

Urban Scaling and the Benefits of Living in Cities

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Abstract: Complexity science offers tools to better understand the processes underlying city development, which is particularly important for building cities that are environmentally, socially, and economically sustainable. Urban scaling laws are empirically observed power-law relationships between population (i.e., city size) and other characteristics of cities in the same country. The theoretical underpinnings of these relationships have been proposed in *The Origins of Scaling in Cities* model, which derives scaling exponents from fundamental concepts in economic geography and general first principles of physics. The model introduces a parameter called *net urban benefit*, defined as a city's social output minus the dissipative costs of infrastructure, representing a new metric relevant to city sustainability. Here we present a new derivation for a functional form of the metric, as well as new derivations that reveal how cities behave when the metric is maximized. We show results for U.S. cities and discuss the implications for the sustainability of the U.S. urban system, including the feedback mechanisms that keep cities on their current trajectories toward maximizing their socioeconomic output while minimizing infrastructure costs.

Keywords: social benefits; infrastructure costs; science of cities

Introduction

In order to develop sustainable cities of the future, we need to better understand the processes that have built the cities we have now. Cities are humanity's greatest invention: self-built habitats that not only provide access to food and shelter, but social, cultural, and intellectual opportunities as well. The existence of cities is driven by humanity's imperative to meet our most fundamental needs, which are most effectively provided for collectively rather than individually. The evolution and nature of cities has been described by a multitude of disciplines, including architecture (Kostof, 1991); economics (Von Thünen, 1826; Glaeser, 2011; Kennedy, 2011); history (Mumford, 1961; Hall, 1998); physics (Bettencourt et al. 2007; Bettencourt, 2013; Bristow & Kennedy, 2015; Kennedy et al. 2015); planning theory (Jacobs, 1969; Lynch, 1981) and sociology (Wirth 1938). Cities connect people with various skills and interests in a multi-component, interconnected system that displays properties consistent with other complex

systems observed in nature. Complexity science gives us a unique set of tools to better understand the self-organized patterns that emerge in cities, such as urban scaling. With this insight, we can better understand the systemic impacts, implementation challenges, and potential drawbacks of various city sustainability strategies, particularly those related to urban planning and transportation.

With origins in biology (West et al. 1997) and human ecology (Zipf, 1949), scaling laws can remarkably be understood through a synthesis of economic geography and physics when applied to cities. In the science of cities literature, Bettencourt *et al.* (2007) have established a variety of scaling relationships for cities; namely, the power-law relationships between population (i.e., city size) and other characteristics of cities in the same country (i.e., the same urban system). The generalized form of city scaling relationships is:

$$Y = Y_0 N^\beta \quad (1)$$

where Y is the city indicator (associated with GDP below), Y_0 is the y-axis intercept, N is population, and β is the scaling exponent. Bettencourt *et al.* (2007) show that social phenomena, such as economic activity, patenting, and crime, scale superlinearly with population size (i.e., $\beta > 1$), while infrastructure related quantities, such as length of infrastructure (e.g., road length, electricity cables), scale sublinearly (i.e., $\beta < 1$). Figure 1 shows examples of the relationships for road area and GDP of U.S. cities for the year 2014. These empirical results confirm that cities produce agglomeration effects in terms of social quantities, while maintaining economies of scale for infrastructure. Amongst the scaling relationships observed for cities is the tendency for urban density to increase with city size (Sugar & Kennedy, 2020), which is an important attribute for urban sustainability (Norman et al. 2006; Kennedy et al. 2009, 2015, Seto et al., 2012).

<Fig. 1 here>

Bettencourt (2013) outlined some of the contributing factors to these observed scaling phenomena in his model, *The Origins of Scaling in Cities*. The model starts with a fundamental concept in economic geography where city-dwellers balance income and costs, such as transportation and housing costs, and then derives scaling exponents from general first principles of physics, as well as concepts from particle physics, electromagnetism, and network theory. Bettencourt's derived scaling relationships are very close to those observed in countries around

the world. The model introduces two new parameters with relevance for the development of cities: *net urban benefit*, \mathcal{L} , is defined as a city's social output minus the dissipative costs of infrastructure, and the parameter G , which we call the *scaling balance indicator*, is a city's balance of superlinear and sublinear scaling variables.

Here we present a deeper investigation into the potential to measure net urban benefits using Bettencourt's model. We present new derivations for a functional form of the metric *maximum net urban benefit*, \mathcal{L}^* , which allows us to calculate it for U.S. cities and discuss the implications for the sustainability of the U.S. urban system. Derived from a balance between outcomes of social interactions and transportation/infrastructure costs, net urban benefit gives insight into the drivers that shape the urban landscape—and most importantly, how these drivers can come head-to-head with efforts to promote city sustainability.

Interpretations of the Origins of Scaling in Cities Model

Bettencourt introduces the parameter G , as follows:

$$G = \frac{A_n Y}{N N} \quad (2)$$

where Y is a city's social output (superlinear scaling), measured using GDP, A_n is a city's network area (sublinear scaling), measured using total road volume, N is population. It is important to note that unlike its constituent variables, values for G for all cities in a given country do not scale with population, but instead tend to cluster around an average value for all cities, defined as G^* (Figure 2).

<Fig. 2 here>

We refer to G as the scaling balance indicator and G^* as the average scaling balance indicator because G^* is also the product of the coefficients for superlinear and sublinear scaling relationships. Bettencourt (2013) defines the generalized power-law relationship for A_n as $A_n = A_0 N^{1-\delta}$ and for Y as $Y = Y_0 N^{1+\delta}$, where δ equals to 1/6. Substituting into Equation 2, we get $G = A_0 Y_0$. Like with scaling relationships themselves, the average scaling balance indicator is unique to a particular system of cities, often all the cities in a given country. Scaling relationships

break down when considering data from different countries/city-systems because the underlying processes and design decisions driving their development differs. In this sense, the average scaling balance indicator is a reflection of the status quo of its urban system; that is, a representation of the way its cities are at a given point in time.

Related to G , another parameter introduced in Bettencourt's model is *net urban benefit*, \mathcal{L} , which is defined as follows:

$$\mathcal{L} = Y - W \quad (3)$$

where Y is, again, a city's social output and W is the cost of infrastructure, which is derived in a similar manner to energy dissipation in a circuit. In this sense, the parameter \mathcal{L} goes beyond GDP to consider the dissipative costs of infrastructure in the city. Bettencourt (2013) defines the parameters Y and W as follows:

$$Y = G \frac{N^2}{A_n(N)} \quad (4)$$

$$W = r' \left(\frac{a}{l} \right)^2 \frac{J^2(N)}{A_n(N)} \quad (5)$$

where r' is the resistance of the infrastructure network; a is baseline area, defined as $a = \left(\frac{G}{\epsilon} \right)^\alpha$, where ϵ is the cost (or force) per person spent on transportation needed to keep the city mixed and α is the scaling exponent for area, $2/3$; l is baseline network distance; and J is the "current" of goods and people moving through the city's infrastructure network, defined as the product of an individual current, J_0 , times population, $J = J_0 N$.

Bettencourt (2013) describes the relationship between \mathcal{L} and G as a curve that is concave down, which can be conceptualized by deriving three main points. The maximum and minimum values for G occur when \mathcal{L} is zero (Equation 6), defining a condition for G_{max} (i.e., the point after which the costs of the city outweigh its benefits). The global maxima of the G versus \mathcal{L} curve, when $\frac{d\mathcal{L}}{dG}$ is zero (Equation 7), occurs when $G = G^*$, equivalent to $1/8$ of G_{max} .

$$G|_{\mathcal{L}=0} = \begin{cases} 0 & : G_{min} \\ \left(\frac{\epsilon^{2\alpha} l^2}{J_0^2 r'} \right)^{\frac{1}{2\alpha-1}} & : G_{max} \end{cases} \quad (6)$$

$$G \Big|_{\frac{d\mathcal{L}}{dG}=0} = G^* = \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{2\alpha-1}} G_{max} = \frac{G_{max}}{8} \quad (7)$$

Therefore, we can say that at G^* the city is achieving its maximum net urban benefit. The corresponding value for \mathcal{L} at G^* is derived by Bettencourt (Equation 8), which demonstrates that the maximum net urban benefit for a given city depends on the nature of the city itself (i.e., its N and A_n), as well as the nature of the other cities in the country (i.e., the country's G^*). We denote the value for maximum net urban benefit with \mathcal{L}^* .

$$\mathcal{L} \Big|_{G=G^*} = \mathcal{L}^* = \frac{2\alpha-1}{\alpha} G^* \frac{N^2}{A_n(N)} = \frac{1}{2} G^* \frac{N^2}{A_n} \quad (8)$$

Bettencourt (2013) describes maximum net urban benefit and G^* as idealized quantities that cities are trying to achieve through their urban planning and policies, which have at their roots in the “tension between social interactivity, transportation costs, and spatial settlement patterns” (Bettencourt 2013). We argue that the maximum net urban benefit and G^* are also reflective of the nature of the urban system at a given point in time—that is, how the status quo of the system manages issues related to social interactivity, transport costs, and settlement patterns. Therefore, these metrics give more insight as an indication of the sustainability of the system itself, and how close specific cities are to the status quo.

A Functional Form of Net Urban Benefit, \mathcal{L}

We will now build upon the Bettencourt model and derive a new equation for \mathcal{L} , as a function of G . The full derivation is provided in the Supplementary Information (SI).

First, we combine Equations 3-7 to give a new equation for \mathcal{L} :

$$\mathcal{L} = G \frac{N^2}{A_n} \left(1 - \left(\frac{G}{8G^*} \right)^{2\alpha-1} \right) \quad (9)$$

From this equation, we can show that \mathcal{L} scales superlinearly. Substituting $A_n = A_0 N^{1-\delta}$, we find $\mathcal{L} \propto N^{1+\delta}$, which is the same theoretical scaling exponent as for Y . We can also confirm

Equation 8 by setting $G = G^*$ and $\alpha = 2/3$, as well as show that the units for \mathcal{L} are the same as for Y (see SI).

Next, to show this equation graphically, we define two new ratios:

$$\gamma = \frac{G}{G^*} \quad (10)$$

$$\lambda = \frac{\mathcal{L}}{\mathcal{L}^*} \quad (11)$$

Combining Equations 9-11, and setting $\alpha = 2/3$, we get a new equation that can be plotted on γ versus λ axes:

$$\lambda = \frac{\mathcal{L}}{\mathcal{L}^*} = 2 \gamma^{\frac{1}{3}} \left(1 - \frac{1}{2} \gamma^{\frac{1}{3}}\right) \quad (12)$$

In Figure 3A, we plot Equation 12 between $0 \leq \gamma \leq 8$. Here we see that the shape of the curve is right-skewed. The local maximum occurs when $\lambda = 1$ (i.e., $\mathcal{L} = \mathcal{L}^*$) and $\gamma = 1$ (i.e., $G = G^*$).

Rearranging Equation 12 gives us a new, functional form of \mathcal{L} that we can use with our previously calculated values for \mathcal{L}^* , G , and G^* ($\gamma = \frac{G}{G^*}$):

$$\mathcal{L} = 2 \mathcal{L}^* \gamma^{\frac{1}{3}} \left(1 - \frac{1}{2} \gamma^{\frac{1}{3}}\right) \quad (13)$$

The City at Its Maximum Net Urban Benefit, \mathcal{L}^*

We will now derive new formulations for \mathcal{L} , Y , and W in terms of G , which allows us to better understand the relationships between these variables, including how they are interacting when a city is at maximum net urban benefit (see SI for full derivations). Combining Equations 4, 8 and 10:

$$\frac{Y}{\mathcal{L}^*} = 2\gamma^{\frac{1}{3}} \quad (14)$$

Combining Equations 3, 4, 8, 9, and 10:

$$\frac{W}{\mathcal{L}^*} = \gamma^{\frac{2}{3}} \quad (15)$$

This confirms Bettencourt's (2013) observation that Y is proportional to $G^{1-\alpha}$ and W is proportional to G^α , or when $\alpha = 2/3$, $Y \propto G^{1/3}$ and $W \propto G^{2/3}$. Both Y and W increase with G , though at different rates.

<Fig. 3 here>

We can now plot Equations 12, 14, and 15 on the same axis, as shown in Figure 3B, from which we can make some important observations. The first is that for positive values of \mathcal{L} , Y is greater than W ; however, W catches up with Y as G approaches G_{max} (i.e., $8G^*$). Therefore, simply maximizing Y is not a viable path to \mathcal{L}^* . In other words, according to Bettencourt's model, maximizing GDP does not lead to maximum net urban benefit because increasing urban benefit can also increase dissipative costs. At the value of maximum net urban benefit (i.e., when $\gamma = 1$ and $G = G^*$), Y is twice the value of \mathcal{L}^* , but W is equal to \mathcal{L}^* . In contrast, at the point $\gamma = 8$ (i.e., $G = 8G^* = G_{max}$), Y is four times larger than \mathcal{L}^* , but W catches up with Y and \mathcal{L} goes to zero. Therefore, we can say that cities that are maximizing their net urban benefit, i.e., cities where $\mathcal{L} = \mathcal{L}^*$, have achieved a balance where their dissipative costs, W , are half of their total GDP, Y .

Results: Net Social Benefit for U.S. Cities

Using our derivations from above, we are now able to calculate actual net urban benefit, \mathcal{L} , and maximum net urban benefit, \mathcal{L}^* , for cities in the U.S. We define cities as U.S. Metropolitan Statistical Areas (MSAs) in this analysis, and we use data from 364 MSAs for the year 2014.

First, we can calculate maximum net urban \mathcal{L}^* for each individual city, as per Equation 8. The values scale superlinearly with population, as expected (Figure 4A). Using our previously calculated values for G and G^* (shown in Figure 2 above), we can combine these values with Equation 13 to calculate actual net urban benefit, \mathcal{L} , for the U.S. cities, shown in Figure 4B. As expected, these values also scale superlinearly with population.

<Fig. 4 here>

The ratio between \mathcal{L} and \mathcal{L}^* (i.e., $\lambda = \frac{\mathcal{L}}{\mathcal{L}^*}$) is generally high for U.S. cities, with more than half of the cities at 99% of their \mathcal{L}^* or higher (see Table S2). Table 1 shows the 20 cities that are closest to their \mathcal{L}^* , as well as the 20 cities that are the farthest from their \mathcal{L}^* . Results for all cities are provided in the SI (Table S3).

<Table 1 here>

The five cities that are closest reaching their \mathcal{L}^* are Columbia, MO, Hattiesburg, MS, Burlington, VT, Orlando, FL, and Buffalo, NY. These cities are not the largest nor wealthiest in country, but they are instead small- to medium-sized, secondary cities that are frequently overshadowed by their larger counterparts. It is interesting to note that, apart from New York City, the remaining top 20 cities closest to their \mathcal{L}^* could be described in a similar way.

The cities that are farthest from their \mathcal{L}^* values, Midland, TX, and East Stroudsburg, PA, are also the most extreme outliers on either side of G^* , as shown in Figure 2 above. Both are cities with populations near 160,000, but Midland, TX has the highest GDP per capita in the country at nearly \$145,000, while East Stroudsburg, PA is among the poorest with a GDP at about \$30,000 per capita. Accordingly, Midland, TX is far on the right side of G^* with $G = 4.79$ and \mathcal{L} at 51% of \mathcal{L}^* , and East Stroudsburg, PA is on the left side of G^* with $G = 0.015$ and \mathcal{L} at 44% of \mathcal{L}^* . Midland, TX, part of the Permian Basin, has thriving oil industry that is arguably outpacing its urban growth. In contrast, East Stroudsburg, PA, has seen economic decline and factory closures over the previous decades. Other interesting cases to note on the farthest 20 list are Deltona-Daytona Beach-Ormond Beach, FL (which includes Palm Bay, FL) and The Villages, FL. Palm Bay and The Villages were the two fastest growing Core Based Statistical Areas from 2000-2010, adding 92 percent and 75 percent respectively to their populations in the 10-year period (U.S. Census Bureau 2012). Their farther distance from \mathcal{L}^* may indicate that their population growth was outpacing their economic development and infrastructure.

Discussion

The results provide insights into the fundamental nature of cities, with implications for socio-economic and environmental aspects of urban sustainability. Our findings that over half of U.S. cities are within 99% of their \mathcal{L}^* , and the top five U.S. cities closest to \mathcal{L}^* are medium-sized, secondary cities, warrants a closer look at the concept of net urban benefit. Conceptualizing the

quantities of \mathcal{L}^* and G^* as desirable positions that urban practitioners are aiming to achieve with their policies would indicate that the majority of cities in the U.S. are in a state of idealized balance. However, growing problems related to climate change, resource consumption, and waste production prove otherwise. Our findings support our assertion that the concept of maximum net urban benefit is not necessarily normative, but in fact reflects the nature of the urban system. We can instead view G^* , and hence \mathcal{L}^* , as attractors of the urban system that cities trend towards. As described above, G^* is empirically calculated as the average of all G values in the country—and theoretically as the product of the coefficients of the country’s superlinear and sublinear scaling relationships—making it more indicative of the status quo of the urban system, i.e., a representation of the way cities are at a given point in time. Therefore, the two cities closest to their \mathcal{L}^* —Columbia, MO, and Hattiesburg, MS—are better described as the best representation of a typical city in the United States, not necessarily those with the most “net benefit”.

If we consider G^* and \mathcal{L}^* as attractors, there are various feedback mechanisms keeping cities on their trajectories towards them, which reveals information about the sustainability and agility of the urban system itself. Referring to Figure 3B above, if cities move too far to the right of G^* with rapid economic growth (i.e., rapid growth in Y , which represents GDP), the costs quickly catch up, which can trigger feedback mechanisms (Figure 5). The feedback mechanisms may in fact be recognizable as strategic policies many cities employ as part of sustainability efforts. A city that is rapidly growing in terms of GDP and infrastructure sprawl will confront rising transport congestion and costs with efficiency measures, such as building rapid commuter transit and/or building pockets of densified housing surrounding transit nodes and employment opportunities. Increasing infrastructure complexity, through transit and densification, allows cities to rein in the dissipative cost of infrastructure that would otherwise impede economic growth (see Sugar and Kennedy, 2020). On the left side of G^* , GDP and infrastructure are both underdeveloped compared to average. In this case, the city will respond by investing in infrastructure to attract more businesses and entrepreneurs to grow the economy—an approach that forms the basis of much of the economic development literature published by think tanks and government agencies (e.g., see EPA 2016, Liu 2016, McKinsey & Company 2013).

<Fig. 5 here>

These attractors and feedback mechanisms are important for the sustainability of the urban system because the status quo of the urban system, and hence G^* , corresponds to a specific balance between income and costs that city-dwellers in a specific country strive towards. This can be thought of as a collective paradigm of “ideal city living.” The *Origins of Scaling in Cities* model is rooted in Alonso’s theory of land rents (Alonso 1964), an extension of Johann Heinrich von Thünen’s model of land use (von Thünen, 1826) that is applied to urbanization. While von Thünen considered agricultural land use and modeled how market factors would shape the type and locations of different crops, Alonso’s model employs the concept equilibrium of the household, where a household uses its income to maximize its city-living experience. Households partition their income between three budget items: housing, transportation, and goods and services. A country’s policies, culture, and values all influence how a household partitions its income, and hence the feedback mechanisms in place that shape the nature of its cities. The most obvious example is that of the U.S. and its automobiles. When gasoline is cheap and cars are readily available, more household income can be spent on larger houses and more consumer goods. In turn, cities become more sprawled, suburban houses get larger and larger, and big retail chains sell more and more products. Contrast this with a city like Singapore, where automobile ownership is restricted and 80 percent of the population lives in high-density public housing (Singapore HDB, 2018), and there is a very different urban landscape.

The basin of attraction for U.S. cities—the collective paradigm of “ideal city living”, which we represent mathematically as G^* —relies on cheap sources of energy and an abundance of space, and the feedback mechanisms in place in terms of urban planning keep cities within its range. As we face a probable future with climatic change, resource scarcity, and large-scale migration, the longevity of the current urban system is being called into question. Disruptions and shocks, such as spikes in energy costs and, most recently, economic disruption caused by the COVID-19 pandemic, have the potential to alter the costs-benefit curve and create new definitions of ‘maximum net urban benefit.’ The question is whether the urban system will be agile enough to adjust to a new reality, or whether cities will fall into a new, potentially unknown, attractor basin of human inventions left behind.

Conclusion

The primary contributions of this paper are new derivations for net urban benefits and maximum net urban benefits for cities using *The Origins of Scaling in Cities* model. Using data for 2014, we calculate the actual and maximum net urban benefits for 364 U.S. Metropolitan Statistical Areas (MSAs). The close proximity of net urban benefit to the maximum value for many of the cities suggests that the parameter G^* may be viewed as an attractor of the urban system that cities trend towards, maximizing their socioeconomic output while minimizing infrastructure costs. This has implications for the future sustainability of cities, as many of the city development processes and feedback mechanisms that have built cities today are persistent in their reflection of cultural norms, yet vulnerable to acute shocks and disruption.

Methods

Data are for 364 U.S. Metropolitan Statistical Areas (MSAs) for the year 2014. The methodology for calculating G is consistent with Equation 2 above, using data for GDP and population from the Bureau of Economic Affairs (BEA 2016) and data for road area from the Federal Highway Administration (FHWA 2015b). The methodology for calculating \mathcal{L} and \mathcal{L}^* is consistent with Equations 13 and 8 above, with G^* taken to be 0.979 (the mean value for G across all cities). Full description of methods is provided in SI.

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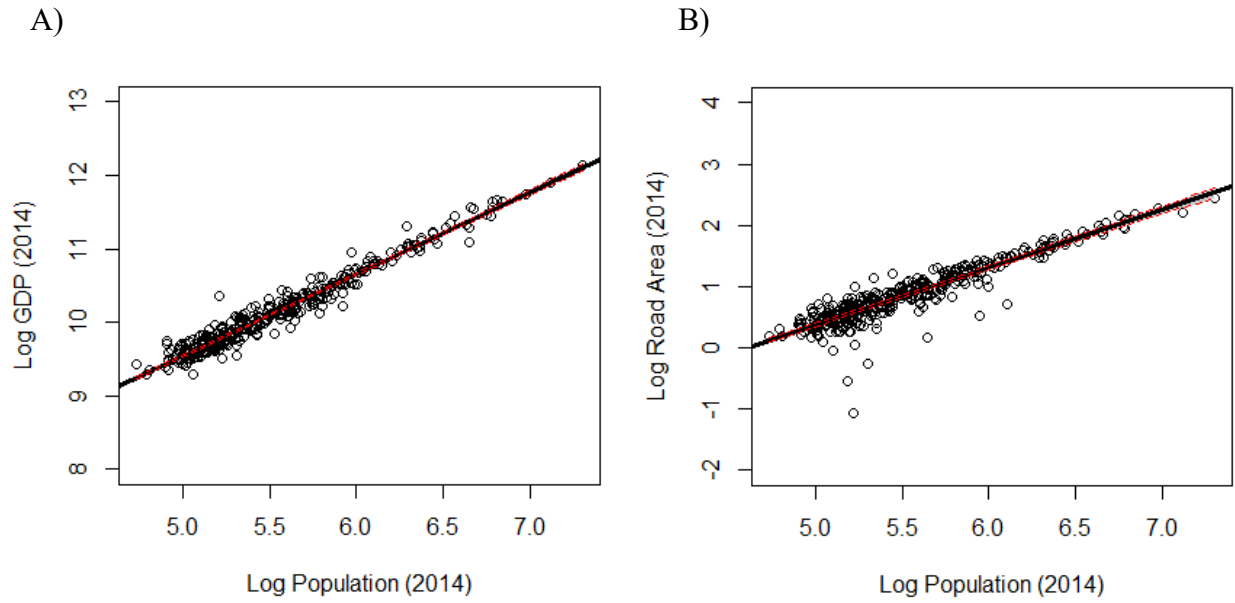


Figure 1: Scaling relationships for U.S. cities for the year 2014 for 364 Metropolitan Statistical Areas: A) GDP (real GDP in 2009 chained dollars) has superlinear scaling ($\beta = 1.11 \pm 0.02$, $R^2 = 0.96$, $n = 364$); B) road area (square miles) has sublinear scaling ($\beta = 0.93 \pm 0.04$, $R^2 = 0.84$, $n = 364$). *Data sources: GDP data is from BEA 2016; road area data is calculated from FHWA 2015.*

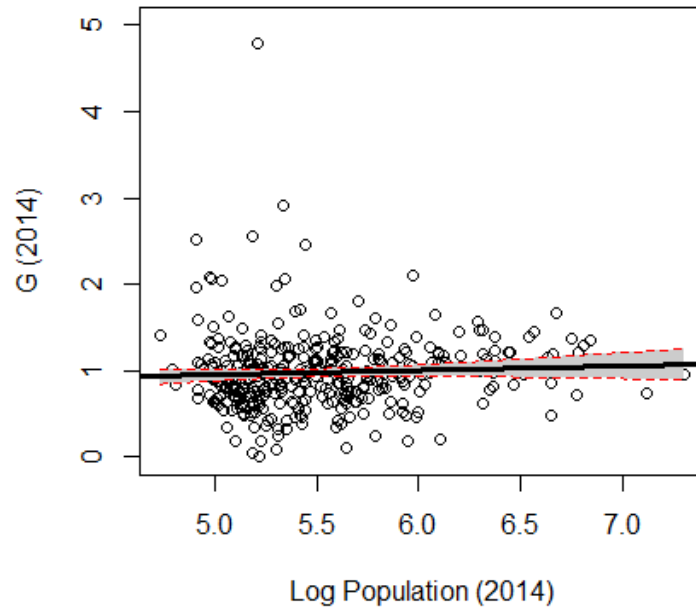


Figure 2: The scaling balance indicator, G , for 364 U.S. Metropolitan Statistical Areas for the year 2014 ($\beta = 0.054 \pm 0.097$, $R^2 = 0.0033$, $n = 364$), confirming that G is independent of population ($\beta = 0$ falls within the confidence interval of the slope). The values cluster around $G^* = 0.979$, which occurs when $\beta = 0$. The methodology for calculating G is provided in the Supplementary Information. *Data sources: GDP data is from BEA 2016; road area data is calculated from FHWA 2015.*

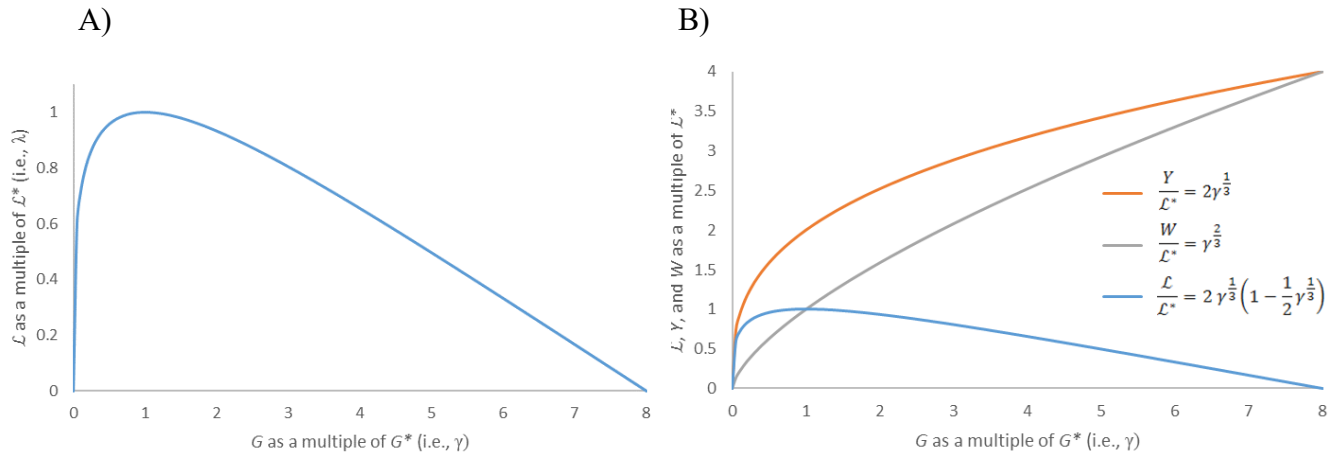


Figure 3: Relationships between G and other model parameters. A) λ versus γ , as described by Equation 12. The y-axis corresponds to the ratio of net urban benefit, L , to maximum net urban benefit, L^* ; the x-axis is G as a multiple of G^* ; B) L , Y , and W versus γ , as described by Equations 12, 14, and 15. The y-axis corresponds to the parameter (L , Y , or W) as a multiple of maximum net urban benefit, L^* ; the x-axis is G as a multiple of G^* .

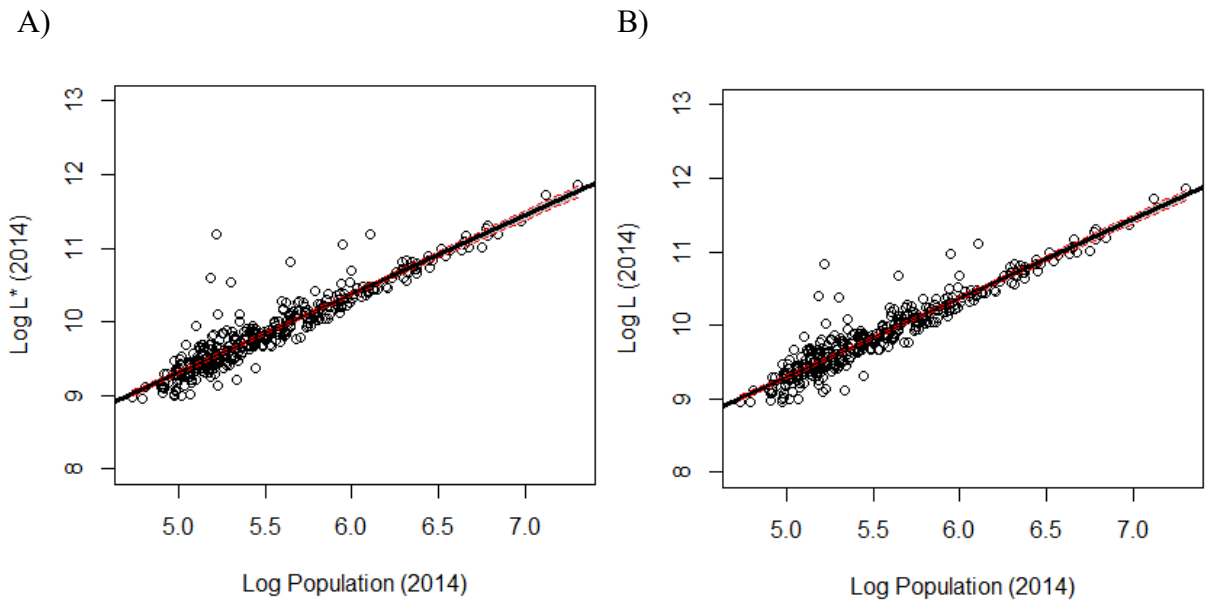


Figure 4: Superlinear scaling relationships for A) maximum urban benefit \mathcal{L}^* ($\beta = 1.065 \pm 0.042$, $R^2 = 0.87$, $n = 364$) and B) urban benefit \mathcal{L} ($\beta = 1.071 \pm 0.039$, $R^2 = 0.89$, $n = 364$), both for the year 2014 for 364 MSAs in the U.S. *Data sources: GDP data is from BEA 2016; road area data is calculated from FHWA 2015.*

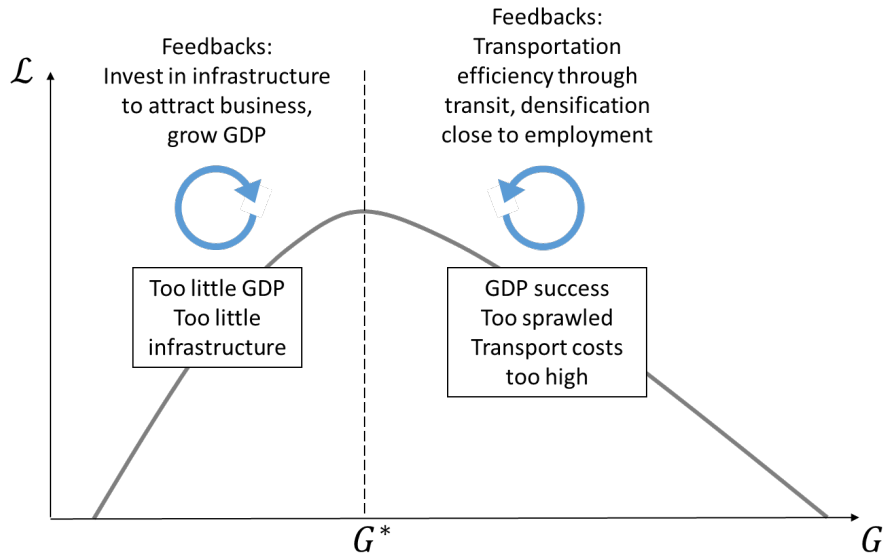


Figure 5: Diagram of feedback mechanisms that tend cities towards G^* . A city that is rapidly growing in terms of GDP and infrastructure sprawl (right side of G^*) will confront rising transportation costs (e.g., congestion) with efficiency measures, such as building rapid commuter transit and densification of housing close to employment opportunities. A city with GDP and infrastructure that is underdeveloped compared to average (left side of G^*) will respond by investing in infrastructure to attract more businesses and grow the economy.

Table 1: The 20 U.S. MSAs that are closest to and farthest from their values of maximum urban benefit \mathcal{L}^* . *Data sources: GDP data is from BEA 2016; road area data is calculated from FHWA 2015.*

Rank	MSA Name	Population	Per-capita GDP	$\mathcal{L}/\mathcal{L}^*$
MSAs closest to their \mathcal{L}^*				
1	Columbia, MO	172,711	42,296	100.000%
2	Hattiesburg, MS	148,660	33,533	100.000%
3	Burlington-South Burlington, VT	216,175	53,961	100.000%
4	Orlando-Kissimmee-Sanford, FL	2,321,415	45,477	100.000%
5	Buffalo-Niagara Falls, NY	1,136,367	43,603	100.000%
6	Palm Bay-Melbourne-Titusville, FL	556,891	31,191	99.999%
7	Battle Creek, MI	134,870	37,577	99.999%
8	Lewiston-Auburn, ME	107,437	35,351	99.999%
9	Hickory-Lenoir-Morganton, NC	362,905	31,055	99.999%
10	Grand Rapids-Wyoming, MI	1,027,694	46,111	99.998%
11	Harrisburg-Carlisle, PA	560,851	53,483	99.998%
12	Albany, GA	154,930	29,665	99.998%
13	San Diego-Carlsbad-San Marcos, CA	3,263,452	59,377	99.997%
14	Danville, IL	79,717	31,549	99.997%
15	Fort Smith, AR-OK	279,605	34,960	99.996%
16	Athens-Clarke County, GA	199,008	34,672	99.996%
17	Cape Coral-Fort Myers, FL	679,519	31,771	99.996%
18	Columbus, OH	1,994,552	53,964	99.995%
19	New York-Newark-Jersey City, NY-NJ-PA	20,092,801	69,105	99.993%
20	Greenville-Anderson-Mauldin, SC	862,452	38,721	99.992%
MSAs farthest from their \mathcal{L}^*				
364	East Stroudsburg, PA	166,320	29,353	43.697%
363	Midland, TX	161,289	144,951	51.303%
362	Chambersburg, PA	152,901	28,986	61.628%
361	Daphne-Fairhope-Foley, AL	200,102	29,530	68.535%
360	Asheville, NC	442,315	32,617	73.191%
359	Barnstable Town, MA	214,912	44,567	80.813%
358	Yuba City, CA	169,831	28,393	81.709%
357	Hammond, LA	127,066	26,073	82.107%
356	Albany-Schenectady-Troy, NY	880,162	50,485	82.260%
355	Louisville/Jefferson County, KY-IN	1,269,703	48,661	82.841%
354	Odessa, TX	153,902	61,039	85.848%
353	Lake Havasu City-Kingman, AZ	203,366	17,407	86.025%
352	Deltona-Daytona Beach-Ormond Beach, FL	609,952	21,564	86.083%
351	Casper, WY	81,629	68,873	86.277%
350	San Luis Obispo-Paso Robles, CA	279,084	42,163	87.070%
349	Chico, CA	224,230	28,560	90.210%
348	Yuma, AZ	203,238	24,149	90.855%
347	The Villages, FL	114,340	17,448	91.123%
346	Madera-Chowchilla, CA	154,537	28,498	91.160%
345	Bridgeport-Stamford-Norwalk, CT	945,438	93,655	91.633%