)	1	45927.98	19	10233.65	89.76	< 0.001*
1	Variance (V)	1	32116.74	19	7055.31	91.04	< 0.001*
1	P X M	1	368.06	19	6039.40	1.22	0.283
1	P X BC	1	209.00	19	9719.30	0.43	0.519
1	M X BC	1	18576.40	19	6235.23	59.59	< 0.001*
1	P X V	1	307.94	19	11159.94	0.55	0.466
1	M X V	1	7666.74	19	11538.64	13.29	0.002*
1	BC X V	1	1865.29	19	7760.93	4.81	0.040*
1	P X M X BC	1	2546.50	19	4585.96	11.11	0.003*
1	P X M X V	1	159.73	19	11520.65	0.28	0.604
1	P X V X BC	1	1276.86	19	9399.36	2.72	0.115
1	M X BC X V	1	1535.72	19	9969.66	3.08	0.095
1	P X M X BC X V	1	12.57	19	4505.31	0.056	0.816

Table 17. Summary of ANOVA effects for cluster 2 on RR trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
2	Probability	1	2850.22	28	33364.37	2.39	0.133
2	Magnitude	1	1046.00	28	18874.14	1.55	0.223
2	Better Choice	1	23186.21	28	16055.46	40.44	< 0.001*
2	Variance	1	4124.16	28	26150.14	4.42	0.045*
2	P X M	1	725.00	28	11006.94	1.84	0.185
2	P X BC	1	3017.84	28	17959.24	4.71	0.039*
2	M X BC	1	4614.97	28	7962.12	16.23	< 0.001*
2	P X V	1	12071.36	28	16466.14	20.53	< 0.001*
2	M X V	1	8055.56	28	10850.00	20.79	< 0.001*
2	BC X V	1	5726.08	28	11673.23	13.73	< 0.001*
2	P X M X BC	1	4.69	28	14003.64	0.0094	0.924
2	P X M X V	1	2083.93	28	15872.32	3.68	0.065
2	P X V X BC	1	98.08	28	8936.64	0.31	0.584
2	M X BC X V	1	1180.17	28	8389.27	3.94	0.057
2	P X M X BC X V	1	733.36	28	13292.34	1.54	0.224

Table 18. Summary of ANOVA effects for cluster 3 on of RR trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
3	Probability	1	35450.87	38	33281.42	40.48	< 0.001*
3	Magnitude	1	13695.32	38	37708.50	13.80	< 0.001*
3	Better Choice	1	94113.27	38	21803.05	164.03	< 0.001*
3	Variance	1	14487.25	38	23758.24	23.17	< 0.001*
3	P X M	1	21.81	38	15315.34	0.054	0.817
3	P X BC	1	1613.27	38	17394.72	3.52	0.068
3	M X BC	1	11481.64	38	22099.26	19.74	< 0.001*
3	P X V	1	1778.06	38	29350.76	2.30	0.137
3	M X V	1	4667.81	38	15872.82	11.17	< 0.001 *
3	BC X V	1	82.34	38	11111.07	0.28	0.599
3	P X M X BC	1	11.13	38	18994.77	0.022	0.882
3	P X M X V	1	192.59	38	21931.37	0.33	0.567
3	P X V X BC	1	1278.92	38	22972.82	2.12	0.154
3	Probability	1	35450.87	38	33281.42	40.48	< 0.001*
3	Magnitude	1	13695.32	38	37708.50	13.80	< 0.001*

Table 19. Summary of ANOVA effects for cluster 1 on AR trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
1	Probability (P)	1	4187.95	18	9188.61	8.20	0.010*
1	Magnitude (M)	1	9940.14	18	9228.09	19.39	<0.001*
1	Better Choice (BC)	1	3754.40	18	8409.67	8.04	0.011*
1	Variance (V)	1	0.082	18	9747.31	0.00015	0.990
1	P X M	1	254.83	18	6217.56	0.74	0.402
1	P X BC	1	454.40	18	5634.67	1.45	0.244
1	M X BC	1	2.06	18	7682.84	0.0048	0.945
1	P X V	1	319.53	18	5761.20	1.00	0.331
1	M X V	1	1908.34	18	4589.05	7.49	0.014*
1	BC X V	1	1650.45	18	2984.45	9.95	0.005*
1	P X M X BC	1	254.83	18	4284.23	1.07	0.314
1	P X M X V	1	4413.61	18	11179.62	7.11	0.016*
1	P X V X BC	1	120.84	18	4214.05	0.52	0.482
1	M X BC X V	1	9.95	18	1812.45	0.099	0.757
1	P X M X BC X V	1	701.11	18	3942.12	3.20	0.090

Table 20. Summary of ANOVA effects for cluster 2 on AR trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
2	Probability	1	3704.22	46	71799.25	2.42	0.126
2	Magnitude	1	1444.68	46	33885.88	2.00	0.164
2	Better Choice	1	70405.61	46	25137.44	131.64	<0.001*
2	Variance	1	67938.21	46	36152.76	88.32	<0.001*
2	P X M	1	2637.86	46	19840.61	6.25	0.016*
2	P X BC	1	356.61	46	18317.69	0.92	0.344
2	M X BC	1	24638.67	46	15058.55	76.90	<0.001*
2	P X V	1	2.08	46	24903.47	0.0039	0.950
2	M X V	1	14237.04	46	21983.10	30.44	<0.001*
2	BC X V	1	1086.17	46	17954.80	2.84	0.098
2	P X M X BC	1	107.00	46	18867.30	0.27	0.608
2	P X M X V	1	955.57	46	18833.32	2.38	0.129
2	P X V X BC	1	0.014	46	30351.37	2.2e-05	0.996
2	M X BC X V	1	43.76	46	21901.37	0.094	0.761
2	P X M X BC X V	1	156.48	46	16411.57	0.45	0.506

Table 21. Summary of ANOVA effects for cluster 3 on AR trials.

Cluster	Effect	DF	SS	DFE	SSE	F	p
3	Probability	1	22087.51	21	38643.92	12.00	<0.001*
3	Magnitude	1	2191.67	21	16964.75	2.71	0.114
3	Better Choice	1	35970.84	21	18545.30	40.73	<0.001*
3	Variance	1	1133.53	21	17859.70	1.33	0.261
3	P X M	1	2343.95	21	11096.50	4.44	0.047*
3	P X BC	1	3082.58	21	13255.09	4.88	0.038*
3	M X BC	1	3323.87	21	8801.30	7.93	0.010*
3	P X V	1	2984.73	21	14992.53	4.18	0.054
3	M X V	1	1691.67	21	9348.08	3.80	0.065
3	BC X V	1	371.60	21	8704.27	0.90	0.354
3	P X M X BC	1	854.17	21	9399.47	1.91	0.182
3	P X M X V	1	2645.53	21	6168.54	9.01	0.007*
3	P X V X BC	1	258.53	21	6029.14	0.90	0.353
3	M X BC X V	1	378.48	21	14884.19	0.53	0.473
3	P X M X BC X V	1	115.54	21	5310.33	0.46	0.506

Appendix C
Mean Parameter Values

Table 22. Mean parameter values obtained for optimization fit across all participants for EUT, CPTU, CPTSm and CPTS and α-maxmin.

Model	Beta (SD)	Lambda (SD)	Gamma (SD)	Alpha (SD)
EUT	0.51 (0.33)	-	-	0.72 (0.22)
CPTU	0.56 (0.31)	2.57 (1.22)	0.73 (0.20)	0.75 (0.21)
CPTSm	0.64 (0.27)	2.65 (1.49)	0.69 (0.25)	0.73 (0.24)
CPTS	0.72 (0.24)	2.69 (1.39)	0.61 (0.28)	0.74 (0.25)

Note. Alpha values were estimated using α-maxmin model and are depicted within the row corresponding to the model parameters that were used to estimate the ambiguity parameter.

Table 23. Mean parameter values for each cluster for all models using optimization method

Model	**Cluster**	**Beta (SD)**	**Lambda (SD)**	**Gamma (SD)**	**Alpha (SD)**
EUT	1	0.19 (0.14)	-	-	0.96 (0.05)
	2	0.49 (0.32)	-	-	0.74 (0.16)
	3	0.71 (0.25)	-	-	0.47 (0.17)
CPTU	1	0.30 (0.20)	2.69 (1.33)	0.73 (0.08)	0.97 (0.05)
	2	0.51 (0.34)	2.25 (1.22)	0.66 (0.25)	0.79 (0.14)
	3	0.73 (0.20)	2.74 (1.15)	0.79 (0.18)	0.47 (0.12)
CPTS	1	0.60 (0.21)	3.68 (0.89)	0.54 (0.29)	0.93 (0.07)
	2	0.70 (0.27)	2.10 (1.19)	0.74 (0.24)	0.74 (0.22)
	3	0.79 (0.20)	1.85 (1.27)	0.55 (0.29)	0.53 (0.24)
CPTSm	1	0.49 (0.19)	4.00 (0.83)	0.61 (0.23)	0.96 (0.08)
	2	0.58 (0.30)	2.94 (1.23)	0.70 (0.28)	0.75 (0.21)
	3	0.76 (0.23)	1.71 (1.31)	0.71 (0.25)	0.51 (0.24)

Table 24. Mean parameter values for all models on risky trials obtained from HBA fit.

Model	**Parameter**	**Mean**	**HDImin**	**HDImax**
EUT	mu.β	0.32	0.27	0.38
	mu.φ	2.54	1.72	3.49
CPTU	mu.β	0.46	0.41	0.51
	mu.λ	0.82	0.10	3.16
	mu.γ	0.74	0.68	0.80
	mu.φ	1.10	0.80	1.43
CPTS	mu.β	0.70	0.62	0.78
	mu.λ	2.48	1.94	3.07
	mu.γ	0.65	0.57	0.75
	mu.φ	0.22	0.16	0.28
CPTSm	mu.β	0.69	0.62	0.76
	mu.λ	2.18	1.72	2.66
	mu.γ	0.70	0.62	0.77
	mu.φ	0.25	0.18	0.31

Table 25. Mean parameter values on ambiguous trials for alpha-maxmin informed by all risky choice models using HBA method

Model	Parameter	Mean	HDImin	HDImax
EUT	mu.α	0.75	0.67	0.82
	mu.φ	2.50	2.19	2.82
CPTU	mu.α	0.95	0.89	0.99
	mu.φ	0.83	0.69	0.97
CPTS	mu.α	0.79	0.71	0.88
	mu.φ	0.13	0.11	0.15
CPTSm	mu.α	0.92	0.85	0.99
	mu.φ	0.13	0.10	0.15

Table 26. Mean cluster level parameters on risky trials for all models obtained from HBA

Model	Cluster	Beta (SD)	Lambda (SD)	Gamma (SD)	Luce (SD)
EUT	1	0.32 (0.003)	-	-	4.44 (1.25)
	2	0.32 (0.004)	-	-	2.54 (0.93)
	3	0.33 (0.01)	-	-	2.55 (1.17)
CPTU	1	0.45 (0.03)	1.06 (0.02)	0.55 (0.18)	2.10 (0.85)
	2	0.44 (0.02)	1.06 (0.01)	0.66 (0.16)	1.13 (0.46)
	3	0.47 (0.03)	1.06 (0.02)	0.82 (0.11)	1.16 (0.54)
CPTS	1	0.62 (0.16)	7.08 (6.71)	0.56 (0.13)	0.49 (0.39)
	2	0.59 (0.15)	4.42 (3.08)	0.68 (0.15)	0.26 (0.24)
	3	0.75 (0.12)	1.90 (1.41)	0.61 (0.16)	0.26 (0.15)
CPTSm	1	0.69 (0.11)	6.17 (4.36)	0.61 (0.12)	0.31 (0.07)
	2	0.60 (0.11)	3.31 (1.76)	0.70 (0.14)	0.27 (0.18)
	3	0.72 (0.10)	1.58 (1.02)	0.67 (0.14)	0.28 (0.10)

Table 27. Mean cluster level parameters on ambiguous trials for alpha-maxmin informed by all risky choice models using HBA method

Model	Cluster	Alpha (SD)	Luce (SD)
EUT	1	0.89 (0.09)	2.74 (1.87)
	2	0.74 (0.12)	2.89 (0.86)
	3	0.40 (0.15)	2.69 (1.24)
CPTU	1	0.96 (0.05)	0.86 (0.46)
	2	0.86 (0.12)	1.08 (0.51)
	3	0.47 (0.20)	1.08 (0.63)
CPTS	1	0.91 (0.05)	0.18 (0.14)
	2	0.75 (0.15)	0.15 (0.11)
	3	0.51 (0.18)	0.20 (0.10)
CPTSm	1	0.95 (0.03)	0.21 (0.29)
	2	0.85 (0.12)	0.12 (0.09)
	3	0.56 (0.19)	0.25 (0.17)

Appendix D
Parameter Plots by Cluster

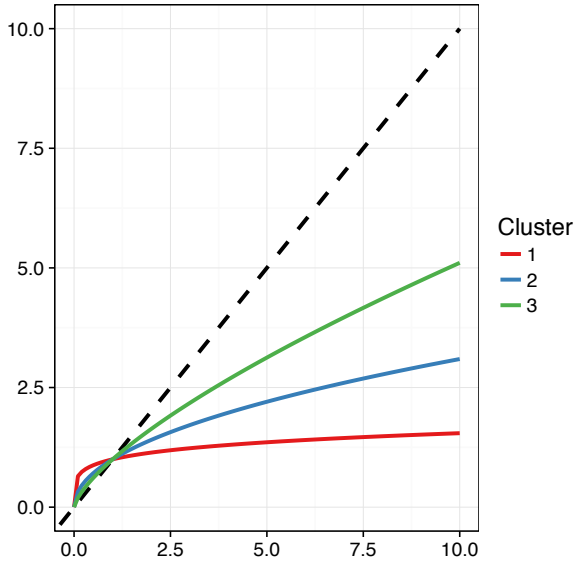

Figure 19. Diminishing marginal utility function plotted by cluster using EUT beta values from optimization fit.

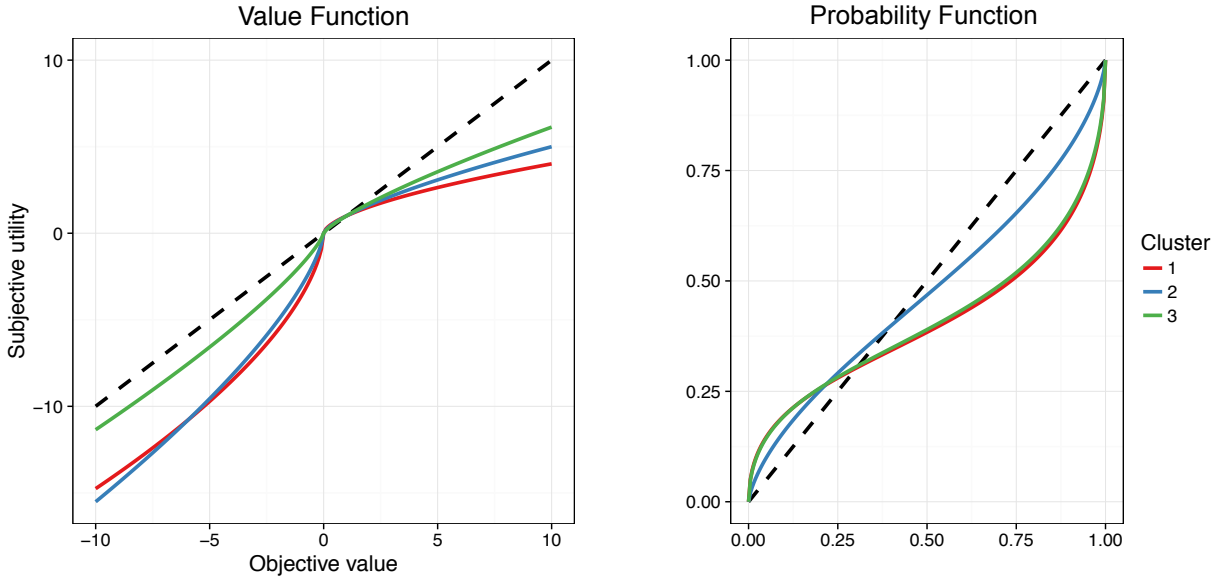

Figure 20. Value and probability functions plotted by cluster using CPTS beta, lambda and gamma parameters from optimization fit.

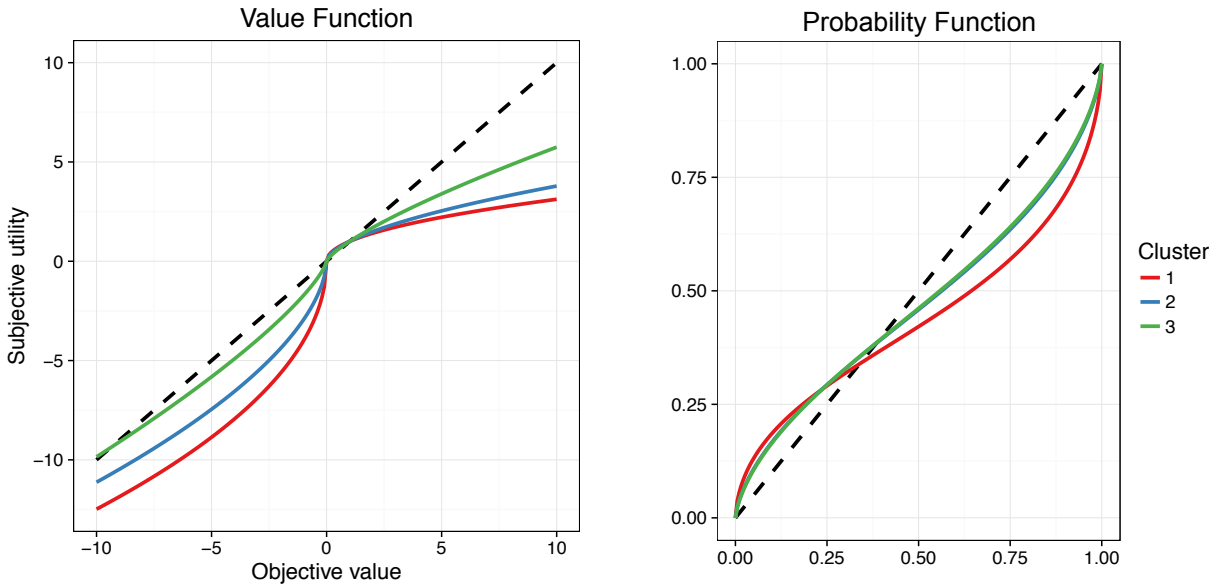

Figure 21. Value and probability functions plotted by cluster using CPTSm beta, lambda and gamma parameters from optimization fit.

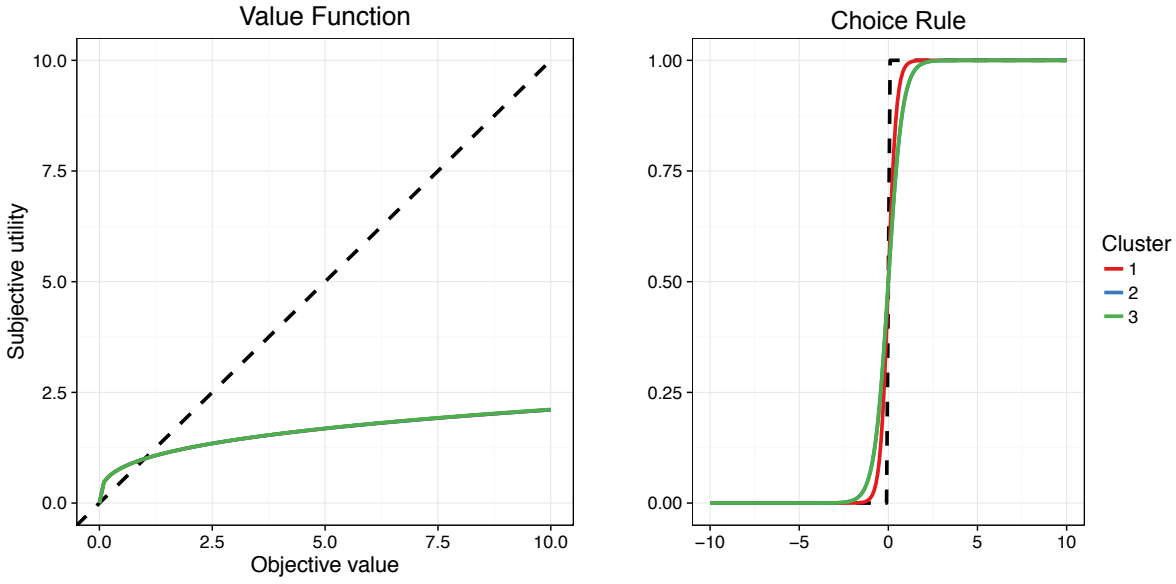

Figure 22. Value and choice rule functions plotted by cluster using EUT beta and luce parameters from HBA fit.

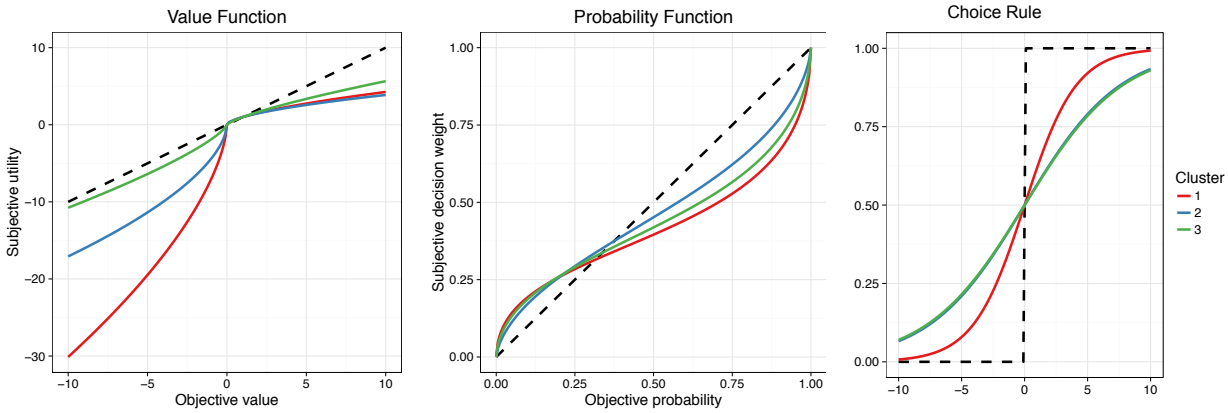

Figure 23. Value, probability, and choice rule functions plotted by cluster using CPTS beta, lambda, gamma and luce parameters from HBA fit.

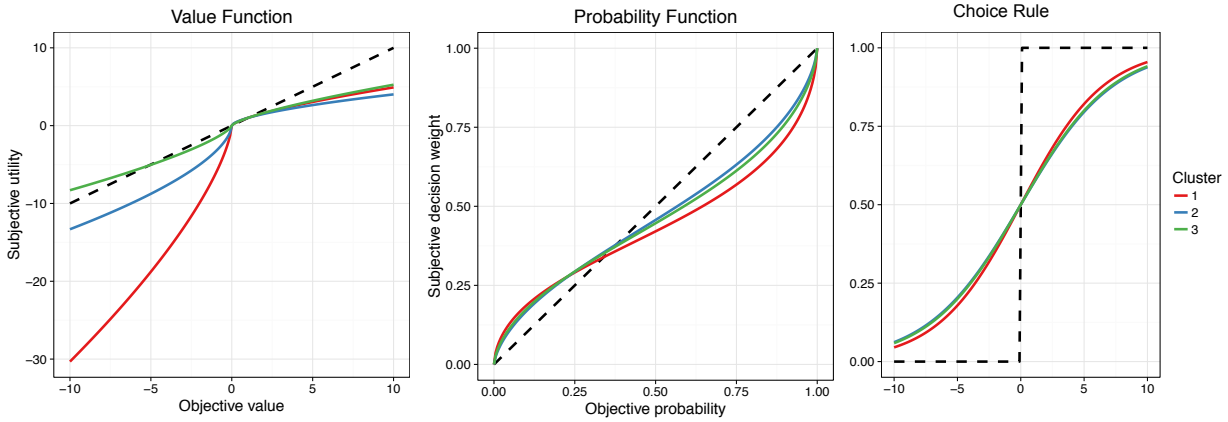

Figure 24. Value, probability, and choice rule functions plotted by cluster using CPTSm beta, lambda, gamma and luce parameters from HBA fit.