Construction of Systematic Binary Self-Complementary Codes

by

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B. Tech., Maulana Abul Kalam Azad University of Technology, India, 2017

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ABSTRACT

Efficient transmission of data through channels is of utmost importance in modern digital communication systems. In order to reduce errors due to impairments such as noise in the channel, error control coding is used for reliable transmission of data. This work is focused on a class of linear block codes called binary self-complementary codes which contain the all-ones codeword. The construction of binary self-complementary codes and their weight distributions are considered. The Magma Computational Algebra System is used to generate the weight distributions of the best codes in terms of the minimum Hamming distance. General expressions for the Hamming weight distributions of the best codes for small dimensions are obtained and the best self-complementary codes are compared with the best possible linear codes based on the minimum Hamming distance. The results indicate that the best self-complementary codes have minimum distances which are similar to those of the best binary linear codes with differences of 1, 2 and 3 for 34, 7 and 1 values of n, respectively, for $n \leq 16$.

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DEDICATION

I dedicate this work to my parents for their continuous support and guidance. Thank you for believing in me.

Chapter 1

Introduction

The demand for efficient and reliable transmission of data through communication channels is increasing. However, these channels are prone to errors due to impairments such as noise which can corrupt the data. Interference and noise can make the transmission of data from source to receiver challenging, hence error control is required in all communication systems. Error control coding is employed in data communications to correct channel errors. Error control strategies include Automatic Repeat Request (ARQ) and Forward Error Correction (FEC). For a one-way system, FEC can be used to detect and correct errors at the receiver by adding redundancy. The receiver uses the redundancy to detect and correct errors. Most communication systems in use today employ FEC. ARQ is used in two-way systems where data can be sent in both directions so the transmitter also acts as a receiver. In this case, when errors are detected at the receiver, the transmitter is sent a signal from the receiver to retransmit the data. This process continues until the message is received correctly [8].

Hamming introduced the Hamming codes as an FEC solution to control errors by adding redundancy [7]. Since then, there has been a great deal of effort to develop efficient encoding and decoding techniques for error control. Many efficient and reliable FEC codes have been developed such as the Golay codes [5].

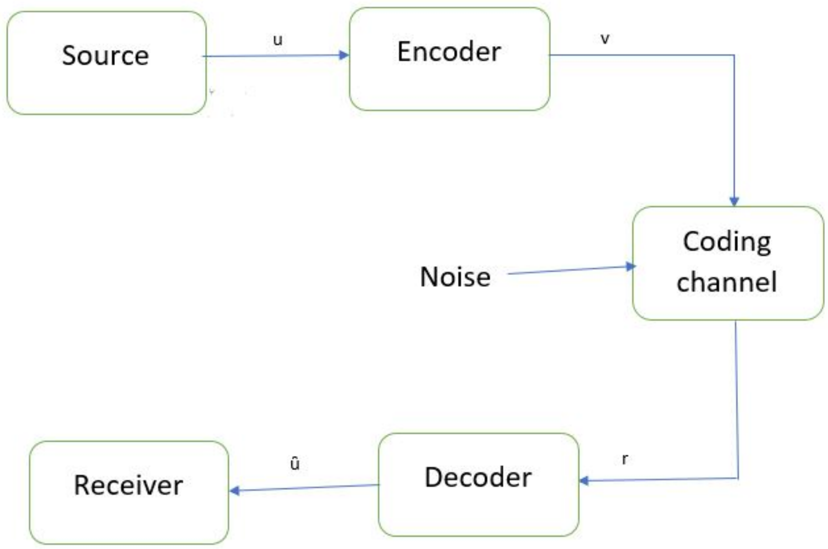

Figure 1.1: Model of a communication system [8].

Figure 1.1 shows the communication system model which has three main parts

(i) an encoder, to encode the source message u into a codeword v,

(ii) the channel through which v is transmitted, and

(iii) a decoder to decode the received word r to a word $\hat{u}$.

The goal is to have $u = \hat{u}$.

The remainder of this chapter presents Galois fields and vector spaces. Chapter 2 gives an overview of binary linear block codes including generator and parity check matrices, minimum distance, equivalent binary linear codes and codes in systematic form, weight enumerators, and self-complementary codes. Chapter 3 presents several classes of linear block codes, namely the repetition codes, single parity check codes, Hamming codes, and Reed-Muller codes. Chapter 4 gives the best self-complementary codes up to length $n = 16$, general expressions for the weight enumerators of codes with small dimensions and a comparison of the best binary self-complementary codes with the best binary linear codes. Finally, Chapter 5 gives the conclusion and future work that can be done in this area.

1.1 Galois Fields

A Galois field contains a finite set of elements on which the mathematical operations addition and multiplication are defined. A field must contain at least 2 elements and hence the smallest Galois field is GF(2) which contains the elements 0 and 1. Hence, GF(2) is also called the binary field. In GF(2), the addition operation is modulo-2 addition and multiplication is modulo-2 multiplication. This field is widely used in digital communication systems [8]. The Galois field GF(q) exists for $q = p^m$, p prime, $m \geq 1$, but only GF(2) is considered here. The modulo-2 addition and multiplication tables are

Modulo-2 addition

$+$	0 1
0	0 1
1	1 0

Modulo-2 multiplication

$\times$	0 1
0	0 0
1	0 1

1.2 Vector Spaces

A vector space V contains a set of elements which are vectors on which the operations vector addition $(+)$, scalar multiplication $(\cdot)$ and inner product $(*)$ can be performed [6]. The vector space V_n contains all vectors of length n over the binary field GF(2).

1.2.1 Operations on Vectors

Vector addition

The addition of two vectors $u = (u_1, u_2, \ldots, u_n)$ and $v = (v_1, v_2, \ldots, v_n)$ in V_n is defined as

$$u + v = (u_1 + v_1, u_2 + v_2, \ldots, u_n + v_n) \tag{1.1}$$

where $u_i + v_i$ is modulo-2 addition. A vector space is closed under vector addition since addition of any two vectors in V_n will give another vector in V_n. The set of all vectors over GF(2) forms a vector space over GF(2) [8].

Example 1: Consider the vector space V_3 so that $n = 3$. Thus, V_3 consists of $2^3 = 8$ vectors

$$V_3 = \{(000), (001), (010), (011), (100), (101), (110), (111)\}$$

The addition of any two vectors produces another vector in the vector space, for example

$$(010) + (101) = (111)$$

Scalar multiplication

The scalar multiplication of any vector v in V_n by an element a from GF(2) is given by $a \cdot v$ where a can be either 0 or 1 since it is a binary field.

Example 2: For a vector $v = 101$ in V_3

$$0 \cdot (101) = (000)$$

$$1 \cdot (101) = (101)$$

The scalar multiplication of a vector from the vector space V_n by an element from GF(2) generates another vector from V_n. Hence, V_n is closed under scalar multiplication.

1.2.2 Vector Subspaces

A vector subspace S is a subset of vectors from V_n that is closed under vector addition and scalar multiplication.

Example 3: A subspace of the vector space V_3 is

$$S = \{(000), (101), (110), (011)\}$$

A vector subspace must have the all-zero vector, i.e. (000) for V_3.

1.2.3 Basis

A basis of a vector subspace S is a set of linearly independent vectors which span S. The number of vectors in the basis is called the dimension of the vector space and is denoted by k. A basis is a simple means of representing a vector space and it cannot have the all-zeros vector [6].

Example 4: Consider the following subspace of V_5

$$S = \{(00000), (00111), (11100), (11011)\}$$

The vectors (00111) and (11100) in S form a basis of S as the linear combinations of these two vectors give the vector subspace

$$0 \cdot (00111) + 0 \cdot (11100) = (00000)$$

$$0 \cdot (00111) + 1 \cdot (11100) = (11100)$$

$$1 \cdot (00111) + 0 \cdot (11100) = (00111)$$

$$1 \cdot (00111) + 1 \cdot (11100) = (11011)$$

The dimension of S is the number of vectors in the basis which is $k = 2$.

1.2.4 Dual Spaces

Every vector space has a corresponding dual space. If S is a k-dimensional subspace of the n-dimensional vector space V_n, then the set of vectors orthogonal to S is known as the dual space $S^{\perp}$ of S [9].

Example 5: Consider the following vector subspace S of V_5

$$00000$$
$$00111$$
$$11100$$
$$11011$$

The dual space $S^{\perp}$ is

$$00000$$
$$00011$$
$$11000$$
$$10101$$
$$11011$$

$$01110$$

$$10110$$

$$01101$$

The sum of the dimensions of the subspace S and dual space $S^{\perp}$ is equal to the dimension of the vector space V_n, that is

$$dim(S) + dim(S^{\perp}) = dim(V_n)$$

In the above example, the dimension of S is 2 and the dimension of $S^{\perp}$ is 3. Hence, the dimension of V_5 is 5.

Chapter 2

Binary Linear Block Codes

Encoding a message means adding redundancy to the message at the source. It is important to keep the redundancy as low as possible since it is overhead. Block coding means a message to be transmitted through the channel is a block of k bits, known as the message bits, constituting a set of 2^k possible messages. Each block of message bits is passed through an encoder which adds redundancy to these bits to form an n-bit codeword where $n > k$. Hence, there are also 2^k codewords. The $n - k$ bits added to the message by the encoder are called the redundant or parity bits. At the receiver, a decoder is used to recover the message from the n-bit received word. Hence, these codes are called block codes and they are denoted as (n, k) codes [9]. A message can be denoted by $m = (m_0, m_1, \ldots, m_{k-1})$. The encoder uses this message to generate a codeword $c = (c_0, c_1, \ldots, c_{n-1})$ of n bits. The code rate R of an (n, k) block code is defined as the ratio of the length of the message to the length of the codeword, i.e. $R = k/n$. The code rate is 1 when no redundant bits are added to the message bits. A block code of length n with 2^k codewords is called an (n, k) linear block code if the 2^k codewords form a k-dimensional subspace of the vector space V_n. A binary block code is linear if and only if the modulo-2 sum of any two codewords is another codeword [8].

2.1 Generator Matrix

Since an (n, k) linear block code C is a k-dimensional subspace of the vector space V_n, it is possible to find k linearly independent codewords $(g_0, g_1, \ldots, g_{k-1})$ in C such that each codeword in C is a linear combination of these k codewords [8]. Thus, these k linearly independent vectors form a basis. Thus, an (n, k) linear block code can be defined by a generator matrix G which is a basis of the vector space. A codeword can then be written as

$$c = mG$$

where c is the n-bit codeword, m is the k bit message, and G is the generator matrix.

The generator matrix G of an (n, k) linear block code is a $k \times n$ matrix whose k linearly independent rows are codewords and is given by

$$G = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0,n-1} \\ g_{10} & g_{11} & \cdots & g_{1,n-1} \\ \vdots & \vdots & & \vdots \\ g_{k-1,0} & g_{k-1,1} & \cdots & g_{k-1,n-1} \end{bmatrix} \quad (2.1)$$

2.2 Parity Check Matrix

The parity check matrix H of an (n, k) linear block code C is an $(n - k) \times n$ matrix whose rows are linearly independent. A generator matrix G is a basis of the subspace S and the parity check matrix H is a basis of the dual space $S^{\perp}$. For each code C there is a corresponding parity check matrix. The parity check matrix H of a code C is given by

$$H = \begin{bmatrix} h_{00} & h_{01} & \dots & h_{0,n-1} \\ h_{10} & h_{11} & \dots & h_{1,n-1} \\ \vdots & \vdots & & \vdots \\ h_{n-k-1,0} & h_{n-k-1,1} & \dots & h_{n-k-1,n-1} \end{bmatrix} \tag{2.2}$$

2.3 Minimum Distance

Minimum distance is an important parameter of a binary linear block code which determines the error detection and error correction capabilities of the code [8].

The Hamming weight of a codeword is defined as the number of nonzero elements in the codeword. For binary codes, the Hamming weight is the number of ones in the codeword. For example, if $c = (01110100)$ is a codeword, then the Hamming weight of c is $w(c) = 4$.

The Hamming distance between two codewords x and y, denoted by $d(x, y)$, is defined as the number of positions in which they differ. For example, if $x = (001111)$ and $y = (111000)$, then the Hamming distance between x and y is $d(x, y) = 5$. The Hamming distance between two codewords x and y is equal to the Hamming weight of their sum

$$d(x, y) = w(x + y)$$

which is another codeword since for linear block codes the sum of two codewords is another codeword. For the example, $(x + y) = (001111) + (111000) = (110111) = z$ and $w(z) = 5$. Therefore, $d(x, y) = w(x + y) = w(z) = 5$.

The minimum Hamming distance d_{min} of a linear block code C is defined as the smallest Hamming distance between all pairs of codewords in the code

$$d_{min} = \min\{d(x, y) : x, y \in C, x \neq y\} \tag{2.3}$$

Since for a linear block code the sum of two codewords is another codeword, the minimum Hamming distance of a linear block code is given by

$$d_{min} = \min\{w(z) : z \in C, z \neq 0\} \tag{2.4}$$

Thus, the minimum Hamming distance d_{min} of a linear block code C is the minimum weight of the non-zero codewords [8].

A binary linear block code of length n, dimension k and minimum Hamming distance d_{min} is called an (n, k, d) code. Such a code can detect $d_{min} - 1$ errors and correct $\lfloor (d_{min} - 1)/2 \rfloor$ errors [6].

2.4 Equivalent Binary Linear Codes

Two binary linear codes are said to be equivalent if one can be obtained from the other using one or more of the following operations [6]

1. permuting the rows of the generator matrix,
2. adding one row to another row in the generator matrix, and
3. permuting the columns of the generator matrix.

Example 6: Consider two (n, k) codes C_1 and C_2 with generator matrices

$$G_1 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

respectively. C_1 and C_2 are equivalent codes because G_2 can be obtained from G_1 by permuting the 1st and 5th columns.

2.5 Codes in Systematic Form

Any linear block code C can be arranged in systematic form in which the codewords can be divided into two parts, namely message and parity, as shown in Figure 2.1. There are k message bits and $n - k$ parity bits which are the redundancy. The positions of the parity and message parts is interchangeable, so the parity bits can be on the left and the message bits on the right. A linear block code in systematic form is called a systematic linear block code. All codes constructed in this report are systematic. The generator matrix G in systematic form is

$$G = [I_k \mid P]$$

or

$$G = [P \mid I_k]$$

where I_k is the $k \times k$ identity matrix and P is the $k \times (n - k)$ parity matrix.

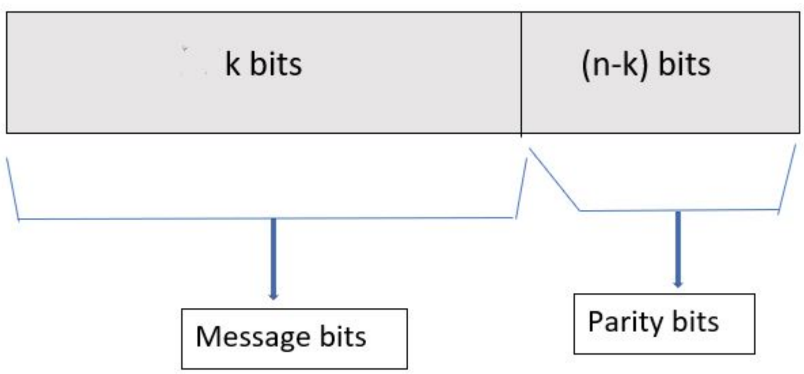

Figure 2.1: Structure of a systematic codeword [3].

2.6 Weight Enumerator

For an (n, k, d) code C, the coefficients $A_0, A_1, \ldots, A_n$ are called the weight distribution of C, where A_i is the number of codewords of weight i in C [8]. The weight distribution is usually represented in polynomial form which is called the weight enumerator

$$A(x) = A_0 + A_1x + \ldots + A_nx^n \tag{2.5}$$

A_0 is the number of codewords of weight 0 which should always be 1 since all (n, k, d) codes must have the all-zero codeword.

Example 7: Let the generator matrix of an (n, k, d) code C be

$$G = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The weight distribution of C is $A_0 = A_4 = 1$, $A_3 = 2$, $A_1 = A_2 = 0$. Hence the weight enumerator of C is

$$1 + 2x^3 + x^4$$

2.7 Magma Computational Algebraic System

Magma is a software package designed for algebraic computations using the Magma language. It also has an online calculator [2] which is available free of cost to execute code in the Magma language. It is used here to obtain the weight enumerators.

Example 8: The weight enumerator obtained in Example 7 can be obtained using Magma. Figure 2.2 shows the Magma code in the calculator for the code with parameters $n = 5$ and $k = 2$. In the code, the binary field GF(2) has to be specified and the generator matrix G entered. The function LinearCode(G) generates the (n, k, d) code, which is a $(5, 2, 3)$ code in this case, from the generator matrix G and the function WeightDistribution(C) generates the weight distribution.

Figure 2.3 shows the output window which displays the generator matrix G, parameters $[5, 2, 3]$ and weight distribution $[< 0, 1 >, < 3, 2 >, < 4, 1 >]$ of the $(5, 2, 3)$ code. Hence, the weight enumerator is $A(x) = 1 + 2x^3 + x^4$.

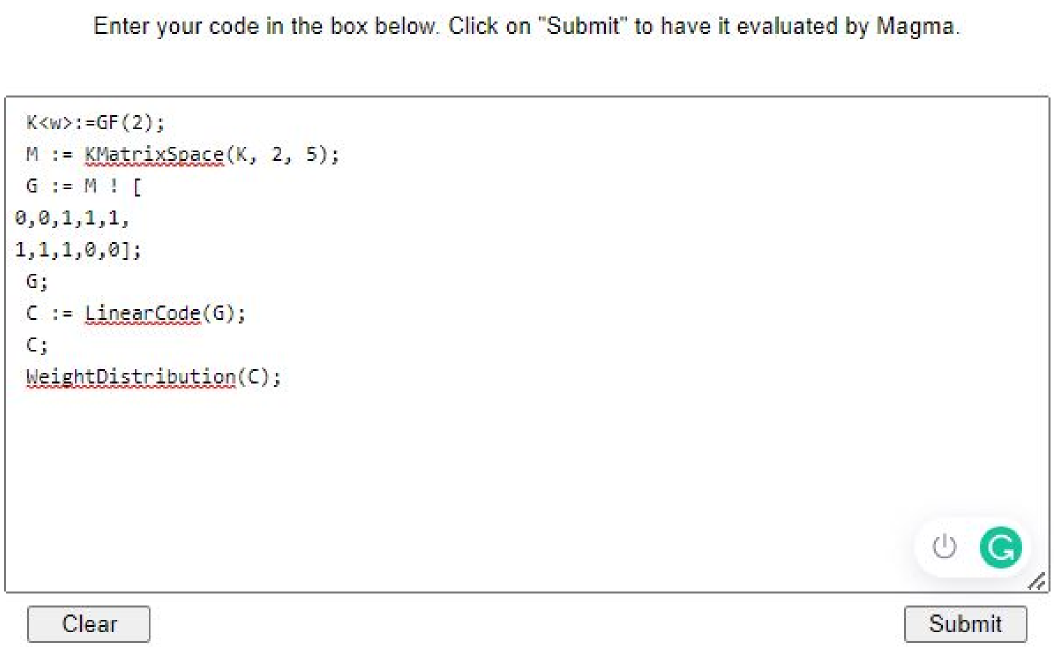

Figure 2.2: The online Magma calculator (input window) [2].

```
[0 0 1 1 1]
[1 1 1 0 0]
[5, 2, 3] Linear Code over GF(2)
Generator matrix:
[1 1 0 1 1]
[0 0 1 1 1]
[ <0, 1>, <3, 2>, <4, 1> ]
```

Figure 2.3: The online Magma calculator (output window) [2].

2.8 Self-Complementary Codes

A binary code C is said to be self-complementary if the complement of a codeword in C is another codeword in C. For example, if a codeword $c_1 = (c_1, c_2, \ldots, c_n) \in C$ and the complement of c_1, $\bar{c}_1 = (1 + c_1, 1 + c_2, \ldots, 1 + c_n) \in C$, then the code C is self-complementary. This means that the all-ones codeword $(11 \ldots 1)$ must belong to C for it to be self-complementary since a binary linear code must have the all-zeros codeword whose complement is the all-one codeword [4].

The weight enumerator of a self-complementary code must have $A_n = 1$. An important point to consider for constructing binary systematic self-complementary codes is that each column in the generator matrix must have a odd number of ones as the modulo-2 sum of all the rows of the generator matrix must give the all-ones codeword.

Chapter 3

Classes of Linear Block Codes

3.1 Repetition Codes

Repetition codes are the simplest linear block codes as each codeword consists of a single message bit repeated several times. The single message bit when passed through an encoder creates a block of n identical bits, resulting in an $(n, 1, n)$ block code. It only contains the all-zeros codeword $(00 \ldots 0)$ and the all-ones codeword $(11 \ldots 1)$, so the minimum distance of a repetition code of length n is n. Repetition codes are always self-complementary because they contain the all-ones codeword.

Example 9: The generator matrix of the $(3, 1, 3)$ repetition code is

$$G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

so the codewords are (000) and (111).

3.2 Single Parity Check Codes

A Single Parity Check (SPC) code is an $(n, n-1, 2)$ linear block code with a single parity bit p which is the modulo-2 sum of the message bits. Thus, an SPC code is a $(k+1, k)$ linear block code [8].

Example 10: For the $(3, 2, 2)$ SPC code, if $m = 00$, the parity bit is $p = 0 + 0 = 0$ and the codeword c is (000), and if $m = 01$, $p = 0 + 1 = 1$ and the codeword c is (011).

In an SPC code, p is 0 if the weight of the message is even and 1 if the weight of the message is odd. Hence, all codewords of an SPC code have even weight and the minimum distance is 2. The $(n, n-1, 2)$ SPC codes and $(n, 1, n)$ repetition codes are duals of each other [8].

Since SPC codes always have even weight codewords, an SPC code cannot be self-complementary if n is odd, because in this case the all-ones codeword will have odd weight. Hence, SPC codes are self-complementary only when n is even.

Example 11: The generator matrix of the $(3, 2, 2)$ SPC code C is

$$G = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

so the codewords are (000), (011), (101) and (110). C does not contain the all-ones codeword because n is odd, hence C is not self complementary.

3.3 Hamming Codes

Hamming codes are single error correcting codes as they have $d_{min} = 3$. They are widely used for error control in digital communications due to their easy encoding and decoding. Hamming codes have length $n = 2^t - 1$ and dimension $k = 2^t - t - 1$, where k is the number of message bits and t is the number of parity bits and $t = n - k$. The weight enumerator of a Hamming code of length $n = 2^t - 1$ is [4]

$$A(x) = \frac{1}{(n+1)}\left\{(1+x)^n + n(1-x)(1-x^2)^{(n-1)/2}\right\} \tag{3.1}$$

Example 12: For the $(7,4,3)$ Hamming code, the weight enumerator is

$$A(x) = \tfrac{1}{(7+1)}\{(1+x)^7 + 7(1-x)(1-x^2)^{(7-1)/2}\}$$

$$= \tfrac{1}{8}\{(1+x)^7 + 7(1-x)(1-x^2)^3\}$$

$$= 1 + 7x^3 + 7x^4 + x^7$$

Since $A_7 = 1$, the $(7,4,3)$ Hamming code is self-complementary. For all Hamming codes, $A_n = 1$, hence Hamming codes are always self-complementary. The $(2^t, 2^t - t - 1, 4)$ extended Hamming codes are obtained by adding an even parity bit to each codeword in the Hamming code. Hence, all extended Hamming codes are self-complementary [4].

3.4 Reed-Muller Codes

Another important class of linear block codes used for error correction and detection is the Reed-Muller (RM) codes. An RM code is denoted by RM(r, s) where $n = 2^s$ and r is the order of the code with $0 \leq r \leq s$. The minimum distance is $d_{min} = 2^{s-r}$ and the dimension is [8]

$$k = 1 + \binom{s}{1} + \binom{s}{2} + \ldots + \binom{s}{r}$$

The repetition code of length $n = 2^s$ is RM$(0, s)$ and the single parity check code with length $n = 2^s$ is RM$(s-1, s)$. RM codes always have the all-ones codeword and hence all RM codes are self-complementary [4].

Chapter 4

Results and Discussions

Binary self-complementary codes with different values of n (length), k (dimension) and d_{min} (minimum distance) are presented in this chapter. Only the best binary self-complementary codes are considered, so the codes have the highest d_{min}. The generator matrix and the weight enumerator of each code are provided. For k = 2 and k = 3, general formulae are derived for the weight enumerators. The results obtained indicate that for some parameters there is more than one weight enumerator for best self-complementary codes. Since the codes are in systematic form, addition of all the rows of the generator matrix must give the all-ones codeword and hence each column of this matrix must have odd weight. This aids in constructing good codes because even weight columns need not be considered.

In [4], binary self-complementary codes in non-systematic form up to length $n = 12$ were given. This report extends these results and presents self-complementary codes in systematic form up to length $n = 16$. For $k = 2$ and $k = 3$, codes are given starting from $n = 13$ since codes up to $n = 12$ were given in [4] and general formulae for the weight enumerators are derived. Further, $(10, 5, 4)$ and $(11, 5, 4)$ codes are given which improve on the $(10, 5, 3)$ and $(11, 5, 3)$ codes in [4].

Constructing the best codes requires finding all possible odd weight columns for a given k. The number of odd weight columns is $\binom{k}{1} + \binom{k}{3} + \ldots + \binom{k}{k-1}$ if k is even and $\binom{k}{1} + \binom{k}{3} + \ldots + \binom{k}{k}$ if k is odd.

Example 13: For $k = 3$, the generator matrix must be composed of the following four odd weight columns

$$1001$$
$$0101$$
$$0011$$

For instance

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

generates a $(4, 3, 2)$ self-complementary code. These four columns can be used to construct the generator matrix of codes with $n > 4$ and $k = 3$. For example, an $(8, 3, 4)$ code can be constructed by combining two copies of the four columns

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Note that the columns can be interchanged and d_{min} is the same as the corresponding codes are equivalent.

For $n - k = 0$, the best self-complementary codes have parameters $(n, n, 1)$ and for $n - k = 1$, the best self-complementary codes have parameters $(n, n-1, 1)$ for n odd and $(n, n-1, 2)$ for n even. In the following sections, the best self-complementary codes for $k = 2$ to 14 up to $n = 16$ are given. A comparison of the best binary linear block codes and the best binary self-complementary codes is given in Section 4.14.

4.1 Best Binary Self-Complementary Codes for $k = 2$

(13,2,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + x^6 + x^7 + x^{13}$

(14,2,7)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 2x^7 + x^{14}$

(15,2,7)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + x^7 + x^8 + x^{15}$

(16,2,8)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 2x^8 + x^{16}$

From these examples, a general formula for the weight enumerator of a self-complementary code when $k = 2$ is

$A(x) = 1 + x^{(n-1)/2} + x^{(n+1)/2} + x^n$ n odd

$A(x) = 1 + 2x^{n/2} + x^n$ n even

4.2 Best Binary Self-Complementary Codes for $k = 3$

(12,3,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 6x^6 + x^{12}$

(13,3,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 3x^6 + 3x^7 + x^{13}$

(14,3,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 3x^6 + 3x^8 + x^{14}$

Another $(14, 3, 6)$ code is given below which has a different weight enumerator

(14,3,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + x^6 + 4x^7 + x^8 + x^{14}$

(15,3,7)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 3x^7 + 3x^8 + x^{15}$

(16,3,8)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 6x^8 + x^{16}$

(18,3,8)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + x^8 + 4x^9 + x^{10} + x^{18}$

From these examples, a general formula for the weight enumerator of a self-complementary code when $k = 3$ is

$A(x) = 1 + 3x^{(n-1)/2} + 3x^{(n+1)/2} + x^n$ n odd

$A(x) = 1 + 3x^{(n/2)-1} + 3x^{(n/2)+1} + x^n$ n even

and when $n = 2 \bmod 4$ there is also the weight enumerator

$A(x) = 1 + x^{(n/2)-1} + 4x^{n/2} + x^{(n/2)+1} + x^n$

4.3 Best Binary Self-Complementary Codes for $k = 4$

In this section, there is more than one weight enumerator for some of the code parameters. In these cases, a code is given for each weight enumerator.

(6,4,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 3x^2 + 8x^3 + 3x^4 + x^6$

(6,4,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 7x^2 + 7x^4 + x^6$

(7,4,3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 7x^3 + 7x^4 + x^7$

(8,4,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 14x^4 + x^8$

(9,4,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 7x^4 + 7x^5 + x^9$

(10,4,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 3x^4 + 8x^5 + 3x^6 + x^{10}$

(10,4,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 7x^4 + 7x^6 + x^{10}$

(11,4,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + x^4 + 6x^5 + 6x^6 + x^7 + x^{11}$

(11,4,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 5x^4 + 2x^5 + 2x^6 + 5x^7 + x^{11}$

(12,4,5)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 4x^5 + 6x^6 + 4x^7 + x^{12}$

(13,4,5)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + x^5 + 6x^6 + 6x^7 + x^8 + x^{13}$

(13,4,5)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 3x^5 + 4x^6 + 4x^7 + 3x^8 + x^{13}$

(14,4,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 3x^6 + 8x^7 + 3x^8 + x^{14}$

(14,4,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 7x^6 + 7x^8 + x^{14}$

(15,4,7)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 7x^7 + 7x^8 + x^{15}$

(16,4,8)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 14x^8 + x^{16}$

4.4 Best Binary Self-Complementary Codes for $k = 5$

(7,5,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 10x^2 + 5x^3 + 5x^4 + 10x^5 + x^7$

(7,5,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 6x^2 + 9x^3 + 9x^4 + 6x^5 + x^7$

(8,5,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 4x^2 + 8x^3 + 6x^4 + 8x^5 + 4x^6 + x^8$

(8,5,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 6x^2 + 8x^3 + 2x^4 + 8x^5 + 6x^6 + x^8$

(8,5,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 10x^2 + 10x^4 + 10x^6 + x^8$

(8,5,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 2x^2 + 8x^3 + 10x^4 + 8x^5 + 2x^6 + x^8$

(9,5,3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 6x^3 + 9x^4 + 9x^5 + 6x^6 + x^9$

(10,5,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 15x^4 + 15x^6 + x^{10}$

(11,5,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 9x^4 + 6x^5 + 6x^6 + 9x^7 + x^{11}$

(11,5,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 5x^4 + 10x^5 + 10x^6 + 5x^7 + x^{11}$

(12,5,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 3x^4 + 8x^5 + 8x^6 + 8x^7 + 3x^8 + x^{12}$

(12,5,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + x^4 + 8x^5 + 12x^6 + 8x^7 + x^8 + x^{12}$

(12,5,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 5x^4 + 20x^6 + 5x^8 + x^{12}$

(13,5,5)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 7x^5 + 8x^6 + 8x^7 + 7x^8 + x^{13}$

(14,5,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 15x^6 + 15x^8 + x^{14}$

(15,5,7)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 15x^7 + 15x^8 + x^{15}$

(16,5,8)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 30x^8 + x^{16}$

4.5 Best Binary Self-Complementary Codes for $k = 6$

(8,6,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 8x^2 + 16x^3 + 14x^4 + 16x^5 + 8x^6 + x^8$

(8,6,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 16x^2 + 30x^4 + 16x^6 + x^8$

(9,6,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 7x^2 + 9x^3 + 15x^4 + 15x^5 + 9x^6 + 7x^7 + x^9$

(10,6,3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 10x^3 + 15x^4 + 12x^5 + 15x^6 + 10x^7 + x^{10}$

(11,6,3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 3x^3 + 16x^4 + 12x^5 + 12x^6 + 16x^7 + 3x^8 + x^{11}$

(12,6,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 11x^4 + 16x^5 + 8x^6 + 16x^7 + 11x^8 + x^{12}$

(13,6,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 4x^4 + 15x^5 + 12x^6 + 12x^7 + 15x^8 + 4x^9 + x^{13}$

(14,6,5)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 12x^5 + 15x^6 + 8x^7 + 15x^8 + 12x^9 + x^{14}$

(15,6,5)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 8x^5 + 12x^6 + 11x^7 + 11x^8 + 12x^9 + 8x^{10} + x^{15}$

(16,6,6)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 20x^6 + 22x^8 + 20x^{10} + x^{16}$

4.6 Best Binary Self-Complementary Codes for $k = 7$

(9,7,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 21x^2 + 7x^3 + 35x^4 + 35x^5 + 7x^6 + 21x^7 + x^9$

(10,7,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 21x^2 + 42x^4 + 42x^6 + 21x^8 + x^{10}$

(11,7,3)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&1&1 \\ 0&1&0&0&0&0&0&0&1&0&1 \\ 0&0&1&0&0&0&0&0&1&1&0 \\ 0&0&0&1&0&0&0&1&1&1&0 \\ 0&0&0&0&1&0&0&1&1&0&1 \\ 0&0&0&0&0&1&0&1&0&1&1 \\ 0&0&0&0&0&0&1&0&1&1&1 \end{bmatrix}$$

$A(x) = 1 + 13x^3 + 26x^4 + 24x^5 + 24x^6 + 26x^7 + 13x^8 + x^{11}$

(12,7,4)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&1&0&1&1&0 \\ 0&1&0&0&0&0&0&1&0&1&0&1 \\ 0&0&1&0&0&0&0&1&0&0&1&1 \\ 0&0&0&1&0&0&0&0&1&1&1&0 \\ 0&0&0&0&1&0&0&0&1&1&0&1 \\ 0&0&0&0&0&1&0&0&1&0&1&1 \\ 0&0&0&0&0&0&1&0&0&1&1&1 \end{bmatrix}$$

$A(x) = 1 + 39x^4 + 48x^6 + 39x^8 + x^{12}$

(13,7,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 18x^4 + 21x^5 + 24x^6 + 24x^7 + 21x^8 + 18x^9 + x^{13}$

(14,7,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 18x^4 + 45x^6 + 45x^8 + 18x^{10} + x^{14}$

(15,7,4)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$A(x) = 1 + 2x^4 + 16x^5 + 24x^6 + 21x^7 + 21x^8 + 24x^9 + 16x^{10} + 2x^{11} + x^{15}$

(16,7,5)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 10x^5 + 20x^6 + 22x^7 + 22x^8 + 22x^9 + 20x^{10} + 10x^{11} + x^{16}$

4.7 Best Binary Self-Complementary Codes for $k = 8$

(10,8,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 13x^2 + 32x^3 + 50x^4 + 64x^5 + 50x^6 + 32x^7 + 13x^8 + x^{10}$

(11,8,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$A(x) = 1 + 6x^2 + 23x^3 + 40x^4 + 58x^5 + 58x^6 + 40x^7 + 23x^8 + 6x^9 + x^{11}$

(12,8,3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 16x^3 + 39x^4 + 48x^5 + 48x^6 + 48x^7 + 39x^8 + 16x^9 + x^{12}$

(13,8,3)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 10x^3 + 24x^4 + 39x^5 + 54x^6 + 54x^7 + 39x^8 + 24x^9 + 10x^{10} + x^{13}$

(14,8,3)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&1&0&0&0&0&0&0&0&1&0&1&1&1 \\ 0&0&1&0&0&0&0&0&0&0&1&1&0&1 \\ 0&0&0&1&0&0&0&0&1&1&1&1&0&1 \\ 0&0&0&0&1&0&0&0&1&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&1&0&1&0&0&1 \\ 0&0&0&0&0&0&1&0&1&1&0&0&0&1 \\ 0&0&0&0&0&0&0&1&0&1&1&1&1&0 \end{bmatrix}$$

$A(x) = 1 + 2x^3 + 22x^4 + 32x^5 + 41x^6 + 60x^7 + 41x^8 + 32x^9 + 22x^{10} + 2x^{11} + x^{14}$

(15,8,4)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&1&1&1&0&0&0&1 \\ 0&1&0&0&0&0&0&0&0&1&0&1&1&0&1 \\ 0&0&1&0&0&0&0&0&0&0&1&1&0&1&1 \\ 0&0&0&1&0&0&0&0&1&1&1&1&0&0&0 \\ 0&0&0&0&1&0&0&0&1&0&0&1&1&0&0 \\ 0&0&0&0&0&1&0&0&1&0&1&0&0&1&0 \\ 0&0&0&0&0&0&1&0&1&1&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&1&1&1&1&0&0 \end{bmatrix}$$

$A(x) = 1 + 12x^4 + 30x^5 + 34x^6 + 51x^7 + 51x^8 + 34x^9 + 30x^{10} + 12x^{11} + x^{15}$

(16,8,4)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&1&1&1&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&0&1&0&1&1&1&1&0 \\ 0&0&1&0&0&0&0&0&0&0&1&1&0&1&1&0 \\ 0&0&0&1&0&0&0&0&1&1&1&1&0&1&0&0 \\ 0&0&0&0&1&0&0&0&1&0&0&1&1&0&0&1 \\ 0&0&0&0&0&1&0&0&1&0&1&0&0&1&0&1 \\ 0&0&0&0&0&0&1&0&1&1&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0&1&1&1&1&0&0&1 \end{bmatrix}$$

$A(x) = 1 + 5x^4 + 24x^5 + 28x^6 + 40x^7 + 60x^8 + 40x^9 + 28x^{10} + 24x^{11} + 5x^{12} + x^{16}$

4.8 Best Binary Self-Complementary Codes for $k = 9$

(11,9,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&1&1 \\ 0&1&0&0&0&0&0&0&0&1&1 \\ 0&0&1&0&0&0&0&0&0&1&1 \\ 0&0&0&1&0&0&0&0&0&1&1 \\ 0&0&0&0&1&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&0&1&0 \\ 0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&1&1&1 \end{bmatrix}$$

$A(x) = 1 + 20x^2 + 25x^3 + 110x^4 + 100x^5 + 100x^6 + 110x^7 + 25x^8 + 20x^9 + x^{11}$

(12,9,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A(x) = 1 + 12x^2 + 16x^3 + 71x^4 + 112x^5 + 88x^6 + 112x^7 + 71x^8 + 16x^9 + 12x^{10} + x^{12}$

(13,9,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + x^2 + 21x^3 + 50x^4 + 77x^5 + 106x^6 + 106x^7 + 77x^8 + 50x^9 + 21x^{10} + x^{11} + x^{13}$

(14,9,3)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1&1 \\ 0&1&0&0&0&0&0&0&0&0&1&0&1&1 \\ 0&0&1&0&0&0&0&0&0&0&1&1&0&1 \\ 0&0&0&1&0&0&0&0&0&0&1&1&1&1 \\ 0&0&0&0&1&0&0&0&0&1&0&0&1&1 \\ 0&0&0&0&0&1&0&0&0&1&0&1&0&1 \\ 0&0&0&0&0&0&1&0&0&1&0&1&1&0 \\ 0&0&0&0&0&0&0&1&0&1&1&0&1&1 \\ 0&0&0&0&0&0&0&0&1&1&1&0&1&0 \end{bmatrix}$$

$A(x) = 1 + 8x^3 + 42x^4 + 64x^5 + 85x^6 + 112x^7 + 85x^8 + 64x^9 + 42x^{10} + 8x^{11} + x^{14}$

(15,9,4)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1&1&1 \\ 0&1&0&0&0&0&0&0&0&0&1&0&1&1&1 \\ 0&0&1&0&0&0&0&0&0&0&1&1&0&1&1 \\ 0&0&0&1&0&0&0&0&0&0&1&1&1&1&0 \\ 0&0&0&0&1&0&0&0&0&1&0&0&1&1&1 \\ 0&0&0&0&0&1&0&0&0&1&0&1&0&1&0 \\ 0&0&0&0&0&0&1&0&0&1&0&1&1&0&0 \\ 0&0&0&0&0&0&0&1&0&1&1&0&1&1&0 \\ 0&0&0&0&0&0&0&0&1&1&1&0&1&0&1 \end{bmatrix}$$

$A(x) = 1 + 30x^4 + 60x^5 + 60x^6 + 105x^7 + 105x^8 + 60x^9 + 60x^{10} + 30x^{11} + x^{15}$

(16,9,4)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1 \\ 0&1&0&0&0&0&0&0&0&0&1&0&1&1&1&1 \\ 0&0&1&0&0&0&0&0&0&0&1&1&0&1&1&1 \\ 0&0&0&1&0&0&0&0&0&0&1&1&1&1&0&1 \\ 0&0&0&0&1&0&0&0&0&1&0&0&1&1&1&1 \\ 0&0&0&0&0&1&0&0&0&1&0&1&0&1&0&1 \\ 0&0&0&0&0&0&1&0&0&1&0&1&1&0&0&1 \\ 0&0&0&0&0&0&0&1&0&1&1&0&1&1&0&1 \\ 0&0&0&0&0&0&0&0&1&1&1&0&1&0&1&1 \end{bmatrix}$$

$A(x) = 1 + 16x^4 + 50x^5 + 64x^6 + 88x^7 + 94x^8 + 88x^9 + 64x^{10} + 40x^{11} + 16x^{12} + x^{16}$

4.9 Best Binary Self-Complementary Codes for $k = 10$

(12,10,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&0&0&1&1 \\ 0&0&1&0&0&0&0&0&0&0&1&1 \\ 0&0&0&1&0&0&0&0&0&0&1&1 \\ 0&0&0&0&1&0&0&0&0&0&1&1 \\ 0&0&0&0&0&1&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&1 \end{bmatrix}$$

$A(x) = 1 + 22x^2 + 48x^3 + 127x^4 + 208x^5 + 212x^6 + 208x^7 + 127x^8 + 48x^9 + 22x^{10} + x^{12}$

(13,10,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&1&0&1\\ 0&1&0&0&0&0&0&0&0&0&1&1&1\\ 0&0&1&0&0&0&0&0&0&0&1&1&0\\ 0&0&0&1&0&0&0&0&0&0&1&1&0\\ 0&0&0&0&1&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&1&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&1&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&1&1 \end{bmatrix}$$

$A(x) = 1 + 9x^2 + 37x^3 + 90x^4 + 165x^5 + 210x^6 + 210x^7 + 165x^8 + 90x^9 + 37x^{10} + 9x^{11} + x^{13}$

(14,10,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1&0\\ 0&1&0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&1&0&0&0&0&0&0&0&0&0&1&1\\ 0&0&0&1&0&0&0&0&0&0&1&0&1&0\\ 0&0&0&0&1&0&0&0&0&0&1&0&0&1\\ 0&0&0&0&0&1&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&0&1&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&0&1&0&0&1&0&0&1\\ 0&0&0&0&0&0&0&0&1&0&1&1&0&1\\ 0&0&0&0&0&0&0&0&0&1&1&1&1&0 \end{bmatrix}$$

$A(x) = 1+2x^2+26x^3+68x^4+120x^5+185x^6+220x^7+185x^8+120x^9+68x^{10}+26x^{11}+2x^{12}+x^{14}$

(15,10,3)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1&0&1\\ 0&1&0&0&0&0&0&0&0&0&0&1&0&1&1\\ 0&0&1&0&0&0&0&0&0&0&0&0&1&1&1\\ 0&0&0&1&0&0&0&0&0&0&1&0&1&0&1\\ 0&0&0&0&1&0&0&0&0&0&1&0&0&1&1\\ 0&0&0&0&0&1&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&1&0&0&0&1&1&0&0&1\\ 0&0&0&0&0&0&0&1&0&0&1&0&0&1&0\\ 0&0&0&0&0&0&0&0&1&0&1&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&1&1&1&0&1 \end{bmatrix}$$

$A(x) = 1 + 13x^3 + 51x^4 + 96x^5 + 144x^6 + 207x^7 + 207x^8 + 144x^9 + 96x^{10} + 51x^{11} + 13x^{12} + x^{15}$

(16,10,4)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1&0&1&0\\ 0&1&0&0&0&0&0&0&0&0&0&1&0&1&1&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&1&1&1&0\\ 0&0&0&1&0&0&0&0&0&0&1&0&1&0&1&0\\ 0&0&0&0&1&0&0&0&0&0&1&0&0&1&1&0\\ 0&0&0&0&0&1&0&0&0&0&1&0&1&0&0&1\\ 0&0&0&0&0&0&1&0&0&0&1&1&0&0&1&0\\ 0&0&0&0&0&0&0&1&0&0&1&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&1&0&1&1&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&1&1&1&0&1&1 \end{bmatrix}$$

$A(x) = 1 + 64x^4 + 240x^6 + 414x^8 + 240x^{10} + 64x^{12} + x^{16}$

4.10 Best Binary Self-Complementary Codes for $k = 11$

(13,11,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1 \\ 0&1&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&1&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&1&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&1&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1&1&0 \end{bmatrix}$$

$A(x) = 1 + 55x^2 + 11x^3 + 330x^4 + 165x^5 + 462x^6 + 462x^7 + 165x^8 + 330x^9 + 11x^{10} + 55x^{11} + x^{13}$

(14,11,2)

$$G = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1
\end{bmatrix}$$

$A(x) = 1 + 45x^2 + 20x^3 + 211x^4 + 240x^5 + 255x^6 + 504x^7 + 255x^8 + 240x^9 + 211x^{10} + 20x^{11} + 45x^{12} + x^{14}$

(15,11,3)

$$G = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}$$

$A(x) = 1+35x^3+105x^4+168x^5+280x^6+435x^7+435x^8+280x^9+168x^{10}+105x^{11}+35x^{12}+x^{15}$

(16,11,4)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\ 0&1&0&0&0&0&0&0&0&0&0&0&1&0&1&1\\ 0&0&1&0&0&0&0&0&0&0&0&1&1&0&1\\ 0&0&0&1&0&0&0&0&0&0&0&1&0&0&1&1\\ 0&0&0&0&1&0&0&0&0&0&0&1&0&1&0&1\\ 0&0&0&0&0&1&0&0&0&0&1&1&0&0&1\\ 0&0&0&0&0&0&1&0&0&0&0&1&1&1&1&1\\ 0&0&0&0&0&0&0&1&0&0&0&1&1&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&1&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&1&0&1&1&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&0 \end{bmatrix}$$

$A(x) = 1 + 140x^4 + 448x^6 + 870x^8 + 448x^{10} + 140x^{12} + x^{16}$

4.11 Best Binary Self-Complementary Codes for $k = 12$

(14,12,2)

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A(x) = 1 + 67x^2 + 561x^4 + 1419x^6 + 1419x^8 + 561x^{10} + 67x^{12} + x^{14}$

(15,12,2)

$$G = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}$$

$A(x) = 1 + 46x^2 + 21x^3 + 275x^4 + 286x^5 + 660x^6 + 759x^7 + 759x^8 + 660x^9 + 286x^{10} + 275x^{11} + 21x^{12} + 46x^{13} + x^{15}$

(16,12,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&1&0&1&1 \\ 0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&1 \\ 0&0&0&0&0&1&0&0&0&0&0&0&1&0&1&1 \\ 0&0&0&0&0&0&1&0&0&0&0&0&1&0&1&1 \\ 0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&1&0 \end{bmatrix}$$

$A(x) = 1 + 21x^2 + 36x^3 + 150x^4 + 276x^5 + 475x^6 + 712x^7 + 754x^8 + 712x^9 + 475x^{10} + 276x^{11} + 150x^{12} + 36x^{13} + 21x^{14} + x^{16}$

4.12 Best Binary Self-Complementary Codes for $k = 13$

(15,13,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&1&1\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&1&1\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&1&1\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&1&1\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&1&1\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&1&1\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&1&1\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&1&1\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&1&1\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&1&1\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&1&1\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&1\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&1 \end{bmatrix}$$

$A(x) = 1 + 78x^2 + 13x^3 + 715x^4 + 286x^5 + 1716x^6 + 1287x^7 + 1287x^8 + 1716x^9 + 286x^{10} + 715x^{11} + 13x^{12} + 78x^{13} + x^{15}$

(16,13,2)

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&1&1&1\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&1&1&1\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&1&1&1\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&1&1&1\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&1&1\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1 \end{bmatrix}$$

$A(x) = 1 + 78x^2 + 728x^4 + 2002x^6 + 2574x^8 + 2002x^{10} + 728x^{12} + 78x^{14} + x^{16}$

4.13 Best Binary Self-Complementary Codes for $k = 14$

(16,14,2)

$$G = \begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1
\end{bmatrix}$$

$$A(x) = 1 + 92x^2 + 1092x^4 + 4004x^6 + 6006x^8 + 4004x^{10} + 1092x^{12} + 92x^{14} + x^{16}$$

The weight enumerators of the best self-complementary codes for $k = 1, 2, 3$ are given in Table 4.1.

k	Weight enumerator for n odd	Weight enumerator for n even
1	$1 + x^n$	$1 + x^n$
2	$1 + x^{(n-1)/2} + x^{(n+1)/2} + x^n$	$1 + 2x^{n/2} + x^n$
3	$1 + 3x^{(n-1)/2} + 3x^{(n+1)/2} + x^n$	$1 + 3x^{(n/2)-1} + 3x^{(n/2)+1} + x^n$
3, $n = 2 \bmod 4$	-	$1 + x^{(n/2)-1} + 4x^{n/2} + x^{(n/2)+1} + x^n$

Table 4.1: Weight enumerators of the best self-complementary codes for $k = 1, 2, 3$.

4.14 Comparison of Best Binary Linear Codes with Best Binary Self-Complementary Codes

A binary linear (n, k) code C is a best known code if it has the highest minimum distance d among all binary (n, k) codes. Similarly, a binary self-complementary (n, k) code C is a best code if it has the highest minimum distance d among all binary self-complementary (n, k) codes. Table 4.2 presents the minimum distances of the best binary linear codes up to length $n = 16$ from [1] and Table 4.3 presents the minimum distances of the best binary self-complementary codes up to length $n = 16$. In these tables, the first column gives the value of n, the first row gives the value of k and the remaining entries give the value of d for the corresponding best (n, k) codes. The columns for $k = 1$ correspond to the repetition codes so the minimum distances are the same in both tables since repetition codes are always self-complementary. The far right diagonal of both tables $(n = k)$ has the same minimum distance which is 1. The next diagonal in Table 4.2 corresponds to the SPC codes so the values on this diagonal in Table 4.3 are the same when n is even. It can be observed from these tables that for $k = 1, 5, 11, 13, 15, 16$, the best self-complementary codes have the same minimum distance as the best binary linear codes. The differences are greatest with $k = 2$ because the generator matrices have only two rows and the columns must have odd weight. The difference in d_{min} is 1 for $n = 3, 5, 6, 7, 8, 10$, 2 for $n = 9, 11, 12, 13, 14, 16$, and for $n = 15$ it is 3.

For $k = 3$, the difference in d_{min} is 1 for $n = 6, 7, 10, 11, 13, 15$ and for $n = 14$ it is 2.

For $k = 4$, the difference in d_{min} is 1 for $n = 5, 11, 12, 13, 14, 15$.

For $k = 6$, the difference in d_{min} is 1 for $n = 7, 11, 15$.

For $k = 7$, the difference in d_{min} is 1 for $n = 15, 16$.

For $k = 8$, the difference in d_{min} is 1 for $n = 9, 13, 14, 16$.

For $k = 9$, the difference in d_{min} is 1 for $n = 13, 14$.

For $k = 10$, the difference in d_{min} is 1 for $n = 11, 14, 15$.

For $k = 12$, the difference in d_{min} is 1 for $n = 13$.

For $k = 14$, the difference in d_{min} is 1 for $n = 15$.

These results indicate that the best self-complementary codes have minimum distances which are similar to those of the best binary linear codes with differences of 1, 2 and 3 for 34, 7 and 1 values of n, respectively. These differences are shown in Table 4.4.

n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	3	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-
4	4	2	2	1	-	-	-	-	-	-	-	-	-	-	-	-
5	5	3	2	2	1	-	-	-	-	-	-	-	-	-	-	-
6	6	4	3	2	2	1	-	-	-	-	-	-	-	-	-	-
7	7	4	4	3	2	2	1	-	-	-	-	-	-	-	-	-
8	8	5	4	4	2	2	2	1	-	-	-	-	-	-	-	-
9	9	6	4	4	3	2	2	2	1	-	-	-	-	-	-	-
10	10	6	5	4	4	3	2	2	2	1	-	-	-	-	-	-
11	11	7	6	5	4	4	3	2	2	2	1	-	-	-	-	-
12	12	8	6	6	4	4	4	3	2	2	2	1	-	-	-	-
13	13	8	7	6	5	4	4	4	3	2	2	2	1	-	-	-
14	14	9	8	7	6	5	4	4	4	3	2	2	2	1	-	-
15	15	10	8	8	7	6	5	4	4	4	3	2	2	2	1	-
16	16	10	8	8	8	6	6	5	4	4	4	2	2	2	2	1

Table 4.2: Minimum distances of the best binary linear codes up to length $n = 16$ [1].

n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	2	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	3	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-
4	4	2	2	1	-	-	-	-	-	-	-	-	-	-	-	-
5	5	2	2	1	1	-	-	-	-	-	-	-	-	-	-	-
6	6	3	2	2	2	1	-	-	-	-	-	-	-	-	-	-
7	7	3	3	3	2	1	1	-	-	-	-	-	-	-	-	-
8	8	4	4	4	2	2	2	1	-	-	-	-	-	-	-	-
9	9	4	4	4	3	2	2	1	1	-	-	-	-	-	-	-
10	10	5	4	4	4	3	2	2	2	1	-	-	-	-	-	-
11	11	5	5	4	4	3	3	2	2	1	1	-	-	-	-	-
12	12	6	6	5	4	4	4	3	2	2	2	1	-	-	-	-
13	13	6	6	5	5	4	4	3	2	2	2	1	1	-	-	-
14	14	7	6	6	6	5	4	3	3	2	2	2	2	1	-	-
15	15	7	7	7	7	5	4	4	4	3	3	2	2	1	1	-
16	16	8	8	8	8	6	5	4	4	4	4	2	2	2	2	1

Table 4.3: Minimum distances of the best binary self-complementary codes up to length $n = 16$.

n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0	0	0	0	-	-	-	-	-	-	-	-	-	-	-	-
5	0	1	0	1	0	-	-	-	-	-	-	-	-	-	-	-
6	0	1	1	0	0	0	-	-	-	-	-	-	-	-	-	-
7	0	1	1	0	0	1	0	-	-	-	-	-	-	-	-	-
8	0	1	0	0	0	0	0	0	-	-	-	-	-	-	-	-
9	0	2	0	0	0	0	0	1	0	-	-	-	-	-	-	-
10	0	1	1	0	0	0	0	0	0	0	-	-	-	-	-	-
11	0	2	1	1	0	1	0	0	0	1	0	-	-	-	-	-
12	0	2	0	1	0	0	0	0	0	0	0	0	-	-	-	-
13	0	2	1	1	0	0	0	1	1	0	0	1	0	-	-	-
14	0	2	2	1	0	0	0	1	1	1	0	0	0	0	-	-
15	0	3	1	1	0	1	1	0	0	1	0	0	0	1	0	-
16	0	2	0	0	0	0	1	1	0	0	0	0	0	0	0	0

Table 4.4: Differences in minimum distance between the best binary self-complementary codes and the best binary linear codes up to length $n = 16$.

Chapter 5

Conclusion and Future Work

Binary self-complementary codes up to length $n = 16$ were constructed in this project and their weight enumerators were generated. All codes given are in systematic form. The single parity-check, repetition, Hamming and Reed-Muller codes that are self-complementary were discussed. The Magma computational algebraic system was used to generate the weight enumerators of the codes. It was found that for some parameters there are multiple weight enumerators for the best self-complementary codes. Finally, a comparison was made between the best binary linear codes and the best binary self-complementary codes based on their minimum distances. It was found that many of the best binary self-complementary codes have the same minimum distances as the best binary linear codes.

This project considered binary codes since most digital communication systems use binary codes. Future work can examine non-binary self-complementary codes. Further, binary codes of length $n > 16$ can be explored.

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