COMPUTING THE HORIZONTAL PRESSURE GRADIENT FORCE IN SIGMA COORDINATES

by

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Abstract

In this thesis we consider the problem of computing the horizontal pressure gradient force under σ coordinates, within a discrete numerical model. This calculation has previously been found to be accompanied by large truncation error when performed in regions of steep (mountainous) terrain. Corby et al.(1972) proposed a limited solution by showing that if one considered an atmosphere where temperature varied linearly with $\log p$, a certain finite difference scheme could be used for which the discretization error vanished when no geostrophic wind existed.

We take this as a starting point, and extend the result of Corby et al.(1972) to a case where a simple wind exists by requiring that isobaric surfaces be tilted rather than horizontal isothermal planes, as was true in his study, as well as to a more complicated case, in which isobaric planes carry a temperature variation, and for which we have derived an accompanying truncation error. In addition we show how the selection of temperature and finite difference scheme by Corby et al.(1972) can be seen to be a consequence of an attempt to transform the continuous hydrostatic equation into a discrete version of it, without loss of accuracy. Finally, we show the importance of employing data which is consistent with both the finite difference and continuous equations by conducting a numerical experiment.

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Table of Contents

List of Symbols and Abbreviations

p	pressure
σ	sigma, ratio of pressure to surface pressure
ρ	density
R	the ideal gas constant
T	temperature
γ	lapse rate of temperature with respect to height
ϕ	geopotential
$\vec{V}_g$	geostrophic wind vector
Δ	central differencing operator
δ	central differencing operator with divsion by the interval considered
$\overline{F}$	averaging operator
$F^{\prime}(x)$	differentiation of F with respect to x
∇	gradient operator
ln	logarithm to base e
P	a fixed point of observation at sea level
x	variable that increases toward the east with respect to P
y	variable that increases toward the north with respect to P
z	variable that increases skyward toward the zenith with respect to P
$Z(x)$	mountain surface height at x in the x - z plane

List of Tables

List of Tables in Appendix B

CASE I.B - Horizontal isobaric planes under $T = Az + B$

CASE II.A - Barotropic atmosphere - tilted (parallel) isobaric planes under $\nabla_p T = \vec{0}$, under $T = A \ln p + B$.

CASE II.B - Barotropic atmosphere - tilted (parallel) isobaric planes under $\nabla_p T = \vec{0}$, under $T = Az + B$.

CASE III.A.1 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 1.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = A \ln p + B_o + C_o(x - x^{(R)})$.

CASE III.B.2 - Baroclinic atmosphere - under $\nabla_p T = \begin{bmatrix} 2.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under

$T = Az + B_o + C_o(x - x^{(R)})$.

List of Figures

Chapter 1

Introduction

One of the central problems in numerical modelling is to find finite difference analogs for continuous equations in such a way as to maintain consistency within the discrete world, as much as possible (Arakawa and Suarez, 1983). Computation of the horizontal pressure gradient force (HPGF) within a discrete model under σ coordinates has been reported to be accompanied by a large truncation error, when performed in the vicinity of steep mountainous regions, by a number of reasearchers: Kurihara (1968), Kasahara (1974), Tokioka (1978), Simmons and Burridge (1981). Several attempts to approach the problem assuming an atmosphere where temperature varies under a constant lapse rate $T = Az + B$ (Gary, 1973; Nakamura, 1978), were unable to show substantially satisfactory results. On the other hand, Corby et al.(1972) proposed a temperature structure ($T = A \ln p + B$) and finite-difference scheme for which no ensuing truncation error was found when calculating the HPGF, but only in a simple case when there was no geostrophic wind present. Danard (1989), and Danard et al.(1993) also assumed a linear variation with $\ln p$ in calculating the HPGF from surface observations of temperature and pressure.

In this thesis, we will show the importance of choosing data extrapolated from functions that are consistent with the discrete version of the basic driving equation of an atmospheric model. After introducing pertinent meteorological background for discussion of this problem including the sigma coordinate in Chapter 2, we will then proceed in Chapter 3 to show that if we wish to transform the continuous hydrostatic equation into a discrete analog of it, we can determine what a suitable temperature structure (henceforth T-structure or T-function) for doing so might be. Following this will be a review of the finite difference scheme recommended by Corby et al.(1972), to be referred to as the Corby finite difference scheme for simplicity throughout the rest of the thesis, showing that no discretization error should result when an atmosphere is at rest.

In Chapter 4 we compare data (surface temperature and pressure) extrapolated from a T-function consistent with the discrete hydrostatic equation we have chosen, to that of a T-function not consistent with it, although consistent with the continuous version of the hydrostatic equation. This will be carried out by means of a series of numerical experiments computing the HPGF from these surface temperature and pressure values, along terrain of a cosine shaped hill. We further show that although the non-consistent T-function ($T = Az + B$) discrete model experiences truncation error in calculation of the HPGF, in the limit as the grid size goes to zero the HPGF converges to the correct value.

Chapter 5 reports on the numerical results of the comparison between the two different T-structure models ($T = Az + B$ and $T = A \ln p + B$) when no geostrophic wind is present. Corby's claim (see first paragraph of this Introduction) is empirically verified. However, when $T = Az + B$ is employed, the calculated surface geostrophic wind differs from its expected value. A derivation for this truncation error is also carried out, using Taylor's series.

In Chapter 6, the author's own geometrical argument is presented which shows why Corby's finite-difference scheme of twenty years ago will be applicable to a more complex case - a barotropic atmosphere in which isobaric planes are parallel, and tilted with respect to the horizontal (but carry no temperature variation). In addition, it is shown that the scheme is equally valid on any sigma level above that of the surface, although we are primarily interested in the surface sigma level. Once more we show that when $T = Az + B$, calculation of the HPGF has an accompanying truncation error and we derive its formula.

The most complex of the three atmospheres we consider in this thesis is presented in Chapter 7. It is a simple linear baroclinic atmosphere where the temperature varies according to $B_o + C_o(x - x^{(R)})$ on constant pressure planes (with y fixed for clarity of discussion). In this case the geostrophic wind vector varies in the vertical as given by the thermal wind equation. For the case of $T = A \ln p + B$, this vertical variation is put to use in calculating the truncation error for the HPGF. It is found that this discretization error has a Laplacian form to it and an exact expression for this truncation error is derived, for any arbitrary σ level. The case when $T = Az + B_o + C_o(x - x^{(R)})$ occurs in a baroclinic atmosphere is analyzed as well.

Chapter 2

Relevant meteorological equations

The governing meteorological principles that will serve as a basis for the derivations in this thesis are presented in this chapter.

As a preliminary step to the study of the pertinent meterology related to the problem of computing horizontal pressure gradient force, it will be convenient to place a cartesian coordinate system at a fixed point of observation at sea level. For the sake of definiteness, we will choose this point P as part of the Northern hemisphere, and take positive x to be directed eastward from this point, positive y to point northward from it, and positive z to point skyward toward the zenith with respect to P. The x - y plane is chosen tangent to the spherical earth at P. We will call the unit vectors in the positive x, y and z directions $\vec{i}$, $\vec{j}$, and $\vec{k}$ respectively so an arbitrary vector $\vec{r}$ will be written as

$$\vec{r} = r_x\vec{i} + r_y\vec{j} + r_z\vec{k} \tag{2.0.1}$$

or as

$$\vec{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \tag{2.0.2}$$

where

$$\vec{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{2.0.3}$$

All derivations presented in this chapter are standard ones in meteorology, and are adapted from Holton (1979), unless it is noted otherwise. A discussion of the pressure gradient force including its definition, derivation and several of its uses, will now follow.

2.1 Pressure gradient force

The force which arises due to spatial variation in pressure is called the pressure gradient force (PGF). Let us consider the cube of dimensions Δx, Δy and Δz with center $\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}$ in the pressure field $p\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If we denote by ${F_1}^{x_o-\frac{\Delta x}{2}}$ the normal force acting on the left face of the cube, at $\begin{bmatrix} x_o-\frac{\Delta x}{2} \\ y_o \\ z_o \end{bmatrix}$, in the positive x direction, and by $-{F_2}^{x_o+\frac{\Delta x}{2}}$, the normal force acting on the right face of the cube, at $\begin{bmatrix} x_o+\frac{\Delta x}{2} \\ y_o \\ z_o \end{bmatrix}$, in the negative x direction, then as a result the net pressure gradient force acting on the cube over the distance Δx is

$$PGF^{x_o} = {F_1}^{x_o-\frac{\Delta x}{2}} - {F_2}^{x_o+\frac{\Delta x}{2}}. \tag{2.1.1}$$

But because force is pressure times area this can be rewritten as follows

$$PGF^{x_o} = p\begin{bmatrix} x_o-\frac{\Delta x}{2} \\ y_o \\ z_o \end{bmatrix} * [\Delta y] * [\Delta z] - p\begin{bmatrix} x_o+\frac{\Delta x}{2} \\ y_o \\ z_o \end{bmatrix} * [\Delta y] * [\Delta z]. \tag{2.1.2}$$

The pressure functions on the right hand side (RHS) of the above equation can be expanded by the following Taylor's series

$$p\begin{bmatrix} x_o-\frac{\Delta x}{2} \\ y_o \\ z_o \end{bmatrix} = p\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} - \frac{1}{1!}\{\frac{\partial p}{\partial x}\begin{bmatrix} x \\ y_o \\ z_o \end{bmatrix}_{x=x_o}\} * \left[\frac{\Delta x}{2}\right] + \frac{1}{2!}\{\frac{\partial^2 p}{\partial x^2}\begin{bmatrix} x \\ y_o \\ z_o \end{bmatrix}_{x=x_o}\} * \left[\frac{\Delta x}{2}\right]^2 - ..., \tag{2.1.3}$$

and

$$p\begin{bmatrix} x_o+\frac{\Delta x}{2} \\ y_o \\ z_o \end{bmatrix} = p\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} + \frac{1}{1!}\{\frac{\partial p}{\partial x}\begin{bmatrix} x \\ y_o \\ z_o \end{bmatrix}_{x=x_o}\} * \left[\frac{\Delta x}{2}\right] + \frac{1}{2!}\{\frac{\partial^2 p}{\partial x^2}\begin{bmatrix} x \\ y_o \\ z_o \end{bmatrix}_{x=x_o}\} * \left[\frac{\Delta x}{2}\right]^2 + ... \tag{2.1.4}$$

We now introduce the mass M, of this volume element as

$$M = \int_{z=z_o-\frac{\Delta z}{2}}^{z=z_o+\frac{\Delta z}{2}} \int_{y=y_o-\frac{\Delta y}{2}}^{y=y_o+\frac{\Delta y}{2}} \int_{x=x_o-\frac{\Delta x}{2}}^{x=x_o+\frac{\Delta x}{2}} \rho\begin{bmatrix} x \\ y \\ z \end{bmatrix} dx\ dy\ dz, \tag{2.1.5}$$

where $\rho\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is the density at point $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If we require that each of $\Delta x, \Delta y, \Delta z$ be sufficiently small so that the density is uniform throughout the cube, the mass can then be written as

$$M = \rho\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} * [\Delta x] * [\Delta y] * [\Delta z]. \tag{2.1.6}$$

The PGF per unit mass in the x direction can then be given by

$$\frac{PGF^{x_o}}{M} = -\frac{1}{\rho}\{\frac{\partial p}{\partial x}\begin{bmatrix} x \\ y_o \\ z_o \end{bmatrix}_{x=x_o}\} - \frac{1}{2*3!}\frac{1}{\rho}\{\frac{\partial^3 p}{\partial x^3}\begin{bmatrix} x \\ y_o \\ z_o \end{bmatrix}_{x=x_o}\} * \left[\frac{\Delta x}{2}\right]^2 - ... \tag{2.1.7}$$

from (2.1.2), (2.1.3), (2.1.4) and (2.1.6). Taking the limit of (2.1.7) as $\Delta x \to 0$ yields the pressure gradient force per unit mass in the x direction

$$\frac{PGF^{x_o}}{M} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{2.1.8}$$

where it is understood that the partial derivative with respect to x written in this way is taken at constant y and constant z. (Of course, analogously $\frac{\partial}{\partial y}$ will be taken at constant x and z, and $\frac{\partial}{\partial z}$ at constant x and y.) Observing first that the argument (2.1.1) through (2.1.8) is valid for arbitrary x_o, and secondly that this argument could be reproduced for the y-directed PGF and z-directed PGF, we conclude that the pressure gradient force per unit mass in three dimensions will be

$$\frac{P\vec{G}F}{M} = \begin{bmatrix} -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ -\frac{1}{\rho}\frac{\partial p}{\partial z} \end{bmatrix} = -\frac{1}{\rho}\nabla p. \tag{2.1.9}$$

2.2 Hydrostatic equation

An important application of pressure gradient force is the role it has in vertically offsetting the downward pull of gravitational force on a parcel of air. An atmosphere is said to be in hydrostatic balance if the pressure gradient force acting upward on a volume V of air with mass M exactly matches the gravitational force acting downward on it. This condition can be stated as

$$-V\frac{\partial p}{\partial z} = M g, \tag{2.2.1}$$

which can be rewritten as follows

$$-\frac{\partial p}{\partial z} = \rho\, g. \tag{2.2.2}$$

By the inverse function theorem, one can then obtain from (2.2.2)

$$g\,\frac{\partial z}{\partial p} = -\frac{1}{\rho}. \tag{2.2.3}$$

We will assume that our atmosphere follows the ideal gas law

$$p\,\frac{1}{\rho} = R\,T, \tag{2.2.4}$$

where R is the ideal gas constant (see Appendix D) and T is temperature. We therefore have from (2.2.3) and (2.2.4)

$$g\frac{\partial z}{\partial p} = -\frac{R\,T}{p}. \tag{2.2.5}$$

Defining the geopotential ϕ of an element of mass under consideration by

$$\phi = g\ z, \tag{2.2.6}$$

the product of the gravitation constant g and height z of this mass above sea level, allows (2.2.5) to be rewritten as

$$\frac{\partial \phi}{\partial p} = -\frac{R\,T}{p}. \tag{2.2.7}$$

This is known as the hydrostatic equation.

If we now consider the pressure gradient force in the horizontal, and on the basis of (2.1.9) define

$$-\frac{1}{\rho}\,\nabla_h\, p = \begin{bmatrix} -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ 0 \end{bmatrix} \tag{2.2.8}$$

as the horizontal pressure gradient force per unit mass, we see that knowledge of the density field is required for evaluation, and such information may not always be easily available. However, if we assume that constant pressure surfaces are continuous, an attempt to measure the discrepancy in pressure between two such surfaces a distance Δx apart, could instead be replaced by an effort to measure the pressure difference between these same two surfaces in the vertical, after multiplication by an appropriate transformation factor. The density term could perhaps be absorbed into another factor for the new expression that develops from an application of the hydrostatic equation. Such an approach is now presented in detail below.

2.3 Horizontal pressure gradient force evaluated on a constant pressure surface

Using geometric considerations it is now shown that the horizontal pressure gradient force is actually equivalent to the geopotential gradient on a constant pressure, that is isobaric, surface as indicated below :

$$\begin{bmatrix} -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ 0 \end{bmatrix} = \begin{bmatrix} -\left(\frac{\partial \phi}{\partial x}\right)_p \\ -\left(\frac{\partial \phi}{\partial y}\right)_p \\ 0 \end{bmatrix}. \tag{2.3.1}$$

Focusing attention on the x component of the above, and referring to Figure 1 on the next page, one may note that the difference in pressure between isobaric planes p_2 and p_1 is

$$p_2^B - p_1^A = p_2^B - p_1^C. \tag{2.3.2}$$

The pressure change over Δx is then

$$\left(\frac{p_2^B - p_1^A}{\Delta x}\right) = \left(-\frac{p_1^C - p_2^B}{\Delta z}\right) * \left(\frac{\Delta z}{\Delta x}\right), \tag{2.3.3}$$

where the - sign has been introduced to account for the fact that the direction of increasing positive z corresponds to a direction of decreasing pressure. Taking the limit of (2.3.3) as $\Delta x \longrightarrow 0$ yields

$$\left(\frac{\partial p}{\partial x}\right)_z = -\left(\frac{\partial p}{\partial z}\right) * \left(\frac{\partial z}{\partial x}\right)_p. \tag{2.3.4}$$

However, recalling (2.2.2)

$$\left(\frac{\partial p}{\partial z}\right) = -g\ \rho,$$

and insering it into (2.3.4) provides the following conclusion, with the use of (2.2.6)

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -g\left(\frac{\partial z}{\partial x}\right)_p = -\left(\frac{\partial \phi}{\partial x}\right)_p \tag{2.3.5}$$

and this is what was to be shown. Since a similar argument could be made for the y directed PGF, we are assured that (2.3.1) holds.

2.4 Horizontal pressure gradient force under σ coordinates

Digressing now for a moment, this opportunity is taken to make a few comments about the x component of the HPGF presented in (2.3.5). We point out that when this expression

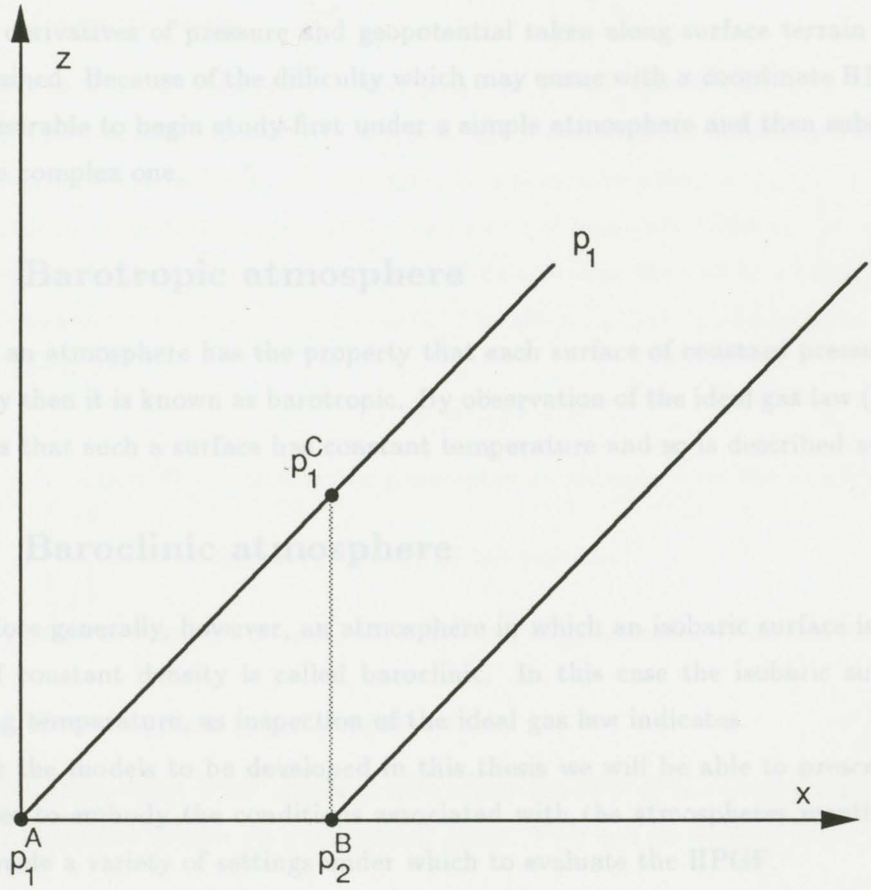

Figure 1: Pressure gradients illustrated.
The pressure gradient with fixed z is $p_2^B - p_1^A$, while the pressure gradient at fixed x is $p_2^B - p_1^C$.

is derived with σ coordinates, in which the vertical measure given by pressure is replaced by the ratio σ of this pressure to surface pressure, that is $\sigma = \frac{p}{p_s}$, it will expand into a sum of two terms each of which will give rise to truncation error in a finite difference formulation (a detailed discussion follows in Section 3.1). In the process, values for temperature as well as for derivatives of pressure and geopotential taken along surface terrain will need to be determined. Because of the difficulty which may ensue with σ coordinate HPGF evaluation, it is desirable to begin study first under a simple atmosphere and then subsequently under a more complex one.

2.5 Barotropic atmosphere

If an atmosphere has the property that each surface of constant pressure has constant density then it is known as barotropic. By observation of the ideal gas law (2.2.4), this then implies that such a surface has constant temperature and so is described as isothermal.

2.6 Baroclinic atmosphere

More generally, however, an atmosphere in which an isobaric surface is not necessarily one of constant density is called baroclinic. In this case the isobaric surfaces can have varying temperature, as inspection of the ideal gas law indicates.

In the models to be developed in this thesis we will be able to prescribe the isobaric surfaces to embody the conditions associated with the atmospheres mentioned above and so provide a variety of settings under which to evaluate the HPGF.

2.7 Geostrophic wind

We will show in this section that once a method to compute the HPGF is developed, it will also enable us to calculate what is known as the geostrophic wind with little extra effort. This is a constant velocity wind that results when the HPGF is exactly opposed by the Coriolis force (discussed below) on a constant z plane. Meteorologists have found that the geostrophic wind approximates the real wind to within about 15 per cent in mid to high latitudes, above the 1 km level where frictional force plays a more negligible role (Wallace and Hobbs, 1977). Hence in conjunction with our investigation of the HPGF we will be able to develop a simple interesting wind model as well. In this regard, we note that although a person may be interested in knowing the motion of a wind relative to certain fixed points, for example several cities, account must be taken of the fact that the earth itself is turning. This aspect is now considered.

2.7.1 Coriolis force

Air motion as viewed by a stationary observer, who is part of a rotating frame of reference (the earth), appears to be under the influence of a deflecting force. This force is known as the Coriolis force. For instance, to a person standing in the Northern hemisphere, an incoming wind travelling straight down a line of longitude looks as if it is being deflected to its right, that is to the west, because at the same time the viewer is being moved eastward by the counterclockwise spin of the earth. The effect of the Coriolis force on horizontal wind flow is given by

$$\vec{F_C} = -f\vec{k} \times \vec{V}, \tag{2.7.1}$$

where $f = 2\Omega sin(\theta)$ is the Coriolis parameter at latitude θ, Ω the earth's angular speed, and $\vec{V} = \begin{bmatrix} u \\ v \\ 0 \end{bmatrix}$ is the horizontal velocity of the particle.

With this in place an operational definition of geostrophic wind can be given.

2.7.2 Geostrophic wind equation

The geostrophic wind vector results when the sum of the pressure gradient force and the Coriolis force is zero on a horizontal plane. This can be written as

$$-\frac{1}{\rho}\nabla_h\, p - f\vec{k} \times \vec{V_g} = \vec{0} \tag{2.7.2}$$

where $\vec{V_g} = \begin{bmatrix} u_g \\ v_g \\ 0 \end{bmatrix}$. Moving the pressure gradient term to the RHS, multiplying by -1, and then taking a cross product with $\vec{k}$ yields

$$f\vec{k} \times \vec{V_g} \times \vec{k} = -\frac{1}{\rho}\nabla_h\, p \times \vec{k}, \tag{2.7.3}$$

which then implies for non zero f (that is, for a latitude other than that at the equator)

$$\vec{V_g} = \frac{1}{f}\left[-\frac{1}{\rho}\nabla_h\, p \times \vec{k}\right]. \tag{2.7.4}$$

This is known as the geostrophic wind equation. Upon evaluation of the cross product in (2.7.4) we find that

$$\vec{V}_g = \begin{bmatrix} -\frac{1}{f}\frac{1}{\rho}\frac{\partial p}{\partial y} \\ \frac{1}{f}\frac{1}{\rho}\frac{\partial p}{\partial x} \\ 0 \end{bmatrix} \tag{2.7.5}$$

or by (2.3.1)

$$\vec{V}_g = \begin{bmatrix} -\frac{g}{f}\left(\frac{\partial z}{\partial y}\right)_p \\ \frac{g}{f}\left(\frac{\partial z}{\partial x}\right)_p \\ 0 \end{bmatrix}. \tag{2.7.6}$$

By comparing (2.7.5) with (2.2.8) one sees that if one can compute HPGF components, then by simply dividing by a (non-zero) Coriolis parameter f one can obtain the geostrophic wind vector components, up to a factor of -1.

2.7.3 Thermal wind equation

Given that we have an expression for the geostrophic wind, we may wish to examine how it varies in the vertical. We start by applying partial differentiation with respect to p to (2.7.6) obtaining

$$\frac{\partial \vec{V}_g}{\partial p} = \frac{g}{f}\begin{bmatrix} -\frac{\partial}{\partial p}\left[\left(\frac{\partial z}{\partial y}\right)_p\right] \\ \frac{\partial}{\partial p}\left[\left(\frac{\partial z}{\partial x}\right)_p\right] \\ 0 \end{bmatrix}. \tag{2.7.7}$$

Since it is assumed that the second order partial derivatives are continouous, one can then interchange their order, and so conclude that

$$\frac{\partial \vec{V}_g}{\partial p} = \frac{g}{f}\begin{bmatrix} -\left(\frac{\partial}{\partial y}\left[\frac{\partial z}{\partial p}\right]\right)_p \\ \left(\frac{\partial}{\partial x}\left[\frac{\partial z}{\partial p}\right]\right)_p \\ 0 \end{bmatrix}. \tag{2.7.8}$$

Employing the hydrostatic equation as written in (2.2.5) yields

$$\frac{\partial \vec{V}_g}{\partial p} = \frac{g}{f}\begin{bmatrix} -\left(\frac{\partial}{\partial y}\left[\frac{-RT}{pg}\right]\right)_p \\ \left(\frac{\partial}{\partial x}\left[\frac{-RT}{pg}\right]\right)_p \\ 0 \end{bmatrix}. \tag{2.7.9}$$

Since evaluation is done on a constant pressure surface, then

$$\frac{\partial \vec{V}_g}{\partial p} = \frac{g}{f}\begin{bmatrix} \frac{R}{p\ g}\left(\frac{\partial T}{\partial y}\right)_p \\ \frac{-R}{p\ g}\left(\frac{\partial T}{\partial x}\right)_p \\ 0 \end{bmatrix}. \tag{2.7.10}$$

Bringing the factor p over to the left hand side (LHS) provides the conclusion

$$\frac{\partial \vec{V}_g}{\partial \ln p} = \frac{R}{f}\begin{bmatrix} \left(\frac{\partial T}{\partial y}\right)_p \\ -\left(\frac{\partial T}{\partial x}\right)_p \\ 0 \end{bmatrix} \tag{2.7.11}$$

which is known as the thermal wind equation.

We recall from Section 2.5 that in a barotropic atmosphere temperature stays constant on an isobaric surface, that is $\begin{bmatrix} \left(\frac{\partial T}{\partial y}\right)_p \\ \left(\frac{\partial T}{\partial x}\right)_p \\ 0 \end{bmatrix} = \vec{0}$. This in conjunction with (2.7.11) shows that the geostrophic wind vector remains constant with height in such an atmosphere and hence, as pointed out at the end of Section 2.7.2, so does the HPGF.

Chapter 3

Discrete model calculation of horizontal pressure gradient in sigma coordinates

3.1 Isobaric geopotential gradient in sigma coordinates

In a 1957 paper Phillips suggested that in order to capture some physically significant measure in a vertical coordinate, of a space field F, one should replace the height of each point in a vertical column of air, by the corresponding ratio σ, of air pressure at that level with respect to that at the surface, that is, $\sigma = \frac{p}{p_s}$. This recommendation has become a standard practise in meteorology (Mihailovic and Janjic, 1986), as the earth's surface then coincides with a coordinate surface (Kurihara, 1968).

One consequence is that the (previously) single vertical variable z, now becomes represented by the product σp_s, where p_s is the surface pressure and σ is the ratio just cited. More significantly, differentiation of the field F, with respect to x on a constant p surface produces two terms on a constant σ surface since the vertical coordinate is now a dependant variable.

For $F\,(x, \sigma(x,p))$ we have

$$\left(\frac{\partial F\,(x,\ \sigma(x,p))}{\partial x}\right)_p = \left(\frac{\partial F\,(x, \sigma(x,p))}{\partial x}\right)_\sigma + \left(\frac{\partial F\,(x, \sigma(x,p))}{\partial \sigma}\right) * \left(\frac{\partial\ \sigma(x,p)}{\partial x}\right)_p . \quad (3.1.1)$$

If now $F(x, \sigma(x,p)) = p = \sigma\, p_s(x)$, then

$$\left(\frac{\partial p}{\partial x}\right)_p = \left(\frac{\partial (\sigma p_s(x))}{\partial x}\right)_p = \left(\frac{\partial (\sigma p_s(x))}{\partial x}\right)_\sigma + \left(\frac{\partial (\sigma p_s(x))}{\partial \sigma}\right) * \left(\frac{\partial \sigma}{\partial x}\right)_p, \quad (3.1.2)$$

or

$$0 = \sigma * \left(\frac{\partial p_s}{\partial x}\right)_\sigma + p_s(x) * \left(\frac{\partial \sigma}{\partial x}\right)_p, \quad (3.1.3)$$

that is

$$-\left[\frac{\sigma}{p_s(x)}\right] * \left(\frac{\partial p_s}{\partial x}\right)_\sigma = \left(\frac{\partial \sigma}{\partial x}\right)_p. \quad (3.1.4)$$

Upon inserting this expression into (3.1.1), we find that

$$\left(\frac{\partial F(x,\ \sigma(x,p))}{\partial x}\right)_p = \left(\frac{\partial F(x, \sigma(x,p))}{\partial x}\right)_\sigma - \left(\frac{\partial F(x, \sigma(x,p))}{\partial \sigma}\right) * \left[\frac{\sigma}{p_s(x)}\right] * \left(\frac{\partial p_s}{\partial x}\right)_\sigma. \quad (3.1.5)$$

We now recall the hydrostatic equation (2.2.7)

$$\frac{\partial \phi}{\partial p} = -\frac{R\,T}{p}$$

and then restrict our attention to a vertical column, and so x is fixed, hence p_s is constant. Now because $p = \sigma\, p_s$, we have

$$\frac{\partial \phi}{\partial \sigma} = -\frac{R\,T}{\sigma}. \quad (3.1.6)$$

Appying (3.1.5) to ϕ yields

$$\left(\frac{\partial \phi(x,\ \sigma(x,p))}{\partial x}\right)_p = \left(\frac{\partial \phi(x, \sigma(x,p))}{\partial x}\right)_\sigma - \left[\frac{\partial \phi(x,\sigma)}{\partial \sigma}\right] * \left[\frac{\sigma}{p_s(x)}\right] * \left(\frac{\partial p_s}{\partial x}\right)_\sigma, \quad (3.1.7)$$

and thus

$$\left(\frac{\partial \phi(x,\ \sigma(x,p))}{\partial x}\right)_p = \left(\frac{\partial \phi(x, \sigma(x,p))}{\partial x}\right)_\sigma - \left[-\frac{R\,T(x,\sigma)}{\sigma}\right] * \left[\frac{\sigma}{p_s(x)}\right] * \left(\frac{\partial p_s}{\partial x}\right)_\sigma, \quad (3.1.8)$$

or

$$\left(\frac{\partial \phi(x,\ \sigma(x,p))}{\partial x}\right)_p = \left(\frac{\partial \phi(x,\sigma)}{\partial x}\right)_\sigma + \left[\frac{R\,T(x,\sigma)}{p_s(x)}\right] * \left(\frac{\partial p_s}{\partial x}\right)_\sigma. \quad (3.1.9)$$

This can be rewritten as

$$\left(\frac{\partial \phi(x,\sigma(x,p))}{\partial x}\right)_p = \left(\frac{\partial \phi(x,\sigma)}{\partial x}\right)_\sigma + R\,T(x,\sigma) * \left(\frac{\partial \ln p(x,\sigma_s(x))}{\partial x}\right). \quad (3.1.10)$$

Derivation (3.1.1) through (3.1.10) is due to Haltiner and Williams (1980).

Computation of $\left(\frac{\partial \phi}{\partial x}\right)_p$ is of fundamental importance in determining wind flow in meteorology as it is effectively a calculation of the HPGF, which was discussed in Section 2.3.

From (3.1.10) we see that after transforming to σ coordinates, $\left(\frac{\partial \phi}{\partial x}\right)_p$ appears as the sum of two terms. However, it has been found that when one computes and then adds these two components in a discrete numerical model, large truncation errors result, especially over mountainous regions (Kurihara,1968; Nakamura,1978). In such a case, the two terms may individually be 10 to 20 times larger in magnitude than a typical value of their sum (Sundqvist, 1975), but of opposite sign (Arakawa and Suarez, 1983). Hence the gradient sought is a small 'residual' sum (Corby et al., 1972) and evidently quite sensitive to inconsistent truncation in the two components (Sundqvist, 1975; Mesinger, 1982).

3.2 Continuous vs. discrete equation considerations

A continuous equation such as (3.1.10) holds exactly, for all real-valued x in the range of discussion. However, the two terms contain pure derivatives, for instance :

$$\left(\frac{\partial \ln p\,(x,\sigma_s)}{\partial x}\right) = \lim_{\Delta x \to 0} \frac{\ln p(x+\frac{\Delta x}{2},\sigma_s) - \ln p(x-\frac{\Delta x}{2},\sigma_s)}{\Delta x} \tag{3.2.1}$$

and this presents a problem in a discrete model of an atmosphere. To calculate a number which is to represent a derivative in the model setting, our only resort is to perform some sort of differencing on the finite set of values at our disposal. For example

$$\frac{\ln p(x^{(i)}+\frac{\Delta x}{2},\sigma_s) - \ln p(x^{(i)}-\frac{\Delta x}{2},\sigma_s)}{\Delta x} \tag{3.2.2}$$

might represent $\left(\frac{\partial \ln p(x,\sigma_s)}{\partial x}\right)$ evaluated at $x = x^{(i)}$ where $\frac{\Delta x}{2}$ is some multiple of the x-directed grid unit involved. (The notation to be followed throughout this thesis will have vertical indices carried in subscripts, and horizontal ones in supercripts.) Clearly there will be a discrepancy between the true value and the finite-difference approximation in all but a few select cases. This is known as the truncation error due to discretization.

Hence great care should be taken to approximate (non-linear) functions to as high a degree of accuracy as is possible with the finite amount of data available. To this extent, we may be aided (or harmed) by the selection of functions we cite from which to extrapolate values, such as temperature and pressure, for the large number of points which comprise the entire grid.

The problem to be addressed then, is to define a model atmosphere and finite-difference approximations for the two components of the isobaric geopotential gradient, shown in (3.1.10), such that the discretization error which arises is as small as possible. As has been noted by Arakawa and Suarez (1983) and also by Mesinger et al.(1988), such errors do not appear to be susceptible to elimination in general, however they might be required to vanish for particular atmospheres. So, in an appropriately chosen setting, since the two terms are of opposite sign, one could hope that the truncation error which results from the first term might match that which arises in the second, so that there would be no net analytic truncation error in calculating the sum (Corby et al.,1972).

Let us recall the hydrostatic equation (2.2.5). We have two variables which vary in the vertical, p and z. In addition, we have a function T, which also varies in the vertical. We will now study several consequences of allowing T to be a function of p (which we will find leads us to our desired goal), and will later return to consider a case where T is dependant on z, by way of comparison.

We take some guidance for our investigation into a $T(p)$, or $T(\sigma)$, atmosphere by noting a warning from Sundqvist's (1975) research: if we had a finite difference representation of the hydrostatic relation it would (in general) yield $T(\sigma)$ values different from the ones we would obtain from an analytic representation, on account of vertical truncation (please note Appendix A.1). Considering that T values are of essential importance to our calculation of isobaric geopotential gradient, we should guard against this occurrence, if at all possible.

We are then left with the task of arriving at a finite difference equation to represent the continuous hydrostatic equation (as exactly as possible) in a discrete world model. We start by inspecting (3.1.6), rewritten as

$$\frac{\partial \phi(\sigma)}{\partial \ln \sigma} = -R\,T(\sigma) \tag{3.2.3}$$

Now, a finite difference form representing (3.2.3) might take on a form similar to

$$\frac{\phi\left(\sigma_{i+\frac{1}{2}}\right) - \phi\left(\sigma_{i-\frac{1}{2}}\right)}{\Delta \ln \sigma_i} = -R\,T(\sigma_i) \tag{3.2.4}$$

where $\Delta \ln \sigma_i = \ln \sigma_{i+\frac{1}{2}} - \ln \sigma_{i-\frac{1}{2}}$.

Following the σ surface model used by Corby et al.(1972), we will think of the hydrostatic equation as being centered about $\sigma_{[i+\frac{1}{2}]}$ in the vertical. This will allow us to make use of the values for σ_i and $T(\sigma_i)$ which are stored on full σ levels. So we will rewrite the above as

$$\phi\left(\sigma_{[i+\frac{1}{2}]+\frac{1}{2}}\right) - \phi\left(\sigma_{[i+\frac{1}{2}]-\frac{1}{2}}\right) = -R\,T\left(\sigma_{i+\frac{1}{2}}\right) * \Delta \ln \sigma_{i+\frac{1}{2}} \tag{3.2.5}$$

From this last equation, we see that one way of assigning a value to T (at a half σ level) would be to calculate the average of the temperature at the full σ levels just above it and just below it.

With this in mind, let us see whether we can somehow preserve the continuous form of the hydrostatic equation (3.1.6) within a vertical interval $[s_B, s]$ (we will think of s_B as fixed and s as varying). We now investigate whether there is a class of functions T that can satisfy

$$\int_{\sigma=s_B}^{\sigma=s} -\frac{R\,T(\sigma)}{\sigma}\,d\sigma = -R\left[\frac{T(s_B)\,+\,T(s)}{2}\right] * \int_{\sigma=s_B}^{\sigma=s} \frac{1}{\sigma}\,d\sigma \tag{3.2.6}$$

where T is a function of σ alone. We are asking the mathematical question: Is there some T for which the integral of $T(\sigma)$ divided by σ over the (real line) $[s_B, s]$ will exactly equal the average value of $T(\sigma)$ over that interval, times the difference of the logarithms of the endpoints of that interval ?

We note that if there is such a T, it will be a temperature function because it will satisfy the hydrostatic equation (3.1.6), and furthermore, because

$$\int_{\sigma=s_B}^{\sigma=s} \frac{\partial\phi(\sigma)}{\partial\sigma}\,d\sigma = \int_{\sigma=s_B}^{\sigma=s} -\frac{R\,T(\sigma)}{\sigma}\,d\sigma, \tag{3.2.7}$$

we would then have

$$\phi(s) - \phi(s_B) = -R\left[\frac{T(s)\,+\,T(s_B)}{2}\right] * [\,\ln s - \ln s_B\,] \tag{3.2.8}$$

as an exact equation, and could choose the endpoints to be appropriate σ levels.

We are in effect asking : For what class of functions T, will there be no loss of accuracy by transforming the hydrostatic equation from the continuous model to the discrete one (that is, no truncation error) ?

We will study the following equation

$$\int_{u=y_o}^{u=y} \left(\frac{T(u)}{u}\right) du = \left[\frac{T(y)\,+\,T(y_o)}{2}\right] * [\ln y - \ln y_o]\,. \tag{3.2.9}$$

First we differentiate

$$\frac{d}{dy}\left[\int_{u=y_o}^{u=y} \left(\frac{T(u)}{u}\right) du\right] = \frac{d}{dy}\left[\left[\frac{T(y)\,+\,T(y_o)}{2}\right] * [\ln y - \ln y_o]\right] \tag{3.2.10}$$

and then use the fundamental theorem of calculus

$$\frac{d}{dy}\left[\int_{u=y_o}^{u=y} \left(\frac{T(u)}{u}\right) du\right] = \frac{T(y)}{y} \tag{3.2.11}$$

to obtain

$$\frac{T(y)}{y} = \left[\frac{1}{2}\,\frac{dT(y)}{dy}\right] * [\ln y - \ln y_o] + \frac{1}{y} * \left[\frac{T(y) + T(y_o)}{2}\right]. \tag{3.2.12}$$

We collect like terms

$$\frac{1}{2}\left[\frac{T(y) - T(y_o)}{y}\right] = \left[\frac{1}{2}\,T^{/}(y)\right] * [\ln y - \ln y_o], \tag{3.2.13}$$

and are left with

$$\frac{T(y) - T(y_o)}{y * [\ln y - \ln y_o]} = T^{/}(y), \tag{3.2.14}$$

so

$$\frac{1}{y * [\ln y - \ln y_o]} = \frac{T^{/}(y)}{T(y) - T(y_o)}. \tag{3.2.15}$$

Integrating we see that

$$\int_{y=w_o}^{y=w}\left[\frac{1}{y * [\ln y - \ln y_o]}\right] dy = \int_{y=w_o}^{y=w}\left[\frac{T^{/}(y)}{T(y) - T(y_o)}\right] dy. \tag{3.2.16}$$

Since the argument is increasing, we may bypass absolute value signs and write

$$\ln[\,\ln w - \ln w_o\,] = \ln[\,T(w) - T(w_o)\,] + C_1, \tag{3.2.17}$$

which implies

$$[\,\ln w - \ln w_o\,] = C_2\,[\,T(w) - T(w_o)\,], \tag{3.2.18}$$

or

$$T(w) = C_3\,[\,\ln w\,] + C_4 \tag{3.2.19}$$

where each C_i is a constant (i=1,...,4). Thus, if we want our discrete hydrostatic equation to exactly equal our continuous one, over the interval $[\sigma, \sigma + \Delta\sigma]$, given that we assume T is a function of σ, one way to do it would be to choose $T(\sigma) = A \ln \sigma + B$ (with A and B constant), and to choose (on the basis of (3.2.8))

$$\phi(\sigma_{i+1}) - \phi(\sigma_i) = -\frac{R\,[T(\sigma_{i+1}) + T(\sigma_i)]}{2} * [\ln \sigma_{i+1} - \ln \sigma_i] \tag{3.2.20}$$

or, in shorthand, with $\sigma = \sigma_{i+\frac{1}{2}}$,

$$\delta_{\,\ln\sigma}\,(\phi(\sigma)) = -R\,T(\sigma), \tag{3.2.21}$$

where we define the difference operator as

$$\delta_{y(x)}\,(\,F(x)\,) = \frac{F(x + \frac{\Delta x}{2}) - F(x - \frac{\Delta x}{2})}{\Delta y(x)} \tag{3.2.22}$$

with $\Delta y(x) \;=\; y(x+\frac{\Delta x}{2}) - y(x-\frac{\Delta x}{2})$, and the average operator as

$$\overline{F(x)} \;=\; \frac{F(x+\frac{\Delta x}{2}) + F(x-\frac{\Delta x}{2})}{2}. \tag{3.2.23}$$

Clearly, because $p = \sigma\ p_s$ where p_s is constant for fixed x, this entire argument could have been performed for pressure p, instead of for σ; this would have left us with the conclusion

$$g\ z_U - g\ z_L \;=\; \phi(p_U) - \phi(p_L) \;=\; -\frac{R\,[T(p_U) + T(p_L)]}{2} \;*\; [\ln p_U - \ln p_L] \tag{3.2.24}$$

for an U(pper) and L(ower) pressure level, where we have used (2.2.6) in the first equality.

3.3 Finding of Corby et al. (1972)

We will follow the convention adopted by Corby et al.(1972) in describing a σ model of n levels by placing $\sigma_{\frac{1}{2}}$ at the top of the atmosphere and $\sigma_{n+\frac{1}{2}}$ at the earth's surface, in deriving a formula for the geopotential at the kth σ level. (Please see Figure 2, where n=5.)

We first write

$$\phi_k \;-\; \phi_{k+1} \;=\; \frac{R\,(T(\sigma_k) + T(\sigma_{k+1}))}{2} \;*\; [\ln\sigma_{k+1} - \ln\sigma_k] \tag{3.3.1}$$

$$\phi_{k+1} \;-\; \phi_{k+2} \;=\; \frac{R\,(T(\sigma_{k+1}) + T(\sigma_{k+2}))}{2} \;*\; [\ln\sigma_{k+2} - \ln\sigma_{k+1}] \tag{3.3.2}$$

....

$$\phi_{n-1} \;-\; \phi_n \;=\; \frac{R\,(T(\sigma_{n-1}) + T(\sigma_n))}{2} \;*\; [\ln\sigma_n - \ln\sigma_{n-1}] \tag{3.3.3}$$

$$\phi_n \;-\; \phi_s \;=\; -\;R\;T(\sigma_n) \;*\; \ln\sigma_n \tag{3.3.4}$$

and then sum, to obtain

$$\phi_k - \phi_s \;=\; -R\left[\sum_{i=k}^{i=n-1} \frac{R\,(T(\sigma_i) + T(\sigma_{i+1}))}{2} \;*\; [\ln\sigma_i - \ln\sigma_{i+1}]\right] - R\,T(\sigma_n) * \ln\sigma_n \tag{3.3.5}$$

where the σ_n terms represent the hydrostatic equation (derived above) taken between the surface $\sigma_{n+\frac{1}{2}}$, and the σ_n th level. This can be rewritten as

$$\phi_k \;-\; \phi_s \;=\; \sum_{l=k}^{l=n-1} \frac{R\,(T_{l+1} + T_l)}{2} \;*\; \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right) \;+\; R\;T_n \;*\; \ln\left(\frac{1}{\sigma_n}\right) \tag{3.3.6}$$

where n is the number of levels in the model.

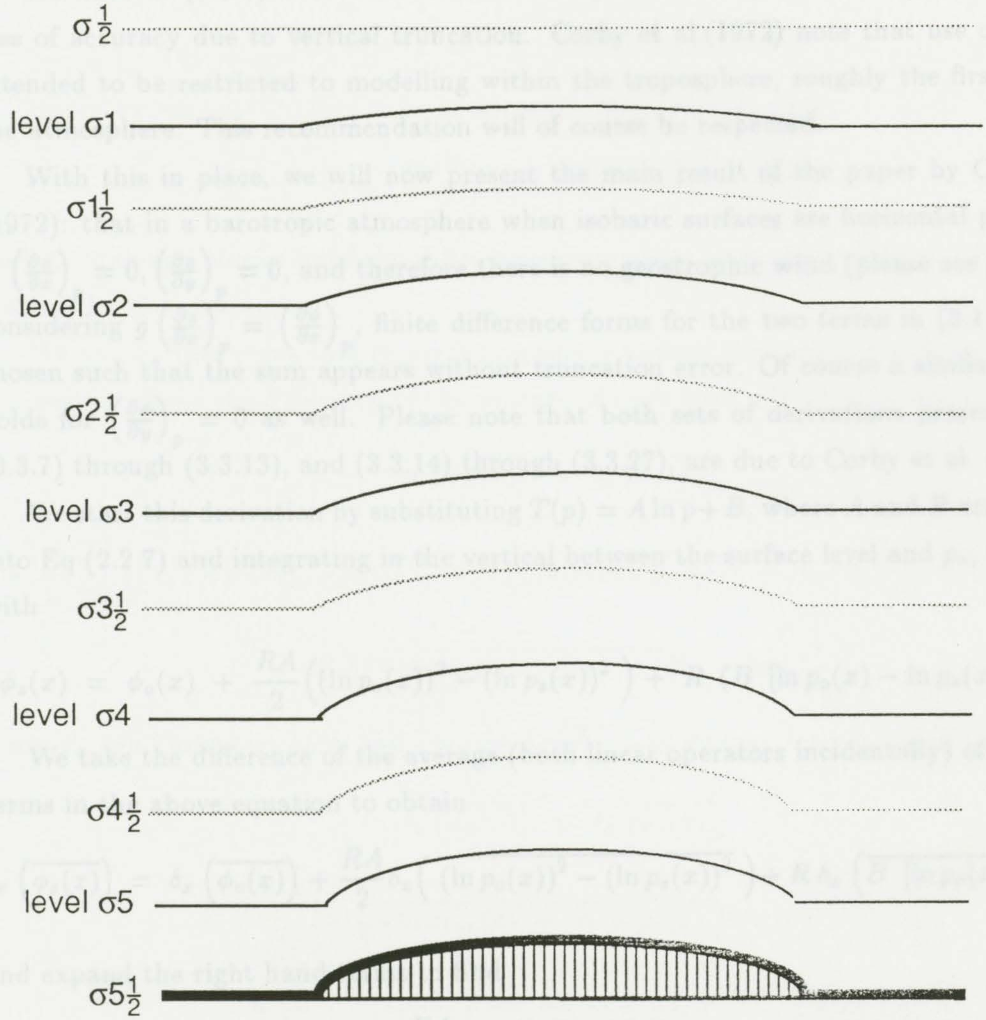

Figure 2: An example of σ surface configuration over a mountain.

A $\sigma = Q$ surface is comprised of points in the atmosphere where the ratio of air pressure to surface pressure is Q (a fixed number), where of course, $Q \epsilon [0, 1]$. For instance, in the illustration above, $\sigma = 1$ is given by $\sigma_{5\frac{1}{2}}$, $\sigma = 0.5$ by σ_3, and and $\sigma = 0$ by $\sigma_{\frac{1}{2}}$, with the labelling of σ levels following the convention established in the Corby model.

We have in (3.3.6) a discrete form of the hydrostatic equation which experiences no loss of accuracy due to vertical truncation. Corby et al.(1972) note that use of (3.3.6) is intended to be restricted to modelling within the troposphere, roughly the first 10 km of the atmosphere. This recommendation will of course be respected.

With this in place, we will now present the main result of the paper by Corby et al. (1972): that in a barotropic atmosphere when isobaric surfaces are horizontal planes, that is $\left(\frac{\partial z}{\partial x}\right)_p = 0, \left(\frac{\partial z}{\partial y}\right)_p = 0$, and therefore there is no geostrophic wind (please see (2.7.6)), by considering $g\left(\frac{\partial z}{\partial x}\right)_p = \left(\frac{\partial \phi}{\partial x}\right)_p$, finite difference forms for the two terms in (3.1.10) can be chosen such that the sum appears without truncation error. Of course a similar argument holds for $\left(\frac{\partial \phi}{\partial y}\right)_p = 0$ as well. Please note that both sets of derivations presented below, (3.3.7) through (3.3.13), and (3.3.14) through (3.3.27), are due to Corby et al. (1972).

We start this derivation by substituting $T(p) = A \ln p + B$, where A and B are constants, into Eq (2.2.7) and integrating in the vertical between the surface level and p_o, and are left with

$$\phi_s(x) = \phi_o(x) + \frac{RA}{2}\left((\ln p_o(x))^2 - (\ln p_s(x))^2\right) + R\left(B\left[\ln p_o(x) - \ln p_s(x)\right]\right). \quad (3.3.7)$$

We take the difference of the average (both linear operators incidentally) of each of the terms in the above equation to obtain

$$\delta_x\left(\overline{\phi_s(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) + \frac{RA}{2}\delta_x\left(\overline{(\ln p_o(x))^2 - (\ln p_s(x))^2}\right) + R\,\delta_x\left(\overline{B\left[\ln p_o(x) - \ln p_s(x)\right]}\right), \quad (3.3.8)$$

and expand the right hand terms to find

$$\delta_x\left(\overline{\phi_s(x)}\right) = -\frac{RA}{2}\delta_x\left(\overline{-(\ln p_s(x))^2}\right) - R\,B\,\delta_x\left(\overline{\ln p_s(x)}\right) \quad (3.3.9)$$

where we have used the fact that $\delta_x\left(\overline{\phi_o(x)}\right) = 0$ and $\delta_x\left(\overline{(\ln p_o(x))^2}\right) = 0$ since Corby et al. (1972) required that the initializing constant pressure surface be a horizontal plane, and clearly δ_x applied to any function of a constant number is zero. We now have

$$\delta_x\left(\overline{\phi_s(x)}\right) = -RA\left(\overline{\overline{\ln p_s(x)} * \delta_x\left(\ln p_s(x)\right)}\right) - R\overline{B}\delta_x\left(\overline{\ln p_s(x)}\right) \quad (3.3.10)$$

which follows from a lemma concerning differences of averages of products proved in Appendix C. After absorbing terms we obtain

$$\delta_x\left(\overline{\phi_s(x)}\right) = -R\left(\overline{\overline{A \ln p_s(x) + B} * \delta_x\left(\ln p_s(x)\right)}\right) \quad (3.3.11)$$

or

$$\delta_x\left(\overline{\phi_s(x)}\right) \;=-\; R\left(\overline{\overline{T_s(x)}*\delta_x\left(\ln p_s(x)\right)}\right). \tag{3.3.12}$$

Hence we see that if there is no geostrophic wind, we can write

$$0 \;=\; \delta_x\left(\overline{\phi_s(x)}\right)+R\left(\overline{\overline{T_s(x)}*\delta_x\left(\ln p_s(x)\right)}\right), \tag{3.3.13}$$

and so we should choose our finite difference components as the two terms on the right hand side of the above equation.

We may further prove that no error propagates upward through any of the σ levels above the surface. Take the difference of the average of each term of (3.3.6) to obtain

$$\delta_x\left(\overline{\phi_k(x)}\right) \;=\; \delta_x\left(\overline{\phi_s(x)}\right) \tag{3.3.14}$$

$$+\frac{R}{2}\sum_{l=k}^{n-1}\left[\left(\delta_x\left(\overline{A\ln(\sigma_{l+1}*p_s)(x)}\right)+\delta_x\left(\overline{A\ln(\sigma_l*p_s)(x)}\right)+\delta_x\left(\overline{2\;B}\right)\right)\;*\;\ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+\;R\;\delta_x\left(\overline{A\;\ln(\sigma_n*p_s)(x)\;+\;B}\right)\;*\;\ln\left(\frac{1}{\sigma_n}\right).$$

Since δ_x is a linear operator, and because $\delta_x\left(\overline{B}\right)=0$, we may rewrite Eq (3.3.14) as

$$\delta_x\left(\overline{\phi_k(x)}\right) \;=\; \delta_x\left(\overline{\phi_s(x)}\right) \tag{3.3.15}$$

$$+\;\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{A\ln\sigma_{l+1}(x)\;+\;A\;\ln p_s(x)}\right)\;*\;\ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+\;\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{A\ln\sigma_l(x)\;+\;A\;\ln p_s(x)}\right)\;*\;\ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+\;R\;\delta_x\left(\overline{A\ln\sigma_n(x)\;+\;A\;\ln p_s(x)}\right)\;*\;\ln\left(\frac{1}{\sigma_n}\right).$$

However since a difference operator of a function evaluated on a constant σ surface yields a zero result, we will be able to drop several terms

$$\delta_x\left(\overline{\phi_k(x)}\right) \;=\; \delta_x\left(\overline{\phi_s(x)}\right) \tag{3.3.16}$$

$$+\;\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{A\;\ln p_s(x)}\right)\;*\;\ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]+\frac{R}{2}\sum_{l=k}^{n-1}\left[\;\delta_x\left(\overline{A\;\ln p_s(x)}\right)\;*\;\ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+\;R\;A\;\left(\delta_x\left(\overline{\ln p_s(x)}\right)\right)\;*\;\ln\left(\frac{1}{\sigma_n}\right).$$

We now have

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) + RA\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{\ln p_s(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right] + R\,A\left(\delta_x\left(\overline{\ln p_s(x)}\right)\right) * \ln\left(\frac{1}{\sigma_n}\right), \tag{3.3.17}$$

and so can express this as

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) + RA\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{\ln p_s(x)}\right) * (\ln\sigma_{l+1} - \ln\sigma_l)\right] - R\,A\left(\delta_x\left(\overline{\ln p_s(x)}\right)\right) * \ln\sigma_n. \tag{3.3.18}$$

The telescoping sum leaves us with

$$\begin{aligned}\delta_x\left(\overline{\phi_k(x)}\right) &= \delta_x\left(\overline{\phi_s(x)}\right) \\ &+ RA\;\delta_x\left(\overline{\ln p_s(x)}\right) * \ln\sigma_n(x) - RA\;\delta_x\left(\overline{\ln p_s(x)}\right) * \ln\sigma_k(x) \\ &- RA\;\delta_x\left(\overline{\ln p_s(x)}\right) * \ln\sigma_n(x),\end{aligned} \tag{3.3.19}$$

which we can state as

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) - RA\;\ln\sigma_k(x) * \delta_x\left(\overline{\ln p_s(x)}\right). \tag{3.3.20}$$

At this point we first use the fact that (1) $\delta_x\left(\overline{f(x)}\right) = \overline{\delta_x\left(f(x)\right)}$ which can be shown by taking $T(x) = 1$ in the derivation of Appendix C, and then the fact that (2) on a constant σ surface σ_k, $\ln\left[\sigma_k(x)\right] * \overline{g(x)} = \ln\left[\sigma_k\right] * \overline{g(x)} = \overline{\ln\left[\sigma_k(x)\right] * g(x)}$ for any $g(x)$, in order to be able to rewrite Eq (3.3.20) as

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) - \overline{RA\ln\left[\sigma_k(x)\right] * \delta_x\left(\ln p_s(x)\right)}. \tag{3.3.21}$$

But on this same constant σ surface, we also have $\ln\left[\sigma_k(x)\right] = \ln\left[\sigma_k\right] = \overline{\ln\left[\sigma_k(x)\right]}$ for any x, hence by substitution we conclude that

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) - \overline{RA\overline{\ln\left[\sigma_k(x)\right]} * \delta_x\left(\ln p_s(x)\right)}. \tag{3.3.22}$$

Now we insert our expression for $\delta_x\left(\overline{\phi_s(x)}\right)$ from (3.3.11) to obtain

$$\delta_x\left(\overline{\phi_k(x)}\right) = -R\left(\overline{\overline{A\ln p_s(x) + B} * \delta_x\left(\ln p_s(x)\right)}\right) - \overline{RA\overline{\ln\sigma_k(x)} * \delta_x\left(\ln p_s(x)\right)}, \tag{3.3.23}$$

and absorb under the (linear) averaging operator to arrive at

$$\delta_x\left(\overline{\phi_k(x)}\right) = -R\left(\overline{\overline{A\ln(p_s * \sigma_k)(x) + B} * \delta_x\left(\ln p_s(x)\right)}\right). \tag{3.3.24}$$

This leaves us with

$$\delta_x \left(\overline{\phi_k(x)}\right) \;=\; -\; R\left(\overline{A \ln p_k(x) + B * \delta_x\left(\ln p_s(x)\right)}\right), \tag{3.3.25}$$

that is

$$\delta_x \left(\overline{\phi_k(x)}\right) \;=\; -\; R\left(\overline{\overline{T_k(x)} * \delta_x\left(\ln p_s(x)\right)}\right), \tag{3.3.26}$$

or

$$0 = \delta_x \left(\overline{\phi_k(x)}\right) + \; R\left(\overline{\overline{T_k(x)} * \delta_x\left(\ln p_s(x)\right)}\right). \tag{3.3.27}$$

Hence the Corby finite difference scheme holds for any σ_k level. However, we will predominantly be interested in the surface level. In this regard, Corby et al.(1972) have shown that when there is no geostrophic wind, if one were to choose $\delta_x\left(\overline{\phi_s(x)}\right)$ as the finite difference expression for $\left(\frac{\partial \phi_s(x)}{\partial x}\right)$, and then choose $R\left(\overline{\overline{T_s(x)} * \delta_x\left(\ln p_s(x)\right)}\right)$ to represent $R\,T\left[\sigma_s(x)\right] \;*\; \left(\frac{\partial \ln p_s(x)}{\partial x}\right)$, then the individual truncation errors for (3.1.10) would be of equal magnitude but of opposite sign and so cancel exactly (mutually compensating). We think of this as the simplest possible case, and as necessary conditions to expand upon, in order to expect to avoid large truncation error in more general settings, an observation Mihailovic and Janjic (1986) also make in reviewing the value of the work by Corby et al.(1972).

Chapter 4

Numerical experiments

4.1 Motivation

We have seen that by proposing a suitably selected temperature structure $T = A \ln p + B$, the continuous hydrostatic equation can be transformed exactly into a discrete version, and can also be algebraically manipulated to determine

$$\delta_x\left(\overline{\phi_s(x)}\right) + R\left(\overline{\overline{T_s(x)} * \delta_x\left(\ln p_s(x)\right)}\right) \tag{4.1.1}$$

as a precise representation to $\left(\frac{\partial \phi_s}{\partial x}\right) + R\,T\,[\sigma_s] * \left(\frac{\partial \ln p_s(x)}{\partial x}\right)$ in a case where isobaric surfaces are horizontal planes.

We will now examine the above finding in a numerical experiment. In addition, we wish to test the sensitivity of (4.1.1), to another set of data, likewise derived from the analytic hydrostatic equation (and for which (4.1.1) will converge to a correct solution as the grid interval $|\Delta x| \to 0$) but which will however, be governed by a different temperature structure ($T = Az + B$). Such a comparison may give an indication of the critical importance of choosing grid point values of T,p and z so as to reduce horizontal truncation error (Kasahara, 1977), for this problem.

4.2 Experiment design

A numerical study was done to calculate the initial geostrophic wind at the surface of a cosine shaped mountain under the two different temperature structures just mentioned, using the Corby finite difference scheme. The computer programs used were adapted from

000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000
000	000	000	000	000	003	010	016	019	010	003	000	000	000	000	000	000
000	000	000	001	012	031	052	068	073	068	052	031	012	001	000	000	000
000	000	001	016	048	085	121	145	154	145	121	085	048	016	001	000	000
000	000	012	048	099	154	204	238	250	238	204	154	099	048	012	000	000
000	003	031	085	154	226	289	331	346	331	289	226	154	085	031	003	000
000	010	052	121	204	289	361	410	427	410	361	289	204	121	052	010	000
000	016	068	145	238	331	410	462	481	462	410	331	238	145	068	016	000
000	019	073	154	250	346	427	481	500	481	427	346	250	154	073	019	000
000	016	068	145	238	331	410	462	481	462	410	331	238	145	068	016	000
000	010	052	121	204	289	361	410	427	410	361	289	204	121	052	010	000
000	003	031	085	154	226	289	331	346	331	289	226	154	085	031	003	000
000	000	012	048	099	154	204	238	250	238	204	154	099	048	012	000	000
000	000	001	016	048	085	121	145	154	145	121	085	048	016	001	000	000
000	000	000	001	012	031	052	068	073	068	052	031	012	001	000	000	000
000	000	000	000	000	003	010	016	019	010	003	000	000	000	000	000	000
000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000

Table 1: Mountain surface as seen from above (with height in tens of m)

the initialization routine of the wind model program IDEAL85 (Danard and Galbraith, 1985) prepared by the Atmospheric Dynamics Corporation. However, all details related to the problem with which we are concerned are presented in this chapter, and the source code appears in Appendix E.

The programs were compiled and executed under two different methods for computing surface temperature and pressure (following from the two different temperature profiles) with the aim of calculating the isobaric geopotential gradient, used to determine the initial geostrophic wind, by (2.7.6). The truncation errors which resulted from using the two temperature profiles were compared. Details are given in Appendix B.

An ideal hill was constructed over a 17 by 17 square grid, which had a cosine profile when projected onto any plane containing $\vec{k}$ and passing through the center, with the elevation tailing off to zero within 1 unit of the boundary. (A grid size 'unit' represented 5000m and the hill apex varied from 1000m to 5000m.) Please see Table 1 above, with mountain top 5000m.

4.3 Atmospheric Model Description

We can define a model atmosphere by assuming an initial constant pressure surface of say 850 mb, that is $p_o = p_{850}$. (One millibar is defined as 1 mb = 10^{-3} bar = 10^3 dynes / cm ^{2}.) The height of the p_{850} surface is typically approximately 1500m. Also it is pointed out that the ϕ_o referred to in Eq's (3.3.7), (6.1.23) and (7.4.9) is the geopotential associated with this p_o.

Further, we take as given, both the 850 mb height $z_{850}^{x^{(R)}}$ and temperature $T_{850}^{x^{(R)}}$ directly above a fixed surface reference station, whose elevation $z_s^{x^{(R)}}$ and temperature $T_s^{x^{(R)}}$ are also known. Since the distance between neighboring radiosonde observations of isobaric height far exceeds the width of the hill, which is 80 km, this problem falls into the category of meso-beta modelling (model scale on the order of 20 km to 200 km, Orlanski (1975)) and so the isobaric surfaces are taken to be planes.

Given these premises, we can then determine the constant pressure planes above and below, p_{850} by either procedure (1) or (2) below, depending on whether we choose our vertical coordinate to be height z (as in (1)), or pressure p (as in (2)).

(1) We assume a fixed lapse rate of temperature with respect to height (based on $T_{850}^{x^{(R)}}$ and $T_s^{x^{(R)}}$) and then employ the hydrostatic equation to pick up associated pressure for all points in the vertical column $x^{(R)}$.

(2) We assume that temperature is a function of pressure, and it increases between the points $z_{850}^{x^{(R)}}$ and $z_s^{x^{(R)}}$ according to this function. (Of course we will be interested in studying $T(p)$ as indicated in (3.2.19) by the reasoning of Section 3.2.) Since $T_{850}^{x^{(R)}}$ and $T_s^{x^{(R)}}$ are known, a $p_s^{x^{(R)}}$ can be found by using the hydrostatic equation. Hence once again all the temperature and pressure values can be determined within the vertical column $x^{(R)}$.

In either case, the pressure and temperature distribution of the entire atmosphere can be specified, given that we state the temperature variation on p_{850} we wish to study, since we can extrapolate within any column x to find T and p values, using the parameters determined by (1) or (2) as the case may be.

4.4 Three atmospheres to be tested

We undertake our numerical study over a series of three atmospheres of increasing complexity, and compare the resulting mathematical noise that ensues in the case of $T = Az + B$ with that for $T = A \ln p + B$ (each alluded to in Section 4.3), when calculating initial geostrophic wind at the surface of the mountain. Please note that in our analyses we will consider only the x-component (so study will take place in the x - z plane), however the experiments will make use of both x and y components.

In the first two cases the atmosphere will be assumed barotropic, so there will be no temperature variation on isobaric surfaces, while in the third case the atmosphere will be baroclinic, hence a temperature distribution can and will be prescribed for these surfaces.

In Case I we assume the $p_o = p_{850}$ surface to be a horizontal plane and so no geostrophic wind is present. In the x - z plane p_{850} will appear as a line. The slope of this line will be denoted by m_{850}, hence the fact that p_{850} is horizontal will be written $m_{850} = 0$. We then have for any x

$$z_{850}^{x} = z_{850}^{x^{(R)}} . \tag{4.4.1}$$

If we call the temperature gradient on the p_{850} plane k_{850}, where k_{850} is unrelated to $\vec{k}$, then we can write the fact that temperature remains constant on this plane as $k_{850} = 0$. We have for arbitrary x

$$T_{850}^{x} = T_{850}^{x^{(R)}} . \tag{4.4.2}$$

In Case II the p_{850} surface will be a tilted plane with slope m_{850}, a fixed number, so for each x we will have

$$z_{850}^{x} = z_{850}^{x^{(R)}} + m_{850}\, x . \tag{4.4.3}$$

However, we will keep the temperature gradient on this plane to be zero ($k_{850} = 0$), and so once more (4.4.2) will hold.

Finally, in Case III a (non-zero) slope m_{850} remains in the p_{850} surface, hence we will have (4.4.3) again, but we will set the temperature gradient on this plane to be a fixed number k_{850}, and so we will have

$$T_{850}^{x} = T_{850}^{x^{(R)}} + k_{850}\, x . \tag{4.4.4}$$

These conditions will be true regardless of whether the atmosphere is defined with the first temperature structure mentioned above, or the second. We will therefore be looking at six different cases.

4.5 Surface temperature and pressure under $T = A \ln p + B$

We begin with the case where temperature follows the rule $T = A \ln p + B$, and proceed then to determine the surface temperature and pressure.

We must first calculate A and B. To do so, we consider the column of air over the reference station at $x^{(R)}$, and will denote the temperature at the 850 mb level by $T_{850}^{x^{(R)}}$ (and pressure by $p_{850}^{x^{(R)}}$), while at the reference station level we will call the temperature $T_s^{x^{(R)}}$ (and the pressure $p_s^{x^{(R)}}$). We can now write

$$T_s^{x^{(R)}} = A \ln p_s^{x^{(R)}} + B \tag{4.5.1}$$

and

$$T_{850}^{x^{(R)}} = A \ln p_{850}^{x^{(R)}} + B. \tag{4.5.2}$$

We subtract (4.5.2) from (4.5.1) to find that

$$A = \frac{T_s^{x^{(R)}} - T_{850}^{x^{(R)}}}{\ln p_s^{x^{(R)}} - \ln p_{850}^{x^{(R)}}}, \tag{4.5.3}$$

and then solve for B

$$B = \frac{T_{850}^{x^{(R)}} \ln p_s^{x^{(R)}} - T_s^{x^{(R)}} \ln p_{850}^{x^{(R)}}}{\ln p_s^{x^{(R)}} - \ln p_{850}^{x^{(R)}}}. \tag{4.5.4}$$

However since $\ln p_s^{x^{(R)}}$ is not yet known, we must integrate the hydrostatic equation (2.2.7) for fixed $x = x^{(R)}$ between the surface level and the 850 mb level

$$\int_{z=z_s}^{z=z_{850}} d\phi = -R \int_{p=p_s}^{p=p_{850}} (A \ln p + B)\ d\ln p \tag{4.5.5}$$

to arrive at

$$g z_{850}^{x^{(R)}} - g z_s^{x^{(R)}} = -\frac{RA}{2}\left[\ln^2 p_{850}^{x^{(R)}} - \ln^2 p_s^{x^{(R)}}\right] - R\,B\left[\ln p_{850}^{x^{(R)}} - \ln p_s^{x^{(R)}}\right], \tag{4.5.6}$$

where we have used (2.2.6). Substituting for A and B from above we find, after certain terms cancel, that

$$\ln p_s^{x^{(R)}} = \frac{g(z_{850}^{x^{(R)}} - z_s^{x^{(R)}})}{\frac{1}{2}R(T_{850}^{x^{(R)}} + T_s^{x^{(R)}})} + \ln p_{850}^{x^{(R)}}. \tag{4.5.7}$$

We now use this expression and substitute it into (4.5.3) to obtain

$$A = \frac{R}{2g}\,\frac{(T_s^{x^{(R)}})^2 - (T_{850}^{x^{(R)}})^2}{z_{850}^{x^{(R)}} - z_s^{x^{(R)}}}. \tag{4.5.8}$$

Derivation (4.5.1) through (4.5.8) is valid for an arbitrary point x, not just $x^{(R)}$, since we have assumed that $T = A \ln p + B$ describes the entire atmosphere. Hence we have

$$\ln p_s^x = \frac{g(z_{850}^x - z_s^x)}{\frac{1}{2}R(T_{850}^x + T_s^x)} + \ln p_{850}^x. \tag{4.5.9}$$

However, by subtracting (4.5.2) from (4.5.1) (each, with argument x) we notice that

$$\ln p_s^x - \ln p_{850}^x = \frac{T_s^x - T_{850}^x}{A}, \tag{4.5.10}$$

hence

$$\frac{T_s^x - T_{850}^x}{A} = \frac{g(z_{850}^x - z_s^x)}{\frac{1}{2}R(T_{850}^x + T_s^x)}, \tag{4.5.11}$$

and therefore

$$(T_s^x)^2 - (T_{850}^x)^2 = \frac{2Ag(z_{850}^x - z_s^x)}{R}. \tag{4.5.12}$$

We conclude that

$$T_s^x = \sqrt{\frac{2Ag(z_{850}^x - z_s^x)}{R} + (T_{850}^x)^2}, \tag{4.5.13}$$

and once T_s^x is known we can then find $\ln p_s^x$ from equation (4.5.9). Of course, B can now be found from (4.5.1), (4.5.8) and (4.5.13).

4.6 Surface temperature and pressure under $T = Az + B$

If temperature varies linearly with height, then computing the surface temperature is straightforward, once the surface height is given.

The temperature at the surface of the terrain over point z_s^x is provided by the linear function

$$T(z_s^x) = T_s^x = T_{850}^x + \gamma\,(z_{850}^x - z_s^x) \tag{4.6.1}$$

where the lapse rate of temperature with respect to height is

$$\gamma = -\frac{T_{850}^{x(R)} - T_s^{x(R)}}{z_{850}^{x(R)} - z_s^{x(R)}}. \tag{4.6.2}$$

To compute the surface pressure requires that we integrate the hydrostatic equation (2.2.2) between the surface level and the 850 mb pressure level. We will write the integrand as follows (for x fixed)

$$-\,g dz = RT^x d(\ln p^x), \tag{4.6.3}$$

and so, obtain

$$-\int_{z=z_s}^{z=z_{850}} \frac{g}{RT^x}\, dz = \int_{p=p_s}^{p=p_{850}} d(\ln p^x). \tag{4.6.4}$$

Now we evaluate the right hand side

$$-\frac{g}{R}\int_{z=z_s}^{z=z_{850}} \frac{1}{T^x}\, dz = \ln p_{850}^x - \ln p_s^x. \tag{4.6.5}$$

Using (4.6.1) leaves us with the following evaluation

$$-\frac{g}{R}\int_{T=T_s}^{T=T_{850}} \frac{1}{-\gamma T^x}\, dT^x \;=\; \frac{g}{R\gamma}\ln\left(\frac{T^x_{850}}{T^x_s}\right) \;=\; \ln p^x_{850} - \ln p^x_s. \tag{4.6.6}$$

We finally arrive at

$$p^x_s = p^x_{850}\left(\frac{T^x_s}{T^x_{850}}\right)^{\frac{g}{R\gamma}}, \tag{4.6.7}$$

hence the temperature and pressure at the surface are readily found (where of course $p^x_{850} =$ 850 mb for any x).

This formula can be evaluated as follows for the three cases under investigation. For Case I (where $k_{850} = 0$ and $m_{850} = 0$), we have

$$T^x_s = T^{x(R)}_{850} + \gamma\ \left[z^{x(R)}_{850} - z^x_s\right]. \tag{4.6.8}$$

In Case II ($k_{850} = 0$ and m_{850} is non zero) surface temperature is given by

$$T^x_s = T^{x(R)}_{850} + \gamma\ \left[(z^{x(R)}_{850} + m_{850}\ x) - z^x_s\right]. \tag{4.6.9}$$

Finally, in Case III (where both k_{850} and m_{850} are each non zero) surface temperature is provided by

$$T^x_s = T^x_{850} + \gamma\ \left[(z^{x(R)}_{850} + m_{850}\ x) - z^x_s\right] \tag{4.6.10}$$

where of course $T^x_{850} = T^{x(R)}_{850} + k_{850}\ x$ as was pointed out in (4.4.4).

4.7 Limiting value for Corby's scheme under $T = Az + B$

In this section we will show that the Corby finite difference scheme will converge to the expected result in the limit as the grid interval $\Delta x \to 0$ when the $T = Az + B$ profile is used and the atmosphere is assumed barotropic, as is true in Case I and Case II.

Let us first consider the Corby finite difference scheme given by (4.1.1)

$$\overline{\delta_x\left(gZ(x)\right) + R\overline{T_s(x)} * \delta_x\left(\ln p_s(x)\right)}$$

with surface terrain height given by $Z(x) = z_s^x$, which for the model with which we are concerned is

$$Z(x) = \frac{H_{apex}}{2}\left(1 + cos\left(\frac{2\pi x}{L}\right)\right) \tag{4.7.1}$$

where L is the period, which we take to be the length of the base of the mountain, and H_{apex} is the height of the mountain top.

If we expand (4.1.1) according to (3.2.22) and (3.2.23) we will find that we can rewrite it as

$$C = \frac{C_+ + C_-}{2} \tag{4.7.2}$$

where

$$C_+ = \left[g\left[\frac{Z(x+\Delta x) - Z(x)}{\Delta x}\right] + R\left[\frac{T_s(x+\Delta x) + T_s(x)}{2} * \frac{\ln p_s(x+\Delta x) - \ln p_s(x)}{\Delta x}\right]\right], \tag{4.7.3}$$

and

$$C_- = \left[g\left[\frac{Z(x) - Z(x-\Delta x)}{\Delta x}\right] + R\left[\frac{T_s(x) + T_s(x-\Delta x)}{2} * \frac{\ln p_s(x) - \ln p_s(x-\Delta x)}{\Delta x}\right]\right]. \tag{4.7.4}$$

If we suppose that T_s is a continuous function, and that p_s is a continuously differentiable function, then we can take the limit

$$\lim_{\Delta x \to 0} R\left[\frac{T_s(x+\Delta x) + T_s(x)}{2}\right] * \left[\frac{\ln p_s(x+\Delta x) - \ln p_s(x)}{\Delta x}\right] = R\, T_s\, \frac{dp_s}{dx}. \tag{4.7.5}$$

If we further assume that $Z(x)$ is a continuously differentiable function (true by (4.7.1)), then we can take the limit

$$\lim_{\Delta x \to 0} g\left[\frac{Z(x+\Delta x) - Z(x)}{\Delta x}\right] = g\frac{dZ}{dx} \tag{4.7.6}$$

and obtain a continuous function as a result.

Now combining (4.7.5), (4.7.6) and the definition of C_+ from (4.7.3) gives

$$\lim_{\Delta x \to 0} C_+ \;=\; g\frac{dZ}{dx} + R\,T_s\,\frac{dp_s}{dx}, \tag{4.7.7}$$

and by an analogous argument, C_- converges to the same limit. Hence from (4.7.2) we have

$$\lim_{\Delta x \to 0} C \;=\; g\frac{dZ}{dx} + R\,T_s\,\frac{dp_s}{dx}. \tag{4.7.8}$$

When $T = A\ln p + B$ in Case I, the sequence (3.3.7) through (3.3.13) shows that (4.7.2) has value zero, and so certainly $\lim_{\Delta x \to 0} C = 0$. The continuity and differentiability conditions are satisfied because of (4.5.13), (4.5.9) and (4.6.8).

However when $T = Az + B$ for Case I, we proceed as follows. From (4.6.7) we have

$$\ln\left[\frac{p_s^x}{p_{850}^x}\right] = \frac{g}{R\gamma}\ln\left[\frac{T_s^x}{T_{850}^x}\right]. \tag{4.7.9}$$

Now, consider (4.7.9) with argument $x + \Delta x$ and subtract from it (4.7.9) with argument x. Then note that p_{850}^x is constant for all x (because it is an 850 mb surface) and that T_{850}^x is constant for all x (with value $T_{850}^{x(R)}$) for a barotropic atmosphere (Cases I and II). We can then conclude that

$$\frac{\ln p_s^{x+\Delta x} - \ln p_s^x}{\Delta x} = \frac{g}{R\gamma}\,\frac{\ln T_s^{x+\Delta x} - \ln T_s^x}{\Delta x}, \tag{4.7.10}$$

and so

$$\delta_x\left(\ln p_s(x)\right) \;=\; \frac{g}{R\gamma}\delta_x\left(\ln T_s(x)\right). \tag{4.7.11}$$

We then know that (4.1.1) becomes

$$\overline{\delta_x\left(gZ(x)\right)} + \overline{RT_s(x)} * \overline{\frac{g}{R\gamma}\delta_x\left(\ln T_s(x)\right)}. \tag{4.7.12}$$

By analogy to the derivation of (4.7.5) through (4.7.8), we have

$$\lim_{\Delta x \to 0} C = g\frac{dZ}{dx} + R\,T_s\,\frac{d\ln T_s}{dx} = g\frac{dZ}{dx} + \frac{g}{\gamma}\frac{dT_s}{dx} \tag{4.7.13}$$

where the continuity and differentiability conditions previously mentioned are satisfied because of (4.7.4), (4.6.1) and (4.6.8).

Eq (4.6.8) shows that $\frac{dT_s}{dx} = -\gamma$ for Case I, hence we have

$$\lim_{\Delta x \to 0} C \;=\; g\frac{dZ}{dx} - \frac{g}{\gamma}\,\gamma\frac{dZ}{dx} \;=\; 0. \tag{4.7.14}$$

This shows that when temperature is given by the distribution $T(z) = Az + B$, the Corby finite difference scheme (designed for $T(p) = A\ln p + B$) converges to the correct solution as $\Delta x \to 0$. Recall that in the case we are studying we have no geostrophic wind, and so (3.1.10) is zero, hence we expect our finite difference expression to tend to zero in the limit.

Chapter 5

Barotropic atmosphere with horizontal isobaric surfaces

5.1 Numerical results

We now consider numerical results of the experiment testing the claim of Corby et al. (1972): that when constant pressure planes are horizontal, the HPGF (3.1.10) will be zero when computed with the finite difference expression (4.1.1), and hence so will the geostrophic wind components (2.7.6).

The reader is directed to Appendix B, where each section is comprised of nine tables: the first three concern the u component of geostrophic wind (with a display of ideal values, results calculated from Corby's scheme (4.1.1), and the difference between the two), the next three - the v component, followed by three (in the same format as that just mentioned above) dealing with the magnitude of the geostrophic wind (g-wind) vector.

The information reported in Appendix B is for a mountain of apex 5000m. This figure (height) draws its importance from the fact that as far as modelling elevated terrain on the earth's surface is concerned, for all but possibly a small number of exceptions, one can think of mountains as rising above their immediately surrounding terrain by no greater than 5000m. However, information in Tables 2 through 5 shown on P.35, P.48 and P.80 are for mountains of apex 1000m through 5000m. All calculations were performed in double precision arithmetic (16 digits) by the IDEAL programs of Appendix E.

At present we are interested in Cases I.A and I.B in Appendix B, which show a contrast between using $T = A \ln p + B$ (Tables B.1 - B.9) versus $T = Az + B$ (Tables B.10 - B.18) as

Mt Top	MVD $T = Az + B$	MAX $T = Az + B$	MVD $T = A\ln p + B$	MAX $T = A\ln p + B$
5000 m	0.01326522	0.60260431	$0.1121 * 10^{-12}$	$7.8771 * 10^{-12}$
4000 m	0.00666070	0.30216594	$0.0872 * 10^{-12}$	$4.6794 * 10^{-12}$
3000 m	0.00275663	0.12487604	$0.0824 * 10^{-12}$	$3.0822 * 10^{-12}$
2000 m	0.00080152	0.03625426	$0.0652 * 10^{-12}$	$2.6838 * 10^{-12}$
1000 m	0.00009835	0.00444144	$0.0950 * 10^{-12}$	$3.0078 * 10^{-12}$

Table 2: Mean magnitude (m/sec) and maximum vector deviation from expected $\vec{V}_{g_s}$ for barotropic case with horizontal isobaric surfaces

a temperature profile, when computing $\vec{V}_{g_s}$, for this simplest of cases - when an atmosphere is completely at rest. (This means that the geostrophic wind components are expected to be identically equal to zero - as is indicated in B.1, B.4, B.7 as well as B.10, B.13 and B.16.) Comparing the u component deviation from expected value (B.3 and B.12), v component deviation (B.6 and B.15) or g-wind velocity deviation (B.9 and B.18), one sees that the $T = A\ln p + B$ temperature profile exhibits considerably smaller departures from expected values than its $T = Az + B$ counterpart. (The analytic truncation error when $T = A\ln p + B$ is zero - as indicated by (3.3.13) - but is non-zero (see (5.2.18)) when $T = Az + B$.)

Table 2 above summarizes the mean value of vector magnitude deviation, MVD, of Corby finite-difference calculated g-wind, $\vec{V}_{g_s}^{fd}$, from expected (analytic) g-wind, $\vec{V}_{g_s}^{analytic}$:

$$MVD = \frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n}\sqrt{\left[u^{analytic}(i,j) - u^{fd}(i,j)\right]^2 + \left[v^{analytic}(i,j) - v^{fd}(i,j)\right]^2}, \qquad (5.1.1)$$

and the maximum discrepancy between $\vec{V}_{g_s}^{analytic}$ and $\vec{V}_{g_s}^{fd}$ experienced over the entire grid, denoted MAX.

The results for five mountains with apex varying from 1000m through 5000m are presented in Table 2 at the top of this page, as captured in MVD and MAX measures (both shown in m/sec). The ratio of MVD value for $T = Az + B$ to that for $T = A\ln p + B$ at the corresponding hill height is on the order of 10^{11} : 1. So the key conclusion to be drawn is the preferability of selection of $T = A\ln p + B$ for the finite difference scheme (4.1.1) used.

The result of Corby et al.(1972) as summarized in (3.3.13) showed that when $T = A\ln p + B$, since the analytic discretization error is zero, the only truncation error that will occur is that due to accumulation of roundoff from machine calculation and function evaluation. Hence the numbers which appear in Table 2 for MVD and MAX for this temperature structure (T-structure) must be given this interpretation. We note that this type of mathematical noise in MAX (last column in Table 2) is on the order of 10^{-11} m/sec. Taking differences of logarithms of nearly equal numbers has been found to be an especially suspect source for loss of accuracy in machine compuation. For example suppose two adjacent grid points have pressures of 1000 mb ($1.000*10^5$ Pa) and 1005 mb ($1.005*10^5$ Pa). Suppose further that the logarithm's intrinsic function gives 16 significant figures. However $\ln(1.005*10^5) - \ln(1.000*10^5) = 11.51791300648126 - 11.51292546497022 = 0.00498754151104$ which has only twelve significant figures. Hence a loss of four significant figures has been experienced in this instance due to finite digit computation.

By comparison, when $T = Az + B$ the MVD and MAX errors which appear have contributions from both machine roundoff (and function evaluation) as well as analytic truncation error. The last is due to inconsistency between the discrete hydrostatic equation and this temperature profile. As this seems to be a major factor, we will analyze it more thoroughly in Sections 5.2, 6.5 and Appendix A.1.

5.2 Truncation error for $T = Az + B$

Recalling the basic equation with which we are concerned (3.1.10), we first note that it is a statement concerning $T(x,\sigma)$ and $p(x,\sigma)$. However since we are only interested in studying the geostrophic wind at the surface (of mountain terrain $Z(x)$), we can think of $T(x,\sigma_s(x))$ and $p(x,\sigma_s(x))$ as functions of only one variable x (and in fact these are given by (4.6.7) and (4.6.8)).

Let us recall the pressure-temperature correspondence for the case we are analyzing from Section 4.6, by noting (4.6.7) from which we can ascertain

$$\frac{d\ln p_s^x}{dx} = \frac{g}{R\gamma}\frac{d\ln T_s^x}{dx}. \tag{5.2.1}$$

We will now begin our study of the truncation error associated with the Corby finite difference scheme, by examining the second term on the right hand side of (3.1.10). That is, we will attempt to find an expression for

$$RT_s(x) * \frac{d\ln p_s^x}{dx} - \overline{\overline{RT_s(x)} * \delta_x\left(\ln p_s(x)\right)} \tag{5.2.2}$$

which using (4.6.7) and (4.7.11), can be expresssed as

$$T_s(x) * \frac{g}{\gamma}\frac{d\ln T_s^x}{dx} - \frac{g}{\gamma}\overline{\overline{T_s(x)} * \delta_x (\ln T_s(x))} \tag{5.2.3}$$

where

$$\overline{\overline{T_s(x)} * \delta_x (\ln T_s(x))} = \tag{5.2.4}$$

$$\frac{1}{2}[\frac{T_s(x+\Delta x) + T_s(x)}{2} * \frac{\ln T_s(x+\Delta x) - \ln T_s(x)}{\Delta x} +$$

$$\frac{T_s(x) + T_s(x-\Delta x)}{2} * \frac{\ln T_s(x) - \ln T_s(x-\Delta x)}{\Delta x}].$$

First consider the Taylor's series

$$T_s(x+\Delta x) = T_s(x) + \frac{1}{1!}T_s^{|}(x)\,[\Delta x] + \frac{1}{2!}\Big[T_s^{||}(x)\Big] * [\Delta x]^2 + \frac{1}{3!}\Big[T_s^{|||}(x)\Big]\,[\Delta x]^3 + ... \tag{5.2.5}$$

Then after division by T_s we use the resulting expression as an argument to the ln function, we will have

$$\ln\left[\frac{T_s(x+\Delta x)}{T_s(x)}\right] = \ln\left[1 \ + \ \frac{1}{1!}\left[\frac{T_s^{|}}{T_s}\right][\Delta x] \ + \ \frac{1}{2!}\left[\frac{T_s^{||}}{T_s}\right] * [\Delta x]^2 \ + \ \frac{1}{3!}\left[\frac{T_s^{|||}}{T_s}\right][\Delta x]^3 \ + \ ... \ \right]. \tag{5.2.6}$$

If we recall that

$$\ln[1 \ + \ \alpha] = \alpha - \frac{\alpha^2}{2} + \frac{\alpha^3}{3} - \frac{\alpha^4}{4} + \ ... \ , \tag{5.2.7}$$

then by setting $\alpha = \sum_{i=1}^{i=\infty} \frac{1}{i!}\frac{T_s^{[i]}}{T_s}[\Delta x]^i$ and calculating the first several low order terms we obtain

$$\frac{1}{\Delta x}\ln\left[\frac{T_s(x+\Delta x)}{T_s(x)}\right] = \left[\frac{T_s^{|}}{T_s}\right] + \frac{1}{2}\left[\frac{T_s^{||}}{T_s} - \left[\frac{T_s^{|}}{T_s}\right]^2\right][\Delta x] + \left[\frac{1}{6}\frac{T_s^{|||}}{T_s} - \frac{1}{2}\frac{T_s^{||}T_s^{|}}{T_s^2} + \frac{1}{3}\left[\frac{T_s^{|}}{T_s}\right]^3\right][\Delta x]^2 + \dots . \tag{5.2.8}$$

We also have from (5.2.5)

$$\frac{T_s(x+\Delta x) + T_s(x)}{2} = T_s \ + \ \frac{1}{2}T_s^{|}[\Delta x] \ + \ \frac{1}{2*2!}\Big[T_s^{||}\Big] * [\Delta x]^2 \ + \ \frac{1}{2*3!}\Big[T_s^{|||}\Big][\Delta x]^3 \ + \ .... \tag{5.2.9}$$

Hence multiplying (5.2.8) by (5.2.9) yields the following

$$\frac{T_s(x+\Delta x)+T_s(x)}{2} * \frac{\ln T_s(x+\Delta x) - \ln T_s(x)}{\Delta x} = \tag{5.2.10}$$

$$T_s^{|} + \left[\frac{1}{2}T_s^{||} - \frac{\left[T_s^{|}\right]^2}{T_s}\right][\Delta x]^1 + \left[\frac{1}{6}T_s^{|||} - \frac{3}{4}\frac{T_s^{||}T_s^{|}}{T_s} + \frac{7}{12}\frac{\left[T_s^{|}\right]^3}{[T_s]^2}\right][\Delta x]^2 +$$

$$\left[-\frac{1}{24}T_s^{[4]} + \frac{1}{3}\frac{T_s^{|||}T_s^{|}}{T_s} - \frac{1}{2}\frac{T_s^{||}\left[T_s^{|}\right]^2}{[T_s]^2} + \frac{5}{12}\frac{\left[T_s^{|}\right]^4}{[T_s]^3}\right][\Delta x]^3 +$$

$$\left[\frac{1}{120}T_s^{[5]} - \frac{1}{24}\frac{T_s^{[4]}T_s^{|}}{T_s} - \frac{1}{12}\frac{T_s^{|||}T_s^{||}}{T_s} + \frac{7}{24}\frac{T_s^{|||}\left[T_s^{|}\right]^2}{[T_s]^2} + \frac{3}{16}\frac{T_s^{|}\left[T_s^{||}\right]^2}{[T_s]^2} - \frac{2}{3}\frac{T_s^{||}\left[T_s^{|}\right]^3}{[T_s]^3} + \frac{13}{40}\frac{\left[T_s^{|}\right]^5}{[T_s]^4}\right][\Delta x]^4 + \ldots$$

We can show in a similar manner that

$$\frac{T_s(x)+T_s(x-\Delta x)}{2} * \frac{\ln T_s(x) - \ln T_s(x-\Delta x)}{\Delta x} = \tag{5.2.11}$$

$$T_s^{|} + \left[-\frac{1}{2}T_s^{||} + \frac{\left[T_s^{|}\right]^2}{T_s}\right][\Delta x]^1 + \left[\frac{1}{6}T_s^{|||} - \frac{1}{2}\frac{T_s^{||}T_s^{|}}{T_s} + \frac{7}{12}\frac{\left[T_s^{|}\right]^3}{[T_s]^2}\right][\Delta x]^2 +$$

$$\left[\frac{1}{24}T_s^{[4]} - \frac{1}{3}\frac{T_s^{|||}T_s^{|}}{T_s} + \frac{1}{2}\frac{T_s^{||}\left[T_s^{|}\right]^2}{[T_s]^2} - \frac{5}{12}\frac{\left[T_s^{|}\right]^4}{[T_s]^3}\right][\Delta x]^3 +$$

$$\left[\frac{1}{120}T_s^{[5]} - \frac{1}{24}\frac{T_s^{[4]}T_s^{|}}{T_s} - \frac{1}{12}\frac{T_s^{|||}T_s^{||}}{T_s} + \frac{7}{24}\frac{T_s^{|||}\left[T_s^{|}\right]^2}{[T_s]^2} - \frac{3}{16}\frac{T_s^{|}\left[T_s^{||}\right]^2}{[T_s]^2} - \frac{13}{24}\frac{T_s^{||}\left[T_s^{|}\right]^3}{[T_s]^3} + \frac{13}{40}\frac{\left[T_s^{|}\right]^5}{[T_s]^4}\right][\Delta x]^4 + \ldots$$

If we subtract from $\left[T_s(x) * \frac{d\ln T_s(x)}{dx}\right] = \left[\frac{dT_s(x)}{dx}\right]$ the average of (5.2.10) and (5.2.11) (and multiply through by $\frac{g}{\gamma}$) we will obtain as the truncation error due to the second term of (3.1.10)

$$T_s(x) * \frac{g}{\gamma}\frac{d\ln T_s^x}{dx} - \frac{g}{\gamma}T_s(x) * \delta_x\left(\ln T_s(x)\right) = \tag{5.2.12}$$

$$-\frac{g}{\gamma}\left[\frac{1}{6}T_s^{|||} - \frac{5}{8}\frac{T_s^{||}T_s^{|}}{T_s} + \frac{7}{12}\frac{\left[T_s^{|}\right]^3}{[T_s]^2}\right][\Delta x]^2$$

$$-\frac{g}{\gamma}\left[\frac{1}{120}T_s^{[5]} - \frac{1}{24}\frac{T_s^{[4]}T_s^{|}}{T_s} - \frac{1}{12}\frac{T_s^{|||}T_s^{||}}{T_s} + \frac{7}{24}\frac{T_s^{|||}\left[T_s^{|}\right]^2}{[T_s]^2} - \frac{29}{48}\frac{T_s^{||}\left[T_s^{|}\right]^3}{[T_s]^3} + \frac{13}{40}\frac{\left[T_s^{|}\right]^5}{[T_s]^4}\right][\Delta x]^4 + \ldots$$

To calculate the contribution to the truncation error from the first term of (3.1.10), we add the following two series where $Z(x)$ is the height of the surface terrain (by Eq (4.7.1)).

$$\frac{dZ(x)}{dx} - \left[\frac{Z(x+\Delta x) - Z(x)}{\Delta x}\right] = -\frac{1}{2!}\left[Z^{||}\right]*[\Delta x] - \frac{1}{3!}\left[Z^{|||}\right][\Delta x]^2 - \ldots \tag{5.2.13}$$

$$\frac{dZ(x)}{dx} - \left[\frac{Z(x) - Z(x-\Delta x)}{\Delta x}\right] = +\frac{1}{2!}\left[Z^{||}\right]*[\Delta x] - \frac{1}{3!}\left[Z^{|||}\right][\Delta x]^2 + \ldots \tag{5.2.14}$$

and divide by 2 to arrive at

$$\frac{dZ(x)}{dx} - \left[\left[\frac{Z(x+\Delta x) - Z(x-\Delta x)}{2\Delta x}\right]\right] = -\frac{1}{3!}\left[Z^{|||}\right][\Delta x]^2 - \frac{1}{5!}\left[Z^{(V)}\right]*[\Delta x]^4 - \ldots. \tag{5.2.15}$$

After we multiply by g we have

$$\left[\frac{d\phi_s(x)}{dx}\right] - \left[g\left[\frac{Z(x+\Delta x) - Z(x-\Delta x)}{2\Delta x}\right]\right] = -\frac{g}{3!}\left[Z^{|||}\right][\Delta x]^2 - \frac{g}{5!}\left[Z^{(V)}\right]*[\Delta x]^4 - \ldots \tag{5.2.16}$$

From (4.6.8) we have

$$Z^{|} = \frac{1}{-\gamma}T^{|}, \tag{5.2.17}$$

and so by adding (5.2.16) and (5.2.12) we obtain

$$\left[\left[\frac{d\phi_s(x)}{dx}\right] + RT_s(x)*\frac{d\ln p_s(x)}{dx}\right] - \left[\left[\delta_x(g\ Z(x))\right] + \overline{\overline{RT_s(x)}*\delta_x\left(\ln p_s(x)\right)}\right] \tag{5.2.18}$$

$$= -\frac{g}{\gamma}\left[+\frac{5}{8}\frac{T_s^{||}T_s^{|}}{T_s} - \frac{7}{12}\frac{\left[T_s^{|}\right]^3}{[T_s]^2}\right][\Delta x]^2$$

$$-\frac{g}{\gamma}\left[-\frac{1}{24}\frac{T_s^{[4]}T_s^{|}}{T_s} - \frac{1}{12}\frac{T_s^{|||}T_s^{||}}{T_s} + \frac{7}{24}\frac{T_s^{|||}\left[T_s^{|}\right]^2}{[T_s]^2} - \frac{29}{48}\frac{T_s^{||}\left[T_s^{|}\right]^3}{[T_s]^3} + \frac{13}{40}\frac{\left[T_s^{|}\right]^5}{[T_s]^4}\right][\Delta x]^4 + \ldots.$$

We have arrived at an expression for the truncation error in Case I when $T = Az + B$. Of course, Eq (5.2.18) could be reported in terms of $Z(x)$, $Z^{|}(x)$, and higher order derivatres by (5.2.17).

We make two further observations about this result at this point. First, if we take the limit as Δx goes to zero, we see that the Corby finite difference scheme tends to the analytic result, as was shown previously in Section 4.7.

Secondly, one might ask under what condition, if any, does the truncation error vanish. Setting Eq (5.2.18) to 0, implies that all the coefficents of powers of Δx would have to be zero. For instance, this would require that

$$\frac{5}{8}\frac{T_s^{||}T_s^{|}}{T_s} - \frac{7}{12}\frac{\left[T_s^{|}\right]^3}{[T_s]^2} = 0. \tag{5.2.19}$$

If we consider an interval on which $T_s(x)$ is not zero, this would leave us with

$$\frac{5}{8}T_s''T_s' = \frac{7}{12}\frac{\left[T_s'\right]^3}{[T_s]}. \tag{5.2.20}$$

Now either $T_s' = 0$, or we would have

$$\frac{T_s''}{T_s'} = \frac{14}{15}\frac{T_s'}{T_s}, \tag{5.2.21}$$

which would lead us to

$$T_s(x) = \left[\frac{K_o}{15}x\right]^{15}. \tag{5.2.22}$$

where K_o is a constant of integration. If now, we were to substitute this result into the coefficient for $[\Delta x]^4$ in Eq (5.2.18) we obtain a non-zero result. Hence for the truncation error to vanish we must have $T_s' = 0$ as mentioned above. This means that the temperature along the surface of the mountain must be constant, and by (4.6.8) that the lapse rate γ is zero. We conclude that the atmosphere must be isothermal for this to happen.

Chapter 6

Barotropic atmosphere with isobaric planes inclined to the horizontal

We consider in Case II a barotropic atmosphere in which the p_{850} plane is set tilted with respect to the horizontal (so that a vector normal to p_{850} will have zero y component), and hence the slope of p_{850} in the x - z plane, m_{850}, will be a fixed non-zero number. The temperature gradient on p_{850} is of course zero ($k_{850} = 0$). In a barotropic atmosphere the geostrophic wind vector remains constant with height (as was discussed in Section 2.7.3), and so all isobaric planes will then be parallel, because (2.7.6) will be the same at any pressure, that is height, level.

6.1 Analysis for $T = A \ln p + B$

Here the experiment revealed that exactly the same finite difference approximations used in Case I could be used to obtain a similar result, that is, mutually compensating analytic truncation errors in the two terms, yielding a zero net (analytic) truncation error. (Please see Tables B.19 through B.27, with particular attention to B.21 and B.24 which reflect the precision of the calculation.)

We will show that this result was to be expected. To do this a somewhat different approach to that used in Section 3.3 for the problem of computing $\left(\frac{\partial \phi}{\partial x}\right)_p$ will now be undertaken. We will investigate the problem graphically, and use a geometrical argument.

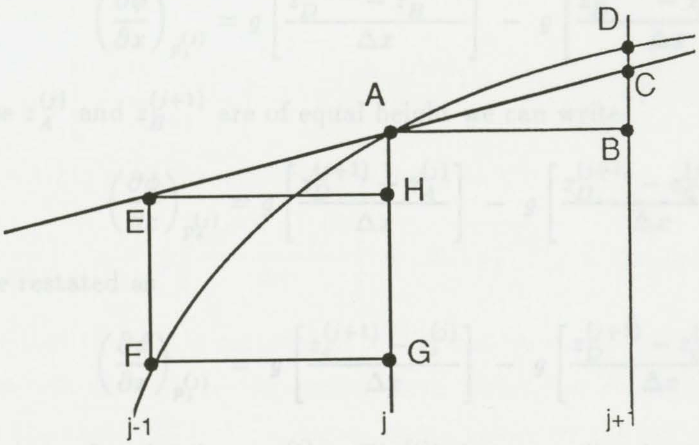

Figure 3: Geometrical calculation of isobaric geopotential gradient. The isobaric plane which intersects mountain segment FAD at point A has slope $\frac{CB}{BA} = \frac{AH}{HE}$. We will show that when $T = A\ln(p) + B$, this can be computed solely as a function of temperature and ln (pressure) values, taken at surface points F, A, and D.

Figure 3 on P.42 shows the isobaric plane (EC) which cuts the terrain surface (FAD) at A, or $z_s^{(j)}$. We will derive a finite difference expression for $\left(\frac{\partial \phi}{\partial x}\right)_{p_s(x^{(j)})}$ based on this assumption. However, if the isobaric plane were to pass through point A at a different angle, perhaps in a way such that C would be above surface point D, an analogous derivation could be put forth which would be the same as the one we will present, differing only in the placement of several plus and minus signs but reaching the same final result. We write

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s(x^{(j)})} = g m_{850} = g\left[\frac{z_C^{(j+1)} - z_B^{(j+1)}}{\Delta x}\right], \tag{6.1.1}$$

or, by adding and subtracting $\frac{g z_D^{(j+1)}}{\Delta x}$, and then using the notation $p_s(x^{(j)}) = p_s^{(j)}$, we have

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_D^{(j+1)} - z_B^{(j+1)}}{\Delta x}\right] - g\left[\frac{z_D^{(j+1)} - z_C^{(j+1)}}{\Delta x}\right], \tag{6.1.2}$$

and because $z_A^{(j)}$ and $z_B^{(j+1)}$ are of equal height we can write

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_D^{(j+1)} - z_A^{(j)}}{\Delta x}\right] - g\left[\frac{z_D^{(j+1)} - z_C^{(j+1)}}{\Delta x}\right]. \tag{6.1.3}$$

This can be restated as

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j+1)} - z_s^{(j)}}{\Delta x}\right] - g\left[\frac{z_D^{(j+1)} - z_C^{(j+1)}}{\Delta x}\right]. \tag{6.1.4}$$

The mountain surface height at $x^{(j)}$ is $Z(x^{(j)})$, but we will write it as $z_s^{(j)}$ here and for the rest of the thesis when it is used as part of a finite difference expression, since it is a more compact notation and because $p_s^{(j)}$ and $T_s^{(j)}$ terms will also appear in such an expression.

We now integrate the hydrostatic equation (3.2.24) with $T = A \ln p + B$ between an U(pper) and L(ower) pressure level to obtain

$$g z_U - g z_L = -\frac{RA}{2}\left[(\ln p_U)^2 - (\ln p_L)^2\right] - R\,B\,[\ln p_U - \ln p_L]. \tag{6.1.5}$$

Factoring provides us with

$$g z_U - g z_L = -\frac{RA}{2}\left[(\ln p_U - \ln p_L) * (\ln p_U + \ln p_L)\right] - R\,B\,[\ln p_U - \ln p_L]. \tag{6.1.6}$$

Recalling that $T = A \ln p + B$ allows (6.1.6) to be rewritten as

$$g z_U - g z_L = -R\left[\frac{T(p_U) + T(p_L)}{2}\right] * [\ln p_U - \ln p_L] \tag{6.1.7}$$

and we can now apply (6.1.7) to line segment DC as follows

$$g\left[z_D^{(j+1)}\right] - g\left[z_C^{(j+1)}\right] = -R\left[\frac{T\left(p_D^{(j+1)}\right) + T\left(p_C^{(j+1)}\right)}{2}\right] * \left[\ln p_D^{(j+1)} - \ln p_C^{(j+1)}\right]. \tag{6.1.8}$$

Thus

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j+1)} - z_s^{(j)}}{\Delta x}\right] + R\left[\frac{T(p_D^{(j+1)}) + T(p_C^{(j+1)})}{2}\right] * \left[\frac{\ln p_D^{(j+1)} - \ln p_C^{(j+1)}}{\Delta x}\right], \tag{6.1.9}$$

and since EAC represents a constant pressure plane (line) we have

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j+1)} - z_s^{(j)}}{\Delta x}\right] + R\left[\frac{T(p_D^{(j+1)}) + T(p_A^{(j)})}{2}\right] * \left[\frac{\ln p_D^{(j+1)} - \ln p_A^{(j)}}{\Delta x}\right], \tag{6.1.10}$$

or

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j+1)} - z_s^{(j)}}{\Delta x}\right] + R\left[\frac{T(p_s^{(j+1)}) + T(p_s^{(j)})}{2}\right] * \left[\frac{\ln p_s^{(j+1)} - \ln p_s^{(j)}}{\Delta x}\right]. \tag{6.1.11}$$

Rewriting (6.1.11) with the average operator yields

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j+1)} - z_s^{(j)}}{\Delta x}\right] + \overline{RT(p_s^{(j+\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j+\frac{1}{2})}\right). \tag{6.1.12}$$

We note that (6.1.12) is centered at $x^{(j+\frac{1}{2})}$ but we would rather have it centered at $x^{(j)}$, therefore we will derive a corresponding form where this is so. We start by developing a formula for the same $\left(\frac{\partial\phi}{\partial x}\right)_{p_s(j)}$, however, using values on the left side :

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{\left(z_A^{(j)} - z_G^{(j)}\right) - \left(z_H^{(j)} - z_G^{(j)}\right)}{\Delta x}\right] = g\left[\frac{z_A^{(j)} - z_G^{(j-1)}}{\Delta x}\right] - g\left[\frac{z_H^{(j)} - z_G^{(j)}}{\Delta x}\right]. \tag{6.1.13}$$

Thus we have

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_A^{(j)} - z_F^{(j-1)}}{\Delta x}\right] - g\left[\frac{z_E^{(j-1)} - z_F^{(j-1)}}{\Delta x}\right] \tag{6.1.14}$$

and

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j)} - z_s^{(j-1)}}{\Delta x}\right] - g\left[\frac{z_E^{(j-1)} - z_F^{(j-1)}}{\Delta x}\right]. \tag{6.1.15}$$

With a similar argument to the one used previously, we now have

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j)} - z_s^{(j-1)}}{\Delta x}\right] + R\left[\frac{T(p_E^{(j-1)}) + T(p_F^{(j-1)})}{2}\right] * \left[\frac{\ln p_E^{(j-1)} - \ln p_F^{(j-1)}}{\Delta x}\right] \tag{6.1.16}$$

and so

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j)} - z_s^{(j-1)}}{\Delta x}\right] + R\left[\frac{T(p_A^{(j)}) + T(p_F^{(j-1)})}{2}\right] * \left[\frac{\ln p_A^{(j)} - \ln p_F^{(j-1)}}{\Delta x}\right], \quad (6.1.17)$$

or

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j)} - z_s^{(j-1)}}{\Delta x}\right] + R\left[\frac{T(p_s^{(j)}) + T(p_s^{(j-1)})}{2}\right] * \left[\frac{\ln p_s^{(j)} - \ln p_s^{(j-1)}}{\Delta x}\right]. \quad (6.1.18)$$

In averaging notation we then obtain

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j)} - z_s^{(j-1)}}{\Delta x}\right] + \overline{RT(p_s^{(j-\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j-\frac{1}{2})}\right). \quad (6.1.19)$$

We now take one half the sum of (6.1.12) and (6.1.19) to obtain the expression

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j+1)} - z_s^{(j-1)}}{2\Delta x}\right] + \frac{R}{2}\left[\overline{T(p_s^{(j+\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j+\frac{1}{2})}\right) + \overline{T(p_s^{(j-\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j-\frac{1}{2})}\right)\right]. \quad (6.1.20)$$

and so we have

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\left[\frac{z_s^{(j+1)} - z_s^{(j-1)}}{2\Delta x}\right] + R\left[\overline{\overline{T(p_s^{(j)})} * \delta_x\left(\ln p_s^{(j)}\right)}\right], \quad (6.1.21)$$

or

$$gm_{850} = \left(\frac{\partial \phi}{\partial x}\right)_{p_s^{(j)}} = g\,\delta_x\left(\overline{z_s^{(j)}}\right) + R\left[\overline{\overline{T(p_s^{(j)})} * \delta_x\left(\ln p_s^{(j)}\right)}\right] \quad (6.1.22)$$

where we have used (6.1.1) on the left hand side,g times the slope of the isobaric plane intersecting the mountain at point A. (As mentioned on P.41, this slope is the same as that of the initialising p_{850} plane in a barotropic atmosphere, m_{850}.)

Incidentally, we note that references to Δx are absorbed within the calculation of the horizontal pressure gradient force, that is, regardless of what size Δx is, the result is still gm_{850}. Hence our numerical experiment loses no generality by employing only one grid size.

We will now show that (6.1.22) will hold on any of the σ levels above the surface. We start in the same manner as was done previously in Chapter 3, and indeed, much of that derivation follows in the same way. Equation (3.3.8) still holds so we expand its right hand side to give

$$\delta_x\left(\overline{\phi_s(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) - \frac{RA}{2}\delta_x\left(\overline{-(\ln p_s)^2}\right) - RB\delta_x(\overline{\ln p_s}) \quad (6.1.23)$$

By analogy to (3.3.10), (6.1.23) may be written

$$\delta_x\left(\overline{\phi_s(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) - RA\left(\overline{\overline{\ln p_s} * \delta_x(\ln p_s)}\right) - R\overline{\overline{B}\delta_x(\ln p_s)}. \qquad (6.1.24)$$

We gather terms to obtain

$$\delta_x\left(\overline{\phi_s(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{A\ln(p_s) + B} * \delta_x(\ln p_s)\right). \qquad (6.1.25)$$

This means that

$$\delta_x\left(\overline{\phi_s(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{\overline{T_s} * \delta_x(\ln p_s)}\right). \qquad (6.1.26)$$

Substituting (6.1.26) into (3.3.22) gives

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{\overline{T_s} * \delta_x(\ln p_s)}\right) - R\,\overline{\overline{A\ln\sigma_k} * \delta_x(\ln p_s)}. \qquad (6.1.27)$$

That is

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{\overline{T_k} * \delta_x(\ln p_s)}\right), \qquad (6.1.28)$$

or

$$\delta_x\left(\overline{\phi_k(x)}\right) + R\left(\overline{\overline{T_k} * \delta_x(\ln p_s)}\right) = \delta_x\left(\overline{\phi_o(x)}\right). \qquad (6.1.29)$$

From equation (3.1.10) and (2.3.1), the left hand side of equation (6.1.29) is the negative of the horizontal pressure gradient force at the kth σ level. Since the right hand side of equation (6.1.29) is constant in the vertical, the HPGF is independant of height. This is what one would expect in a barotropic atmosphere, as was pointed out earlier in Section 2.7.3.

6.2 Analysis for $T = Az + B$

Given that the p_{850} surface is a tilted plane with slope m_{850} and zero temperature gradient (hence $T_{850}^x = T_{850}^{x(R)}$), we know that the temperature T_z^x at an arbitrary point z^x is

$$T_z^x = T_{850}^{x(R)} + \gamma\ \left[(z_{850}^{x(R)} + m_{850}\ x) - z^x\right] \tag{6.2.1}$$

by analogy to (4.6.9). Let us determine conditions for a constant pressure surface

$$p_z^x = p_{z+\Delta z}^{x+\Delta x}. \tag{6.2.2}$$

If we replace p_s^x and T_s^x in equation (4.6.7) by p_z^x and T_z^x, and make use of equation (6.2.1), we obtain

$$p_z^x = p_{850}\left[\frac{T_{850}^{x(R)} + \gamma\ \left[(z_{850}^{x(R)} + m_{850}\ x) - z^x\right]}{T_{850}^{x(R)}}\right]^{\frac{g}{R\gamma}}. \tag{6.2.3}$$

Similarly,

$$p_{z+\Delta z}^{x+\Delta x} = p_{850}\left[\frac{T_{850}^{x(R)} + \gamma\ \left[(z_{850}^{x(R)} + m_{850}\ (x+\Delta x)) - z^{x+\Delta x}\right]}{T_{850}^{x(R)}}\right]^{\frac{g}{R\gamma}}. \tag{6.2.4}$$

Equating (6.2.3) and (6.2.4) we have

$$\frac{T_{850}^{x(R)} + \gamma\left[z_{850}^{x(R)} + m_{850}x - z^x\right]}{T_{850}^{x(R)}} = \frac{T_{850}^{x(R)} + \gamma\left[z_{850}^{x(R)} + m_{850}\ (x+\Delta x) - z^{x+\Delta x}\right]}{T_{850}^{x(R)}}. \tag{6.2.5}$$

After simplification this reduces to the condition

$$z^{x+\Delta x} - z^x = m_{850}\ \Delta x \tag{6.2.6}$$

which we note is independant of pressure value p. Since the increment in z on any constant pressure surface is a fixed multiple (m_{850}) of the increment in x, this means that constant pressure surfaces are parallel planes. This result is to be expected in a barotropic case.

6.3 Limit value for finite difference scheme when $T = Az + B$

The derivation of Section 4.7, (4.7.9) through (4.7.13), can be repeated for the problem of determining the limit of the Corby finite difference scheme (4.1.1) for Case II, as the atmosphere is barotropic here as well, with T^x_{850} constant and T^x_s given by (4.6.9). Recall (4.7.13)

$$\lim_{\Delta x \to 0} C = g\frac{dZ}{dx} + \frac{g}{\gamma}\frac{dT_s}{dx}.$$

Inspection of (4.6.9) shows that

$$\lim_{\Delta x \to 0} C = g\frac{dZ}{dx} + \frac{g}{\gamma}\left[\gamma m_{850} - \gamma\frac{dZ}{dx}\right] = g m_{850}. \quad (6.3.1)$$

When the Corby finite difference scheme is supplied with $T = Az + B$ data, it will converge to the correct value in the barotropic case as the grid interval tends to 0. That is, (6.3.1) yields the same value as (6.1.1) or (6.1.29).

6.4 Numerical results

Mt	MVD	MAX	MVD	MAX
Top	$T = Az + B$	$T = Az + B$	$T = A\ln p + B$	$T = A\ln p + B$
5000 m	0.01326564	0.54579312	$0.1520 * 10^{-12}$	$4.9258 * 10^{-12}$
4000 m	0.00666091	0.27927330	$0.1439 * 10^{-12}$	$4.6185 * 10^{-12}$
3000 m	0.00275672	0.11267600	$0.1395 * 10^{-12}$	$4.7979 * 10^{-12}$
2000 m	0.00080155	0.03272225	$0.1414 * 10^{-12}$	$4.1478 * 10^{-12}$
1000 m	0.00009835	0.00402228	$0.1457 * 10^{-12}$	$4.4551 * 10^{-12}$

Table 3: Mean magnitude (m/sec) and maximum of vector deviation from expected $\vec{V}_{g_s}$ for barotropic case with tilted surfaces

Since in this chapter we have set the p_{850} plane to be tilted with respect to the horizontal (with slope m_{850}), and have further required that there be no temperature gradient on this plane, we then know that the expected u (respectively v) geostrophic wind component can be calculated from (6.1.22) with no accompanying analytic discretization error when $T = A\ln p + B$. This was found to be supported by the numerical experiment, as one may see by inspecting tables B.19 and B.22 (ideal u and v) and comparing them with B.20 and B.23 (finite-difference computed u and v) repectively. But as before, when $T = Az + B$, there is a noticeable truncation error as seen in the departure (B.30 and B.33) of the u and v Corby finite difference components from their ideal values (B.28 and B.31).

From Table 3 above, one can see that when $T = A\ln p + B$, the truncation error is very small (MVD is on the order of 10^{-12} m/sec). But on account of the derivation of Section

6.1, culminating in (6.1.22), we are assured that the mathematical noise in evidence under MVD and MAX for this T-structure can be attributed solely to accumulation of machine roundoff error and function evaluation error over the entire 17 by 17 grid.

We further note from Table 3, that both MVD and MAX errors for $T = Az + B$ are on the order of 10^{11} times greater than that for $T = A\ln p + B$. (These figures are comparable to those found for Case I in Table 2, and this is evidently so because there is no analytic truncation error when $T = A\ln p + B$ in either Case I or II, but there is when $T = Az + B$.) The experiment indicates empirical evidence to suggest a strong preference for using the T-structure consistent with the discrete hydrostatic equation, in conjunction with (4.1.1) to calculate surface geostrophic wind. In the next section we will derive a formula for the discrepancy between the Corby finite-difference calculated geostrophic wind component and its expected value in a barotropic case when $T = A\ln p + B$. This will be (6.5.28),(6.5.21) and (6.5.24). After we review these equations from the perspective of $T = A\ln p + B$, and in particular (6.1.7), then it will be evident that the analytic truncation error, (6.5.21) and (6.5.24), would vanish for this T-structure. This is reviewed in Appendix A.1.

6.5 Truncation error for $T = Az + B$ in Case II

We can apply the argument of Section 5.2 to obtain the truncation error when $T = Az+B$ for Case II since T^x_{850} is constant by (4.4.2) and so (4.7.11) will still hold. Since we have from (4.6.9) that

$$\frac{dT_s(x)}{dx} = \gamma\left[m_{850} - \frac{dZ}{dx}\right] \tag{6.5.1}$$

where $Z(x) = z^x_s$, we can cite (5.2.18) as the desired error with $T^{|}(x)$ as just given. This truncation error is reported in terms of $T(x)$ and its derivatives (or equivalently $Z(x)$ and its derivatives). However, we will now take another approach to the study of this problem, motivated by the geometrical analysis of Section 6.1 and Figure 3. We will show that an evaluation of Corby's finite difference expression on $T = Az + B$ data will yield the slope of the isobaric plane intersecting the surface point $\begin{bmatrix} x \\ 0 \\ z^x_A \end{bmatrix}$, plus remaining terms. (Whereas previously it was desirable to refer to grid points with subscripts $(j-1, j, j+1)$ to present formulas such as (6.1.20) and (6.1.21) in a natural and unambiguous way, the grid interval Δx will play a more prominent role in the following discussion, and so surface heights, temperatures and pressures will use $(x, \Delta x)$ notation.)

We consider the Corby expression for the horizontal pressure gradient force (cf. the right hand side of equation (4.1.1))

$$c = \left[\frac{c_+ + c_-}{2}\right] \tag{6.5.2}$$

where

$$c_+ = g\left[\frac{z_D^{x+\Delta x} - z_A^x}{\Delta x}\right] + R\left[\left[\frac{T_D^{x+\Delta x} + T_A^x}{2}\right] * \left[\frac{\ln p_D^{x+\Delta x} - \ln p_A^x}{\Delta x}\right]\right] \tag{6.5.3}$$

and

$$c_- = g\left[\frac{z_A^x - z_F^{x-\Delta x}}{\Delta x}\right] + R\left[\left[\frac{T_A^x + T_F^{x-\Delta x}}{2}\right] * \left[\frac{\ln p_A^x - \ln p_F^{x-\Delta x}}{\Delta x}\right]\right], \tag{6.5.4}$$

with superscipt notation referring to Figure 3. (Note that c is a geometric equivalent of C from (4.7.2).) We focus our attention solely on c_+ for the moment, and obtain the following (by adding and subtracting appropriate terms, and using the fact that $p_A^x = p_C^{x+\Delta x}$)

$$c_+ = g\left[\frac{z_D^{x+\Delta x} - z_A^x}{\Delta x}\right] + g\left[\frac{-z_D^{x+\Delta x} + z_C^{x+\Delta x}}{\Delta x}\right] - g\left[\frac{-z_D^{x+\Delta x} + z_C^{x+\Delta x}}{\Delta x}\right] + E_1 + E_2 \tag{6.5.5}$$

with

$$E_1 = R\left[\frac{T_D^{x+\Delta x} + T_C^{x+\Delta x}}{2}\right] * \left[\frac{\ln p_D^{x+\Delta x} - \ln p_C^{x+\Delta x}}{\Delta x}\right] \tag{6.5.6}$$

and

$$E_2 = R\left[\frac{-T_C^{x+\Delta x} + T_A^x}{2}\right] * \left[\frac{\ln p_D^{x+\Delta x} - \ln p_C^{x+\Delta x}}{\Delta x}\right]. \tag{6.5.7}$$

We now have

$$c_+ = g\left[\frac{z_C^{x+\Delta x} - z_A^x}{\Delta x}\right] + \left[-g\left[\frac{-z_D^{x+\Delta x} + z_C^{x+\Delta x}}{\Delta x}\right] + E_1\right] + E_2. \tag{6.5.8}$$

Since we are analyzing $T = Az + B$ we have

$$E_1 = R\left[\frac{T_D^{x+\Delta x} + T_C^{x+\Delta x}}{2}\right] * \frac{g}{R\gamma}\left[\frac{\ln T_D^{x+\Delta x} - \ln T_C^{x+\Delta x}}{\Delta x}\right]. \tag{6.5.9}$$

Now if we set

$$T_{sl}^{x+\Delta x} = T_{850}^{x+\Delta x} + \gamma\left[z_{850}^{x^{(R)}} + m_{850}\left(x + \Delta x - x^{(R)}\right)\right] \tag{6.5.10}$$

where

$$T_{850}^{x+\Delta x} = T_{850}^{x^{(R)}} + k_{850}\left(x + \Delta x - x^{(R)}\right), \tag{6.5.11}$$

then we can write

$$T_D^{x+\Delta x} = T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}, \tag{6.5.12}$$

and

$$T_C^{x+\Delta x} = T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x}. \tag{6.5.13}$$

Hence E_1 is

$$\frac{g}{\gamma}\left[\frac{(T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}) + (T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x})}{2}\right] * \left[\frac{\ln\left(T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}\right) - \ln\left(T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x}\right)}{\Delta x}\right]. \tag{6.5.14}$$

We note here that E_1 would have been $-g\left[\frac{z_D^{x+\Delta x} - z_C^{x+\Delta x}}{\Delta x}\right]$ under the $T = A \ln p + B$ structure, but is not so for the temperature profile we are considering now.

We turn to investigate E_2. Eq (6.5.7) can be rewritten as

$$E_2 = R\left[\frac{-T_C^{x+\Delta x} + T_A^x}{\Delta x}\right] * \left[\frac{\ln p_D^{x+\Delta x} - \ln p_A^x}{2}\right] \tag{6.5.15}$$

where we have used the fact that $p_C^{x+\Delta x} = p_A^x$. Let us evaluate the second factor, by considering the two points

$$T_A^x = T_{850}^{x+\Delta x} - k_{850}\,\Delta x + \gamma\left[z_{850}^{x(R)} + m_{850}\,x - z_A^x\right] \tag{6.5.16}$$

and

$$T_C^{x+\Delta x} = T_{850}^{x+\Delta x} \qquad + \gamma\left[z_{850}^{x(R)} + m_{850}\,(x+\Delta x) - z_C^{x+\Delta x}\right] \tag{6.5.17}$$

We first subtract, and then divide by Δx, to find that

$$\left[\frac{T_A^x - T_C^{x+\Delta x}}{\Delta x}\right] = -k_{850} + \gamma\left[-m_{850} - \frac{z_A^x - z_C^{x+\Delta x}}{\Delta x}\right], \tag{6.5.18}$$

and an inspection of Figure 3 reveals that

$$\frac{z_A^x - z_C^{x+\Delta x}}{\Delta x} = -\, m_{p^x} \tag{6.5.19}$$

is the negative of the slope of the constant pressure plane which passes through the surface point $\begin{bmatrix} x \\ 0 \\ z_x^A \end{bmatrix}$. In Case I, $m_{850} = m_{p^x} = 0$, and $k_{850} = 0$, hence $\left[\frac{T_A^x - T_C^{x+\Delta x}}{\Delta x}\right] = 0$, while in Case II, $m_{850} = m_{p^x}$, and $k_{850} = 0$ so $\left[\frac{T_A^x - T_C^{x+\Delta x}}{\Delta x}\right] = 0$ once more. So by (6.5.15), $E_2 = 0$, and hence the evaluation of (6.5.2) for the two barotropic Cases (I and II) is summarized by (6.5.28). However, for the general result, we refer back to equation (6.5.8), (6.5.14), (6.5.15), and (6.5.18) and write

$$c_+ = gm_{p^x} + H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x}) + M_+ \tag{6.5.20}$$

where

$$H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x}) = -g\left[\frac{-z_D^{x+\Delta x} + z_C^{x+\Delta x}}{\Delta x}\right] + \tag{6.5.21}$$

$$\frac{g}{\gamma}\left[\frac{(T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}) + (T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x})}{2}\right]\left[\frac{\ln\left(T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}\right) - \ln\left(T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x}\right)}{\Delta x}\right]$$

and

$$M_+ = R\,[-k_{850} + \gamma\,[m_{p^x} - m_{850}]] * \left[\frac{\ln p_D^{x+\Delta x} - \ln p_A^x}{2}\right]. \tag{6.5.22}$$

We find after a similar derivation for c_- that

$$c_- = g\ m_{p^x} + H_-(z_E^{x-\Delta x}, z_F^{x-\Delta x}) + M_- \tag{6.5.23}$$

where

$$H_-(z_E^{x-\Delta x}, z_F^{x-\Delta x}) = -g\left[\frac{-z_E^{x-\Delta x} + z_F^{x-\Delta x}}{\Delta x}\right] + \tag{6.5.24}$$

$$\frac{g}{\gamma}\left[\frac{(T_{sl}^{x-\Delta x} - \gamma z_E^{x-\Delta x}) + (T_{sl}^{x-\Delta x} - \gamma z_F^{x-\Delta x})}{2}\right]\left[\frac{\ln\left(T_{sl}^{x-\Delta x} - \gamma z_E^{x-\Delta x}\right) - \ln\left(T_{sl}^{x-\Delta x} - \gamma z_F^{x-\Delta x}\right)}{\Delta x}\right]$$

and

$$M_- = R\ [-k_{850} + \gamma\,[m_{p^x} - m_{850}]] * \left[\frac{-\ln p_A^x + \ln p_F^{x-\Delta x}}{2}\right]. \tag{6.5.25}$$

In order to reach this last result we made use of

$$T_{sl}^{x-\Delta x} = T_{850}^{x-\Delta x-x^{(R)}} + \gamma\left[z_{850}^{x^{(R)}} + m_{850}\ (x - \Delta x - x^{(R)})\right] \tag{6.5.26}$$

and

$$T_{850}^{x-\Delta x} = T_{850}^{x^{(R)}} + k_{850}\ (x - \Delta x - x^{(R)}). \tag{6.5.27}$$

In conclusion, we have found that when $T = Az + B$ the Corby expression expands as follows

$$\frac{c_+ + c_-}{2} = gm_{p^x} + \frac{1}{2}H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x}) + \frac{1}{2}H_-(z_E^{x-\Delta x}, z_F^{x-\Delta x}) \tag{6.5.28}$$

for Case I and II; and in general it can be written as

$$\frac{c_+ + c_-}{2} = gm_{p^x} \tag{6.5.29}$$

$$+\frac{1}{2}H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x}) + \frac{1}{2}H_-(z_E^{x-\Delta x}, z_F^{x-\Delta x})$$

$$+ R\left[-k_{850} + \gamma\left[m_{p^x} - m_{850}\right]\right] * \left[\left[\frac{\ln p_D^{x+\Delta x} - 2\ln p_A^x + \ln p_F^{x-\Delta x}}{4}\right]\right].$$

(Eq (6.5.29) will be of use when investigating Case III.) To interpret these formulas in terms of the surface terrain and the point $\begin{bmatrix} x \\ 0 \\ z_A^x \end{bmatrix}$, one can employ the following $z_E^{x-\Delta x} = z_A^x - m_{p^x}\Delta x$, and $z_C^{x+\Delta x} = z_A^x + m_{p^x}\Delta x$. Note that (6.5.28) can be viewed as the sum of the ideal result gm_{p^x} plus a truncation error for Case I and II. (Please see Appendix A.1 for further discussion.) We will show later on in Section 7.7 that (6.5.29) can be similarly viewed as ideal plus truncation error for Case III.

Chapter 7

Simple Linear Baroclinic Atmosphere

In this chapter we will investigate the consequences of allowing temperature to vary under $T = A\ln p + B_o + C_o(x - x^{(R)})$ in Sections 7.1 through 7.6, and then we will consider $T = Az + B_o + C_o(x - x^{(R)})$ in Section 7.7 for comparison.

7.1 Linear Baroclinic case for $T = A\ln p + B_o + C_o(x - x^{(R)})$

If temperature varies on the constant pressure planes, we will have what is known as geostrophic shear which means that the geostrophic wind vector will be different at different heights. This implies that an isobaric plane with pressure value q, for instance, which cuts the mountain surface at point $\begin{bmatrix} x^{(j)} \\ 0 \\ Z(x^{(j)}) \end{bmatrix}$ will in general have a different slope from that of the p_{850} plane at $\begin{bmatrix} x^{(j)} \\ 0 \\ z_{850}(x^{(j)}) \end{bmatrix}$. We will use $g\left[\frac{z^{(j+1)}-z^{(j-1)}}{2\Delta x}\right]_{p_{850}(x^{(j)})}$ to represent the slope of the p_{850} plane, and $g\left[\frac{z_q^{(j+1)}-z_q^{(j-1)}}{2\Delta x}\right]_{p_s(x^{(j)})}$ to stand for the slope of the constant pressure surface intersecting the mountain in the column of air over $x^{(j)}$. We show that

$$g\left[\frac{z_q^{(j+1)} - z_q^{(j-1)}}{2\Delta x}\right]_{p_s(x^{(j)})} = g\left[\frac{z_s^{(j+1)} - z_s^{(j-1)}}{2\Delta x}\right] + \tag{7.1.1}$$

$$R\left[\overline{T(p_s^{(j)})} * \delta_x\left(\ln p_s^{(j)}\right)\right] + RC_0\left[\frac{\ln p_s^{(j+1)} - 2\ln p_s^{(j)} + \ln p_s^{(j-1)}}{4}\right],$$

where $z_q^{(j+1)}$ represents the height of the isobaric plane that intersects $\begin{bmatrix} x^{(j)} \\ 0 \\ Z(x^{(j)}) \end{bmatrix}$, over the point $x^{(j+1)}$, and $z_q^{(j-1)}$ for the height of this same plane over the point $x^{(j-1)}$. Of course, $z_s^{(j+1)}$ is the height of the mountain at the point $x^{(j+1)}$ and $z_s^{(j-1)}$ the terrain height at the point $x^{(j-1)}$. We begin by decomposing (7.1.1) into a 'right' and 'left' half

$$g\left[\frac{z_q^{(j+1)} - z_q^{(j)}}{\Delta x}\right]_{p_s(x^{(j)})} = \tag{7.1.2}$$

$$g\left[\frac{z_s^{(j+1)} - z_s^{(j)}}{\Delta x}\right] + R\overline{T(p_s^{(j+\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j+\frac{1}{2})}\right) + RC_0\left[\frac{\ln p_s^{(j+1)} - \ln p_s^{(j)}}{2}\right],$$

$$g\left[\frac{z_q^{(j)} - z_q^{(j-1)}}{\Delta x}\right]_{p_s(x^{(j)})} = \tag{7.1.3}$$

$$g\left[\frac{z_s^{(j)} - z_s^{(j-1)}}{\Delta x}\right] + R\overline{T(p_s^{(j-\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j-\frac{1}{2})}\right) + RC_0\left[\frac{-\ln p_s^{(j)} + \ln p_s^{(j-1)}}{2}\right].$$

Recall that to find an expression for $z_U - z_L$, the vertical line segment between U(pper) and L(ower) heights, we can make use of (3.2.24)

$$gz_U - gz_L = -R * \left[\frac{T(p_U) + T(p_L)}{2}\right] * \left[\ln p_U - \ln p_L\right].$$

We direct the reader to Figure 3, and apply this to line segment DC, by recalling (6.1.8)

$$g\left[z_D^{(j+1)} - z_C^{(j+1)}\right] = -R\left[\frac{T\left(p_D^{(j+1)}\right) + T\left(p_C^{(j+1)}\right)}{2}\right] * \left[\ln p_D^{(j+1)} - \ln p_C^{(j+1)}\right].$$

But note that in this linear baroclinic case the temperature will depend on the x position on the constant pressure surface $p_s^{(j)}$. We see from Figure 3 that $p_C^{(j+1)} = p_A^{(j)} = p_s^{(j)}$. Similarly, $p_D^{(j+1)} = p_s^{(j+1)}$. Thus we rewrite the above eq (6.1.8) as

$$g\left[z_D^{(j+1)} - z_C^{(j+1)}\right] = -R\left[\frac{T\left(p_D^{(j+1)}\right) + T\left(p_C^{(j+1)}\right)}{2}\right] * \left[\ln p_s^{(j+1)} - \ln p_s^{(j)}\right]. \tag{7.1.4}$$

But, since we are currently investigating $T = A\ln p + \left[B(x) = B_o + C_o(x - x^{(R)})\right]$, where the constant $C_o = \left(\frac{\partial T}{\partial x}\right)_p$, we see that

$$T\left(p_C^{(j+1)}\right) = A\ln p_q^{x^{(R)}} + \left[B_o + C_o\left(\left(x^{(j)} + \Delta x - x^{(R)}\right) - \Delta x\right)\right] + C_o\,\Delta x, \qquad (7.1.5)$$

which holds since $p_C^{(j+1)} = p_A^{(j)}$ on the constant pressure plane; this reduces to

$$T\left(p_C^{(j+1)}\right) = A\ln p_q^{x^{(R)}} + \left[B_o + C_o\left(x^{(j)} - x^{(R)}\right)\right] + C_o\,\Delta x, \qquad (7.1.6)$$

or

$$T\left(p_C^{(j+1)}\right) = T\left(p_q^{x^{(R)}}\right) + C_o\left(\Delta x\right) = T\left(p_s^{(j)}\right) + C_o\,\Delta x. \qquad (7.1.7)$$

Of course, we also see from Figure 3 that

$$T\left(p_D^{(j+1)}\right) = T\left(p_s^{(j+1)}\right), \qquad (7.1.8)$$

and so we have

$$\left[\frac{T\left(p_D^{(j+1)}\right) + T\left(p_C^{(j+1)}\right)}{2}\right] = \left[\frac{T\left(p_s^{(j)}\right) + T\left(p_s^{(j+1)}\right)}{2}\right] + \left[\frac{C_0\,\Delta x}{2}\right]. \qquad (7.1.9)$$

Now

$$\left[\frac{z_D^{(j+1)} - z_C^{(j+1)}}{\Delta x}\right] = \qquad (7.1.10)$$

$$-\frac{R}{g}\left[\left[\frac{T(p_s^{(j)}) + T(p_s^{(j+1)})}{2}\right] * \left[\frac{\ln p_s^{(j+1)} - \ln p_s^{(j)}}{\Delta x}\right]\right] - \frac{R}{g}\left[\left[\frac{C_0\,\Delta x}{2}\right] * \left[\frac{\ln p_s^{(j+1)} - \ln p_s^{(j)}}{\Delta x}\right]\right].$$

But we can write this as

$$\left[\frac{z_D^{(j+1)} - z_C^{(j+1)}}{\Delta x}\right] = -\frac{R}{g}\overline{T(p_s^{(j+\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j+\frac{1}{2})}\right) - \frac{R}{g}C_0\left[\frac{\ln p_s^{(j+1)} - \ln p_s^{(j)}}{2}\right]. \qquad (7.1.11)$$

If we let $m_{x^{(j)}}$ be the slope of the constant pressure surface which intersects $\begin{bmatrix} x^{(j)} \\ 0 \\ Z(x^{(j)}) \end{bmatrix}$, the contour of the hill, at $x^{(j)}$ then

$$m_{x^{(j)}} = \left[\frac{z_q^{(j+1)} - z_q^{(j)}}{\Delta x}\right]_{p_s(x^{(j)})} = \left[\frac{z_C^{(j+1)} - z_B^{(j+1)}}{\Delta x}\right] = \left[\frac{z_D^{(j+1)} - z_B^{(j+1)}}{\Delta x}\right] - \left[\frac{z_D^{(j+1)} - z_C^{(j+1)}}{\Delta x}\right], \qquad (7.1.12)$$

or by interchanging the order of the summands, and multiplying by g, we obtain

$$g\left[\frac{z_q^{(j+1)}-z_q^{(j)}}{\Delta x}\right]_{p_s(x^{(j)})}=g\left[\frac{z_C^{(j+1)}-z_B^{(j+1)}}{\Delta x}\right]=g\left[\frac{z_s^{(j+1)}-z_s^{(j)}}{\Delta x}\right]-g\left[\frac{z_D^{(j+1)}-z_C^{(j+1)}}{\Delta x}\right], \tag{7.1.13}$$

and so

$$g\left[\frac{z_q^{(j+1)}-z_q^{(j)}}{\Delta x}\right]_{p_s(x^{(j)})}=g\left[\frac{z_s^{(j+1)}-z_s^{(j)}}{\Delta x}\right]+RT(p_s^{(j+\frac{1}{2})})*\delta_x\left(\ln p_s^{(j+\frac{1}{2})}\right)+RC_0\left[\frac{\ln p_s^{(j+1)}-\ln p_s^{(j)}}{2}\right. \tag{7.1.14}$$

We show in a similar manner, the corresponding result for calculating the slope from the left hand side of $\begin{bmatrix} x^{(j)} \\ 0 \\ Z(x^{(j)}) \end{bmatrix}$. We see from Figure 3 that

$$g\left[\frac{z_A^{(j)}-z_H^{(j)}}{\Delta x}\right]=g\left[\frac{z_A^{(j)}-z_G^{(j)}}{\Delta x}\right]-g\left[\frac{z_H^{(j)}-z_G^{(j)}}{\Delta x}\right], \tag{7.1.15}$$

but this can be rewritten as

$$g\left[\frac{z_A^{(j)}-z_H^{(j)}}{\Delta x}\right]=g\left[\frac{z_A^{(j)}-z_F^{(j-1)}}{\Delta x}\right]-g\left[\frac{z_E^{(j-1)}-z_F^{(j-1)}}{\Delta x}\right], \tag{7.1.16}$$

or

$$g\left[\frac{z_A^{(j)}-z_H^{(j)}}{\Delta x}\right]=g\left[\frac{z_s^{(j)}-z_s^{(j-1)}}{\Delta x}\right]-g\left[\frac{z_E^{(j-1)}-z_F^{(j-1)}}{\Delta x}\right]. \tag{7.1.17}$$

We expand the temperature terms as follows

$$T\left(p_E^{(j-1)}\right)=A\ln p_q^{x^{(R)}}+\left[B_o+C_o\left(x^{(j)}-\Delta x-x^{(R)}\right)\right] \tag{7.1.18}$$

which holds since $p_E^{(j-1)}=p_A^{(j)}$ on the constant pressure plane. Adding and subtracting Δx we find

$$T\left(p_E^{(j-1)}\right)=A\ln p_q^{x^{(R)}}+\left[B_o+C_o*\left(\left(x^{(j)}-\Delta x-x^{(R)}\right)+\Delta x\right)\right]-C_o\,\Delta x, \tag{7.1.19}$$

and this is then

$$T\left(p_E^{(j-1)}\right)=A\ln p_q^{x^{(R)}}+\left[B_o+C_o\left(x^{(j)}-x^{(R)}\right)\right]-C_o\Delta x, \tag{7.1.20}$$

and so

$$T\left(p_E^{(j-1)}\right)=T\left(p_q^{x^{(R)}}\right)-C_o\,\Delta x=T\left(p_s^{(j)}\right)-C_o\,\Delta x. \tag{7.1.21}$$

We find from Figure 3 that we also have

$$T\left(p_F^{(j-1)}\right) = T\left(p_s^{(j-1)}\right). \tag{7.1.22}$$

Hence we may write

$$\left[\frac{T\left(p_E^{(j-1)}\right) + T\left(p_F^{(j-1)}\right)}{2}\right] = \left[\frac{T\left(p_s^{(j)}\right) + T\left(p_s^{(j-1)}\right)}{2}\right] - \left[\frac{C_o\ \Delta x}{2}\right]. \tag{7.1.23}$$

We now continue by applying (7.1.4) and (7.1.23) to the particular case

$$g\left[\frac{z_E^{(j-1)} - z_F^{(j-1)}}{\Delta x}\right] = \tag{7.1.24}$$

$$-R\left[\left[\frac{T(p_s^{(j)}) + T(p_s^{(j-1)})}{2}\right] * \left[\frac{\ln p_s^{(j)} - \ln p_s^{(j-1)}}{\Delta x}\right]\right] + R\left[\left[\frac{C_o\ \Delta x}{2}\right] * \left[\frac{\ln p_s^{(j)} - \ln p_s^{(j-1)}}{\Delta x}\right]\right], \tag{7.1.25}$$

and this can be rewritten as

$$g\left[\frac{z_E^{(j-1)} - z_F^{(j-1)}}{\Delta x}\right] = -R\overline{T(p_s^{(j-\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j-\frac{1}{2})}\right) + RC_o\left[\frac{\ln p_s^{(j)} - \ln p_s^{(j-1)}}{2}\right]. \tag{7.1.26}$$

Now we use the above result to conclude

$$g\left[\frac{z_q^{(j)} - z_q^{(j-1)}}{\Delta x}\right]_{p^s(x^{(j)})} = \tag{7.1.27}$$

$$g\left[\frac{z_s^{(j)} - z_s^{(j-1)}}{\Delta x}\right] + R\overline{T(p_s^{(j-\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j-\frac{1}{2})}\right) + RC_0\left[\frac{\ln p_s^{(j-1)} - \ln p_s^{(j)}}{2}\right].$$

In summary, we add a half of each of equations (7.1.14) and (7.1.27) to find that

$$g\left[\frac{z_q^{(j+1)} - z_q^{(j-1)}}{2\Delta x}\right]_{p_s(x^{(j)})} = g\left[\frac{z_s^{(j+1)} - z_s^{(j-1)}}{2\Delta x}\right] + \tag{7.1.28}$$

$$R\overline{T(p_s^{(j+\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j+\frac{1}{2})}\right) + R\overline{T(p_s^{(j-\frac{1}{2})})} * \delta_x\left(\ln p_s^{(j-\frac{1}{2})}\right) + RC_0\left[\frac{\ln p_s^{(j+1)} - 2\ln p_s^{(j)} + \ln p_s^{(j-1)}}{4}\right],$$

and this can be rewritten to yield the desired result (7.1.1).

7.2 Isobaric slope difference

In order to calculate the discrepancy between the slopes of the constant pressure surfaces at the 850 mb level and that at the surface, over point x_j, we differentiate the hydrostatic equation with respect to x, on a constant p surface (plane)

$$\frac{\partial}{\partial p}\left(\frac{\partial \phi}{\partial x}\right)_p = \left(\frac{\partial}{\partial x}\right)_p \frac{\partial \phi}{\partial p} = \left(\frac{\partial}{\partial x}\right)_p \frac{-RT}{p}. \tag{7.2.1}$$

We interchange the order of differentiation (as we are assuming continuous second order partial derivates)

$$\frac{\partial}{\partial p}\left(\frac{\partial \phi}{\partial x}\right)_p = -R\left(\frac{\partial T}{\partial x}\right)_p * \frac{1}{p}, \tag{7.2.2}$$

and integrate in the vertical column of air over point x

$$\int_{p=p_B}^{p=p_T} d\left(\frac{\partial \phi\,[x,p]}{\partial x}\right)_p = -R\int_{p=p_B}^{p=p_T}\left(\frac{\partial T\,[x,p]}{\partial x}\right)_p * \left[\frac{dp}{p}\right] \tag{7.2.3}$$

where we use the fact that the temperature gradient is constant, say C_0, on the constant p-surface. Now

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_T} - \left(\frac{\partial \phi}{\partial x}\right)_{p_B} = -R\left(\frac{\partial T}{\partial x}\right)_p * \left[\ln p_T - \ln p_B\right], \tag{7.2.4}$$

and so

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_T} + R\left(\frac{\partial T}{\partial x}\right)_p * \left[\ln p_T - \ln p_B\right] = \left(\frac{\partial \phi}{\partial x}\right)_{p_B}. \tag{7.2.5}$$

This becomes

$$\left(\frac{\partial \phi}{\partial x}\right)_{p_{850}(x^{(j)})} + R\left(\frac{\partial T}{\partial x}\right)_p * \left[\ln p_{850}(x^{(j)}) - \ln p_s(x^{(j)})\right] = \left(\frac{\partial \phi}{\partial x}\right)_{p_s(x^{(j)})}. \tag{7.2.6}$$

Although this is true in the continuous (or analytic case), we will show in the next section that the analog holds (exactly) as well, in the finite difference case, that is

$$g\left[\frac{z^{(j+1)} - z^{(j-1)}}{2\Delta x}\right]_{p_{850}(x^{(j)})} + RC_0\left[\ln p_{850}^{(j)} - \ln p_s^{(j)}\right] = g\left[\frac{z_q^{(j+1)} - z_q^{(j-1)}}{2\Delta x}\right]_{p_s(x^{(j)})}, \tag{7.2.7}$$

where we use $\left(\frac{\partial T}{\partial x}\right)_p = C_0$. But since we have seen in Eq (7.1.1) that

$$g\left[\frac{z_q^{(j+1)}-z_q^{(j-1)}}{2\Delta x}\right]_{p_s(x_j)}=g\left[\frac{z_s^{(j+1)}-z_s^{(j-1)}}{2\Delta x}\right]+$$

$$R\left[\overline{T(p_s^{(j)})}\ *\ \delta_x\left(\ln p_s^{(j)}\right)\right]+RC_0\left[\frac{\ln p_s^{(j+1)}-2\ln p_s^{(j)}+\ln p_s^{(j-1)}}{4}\right],$$

we can combine these two equations into

$$g\left[\frac{z^{(j+1)}-z^{(j-1)}}{2\Delta x}\right]_{p_{850}(x^{(j)})}+RC_0*\left[\ln p_{850}^{(j)}-\frac{4\ln p_s^{(j)}}{4}\right]-RC_0\left[\frac{\ln p_s^{(j+1)}-2\ln p_s^{(j)}+\ln p_s^{(j-1)}}{4}\right]= \tag{7.2.8}$$

$$g\left[\frac{z_q^{(j+1)}-z_q^{(j-1)}}{2\Delta x}\right]_{p_s(x_j)}+R\left[\overline{T(p_s^{(j)})}\ *\ \delta_x\left(\ln p_s^{(j)}\right)\right],$$

and so

$$\delta_x\left(\overline{\phi_{850}^{(j)}}\right)+RC_0\ln p_{850}^{(j)}-RC_0\left[\frac{\ln p_s^{(j+1)}+2\ln p_s^{(j)}+\ln p_s^{(j-1)}}{4}\right]= \tag{7.2.9}$$

$$g\ \delta_x\left(z_s^{(j)}\right)\ +\ R\left[\overline{T(p_s^{(j)})}\ *\ \delta_x\left(\ln p_s^{(j)}\right)\right].$$

However, since p_{850} is a constant pressure plane, we can use the fact that the average of a constant number is that number itself, and so $\overline{\ln p_{850}(x)}=\ln p_{850}(x)$ to conclude that

$$\delta_x\left(\overline{\phi_{850}^{(j)}}\right)+RC_0\left[\overline{\ln p_{850}^{(j)}}\right]-RC_0\left[\overline{\ln p_s^{(j)}}\right]=g\ \delta_x\left(z_s^{(j)}\right)\ +\ R\left[\overline{T(p_s^{(j)})}\ *\ \delta_x\left(\ln p_s^{(j)}\right)\right]. \tag{7.2.10}$$

We summarize this as

$$\delta_x\left(\phi_{850}(x)\right)+RC_o\left[\overline{\ln\frac{p_{850}(x)}{p_s(x)}}\right]=\delta_x\left(\phi_s(x)\right)+\ R\ \left(\overline{[T_s(x)]}*\delta_x\left(\ln p_s(x)\right)\right), \tag{7.2.11}$$

where we denote the initializing plane p_{850}. If we rewrite (7.2.6) with the right hand side expanded with σ coordinates we will have

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_{850(x^{(j)})}}+RC_0\left[\ln p_{850}(x^{(j)})-\ln p_s(x^{(j)})\right]=\left(\frac{\partial\phi_s(x^{(j)})}{\partial x}\right)+R\ T\left[\sigma_s(x^{(j)})\right]*\left(\frac{\partial\ln p_s(x^{(j)})}{\partial x}\right) \tag{7.2.12}$$

Subtracting (7.2.11) from (7.2.12) produces the truncation error, since (7.2.12) is an identity for the continuous version, and (7.2.11) is an identity for the discrete analog of it. Because the 850 mb surface is a plane, we know that $\left(\frac{\partial\phi}{\partial x}\right)_{p_{850(x^{(j)})}} - \delta_x\left(\overline{\phi_{850}(x)}\right) = 0$. Thus the truncation error is

$$R\,C_o\left(\frac{\ln p_s(x+\Delta x)\ -\ 2\ \ln p_s(x)\ +\ \ln p_s(x-\Delta x)}{4}\right) \tag{7.2.13}$$

which for fixed Δx is roughly proportional to the curvature of $\ln p_s$. We can see one effect of this by inspecting Tables B.41 and B.59. Evidently, the discrepancy between the analytic and finite difference computations of the v-component (ie Eq (7.2.13)) is smallest halfway up the hill, where the curvature or second derivative of a cosine hill changes sign and p_s is approximately a linear function of x.

7.3 A finite difference analog

At this point we wish to show that the analytic equation

$$\left(\frac{\partial\phi}{\partial x}\right)_{p_{850}} - \left(\frac{\partial\phi}{\partial x}\right)_{p_s} = -R\left(\frac{\partial T}{\partial x}\right)_{p_s} * [\ln p_o - \ln p_s]$$

has a corresponding finite difference form. We denote 850 mb values by $[\quad]_{p_{850}}$, and values taken on the tangent to the mountain at point $\begin{bmatrix} x^{(j)} \\ 0 \\ Z(x^{(j)}) \end{bmatrix}$ by $[\quad]_{p_s}$.

We will now derive the following equality, which can be viewed as a finite difference analog of Eq (7.2.6)

$$g\left[\frac{z_U^{(j+1)} - z_U^{(j-1)}}{2\ \Delta x}\right]_{p_{850}(x_j)} - g\left[\frac{z_L^{(j+1)} - z_L^{(j-1)}}{2\ \Delta x}\right]_{p_s(x^{(j)})} = -R\ C_0\ \Delta x * \left[\frac{\ln p_U - \ln p_L}{\Delta x}\right]_{p_s(x^{(j)})}. \tag{7.3.1}$$

It is an expression for the difference between the slopes $U_4\ U_1\ U_3$ and $L_4\ L_1\ L_3$ when temperature on constant pressure planes is given by $T(x) = A\ln(p) + B_o + C_o(x - x^{(R)})$).

We split the left hand side of (7.3.1) into two parts

$$\frac{1}{2}\left[g\left[\frac{z_U^{(j+1)} - z_U^{(j)}}{\Delta x}\right]_{p_{850}(x^{(j)})} + g\left[\frac{z_U^{(j)} - z_U^{(j-1)}}{\Delta x}\right]_{p_{850}(x^{(j)})}\right] - \tag{7.3.2}$$

$$\frac{1}{2}\left[g\left[\frac{z_L^{(j+1)} - z_L^{(j)}}{\Delta x}\right]_{p_s(x_j)} + g\left[\frac{z_L^{(j)} - z_L^{(j-1)}}{\Delta x}\right]_{p_s(x^{(j)})}\right],$$

and recombine terms to obtain

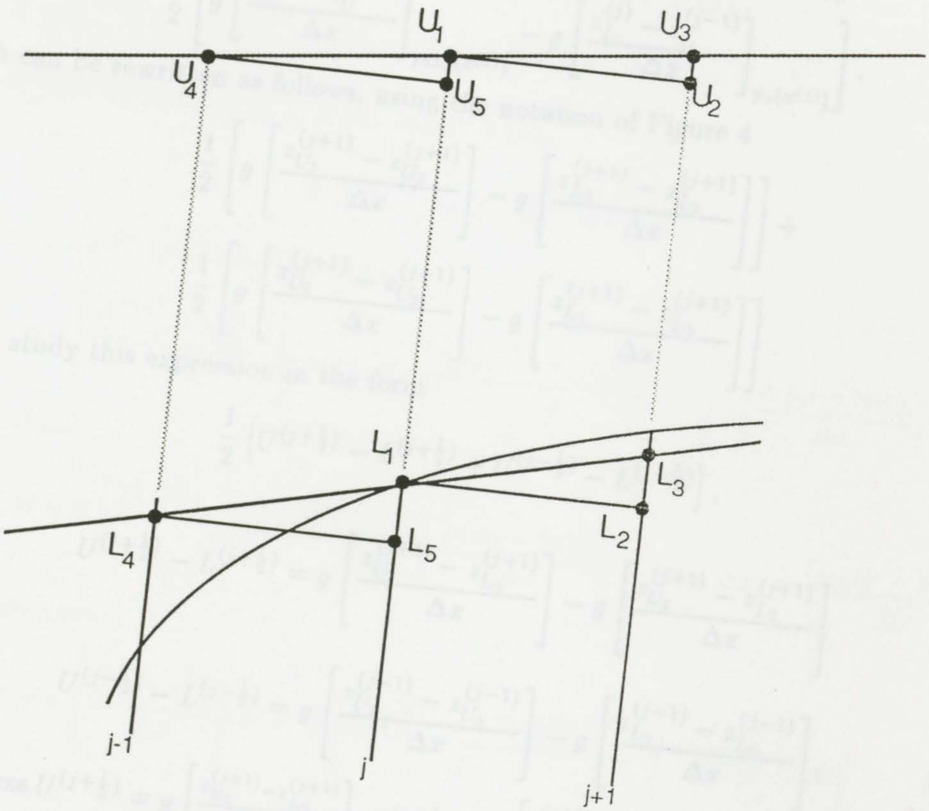

Figure 4: Measurement of constant pressure plane slope difference due to $\left(\frac{\partial T}{\partial x}\right)_p = C_o$. In a baroclinic atmosphere, an isobaric geopotential gradient at one height, say $L_1 \left[= \begin{bmatrix} x_j \\ 0 \\ z_j^L \end{bmatrix} \right]$, in general will not be the same as that at another height, for instance, $U_1 \left[= \begin{bmatrix} x_j \\ 0 \\ z_j^U \end{bmatrix} \right]$, for a fixed x.

$$\frac{1}{2}\left[g\left[\frac{z_U^{(j+1)}-z_U^{(j)}}{\Delta x}\right]_{p850(x^{(j)})}-g\left[\frac{z_L^{(j+1)}-z_L^{(j)}}{\Delta x}\right]_{p_s(x^{(j)})}\right]+ \tag{7.3.3}$$

$$\frac{1}{2}\left[g\left[\frac{z_U^{(j)}-z_U^{(j-1)}}{\Delta x}\right]_{p850(x^{(j)})}-g\left[\frac{z_L^{(j)}-z_L^{(j-1)}}{\Delta x}\right]_{p_s(x^{(j)})}\right],$$

which can be rewritten as follows, using the notation of Figure 4

$$\frac{1}{2}\left[g\left[\frac{z_{U_3}^{(j+1)}-z_{U_2}^{(j+1)}}{\Delta x}\right]-g\left[\frac{z_{L_3}^{(j+1)}-z_{L_2}^{(j+1)}}{\Delta x}\right]\right]+$$

$$\frac{1}{2}\left[g\left[\frac{z_{U_1}^{(j+1)}-z_{U_5}^{(j+1)}}{\Delta x}\right]-g\left[\frac{z_{L_1}^{(j+1)}-z_{L_5}^{(j+1)}}{\Delta x}\right]\right]. \tag{7.3.4}$$

We will study this expression in the form

$$\frac{1}{2}\left[U^{(j+\frac{1}{2})}-L^{(j+\frac{1}{2})}+U^{(j-\frac{1}{2})}-L^{(j-\frac{1}{2})}\right], \tag{7.3.5}$$

where

$$U^{(j+\frac{1}{2})}-L^{(j+\frac{1}{2})}=g\left[\frac{z_{U_3}^{(j+1)}-z_{L_3}^{(j+1)}}{\Delta x}\right]-g\left[\frac{z_{U_2}^{(j+1)}-z_{L_2}^{(j+1)}}{\Delta x}\right] \tag{7.3.6}$$

and

$$U^{(j-\frac{1}{2})}-L^{(j-\frac{1}{2})}=g\left[\frac{z_{U_1}^{(j-1)}-z_{U_5}^{(j-1)}}{\Delta x}\right]-g\left[\frac{z_{L_1}^{(j-1)}-z_{L_5}^{(j-1)}}{\Delta x}\right], \tag{7.3.7}$$

with of course $U^{(j+\frac{1}{2})}=g\left[\frac{z_{U_3}^{(j+1)}-z_{L_3}^{(j+1)}}{\Delta x}\right]$, $L^{(j+\frac{1}{2})}=g\left[\frac{z_{U_2}^{(j+1)}-z_{L_2}^{(j+1)}}{\Delta x}\right]$, $U^{(j-\frac{1}{2})}=g\left[\frac{z_{U_1}^{(j-1)}-z_{U_5}^{(j-1)}}{\Delta x}\right]$ and $L^{(j-\frac{1}{2})}=g\left[\frac{z_{L_1}^{(j-1)}-z_{L_5}^{(j-1)}}{\Delta x}\right]$. We will establish the main result of this section by deriving

$$\frac{1}{2}\left[U^{(j-\frac{1}{2})}-L^{(j-\frac{1}{2})}\right]=-R\left[\frac{C_o\,\Delta x}{2}\right]*\left[\frac{\ln(p_U)-\ln(p_L)}{\Delta x}\right]_{p_s(x^{(j)})}=\frac{1}{2}\left[U^{(j+\frac{1}{2})}-L^{(j+\frac{1}{2})}\right], \tag{7.3.8}$$

after which we will add the average of (7.3.6) and (7.3.7) to obtain (7.3.1). We start by noting that $p_{U_3}^{(j+1)}=p_{U_1}^{(j)}=p^U$ on the constant p plane (in Figure 4) and so

$$T\left(p_{U_3}^{(j+1)}\right)=A\ln\left(p_U^{x^{(R)}}\right)+\left[B_o+C_o\left(\left(x^{(j)}+\Delta x-x^{(R)}\right)-\Delta x\right)\right]+C_o\,\Delta x \tag{7.3.9}$$

becomes

$$T\left(p_{U_3}^{(j+1)}\right)=A\ln\left(p_U^{x^{(R)}}\right)+\left[B_o+C_o\left(x^{(j)}-x^{(R)}\right)\right]+C_o\,\Delta x, \tag{7.3.10}$$

or

$$T\left(p_{U_3}^{(j+1)}\right) = T\left(p_U^{(j)}\right) + C_o\, \Delta x. \tag{7.3.11}$$

However, since $p_{L_3}^{(j+1)} = p_{L_1}^{(j)} = p_L$ on the lower constant pressure plane,

$$T\left(p_{L_3}^{(j+1)}\right) = A \ln p_L^{x^{(R)}} + \left[B_o + C_o\left(\left(x^{(j)} + \Delta x - x^{(R)}\right) - \Delta x\right)\right] + C_o\, \Delta x, \tag{7.3.12}$$

and we have by analogy to (7.3.10) and (7.3.11)

$$T\left(p_{L_3}^{(j+1)}\right) = T\left(p_L^{(j)}\right) + C_o\, \Delta x. \tag{7.3.13}$$

We now apply (7.3.11) and (7.3.13) to

$$U^{(j+\frac{1}{2})} = g\left[\frac{z_{U_3}^{(j+1)} - z_{L_3}^{(j+1)}}{\Delta x}\right] = -R\left[\frac{T(p_{U_3}^{(j+1)}) + T(p_{L_3}^{(j+1)})}{2}\right] * \left[\frac{\ln p_U - \ln p_L}{\Delta x}\right], \tag{7.3.14}$$

and so obtain

$$U^{(j+\frac{1}{2})} = -R\left[\frac{T(p_U^{(j)})\ +\ T(p_L^{(j)})}{2}\right] * \left[\frac{\ln p_U - \ln p_L}{\Delta x}\right] - RC_o\, \Delta x\ *\ \left[\frac{\ln p_U - \ln p_L}{\Delta x}\right]. \tag{7.3.15}$$

But we also have

$$L^{(j+\frac{1}{2})} = g\left[\frac{z_{U_1}^{(j)} - z_{L_1}^{(j)}}{\Delta x}\right] = -R\left[\frac{T(p_U^{(j)})\ +\ T(p_L^{(j)})}{2}\right] * \left[\frac{\ln p_U - \ln p_L}{\Delta x}\right]. \tag{7.3.16}$$

Therefore we can calculate the difference as

$$\frac{1}{2}\left[U^{(j+\frac{1}{2})} - L^{(j+\frac{1}{2})}\right] = g\left[\frac{z_{U_3}^{(j+1)} - z_{U_2}^{(j+1)}}{2\,\Delta x}\right] - g\left[\frac{z_{L_3}^{(j+1)} - z_{L_2}^{(j+1)}}{2\,\Delta x}\right] =$$

$$\left[\frac{-RC_o\, \Delta x}{2}\right] * \left[\frac{\ln p_U - \ln p_L}{\Delta x}\right], \tag{7.3.17}$$

and can also show in a parallel manner, that

$$\frac{1}{2}\left[U^{(j-\frac{1}{2})} - L^{(j-\frac{1}{2})}\right] = g\left[\frac{z_{U_1}^{(j-1)} - z_{U_5}^{(j-1)}}{2\,\Delta x}\right] - g\left[\frac{z_{L_1}^{(j-1)} - z_{L_5}^{(j-1)}}{2\,\Delta x}\right] =$$

$$\left[\frac{-RC_o\, \Delta x}{2}\right] * \left[\frac{\ln p_U - \ln p_L}{\Delta x}\right], \tag{7.3.18}$$

and so (7.3.1) follows by adding (7.3.17) and (7.3.18).

7.4 Analysis for σ levels above the surface

We consider the temperature structure $T = A\ln p + \left[B(x) = B_o + C_o(x - x^{(R)})\right]$ where

$$\left(\frac{\partial B(x)}{\partial x}\right)_p = C_o = \left(\frac{\partial T}{\partial x}\right)_p . \tag{7.4.1}$$

We now derive what this implies for ϕ on the σ_k surfaces, by starting with (3.3.14), namely

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right)$$

$$+\frac{R}{2}\sum_{l=k}^{n-1}\left[\left(\delta_x\left(\overline{A\ln\left[(\sigma_{l+1} * p_s)(x)\right]}\right) + \delta_x\left(\overline{A\ln\left[(\sigma_l * p_s)(x)\right]}\right) + \delta_x\left(\overline{2\ B(x)}\right)\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+ R\ \delta_x\left(\overline{A\ \ln\left[(\sigma_n * p_s)(x)\right] + B(x)}\right) * \ln\left(\frac{1}{\sigma_n}\right).$$

However, since B is now a function of x, we expand the above equation, (3.3.14), to obtain, instead of (3.3.15),

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) \tag{7.4.2}$$

$$+\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{A\ln\sigma_{l+1}(x) + A\ \ln p_s(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{A\ln\sigma_l(x) + A\ \ln p_s(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{2\ B\ (x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+ R\ \delta_x\left(\overline{A\ \ln\sigma_n(x) + A\ \ln p_s(x) + B\ (x)}\right) * \ln\left(\frac{1}{\sigma_n}\right).$$

But because a difference operator of a constant evaluated on a constant σ surface yields a zero result, we can rewrite this as

$$\delta_x\left(\ \overline{\phi_k(x)}\ \right) = \delta_x\left(\ \overline{\phi_s(x)}\ \right) \tag{7.4.3}$$

$$+\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{A\ \ln p_s(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right] + \frac{R}{2}\sum_{l=k}^{n-1}\left[\ \delta_x\left(\overline{A\ \ln p_s(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+\frac{R}{2}\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{\ 2\ B(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right] + R\left(\overline{A\ \delta_x\left(p_s(x)\right) + \delta_x\left(B(x)\right)}\right) * \ln\left(\frac{1}{\sigma_n}\right).$$

We combine the second and third terms on the right, to arrive at

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) \tag{7.4.4}$$

$$+ RA\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{\ln p_s(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+ R\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{B(x)}\right) * \ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right)\right]$$

$$+ R\left(\overline{A\delta_x\left(\ln p_s(x)\right) + \delta_x\left(B(x)\right)}\right) * \ln\left(\frac{1}{\sigma_n}\right).$$

Since $\ln\left(\frac{\sigma_{l+1}}{\sigma_l}\right) = \ln\left(\sigma_{l+1}\right) - \ln\left(\sigma_l\right)$, the above equation may be written

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) \tag{7.4.5}$$

$$+ RA\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{\ln\left[p_s(x)\right]}\right) * \left(\ln\left(\sigma_{l+1}\right) - \ln\left(\sigma_l\right)\right)\right]$$

$$+ R\sum_{l=k}^{n-1}\left[\delta_x\left(\overline{B(x)}\right) * \left(\ln\left(\sigma_{l+1}\right) - \ln\left(\sigma_l\right)\right)\right]$$

$$- R\left(\overline{A\delta_x\left(\ln p_s(x)\right) + \delta_x\left(B(x)\right)}\right) * \ln\left(\sigma_n\right).$$

The telescoping nature of the sums leaves us with

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) \tag{7.4.6}$$

$$+ RA\ \delta_x\left(\overline{\ln p_s(x)}\right) * \ln\sigma_n(x) - RA\ \delta_x\left(\overline{\ln p_s(x)}\right) * \ln\sigma_k(x)$$

$$+ R\ \delta_x\left(\overline{B\ (x)}\right) * \ln\sigma_n(x) - R\ \delta_x\left(\overline{B\ (x)}\right) * \ln\sigma_k(x)$$

$$- RA\ \delta_x\left(\overline{\ln p_s(x)}\right) * \ln\sigma_n(x) - R\ \delta_x\left(\overline{B\ (x)}\right) * \ln\sigma_n(x),$$

which can be compactly written as

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) - RA\ \ln\sigma_k(x) * \delta_x\left(\overline{\ln\left[p_s(x)\right]}\right) - R\ \ln\left[\sigma_k(x)\right] * \delta_x\left(\overline{B\ (x)}\right). \tag{7.4.7}$$

But because evaluation is on a constant σ surface we have

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_s(x)}\right) - \overline{RA\ln\left[\sigma_k(x)\right] * \delta_x\left(\ln\left[p_s(x)\right]\right)} - R\ \overline{\ln\left[\sigma_k(x)\right] * \delta_x\left(B(x)\right)}. \tag{7.4.8}$$

We now take a moment to show that

$$\delta_x \left(\overline{\phi_s(x)}\right) \;=\; \delta_x \left(\overline{\phi_o(x)}\right) - R \;\left(\overline{[T_s(x)] * \delta_x \left(\ln p_s(x)\right)}\right) + R\ C_o\ \left[\overline{\ln \frac{p_o(x)}{p_s(x)}}\right]. \qquad (7.4.9)$$

We start by applying the average operator and the difference operator to the expression for $\phi_s(x)$, given by equation (3.3.7), but with $B(x)$ in place of B,

$$\delta_x \left(\overline{\phi_o(x)}\right) \;+\; \frac{1}{2} RA\ \delta_x \left(\overline{\ln^2 p_o(x) \;-\; \ln^2 p_s(x)}\right) +\; R\ \delta_x \left(\overline{B(x) \;*\; \left(\ln p_o(x) - \ln p_s(x)\right)}\right). \qquad (7.4.10)$$

Equations (7.4.10) through (7.4.17) are all expressions for $\delta_x\left(\phi_s(x)\right)$. Since p_o is constant (7.4.10) becomes

$$\delta_x \left(\overline{\phi_o(x)}\right) - \frac{1}{2} RA\ \delta_x \left(\overline{\ln^2 p_s(x)}\right) + R\ \delta_x \left(\overline{B(x)}\right) * \ln p_o(x) - R\ \delta_x \left(\overline{B(x) * \ln p_s(x)}\right). \qquad (7.4.11)$$

We make use of Appendix C in the second term and Proposition 1 (derived in Section 7.5) in the last term to obtain

$$\begin{aligned} &\delta_x \left(\overline{\phi_o(x)}\right) - R\ A\ \left(\overline{\ln\left[p_s(x)\right] * \delta_x \left(\ln p_s(x)\right)}\right) + R\ \delta_x \left(\overline{B(x)}\right) * \ln p_o(x) \qquad (7.4.12) \\ &\qquad -R\ \left(B(x) \;*\; \delta_x \left(\overline{\ln p_s(x)}\right)\right) \\ &\qquad -R\ C_o\ \left(\frac{\ln p_s(x + \Delta x) \;+\; \ln p_s(x - \Delta x)}{2}\right). \end{aligned}$$

At this point we split the last term into two halves, and then add and subtract another selectively chosen expression to arrive at

$$\begin{aligned} &\delta_x \left(\overline{\phi_o(x)}\right) - R\ A\ \left(\overline{\ln\left[p_s(x)\right] * \delta_x \left(\ln p_s(x)\right)}\right) + R\ \delta_x \left(\overline{B(x)}\right) * \ln p_o(x) \qquad (7.4.13) \\ &\qquad -R\ \left(B(x) \;*\; \delta_x \left(\overline{\ln p_s(x)}\right)\right) \\ &\qquad -R\ \frac{C_o}{2} \left(\frac{\ln p_s(x + \Delta x) \;+\; \ln p_s(x - \Delta x)}{2}\right) \\ &\qquad -R\ \frac{C_o}{2}\ \left(\frac{\ln p_s(x + \Delta x) \;+\; \ln p_s(x - \Delta x)}{2}\right) \\ &\qquad +2R\ \frac{C_o}{2}\ \left(\frac{\ln p_s(x)}{2}\right) \\ &\qquad -2R\ \frac{C_o}{2}\ \left(\frac{\ln p_s(x)}{2}\right) \end{aligned}$$

After gathering like terms, we find that

$$\delta_x\left(\overline{\phi_o(x)}\right) - R\,A\,\left(\overline{\overline{\ln[p_s(x)]} * \delta_x(\ln[p_s(x)])}\right) + R\,\delta_x\left(\overline{B(x)}\right) * \ln p_o(x) \tag{7.4.14}$$

$$-R\left(B(x) * \delta_x\left(\overline{\ln p_s(x)}\right)\right)$$

$$-R\,\frac{C_o}{2}\,\left(\frac{\ln p_s(x+\Delta x)\; - \;2\ln p_s(x)\; + \;\ln p_s(x-\Delta x)}{2}\right)$$

$$-R\,C_o\left(\frac{\ln p_s(x+\Delta x)\; + \;2\ln p_s(x)\; + \;\ln p_s(x-\Delta x)}{4}\right).$$

We now make use of Proposition 2 derived in Section 7.5 in absorbing the middle two terms from above, and then rewrite the last term to arrive at

$$\delta_x\left(\overline{\phi_o(x)}\right) - R\,A\,\left(\overline{\overline{\ln[p_s(x)]} * \delta_x(\ln p_s(x))}\right) + R\,C_o\,\ln p_o(x) \tag{7.4.15}$$

$$-R\,\left(\overline{\overline{B(x)} * \delta_x(\ln p_s(x))}\right)$$

$$-R\,C_o\,\left(\overline{\overline{\ln p_s(x)}}\right).$$

This yields the following by use of the fact that p_o is constant

$$\delta_x\left(\overline{\phi_o(x)}\right) - R\,\left(\overline{\overline{[A\ln p_s(x) + B(x)]} * \delta_x(\ln p_s(x))}\right) + RC_o\left(\overline{\overline{\ln p_o(x)}}\right) - RC_o\left(\overline{\overline{\ln p_s(x)}}\right). \tag{7.4.16}$$

We now employ the definition of T, and combine the last two terms to find

$$\delta_x\left(\overline{\phi_o(x)}\right) - R\,\left(\overline{\overline{[T_s(x)]} * \delta_x(\ln p_s(x))}\right) + R\,C_o\,\left(\overline{\overline{\ln p_o(x) - \ln p_s(x)}}\right). \tag{7.4.17}$$

Finally, we use a log property to arrive at (7.4.9)

$$\delta_x\left(\overline{\phi_s(x)}\right) \;=\; \delta_x\left(\overline{\phi_o(x)}\right) - R\,\left(\overline{\overline{[T_s(x)]} * \delta_x(\ln p_s(x))}\right) + R\,C_o\,\left[\overline{\overline{\ln\frac{p_o(x)}{p_s(x)}}}\right],$$

which is the conclusion we sought.

We recall equation (7.4.7), rewritten as

$$\delta_x\left(\overline{\phi_k(x)}\right) \;=\; \delta_x\left(\overline{\phi_s(x)}\right) \;-\; \overline{RA\ln[\sigma_k(x)] \;*\; \delta_x(\ln p_s(x))} - R\,C_o\,\overline{\ln\sigma_k(x)}, \tag{7.4.18}$$

or more usefully as

$$\delta_x\left(\overline{\phi_k(x)}\right) \;=\; \delta_x\left(\overline{\phi_s(x)}\right) \;-\; \overline{RA\ln[\sigma_k(x)] \;*\; \delta_x(\ln p_s(x))} - R\,C_o\,\overline{\overline{\ln[\sigma_k(x)]}}. \tag{7.4.19}$$

Substituting (7.4.9) in the first term on the right side of (7.4.19), we now find that $\delta_x\left(\overline{\phi_k(x)}\right)$ is

$$\delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{[A(\ln p_s(x) + \ln \sigma_k(x)) + B(x)] * \delta_x(\ln p_s(x))}\right) + R\,C_o\left[\overline{\overline{\ln \frac{p_o(x)}{p_s(x)} - \ln \sigma_k(x)}}\right]. \tag{7.4.20}$$

This is

$$\delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{[A(\ln[p_s * \sigma_k](x)) + B(x)] * \delta_x(\ln p_s(x))}\right) + R\,C_o\left[\overline{\overline{\ln p_o(x) - \ln(p_s * \sigma_k)(x)}}\right], \tag{7.4.21}$$

which is

$$\delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{[A(\ln p_k(x)) + B(x)] * \delta_x(\ln p_s(x))}\right) + R\,C_o\left[\overline{\overline{\ln \frac{p_o(x)}{p_k(x)}}}\right]. \tag{7.4.22}$$

Finally

$$\delta_x\left(\overline{\phi_k(x)}\right) = \delta_x\left(\overline{\phi_o(x)}\right) - R\left(\overline{[\overline{T_k(x)}] * \delta_x(\ln p_s(x))}\right) + R\,C_o\left[\overline{\overline{\ln \frac{p_o(x)}{p_k(x)}}}\right]. \tag{7.4.23}$$

We conclude that, in a linear baroclinic atmosphere we have on σ level k

$$\delta_x\left(\overline{\phi_k(x)}\right) + R\left(\overline{[\overline{T_k(x)}] * \delta_x(\ln p_s(x))}\right) = \delta_x\left(\overline{\phi_o(x)}\right) + R\,C_o\left[\overline{\overline{\ln \frac{p_o(x)}{p_k(x)}}}\right]. \tag{7.4.24}$$

7.5 Supporting propositions

Proposition 1

$$\delta_x \left(\overline{(B_o + C_o(x - x_o)) \, * \, \ln p_s(x)}\right) \; = \tag{7.5.1}$$

$$(B_o + C_o(x - x_o)) \, * \, \delta_x\left(\overline{\ln p_s(x)}\right) \; + \; C_o \; \left(\frac{\ln p_s(x + \Delta x) + \ln p_s(x - \Delta x)}{2}\right).$$

We start with

$$\delta_x \left(\overline{B_o \; \ln p_s(x) \; + \; C_o \; (x - x_o) * \ln p_s(x)}\right) = \tag{7.5.2}$$

$$B_o \; \delta_x \left(\overline{\ln p_s(x)}\right) +$$

$$\delta_x \left(\frac{C_o}{2}(x + \frac{\Delta x}{2} - x_o) \; * \; \ln p_s(x + \frac{\Delta x}{2}) \; + \; \frac{C_o}{2}(x - \frac{\Delta x}{2} - x_o) \; * \; \ln p_s(x - \frac{\Delta x}{2}) \right).$$

Expanding the difference operator as it pertains to the second term of the RHS of (7.5.2) yields

$$\frac{C_o}{2} \; \left(\frac{(x + \Delta x - x_o) * \ln p_s(x - \Delta x) \; + \; (x - x_o) * \ln p_s(x)}{\Delta x} \right) \tag{7.5.3}$$

$$+ \frac{C_o}{2} \left(\frac{- \; (x - x_o) * \; \ln p_s(x) \; - \; (x - \Delta x - x_o) \; * \; \ln p_s(x - \Delta x)}{\Delta x} \right).$$

Now collecting the $x - x_o$ terms implies that

$$\frac{C_o}{2} \, (x - x_o) \, * \, \left(\frac{\ln p_s(x + \Delta x) \; + \; \ln p_s(x) \; - \; \ln p_s(x) \; - \; \ln p_s(x - \Delta x)}{\Delta x} \right), \tag{7.5.4}$$

or

$$C_o \; (x - x_o) * \delta_x \left(\; \overline{\ln p_s(x)} \right). \tag{7.5.5}$$

Gathering the Δx terms in (7.5.3) gives the expression

$$C_o \; \left((\Delta x) * \frac{\ln p_s(x + \Delta x) \; + \ln p_s(x - \Delta x)}{2 \, \Delta x} \right). \tag{7.5.6}$$

Observation of (7.5.2), (7.5.5) and (7.5.6) allows us to conclude that Proposition 1 is valid.

Proposition 2

$$\overline{\overline{(B_o + C_o(x - x_o))} * \delta_x \left(\ln p_s(x)\right)} = \tag{7.5.7}$$

$$(B_o + C_o(x - x_o)) \, * \, \delta_x \left(\overline{\ln p_s(x)}\right) + C_o\left(\frac{\ln p_s(x+\Delta x) - 2\, \ln p_s(x) \, + \, \ln p_s(x - \Delta x)}{4}\right).$$

We start by noting that

$$\overline{\overline{\left(B_o + C_o\,(x - x_o)\right)} \;\; * \;\; \delta_x \left(\ln p_s(x)\right)} \; = \; \overline{(B_o + C_o\;(x - x_o)) \;\; * \;\; \delta_x \left(\ln p_s(x)\right)}\,, \tag{7.5.8}$$

which is

$$\overline{B_o \;\delta_x \left(\ln p_s(x)\right) \, + \, C_o(x - x_o) * \delta_x \left(\ln p_s(x)\right)}. \tag{7.5.9}$$

The first term can be written as

$$B_o \;\delta_x \left(\overline{\ln p_s(x)}\right), \tag{7.5.10}$$

however the second term

$$\overline{C_o(x - x_o) * \delta_x \left(\ln p_s(x)\right)} \tag{7.5.11}$$

requires applying the difference operator, which yields

$$\overline{C_o(x - x_o) \, * \, \frac{\ln p_s(x \, + \, \frac{\Delta x}{2}) \; - \; \ln p_s(x \; - \; \frac{\Delta x}{2})}{\Delta x}}. \tag{7.5.12}$$

Now expanding the average operator in (7.5.12) leaves the following

$$\frac{C_o\left(x + \frac{\Delta x}{2} - x_o\right)}{2} * \frac{\ln p_s(x + \Delta x) - \ln p_s(x)}{\Delta x} + \frac{C_o\left(x - \frac{\Delta x}{2} - x_o\right)}{2} * \frac{\ln p_s(x) \; - \; \ln p_s(x - \Delta x)}{\Delta x} \tag{7.5.13}$$

Collecting $x - x_o$ terms, we have

$$\frac{C_o}{2}\,(x - x_o) \, * \left(\frac{\ln p_s(x + \Delta x) \, - \, \ln p_s(x) \, + \, \ln p_s(x) \, - \, \ln p_s(x - \Delta x)}{\Delta x}\right), \tag{7.5.14}$$

which is

$$C_o \; (x \; - \; x_o) \, * \, \delta_x \left(\overline{\ln p_s(x)}\right). \tag{7.5.15}$$

We gather the $\frac{\Delta x}{2}$ terms to find

$$\frac{C_o}{2}\,(\frac{\Delta x}{2}) \, * \, \left(\frac{\ln p_s(x \, + \, \Delta x) \, - \, 2\, \ln p_s(x) \, + \, \ln p_s(x \, - \, \Delta x)}{\Delta x}\right), \tag{7.5.16}$$

and this completes the proof of Proposition 2 on the basis of (7.5.10), (7.5.15) and (7.5.16).

7.6 Analytic $\vec{V}_g$ when $T = A\ln p + B_o + C_o(x - x^{(R)})$

Since we wish to compute the surface geostrophic wind via finite differences of surface temperature and pressure values, we should have, in addition, a method to calculate an expected geostrophic wind as well. Although it is not possible to have one analytic geostrophic wind (g-wind) vector to which the Corby finite difference scheme will converge under both temperature profiles (as we will show in Sections 7.6 and 7.7), we can derive the analytic g-wind vector for the $T = A\ln p + B_o + C_o(x - x^{(R)})$ temperature structure based on the thermal wind equation (2.7.11), and will do so below. We will then derive the analytic g-wind vector for the $T = Az + B + C_o(x - x^{(R)})$ atmosphere in the next section, and in an experiment will compare the discrepancy between the analytic and finite-difference computed g-wind components for each of these respective temperature profiles. We start with the thermal wind equation (2.7.11)

$$\frac{\partial \vec{V}_g}{\partial \ln p} = \frac{R}{f}\begin{bmatrix} \left(\frac{\partial T}{\partial y}\right)_p \\ -\left(\frac{\partial T}{\partial x}\right)_p \\ 0 \end{bmatrix}.$$

Now because $T = A\ln p + B_o + C_o(x - x^{(R)})$, then $\left(\frac{\partial T}{\partial x}\right)_p = C_o$. Furthermore because we have been isolating our analysis throughout the thesis to the x - z plane, we set $\left(\frac{\partial T}{\partial y}\right)_p = 0$. (Note however that computations in the experiments are performed for x - y - z space.) We can easily integrate (2.7.11) from the surface to the 850 mb pressure level to obtain

$$\vec{V}_{g850} - \vec{V}_{g_s} = \frac{R}{f}\begin{bmatrix} 0 \\ -\left(\frac{\partial T}{\partial x}\right)_p \\ 0 \end{bmatrix} * \ln\frac{p_{850}}{p_s} \tag{7.6.1}$$

We see incidentally that the effect of letting temperature vary with x appears in the v_g component. $\vec{V}_{g_s}$ can now be calculated by

$$\vec{V}_{g_s} = \begin{bmatrix} u_{g_s} \\ v_{g_s} \\ 0 \end{bmatrix} = \begin{bmatrix} u_{g850} \\ v_{g850} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p * \ln\frac{p_{850}}{p_s} \\ 0 \end{bmatrix} \tag{7.6.2}$$

We state the following as the analytic surface geostrophic wind components :

$$u_{g_s} = u_{g850}, \tag{7.6.3}$$

$$v_{g_s} = v_{g850} + \frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p * \ln\frac{p_{850}}{p_s}. \tag{7.6.4}$$

7.7 Analytic $\vec{V}_g$ when $T = Az + B_o + C_o(x - x^{(R)})$

Let us determine the shape of a constant pressure surface of value Q mb, say p_Q, above and below that of p_{850} when temperature varies linearly with height, in the vertical. For every x on such a surface the ratio $\left[\frac{p_Q^x}{p_{850}^x}\right]$ clearly must be constant. Hence we can let

$$j_1 = \ln\left[\frac{p_Q^x}{p_{850}^x}\right] = \frac{g}{R\gamma}\ln\left[\frac{T_Q^x}{T_{850}^x}\right] \tag{7.7.1}$$

where the second equality follows by analogy to (4.7.9). Employing (4.4.4) and an equation similar to (4.6.1) with z_s^x replaced by $z_Q(x)$, we may write (7.7.1) as

$$j_1 = \frac{g}{R\gamma}\ln\left[\frac{\left[T_{850}^{x(R)} + k_{850}\ x\right] + \gamma\ \left[\left(z_{850}^{x(R)} + m_{850}\ x\right) - z_Q(x)\right]}{T_{850}^{x(R)} + k_{850}\ x}\right]. \tag{7.7.2}$$

If we set

$$k_1 = e^{\frac{R\gamma}{g}j_1}, \tag{7.7.3}$$

where k_1 is unrelated to k_{850} or $\vec{k}$, then by multiplying (7.7.2) by $\frac{R\gamma}{g}$ and taking the exponential we obtain

$$k_1\ \left[T_{850}^{x(R)} + k_{850}\ x\right] = \left[T_{850}^{x(R)} + \gamma\ z_{850}^{x(R)}\right] + \left[k_{850} + \gamma\ m_{850}\right]\ x - \gamma\ z_Q(x). \tag{7.7.4}$$

Solving (7.7.4) for $z_Q(x)$ yields

$$z_Q(x) = \left[\left[\frac{1-k_1}{\gamma}\right]T_{850}^{x(R)} + z_{850}^{x(R)}\right] + \left[m_{850} + \left[\frac{1-k_1}{\gamma}\right]k_{850}\right]\ x, \tag{7.7.5}$$

and we see that the constant pressure surface p_Q intersects the x - z plane in a straight line, with the slope of this isobar being $m_{850} + \left[\frac{1-k_1}{\gamma}\right]\ k_{850}$ where $k_1 = \left[\frac{p_Q^x}{p_{850}^x}\right]^{\frac{R\gamma}{g}}$ from (7.7.1) and (7.7.3). (Since this argument is independant of the y-component it will hold in the $\begin{bmatrix} x \\ y_f \\ z \end{bmatrix}$ plane for any fixed y_f, and because the p_{850} surface is itself a plane, we are then assured that the p_Q surface will be a plane as well.)

We will now show that the limit to which the Corby expression converges under the temperature profile discussed in this section is gm_{p^x} where m_{p^x} is the slope of the isobar which passes through the point $\begin{bmatrix} x \\ y_f \\ Z(x) \end{bmatrix}$ with $Z(x)$ the height of the mountain at x. We refer the reader to Figure 3 (where we will focus our derivation around surface point z_A^x), and begin the proof by restating the result found in (6.5.20) written, with the substitution of (6.5.22), as

$$c_+ = g\ m_{p^x} + H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x})\ +\ R\ [-k_{850} + \gamma\,[m_{p^x} - m_{850}]] * \left[\frac{\ln p_D^{x+\Delta x} - \ln p_A^x}{2}\right].$$

For convenience we repeat (6.5.21)

$$H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x}) = -g \left[\frac{-z_D^{x+\Delta x} + z_C^{x+\Delta x}}{\Delta x}\right] +$$

$$\frac{g}{\gamma}\left[\frac{(T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}) + (T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x})}{2}\right]\left[\frac{\ln\left(T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}\right) - \ln\left(T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x}\right)}{\Delta x}\right].$$

Clearly from Figure 3

$$\lim_{\Delta x \to 0}\left[\frac{\ln p_D^{x+\Delta x} - \ln p_A^x}{2}\right] = 0, \tag{7.6}$$

and so the last term of Eq (6.5.20) vanishes in the limit. Thus our main concern will be the second term of (6.5.20), which we will study as it is expressed in (6.5.21) and will use the following notation

$$\lim_{\Delta x \to 0} H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x}) = \lim_{\Delta x \to 0} H_1 + \lim_{\Delta x \to 0} H_{21} * \lim_{\Delta x \to 0} H_{22}. \tag{7.7}$$

We note from Figure 3 that that $z_A^x = Z(x)$, $z_D^{x+\Delta x} = Z(x+\Delta x)$ and $z_C^{x+\Delta x} = Z(x) + m_{p_s^x}\ \Delta x$, and so we can calculate

$$\lim_{\Delta x \to 0} H_1 = \lim_{\Delta x \to 0} -g\frac{[-Z(x+\Delta x) + Z(x)] + [Z(x) + m_{p_s^x}\ \Delta x - Z(x)]}{\Delta x} = g\left[Z^{|}(x) - m_{p_s^x}\right]. \tag{7.8}$$

We investigate the factors comprising the second term of Eq (7.7.7) :

$$\lim_{\Delta x \to 0} H_{21} = \frac{g}{\gamma}\lim_{\Delta x \to 0}\frac{\left[(T_{sl}^{x+\Delta x} - \gamma z_D^{x+\Delta x}) + (T_{sl}^{x+\Delta x} - \gamma z_C^{x+\Delta x})\right]}{2} = \frac{g}{\gamma}T_s^x, \tag{7.9}$$

where Eq (6.5.10) has been used to derive the limit of Eq (7.7.9). Also

$$\lim_{\Delta x \to 0} H_{22} = \lim_{\Delta x \to 0}\left[\frac{\ln[Q(x+\Delta x) - \ln Q(x)] - \ln[R(x+\Delta x) - \ln Q(x)]}{\Delta x}\right] \tag{7.10}$$

where

$$Q(x+\Delta x) = T_{850}^{x(R)} + k_{850}\ (x+\Delta x) + \gamma\ \left[z_{850}^{x(R)} + m_{850}\ (x+\Delta x) - Z(x+\Delta x)\right], \quad (7.7.11)$$

and of course

$$Q(x) = T_{850}^{x(R)} + k_{850}\ x + \gamma\ \left[z_{850}^{x(R)} + m_{850}\ x - Z(x)\right], \quad (7.7.12)$$

with

$$R(x+\Delta x) = T_{850}^{x(R)} + k_{850}\ (x+\Delta x) + \gamma\ \left[z_{850}^{x(R)} + m_{850}\ (x+\Delta x) - \left[Z(x) + m_{p_s^x}\ \Delta x\right]\right]. \quad (7.7.13)$$

We use the fact that the ln function is continuously differentiable, along with an application of the chain rule, to conclude that

$$\lim_{\Delta x\to 0}\left[\frac{\ln Q(x+\Delta x) - \ln Q(x)}{\Delta x}\right] = \frac{k_{850} + \gamma\ \left[m_{850} - Z^{\mathsf{I}}(x)\right]}{T_{850}^{x(R)} + k_{850}\ x + \gamma\ \left[z_{850}^{x(R)} + m_{850}\ x - Z(x)\right]}. \quad (7.7.14)$$

By a similar argument we can state that

$$\lim_{\Delta x\to 0}\left[\frac{\ln R(x+\Delta x) - \ln Q(x)}{\Delta x}\right] = \frac{k_{850} + \gamma\ \left[m_{850} - m_{p_s^x}\right]}{T_{850}^{x(R)} + k_{850}\ x + \gamma\ \left[z_{850}^{x(R)} + m_{850}\ x - Z(x)\right]}. \quad (7.7.15)$$

Hence

$$\lim_{\Delta x\to 0} H_{22} = \frac{\gamma\ \left[m_{850} - Z^{\mathsf{I}}(x)\right]}{T_s^x} - \frac{\gamma\ \left[m_{850} - m_{p_s^x}\right]}{T_x^s} = \frac{\gamma\ \left[-Z^{\mathsf{I}}(x) + m_{p_s^x}\right]}{T_s^x}, \quad (7.7.16)$$

and so we have obtained from (7.7.9) and (7.7.16)

$$\lim_{\Delta x\to 0} H_{21} * H_{22} = g\left[-Z^{\mathsf{I}}(x) + m_{p_s^x}\right]. \quad (7.7.17)$$

Overall, we then have from (7.7.7), (7.7.8) and (7.7.17)

$$\lim_{\Delta x\to 0} H_+(z_D^{x+\Delta x}, z_C^{x+\Delta x}) = \lim_{\Delta x\to 0} H_1 + \lim_{\Delta x\to 0} H_{21} * \lim_{\Delta x\to 0} H_{22} =$$

$$g\left[Z^{\mathsf{I}}(x) - m_{p_s^x}\right] + g\left[-Z^{\mathsf{I}}(x) + m_{p_s^x}\right] = 0, \quad (7.7.18)$$

and referring back to (6.5.20) we conclude that

$$\lim_{\Delta x\to 0} c_+ = g m_{p^x}. \quad (7.7.19)$$

It can likewise be shown that $\lim_{\Delta x \to 0} c_- = gm_{p^x}$, and so

$$\lim_{\Delta x \to 0} \frac{c_+ + c_-}{2} = gm_{p_s^x}. \tag{7.7.20}$$

We will now find an expression for the $m_{p_s^x}$ reported in terms of surface temperature and then one based on surface pressure. In the process we will verify what was suggested in (7.7.5) regarding this slope.

Since we are studying the constant pressure plane p_Q, we have

$$j_1 = \ln\left[\frac{p_Q^x}{p_{850}^x}\right] = \frac{g}{R\gamma}\ln\left[\frac{T_Q^x}{T_{850}^x}\right] = \frac{g}{R\gamma}\ln\left[\frac{T_Q^{x+\Delta x}}{T_{850}^{x+\Delta x}}\right]. \tag{7.7.21}$$

This tells us that

$$j_1 = \frac{g}{R\gamma}\ln\left[\frac{\left[T_{850}^{x(R)} + k_{850}\ x\right] + \gamma\ \left[(z_{850}^{x(R)} + m_{850}\ x) - z_Q(x)\right]}{T_{850}^{x(R)} + k_{850}\ x}\right] =$$

$$\frac{g}{R\gamma}\ln\left[\frac{\left[T_{850}^{x(R)} + k_{850}\ (x+\Delta x)\right] + \gamma\ \left[z_{850}^{x(R)} + m_{850}\ (x+\Delta x) - z_Q(x+\Delta x)\right]}{T_{850}^{x(R)} + k_{850}\,(x+\Delta x)}\right]. \tag{7.7.22}$$

Equating the arguments of the ln functions, and letting $z_Q(x+\Delta x) = Z(x) + m_{p_Q^x}\Delta x$ where $Z(x)$ is the mountain height, and p_Q^x is the pressure at $\begin{bmatrix} x \\ 0 \\ z_A^x \end{bmatrix}$, leaves us with

$$\gamma\ \left[T_{x(R)}^{850} + k_{850}\ x\right] * \left[m_{850} - m_{p_Q^x}\right]\ \ \Delta x =$$

$$\gamma\ k_{850}\ \ \left[(z_{850}^{x(R)} + m_{850}\ x) - F(x)\right]\ \ \Delta x. \tag{7.7.23}$$

We now find that

$$m_{850} - m_{p_Q^x} = \left[\frac{k_{850}}{T_{850}^{x(R)} + k_{850}\ x}\right]\left[z_{850}^{x(R)} + m_{850}\ x - F(x)\right]. \tag{7.7.24}$$

If we now write p_Q^x as p_s^x, we then are left with

$$m_{850} - m_{p_s^x} = \frac{1}{\gamma}\left[\frac{k_{850}}{T_{850}^{x(R)} + k_{850}\ x}\right]\left[\gamma z_{850}^x - \gamma z_s^x\right]. \tag{7.7.25}$$

Making use of the definition of T_{850}^x, and then adding and subtracting appropriate terms, we obtain

$$m_{850} - m_{p_s^x} =$$

$$\frac{1}{\gamma}\left[\frac{k_{850}}{T^x_{850}}\right] * \left[\left[T^x_{850} + \gamma\ \left[z^{x(R)}_{850} + m_{850}\ x - z^x_s\right]\right] - \left[T^x_{850} + \gamma\ \left[z^{x(R)}_{850} + m_{850}\ x - z^x_{850}\right]\right]\right]. \tag{7.7.26}$$

We see that

$$m_{p^x_s} = m_{850} - \frac{1}{\gamma}\left[\frac{k_{850}}{T^x_{850}}\right] * \left[T^x_s - T^x_{850}\right], \tag{7.7.27}$$

or multiplying by g we obtain

$$g m_{p^x_s} = g m_{850} + \frac{g}{\gamma}\ k_{850}\ \left[1 - \frac{T^x_s}{T^x_{850}}\right], \tag{7.7.28}$$

and when converted to pressure terms we have

$$g m_{p^x_s} = g m_{850} + \frac{g}{\gamma}\ k_{850}\ \left[1 - \left[\frac{p^x_s}{p^x_{850}}\right]^{\frac{R\gamma}{g}}\right]. \tag{7.7.29}$$

Putting together our last two results: (7.7.20) and the preceding equation, shows that the Corby formula will converge to $g m_{p^x_s}$, with numerical value provided by the right hand side of (7.7.29). We also note that this analytical result might have been inferred from (7.7.5). The reader is referred to Appendix A.2 for another derivation of the fact that the Corby scheme converges to (7.7.28) .

Incidentally, dividing (7.7.29) through by f will yield the v-geostrophic wind component at $\begin{bmatrix} x \\ 0 \\ Z(x) \end{bmatrix}$

$$\frac{g}{f} m_{p^x_s} = \frac{g}{f} m_{850} + \frac{g}{\gamma f}\ k_{850}\ \left[1 - \frac{T^x_s}{T_x^{850}}\right]. \tag{7.7.30}$$

This is

$$v_{g^s_x} = v_{g^x_{850}} + \frac{g}{\gamma f}\left(\frac{\partial T}{\partial x}\right)_{p_{850}} * \left[1 - \frac{T^x_s}{T^x_{850}}\right] \tag{7.7.31}$$

or, in term of pressure

$$v_{g^x_s} = v_{g^x_{850}} + \frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_{p_{850}} * \left[1 - \left[\frac{p^x_s}{p^x_{850}}\right]^{\frac{R\gamma}{g}}\right]. \tag{7.7.32}$$

A similar result can be found for u_{g_s} where (7.7.32) will have an equivalent form except the v's will be replaced by u's and the x's replaced by y's. Since we have decided to set $\left(\frac{\partial T}{\partial y}\right)_{p_{850}} = 0$ we will obtain for u_{g_s},

$$u_{g_s} = u_{g_{850}}. \tag{7.7.33}$$

At this point we will show that indeed, these are the analytic surface geostrophic wind components for the case we are studying in this section.

To find the temperature variation with x along an isobar $p = p_Q$, we differentiate (7.7.4) and then use (7.7.1), (7.7.2) and (7.7.3) to find

$$\left(\frac{\partial T_Q^x}{\partial x}\right)_{p=p_Q} = k_1\, k_{850} = k_{850}\left[\frac{p_Q^x}{p_{850}^x}\right]^{\frac{R\gamma}{g}} \tag{7.7.34}$$

Therefore, by integrating the thermal wind equation (2.7.11) (as it pertains to the v-component only) we have

$$v_{g_s^x} = v_{g_{850}^x} + \frac{R}{f}\int_{p=p_s}^{p=p_{850}} \left[\frac{\partial T}{\partial x}\right]_p \frac{dp}{p} = v_{g_{850}^x} + \frac{R}{f}\int_{p=p_s}^{p=p_{850}} k_{850}\left[\frac{p}{p_{850}^x}\right]^{\frac{R\gamma}{g}} \frac{dp}{p} \tag{7.7.35}$$

and so

$$v_{g_s^x} = v_{g_{850}^x} + \frac{R}{f} k_{850}\, [p_{850}^x]^{\frac{-R\gamma}{g}} * \int_{p=p_s}^{p=p_{850}} [p]^{\frac{R\gamma}{g}-1}\, dp. \tag{7.7.36}$$

This simplifies to the following

$$v_{g_s^x} = v_{g_{850}^x} + \frac{R}{f} k_{850}\, [p_{850}^x]^{\frac{-R\gamma}{g}} * \left[\frac{[p_{850}]^{\frac{R\gamma}{g}} - [p_s^x]^{\frac{R\gamma}{g}}}{\frac{R\gamma}{g}}\right]. \tag{7.7.37}$$

Finally, we have

$$v_{g_s^x} = v_{g_{850}^x} + \frac{g}{\gamma f}\, k_{850}\left[1 - \left[\frac{p_s^x}{p_{850}^x}\right]^{\frac{R\gamma}{g}}\right], \tag{7.7.38}$$

and so the analytic result for this case is then

$$v_{g_s^x} = v_{g_{850}^x} + \frac{g}{\gamma f}\, k_{850}\left[1 - \left[\frac{T_s^x}{T_{850}^x}\right]\right]. \tag{7.7.39}$$

7.8 Numerical results

From Tables 4 and 5 below, one can see that the truncation error is smaller under the $T = A\ln p + B_o + C_o(x - x^{(R)})$ profile than that for $T = Az + B_o + C_o(x - x^{(R)})$, at each height, and for both k_{850} factors.

However, the error roughly doubles (for either distribution) when the baroclinity factor k_{850} doubles.We take note that, as was shown previously, there is an analytic truncation error for the $T = A\ln p + B_o + C_o(x - x^{(R)})$ case, here (see (7.2.13)).

Mt	MVD	MAX	MVD	MAX
Top	$T = Az + B$	$T = Az + B$	$T = A\ln p + B$	$T = A\ln p + B$
5000 m	0.01542758	0.55813387	0.00800488	0.27240320
4000 m	0.00917657	0.31426375	0.00634361	0.21759538
3000 m	0.00548133	0.18007625	0.00471451	0.16294712
2000 m	0.00325935	0.11465569	0.00311544	0.11007571
1000 m	0.00158277	0.05851451	0.00154451	0.05786726

Table 4: Mean magnitude (m/sec) and maximum of vector deviation from expected $\vec{V}_{g_s}$ for baroclinic case with $\left(\frac{\partial T}{\partial x}\right)_p = 1.0 * 10^{-5} C^o/m$
Note that $B(x) = B_o + C_o(x - x^{(R)})$ with $C_o = \left(\frac{\partial T}{\partial x}\right)_p = k_{850}$

Mt	MVD	MAX	MVD	MAX
Top	$T = Az + B$	$T = Az + B$	$T = A\ln p + B$	$T = A\ln p + B$
5000 m	0.02059560	0.72353997	0.01598677	0.65080601
4000 m	0.01426472	0.48102181	0.01266937	0.45898628
3000 m	0.00985721	0.35201274	0.00941601	0.33273572
2000 m	0.00636065	0.23045890	0.00622244	0.22144827
1000 m	0.00315656	0.11912680	0.00308490	0.11780949

Table 5: Mean magnitude (m/sec) and maximum of vector deviation from expected $\vec{V}_{g_s}$ for baroclinic case with $\left(\frac{\partial T}{\partial x}\right)_p = 2.0 * 10^{-5} C^o/m$
Note that $B(x) = B_o + C_o(x - x^{(R)})$ with $C_o = \left(\frac{\partial T}{\partial x}\right)_p = k_{850}$

Tables B.37 - B.45 (in Appendix B) detail computations of surface geostrophic wind vectors for $\left(\frac{\partial T}{\partial x}\right)_p = 1.0 * 10^{-5} C^o/m$ when $T = A\ln p + B_o + C_o(x - x^{(R)})$ and tables B.46 through B.54 for $T = Az + B_o + C_o(x - x^{(R)})$. Note that since we set $\left(\frac{\partial T}{\partial y}\right)_p = 0$, there was no accompanying analytic truncation error (although there was some due to calculation performed on a finite precision machine, or machine error for short) when $T = A \ln p + B_o + C_o(x - x^{(R)})$ in the u_g component (Table B.39), whereas there was analytic (as suggested in (6.5.29)) in addition to machine error for $T = Az + B_o + C_o(x - x^{(R)})$, shown in Table B.48.

Tables B.55 - B.63 show results for $\left(\frac{\partial T}{\partial x}\right)_p = 2.0 * 10^{-5} C^o/m$ when $T = A \ln p + B_o + C_o(x - x^{(R)})$, and B.64 through B.72 for $T = Az + B_o + C_o(x - x^{(R)})$. Note that once again when the baroclinity factor is zero in the y direction, analytic truncation error is non-existant when $T = A \ln p + B_o + C_o(x - x^{(R)})$. Hence Table B.57 exhibits only machine error. However when $T = Az + B_o + C_o(x - x^{(R)})$ the results shown in Table B.66 have a contribution from both of these types of error.

The analytic geostrophic wind components for $T = A \ln p + B_o + C_o(x - x^{(R)})$ are given by (7.6.3) for u and (7.6.4) for v, and for $T = Az + B_o + C_o(x - x^{(R)})$ by (7.7.33) for u and (7.7.32) for v repectively. These equations are used to generate tables B.37 and B.55 for u, and B.40, B.58 for v if T is a function of p in the manner indicated above. However, when T is a function of z (as previously specified), the associated u components generated, may be found in B.46 and B.64, while the associated v components in B.49 and B.67.

Since there is a temperature gradient on the p_{850} plane, the thermal wind equation (2.7.11) has been used to show that there will be different analytic targets for the two temperature structures. In comparing the two we conclude that when $T = A \ln p + B_o + C_o(x - x^{(R)})$ the Corby calculated geostrophic wind components are closer to their expected values than is the case when $T = Az + B_o + C_o(x - x^{(R)})$.

Chapter 8

Summary and Conclusions

In this thesis we have developed a derivation that shows if one wished to pass from the continuous hydrostatic equation to a discrete version of it, one method of doing so would be to use a temperature structure of $T = A \ln p + B$ along with the discrete hydrostatic equation (3.2.24)

$$\phi(p_U) - \phi(p_L) = -\frac{R\,[T(p_U) + T(p_L)]}{2} * [\ln p_U - \ln p_L]$$

where p_U and p_L are upper and lower pressure levels. This combination was used to verify that the Corby finite difference scheme for computing the horizontal pressure gradient force in σ coordinates is accompanied by zero analytic truncation error when an atmosphere is at rest.

We extended this result to a barotropic atmosphere in which a wind was present because of the tilt in the slope of the initializing p_o plane, and explained with a geometrical argument how a finite difference scheme making use of only surface temperatures and pressures could represent the isobaric geopotential gradient (equivalent to the horizontal pressure gradient force). We further derived an expression for the analytic truncation error that ensues for $T = Az + B$ computation of HPGF, originating from a geometrical analysis.

We concurrently conducted a numerical experiment in which we compared the computation of geostrophic wind under the temperature profile mentioned above, with that of $T = Az + B$, which is an ostensibly legitimate alternative in that it satisfies the analytic hydrostatic equation, and was shown to converge in the limit to the same result that $T = A \ln p + B$ geostrophic wind components did. However, since this temperature structure was not consistent with the discrete version of the hydrostatic equation which was implicit in the Corby scheme used for this calculation, it produced worse net (analytic plus roundoff) truncation error than $T = A \ln p + B$ did in each case considered. An expression representing the analytic part of this error was derived, both with a Taylor series expansion, and separately by a geometrically motivated study.

Furthermore, we showed that the combination of $A \ln p + B$ temperature structure with the Corby finite difference scheme could be used in a simple linear baroclinic atmosphere, but for which there was some associated analytic discretization error, and for which we gave a formula which exhibited a Laplacian form.

Overall, the results of our analysis supported by the outcome of the numerical experiments, indicated the necessity of choosing data consistent with one's finite difference equations and not just consistent with the continuous equations. In particular, it was shown that $T = A \ln p + B$ is a more suitable temperature structure for the Corby finite difference scheme than is $T = Az + B$ to compute HPGF in σ coordinates.

Appendix A

Supplementary derivations

A.1 $T = Az + B$ is not consistent with the discrete hydrostatic equation

We have from (3.2.24) that when $T = A \ln p + B$

$$g\,[z_U^x - z_L^x] = -R\left[\frac{T_U^x + T_L^x}{2}\right] * [\ln p_U^x - \ln p_L^x]$$

for upper and lower levels, U and L. Furthermore, if we choose U to be the 850mb level, and L to be arbitrary, we then have

$$g z_{850}^x \; - \; g z^x \; = \; -\frac{RA}{2}\;\left[\;(\ln p_{850}^x)^2 - (\ln p^x)^2\;\right] \; - \; R\,B\;[\ln p_{850}^x \; - \; \ln p^x] \qquad \text{(A.1.1)}$$

for x fixed. This then implies

$$0 = -\frac{RA}{2g}\;\left[\;(\ln p^x)^2\;\right] - \frac{RB}{g}\;[\ln p^x] + \left[\left[\frac{RA}{2g}\left[\;(\ln p_{850}^x)^2\;\right] \; + \frac{RB}{g}\,[\ln p_{850}^x] + z_{850}^x\right] - z^x\right] \qquad \text{(A.1.2)}$$

or

$$0 = c_1\;[\ln p^x]^2 \; + \; c_2\;\;[\ln p^x] + c_3(z^x) \qquad \text{(A.1.3)}$$

where

$$c_1 = -\frac{RA}{2g}, \qquad \text{(A.1.4)}$$

$$c_2 = -\frac{RB}{g} \qquad \text{(A.1.5)}$$

and

$$c_3\,(z) \;=\; \left[\left[\frac{RA}{2g}\left[(\ln p^x_{850})^2\right] + \frac{RB}{g}\left[\ln p^x_{850}\right] + z^x_{850}\right] - z^x\right]. \tag{A.1.6}$$

Solving this quadratic equation yields

$$\ln p^x_1 = \frac{-c_2 + \left[{c_2}^2 - 4c_1 c_3(z)\right]^{\frac{1}{2}}}{2c_1} \tag{A.1.7}$$

$$\ln p^x_2 = \frac{-c_2 - \left[{c_2}^2 - 4c_1 c_3(z)\right]^{\frac{1}{2}}}{2c_1}. \tag{A.1.8}$$

However, by comparing temperatures at the 850 mb level to those below it, it can be shown with simple algebra that the appropriate branch to choose is the negative one. Hence

$$p^x(z) = e^{\frac{-c_2 - \left[{c_2}^2 - 4c_1 c_3(z)\right]^{\frac{1}{2}}}{2c_1}}, \tag{A.1.9}$$

and differentiation with respect to z (with x fixed) produces the pressure profile for this case,

$$\frac{\partial p^x(z)}{\partial z} = \left[-\left[{c_2}^2 - 4c_1 c_3(z)\right]^{\frac{-1}{2}}\right] * e^{\frac{-c_2 - \left[{c_2}^2 - 4c_1 c_3(z)\right]^{\frac{1}{2}}}{2c_1}}. \tag{A.1.10}$$

However, when $T = Az + B$, we can write the temperature as

$$T^x(z) = \left[T^{x(R)}_{850} + \gamma\, z^{x(R)}_{850}\right] + \left[k_{850} + \gamma * m_{850}\right]\; x - \gamma\; z^x \tag{A.1.11}$$

which for fixed x can be summarized as

$$T^x(z) \;=\; \left[k^x - \gamma z^x\right] \tag{A.1.12}$$

where $k^x = \left[T^{x(R)}_{850} + \gamma\; z^{x(R)}_{850}\right] + \left[k_{850} + \gamma * m_{850}\right]\; x$ is constant.

The pressure is (by (4.6.7)

$$p^x_z = p^x_{850}\;\left[\frac{k^x - \gamma z^x}{T^x_{850}}\right]^{\frac{g}{R\gamma}}, \tag{A.1.13}$$

or by taking logarithms

$$\ln p^x_z - \ln p^x_{850} = \frac{g}{R\gamma}\left[\ln T^x - \ln T^x_{850},\right]. \tag{A.1.14}$$

When differentiated with respect to z (with x fixed) yields

$$\frac{1}{p^x}\frac{\partial p^x}{\partial z} = \frac{g}{R\gamma}\;\frac{1}{T^x}\frac{\partial T^x}{\partial z}. \tag{A.1.15}$$

We then have

$$\frac{\partial p^x(z)}{\partial z} = \frac{g}{R\gamma}\,\frac{p^x}{[k^x - \gamma z^x]} * -\frac{\gamma}{1} \tag{A.1.16}$$

which implies

$$\frac{\partial p^x(z)}{\partial z} = -\frac{g}{R}\frac{p^x_{850}}{T^x_{850}}\left[\frac{k^x - \gamma z^x}{T^x_{850}}\right]^{\frac{g}{R\gamma}-1} \tag{A.1.17}$$

Note that this is a different pressure profile from that determined by $T = A\ln p + B$ (which satisfies the (discrete) hydrostatic equation presented at the start of this section). We recall that whether $T = Az + B$ or $T = A\ln p + B$, the point $\begin{bmatrix} x \\ 0 \\ z^x_{850} \end{bmatrix}$ on the 850 mb intializing plane, carries the same value for pressure. And so, the fact that $\frac{\partial p^x}{\partial z}$ is different in the two cases, leads us to suspect that (3.2.24) is not satisfied when $T = Az + B$. Indeed, we can show this in a few lines. We have from (4.6.7), for upper and lower levels

$$\ln p^x_U - \ln p^x_L = \frac{g}{R\gamma}\left[\ln T^x_U - \ln T^x_L\right],$$

and so

$$-R\left[\frac{T^x_U + T^x_L}{2}\right] * \left[\ln p^x_U - \ln p^x_L\right] = -R\left[\frac{T^x_U + T^x_L}{2}\right] * \frac{g}{R\gamma}\left[\ln T^x_U - \ln T^x_L\right], \tag{A.1.18}$$

but from (A.1.12) we see that the right hand side is just

$$-\frac{g}{\gamma}\left[\frac{(k^x - \gamma z^x_U) + (k^x - \gamma z^x_L)}{2}\right] * \left[\ln(k^x - \gamma z^x_U) - \ln(k^x - \gamma z^x_L)\right], \tag{A.1.19}$$

which is certainly not $g\,[z^x_U - z^x_L]$, as it would have to be, if one hoped for it to experience no vertical truncation. (Please compare the first equation of this section (Eq (3.2.24)) for the T-structure $T = A\ln p + B$, with (A.1.18) and (A.1.19) which follow when $T = Az + B$.)

A.2 Case III Finite difference scheme limit for $T = Az + B$

We can rewrite (4.6.7) for Case III with $T = Az + B$ as

$$\left[\frac{p_s^x}{p_{850}^x}\right] = \left[\frac{T_s^x}{T_{850}^x}\right]^{\frac{g}{R\gamma}}$$

since x is fixed for this equation, which was derived from vertical integration. Now using (4.6.10) and (4.4.4) we have for $Z(x) = z_s^x$

$$\left[\frac{T_s^x}{T_{850}^x}\right] = \left[\frac{\left[T_{850}^{x(R)} + k_{850}\ x\right] + \gamma\left[z_{850}^{x(R)} + m_{850}\ x - Z(x)\right]}{T_{850}^{x(R)} + k_{850}\ x}\right]. \tag{A.2.1}$$

Hence, recombining terms and recalling (4.6.7) leaves us with

$$\left[\frac{p_s^x}{p_{850}^x}\right] = \left[\frac{\left[T_{850}^{x(R)} + \gamma\ z_{850}^{x(R)}\right] + \left[k_{850} + \gamma\ m_{850}\right]\ x - \gamma\ Z(x)}{T_{850}^{x(R)} + k_{850}\ x}\right]^{\frac{g}{R\gamma}} \tag{A.2.2}$$

which by taking logarithms we can write as

$$\ln\left[p_s^x\right] = \ln\left[p_{850}^x\right] + \frac{g}{R\gamma}\ \ln\left[\frac{T_0 + T_g\ x - \gamma\ Z(x)}{T_{850}^{x(R)} + k_{850}\ x}\right] \tag{A.2.3}$$

where $T_0 = T_{850}^{x(R)} + \gamma\ z_{850}^{x(R)}$ and $T_g = k_{850} + \gamma\ m_{850}$. Subtracting (A.2.3) with argument x from (A.2.3) with argument $x + \Delta x$ leaves us with

$$\ln\left[p_s^{x+\Delta x}\right] - \ln\left[p_s^x\right] = \tag{A.2.4}$$

$$\frac{g}{R\gamma}\left[\ln\left[\frac{T_0 + T_g\ [x + \Delta x] - \gamma\ Z(x + \Delta x)}{T_{850}^{x(R)} + k_{850}\ [x + \Delta x]}\right] - \ln\left[\frac{T_0 + T_g\ x - \gamma\ Z(x)}{T_{850}^{x(R)} + k_{850}\ x}\right]\right],$$

which we can rewrite as $w_1(x) - w_2(x)$ where

$$w_1(x) = \frac{g}{R\gamma}\left[\ln\left[T_0 + T_g\ [x + \Delta x] - \gamma Z(x + \Delta x)\right] - \ln\left[T_0 + T_g\ x - \gamma\ Z(x)\right]\right] \tag{A.2.5}$$

and

$$w_2(x) = \frac{g}{R\gamma}\ \left[\ln\left[T_{850}^{x(R)} + k_{850}\ [x + \Delta x]\right] - \ln\left[T_{850}^{x(R)} + k_{850}\ x\right]\right]. \tag{A.2.6}$$

We now have

$$\lim_{\Delta x \to 0} \frac{\ln p_s^{x+\Delta x} - \ln p_s^x}{\Delta x} = \lim_{\Delta x \to 0} \frac{w_1(x)}{\Delta x} - \lim_{\Delta x \to 0} \frac{w_2(x)}{\Delta x} \tag{A.2.7}$$

where

$$\lim_{\Delta x \to 0} \frac{w_1(x)}{\Delta x} = \frac{g}{R\gamma}\left[\frac{T_g - \gamma\ Z^{\prime}(x)}{T_0 + T_1\ x - \gamma\ \ Z(x)}\right] \tag{A.2.8}$$

and

$$\lim_{\Delta x \to 0} \frac{w_2(x)}{\Delta x} = \frac{g}{R\gamma}\left[\frac{k_{850}}{T_{850}^{x(R)} + k_{850} * x}\right]. \tag{A.2.9}$$

We also note that

$$\lim_{\Delta x \to 0}\left[\frac{T_s^{x+\Delta x} + T_s^x}{2}\right] = T_s^x = T_0 + T_g\ x - \gamma\ Z(x). \tag{A.2.10}$$

Now, recalling the Corby finite difference scheme presented in (4.7.1) through (4.7.3), we continue by evaluating the limit of (4.7.2), written as

$$\lim_{\Delta x \to 0}\left[g\left[\frac{Z(x+\Delta x) + Z(x)}{2}\right] + R\left[\frac{T_s^{x+\Delta x} + T_s^x}{2}\right] * \left[\frac{\ln p_s^{x+\Delta x} - \ln p_s^x}{\Delta x}\right]\right], \tag{A.2.11}$$

which is

$$g\ Z'(x) + \frac{g}{\gamma}\left[\frac{T_g - \gamma\ Z'(x)}{1}\right] - \frac{g}{\gamma}k_{850}\left[\frac{T_0 + T_g\ x - \gamma\ Z(x)}{T_{850}^{x(R)} + k_{850}\ x}\right], \tag{A.2.12}$$

or

$$\frac{g}{\gamma}\left[k_{850} + \gamma\ m_{850}\right] - \frac{g}{\gamma}k_{850}\left[\frac{T_0 + T_g x - \gamma\ Z(x)}{T_{850}^{x(R)} + k_{850}\ x}\right] \tag{A.2.13}$$

We can rewrite this as follows

$$\frac{g}{\gamma}k_{850} + g m_{850} - \frac{g}{\gamma}k_{850} * \left[\frac{T_0 + T_g\ x - \gamma\ Z(x)}{T_{850}^{x(R)} + k_{850}\ x}\right]. \tag{A.2.14}$$

But if we note that

$$\left[\frac{T_0 + T_g\ x - \gamma\ Z(x)}{T_{850}^{x(R)} + k_{850}\ x}\right] = \left[1 + \frac{\gamma\ \left[z_{850}^{x(R)} + m_{850}\ x - Z(x)\right]}{\left[T_{850}^{x(R)} + k_{850}\ x\right]}\right], \tag{A.2.15}$$

then we can write (A.5.14) as

$$g m_{850} - \frac{g}{\gamma}k_{850}\ \left[\frac{\gamma\ \left[z_{850}^{x(R)} + m_{850}\ x - Z(x)\right]}{T_{850}^{x(R)} + k_{850}\ x}\right]. \tag{A.2.16}$$

We finally obtain as the limit of (A.2.11) the right hand side of (7.7.28)

$$g m_{850} + k_{850}\ \frac{g}{\gamma}\left[\frac{T_{850}^x - T_s^x}{T_{850}^x}\right].$$

It can be shown in an analogous manner that the limit of (4.7.2) is also (7.7.28), and so (4.7.1) has this limit as well.

Appendix B

Tables of Numerical Results

The tables on the following pages are the results of calculations performed by the CASE programs in Appendix E to compute surface geostrophic wind (components) under two different temperature profiles $T = A \ln p + B$ and $T = Az + B$ in each of three different cases: (I) when isobaric planes are horizontal, and so, no (geostrophic) wind is present; (II) when isobaric planes carry no temperature variation, and are parallel to each other, but inclined with respect to the horizontal (geostrophic wind present); and (III) when isobaric planes are such that $\nabla_p T = \begin{bmatrix} C_o \\ 0 \\ 0 \end{bmatrix}$ for C_o fixed (and so are not parallel one to the other), but are inclined with respect to the horizontal (geostrophic wind present). The results in Appendix B are for a hill with apex 5000m at latitude 48 ^{o}, and computations have been performed in double precision.

CASE I.A - Horizontal isobaric planes with $T = A \ln p + B$

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.1 Analytic u component of surface geostrophic wind (10^{-11}m/sec).

-0.0000	-0.0000	-0.0000	0.0266	-0.1653	-0.1479	-0.1151	-0.1479	-0.1653	0.0266	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0505	-0.1595	-0.0294	0.0303	0.0206	-0.1676	0.0206	0.0303	-0.0294	-0.1595	-0.0505	-0.0000	-0.0000
-0.0505	-0.1368	-0.2114	-0.0653	0.1112	0.0973	0.0701	0.0973	0.1112	-0.0653	-0.2114	-0.1368	-0.0505	-0.0000
-0.1485	-0.0520	-0.0984	-0.0120	-0.2134	-0.0263	0.0066	-0.0263	-0.2134	-0.0120	-0.0984	-0.0520	-0.1485	-0.0000
0.0248	0.1437	0.1101	0.0260	0.1121	0.0922	0.1129	0.0922	0.1121	0.0260	0.1101	0.1437	0.0248	0.0266
0.1835	0.1855	0.1375	0.0930	0.2652	-0.2607	-0.0804	-0.2607	0.2652	0.0930	0.1375	0.1855	0.1835	-0.1771
-0.0081	-0.0246	0.0266	0.1143	-0.3743	-0.1878	0.3678	-0.1878	-0.3743	0.1143	0.0266	-0.0246	-0.0081	-0.1917
-0.1163	-0.0635	-0.0451	-0.1888	-0.1820	0.5570	-0.1567	0.5570	-0.1820	-0.1888	-0.0451	-0.0635	-0.1163	0.0271
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.1163	0.0635	0.0451	0.1888	0.1820	-0.5570	0.1567	-0.5570	0.1820	0.1888	0.0451	0.0635	0.1163	-0.0271
0.0081	0.0246	-0.0266	-0.1143	0.3743	0.1878	-0.3678	0.1878	0.3743	-0.1143	-0.0266	0.0246	0.0081	0.1917
-0.1835	-0.1855	-0.1375	-0.0930	-0.2652	0.2607	0.0804	0.2607	-0.2652	-0.0930	-0.1375	-0.1855	-0.1835	0.1771
-0.0248	-0.1437	-0.1101	-0.0260	-0.1121	-0.0922	-0.1129	-0.0922	-0.1121	-0.0260	-0.1101	-0.1437	-0.0248	-0.0266
0.1485	0.0520	0.0984	0.0120	0.2134	0.0263	-0.0066	0.0263	0.2134	0.0120	0.0984	0.0520	0.1485	-0.0000
0.0505	0.1368	0.2114	0.0653	-0.1112	-0.0973	-0.0701	-0.0973	-0.1112	0.0653	0.2114	0.1368	0.0505	-0.0000
-0.0000	0.0505	0.1595	0.0294	-0.0303	-0.0206	0.1676	-0.0206	-0.0303	0.0294	0.1595	0.0505	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0266	0.1653	0.1479	0.1151	0.1479	0.1653	-0.0266	-0.0000	-0.0000	-0.0000	-0.0000

Table B.2 Finite Difference u component of surface geostrophic wind (10^{-11}m/sec).

0.0000	0.0000	0.0000	-0.0266	0.1653	0.1479	0.1151	0.1479	0.1653	-0.0266	0.0000	0.0000	0.0000	0.0000
0.0000	0.0505	0.1595	0.0294	-0.0303	-0.0206	0.1676	-0.0206	-0.0303	0.0294	0.1595	0.0505	0.0000	0.0000
0.0505	0.1368	0.2114	0.0653	-0.1112	-0.0973	-0.0701	-0.0973	-0.1112	0.0653	0.2114	0.1368	0.0505	0.0000
0.1485	0.0520	0.0984	0.0120	0.2134	0.0263	-0.0066	0.0263	0.2134	0.0120	0.0984	0.0520	0.1485	0.0000
-0.0248	-0.1437	-0.1101	-0.0260	-0.1121	-0.0922	-0.1129	-0.0922	-0.1121	-0.0260	-0.1101	-0.1437	-0.0248	-0.0266
-0.1835	-0.1855	-0.1375	-0.0930	-0.2652	0.2607	0.0804	0.2607	-0.2652	-0.0930	-0.1375	-0.1855	-0.1835	0.1771
0.0081	0.0246	-0.0266	-0.1143	0.3743	0.1878	-0.3678	0.1878	0.3743	-0.1143	-0.0266	0.0246	0.0081	0.1917
0.1163	0.0635	0.0451	0.1888	0.1820	-0.5570	0.1567	-0.5570	0.1820	0.1888	0.0451	0.0635	0.1163	-0.0271
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.1163	-0.0635	-0.0451	-0.1888	-0.1820	0.5570	-0.1567	0.5570	-0.1820	-0.1888	-0.0451	-0.0635	-0.1163	0.0271
-0.0081	-0.0246	0.0266	0.1143	-0.3743	-0.1878	0.3678	-0.1878	-0.3743	0.1143	0.0266	-0.0246	-0.0081	-0.1917
0.1835	0.1855	0.1375	0.0930	0.2652	-0.2607	-0.0804	-0.2607	0.2652	0.0930	0.1375	0.1855	0.1835	-0.1771
0.0248	0.1437	0.1101	0.0260	0.1121	0.0922	0.1129	0.0922	0.1121	0.0260	0.1101	0.1437	0.0248	0.0266
-0.1485	-0.0520	-0.0984	-0.0120	-0.2134	-0.0263	0.0066	-0.0263	-0.2134	-0.0120	-0.0984	-0.0520	-0.1485	0.0000
-0.0505	-0.1368	-0.2114	-0.0653	0.1112	0.0973	0.0701	0.0973	0.1112	-0.0653	-0.2114	-0.1368	-0.0505	0.0000
0.0000	-0.0505	-0.1595	-0.0294	0.0303	0.0206	-0.1676	0.0206	0.0303	-0.0294	-0.1595	-0.0505	0.0000	0.0000
0.0000	0.0000	0.0000	0.0266	-0.1653	-0.1479	-0.1151	-0.1479	-0.1653	0.0266	0.0000	0.0000	0.0000	0.0000

Table B.3 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (10^{-11}m/sec).

CASE I.A - Horizontal isobaric planes with $T = A \ln p + B$

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.4 Analytic v component of surface geostrophic wind (10^{-11}m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0266	-0.1771	-0.1917	0.0271	0.0000	-0.0271	0.1917	0.1771	-0.0266	0.0000	0.0000	0.0000
-0.0505	-0.1485	0.0248	0.1835	-0.0081	-0.1163	0.0000	0.1163	0.0081	-0.1835	-0.0248	0.1485	0.0505	0.0000
-0.1368	-0.0520	0.1437	0.1855	-0.0246	-0.0635	0.0000	0.0635	0.0246	-0.1855	-0.1437	0.0520	0.1368	0.0505
-0.2114	-0.0984	0.1101	0.1375	0.0266	-0.0451	0.0000	0.0451	-0.0266	-0.1375	-0.1101	0.0984	0.2114	0.1595
-0.0653	-0.0120	0.0260	0.0930	0.1143	-0.1888	0.0000	0.1888	-0.1143	-0.0930	-0.0260	0.0120	0.0653	0.0294
0.1112	-0.2134	0.1121	0.2652	-0.3743	-0.1820	0.0000	0.1820	0.3743	-0.2652	-0.1121	0.2134	-0.1112	-0.0303
0.0973	-0.0263	0.0922	-0.2607	-0.1878	0.5570	0.0000	-0.5570	0.1878	0.2607	-0.0922	0.0263	-0.0973	-0.0206
0.0701	0.0066	0.1129	-0.0804	0.3678	-0.1567	0.0000	0.1567	-0.3678	0.0804	-0.1129	-0.0066	-0.0701	0.1676
0.0973	-0.0263	0.0922	-0.2607	-0.1878	0.5570	0.0000	-0.5570	0.1878	0.2607	-0.0922	0.0263	-0.0973	-0.0206
0.1112	-0.2134	0.1121	0.2652	-0.3743	-0.1820	0.0000	0.1820	0.3743	-0.2652	-0.1121	0.2134	-0.1112	-0.0303
-0.0653	-0.0120	0.0260	0.0930	0.1143	-0.1888	0.0000	0.1888	-0.1143	-0.0930	-0.0260	0.0120	0.0653	0.0294
-0.2114	-0.0984	0.1101	0.1375	0.0266	-0.0451	0.0000	0.0451	-0.0266	-0.1375	-0.1101	0.0984	0.2114	0.1595
-0.1368	-0.0520	0.1437	0.1855	-0.0246	-0.0635	0.0000	0.0635	0.0246	-0.1855	-0.1437	0.0520	0.1368	0.0505
-0.0505	-0.1485	0.0248	0.1835	-0.0081	-0.1163	0.0000	0.1163	0.0081	-0.1835	-0.0248	0.1485	0.0505	0.0000
0.0000	0.0000	0.0266	-0.1771	-0.1917	0.0271	0.0000	-0.0271	0.1917	0.1771	-0.0266	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.5 Finite Difference v component of surface geostrophic wind (10^{-11}m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0266	0.1771	0.1917	-0.0271	0.0000	0.0271	-0.1917	-0.1771	0.0266	0.0000	0.0000	0.0000
0.0505	0.1485	-0.0248	-0.1835	0.0081	0.1163	0.0000	-0.1163	-0.0081	0.1835	0.0248	-0.1485	-0.0505	0.0000
0.1368	0.0520	-0.1437	-0.1855	0.0246	0.0635	0.0000	-0.0635	-0.0246	0.1855	0.1437	-0.0520	-0.1368	-0.0505
0.2114	0.0984	-0.1101	-0.1375	-0.0266	0.0451	0.0000	-0.0451	0.0266	0.1375	0.1101	-0.0984	-0.2114	-0.1595
0.0653	0.0120	-0.0260	-0.0930	-0.1143	0.1888	0.0000	-0.1888	0.1143	0.0930	0.0260	-0.0120	-0.0653	-0.0294
-0.1112	0.2134	-0.1121	-0.2652	0.3743	0.1820	0.0000	-0.1820	-0.3743	0.2652	0.1121	-0.2134	0.1112	0.0303
-0.0973	0.0263	-0.0922	0.2607	0.1878	-0.5570	0.0000	0.5570	-0.1878	-0.2607	0.0922	-0.0263	0.0973	0.0206
-0.0701	-0.0066	-0.1129	0.0804	-0.3678	0.1567	0.0000	-0.1567	0.3678	-0.0804	0.1129	0.0066	0.0701	-0.1676
-0.0973	0.0263	-0.0922	0.2607	0.1878	-0.5570	0.0000	0.5570	-0.1878	-0.2607	0.0922	-0.0263	0.0973	0.0206
-0.1112	0.2134	-0.1121	-0.2652	0.3743	0.1820	0.0000	-0.1820	-0.3743	0.2652	0.1121	-0.2134	0.1112	0.0303
0.0653	0.0120	-0.0260	-0.0930	-0.1143	0.1888	0.0000	-0.1888	0.1143	0.0930	0.0260	-0.0120	-0.0653	-0.0294
0.2114	0.0984	-0.1101	-0.1375	-0.0266	0.0451	0.0000	-0.0451	0.0266	0.1375	0.1101	-0.0984	-0.2114	-0.1595
0.1368	0.0520	-0.1437	-0.1855	0.0246	0.0635	0.0000	-0.0635	-0.0246	0.1855	0.1437	-0.0520	-0.1368	-0.0505
0.0505	0.1485	-0.0248	-0.1835	0.0081	0.1163	0.0000	-0.1163	-0.0081	0.1835	0.0248	-0.1485	-0.0505	0.0000
0.0000	0.0000	-0.0266	0.1771	0.1917	-0.0271	0.0000	0.0271	-0.1917	-0.1771	0.0266	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.6 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (10^{-11}m/sec).

CASE I.A - Horizontal isobaric planes with $T = A \ln p + B$

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.7 Analytic surface geostrophic wind velocity (10^{-11} m/sec).

0.0000	0.0000	0.0000	0.0266	0.1653	0.1479	0.1151	0.1479	0.1653	0.0266	0.0000	0.0000	0.0000	0.0000
0.0000	0.0505	0.1617	0.1796	0.1941	0.0341	0.1676	0.0341	0.1941	0.1796	0.1617	0.0505	0.0000	0.0000
0.0715	0.2019	0.2129	0.1947	0.1115	0.1516	0.0701	0.1516	0.1115	0.1947	0.2129	0.2019	0.0715	0.0000
0.2019	0.0735	0.1742	0.1859	0.2148	0.0687	0.0066	0.0687	0.2148	0.1859	0.1742	0.0735	0.2019	0.0505
0.2129	0.1742	0.1558	0.1399	0.1152	0.1026	0.1129	0.1026	0.1152	0.1399	0.1558	0.1742	0.2129	0.1617
0.1947	0.1859	0.1399	0.1315	0.2887	0.3219	0.0804	0.3219	0.2887	0.1315	0.1399	0.1859	0.1947	0.1796
0.1115	0.2148	0.1152	0.2887	0.5293	0.2615	0.3678	0.2615	0.5293	0.2887	0.1152	0.2148	0.1115	0.1941
0.1516	0.0687	0.1026	0.3219	0.2615	0.7877	0.1567	0.7877	0.2615	0.3219	0.1026	0.0687	0.1516	0.0341
0.0701	0.0066	0.1129	0.0804	0.3678	0.1567	0.0000	0.1567	0.3678	0.0804	0.1129	0.0066	0.0701	0.1676
0.1516	0.0687	0.1026	0.3219	0.2615	0.7877	0.1567	0.7877	0.2615	0.3219	0.1026	0.0687	0.1516	0.0341
0.1115	0.2148	0.1152	0.2887	0.5293	0.2615	0.3678	0.2615	0.5293	0.2887	0.1152	0.2148	0.1115	0.1941
0.1947	0.1859	0.1399	0.1315	0.2887	0.3219	0.0804	0.3219	0.2887	0.1315	0.1399	0.1859	0.1947	0.1796
0.2129	0.1742	0.1558	0.1399	0.1152	0.1026	0.1129	0.1026	0.1152	0.1399	0.1558	0.1742	0.2129	0.1617
0.2019	0.0735	0.1742	0.1859	0.2148	0.0687	0.0066	0.0687	0.2148	0.1859	0.1742	0.0735	0.2019	0.0505
0.0715	0.2019	0.2129	0.1947	0.1115	0.1516	0.0701	0.1516	0.1115	0.1947	0.2129	0.2019	0.0715	0.0000
0.0000	0.0505	0.1617	0.1796	0.1941	0.0341	0.1676	0.0341	0.1941	0.1796	0.1617	0.0505	0.0000	0.0000
0.0000	0.0000	0.0000	0.0266	0.1653	0.1479	0.1151	0.1479	0.1653	0.0266	0.0000	0.0000	0.0000	0.0000

Table B.8 Finite Difference surface geostrophic wind velocity (10^{-11} m/sec).

0.0000	0.0000	0.0000	-0.0266	-0.1653	-0.1479	-0.1151	-0.1479	-0.1653	-0.0266	0.0000	0.0000	0.0000	0.0000
0.0000	-0.0505	-0.1617	-0.1796	-0.1941	-0.0341	-0.1676	-0.0341	-0.1941	-0.1796	-0.1617	-0.0505	0.0000	0.0000
-0.0715	-0.2019	-0.2129	-0.1947	-0.1115	-0.1516	-0.0701	-0.1516	-0.1115	-0.1947	-0.2129	-0.2019	-0.0715	0.0000
-0.2019	-0.0735	-0.1742	-0.1859	-0.2148	-0.0687	-0.0066	-0.0687	-0.2148	-0.1859	-0.1742	-0.0735	-0.2019	-0.0505
-0.2129	-0.1742	-0.1558	-0.1399	-0.1152	-0.1026	-0.1129	-0.1026	-0.1152	-0.1399	-0.1558	-0.1742	-0.2129	-0.1617
-0.1947	-0.1859	-0.1399	-0.1315	-0.2887	-0.3219	-0.0804	-0.3219	-0.2887	-0.1315	-0.1399	-0.1859	-0.1947	-0.1796
-0.1115	-0.2148	-0.1152	-0.2887	-0.5293	-0.2615	-0.3678	-0.2615	-0.5293	-0.2887	-0.1152	-0.2148	-0.1115	-0.1941
-0.1516	-0.0687	-0.1026	-0.3219	-0.2615	-0.7877	-0.1567	-0.7877	-0.2615	-0.3219	-0.1026	-0.0687	-0.1516	-0.0341
-0.0701	-0.0066	-0.1129	-0.0804	-0.3678	-0.1567	0.0000	-0.1567	-0.3678	-0.0804	-0.1129	-0.0066	-0.0701	-0.1676
-0.1516	-0.0687	-0.1026	-0.3219	-0.2615	-0.7877	-0.1567	-0.7877	-0.2615	-0.3219	-0.1026	-0.0687	-0.1516	-0.0341
-0.1115	-0.2148	-0.1152	-0.2887	-0.5293	-0.2615	-0.3678	-0.2615	-0.5293	-0.2887	-0.1152	-0.2148	-0.1115	-0.1941
-0.1947	-0.1859	-0.1399	-0.1315	-0.2887	-0.3219	-0.0804	-0.3219	-0.2887	-0.1315	-0.1399	-0.1859	-0.1947	-0.1796
-0.2129	-0.1742	-0.1558	-0.1399	-0.1152	-0.1026	-0.1129	-0.1026	-0.1152	-0.1399	-0.1558	-0.1742	-0.2129	-0.1617
-0.2019	-0.0735	-0.1742	-0.1859	-0.2148	-0.0687	-0.0066	-0.0687	-0.2148	-0.1859	-0.1742	-0.0735	-0.2019	-0.0505
-0.0715	-0.2019	-0.2129	-0.1947	-0.1115	-0.1516	-0.0701	-0.1516	-0.1115	-0.1947	-0.2129	-0.2019	-0.0715	0.0000
0.0000	-0.0505	-0.1617	-0.1796	-0.1941	-0.0341	-0.1676	-0.0341	-0.1941	-0.1796	-0.1617	-0.0505	0.0000	0.0000
0.0000	0.0000	0.0000	-0.0266	-0.1653	-0.1479	-0.1151	-0.1479	-0.1653	-0.0266	0.0000	0.0000	0.0000	0.0000

Table B.9 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (10^{-11} m/sec).

CASE I.B - Horizontal isobaric planes with $T = Az + B$

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.10 Analytic u component of surface geostrophic wind (m/sec).

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0003	-0.0014	-0.0021	-0.0014	-0.0003	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0005	-0.0074	-0.0241	-0.0436	-0.0525	-0.0436	-0.0241	-0.0074	-0.0005	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0012	-0.0148	-0.0570	-0.1268	-0.1952	-0.2239	-0.1952	-0.1268	-0.0570	-0.0148	-0.0012	-0.0000	-0.0000
-0.0000	-0.0004	-0.0107	-0.0567	-0.1567	-0.2948	-0.4187	-0.4688	-0.4187	-0.2948	-0.1567	-0.0567	-0.0107	-0.0004	-0.0000
-0.0000	-0.0028	-0.0267	-0.0991	-0.2317	-0.4001	-0.5450	-0.6026	-0.5450	-0.4001	-0.2317	-0.0991	-0.0267	-0.0028	-0.0000
-0.0001	-0.0052	-0.0314	-0.0974	-0.2083	-0.3433	-0.4570	-0.5017	-0.4570	-0.3433	-0.2083	-0.0974	-0.0314	-0.0052	-0.0001
-0.0002	-0.0040	-0.0192	-0.0541	-0.1102	-0.1768	-0.2324	-0.2541	-0.2324	-0.1768	-0.1102	-0.0541	-0.0192	-0.0040	-0.0002
-0.0001	-0.0012	-0.0052	-0.0140	-0.0277	-0.0438	-0.0571	-0.0623	-0.0571	-0.0438	-0.0277	-0.0140	-0.0052	-0.0012	-0.0001
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.0001	0.0012	0.0052	0.0140	0.0277	0.0438	0.0571	0.0623	0.0571	0.0438	0.0277	0.0140	0.0052	0.0012	0.0001
0.0002	0.0040	0.0192	0.0541	0.1102	0.1768	0.2324	0.2541	0.2324	0.1768	0.1102	0.0541	0.0192	0.0040	0.0002
0.0001	0.0052	0.0314	0.0974	0.2083	0.3433	0.4570	0.5017	0.4570	0.3433	0.2083	0.0974	0.0314	0.0052	0.0001
0.0000	0.0028	0.0267	0.0991	0.2317	0.4001	0.5450	0.6026	0.5450	0.4001	0.2317	0.0991	0.0267	0.0028	0.0000
-0.0000	0.0004	0.0107	0.0567	0.1567	0.2948	0.4187	0.4688	0.4187	0.2948	0.1567	0.0567	0.0107	0.0004	-0.0000
-0.0000	0.0000	0.0012	0.0148	0.0570	0.1268	0.1952	0.2239	0.1952	0.1268	0.0570	0.0148	0.0012	0.0000	-0.0000
-0.0000	-0.0000	0.0000	0.0005	0.0074	0.0241	0.0436	0.0525	0.0436	0.0241	0.0074	0.0005	0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0003	0.0014	0.0021	0.0014	0.0003	0.0000	-0.0000	-0.0000	-0.0000	-0.0000

Table B.11 Finite Difference u component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0014	0.0021	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0005	0.0074	0.0241	0.0436	0.0525	0.0436	0.0241	0.0074	0.0005	0.0000	0.0000	0.0000
0.0000	0.0000	0.0012	0.0148	0.0570	0.1268	0.1952	0.2239	0.1952	0.1268	0.0570	0.0148	0.0012	0.0000	0.0000
0.0000	0.0004	0.0107	0.0567	0.1567	0.2948	0.4187	0.4688	0.4187	0.2948	0.1567	0.0567	0.0107	0.0004	0.0000
0.0000	0.0028	0.0267	0.0991	0.2317	0.4001	0.5450	0.6026	0.5450	0.4001	0.2317	0.0991	0.0267	0.0028	0.0000
0.0001	0.0052	0.0314	0.0974	0.2083	0.3433	0.4570	0.5017	0.4570	0.3433	0.2083	0.0974	0.0314	0.0052	0.0001
0.0002	0.0040	0.0192	0.0541	0.1102	0.1768	0.2324	0.2541	0.2324	0.1768	0.1102	0.0541	0.0192	0.0040	0.0002
0.0001	0.0012	0.0052	0.0140	0.0277	0.0438	0.0571	0.0623	0.0571	0.0438	0.0277	0.0140	0.0052	0.0012	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0001	-0.0012	-0.0052	-0.0140	-0.0277	-0.0438	-0.0571	-0.0623	-0.0571	-0.0438	-0.0277	-0.0140	-0.0052	-0.0012	-0.0001
-0.0002	-0.0040	-0.0192	-0.0541	-0.1102	-0.1768	-0.2324	-0.2541	-0.2324	-0.1768	-0.1102	-0.0541	-0.0192	-0.0040	-0.0002
-0.0001	-0.0052	-0.0314	-0.0974	-0.2083	-0.3433	-0.4570	-0.5017	-0.4570	-0.3433	-0.2083	-0.0974	-0.0314	-0.0052	-0.0001
-0.0000	-0.0028	-0.0267	-0.0991	-0.2317	-0.4001	-0.5450	-0.6026	-0.5450	-0.4001	-0.2317	-0.0991	-0.0267	-0.0028	-0.0000
0.0000	-0.0004	-0.0107	-0.0567	-0.1567	-0.2948	-0.4187	-0.4688	-0.4187	-0.2948	-0.1567	-0.0567	-0.0107	-0.0004	0.0000
0.0000	-0.0000	-0.0012	-0.0148	-0.0570	-0.1268	-0.1952	-0.2239	-0.1952	-0.1268	-0.0570	-0.0148	-0.0012	-0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0005	-0.0074	-0.0241	-0.0436	-0.0525	-0.0436	-0.0241	-0.0074	-0.0005	-0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0003	-0.0014	-0.0021	-0.0014	-0.0003	-0.0000	0.0000	0.0000	0.0000	0.0000

Table B.12 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (m/sec).

CASE I.B - Horizontal isobaric planes with $T = Az + B$

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.13 Analytic v component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0001	-0.0002	-0.0001	0.0000	0.0001	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
-0.0000	-0.0004	-0.0028	-0.0052	-0.0040	-0.0012	0.0000	0.0012	0.0040	0.0052	0.0028	0.0004	0.0000	0.0000
-0.0012	-0.0107	-0.0267	-0.0314	-0.0192	-0.0052	0.0000	0.0052	0.0192	0.0314	0.0267	0.0107	0.0012	0.0000
-0.0148	-0.0567	-0.0991	-0.0974	-0.0541	-0.0140	0.0000	0.0140	0.0541	0.0974	0.0991	0.0567	0.0148	0.0005
-0.0570	-0.1567	-0.2317	-0.2083	-0.1102	-0.0277	0.0000	0.0277	0.1102	0.2083	0.2317	0.1567	0.0570	0.0074
-0.1268	-0.2948	-0.4001	-0.3433	-0.1768	-0.0438	0.0000	0.0438	0.1768	0.3433	0.4001	0.2948	0.1268	0.0241
-0.1952	-0.4187	-0.5450	-0.4570	-0.2324	-0.0571	0.0000	0.0571	0.2324	0.4570	0.5450	0.4187	0.1952	0.0436
-0.2239	-0.4688	-0.6026	-0.5017	-0.2541	-0.0623	0.0000	0.0623	0.2541	0.5017	0.6026	0.4688	0.2239	0.0525
-0.1952	-0.4187	-0.5450	-0.4570	-0.2324	-0.0571	0.0000	0.0571	0.2324	0.4570	0.5450	0.4187	0.1952	0.0436
-0.1268	-0.2948	-0.4001	-0.3433	-0.1768	-0.0438	0.0000	0.0438	0.1768	0.3433	0.4001	0.2948	0.1268	0.0241
-0.0570	-0.1567	-0.2317	-0.2083	-0.1102	-0.0277	0.0000	0.0277	0.1102	0.2083	0.2317	0.1567	0.0570	0.0074
-0.0148	-0.0567	-0.0991	-0.0974	-0.0541	-0.0140	0.0000	0.0140	0.0541	0.0974	0.0991	0.0567	0.0148	0.0005
-0.0012	-0.0107	-0.0267	-0.0314	-0.0192	-0.0052	0.0000	0.0052	0.0192	0.0314	0.0267	0.0107	0.0012	0.0000
-0.0000	-0.0004	-0.0028	-0.0052	-0.0040	-0.0012	0.0000	0.0012	0.0040	0.0052	0.0028	0.0004	0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0001	-0.0002	-0.0001	0.0000	0.0001	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.14 Finite Difference v component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0001	0.0002	0.0001	0.0000	-0.0001	-0.0002	-0.0001	-0.0000	0.0000	0.0000	0.0000
0.0000	0.0004	0.0028	0.0052	0.0040	0.0012	0.0000	-0.0012	-0.0040	-0.0052	-0.0028	-0.0004	-0.0000	0.0000
0.0012	0.0107	0.0267	0.0314	0.0192	0.0052	0.0000	-0.0052	-0.0192	-0.0314	-0.0267	-0.0107	-0.0012	-0.0000
0.0148	0.0567	0.0991	0.0974	0.0541	0.0140	0.0000	-0.0140	-0.0541	-0.0974	-0.0991	-0.0567	-0.0148	-0.0005
0.0570	0.1567	0.2317	0.2083	0.1102	0.0277	0.0000	-0.0277	-0.1102	-0.2083	-0.2317	-0.1567	-0.0570	-0.0074
0.1268	0.2948	0.4001	0.3433	0.1768	0.0438	0.0000	-0.0438	-0.1768	-0.3433	-0.4001	-0.2948	-0.1268	-0.0241
0.1952	0.4187	0.5450	0.4570	0.2324	0.0571	0.0000	-0.0571	-0.2324	-0.4570	-0.5450	-0.4187	-0.1952	-0.0436
0.2239	0.4688	0.6026	0.5017	0.2541	0.0623	0.0000	-0.0623	-0.2541	-0.5017	-0.6026	-0.4688	-0.2239	-0.0525
0.1952	0.4187	0.5450	0.4570	0.2324	0.0571	0.0000	-0.0571	-0.2324	-0.4570	-0.5450	-0.4187	-0.1952	-0.0436
0.1268	0.2948	0.4001	0.3433	0.1768	0.0438	0.0000	-0.0438	-0.1768	-0.3433	-0.4001	-0.2948	-0.1268	-0.0241
0.0570	0.1567	0.2317	0.2083	0.1102	0.0277	0.0000	-0.0277	-0.1102	-0.2083	-0.2317	-0.1567	-0.0570	-0.0074
0.0148	0.0567	0.0991	0.0974	0.0541	0.0140	0.0000	-0.0140	-0.0541	-0.0974	-0.0991	-0.0567	-0.0148	-0.0005
0.0012	0.0107	0.0267	0.0314	0.0192	0.0052	0.0000	-0.0052	-0.0192	-0.0314	-0.0267	-0.0107	-0.0012	-0.0000
0.0000	0.0004	0.0028	0.0052	0.0040	0.0012	0.0000	-0.0012	-0.0040	-0.0052	-0.0028	-0.0004	-0.0000	0.0000
0.0000	0.0000	0.0000	0.0001	0.0002	0.0001	0.0000	-0.0001	-0.0002	-0.0001	-0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.15 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (m/sec).

CASE I.B - Horizontal isobaric planes with $T = Az + B$

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.16 Analytic surface geostrophic wind velocity (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0014	0.0021	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0005	0.0074	0.0241	0.0436	0.0525	0.0436	0.0241	0.0074	0.0005	0.0000	0.0000	0.0000
0.0000	0.0000	0.0013	0.0151	0.0573	0.1269	0.1952	0.2239	0.1952	0.1269	0.0573	0.0151	0.0013	0.0000	0.0000
0.0000	0.0013	0.0151	0.0627	0.1599	0.2954	0.4188	0.4688	0.4188	0.2954	0.1599	0.0627	0.0151	0.0013	0.0000
0.0005	0.0151	0.0627	0.1402	0.2513	0.4037	0.5452	0.6026	0.5452	0.4037	0.2513	0.1402	0.0627	0.0151	0.0005
0.0074	0.0573	0.1599	0.2513	0.2946	0.3605	0.4578	0.5017	0.4578	0.3605	0.2946	0.2513	0.1599	0.0573	0.0074
0.0241	0.1269	0.2954	0.4037	0.3605	0.2501	0.2364	0.2541	0.2364	0.2501	0.3605	0.4037	0.2954	0.1269	0.0241
0.0436	0.1952	0.4188	0.5452	0.4578	0.2364	0.0808	0.0623	0.0808	0.2364	0.4578	0.5452	0.4188	0.1952	0.0436
0.0525	0.2239	0.4688	0.6026	0.5017	0.2541	0.0623	0.0000	0.0623	0.2541	0.5017	0.6026	0.4688	0.2239	0.0525
0.0436	0.1952	0.4188	0.5452	0.4578	0.2364	0.0808	0.0623	0.0808	0.2364	0.4578	0.5452	0.4188	0.1952	0.0436
0.0241	0.1269	0.2954	0.4037	0.3605	0.2501	0.2364	0.2541	0.2364	0.2501	0.3605	0.4037	0.2954	0.1269	0.0241
0.0074	0.0573	0.1599	0.2513	0.2946	0.3605	0.4578	0.5017	0.4578	0.3605	0.2946	0.2513	0.1599	0.0573	0.0074
0.0005	0.0151	0.0627	0.1402	0.2513	0.4037	0.5452	0.6026	0.5452	0.4037	0.2513	0.1402	0.0627	0.0151	0.0005
0.0000	0.0013	0.0151	0.0627	0.1599	0.2954	0.4188	0.4688	0.4188	0.2954	0.1599	0.0627	0.0151	0.0013	0.0000
0.0000	0.0000	0.0013	0.0151	0.0573	0.1269	0.1952	0.2239	0.1952	0.1269	0.0573	0.0151	0.0013	0.0000	0.0000
0.0000	0.0000	0.0000	0.0005	0.0074	0.0241	0.0436	0.0525	0.0436	0.0241	0.0074	0.0005	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0014	0.0021	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.17 Finite Difference surface geostrophic wind velocity (m/sec).

0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0003	-0.0014	-0.0021	-0.0014	-0.0003	-0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0005	-0.0074	-0.0241	-0.0436	-0.0525	-0.0436	-0.0241	-0.0074	-0.0005	-0.0000	0.0000	0.0000
0.0000	-0.0000	-0.0013	-0.0151	-0.0573	-0.1269	-0.1952	-0.2239	-0.1952	-0.1269	-0.0573	-0.0151	-0.0013	-0.0000	0.0000
-0.0000	-0.0013	-0.0151	-0.0627	-0.1599	-0.2954	-0.4188	-0.4688	-0.4188	-0.2954	-0.1599	-0.0627	-0.0151	-0.0013	-0.0000
-0.0005	-0.0151	-0.0627	-0.1402	-0.2513	-0.4037	-0.5452	-0.6026	-0.5452	-0.4037	-0.2513	-0.1402	-0.0627	-0.0151	-0.0005
-0.0074	-0.0573	-0.1599	-0.2513	-0.2946	-0.3605	-0.4578	-0.5017	-0.4578	-0.3605	-0.2946	-0.2513	-0.1599	-0.0573	-0.0074
-0.0241	-0.1269	-0.2954	-0.4037	-0.3605	-0.2501	-0.2364	-0.2541	-0.2364	-0.2501	-0.3605	-0.4037	-0.2954	-0.1269	-0.0241
-0.0436	-0.1952	-0.4188	-0.5452	-0.4578	-0.2364	-0.0808	-0.0623	-0.0808	-0.2364	-0.4578	-0.5452	-0.4188	-0.1952	-0.0436
-0.0525	-0.2239	-0.4688	-0.6026	-0.5017	-0.2541	-0.0623	0.0000	-0.0623	-0.2541	-0.5017	-0.6026	-0.4688	-0.2239	-0.0525
-0.0436	-0.1952	-0.4188	-0.5452	-0.4578	-0.2364	-0.0808	-0.0623	-0.0808	-0.2364	-0.4578	-0.5452	-0.4188	-0.1952	-0.0436
-0.0241	-0.1269	-0.2954	-0.4037	-0.3605	-0.2501	-0.2364	-0.2541	-0.2364	-0.2501	-0.3605	-0.4037	-0.2954	-0.1269	-0.0241
-0.0074	-0.0573	-0.1599	-0.2513	-0.2946	-0.3605	-0.4578	-0.5017	-0.4578	-0.3605	-0.2946	-0.2513	-0.1599	-0.0573	-0.0074
-0.0005	-0.0151	-0.0627	-0.1402	-0.2513	-0.4037	-0.5452	-0.6026	-0.5452	-0.4037	-0.2513	-0.1402	-0.0627	-0.0151	-0.0005
-0.0000	-0.0013	-0.0151	-0.0627	-0.1599	-0.2954	-0.4188	-0.4688	-0.4188	-0.2954	-0.1599	-0.0627	-0.0151	-0.0013	-0.0000
0.0000	-0.0000	-0.0013	-0.0151	-0.0573	-0.1269	-0.1952	-0.2239	-0.1952	-0.1269	-0.0573	-0.0151	-0.0013	-0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0005	-0.0074	-0.0241	-0.0436	-0.0525	-0.0436	-0.0241	-0.0074	-0.0005	-0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0003	-0.0014	-0.0021	-0.0014	-0.0003	-0.0000	0.0000	0.0000	0.0000	0.0000

Table B.18 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (m/sec).

CASE II.A - Barotropic atmosphere - tilted (parallel) isobaric planes with $\nabla_p(T) = \vec{0}$, under $T = A \ln p + B$

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.19 Analytic u component of surface geostrophic wind (m/sec).

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.387[illegible]	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.20 Finite Difference u component of surface geostrophic wind (m/sec).

0.3478	0.3035	0.1577	-0.0947	0.2702	0.2075	0.1215	0.1774	0.0956	0.3119	-0.1707	0.0516	-0.2505	0.2026	-0.1925
0.1901	-0.0177	-0.3745	0.1771	-0.1976	-0.0391	-0.1828	0.0528	-0.2063	0.1144	-0.3038	-0.0016	-0.4247	0.0078	-0.3550
-0.0654	-0.1949	0.2454	0.0021	0.1156	-0.2025	0.0945	-0.1614	0.1889	-0.2970	0.1156	-0.2339	0.2291	-0.2338	0.2807
-0.1909	-0.0350	0.1023	-0.0401	0.1095	0.0276	0.1238	-0.2867	0.1238	-0.0668	0.2038	-0.0401	0.1967	-0.5100	0.3442
-0.0719	0.0491	-0.2822	0.1762	0.0716	0.1742	-0.2780	-0.1098	0.0997	0.2686	-0.0229	0.1762	-0.2350	0.2144	-0.4536
0.2333	0.1028	0.1481	0.0544	0.2878	0.4933	-0.2082	0.1780	0.1694	0.3044	0.0990	0.0544	0.1481	0.2680	0.0150
0.0414	-0.1305	0.1220	-0.1651	0.3608	-0.1891	0.0965	-0.5536	-0.3755	-0.0946	0.1721	-0.0707	0.2165	-0.2248	0.0414
-0.0771	0.0485	-0.1222	0.1662	-0.1622	-0.3106	-0.0819	-0.0290	-0.6247	0.0906	-0.2566	0.0482	-0.1104	0.0840	-0.0889
-0.2063	0.1447	-0.1621	0.1919	-0.4925	0.3807	-0.0677	0.2863	-0.1385	0.0975	-0.2093	0.0503	-0.1739	0.0975	0.0326
0.0155	-0.2076	0.1048	-0.1600	0.0975	0.0335	0.3005	-0.2481	0.4893	-0.3677	0.2391	-0.0420	0.0812	-0.1486	0.1128
-0.0352	0.2547	0.0730	0.0769	0.2590	-0.3712	0.0985	0.2765	0.1928	0.1008	0.0230	0.0297	0.0258	0.0186	0.1182
-0.1799	0.2574	-0.0946	0.1406	-0.1871	-0.2038	0.1201	0.3002	-0.1633	0.1738	-0.1871	-0.0481	-0.0003	0.1630	-0.1268
0.1248	-0.2082	0.0524	-0.1700	0.0291	-0.1680	0.2842	0.1160	0.2842	-0.3569	0.1236	-0.1700	-0.0420	-0.2553	0.1731
0.3163	-0.1690	-0.0253	-0.1426	0.0855	-0.3991	0.1656	-0.1791	0.2601	-0.2103	0.0855	-0.2369	-0.2141	-0.1439	-0.0274
-0.0178	0.1494	0.0846	0.2873	-0.0622	0.0671	0.0061	0.1676	-0.4660	-0.1689	-0.1565	-0.0667	0.1621	0.1239	-0.3611
-0.4575	-0.1135	0.0243	-0.0087	-0.3508	0.2576	-0.3067	0.0243	-0.3067	-0.1436	0.0858	0.1917	0.1875	-0.0925	0.3240
-0.0900	-0.1561	-0.1356	-0.4902	0.1448	-0.0595	-0.0232	0.0618	0.4313	-0.1157	0.2007	0.1311	0.0500	-0.0011	0.2009

Table B.21 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (10^{-11} m/sec).

CASE II.A - Barotropic atmosphere - tilted (parallel) isobaric planes with $\nabla_p(T) = \vec{0}$, under $T = A \ln p + B$

-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963

Table B.22 Analytic v component of surface geostrophic wind (m/sec).

-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963

Table B.23 Finite Difference v component of surface geostrophic wind (m/sec).

0.1808	0.0773	-0.3100	0.1502	0.0777	-0.0818	-0.1104	-0.0188	0.1825	-0.2020	-0.0412	0.1974	0.0470	-0.0466
0.0707	0.2416	-0.0938	-0.0918	-0.0477	-0.0452	0.1677	0.1860	-0.1242	-0.1097	0.0273	-0.1426	0.1532	0.0442
0.1325	-0.0533	0.0309	-0.0570	-0.1251	0.0185	-0.0742	0.0574	-0.1176	0.1920	0.2220	-0.1651	0.0318	-0.1006
-0.2287	-0.0812	0.0535	-0.0826	0.1274	0.0011	-0.0034	0.0039	0.0783	-0.0421	-0.1783	0.4521	0.1217	-0.4088
0.1255	0.0124	-0.0545	0.1069	0.0762	-0.0409	-0.0860	-0.0367	-0.0121	-0.0900	-0.0230	-0.0900	-0.2030	0.0102
-0.1151	0.0676	0.1241	0.1516	-0.1530	-0.2748	0.1737	0.0793	0.0284	0.1486	-0.0128	0.0437	0.0376	-0.0387
-0.0084	-0.0142	0.0380	-0.1151	0.1467	-0.0692	-0.0624	0.2513	-0.0354	-0.1514	-0.2099	-0.1576	-0.1635	-0.1383
-0.1361	-0.0124	-0.0366	0.2220	0.4323	-0.1709	0.0556	0.3766	-0.3210	-0.2050	-0.0410	-0.1594	-0.1302	-0.0181
0.0800	0.0491	0.1315	0.2305	-0.1234	-0.1889	-0.0152	0.1350	0.0459	-0.0248	0.1685	-0.0321	0.1257	0.1240
-0.0417	0.0819	-0.0366	0.0331	0.3379	0.0414	-0.3927	-0.0010	0.3398	0.0782	-0.0410	0.0294	-0.0358	-0.0890
-0.0555	0.2690	0.1324	-0.0206	0.0522	0.2612	0.2445	-0.3387	-0.3186	-0.0569	-0.1154	-0.0634	0.0253	0.0505
-0.0679	0.0676	0.1241	0.1516	0.0829	0.0320	0.1500	0.2210	-0.2078	-0.0402	-0.0128	0.1381	-0.0096	-0.2452
-0.2049	0.1069	-0.0545	0.0126	0.0762	0.0063	0.0085	0.0105	-0.0121	0.0044	-0.0230	0.1460	-0.1086	-0.1070
0.3186	0.0368	-0.2769	-0.1298	0.0330	0.0484	-0.0034	0.0039	-0.0633	0.0995	0.1050	-0.0908	-0.2699	0.2358
0.4209	-0.1566	-0.2051	-0.0570	0.1817	-0.0524	-0.0624	0.0220	-0.0469	-0.1385	0.1513	0.0975	-0.0938	0.1366
-0.0223	-0.1508	0.1829	0.2859	-0.2365	-0.1780	0.1943	0.1211	-0.0652	-0.1215	-0.0190	0.2631	0.1505	-0.2109
-0.1048	0.0038	-0.0253	-0.2025	0.1461	0.0974	-0.1315	0.1340	-0.0298	-0.1771	0.1383	-0.0074	-0.1681	0.1020

Table B.24 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (10^{-11}m/sec).

CASE II.A - Barotropic atmosphere - tilted (parallel) isobaric planes with $\nabla_p(T) = \vec{0}$, under $T = A \ln p + B$

10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093

Table B.25 Analytic surface geostrophic wind velocity (m/sec).

10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093

Table B.26 Finite Difference surface geostrophic wind velocity (m/sec).

0.0423	-0.2956	-0.1386	0.3201	-0.2535	-0.1608	0.0203	0.0215	-0.0250	-0.3007	0.2563	0.0144	-0.0675	-0.1311	0.1263
0.0517	-0.0558	-0.0529	0.0068	0.1691	0.0600	0.1208	-0.1739	-0.0767	0.0615	0.2318	-0.0238	0.3144	-0.1410	0.1160
-0.1267	-0.0336	-0.0597	-0.0286	0.0005	0.2013	-0.0581	0.1375	-0.1343	0.2359	-0.2231	-0.0970	0.0480	0.0739	-0.0327
0.0309	0.2210	0.0282	-0.0306	0.0263	-0.1267	-0.0552	0.1288	-0.0577	-0.0410	-0.0515	0.1778	-0.4926	0.1142	0.2165
0.1609	-0.1343	0.1126	-0.0282	-0.1274	-0.1448	0.1586	0.1254	-0.0107	-0.1069	0.0909	-0.0565	0.1839	0.0885	0.1897
-0.0938	0.0584	-0.1256	-0.1354	-0.2624	-0.0787	0.3384	-0.2341	-0.1455	-0.1590	-0.1769	-0.0123	-0.1041	-0.1512	0.0282
-0.1682	0.0647	-0.0407	0.0382	-0.0547	-0.0488	0.0199	0.2988	-0.0613	0.0734	0.0606	0.2197	0.0469	0.2455	0.1062
-0.0824	0.1011	0.0648	-0.0400	-0.1284	-0.2522	0.1895	-0.0373	-0.0647	0.2489	0.2968	0.0158	0.1918	0.0803	0.0552
-0.1526	-0.1354	0.0270	-0.2023	0.0087	-0.0560	0.1995	-0.1119	-0.0606	-0.0838	0.1140	-0.1736	0.1052	-0.1556	-0.1256
0.1528	0.1284	-0.1195	0.1030	-0.0725	-0.3183	-0.1689	0.4617	-0.2135	-0.1442	-0.1750	0.0552	-0.0620	0.0973	0.0306
-0.0179	-0.0616	-0.2737	-0.1526	-0.0950	0.1158	-0.2778	-0.3409	0.2199	0.2421	0.0410	0.0908	0.0457	-0.0309	-0.0972
0.0449	-0.0517	-0.0192	-0.1732	-0.0542	0.0149	-0.0814	-0.2665	-0.1270	0.1105	0.1181	0.0327	-0.1240	-0.0627	0.2760
-0.0835	0.2755	-0.1190	0.1235	-0.0240	0.0052	-0.1302	-0.0584	-0.1339	0.1673	-0.0581	0.0952	-0.1128	0.2096	0.0204
-0.0787	-0.2123	-0.0218	0.3114	0.0792	0.1453	-0.1160	0.0815	-0.1174	0.1490	-0.1270	0.0096	0.1755	0.3057	-0.2000
-0.0744	-0.4437	0.1037	0.0584	0.0785	-0.1927	0.0444	-0.0174	0.1846	0.1162	0.1931	-0.1066	-0.1586	0.0300	0.0355
0.2149	0.0698	0.1249	-0.1606	-0.1030	0.0997	0.2945	-0.1853	0.0256	0.1215	0.0716	-0.0670	-0.3187	-0.0947	0.0476
-0.2700	0.1627	0.0561	0.2377	0.1187	-0.1053	-0.0773	0.0911	-0.3094	0.0776	0.0712	-0.1817	-0.0153	0.1515	-0.1798

Table B.27 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (10^{-11} m/sec).

CASE II.B - Barotropic atmosphere - tilted (parallel) isobaric planes with $\nabla_p(T) = \vec{0}$, under $T = Az + B$

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.28 Analytic u component of surface geostrophic wind (m/sec).

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3881	-4.3892	-4.3900	-4.3892	-4.3881	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3883	-4.3952	-4.4119	-4.4315	-4.4404	-4.4315	-4.4119	-4.3952	-4.3883	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3890	-4.4026	-4.4449	-4.5148	-4.5832	-4.6119	-4.5832	-4.5148	-4.4449	-4.4026	-4.3890	-4.3878	-4.3878
-4.3878	-4.3883	-4.3985	-4.4446	-4.5447	-4.6828	-4.8068	-4.8569	-4.8068	-4.6828	-4.5447	-4.4446	-4.3985	-4.3883	-4.3878
-4.3878	-4.3906	-4.4145	-4.4870	-4.6197	-4.7881	-4.9332	-4.9908	-4.9332	-4.7882	-4.6197	-4.4871	-4.4145	-4.3906	-4.3878
-4.3879	-4.3930	-4.4193	-4.4853	-4.5963	-4.7313	-4.8451	-4.8898	-4.8451	-4.7313	-4.5963	-4.4853	-4.4193	-4.3930	-4.3879
-4.3880	-4.3918	-4.4071	-4.4420	-4.4981	-4.5648	-4.6204	-4.6421	-4.6204	-4.5648	-4.4981	-4.4420	-4.4071	-4.3918	-4.3880
-4.3879	-4.3890	-4.3931	-4.4018	-4.4155	-4.4317	-4.4450	-4.4502	-4.4450	-4.4317	-4.4155	-4.4018	-4.3931	-4.3890	-4.3879
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3877	-4.3866	-4.3826	-4.3739	-4.3602	-4.3441	-4.3308	-4.3256	-4.3308	-4.3441	-4.3602	-4.3739	-4.3826	-4.3866	-4.3877
-4.3876	-4.3839	-4.3686	-4.3338	-4.2778	-4.2112	-4.1557	-4.1340	-4.1557	-4.2112	-4.2778	-4.3338	-4.3686	-4.3839	-4.3876
-4.3877	-4.3827	-4.3565	-4.2906	-4.1797	-4.0449	-3.9313	-3.8865	-3.9313	-4.0449	-4.1797	-4.2906	-4.3565	-4.3827	-4.3877
-4.3878	-4.3850	-4.3612	-4.2888	-4.1564	-3.9881	-3.8432	-3.7857	-3.8432	-3.9881	-4.1563	-4.2888	-4.3612	-4.3850	-4.3878
-4.3878	-4.3874	-4.3771	-4.3312	-4.2313	-4.0933	-3.9695	-3.9194	-3.9694	-4.0933	-4.2312	-4.3312	-4.3771	-4.3874	-4.3878
-4.3878	-4.3878	-4.3866	-4.3731	-4.3309	-4.2611	-4.1928	-4.1641	-4.1928	-4.2611	-4.3309	-4.3731	-4.3866	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3873	-4.3805	-4.3638	-4.3443	-4.3354	-4.3443	-4.3638	-4.3805	-4.3873	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3875	-4.3864	-4.3857	-4.3864	-4.3875	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.29 Finite Difference u component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0014	0.0022	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0005	0.0074	0.0241	0.0437	0.0526	0.0437	0.0241	0.0074	0.0005	0.0000	0.0000	0.0000
0.0000	0.0000	0.0012	0.0148	0.0571	0.1270	0.1954	0.2241	0.1954	0.1270	0.0571	0.0148	0.0012	0.0000	0.0000
0.0000	0.0004	0.0107	0.0568	0.1569	0.2950	0.4190	0.4691	0.4190	0.2950	0.1569	0.0568	0.0107	0.0004	0.0000
0.0000	0.0028	0.0267	0.0992	0.2319	0.4003	0.5453	0.6029	0.5454	0.4004	0.2319	0.0992	0.0267	0.0028	0.0000
0.0001	0.0052	0.0315	0.0975	0.2085	0.3435	0.4572	0.5020	0.4573	0.3435	0.2085	0.0975	0.0315	0.0052	0.0001
0.0002	0.0040	0.0193	0.0542	0.1103	0.1770	0.2325	0.2543	0.2326	0.1770	0.1103	0.0542	0.0193	0.0040	0.0002
0.0001	0.0012	0.0052	0.0140	0.0277	0.0438	0.0572	0.0624	0.0572	0.0439	0.0277	0.0140	0.0053	0.0012	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0001	-0.0012	-0.0052	-0.0139	-0.0276	-0.0437	-0.0570	-0.0622	-0.0570	-0.0437	-0.0276	-0.0139	-0.0052	-0.0012	-0.0001
-0.0002	-0.0039	-0.0192	-0.0540	-0.1100	-0.1766	-0.2321	-0.2538	-0.2321	-0.1766	-0.1100	-0.0540	-0.0192	-0.0039	-0.0002
-0.0001	-0.0051	-0.0314	-0.0972	-0.2081	-0.3429	-0.4565	-0.5013	-0.4566	-0.3429	-0.2081	-0.0973	-0.0314	-0.0051	-0.0001
-0.0000	-0.0028	-0.0266	-0.0990	-0.2315	-0.3997	-0.5446	-0.6021	-0.5446	-0.3997	-0.2315	-0.0990	-0.0266	-0.0028	-0.0000
0.0000	-0.0004	-0.0107	-0.0566	-0.1566	-0.2945	-0.4184	-0.4684	-0.4184	-0.2945	-0.1566	-0.0566	-0.0107	-0.0004	0.0000
0.0000	-0.0000	-0.0012	-0.0148	-0.0569	-0.1267	-0.1950	-0.2237	-0.1950	-0.1267	-0.0570	-0.0148	-0.0012	-0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0005	-0.0073	-0.0240	-0.0435	-0.0525	-0.0435	-0.0240	-0.0073	-0.0005	-0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0003	-0.0014	-0.0021	-0.0014	-0.0003	-0.0000	0.0000	0.0000	0.0000	0.0000

Table B.30 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (m/sec).

CASE II.B - Barotropic atmosphere - tilted (parallel) isobaric planes with $\nabla_p(T) = \vec{0}$, under $T = Az + B$

-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963

Table B.31 Analytic v component of surface geostrophic wind (m/sec).

-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9965	-8.9965	-8.9964	-8.9963	-8.9962	-8.9961	-8.9962	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9968	-8.9991	-9.0015	-9.0003	-8.9976	-8.9963	-8.9951	-8.9924	-8.9912	-8.9936	-8.9959	-8.9963	-8.9963
-8.9976	-9.0071	-9.0231	-9.0279	-9.0156	-9.0016	-8.9963	-8.9911	-8.9772	-8.9650	-8.9698	-8.9857	-8.9951	-8.9963
-9.0112	-9.0532	-9.0957	-9.0940	-9.0506	-9.0104	-8.9963	-8.9824	-8.9424	-8.8992	-8.8974	-8.9398	-8.9816	-8.9958
-9.0535	-9.1534	-9.2285	-9.2051	-9.1068	-9.0241	-8.9964	-8.9687	-8.8864	-8.7884	-8.7650	-8.8399	-8.9394	-8.9890
-9.1235	-9.2917	-9.3971	-9.3402	-9.1736	-9.0403	-8.9964	-8.9527	-8.8199	-8.6536	-8.5968	-8.7020	-8.8698	-8.9724
-9.1920	-9.4158	-9.5422	-9.4541	-9.2292	-9.0536	-8.9964	-8.9394	-8.7644	-8.5400	-8.4520	-8.5782	-8.8014	-8.9528
-9.2207	-9.4659	-9.5999	-9.4989	-9.2510	-9.0589	-8.9964	-8.9342	-8.7427	-8.4953	-8.3945	-8.5281	-8.7728	-8.9439
-9.1920	-9.4158	-9.5423	-9.4541	-9.2292	-9.0536	-8.9964	-8.9394	-8.7644	-8.5400	-8.4520	-8.5782	-8.8014	-8.9528
-9.1235	-9.2917	-9.3971	-9.3403	-9.1736	-9.0403	-8.9964	-8.9527	-8.8198	-8.6536	-8.5968	-8.7020	-8.8698	-8.9724
-9.0535	-9.1534	-9.2285	-9.2051	-9.1068	-9.0241	-8.9964	-8.9687	-8.8864	-8.7884	-8.7650	-8.8399	-8.9394	-8.9890
-9.0112	-9.0532	-9.0957	-9.0940	-9.0506	-9.0104	-8.9963	-8.9824	-8.9424	-8.8992	-8.8974	-8.9398	-8.9816	-8.9958
-8.9976	-9.0071	-9.0231	-9.0279	-9.0156	-9.0016	-8.9963	-8.9911	-8.9772	-8.9650	-8.9698	-8.9857	-8.9951	-8.9963
-8.9963	-8.9968	-8.9991	-9.0015	-9.0003	-8.9976	-8.9963	-8.9951	-8.9924	-8.9912	-8.9936	-8.9959	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9965	-8.9965	-8.9964	-8.9963	-8.9962	-8.9961	-8.9962	-8.9963	-8.9963	-8.9963	-8.9963
-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963	-8.9963

Table B.32 Finite Difference v component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0001	0.0002	0.0001	0.0000	-0.0001	-0:0002	-0.0001	-0.0000	0.0000	0.0000	0.0000
0.0000	0.0004	0.0028	0.0052	0.0040	0.0012	0.0000	-0.0012	-0.0039	-0.0051	-0.0028	-0.0004	-0.0000	0.0000
0.0012	0.0108	0.0268	0.0315	0.0193	0.0053	0.0000	-0.0052	-0.0191	-0.0313	-0.0266	-0.0106	-0.0012	-0.0000
0.0149	0.0569	0.0994	0.0976	0.0543	0.0140	0.0000	-0.0139	-0.0539	-0.0971	-0.0989	-0.0566	-0.0147	-0.0005
0.0572	0.1571	0.2322	0.2088	0.1104	0.0278	0.0000	-0.0276	-0.1099	-0.2079	-0.2313	-0.1564	-0.0569	-0.0073
0.1271	0.2953	0.4008	0.3439	0.1772	0.0439	0.0000	-0.0437	-0.1765	-0.3427	-0.3995	-0.2943	-0.1266	-0.0240
0.1956	0.4194	0.5459	0.4578	0.2329	0.0573	0.0000	-0.0570	-0.2319	-0.4563	-0.5443	-0.4181	-0.1949	-0.0435
0.2244	0.4696	0.6036	0.5026	0.2546	0.0625	0.0000	-0.0622	-0.2537	-0.5010	-0.6019	-0.4682	-0.2235	-0.0524
0.1956	0.4195	0.5459	0.4578	0.2329	0.0573	0.0000	-0.0570	-0.2319	-0.4563	-0.5443	-0.4182	-0.1949	-0.0435
0.1271	0.2954	0.4008	0.3439	0.1772	0.0439	0.0000	-0.0437	-0.1765	-0.3427	-0.3995	-0.2943	-0.1266	-0.0240
0.0572	0.1571	0.2322	0.2088	0.1104	0.0278	0.0000	-0.0276	-0.1099	-0.2079	-0.2313	-0.1564	-0.0569	-0.0073
0.0149	0.0569	0.0994	0.0977	0.0543	0.0140	0.0000	-0.0139	-0.0539	-0.0971	-0.0989	-0.0566	-0.0147	-0.0005
0.0012	0.0108	0.0268	0.0315	0.0193	0.0053	0.0000	-0.0052	-0.0191	-0.0313	-0.0266	-0.0107	-0.0012	-0.0000
0.0000	0.0004	0.0028	0.0052	0.0040	0.0012	0.0000	-0.0012	-0.0039	-0.0051	-0.0028	-0.0004	-0.0000	0.0000
0.0000	0.0000	0.0000	0.0001	0.0002	0.0001	0.0000	-0.0001	-0.0002	-0.0001	-0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.33 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (m/sec).

CASE II.B - Barotropic atmosphere - tilted (parallel) isobaric planes with $\nabla_p(T) = \vec{0}$, under $T = Az + B$

10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093	10.0093

Table B.34 Analytic surface geostrophic wind velocity (m/sec).

10.0093	10.0093	10.0093	10.0093	10.0093	10.0095	10.0100	10.0103	10.0100	10.0095	10.0093	10.0093	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0096	10.0127	10.0201	10.0286	10.0325	10.0285	10.0198	10.0125	10.0096	10.0093	10.0093	10.0093
10.0093	10.0093	10.0103	10.0184	10.0392	10.0692	10.0976	10.1096	10.0954	10.0621	10.0299	10.0134	10.0095	10.0093	10.0093
10.0093	10.0106	10.0237	10.0584	10.1073	10.1593	10.2046	10.2237	10.1954	10.1252	10.0512	10.0106	10.0045	10.0084	10.0093
10.0098	10.0239	10.0722	10.1423	10.2001	10.2391	10.2724	10.2880	10.2479	10.1436	10.0268	9.9649	9.9703	9.9973	10.0089
10.0160	10.0630	10.1644	10.2608	10.2888	10.2625	10.2425	10.2394	10.1938	10.0675	9.9178	9.8460	9.8830	9.9605	10.0028
10.0311	10.1255	10.2839	10.3941	10.3669	10.2465	10.1525	10.1234	10.0746	9.9311	9.7528	9.6766	9.7543	9.8975	9.9879
10.0487	10.1861	10.3902	10.5086	10.4344	10.2380	10.0859	10.0369	9.9835	9.8211	9.6140	9.5295	9.6376	9.8351	9.9703
10.0567	10.2115	10.4335	10.5551	10.4634	10.2388	10.0656	10.0094	9.9535	9.7820	9.5615	9.4721	9.5907	9.8089	9.9623
10.0486	10.1850	10.3858	10.4969	10.4111	10.2004	10.0361	9.9822	9.9332	9.7819	9.5887	9.5167	9.6329	9.8340	9.9702
10.0310	10.1221	10.2675	10.3483	10.2733	10.0940	9.9497	9.9007	9.8702	9.7736	9.6532	9.6274	9.7370	9.8940	9.9877
10.0159	10.0586	10.1373	10.1772	10.1096	9.9647	9.8432	9.8000	9.7925	9.7637	9.7317	9.7588	9.8551	9.9560	10.0027
10.0098	10.0215	10.0489	10.0562	9.9988	9.8903	9.7958	9.7604	9.7701	9.7914	9.8219	9.8772	9.9468	9.9949	10.0089
10.0093	10.0103	10.0143	10.0088	9.9703	9.9014	9.8380	9.8130	9.8284	9.8664	9.9134	9.9607	9.9951	10.0081	10.0093
10.0093	10.0093	10.0092	10.0054	9.9892	9.9581	9.9265	9.9133	9.9243	9.9509	9.9799	10.0004	10.0084	10.0093	10.0093
10.0093	10.0093	10.0093	10.0091	10.0062	9.9990	9.9904	9.9865	9.9902	9.9987	10.0060	10.0091	10.0093	10.0093	10.0093
10.0093	10.0093	10.0093	10.0093	10.0093	10.0092	10.0087	10.0084	10.0087	10.0092	10.0093	10.0093	10.0093	10.0093	10.0093

Table B.35 Finite Difference surface geostrophic wind velocity (m/sec).

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0001	-0.0006	-0.0009	-0.0006	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0002	-0.0033	-0.0108	-0.0193	-0.0232	-0.0191	-0.0104	-0.0031	-0.0002	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0009	-0.0090	-0.0298	-0.0599	-0.0883	-0.1002	-0.0861	-0.0528	-0.0206	-0.0040	-0.0001	0.0000	-0.0000
-0.0000	-0.0013	-0.0144	-0.0491	-0.0979	-0.1499	-0.1953	-0.2144	-0.1860	-0.1158	-0.0418	-0.0012	0.0049	0.0009	0.0000
-0.0005	-0.0146	-0.0629	-0.1329	-0.1908	-0.2298	-0.2631	-0.2786	-0.2386	-0.1343	-0.0175	0.0445	0.0390	0.0120	0.0005
-0.0067	-0.0537	-0.1551	-0.2514	-0.2795	-0.2531	-0.2332	-0.2300	-0.1844	-0.0581	0.0916	0.1633	0.1263	0.0488	0.0065
-0.0218	-0.1161	-0.2745	-0.3847	-0.3576	-0.2372	-0.1432	-0.1141	-0.0653	0.0782	0.2565	0.3327	0.2550	0.1118	0.0215
-0.0394	-0.1767	-0.3808	-0.4992	-0.4251	-0.2287	-0.0766	-0.0276	0.0258	0.1882	0.3953	0.4798	0.3717	0.1742	0.0390
-0.0474	-0.2021	-0.4241	-0.5458	-0.4540	-0.2295	-0.0562	-0.0000	0.0558	0.2274	0.4478	0.5373	0.4186	0.2004	0.0471
-0.0393	-0.1757	-0.3764	-0.4876	-0.4018	-0.1911	-0.0268	0.0271	0.0762	0.2274	0.4207	0.4927	0.3765	0.1753	0.0391
-0.0216	-0.1127	-0.2581	-0.3390	-0.2639	-0.0846	0.0596	0.1086	0.1392	0.2357	0.3561	0.3819	0.2723	0.1154	0.0216
-0.0066	-0.0492	-0.1279	-0.1678	-0.1003	0.0447	0.1661	0.2094	0.2168	0.2457	0.2776	0.2505	0.1543	0.0534	0.0066
-0.0005	-0.0121	-0.0396	-0.0468	0.0105	0.1190	0.2136	0.2489	0.2393	0.2179	0.1874	0.1322	0.0625	0.0145	0.0005
-0.0000	-0.0009	-0.0050	0.0006	0.0391	0.1080	0.1714	0.1963	0.1810	0.1430	0.0960	0.0486	0.0143	0.0013	0.0000
-0.0000	-0.0000	0.0001	0.0039	0.0202	0.0513	0.0828	0.0960	0.0850	0.0584	0.0295	0.0090	0.0009	0.0000	-0.0000
-0.0000	-0.0000	0.0000	0.0002	0.0031	0.0103	0.0189	0.0229	0.0191	0.0107	0.0033	0.0002	0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0001	0.0006	0.0009	0.0006	0.0001	0.0000	-0.0000	-0.0000	-0.0000	-0.0000

Table B.36 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (m/sec).

CASE III.A.1 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 1.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = A \ln p + B_O + C_O(x - x^{(R)})$

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.37 Analytic u component of surface geostrophic wind (m/sec).

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.387	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.38 Finite Difference u component of surface geostrophic wind (m/sec).

0.1406	0.0825	-0.0347	-0.0990	0.0349	0.2921	0.3813	0.1480	-0.0452	0.3119	-0.0410	-0.0059	-0.2183	0.1458	-0.2005
-0.1820	0.0258	-0.0543	0.0958	-0.1858	0.2678	-0.1355	0.1471	-0.1120	0.1144	-0.4749	0.0601	-0.2014	0.0165	-0.4824
0.1128	-0.1779	0.2543	0.0493	0.1627	-0.2497	0.0001	-0.2559	0.2834	-0.2970	0.0684	-0.1867	0.1362	-0.1688	0.3720
0.1123	0.0003	-0.0393	0.0543	0.2983	-0.0668	0.1238	-0.1924	0.1238	-0.0668	0.2038	-0.1345	0.1967	-0.1029	0.3059
-0.1691	0.0728	-0.3766	0.0817	-0.0229	-0.0146	0.0997	0.0790	-0.0891	0.2686	-0.1173	0.0817	-0.1406	0.2379	-0.6297
-0.1030	0.1028	0.0537	0.0544	0.1934	0.2100	-0.1139	-0.1997	0.1694	0.3044	0.0046	0.1488	0.1008	0.0084	0.0504
-0.0176	-0.2013	0.2165	-0.1651	0.2192	0.0941	0.0022	-0.3647	0.0965	-0.0946	0.2664	-0.0235	0.1692	-0.2248	0.3128
-0.0122	0.0014	-0.1104	0.2134	-0.3274	0.1378	0.2249	0.3250	-0.2471	0.0906	-0.1858	-0.0226	-0.1104	0.0721	-0.2335
-0.0883	0.2273	-0.1621	0.2155	-0.3037	0.0503	-0.4453	-0.0441	-0.2329	0.0975	-0.5870	0.1683	-0.1739	0.1211	-0.2005
0.0095	-0.2429	0.0458	-0.1837	0.2156	-0.0844	0.0881	-0.2716	0.3949	-0.3677	0.2628	0.0052	0.0812	-0.1132	0.2957
-0.0824	0.0894	0.1202	0.0769	0.1646	0.3841	0.3817	0.2765	0.0040	0.1008	0.2118	-0.0647	0.0730	-0.0286	0.2834
0.0443	0.0686	-0.0003	0.1406	-0.1871	0.1738	-0.2576	0.2059	-0.4465	0.1738	-0.2816	-0.0481	-0.0475	0.0214	-0.1268
0.2643	-0.1373	0.0996	-0.0756	0.0291	-0.3569	-0.0935	-0.0728	0.2842	-0.3569	0.1236	-0.2643	0.0052	-0.2082	-0.0271
0.1538	0.0117	0.1163	-0.1897	0.2743	-0.3991	0.0712	-0.1791	0.2601	-0.2103	0.1800	-0.1426	0.1163	0.0147	0.0633
0.0580	0.0621	0.1480	0.0513	-0.1094	0.1615	-0.2772	0.2619	-0.3716	-0.1689	-0.0149	-0.0196	-0.0216	0.3744	-0.1149
-0.3402	0.0660	-0.1044	0.1200	-0.4629	0.1987	-0.1651	0.0243	-0.0943	-0.1436	-0.1797	0.0970	0.0859	-0.0552	0.1501
-0.1294	-0.0584	-0.0735	-0.2846	0.2020	-0.3677	0.0813	-0.0267	0.4603	-0.1157	0.2134	-0.0246	0.1565	-0.1885	0.1722

Table B.39 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (10^{-11} m/sec).

CASE III.A.1 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 1.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = A \ln p + B_O + C_O(x - x^{(R)})$

-14.0364	-14.0341	-14.0317	-14.0293	-14.0269	-14.0246	-14.0222	-14.0198	-14.0174	-14.0151	-14.0127	-14.0103	-14.0080	-14.0056
-14.0357	-14.0333	-14.0309	-13.9371	-13.7063	-13.4933	-13.4077	-13.4888	-13.6970	-13.9229	-14.0120	-14.0096	-14.0072	-14.0049
-14.0349	-14.0102	-13.6464	-13.0108	-12.3364	-11.8331	-11.6466	-11.8290	-12.3279	-12.9975	-13.6279	-13.9865	-14.0065	-14.0041
-14.0118	-13.5010	-12.4902	-11.2555	-10.0940	-9.2722	-8.9751	-9.2690	-10.0870	-11.2438	-12.4731	-13.4781	-13.9834	-14.0034
-13.6495	-12.4916	-10.8225	-8.9791	-7.3221	-6.1757	-5.7658	-6.1735	-7.3169	-8.9697	-10.8075	-12.4702	-13.6218	-14.0026
-13.0152	-11.2578	-8.9799	-6.5723	-4.4544	-3.0049	-2.4896	-3.0038	-4.4512	-6.5653	-8.9674	-11.2385	-12.9886	-13.9105
-12.3418	-10.0970	-7.3231	-4.4544	-1.9580	-0.2588	0.3436	-0.2587	-1.9566	-4.4496	-7.3128	-10.0795	-12.3165	-13.6817
-11.8395	-9.2756	-6.1766	-3.0044	-0.2580	1.6067	2.2669	1.6061	-0.2579	-3.0011	-6.1678	-9.2595	-11.8151	-13.4706
-11.6541	-8.9791	-5.7668	-2.4885	0.3454	2.2682	2.9488	2.2673	0.3451	-2.4858	-5.7586	-8.9635	-11.6301	-13.3866
-11.8379	-9.2741	-6.1751	-3.0028	-0.2564	1.6083	2.2686	1.6077	-0.2562	-2.9995	-6.1663	-9.2579	-11.8136	-13.4691
-12.3388	-10.0939	-7.3200	-4.4512	-1.9548	-0.2556	0.3469	-0.2555	-1.9534	-4.4464	-7.3097	-10.0765	-12.3135	-13.6788
-13.0107	-11.2533	-8.9753	-6.5676	-4.4496	-3.0001	-2.4848	-2.9990	-4.4464	-6.5606	-8.9628	-11.2339	-12.9841	-13.9061
-13.6436	-12.4856	-10.8164	-8.9729	-7.3159	-6.1694	-5.7595	-6.1672	-7.3108	-8.9636	-10.8015	-12.4643	-13.6159	-13.9967
-14.0044	-13.4936	-12.4827	-11.2479	-10.0864	-9.2646	-8.9675	-9.2613	-10.0794	-11.2363	-12.4656	-13.4707	-13.9760	-13.9960
-14.0260	-14.0013	-13.6375	-13.0018	-12.3274	-11.8240	-11.6375	-11.8200	-12.3189	-12.9885	-13.6190	-13.9776	-13.9976	-13.9952
-14.0253	-14.0229	-14.0205	-13.9267	-13.6959	-13.4829	-13.3973	-13.4783	-13.6866	-13.9125	-14.0016	-13.9992	-13.9968	-13.9945
-14.0245	-14.0222	-14.0198	-14.0174	-14.0150	-14.0127	-14.0103	-14.0079	-14.0056	-14.0032	-14.0008	-13.9985	-13.9961	-13.9937

Table B.40 Analytic v component of surface geostrophic wind (m/sec).

-14.0364	-14.0341	-14.0317	-14.0293	-14.0269	-14.0246	-14.0222	-14.0198	-14.0174	-14.0151	-14.0127	-14.0103	-14.0080	-14.0056
-14.0357	-14.0333	-14.0081	-13.9028	-13.7108	-13.5252	-13.4494	-13.5206	-13.7014	-13.8887	-13.9891	-14.0096	-14.0072	-14.0049
-14.0294	-13.9254	-13.5784	-13.0011	-12.3791	-11.9123	-11.7388	-11.9081	-12.3706	-12.9877	-13.5600	-13.9019	-14.0009	-14.0041
-13.8903	-13.3760	-12.4342	-11.2738	-10.1789	-9.4034	-9.1229	-9.4000	-10.1717	-11.2620	-12.4171	-13.3532	-13.8621	-13.9978
-13.4566	-12.3638	-10.7789	-9.0257	-7.4497	-6.3598	-5.9702	-6.3574	-7.4443	-9.0160	-10.7638	-12.3425	-13.4291	-13.9068
-12.8079	-11.1277	-8.9475	-6.6447	-4.6215	-3.2384	-2.7470	-3.2371	-4.6179	-6.6373	-8.9346	-11.1082	-12.7816	-13.7023
-12.1237	-9.9647	-7.2994	-4.5475	-2.1573	-0.5330	0.0424	-0.5326	-2.1554	-4.5421	-7.2887	-9.9471	-12.0986	-13.4197
-11.6143	-9.1418	-6.1583	-3.1109	-0.4785	1.3056	1.9367	1.3053	-0.4777	-3.1070	-6.1491	-9.1255	-11.5901	-13.1886
-11.4264	-8.8448	-5.7503	-2.5996	0.1176	1.9576	2.6083	1.9571	0.1179	-2.5963	-5.7416	-8.8289	-11.4026	-13.1001
-11.6128	-9.1403	-6.1567	-3.1093	-0.4768	1.3072	1.9383	1.3069	-0.4761	-3.1054	-6.1475	-9.1239	-11.5886	-13.1871
-12.1207	-9.9617	-7.2963	-4.5443	-2.1541	-0.5298	0.0457	-0.5294	-2.1522	-4.5390	-7.2856	-9.9440	-12.0956	-13.4167
-12.8034	-11.1231	-8.9429	-6.6400	-4.6168	-3.2337	-2.7422	-3.2323	-4.6131	-6.6326	-8.9300	-11.1037	-12.7771	-13.6978
-13.4507	-12.3578	-10.7728	-9.0196	-7.4436	-6.3536	-5.9639	-6.3512	-7.4381	-9.0098	-10.7577	-12.3365	-13.4232	-13.9009
-13.8829	-13.3686	-12.4267	-11.2662	-10.1713	-9.3958	-9.1152	-9.3924	-10.1641	-11.2544	-12.4096	-13.3458	-13.8547	-13.9904
-14.0204	-13.9165	-13.5695	-12.9921	-12.3701	-11.9032	-11.7298	-11.8991	-12.3616	-12.9787	-13.5510	-13.8930	-13.9920	-13.9952
-14.0253	-14.0229	-13.9977	-13.8924	-13.7004	-13.5148	-13.4390	-13.5102	-13.6910	-13.8783	-13.9787	-13.9992	-13.9968	-13.9945
-14.0245	-14.0222	-14.0198	-14.0174	-14.0150	-14.0127	-14.0103	-14.0079	-14.0056	-14.0032	-14.0008	-13.9985	-13.9961	-13.9937

Table B.41 Finite Difference v component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0229	-0.0342	0.0044	0.0318	0.0417	0.0318	0.0044	-0.0342	-0.0229	0.0000	0.0000	0.0000
-0.0056	-0.0848	-0.0680	-0.0097	0.0428	0.0792	0.0922	0.0791	0.0427	-0.0098	-0.0680	-0.0847	-0.0056	0.0000
-0.1215	-0.1250	-0.0560	0.0183	0.0849	0.1312	0.1477	0.1311	0.0847	0.0181	-0.0561	-0.1249	-0.1213	-0.0056
-0.1929	-0.1278	-0.0436	0.0466	0.1276	0.1841	0.2044	0.1839	0.1273	0.0463	-0.0438	-0.1278	-0.1927	-0.0958
-0.2072	-0.1302	-0.0324	0.0724	0.1671	0.2335	0.2574	0.2333	0.1667	0.0720	-0.0327	-0.1302	-0.2070	-0.2082
-0.2181	-0.1323	-0.0237	0.0931	0.1993	0.2742	0.3012	0.2739	0.1988	0.0926	-0.0241	-0.1324	-0.2180	-0.2620
-0.2252	-0.1338	-0.0183	0.1065	0.2204	0.3011	0.3303	0.3008	0.2198	0.1059	-0.0188	-0.1340	-0.2250	-0.2820
-0.2276	-0.1344	-0.0165	0.1111	0.2278	0.3106	0.3405	0.3102	0.2272	0.1105	-0.0170	-0.1346	-0.2275	-0.2864
-0.2252	-0.1338	-0.0183	0.1065	0.2204	0.3011	0.3303	0.3008	0.2198	0.1059	-0.0188	-0.1340	-0.2251	-0.2820
-0.2181	-0.1323	-0.0237	0.0931	0.1993	0.2742	0.3012	0.2739	0.1988	0.0926	-0.0241	-0.1324	-0.2180	-0.2620
-0.2072	-0.1302	-0.0324	0.0724	0.1671	0.2335	0.2574	0.2333	0.1667	0.0720	-0.0327	-0.1303	-0.2071	-0.2082
-0.1929	-0.1278	-0.0436	0.0466	0.1276	0.1842	0.2044	0.1840	0.1273	0.0463	-0.0438	-0.1278	-0.1927	-0.0958
-0.1215	-0.1250	-0.0560	0.0183	0.0849	0.1312	0.1477	0.1311	0.0847	0.0181	-0.0561	-0.1249	-0.1213	-0.0056
-0.0056	-0.0848	-0.0680	-0.0097	0.0428	0.0792	0.0922	0.0791	0.0427	-0.0098	-0.0680	-0.0847	-0.0056	0.0000
0.0000	0.0000	-0.0229	-0.0342	0.0044	0.0318	0.0417	0.0318	0.0044	-0.0342	-0.0229	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.42 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (m/sec).

CASE III.A.1 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 1.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = A \ln p + B_o + C_o(x - x^{(R)})$

14.7085	14.7063	14.7040	14.7017	14.6995	14.6972	14.6949	14.6927	14.6904	14.6881	14.6859	14.6836	14.6814	14.6791	14.6768
14.7078	14.7056	14.7033	14.7010	14.6115	14.3915	14.1888	14.1074	14.1845	14.3827	14.5980	14.6829	14.6807	14.6784	14.6761
14.7071	14.7048	14.6813	14.3345	13.7307	13.0935	12.6204	12.4457	12.6166	13.0855	13.7181	14.3169	14.6587	14.6777	14.6754
14.7064	14.6828	14.1961	13.2385	12.0805	11.0065	10.2580	9.9903	10.2551	11.0000	12.0697	13.2224	14.1744	14.6557	14.6747
14.7057	14.3375	13.2398	11.6781	9.9938	8.5362	7.5757	7.2455	7.5739	8.5317	9.9854	11.6643	13.2197	14.3111	14.6740
14.6176	13.7349	12.0827	9.9946	7.9024	6.2525	5.3181	5.0449	5.3175	6.2503	7.8966	9.9833	12.0647	13.7097	14.5861
14.3991	13.0986	11.0092	8.5371	6.2525	4.8049	4.3954	4.4012	4.3954	4.8043	6.2491	8.5282	10.9931	13.0748	14.3681
14.1977	12.6264	10.2611	7.5765	5.3178	4.3954	4.6727	4.9388	4.6725	4.3954	5.3160	7.5694	10.2465	12.6036	14.1672
14.1176	12.4527	9.9939	7.2463	5.0444	4.4014	4.9394	5.2866	4.9390	4.4014	5.0430	7.2397	9.9798	12.4303	14.0874
14.1962	12.6250	10.2597	7.5752	5.3169	4.3953	4.6733	4.9396	4.6731	4.3953	5.3151	7.5681	10.2451	12.6022	14.1658
14.3962	13.0958	11.0064	8.5344	6.2503	4.8036	4.3953	4.4015	4.3952	4.8030	6.2469	8.5256	10.9904	13.0719	14.3653
14.6134	13.7306	12.0785	9.9904	7.8985	6.2492	5.3154	5.0425	5.3148	6.2469	7.8927	9.9792	12.0605	13.7055	14.5819
14.7000	14.3318	13.2341	11.6725	9.9883	8.5309	7.5706	7.2405	7.5689	8.5264	9.9799	11.6587	13.2140	14.3054	14.6684
14.6993	14.6757	14.1891	13.2314	12.0735	10.9995	10.2511	9.9834	10.2482	10.9931	12.0626	13.2153	14.1673	14.6486	14.6676
14.6986	14.6963	14.6727	14.3260	13.7222	13.0850	12.6119	12.4373	12.6081	13.0770	13.7096	14.3084	14.6502	14.6692	14.6669
14.6979	14.6956	14.6934	14.6911	14.6015	14.3816	14.1789	14.0975	14.1746	14.3728	14.5880	14.6730	14.6707	14.6685	14.6662
14.6972	14.6949	14.6927	14.6904	14.6881	14.6859	14.6836	14.6813	14.6791	14.6768	14.6746	14.6723	14.6700	14.6678	14.6655

Table B.43 Analytic surface geostrophic wind velocity (m/sec).

14.7085	14.7063	14.7040	14.7017	14.6995	14.6972	14.6949	14.6927	14.6904	14.6881	14.6859	14.6836	14.6814	14.6791	14.6768
14.7078	14.7056	14.7033	14.6792	14.5788	14.3958	14.2191	14.1470	14.2147	14.3869	14.5653	14.6611	14.6807	14.6784	14.6761
14.7071	14.6995	14.6004	14.2698	13.7215	13.1338	12.6947	12.5321	12.6908	13.1257	13.7089	14.2522	14.5779	14.6724	14.6754
14.7011	14.5669	14.0773	13.1857	12.0976	11.0844	10.3767	10.1232	10.3737	11.0778	12.0865	13.1695	14.0556	14.5400	14.6694
14.6141	14.1539	13.1193	11.6378	10.0357	8.6459	7.7266	7.4092	7.7246	8.6412	10.0270	11.6237	13.0992	14.1278	14.5826
14.4188	13.5387	11.9615	9.9654	7.9627	6.3727	5.4535	5.1767	5.4527	6.3700	7.9565	9.9539	11.9434	13.5137	14.3877
14.1493	12.8933	10.8880	8.5167	6.3192	4.8895	4.4201	4.3880	4.4200	4.8886	6.3154	8.5075	10.8719	12.8697	14.1188
13.9293	12.4155	10.1403	7.5616	5.3787	4.4138	4.5779	4.7962	4.5779	4.4137	5.3765	7.5541	10.1256	12.3929	13.8993
13.8452	12.2399	9.8734	7.2332	5.1001	4.3894	4.8047	5.1045	4.8045	4.3894	5.0984	7.2263	9.8591	12.2177	13.8155
13.9279	12.4141	10.1389	7.5603	5.3778	4.4136	4.5784	4.7969	4.5783	4.4136	5.3755	7.5528	10.1242	12.3914	13.8979
14.1465	12.8905	10.8852	8.5141	6.3169	4.8881	4.4197	4.3881	4.4196	4.8872	6.3131	8.5049	10.8691	12.8668	14.1160
14.4145	13.5344	11.9573	9.9613	7.9588	6.3692	5.4506	5.1742	5.4498	6.3666	7.9526	9.9498	11.9392	13.5095	14.3835
14.6084	14.1483	13.1136	11.6321	10.0302	8.6406	7.7215	7.4041	7.7195	8.6358	10.0215	11.6181	13.0936	14.1221	14.5770
14.6940	14.5598	14.0702	13.1786	12.0905	11.0774	10.3698	10.1163	10.3668	11.0708	12.0795	13.1625	14.0486	14.5329	14.6623
14.6986	14.6910	14.5919	14.2613	13.7130	13.1253	12.6862	12.5236	12.6823	13.1172	13.7004	14.2437	14.5694	14.6639	14.6669
14.6979	14.6956	14.6934	14.6693	14.5689	14.3858	14.2092	14.1371	14.2048	14.3770	14.5554	14.6512	14.6707	14.6685	14.6662
14.6972	14.6949	14.6927	14.6904	14.6881	14.6859	14.6836	14.6813	14.6791	14.6768	14.6746	14.6723	14.6700	14.6678	14.6655

Table B.44 Finite Difference surface geostrophic wind velocity (m/sec).

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	0.0218	0.0326	-0.0042	-0.0303	-0.0396	-0.0303	-0.0042	0.0326	0.0218	-0.0000	-0.0000	-0.0000
-0.0000	0.0053	0.0809	0.0647	0.0092	-0.0403	-0.0743	-0.0863	-0.0742	-0.0402	0.0093	0.0647	0.0808	0.0053	-0.0000
0.0053	0.1159	0.1188	0.0528	-0.0171	-0.0779	-0.1187	-0.1329	-0.1186	-0.0777	-0.0169	0.0529	0.1187	0.1157	0.0053
0.0916	0.1835	0.1205	0.0404	-0.0419	-0.1097	-0.1508	-0.1637	-0.1507	-0.1094	-0.0416	0.0406	0.1205	0.1833	0.0914
0.1989	0.1962	0.1212	0.0291	-0.0603	-0.1201	-0.1354	-0.1319	-0.1352	-0.1198	-0.0599	0.0294	0.1212	0.1960	0.1985
0.2498	0.2053	0.1212	0.0203	-0.0667	-0.0846	-0.0246	0.0132	-0.0246	-0.0843	-0.0662	0.0207	0.1213	0.2051	0.2493
0.2684	0.2109	0.1208	0.0149	-0.0609	-0.0184	0.0948	0.1426	0.0947	-0.0184	-0.0605	0.0153	0.1209	0.2107	0.2679
0.2724	0.2128	0.1205	0.0131	-0.0557	0.0120	0.1347	0.1821	0.1345	0.0120	-0.0554	0.0135	0.1207	0.2126	0.2719
0.2684	0.2109	0.1208	0.0149	-0.0608	-0.0183	0.0949	0.1427	0.0948	-0.0183	-0.0605	0.0153	0.1209	0.2107	0.2679
0.2498	0.2053	0.1212	0.0203	-0.0666	-0.0845	-0.0244	0.0135	-0.0244	-0.0842	-0.0662	0.0207	0.1213	0.2051	0.2493
0.1989	0.1962	0.1212	0.0291	-0.0603	-0.1201	-0.1352	-0.1317	-0.1350	-0.1197	-0.0599	0.0294	0.1212	0.1960	0.1985
0.0916	0.1835	0.1205	0.0404	-0.0419	-0.1097	-0.1508	-0.1636	-0.1506	-0.1094	-0.0416	0.0406	0.1205	0.1833	0.0914
0.0053	0.1159	0.1188	0.0528	-0.0171	-0.0779	-0.1187	-0.1329	-0.1186	-0.0777	-0.0169	0.0529	0.1187	0.1157	0.0053
-0.0000	0.0053	0.0809	0.0647	0.0092	-0.0403	-0.0743	-0.0863	-0.0742	-0.0402	0.0093	0.0647	0.0808	0.0053	-0.0000
-0.0000	-0.0000	-0.0000	0.0218	0.0326	-0.0042	-0.0303	-0.0396	-0.0303	-0.0042	0.0326	0.0218	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000

Table B.45 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (m/sec).

CASE III.B.1 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 1.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$,under $T = Az + B_o + C_o(x - x^{(R)})$

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.46 Analytic u component of surface geostrophic wind (m/sec).

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3881	-4.3892	-4.3900	-4.3892	-4.3881	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3883	-4.3952	-4.4119	-4.4315	-4.4404	-4.4315	-4.4119	-4.3952	-4.3883	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3890	-4.4027	-4.4450	-4.5149	-4.5834	-4.6121	-4.5833	-4.5148	-4.4449	-4.4026	-4.3890	-4.3878	-4.3878
-4.3878	-4.3883	-4.3986	-4.4447	-4.5450	-4.6832	-4.8072	-4.8573	-4.8070	-4.6828	-4.5447	-4.4446	-4.3985	-4.3883	-4.3878
-4.3878	-4.3906	-4.4146	-4.4873	-4.6201	-4.7887	-4.9337	-4.9912	-4.9334	-4.7882	-4.6197	-4.4870	-4.4145	-4.3906	-4.3878
-4.3879	-4.3930	-4.4194	-4.4855	-4.5967	-4.7318	-4.8456	-4.8902	-4.8452	-4.7313	-4.5962	-4.4852	-4.4193	-4.3930	-4.3879
-4.3880	-4.3918	-4.4071	-4.4421	-4.4983	-4.5651	-4.6206	-4.6423	-4.6205	-4.5648	-4.4981	-4.4419	-4.4071	-4.3918	-4.3880
-4.3879	-4.3890	-4.3931	-4.4018	-4.4156	-4.4317	-4.4451	-4.4503	-4.4450	-4.4317	-4.4155	-4.4018	-4.3931	-4.3890	-4.3879
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3877	-4.3866	-4.3826	-4.3738	-4.3601	-4.3440	-4.3307	-4.3255	-4.3308	-4.3441	-4.3602	-4.3739	-4.3826	-4.3866	-4.3877
-4.3876	-4.3839	-4.3686	-4.3337	-4.2776	-4.2109	-4.1555	-4.1338	-4.1556	-4.2112	-4.2778	-4.3339	-4.3687	-4.3839	-4.3876
-4.3877	-4.3827	-4.3564	-4.2904	-4.1794	-4.0444	-3.9308	-3.8862	-3.9311	-4.0449	-4.1798	-4.2906	-4.3565	-4.3827	-4.3877
-4.3878	-4.3850	-4.3611	-4.2886	-4.1559	-3.9875	-3.8427	-3.7853	-3.8430	-3.9881	-4.1564	-4.2889	-4.3612	-4.3850	-4.3878
-4.3878	-4.3874	-4.3771	-4.3311	-4.2310	-4.0929	-3.9690	-3.9190	-3.9693	-4.0933	-4.2313	-4.3312	-4.3772	-4.3874	-4.3878
-4.3878	-4.3878	-4.3866	-4.3730	-4.3308	-4.2610	-4.1926	-4.1640	-4.1927	-4.2611	-4.3309	-4.3731	-4.3866	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3873	-4.3805	-4.3638	-4.3442	-4.3353	-4.3443	-4.3638	-4.3805	-4.3873	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3875	-4.3864	-4.3857	-4.3864	-4.3875	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.47 Finite Difference u component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0014	0.0022	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0005	0.0074	0.0241	0.0437	0.0526	0.0437	0.0241	0.0074	0.0005	0.0000	0.0000	0.0000
0.0000	0.0000	0.0012	0.0148	0.0572	0.1271	0.1956	0.2242	0.1955	0.1270	0.0571	0.0148	0.0012	0.0000	0.0000
0.0000	0.0004	0.0108	0.0569	0.1572	0.2954	0.4194	0.4694	0.4191	0.2950	0.1568	0.0568	0.0107	0.0004	0.0000
0.0000	0.0028	0.0268	0.0994	0.2323	0.4009	0.5459	0.6034	0.5456	0.4004	0.2318	0.0992	0.0267	0.0028	0.0000
0.0001	0.0052	0.0315	0.0977	0.2088	0.3440	0.4577	0.5024	0.4574	0.3435	0.2084	0.0974	0.0314	0.0052	0.0001
0.0002	0.0040	0.0193	0.0543	0.1105	0.1772	0.2328	0.2545	0.2326	0.1770	0.1102	0.0541	0.0192	0.0040	0.0002
0.0001	0.0012	0.0053	0.0140	0.0278	0.0439	0.0573	0.0625	0.0572	0.0439	0.0277	0.0140	0.0052	0.0012	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0001	-0.0012	-0.0052	-0.0140	-0.0277	-0.0438	-0.0571	-0.0623	-0.0571	-0.0437	-0.0276	-0.0139	-0.0052	-0.0012	-0.0001
-0.0002	-0.0040	-0.0192	-0.0541	-0.1102	-0.1769	-0.2324	-0.2540	-0.2322	-0.1766	-0.1100	-0.0540	-0.0192	-0.0039	-0.0002
-0.0001	-0.0052	-0.0314	-0.0974	-0.2084	-0.3434	-0.4570	-0.5016	-0.4567	-0.3429	-0.2080	-0.0972	-0.0313	-0.0051	-0.0001
-0.0000	-0.0028	-0.0267	-0.0992	-0.2319	-0.4003	-0.5452	-0.6026	-0.5448	-0.3997	-0.2314	-0.0989	-0.0266	-0.0028	-0.0000
0.0000	-0.0004	-0.0107	-0.0568	-0.1568	-0.2949	-0.4188	-0.4688	-0.4185	-0.2945	-0.1565	-0.0566	-0.0107	-0.0004	0.0000
0.0000	-0.0000	-0.0012	-0.0148	-0.0570	-0.1269	-0.1952	-0.2238	-0.1951	-0.1267	-0.0569	-0.0147	-0.0012	-0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0005	-0.0074	-0.0240	-0.0436	-0.0525	-0.0436	-0.0240	-0.0073	-0.0005	-0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0003	-0.0014	-0.0021	-0.0014	-0.0003	-0.0000	0.0000	0.0000	0.0000	0.0000

Table B.48 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (m/sec).

CASE III.B.1 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 1.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$,under $T = Az + B_o + C_o(x - x^{(R)})$

-14.1156	-14.1132	-14.1107	-14.1083	-14.1058	-14.1033	-14.1009	-14.0984	-14.0960	-14.0935	-14.0910	-14.0886	-14.0861	-14.0837
-14.1149	-14.1124	-14.1099	-14.0131	-13.7755	-13.5563	-13.4683	-13.5516	-13.7659	-13.9985	-14.0903	-14.0878	-14.0854	-14.0829
-14.1141	-14.0886	-13.7138	-13.0610	-12.3711	-11.8581	-11.6685	-11.8540	-12.3625	-13.0473	-13.6947	-14.0640	-14.0846	-14.0821
-14.0903	-13.5643	-12.5282	-11.2714	-10.0978	-9.2724	-8.9751	-9.2692	-10.0907	-11.2596	-12.5108	-13.5406	-14.0608	-14.0814
-13.7171	-12.5297	-10.8329	-8.9791	-7.3308	-6.2004	-5.7983	-6.1983	-7.3257	-8.9697	-10.8178	-12.5078	-13.6884	-14.0806
-13.0655	-11.2738	-8.9799	-6.5906	-4.5186	-3.1167	-2.6214	-3.1157	-4.5155	-6.5837	-8.9674	-11.2541	-13.0382	-13.9857
-12.3767	-10.1008	-7.3319	-4.5187	-2.1124	-0.4967	0.0719	-0.4965	-2.1109	-4.5139	-7.3216	-10.0832	-12.3508	-13.7500
-11.8647	-9.2759	-6.2014	-3.1163	-0.4960	1.2564	1.8718	1.2560	-0.4956	-3.1130	-6.1927	-9.2597	-11.8398	-13.5329
-11.6761	-8.9791	-5.7993	-2.6205	0.0734	1.8728	2.5043	1.8722	0.0734	-2.6178	-5.7912	-8.9635	-11.6517	-13.4466
-11.8631	-9.2743	-6.1999	-3.1148	-0.4945	1.2579	1.8733	1.2575	-0.4941	-3.1115	-6.1912	-9.2582	-11.8383	-13.5313
-12.3736	-10.0977	-7.3288	-4.5156	-2.1093	-0.4936	0.0749	-0.4934	-2.1078	-4.5109	-7.3186	-10.0801	-12.3478	-13.7470
-13.0609	-11.2692	-8.9753	-6.5860	-4.5141	-3.1121	-2.6168	-3.1111	-4.5109	-6.5791	-8.9628	-11.2495	-13.0336	-13.9811
-13.7109	-12.5235	-10.8267	-8.9730	-7.3247	-6.1943	-5.7922	-6.1922	-7.3196	-8.9636	-10.8116	-12.5017	-13.6823	-14.0745
-14.0826	-13.5566	-12.5206	-11.2637	-10.0901	-9.2648	-8.9675	-9.2616	-10.0831	-11.2519	-12.5031	-13.5330	-14.0532	-14.0737
-14.1049	-14.0794	-13.7046	-13.0518	-12.3619	-11.8490	-11.6593	-11.8448	-12.3533	-13.0381	-13.6855	-14.0549	-14.0754	-14.0730
-14.1041	-14.1017	-14.0992	-14.0024	-13.7647	-13.5456	-13.4576	-13.5409	-13.7551	-13.9878	-14.0796	-14.0771	-14.0747	-14.0722
-14.1034	-14.1009	-14.0985	-14.0960	-14.0935	-14.0911	-14.0886	-14.0862	-14.0837	-14.0813	-14.0788	-14.0763	-14.0739	-14.0714

Table B.49 Analytic v component of surface geostrophic wind (m/sec).

-14.1156	-14.1132	-14.1107	-14.1083	-14.1058	-14.1033	-14.1009	-14.0984	-14.0960	-14.0935	-14.0910	-14.0886	-14.0861	-14.0837
-14.1149	-14.1124	-14.0864	-13.9780	-13.7802	-13.5892	-13.5112	-13.5842	-13.7702	-13.9631	-14.0667	-14.0878	-14.0854	-14.0829
-14.1083	-14.0016	-13.6467	-13.0563	-12.4188	-11.9400	-11.7622	-11.9333	-12.4021	-13.0322	-13.6220	-13.9763	-14.0788	-14.0821
-13.9662	-13.4465	-12.4978	-11.3215	-10.2026	-9.4091	-9.1229	-9.3953	-10.1569	-11.2468	-12.4270	-13.4016	-13.9345	-14.0756
-13.5334	-12.4560	-10.8881	-9.1232	-7.5113	-6.3954	-5.9986	-6.3652	-7.3980	-8.9193	-10.6751	-12.3210	-13.4755	-13.9813
-12.9103	-11.2988	-9.1795	-6.8704	-4.7913	-3.3693	-2.8684	-3.3128	-4.5681	-6.4476	-8.7044	-10.9664	-12.7694	-13.7638
-12.2812	-10.2628	-7.7088	-4.9526	-2.4799	-0.7998	-0.2118	-0.7120	-2.1253	-4.2627	-6.9001	-9.6570	-12.0025	-13.4565
-11.8311	-9.5610	-6.7288	-3.6760	-0.9369	0.9180	1.5646	1.0318	-0.4726	-2.7607	-5.6325	-8.7094	-11.4170	-13.1998
-11.6690	-9.3141	-6.3861	-3.2290	-0.3953	1.5217	2.1891	1.6456	0.1120	-2.2250	-5.1755	-8.3630	-11.1980	-13.1002
-11.8296	-9.5595	-6.7273	-3.6744	-0.9354	0.9195	1.5661	1.0333	-0.4711	-2.7591	-5.6310	-8.7078	-11.4154	-13.1982
-12.2781	-10.2597	-7.7057	-4.9496	-2.4769	-0.7967	-0.2088	-0.7090	-2.1222	-4.2596	-6.8970	-9.6539	-11.9994	-13.4535
-12.9057	-11.2942	-9.1749	-6.8659	-4.7867	-3.3647	-2.8638	-3.3082	-4.5635	-6.4430	-8.6998	-10.9618	-12.7648	-13.7592
-13.5273	-12.4499	-10.8820	-9.1171	-7.5052	-6.3892	-5.9925	-6.3590	-7.3919	-8.9131	-10.6689	-12.3149	-13.4694	-13.9752
-13.9585	-13.4389	-12.4902	-11.3138	-10.1949	-9.4015	-9.1152	-9.3877	-10.1493	-11.2391	-12.4194	-13.3940	-13.9269	-14.0680
-14.0992	-13.9924	-13.6375	-13.0471	-12.4096	-11.9308	-11.7530	-11.9241	-12.3929	-13.0230	-13.6128	-13.9671	-14.0697	-14.0730
-14.1042	-14.1017	-14.0756	-13.9673	-13.7695	-13.5785	-13.5004	-13.5735	-13.7595	-13.9524	-14.0560	-14.0771	-14.0747	-14.0722
-14.1034	-14.1009	-14.0985	-14.0960	-14.0935	-14.0911	-14.0886	-14.0862	-14.0837	-14.0813	-14.0788	-14.0763	-14.0739	-14.0714

Table B.50 Finite Difference v component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0236	-0.0351	0.0048	0.0328	0.0428	0.0326	0.0043	-0.0354	-0.0236	0.0000	0.0000	0.0000
-0.0058	-0.0870	-0.0671	-0.0047	0.0477	0.0818	0.0938	0.0793	0.0396	-0.0151	-0.0727	-0.0877	-0.0058	0.0000
-0.1241	-0.1178	-0.0304	0.0501	0.1048	0.1367	0.1477	0.1261	0.0662	-0.0128	-0.0837	-0.1390	-0.1263	-0.0058
-0.1837	-0.0737	0.0553	0.1442	0.1805	0.1949	0.2003	0.1669	0.0723	-0.0504	-0.1427	-0.1868	-0.2129	-0.0993
-0.1552	0.0250	0.1996	0.2798	0.2727	0.2526	0.2470	0.1971	0.0526	-0.1361	-0.2630	-0.2877	-0.2688	-0.2219
-0.0955	0.1620	0.3769	0.4339	0.3675	0.3031	0.2837	0.2155	0.0144	-0.2512	-0.4215	-0.4262	-0.3484	-0.2935
-0.0336	0.2851	0.5274	0.5597	0.4409	0.3384	0.3072	0.2242	-0.0230	-0.3524	-0.5602	-0.5503	-0.4229	-0.3331
-0.0071	0.3349	0.5868	0.6085	0.4688	0.3512	0.3152	0.2266	-0.0386	-0.3928	-0.6157	-0.6005	-0.4536	-0.3464
-0.0336	0.2852	0.5274	0.5597	0.4409	0.3384	0.3072	0.2242	-0.0230	-0.3524	-0.5602	-0.5503	-0.4229	-0.3331
-0.0955	0.1620	0.3769	0.4340	0.3675	0.3031	0.2837	0.2155	0.0144	-0.2513	-0.4215	-0.4262	-0.3484	-0.2935
-0.1552	0.0251	0.1996	0.2799	0.2727	0.2526	0.2470	0.1971	0.0526	-0.1361	-0.2630	-0.2877	-0.2688	-0.2219
-0.1837	-0.0737	0.0553	0.1442	0.1805	0.1949	0.2003	0.1669	0.0723	-0.0504	-0.1427	-0.1868	-0.2129	-0.0993
-0.1241	-0.1178	-0.0304	0.0501	0.1048	0.1367	0.1477	0.1261	0.0662	-0.0128	-0.0837	-0.1390	-0.1263	-0.0058
-0.0058	-0.0870	-0.0671	-0.0047	0.0477	0.0818	0.0938	0.0793	0.0396	-0.0151	-0.0727	-0.0877	-0.0058	0.0000
0.0000	0.0000	-0.0236	-0.0351	0.0048	0.0328	0.0428	0.0326	0.0043	-0.0354	-0.0236	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.51 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (m/sec).

CASE III.B.1 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 1.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = Az + B_O + C_O(x - x^{(R)})$

14.7842	14.7819	14.7795	14.7772	14.7748	14.7725	14.7701	14.7678	14.7654	14.7631	14.7607	14.7584	14.7560	14.7537	14.7514
14.7835	14.7812	14.7788	14.7765	14.6840	14.4574	14.2488	14.1651	14.2443	14.4482	14.6701	14.7577	14.7553	14.7530	14.7506
14.7828	14.7804	14.7561	14.3987	13.7783	13.1262	12.6439	12.4662	12.6400	13.1181	13.7654	14.3804	14.7326	14.7522	14.7499
14.7821	14.7577	14.2563	13.2744	12.0953	11.0099	10.2582	9.9903	10.2553	11.0034	12.0843	13.2579	14.2338	14.7296	14.7492
14.7813	14.4018	13.2757	11.6878	9.9938	8.5437	7.5959	7.2714	7.5942	8.5393	9.9854	11.6738	13.2551	14.3744	14.7484
14.6905	13.7826	12.0976	9.9946	7.9176	6.2985	5.3821	5.1112	5.3815	6.2962	7.9119	9.9833	12.0792	13.7567	14.6578
14.4652	13.1315	11.0126	8.5445	6.2985	4.8698	4.4158	4.3884	4.4158	4.8692	6.2951	8.5358	10.9965	13.1071	14.4332
14.2579	12.6500	10.2613	7.5967	5.3818	4.4158	4.5642	4.7704	4.5640	4.4157	5.3800	7.5897	10.2467	12.6268	14.2264
14.1756	12.4733	9.9939	7.2722	5.1108	4.3884	4.7708	5.0522	4.7705	4.3884	5.1094	7.2657	9.9798	12.4505	14.1444
14.2565	12.6486	10.2599	7.5955	5.3810	4.4156	4.5646	4.7710	4.5645	4.4155	5.3791	7.5884	10.2453	12.6253	14.2250
14.4623	13.1286	11.0098	8.5419	6.2963	4.8685	4.4155	4.3885	4.4155	4.8678	6.2929	8.5331	10.9937	13.1042	14.4303
14.6861	13.7783	12.0933	9.9904	7.9138	6.2952	5.3794	5.1089	5.3788	6.2929	7.9081	9.9792	12.0750	13.7524	14.6534
14.7755	14.3959	13.2699	11.6821	9.9883	8.5384	7.5909	7.2665	7.5892	8.5340	9.9799	11.6681	13.2493	14.3686	14.7426
14.7747	14.7504	14.2490	13.2672	12.0882	11.0029	10.2513	9.9834	10.2484	10.9964	12.0772	13.2507	14.2266	14.7223	14.7419
14.7740	14.7716	14.7473	14.3899	13.7696	13.1176	12.6353	12.4576	12.6314	13.1094	13.7567	14.3717	14.7239	14.7435	14.7411
14.7733	14.7709	14.7686	14.7662	14.6738	14.4472	14.2386	14.1549	14.2341	14.4380	14.6598	14.7474	14.7451	14.7428	14.7404
14.7725	14.7702	14.7678	14.7655	14.7631	14.7608	14.7584	14.7561	14.7537	14.7514	14.7491	14.7467	14.7444	14.7420	14.7397

Table B.52 Analytic surface geostrophic wind velocity (m/sec).

14.7842	14.7819	14.7795	14.7772	14.7748	14.7726	14.7705	14.7684	14.7658	14.7632	14.7607	14.7584	14.7560	14.7537	14.7514
14.7835	14.7812	14.7788	14.7541	14.6527	14.4693	14.2935	14.2221	14.2888	14.4597	14.6385	14.7353	14.7553	14.7530	14.7506
14.7828	14.7749	14.6734	14.3393	13.7922	13.2140	12.7895	12.6341	12.7832	13.1983	13.7694	14.3158	14.6493	14.7468	14.7499
14.7765	14.6394	14.1477	13.2647	12.1997	11.2261	10.5660	10.3354	10.5536	11.1845	12.1303	13.1979	14.1050	14.6092	14.7437
14.6873	14.2278	13.2152	11.7765	10.2264	8.9080	8.0773	7.8035	8.0532	8.8123	10.0446	11.5797	13.0880	14.1727	14.6537
14.4925	13.6372	12.1324	10.2168	8.2663	6.7340	5.9018	5.6694	5.8695	6.5767	7.9181	9.7920	11.8234	13.5039	14.4463
14.2310	13.0428	11.1690	8.8971	6.6905	5.1952	4.6893	4.6471	4.6750	5.0353	6.1970	8.2062	10.6150	12.7807	14.1539
14.0235	12.6190	10.5220	8.0407	5.7454	4.5297	4.5389	4.7173	4.5632	4.4568	5.2075	7.1485	9.7546	12.2316	13.9100
13.9456	12.4667	10.2959	7.7482	5.4479	4.4056	4.6442	4.9036	4.6863	4.3893	4.9197	6.7852	9.4442	12.0270	13.8155
14.0220	12.6167	10.5162	8.0241	5.7020	4.4436	4.4273	4.6003	4.4523	4.3696	5.1599	7.1301	9.7485	12.2293	13.9085
14.2280	13.0373	11.1511	8.8408	6.5419	4.8854	4.2311	4.1391	4.2157	4.7157	6.0369	8.1456	10.5964	12.7751	14.1509
14.4880	13.6296	12.1053	10.1285	8.0379	6.2666	5.1742	4.8274	5.1378	6.0980	7.6800	9.7003	11.7958	13.4963	14.4419
14.6815	14.2203	13.1916	11.6966	10.0197	8.4988	7.4558	7.0879	7.4301	8.3991	9.8346	11.4987	13.0643	14.1652	14.6478
14.7692	14.6318	14.1337	13.2198	12.0791	10.9858	10.2049	9.9220	10.1923	10.9436	12.0092	13.1530	14.0911	14.6016	14.7364
14.7740	14.7661	14.6639	14.3215	13.7470	13.1207	12.6460	12.4689	12.6398	13.1050	13.7243	14.2980	14.6398	14.7380	14.7411
14.7733	14.7709	14.7686	14.7435	14.6381	14.4444	14.2565	14.1794	14.2518	14.4349	14.6239	14.7248	14.7451	14.7428	14.7404
14.7725	14.7702	14.7678	14.7655	14.7631	14.7607	14.7580	14.7555	14.7533	14.7513	14.7491	14.7467	14.7444	14.7420	14.7397

Table B.53 Finite Difference surface geostrophic wind velocity (m/sec).

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0001	-0.0004	-0.0006	-0.0004	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	0.0224	0.0313	-0.0119	-0.0447	-0.0571	-0.0445	-0.0115	0.0315	0.0223	-0.0000	-0.0000	-0.0000
-0.0000	0.0055	0.0827	0.0594	-0.0138	-0.0878	-0.1456	-0.1679	-0.1432	-0.0802	-0.0040	0.0646	0.0834	0.0055	-0.0000
0.0055	0.1183	0.1087	0.0097	-0.1044	-0.2162	-0.3078	-0.3451	-0.2983	-0.1810	-0.0460	0.0600	0.1288	0.1204	0.0055
0.0940	0.1740	0.0606	-0.0888	-0.2325	-0.3643	-0.4813	-0.5321	-0.4590	-0.2731	-0.0592	0.0940	0.1672	0.2017	0.0948
0.1980	0.1454	-0.0348	-0.2222	-0.3487	-0.4355	-0.5197	-0.5581	-0.4880	-0.2804	-0.0063	0.1913	0.2559	0.2528	0.2115
0.2342	0.0887	-0.1564	-0.3525	-0.3920	-0.3253	-0.2735	-0.2587	-0.2592	-0.1661	0.0981	0.3295	0.3815	0.3264	0.2793
0.2344	0.0311	-0.2607	-0.4440	-0.3636	-0.1139	0.0253	0.0531	0.0008	-0.0411	0.1724	0.4412	0.4921	0.3952	0.3164
0.2300	0.0066	-0.3020	-0.4761	-0.3371	-0.0172	0.1266	0.1486	0.0843	-0.0008	0.1897	0.4805	0.5356	0.4235	0.3289
0.2345	0.0319	-0.2563	-0.4287	-0.3210	-0.0280	0.1373	0.1706	0.1121	0.0460	0.2192	0.4583	0.4968	0.3961	0.3165
0.2343	0.0913	-0.1412	-0.2989	-0.2456	-0.0169	0.1843	0.2494	0.1998	0.1521	0.2560	0.3875	0.3973	0.3291	0.2794
0.1981	0.1487	-0.0120	-0.1380	-0.1241	0.0286	0.2053	0.2815	0.2410	0.1949	0.2280	0.2789	0.2792	0.2561	0.2116
0.0940	0.1757	0.0783	-0.0145	-0.0313	0.0397	0.1352	0.1786	0.1591	0.1349	0.1453	0.1694	0.1851	0.2034	0.0948
0.0055	0.1186	0.1153	0.0474	0.0091	0.0171	0.0464	0.0614	0.0561	0.0528	0.0680	0.0977	0.1355	0.1206	0.0055
-0.0000	0.0055	0.0834	0.0684	0.0226	-0.0032	-0.0107	-0.0113	-0.0084	0.0044	0.0324	0.0737	0.0841	0.0055	-0.0000
-0.0000	-0.0000	-0.0000	0.0227	0.0357	0.0027	-0.0179	-0.0246	-0.0177	0.0032	0.0359	0.0227	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0001	0.0004	0.0006	0.0004	0.0001	0.0000	-0.0000	-0.0000	-0.0000	-0.0000

Table B.54 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (m/sec).

CASE III.A.2 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 2.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = A \ln p + B_O + C_O(x - x^{(R)})$

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.55 Analytic u component of surface geostrophic wind (m/sec).

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.38[illegible]	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.56 Finite Difference u component of surface geostrophic wind (m/sec).

0.2394	0.1909	-0.0474	0.2264	0.2306	0.0163	0.3013	0.3860	0.2387	0.3119	-0.2412	-0.0852	-0.2437	0.1659	-0.2795
0.1669	-0.0580	-0.2403	0.2164	-0.3864	-0.0626	-0.0412	0.1707	-0.2063	0.1144	-0.5458	0.0747	-0.2241	-0.0959	-0.3364
0.0571	-0.4474	0.2631	-0.1631	0.0684	-0.1553	0.0001	-0.2559	0.0945	-0.2970	0.1156	-0.1395	0.1451	-0.2381	0.1667
-0.0907	-0.1575	0.2675	-0.1345	0.1095	0.0276	0.0294	-0.2867	0.1238	-0.0668	0.2983	-0.1345	0.2203	-0.2253	0.3143
-0.2318	0.2615	-0.3294	0.0817	-0.0229	0.0799	-0.0891	0.0790	-0.0891	0.2686	0.0716	-0.0127	-0.2822	0.0491	-0.3420
0.2333	0.1264	-0.0880	0.0544	0.2878	0.1156	-0.3026	0.3668	0.1694	0.3044	0.1934	0.1488	0.0537	0.1500	-0.0440
0.2184	-0.1540	0.1692	-0.1651	0.4553	-0.0946	-0.2810	-0.3647	0.1909	-0.0946	0.2664	0.0709	0.2165	-0.0832	0.1358
-0.2777	0.0485	-0.1104	0.1190	-0.3038	-0.0510	0.0361	0.1598	-0.1055	0.0906	-0.3982	0.1190	-0.0986	0.1666	-0.2187
-0.3155	0.0621	-0.1621	0.1683	-0.4453	0.2391	-0.4690	0.4515	-0.4453	0.0975	-0.3273	0.0503	-0.1621	0.0385	-0.1651
0.0951	-0.1958	0.0930	-0.1364	0.3808	-0.1788	0.0645	-0.2716	-0.0535	-0.3677	0.2628	-0.1600	0.1284	-0.2194	0.2337
0.0710	0.1367	0.1674	0.0769	0.1646	-0.1824	0.6650	0.2765	0.0040	0.1008	-0.0714	-0.0647	0.0258	0.0422	0.2834
-0.2036	0.1158	-0.0946	0.1406	-0.3760	0.1738	0.1201	0.3002	-0.2576	0.1738	-0.0928	-0.0481	-0.0003	0.2102	-0.1681
0.3199	0.0750	0.0052	-0.0756	0.0291	-0.2624	0.0953	-0.0728	0.2842	-0.3569	0.2179	-0.0756	0.2884	-0.1373	0.0055
0.3126	-0.1468	-0.0725	-0.1426	0.0855	-0.3991	-0.0232	-0.1791	0.2601	-0.2103	-0.0089	-0.0481	-0.1197	-0.1918	0.1786
-0.2919	-0.1921	-0.1057	0.1457	0.0322	0.1615	-0.1828	0.2619	-0.4660	-0.1689	0.0322	-0.2792	-0.2407	0.2141	-0.4208
-0.2588	-0.0295	0.1079	0.0568	-0.3744	0.2340	-0.0943	0.1187	-0.0235	-0.1436	0.1153	-0.0632	0.2771	0.0223	0.2179
0.1840	-0.0286	0.1125	-0.4571	-0.0417	-0.0047	0.0277	-0.1372	0.3201	-0.1157	0.0461	0.0795	-0.0662	-0.0819	0.4937

Table B.57 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (10^{-11}m/sec).

CASE III.A.2 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 2.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = A \ln p + B_O + C_O(x - x^{(R)})$

-19.0731	-19.0667	-19.0602	-19.0538	-19.0473	-19.0409	-19.0345	-19.0280	-19.0216	-19.0152	-19.0088	-19.0023	-18.9959	-18.9895
-19.0716	-19.0652	-19.0587	-18.8694	-18.4068	-17.9797	-17.8071	-17.9675	-18.3819	-18.8312	-19.0073	-19.0009	-18.9945	-18.9881
-19.0701	-19.0190	-18.2902	-17.0183	-15.6696	-14.6630	-14.2896	-14.6530	-15.6482	-16.9837	-18.2407	-18.9549	-18.9930	-18.9866
-19.0240	-18.0011	-15.9792	-13.5107	-11.1894	-9.5474	-8.9540	-9.5408	-11.1739	-13.4829	-15.9357	-17.9402	-18.9470	-18.9851
-18.2996	-15.9832	-12.6461	-8.9619	-5.6514	-3.3620	-2.5444	-3.3596	-5.6435	-8.9431	-12.6113	-15.9288	-18.2254	-18.9836
-17.0312	-13.5170	-8.9635	-4.1525	0.0778	2.9714	3.9983	2.9692	0.0777	-4.1437	-8.9385	-13.4706	-16.9619	-18.7999
-15.6850	-11.1965	-5.6523	0.0794	5.0649	8.4562	9.6559	8.4499	5.0575	0.0793	-5.6364	-11.1578	-15.6209	-18.3434
-14.6806	-9.5546	-3.3609	2.9767	8.4609	12.1820	13.4965	12.1729	8.4484	2.9702	-3.3514	-9.5214	-14.6204	-17.9221
-14.3100	-8.9620	-2.5418	4.0075	9.6663	13.5032	14.8579	13.4930	9.6520	3.9987	-2.5346	-8.9308	-14.2512	-17.7546
-14.6776	-9.5515	-3.3578	2.9799	8.4641	12.1853	13.4997	12.1762	8.4517	2.9734	-3.3483	-9.5184	-14.6174	-17.9192
-15.6790	-11.1904	-5.6461	0.0857	5.0713	8.4626	9.6624	8.4564	5.0639	0.0855	-5.6302	-11.1517	-15.6149	-18.3375
-17.0223	-13.5079	-8.9543	-4.1432	0.0873	2.9809	4.0079	2.9787	0.0872	-4.1344	-8.9293	-13.4616	-16.9530	-18.7910
-18.2877	-15.9712	-12.6340	-8.9496	-5.6391	-3.3495	-2.5319	-3.3472	-5.6311	-8.9309	-12.5992	-15.9168	-18.2135	-18.9718
-19.0091	-17.9862	-15.9643	-13.4956	-11.1742	-9.5321	-8.9387	-9.5255	-11.1588	-13.4678	-15.9208	-17.9253	-18.9322	-18.9703
-19.0523	-19.0012	-18.2723	-17.0004	-15.6516	-14.6450	-14.2715	-14.6349	-15.6302	-16.9658	-18.2229	-18.9371	-18.9752	-18.9688
-19.0508	-19.0444	-19.0379	-18.8486	-18.3860	-17.9588	-17.7863	-17.9467	-18.3611	-18.8104	-18.9865	-18.9801	-18.9737	-18.9673
-19.0493	-19.0429	-19.0365	-19.0300	-19.0236	-19.0172	-19.0107	-19.0043	-18.9979	-18.9915	-18.9851	-18.9787	-18.9723	-18.9659

Table B.58 Analytic v component of surface geostrophic wind (m/sec).

-19.0731	-19.0667	-19.0602	-19.0538	-19.0473	-19.0409	-19.0345	-19.0280	-19.0216	-19.0152	-19.0088	-19.0023	-18.9959	-18.9895
-19.0716	-19.0652	-19.0130	-18.8011	-18.4157	-18.0433	-17.8904	-18.0310	-18.3906	-18.7629	-18.9617	-19.0009	-18.9945	-18.9881
-19.0590	-18.8496	-18.1544	-16.9991	-15.7551	-14.8213	-14.4738	-14.8109	-15.7332	-16.9640	-18.1050	-18.7858	-18.9818	-18.9866
-18.7810	-17.7514	-15.8676	-13.5475	-11.3592	-9.8096	-9.2491	-9.8024	-11.3429	-13.5189	-15.8236	-17.6908	-18.7048	-18.9740
-17.9140	-15.7280	-12.5593	-9.0553	-5.9067	-3.7300	-2.9526	-3.7268	-5.8974	-9.0353	-12.5236	-15.6736	-17.8408	-18.7925
-16.6171	-13.2572	-8.8991	-4.2977	-0.2564	2.5047	3.4843	2.5036	-0.2548	-4.2871	-8.8728	-13.2104	-16.5486	-18.3843
-15.2493	-10.9326	-5.6054	-0.1071	4.6664	7.9083	9.0545	7.9033	4.6611	-0.1051	-5.5878	-10.8932	-15.1857	-17.8205
-14.2308	-9.2877	-3.3249	2.7633	8.0201	11.5803	12.8370	11.5727	8.0100	2.7594	-3.3135	-9.2537	-14.1711	-17.3594
-13.8553	-8.6939	-2.5095	3.7849	9.2108	12.8826	14.1780	12.8740	9.1990	3.7787	-2.5003	-8.6619	-13.7970	-17.1829
-14.2278	-9.2846	-3.3218	2.7665	8.0234	11.5836	12.8402	11.5759	8.0132	2.7626	-3.3104	-9.2506	-14.1681	-17.3564
-15.2432	-10.9265	-5.5992	-0.1008	4.6727	7.9148	9.0610	7.9098	4.6674	-0.0988	-5.5817	-10.8872	-15.1798	-17.8145
-16.6082	-13.2481	-8.8899	-4.2883	-0.2469	2.5142	3.4938	2.5131	-0.2453	-4.2778	-8.8637	-13.2014	-16.5396	-18.3755
-17.9021	-15.7160	-12.5472	-9.0431	-5.8943	-3.7175	-2.9401	-3.7143	-5.8851	-9.0230	-12.5115	-15.6616	-17.8289	-18.7806
-18.7662	-17.7365	-15.8526	-13.5324	-11.3440	-9.7943	-9.2338	-9.7871	-11.3277	-13.5038	-15.8087	-17.6759	-18.6900	-18.9592
-19.0412	-18.8318	-18.1366	-16.9812	-15.7371	-14.8033	-14.4557	-14.7929	-15.7153	-16.9462	-18.0872	-18.7681	-18.9641	-18.9688
-19.0508	-19.0444	-18.9922	-18.7803	-18.3949	-18.0225	-17.8695	-18.0102	-18.3698	-18.7421	-18.9409	-18.9801	-18.9737	-18.9673
-19.0494	-19.0429	-19.0365	-19.0300	-19.0236	-19.0172	-19.0107	-19.0043	-18.9979	-18.9915	-18.9851	-18.9787	-18.9723	-18.9659

Table B.59 Finite Difference v component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0457	-0.0683	0.0089	0.0636	0.0832	0.0635	0.0087	-0.0683	-0.0456	0.0000	0.0000	0.0000
-0.0112	-0.1694	-0.1357	-0.0192	0.0856	0.1583	0.1842	0.1580	0.0851	-0.0196	-0.1357	-0.1690	-0.0111	0.0000
-0.2429	-0.2498	-0.1117	0.0368	0.1698	0.2621	0.2950	0.2616	0.1689	0.0360	-0.1121	-0.2494	-0.2422	-0.0111
-0.3856	-0.2552	-0.0868	0.0935	0.2553	0.3680	0.4082	0.3672	0.2540	0.0921	-0.0877	-0.2552	-0.3846	-0.1912
-0.4141	-0.2598	-0.0643	0.1451	0.3342	0.4666	0.5140	0.4656	0.3325	0.1433	-0.0657	-0.2602	-0.4133	-0.4155
-0.4357	-0.2639	-0.0469	0.1866	0.3986	0.5479	0.6014	0.5466	0.3965	0.1844	-0.0486	-0.2646	-0.4351	-0.5229
-0.4498	-0.2669	-0.0360	0.2133	0.4408	0.6017	0.6595	0.6002	0.4384	0.2108	-0.0379	-0.2678	-0.4493	-0.5628
-0.4547	-0.2680	-0.0323	0.2226	0.4555	0.6205	0.6799	0.6190	0.4531	0.2200	-0.0343	-0.2690	-0.4543	-0.5716
-0.4498	-0.2669	-0.0360	0.2133	0.4408	0.6017	0.6595	0.6002	0.4384	0.2109	-0.0379	-0.2678	-0.4493	-0.5628
-0.4357	-0.2639	-0.0469	0.1866	0.3986	0.5479	0.6015	0.5466	0.3965	0.1844	-0.0486	-0.2646	-0.4351	-0.5229
-0.4141	-0.2598	-0.0643	0.1451	0.3342	0.4667	0.5140	0.4656	0.3325	0.1433	-0.0657	-0.2602	-0.4133	-0.4155
-0.3856	-0.2552	-0.0868	0.0935	0.2553	0.3680	0.4082	0.3672	0.2540	0.0921	-0.0877	-0.2552	-0.3846	-0.1912
-0.2429	-0.2498	-0.1117	0.0368	0.1698	0.2622	0.2951	0.2616	0.1690	0.0360	-0.1121	-0.2494	-0.2422	-0.0111
-0.0112	-0.1694	-0.1358	-0.0192	0.0856	0.1583	0.1842	0.1580	0.0851	-0.0196	-0.1357	-0.1690	-0.0111	0.0000
0.0000	0.0000	-0.0457	-0.0683	0.0089	0.0636	0.0832	0.0635	0.0087	-0.0683	-0.0456	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.60 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (m/sec).

CASE III.A.2 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 2.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = A \ln p + B_o + C_o(x - x^{(R)})$

19.5776	19.5713	19.5650	19.5587	19.5525	19.5462	19.5399	19.5336	19.5274	19.5211	19.5149	19.5086	19.5024	19.4961	19.4899
19.5762	19.5699	19.5636	19.5573	19.3729	18.9225	18.5073	18.3398	18.4955	18.8983	19.3356	19.5072	19.5009	19.4947	19.4884
19.5747	19.5684	19.5186	18.8091	17.5749	16.2723	15.3054	14.9481	15.2958	16.2517	17.5413	18.7610	19.4561	19.4932	19.4870
19.5733	19.5234	18.5282	16.5707	14.2054	12.0190	10.5074	9.9713	10.5014	12.0046	14.1789	16.5288	18.4690	19.4484	19.4856
19.5718	18.8183	16.5746	13.3857	9.9784	7.1548	5.5277	5.0722	5.5263	7.1485	9.9616	13.3528	16.5221	18.7461	19.4841
19.3920	17.5874	14.2113	9.9798	6.0412	4.3885	5.2992	5.9363	5.2980	4.3885	6.0352	9.9574	14.1672	17.5203	19.3051
18.9457	16.2872	12.0256	7.1555	4.3885	6.7012	9.5268	10.6061	9.5213	6.6956	4.3885	7.1430	11.9896	16.2254	18.8609
18.5343	15.3223	10.5140	5.5271	5.3022	9.5310	12.9481	14.1918	12.9396	9.5199	5.2986	5.5213	10.4838	15.2647	18.4514
18.3708	14.9676	9.9785	5.0709	5.9425	10.6156	14.1982	15.4922	14.1886	10.6026	5.9366	5.0672	9.9505	14.9114	18.2887
18.5314	15.3194	10.5112	5.5252	5.3040	9.5339	12.9512	14.1949	12.9427	9.5228	5.3004	5.5194	10.4811	15.2618	18.4486
18.9399	16.2814	12.0199	7.1506	4.3887	6.7060	9.5325	10.6120	9.5270	6.7005	4.3886	7.1381	11.9839	16.2197	18.8551
19.3833	17.5787	14.2027	9.9715	6.0348	4.3887	5.3046	5.9427	5.3034	4.3887	6.0288	9.9492	14.1586	17.5116	19.2965
19.5602	18.8067	16.5630	13.3742	9.9674	7.1451	5.5202	5.0659	5.5187	7.1388	9.9506	13.3414	16.5106	18.7346	19.4726
19.5588	19.5090	18.5137	16.5563	14.1910	12.0048	10.4936	9.9576	10.4875	11.9904	14.1646	16.5143	18.4546	19.4340	19.4711
19.5573	19.5511	19.5012	18.7918	17.5575	16.2550	15.2882	14.9308	15.2786	16.2344	17.5240	18.7437	19.4388	19.4759	19.4697
19.5559	19.5496	19.5433	19.5370	19.3526	18.9023	18.4871	18.3195	18.4753	18.8781	19.3154	19.4870	19.4807	19.4745	19.4683
19.5544	19.5482	19.5419	19.5356	19.5293	19.5231	19.5168	19.5105	19.5043	19.4980	19.4918	19.4855	19.4793	19.4730	19.4668

Table B.61 Analytic surface geostrophic wind velocity (m/sec).

19.5776	19.5713	19.5650	19.5587	19.5525	19.5462	19.5399	19.5336	19.5274	19.5211	19.5149	19.5086	19.5024	19.4961	19.4899
19.5762	19.5699	19.5636	19.5128	19.3063	18.9312	18.5692	18.4206	18.5572	18.9068	19.2691	19.4627	19.5009	19.4947	19.4884
19.5747	19.5575	19.3535	18.6772	17.5563	16.3547	15.4571	15.1243	15.4472	16.3336	17.5223	18.6291	19.2915	19.4824	19.4870
19.5624	19.2868	18.2856	16.4631	14.2404	12.1772	10.7462	10.2371	10.7396	12.1620	14.2131	16.4207	18.2268	19.2126	19.4747
19.3849	18.4436	16.3286	13.3037	10.0624	7.3581	5.7589	5.2887	5.7569	7.3507	10.0443	13.2700	16.2762	18.3724	19.2979
18.9860	17.1867	13.9644	9.9220	6.1419	4.3953	5.0524	5.6030	5.0518	4.3952	6.1345	9.8985	13.9201	17.1204	18.9007
18.4356	15.8680	11.7802	7.1185	4.3891	6.4053	9.0440	10.0617	9.0397	6.4014	4.3891	7.1047	11.7437	15.8069	18.3527
17.9862	14.8919	10.2720	5.5053	5.1855	9.1420	12.3837	13.5661	12.3766	9.1331	5.1834	5.4984	10.2413	14.8349	17.9053
17.8144	14.5334	9.7385	5.0548	5.7947	10.2026	13.6094	14.8414	13.6012	10.1919	5.7907	5.0502	9.7098	14.4779	17.7343
17.9833	14.8890	10.2692	5.5034	5.1872	9.1448	12.3868	13.5692	12.3796	9.1359	5.1850	5.4965	10.2385	14.8320	17.9024
18.4298	15.8622	11.7746	7.1137	4.3890	6.4100	9.0497	10.0675	9.0453	6.4061	4.3889	7.0999	11.7381	15.8012	18.3470
18.9773	17.1780	13.9558	9.9138	6.1354	4.3948	5.0571	5.6089	5.0566	4.3947	6.1280	9.8903	13.9115	17.1118	18.8921
19.3733	18.4320	16.3171	13.2923	10.0514	7.3482	5.7509	5.2818	5.7488	7.3408	10.0334	13.2586	16.2646	18.3609	19.2864
19.5479	19.2723	18.2711	16.4486	14.2260	12.1631	10.7323	10.2233	10.7257	12.1478	14.1988	16.4063	18.2124	19.1982	19.4603
19.5573	19.5402	19.3362	18.6598	17.5389	16.3374	15.4399	15.1070	15.4299	16.3163	17.5050	18.6118	19.2742	19.4651	19.4697
19.5559	19.5496	19.5433	19.4925	19.2861	18.9109	18.5489	18.4004	18.5370	18.8866	19.2489	19.4425	19.4807	19.4745	19.4683
19.5544	19.5482	19.5419	19.5356	19.5293	19.5231	19.5168	19.5105	19.5043	19.4980	19.4918	19.4855	19.4793	19.4731	19.4668

Table B.62 Finite Difference surface geostrophic wind velocity (m/sec).

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	0.0445	0.0666	-0.0086	-0.0618	-0.0808	-0.0617	-0.0085	0.0665	0.0445	-0.0000	-0.0000	-0.0000
-0.0000	0.0109	0.1651	0.1320	0.0186	-0.0824	-0.1517	-0.1762	-0.1514	-0.0819	0.0190	0.1319	0.1646	0.0108	-0.0000
0.0109	0.2366	0.2426	0.1077	-0.0350	-0.1583	-0.2388	-0.2658	-0.2382	-0.1574	-0.0342	0.1080	0.2422	0.2358	0.0108
0.1870	0.3747	0.2459	0.0820	-0.0840	-0.2033	-0.2312	-0.2166	-0.2306	-0.2021	-0.0828	0.0828	0.2459	0.3737	0.1862
0.4060	0.4007	0.2469	0.0578	-0.1007	-0.0068	0.2469	0.3333	0.2462	-0.0067	-0.0993	0.0589	0.2472	0.3998	0.4044
0.5101	0.4192	0.2453	0.0370	-0.0006	0.2959	0.4828	0.5445	0.4816	0.2942	-0.0005	0.0383	0.2458	0.4185	0.5082
0.5481	0.4304	0.2420	0.0218	0.1168	0.3890	0.5644	0.6257	0.5630	0.3869	0.1153	0.0229	0.2426	0.4298	0.5461
0.5564	0.4341	0.2400	0.0161	0.1478	0.4130	0.5888	0.6508	0.5873	0.4107	0.1459	0.0171	0.2407	0.4335	0.5544
0.5481	0.4304	0.2420	0.0218	0.1169	0.3891	0.5644	0.6257	0.5630	0.3869	0.1153	0.0229	0.2426	0.4298	0.5461
0.5101	0.4192	0.2453	0.0370	-0.0003	0.2961	0.4829	0.5446	0.4817	0.2944	-0.0003	0.0382	0.2458	0.4185	0.5082
0.4060	0.4007	0.2469	0.0577	-0.1006	-0.0061	0.2475	0.3338	0.2468	-0.0060	-0.0992	0.0589	0.2472	0.3998	0.4044
0.1870	0.3747	0.2459	0.0819	-0.0840	-0.2031	-0.2307	-0.2159	-0.2301	-0.2020	-0.0828	0.0828	0.2459	0.3737	0.1862
0.0109	0.2366	0.2426	0.1077	-0.0350	-0.1582	-0.2387	-0.2657	-0.2382	-0.1574	-0.0342	0.1080	0.2422	0.2358	0.0108
-0.0000	0.0109	0.1651	0.1320	0.0186	-0.0824	-0.1517	-0.1762	-0.1514	-0.0819	0.0190	0.1319	0.1646	0.0108	-0.0000
-0.0000	-0.0000	-0.0000	0.0445	0.0666	-0.0086	-0.0618	-0.0808	-0.0617	-0.0085	0.0665	0.0445	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000

Table B.63 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (m/sec).

CASE III.B.2 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 2.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = Az + B_O + C_O(x - x^{(R)})$

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.64 Analytic u component of surface geostrophic wind (m/sec).

-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3881	-4.3892	-4.3900	-4.3892	-4.3881	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3883	-4.3952	-4.4120	-4.4316	-4.4405	-4.4315	-4.4119	-4.3952	-4.3883	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3890	-4.4027	-4.4451	-4.5151	-4.5836	-4.6122	-4.5833	-4.5148	-4.4449	-4.4026	-4.3890	-4.3878	-4.3878
-4.3878	-4.3883	-4.3986	-4.4449	-4.5452	-4.6836	-4.8077	-4.8576	-4.8071	-4.6828	-4.5446	-4.4445	-4.3985	-4.3883	-4.3878
-4.3878	-4.3906	-4.4147	-4.4875	-4.6205	-4.7893	-4.9343	-4.9916	-4.9336	-4.7882	-4.6196	-4.4869	-4.4145	-4.3906	-4.3878
-4.3879	-4.3930	-4.4194	-4.4857	-4.5970	-4.7323	-4.8461	-4.8906	-4.8454	-4.7313	-4.5962	-4.4852	-4.4192	-4.3930	-4.3879
-4.3880	-4.3918	-4.4072	-4.4422	-4.4985	-4.5653	-4.6209	-4.6425	-4.6205	-4.5648	-4.4980	-4.4419	-4.4070	-4.3918	-4.3880
-4.3879	-4.3890	-4.3931	-4.4019	-4.4156	-4.4318	-4.4451	-4.4503	-4.4451	-4.4317	-4.4155	-4.4018	-4.3931	-4.3890	-4.3879
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878
-4.3877	-4.3866	-4.3826	-4.3738	-4.3601	-4.3440	-4.3306	-4.3255	-4.3307	-4.3441	-4.3602	-4.3739	-4.3826	-4.3866	-4.3877
-4.3876	-4.3838	-4.3685	-4.3336	-4.2774	-4.2107	-4.1552	-4.1336	-4.1555	-4.2112	-4.2779	-4.3339	-4.3687	-4.3839	-4.3876
-4.3877	-4.3826	-4.3563	-4.2902	-4.1790	-4.0439	-3.9303	-3.8858	-3.9309	-4.0449	-4.1799	-4.2907	-4.3565	-4.3827	-4.3877
-4.3878	-4.3850	-4.3611	-4.2884	-4.1555	-3.9870	-3.8421	-3.7848	-3.8428	-3.9881	-4.1565	-4.2890	-4.3612	-4.3850	-4.3878
-4.3878	-4.3874	-4.3771	-4.3309	-4.2307	-4.0925	-3.9686	-3.9187	-3.9692	-4.0933	-4.2313	-4.3312	-4.3772	-4.3874	-4.3878
-4.3878	-4.3878	-4.3866	-4.3730	-4.3307	-4.2608	-4.1924	-4.1638	-4.1927	-4.2611	-4.3309	-4.3731	-4.3866	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3873	-4.3804	-4.3637	-4.3442	-4.3353	-4.3442	-4.3638	-4.3805	-4.3873	-4.3878	-4.3878	-4.3878
-4.3878	-4.3878	-4.3878	-4.3878	-4.3878	-4.3875	-4.3864	-4.3857	-4.3864	-4.3875	-4.3878	-4.3878	-4.3878	-4.3878	-4.3878

Table B.65 Finite Difference u component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0014	0.0022	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0005	0.0074	0.0242	0.0438	0.0527	0.0437	0.0241	0.0074	0.0005	0.0000	0.0000	0.0000
0.0000	0.0000	0.0012	0.0149	0.0573	0.1273	0.1958	0.2244	0.1955	0.1270	0.0571	0.0148	0.0012	0.0000	0.0000
0.0000	0.0004	0.0108	0.0570	0.1574	0.2958	0.4199	0.4698	0.4193	0.2950	0.1568	0.0567	0.0107	0.0004	0.0000
0.0000	0.0028	0.0269	0.0996	0.2327	0.4015	0.5465	0.6038	0.5458	0.4004	0.2318	0.0991	0.0267	0.0028	0.0000
0.0001	0.0052	0.0316	0.0979	0.2092	0.3445	0.4582	0.5027	0.4576	0.3435	0.2084	0.0974	0.0314	0.0052	0.0001
0.0002	0.0040	0.0194	0.0544	0.1107	0.1775	0.2331	0.2547	0.2327	0.1770	0.1102	0.0541	0.0192	0.0040	0.0002
0.0001	0.0012	0.0053	0.0141	0.0278	0.0440	0.0573	0.0625	0.0572	0.0439	0.0277	0.0140	0.0052	0.0012	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0001	-0.0012	-0.0052	-0.0140	-0.0277	-0.0438	-0.0572	-0.0623	-0.0571	-0.0437	-0.0276	-0.0139	-0.0052	-0.0012	-0.0001
-0.0002	-0.0040	-0.0193	-0.0542	-0.1104	-0.1771	-0.2326	-0.2542	-0.2323	-0.1766	-0.1099	-0.0539	-0.0191	-0.0039	-0.0002
-0.0001	-0.0052	-0.0315	-0.0976	-0.2088	-0.3439	-0.4575	-0.5020	-0.4569	-0.3429	-0.2079	-0.0971	-0.0313	-0.0051	-0.0001
-0.0000	-0.0028	-0.0268	-0.0994	-0.2323	-0.4008	-0.5457	-0.6030	-0.5450	-0.3997	-0.2313	-0.0989	-0.0266	-0.0028	-0.0000
0.0000	-0.0004	-0.0107	-0.0569	-0.1571	-0.2953	-0.4192	-0.4691	-0.4187	-0.2945	-0.1565	-0.0566	-0.0107	-0.0004	0.0000
0.0000	-0.0000	-0.0012	-0.0148	-0.0571	-0.1270	-0.1954	-0.2240	-0.1952	-0.1267	-0.0569	-0.0147	-0.0012	-0.0000	0.0000
0.0000	0.0000	-0.0000	-0.0005	-0.0074	-0.0241	-0.0436	-0.0525	-0.0436	-0.0240	-0.0073	-0.0005	-0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-0.0000	-0.0003	-0.0014	-0.0021	-0.0014	-0.0003	-0.0000	0.0000	0.0000	0.0000	0.0000

Table B.66 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (m/sec).

CASE III.B.2 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 2.0 * 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = Az + B_O + C_O(x - x^{(R)})$

-19.2314	-19.2247	-19.2180	-19.2113	-19.2046	-19.1979	-19.1912	-19.1845	-19.1778	-19.1711	-19.1645	-19.1578	-19.1511	-19.1445
-19.2299	-19.2231	-19.2164	-19.0212	-18.5446	-18.1052	-17.9279	-18.0926	-18.5188	-18.9815	-19.1629	-19.1563	-19.1496	-19.1430
-19.2283	-19.1755	-18.4247	-17.1185	-15.7389	-14.7130	-14.3332	-14.7027	-15.7170	-17.0828	-18.3734	-19.1088	-19.1481	-19.1414
-19.1807	-18.1275	-16.0552	-13.5424	-11.1969	-9.5479	-8.9540	-9.5412	-11.1813	-13.5142	-16.0105	-18.0644	-19.1007	-19.1399
-18.4345	-16.0593	-12.6668	-8.9619	-5.6688	-3.4114	-2.6092	-3.4090	-5.6609	-8.9431	-12.6316	-16.0034	-18.3576	-19.1384
-17.1319	-13.5488	-8.9635	-4.1891	-0.0503	2.7485	3.7357	2.7466	-0.0503	-4.1803	-8.9385	-13.5017	-17.0604	-18.9490
-15.7547	-11.2040	-5.6697	-0.0488	4.7572	7.9823	9.1148	7.9767	4.7505	-0.0487	-5.6539	-11.1651	-15.6890	-18.4790
-14.7310	-9.5551	-3.4104	2.7535	7.9866	11.4841	12.7095	11.4761	7.9754	2.7477	-3.4009	-9.5219	-14.6695	-18.0457
-14.3540	-8.9620	-2.6067	3.7442	9.1242	12.7155	13.9728	12.7066	9.1115	3.7364	-2.5994	-8.9308	-14.2941	-17.8736
-14.7279	-9.5520	-3.4073	2.7565	7.9896	11.4872	12.7126	11.4792	7.9785	2.7508	-3.3978	-9.5188	-14.6665	-18.0427
-15.7486	-11.1979	-5.6636	-0.0427	4.7633	7.9884	9.1209	7.9828	4.7566	-0.0426	-5.6478	-11.1590	-15.6829	-18.4729
-17.1227	-13.5396	-8.9543	-4.1799	-0.0412	2.7577	3.7449	2.7558	-0.0411	-4.1711	-8.9293	-13.4925	-17.0512	-18.9398
-18.4222	-16.0470	-12.6546	-8.9496	-5.6566	-3.3991	-2.5969	-3.3968	-5.6487	-8.9309	-12.6194	-15.9912	-18.3454	-19.1262
-19.1654	-18.1121	-16.0399	-13.5271	-11.1816	-9.5326	-8.9387	-9.5260	-11.1660	-13.4989	-15.9952	-18.0491	-19.0854	-19.1247
-19.2099	-19.1572	-18.4063	-17.1002	-15.7205	-14.6946	-14.3148	-14.6844	-15.6986	-17.0645	-18.3551	-19.0905	-19.1298	-19.1231
-19.2084	-19.2017	-19.1950	-18.9998	-18.5232	-18.0838	-17.9065	-18.0712	-18.4974	-18.9601	-19.1416	-19.1349	-19.1283	-19.1216
-19.2069	-19.2002	-19.1935	-19.1868	-19.1801	-19.1734	-19.1667	-19.1600	-19.1534	-19.1467	-19.1400	-19.1334	-19.1267	-19.1201

Table B.67 Analytic v component of surface geostrophic wind (m/sec).

-19.2314	-19.2247	-19.2180	-19.2113	-19.2046	-19.1979	-19.1912	-19.1845	-19.1778	-19.1711	-19.1645	-19.1578	-19.1511	-19.1445
-19.2299	-19.2231	-19.1693	-18.9509	-18.5540	-18.1708	-18.0134	-18.1578	-18.5276	-18.9110	-19.1159	-19.1563	-19.1496	-19.1430
-19.2168	-19.0013	-18.2878	-17.1040	-15.8302	-14.8753	-14.5204	-14.8623	-15.7999	-17.0576	-18.2311	-18.9342	-19.1366	-19.1414
-18.9314	-17.8815	-15.9679	-13.6113	-11.3872	-9.8157	-9.2490	-9.7981	-11.3324	-13.5196	-15.8697	-17.7975	-18.8498	-19.1284
-18.0526	-15.8554	-12.6783	-9.1527	-5.9756	-3.7869	-3.0092	-3.7560	-5.8588	-8.9390	-12.4450	-15.6867	-17.9474	-18.9408
-16.7646	-13.4423	-9.1309	-4.5403	-0.4852	2.2716	3.2424	2.3257	-0.2644	-4.1154	-8.6437	-13.0830	-16.5805	-18.5136
-15.4370	-11.2333	-6.0233	-0.5731	4.1994	7.4203	8.5482	7.5030	4.5463	0.1118	-5.2100	-10.6072	-15.1196	-17.9172
-14.4688	-9.7066	-3.9198	2.0916	7.3376	10.8650	12.0961	10.9719	7.7907	2.9970	-2.8243	-8.8395	-14.0194	-17.4244
-14.1160	-9.1629	-3.1774	3.0294	8.4413	12.0761	13.3433	12.1925	8.9361	4.0218	-1.9695	-8.1982	-13.6111	-17.2346
-14.4657	-9.7035	-3.9168	2.0946	7.3407	10.8681	12.0992	10.9749	7.7937	3.0001	-2.8212	-8.8365	-14.0164	-17.4213
-15.4309	-11.2272	-6.0172	-0.5670	4.2055	7.4265	8.5543	7.5091	4.5524	0.1179	-5.2039	-10.6011	-15.1135	-17.9111
-16.7554	-13.4331	-9.1217	-4.5311	-0.4760	2.2807	3.2516	2.3348	-0.2553	-4.1062	-8.6345	-13.0738	-16.5714	-18.5044
-18.0403	-15.8432	-12.6661	-9.1405	-5.9633	-3.7747	-2.9970	-3.7437	-5.8466	-8.9267	-12.4327	-15.6745	-17.9352	-18.9286
-18.9161	-17.8661	-15.9526	-13.5960	-11.3719	-9.8005	-9.2337	-9.7828	-11.3171	-13.5043	-15.8544	-17.7822	-18.8346	-19.1132
-19.1984	-18.9829	-18.2695	-17.0857	-15.8118	-14.8569	-14.5020	-14.8439	-15.7815	-17.0393	-18.2127	-18.9159	-19.1183	-19.1231
-19.2084	-19.2017	-19.1479	-18.9295	-18.5325	-18.1493	-17.9920	-18.1364	-18.5062	-18.8896	-19.0945	-19.1349	-19.1283	-19.1216
-19.2069	-19.2002	-19.1935	-19.1868	-19.1801	-19.1734	-19.1667	-19.1600	-19.1534	-19.1467	-19.1400	-19.1334	-19.1267	-19.1201

Table B.68 Finite Difference v component of surface geostrophic wind (m/sec).

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0471	-0.0703	0.0094	0.0655	0.0855	0.0652	0.0088	-0.0705	-0.0471	0.0000	0.0000	0.0000
-0.0115	-0.1743	-0.1369	-0.0145	0.0913	0.1623	0.1872	0.1595	0.0829	-0.0252	-0.1424	-0.1747	-0.0115	0.0000
-0.2493	-0.2460	-0.0873	0.0688	0.1903	0.2679	0.2950	0.2569	0.1511	0.0055	-0.1408	-0.2669	-0.2508	-0.0115
-0.3819	-0.2039	0.0115	0.1909	0.3067	0.3755	0.4000	0.3470	0.1979	-0.0042	-0.1866	-0.3167	-0.4102	-0.1976
-0.3673	-0.1065	0.1674	0.3512	0.4349	0.4770	0.4933	0.4209	0.2142	-0.0649	-0.2948	-0.4188	-0.4799	-0.4354
-0.3177	0.0293	0.3536	0.5243	0.5578	0.5619	0.5666	0.4737	0.2042	-0.1605	-0.4439	-0.5579	-0.5694	-0.5618
-0.2622	0.1515	0.5095	0.6619	0.6490	0.6191	0.6134	0.5043	0.1848	-0.2493	-0.5766	-0.6823	-0.6501	-0.6214
-0.2380	0.2010	0.5707	0.7148	0.6829	0.6394	0.6295	0.5142	0.1754	-0.2854	-0.6299	-0.7326	-0.6830	-0.6390
-0.2622	0.1515	0.5095	0.6619	0.6490	0.6191	0.6134	0.5043	0.1848	-0.2493	-0.5766	-0.6823	-0.6501	-0.6214
-0.3177	0.0293	0.3536	0.5243	0.5578	0.5619	0.5666	0.4737	0.2042	-0.1606	-0.4439	-0.5579	-0.5694	-0.5618
-0.3673	-0.1065	0.1675	0.3512	0.4349	0.4770	0.4933	0.4209	0.2142	-0.0649	-0.2949	-0.4188	-0.4799	-0.4354
-0.3819	-0.2039	0.0115	0.1909	0.3067	0.3755	0.4000	0.3470	0.1978	-0.0042	-0.1866	-0.3167	-0.4102	-0.1976
-0.2493	-0.2460	-0.0873	0.0688	0.1903	0.2679	0.2950	0.2569	0.1511	0.0054	-0.1408	-0.2669	-0.2508	-0.0115
-0.0115	-0.1743	-0.1369	-0.0145	0.0913	0.1623	0.1872	0.1595	0.0829	-0.0252	-0.1424	-0.1747	-0.0115	0.0000
0.0000	0.0000	-0.0471	-0.0703	0.0094	0.0655	0.0855	0.0652	0.0088	-0.0705	-0.0471	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table B.69 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (m/sec).

CASE III.B.2 - Baroclinic atmosphere - with $\nabla_p T = \begin{bmatrix} 2.0 \cdot 10^{-5} C^o/m \\ 0 \\ 0 \end{bmatrix}$, under $T = Az + B_O + C_O(x - x^{(R)})$

19.7321	19.7256	19.7190	19.7125	19.7060	19.6994	19.6929	19.6864	19.6799	19.6734	19.6669	19.6604	19.6539	19.6474	19.6409
19.7306	19.7241	19.7176	19.7110	19.5207	19.0566	18.6293	18.4570	18.6171	19.0315	19.4820	19.6589	19.6524	19.6459	19.6394
19.7292	19.7226	19.6711	18.9400	17.6719	16.3391	15.3533	14.9897	15.3435	16.3180	17.6373	18.8901	19.6061	19.6444	19.6379
19.7277	19.6762	18.6509	16.6440	14.2355	12.0259	10.5079	9.9713	10.5018	12.0114	14.2086	16.6009	18.5897	19.5982	19.6364
19.7262	18.9495	16.6479	13.4053	9.9784	7.1686	5.5579	5.1050	5.5565	7.1623	9.9616	13.3720	16.5940	18.8747	19.6349
19.5407	17.6849	14.2416	9.9798	6.0664	4.3881	5.1776	5.7627	5.1766	4.3881	6.0604	9.9574	14.1968	17.6156	19.4504
19.0807	16.3543	12.0326	7.1693	4.3881	6.4717	9.1087	10.1160	9.1039	6.4669	4.3881	7.1568	11.9963	16.2910	18.9928
18.6573	15.3706	10.5144	5.5573	5.1802	9.1125	12.2938	13.4457	12.2864	9.1028	5.1772	5.5515	10.4842	15.3117	18.5715
18.4892	15.0097	9.9785	5.1037	5.7682	10.1245	13.4513	14.6456	13.4429	10.1130	5.7631	5.1000	9.9505	14.9524	18.4043
18.6543	15.3676	10.5116	5.5554	5.1818	9.1152	12.2967	13.4485	12.2892	9.1055	5.1788	5.5496	10.4814	15.3088	18.5686
19.0747	16.3484	12.0269	7.1644	4.3880	6.4762	9.1141	10.1215	9.1092	6.4714	4.3880	7.1520	11.9906	16.2851	18.9868
19.5317	17.6759	14.2329	9.9715	6.0601	4.3880	5.1824	5.7686	5.1814	4.3880	6.0540	9.9492	14.1881	17.6068	19.4415
19.7142	18.9376	16.6361	13.3937	9.9674	7.1589	5.5504	5.0987	5.5490	7.1527	9.9506	13.3604	16.5023	18.8628	19.6230
19.7127	19.6612	18.6361	16.6292	14.2210	12.0117	10.4940	9.9576	10.4879	11.9972	14.1941	16.5862	18.5748	19.5833	19.6216
19.7112	19.7047	19.6532	18.9221	17.6541	16.3214	15.3357	14.9722	15.3259	16.3003	17.6196	18.8723	19.5883	19.6265	19.6201
19.7097	19.7032	19.6967	19.6901	19.4999	19.0358	18.6085	18.4362	18.5963	19.0107	19.4612	19.6380	19.6315	19.6251	19.6186
19.7082	19.7017	19.6952	19.6886	19.6821	19.6756	19.6691	19.6626	19.6560	19.6495	19.6430	19.6365	19.6301	19.6236	19.6171

Table B.70 Analytic surface geostrophic wind velocity (m/sec).

19.7321	19.7256	19.7190	19.7125	19.7060	19.6995	19.6932	19.6869	19.6802	19.6734	19.6669	19.6604	19.6539	19.6474	19.6409
19.7306	19.7241	19.7176	19.6652	19.4539	19.0713	18.7033	18.5526	18.6908	19.0456	19.4150	19.6131	19.6524	19.6459	19.6394
19.7292	19.7114	19.5016	18.8103	17.6722	16.4615	15.5654	15.2353	15.5529	16.4323	17.6272	18.7551	19.4362	19.6332	19.6379
19.7164	19.4334	18.4145	16.5750	14.3501	12.3127	10.9299	10.4470	10.9138	12.2618	14.2630	16.4803	18.3330	19.3539	19.6252
19.5338	18.5789	16.4585	13.4491	10.2529	7.6580	6.2200	5.8285	6.2006	7.5665	10.0621	13.2291	16.2960	18.4767	19.4424
19.1292	17.3306	14.1502	10.1732	6.4612	4.7571	5.3520	5.8678	5.3746	4.7387	6.1694	9.7381	13.8092	17.1526	19.0265
18.5789	16.0496	12.0669	7.4842	4.5348	6.2030	8.7415	9.7275	8.8116	6.4426	4.4994	6.8465	11.4863	15.7445	18.4467
18.1361	15.1198	10.6544	5.8942	4.8859	8.5721	11.7392	12.8888	11.8381	8.9629	5.3366	5.2300	9.8710	14.6904	17.9684
17.9684	14.7822	10.1594	5.4174	5.3320	9.5136	12.8486	14.0463	12.9580	9.9553	5.9521	4.8096	9.2986	14.3009	17.7044
18.1331	15.1162	10.6473	5.8712	4.8371	8.5297	11.6991	12.8491	11.7985	8.9226	5.2926	5.2048	9.8636	14.6868	17.9654
18.5729	16.0415	12.0471	7.4153	4.3148	5.9511	8.5099	9.5007	8.5823	6.2015	4.2795	6.7723	11.4660	15.7365	18.4406
19.1202	17.3191	14.1218	10.0802	6.1640	4.0718	4.5441	5.0668	4.5720	4.0529	5.8594	9.6418	13.7805	17.1411	19.0175
19.5218	18.5656	16.4324	13.3724	10.0408	7.1734	5.3861	4.8277	5.3650	7.0772	9.8470	13.1517	16.2699	18.4635	19.4305
19.7015	19.4182	18.3945	16.5300	14.2390	12.0858	10.5735	10.0309	10.5574	12.0347	14.1517	16.4354	18.3130	19.3388	19.6104
19.7112	19.6935	19.4831	18.7856	17.6260	16.3759	15.4371	15.0880	15.4247	16.3467	17.5811	18.7304	19.4178	19.6154	19.6201
19.7097	19.7032	19.6967	19.6441	19.4297	19.0394	18.6620	18.5069	18.6495	19.0137	19.3908	19.5921	19.6315	19.6251	19.6186
19.7082	19.7017	19.6952	19.6886	19.6821	19.6755	19.6688	19.6621	19.6557	19.6495	19.6430	19.6365	19.6301	19.6236	19.6171

Table B.71 Finite Difference surface geostrophic wind velocity (m/sec).

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0001	-0.0003	-0.0005	-0.0003	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	0.0458	0.0668	-0.0147	-0.0740	-0.0956	-0.0737	-0.0141	0.0670	0.0457	-0.0000	-0.0000	-0.0000
-0.0000	0.0112	0.1696	0.1296	-0.0003	-0.1225	-0.2121	-0.2456	-0.2094	-0.1143	0.0101	0.1350	0.1699	0.0112	-0.0000
0.0112	0.2428	0.2364	0.0690	-0.1146	-0.2868	-0.4220	-0.4757	-0.4120	-0.2504	-0.0544	0.1206	0.2567	0.2443	0.0112
0.1924	0.3706	0.1894	-0.0438	-0.2745	-0.4894	-0.6621	-0.7235	-0.6441	-0.4042	-0.1005	0.1429	0.2980	0.3980	0.1925
0.4115	0.3542	0.0915	-0.1934	-0.3948	-0.3690	-0.1745	-0.1051	-0.1981	-0.3506	-0.1090	0.2193	0.3876	0.4630	0.4239
0.5018	0.3048	-0.0343	-0.3149	-0.1468	0.2688	0.3672	0.3884	0.2923	0.0243	-0.1113	0.3103	0.5100	0.5465	0.5461
0.5212	0.2508	-0.1400	-0.3369	0.2943	0.5404	0.5547	0.5568	0.4483	0.1398	-0.1594	0.3215	0.6132	0.6213	0.6032
0.5208	0.2274	-0.1809	-0.3137	0.4362	0.6108	0.6027	0.5993	0.4849	0.1577	-0.1890	0.2904	0.6519	0.6515	0.6199
0.5212	0.2515	-0.1357	-0.3158	0.3447	0.5855	0.5976	0.5994	0.4907	0.1828	-0.1138	0.3448	0.6179	0.6220	0.6032
0.5018	0.3069	-0.0203	-0.2509	0.0732	0.5251	0.6042	0.6208	0.5270	0.2699	0.1085	0.3797	0.5247	0.5487	0.5462
0.4115	0.3568	0.1110	-0.1087	-0.1039	0.3162	0.6384	0.7018	0.6094	0.3351	0.1947	0.3074	0.4075	0.4656	0.4239
0.1924	0.3720	0.2037	0.0213	-0.0734	-0.0145	0.1644	0.2710	0.1840	0.0755	0.1036	0.2087	0.3124	0.3993	0.1925
0.0112	0.2430	0.2415	0.0992	-0.0180	-0.0741	-0.0795	-0.0733	-0.0694	-0.0374	0.0424	0.1508	0.2618	0.2445	0.0112
-0.0000	0.0112	0.1701	0.1365	0.0282	-0.0545	-0.1014	-0.1158	-0.0987	-0.0464	0.0385	0.1419	0.1705	0.0112	-0.0000
-0.0000	-0.0000	-0.0000	0.0461	0.0702	-0.0036	-0.0535	-0.0707	-0.0532	-0.0030	0.0704	0.0460	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0001	0.0003	0.0005	0.0003	0.0001	0.0000	-0.0000	-0.0000	-0.0000	-0.0000

Table B.72 Deviation (Analytic - Finite Difference) in calculation of surface geostrophic wind velocity (m/sec).

Appendix C

Derivation of a Product Rule

To show that (C.1) is true, we expand both terms on its RHS, and then add the two.

$$\delta_x\left(\overline{T(x) \; * \; U(x)}\right) \; = \; \overline{\overline{T(x)} \; * \; \delta_x\left(U(x)\right)} \; + \; \overline{\overline{U(x)} \; * \; \delta_x\left(T(x)\right)} \tag{C.1}$$

Observe that

$$\overline{\frac{T\left(x+\frac{\Delta x}{2}\right) \; + \; T\left(x-\frac{\Delta x}{2}\right)}{2} \; * \; \frac{U\left(x+\frac{\Delta x}{2}\right) \; - \; U\left(x-\frac{\Delta x}{2}\right)}{\Delta x}} \tag{C.2}$$

is actually

$$\frac{\overline{TU\left(x+\frac{\Delta x}{2}\right) - T\left(x+\frac{\Delta x}{2}\right)U\left(x-\frac{\Delta x}{2}\right) + T\left(x-\frac{\Delta x}{2}\right)U\left(x+\frac{\Delta x}{2}\right) - TU\left(x-\frac{\Delta x}{2}\right)}}{2\Delta x}. \tag{C.3}$$

Now expand the averaging operator to obtain the following two terms

$$\frac{TU\left(x+\Delta x\right) \; - \; T\left(x+\Delta x\right)U\left(x\right) + T\left(x\right)U\left(x+\Delta x\right) - TU\left(x\right)}{4\Delta x} \tag{C.4}$$

$$+\frac{TU\left(x\right) \; + \; T\left(x\right)U\left(x-\Delta x\right) - T\left(x-\Delta x\right)U\left(x\right) + TU\left(x-\Delta x\right)}{4\Delta x}. \tag{C.5}$$

We introduce the corresponding terms from the companion expression (i.e. second term on RHS of (C.1))

$$\frac{UT\left(x+\Delta x\right) \; - \; U\left(x+\Delta x\right)T\left(x\right) + U\left(x\right)T\left(x+\Delta x\right) - UT\left(x\right)}{4\Delta x} \tag{C.6}$$

$$+\frac{UT\left(x\right) \; + \; U\left(x\right)T\left(x-\Delta x\right) - U\left(x-\Delta x\right)T\left(x\right) + UT\left(x-\Delta x\right)}{4\Delta x}, \tag{C.7}$$

and then sum to end up with

$$\frac{TU\left(x+\Delta x\right)-TU\left(x-\Delta x\right)}{2\Delta x}. \tag{C.8}$$

We now expand LHS side of (C.1.1) to find

$$\delta_x\left(\frac{TU\left(x+\frac{\Delta x}{2}\right)\;+\;TU\left(x-\frac{\Delta x}{2}\right)}{2}\right) \tag{C.9}$$

and this is

$$\frac{TU\left(x+\Delta x\right)\;+TU\left(x\right)-\;TU\left(x\right)\;-\;TU\left(x-\Delta x\right)}{2\Delta x}. \tag{C.10}$$

Because (C.10) equals (C.8), then (C.1) holds.

Appendix D

Ideal gas constant R

The value of the gas constant R for different gases is determined using Avagadro's hypothesis that at a given temperature and pressure gases containing the same number of molecules occupy the same volume. Experimental work has shown that at a pressure of $P_o = 1014mb$ and temperature $T_o = 0C^o$, 22.4 kliter ($= V_o$) of a gas will have a mass in kilograms equal to the molecular weight of the gas μ. This quantity of gas is defined as 1 kmol. Hence by the ideal gas law, $\frac{P_o}{\mu}\frac{V_o}{T_o} = R$, but $R = \frac{R^*}{\mu}$,where R^* is the universal gas constant and μ has units of kilograms per kilomole. From experiments $R^* = 8314.3\ J/C^o\ kmol$. In the atmosphere, the apparent molecular weight of air μ_{atm} is determined by $\mu_{atm} = \frac{\sum_{i=N}^{i=1} m_i}{\sum_{i=N}^{i=1} m_i * \mu_i^{-1}}$ where the m_i represent the fractional contribution by mass ($\sum_{i=N}^{i=1} m_i = 1$) of the N individual gases comprising the atmosphere, and the μ_i are the respective repective molecular weights. The predominant gases in the atmosphere are nitrogen ($m_1 = 0.7551$), oxygen ($m_2 = 0.2314$), and argon ($m_3 = 0.0128$), having respective molecular weights $\mu_1 = 28.02$, $\mu_2 = 32.00$, and $\mu_3 = 39.94$, in kilograms per kilomole. One then finds that $\mu_{atm} = 28.98$ by the formula just cited. Therefore, the dry gas constant of the atmosphere $R_d = 287J/C^o kilogram = 2870000dyne\ cm/C^o\ g$. This is the value of R used in the CASE programs. (The explanation of R is due to Pielke (1984).)

Appendix E

Computer program listings

The programs presented in this Appendix, called CASEip.c and CASEiz.c (where i = I, II, or III, with p representing $T = A\ln p + B$ and z representing $T = Az + B$), comprise the source code for the numerical experiments discussed in Sections 4.4, 4.5 and 4.6 on pages 27 through 31. The main purpose of the numerical study was to compare geostrophic wind components computed by the Corby finite difference scheme (4.1.1) under the two different temperature structures. Since the central results of Chapters 6 and 7 (namely (6.1.22) and (7.2.10)) were shown to be independant of initial geostrophic wind angle and magnitude, as well as independant of grid size interval, the corresponding parameters were fixed in the programs without loss of generality to what was to be investigated.

The latitude taken was 48 ^{o}, however this only affects the geostrophic wind component calculation in the factor $\frac{1}{f}$ where $f = 2\Omega sin(\theta)$ with θ the latitude, and Ω the earth's angular speed. The reciprocal of the Coriolis parameter, $\frac{1}{f}$, actually serves only as a scaling factor after the finite difference scheme has been applied, as noted from (2.7.6). Hence selecting one particular latitude is not a significant restriction to our investigation, notwithstanding the facts that (1) f is zero at the equator and so $\vec{V_g}$ is not defined there, and (2) because $sin(\theta)$ is nearly zero for low latitudes, clearly $\frac{1}{f}$ will magnify whatever truncation error results in the $\left(\frac{\partial\phi}{\partial x}\right)_p$ calculation by a large amount. These problems would occur however for any type of scheme used to compute the HPGF at low latitudes.

The CASE programs were adapted from IDEAL85 (Danard and Galbraith, 1985). They compute the surface coordinates of a cosine shaped hill, as well as the temperature and pressure at those points based on the model in Chapter 4. The Corby finite difference scheme (4.1.1) is then used on this data to compute surface geostrophic wind components.

```
#include    <StdIO.h>
#include    <math.h>
#include    <string.h>

/*  Case I with T = A ln p + B.
    Barotropic atmosphere with horizontal isobaric planes.
    The program is written in C.                                                          */

main ()
{
#define SIZEP      21
    double  ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,F,LEN,uansc,ufdsc,difxsc,vansc,vfdsc,difysc,fdvelsc,anvelsc,difvelsc;
    double  GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,L,VDIFTOT,UDIFTOT,FDVEL;
    double  X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP],DIFVEL[SIZEP][SIZEP];
    double  PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int     i,j;
    //
    R     = 2870000.0;
    G     = 980.0;
    F     = 0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.  */
    ZREF =  0;
    TREF =  295.2;
    Z85R =  154000;
    T85R  = 286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =  0;
    DZDY =  0;
    for(i=1;i<SIZEP ;i++)
    { for(j=1;j<SIZEP  ;j++) {
/*        Initialize matrix ZS with mountain surface heights.
          The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.  */
          LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11)) )  ;
          ZS[ i][j ] =  (H/2)  * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )   );
          if ( LEN >  8.0   ) ZS[i][j] = 0;
      }
    }
    fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                            */
/*  The reference station is at (X[3], Y[13], 0)                                          */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
/*  Formulas below have been derived in Section 4.5                                       */
    AAA1 = 980.0 * (Z85R - ZREF) ;
    AAA2 = ( ((TREF * TREF) - (T85R * T85R)) * R ) / (2.0 * AAA1) ;
    fprintf(stdout,"\n");
    for(i=1;i<SIZEP;i++) {
        for(j=1;j<SIZEP;j++) {
/*      Initialize 850mb plane with height and temperature values.                        */
            ZZ85 = Z85R + ( Y[i] * DZDY + X[j] * DZDX );
            TT85 = T85R;
            GDEL = 980.0 * (ZZ85 - ZS [i][j]);
            HOLD = (((2 * AAA2) * GDEL) / R )  + ( TT85 * TT85 );
/*      Initialize (mountain) surface temperature values.                                 */
        TTS[i][j] = sqrt ( HOLD )    ;
/*      Initialize ln surface pressure values.                                            */
        LNPS[i][j] = ( (2 * GDEL) / (R * (TT85 + TTS[i][j] )) )  ;
/*      Analytic surface geostrophic wind components  u_{a} and v_{a}.                   */
            UBS = 0;
            VBS = 0;
/*      Analytic surface geostrophic wind magnitude                                       */
            QQ[i][j] = sqrt ((UBS * UBS) + (VBS * VBS));
        }
    }
    for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"                         CASE I.A - Horizontal isobaric planes with T = A ln p + B " );
    fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout,"         " );
    for (i=19;i>=3;i--) {
        for(j=3;j<20;j++) {
          uansc = UBS * 1e11 ;
          fprintf(stdout,"%7.4f ", uansc );
        }fprintf(stdout,"\n");fprintf(stdout,"         " );
    }
    fprintf(stdout,"\n                    Table B.1 Analytic u component of surface geostrophic wind ( 10^{-11} m/sec)." );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    VDIFTOT = 0.0;
    for(i=3;i<20;i++) {
         for(j=3;j<20;j++) {
/*       Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
         the Y directed pressure gradient by - { 1/F }.                                   */
         PY [i][j] = - ( (  G * (   ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)   ) )
                        +  R *
                        (       ( ( TTS[i+1][j]   + TTS[i][j] )     / 4.0  )
                              * ( ( LNPS[i+1][j] - LNPS[i][j])      / DELL )
                              + ( ( TTS[i][j]     + TTS[i-1][j] )  / 4.0  )
                              * ( ( LNPS[i][j]    - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*       Difference between analytic and Corby finite difference v surface geostrophic wind components  */
         DIFY[i][j] = VBS - (PX[i][j] / 100);
/*       VDIFTOT is an accumulator for the MVD error                                      */
         VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
         }
    }
```

```
   fprintf(stdout,"              " );
    for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
          PYS  =  PY [i][j] / 100 ;
          ufdsc = PYS *  lell;
          fprintf(stdout,"%7.4f ", ufdsc );
      }fprintf(stdout,"\n"); fprintf(stdout,"            " );
    }
   fprintf(stdout,"\n");
   fprintf(stdout,"                          Table B.2 Finite Difference u component of surface geostrophic wind (10^{-11} m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"              " );
     for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
           difxsc = DIFX[i][j] * lell;
           fprintf(stdout,"%7.4f ", difxsc );
        }fprintf(stdout,"\n");fprintf(stdout,"            " );}
   fprintf(stdout,"\n                                     Table B.3 Deviation (Analytic - Finite Difference) in calculation   ");
   fprintf(stdout,"\n                                      of the u component of surface geostrophic wind (10^{-11} m/sec). ");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"                                       CASE I.A - Horizontal isobaric planes with T = A ln p + B " );
   fprintf(stdout,"\n");  fprintf(stdout,"\n");fprintf(stdout,"          " );
    for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
          VBS = 0  ;vansc = VBS * lell ;
          fprintf(stdout,"%7.4f ", vansc );
      }fprintf(stdout,"\n");fprintf(stdout,"        " );}
   fprintf(stdout,"\n                              Table B.4 Analytic v component of surface geostrophic wind (10^{-11} m/sec). ");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   for(i=3;i<20;i++) {
       for(j=3;j<20;j++) {
/*     Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
       X directed pressure gradient by { 1/F }.                                                              */
       PX[i][j] =    ( (  G * ( ( ZS[i][j+1] - ZS[i][j-1]) / (2.0 * DELL)  ) )
                        + R *
                        (       ( ( TTS[i][j+1]  + TTS[i][j] )   / 4.0  )
                              * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                              + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                              * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components        */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                          */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
        }
    }
    fprintf(stdout,"          " );
    for(i=19;i>=3;i--){
    for(j=3;  j<20 ;j++) {
       PXS  =  PX [i][j]  / 100  ; vfdsc  = PXS  * lell;
        fprintf(stdout,"%7.4f ", vfdsc );
     }fprintf(stdout,"\n");fprintf(stdout,"       " );}
   fprintf(stdout,"\n                Table B.5 Finite Difference v component of surface geostrophic wind (10^{-11} m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"          " );
     for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
         difysc = DIFY[i][j] * lell;
         fprintf(stdout,"%7.4f ", difysc );
       }fprintf(stdout,"\n");fprintf(stdout,"       " );}
   fprintf(stdout,"\n                                     Table B.6 Deviation (Analytic - Finite Difference) in calculation " );
   fprintf(stdout,"\n                                       of the v component of surface geostrophic wind (10^{-11} m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"                                      CASE I.A - Horizontal isobaric planes with T = A ln p + B " );
   fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout," " );
    for(i=3;i< 20;i++) {
       for(j=3;j<20;j++) {
/*        Analytic surface geostrophic wind magnitude                                                        */
          anvelsc = QQ[i][j] * lell;
          fprintf(stdout,"%7.4f ", anvelsc );
      }fprintf(stdout,"\n");fprintf(stdout," " ); }
   fprintf(stdout,"\n");
   fprintf(stdout,"                     Table B.7 Analytic calculation of surface geostrophic wind (10^{-11} m/sec).  " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    for (i=19;i>=3;i--) {
       for (j=3;j<20;j++) {
/*        Corby finite difference surface geostrophic wind magnitude                                         */
          FDVEL =  0.01 * sqrt ((PX[i][j] * PX[i ][j]) + (PY [i][j] * PY [i][j])) ;
          fdvelsc = FDVEL * lell;
          fprintf(stdout,"%7.4f ",fdvelsc);
          DIFVEL[i][j] = QQ[i][j]  - FDVEL  ;
      }fprintf(stdout,"\n");}
   fprintf(stdout,"\n");
   fprintf(stdout,"                  Table B.8 Finite Difference calculation of surface geostrophic wind (10^{-11} m/sec). ");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   MAXDIF = 0.0;
    for (i=19;i>=3;i--){
       for(j=3;j<20;j++) { */
/*          Find maximum (MAX) error                                                                         */
            if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*          Difference between analytic and Corby finite difference surface geostrophic wind magnitude       */
            difvelsc =  DIFVEL[i][j] * lell ;
            fprintf(stdout,"%7.4f ", difvelsc ); }fprintf(stdout,"\n");}
   fprintf(stdout,"\n                                       Table B. 9 Difference (Analytic - Finite Difference) in calculation");
   fprintf(stdout,"\n                                               of surface geostrophic wind (10^{-11}m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/* Compute MVD  error                                                                                        */
   VDIFTOT = sqrt (VDIFTOT) / 289; ;
   fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
   fprintf(stdout,"%20.16f", MAXDIF  );
   //
   return 0;
 }
```

```
#include    <StdIO.h>
#include    <math.h>
#include    <string.h>

/* Case I with T = A z + B.
   Barotropic atmosphere with horizontal isobaric planes.
   The program is written in C.                                                                              */

 main ()
{
#define     SIZEP        21
    double  ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,DTDXP,DTDYP,F,LEN,EXTRA,GAMMA,UBS,VBS;
    double  GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,VDIFTOT,UDIFTOT,FDVEL;
    double  X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP];
    double  PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int     i,j;
    //
    R     =  2870000.0;
    G     =  980.0;
    F     =  0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.                    */
    ZREF =  0;
    TREF =  295.2;
    Z85R =  154000;
    T85R =  286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =  0 ;
    DZDY =  0 ;
    for(i=1;i<SIZEP ;i++)
    {
       for(j=1;j<SIZEP   ;j++) {
/*        Initialize matrix ZS with mountain surface heights.
          The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.    */
          LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11))  )  ;
          ZS[i][j] =  (H/2)  * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )   );
          if ( LEN >  8.0   ) ZS[ i][j ] =   0 ;
     }
    }
    fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                                               */
/*  The reference station is at (X[3], Y[13], 0)                                                             */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
    for(i=1;i<SIZEP;i++) {
       for(j=1;j<SIZEP;j++) {
/*     Initialize 850mb plane with height and temperature values.                                            */
       ZZ85 = Z85R + ( Y[i] * DZDY  + X[j] * DZDX );
       TT85 = T85R ;
/*     Formulas below have been derived in Section 4.6                                                       */
       GAMMA = 9.0 / 154000 ;
/*     Initialize (mountain) surface temperature values.                                                     */
       TTS[i][j] = TT85 + GAMMA * (ZZ85 - ZS [i][j])      ;
/*     Initialize ln surface pressure values.                                                                */
       LNPS[i][j] = ( G / (R * GAMMA) ) * log ( 1 + ((GAMMA * (ZZ85 - ZS[i][j])) / TT85) )  ;
/*     Analytic surface geostrophic wind components  u_{a} and v_{a}.                                        */
       UBS =   0 ;
       VBS =   0 ;
/*     Analytic surface geostrophic wind magnitude                                                           */
       QQ[i][j] =   sqrt  ((UBS * UBS) + (VBS * VBS)) ;
       }
    }
    for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"                                  CASE I.B - Horizontal isobaric planes with T = Az + B  " );
    fprintf(stdout,"\n");  fprintf(stdout,"\n");fprintf(stdout,"  " );
    for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
           fprintf(stdout,"%7.4f ",UBS );
       }fprintf(stdout,"\n");fprintf(stdout,"     " );
    }
    fprintf(stdout,"\n                                       Table B.10 Analytic u component of surface geostrophic wind (m/sec).");
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
     VDIFTOT = 0.0;
     for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*      Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
        the Y directed pressure gradient by - { 1/F }.                                                       */
        PY [i][j] = - ( (  G * (  ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)  ) )
                           + R *
                  (         ( ( TTS[i+1][j]  + TTS[i][j] )    / 4.0  )
                          * ( ( LNPS[i+1][j] - LNPS[i][j])    / DELL )
                          + ( ( TTS[i][j]    + TTS[i-1][j] )  / 4.0  )
                          * ( ( LNPS[i][j]   - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components         */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                          */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
         }
     }
```

```
    fprintf(stdout,"     " );
    for (i=19;i>=3;i--) {
       for(j=3;  j<20 ;j++) {
          PYS  =  PY [i][j]  / 100  ;
         fprintf(stdout,"%7.4f ", PYS  );
         }fprintf(stdout,"\n");fprintf(stdout,"        " );}
   fprintf(stdout,"\n                              Table B.11 Finite Difference u component of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"     " );
   for (i=19;i>=3;i--){
      for(j=3;j<20;j++) {
        fprintf(stdout,"%7.4f ",DIFX[i][j]);
      }
      fprintf(stdout,"\n");fprintf(stdout,"      " );
   }
   fprintf(stdout,"\n                                        Table B.12 Deviation (Analytic - Finite Difference) in calculation");
   fprintf(stdout,"\n                                             of the u component of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"                                          CASE I.B - Horizontal isobaric planes with T = Az + B");
   fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout,"  " );
   for(i=3;i<20;i++) {
      for(j=3;j<20;j++) {
         fprintf(stdout,"%8.4f ", VBS);
      }fprintf(stdout,"\n");fprintf(stdout,"  " );}
    fprintf(stdout,"\n                                     Table B.13 Analytic v component of surface geostrophic wind (m/sec).");
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    for(i=3;i< 20;i++) {
      for (j=3;j<20;j++) {
/*    Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
      X directed pressure gradient by { 1/F }.                                                                   */
      PX[i][j] =    ( (  G * ( ( ZS[i][j+1] - ZS[i][j-1]) / (2.0 * DELL)  ) )
                     + R *
                    (      ( ( TTS[i][j+1]  + TTS[i][j] )   / 4.0  )
                         * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                         + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                         * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*       Difference between analytic and Corby finite difference v surface geostrophic wind components           */
         DIFY[i][j] = VBS - (PX[i][j] / 100);
/*       VDIFTOT is an accumulator for the MVD error                                                             */
         VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
         }
    }
    fprintf(stdout,"     " );
    for (i=19;i>=3;i--) {
     for(j=3;  j<20 ;j++) {
       PXS  =  PX [i][j]  / 100  ;
       fprintf(stdout,"%7.4f ", PXS  );
     }fprintf(stdout,"\n"); fprintf(stdout,"      " );}
   fprintf(stdout,"\n                              Table B.14 Finite Difference v component of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout," " );
     for (i=19;i>=3;i--){
      for(j=3;j<20;j++) {
         fprintf(stdout,"%8.4f ",DIFY[i][j]);
      }fprintf(stdout,"\n");fprintf(stdout,"  " );}
   fprintf(stdout,"\n                                     Table B.15 Deviation (Analytic - Finite Difference) in calculation");
   fprintf(stdout,"\n                                           of the v component of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"                                 CASE I.B - Horizontal isobaric planes with T = Az + B " );
   fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout," " );
   for (i=19;i>=3;i--) {
      for(j=3;j<20;j++) {
/*         Analytic surface geostrophic wind magnitude                                                          */
           fprintf(stdout,"%7.4f ",QQ[i][j]);
      }fprintf(stdout,"\n");fprintf(stdout," " ); }
    fprintf(stdout,"\n                                 Table B.16 Analytic calculation of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    for (i=19;i>=3;i--){
      for (j=3;j<20;j++) {
 /*        Corby finite difference surface geostrophic wind magnitude                                           */
           FDVEL =  0.01 * sqrt ((PX[i][j] * PX[i][j]) + (PY[i][j] * PY[i][j])) ;
           fprintf(stdout,"%7.4f ",FDVEL);
           DIFY[k][j] = QQ[i][j]  - FDVEL  ;
      }
      fprintf(stdout,"\n");}
   fprintf(stdout,"\n                              Table B.17 Finite Difference calculation of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   MAXDIF = 0.0;
    for (i=19;i>=3;i--){
      for(j=3;j<20;j++) {
/*         Find maximum (MAX) error                                                                              */
           if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*         Difference between analytic and Corby finite difference surface geostrophic wind magnitude            */
           fprintf(stdout,"%7.4f ",DIFY[i][j]);
         }fprintf(stdout,"\n");}
   fprintf(stdout,"                                 Table B.18 Difference (Analytic - Finite Difference) in calculation  " );
   fprintf(stdout,"\n                                                  of surface geostrophic wind (m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/* Compute MVD  error                                                                                            */
   VDIFTOT = sqrt (VDIFTOT) / 289; ;
   fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
   fprintf(stdout,"%20.16f", MAXDIF  );
   //
   return 0;
}
```

```
#include    <StdIO.h>
#include    <math.h>
#include    <string.h>

/*  Case II with T = A ln p + B.
    Barotropic atmosphere with parallel isobaric planes inclined with respect to the horizontal.
    The program is written in C.                                                                          */

main ()
{
#define     SIZEP        21
    double  ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,DTDXP,DTDYP,F,LEN,difxsc,difysc,difvelsc;
    double  GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,VDIFTOT,UDIFTOT,FDVEL;
    double  X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP],DIFVEL[SIZEP][SIZEP];
    double  PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int     i,j;
    //
    R    =  2870000.0;
    G    =  980.0;
    F    =  0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.                  */
    ZREF =  0;
    TREF =  295.2;
    Z85R =  154000;
    T85R  = 286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =  - 9.45532738115e-5 ;
    DZDY =    4.61168237962e-5 ;
    for(i=1;i<SIZEP ;i++)
    {
       for(j=1;j<SIZEP  ;j++) {
/*        Initialize matrix ZS with mountain surface heights.
          The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.  */
          LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11))  )   ;
          ZS[ i][j ] =  (H/2)  * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )   );
          if ( LEN >  8.0   ) ZS[i][j] = 0;
       }
     }
    fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                                            */
/*  The reference station is at (X[3], Y[13], 0)                                                          */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
/*  Formulas below have been derived in Section 4.5                                                       */
    AAA1 = 980.0 * (Z85R - ZREF) ;
    AAA2 = ( ((TREF * TREF) - (T85R * T85R)) * R ) / (2.0 * AAA1) ;
    fprintf(stdout,"\n");
    for(i=1;i<SIZEP;i++) {
       for(j=1;j<SIZEP;j++) {
/*      Initialize 850mb plane with height and temperature values.                                        */
        ZZ85 = Z85R + ( Y[i] * DZDY  + X[j] * DZDX );
        TT85 = T85R ;
        GDEL = 980.0 * (ZZ85 - ZS [i][j]) ;
        HOLD = (((2 * AAA2) * GDEL) / R )  + ( TT85 * TT85 )  ;
/*      Initialize (mountain) surface temperature values.                                                 */
        TTS[i][j] = sqrt ( HOLD )    ;
/*      Initialize ln surface pressure values.                                                            */
        LNPS[i][j] = ( (2 * GDEL) / (R * (TT85 + TTS[i][j] )) )  ;
/*      Analytic surface geostrophic wind components  u_{a} and v_{a}.                                    */
        UBS = - 0.01 * ( (G * DZDY) / F ) ;
        VBS =   0.01 * ( (G * DZDX) / F ) ;
/*      Analytic surface geostrophic wind magnitude                                                       */
        QQ[i][j] =   sqrt ((UBS * UBS) + (VBS * VBS)) ;
        }
    }
    for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"  CASE II.A Barotropic atmosphere - tilted (parallel) isobaric planes with delp T = 0, under T= A ln p + B " );
    fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout,"\n"); fprintf(stdout,"\n");fprintf(stdout,"    " );
     for (i=19;i>=3;i--){
      for(j=3;j<20;j++) {
           fprintf(stdout,"%7.4f ",UBS  );
       }fprintf(stdout,"\n");fprintf(stdout,"     " );
     }
    fprintf(stdout,"\n");
    fprintf(stdout,"             Table B.19 Analytic u component of surface geostrophic wind (m/sec).   " );
    for(i=1;i<26;i++) { fprintf(stdout,"\n"); }
    for(i=3;i<20;i++) {
        for (j=3;j<20;j++) {
/*      Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
        the Y directed pressure gradient by - { 1/F }.                                                    */
        PY [i][j] = - ( (  G * (  ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)  ) )
                         +  R *
                        (        ( ( TTS[i+1][j]  + TTS[i][j] )    / 4.0  )
                               * ( ( LNPS[i+1][j] - LNPS[i][j])    / DELL )
                               + ( ( TTS[i][j]    + TTS[i-1][j] )  / 4.0  )
                               * ( ( LNPS[i][j]   - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components     */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                       */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
        }
    }
```

```
fprintf(stdout,"     " );
 for (i=19;i>=3;i--){
   for(j=3;  j<20 ;j++) {
     PYS  =  PY [i][j]  / 100  ;
    fprintf(stdout,"%7.4f ", PYS  );
   }fprintf(stdout,"\n"); fprintf(stdout,"     " );
}
fprintf(stdout,"\n                              Table B.20 Finite Difference u component of surface geostrophic wind (m/sec).  ");
for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
fprintf(stdout,"     " );
 for (i=19;i>=3;i--) {
   for(j=3;j<20;j++) {
       difxsc  =  DIFX[i][j] * 1ell ;
       fprintf(stdout,"%7.4f ", difxsc );
   }fprintf(stdout,"\n");fprintf(stdout,"     " );}
fprintf(stdout,"\n                              Table B.21 Deviation (Analytic - Finite Difference) in calculation " );
fprintf(stdout,"\n                                    of the u component of surface geostrophic wind (10^{-11} m/sec)." );
for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
fprintf(stdout," CASE II.A Barotropic atmosphere - tilted (parallel) isobaric planes with delp T = 0, under T = A ln p + B");
fprintf(stdout,"\n");
fprintf(stdout,"\n");
fprintf(stdout,"     " );
for(i=3;i<20;i++) {
 for(j=3;j<20;j++) {
       fprintf(stdout,"%7.4f ", VBS );
 }fprintf(stdout,"\n");fprintf(stdout," " );}
fprintf(stdout,"\n                                    Table B.22 Analytic v component of surface geostrophic wind (m/sec).");
for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
VDIFTOT = 0.0;
for(i=3;i< 20;i++) {
   for (j=3;j<20;j++) {
/*    Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
      X directed pressure gradient by { 1/F }.                                                          */
      PX[i][j] =    ( (  G * ( ( ZS[i][j+1]  - ZS[i][j-1]) / (2.0 * DELL)  ) )
                       + R *
                       (      ( ( TTS[i][j+1]  + TTS[i][j] )   / 4.0  )
                            * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                            + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                            * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*       Difference between analytic and Corby finite difference v surface geostrophic wind components   */
         DIFY[i][j] = VBS - (PX[i][j] / 100);
/*       VDIFTOT is an accumulator for the MVD error                                                     */
         VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
         }
  }
 fprintf(stdout,"     " );
 for (i=19;i>=3;i--) {
   for(j=3;j<20;j++) {
      PXS  =  PX [i][j]  / 100 ;
     fprintf(stdout,"%7.4f ",PXS );
   }fprintf(stdout,"\n"); fprintf(stdout,"     " );}
fprintf(stdout,"\n");
fprintf(stdout,"                           Table B.23  Finite Difference v component of surface geostrophic wind (m/sec).   " );
for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
fprintf(stdout,"     " );
 for (i=19;i>=3;i--) {
    for(j=3;j<20;j++) {
      difysc =  DIFY[i][j] * 1ell ;
      fprintf(stdout,"%7.4f ",difysc);
    } fprintf(stdout,"\n");fprintf(stdout,"     " );
 }
fprintf(stdout,"\n                              Table B.24 Deviation (Analytic - Finite Difference) in calculation " );
fprintf(stdout,"\n                                    of the v component of surface geostrophic wind (10^{-11} m/sec). " );
for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
fprintf(stdout,"     CASE II.A Barotropic atmosphere - tilted (parallel) isobaric planes with delp T = 0, under T=A ln p + B");
fprintf(stdout,"\n");
fprintf(stdout,"\n");    fprintf(stdout,"     " );
for (i=19;i>=3;i--) {
    for(j=3;j<20;j++) {
/*        Analytic surface geostrophic wind magnitude                                                     */
          fprintf(stdout,"%7.4f ",QQ[i][j]);
    }fprintf(stdout,"\n");fprintf(stdout,"     " );}
fprintf(stdout,"\n                                Table B.25 Analytic calculation of surface geostrophic wind (m/sec).   ");
for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
 fprintf(stdout,"     " );
 for (i=19;i>=3;i--) {
    for (j=3;j<20;j++) {
/*      Corby finite difference surface geostrophic wind magnitude                                       */
        FDVEL =  0.01 *   sqrt ((PX[i][j] * PX[i ][j]) + (PY [i][j] * PY [i][j])) ;
        fprintf(stdout,"%7.4f ",FDVEL);
        DIFVEL[i][j] = QQ[i][j]  - FDVEL  ;
   }fprintf(stdout,"\n");fprintf(stdout,"     " );
}
fprintf(stdout,"\n                        Table B.26 Finite Difference calculation of surface geostrophic wind (m/sec).   ");
for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
fprintf(stdout,"     " );
MAXDIF = 0.0;
 for (i=19;i>=3;i--) {
    for(j=3;j<20;j++) {
/*        Find maximum (MAX) error                                                                        */
          if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*        Difference between analytic and Corby finite difference surface geostrophic wind magnitude     */
          difvelsc =  DIFVEL[i][j] * 1ell ;
          fprintf(stdout,"%7.4f ", difvelsc );
         }fprintf(stdout,"\n"); fprintf(stdout,"     " );
 }
fprintf(stdout,"\n                                    Table B.27 Difference (Analytic - Finite Difference) in calculation " );
fprintf(stdout,"\n                                          of surface geostrophic wind (10^{-11} m/sec).  " );
for (i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/* Compute MVD  error                                                                                     */
VDIFTOT = sqrt (VDIFTOT) / 289; ;
fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
fprintf(stdout,"%20.16f", MAXDIF  );
//
return 0;
}
```

```
#include    <StdIO.h>
#include    <math.h>
#include    <string.h>

 /* Case II with T = A z + B.
    Barotropic atmosphere with parallel isobaric planes inclined with respect to the horizontal.
    The program is written in C.                                                                   */

 main ()
{
#define     SIZEP          21
    double  ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,DTDXP,DTDYP,F,LEN,EXTRA,GAMMA,UBS,VBS;
    double  GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,VDIFTOT,UDIFTOT,FDVEL;
    double  X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP];
    double  PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int     i,j;
    //
    R    =  2870000.0;
    G    =  980.0;
    F    =  0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.           */
    ZREF =  0;
    TREF =  295.2;
    Z85R =  154000;
    T85R  = 286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =  - 9.45532738115e-5 ;
    DZDY =    4.61168237962e-5  ;
    for(i=1;i<SIZEP ;i++)
    {
       for(j=1;j<SIZEP  ;j++) {
/*        Initialize matrix ZS with mountain surface heights.
          The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.   */
          LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11))  )  ;
          ZS[ i][j ] =  (H/2)  * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )    );
          if ( LEN >  8.0    ) ZS[i][j] =   0 ;
       }
     }
    fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                                     */
/*  The reference station is at (X[3], Y[13], 0)                                                   */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
    fprintf(stdout,"\n");fprintf(stdout,"\n");
    for(i=1;i<SIZEP;i++) {
       for(j=1;j<SIZEP;j++) {
/*     Initialize 850mb plane with height and temperature values.                                  */
       ZZ85 = Z85R + ( Y[i] * DZDY  + X[j] * DZDX );
       TT85 = T85R ;
/*     Formulas below have been derived in Section 4.6                                             */
       GAMMA = 9.0 / 154000 ;
/*     Initialize (mountain) surface temperature values.                                           */
       TTS[i][j] = TT85 + GAMMA * (ZZ85 - ZS [i][j])      ;
/*     Initialize ln surface pressure values.                                                      */
       LNPS[i][j] = ( G / (R * GAMMA) ) * log ( 1 + ((GAMMA * (ZZ85 - ZS[i][j])) / TT85) )  ;
/*     Analytic surface geostrophic wind components  u_{a} and v_{a}.                              */
       UBS = -  0.01 * ( G * DZDY / F ) ;
       VBS =    0.01 * ( G * DZDX / F ) ;
/*     Analytic surface geostrophic wind magnitude                                                 */
       QQ[i][j] =   sqrt ((UBS * UBS) + (VBS * VBS)) ;
       }
    }
    for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"        CASE II.B Barotropic atmosphere - tilted (parallel) isobaric planes with delp T = 0 under T = Az + B");
    fprintf(stdout,"\n");  fprintf(stdout,"\n");fprintf(stdout,"       " );
    for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
          fprintf(stdout,"%7.4f ",UBS );
       }fprintf(stdout,"\n");fprintf(stdout,"      " );}
    fprintf(stdout,"\n                              Table B.28 Analytic u component of surface geostrophic wind (m/sec).");
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    VDIFTOT = 0.0;
    for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*      Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
        the Y directed pressure gradient by - { 1/F }.                                              */
        PY [i][j] = - ( (  G * (   ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)   ) )
                       + R *
                  (       ( ( TTS[i+1][j]  + TTS[i][j] )    / 4.0  )
                        * ( ( LNPS[i+1][j] - LNPS[i][j])     / DELL )
                        + ( ( TTS[i][j]    + TTS[i-1][j] )  / 4.0  )
                        * ( ( LNPS[i][j]   - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components  */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                 */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
        }
   }
```

```
   fprintf(stdout,"    " );
   for (i=19;i>=3;i--){
     for(j=3;  j<20 ;j++) {
        PYS  =  PY [i][j]  / 100  ;
        fprintf(stdout,"%7.4f ", PYS  );
        }fprintf(stdout,"\n");fprintf(stdout,"     " );}
  fprintf(stdout,"\n");
  fprintf(stdout,"                    Table B.29 Finite Difference u component of surface geostrophic wind (m/sec). ");
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  fprintf(stdout,"    " );
   for (i=19;i>=3;i--){
     for(j=3;j<20;j++) {
         fprintf(stdout,"%7.4f ",DIFX[i][j]);
     }fprintf(stdout,"\n");fprintf(stdout,"     " );}
  fprintf(stdout,"\n                                   Table B.30 Deviation (Analytic - Finite Difference) in calculation " );
  fprintf(stdout,"\n                                        of the u component of surface geostrophic wind (m/sec). " );
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  fprintf(stdout,"        CASE II.B Barotropic atmosphere - tilted (parallel) isobaric planes with delp T =0, under T = Az + B");
  fprintf(stdout,"\n");fprintf(stdout,"\n");
  fprintf(stdout," " );
  for (i=19;i>=3;i--) {
     for(j=3;j<20;j++) {
         fprintf(stdout,"%8.4f ", VBS);
     }fprintf(stdout,"\n");fprintf(stdout,"  " );}
   fprintf(stdout,"\n                                  Table B.31  Analytic v component of surface geostrophic wind  (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   for(i=3;i< 20;i++) {
     for (j=3;j<20;j++) {
/*   Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
     X directed pressure gradient by { 1/F }.                                                          */
     PX[i][j] =   ( (  G * ( ( ZS[i][j+1] - ZS[i][j-1]) / (2.0 * DELL)  ) )
                     + R *
                    (      ( ( TTS[i][j+1]  + TTS[i][j] )   / 4.0  )
                         * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                         + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                         * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*    Difference between analytic and Corby finite difference v surface geostrophic wind components    */
      DIFY[i][j] = VBS - (PX[i][j] / 100);
/*    VDIFTOT is an accumulator for the MVD error                                                      */
      VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
      }
   }
   fprintf(stdout,"      " );
   for (i=19;i>=3;i--) {
    for(j=3; j<20 ;j++) {
      PXS  =  PX [i][j]  / 100  ;
      fprintf(stdout,"%7.4f ", PXS  );
     }fprintf(stdout,"\n"); fprintf(stdout,"    " );}
  fprintf(stdout,"\n                       Table B.32 Finite Difference v component of surface geostrophic wind  (m/sec).");
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  fprintf(stdout," " );
   for (i=19;i>=3;i--) {
     for(j=3;j<20;j++) {
         fprintf(stdout,"%8.4f ",DIFY[i][j]);
      }fprintf(stdout,"\n");fprintf(stdout," " );}
  fprintf(stdout,"\n                                  Table B.33 Deviation (Analytic - Finite Difference) in calculation ");
  fprintf(stdout,"\n                                       of the v component of surface geostrophic wind (m/sec). " );
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  fprintf(stdout,"        CASE II.B Barotropic atmosphere - tilted (parallel) isobaric planes with delp T = 0, under T = Az + B");
  fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout," " );
  for (i=19;i>=3;i--){
     for(j=3;j<20;j++) {
/*        Analytic surface geostrophic wind magnitude                                                  */
          fprintf(stdout,"%7.4f ",QQ[i][j]);
     }fprintf(stdout,"\n");fprintf(stdout," " ); }
  fprintf(stdout,"\n                                 Table B.34 Analytic calculation of surface geostrophic wind (m/sec).");
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  for (i=19;i>=3;i--){
     for (j=3;j<20;j++) {
 /*       Corby finite difference surface geostrophic wind magnitude                                   */
          FDVEL =  0.01 * sqrt  ((PX[i][j] * PX[i][j]) + (PY[i][j] * PY[i][j])) ;
          fprintf(stdout,"%7.4f ",FDVEL);
          DIFY[i][j] = QQ[i][j]  - FDVEL  ;
     }fprintf(stdout,"\n");}
  fprintf(stdout,"\n");
  fprintf(stdout,"                          Table B.35 Finite Difference calculation of surface geostrophic wind  (m/sec).");
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  MAXDIF = 0.0;
   for (i=19;i>=3;i--){
     for(j=3;j<20;j++) {
/*         Find maximum (MAX) error                                                                    */
           if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*         Difference between analytic and Corby finite difference surface geostrophic wind magnitude  */
           fprintf(stdout,"%7.4f ",DIFY[i][j]);
        }fprintf(stdout,"\n");}
  fprintf(stdout,"\n                                 Table B.36 Difference (Analytic - Finite Difference) in calculation ");
  fprintf(stdout,"\n                                          of surface geostrophic wind (m/sec). " );
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/* Compute MVD  error                                                                                  */
  VDIFTOT = sqrt (VDIFTOT) / 289; ;
  fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
  fprintf(stdout,"%20.16f", MAXDIF  );
  //
  return 0;
}
```

```
#include     <StdIO.h>
#include     <math.h>
#include     <string.h>

/*  Case III with T = A ln p + B + C (x - xref).
    Baroclinic atmosphere with x-directed temperature gradient (1 * 10^{-5} degrees Kelvin/m),
    but no y-directed temperature gradient. The program is written in C.                                  */

main ()
{
#define     SIZEP           21
    double  ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,DTDXP,DTDYP,F,LEN;
    double  GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,VDIFTOT,UDIFTOT,FDVEL,difxsc;
    double  X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP],UBS[SIZEP][SIZEP],VBS[SIZEP][SIZEP];
    double  PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int     i,j;
    //
    R    =  2870000.0;
    G    =  980.0;
    F    =  0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.                 */
    ZREF =  0;
    TREF =  295.2;
    Z85R =  154000;
    T85R  = 286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =   - 9.45532738115e-5 ;
    DZDY =     4.61168237962e-5  ;
    DTDXP =    1.0e-7 ;
    DTDYP =    0.0  ;
    for(i=1;i<SIZEP ;i++)
    {
       for(j=1;j<SIZEP  ;j++) {
/*        Initialize matrix ZS with mountain surface heights.
          The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.   */
          LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11))  )  ;
          ZS[ i][j ] =  (H/2)  * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )   );
          if ( LEN >  8.0   ) ZS[ i][j ] =   0 ;
       }
     }
    fprintf(stdout,"\n"); fprintf(stdout,"\n");fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                                            */
/*  The reference station is at (X[3], Y[13], 0)                                                          */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
/*  Formulas below have been derived in Section 4.5                                                       */
    AAA1 = 980   * (Z85R - ZREF) ;
    AAA2 = ( ((TREF * TREF) - (T85R * T85R)) * R ) / (2.0 * AAA1) ;
    for(i=1;i<SIZEP;i++) {
       for(j=1;j<SIZEP;j++) {
/*     Initialize 850mb plane with height and temperature values.                                         */
       ZZ85 = Z85R + ( Y[i] * DZDY  + X[j] * DZDX  );
       TT85 = T85R + ( Y[i] * DTDYP + X[j] * DTDXP );
       GDEL = 980 * (ZZ85 - ZS [i][j]) ;
       HOLD = (((2 * AAA2) * GDEL) / R )  + ( TT85 * TT85 )   ;
/*     Initialize (mountain) surface temperature values.                                                  */
       TTS[i][j] = sqrt  ( HOLD )     ;
/*     Initialize ln surface pressure values. Note from the derivations                                   */
       of Sections 4.5 and 7.6  that - LNPS[i][j] = - ( lnps[i][j] - lnp850[i][j] )  = lnp850 - lnps[i][j]
       which is what is required                                                                          */
       LNPS[i][j] = ( (2 * GDEL) / (R * (TT85 + TTS[i][j] )) )  ;
/*     Analytic surface geostrophic wind components  u_{a} and v_{a}.                                     */
       UBS[i][j] = - 0.01 * (G * DZDY / F) ;
       VBS[i][j] =   0.01 * (G * DZDX / F)  -   ((R * LNPS[i][j] * DTDXP)/F)   )  ;
/*     Analytic surface geostrophic wind magnitude                                                        */
       QQ[i][j]   =   sqrt ((UBS[i][j] * UBS[i][j]) + (VBS[i][j] * VBS[i][j])) ;
       }
    }
   for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"     CASE III.A.I Baroclinic atmosphere - with  delp T =(1.0 * 10-5,0,0), under T = A ln p + B + C (x-xr)" );
    fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout,"         " );
    for (i=19;i>=3;i--)  {
       for(j=3;j<20;j++) {
          fprintf(stdout,"%7.4f ",UBS[i][j] );
        }
       fprintf(stdout,"\n"); fprintf(stdout,"        " );
    }
    fprintf(stdout,"\n");
    fprintf(stdout,"                                      Table B.37 Analytic u component of surface geostrophic wind (m/sec)." );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    VDIFTOT = 0.0;
     for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*      Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
        the Y directed pressure gradient by - { 1/F }.                                                    */
        PY [i][j] = - ( ( G * (  ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)  ) )
                       +  R *
                       (       ( ( TTS[i+1][j]  + TTS[i][j] )     / 4.0  )
                             * ( ( LNPS[i+1][j] - LNPS[i][j])     / DELL )
                             + ( ( TTS[i][j]     + TTS[i-1][j] )  / 4.0  )
                             * ( ( LNPS[i][j]    - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components    */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                       */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
         }
    }
```

```
 fprintf(stdout,"         " );
  for (i=19;i>=3;i--) {
     for(j=3;j<20;j++) {
        PYS  =  PY [i][j]  / 100  ;
          fprintf(stdout,"%7.4f ", PYS  );
     }fprintf(stdout,"\n"); fprintf(stdout,"          " );
   }
   fprintf(stdout,"\n");
   fprintf(stdout,"                              Table B.38 Finite Difference u component of surface geostrophic wind (m/sec).   ");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"     " );
    for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
          difxsc = DIFX[i][j] * lell ;
           fprintf(stdout,"%7.4f ", difxsc  );
       }fprintf(stdout,"\n");fprintf(stdout," " );
   }
   fprintf(stdout,"\n                                    Table B.39 Deviation (Analytic - Finite Difference) in calculation " );
   fprintf(stdout,"\n                                         of the u component of surface geostrophic wind ( 10^(-11) m/sec). " );
   for(i=1;i< 24;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"         CASE III.A.I Baroclinic atmosphere - with  delp T =(1.0 * 10-5,0,0), under T= A ln p + B + C (x-xr)" );
   fprintf(stdout,"\n");
   fprintf(stdout,"\n");fprintf(stdout,"  " );
   for (i=19;i>=3;i--) {
      for(j=3;j<20;j++) {
         fprintf(stdout,"%7.4f ", VBS[i][j] );
      }fprintf(stdout,"\n");fprintf(stdout," " );
   }
    fprintf(stdout,"\n");
    fprintf(stdout,"                                Table B.40 Analytic v component of surface geostrophic wind (m/sec).   " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    for(i=3;i< 20;i++) {
      for (j=3;j<20;j++) {
/*    Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
      X directed pressure gradient by ( 1/F ).                                                                                  */
      PX[i][j] =    ( (  G * ( ( ZS[i][j+1] - ZS[i][j-1]) / (2.0 * DELL)  ) )
                    + R *
             (         ( ( TTS[i][j+1]  + TTS[i][j] )   / 4.0  )
                     * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                     + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                     * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*       Difference between analytic and Corby finite difference v surface geostrophic wind components                  */
         DIFY[i][j] = VBS - (PX[i][j] / 100);
/*       VDIFTOT is an accumulator for the MVD error                                                                     */
         VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
      }
   }
   fprintf(stdout," " );
   for (i=19;i>=3;i--) {
    for(j=3;j<20 ;j++) {
       PXS  =  PX [i][j]  / 100 ;
     fprintf(stdout,"%7.4f ", PXS );
    }fprintf(stdout,"\n"); fprintf(stdout," " );}
    fprintf(stdout,"\n");
   fprintf(stdout,"                  Table B.41 Finite Difference v component of surface geostrophic wind (m/sec).   " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout," " );
   for (i=19;i>=3;i--) {
      for(j=3;j<20;j++) {
         fprintf(stdout,"%7.4f ",DIFY[i][j]);
      }fprintf(stdout,"\n");fprintf(stdout,"     " );}
   fprintf(stdout,"\n                              Table B.42 Deviation (Analytic - Finite Difference) in calculation " );
   fprintf(stdout,"\n                                  of the v component of surface geostrophic wind (m/sec). " );
   for(i=1;i< 24;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"                " );
   fprintf(stdout,"  CASE III.A.I Baroclinic atmosphere - with  delp T = (1.0 * 10-5,0,0), under T = A ln p + B + C (x-xr)" );
   fprintf(stdout,"\n"); fprintf(stdout,"\n");fprintf(stdout," " );
    for (i=19;i>=3;i--) {
      for(j=3;j<20;j++) {
/*        Analytic surface geostrophic wind magnitude                                                                    */
          fprintf(stdout,"%7.4f ",QQ[i][j]);}
      fprintf(stdout,"\n");fprintf(stdout," " ); }
   fprintf(stdout,"\n                              Table B.43 Analytic calculation of surface geostrophic wind (m/sec).   " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   for (i=19;i>=3;i--) {
      for (j=3;j<20;j++) {
/*       Corby finite difference surface geostrophic wind magnitude                                                      */
         FDVEL = 0.01 * sqrt ((PX[i][j] * PX[i][j]) + (PY[i][j] * PY[i][j])) ;
         fprintf(stdout,"%7.4f ",FDVEL);
            DIFY[i][j] = QQ[i][j] - FDVEL  ;
      }fprintf(stdout,"\n");}
   fprintf(stdout,"\n            Table B.44 Finite Difference calculation of surface geostrophic wind (m/sec).   " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   MAXDIF = 0.0;
   for (i=19;i>=3;i--) {
      for(j=3;j<20;j++) {
/*        Find maximum (MAX) error                                                                                       */
          if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*        Difference between analytic and Corby finite difference surface geostrophic wind magnitude                     */
          fprintf(stdout,"%7.4f ",DIFY[i][j]);
        }fprintf(stdout,"\n"); }
   fprintf(stdout,"\n                              Table B.45 Difference (Analytic - Finite Difference ) in calculation " );
   fprintf(stdout,"\n                                          of surface geostrophic wind (m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/* Compute MVD  error                                                                                                    */
   VDIFTOT = sqrt (VDIFTOT) / 289; ;
   fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
   fprintf(stdout,"%20.16f", MAXDIF );
   //
   return 0;
}
```

```
#include    <StdIO.h>
#include    <math.h>
#include    <string.h>

/*  Case III with T = A z + B + C (x - xref).
    Baroclinic atmosphere with x-directed temperature gradient (1 * 10^{-5} degrees Kelvin/m),
    but no y-directed temperature gradient. The program is written in C.                                  */
  main ()
{
#define     SIZEP       21
    double  ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,DTDXP,DTDYP,F,LEN,EXTRA,GAMMA;
    double  GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,VDIFTOT,UDIFTOT,FDVEL;
    double  X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP],UBS[SIZEP][SIZEP],VBS[SIZEP][SIZEP];
    double  PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int     i,j ;
    //
    R   =  2870000.0;
    G   =  980.0;
    F   =  0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.                 */
    ZREF =  0;
    TREF =  295.2;
    Z85R = 154000;
    T85R  = 286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =  - 9.45532738115e-5 ;
    DZDY =     4.61168237962e-5  ;
    DTDXP =   1.0e-7 ;
    DTDYP =   0.0  ;
    for(i=1;i<SIZEP ;i++)
    {
       for(j=1;j<SIZEP  ;j++) {
/*        Initialize matrix ZS with mountain surface heights.
          The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.    */
          LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11))  )  ;
          ZS[ i][j ] =  (H/2)  * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )   );
          if ( LEN >  8.0   ) ZS[ i][j ] =   0 ;
      }
    }
    fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                                              */
/*  The reference station is at (X[3], Y[13], 0)                                                           */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
    fprintf(stdout,"\n");fprintf(stdout,"\n");
    for(i=1;i<SIZEP;i++) {
       for(j=1;j<SIZEP;j++) {
/*     Initialize 850mb plane with height and temperature values.                                           */
       ZZ85 = Z85R + ( Y[i] * DZDY  + X[j] * DZDX  );
       TT85 = T85R + ( Y[i] * DTDYP + X[j] * DTDXP );
/*     Formulas below have been derived in Section 4.6                                                      */
       GAMMA = 9.0 / 154000 ;
/*     Initialize (mountain) surface temperature values.                                                    */
       TTS[i][j] = TT85 + GAMMA * (ZZ85 - ZS [i][j])       ;
/*     Initialize ln surface pressure values.                                                               */
       Note from the derivation of Section 7.7 that EXTRAX = ((DTDXP * G)/(GAMMA * F)) * ( 1 - (TTS[i][j]/TT85) )   */
       LNPS[i][j] = ( G / (R * GAMMA) ) * log ( 1 + ((GAMMA * (ZZ85 - ZS[i][j])) / TT85) )  ;
       EXTRAX = ((DTDXP * G)/(GAMMA * F)) * ( 1 - (TTS[i][j]/TT85) );
/*     Analytic surface geostrophic wind components  u_{a} and v_{a}.                                       */
       UBS[i][j] = - 0.01 * ( G * DZDY / F ) ;
       VBS[i][j] =   0.01 * ((G * DZDX / F ) + EXTRAX) ;
/*     Analytic surface geostrophic wind magnitude                                                          */
       QQ[i][j]    =     sqrt ((UBS[i][j] * UBS[i][j]) + (VBS[i][j] * VBS[i][j])) ;
       }
    }
    for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"        CASE III.B.1 Baroclinic atmosphere - with  delp T = (1.0 * 10-5,0,0), under T = Az+B+C(x-xr)" );
    fprintf(stdout,"\n"); fprintf(stdout,"\n");fprintf(stdout,"     " );
    for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
           fprintf(stdout,"%7.4f ",UBS[i][j] );
       }
       fprintf(stdout,"\n");fprintf(stdout,"     " );
    }
    fprintf(stdout,"\n                                        Table B.46 Analytic u component of surface geostrophic wind (m/sec).   " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
     VDIFTOT = 0.0;
     for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*      Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
        the Y directed pressure gradient by - { 1/F }.                                                      */
        PY [i][j] = - ( (  G * (  ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)  ) )
                    +  R *
                    (        ( ( TTS[i+1][j]  + TTS[i][j] )    / 4.0  )
                           * ( ( LNPS[i+1][j] - LNPS[i][j])    / DELL )
                           + ( ( TTS[i][j]    + TTS[i-1][j] )  / 4.0  )
                           * ( ( LNPS[i][j]   - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components       */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                         */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
         }
     }
```

Table B.12 Deviation (Analytic - Finite Difference) in calculation of the u component of surface geostrophic wind (m/sec).

```
  fprintf(stdout,"     " );
  for (i=19;i>=3;i--){
    for(j=3;  j<20 ;j++) {
      k = 22 - i;
      PYS  =  PY [i][j]  / 100  ;  fprintf(stdout,"%7.4f ", PYS  );}
    fprintf(stdout,"\n");fprintf(stdout,"      " );
  }
 fprintf(stdout,"\n                          Table B.47 Finite Difference u component of surface geostrophic wind (m/sec).   " );
 for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
 fprintf(stdout,"     " );
  for (i=19;i>=3;i--){
    for(j=3;j<20;j++) {
       fprintf(stdout,"%7.4f ",DIFX[i][j]);}
    fprintf(stdout,"\n");fprintf(stdout,"      " );
 }
 fprintf(stdout,"\n                                  Table B.48 Deviation (Analytic - Finite Difference) in calculation " );
 fprintf(stdout," \n                                         of the u component of surface geostrophic wind (m/sec). " );
 for(i=1;i<26;i++) { fprintf(stdout,"\n"); }
 fprintf(stdout,"          CASE III.B.1 Baroclinic atmosphere - with  delp T = (1.0 * 10-5,0,0), under T = Az+B+C(x-xr)" );
 fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout,"  " );
 for (i=19;i>=3;i--){
   for(j=3;j<20;j++) {
      fprintf(stdout,"%8.4f ",VBS[i][j] );}
   fprintf(stdout,"\n");fprintf(stdout,"  " );}
  fprintf(stdout,"\n                                 Table B.49 Analytic v component of surface geostrophic wind (m/sec). " );
  for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  for(i=3;i< 20;i++) {
    for (j=3;j<20;j++) {
/*    Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
      X directed pressure gradient by ( 1/F ).                                                           */
      PX[i][j] =   ( ( G * ( ( ZS[i][j+1] - ZS[i][j-1]) / (2.0 * DELL)  ) )
                   + R *
                   (      ( ( TTS[i][j+1]  + TTS[i][j] )   / 4.0  )
                        * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                        + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                        * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*     Difference between analytic and Corby finite difference v surface geostrophic wind components      */
       DIFY[i][j] = VBS - (PX[i][j] / 100);
/*     VDIFTOT is an accumulator for the MVD error                                                        */
       VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
       }
   }
  fprintf(stdout,"     " );
  for (i=19;i>=3;i--){
   for(j=3; j<20 ;j++) {
     PXS  =  PX [i][j]  / 100  ;
     fprintf(stdout,"%7.4f ", PXS  );
   }fprintf(stdout,"\n"); fprintf(stdout,"      " );
  }
 fprintf(stdout,"\n                          Table B.50 Finite Difference v component of surface geostrophic wind (m/sec).   " );
 for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
 fprintf(stdout," " );
  for (i=19;i>=3;i--){
    for(j=3;j<20;j++) {
       fprintf(stdout,"%8.4f ",DIFY[i][j]);
    } fprintf(stdout,"\n");fprintf(stdout,"  " );}
 fprintf(stdout,"\n                             Table B.51 Deviation (Analytic - Finite Difference) in calculation " );
 fprintf(stdout,"\n                                    of the v component of surface geostrophic wind (m/sec). " );
 for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
 fprintf(stdout,"          CASE III.B.1 Baroclinic atmosphere - with  delp T = (1.0 * 10-5,0,0), under T = Az+B+C(x-xr)" );
 fprintf(stdout,"\n");fprintf(stdout,"\n");
 fprintf(stdout," " );
  for (i=19;i>=3;i--){
     for(j=3;j<20;j++) {
/*        Analytic surface geostrophic wind magnitude                                                      */
          fprintf(stdout,"%7.4f ",QQ[i][j]);
     }fprintf(stdout,"\n");fprintf(stdout," " ); }
 fprintf(stdout,"\n");
 fprintf(stdout,"                         Table B.52 Analytic calculation of surface geostrophic wind (m/sec).  " );
 for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
  for (i=19;i>=3;i--) {
     for (j=3;j<20;j++) {
/*       Corby finite difference surface geostrophic wind magnitude                                       */
          FDVEL =  0.01 * sqrt ((PX[i][j] * PX[i][j]) + (PY[i][j] * PY[i][j])) ;
         fprintf(stdout,"%7.4f ",FDVEL);
         DIFY[i][j] = QQ[i][j]  - FDVEL  ;
     }fprintf(stdout,"\n");
  }
 fprintf(stdout,"\n                         Table B.53 Finite Difference calculation of surface geostrophic wind (m/sec). " );
 for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
 MAXDIF = 0.0;
 for (i=19;i>=3;i--){
    for(j=3;j<20;j++) {
/*        Find maximum (MAX) error                                                                          */
          if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*        Difference between analytic and Corby finite difference surface geostrophic wind magnitude      */
          fprintf(stdout,"%7.4f ",DIFY[i][j]);
       }fprintf(stdout,"\n");
  }
 fprintf(stdout,"\n                              Table B.54 Difference (Analytic - Finite Difference) in calculation  " );
 fprintf(stdout,"\n                                        of surface geostrophic wind (m/sec). " );
 for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/* Compute MVD  error                                                                                      */
 VDIFTOT = sqrt (VDIFTOT) / 289; ;
 fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
 fprintf(stdout,"%20.16f", MAXDIF  );
 //
 return 0;
}
```

```
#include    <StdIO.h>
#include    <math.h>
#include    <string.h>

/*  Case III with T = A ln p + B + C (x - xref).
    Baroclinic atmosphere with x-directed temperature gradient (2 * 10^(-5) degrees Kelvin/m),
    but no y-directed temperature gradient. The program is written in C.                                   */

main ()
{
#define     SIZEP          21
    double  ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,DTDXP,DTDYP,F,LEN;
    double  GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,L,VDIFTOT,UDIFTOT,FDVEL,difxsc;
    double  X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP],UBS[SIZEP][SIZEP],VBS[SIZEP][SIZEP];
    double  PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int     i,j;
    //
    R    =  2870000.0;
    G    =  980.0;
    F    =  0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.                   */
    ZREF =  0;
    TREF =  295.2;
    Z85R =  154000;
    T85R  = 286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =  - 9.45532738115e-5 ;
    DZDY =    4.61168237962e-5  ;
    DTDXP =   2.0e-7 ;
    DTDYP =   0.0  ;
    for(i=1;i<SIZEP ;i++)
    {
       for(j=1;j<SIZEP  ;j++) {
/*          Initialize matrix ZS with mountain surface heights.
            The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.   */
            LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11))  )   ;
            ZS[ i][j ] =  (H/2)   * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )   );
            if ( LEN >  8.0   ) ZS[ i][j ] =    0 ;
       }
     }
    fprintf(stdout,"\n"); fprintf(stdout,"\n");fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                                             */
/*  The reference station is at (X[3], Y[13], 0)                                                          */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
/*  Formulas below have been derived in Section 4.5                                                        */
    AAA1 = 980 * (Z85R - ZREF) ;
    AAA2 = ( ((TREF * TREF) - (T85R * T85R)) * R ) / (2.0 * AAA1) ;
    fprintf(stdout,"\n");
    for(i=1;i<SIZEP;i++) {
       for(j=1;j<SIZEP;j++) {
/*     Initialize 850mb plane with height and temperature values.                                          */
       ZZ85 = Z85R + ( Y[i] * DZDY  + X[j] * DZDX  );
       TT85 = T85R + ( Y[i] * DTDYP + X[j] * DTDXP );
       GDEL = 980  * (ZZ85 - ZS [i][j]) ;
       HOLD = (((2 * AAA2) * GDEL) / R )  + ( TT85 * TT85 )  ;
/*     Initialize (mountain) surface temperature values.                                                   */
       TTS[i][j] = sqrt ( HOLD )   ;
/*     Initialize ln surface pressure values. Note from the derivations                                    */
       of Sections 4.5 and 7.6  that - LNPS[i][j] = - ( lnps[i][j] - lnp850[i][j] )  = lnp850 - lnps[i][j]
       which is what is required                                                                           */
       LNPS[i][j] = ( (2 * GDEL) / (R * (TT85 + TTS[i][j] )) )  ;
/*     Analytic surface geostrophic wind components  u_{a} and v_{a}.                                      */
       UBS[i][j] = - 0.01 * (G * DZDY / F) ;
       VBS[i][j] =   0.01 * (G * DZDX / F)  -  ((R * LNPS[i][j] * DTDXP)/F)  )  ;
/*     Analytic surface geostrophic wind magnitude                                                         */
       QQ[i][j]   = sqrt ((UBS[i][j] * UBS[i][j]) + (VBS[i][j] * VBS[i][j])) ;
       }
    }
    for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"     CASE III.A.I Baroclinic atmosphere - with  delp T = (2.0 * 10-5,0,0), under T = A ln p + B + C (x-xr)" );
    fprintf(stdout,"\n");fprintf(stdout,"\n");fprintf(stdout,"         " );
    for (i=19;i>=3;i--)  {
       for(j=3;j<20;j++) {
          fprintf(stdout,"%7.4f ",UBS[i][j] );
       }
       fprintf(stdout,"\n"); fprintf(stdout,"       " );
    }
    fprintf(stdout,"\n");
    fprintf(stdout,"                                   Table B.55 Analytic u component of surface geostrophic wind (m/sec)." );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    VDIFTOT = 0.0;
     for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*      Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
        the Y directed pressure gradient by - { 1/F }.                                                     */
        PY [i][j] = - ( (  G * (  ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)  ) )
                        +  R *
                        (          ( ( TTS[i+1][j]  + TTS[i][j] )    / 4.0  )
                                 * ( ( LNPS[i+1][j] - LNPS[i][j])    / DELL )
                                 + ( ( TTS[i][j]    + TTS[i-1][j] )  / 4.0  )
                                 * ( ( LNPS[i][j]   - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components     */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                        */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
         }
     }
```

```
    fprintf(stdout,"        " );
    for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
         PYS  =  PY [i][j]  / 100  ;
         fprintf(stdout,"%7.4f ", PYS );
       }fprintf(stdout,"\n"); fprintf(stdout,"           " );
    }
    fprintf(stdout,"\n");
    fprintf(stdout,"                                Table B.56 Finite Difference u component of surface geostrophic wind (m/sec).   " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
     fprintf(stdout,"      " );
     for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
           difxsc = DIFX[i][j] * lell ;
           fprintf(stdout,"%7.4f ", difxsc );
        }fprintf(stdout,"\n");fprintf(stdout,"      " );
    }
    fprintf(stdout,"\n                               Table B.57 Deviation (Analytic - Finite Difference) in calculation " );
    fprintf(stdout,"\n                                     of the u component of surface geostrophic wind ( 10^{-11} m/sec). ");
    for(i=1;i< 24;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"        CASE III.A.I Baroclinic atmosphere - with  delp T =(2.0 * 10-5,0,0), under T = A ln p + B + C (x-xr)" );
    fprintf(stdout,"\n");
    fprintf(stdout,"\n");fprintf(stdout,"    " );
    for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
           fprintf(stdout,"%7.4f ", VBS[i][j] );
       }fprintf(stdout,"\n");fprintf(stdout,"      " );
    }
     fprintf(stdout,"\n                              Table B.58 Analytic v component of surface geostrophic wind (m/sec).   " );
     for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
     for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*     Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
       X directed pressure gradient by { 1/F }.                                                                  */
       PX[i][j] =  ( (  G * ( ( ZS[i][j+1] - ZS[i][j-1]) / (2.0 * DELL)  ) )
                       + R *
                       (      ( ( TTS[i][j+1]  + TTS[i][j] )    / 4.0  )
                            * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                            + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                            * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*        Difference between analytic and Corby finite difference v surface geostrophic wind components         */
          DIFY[i][j] = VBS - (PX[i][j] / 100);
/*        VDIFTOT is an accumulator for the MVD error                                                           */
          VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
          }
     }
    fprintf(stdout,"     " );
    for (i=19;i>=3;i--) {
     for(j=3;  j<20 ;j++) {
        PXS  =  PX [i][j]  / 100  ;
        fprintf(stdout,"%7.4f ", PXS  );
     }fprintf(stdout,"\n"); fprintf(stdout,"    " );}
     fprintf(stdout,"\n");
    fprintf(stdout,"                                Table B.59 Finite Difference v component of surface geostrophic wind (m/sec).   " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"   " );
    for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
          fprintf(stdout,"%7.4f ",DIFY[i][j]);
       }fprintf(stdout,"\n");fprintf(stdout,"   " );}
    fprintf(stdout,"\n                                            Table B.60 Deviation (Analytic - Finite Difference) in calculation " );
    fprintf(stdout,"\n                                                  of the v component of surface geostrophic wind (m/sec). " );
    for(i=1;i< 24;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"      " );
    fprintf(stdout,"      CASE III.A.I Baroclinic atmosphere - with  delp T =(2.0 * 10-5,0,0), under T=A ln p + B + C (x-xr)" );
    fprintf(stdout,"\n"); fprintf(stdout,"\n");fprintf(stdout," " );
     for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
/*         Analytic surface geostrophic wind magnitude                                                           */
          fprintf(stdout,"%7.4f ",QQ[i][j]);
       }fprintf(stdout,"\n");fprintf(stdout," " ); }
    fprintf(stdout,"\n                                   Table B.61 Analytic calculation of surface geostrophic wind (m/sec).   " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    for (i=19;i>=3;i--) {
       for (j=3;j<20;j++) {
/*        Corby finite difference surface geostrophic wind magnitude                                             */
          FDVEL =  0.01 * sqrt ((PX[i][j] * PX[i][j]) + (PY[i][j] * PY[i][j])) ;
          fprintf(stdout,"%7.4f ",FDVEL);
          DIFY[i][j] = QQ[i][j] - FDVEL  ;
       fprintf(stdout,"\n");}
    fprintf(stdout,"\n");
    fprintf(stdout,"                          Table B.62 Finite Difference calculation of surface geostrophic wind (m/sec).   " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    MAXDIF = 0.0;
    for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
/*          Find maximum (MAX) error                                                                             */
            if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*          Difference between analytic and Corby finite difference surface geostrophic wind magnitude          */
          fprintf(stdout,"%7.4f ",DIFY[i][j]);
          }fprintf(stdout,"\n");
     }
    fprintf(stdout,"\n                                         Table B.63 Difference (Analytic - Finite Difference ) in calculation " );
    fprintf(stdout,"\n                                              of surface geostrophic wind (m/sec). " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/*  Compute MVD  error                                                                                           */
    VDIFTOT = sqrt (VDIFTOT) / 289; ;
    fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
    fprintf(stdout,"%20.16f", MAXDIF  );
    //
    return 0;
}
```

```
#include     <StdIO.h>
#include     <math.h>
#include     <string.h>

/*  Case III with T = A z + B + C (x - xref).
    Baroclinic atmosphere with x-directed temperature gradient (2 * 10^{-5} degrees Kelvin/m),
    but no y-directed temperature gradient. The program is written in C.                                  */
 main ()
{
 #define     SIZEP         21
    double   ZZ85,TT85,AAA1,AAA2,DZDX,DZDY,DTDXP,DTDYP,F,LEN,EXTRA,GAMMA;
    double   GDEL,HOLD,UBS,VBS,MAXDIF,G,PXS,PYS,TREF,ZREF,T85R,Z85R,R,DELL,H,L,VDIFTOT,UDIFTOT,FDVEL;
    double   X[SIZEP],Y[SIZEP],ZS[SIZEP][SIZEP],TTS[SIZEP][SIZEP],LNPS[SIZEP][SIZEP],UBS[SIZEP][SIZEP],VBS[SIZEP][SIZEP];
    double   PX[SIZEP][SIZEP],PY[SIZEP][SIZEP],DIFY[SIZEP][SIZEP],DIFX[SIZEP][SIZEP],QQ[SIZEP][SIZEP];
    int      i,j;
    //
    R    =   2870000.0;
    G    =   980.0;
    F    =   0.000103;
/*  Pressure, temperature and height values are written in cgs (cm,grams,sec) units.
    Temperature at the (sea level, ZREF = 0) reference station is 295.2 degrees Kelvin.
    Over the reference station, the 850 mb plane is at height 1540m, and the temperature there is 286.2 degrees Kelvin.
    The mountain apex H is 5000m above sea level. The grid interval DELL is 5000m.
    DZDY is the (Z - Y) slope of the 850mb plane. DZDX is the (Z - X) slope of the 850mb plane.
    DZDYP is the (Z - Y) temperature gradient DZDXP is the (Z - X) temperature gradient.                 */
    ZREF =  0;
    TREF =  295.2;
    Z85R =  154000;
    T85R  = 286.2 ;
    H    =  500000;
    DELL =  500000;
    DZDX =   - 9.45532738115e-5 ;
    DZDY =     4.61168237962e-5  ;
    DTDXP =   2.0e-7 ;
    DTDYP =   0.0  ;
    for(i=1;i<SIZEP ;i++)
    {
       for(j=1;j<SIZEP  ;j++)
       {
/*        Initialize matrix ZS with mountain surface heights.
          The mountain's shape is such that it exhibits a cosine profile within any vertical plane containing the apex.   */
          LEN = sqrt ( ((i - 11) * (i - 11)) +  ((j - 11) * (j - 11))  )  ;
          ZS[ i][j ] =  (H/2)  * ( 1 + cos ( LEN  * ( 3.141592653589793 / 8.00 )  )   );
          if ( LEN >  8.0   ) ZS[ i][j ] =   0 ;
       }
     }
    fprintf(stdout,"\n");
/*  Initialize arrays X and Y to hold grid point positions. Index j of array X increases eastward
    and index i of array Y increases northward                                                            */
/*  The reference station is at (X[3], Y[13], 0)                                                          */
    for(j=1;j<SIZEP;j++) { X[j]  = ( j - 3  ) * 500000 ; }
    for(i=1;i<SIZEP;i++) { Y[i]  = ( i - 13 ) * 500000 ; }
    fprintf(stdout,"\n");fprintf(stdout,"\n");
    for(i=1;i<SIZEP;i++) {
        for(j=1;j<SIZEP;j++) {
/*      Initialize 850mb plane with height and temperature values.                                        */
        ZZ85 = Z85R + ( Y[i] * DZDY  + X[j] * DZDX  );
        TT85 = T85R + ( Y[i] * DTDYP + X[j] * DTDXP );
/*      Formulas below have been derived in Section 4.6                                                   */
        GAMMA = 9.0 / 154000 ;
/*      Initialize (mountain) surface temperature values.                                                 */
        TTS[i][j] = TT85 + GAMMA * (ZZ85 - ZS [i][j])      ;
/*      Initialize ln surface pressure values.                                                            */
        Note from the derivation of Section 7.7 that EXTRAX = ((DTDXP * G)/(GAMMA * F)) * ( 1 - (TTS[i][j]/TT85) )   */
        LNPS[i][j] = ( G / (R * GAMMA) ) * log ( 1 + ((GAMMA * (ZZ85 - ZS[i][j])) / TT85) )  ;
        EXTRAX = ((DTDXP * G)/(GAMMA * F)) * ( 1 - (TTS[i][j]/TT85) );
/*      Analytic surface geostrophic wind components  u_{a} and v_{a}.                                    */
        UBS[i][j] = - 0.01 * ( G * DZDY / F ) ;
        VBS[i][j] =   0.01 * ((G * DZDX / F ) + EXTRAX) ;
/*      Analytic surface geostrophic wind magnitude                                                       */
        QQ[i][j]   =    sqrt ((UBS[i][j] * UBS[i][j]) + (VBS[i][j] * VBS[i][j])) ;
        }
    }
    for(i=1;i< 14;i++) { fprintf(stdout,"\n"); }
    fprintf(stdout,"             CASE III.B.2 Baroclinic atmosphere - with delp T = (2.0 * 10-5,0,0), under T = Az+B+C(x-xr)" );
    fprintf(stdout,"\n");  fprintf(stdout,"\n");fprintf(stdout,"     " );
     for (i=19;i>=3;i--)  {
       for(j=3;j<20;j++) {
            fprintf(stdout,"%7.4f ",UBS[i][j] );
       }
       fprintf(stdout,"\n");fprintf(stdout,"     " );
    }
    fprintf(stdout,"\n");
    fprintf(stdout,"                         Table B.64 Analytic u component of surface geostrophic wind (m/sec).");
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
     VDIFTOT = 0.0;
     for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*       Corby finite difference surface geostrophic wind component u_{fd} is calculated by multiplying
         the Y directed pressure gradient by - { 1/F }.                                                  */
         PY [i][j] = - ( (  G * (   ( ZS[i+1][j] - ZS[i-1][j]) / (2.0 * DELL)  ) )
                        + R *
                        (         ( ( TTS[i+1][j]  + TTS[i][j] )    / 4.0  )
                                * ( ( LNPS[i+1][j] - LNPS[i][j])    / DELL )
                                + ( ( TTS[i][j]    + TTS[i-1][j] )  / 4.0  )
                                * ( ( LNPS[i][j]   - LNPS[i-1][j] ) / DELL ) ) ) / F ;
/*       Difference between analytic and Corby finite difference v surface geostrophic wind components   */
         DIFY[i][j] = VBS - (PX[i][j] / 100);
/*       VDIFTOT is an accumulator for the MVD error                                                      */
         VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
          }
     }
```

```
    fprintf(stdout,"     " );
    for (i=19;i>=3;i--) {
       for(j=3;  j<20 ;j++) {
          PYS  =  PY [i][j]  / 100  ;
          fprintf(stdout,"%7.4f ", PYS  );}
       fprintf(stdout,"\n");fprintf(stdout,"     " );
    }
   fprintf(stdout,"\n                         Table B.65 Finite Difference u component of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"     " );
    for(i=3;i< 20;i++) {
       for(j=3;j<20;j++) {
          k = 22 - i;
          fprintf(stdout,"%7.4f ",DIFX[k][j]);}
       fprintf(stdout,"\n");fprintf(stdout,"     " );}
   fprintf(stdout,"\n                              Table B.66 Deviation (Analytic - Finite Difference) in calculation " );
   fprintf(stdout,"\n                                     of the u component of surface geostrophic wind (m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"              CASE III.B.2 Baroclinic atmosphere - with delp T = (2.0 * 10-5,0,0), under T = Az+B+C(x-xr)");
   fprintf(stdout,"\n");fprintf(stdout,"\n");
   fprintf(stdout,"  " );
   for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
          fprintf(stdout,"%8.4f ",VBS[i][j] );}
       fprintf(stdout,"\n");fprintf(stdout,"  " );}
    fprintf(stdout,"\n                             Table B.67 Analytic v component of surface geostrophic wind (m/sec).");
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    for(i=3;i< 20;i++) {
       for (j=3;j<20;j++) {
/*     Corby finite difference surface geostrophic wind component v_{fd} is calculated by multiplying the
       X directed pressure gradient by { 1/F }.                                                        */
       PX[i][j] =   ( (  G * ( ( ZS[i][j+1] - ZS[i][j-1]) / (2.0 * DELL)  ) )
                     + R *
                     (     ( ( TTS[i][j+1]  + TTS[i][j] )   / 4.0  )
                         * ( ( LNPS[i][j+1] - LNPS[i][j] )  / DELL )
                         + ( ( TTS[i][j]    + TTS[i][j-1] ) / 4.0  )
                         * ( ( LNPS[i][j]   - LNPS[i][j-1]) / DELL ) ) ) / F ;
/*      Difference between analytic and Corby finite difference v surface geostrophic wind components      */
        DIFY[i][j] = VBS - (PX[i][j] / 100);
/*      VDIFTOT is an accumulator for the MVD error                                                        */
        VDIFTOT = VDIFTOT + ( DIFY[i][j] * DIFY[i][j] );
        }
    }
    fprintf(stdout,"     " );
    for (i=19;i>=3;i--){
      for(j=3;  j<20 ;j++) {
         PXS  =  PX [i][j]  / 100  ;
         fprintf(stdout,"%7.4f ", PXS  );}
      fprintf(stdout,"\n"); fprintf(stdout,"     " );}
   fprintf(stdout,"\n                         Table B.68 Finite Difference v component of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout," " );
    for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
          fprintf(stdout,"%8.4f ",DIFY[i][j]);}
      fprintf(stdout,"\n");fprintf(stdout,"  " );}
   fprintf(stdout,"\n                           Table B.69 Deviation (Analytic - Finite Difference) in calculation " );
   fprintf(stdout,"\n                                  of the v component of surface geostrophic wind (m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   fprintf(stdout,"            CASE III.B.2 Baroclinic atmosphere - with  delp T = (2.0 * 10-5,0,0), under T = Az+B+C(x-xr)" );
   fprintf(stdout,"\n");fprintf(stdout,"\n");
   fprintf(stdout," " );
   for (i=19;i>=3;i--){
       for(j=3;j<20;j++) {
/*        Analytic surface geostrophic wind magnitude                                                       */
          fprintf(stdout,"%7.4f ",QQ[i][j]);}
       fprintf(stdout,"\n");fprintf(stdout," " );  }
    fprintf(stdout,"\n                         Table B.70 Analytic calculation of surface geostrophic wind (m/sec).  " );
    for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
    for (i=19;i>=3;i--){
       for (j=3;j<20;j++) {
/*        Corby finite difference surface geostrophic wind magnitude                                        */
          FDVEL =  0.01 * sqrt ((PX[i][j] * PX[i][j]) + (PY [i][j] * PY [i][j])) ;
          fprintf(stdout,"%7.4f ",FDVEL);
          DIFY[i][j] = QQ[i][j]  - FDVEL  ;
       } fprintf(stdout,"\n");
   }
   fprintf(stdout,"\n                         Table B.71 Finite Difference calculation of surface geostrophic wind (m/sec).");
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
   MAXDIF = 0.0;
   for (i=19;i>=3;i--) {
       for(j=3;j<20;j++) {
/*           Find maximum (MAX) error                                                                       */
             if ( MAXDIF <  fabs ( DIFY[i][j] )  ) MAXDIF = fabs( DIFY[i][j] );
/*           Difference between analytic and Corby finite difference surface geostrophic wind magnitude     */
             fprintf(stdout,"%7.4f ",DIFY[i][j]);
            }fprintf(stdout,"\n");
    }
   fprintf(stdout,"\n                           Table B.72 Difference (Analytic - Finite Difference) in calculation");
   fprintf(stdout,"\n                                    of surface geostrophic wind (m/sec). " );
   for(i=1;i< 26;i++) { fprintf(stdout,"\n"); }
/* Compute MVD  error                                                                                       */
   VDIFTOT = sqrt (VDIFTOT) / 289; ;
   fprintf(stdout,"\n"); fprintf(stdout,"%20.16f", VDIFTOT );
   fprintf(stdout,"%20.16f", MAXDIF  );
  //
   return 0;
}
```

Table B.15 Deviation (Analytic - Finite Difference) in calculation of the v component of surface geostrophic wind (m/sec).

References

Arakawa A., and V. R. Lamb, 1977: Computational design of the basic dynamical processes of the UCLA General Circulation Model. Methods in Computational Physics, Vol. 17, J. Chang. Ed., Academic Press, 173-265.

—, and M. J. Suarez, 1983: Vertical differencing of the primitive equations in sigma coordinates, Monthly Weather Review, 111, 34-56.

Corby, G. A., A. Gilchrist and R. L. Newson,1972: A general circulation model of the atmosphere suitable for long period integration. Quart. J. Roy. Meteor. Soc., 98, 809-832.

Danard, M. B., 1989: On computing the surface horizontal pressure gradient over elevated terrain. Monthly Weather Review, 117, 1344-1350.

—, and J. Galbraith, 1985: Supplement to sensitivity test with the small area model. Unpublished report prepared by Atmospheric Dynamics Corporation, Victoria, B.C., June 1985, 22 pp. (IDEAL85)

—, Q. Zhang, and J. Kozlowski, 1993: On computing the horizontal pressure gradient force in sigma coordinates. Submitted to Monthly Weather Review.

Gary, J. M., 1973: Estimation of truncation error in transformed coordinate primitve equation atmospheric models. Journal of Atmospheric Sciences, 30, 223-233.

Holton, J., 1979: An Introduction to Dynamic Meteorology. Academic Press. 2nd Edition, 391 pp.

Haltiner, G. J., and R.T. Williams, 1980: Numerical Prediction and Dynamic Meteorology. John Wiley and Sons, 2nd Edition, 477 pp.

Janjic, Z., 1989: On the pressure gradient force error in sigma-coordinate spectral models. Monthly Weather Review, 117, 2285 - 2292.

Lindzen, R., 1990: Dynamics in Atmospheric Physics. Cambridge University Press, 1st Edition. 352 pp.

Kasahara, A., 1974: Various vertical coordinate systems used for numerical weather prediction. Monthly Weather Review, 102, 509 - 522.

Kasahara, A., 1977: Computational aspects of atmospheric simulation. Methods in Computational Physics. Vol. 17, J. Chang. Ed., Academic Press, 1-55.

Kurihara, Y., 1968: Note on the finite difference expressions for the hydrostatic relation and pressure gradient force. Monthly Weather Review, 96, 654-656.

Mahrer, Y., 1984. An improved numerical approximation of the horizontal gradients in a terrain-following coordinate system. Monthly Weather Reivew, 112, 918 - 922.

Mesinger, F., 1982: On the convergence and error problems of the calculation of the pressure gradient force in sigma coordinate models Geophys. Astrophys. Fluid Dyn., 19, 105 - 117.

Mesinger, F., Z. Janjic, S. Nickovic, D. Gavrilov and D. Deaven, 1988: The step-mountain coordinate: model description and performance for cases of alpine lee cyclogenisesis and for a case of an appalachian redevelopment. Monthly Weather Review, 116, 1493 - 1518.

Mihailovic, D., and Z. Janjic, 1986: Comparison of methods for reducing the error of the pressure gradient force in sigma coordinate models. Meteorology and Atmospheric Physics, 35, 177 - 184.

Mihailovic, D., 1986: Pressure gradient force and calculation of initial temperature in terrain-following coordinate models. Meteorology and atmospheric physics, 35, 53 - 58.

Nakamura, H., 1978: Dynamical effects of mountains on the general circulation of the atmosphere: I. Development of finite difference schemes suitable for incorporating mountains. J. Meteor. Soc. Japan, 56, 317-339.

Orlanski, I., 1975: A rational subdivision of scales for for atmospheric processes. Bulletin of the American Meteroroligical Society., 56, 527-530.

Phillips, N. A., 1957: A coordinate system having some special advantages for numerical forecasting. J. Meteor., 14, 184-185.

Pielke, R. A. 1984. Mesoscale Meteorological Modeling. Academic Press, Orlando, Florida. 450 pp.

Press, W., B. Flannery, S. Teukolsky, and W. Vetterling, 1988. Numerical Recipies in C. Cambridge University Press, Cambridge. 735 pp.

Simmons, A.J., and D. Burridge, 1981: An energy and angular-momentum conserving vertical finite-difference scheme and hybrid vertical coordinates. Monthly Weather Review. 109, 758-770.

Sundqvist, H. 1975: On truncation errors in the sigma-system models, Atmosphere, 13, 81-95.

——, 1976: On vertical interpolation and truncation in connexion with use of sigma-system models. Atmosphere, 14, 37-52.

Tokioka, T., 1978: Some considerations on vertical differencing. J. Meteor. Soc. Japan, 56, 98 - 111.

Tomine, K., and S. Abe, 1982: A trial to reduce truncation errors of the pressure gradient force in the sigma coordinate systems. J. Meteor. Soc. Japan, 60, 709 - 716.

Wallace, J., and P. Hobbs, 1977. Atmospheric Science. Academic Press. New York. 467 pp.

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