

Robust Model Predictive Control for Switched Dynamic Systems: Asynchronous
Switching, Stability and Feasibility, and Fault-Tolerance

by

Tianyu Tan

B.Sc., Harbin Institute of Technology, 2015

M.Sc., University of Birmingham, 2017

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University of Victoria

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ABSTRACT

Switched systems, as a superior modeling tool, are generally composed of a finite number of subsystems and a logical principle interpreted by switching signals that regulate the mode transitions between subsystems. Throughout the past few decades, switched systems have achieved phenomenal success in a wide range of engineering industries, such as the chemical process industry, robotics field, power electronics industry, generic engineering, smart automotive industry, waste treatment industry, etc. In addition to the prevalent industrial applications, switching dynamics has also stimulated broad interest in academia since the transient responses induced by mode transitions may introduce instability factors into the control synthesis and analysis even with all subsystems operating stably.

Considering the exposure to the challenging and volatile industry environment, switching control systems may also face threats to numerous inevitable failures, e.g., asynchronous switching, unconstrained switching, and controller failures, and many switched systems may also encounter physical limitations owing to spatial and system constraints as well as external disturbances. To this end, majority of previous studies have concentrated on switching control design against an individual fault, but few results are devoted to investigating the attenuation of the combined effect of multiple faults which happen simultaneously while fulfilling system constraints. Switched model predictive control (MPC), as an optimal control methodology of switched systems, can effectively incorporate system constraints into the optimization problem while providing optimal control actions with a certain degree of inherent robustness. However, how to ensure the closed-loop stability and recursive feasibility of the switched MPC algorithm is still an open problem nowadays. Therefore, to achieve the goal of reliable and executable switching controller design, this dissertation studies three problems in switched MPC and one switching stabilization control problem for a class of constrained switched systems from a theoretical context. Effective switched MPC algorithms are designed with guaranteed closed-loop stability and recursive feasibility. Additionally, a novel robust stability criterion for switched systems is explored subject to the aforementioned faults.

In Chapter 1, we present a comprehensive literature review of state-of-the-art switching control techniques, fault-tolerant switching control design, and switched MPC synthesis and analysis as well as the motivations and objectives of this dissertation. Chapter 2 provides some notations and preliminaries which are useful in

succeeding chapters. In Chapter 3, we study the switched MPC problem without using terminal constraints. With the prescribed switching sequences information, a sufficient condition on the prediction horizon that guarantees the closed-loop stability and feasibility of the switched MPC design is proposed based on reasonable assumptions. In addition, the length of the prediction horizon is quantitatively determined based on the estimated suboptimal parameters.

Chapter 4 concerns the asynchronously switched MPC problem with mode-dependent dwell-time (MDT) constraints. In the light of the proposed strategy, the lower bound of MDTs that ensures the persistent feasibility of the switched MPC in the presence of asynchronous switching is determined by letting the evolved reachable set be included in a target feasible region. Then, a common terminal set is designed and a superior stability property with respect to this terminal set is found. Two stability criteria are claimed by driving state trajectories into the devised terminal set.

In Chapter 5, the stabilization problem for a class of constrained switched linear systems is investigated subject to multiple faults. To mitigate the negative effect of multiple faults while fulfilling system constraints, a contractive set for initial states is established with MDT restriction and a non-conservative uniformly asymptotic stability condition is developed regarding the contractive set. Then, the necessary and sufficient stability condition is further extended to perturbed switched systems.

In Chapter 6, we investigate the robust MPC (RMPC) for asynchronously switched linear systems in the presence of joint effects of controller failures and additive disturbances. In order to eliminate the adverse impact of external disturbances, the tube-based MPC technique is employed so as to let the nominal switched systems satisfy the tightened mode-dependent constraints. Inspired by Chapter 5, a fault-tolerant MDT contractive set is constructed serving as the common terminal set for all modes. The closed-loop stability is guaranteed by forcing the state trajectories into this target set.

Chapter 7 concludes this dissertation and provides some promising future research directions.

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Acronyms

ADT average dwell time

AMI asynchronous switching MDT invariant

DC direct current

DMC dynamic matrix control

DMPC Distributed MPC

DoS denial-of-service

DT dwell time

ISpS input-to-state practical stability

ISS input-to-state stability

LMIs linear matrix inequalities

LTI linear time invariant

MAC model algorithmic control

MDADT mode-dependent average dwell time

MDPDT mode-dependent persistent dwell time

MDT mode-dependent dwell time

MPC model predictive control

PDT persistent dwell time

PI proportional integral

PWM pulse-width-modulation

QP quadratic programming

RHC receding horizon control

RMPC robust MPC

RPI robust positive invariant

SISO single-input-single-output

SSG smart switched signals generator

T-S Takagi-Sugeno

TAS truncated admissible switching

UAS uniform asymptotic stability

UAV unmanned aerial vehicle

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Victoria, B.C., Canada

March 2023

To my beloved parents

Chapter 1

Introduction

In this chapter, the introductory information of switched systems and stability analyzing tools with respect to switched systems is first depicted. Then, typical faults of switched systems, i.e., asynchronous switching, unconstrained switching, and controller failures, are introduced and the stabilization techniques in dealing with each individual fault are presented with the detailed literature review. Thereafter, the preliminary knowledge of [model predictive control \(MPC\)](#), [robust MPC \(RMPC\)](#), and switched MPC is given, followed by a comprehensive review regarding the recent development of switched MPC. Finally, the research motivations and contributions are presented for each completed work.

1.1 Switched Systems

1.1.1 Overview

With the rapid development of advanced information technologies including wireless information transmission and information processing techniques, complex system dynamics may encounter the coupling between continuous processes and isolated events [2]. Typical examples of such kind of systems are ubiquitous exist in control engineering, such as chemical process control systems [3], complex network control systems [4], intelligent transportation systems [5], and so on. In order to achieve better overall performance, employing a single continuous-time system or a discrete-time system to depict such completed and diverse system behavior is no longer appropriate. To this end, the concept of *hybrid systems* which refers to a dynamic system demonstrating the interaction and coexistence of discrete-time systems and continu-

ous system is proposed in [6]. Serving as a powerful modeling tool, hybrid systems have received enormous research attention in academia since the transitions between different natures of the system over time may lead to challenging theoretical problems. Many important works have demonstrated their efforts in studying hybrid systems from control synthesis [7, 8] and analyses [9]. However, in a certain amount of instances, the continuous process is the main interest of their study rather than the detailed behavior of the hybrid dynamics which motivates the investigation of switched systems [1].

As an imperative class of hybrid systems, *switched systems* arise when a continuous-time system isolated by several discrete events [2]. Specifically, switched systems usually consist of a group of subsystems or modes orchestrated by control decisions, e.g., switching signals [1]. Figure 1.1 displays a commonly adopted switching architecture [1]. The control decisions, e.g., switching signals, will be transmitted from the plant side to the mode-dependent controller side. Then, the corresponding controller will generate the control actions for the activated subsystem. Compared with hybrid systems, such a hierarchical architecture of switching dynamics naturally brings autonomy and adaptability since the control decisions can be appropriately designed depending on the system states, time scales, fuzzy policies, etc. Moreover, as a prominent modeling tool, switching dynamics also provide a feasible framework to achieve tailored controller design for each subsystem.

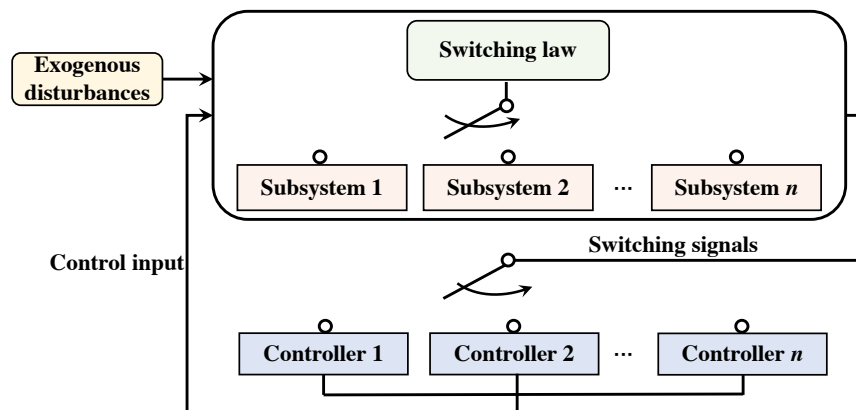


Figure 1.1: The schematic diagram of a switched system in the closed-loop [1].

In a broad spectrum of engineering fields, the application of switched systems has achieved great success over the past decades. In [10], switched systems are employed to model the process of a chemical reactor recycle system with [Takagi-Sugeno](#)

(T-S) fuzzy approach. By using the state-feedback controller, the closed-loop exponential stability is guaranteed. In [11], switching dynamics are serving as a “medical weapon” to conduct genetic therapy for mice with diabetes and the next generation of cell-based therapies has achieved positive protein expression. Switching dynamics also play an instrumental role in designing the powertrain system of a fuel-cell vehicle [12]. The optimal configurations to attain fuel efficiency heavily depend on the switching law between each powertrain subsystem, e.g., fuel-cell power systems or battery power systems. For unmanned combat air vehicles, in [13], the aircraft dynamics are decomposed into a set of simplified predefined maneuvers. Thus, the complexity of the control design process for the overall maneuver of air vehicles can be reduced to a great extent while presenting better control performance. In [14], the dynamic process of social network links changing with time is also modeled as a switched system and the internal characteristic of the switching topologies is explored therein. Figure 1.2 displays a practical illustration of the above-mentioned applications of the switched systems. All these real-world applications actively motivate the academic research on switched systems.

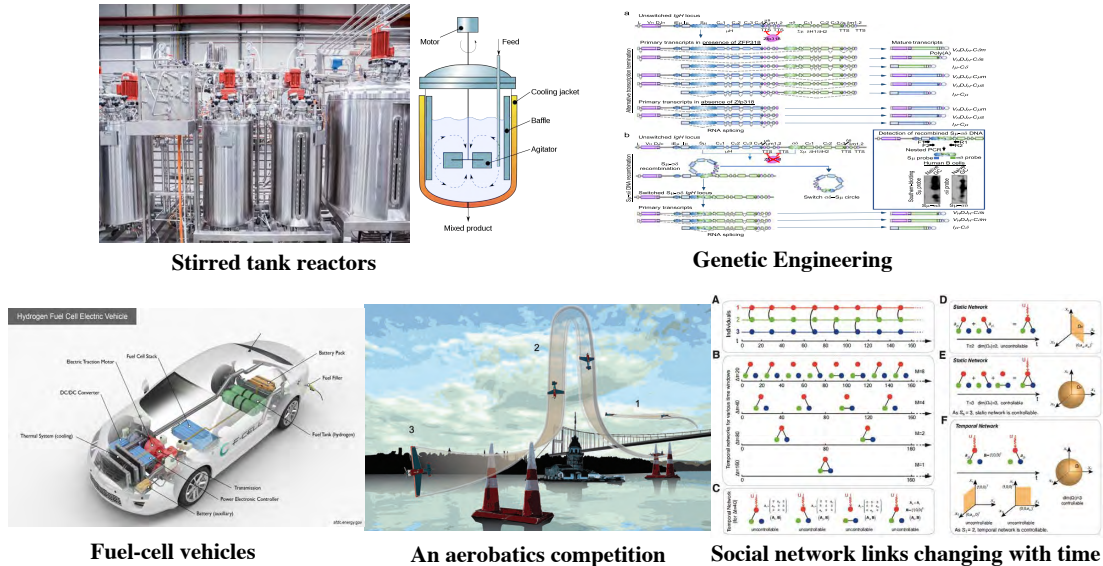


Figure 1.2: Practical illustrations of the switched systems.

In order to have a comprehensive literature review of control synthesis and analysis for switched systems, we first introduce the general classifications of switched systems. Based on the switching events, switched systems can be categorized as:

- Time-dependent switched systems in which the active subsystem depend on the switching signals over time;
- State-dependent switched systems where the resulting operating mode depends on the family of switching surfaces [2].

From the perspective of the nature of system behaviors, switched systems can also be classified as:

- Deterministic switched systems if the switching rule and the dynamics of each mode are known with certainty;
- Stochastic switched systems where the enabled subsystems are determined by the probabilities of a stochastic process.

Figure 1.3 summarizes the classifications of switched systems. Among all these categories, in this dissertation, we mainly concentrate on studying deterministic switching dynamics since it can effectively capture the abrupt changing of system behaviors over time or partitions of space without unpredictable mode transition uncertainties. In the following section, a thorough literature review is given to present state-of-the-art research regarding the control design of switched systems.

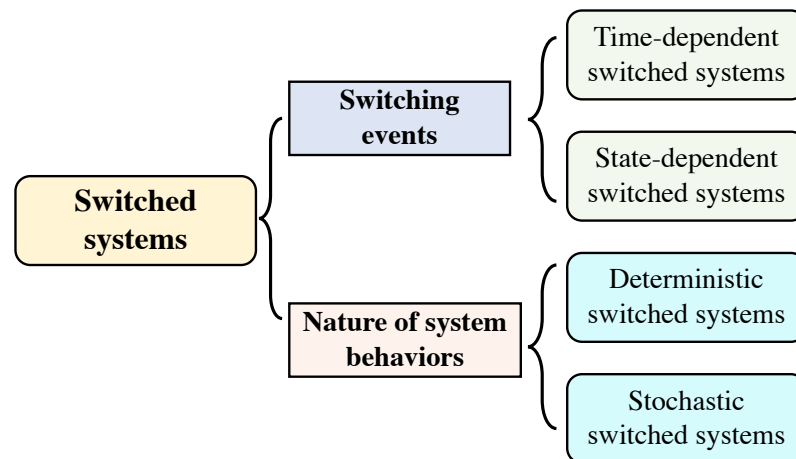


Figure 1.3: Classifications of switched systems.

1.1.2 Literature review

The stabilization problem of switched systems is still a vibrant research topic now a day since the switched systems may encounter the instability issue even if all the

subsystems are stable. A comprehensive literature review with respect to the stabilization problem of switched systems is given from three aspects:

Stability with arbitrary switching

In [15], a *common Lyapunov function* method is proposed for a class of switched linear systems under all subsystems to be stable with identical dimensions in Euclidean space. However, for switched systems with more than two subsystems, the sufficient condition to acquire a common Lyapunov function is every two subsystems must share a Lyapunov function. In [16], the existence of a common Lyapunov function is guaranteed by another conservative condition that the system matrices are required to satisfy the commutative law. Some research interests are focused on using Lie Algebraic conditions to construct common Lyapunov functions whereas solving the Lie algebras naturally brings the difficulties [17]. Later in [18], a gradient iteration algorithm is presented so as to obtain a quadratic common Lyapunov function for the switched systems with multiple stable modes. Nonetheless, for the majority of switched systems, a common Lyapunov function may not exist which makes this approach appear to be conservative.

In order to reduce conservatism, constructing *switched Lyapunov functions* are an alternative way for the stability study of switched systems. In [19], a switched Lyapunov function is established by considering the linear combination of all individual Lyapunov functions for each mode. The resulting system is proved to be asymptotic stable. However, for nonlinear switched systems, the quadratic form Lyapunov function might be challenging to construct. Thus, in [20], the *converse Lyapunov theorem* is presented which ensures the infinitesimal increasing rate of the Lyapunov function.

Stability with local constraints of Lyapunov functions

Although the aforementioned approaches can ensure the closed-loop stability of switched systems with arbitrary switching features, the decay rate design of the global Lyapunov function might be conservative for the local Lyapunov function of each individual subsystem. With this concern, a *multiple Lyapunov function* method is proposed in [21]. This criterion requires that the value of each subsystem's Lyapunov function is non-increasing in any adjacent switching instant for an identical subsystem in the closed-loop. Additionally, for each subsystem in the closed-loop, the value of the mode-dependent Lyapunov function requires decreasing. In [22], the non-increasing

condition of the multiple Lyapunov function in [21] is relaxed and the resulting system is asymptotic stable with the generalized condition. Later in [23], a modified multiple discontinuous Lyapunov function is employed to tackle the stabilization problem for switched systems with slow switching characteristics. The presented stability condition is accommodated to switched systems with unstable modes. In [24], the switching control problem with white noise is converted to a stochastic switching problem. With the assistance of multiple Lyapunov functions, the closed-loop system is demonstrated to be globally uniformly asymptotically stable. For nonlinear switched systems, in [25], the integral-type multiple Lyapunov function is also demonstrated to be effective to confront the stabilization issue by designing the feedback controllers with more stringent restrictions.

Except for the above-mentioned representative works, some other important works concentrate on developing less conservative stability analyzing tools. In [26, 27], a stability condition based on *multiple weak Lyapunov functions* is developed. In contrast with the multiple Lyapunov function approach, the monotonically decreasing requirement of mode-dependent Lyapunov function for a certain subsystem in the closed-loop is relaxed. Nevertheless, it demands, at the next activation switching instant of the identical subsystem, the value of the Lyapunov function is supposed to be less than the value at the time instant when this mode is lastly enabled. For easy understanding, Figure 1.4 illustrates three examples to indicate the basic ideas of stability strategies with local constraints of Lyapunov functions. It is observed that the value of Lyapunov functions V or mode-dependent Lyapunov functions V_q satisfy the corresponding stability criteria.

Stability with scheduling of switching signals

In addition to the stability strategies of state sensitive switched systems, many switching dynamics with slow switching features may allow the appropriately designed switching sequences under residence time restrictions to attenuate the transient effects of switchings. To this end, the concept of **dwelling time (DT)** which refers to the amount of time that the subsystem resides in each activated mode is introduced to address the switching control problem [2]. In [28], the DT constraint is initially employed to stabilize the time-dependent switched systems. The DT-based approach claims that, for each subsystem in the closed-loop, the value of the Lyapunov function is monotonically decreasing whereas, at any switching instants, the incremental value

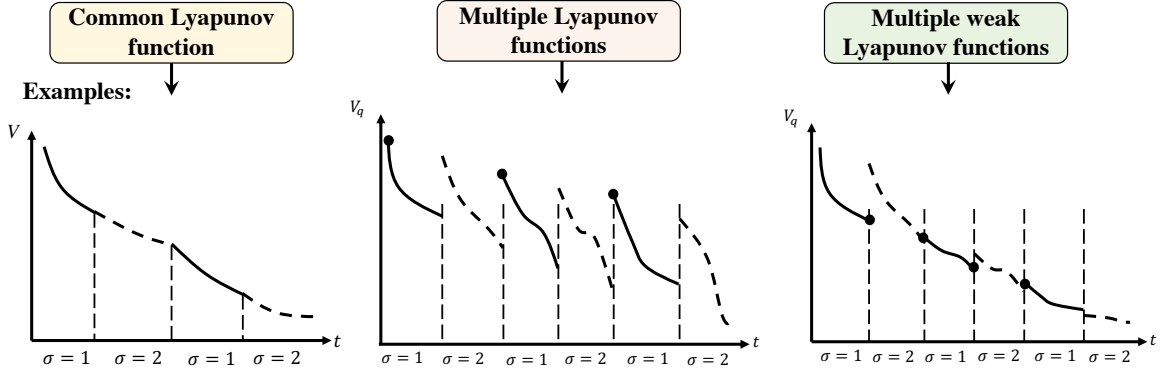


Figure 1.4: Three examples to respectively illustrate the basic ideas of stability strategies with local constraints of Lyapunov functions where mode $q \in \mathcal{Q} := \{1, 2\}$ and σ denotes the switching signals.

of the Lyapunov function is bounded. The main purpose to design a sufficient length of DT is to mitigate the negative effect of the switching-induced increasing value of the Lyapunov function. Then in [29], the Lyapunov-Metzler inequality is employed for seeking the minimum DT in order to attain the asymptotic stability of switched closed-loop systems. In [30], the stabilization problem of switched systems with parametric uncertainties is considered. By setting the DT constraint, the time-varying state-feedback gains are obtained such that the convergence of state trajectories is ensured. Some interesting results are also presented to study the closed-loop stability of Markov jump systems subject to DT constraints [31–33].

Afterward in [34], a more general **average dwell time (ADT)** approach is proposed by assigning the upper bound of switching times during a finite interval. The exponential stability is guaranteed as long as the ADT is sufficiently long. Inspired by [34], in [35], the stability result based on the ADT condition has been extended to switched positive systems in which the system state always stays in the positive orthant. The developed results are accommodated with both continuous-time and discrete-time switching dynamics. To facilitate the controller design, in [36], the switched system model reduction problem is studied with ADT constraints. The exponential stability of the resulting error system is successfully guaranteed. From the noise attenuation aspect, the H_∞ filtering problem for two-dimensional switched systems with ADT restriction is explored in [37] and the resulting filtering error converges at an exponential speed.

Considering the specific ADT constraint for each mode, a more applicable **mode-**

dependent average dwell time (MDADT) technique of stabilizing switched systems is developed in [38]. The simulation results verify that the system performance of switched systems can be improved to some extent with a more flexible MDADT restriction compared with directly applying ADT constraints. In [39], by constructing a quasi-time-dependent Lyapunov function with detailed increment value requirement of either closed-loop system within subsystems or at switching instants, the global uniform asymptotic stability is ensured for switched linear systems. Moreover, contraction analysis of switched systems is conducted via MDADT constraints in [40]. Necessary and sufficient stability conditions are presented so as to guarantee the exponential stability of switched systems with the designed controller. For the sake of clarity, Figure 1.5 shows two examples to respectively illustrate the basic ideas of stability strategies with ADT and MDADT constraints.

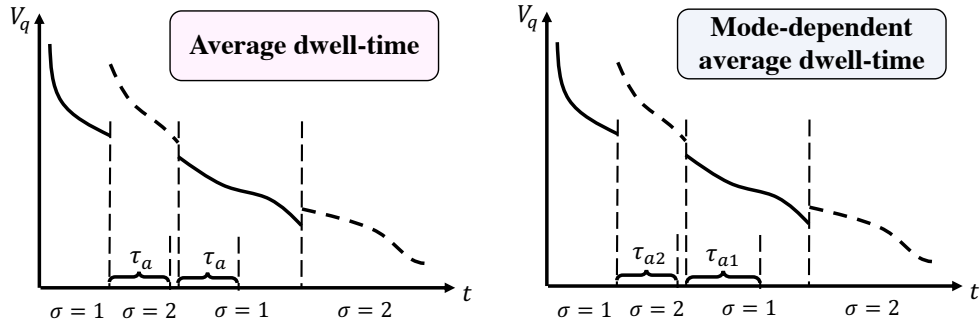


Figure 1.5: Two examples to respectively illustrate the basic ideas of stability strategies with ADT and MDADT constraints where τ_a is ADT and τ_{a1} as well as τ_{a2} are MDADTs of mode 1 and mode 2, respectively.

1.2 Switched Systems with Typical Faults

In addition to the fundamental stability requirement of switching control, with the rapidly growing demand for safety and reliability in engineering practice, resilient and robust control synthesis is almost the prerequisite to meet this requirement. In this section, several typical encountered faults, e.g., asynchronous switching, unconstrained switching, and controller failures, with respect to switching dynamics are presented and then a detailed literature review is given to exhibit the state-of-the-art control technique in dealing with these malfunctions, respectively.

1.2.1 Asynchronous switching

Asynchronous switching which refers to the mismatch between the desired controller and currently enabled subsystem is one of the common failures of switching systems [41]. The main reason for this phenomenon is the generated switching signals cannot be immediately detected by the mode-dependent controllers. Thereby, the lagged controller of the last activated subsystem is still applied in the currently enabled mode which may result in diverging of state trajectories. The asynchronous switching phenomenon is intuitively demonstrated in Figure 1.6. In real-world applications, asynchronous switching ubiquitously exists. In [42], a robot manipulator with load shift is modeled as a switched system. The asynchronous switching happens when the load change is not detectable in real time. A payload transport problem by employing drones with electromagnetic grippers is studied in [43]. By taking the pick-up and drop-off maneuvers as two subsystems, asynchronous switching may encounter when the electromagnetic inference affects the transmission of switching signals.

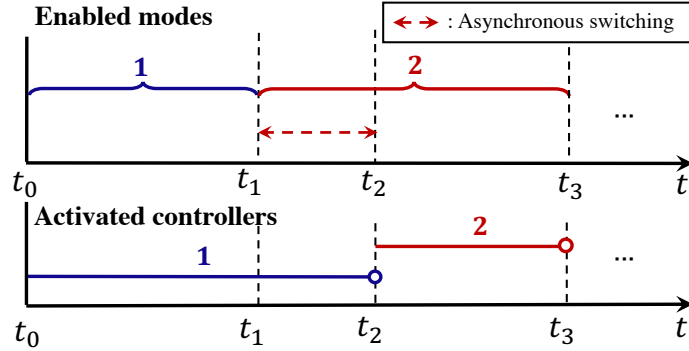


Figure 1.6: Illustration of all asynchronous switching where $\mathcal{Q} = \{1, 2\}$ denotes the set of admissible modes.

From theoretical perspective, many important results have been reported on studying the switching control problem in the presence of asynchronous switching [41, 44–52]. For a constant length of asynchronous switching periods, the switching control problem is initially explored in [47]. Both the online and offline state feedback gains design methods which satisfy the asymptotic stability and exponential stability conditions are presented, respectively. Then, the assumption of fixed length asynchronous switching duration is relaxed in [46] by conducting a more precise estimation of the overshoots regarding transition matrices.

With the assistance of ADT constraints, in [41], the asynchronous switching control problem for a class of switched linear systems is investigated. By providing proper

increasing and decreasing rates of Lyapunov functions, the resulting system is globally uniformly exponentially stable with the maximal length of asynchronous switching. In [44], the asynchronous H_∞ control technique is employed and this method is proved to be effective in stabilizing switched systems under the joint effects of asynchronous switching and external disturbances. In [45], the stabilization problem for asynchronously switched systems with time delays in both states and the state derivatives is studied. A sufficient exponential stability condition is derived with ADT constraints and the [linear matrix inequalities \(LMIs\)](#) solvability condition for computing feedback gains is also claimed. Subsequently, with less conservative MDADT constraints, the asynchronous switching control problem for two-dimensional switched systems is studied and H_∞ controller design is discussed for noise attenuation [50].

Rather than enforcing DT restrictions, in the light of a modified Lyapunov function method, global asymptotic stability of asynchronously switched systems is ensured and the detrimental impact of exogenous disturbances is eliminated with the designed non-weighted l_2 gains [49]. In [48], the control issue of switched systems under asynchronous [denial-of-service \(DoS\)](#) attacks is converted to the stabilization problem of asynchronously switched systems. In order to mitigate the adverse effect of asynchronous switching, the authors devise an active strategy based on predictor and buffer mechanisms to compensate for the unreceived data such that the estimation error between practical states and real states is bounded. Other research interests are concentrating on developing control strategies for stochastic switched systems with asynchronous switching [51, 52].

1.2.2 Unconstrained switching

Apart from asynchronous switching issues, intermittent unconstrained mode transitions, i.e., *unconstrained switching*, may also pose threat to the reliability of switched systems. The terminology *unconstrained switching* is used to describe the situation that the switching signals may intermittently disordered switch to other admissible modes without obeying the prescribed DT constraint. Figure 1.7 demonstrates the disordering switching signals compared with the residence period which satisfies the DT constraint. Note that many engineering control systems may encounter unconstrained switching issues. In [53], a tunnel diode circuit system is modeled as a switched system. The disordered fast switching between two parallel resistors with different values can cause the unconstrained switching issue which may result in

diode failure due to the rapidly changing current levels. Another practical example of switching dynamics with disordered switching appears in [54]. To regulate the output voltage at a desired level, the rapid switching scheme of a physical switch of a [pulse-width-modulation \(PWM\)](#)-driven boost converter is adopted. Thereby, it may lead to the problem of intermittent unconstrained switching.

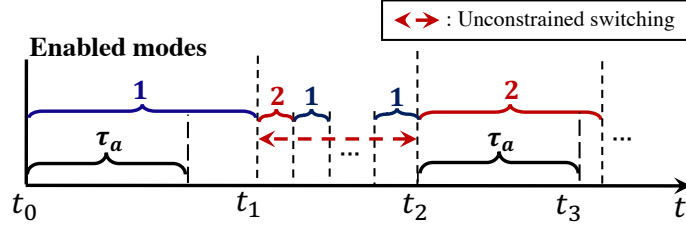


Figure 1.7: Illustration of unconstrained switching where $\mathcal{Q} = \{1, 2\}$ denotes the set of all admissible modes and τ_a is the lower bound of DT.

In order to eliminate the negative effect of unconstrained switching, many works are dedicated to studying the switching control problem from the theoretical perspective [55–58]. In [55], the concept of [persistent dwell time \(PDT\)](#) is introduced to deal with the switching dynamics with intermittently disordered switching. The switched system which satisfies the PDT constraint requires that the length of unconstrained switching interval cannot exceed a certain threshold. With the PDT constraint, uniform exponential stability is achieved of the switched closed-loop system. Thereafter in [56], adaptive output feedback controller design regarding switched nonlinear systems with PDT constraints is conducted. The output feedback stabilization problem is tackled by establishing the error dynamics for the observed state trajectory and real system states and the convergence of the resulting error system is obtained. For the perturbed switched linear systems, a quasi-time varying stabilizing controller is devised and a [robust positive invariant \(RPI\)](#) set is constructed with the [mode-dependent persistent dwell time \(MDPDT\)](#) constraint [57]. Furthermore, the asymptotic stability of the resulting system is guaranteed by letting the state trajectories converge into the RPI set. Later in [58], a generalized non-conservative stability criterion is proposed by precisely characterizing PDT constraints. Removing the inadmissible or redundant switching sequences is the main advantage of the control scheme therein.

Several critical results are focusing on discussing the filter design problem of switched systems for noise attenuation purposes. In [59], the H_∞ filtering problem is considered for disturbed switched systems in the presence of unconstrained

switching. The global uniform asymptotic stability of the resulting filtering error system is attained by adopting the quasi-time-dependent Lyapunov stability criterion. The filtering problem is also explored in [54] by taking into consideration all possible **truncated admissible switching (TAS)** sequences with PDT constraints. Instead of utilizing the quasi-time-dependent Lyapunov technique, the novel Lyapunov functions in the light of the admissible switching sequences are established such that the closed-loop global uniform asymptotic stability is achieved.

1.2.3 Controller failures

Another frequently encountered failure for switched systems is the occasional disconnection of controllers, i.e., *controller failures*. Specifically, during the period of controller failures, the systems in the closed-loop will not receive any control inputs, i.e., zero control input signal. Figure 1.8 provides an intuitive illustration of controller failures. It is worth mentioning that many environmental factors may result in controller failures, such as data transmission congestions, packet dropouts, and unreliable communications [60]. Exposure to a complex environment, various engineering control industries may face the potential threat of controller failures. In [61], the control failures issue is investigated with respect to magnetic bearings. The hierarchy distributed control structure is devised so that the adaptability is improved in case of the happening of controller failures. A resilient distributed control scheme of a power distribution system is proposed to mitigate the detrimental effect of centralized controller failures caused by cyber attacks [62].

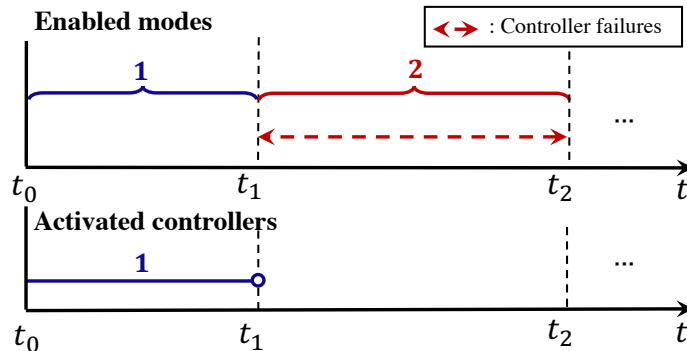


Figure 1.8: Illustration of controller failures where $\mathcal{Q} = \{1, 2\}$ denotes the set of all admissible modes.

Switched systems, as a powerful modeling tool, are capable of depicting a control system with controller failures by supplementing additional faulty modes. In [63], a

linear time invariant (LTI) system with controller failures is modeled as a switched system. By employing a piecewise Lyapunov function, the authors show that if the duration of controller failures is upper bounded and ADT is sufficiently long, the global exponential stability of the LTI system is attained. Thereafter, the switching control problem under the joint effects of controller failures and time-varying delay is explored in [64]. The stabilization problem is tackled by using the predefined frequency of controller failures with the piecewise Lyapunov function method. In [65], an LTI system in the presence of controller failures is further investigated in the light of time-scheduled Lyapunov functions. A convex stability condition is claimed which guarantees the Lyapunov function to be monotonically decreasing during both DT and controller faulty portions.

Since the intermittent failure to receive the input signal may lead to state trajectories diverging, an alternative way to study the controller failure problem for switched systems is to explore the stabilization problem for switching dynamics with unstable modes. In [66], the stability properties of switched linear systems with unstable modes are examined by taking the advantage of ADT methods. As long as the ADT constraint is satisfied, exponential stability is achieved with the bounded operation time of unstable modes. The stabilization of switched linear systems with all modes unstable is studied in [67] based on the discretized Lyapunov functions. An algorithm is provided to determine the stability region of maximal and minimal DTs. In [68], the theoretical results of discussing the stabilization problem for switched nonlinear systems with unstable subsystems are presented. By appropriately scheduling the switching sequences, the diverging of state trajectories caused by the unstable mode in the closed-loop is effectively attenuated such that the global converging of system states is ensured. Inspired by the aforementioned works, the stabilizing controller design of two-dimensional switched systems with unstable modes is presented in [69, 70]. The ADT approach and admissible switching sequences-based Lyapunov function method are respectively utilized to conduct the control synthesis and analysis so as to eliminate the activation of unstable modes-induced instability issues.

Table 1.1 summarizes the representative works on studying switching control problems with typical faults. It is observed that more research attentions are still in great desire to explore switching nonlinear control design for fault tolerant control purposes. Moreover, to realize reliable control of switched linear systems, less conservative stability criteria are still worth investing efforts to explore.

Table 1.1: The representative works on studying switching control with typical faults.

	Asynchronous switching		Unconstrained switching		Controller failures	
	Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
ADT	[41, 44, 45]				[63, 66, 69]	
MDADT	[50]		MDPDT: [57]			
DT	[48]				[65, 67]	[68]
PDT			[54–56, 58]			
Lyapunov functions	[46, 47, 49, 52]	[51]	[54, 57–59]		[63, 64, 67, 70]	

1.3 Model Predictive Control

MPC first presented in the late 1970s, such as [dynamic matrix control \(DMC\)](#) or [model algorithmic control \(MAC\)](#), has witnessed great success over the past decades in a broad spectrum of engineering fields [71], e.g., power electronics [72], robotics [73], intelligent transportation systems [74], smart building operations [75], metallurgical industries [76], and pollutant treatment industries [77]. The reason behind is MPC, as an advanced and prominent optimal control methodology, provides a systematic way to predict the future behaviors of system states and generate optimal control actions based on prescribed performance index and system constraints [78]. Compared with conventional control schemes, the primary advantage of MPC is that it can take the system dynamics and constraints explicitly into consideration which essentially grants the MPC the ability to forecast the future events and control commands by solving the optimization control problem. Since the control implementation of MPC is conducted with a receding horizon manner in which the finite horizon optimal control problem is repeatedly solved at each sampling instant, MPC is also known as [receding horizon control \(RHC\)](#).

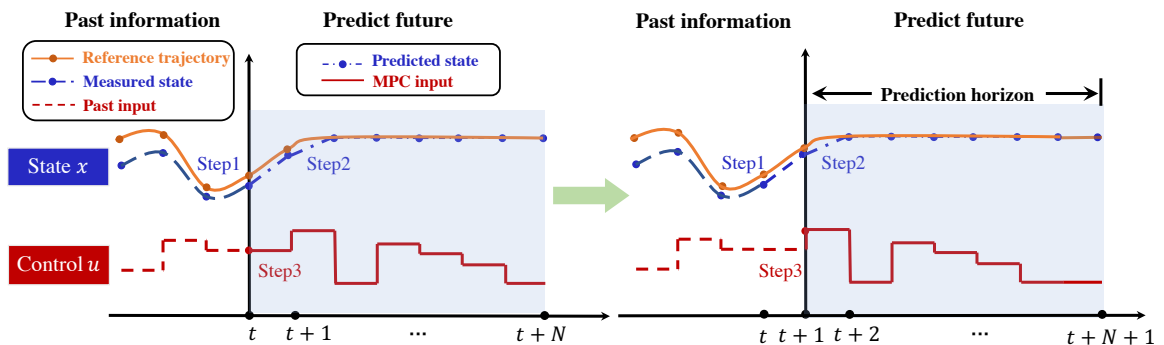


Figure 1.9: The schematic diagram of MPC.

To intuitively demonstrate the philosophy of MPC, in Figure 1.9, the schematic diagram shows the trajectories of system states and the control input of a reference path tracking problem with the MPC scheme. For standard MPC:

- In Step 1, at time instant t , the system state is measured by sensors or estimated by observers. The past information of system states is the utilized in this step.
- In Step 2, a prediction horizon length control sequence is acquired by solving the finite horizon optimal control problem in the light of the current measurement of system states.
- In Step 3, according to the MPC law, only the first element of the optimal control sequence is implemented while the other elements are discarded. Thereby, the feedback mechanism is introduced into this scheme in essence [79].

Then, at the next time instant $t + 1$, the optimization procedure from Step 1 to 3 will be conducted again based on the new measurement of state information. The gap between the actual states and predicted states is essentially reduced through the repetition process and it grants the inherent robustness of the MPC scheme to some extent regarding multiplicative uncertainty and additive disturbances [79].

1.3.1 Classic MPC

Inspired by the wide range of applications, some early research works focus on studying the stabilization issue which is an overriding requirement for the closed-loop system with an MPC scheme [80–85]. In [80], the RHC scheme with an infinite prediction horizon is proposed. As long as the prediction horizon is designed sufficiently long, the closed-loop stability is ensured without constraint violation. However, prolonging the prediction horizon will directly increase the computational burden which leads to a shift toward control solutions with the finite horizon. With a finite prediction horizon, the equality terminal constraint is designed in [81] by forcing the terminal state to reach the equilibrium point. Nonetheless, such a stringent constraint may result in the optimal control problem being unsolvable since a fast convergence rate of system states is required over a finite prediction horizon.

In [82], a dual-mode RHC scheme is presented by introducing the *terminal region* or *terminal constraint*. When the terminal state enters the terminal region, a local stabilizing state feedback law is activated. Therefore, the mechanism of switching

between two control laws is named *dual-mode* RHC scheme. Motivated by [82], with the terminal constraint, the authors in [83] propose a nonlinear MPC scheme by supplementing a terminal cost to penalize the terminal states. Under the assumption of initially solvable condition with respect to the optimal control problem, the *recursive feasibility* which guarantees the optimal control problem to be continuously solvable as well as closed-loop stability are ensured by constructing a candidate input profile with the determined decremental lower bound of the terminal cost function. In [84], the MPC problem is discussed in a practical way without considering the terminal constraint. The authors show that if the terminal cost function is weighted by a decent positive constant, the terminal constraint is redundant. In addition, the asymptotic stability of the closed-loop system is also ensured by constructing the domain of attraction. Later, the stability results in [84] have been extended to continuous-time nonlinear system [85]. The lower bound of prediction horizon in each sampling instant is devised such that the exponential stability of the closed-loop system is ensured without terminal constraints. All these aforementioned works lay the theoretical foundation to investigate the following **RMPC** and switched MPC issues.

1.3.2 Robust MPC

Although the RHC scheme naturally possesses certain degree of inherent robustness, it only has the capability to tolerate sufficiently small external disturbances [86]. An example in [87] claims that MPC for linear system with convex constraints may preserve robustness with relatively small disturbances. However, for nonlinear system with both state and terminal constraints, the robustness of MPC scheme is never retained even if in the presence of arbitrarily small disturbances. Based on the above observations, in this section, we aim to present a thorough literature review to introduce two classic RMPC techniques to handle the exogenous disturbances.

Min-max MPC

The idea of min-max strategy originates from the game theory [88]. Specifically, the admissible control actions and time-varying disturbance vectors are regarded as opposing players, i.e., the minimizing control input is obtained by considering the worst case of external disturbances [88]. The min-max strategy is initially proposed in [89] by recasting the minimax problem into a linear programming problem for a **single-input-single-output (SISO)** system. Then, the min-max control strategy is further

extended by considering the MPC problem with time-varying prediction horizons [90]. For the nonlinear MPC problem, the min-max control strategy is implemented with an auxiliary H_∞ control law and, under the proposed robust control law, the stability margin is guaranteed [91]. A more general min-max MPC framework is presented in [92] for a class of nonlinear systems. **Input-to-state practical stability (ISpS)** for closed-loop system has been ensured by constructing the ISpS Lyapunov functions. In [93], a sufficient stability condition which ensures **input-to-state stability (ISS)** of the nonlinear closed-loop system is derived. A detailed summary of stability issues and robustness of min-max MPC design for nonlinear systems is provided in [94].

Tube-based MPC

In spite of the fact that the min-max strategy can effectively tackle external disturbances for both linear and nonlinear system dynamics, the computational load is heavily demanding for the online implementation of such a control strategy. Therefore, many research interests convert to finding an offline-based approach to deal with the RMPC problem [95–100]. The tube-based technique is another alternative way to eliminate the negative impact of additive disturbances. In particular, the term *tube* provides an intuitive geometric description for the tightened admissible constraint [88].

In [95], the MPC problem for linear systems subject to persistent bounded disturbances is studied. In order to retain the system constraints satisfaction, a sequence of disturbance invariant sets $\{\mathcal{Z}_j\}$ [96], $j \in \mathbb{Z}^+$, is employed to tighten the system constraint iteratively, i.e., $\mathcal{X} \ominus \mathcal{Z}_j$ and $\mathcal{U} \ominus K\mathcal{Z}_j$, where \mathcal{X} as well as \mathcal{U} respectively denote the state constraint and input constraint and K is the state feedback gain and \ominus represent the operation of Pontryagin set difference. Thus, the asymptotic stability of the nominal system is ensured with $x_t \in \mathcal{Z}_\infty$, $j \rightarrow \infty$. Then, in [97], instead of computing the tightened constraint stepwise, the disturbance invariant set \mathcal{Z}_∞ is directly subtracted by original constraints, i.e., $\mathcal{X} \ominus \mathcal{Z}_\infty$ and $\mathcal{U} \ominus K\mathcal{Z}_\infty$. By considering the initial condition identical to the nominal system, the robust exponential stability of the closed-loop system is claimed. Later, the tube-based MPC technique is further extended to the nonlinear system scope [98]. In addition to the aforementioned rigid tube technology which the shape of disturbance invariant set is fixed and determined offline, a dynamic homothetic tube-based MPC design is explored in [99, 100]. The cross-section of each tube is decided by an online determined scalar obtained via solv-

ing the MPC optimization problem. Table 1.2 concludes the representative works on investigating classic MPC and RMPC.

Table 1.2: The representative works on studying classic MPC and RMPC.

	Methodology	Linear system	Nonlinear system
Classic MPC	With terminal constraints	[81, 82]	[83]
	Without terminal constraints	[80]	[84, 85]
RMPC	Min-max MPC	[89, 90]	[91–94]
	Tube-based MPC	[95, 97, 99]	[98, 100]

1.4 Switched MPC

As shown in Figure 1.2, switched systems plays an instrumental role in modeling complex dynamical process in real engineering practice. Switched MPC, being an optimization-based control method, can explicitly incorporate dynamical systems with switching features (or a continuous process with sudden changes) as well as system constraints into the control design phase. Such a seamless integration of powerful modeling tools and advanced control techniques naturally infuses autonomy and flexibility with the optimization-based control framework, thereby improving the system performance to a great extent.

1.4.1 Applications of switched MPC

Considering the merits of switched MPC, enormous real-world applications with hierarchical switching logic regulations adopt switched MPC method to realize the optimum performance-oriented control synthesis. In [101], the switched model predictive attitude control design for an **unmanned aerial vehicle (UAV)** subject to bounded wind disturbances is studied. The experimental results verify the good attitude-tracking performance of UAVs under switching logic determined by the rate of rotation angles. In [102], the vehicle steering control problem is investigated via switched MPC strategy. Mode-dependent MPC controllers based on the current tire force conditions are devised and the convergence of system states is ensured in terms of the steady state. A path-tracking problem of autonomous vehicles is explored in [103]. By establishing the simplified linear dynamic model of the vehicle, the optimal control actions of steering angles and angular velocities can be successfully obtained by employing

switched MPC with a fuzzy switching law. Furthermore, in [104], the vehicle cruise control problem which aims to adjust appropriate velocity for autonomous vehicles in the light of surroundings is studied via the MPC design. A state-dependent switching law is considered between acceleration mode and braking mode. Moreover, a fast nonlinear MPC algorithm for automotive dynamics with maneuver mode switching is designed in [105] for real-time application purposes. Except for automotive switched MPC issues, in order to achieve the energy efficient and time-saving railway vehicles traveling target, the authors in [106] discuss the switched nonlinear MPC design under the velocity and journey time constraints. The case study demonstrates the improvement of energy efficiency by adopting the switching control scheme.

Switched MPC has also been an attractive control method in the power electronics field over recent years. In [107], the switched MPC scheme is used in power switches fault-sensitive rectifiers. By employing such a method, a more reliable current value is achieved with guaranteed suppressed offset voltage compared with [proportional integral \(PI\)](#) controller. A constant switching frequency MPC strategy is proposed in [108] to optimize the distribution of the output current harmonics. The filtering design facilitated higher-order harmonics of the output current is manifested in contrast with the conventional MPC approach. In addition, switched MPC has also proved to be effective for realizing the optimal control of [direct current \(DC\)](#) to DC converters [109].

Switched MPC has also seen many successful applications in some promising industries. In [110], the switched model predictive controller design is discussed for the purpose to achieve pneumatic artificial muscles' optimal motion control. In addition, switched MPC is applied to a drinking water distribution network for regulating water storage in each tank [111]. The water consumption is guaranteed without violating the volume capacity and flow constraints. Wastewater treatment industries find that the switched MPC method can effectively control the amount of coagulation chemicals to meet the desired water quality [112]. The fuzzy logic of operation regimes is described as switching dynamics therein. Furthermore, a softly switching MPC algorithm is proposed in [113] to obtain acceptable transient behaviors of aero-engines caused by system parameter changing.

1.4.2 Theoretical results of switched MPC

In view of the fruitful practical applications of switched MPC, in this section, we intend to introduce the switched MPC strategy intrinsically from a theoretical per-

spective. Compared with the conventional MPC strategy, in Step 1, except for the initial condition of system states, the information of the switching signal at each sampling instant is also required for switched MPC scheme. Thus, considering the state-dependent switched system, the switching signal σ depends on the partition of space, i.e., $\sigma := q$, if $x \in \mathcal{X}_q$, where \mathcal{X}_q denotes a non-overlapping partition of state constraint \mathcal{X} . For time-dependent switching dynamics, the switching signals are determined by a piecewise constant function over time t , i.e., $\sigma(t) : \mathbb{N} \rightarrow \mathcal{Q}$, where \mathcal{Q} denotes the set of admissible modes. Then, in Step 2, based on the detected switching signals, the performance indices of the objective function and mode-dependent constraints are determined. In Step 3, by solving the optimal control problem, the first element of the obtained optimal control sequence is applied to the currently activated mode and the optimization procedure from Step 1 to 3 will be repeated at the next sampling instant. A schematic diagram of switched MPC is shown in Figure 1.10.

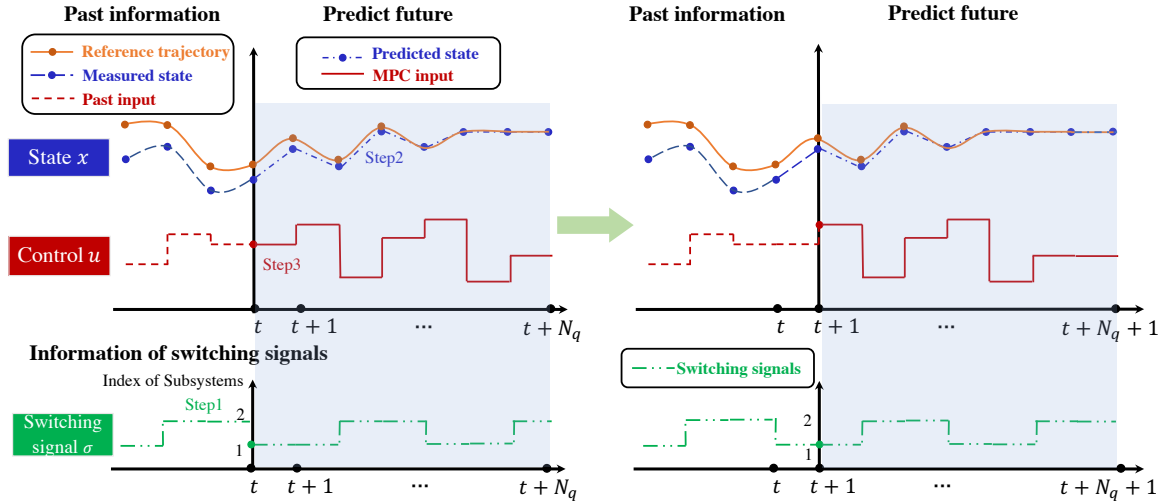


Figure 1.10: The schematic diagram of switched MPC.

Many academic works demonstrate their enthusiasm to provide theoretical support for applying switched MPC since the MPC controller design with abrupt changing of switching system behaviors is challenging or even intractable. Lyapunov-based switched MPC approaches receive the research attention first. In [114], the switched MPC problem is discussed under the Lyapunov-based MPC framework with the known switching pattern. The mode transition constraint is designed to retain the recursive feasibility of switched MPC algorithm and multiple Lyapunov constraints are imposed for the convergence of system states. In [115], the MPC design for

switched linear systems is considered as a combinatorial optimization problem. The regional control law based on the piecewise Lyapunov function is explored for stability concerns. Later in [116], the MPC problem for switched nonlinear systems which decompose into the linear model and the nonlinear part is tackled via a Lyapunov-based approach. The upper bound for the nonlinear term is constructed and the ISS criterion is proposed to guarantee closed-loop stability.

Numerous efforts have been dedicated to study the In [117], the ADT approach is employed to tackle the nonlinear switched MPC problem. The authors show that as long as the ADT constraint is satisfied, even if switching signals are delayed detected within a small range by the mode-dependent controller, the convergence of state trajectories is guaranteed. Without terminal constraints, the stabilization problem of switched MPC is studied in [118] with ADT constraints. The closed-loop asymptotic stability is ensured with controllability assumptions. In [119], the MPC design for the Markov jump system with ADT constraints is investigated. Computational efficiency pursued control strategy is their main interest. Considering all possible truncated switching sequences in the prediction scheme, the switched MPC design is conducted with DT constraints [120]. The recursive feasibility is ensured by adding an additional input constraint. In [121,122], a non-conservative condition of enforcing the constraint satisfaction for switched systems with DT restrictions is first claimed. Then, an algorithm to calculate the switched-robust invariant set is introduced to ensure the recursive feasibility of switched MPC algorithm. The proposed switched MPC scheme is further extended to nonlinear switched systems [122]. Novel optimization techniques are also employed to deal with the switched MPC problem by aligning optimal switching sequences with DT constraints [123,124]. The mixed-integer linear programming is used to develop efficient MPC algorithms for switched systems with DT restrictions. However, no theoretical proof is given. Subsequently, to reduce the computational load of solving the mixed-integer optimal control problem, suboptimal solutions of the nonlinear switched MPC problem are obtained with the decomposition method [125]. Recursive feasibility is proven by establishing a control invariant set under the DT constraint.

A less conservative MPC strategy for switched systems is discussed with [mode-dependent dwell time \(MDT\)](#) constraints [126,127]. The lower bound of MDT is determined with the persistent feasibility retained by letting the reachable set into the common feasible region. Moreover, the proposed control scheme is shown to be effective to tackle the RMPC problem for switched linear systems combing with

tube-based techniques. The RMPC problem for switched linear systems is further investigated in the presence of mode-dependent disturbances constraints [128]. MDTs are also a decision variable to guarantee closed-loop exponential stability. Later, the RMPC result is extended to switched nonlinear systems with mode-dependent equilibria [129]. In [130], the PDT constraint is utilized to address the MPC controller synthesis for switched linear systems under intermittent switching disordering. The mixed-integer nonlinear programming optimization problem is formulated to find cost-effective switching sequences.

Some other imperative issues regarding switched MPC have also been explored. **Distributed MPC (DMPC)** for switched nonlinear system is studied via Lyapunov-based MPC approach [131]. The theoretical results are validated in a multiple-stirred tank reactor network. For saving communication resources purposes, the event-triggered mechanism is introduced in the MPC design for switched linear systems with ADT restrictions [132, 133]. For constrained Markov jump systems, the stochastic MPC strategy is developed [134] and the mean square stability is achieved by employing the robustness constraint. [135] provides the theoretical basis for MPC design with switching cost functions. Table 1.3 summarizes the representative works on exploring switched MPC.

Table 1.3: The representative works on exploring switched MPC.

		Linear system	Nonlinear system
Switched MPC	Lyapunov-based approach	[115, 134]	[114, 116]
	ADT	[118, 119]	[117]
	DT	[120, 121]	[122–125, 129]
	MDT	[126, 128]	
	PDT	[130]	
Others	Distributed control		[131]
	Event-triggered control	[132, 133]	
	Switching cost functions		[135]

1.5 Research Motivations and Contributions

Based on the observations from the comprehensive literature review, there are still numerous open problems that remain to be conquered. As shown in Table 1.1, although many theoretical results are devoted to considering the stabilization problem

for either switched linear or nonlinear systems under one of the typical faults, rare results have shown their interest to address the stabilization problem for constrained switched systems with multiple faults. Furthermore, non-conservative and effective stability conditions to stabilize switched systems are still in great desire. For switched MPC design, it can be observed from Table 1.3 that few results concern finding an executable and straightforward way to ensure the closed-loop stability and recursive feasibility of the switched MPC algorithm without employing terminal constraints. Additionally, for switched systems in the presence of typical faults or additive disturbances, the computationally efficient switched MPC algorithm, to the best of our knowledge, has not been discussed from the theoretical perspective with rigorous proof. Therefore, several questions naturally arise:

- How to design a switched MPC algorithm for all admissible switching sequences without terminal constraints?
- How to attenuate the negative effect of typical faults as well as external disturbances while retaining the closed-loop stability and recursive feasibility of the switched MPC design?
- How to establish a necessary and sufficient stability condition regarding switched systems with multiple faults?

To this end, in the dissertation, we will answer these questions to some extent from a theoretical point of view. Figure 1.11 displays the overall structure of this dissertation. The objectives and motivations of each chapter are detailed as follows:

- **Chapter 2** presents the useful notations and preliminaries throughout the dissertation.
- In **Chapter 3**, the switched MPC problem is studied for a class of switched linear systems with DT constraints. Considering the prescribed switching sequences are prevalent in real-world applications, e.g., stirred tank reactors with predefined mixing rates [117] and stick shift actions for aerobatics competitions [13]. The designed control strategy with prescribed switching sequences is particularly demanded. In addition, to avoid the complexity of designing the terminal constraints for all subsystems, an alternative stability condition without using the terminal constraint is urgently in need of resolution. To conquer the above issues, we propose a sufficient stability condition on prediction

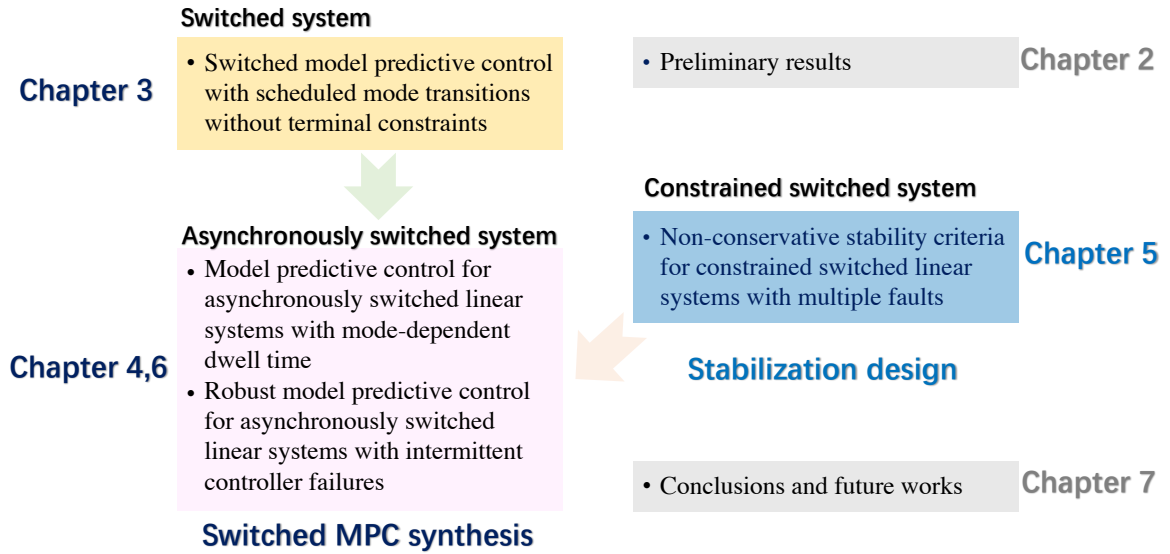


Figure 1.11: The overall structure of the dissertation.

horizons under reasonable assumptions and the corresponding suboptimal parameters are quantitatively estimated [136]. The simulation results are given to verify the theoretical results.

- In **Chapter 4**, the MPC problem for a class of asynchronously switched systems is explored with MDT constraints. Since the mismatch between lagged mode-dependent controllers and the enabled subsystems may result in detrimental effects on the closed-loop stability and recursive feasibility of the switched MPC design. To address this issue, we use the reachable set technique to ensure that all the state trajectories are included in a target feasible region. Furthermore, through the determination of the lower bound with respect to MDTs, two executable asymptotic stability strategies are proposed by driving the system states into the constructed common terminal set. Numerical examples validate the advantages of the proposed control scheme.
- **Chapter 5** investigates the necessary and sufficient stability criteria for constrained switched linear systems with multiple faults. The terminology *multiple faults* refers to the situation that the three typical faults, i.e., asynchronous switching, unconstrained switching, and controller failures happen individually or simultaneously. Therefore, it may introduce additional vulnerability and reliability risks to the switching control problem. To mitigate the negative impact

of multiple faults, a fault-tolerant contractive set is designed under MDT restrictions and we prove the existence of such a set is equivalent to the uniformly asymptotic stability of switched closed-loop systems. Moreover, the stability results are further extended to disturbed switched systems. The simulation results are given to show the effectiveness of the proposed control strategy.

- In **Chapter 6**, the RMPC problem for asynchronously switched systems with controller failures is addressed with MDT constraints. As mentioned earlier, the RHC scheme has a limited degree of inherent robustness to tolerant more than sufficient small external disturbances [86]. Even worse, the controller failures phenomenon may introduce unstable modes into the switching dynamics. Hence, the joint effects of additive disturbances and controller faults as well as the asynchronous switching issue may cause severe diverge of system states and control performance degradation. To tackle this issue, we adopt the tube-based MPC technique to eliminate the negative impact of exogenous disturbances. Moreover, inspired by **Chapter 5**, the non-conservative stability condition is employed to guarantee the closed-loop uniformly asymptotic stability. The efficacy of the RMPC scheme is illustrated via a numerical study.
- **Chapter 7** concludes this dissertation and presents some potential future research directions.

Chapter 2

Background Theories

The commonly used notations and some preliminary results which are useful in this dissertation are presented in this chapter.

2.1 Notations

The notations used in this dissertation are standard. We respectively define the set of all real numbers and non-negative integers as \mathbb{R} and \mathbb{N} . The real number set from a to b is denoted by $\mathbb{R}_{[a,b]}$ and \mathbb{R}^+ is the set of positive real numbers. The sets of positive integers is denoted by \mathbb{Z}^+ . The positive integer set from a to b is represented by $\mathbb{Z}_{[a,b]}^+$. The n -dimensional Euclidean space is represented as \mathbb{R}^n .

Given a matrix X , X^T represents the transpose of X . $\rho(X)$ is used to denote the spectral radius of matrix X . $X \succ 0$ and $X \succeq 0$ mean that the matrix X is positive definite and semi-positive definite. Given a vector z and a matrix Q , we define $\|z\|_Q := \sqrt{z^T Q z}$. The Euclidean norm and infinity norm of z are represented as $\|z\|$ and $\|z\|_\infty$, respectively.

Suppose $\mathcal{X}, \mathcal{Y} \subseteq \mathbb{R}^n$ are two compact polyhedral sets, the Minkowski sum operation is defined as $\mathcal{X} \oplus \mathcal{Y} := \{z \in \mathbb{R}^n \mid z = x + y, x \in \mathcal{X}, y \in \mathcal{Y}\}$ and the Pontryagin difference operation is $\mathcal{X} \ominus \mathcal{Y} := \{z \in \mathbb{R}^n \mid z + y \in \mathcal{X}, \forall y \in \mathcal{Y}\}$. The distance between a vector $z \subseteq \mathbb{R}^n$ and a set \mathcal{Y} is defined as $D(z, \mathcal{Y}) := \inf_{y \in \mathcal{Y}} \|z - y\|$. The Hausdroff distance between set \mathcal{X} and \mathcal{Y} is $\mathbb{D}(\mathcal{X}, \mathcal{Y}) := \max\{\sup_{y \in \mathcal{Y}} D(y, \mathcal{X}), \sup_{x \in \mathcal{X}} D(x, \mathcal{Y})\}$.

The convex hull operation is denoted by $Co\{\cdot\}$. Boldface $\mathbf{0}$ refers to the origin and the vector $\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ with appropriate dimensions is denoted by $\mathbf{1}$. The floor function, $\lfloor a \rfloor$, maps a to the greatest integer less than or equal to scalar a

2.2 Basic Stability Concepts

Consider a general discrete-time autonomous system as follows:

$$x_{k+1} = f(x_k). \quad (2.1)$$

where $x_k \in \mathbb{R}^n$ is the system state and $k \in \mathbb{N}$ denotes the sampling time instant. x_0 is the initial state. The definition of a positive invariant set \mathcal{Y} is recalled.

Definition 2.1. [79] *A set $\mathcal{Y} \subseteq \mathbb{R}^n$ is said to be a positive invariant set for the system dynamics (2.1), if $x_k \in \mathcal{Y}$ implies $f(x_t) \in \mathcal{Y}$ for $t \geq k$.*

The concepts of stability, attractivity, and asymptotic stability with respect to a positive invariant set \mathcal{Y} and system (2.1) are given.

Definition 2.2. [79] *Suppose $\mathcal{Y} \subseteq \mathbb{R}^n$ is positive invariant for the system (2.1). Define \mathcal{B} as the ball in \mathbb{R}^n with unit radius. Then, the origin of the system is (a) stable in \mathcal{Y} if for all $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x_0 \in \mathcal{Y} \cap \delta\mathcal{B}$ implies that $\|x_k\| \leq \epsilon$ for all $k \geq 0$; (b) attractive in \mathcal{Y} if $\|x_k\| \rightarrow 0$ as $k \rightarrow \infty$ for all $x_0 \in \mathcal{Y}$; (c) asymptotically stable in \mathcal{Y} if it is stable in \mathcal{Y} and attractive in \mathcal{Y} .*

Then, we consider a constrained discrete-time system

$$x_{k+1} = f(x_k, u_k), \quad (2.2)$$

where $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}^m$ are the system state and control input. In addition, the system (2.2) satisfies the following compact polyhedral state and input constraints containing the origin:

$$x_k \in \mathcal{X}, u_k \in \mathcal{U}. \quad (2.3)$$

The concept of a control invariant set \mathcal{Y} is presented

Definition 2.3. [79] *A set $\mathcal{Y} \subseteq \mathbb{R}^n$ is said to be control invariant with respect to the system dynamics (2.2), $u_k \in \mathcal{U}$, if for all $x_k \in \mathcal{Y}$ there exists a $u_k \in \mathcal{U}$ such that $f(x_k, u_k) \in \mathcal{Y}$.*

2.3 Definitions of Switched Systems

2.3.1 Set-theoretic definitions of switched systems

Consider a class of constrained discrete-time switched linear systems

$$x_{k+1} = A_{\sigma(k)}x_k + B_{\sigma(k)}u_k, \quad \sigma(k) \in \mathcal{Q}, \quad (2.4)$$

where the switching signal $\sigma(\cdot) : \mathbb{N} \rightarrow \mathcal{Q}$ is a piecewise constant function of time k . The set $\mathcal{Q} := \{1, 2, \dots, q, \dots, \mathcal{Q}\}$, $q \in \mathcal{Q}$, collects all the admissible switching subsystems and $\mathcal{Q} \in \mathbb{Z}^+$ denotes the total number of subsystems. The switching dynamics (2.4) is subject to the compact polyhedral constraint containing the origin

$$x_k \in \mathcal{X} \subseteq \mathbb{R}^n, u_k \in \mathcal{U} \subseteq \mathbb{R}^m. \quad (2.5)$$

We also define the switching instants where the mode transitions happen as $k_s \in \{k_0, k_1, \dots, k_{s-1}, \dots\}$, $s \in \mathbb{Z}_{[1, \infty)}^+$, with $k_0 = 0$. $k \in [k_{s-1}, k_s)$ indicates that the state trajectory is evolved with subsystem $\sigma(k_{s-1})$. Based on the defined switching instants, the concept of **MDT** is presented as follows:

Definition 2.4. [137] *The MDT $\tau_q \in \mathbb{Z}^+$, associated with mode $q \in \mathcal{Q}$, is the minimum amount of time that the system (2.4) has to stay in mode q . Thus, the dwell time τ_q imposes the inequality $k_s - k_{s-1} \geq \tau_q$ when $\sigma(k) = q$ for $k \in [k_{s-1}, k_s)$, $s \in \mathbb{Z}^+$.*

Then, the definition of admissible switching sequences with MDT constraints is given below.

Definition 2.5. *A switching sequence $\zeta(k) := \{\sigma(0), \sigma(1), \dots, \sigma(k-1)\}$ is said to be admissible if the MDT constraint $k_s - k_{s-1} \geq \tau_q$ fulfills for all $q \in \mathcal{Q}$.*

In the following, the constrained switched system with the state feedback control law $\mu_q := K_q x_k$ from (2.4) is considered

$$\begin{aligned} x_{k+1} &= (A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)})x_k = \mathcal{A}_{\sigma(k)}x_k, \quad \sigma(k) \in \mathcal{Q}, \\ x_k &\in \mathcal{X}, \end{aligned} \quad (2.6)$$

where K_q , $q \in \mathcal{Q}$, is the mode-dependent state feedback gain. We recall the definitions of an MDT contractive set and a constraint admissible MDT contractive set.

Definition 2.6. [137] Given a contractive factor $\lambda \in (0, 1)$, a set $\mathcal{Y} \subseteq \mathbb{R}^n$ is said to be MDT contractive regarding system (2.6) for all admissible switching sequences $\zeta(k)$, if $x_k \in \mathcal{Y}$ implies that $\prod_{n=0}^{k-1} \mathcal{A}_{\sigma(n)} x_k \in \lambda \mathcal{Y}$.

Definition 2.7. [137] Given a contractive factor $\lambda \in (0, 1)$, a set $\mathcal{Y} \subseteq \mathbb{R}^n$ is said to be constraint admissible MDT contractive regarding system (2.6) for all admissible switching sequences $\zeta(k)$, if the set \mathcal{Y} is MDT contractive and $x_k \in \mathcal{X}$.

The definition of **uniform asymptotic stability (UAS)** is presented regarding (2.6).

Definition 2.8. [2] Given a constant $\delta > 0$, the system (2.6) is said to be uniformly asymptotically stable for all admissible switching sequences $\zeta(k)$, if for $\|x_0\| \leq \delta$, x_k fulfills $x_k \rightarrow \mathbf{0}$ when $k \rightarrow \infty$.

Consider the following constrained discrete-time disturbed switched systems with feedback control law μ_q , $q \in \mathcal{Q}$

$$\begin{aligned} x_{k+1} &= (A_{\sigma(k)} + B_{\sigma(k)} K_{\sigma(t)}) x_k + w_k = \mathcal{A}_{\sigma(k)} x_k + w_k, \quad \sigma(k) \in \mathcal{Q}, \\ x_k &\in \mathcal{X}, w_k \in \mathcal{W}, \end{aligned} \quad (2.7)$$

where $w_k \in \mathbb{R}^n$ denotes the time-varying disturbance signal and $\mathcal{W} \subseteq \mathbb{R}^n$ is a compact disturbance set. Regarding Definition 2.5, the concept of an allowable disturbance sequence is presented.

Definition 2.9. A disturbance sequence $\zeta^w(k) := \{w_0, w_1, \dots, w_{k-1}\}$ with respect to (2.7) is said to be allowable if all $w_k \in \mathcal{W}$, $k \in \mathbb{N}$.

The definitions of a disturbance MDT invariant set and a constraint admissible disturbance MDT invariant set are introduced below.

Definition 2.10. [138] A set $\mathcal{Y} \subseteq \mathbb{R}^n$ is said to be disturbance MDT invariant regarding system (2.7) for all admissible switching sequences $\zeta(k)$, if $x_k \in \mathcal{Y}$ implies that $x_t \in \mathcal{Y}$ for all $t \geq k$ with every allowable disturbance sequence $\zeta^w(t)$.

Definition 2.11. [138] A set $\mathcal{Y} \subseteq \mathbb{R}^n$ is said to be constraint admissible disturbance MDT invariant regarding system (2.7) for all admissible switching sequences $\zeta(k)$, if the set \mathcal{Y} is disturbance MDT invariant and $x_k \in \mathcal{X}$, $\forall k \in \mathbb{N}$.

2.3.2 Reachable/backward set computation

Consider the system (2.6). Given a set $\mathcal{Y} \subseteq \mathbb{R}^n$, the r -step reachable set of the system (2.6), $r \in \mathbb{N}$, is computed as

$$\mathcal{R}_q^r(\mathcal{Y}) := \{\mathcal{A}_q^r x \mid x \in \mathcal{Y}, q \in \mathcal{Q}\}, \quad (2.8)$$

with $\mathcal{R}_q^0(\mathcal{Y}) = \mathcal{Y}$. Define a r -step backward set for the system (2.6) as

$$\mathcal{P}_q^r(\Xi) := \{x \mid \mathcal{A}_q^r x \in \mathcal{Y}, q \in \mathcal{Q}\}, \quad (2.9)$$

where the number of steps $r \in \mathbb{N}$ with $\mathcal{P}_q^0(\mathcal{Y}) = \{x \mid x \in \mathcal{Y}\}$.

2.4 Standard MPC Setup

In this subsection, we consider system in (2.2) subject to (2.3). The finite horizon cost function to be optimized is defined as

$$J(x_k, \hat{\mathbf{u}}_k) = \sum_{i=0}^{N-1} (\|\hat{x}_{k+i|k}\|_Q^2 + \|\hat{u}_{k+i|k}\|_R^2) + T(\hat{x}_{k+N|k}), \quad (2.10)$$

where $\hat{\mathbf{u}}_k = \{\hat{u}_{k|k}, \hat{u}_{k+1|k}, \dots, \hat{u}_{k+N-1|k}\}$ represents the i th predicted control sequence at sampling instant k . $\hat{x}_{k+i|k}$ and $\hat{u}_{k+i|k}$, $i \in \mathbb{N}$, represent the predicted state and the control input, respectively. Q and R are weighting matrices and terminal cost function is $T(\hat{x}_{k+N|k}) := \|\hat{x}_{k+N|k}\|_P^2$ where P is the terminal penalty matrix. N denotes the length of the prediction horizon. $x_k = \hat{x}_{k|k}$ denotes the initial state. Then, the standard MPC problem is formulated as

$$\hat{\mathbf{u}}_k^* = \arg \min_{\hat{\mathbf{u}}_k} J(x_k, \hat{\mathbf{u}}_k) \quad (2.11a)$$

$$\text{s.t. } \hat{x}_{k+i+1|k} = f(\hat{x}_{k+i|k}, \hat{u}_{k+i|k}), i \in \mathbb{Z}_{[0, N-1]} \quad (2.11b)$$

$$\hat{x}_{k|k} = x_k \quad (2.11c)$$

$$\hat{x}_{k+i|k} \in \mathcal{X}, i \in \mathbb{Z}_{[0, N-1]} \quad (2.11d)$$

$$\hat{u}_{k+i|k} \in \mathcal{U}, i \in \mathbb{Z}_{[0, N-1]} \quad (2.11e)$$

$$\hat{x}_{k+N|k} \in \Omega \quad (2.11f)$$

where the optimal control sequence is $\hat{\mathbf{u}}_k^* = \{\hat{u}_{k|k}^*, \hat{u}_{k+1|k}^*, \dots, \hat{u}_{k+N-1|k}^*\}$ and $\Omega \subseteq \mathbb{R}^n$ is the terminal set which is designed for closed-loop stability consideration. The receding horizon implementation of MPC stipulates that, at time instant k , the optimal control sequence $\hat{\mathbf{u}}_k^*$ is computed and only the first element in $\hat{\mathbf{u}}_k^*$ is implemented, i.e., $\hat{u}_k = \hat{u}_{k|k}^*$. Then, at the next sampling instant, the optimal control problem (2.11) is solved again to obtain \hat{u}_{k+1} .

2.5 General Switched MPC Setup

Consider the switching dynamics (2.4) subject to mode-dependent system constraints

$$x_k \in \mathcal{X}_q, u_k \in \mathcal{U}_q, \quad (2.12)$$

where $\mathcal{X}_q \in \mathbb{R}^n$ and $\mathcal{U}_q \in \mathbb{R}^m$ are the compact state and input constraint sets with origin in each interior, respectively. The mode-dependent finite horizon cost function $J_q : \mathbb{R}^m \times \mathbb{R}^n \rightarrow \mathbb{R}$, $q \in \mathcal{Q}$, is defined as

$$J_q(x_k, \hat{\mathbf{u}}_k) = \sum_{i=0}^{N_q-1} (\|\hat{x}_{k+i|k}\|_{Q_q}^2 + \|\hat{u}_{k+i|k}\|_{R_q}^2) + T_q(\hat{x}_{k+N|k}), \quad (2.13)$$

where Q_q and R_q , $q \in \mathcal{Q}$, are mode-dependent weighting matrices and terminal cost function is $T_q(\hat{x}_{k+N|k}) := \|\hat{x}_{k+N|k}\|_{P_q}^2$ where P_q is the mode-dependent terminal penalty matrix. N_q denotes the length of the mode-dependent prediction horizon. The switched MPC problem is formulated as

$$\hat{\mathbf{u}}_k^* = \arg \min_{\hat{\mathbf{u}}_k} J_{\sigma(k)}(x_k, \hat{\mathbf{u}}_k) \quad (2.14a)$$

$$\text{s.t. } \hat{x}_{k+i+1|k} = A_{\sigma(t)}\hat{x}_{k+i|k} + B_{\sigma(t)}\hat{u}_{k+i|k}, i \in \mathbb{Z}_{[0, N_{\sigma(k)}-1]} \quad (2.14b)$$

$$\hat{x}_{k|k} = x_k \quad (2.14c)$$

$$\hat{x}_{k+i|k} \in \mathcal{X}_{\sigma(k)}, i \in \mathbb{Z}_{[0, N_{\sigma(k)}-1]} \quad (2.14d)$$

$$\hat{u}_{k+i|k} \in \mathcal{U}_{\sigma(k)}, i \in \mathbb{Z}_{[0, N_{\sigma(k)}-1]} \quad (2.14e)$$

$$\hat{x}_{k+N|k} \in \Omega_{\sigma(k)} \quad (2.14f)$$

where $\Omega_q \subseteq \mathcal{X}_q$ denotes the terminal constraint regarding the detected mode. Similar to standard MPC implementation, only the first element in $\hat{\mathbf{u}}_k^*$ is implemented at time k , i.e., $\hat{u}_k = \hat{u}_{k|k}^*$. Then, the optimal control problem (2.14) is solved repeatedly

to obtain \hat{u}_{k+1} at each sampling instant after k .

Chapter 3

Switched Model Predictive Control with Scheduled Mode Transitions without Terminal Constraints

This chapter provides a stability and recursive feasibility criterion for the switched MPC design without employing terminal constraints. With the known information of switching actions subject to DT constraints, the appropriate length of prediction horizon can be designed offline. Thus, the challenge of devising a common terminal region is circumvented.

3.1 Introduction

As aforementioned in Chapter 1, in the past two decades, the switched MPC problem has been discussed mainly via two distinctive tools, i.e., the Lyapunov-based MPC method and various DT-based strategies [114–126, 128–130]. However, compared with the Lyapunov-based method, directly assigning the minimum DT restriction for regulating switching sequences to stabilize the switching systems would be a more efficient and executable way in practical assignments allocation [117]. Hence, in this work, minimum DT constraints are used in the design of future possible switching sequences. In addition, considering the theoretical difficulties of designing the terminal constraints to stabilize the switched closed-loop system, the work in this chapter will further extend the theoretical results of MPC without terminal constraints in [139] to discrete-time switched linear systems by employing the information of all admissible

switching sequences. Under the proposed switched MPC framework, the performance degradation induced by introducing terminal constraints for closed-loop stability consideration can be reduced to some extent.

Note that the scheduled mode transitions are assumed to be known *a priori* in this work. It is necessary to indicate that the prescribed operating schedules universally exist in real-world applications, for instance, a stirred tank reactor with fixed mixing rates to trigger the chemical reactions [10], the maneuver actions executed for an aerobatics competition [13], or a series of operations with a stick shift of a manual-transmission vehicle driving on a designated route. They are all typical switching system models with scheduled mode transitions. Consequently, demanding a pre-designed switching sequence is a reasonable assumption.

Motivated by the above practical problems with targeted switching sequences requirement, in this work, we aim to study the switched MPC problem with scheduled mode transitions without considering terminal constraints. The **main contributions** of this work are threefold:

- Comparing with the conventional switched MPC algorithm that performs the prediction based on the detected switching mode, we propose a switched MPC method by taking into consideration the targeted switching sequences within the prediction horizon to obtain the control input actions.
- To reduce the complexity to design the terminal constraint, we propose the switched MPC scheme without the terminal constraints. Under certain reasonable assumptions, the stability analysis for the closed-loop system and the feasibility of the developed switched MPC algorithm are rigorously studied.
- Based on the modified algorithm targeting the constrained DT invariant (CDI) set, some necessary parameters under the advisable assumptions can be quantitatively evaluated. In addition, the lower bound of the prediction horizon is calculated through the established inequality and the suboptimal estimated function.

The rest of the work is organized as follows. Section 3.2 presents the problem formulation. The stability and feasibility analyses of the proposed switched MPC algorithm are conducted in Section 3.3. Section 3.4 provides a method to evaluate the essential parameters to support the derived results. Simulation results are provided in Section 3.5. Section 3.6 concludes this work.

3.2 Preliminaries and Problem Formulation

3.2.1 Preliminaries

To begin with, several standard definitions for the switching system are presented in advance. Consider a family of constrained discrete-time switched linear systems:

$$x_{k+1} = f_{\sigma(k)}(x_k, u_k) = A_{\sigma(k)}x_k + B_{\sigma(k)}u_k, \quad \sigma(k) \in \mathcal{Q}, \quad (3.1)$$

where the time instant $k \in \mathbb{N}$; $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}^m$ are the state vector and control input vector, respectively; $f_{\sigma} : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is a map which determines the successor state. The switching signal $\sigma(\cdot) : \mathbb{N} \rightarrow \mathcal{Q}$, is generated by a well-designed [smart switched signals generator \(SSG\)](#) and the set of switching modes $\mathcal{Q} := \{1, 2, \dots, q, \dots, \mathcal{Q}\}$ in which $\mathcal{Q} \in \mathbb{Z}^+$ represents the number of modes in system (3.1), $q \in \mathcal{Q} \subset \mathbb{Z}^+$. It is assumed that there exists a finite number of modes in \mathcal{Q} for the switching system (3.1). In addition, the switching signal σ belongs to the switching indices set Σ :

$$\Sigma = \{\sigma \mid T(\sigma) \geq \tau_d\},$$

where $T(\cdot) : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ denotes the dwell duration of the signal σ and $T(\sigma) = \{k_s - k_{s-1} \mid s \in \mathbb{Z}^+\}$ where $k_{s-1} \in \mathbb{N}$ is the switching instant satisfying $k_{s-1} < k_s$ with $k_0 = 0$. The next switching instant k_s is defined as $k_s := \min\{k > k_{s-1} \mid \sigma(k) \neq \sigma(k_{s-1})\}$. τ_d is the minimum DT for all modes of the switching system (3.1) such that $T(\sigma) \geq \tau_d$. To clarify the specific utility of SSG, the definitions related to switching sequence are presented as follows.

Definition 3.1. [120] *A switching sequence $\zeta(k)$ is said to be admissible if the switching instants satisfy the inequality $k_s - k_{s-1} \geq \tau_d$ for all $s \in \mathbb{Z}^+$. The switching sequence $\zeta(z)$ emanating from time $z = 0$ with total z consecutive steps is represented as*

$$\zeta(z) := \{\sigma(0), \sigma(1), \dots, \sigma(z-1)\}, \quad z \in \mathbb{Z}^+. \quad (3.2)$$

Based on Definition 3.1, we assume that the switching sequences generated by SSG are well-designed offline and the sequences are admissible and adequately long. To make the switching sequence setup consistent with the following switched MPC problem, we now introduce the concept of the [TAS](#) sequence.

Definition 3.2. Suppose a truncated switching sequence $\zeta_{ta}(k)$ is drawn from $\zeta(z)$ from time instant $k \in \mathbb{Z}^+$, $z \gg k$. Then, we define a TAS sequence $\zeta_{ta}(k) = \zeta_{ta}(k, k + m - 1; z)$ from time k as follows:

$$\zeta_{ta}(k) := \{\sigma(k), \sigma(k + 1), \dots, \sigma(k + m - 1) | \zeta_{ta}(k) \subset \zeta(z)\}, \quad (3.3)$$

where $1 \leq m < k$, $m \in \mathbb{Z}^+$ and $\zeta_{ta}(k)$ contains the successive m steps switching signals.

Considering the length of prediction horizon N for the following switched MPC problem formulation, the length of a TAS sequence is designed as $m = N$, $N \in \mathbb{Z}^+$. We also assume that the switching instants and the number of switches with respect to $\zeta_{ta}(k)$, i.e., $\mathcal{P}_k := \mathcal{P}(\zeta_{ta}(k)) \in \mathbb{Z}^+$, can be detected instantly, once $\zeta_{ta}(k)$ is generated. Furthermore, the upper bound $\bar{\mathcal{P}}$ of the number of mode transitions within the TAS sequence $\zeta_{ta}(k)$ is given as

$$\mathcal{P}_k \leq \bar{\mathcal{P}}, \quad \forall t \in \mathbb{N}, \quad (3.4)$$

where $\bar{\mathcal{P}} = \lceil N/\tau_d \rceil$, $\bar{\mathcal{P}} \in \mathbb{Z}^+$. Inequality (3.4) is also employed as one of the switching constraints when the admissible switching sequences are designed offline.

The system state and input vectors of switched system (3.1) are subject to state and input constraints, respectively. Furthermore, the state constraint set \mathcal{X} and the input constraint set \mathcal{U} are applicable to all modes

$$x_k \in \mathcal{X} \subseteq \mathbb{R}^n, \quad (3.5a)$$

$$u_k \in \mathcal{U} \subseteq \mathbb{R}^m. \quad (3.5b)$$

Before introducing the following problem formulation, two standard assumptions for the switching dynamics (3.1) are made.

Assumption 3.1. All the subsystem pairs $\{A_q, B_q\}$ are stabilizable for $q \in \mathcal{Q}$.

Assumption 3.2. The constraint sets \mathcal{X} and \mathcal{U} are compact polyhedral convex sets and the origin is contained in each interior, respectively. In this work, the equilibrium points for all modes of system (3.1) are assumed to be the origin.

3.2.2 Problem Formulation

In this section, the switched MPC problem by taking into consideration the targeted switching sequences within the prediction horizon is presented. The cost function

$J(x_k, \zeta_{ta}(k), \hat{\mathbf{u}}_k)$ to be minimized at time instant k along $\zeta(z)$ is defined as

$$J(x_k, \zeta_{ta}(k), \hat{\mathbf{u}}_k) := \sum_{s=1}^{\mathcal{P}_k-1} \sum_{i=k_{s-1}}^{k_s-1} \ell_{k+i|k} + \sum_{i=k_{\mathcal{P}_k-1}}^{N-1} \ell_{k+i|k}, \quad (3.6)$$

where $\ell_{k+i|k} := \|\hat{x}_{k+i|k}\|_Q^2 + \|\hat{u}_{k+i|k}\|_R^2$ represents the i th step stage cost predicted at time instant k , $i \in \mathbb{N}$. $Q \succ 0$ and $R \succ 0$ are positive definite weighting matrices. $\hat{x}_{k+i|k}$ and $\hat{u}_{k+i|k}$ respectively represent the corresponding predicted state and the control input based on the state information which is measured at time k . $\hat{x}_{k|k} = x_k$ represents the initial condition. N denotes the length of prediction horizon. k_s is the s th switching instant of $\zeta_{ta}(k) \subset \zeta(z)$ with $k_0 = k$ with $s \in \{0, 1, \dots, \mathcal{P}_k - 1\}$. $\hat{\mathbf{u}}_k \in \mathcal{U} = \{\hat{u}_{k|k}, \hat{u}_{k+1|k}, \dots, \hat{u}_{k+N-1|k}\}$ denotes the predicted input sequence. Note that (3.6) consists of the summation of $(\mathcal{P}_k + 1)$ subsections, i.e., $J(x_k, \zeta_{ta}(k), \hat{\mathbf{u}}_k) = \sum_{s=0}^{\mathcal{P}_k} J_s(x_k, \zeta_{ta}(k), \hat{\mathbf{u}}_k)$ and each subsection $J_s(x_k) := J_s(x_k, \zeta_{ta}(k), \hat{\mathbf{u}}_k)$ is defined as

$$J_s(x_k) = \begin{cases} \sum_{i=k_{s-1}}^{k_s-1} \ell_{k+i|k}, & s \in \mathbb{N}_{[0, \mathcal{P}_k-1]} \\ \sum_{i=k_{\mathcal{P}_k-1}}^{N-1} \ell_{k+i|k}, & s = \mathcal{P}_k. \end{cases} \quad (3.7)$$

In addition, the horizon H_s^k of each corresponding subsection is described as

$$H_s^k := \begin{cases} k_s - k_{s-1}, & s \in \mathbb{Z}_{[1, \mathcal{P}_k-1]}^+ \\ N - k_{\mathcal{P}_k}, & s = \mathcal{P}_k. \end{cases}$$

According to the designed cost function (3.6), at sampling instant k , given a sufficiently long admissible switching sequence $\zeta(z)$ generated by SSG, the switched MPC

optimal problem is formulated as follows:

$$\hat{\mathbf{u}}_k^* = \arg \min_{\hat{\mathbf{u}}_k} J(x_k, \zeta_{ta}(k), \hat{\mathbf{u}}_k) \quad (3.8a)$$

$$\text{s.t. } \hat{x}_{k+i+1|k} = A_{\sigma(k+i)}\hat{x}_{k+i|k} + B_{\sigma(k+i)}\hat{u}_{k+i|k}, i \in \mathbb{N}_{[0, N-1]} \quad (3.8b)$$

$$\hat{x}_{k|k} = x_k \quad (3.8c)$$

$$\zeta_{ta}(k) = \zeta_{ta}(k, k + N - 1; z) \quad (3.8d)$$

$$\sigma(k+i) \in \zeta_{ta}(k), i \in \mathbb{N}_{[0, N-1]} \quad (3.8e)$$

$$\hat{x}_{k+i|k} \in \mathcal{X}, i \in \mathbb{N}_{[0, N-1]} \quad (3.8f)$$

$$\hat{u}_{k+i|k} \in \mathcal{U}, i \in \mathbb{N}_{[0, N-1]} \quad (3.8g)$$

where $\hat{\mathbf{u}}_k^* \in \mathcal{U} = \{\hat{u}_{k|k}^*, \hat{u}_{k+1|k}^*, \dots, \hat{u}_{k+N-1|k}^*\}$ is the optimal input sequence obtained by solving the optimization problem at time k . $\sigma(k+i) \in \zeta_{ta}(k)$ represents the switching signal of $(k+i)$ th step based on the information of $\zeta_{ta}(k)$ at time k . The successor optimal state $\hat{x}_{k+i+1|k}^* = f_{\sigma(k+i)}(\hat{x}_{k+i|k}^*, \hat{u}_{k+i|k}^*)$ is gained based on the optimal input sequence $\hat{\mathbf{u}}_k^*$. In addition, let $\ell_{k+i|k}^*$ and $J^*(x_k) := J^*(x_k, \zeta_{ta}(k), \hat{\mathbf{u}}_k) = \sum_{s=0}^{\mathcal{P}_k} J_s^*(x_k)$ denote the optimal stage cost and the optimal value of the cost function, respectively. For optimal control sequence $\hat{\mathbf{u}}_k^*$, only the first element of this control sequence, i.e., $\hat{u}_{k|k}^*$, is implemented.

3.3 Main Results

In this section, the main theoretical results of this work are presented. Instead of using the conventional terminal constraints to ensure the closed-loop stability, a sufficient condition on the prediction horizon is derived under general assumptions to achieve the asymptotic stability and recursive feasibility of the closed-loop system. To begin with, we define the feasible set as follows.

Definition 3.3. [126] *For the fixed prediction horizon N and $\hat{x}_{k|k} = x_k$, the feasible set \mathcal{F}_N is characterized as*

$$\mathcal{F}_N := \{x_k \in \mathcal{X} \mid \hat{x}_{k+i|k} \in \mathcal{X}, \hat{u}_{k+i|k} \in \mathcal{U}, \hat{\mathbf{u}}_k \neq \emptyset, i \in \mathbb{N}_{[0, N-1]}\}.$$

In the light of Definition 3.3, if an initial state $x_k \in \mathcal{F}_N$, the existence of an adequately large constant $\beta > 0$ is guaranteed such that the upper bound of the cost

function satisfies $J(x_k) \leq \beta$. Then, the following modified controllability assumption which is intended to ensure the convergence of state trajectories is employed.

Assumption 3.3. *Suppose that the each subsection of the cost function $J_s(x_k)$, $s \in \mathbb{N}$, and the switching times \mathcal{P}_k exist such that the following inequality holds:*

$$J_s(x_k) \leq \alpha(N)\ell(\hat{x}_{k+i|k}) \leq \alpha(N) \sup \ell(\hat{x}_{k+i|k}) \leq \frac{\beta}{\mathcal{P}_k + 1}, \quad \forall \hat{x}_{k+i|k} \in \mathcal{O} \cap \mathcal{X},$$

where the set \mathcal{O} represents the neighbourhood of origin; $\alpha: \mathbb{Z}^+ \rightarrow \mathbb{R}^+$ is a bounded and non-decreasing suboptimal estimated function, i.e., $\alpha(N) = \frac{\gamma}{\mu}(1 - e^{-\mu N})$ where $\gamma, \mu > 0$ are two constants [140]. $\ell(\hat{x}_{k+i|k}) := \ell(\hat{x}_{k+i|k}, \hat{u}_{k+i|k})$ in which $\hat{u}_{k+i|k} \in \hat{\mathbf{u}}_k$ denotes the predicted stage cost.

According to the suboptimal estimate for $J_s(x_k)$, the following inequality for the cost $J(x_k)$

$$J(x_k) \leq (\mathcal{P}_k + 1)\alpha(N)\ell(\hat{x}_{k+i|k}), \quad \forall \hat{x}_{k+i|k} \in \mathcal{O} \cap \mathcal{X}, \quad (3.9)$$

is derived. Let $\mathcal{L} := \inf \ell(\hat{x}_{k+i|k})$ be the lower bound for the states $\hat{x}_{k+i|k} \in \mathcal{X} \setminus \mathcal{O}$, and then the upper bound of $J(x_k)$ can be derived as

$$J(x_k) \leq \frac{\beta}{\mathcal{L}} \cdot \mathcal{L} \leq \frac{\beta}{\mathcal{L}} \cdot \ell(\hat{x}_{k+i|k}), \quad \forall \hat{x}_{k+i|k} \in \mathcal{X} \setminus \mathcal{O}, \quad (3.10)$$

where $\mathcal{X} \setminus \mathcal{O} := \{\hat{x}_{k+i|k} | \hat{x}_{k+i|k} \in \mathcal{X}, \hat{x}_{k+i|k} \notin \mathcal{O}\}$. Combining the inequalities in (3.9) and (3.10), $J(x_k)$ yields

$$J(x_k) \leq \eta \ell(\hat{x}_{k+i|k}), \quad \forall \hat{x}_{k+i|k} \in \mathcal{X}, \quad (3.11)$$

where $\eta := \max\{(\mathcal{P}_k + 1)\alpha(N), \frac{\beta}{\mathcal{L}}\}$. Considering the reasonable upper limit of the number of switches for TAS sequences with appropriate minimum DT setup, we redefine η as $\eta := \max\{(\bar{\mathcal{P}} + 1)\alpha(N), \frac{\beta}{\mathcal{L}}\}$. Based on Assumption 3.3 and the analytical expression for the upper bound of $J(x_k)$, Theorem 3.1 is proposed to ensure the stability and recursive feasibility of the closed-loop system.

Theorem 3.1. *Suppose that Assumption 3.3 and the inequality (3.11) are fulfilled. Given the well-designed function $\alpha(N)$ satisfying (3.9), there exists a lower bound of prediction horizon $N \in \mathbb{Z}^+$ such that the inequality*

$$b - (\bar{\mathcal{P}} + 1)\alpha(N) > 0 \quad (3.12)$$

holds with the coefficients $b = (\frac{\eta}{\eta-1})^{N-2}$ and $\eta = \max\{(\bar{\mathcal{P}} + 1)\alpha(N), \frac{\beta}{\mathcal{L}}\}$. Therefore, the inequality

$$J(f_{\sigma(k)}(x_k, \hat{u}_{k|k}^*)) \leq J^*(x_k) - (b - (\bar{\mathcal{P}} + 1)\alpha(N))\ell_{k+N-1}^* \quad (3.13)$$

holds for $x_k \in \mathcal{F}_N$ where $\ell_{k+i}^* := \ell_{k+i}(\hat{x}_{k+i|k}^*, \hat{u}_{k+i|k}^*)$ denotes the stage cost for $i \in \{k, k+1, \dots, k+N-1\}$. Thus, the prediction horizon N satisfying (3.12) indicates the asymptotic stability of the closed-loop system and recursive feasibility for the switched MPC design.

Proof. Suppose that there exists an admissible control sequence $\hat{\mathbf{u}}_k^* \in \mathcal{U}$. The cost function $J^*(x_k)$ can be decomposed as

$$J^*(x_k) = \sum_{i=0}^{k_{\mathcal{P}_k}-1} \ell_{k+i}^* + J_{\mathcal{P}_k}^*(x_k), \quad (3.14)$$

where $J_{\mathcal{P}_k}^*(x_k) = \sum_{i=k_{\mathcal{P}_k}}^{N-1} \ell_{k+i}^*$. Meanwhile, from (3.11) we have $J_{\mathcal{P}_k}^*(x_k) \leq \eta \ell_{k_{\mathcal{P}_k}}^*$. Hence, the inequality

$$J^*(x_k) \leq \sum_{i=0}^{k_{\mathcal{P}_k}-1} \ell_{k+i}^* + \eta \ell_{k_{\mathcal{P}_k}}^* \quad (3.15)$$

holds. If $f_{\sigma(k)}(x_k, \hat{u}_{k|k}^*) \in \mathcal{F}_N$ holds, we can have

$$J(f_{\sigma(k)}(x_k, \hat{u}_{k|k}^*)) \leq J^*(x_k) - \ell_k^* - J_{\mathcal{P}_k}^*(x_k) + J(x^*(k_{\mathcal{P}_k}, N; x_k)), \quad (3.16)$$

where the suboptimal solution is $J(x^*(k_{\mathcal{P}_k}, N; x_k)) = \sum_{i=k_{\mathcal{P}_k}}^N \ell_{k+i}(\hat{x}_{k+i|k}^*, \hat{u}_{k+i|k}^*)$. To verify the above inequality, the next step is to evaluate the magnitude of $J(x^*(k_{\mathcal{P}_k}, N; x_k))$. From (3.11), the inequality $J^*(x^*(k_{\mathcal{P}_k} + 1, N - 1; x_k)) \leq (\eta - 1)\ell_{k_{\mathcal{P}_k}}^*$ is obtained. It implies that

$$\sum_{i=k_{\mathcal{P}_k}}^{N-1} \ell_{k+i}^* \geq \left(\frac{\eta}{\eta-1}\right) \sum_{i=k_{\mathcal{P}_k}+1}^{N-1} \ell_{k+i}^* \geq \dots \geq \left(\frac{\eta}{\eta-1}\right)^{N-k_{\mathcal{P}_k}-1} \ell_{k+N-1}^*, \quad (3.17)$$

where $k_{\mathcal{P}_k} \in [1, N - 1]$ and $J_{\mathcal{P}_k}^*(x_k) \geq (\frac{\eta}{\eta-1})^{N-2} \ell_{k+N-1}^*$ holds with $k_{\mathcal{P}_k} = 1$. Therefore, the above inequality implies that $J^*(x_k) > (\frac{\eta}{\eta-1})^{N-2} \ell_{k+N-1}^*$. It can be observed that $\ell_{k+N-1}^* < \mathcal{L}$ contradicts with inequality (3.10). Consequently, the state $\hat{x}_{k+N-1|k}$

belongs to the set $\mathcal{O} \cap \mathcal{X}$. Then the inequality

$$J(x^*(k_{\mathcal{P}_k}, N; x_k)) \leq (\bar{\mathcal{P}} + 1)\alpha(N)\ell_{k+N-1}^*,$$

holds under inequality (3.9). Finally, the asymptotic stability could be achieved by

$$\begin{aligned} J(f_{\sigma(k)}(x_k, \hat{u}_{k|k}^*)) &\leq J^*(x_k) - \ell_k^* - J_{\mathcal{P}_k}^*(x_k) \\ &\quad + (\bar{\mathcal{P}} + 1)\alpha(N)\ell_{k+N-1}^* \\ &\leq J^*(x_k) - (b - (\bar{\mathcal{P}} + 1)\alpha(N))\ell_{k+N-1}^*, \end{aligned}$$

when inequality (3.12) is fulfilled.

Hence, inequality (3.12) indicates that $f_{\sigma(k)}(x_k, \hat{u}_{k|k}^*) \in \mathcal{F}_N$ holds and the optimization problem (3.8) is recursively feasibility if it is feasible at the beginning. Therefore, the prediction horizon N satisfies

$$N > \frac{\ln((\bar{\mathcal{P}} + 1)\alpha(N))}{\ln(\eta) - \ln(\eta - 1)} + 2. \quad (3.18)$$

In summary, the asymptotic stability of the closed-loop system and recursive feasibility for the switched MPC design are achieved. \blacksquare

3.4 Parameters Evaluation

In this section, the algorithm to construct the invariant set \mathcal{O} is presented. Based on the obtained invariant set \mathcal{O} , the parameters β and \mathcal{L} are evaluated by solving the **quadratic programming (QP)** problems. Before introducing the computational algorithm, we first give the definition of a constrained DT invariant set.

Definition 3.4. [138] For all the admissible switching sequences $\zeta(z)$, a CDI set $\mathcal{D} \subset \mathbb{R}^n$ with respect to system (3.1) with a minimum DT τ_d and the constraint set $\tilde{\mathcal{X}} := \{x \in \mathcal{X} \mid x \in \mathcal{X}, K_q x \in \mathcal{U}, \forall q \in \mathcal{Q}\}$ is defined as

$$\mathcal{D} := \{x \in \mathcal{D} \mid \bar{A}_{\zeta(z)} x \in \mathcal{D}, x \in \tilde{\mathcal{X}}\}, \quad (3.19)$$

where $\bar{A}_{\zeta(z)} = \prod_{n=0}^{z-1} \bar{A}_{\sigma(n)}$ represents the product of matrices $\bar{A}_{\sigma(n)}$ with respect to the admissible switching sequence $\zeta(z)$; $\bar{A}_{\sigma(n)} = A_{\sigma(n)} + B_{\sigma(n)}K_{\sigma(n)}$ and $\bar{\mathcal{A}} = \{\bar{A}_1, \bar{A}_2, \dots, \bar{A}_{\mathcal{Q}}\}$; $K_q, q \in \mathcal{Q}$, denotes a state feedback gain for each mode of system (3.1).

To estimate the set \mathcal{O} , we employ the state feedback control law $\mu_q := K_q x, q \in \mathcal{Q}$ for the switching system (3.1) to design such an invariant set \mathcal{D} that the states and input constraints are satisfied. For the state feedback gains $K_q, q \in \mathcal{Q}$, the following assumption is required.

Assumption 3.4. [120] *For the unconstrained switching system $x(k+1) = \bar{A}_{\sigma(k)}x(k)$ with $\bar{A}_{\sigma(k)} = A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)}$ being Schur-stable, there exist state feedback gains K_q such that the system is asymptotically stable with minimum DT τ_d for the switching sequence.*

Following Assumption 3.4, we recall Lemma 3.1 for the convenience to present the computational algorithm of the CDI set \mathcal{D} .

Lemma 3.1. [138] *Suppose that Assumptions 3.1, 3.2, and 3.4 hold and define the set $\mathcal{T}_d = \{\tau_d, \tau_d + 1, \dots, 2\tau_d - 1\}$, a set $\Xi \subset \mathbb{R}^n$ is called a DT invariant for the system $x(k+1) = \bar{A}_{\sigma(k)}x(k)$ with DT τ_d , if and only if for $x \in \Xi$,*

$$\bar{A}_q^{t'} x \in \Xi, \forall t' \in \mathcal{T}_d, \forall q \in \mathcal{Q}, \quad (3.20)$$

where $\bar{A}_q^{t'}$ indicates that the mode q dwells t' -time instants.

3.4.1 Computational Algorithm

In this section, the algorithm to estimate the invariant set \mathcal{O} is presented. We modify the algorithm to compute the constraint admissible DT contractive set mentioned in [138] to accommodate the constraint set $\tilde{\mathcal{X}}$ without considering the contractive factor. For notational convenience, the constraint set $\tilde{\mathcal{X}}$ is rewritten as $\tilde{\mathcal{X}} := \{x | Fx \leq \mathbf{1}\}$, where the matrix F is decided by the state constraint \mathcal{X} and the state feedback matrices $K_q, q \in \mathcal{Q}$, where $\mathbf{1} = [1, 1, \dots, 1]^T$. We define a r -step backward set $\mathcal{C}_q^r(\Xi) = \{x | \bar{A}_q^r x \in \Xi, q \in \mathcal{Q}\}$ where the integer $r \in \mathbb{N}$ and $\mathcal{C}_q^0(\Xi) = \{x | x \in \Xi\}$. Then, the intersection of the r -step backward set $\mathcal{C}_q^r(\Xi)$ for all modes of system (3.1) is denoted as $\mathcal{C}^r(\Xi) = \bigcap_{q \in \mathcal{Q}} \mathcal{C}_q^r(\Xi)$. The calculation procedure is summarized in Algorithm 3.1.

In step 1 of Algorithm 3.1, the computation of the initial set \mathcal{D}_0 is to guarantee that all the states of the first $\tau_d - 1$ steps for the system $x(k+1) = \bar{A}_{\sigma(k)}x(k)$ are included in the constraint set $\tilde{\mathcal{X}}$. The step 2 stringently follows the condition (3.20) in Lemma 3.1 to ensure that the next r -step states, $r \in \mathcal{T}_d$, still remain in the set

Algorithm 3.1 Computation of a CDI set for admissible switching sequences

Require: Constraint set $\tilde{\mathcal{X}}$; system matrices set $\tilde{\mathcal{A}}$; minimum DT τ_d .

- 1: Initialize $k = 0$, calculate the initial set $\mathcal{D}_0 = \tilde{\mathcal{X}} \cap_{r=1,2,\dots,\tau_d-1} \mathcal{C}^r(\tilde{\mathcal{X}})$;
 - 2: Compute the set $\mathcal{D}_{k+1} = \mathcal{D}_0 \cap \Xi_k$, where $\Xi_k = \bigcap_{r=\tau_d, \tau_d+1, \dots, 2\tau_d-1} \mathcal{C}^r(\mathcal{D}_k)$;
 - 3: **if** $\mathcal{D}_{k+1} \equiv \mathcal{D}_k$, end the procedure and obtain $\mathcal{D}_\infty = \mathcal{D}_k$;
 - 4: **else** set $k = k + 1$ and return to step 2;
 - 5: **end.**
-

\mathcal{D}_{k+1} . Therefore, once the invariant set \mathcal{D}_∞ is obtained, all the states evolving along all the feasible admissible switching sequences will stay in the CDI set \mathcal{D}_∞ . Moreover, origin is contained in \mathcal{D}_∞ . Consequently, the CDI set \mathcal{D}_∞ is employed to estimate the neighbourhood of origin \mathcal{O} with constraints satisfaction.

3.4.2 Parameters Evaluation

Based on the information of the invariant set \mathcal{O} , the process to evaluate the parameters β and \mathcal{L} are presented in this section. By solving the optimization problem (3.21), the value of β can be evaluated through the further computation via inequality (3.9).

$$\max \ell(\hat{x}_{k+i|k}) \tag{3.21a}$$

$$\text{s.t. } \hat{x}_{k+i|k} \in \mathcal{O} \cap \tilde{\mathcal{X}} \tag{3.21b}$$

$$\hat{u}_{k+i|k} = K_q \hat{x}_{k+i|k}, q \in \mathcal{Q}. \tag{3.21c}$$

Another parameter \mathcal{L} which is directly related to the suboptimal estimated parameter η is also assessed by the following optimization problem (3.22)

$$\min \ell(\hat{x}_{k+i|k}) \tag{3.22a}$$

$$\text{s.t. } \hat{x}_{k+i|k} \in \tilde{\mathcal{X}} \setminus \mathcal{O} \tag{3.22b}$$

$$\hat{u}_{k+i|k} \in \mathcal{U}. \tag{3.22c}$$

Thus, based on the optimization problems (3.21) and (3.22), the suboptimal estimated parameter η can be calculated such that the minimum length of the prediction horizon to stabilize the closed-loop system can be determined.

3.5 Simulation Results

To illustrate our proposed control scheme, we consider a switched system with the minimum DT $\tau_d = 3$ and there exists four distinct modes. The dynamics of each mode is

$$A_1 = \begin{bmatrix} 0.32 & -0.4 \\ 0.8 & -0.8 \end{bmatrix}, A_2 = \begin{bmatrix} 0.08 & -0.26 \\ 1.2 & -1.12 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.16 & 0.24 \\ -2.4 & -0.96 \end{bmatrix}, A_4 = \begin{bmatrix} 0.2 & 0.18 \\ -1.2 & -0.88 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 2 & 1 \end{bmatrix}^T, B_2 = \begin{bmatrix} 1 & -1 \end{bmatrix}^T; B_3 = B_4 = \begin{bmatrix} 0.8 & -1 \end{bmatrix}^T.$$

The input constraint is $-2.5 \leq u(t) \leq 2.5$ and the state constraint is $-2 \leq x_i(t) \leq 2$, $i = 1, 2$. The weighting matrix of states is set to be $Q = \text{diag}(1, 1)$ and the input weighting matrix is designed as $R = 1$. The initial state of the system is given as $x_0 = [0.5, -0.5]^T$. The suboptimal estimated function $\alpha(H)$ is designed as $\alpha(N) = 0.7(1 - e^{-0.08N})$ with $\mu = 0.08$, $\gamma = 0.056$ and $\alpha(N) \in (0, 0.7)$, $\forall N \in \mathbb{Z}^+$. The number of switching times constraint $\bar{\mathcal{P}}$ for the admissible switching sequences is set to be $\bar{\mathcal{P}} = 4$, i.e., $\mathcal{P}_k \leq 4$, $\forall k \in \mathbb{N}$. The feedback gain of each subsystem is given as $K_1 = [-0.1659 \ 0.1982]$, $K_2 = [0.4866 \ -0.3742]$, $K_3 = [-0.0325 \ -0.2006]$, and $K_4 = [-0.4785 \ -0.3627]$. According to the parameters setup, the prediction horizon satisfies $N \geq 27$. Thus, we set $N = 27$ to verify the corresponding theoretical results. Denote three admissible switching sequences generated by the SSG as $\zeta^1(z)$, $\zeta^2(z)$, and $\zeta^3(z)$, i.e., $\zeta^1(z) = A_1^3 A_2^3 A_3^3 A_4^3 A_1^{55}$, $\zeta^2(z) = A_2^3 A_1^3 A_4^3 A_3^3 A_2^{55}$, and $\zeta^3(z) = A_3^3 A_1^3 A_2^{27} A_4^{20} A_1^{14}$ where A_q^τ indicates that the mode q stays τ -time instants.

The calculated CDI set is achieved by using the MATLAB MPT3 toolbox through Algorithm 3.1, as shown in Figure 3.1. Region I and region II illustrate the polytopes \mathcal{O} of the neighbourhood of origin and the set $\mathcal{X} \setminus \mathcal{O}$, respectively. Based on the computational results of the CDI set \mathcal{O} , the corresponding estimated parameters are obtained by solving the optimization problem (3.21) and (3.22).

Figure 3.2, Figure 3.3, and Figure 3.4 show the optimized control input, system states, and the cost function of the closed-loop system using the designed switched MPC controller for three different cases. It can be observed that the asymptotic stability of the closed-loop system and the feasibility of the developed switched MPC algorithm have been achieved without violating the system constraints.

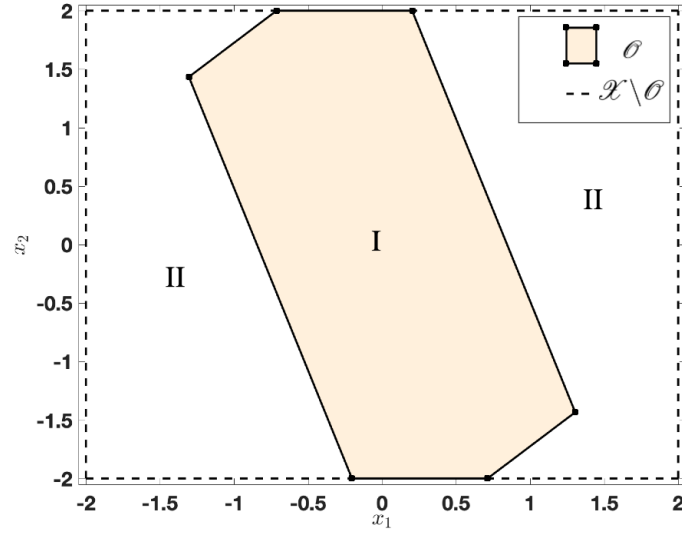


Figure 3.1: The CDI set for example of Section V: I: the invariant set \mathcal{O} of the neighbourhood of origin; II: the set $\mathcal{X} \setminus \mathcal{O}$.

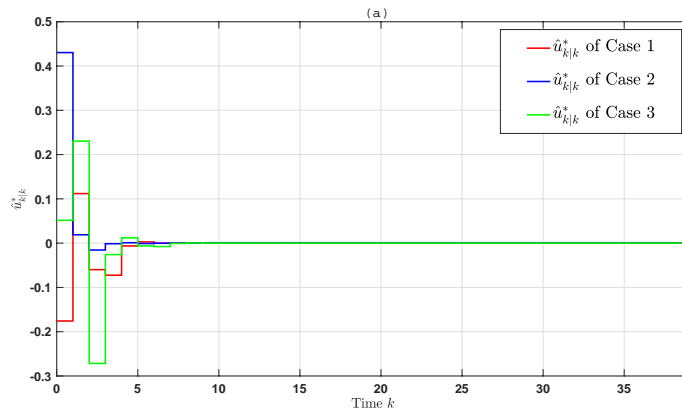


Figure 3.2: Trajectory of control inputs along three admissible switching sequences.

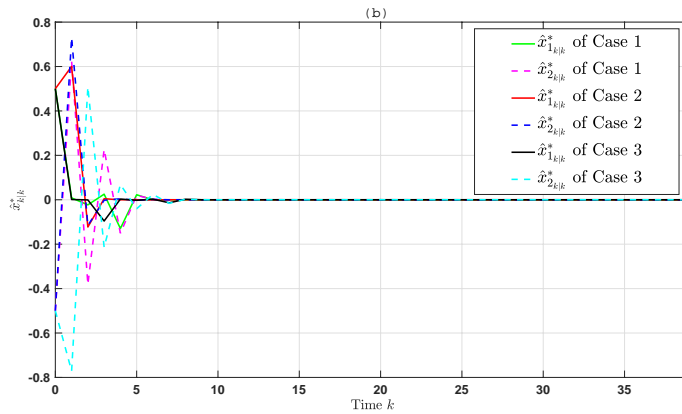


Figure 3.3: Trajectory of system states along three admissible switching sequences.

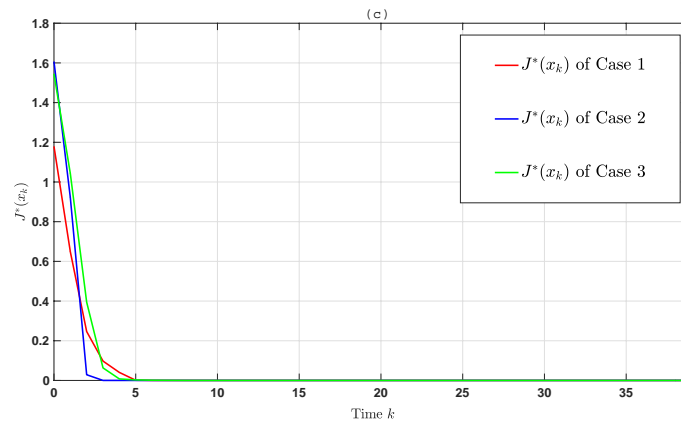


Figure 3.4: Evolution of costs along three admissible switching sequences.

3.6 Conclusion

In this chapter, the switched MPC problem for a class of constrained discrete-time switched linear systems with the known information of the future admissible switching sequences under the minimum DT constraint satisfaction is studied. A sufficient condition which is directly related to the length of the prediction horizon on guaranteeing the closed-loop system stability and the proposed algorithm feasibility is derived. The computational process of the suboptimal estimated parameters by employing the CDI set is presented. Numerical simulations are provided to verify the theoretical results.

Chapter 4

Model Predictive Control for Asynchronously Switched Linear Systems with Mode-Dependent Dwell Time

In Chapter 3, we have investigated the switched MPC strategy without terminal constraint. This chapter presents the result of asynchronously switched MPC design subject to MDT constraints with constructed terminal constraints for all subsystems.

4.1 Introduction

According to the literature review of switched MPC, it is worth mentioning that almost all the existing works related to switched MPC design rely on the assumption of the instantly detected switching signals [114–126, 128–130]. Note that, in practical engineering, aligning the mode-dependent MPC controller to the detected switching mode is unrealistic since the real-time detection of the switching mode by controllers is not practical [41]. This mismatch results in the asynchronous switching issue since there exists a time delay between the practically activated controller and the ideally matched controller. Many practical situations such as networked-induced delays during the signals transmission, long-distance signals transmission, or using the aging physically detected facilities to receive signals, etc. may inevitably cause asynchronous switching phenomena.

However, very few literature explore the control design of switched MPC under asynchronous switching. To the best of our knowledge, only in [117], the asynchronously switched MPC problem is discussed based on the modified ADT condition. However, the asymptotic stability of the closed-loop system can only be achieved when the initial feasible region is contained in a sufficiently small set. Therefore, the determination of DT to ensure the closed-loop stability and the persistent feasibility of the asynchronously switched MPC design has not been fully solved. Moreover, considering the unknown switching pattern and the realistic situation with the switching signal detection delays, a novel framework to study the asynchronously switched MPC problem is in great desire.

To address the above issues, in this chapter, we study the asynchronously switched MPC problem for a class of linear discrete-time switched systems via a more general MDT criterion. Note that if the order of magnitude between the length of MDT and the asynchronous switching period differs a lot, the negative impact of system performance can be neglected [41, 117, 141]. Therefore, this chapter focuses on the case that durations of MDT and the asynchronous switching period are in the same order. The **main contributions** of this chapter are threefold:

- 1) This chapter provides the first attempt to tackle the asynchronously switched MPC problem by using the MDT restriction. A strategy to ensure the persistent feasibility of the asynchronously switched MPC design is proposed. Thereby, the switched MPC problem is continuously solvable at both switching instants or within a mode.
- 2) A constraint admissible contractive [asynchronous switching MDT invariant \(AMI\)](#) set is designed to serve as the common terminal set for all subsystems. Meanwhile, a necessary and sufficient stability condition is proposed based on the AMI set. The rigorous proof is delivered to demonstrate the existence of such an invariant set is equivalent to the asymptotic stability of the closed-loop system.
- 3) In order to guarantee the closed-loop asymptotic stability with the switched MPC design, we present two executable ways to drive the system states into the designed constraint admissible contractive AMI set. Thereby, the minimum value of the admissible stage MDT is acquired offline which significantly reduces the online computational load. Furthermore, detailed proofs are studied.

The remainder of this chapter is organized as follows. Section 4.2 presents the preliminaries and the problem formulation. Section 4.3 is devoted to the persistent feasibility analysis of the derived MPC algorithm. Section 4.4 demonstrates that the stability of the closed-loop system is guaranteed by the proposed algorithm. Simulation results are given in Section 4.5. Section 4.6 concludes this chapter.

4.2 Preliminaries and Problem Formulation

4.2.1 Preliminaries

Consider the following constrained discrete-time switched linear system

$$x_{k+1} = A_{\sigma(k)}x_k + B_{\sigma(k)}u_k, \quad \sigma(k) \in \mathcal{Q}, \quad (4.1)$$

where the time instant $k \in \mathbb{N}$; $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}^m$ are the state vector and control input vector, respectively. The switching signal $\sigma(\cdot) : \mathbb{N} \rightarrow \mathcal{Q}$ is a piecewise constant function of time k and it indicates the currently activated mode of the system in (4.1). Additionally, in this chapter, the switching signal $\sigma(\cdot)$ is considered to be unknown *a priori*. The set of all switching modes \mathcal{Q} is given as $\mathcal{Q} := \{1, 2, \dots, q, \dots, Q\}$, $q \in \mathcal{Q}$, in which $Q \in \mathbb{Z}^+$ represents the total number of modes in system (4.1). The system state and the control input are respectively subject to the following mode-dependent constraints, i.e., $\forall \sigma(k) = q \in \mathcal{Q}$,

$$x_k \in \mathcal{X}_q \subseteq \mathbb{R}^n, \quad (4.2a)$$

$$u_k \in \mathcal{U}_q \subseteq \mathbb{R}^m, \quad (4.2b)$$

where \mathcal{X}_q and \mathcal{U}_q are all compact polyhedral sets that contain the origin in each interior. Define the switching instants $k_{j-1} \in \mathcal{J} := \{k_0, k_1, \dots, k_{j-1}, \dots\}$, $j \in \mathbb{Z}_{[1, \infty)}^+$, with $k_0 = 0$ and $k_{j-1} < k_j$. Hence, $k \in [k_{j-1}, k_j)$ indicates that the system mode or the subsystem $\sigma(k_{j-1})$ is activated.

Based on the definition of switching instants, in the following, the concept of **MDT** is introduced.

Definition 4.1. [137] *The MDT $\tau_q \in \mathbb{Z}^+$, associated with mode $q \in \mathcal{Q}$ with respect to the system dynamics (4.1) and the corresponding switching instants at $k_0, k_1, \dots, k_j, \dots$ with $k_0 = 0$ is the minimum amount of time that the system (4.1)*

has to stay in mode q . Hence, the dwell time τ_q imposes the inequality $k_j - k_{j-1} \geq \tau_q$ when $\sigma(k) = q$ for $k \in [k_{j-1}, k_j)$, $j \in \mathbb{Z}^+$, and $\tau_q \in T := \{\tau_1, \tau_2, \dots, \tau_Q\}$ where T is the set of all the admissible MDTs which satisfy the above inequality.

Define the j th stage of mode transition beginning at switching instants k_{j-1} and ending at k_j . Then, the definition of stage MDT is presented.

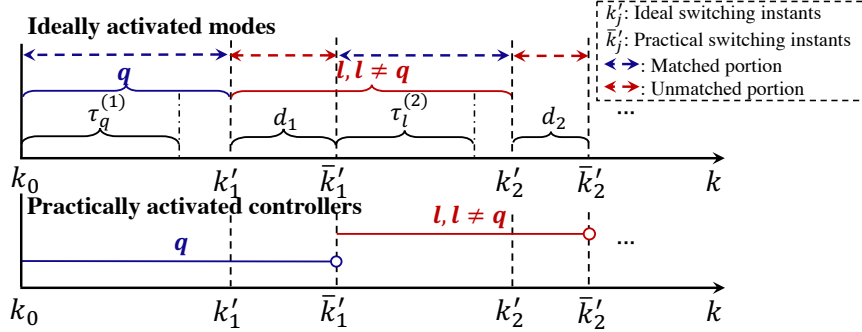


Figure 4.1: Illustration of ideally activated modes and practically employed activated controllers during the mode-unmatched portion.

Definition 4.2. [126] Likewise, $\tau_q^{(j)} \in \mathbb{Z}^+$ associated with the mode $q \in \mathcal{Q}$ is said to be the j th stage MDT, $j \in \mathbb{Z}^+$, with respect to the dynamics (4.1) and the corresponding switching instants at $k_0, k_1, \dots, k_j, \dots$ with $k_0 = 0$ if $k_j - k_{j-1} \geq \tau_q^{(j)}$ when $\sigma(k) = q$ for $k \in [k_{j-1}, k_j)$.

Note that, in most of the existing literatures [120–122, 126, 128], the switching signals are considered to be unknown *a priori* and the currently activated subsystem is dependent on the instantly detected mode. However, the immediate switching detection is difficult to achieve in real time. This may lead to the mismatch between the desired mode-dependent controller and the currently enabled switching mode. Therefore, as shown in Figure 4.1, we first define the ideal switching instants where the desired mode-dependent controller is supposed to be activated with respect to the switched mode as $k'_0, k'_1, \dots, k'_j, \dots$, $j \in \mathbb{N}$, with $k'_0 = k_0$. The corresponding practical switching instants where the desired mode-dependent controller is practically taken into action are denoted as $\bar{k}'_0, \bar{k}'_1, \dots, \bar{k}'_j, \dots$, $j \in \mathbb{N}$, with $\bar{k}'_0 = k_0$. The mode-matched portion refers to the interval on which the activated mode and the mode-dependent controller are perfectly matched. The mode-unmatched portion stands for the interval where the asynchronous switching happens. In addition, \bar{k}'_j satisfies

$$\bar{k}'_j = k'_j + d_j, j \in \mathbb{Z}^+, \quad (4.3)$$

where $d_j \in [0, \bar{d}]$, $d_j \in \mathbb{N}$, represents the length of the j th stage switching signal detection delay; $d_j = 0$ stands for the mode-matched situation; $\bar{d} \geq 1$, $\bar{d} \in \mathbb{Z}^+$, indicates the length of the upper bound of the detection delay. In the following statement, $d \in [0, \bar{d}]$, $d \in \mathbb{N}$, is used to denote the length of detection delay if no specific stage is mentioned.

To have a concrete description of the mode-unmatched portion, we also have the following assumption.

Assumption 4.1. *The length of the mode-unmatched portion is upper bounded by \bar{d} and it has the identical magnitude order with the length of MDT. To simplify the problem setup, we also assume that there are no switches in the interval $[k'_j, k'_j + d_j)$ for $d_j \in [1, \bar{d}]$. In addition, both the ideal switching instants k'_j and the practical switching instants \bar{k}'_j coincide with the sampling instant k .*

Remark 4.1. *Assumption 4.1 ensures that all the switches can be detected in chronological order and there exists an upper bound on the period of asynchronous switching for all the switches. In real practice, the value of \bar{d} can be obtained by measuring the duration for the switching signal to be transmitted from the plant side to the controller side by using stopwatches or utilizing an oscilloscope directly.*

In the following, an admissible MDT switching sequence in the presence of the asynchronous switching is defined.

Definition 4.3. *A switching sequence with the MDT set T , ideal switching instants $k'_0, k'_1, \dots, k'_j, \dots$, and practical switching instants $\bar{k}'_0, \bar{k}'_1, \dots, \bar{k}'_j, \dots$, $j \in \mathbb{N}$ is said to be admissible in the presence of asynchronous switching, if the inequality $k'_j - \bar{k}'_{j-1} \geq \tau_q, j \in \mathbb{Z}^+$ holds for all $q \in \mathcal{Q}$, $d \in [0, \bar{d}]$. This sequence on the interval $[0, k - 1]$ is represented as $\zeta_{T[\bar{d}]}(k) := \{\sigma(0), \sigma(1), \dots, \sigma(k - 1)\}$.*

4.2.2 Problem Formulation

In this section, the switched MPC problem is introduced first and then the objectives for the design of subsequent feasibility and stability strategies are presented. Define the cost function $J_q(\cdot)$ of each subsystem $q \in \mathcal{Q}$ at time instant k as follows

$$J_q(x_k, \hat{\mathbf{u}}_k) := \sum_{i=0}^{N_q-1} \ell_q(\hat{x}_{k+i|k}, \hat{u}_{k+i|k}) + F_q(\hat{x}_{k+N_q|k}), \quad (4.4)$$

where $x_k = \hat{x}_{k|k}$ is the initial state of the predicted state sequence at time instant k and $\hat{\mathbf{u}}_k := \{\hat{u}_{k|k}, \hat{u}_{k+1|k}, \dots, \hat{u}_{k+N_q-1|k}\}$ is the predicted control sequence; $\hat{x}_{k+i|k}$ and $\hat{u}_{k+i|k}$ represent the $(k+i)$ th step predicted state and control input obtained by solving the optimization problem at time k ; $N_q \in \mathbb{Z}^+$ denotes the prediction horizon with respect to mode $q \in \mathcal{Q}$. The stage cost and terminal cost functions of mode q are respectively given as $\ell_q := \|\hat{x}_{k+i|k}\|_{Q_q}^2 + \|\hat{u}_{k+i|k}\|_{R_q}^2$ and $F_q := \|\hat{x}_{k+N_q|k}\|_{P_q}^2$ where $Q_q \succ 0$ and $R_q \succ 0$ are mode-dependent weighting matrices of ℓ_q while $P_q \succ 0$ is the mode-dependent terminal weighting matrix, $q \in \mathcal{Q}$. The following shows the switched MPC problem at time k

$$\hat{\mathbf{u}}_k^* = \arg \min_{\hat{\mathbf{u}}_k} J_{\sigma(k)}(x_k, \hat{\mathbf{u}}_k) \quad (4.5a)$$

$$\text{s.t. } \hat{x}_{i+1+k|k} = A_{\sigma(k)}\hat{x}_{i+k|k} + B_{\sigma(k)}\hat{u}_{i+k|k}, i \in \mathbb{N}_{[0, N_{\sigma(k)}-1]} \quad (4.5b)$$

$$\hat{x}_{k|k} = x_k \quad (4.5c)$$

$$\hat{x}_{k+i|k} \in \mathcal{X}_{\sigma(k)}, i \in \mathbb{N}_{[0, N_{\sigma(k)}-1]} \quad (4.5d)$$

$$\hat{u}_{k+i|k} \in \mathcal{U}_{\sigma(k)}, i \in \mathbb{N}_{[0, N_{\sigma(k)}-1]} \quad (4.5e)$$

$$\hat{x}_{k+N_{\sigma(k)}|k} \in \mathcal{T}_{\sigma(k)} \quad (4.5f)$$

where $\hat{\mathbf{u}}_k^* := \{\hat{u}_{k|k}^*, \hat{u}_{k+1|k}^*, \dots, \hat{u}_{k+N_q-1|k}^*\}$ is the optimal control sequence and only the first element of $\hat{\mathbf{u}}_k^*$ is implemented as the optimal predicted control input, i.e.,

$$u_q^{\text{MPC}}(x_k) := \hat{u}_{k|k}^*. \quad (4.6)$$

The $\mathcal{T}_q \subseteq \mathcal{X}_q$, $q \in \mathcal{Q}$, denotes the terminal set with respect to the detected mode. For notational convenience, we also define the feasible region $\mathcal{X}_q^{N_q}$ associated with mode $q \in \mathcal{Q}$ as

$$\begin{aligned} \mathcal{X}_q^{N_q} := \{ & x_k \in \mathcal{X}_q | \hat{x}_{k+i|k} \in \mathcal{X}_q, \hat{u}_{k+i|k} \in \mathcal{U}_q, \hat{\mathbf{u}}_k \neq \emptyset, \\ & \hat{x}_{k|k} = x_k, \hat{x}_{k+N_q|k} \in \mathcal{T}_q, i \in \mathbb{N}_{[0, N_q-1]} \}. \end{aligned} \quad (4.7)$$

Remark 4.2. *Note that the asynchronous switching is undesired in practice. We do not aim to design the switched MPC controller for the purpose of alleviating the detrimental effect of asynchronous switching, as in the most existing results on this particular case; instead, in this chapter, we utilize another important control tool to keep the system performance, the switching logic, i.e., the dwell time is designed to*

be extended when the asynchronous switchings occur. As a result, the system performance would not be sacrificed to a great extent due to the relatively short asynchronous duration. In addition, the slow switching is the key focus of the control design in (4.5). Hence, the prediction scheme based on the currently activated subsystem is employed.

To ensure the asymptotic stability of the closed-loop system for the special case that the switched system (4.1) stays in one single mode q , $q \in \mathcal{Q}$, in this chapter, we make the following assumption.

Assumption 4.2. *The subsystem matrices pairs $\{A_q, B_q\}$ are stabilizable for all $q \in \mathcal{Q}$. Assuming there exists a mode-dependent feedback control law $\mu_q(\cdot)$, $q \in \mathcal{Q}$, such that the corresponding terminal set \mathcal{T}_q is control invariant regarding the closed-loop system $x_{k+1} = A_q x_k + B_q \mu_q(x_k)$. In addition, the control input obeys $u_k = \mu_q(x_k) \in \mathcal{U}_q$ for $\forall x \in \mathcal{T}_q$ and the following inequality holds*

$$F_q(A_q x + B_q \mu_q(x)) - F_q(x) + \ell_q(x, \mu_q(x)) \preceq 0. \quad (4.8)$$

Remark 4.3. *Assumption 4.2 is a commonly used assumption for analyzing closed-loop stability under the MPC control law [79]. Inequality (4.8) guarantees that the feasible set $\mathcal{X}_q^{N_q}$ for the subsystem q , $q \in \mathcal{Q}$, in closed-loop with the control law (4.6) is a region of attraction.*

The objectives of this section are presented: (i) To determine the lower bound of MDT such that (4.5) is continuously solvable with asynchronous switching; (ii) To compute the admissible stage MDT such that the resulting closed-loop system is asymptotically stable. The skeleton of the following theoretical analysis is shown in Figure 4.2.

4.3 Feasibility Results

The terminology *persistent feasibility* refers to that the switched MPC problem (4.5) is continuously solvable at both switching instants or within a mode [126]. Note that the initial state $x_k \in \mathcal{X}_q^{N_q}$, $q \in \mathcal{Q}$, only ensures the feasibility of the switched MPC problem (4.5) without any mode transitions. The persistent feasibility of the switched MPC problem (4.5) is not guaranteed since the occurrence of a switch may result in the system state being excluded from the feasible region of the target mode, e.g., $\mathcal{X}_l^{N_l}$, $\forall l \neq q, l \in \mathcal{Q}$. What is even worse, the mode-unmatched behavior caused by the

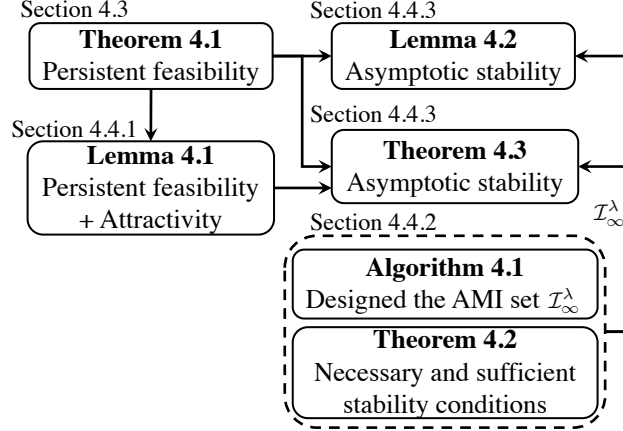


Figure 4.2: Schematic illustrating the relation of the presented algorithms and theories.

asynchronous switching may also lead to infeasibility for the switched MPC problem (4.5).

Hence, to ensure the persistent feasibility of the switched MPC problem (4.5) under the asynchronous switching, in this chapter, we employ the technique of reachable sets computation. Comparing with [126, 128], the set-theoretic-based techniques that we adopted include both synchronous and asynchronous reachable set computation so as to drive the state trajectory in the presence of asynchronous switching into the target region. The definition of the one-step reachable set along the subsystem $q \in \mathcal{Q}$ is recalled [126]

$$\mathcal{R}_q^1(\mathcal{Z}) := \{x | x_0 \in \mathcal{Z}, x = A_q x_0 + B_q u_q^{\text{MPC}}(x_0)\}, \quad (4.9)$$

and the S -step reachable set is given as

$$\mathcal{R}_q^s(\mathcal{Z}) := \mathcal{R}_q^1(\mathcal{R}_q^{s-1}(\mathcal{Z})), \quad s \in \mathbb{Z}_{[1,S]}^+, \quad (4.10)$$

with $\mathcal{R}_q^0(\mathcal{Z}) = \mathcal{Z}$. Considering the mode-unmatched portion, the one-step asynchronous reachable set when a mode transition occurs from mode q to mode l , $q \neq l$, $q \in \mathcal{Q}$, $l \in \mathcal{Q}$, is defined as

$$\mathcal{D}_{q,l}^1(\mathcal{Z}) := \{x | x_0 \in \mathcal{Z}, x = A_l x_0 + B_l u_q^{\text{MPC}}(x_0)\}. \quad (4.11)$$

Likewise, the S -step asynchronous reachable set is given as

$$\mathcal{D}_{q,l}^s(\mathcal{Z}) := \mathcal{D}_{q,l}^1(\mathcal{D}_{q,l}^{s-1}(\mathcal{Z})), \quad s \in \mathbb{Z}_{[1,S]}^+, \quad (4.12)$$

with $\mathcal{D}_{q,l}^0(\mathcal{Z}) = \mathcal{Z}$.

Theorem 4.1. *Consider the switched MPC problem (4.5) under asynchronous switching with $\mathcal{X}_q^{N_q}$, $\forall q \in \mathcal{Q}$, being the feasible region of subsystem q . Suppose that Assumptions 4.1 and 4.2 hold, then the switched MPC problem (4.5) for system (4.1) is persistently feasible, if the MDT $\tau_q^{(j)} \equiv \tau_q$, $j \in \mathbb{Z}^+$, where τ_q fulfills*

$$\bigcup_{l \in \mathcal{Q}, l \neq q} \bigcup_{i=1}^{i=\bar{d}-1} \mathcal{D}_{q,l}^i(\mathcal{R}_q^{\tau_q}(\mathcal{X}_q^{N_q})) \subseteq \mathcal{X}_q^{N_q}, \quad \bar{d} \geq 2, \quad (4.13)$$

and τ_q also satisfies

$$\begin{cases} \mathcal{R}_q^{\tau_q}(\mathcal{X}_q^{N_q}) \subseteq \bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{X}_l^{N_l}, \quad d = 0 \\ \bigcup_{l \in \mathcal{Q}, l \neq q} \mathcal{D}_{q,l}^{\bar{d}}(\mathcal{R}_q^{\tau_q}(\mathcal{X}_q^{N_q})) \subseteq \bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{X}_l^{N_l}, \quad \bar{d} \geq 1. \end{cases} \quad (4.14)$$

Proof. As in Remark 4.3, the feasible set $\mathcal{X}_q^{N_q}$ is positively invariant if the switched system (4.1) in the closed-loop stays in mode q , $q \in \mathcal{Q}$, i.e., $\mathcal{R}_q^r(\mathcal{X}_q^{N_q}) \subseteq \mathcal{X}_q^{N_q}$, $r \in \mathbb{Z}^+$. Hence, $\mathcal{R}_q^{\tau_q}(\mathcal{X}_q^{N_q}) \subseteq \mathcal{X}_q^{N_q}$ can be guaranteed. For $d = 0$, following [126], the reachable set is also required to be contained in the intersection of the feasible sets $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{X}_l^{N_l}$. Then we have $\mathcal{R}_q^{k_j - k_{j-1}}(\mathcal{X}_q^{N_q}) \subseteq \mathcal{R}_q^{\tau_q}(\mathcal{X}_q^{N_q}) \subseteq \bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{X}_l^{N_l}$, $j \in \mathbb{Z}^+$, due to the fact that $k_j - k_{j-1}$ is no less than τ_q . However, in the presence of the asynchronous switching, the mode-unmatched switched MPC controller based on the last activated subsystem $q \in \mathcal{Q}$ still performs in the period of $k \in [k'_j, \bar{k}'_j)$, $j \in \mathbb{Z}^+$, and the predicted control input is implemented in the ideal switching subsystem. Therefore, the asynchronous reachable set is employed to ensure that the evolution of states by using the mode-unmatched controller are contained in the feasible region $\mathcal{X}_q^{N_q}$, $q \in \mathcal{Q}$, when $k \in [k'_j, \bar{k}'_j)$, $j \in \mathbb{Z}^+$, i.e., (4.13). In addition, the reachable set needs to be included in the target region $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{X}_l^{N_l}$ so as to guarantee the switched MPC optimization problem to be persistently feasible when the mode-matched controller is reactivated at instant $k = \bar{k}'_j$, $j \in \mathbb{Z}^+$. Thus, the MDT τ_q , $q \in \mathcal{Q}$, is required to be designed sufficiently long to satisfy both (4.13) and (4.14). ■

Remark 4.4. *Theorem 4.1 provides a strategy to compute the minimum admissible MDT τ_q , $q \in \mathcal{Q}$, to guarantee the switched MPC problem (4.5) to be persistently feasible under the detrimental effect of asynchronous switching. However, such an approach does not guarantee the convergence of all the state trajectories within the union of all feasible regions $\bigcup_{q \in \mathcal{Q}} \mathcal{X}_q^{N_q}$, $q \in \mathcal{Q}$, because the evolution of states may remain in the feasible set. The convergence analysis of system states will be discussed in the following section.*

4.4 Stability Results

In this section, the stability analysis is conducted on the switched MPC problem (4.5) in the presence of asynchronous switching. Initially, a direct criterion with guaranteed convergence characteristics of system states is proposed. Then, in order to design the common terminal set for all subsystems, an algorithm to compute a constraint admissible contractive AMI set is introduced. Finally, two approaches based on the designed AMI set to calculate the admissible value of MDT are presented. The closed-loop system under the asynchronous switching is asymptotically stable as long as the computed MDT restrictions are satisfied.

4.4.1 A Direct Criterion

Lemma 4.1. *Consider the switched MPC problem (4.5) in the presence of asynchronous switching with $\mathcal{X}_q^{N_q}$, $\forall q \in \mathcal{Q}$, being the feasible region of subsystem q . Suppose Assumptions 4.1 and 4.2 hold, the closed-loop system is attractive in the feasible region $\bigcup_{q \in \mathcal{Q}} \mathcal{X}_q^{N_q}$, if the MDT $\tau_q^{(1)}$ fulfills (4.13) and (4.14) and $\tau_q^{(j)}$, $j \in \mathbb{Z}_{[2,+\infty)}^+$, fulfills*

$$\left\{ \begin{array}{l} \bigcup_{n=0}^{\tau_q^{(1)} - \tau_q^{(j)}} \mathcal{R}_q^{\tau_q^{(j)} + n} \left(\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-2}(\mathcal{X}_l^{N_l}) \right) \subseteq \mathcal{X}_q^{N_q}, \bar{d} = 1 \\ \bigcup_{l \in \mathcal{Q}, l \neq q} \bigcup_{i=1}^{i=\bar{d}-1} \mathcal{D}_{q,l}^i \left(\bigcup_{n=0}^{\tau_q^{(1)} - \tau_q^{(j)}} \mathcal{R}_q^{\tau_q^{(j)} + n} \left(\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-2}(\mathcal{X}_l^{N_l}) \right) \right) \\ \subseteq \mathcal{X}_q^{N_q}, \bar{d} \geq 2 \end{array} \right. \quad (4.15)$$

and $\tau_q^{(j)}$, $j \in \mathbb{Z}_{[2,+\infty)}^+$, also satisfies

$$\left\{ \begin{array}{l} \bigcup_{n=0}^{\tau_q^{(1)} - \tau_q^{(j)}} \mathcal{R}_q^{\tau_q^{(j)} + n} \left(\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-2}(\mathcal{X}_l^{N_l}) \right) \subseteq \bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l}), \\ \\ \bigcup_{l \in \mathcal{Q}, l \neq q} \mathcal{D}_{q,l}^{\bar{d}} \left(\bigcup_{n=0}^{\tau_q^{(1)} - \tau_q^{(j)}} \mathcal{R}_q^{\tau_q^{(j)} + n} \left(\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-2}(\mathcal{X}_l^{N_l}) \right) \right) \\ \subseteq \bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l}), \bar{d} \geq 1. \end{array} \right. \quad d = 0 \quad (4.16)$$

Proof. (i) Persistent feasibility: Note that $\tau_q^{(1)}$, $q \in \mathcal{Q}$, fulfills (4.13) and (4.14). It indicates that the state trajectory enters the feasible set $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{X}_l^{N_l}$. The switched MPC problem (4.5) is therefore solvable at the first stage. When the stage index reaches $j \in \mathbb{Z}_{[2,+\infty)}^+$, the reachable set is forced within $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l})$. Due to $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l}) \subseteq \bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{X}_l^{N_l}$, the persistent feasibility of the problem in (4.1) is ensured. (ii) Attractivity: Based on Assumption 4.2, there is no doubt that $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l}) \rightarrow \{\mathbf{0}\}$ as the number of stages $j \rightarrow \infty$ since the inequality (4.8) guarantees that the system states converge to the origin for only a single subsystem q , $q \in \mathcal{Q}$, in closed-loop. Hence, $\|x_{k_j}\| \rightarrow 0$ as $j \rightarrow \infty$ holds which conforms to the definition of attractivity. \blacksquare

Remark 4.5. *It can be observed that the target region $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l})$ in (4.16) contracts one step innerly as j , $j \in \mathbb{Z}_{[2,+\infty)}^+$, increases by one stage. The attractive property of the reachable sets is therefore achieved. It is also worth pointing out that the attractive rate of the target region design can be accelerated by requiring the contraction with every two steps or more.*

Remark 4.6. *As mentioned in Theorem 4.1, the computed $\tau_q^{(1)}$, $q \in \mathcal{Q}$, by using (4.13) and (4.14) is the minimum value of MDT to ensure the switched MPC design under the asynchronous switching to be persistently feasible. Thus, when the value of MDT $\tau_q^{(1)}$ is greater than $\tau_q^{(j)}$, $j \in \mathbb{Z}_{[2,+\infty)}^+$, the calculation of the reachable set with all the possible steps for $\tau_q^{(j)} \leq \tau_q^{(1)}$ needs to be considered for the feasibility concern.*

Lemma 4.1 provides a way to compute the minimum admissible MDT offline in each stage for the attractivity property of the closed-loop system. However, it is not

realistic to consider infinite numbers of switching stages. Hence, in the following two subsections, a more practical way to determine the minimum value of MDT within finite switching stages is proposed.

4.4.2 Computational Algorithm

Before introducing the definition of a constraint admissible contractive AMI set, the constrained switching mode-matched closed-loop system by using the feedback control law $u_k = K_q x_k, q \in \mathcal{Q}$, is firstly defined

$$x_{k+1} = \bar{A}_q x_k, q \in \mathcal{Q}, x_k \in \mathcal{X}, \quad (4.17)$$

where $\bar{A}_q := A_q + B_q K_q$ and $\bar{A}_q \in \bar{\mathcal{A}} := \{\bar{A}_1, \bar{A}_2, \dots, \bar{A}_Q\}$; K_q denotes the feedback gain with respect to the subsystem $q, q \in \mathcal{Q}$, and $\mathcal{X} \subseteq \mathbb{R}^n$ denotes the state constraint. The constrained switching mode-unmatched closed-loop system with the feedback control law $u_k = K_q x_k, q \in \mathcal{Q}$, is given as

$$x_{k+1} = \bar{A}_{q,l} x_k, l \neq q, q \in \mathcal{Q}, l \in \mathcal{Q}, x_k \in \mathcal{X}, \quad (4.18)$$

where $\bar{A}_{q,l} := A_l + B_l K_q$; $\bar{A}_{q,l} \in \bar{\mathcal{A}}_q := \{\bar{A}_{q,1}, \bar{A}_{q,2}, \dots, \bar{A}_{q,Q}\}$ and $\bar{\mathcal{A}} := \bigcup_{q \in \mathcal{Q}} \bar{\mathcal{A}}_q$ represents the union of asynchronous system matrices. The system dynamics (4.18) indicates that the asynchronous switching control law performs in the currently activated switching subsystem $l, l \in \mathcal{Q}$.

Definition 4.4. A set $\Omega \subseteq \mathbb{R}^n$ is said to be a constraint admissible contractive AMI set with respect to a contraction factor $\lambda \in (0, 1)$, the MDT set $\Gamma := \{\gamma_1, \gamma_2, \dots, \gamma_Q\}$, and the closed-loop systems (4.17) and (4.18), if $x_0 \in \Omega$ implies $x_t \in \mathcal{X}$, for $t \in \mathbb{Z}_{[1,k]}^+$ and $\bar{A}_{\zeta_{T[\bar{d}]}(k)} x_0 \in \lambda \Omega$ where the product of the closed-loop system matrices associated with the switching sequence $\zeta_{T[\bar{d}]}(k)$ is

$$\begin{aligned} \bar{A}_{\zeta_{T[\bar{d}]}(k)} &:= \bar{A}_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} \bar{A}_{\sigma(k'_{j-1})}^{t^{(j)}} \cdots \bar{A}_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \bar{A}_{\sigma(k'_1)}^{t^{(2)}} \\ &\quad \bar{A}_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \bar{A}_{\sigma(k'_0)}^{t^{(1)}}, \end{aligned}$$

where $d_j \in [0, \bar{d}]$, $j \in \mathbb{Z}^+$; $t^{(j)}, j \in \mathbb{Z}^+$, represents the stage MDT and $k = \sum_{v=1}^j t^{(v)} + \sum_{v=1}^j d_v, v \in \mathbb{Z}^+$.

Based on Definition 4.4, we aim to modify the algorithm which is designed to

compute the MDT constraint admissible contractive invariant set for all the admissible switching sequences in [137]. The purpose is to make it compatible with the closed-loop systems (4.17) and (4.18) by considering all the admissible switching sequences defined in Definition 4.3 under the asynchronous switching.

The invariant set computation is proceeded in two stages. Let the first stage MDT set be $\tilde{\mathcal{T}}_q := \{1, 2, \dots, \gamma_q - 1\}$, $\gamma_q \in \Gamma$, $q \in \mathcal{Q}$, with $\tilde{\mathcal{T}} := \bigcup_{q \in \mathcal{Q}} \tilde{\mathcal{T}}_q$. The second stage MDT set is denoted as $\mathcal{T}_q := \{\gamma_q, \gamma_q + 1, \dots, 2\gamma_q - 1\}$, $\gamma_q \in \Gamma$, $q \in \mathcal{Q}$, with $\mathcal{T} := \bigcup_{q \in \mathcal{Q}} \mathcal{T}_q$. The reason behind such a division is that the truncated behavior of the switching sequence is demonstrated by using the MDT in the first stage. The evolution of system states under the admissible switching sequence can be manifested with the MDT in the second stage [142].

In order to establish the AMI set, the concept of backward sets is employed. Define a r -step backward set for the mode-matched closed-loop system (4.17) as

$$\theta_q^r(\Xi) := \{x | \bar{A}_q^r x \in \Xi, q \in \mathcal{Q}\}, \quad (4.19)$$

where the number of steps $r \in \mathbb{N}$ with $\theta_q^0(\Xi) = \{x | x \in \Xi\}$. Then, the intersection of the r step backward set for every $r \in \tilde{\mathcal{T}}_q$ is characterized as $\tilde{\Theta}_q(\Xi) := \bigcap_{r \in \tilde{\mathcal{T}}_q} \theta_q^r(\Xi)$. Likewise, the intersection of the reachable set $\theta_q^r(\Xi)$ for every $r \in \mathcal{T}_q$ is denoted as $\Theta_q(\Xi) := \bigcap_{r \in \mathcal{T}_q} \theta_q^r(\Xi)$. For the mode-unmatched closed-loop system (4.18), the i -step backward set is represented as

$$\hat{\theta}_{q,l}^i(\Xi) := \{x | \bar{A}_{q,l}^i x \in \Xi, l \neq q, q \in \mathcal{Q}, l \in \mathcal{Q}\}, \quad (4.20)$$

where the number of steps $i \in \mathbb{N}$ with $\hat{\theta}_{q,l}^0(\Xi) = \{x | x \in \Xi\}$. In addition, we define the asynchronous set which collects all possible length of mode-unmatched portion as $\hat{\mathcal{T}} := \{0, 1, 2, \dots, \bar{d}\}$.

In the following, the computational algorithm of designing a constraint admissible contractive AMI set is presented in Algorithm 4.1.

The set \mathcal{I}_0^λ computed in step 1 collects all the possible state trajectories under the truncated switching sequence for every $r \in \tilde{\mathcal{T}}_q$. The target region that the system states finally converge into is the intersection of the terminal sets for all modes, i.e., $\bigcap_{q \in \mathcal{Q}} \mathcal{T}_q$. The design of steps 2 to 6 considers the backward evolution of system states of the closed-loop systems (4.17) and (4.18) in the presence of asynchronous switching. For every $r \in \mathcal{T}_q$, $q \in \mathcal{Q}$, the operator Θ_q is applied to calculate the intersection of

Algorithm 4.1 Computation of a constraint admissible contractive AMI set

Input: Terminal sets \mathcal{T}_q , $q \in \mathcal{Q}$; system matrices sets $\bar{\mathcal{A}}$ and $\bar{\mathcal{S}}$; upper bound \bar{d} ; contraction factor λ ; MDT sets \mathcal{T} and $\tilde{\mathcal{T}}$.

- 1: Initialize $m = 0$ and $d = 0$ and calculate the initial set $\mathcal{I}_0^\lambda := (\bigcap_{q \in \mathcal{Q}} \mathcal{T}_q) \cap (\bigcap_{q \in \mathcal{Q}} \tilde{\Theta}_q(\bigcap_{q \in \mathcal{Q}} \mathcal{T}_q))$;
- 2: **while** $d \leq \bar{d}$
- 3: Compute $\Xi = \bigcap_{q \in \mathcal{Q}} (\bigcap_{l \in \mathcal{Q}, l \neq q} \Theta_q(\hat{\theta}_{q,l}^d(\lambda \mathcal{I}_m^\lambda)))$;
- 4: Calculate $\mathcal{I}_{m+1}^\lambda = \mathcal{I}_m^\lambda \cap \Xi$;
- 5: Set $d = d + 1$;
- 6: **end while**
- 7: **if** $\mathcal{I}_{m+1}^\lambda \equiv \mathcal{I}_m^\lambda$ **then**
- 8: Set $\mathcal{I}_\infty^\lambda = \mathcal{I}_m^\lambda$ and **Stop**;
- 9: **else**
- 10: Set $m = m + 1$ and return to **step 2**;
- 11: **end if**
- 12: **Output:** An AMI set $\mathcal{I}_\infty^\lambda$.

the backward sets for all possible mode transitions. Furthermore, such an identical procedure will be applied to every mode for $q \in \mathcal{Q}$. Once the condition in step 7 is satisfied, a constrained contractive AMI set is obtained.

Theorem 4.2. *Suppose Assumptions 4.1-4.2 are satisfied. Then, under the asynchronous switching, the closed-loop system (4.1) by using the feedback control laws in (4.17) and (4.18) is asymptotically stable under the MDT $\mathcal{O} := \{o_1, o_2, \dots, o_q, \dots, o_Q\}$ if and only if the constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ generated by Algorithm 4.1 under an MDT set \mathcal{O} and a contraction factor $\lambda \in (0, 1)$ is non-empty.*

Proof. (i) Sufficiency: Consider an admissible MDT switching sequence $\zeta_{T[\bar{d}]}(k)$. The evolution of the system state is $x_k = \bar{A}_{\zeta_{T[\bar{d}]}(k)} x_0$. The existence of such a constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ indicates that $\forall x_0 \in \mathcal{I}_\infty^\lambda$, the evolution of state $x_k = \bar{A}_{\zeta_{T[\bar{d}]}(k)} x_0 \in \lambda^\mu \rho_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \rho_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \cdots \rho_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} \mathcal{I}_\infty^\lambda$, $\mu := \lfloor \frac{t^{(1)}}{\tau_{\sigma(k'_0)}} \rfloor + \lfloor \frac{t^{(2)}}{\tau_{\sigma(k'_1)}} \rfloor + \cdots + \lfloor \frac{t^{(j)}}{\tau_{\sigma(k'_{j-1})}} \rfloor$; $\rho_{\sigma(k'_{j-1}), \sigma(k'_j)}$, $j \in \mathbb{Z}^+$, denotes the maximal Euclidean norm of all eigenvalues with respect to system matrices $\bar{A}_{\sigma(k'_{j-1}), \sigma(k'_j)}$. In order to prove the above statement, it requires that the inequality $\lambda^\mu \rho_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \rho_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \cdots \rho_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} < 1$ holds. Note that we have no additional assumptions requiring the asynchronous system matrices to be stable, i.e., $\rho_{\sigma(k'_{j-1}), \sigma(k'_j)} \geq 1$. This may result in the divergence of the state trajectory. However, according to Assumption 4.2, we have $\rho(\bar{A}_q) < 1$, $q \in \mathcal{Q}$. Thus, λ^μ can be designed sufficiently small since the admissible MDT is the

decision variable. Due to $\lambda \in (0, 1)$, therefore, as $k \rightarrow \infty$, $\mu \rightarrow \infty$, which ensures that the system states converge to the origin, i.e., $\|x_k\| \rightarrow 0$.

The stability proof is given as below. Since the origin is contained in the $\cap_{q \in \mathcal{Q}} \mathcal{T}_q$, we have

$$\delta' \mathcal{B} \subseteq \mathcal{I}_\infty^\lambda \subseteq \cap_{q \in \mathcal{Q}} \mathcal{T}_q \subseteq \epsilon' \mathcal{B},$$

where $\delta' > 0$ and $\epsilon' > 0$ are two positive constants. \mathcal{B} denotes a unit ball in \mathbb{R}^n . Given two constants $\epsilon \geq \epsilon'$ and $\delta \in (0, \delta']$, we have $\|x_0\| \leq \delta$ which indicates that $x_k \in \mathcal{I}_\infty^\lambda \subseteq \epsilon' \mathcal{B} \subseteq \epsilon \mathcal{B}$. Hence, $\|x_k\| \leq \epsilon$ holds. For $\epsilon < \epsilon'$, due to the contractive property of $\mathcal{I}_\infty^\lambda$, there must exist a series of parameters $\lambda^\mu \rho_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \rho_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \cdots \rho_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j}$, $j \in \mathbb{Z}^+$, such that $\lambda^\mu \rho_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \rho_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \cdots \rho_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} \epsilon' < \epsilon$. Likewise, given

$$\delta \in (0, \lambda^\mu \rho_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \rho_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \cdots \rho_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} \delta'] ,$$

we have $\|x_0\| \leq \delta$ which implies that

$$\begin{aligned} x_k &\in \lambda^\mu \rho_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \rho_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \cdots \rho_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} \mathcal{I}_\infty^\lambda \\ &\subseteq \lambda^\mu \rho_{\sigma(k'_0), \sigma(k'_1)}^{d_1} \rho_{\sigma(k'_1), \sigma(k'_2)}^{d_2} \cdots \rho_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} \epsilon' \mathcal{B} \subseteq \epsilon \mathcal{B}, \end{aligned}$$

i.e., $\|x_k\| \leq \epsilon$. Thus, $\forall \epsilon > 0$, there exists a $\delta > 0$ such that $\|x_0\| \leq \delta$ implies that $\|x_k\| \leq \epsilon$. Hence, the asymptotic stability of closed-loop system (4.1) is achieved.

(ii) Necessity: Define the operation $\|\cdot\|_{\mathcal{I}_\infty^\lambda}$ as the norm induced by the AMI set $\mathcal{I}_\infty^\lambda$. In addition, there exists a parameter $\bar{\lambda}_\rho \in (0, 1)$ such that $\|\bar{A}_{\sigma(k'_{j-1}), \sigma(k'_j)}^{d_j} \bar{A}_{\sigma(k'_j), \sigma(k'_{j-1})}^{d_j}\| < \bar{\lambda}_\rho < 1$, $\forall d_j \in \hat{\mathcal{T}}$, $j \in \mathbb{Z}^+$. Then, we reformulate the set \mathcal{I}_0^λ in the form of $\{x | G_m x \leq \mathbf{1}, m \in \mathbb{N}\}$. In the following, inspired by [142], we will show that the constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ can be obtained for a sufficiently large index m . Considering the set evolution of Algorithm 4.1 from $\mathcal{I}_{m-1}^\lambda$ to \mathcal{I}_m^λ with any $\lambda_\rho > \bar{\lambda}_\rho$, $\lambda_\rho \in (0, 1)$, the additional inequality $G_m \bar{A}_{\zeta_{T[\bar{d}]}}(\bar{k}'_m) x \leq \lambda_\rho^m$ is required. The subsequent inequalities are derived along the m th step iteration of Algorithm 4.1

$$\begin{aligned} G_m \bar{A}_{\zeta_{T[\bar{d}]}}(\bar{k}'_m) x &\leq \max_{a \in \mathcal{B}(\|\bar{A}_{\zeta_{T[\bar{d}]}}(\bar{k}'_m) x\|)} G_m a \\ &\leq \max_{b \in \mathcal{B}(\|\bar{A}_{\zeta_{T[\bar{d}]}}(\bar{k}'_m) x\|_{\mathcal{I}_\infty^\lambda})} \beta_1 G_m b \leq \max_{b \in \mathcal{B}(\bar{\lambda}_\rho^m \|x\|_{\mathcal{I}_\infty^\lambda})} \beta_1 G_m b \\ &\leq \max_{\tilde{a} \in \mathcal{B}(\|x\|)} \bar{\lambda}_\rho^m \beta_1 \beta_2 G_m \tilde{a} \leq \bar{\lambda}_\rho^m \beta_1 \beta_2 \|G_m\| \|x\| < \lambda_\rho^m, \end{aligned}$$

where β_1 and β_2 are two positive real numbers. The last inequality holds since $\bar{\lambda}_\rho < \lambda_\rho < 1$. Hence, the additional inequality is redundant for $\mathcal{I}_{m-1}^\lambda$ to obtain \mathcal{I}_m^λ since the convergence of the next step iteration is already guaranteed. The proof is completed. \blacksquare

Remark 4.7. *Theorem 4.2 indicates that once the state trajectories enter the constraint admissible contractive AMI set with the admissible switching sequence, the system states will converge to the origin.*

4.4.3 Stability Criteria Based on the AMI Set

By employing the computed constraint admissible contractive AMI set as the common terminal set for all switching modes, two criteria are proposed to calculate the minimum admissible MDT. The first stability criterion is detailed in the following Theorem.

Theorem 4.3. *Consider the switched MPC problem (4.5) in the presence of asynchronous switching with $\mathcal{X}_q^{N_q}$, $\forall q \in \mathcal{Q}$, being the feasible region of subsystem q . Suppose Assumptions 4.1-4.2 hold and the feedback gain within \mathcal{T}_q is K_q , $q \in \mathcal{Q}$. If a constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ exists for closed-loop systems (4.17) and (4.18) with an MDT set \mathcal{O} and a contraction factor $\lambda \in (0, 1)$, then the closed-loop system (4.1) under the control law (4.6) is asymptotically stable regarding the region of attraction $\bigcup_{q \in \mathcal{Q}} \mathcal{X}_q^{N_q}$ with the MDT $\tau_q^{(j)}$, if $\tau_q^{(1)}$ fulfills (4.13) and (4.14) and $\tau_q^{(j)}$, $j \in \mathbb{Z}_{[2,y]}^+$, $y \in \mathbb{Z}_{[2,+\infty)}^+$, satisfies (4.15) and (4.16). Furthermore, for $j \geq y + 1$, the following inclusion relationship*

$$\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l}) \subseteq \mathcal{I}_\infty^\lambda \quad (4.21)$$

holds and $\tau_q^{(j)} \geq o_q$.

Proof. Assume that a constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ exists with an MDT set \mathcal{O} and contraction factor $\lambda \in (0, 1)$. From Lemma 4.1, it follows that $\bigcap_{l \in \mathcal{Q}, l \neq q} \mathcal{R}_l^{j-1}(\mathcal{X}_l^{N_l}) \rightarrow \{\mathbf{0}\}$. Then, a sufficient large y must exist such that (4.21) holds. The proof is stated from the following three aspects. (i) Feasibility: For $j \in \mathbb{Z}_{[2,y]}^+$, Theorem 4.1 and Lemma 4.1 ensure the persistent feasibility by using the admissible MDT $\tau_q^{(j)}$. When $j \geq y + 1$, the state trajectory along the admissible switching sequence will remain in $\mathcal{I}_\infty^\lambda$ as long as the inequality which ensures the

existence of the AMI set $\tau_q^{(j)} \geq o_q$ holds. Based on the iteration procedure of Algorithm 4.1, it implies that the set inclusion relationship $\mathcal{I}_\infty^\lambda \subseteq \bigcap_{q \in \mathcal{Q}} \mathcal{T}_q \subseteq \bigcap_{q \in \mathcal{Q}} \mathcal{X}_q^{N_q}$ stands. Thus, the persistent feasibility can be guaranteed. (ii) Stability: The stability analysis is similar to the proof of Theorem 4.2 and omitted here. (iii) Attractivity: It can be observed from (4.21) that the system states are forced to converge into $\mathcal{I}_\infty^\lambda$ at the practical switching instants \bar{k}'_j , $j \in \mathbb{Z}^+$. Then for $j \geq y + 1$, if $x_{\bar{k}'_y} \in \mathcal{I}_\infty^\lambda$, it implies that $x_k = \bar{A}_{\zeta_{T[\bar{d}]}(k)} x_{\bar{k}'_y} \in \lambda^c \rho_{\sigma(k'_y), \sigma(k'_{y+1})} \cdots \rho_{\sigma(k'_{y+c-1}), \sigma(k'_{y+c})} \mathcal{I}_\infty^\lambda$ where c represents the number of switching stages after \bar{k}'_y till k . Due to $\lambda \in (0, 1)$ and $\lambda^c \rho_{\sigma(k'_y), \sigma(k'_{y+1})} \cdots \rho_{\sigma(k'_{y+c-1}), \sigma(k'_{y+c})} < 1$, we have $\|x_k\| \rightarrow 0$ as $k \rightarrow \infty$. Hence, the asymptotic stability of the closed-loop system is achieved. \blacksquare

Remark 4.8. *In contrast with the criterion presented in Lemma 4.1, the sufficient condition in Theorem 4.3 is more practical since the computation of the admissible stage MDT can be terminated in finite stages as long as (4.21) satisfies.*

Lemma 4.2. *Consider the switched MPC problem (4.5) in the presence of asynchronous switching with $\mathcal{X}_q^{N_q}$, $\forall q \in \mathcal{Q}$, being the feasible region of subsystem q . Suppose Assumptions 4.1-4.2 hold and the feedback gain within \mathcal{T}_q is K_q , $q \in \mathcal{Q}$. If a constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ exists for closed-loop systems (4.17) and (4.18) with an MDT set \mathcal{O} and a contraction factor $\lambda \in (0, 1)$, then the closed-loop system (4.1) under the control law (4.6) is asymptotically stable regarding the region of attraction $\bigcup_{q \in \mathcal{Q}} \mathcal{X}_q^{N_q}$ with the MDT $\tau_q^{(j)}$, $j \in \mathbb{Z}_{[1, +\infty)}^+$, if $\tau_q^{(1)}$ satisfies (4.13) and*

$$\bigcup_{l \in \mathcal{Q}, l \neq q} \bigcup_{d=0}^{d=\bar{d}} \mathcal{D}_{q,l}^d(\mathcal{R}_q^{\tau_q^{(1)}}(\mathcal{X}_q^{N_q})) \subseteq \mathcal{I}_\infty^\lambda, \quad (4.22)$$

and $\tau_q^{(j)} \equiv o_q$, for all $j \in \mathbb{Z}_{[2, +\infty)}^+$.

The proof of Lemma 4.2 is omitted here. Lemma 4.2 requires the reachable set to be contained in $\mathcal{I}_\infty^\lambda$ in the first stage. For the stage j , $j \in \mathbb{Z}_{[2, +\infty)}^+$, the minimum value of the admissible MDT only needs to guarantee the existence of the AMI set.

4.5 Simulation Results

The simulation is conducted by using the MATLAB MPT3 toolbox [143]. To verify our control scheme, we consider a switched discrete-time linear system in the form of

(4.1) with two modes. The two subsystems are

$$A_1 = \begin{bmatrix} 0.5 & -1 \\ 1 & 0.7 \end{bmatrix}, A_2 = \begin{bmatrix} 1.5 & 1 \\ 0.3 & 0.58 \end{bmatrix}, \quad (4.23a)$$

$$B_1 = \begin{bmatrix} 0.4 & 1 \end{bmatrix}^T, B_2 = \begin{bmatrix} -1 & 2.3 \end{bmatrix}^T. \quad (4.23b)$$

The state constraints are $\mathcal{X}_1 := \{x_k \in \mathbb{R}^2 \mid \|x_k\|_\infty \leq 10\}$ and $\mathcal{X}_2 := \{x_k \in \mathbb{R}^2 \mid \|x_k\|_\infty \leq 8\}$. The input constraints are $\mathcal{U}_1 = \mathcal{U}_2 := \{u_k \in \mathbb{R} \mid \|u_k\|_\infty \leq 2\}$. The prediction horizon is set to be $N_1 = N_2 = 5$. The state weighting matrices are $Q_1 = [10 \ 0; 0 \ 10]$ and $Q_2 = [5 \ 0; 0 \ 5]$ while the input weighting matrix is $R_q = 1$, $q = 1, 2$. $P_1 = [11.0973 \ -0.8344; -0.8344 \ 27.7253]$ and $P_2 = [148.6817 \ 110.5895; 110.5895 \ 90.1414]$ are served as the terminal weighting matrices. The stabilizing feedback gains are $K_1 = [-0.9786 \ -0.5219]$, $K_2 = [-1.5917 \ -1.3731]$ and the corresponding eigenvalues of \bar{A}_1 and \bar{A}_2 are $0.1433 \pm 0.1572i$ and $0.5039, 0.0096$, respectively. The upper bound of the mode-unmatched portion caused by asynchronous switching is set to $\bar{d} = 3$ and $d \in [0, \bar{d}]$.

Table 4.1 shows the calculation results of MDT by employing distinct criteria in the presence of asynchronous switching. The minimum values of admissible MDT $\underline{\tau}_q$ and the stage MDT $\underline{\tau}_q^{(j)}$, $q = 1, 2$, $j \in \mathbb{Z}^+$, are obtained by considering all the possible lengths of the mode-unmatched portion for $d \in [0, \bar{d}]$. The MDT calculated by Theorem 4.2 is to ensure the existence of a constraint admissible contractive AMI set. It can be observed from Table 4.1 that the value of $\underline{\tau}_1^{(2)}$ is greater than $\underline{\tau}_1^{(1)}$ by using the criterion of Theorem 4.3. This is because the target region $\mathcal{B}_2^1(\mathcal{X}_2^{N_2}) \subseteq \mathcal{X}_2^{N_2}$ and the system states need to consume more time to reach the target set. Note that the admissible MDT computation based on Lemma 4.1 is required to consider an infinite number of switching stages. Hence, we only demonstrate the simulation result of other criteria which can be determined in a finite computational time.

The asynchronous reachable sets $\mathcal{D}_{1,2}^1$, $\mathcal{D}_{1,2}^2$, and $\mathcal{D}_{1,2}^3$ which are evolved for the mode transition from mode 1 to mode 2 are illustrated in Figure 4.3. The state trajectory with dot marks is plotted by a feasible MDT $\underline{\tau}_1 = 7$ under the mode-unmatched period $d = 3$. The $x_0 = [-3.2; -4.2]$ is chosen as the initial state. It can be observed that after the state trajectory leaves three asynchronous reachable sets, the system states will stay in the feasible region $\mathcal{X}_2^{N_2}$ which indicates that the persistent feasibility is ensured. The state responses of the closed-loop system based on the switching sequences which obey the minimum MDT requirement of Theorem 4.3 and

Table 4.1: The computational results of MDT based on distinct criteria

Theoretical Criteria	Minimum Values of MDT
Theorem 4.1	$\tau_1 = 7, \tau_2 = 8$
Theorem 4.2	$\tau_1 = 3, \tau_2 = 4$
Theorem 4.3	$\tau_1^{(1)} = 7, \tau_2^{(1)} = 8; \tau_1^{(2)} = 8, \tau_2^{(2)} = 8;$ $\tau_1^{(3)} = 7, \tau_2^{(3)} = 7; \tau_1^{(4)} = 6, \tau_2^{(4)} = 6;$ $\tau_1^{(5)} = 5, \tau_2^{(5)} = 6; \tau_1^{(6)} = 5, \tau_2^{(6)} = 5;$ $\tau_1^{(7)} = 5, \tau_2^{(7)} = 4;$ $\tau_1^{(8)} = 3, \tau_2^{(8)} = 4, \forall j \geq 8$
Lemma 4.2	$\tau_1^{(1)} = 8, \tau_2^{(1)} = 9; \tau_1^{(2)} = 3, \tau_2^{(2)} = 4, \forall j \geq 2$

Lemma 4.2 are shown in Figure 4.4 and Figure 4.5, respectively. The initial state to verify Theorem 4.3 is given as $x_{0T3} = [2.79; 4.66]$. The initial state to validate Lemma 4.2 is chosen as $x_{0L2} = [-3.2; -4.2]$. Both of these two initial states are included in the feasible set $\mathcal{X}_1^{N_1}$. The simulation results clearly demonstrate that asynchronous switching leads to the divergence of system states. Hence, both these two criteria play a significant role in asymptotically stabilizing the closed-loop system.

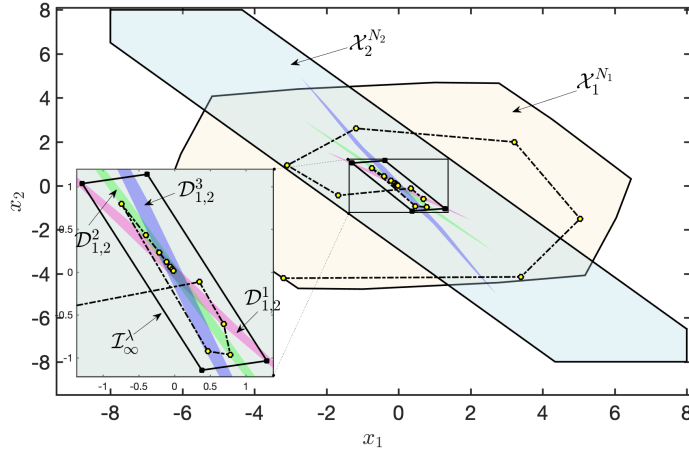


Figure 4.3: Illustration of the asynchronous reachable sets $\mathcal{D}_{1,2}^1$, $\mathcal{D}_{1,2}^2$, and $\mathcal{D}_{1,2}^3$ and a feasible state trajectory.

Figure 4.6 shows the computed constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ by using Algorithm 4.1. The contraction factor is set to $\lambda = 0.999$ and the eigenvalues of asynchronous system matrices $\bar{A}_{1,2}$ and $\bar{A}_{2,1}$ are $0.9291 \pm 0.7536i$ and $-1.3992, 0.5894$, respectively. It is worth knowing that the closed-loop system by using the asynchronous switching control law has not to be stable which is also consistent with the nature of most asynchronous switched systems. The state trajectories 1 to 4 which

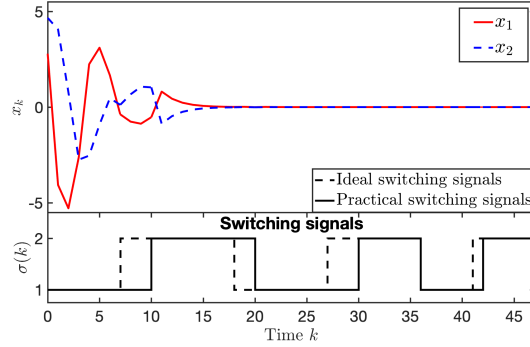


Figure 4.4: State responses which satisfy the MDT constraint in Theorem 4.3.

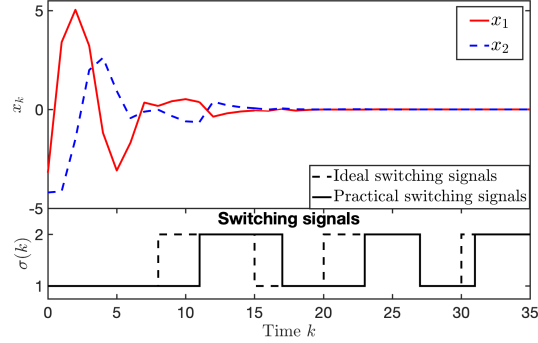


Figure 4.5: State responses which satisfy the MDT constraint in Lemma 4.2.

are denoted by traj1 to traj4 are plotted to intuitively demonstrate the convergence once the states enter the AMI set $\mathcal{I}_\infty^\lambda$. The initial states with respect to traj1 to traj4 are respectively designed to be the four vertices of $\mathcal{I}_\infty^\lambda$: $x_{01} = [1.2911; -1.0354]$, $x_{02} = [-0.3816; 1.145]$, $x_{03} = [0.3816; -1.145]$, and $x_{04} = [-1.2911; 1.0354]$. The corresponding four admissible switching sequences which satisfy the requirement in Definition 4.3 are generated arbitrarily. It can be seen from Figure 4.6 that the system states move out of $\mathcal{I}_\infty^\lambda$ in the first step. The reason for this phenomenon is the truncated behavior of switching sequences with the first stage MDT. By giving the adequate length of admissible MDT, the convergence of state trajectories is guaranteed.

4.6 Conclusions

This chapter provides the first attempt to tackle the asynchronously switched MPC problem by using the MDT approach. The criterion to ensure the persistent feasibility

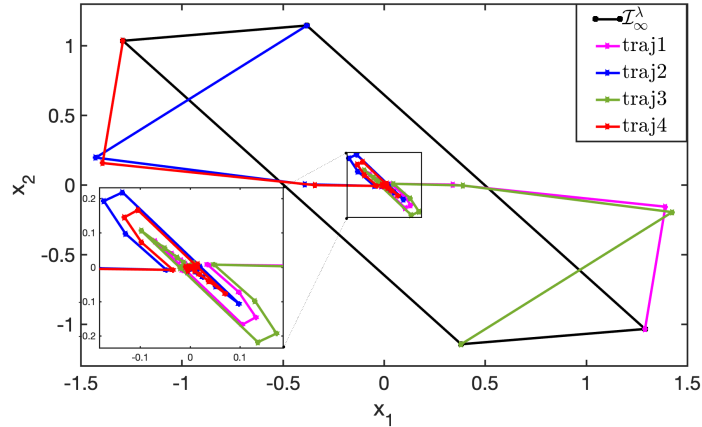


Figure 4.6: Illustration of the constraint admissible contractive AMI set $\mathcal{I}_\infty^\lambda$ and state trajectories.

ity of the switched MPC algorithm has been designed. An algorithm to compute a constraint admissible contractive AMI set has been established. Based on the computed AMI set, a sufficient and necessary condition is proposed with a rigorous proof. Two strategies with respect to the minimum MDT determination are proposed so as to guarantee the asymptotic stability of the closed-loop system. The proposed control scheme has the advantage of reducing the online computational load by determining the MDT restriction offline while preventing significant sacrifice of system performance. Theoretical results are validated and the advantages of the proposed method are highlighted via a numerical example.

Chapter 5

Non-conservative Stability Criteria for Constrained Switched Linear Systems with Multiple Faults

In the previous two chapters, we focus on studying the switched MPC design with or without terminal constraints. In this chapter, we further extend the theoretical results to stabilize the constrained switched system from a set-theoretic perspective.

5.1 Introduction

Considering the possible exposure to the unreliable and changeable industry environment, switched systems may simultaneously encounter *multiple faults*, i.e., lagged mode-dependent controller to be applied to the currently activated mode, controller intermittent disconnection, and unexpected mode transitions disorder. Figure 5.1 shows a schematic diagram of a switched system in a typical stirred-tank reactor under multiple faults. These commonly experienced faults may cause severe adverse effects on reliability and closed-loop stability which makes this issue important yet challenging to be confronted. In the light of the literature review in Chapter 1, some remarkable results have been reported in studying the stabilization problem of switching dynamics in the presence of a single fault [41, 44–52, 54–59, 63–70]. Nonetheless, rare results are devoted to investigating the non-conservative stability strategy for switched systems when multiple faults are encountered concurrently.

Another pivotal issue to address in the field of switched systems is to alleviate

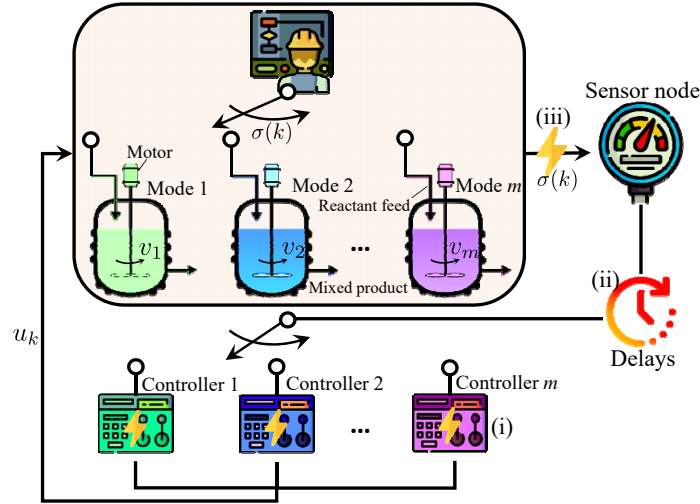


Figure 5.1: Switched systems in a typical stirred-tank reactor with m modes under multiple faults [(i) controller failures; (ii) asynchronous switching; (iii) unconstrained switching].

the adverse effect of additive uncertainties. Although some promising results [54, 57, 59, 63, 128, 144–146] have been dedicated to studying the disturbance attenuation problem of switched systems through the distinct control design, scarcely any results emphasize employing the appropriate MDT constraint individually to alleviate the impact of additive uncertainties. Moreover, it is worth mentioning that the identical magnitude order of the MDT durations and the length of multiple faults are studied in this chapter since the negative impact of the sufficiently short faulty portions on the closed-loop system is negligible.

To conquer these problems, this chapter provides a novel approach to handle the stabilization problem regarding the switched system under multiple faults in the absence or presence of additive disturbances by using the MDT constraint. Despite concentrating on the conventional control design in the majority of works mentioned above, our main interest is employing the powerful switching logic based on the constructed invariant sets. The **main contributions** of this chapter are threefold:

- 1) An algorithm to compute the maximal constraint admissible fault-tolerant-MDT (CAF-MDT) contractive set is designed. Moreover, a necessary and sufficient condition to guarantee the nominal switched closed-loop system under multiple faults to be uniformly asymptotically stable is proposed with a rigorous proof. The lower bound of the MDT is determined.

- 2) For the disturbed switched system with multiple faults, the fault-tolerant disturbance-MDT (FD-MDT) set which can serve as the ‘artificial equilibrium’ for all switching modes is constructed and critical characteristics have been perceived.
- 3) Moreover, the non-conservative stability criterion for nominal switched systems with multiple faults has been further extended to disturbed switched systems based on the designed constraint admissible FD-MDT (CAFD-MDT) invariant set. In addition, the satisfaction of the state constraint is ensured with the determined MDT.

The skeleton of this work is shown in Figure 5.2 and the rest of this chapter is organized as follows: For the nominal switched systems, in Section 5.2, the preliminaries and problem descriptions are presented. Section 5.3 is dedicated to the stability analysis based on the designed CAF-MDT set. Regarding the switched systems exerted by additive disturbances, the system descriptions and notations are stated in Section 5.4. Section 5.5 demonstrates the FD-MDT set design and theoretical results and Section 5.6 is devoted to providing the stability results with respect to the established CAFD-MDT set. Simulation results are reported in Section 5.7. Section 5.8 concludes this chapter.

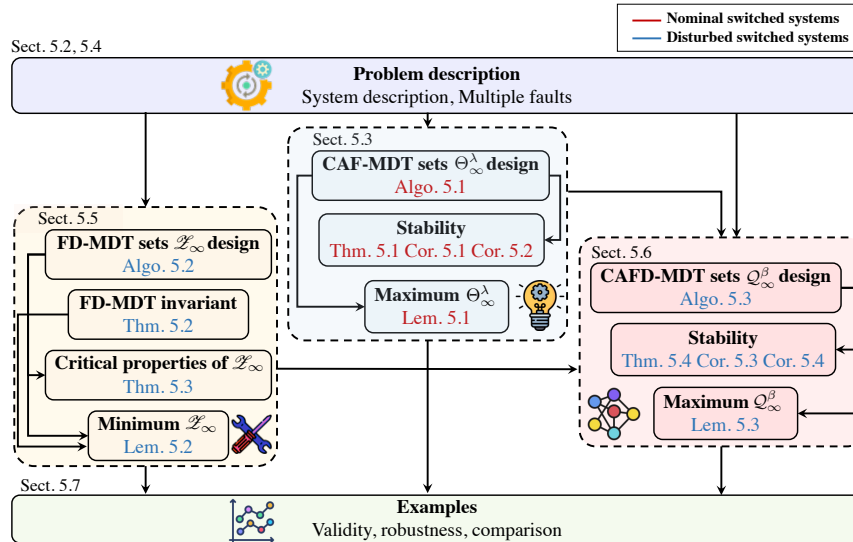


Figure 5.2: Schematic illustrating the relation between each section and the theoretical analysis framework.

5.2 Nominal switched systems

5.2.1 Preliminaries

Consider a class of constrained discrete-time switched linear system with unstable system matrices $A_m \in \mathbb{R}^{n_x \times n_x}$, $m \in \mathcal{M}$, where \mathcal{M} is the set of all admissible switching modes and $\mathcal{M} := \{1, 2, \dots, m, \dots, M\}$; $M \in \mathbb{Z}^+$ denotes the total number of subsystems. Given the input matrices $B_m \in \mathbb{R}^{n_x \times n_u}$, $m \in \mathcal{M}$, assuming the matrix pair (A_m, B_m) is stabilizable and a group of mode-dependent state-feedback controllers can successfully stabilize the corresponding subsystems with feedback gains $K_m \in \mathbb{R}^{n_u \times n_x}$, $m \in \mathcal{M}$, i.e., $\rho(\tilde{A}_m) < 1$, where $\tilde{A}_m = A_m + B_m K_m$ and $\tilde{A}_m \in \tilde{\mathcal{A}} := \{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_M\}$. The constrained switched closed-loop system integrated with the feedback control law $\mu_m := K_m x_k$, $m \in \mathcal{M}$, is given as

$$x_{k+1} = \tilde{A}_{\sigma(k)} x_k, \quad \sigma(k) \in \mathcal{M}, \quad (5.1a)$$

$$x_k \in \mathcal{X} \subseteq \mathbb{R}^{n_x} \quad (5.1b)$$

where $x_k \in \mathbb{R}^{n_x}$ is the system state and x_0 is the initial state. The switching signal $\sigma(\cdot) : \mathbb{N} \rightarrow \mathcal{M}$ is a piecewise constant function of time k and it implies the currently activated mode of system (5.1). The switching signal $\sigma(\cdot)$ is considered to be unknown *a priori*. The set \mathcal{X} denotes the state constraint and we have the succeeding assumption of \mathcal{X} .

Assumption 5.1. *The constraint set \mathcal{X} is a compact non-empty polyhedral region denoted by $\mathcal{X} = \{x | Hx \leq \mathbf{1}\}$ for some proper matrix $H \in \mathbb{R}^{n_z \times n_x}$. In addition, for at least one $\tilde{A}_m \in \tilde{\mathcal{A}}$, the matrix pair (\tilde{A}_m, H) is observable.*

Remark 5.1. *Assumption 5.1 is a mild assumption since it ensures the compactness of the evolved sets [137]. This assumption can be relaxed if only the observable mode \tilde{A}_m in system (5.1) is taken into consideration.*

For simplicity, let the synchronous portion refer to such a mode-matched interval when the mode-dependent controller and the activated subsystem are perfectly aligned. In the following, the characteristics of mode-dependent dwell time (MDT) is introduced. We firstly denote the synchronous portion as $\Omega_{1,s} := [k_{2s}, k_{2s+1})$, $s \in \mathbb{N}$. Thus, $k \in [k_{2s}, k_{2s+1})$ indicates that the subsystem $\sigma(k_{2s})$ is enabled. Moreover, the time instant k_{2s} and k_{2s+1} , $s \in \mathbb{N}$, coincide with the sampling instant k . The concept of MDT is presented as follows:

Definition 5.1. [137] The MDT $\tau_m \in \mathbb{Z}^+$, associated with mode $m \in \mathcal{M}$ and the synchronous portion $\Omega_{1,s}$, $s \in \mathbb{N}$, is the minimum amount of time that the system (5.1) has to stay in mode m . Hence, the dwell time τ_m imposes the MDT constraint $k_{2s+1} - k_{2s} \geq \tau_m$ when $\sigma(k) = m$ for $k \in [k_{2s}, k_{2s+1})$, $s \in \mathbb{N}$, and $\tau_m \in \mathcal{T} := \{\tau_1, \tau_2, \dots, \tau_M\}$ where \mathcal{T} is the set of all the admissible MDTs.

Let us consider the scenario that the mode-dependent controllers may fail to perform at any time k , $k \in \mathbb{N}$, i.e., $u_k = 0$, where $u_k \in \mathbb{R}^{n_u}$ is the control input. Thus, the switching dynamics under the controller failures are reconstructed as the following constrained discrete-time switched linear system

$$x_{k+1} = A_m x_k, \quad m \in \mathcal{M}, x_k \in \mathcal{X}, \quad (5.2)$$

where $A_m \in \mathcal{A} := \{A_1, A_2, \dots, A_M\}$. Meanwhile, the interval when the controller fails to perform is named as the *controller faulty portion* and $h_{[F]} \in \mathbb{N}$ denotes the length of the controller faulty portion.

Since the immediate detection of the switching signal is hard to achieve, the asynchronous switching issue extensively exists in engineering practice. For this concern, the constrained asynchronously switched system with the feedback control law μ_n , $n \in \mathcal{M}$, is given as

$$x_{k+1} = \hat{A}_{m,n} x_k, \quad m, n \in \mathcal{M}, m \neq n, x_k \in \mathcal{X}, \quad (5.3)$$

where $\hat{A}_{m,n} = A_m + B_m K_n$; $\hat{A}_{m,n} \in \hat{\mathcal{A}}_m := \{\hat{A}_{m,1}, \hat{A}_{m,2}, \dots, \hat{A}_{m,M}\}$ and $\hat{\mathcal{A}} := \bigcup_{m \in \mathcal{M}} \hat{\mathcal{A}}_m$ represents the union of asynchronous system matrices. The closed-loop system (5.3) implies that the lagged controller is applied to the currently enabled subsystem. For simplicity, let the *asynchronous portion* denote the mismatch interval between the desired controller and the enabled mode. The variable $h_{[A]} \in \mathbb{N}$ represents the duration of the asynchronous portion.

In addition to the controller failures and the asynchronous switching issues, the unconstrained switching encounters in engineering industries. Rather than satisfying the MDT constraint as in Definition 5.1, during the portion of arbitrary switching, switching signals change frequently. Thus, under the unconstrained switching fault,

switched systems (5.2) and (5.3) are reformulated as

$$x_{k+1} = A_{m_c} x_k, \quad m_c \in \mathcal{M}, x_k \in \mathcal{X}, \quad (5.4a)$$

$$x_{k+1} = \hat{A}_{m_c, n} x_k, \quad m_c, n \in \mathcal{M}, m_c \neq n, x_k \in \mathcal{X}, \quad (5.4b)$$

where $m_c \in \mathcal{M}$ denotes the faulty activated mode induced by arbitrary switching. Define $h_{[I]} \in \mathbb{N}$ by the duration of the *unconstrained switching portion*.

5.2.2 Problem Description

In this work, the stabilization problem for the switched closed-loop system (5.1) under multiple faults is firstly considered. To have a concrete problem description, let $\Omega_{2,s} := [k_{2s+1}, k_{2s+2})$, $s \in \mathbb{N}$, denote the duration of the *faulty portion* where multiple faults may occur. Note that the boundless faulty portion may result in the state trajectories to be divergent and uncontrollable. Therefore, we have the following assumption with respect to the margin of the faulty portion $\Omega_{2,s}$.

Assumption 5.2. *The magnitude order of the length of MDT $\Omega_{1,s}$, $s \in \mathbb{N}$, and duration of the faulty portion $\Omega_{2,s}$, $s \in \mathbb{N}$, are identical. The faulty portion satisfies $0 \leq k_{2s+2} - k_{2s+1} \leq \bar{\Delta}$, $\forall s \in \mathbb{N}$, where $\bar{\Delta} \in \mathbb{Z}^+$ is the upper bound of the faulty portion. Furthermore, both of the times k_{2s+1} and k_{2s+2} coincide with the sampling instant k .*

Remark 5.2. *Assumption 5.2 guarantees that the faulty portion is upper bounded by $\bar{\Delta}$ and it also implies that two faulty portions cannot be adjacent to each other. $\Omega_{2,s} = 0$, $s \in \mathbb{N}$, means that there is no occurrence of multiple faults.*

According to Assumption 5.2, the durations of $h_{[F]}$, $h_{[A]}$, and $h_{[I]}$ are also bounded by $\bar{\Delta}$, i.e.,

$$h_{[F]}, h_{[A]}, h_{[I]} \in [0, \bar{\Delta}], h_{[F]}, h_{[A]}, h_{[I]} \in \mathbb{N}, \forall s \in \mathbb{N}. \quad (5.5)$$

Figure 5.3 demonstrates three types of faulty intervals and enabled modes with the corresponding activated controller of each interval. The $\Omega_{2,0}$ in (1) shows the controller failure scenario while $\Omega_{2,1}$ in (2) displays the asynchronous switching case. The activated subsystems and controllers under the multiple faults are illustrated with $\Omega_{2,2}$ in (3). It can be seen the unconstrained switching together with controller failures and the asynchronous switching have witnessed the negative impact comparing with the synchronous portion $\Omega_{1,2}$.

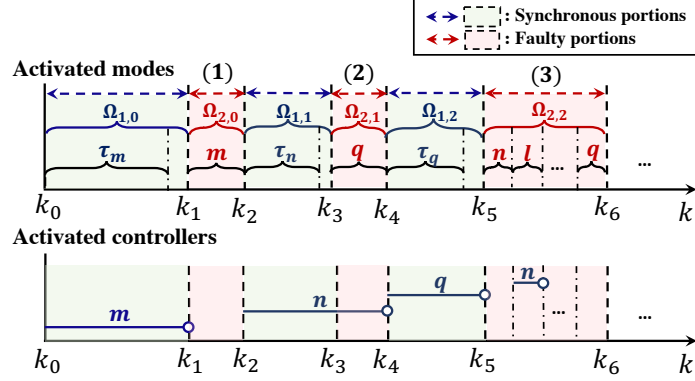


Figure 5.3: Illustration of three types of faulty intervals and the activated controller of each interval, $m, n, l, q \in \mathcal{M}$.

Remark 5.3. *It is also worth pointing out that the superimposed effect of unconstrained switching and asynchronous switching may accidentally result in the synchronization between the enabled modes and controllers. This particular circumstance has been taken into consideration since once the subsystems and controllers are matched the MDT restriction is supposed to be obeyed.*

Remark 5.4. *Due to the impact of the arbitrary switching, the resulting faulty combinations may be expressed in the identical form as the asynchronous switching.*

In the following, the concept of an admissible MDT switching sequence under multiple faults is presented.

Definition 5.2. *A switching sequence with the MDT in set \mathcal{T} , synchronous portions $\Omega_{1,s}$, and faulty portions $\Omega_{2,s}$, $s \in \mathbb{N}$, is said to be admissible in the presence of multiple faults, if the inequalities $k_{2s+1} - k_{2s} \geq \tau_m$ and $0 \leq k_{2s+2} - k_{2s+1} \leq \bar{\Delta}$ hold, $\forall s \in \mathbb{N}$. On the interval $[0, k - 1]$, the admissible switching sequence is denoted as $\xi_{\mathcal{T}[\bar{\Delta}]}(k) := \{\sigma(0), \sigma(1), \dots, \sigma(k - 1)\}$.*

The subsequent example illustrates all the possible admissible switching sequences in the presence of multiple faults by using a directed graph. An automaton type representation is shown in Figure 5.4 [147, 148]. Each node represents the finite length of the sequence and the path which points to the node denoting the admissible switching sequence transitions. Let $\mathcal{M} = \{1, 2\}$ with $\tau_1 = 3$ and $\tau_2 = 2$. The light green block exhibits the admissible switches during the synchronous portion. Define $N \in \mathbb{Z}^+$ as the total number of all the possible faulty combinations. If the upper bound of the faulty portion is $\bar{\Delta} = 2$, the $N = 21$ possible faulty combinations including

the no-fault situation are shown on the right pink ellipse. It is observed that a bi-directional path exists between the synchronous and faulty portions. This connection is equivalent to a single self-looping node which is shown at the bottom of Fig. 5.4. Then, for instance, an admissible switching sequence is $\xi_{\mathcal{T}^{[2]}}(7) = \{\tilde{1}, \tilde{1}, \tilde{1}, 1, 2, \tilde{2}, \tilde{2}\}$ where the notation of each variable is shown in the legend of Figure 5.4. However, $\xi_{\mathcal{T}^{[2]}}(8) = \{\tilde{1}, \tilde{1}, \tilde{1}, \tilde{1}, 1, 2, \hat{2}_1, \tilde{2}\}$ is not admissible since the MDT restriction of this switching sequence is violated and the upper bound of the faulty portion is exceeded.

For notational simplicity, each faulty combination is labeled as c_i , $i \in \mathbb{Z}^+$, and the set $c_i \in \mathcal{C}_N := \{c_1, c_2, \dots, c_N\}$ collects all the possible faulty combinations for every $i \in [1, N]$. As an illustration, c_1 denotes the \emptyset situation while c_2 represents the specific faulty portion of 1 displayed on right pink ellipse, etc.

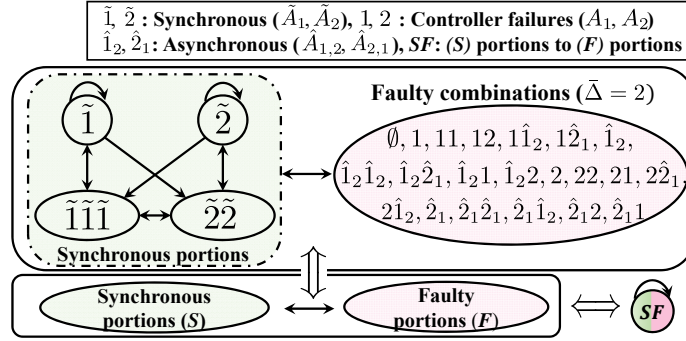


Figure 5.4: An automaton-type representation of admissible switching sequences with multiple faults.

5.3 Stability Results for Nominal Switched Systems

This section begins with the introduction of the computational algorithm to establish a constraint admissible fault-tolerant mode-dependent dwell time (CAF-MDT) contractive set. The definition of a CAF-MDT contractive set is described as follows:

Definition 5.3. A set $\Xi \subseteq \mathbb{R}^{n_x}$ is said to be CAF-MDT contractive with respect to a contraction factor $\lambda \in (0, 1)$, the MDT set $\Phi := \{\phi_1, \phi_2, \dots, \phi_M\}$, and the closed-loop system (5.1) in the presence of multiple faults, if $x_0 \in \Xi$ implies $x_t \in \mathcal{X}$ for $t \in \mathbb{Z}_{[1,k]}^+$ and $\mathcal{A}_{\xi_{\Phi}^{[\Delta]}(k)}x_0 \in \lambda\Xi$ where the product of the system matrices associated with the

switching sequence $\xi_{\Phi[\bar{\Delta}]}(k)$ is

$$\begin{aligned}
\mathcal{A}_{\xi_{\Phi[\bar{\Delta}]}(k)} &:= \hat{A}_{\sigma(k_{2s+1} + \alpha_r^{2s+1} + \hat{\alpha}_r^{2s+1}), \sigma(k_{2s+1} + \alpha_{r-1}^{2s+1} + \hat{\alpha}_{r-1}^{2s+1})}^{\hat{s}_{2s+1,r}} \\
&A_{\sigma(k_{2s+1} + \alpha_{r-1}^{2s+1} + \hat{\alpha}_{r-1}^{2s+1})}^{s_{2s+1,r}} \cdots \hat{A}_{\sigma(k_{2s+1} + s_{2s+1,1}), \sigma(k_{2s+1})}^{\hat{s}_{2s+1,1}} \\
&A_{\sigma(k_{2s+1})}^{s_{2s+1,1}} \tilde{A}_{\sigma(k_{2s})}^{\Omega_{1,s}} \cdots \hat{A}_{\sigma(k_1 + \alpha_r^1 + \hat{\alpha}_r^1), \sigma(k_1 + \alpha_{r-1}^1 + \hat{\alpha}_{r-1}^1)}^{\hat{s}_{1,r}} \\
&A_{\sigma(k_1 + \alpha_{r-1}^1 + \hat{\alpha}_{r-1}^1)}^{s_{1,r}} \cdots \hat{A}_{\sigma(k_1 + s_{1,1}), \sigma(k_1)}^{\hat{s}_{1,1}} A_{\sigma(k_1)}^{s_{1,1}} \tilde{A}_{\sigma(k_0)}^{\Omega_{1,0}}, \tag{5.6}
\end{aligned}$$

where $\hat{s}_{2s+1,r} \in \mathbb{N}$ and $s_{2s+1,r} \in \mathbb{N}$ are the length of the r th asynchronous subsection and the r th controller faulty subsection in the faulty interval $\Omega_{2,s}$, $s \in \mathbb{N}$, respectively. The summation variables are respectively defined as $\alpha_r^{2s+1} := \sum_{v=1}^r s_{2s+1,v}$ and $\hat{\alpha}_r^{2s+1} := \sum_{v=1}^r \hat{s}_{2s+1,v}$, $v \in \mathbb{Z}^+$. Thus, the duration of the faulty portion is $\Omega_{2,s} = \alpha_r^{2s+1} + \hat{\alpha}_r^{2s+1} \in [0, \bar{\Delta}]$ and $k = \sum_{z=0}^s \Omega_{1,z} + \Omega_{2,z}$, $z \in \mathbb{N}$.

The representation in (5.6) indicates that the admissible switching sequence $\xi_{\Phi[\bar{\Delta}]}(k)$ consists of a concatenation of $(s+1)$ -stage subsequences. As an illustration, the first stage is in mode $\sigma(k_0)$ for $\Omega_{1,0}$ steps and then in the mode regarding c_i faulty combination for $\Omega_{2,0}$ steps and the second stage is in mode $\sigma(k_2)$ for $\Omega_{1,1}$ steps and then in the mode with respect to c_i faulty combination for $\Omega_{2,1}$ steps, etc. To be consistent with the expression in (5.6), let $\hat{s}_r^{c_i} \in \mathbb{N}$ and $s_r^{c_i} \in \mathbb{N}$ represent the length of the r th asynchronous subsection and the r th controller faulty subsection with respect to the c_i faulty combination, respectively. Since the unconstrained switching may result in the desired activated mode replaced by a faulty one, we denote $m_{c_i} \in \mathcal{M}$ as the faulty activated mode induced by the c_i faulty combination, $c_i \in \mathcal{C}_N$.

5.3.1 Design of CAF-MDT Sets

In the following, based on Definition 5.3, Algorithm 5.1 is introduced to compute the CAF-MDT contractive set. Inspired by [137], the computational algorithm is modified and the aim is to make it accommodate with switched closed-loop system (5.1) in the presence of multiple faults.

To this end, the computation of the contractive set is proceeded in two phases. Define the MDT set of the first phase as $\mathcal{T}_m^F := \{1, 2, \dots, \tau_m - 1\}$, $\tau_m \in \mathcal{T}$, $m \in \mathcal{M}$, with $\mathcal{S}^F := \cup_{m \in \mathcal{M}} \mathcal{T}_m^F$. The MDT set of the second phase is $\mathcal{T}_m^S := \{\tau_m, \tau_m + 1, \dots, 2\tau_m - 1\}$, $\tau_m \in \mathcal{T}$, $m \in \mathcal{M}$, with $\mathcal{S}^S := \cup_{m \in \mathcal{M}} \mathcal{T}_m^S$. The rationale for designing the first phase MDT set is to collect the truncated behavior of the system states [137].

Algorithm 5.1 Computation of a CAF-MDT set Θ_∞^λ

- 1: **Input:** The state constraint \mathcal{X} ; system matrices sets \mathcal{A} , $\tilde{\mathcal{A}}$, and $\hat{\mathcal{A}}$; upper bound $\bar{\Delta}$; total number N ; contraction factor λ ; MDT sets \mathcal{T}^F and \mathcal{T}^S .
 - 2: Initialize $\ell = 0$ and $p = 1$ and calculate the initial set $\Theta_0^\lambda := \mathcal{X} \cap (\bigcap_{m \in \mathcal{M}} \tilde{\Psi}_m(\mathcal{X}))$.
 - 3: **while** $p \leq N$ **do**
 - 4: Compute $\Xi = \bigcap_{m \in \mathcal{M}} \tilde{\Psi}'_m(\psi_{m_{c_p}}^{s_1^{cp}}(\hat{\psi}_{m_{c_p}, n}^{s_1^{cp}} \cdots \psi_{m_{c_p}}^{s_r^{cp}}(\hat{\psi}_{m_{c_p}, n}^{s_r^{cp}}(\lambda \Theta_\ell^\lambda))))$
 - 5: Calculate $\Theta_{\ell+1}^\lambda = \Theta_\ell^\lambda \cap \Xi$
 - 6: Set $p = p + 1$
 - 7: **end while**
 - 8: **If** $\Theta_{\ell+1}^\lambda \equiv \Theta_\ell^\lambda$, set $\Theta_\infty^\lambda = \Theta_\ell^\lambda$ and **stop**; else set $\ell = \ell + 1$ and goto **step 2**.
-

The state evolution subject to the MDT constraint is reflected on the MDT set of the second phase.

In order to construct the CAF-MDT contractive set, the technique of the backward sets computation is employed. A γ -step backward set for the closed-loop system (5.1) is given as

$$\tilde{\psi}_m^\gamma(\Xi) := \{x | \tilde{A}_m^\gamma x \in \Xi, m \in \mathcal{M}\}, \quad (5.7)$$

where the number of steps $\gamma \in \mathbb{N}$ with $\tilde{\psi}_m^0(\Xi) = \{x | x \in \Xi\}$, $\Xi \subseteq \mathbb{R}^{n_x}$. For every $\gamma \in \mathcal{T}_m^F$, the $\tilde{\Psi}_m(\Xi) := \bigcap_{\gamma \in \mathcal{T}_m^F} \tilde{\psi}_m^\gamma(\Xi)$ denotes the intersection of the backward steps in the first MDT phase. Similarly, $\tilde{\Psi}'_m(\Xi) := \bigcap_{\gamma \in \mathcal{T}_m^S} \tilde{\psi}_m^\gamma(\Xi)$ represents the intersection of the backward set for every $\gamma \in \mathcal{T}_m^S$. Likewise, a γ -step backward set for the system (5.2) under the controller failure is characterized as

$$\psi_m^\gamma(\Xi) := \{x | A_m^\gamma x \in \Xi, m \in \mathcal{M}\}, \quad (5.8)$$

with $\psi_m^0(\Xi) = \{x | x \in \Xi\}$. Furthermore, a γ -step backward set for the asynchronously switched system (5.3) is defined as

$$\hat{\psi}_{m,n}^\gamma(\Xi) := \{x | \hat{A}_{m,n}^\gamma x \in \Xi, m, n \in \mathcal{M}, m \neq n\}, \quad (5.9)$$

with $\hat{\psi}_{m,n}^0(\Xi) = \{x | x \in \Xi\}$.

Subsequently, the Algorithm of designing a CAF-MDT contractive set is proposed.

Within the constraint set, the initial set Θ_0^λ in Step 2 collects all the possible state trajectories under the truncated switching sequences with the MDT in the first phase. All possible faulty combinations c_i , $c_i \in \mathcal{C}_N$, and synchronous portions satisfying the MDT constraint $\Omega_{1,s}$, $s \in \mathbb{N}$, are considered in the backward set computation from

Step 3 to Step 7. Then, the iterative set $\Theta_{\ell+1}^\lambda$ is obtained by intersecting all the backward sets. Once the condition in step 8 is satisfied, a CAF-MDT contractive set is acquired.

Remark 5.5. *For the switched system possessing numerous modes and relatively long faulty portions, the rationality of each possible faulty combination is supposed to be reconsidered.*

5.3.2 Stability Results

To begin with, the definition of uniform asymptotic stability (UAS) is introduced.

Definition 5.4. [2] *The switched system (5.1) under multiple faults is said to be uniformly asymptotically stable along all the admissible switching sequences $\xi_{\mathcal{T}[\bar{\Delta}]}(t)$, if for $\|x_0\| \leq \hat{\delta}$ with a existing positive constant $\hat{\delta}$, x_t satisfies $x_t \rightarrow \mathbf{0}$ as $t \rightarrow \infty$.*

Theorem 5.1. *Suppose Assumptions 5.1 and 5.2 are satisfied. Then, the closed-system (5.1) in the presence of multiple faults with admissible switching sequences $\xi_{\Phi[\bar{\Delta}]}(\cdot)$, is uniformly asymptotically stable if and only if the CAF-MDT set Θ_∞^λ generated by Algorithm 5.1 under the MDT set Φ and a contraction factor $\lambda \in (0, 1)$ is non-empty.*

Proof. (i) Sufficiency: The proof of sufficiency is aimed to show that the existence of such an CAF-MDT set indicates that the closed-loop system (5.1) under multiple faults is uniformly asymptotically stable with an MDT set Φ . Consider an admissible switching sequence $\xi_{\Phi[\bar{\Delta}]}(\cdot)$. For every $x_0 \in \Theta_\infty^\lambda$, we have

$$\begin{aligned}
x_k &= \mathcal{A}_{\xi_{\Phi[\bar{\Delta}]}(k)} x_0 \in \lambda^{\tilde{\ell}} (\tilde{\rho}_{\sigma(k_0)}^{\Omega_{1,0}} \rho_{\sigma(k_1)}^{s_{1,1}} \hat{\rho}_{\sigma(k_1+s_{1,1}),\sigma(k_1)}^{\hat{s}_{1,1}} \cdots \\
&\quad \rho_{\sigma(k_1+\alpha_{r-1}^1+\hat{\alpha}_{r-1}^1)}^{s_{1,r}} \hat{\rho}_{\sigma(k_1+\alpha_r^1+\hat{\alpha}_{r-1}^1),\sigma(k_1+\alpha_{r-1}^1+\hat{\alpha}_{r-1}^1)}^{\hat{s}_{1,r}} \cdots \tilde{\rho}_{\sigma(k_{2s})}^{\Omega_{1,s}} \\
&\quad \rho_{\sigma(k_{2s+1})}^{s_{2s+1,1}} \hat{\rho}_{\sigma(k_{2s+1}+s_{2s+1,1}),\sigma(k_{2s+1})}^{\hat{s}_{2s+1,1}} \cdots \rho_{\sigma(k_{2s+1}+\alpha_{r-1}^{2s+1}+\hat{\alpha}_{r-1}^{2s+1})}^{s_{2s+1,r}} \\
&\quad \hat{\rho}_{\sigma(k_{2s+1}+\alpha_r^{2s+1}+\hat{\alpha}_{r-1}^{2s+1}),\sigma(k_{2s+1}+\alpha_{r-1}^{2s+1}+\hat{\alpha}_{r-1}^{2s+1})}^{\hat{s}_{2s+1,r}}) \Theta_\infty^\lambda, \tag{5.10}
\end{aligned}$$

where $\tilde{\ell} := \lfloor \frac{\Omega_{1,0}}{\phi_{\sigma(k_0)}} \rfloor + \lfloor \frac{\Omega_{1,1}}{\phi_{\sigma(k_2)}} \rfloor + \cdots + \lfloor \frac{\Omega_{1,s}}{\phi_{\sigma(k_{2s})}} \rfloor$, $s \in \mathbb{N}$. The parameters $\tilde{\rho}_{\sigma(\cdot)}$, $\rho_{\sigma(\cdot)}$, and $\hat{\rho}_{\sigma(\cdot),\sigma(\cdot)}$ denote the maximal Euclidean norm of all the corresponding eigenvalues of system matrices $\tilde{A}_{\sigma(\cdot)}$, $A_{\sigma(\cdot)}$ and $\hat{A}_{\sigma(\cdot),\sigma(\cdot)}$, respectively. Let ϱ_k , $k \in \mathbb{Z}^+$, denote the production of the norms in (5.10) within the bracket. In order to force x_k to converge to the origin, it is required that $\lambda^{\tilde{\ell}} \varrho_k < 1$ stands. It is worth pointing out

that ϱ_k is bounded since Assumption 5.2 restricts the duration of the faulty portion, i.e., $\Omega_{2,s} \in [0, \bar{\Delta}]$, $s \in \mathbb{N}$. To mitigate the detrimental effect of multiple faults, the duration of $\Omega_{1,s}$ can be designed sufficiently long such that the inequality $\lambda^{\bar{\ell}} \varrho_k < 1$ holds for all $\xi_{\Phi[\bar{\Delta}]}(k)$. This is because the admissible MDTs are decision variables. Since $\lambda \in (0, 1)$ and $\bar{\ell} \rightarrow \infty$ as $k \rightarrow \infty$, the origin of the closed-loop system (5.1) under multiple faults is attractive in Θ_∞^λ , i.e., $\|x_k\| \rightarrow 0$.

Subsequently, the stability proof is presented. Since the origin is contained in the state constraint \mathcal{X} , it follows that

$$\tilde{\delta}\mathcal{B} \subseteq \Theta_\infty^\lambda \subseteq \mathcal{X} \subseteq \tilde{\epsilon}\mathcal{B}, \quad (5.11)$$

where $\tilde{\delta} > 0$ and $\tilde{\epsilon} > 0$ are two positive constants and $\mathcal{B} \subseteq \mathbb{R}^{n_x}$ is a unit ball. Considering that there exist another two positive constants $\delta \in (0, \tilde{\delta}]$ and $\epsilon \geq \tilde{\epsilon}$. It follows that, for $\|x_0\| \leq \delta$, we have $\|x_k\| \leq \epsilon$ because the inclusion relationship $x_k \in \Theta_\infty^\lambda \subseteq \tilde{\epsilon}\mathcal{B} \subseteq \epsilon\mathcal{B}$ holds. For $\epsilon < \tilde{\epsilon}$, the inequality $\lambda^{\bar{\ell}} \varrho_k \tilde{\epsilon} < \epsilon$ stands due to the attractivity property of Θ_∞^λ . Similarly, for $\delta > \tilde{\delta}$, we have $\delta \in (0, \lambda^{\bar{\ell}} \varrho_k \tilde{\delta}]$. Therefore, for all $\epsilon > 0$, there exists a $\delta > 0$ such that for all $\|x_0\| \leq \delta$ implies that $\|x_k\| \leq \epsilon$, $\forall k \in \mathbb{N}$. Combining with the attractivity result, the UAS of the closed-loop system (5.1) with multiple faults is claimed.

(ii) Necessity: Let $\|\cdot\|_{\Theta_\infty^\lambda}$ denote the norm induced by the CAF-MDT set. There exists a contraction factor $\bar{\lambda}_\varrho \in (0, 1)$ such that $\mathcal{A}_{\xi_{\Phi[\bar{\Delta}]}(k)} \Theta_\infty^\lambda \subset \bar{\lambda}_\varrho \varrho_k \Theta_\infty^\lambda$ stands. Then, it follows that

$$\|\mathcal{A}_{\xi_{\Phi[\bar{\Delta}]}(k)}\|_{\Theta_\infty^\lambda} < \bar{\lambda}_\varrho \varrho_k < 1.$$

According to Assumption 5.1, reformulate the set $\Theta_0^{\bar{\lambda}_\varrho}$ in step 2 of Algorithm 5.1 as $\Theta_0^{\bar{\lambda}_\varrho} := \{x | H_M x \leq \mathbf{1}, \forall M \in \mathbb{Z}^+\}$. Inspired by [142], we will show that a CAF-MDT set can be obtained through finite-time iterations. Considering the iteration from $\Theta_{\ell-1}^{\lambda'_\varrho}$ to $\Theta_\ell^{\lambda'_\varrho}$, for any $\lambda'_\varrho > \bar{\lambda}_\varrho$, an additional inequality $H_M \mathcal{A}'_{\xi_{\Phi[\bar{\Delta}]}(k)} x \leq \lambda'_\varrho \varrho_k \mathbf{1}$ is required where

$$\begin{aligned} \mathcal{A}'_{\xi_{\Phi[\bar{\Delta}]}(k)} &:= \tilde{A}_{\sigma(k_0)}^{\Omega_{1,0}} A_{\sigma(k_1)}^{s_{1,1}} \hat{A}_{\sigma(k_1+s_{1,1}),\sigma(k_1)}^{\hat{s}_{1,1}} \cdots A_{\sigma(k_1+\alpha_{r-1}^1+\hat{\alpha}_{r-1}^1)}^{s_{1,r}} \\ &\quad \hat{A}_{\sigma(k_1+\alpha_r^1+\hat{\alpha}_{r-1}^1),\sigma(k_1+\alpha_{r-1}^1+\hat{\alpha}_{r-1}^1)}^{\hat{s}_{1,r}} \cdots \tilde{A}_{\sigma(k_{2s})}^{\Omega_{1,s}} A_{\sigma(k_{2s+1})}^{s_{2s+1,1}} \\ &\quad \hat{A}_{\sigma(k_{2s+1}+s_{2s+1,1}),\sigma(k_{2s+1})}^{\hat{s}_{2s+1,1}} \cdots A_{\sigma(k_{2s+1}+\alpha_{r-1}^{2s+1}+\hat{\alpha}_{r-1}^{2s+1})}^{s_{2s+1,r}} \\ &\quad \hat{A}_{\sigma(k_{2s+1}+\alpha_r^{2s+1}+\hat{\alpha}_{r-1}^{2s+1}),\sigma(k_{2s+1}+\alpha_{r-1}^{2s+1}+\hat{\alpha}_{r-1}^{2s+1})}^{\hat{s}_{2s+1,r}}. \end{aligned}$$

In order to prove that the additional inequality is redundant, we have the following inequalities at the ℓ th iteration

$$\begin{aligned}
H_M \mathcal{A}'_{\xi_{\Phi[\bar{\Delta}]}(k)} x &\leq \max_{\alpha_1 \in \mathcal{B}(\|\mathcal{A}'_{\xi_{\Phi[\bar{\Delta}]}(k)} x\|)} H_M \alpha_1 \leq \max_{\alpha_2 \in \mathcal{B}(\|\mathcal{A}'_{\xi_{\Phi[\bar{\Delta}]}(k)} x\|_{\Theta_{\infty}^{\bar{\lambda}_{\varrho}}})} \eta_1 H_M \alpha_2 \\
&\leq \max_{\alpha_2 \in \mathcal{B}(\bar{\lambda}_{\varrho}^{\ell} \varrho_k \|x\|_{\Theta_{\infty}^{\bar{\lambda}_{\varrho}}})} \eta_1 H_M \alpha_2 \\
&\leq \max_{\alpha_3 \in \mathcal{B}(\|x\|)} \bar{\lambda}_{\varrho}^{\ell} \varrho_k \eta_1 \eta_2 H_M \alpha_3 \leq \bar{\lambda}_{\varrho}^{\ell} \varrho_k \eta_1 \eta_2 \|H_M\| \|x\| < \lambda'_{\varrho}{}^{\ell},
\end{aligned} \tag{5.12}$$

where η_1 and η_2 are two positive real numbers. Due to $\bar{\lambda}_{\varrho} < \lambda'_{\varrho}$, the last inequality holds. Thus, the additional inequality is redundant. Therefore, the CAF-MDT contractive set $\Theta_{\infty}^{\lambda}$ generated by Algorithm 5.1 under an MDT set Φ and a contraction factor $\lambda \in (0, 1)$ is ensured to be obtained via finite-time iterations. The proof is completed. \blacksquare

Remark 5.6. *Theorem 5.1 implies that once the state trajectory enters $\Theta_{\infty}^{\lambda}$ with the admissible switching sequence $\xi_{\Phi[\bar{\Delta}]}(\cdot)$, the system state will finally converge to the origin.*

Corollary 5.1. *Suppose that the synchronous portion $\Omega_{1,s}$ does not fulfill the MDT constraint and the length of faulty portions $\Omega_{2,s} \equiv 0, \forall s \in \mathbb{N}$. The switched closed-loop system (5.1) is asymptotically stable if the mode transitions are well scheduled, i.e., $\varrho_k < 1$.*

Corollary 5.2. *Suppose the length of synchronous portions $\Omega_{1,s} \equiv 0$ for all $s \in \mathbb{N}$. The asynchronously switched system (5.3) under controller failures and the arbitrary switching fault is asymptotically stable if the switching sequence is elaborately designed, i.e., $\varrho_k < 1$.*

Remark 5.7. *It is worth mentioning that the UAS of the closed-loop system cannot be claimed in both Corollaries 5.1 and 5.2 since only the prescribed switching sequences are considered. However, by employing Algorithm 5.1 with the predefined switching sequence, the constraint satisfaction of the closed-loop system can also be ensured if $x_0 \in \Theta_{\infty}^{\lambda}$.*

5.3.3 Maximum $\Theta_{\infty}^{\lambda}$

The maximal characteristic of the set $\Theta_{\infty}^{\lambda}$ with respect to a contraction factor λ is rigorously proved in Lemma 5.1.

Lemma 5.1. *Suppose Assumptions 5.1 and 5.2 are satisfied. A non-empty Θ_ℓ^λ , $\forall \ell \in \mathbb{N}$, is obtained by using Algorithm 5.1. Then, Θ_∞^λ is the maximal CAF-MDT contractive set with respect to a contraction factor $\lambda \in (0, 1)$.*

Proof. The CAF-MDT contractive property of Θ_∞^λ is directly achieved due to the inclusions $\Theta_{\ell+1}^\lambda \subset \lambda\Theta_\ell^\lambda$ and $\Theta_\ell^\lambda \subset \mathcal{X}$, $\ell \in \mathbb{N}$, from Algorithm 5.1.

The reason behind the acquired Θ_∞^λ is the maximal CAF-MDT set is explained by using contradiction. Suppose Θ_∞^λ is not the largest contractive CAF-MDT invariant set. There must at least exist a set $\Theta' \subseteq \mathcal{X}$ such that $\Theta' \not\subseteq \Theta_\infty^\lambda$. Then, the inclusion $\Theta' \subset \Theta_\infty^\lambda$ holds because the set Θ' is constraint admissible. Let $x_0 \in \Theta'$. Due to the CAF-MDT contractive property of Θ' , it follows that $x_k \in \Theta'$, for all $\nu \in \mathcal{T}_m^S, m \in \mathcal{M}$ and for all $c_i \in \mathcal{C}_N$. Note that $\Theta' \subset \Theta_\infty^\lambda$ and $x_0 \in \Theta'$ indicate that x_0 also belongs to Θ_1^λ , i.e., $x_0 \in \Theta_1^\lambda$. Then, we have $\Theta' \subseteq \Theta_1^\lambda$. Applying the above identical procedure for all $\ell \in \mathbb{N}$, it follows that $\Theta' \subseteq \Theta_\infty^\lambda = \lim_{\ell \rightarrow \infty} \Theta_\ell^\lambda$. It contradicts with the statement $\Theta' \not\subseteq \Theta_\infty^\lambda$. Hence, Θ_∞^λ is the maximal contractive CAF-MDT set with respect to a contraction factor $\lambda \in (0, 1)$. \blacksquare

5.4 Switched Systems with Additive Disturbances

5.4.1 Preliminaries

This section begins with introducing the switching system dynamics exerted by additive uncertainties. Consider the following constrained switched synchronous closed-loop system (5.1) integrated with the feedback control law $\mu_m, m \in \mathcal{M}$, under the additive disturbance

$$\begin{aligned} x_{k+1} &= \tilde{A}_m x_k + w_k, \quad m \in \mathcal{M} \\ x_k &\in \mathcal{X} \subseteq \mathbb{R}^{n_x}, w_k \in \mathcal{W} \subseteq \mathbb{R}^{n_x}, \end{aligned} \quad (5.13)$$

where $w_k \in \mathbb{R}^{n_x}$ is the the unknown and time-varying disturbance vector and \mathcal{W} is the disturbance constraint set. It is assumed that \mathcal{W} is a compact polyhedral region and the origin is required to be contained in \mathcal{W} . The constrained discrete-time switched linear system with additive disturbance in the presence of the controller failure is given as

$$x_{k+1} = A_m x_k + w_k, \quad m \in \mathcal{M}, x_k \in \mathcal{X}, w_k \in \mathcal{W}. \quad (5.14)$$

Furthermore, the constrained asynchronously switched system (5.3) in the presence of the external disturbance is characterized as

$$x_{k+1} = \hat{A}_{m,n}x_k + w_k, \quad m, n \in \mathcal{M}, m \neq n, x_k \in \mathcal{X}, w_k \in \mathcal{W}. \quad (5.15)$$

Likewise, in the presence of the unconstrained switching, perturbed switched systems (5.14) and (5.15) are reformulated as

$$\begin{aligned} x_{k+1} &= A_{m_c}x_k + w_k, \quad m_c \in \mathcal{M}, \\ x_{k+1} &= \hat{A}_{m_c,n}x_k + w_k, \quad m_c, n \in \mathcal{M}, m_c \neq n. \end{aligned} \quad (5.16)$$

Similar to in Section 5.3, in order to stabilize the switched closed-loop system (5.13) with multiple faults, we desire to propose a comparable stability condition regarding the constructed invariant sets designed as follows. To begin with, based on the admissible switching sequence of $\xi_{\mathcal{T}[\bar{\Delta}]}(\cdot)$, the allowable disturbance sequence is defined as $\xi_{\mathcal{T}[\bar{\Delta}]}^w(k) := \{w_0, w_1, \dots, w_{k-1}\}$ with each $w_{k-1} \in \mathcal{W}$. Then, the definitions of the fault-tolerant disturbance-MDT (FD-MDT) invariant set and the constraint admissible FD-MDT (CAFD-MDT) invariant set are presented as follows:

Definition 5.5. [138] *A set $\mathcal{O} \subseteq \mathbb{R}^{n_x}$ is said to be FD-MDT invariant with respect to the closed-loop system (5.13) in the presence of multiple faults with an MDT set $\Phi^w := \{\phi_1^w, \phi_2^w, \dots, \phi_M^w\}$, if $x_0 \in \mathcal{O}$ implies $x_k \in \mathcal{O}$, $k \in \mathbb{Z}^+$, for every admissible switching sequence $\xi_{\Phi^w[\bar{\Delta}]}(k)$ under the allowable disturbance sequence $\xi_{\Phi^w[\bar{\Delta}]}^w(k)$.*

Definition 5.6. [138] *A set $\mathcal{O} \subseteq \mathbb{R}^{n_x}$ is said to be contractive CAFD-MDT invariant with respect to a contraction factor $\beta \in (0, 1)$, an MDT set Φ^w , and the closed-loop system (5.13) in the presence of multiple faults, if $x_0 \in \mathcal{O}$ indicates that $x_t \in \beta\mathcal{O}$ and $x_t \in \mathcal{X}$ for $t \in \mathbb{Z}_{[1,k]}^+$ and every admissible switching sequence $\xi_{\Phi^w[\bar{\Delta}]}(t)$ under the allowable disturbance sequence $\xi_{\Phi^w[\bar{\Delta}]}^w(t)$.*

5.4.2 Techniques

The first technique is the reachable set computation. For a given set $\mathcal{O} \subseteq \mathbb{R}^{n_x}$, the ν -step reachable set of the closed-loop system (5.13), $\nu \in \mathbb{N}$, is given as

$$\begin{aligned} \tilde{\mathcal{R}}_m^\nu(\mathcal{O}) &:= \{\tilde{A}_m^\nu x + \dots + \tilde{A}_m w + w | x \in \mathcal{O}, w \in \mathcal{W}, m \in \mathcal{M}\} \\ &= \tilde{A}_m^\nu \mathcal{O} \oplus \tilde{A}_m^{\nu-1} \mathcal{W} \oplus \dots \oplus \tilde{A}_m \mathcal{W} \oplus \mathcal{W}, \end{aligned} \quad (5.17)$$

with $\tilde{\mathcal{R}}_m^0(\mathcal{O}) = \mathcal{O}$. Likewise, the ν -step reachable set for system (5.14) is

$$\begin{aligned}\mathcal{R}_m^\nu(\mathcal{O}) &:= \{A_m^\nu x + \cdots + A_m w + w \mid x \in \mathcal{O}, w \in \mathcal{W}, m \in \mathcal{M}\} \\ &= A_m^\nu \mathcal{O} \oplus A_m^{\nu-1} \mathcal{W} \oplus \cdots \oplus A_m \mathcal{W} \oplus \mathcal{W},\end{aligned}\quad (5.18)$$

with $\mathcal{R}_m^0(\mathcal{O}) = \mathcal{O}$. In addition, the ν -step reachable set for the asynchronously switched system (5.15) is

$$\begin{aligned}\hat{\mathcal{R}}_{m,n}^\nu(\mathcal{O}) &:= \{\hat{A}_{m,n}^\nu x + \cdots + \hat{A}_{m,n} w + w \mid x \in \mathcal{O}, w \in \mathcal{W}, m, n \in \mathcal{M}, m \neq n\} \\ &= \hat{A}_{m,n}^\nu \mathcal{O} \oplus \hat{A}_{m,n}^{\nu-1} \mathcal{W} \oplus \cdots \oplus \hat{A}_{m,n} \mathcal{W} \oplus \mathcal{W},\end{aligned}\quad (5.19)$$

with $\hat{\mathcal{R}}_{m,n}^0(\mathcal{O}) = \mathcal{O}$. Consider the closed-loop system (5.13) in the presence of multiple faults. The union of one-stage all possible reachable sets for (5.13) with the MDT set \mathcal{T}^S under the total N possible faulty combinations and the allowable disturbance sequence $\xi_{\mathcal{T}[\Delta]}^w(\cdot)$ is defined as

$$\mathcal{R}(\mathcal{O}, \mathcal{W}) := \cup_{m \in \mathcal{M}} (\cup_{\nu \in \mathcal{T}_m^S} (\cup_{i=1}^N \mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})))), \quad (5.20)$$

where $\mathcal{R}^{c_i}(\mathcal{O}) := \hat{\mathcal{R}}_{m_{c_i}, n}^{s_r^{c_i}}(\mathcal{R}_{m_{c_i}}^{s_r^{c_i}} \cdots \hat{\mathcal{R}}_{m_{c_i}, n}^{s_1^{c_i}}(\mathcal{R}_{m_{c_i}}^{s_1^{c_i}}(\mathcal{O})))$ is the operator to compute the reachable set under the c_i th faulty combination.

The other technique is the backward set calculation. The ν -step backward set for the closed-loop system (5.13) and a non-empty set $\mathcal{O} \subseteq \mathbb{R}^{n_x}$ is denoted as

$$\begin{aligned}\tilde{\mathcal{P}}_m^\nu(\mathcal{O}) &:= \{x \mid \tilde{A}_m^\nu x + \cdots + \tilde{A}_m w + w \in \mathcal{O}, w \in \mathcal{W}, m \in \mathcal{M}\} \\ &= \{x \mid \tilde{A}_m^\nu x \in \mathcal{O} \ominus \mathcal{W} \ominus \tilde{A}_m \mathcal{W} \ominus \cdots \ominus \tilde{A}_m^{\nu-1} \mathcal{W}\},\end{aligned}\quad (5.21)$$

with $\tilde{\mathcal{P}}_m^0(\mathcal{O}) = \mathcal{O}$. Similarly, the ν -step backward operator associated with the system (5.14) is represented as

$$\begin{aligned}\mathcal{P}_m^\nu(\mathcal{O}) &:= \{x \mid A_m^\nu x + \cdots + A_m w + w \in \mathcal{O}, w \in \mathcal{W}, m \in \mathcal{M}\} \\ &= \{x \mid A_m^\nu x \in \mathcal{O} \ominus \mathcal{W} \ominus A_m \mathcal{W} \ominus \cdots \ominus A_m^{\nu-1} \mathcal{W}\},\end{aligned}\quad (5.22)$$

with $\mathcal{P}_m^0(\mathcal{O}) = \mathcal{O}$. Likewise, the ν -step backward set for the asynchronously switched

system (5.15) is

$$\begin{aligned}\hat{\mathcal{P}}_{m,n}^\nu(\mathcal{O}) &:= \{x | \hat{A}_{m,n}^\nu x + \cdots + \hat{A}_{m,n} w + w \in \mathcal{O}, w \in \mathcal{W}, m, n \in \mathcal{M}, m \neq n\} \\ &= \{x | \hat{A}_{m,n}^\nu x \in \mathcal{O} \ominus \mathcal{W} \ominus \hat{A}_{m,n} \mathcal{W} \ominus \cdots \ominus \hat{A}_{m,n}^{\nu-1} \mathcal{W}\},\end{aligned}\quad (5.23)$$

with $\hat{\mathcal{P}}_{m,n}^0(\mathcal{O}) = \mathcal{O}$. Based on the definitions from (5.21)-(5.23), the backward set which ensures the state trajectory driving into the set \mathcal{O} in one-stage along the admissible switching sequence $\xi_{\mathcal{T}[\Delta]}(\cdot)$ with the MDT set \mathcal{T}^S under N the possible faulty combinations and the allowable disturbance sequence $\xi_{\mathcal{T}[\Delta]}^w(\cdot)$ is calculated by using the backward operator as follows:

$$\mathcal{P}(\mathcal{O}, \mathcal{W}) := \bigcap_{m \in \mathcal{M}} (\bigcap_{\nu \in \mathcal{T}_m^S} (\bigcap_{i=1}^N \tilde{\mathcal{P}}_m^\nu(\mathcal{P}^{c_i}(\mathcal{O})))), \quad (5.24)$$

where $\mathcal{P}^{c_i}(\mathcal{O}) := \mathcal{P}_{m_{c_i}}^{s_{c_i}^{c_i}}(\hat{\mathcal{P}}_{m_{c_i},n}^{s_{c_i}^{c_i}} \cdots \mathcal{P}_{m_{c_i}}^{s_{r_i}^{c_i}}(\hat{\mathcal{P}}_{m_{c_i},n}^{s_{r_i}^{c_i}}(\mathcal{O})))$ is the operator to calculate the backward set in the presence of the c_i faulty combination.

5.5 FD-MDT Set Design and Analyses

In this section, Theorem 5.2 is firstly proposed to investigate three equivalent statements regarding the FD-MDT invariance.

Theorem 5.2. *Suppose Assumptions 5.1 and 5.2 are satisfied and the operators $\mathcal{R}(\cdot, \cdot)$ and $\mathcal{P}(\cdot, \cdot)$ are defined in (5.20) and (5.24). For a non-empty set $\mathcal{O} \subset \mathbb{R}^{n_x}$, the three equivalent statements are presented as follows: (i) $\mathcal{R}(\mathcal{O}, \mathcal{W}) \subseteq \mathcal{O}$; (ii) $\mathcal{O} \subseteq \mathcal{P}(\mathcal{O}, \mathcal{W})$; (iii) The set \mathcal{O} is an FD-MDT invariant set.*

Proof. (i) \Rightarrow (ii): The condition (i) implies that, for all $\nu \in \mathcal{T}_m^S$, $m \in \mathcal{M}$, and $c_i \in \mathcal{C}_N$ for all faulty combinations, the inclusion $\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})) \subseteq \mathcal{O}$ stands. Thus, applying the operator $\tilde{\mathcal{P}}_m^\nu(\mathcal{P}^{c_i}(\cdot))$ on both sides of $\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})) \subseteq \mathcal{O}$, we have

$$\tilde{\mathcal{P}}_m^\nu(\mathcal{P}^{c_i}(\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})))) \subseteq \tilde{\mathcal{P}}_m^\nu(\mathcal{P}^{c_i}(\mathcal{O})). \quad (5.25)$$

Note that on the left-hand side of (5.25), it directly follows $\tilde{\mathcal{P}}_m^\nu(\mathcal{P}^{c_i}(\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})))) = \mathcal{O}$. Therefore, by taking the intersection for all possible $\nu \in \mathcal{T}_m^S$, $m \in \mathcal{M}$, and $c_i \in \mathcal{C}_N$ for all faulty combinations, the statement (ii) $\mathcal{O} \subseteq \mathcal{P}(\mathcal{O}, \mathcal{W})$ is obtained.

(ii) \Rightarrow (iii): Given $x_0 \in \mathcal{O}$, the inclusion (ii) indicates that $x_0 \in \mathcal{P}(\mathcal{O}, \mathcal{W})$. Thus, considering all the admissible switching sequences $\xi_{\mathcal{T}[\Delta]}(k)$ under the allowable dis-

turbance sequences $\xi_{\mathcal{T}[\Delta]}^w(k)$, the inclusion $\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})) \subseteq \mathcal{O}$ follows for all $\nu \in \mathcal{T}_m^S$, $m \in \mathcal{M}$, and $c_i \in \mathcal{C}_N$, i.e., $x_k \in \mathcal{O}$, $k \in \mathbb{Z}^+$. The FD-MDT invariance of set \mathcal{O} holds.

(iii) \Rightarrow (i): This part of the proof can be completed via contradiction. Assume that the set \mathcal{O} is FD-MDT invariant, but (i) is not fulfilled. Thus, there exists a $\nu \in \mathcal{T}_m^S$, $m \in \mathcal{M}$, or a $c_i \in \mathcal{C}_N$ such that $\tilde{\mathcal{R}}_m^\nu(\mathcal{O}) \not\subseteq \mathcal{R}(\mathcal{O}, \mathcal{W})$ or $\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})) \not\subseteq \mathcal{O}$. Note that the definition of the FD-MDT invariant set requires that, for all $x_0 \in \mathcal{O}$, it implies that $x_k \in \mathcal{O}$, $k \in \mathbb{Z}^+$, with all the admissible switching sequences $\xi_{\mathcal{T}[\Delta]}(k)$ under $\xi_{\mathcal{T}[\Delta]}^w(k)$. However, the inclusion $\tilde{\mathcal{R}}_m^\nu(\mathcal{O}) \not\subseteq \mathcal{R}(\mathcal{O}, \mathcal{W})$ or $\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{O})) \not\subseteq \mathcal{O}$ does not hold which contradicts the statement (i). \blacksquare

Remark 5.8. *Three equivalent statements are presented in Theorem 5.2 for a set \mathcal{O} to be FD-MDT invariant with $\xi_{\mathcal{T}[\Delta]}^w(\cdot)$. However, based on Definition 5.6, for a set \mathcal{O} to be CAFD-MDT invariant, a more stringent condition is required, i.e., $\mathcal{R}(\mathcal{O}, \mathcal{W}) \subseteq \mathcal{X}$ for all $\nu \in \mathcal{T}^S$ as well as $c_i \in \mathcal{C}_N$ and $\nu \in \mathcal{T}^F$ when $\Omega_{2,s} = 0$, $s \in \mathbb{N}$.*

5.5.1 Algorithm Description

The computational algorithm of designing an FD-MDT set is established in this subsection. The aim is to let such a set serve as the ‘artificial origin’ for the disturbed switched closed-loop system (5.13) with multiple faults. Based on the definition (5.20), the j th stage iterative reachable set $\mathcal{Z}_j \subset \mathbb{R}^{n_x}$, $j \in \mathbb{Z}^+$, is initially defined as

$$\begin{aligned} \mathcal{Z}_j &:= \mathcal{R}(\mathcal{Z}_{j-1}, \mathcal{W}) \\ &= \cup_{m \in \mathcal{M}} (\cup_{\nu \in \mathcal{T}_m^S} (\cup_{i=1}^N \mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{Z}_{j-1}))))), \end{aligned} \quad (5.26)$$

with $\mathcal{Z}_0 = \{\mathbf{0}\}$. The \mathcal{Z}_0 is designed as the origin because it is desirable to design a minimal FD-MDT set such that the system states can eventually oscillate in a relatively small region. However, note that the union operation does not retain the convexity of the result set. Thus, the convex hull is exploited so as to obtain a convex boundary of \mathcal{Z}_j . Let \mathcal{L}_j denote the convex hull of \mathcal{Z}_j

$$\mathcal{L}_j := \text{Co}\{\mathcal{Z}_j\}. \quad (5.27)$$

Algorithm 5.2 Computation of a minimal FD-MDT invariant set \mathcal{L}_∞

- 1: **Input:** System matrices sets \mathcal{A} , $\tilde{\mathcal{A}}$, and $\hat{\mathcal{A}}$; the upper bound $\bar{\Delta}$; a total number N ; an MDT set \mathcal{T}^S ; the disturbance constraint set \mathcal{W} .
 - 2: Initialize $j = 0$, $m = 1$, $i = 1$, $\nu = \tau_m$, and $\mathcal{L}_0 = \{\mathbf{0}\}$.
 - 3: **while** $m \leq M$ **do**
 - 4: **while** $\nu \leq 2\tau_m - 1$ **do**
 - 5: **while** $i \leq N$ **do**
 - 6: $\mathcal{L}_{j+1} = \text{Co}\{\mathcal{L}_j, \mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{L}_j))\}$;
 - 7: Set $i = i + 1$;
 - 8: **end while**
 - 9: Set $\nu = \nu + 1$;
 - 10: **end while**
 - 11: Set $m = m + 1$;
 - 12: **end while**
 - 13: **If** $\mathcal{L}_{j+1} \equiv \mathcal{L}_j$, set $\mathcal{L}_\infty = \mathcal{L}_j$ and **stop**; else set $j = j + 1$ and goto **step 2**.
-

Thus, instead of using the union operation, \mathcal{Z}_j defined in (5.26) is transformed into

$$\begin{aligned} \mathcal{Z}_j &= \text{Co}\{\mathcal{R}(\mathcal{Z}_{j-1}, \mathcal{W})\} \\ &= \text{Co}\{\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{Z}_{j-1})), \forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}, c_i \in \mathcal{C}_N\}, \end{aligned} \quad (5.28)$$

by taking its convex hull. The computational algorithm of \mathcal{L}_∞ is shown as follows. In Algorithm 5.2, by collecting all the convex hulls of the reachable set $\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{L}_j))$ for every $\nu \in \mathcal{T}_m^S$, $m \in \mathcal{M}$, and $c_i \in \mathcal{C}_N$, from step 3 to step 12, an FD-MDT invariant set \mathcal{L}_∞ is claimed.

5.5.2 Theoretical Analyses

In Theorem 5.3, some important properties of the computed FD-MDT invariant set and the convergence of Algorithm 5.2 are presented with rigorous proofs.

Theorem 5.3. *Suppose Assumptions 5.1 and 5.2 are satisfied and a minimal FD-MDT \mathcal{Z}_j , $j \in \mathbb{Z}^+$, is computed through Algorithm 5.2. The following statements stand:*

- (i) $\mathcal{Z}_j \equiv \text{Co}\{\mathcal{Z}_j\}$, $\forall j \in \mathbb{N}$;
- (ii) $\{\mathbf{0}\} \subseteq \mathcal{Z}_j$ and $\mathcal{Z}_j \subseteq \mathcal{Z}_{j+1}$, $\forall j \in \mathbb{N}$;
- (iii) The FD-MDT set $\mathcal{L}_\infty := \lim_{j \rightarrow \infty} \mathcal{Z}_j$ exists. Thus, a series of sets \mathcal{Z}_j , $j \in \mathbb{Z}^+$ obtained from Algorithm 5.2 converges to \mathcal{L}_∞ ;

(iv) A bounded set $\mathcal{Z}_\infty := \lim_{j \rightarrow \infty} \mathcal{Z}_j$ exists;

(v) $\mathcal{L}_\infty = \text{Co}\{\mathcal{Z}_\infty\}$.

Proof. (i) For notational convenience, define $\mathcal{D}_{[m,\nu]}^{c_i}$ as

$$\mathcal{D}_{[m,\nu]}^{c_i} = \mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^{\nu-1} \mathcal{W} \oplus \cdots \oplus \mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m \mathcal{W} \oplus \mathcal{A}_{m_{c_i}}^{c_i} \mathcal{W} \oplus \cdots \oplus \hat{A}_{m_{c_i},n} \mathcal{W} \oplus \mathcal{W}. \quad (5.29)$$

where $\mathcal{A}_{m_{c_i}}^{c_i} := \hat{A}_{m_{c_i},n}^{s_r^{c_i}} A_{m_{c_i}}^{s_r^{c_i}} \cdots \hat{A}_{m_{c_i},n}^{s_1^{c_i}} A_{m_{c_i}}^{s_1^{c_i}}$ is the product of system matrices under the c_i th faulty combination. Due to $\mathcal{L}_0 = \mathcal{Z}_0 = \{\mathbf{0}\}$, from (5.28), it directly follows that

$$\mathcal{L}_1 = \text{Co}\{\mathcal{D}_{[m,\nu]}^{c_i}, \forall c_i \in \mathcal{C}_N, \forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}\} = \text{Co}\{\cup_{\nu \in \mathcal{T}_m^S, m \in \mathcal{M}} \cup_{i=1}^N \mathcal{D}_{[m,\nu]}^{c_i}\} = \text{Co}\{\mathcal{Z}_1\}.$$

Suppose that $\mathcal{L}_j = \text{Co}\{\mathcal{Z}_j\}$ holds. In the following, we aim to show that if $\mathcal{L}_{j+1} = \text{Co}\{\mathcal{Z}_{j+1}\}$ can be derived based on the induction approach [138]. For this purpose, the reachable set \mathcal{Z}_{j+1} is expressed by using (5.29)

$$\mathcal{Z}_{j+1} = \cup_{\nu \in \mathcal{T}_m^S, m \in \mathcal{M}} \cup_{i=1}^N (\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \mathcal{Z}_j \oplus \mathcal{D}_{[m,\nu]}^{c_i}). \quad (5.30)$$

Note that, in the process of computing \mathcal{Z}_j , every possible admissible switching sequence $\xi_{\mathcal{T}[\Delta]}(\cdot)$ with multiple faults $\forall \nu \in \mathcal{T}_m^S$, $m \in \mathcal{M}$, and $\forall c_i \in \mathcal{C}_N$ is taken into consideration. The reachable set along each considered switching sequence is denoted as $\Gamma_1, \Gamma_2, \dots, \Gamma_{(NM_\tau)^j}$ where $M_\tau = \sum_{m=1}^M \tau_m$ is the total number of elements in \mathcal{T}^S . Reformulate \mathcal{Z}_j as $\mathcal{Z}_j = \cup_{v=1}^{(NM_\tau)^j} \Gamma_v$. Thus, \mathcal{Z}_{j+1} is transformed into

$$\begin{aligned} \mathcal{Z}_{j+1} &= \cup_{\nu \in \mathcal{T}_m^S, m \in \mathcal{M}} \cup_{i=1}^N (\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu (\cup_{v=1}^{(NM_\tau)^j} \Gamma_v) \oplus \mathcal{D}_{[m,\nu]}^{c_i}) \\ &= \cup_{v=1}^{(NM_\tau)^j} \cup_{\nu \in \mathcal{T}_m^S, m \in \mathcal{M}} \cup_{i=1}^N (\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \Gamma_v \oplus \mathcal{D}_{[m,\nu]}^{c_i}) \\ &= \cup_{f=1}^{(NM_\tau)^{j+1}} \Upsilon_f, \end{aligned} \quad (5.31)$$

where Υ_f represents the reachable set along the f th admissible switching sequence in the $(j+1)$ th iteration of Algorithm 5.2. Following the concept of a convex hull, we have

$$\begin{aligned} \mathcal{L}_{j+1} &= \text{Co}\{\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \mathcal{Z}_j \oplus \mathcal{D}_{[m,\nu]}^{c_i}, \forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}, c_i \in \mathcal{C}_N\} \\ &= \{\sum_{h=1}^{NM_\tau} a_h (\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \mathcal{Z}_j \oplus \mathcal{D}_{[m,\nu]}^{c_i}) \mid \sum_{h=1}^{NM_\tau} a_h = 1\} \end{aligned} \quad (5.32)$$

from (5.28) by using the linear combinations. Due to $\mathcal{L}_j = \text{Co}\{\mathcal{Z}_j\} = \{\sum_{v=1}^{(NM_\tau)^j} b_v \Gamma_v \mid$

$\sum_{v=1}^{(NM_\tau)^j} b_v = 1\}$, substituting \mathcal{Z}_j into \mathcal{Z}_{j+1} , we have

$$\begin{aligned}\mathcal{Z}_{j+1} &= \left\{ \sum_{h=1}^{NM_\tau} a_h (\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu (\sum_{v=1}^{(NM_\tau)^j} b_v \Gamma_v) \oplus \mathcal{D}_{[m,\nu]}^{c_i}) \mid \sum_{h=1}^{NM_\tau} a_h = 1, \sum_{v=1}^{(NM_\tau)^j} b_v = 1 \right\} \\ &= \left\{ \sum_{h=1}^{NM_\tau} \sum_{v=1}^{(NM_\tau)^j} g_{hv} (\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \Gamma_v \oplus \mathcal{D}_{[m,\nu]}^{c_i}) \mid g_{hv} = a_h b_v, \sum_{h=1}^{NM_\tau} \sum_{v=1}^{(NM_\tau)^j} g_{hv} = 1 \right\} \\ &= \left\{ \sum_{f=1}^{(NM_\tau)^{j+1}} g_f \Upsilon_f \mid \sum_{f=1}^{(NM_\tau)^{j+1}} g_f = 1 \right\} \equiv Co\{\mathcal{Z}_{j+1}\}.\end{aligned}$$

Thus, we have $\mathcal{Z}_j \equiv Co\{\mathcal{Z}_j\}$, $\forall j \in \mathbb{N}$.

(ii) Since $\{\mathbf{0}\} \subseteq \mathcal{W}$, from (5.28), it follows that $\{\mathbf{0}\} \subseteq \mathcal{R}(\mathcal{Z}_{j-1}, \mathcal{W})$ and $\{\mathbf{0}\} \subseteq \mathcal{Z}_j \subseteq \mathcal{Z}_j$. Due to $\mathcal{Z}_j \subseteq \mathcal{R}(\mathcal{Z}_j, \mathcal{W})$, we have $\mathcal{Z}_j \subseteq \mathcal{Z}_{j+1}$.

(iii) Consistent with the notations in (i), define $\mathcal{W} := Co\{\mathcal{D}_{[m,\nu]}^{c_i}, \forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}, \forall c_i \in \mathcal{C}_N\}$. Following [96], there must exist a $\lambda_p \in (0, 1)$, a constant $\omega > 0$, and a proper norm ball $\mathcal{B}(e)$ such that $\mathcal{W} \subseteq \omega \mathcal{B}(e)$ and $\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \mathcal{W} \subseteq \lambda_p \omega \mathcal{B}(e)$, $\forall \nu \in \mathcal{T}_m^S$, $m \in \mathcal{M}$, and $\forall c_i \in \mathcal{C}_N$. This is because an MDT is a decision variable. Subsequently, from (5.32), we have

$$\begin{aligned}\mathcal{Z}_{j+1} &= Co\{\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \mathcal{Z}_j \oplus \mathcal{D}_{[m,\nu]}^{c_i}, \forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}, c_i \in \mathcal{C}_N\} \\ &\subseteq Co\{\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \mathcal{Z}_j \oplus \mathcal{W}, \forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}, c_i \in \mathcal{C}_N\}.\end{aligned}$$

Considering the iteration of \mathcal{Z}_j in Algorithm 5.2, due to $\mathcal{Z}_0 = \{\mathbf{0}\}$ and $\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^\nu \mathcal{W} \subseteq \lambda_p \omega \mathcal{B}(e)$, it follows that

$$\begin{aligned}\mathcal{Z}_j &\subseteq Co\{\mathcal{A}_{m_{c_i}^j}^{c_i^j} \tilde{A}_{m^j}^{\nu^j} \cdots \mathcal{A}_{m_{c_i^1}^{c_i^1}} \tilde{A}_{m^1}^{\nu^1} \mathcal{Z}_0 \oplus \mathcal{A}_{m_{c_i^j}^{c_i^j}} \tilde{A}_{m^j}^{\nu^j} \cdots \\ &\quad \mathcal{A}_{m_{c_i^2}^{c_i^2}} \tilde{A}_{m^2}^{\nu^2} \mathcal{W} \oplus \cdots \oplus \mathcal{A}_{m_{c_i^j}^{c_i^j}} \tilde{A}_{m^j}^{\nu^j} \mathcal{W} \oplus \mathcal{W}, \forall \nu \in \mathcal{T}_m^S, \\ &\quad m \in \mathcal{M}, c_i \in \mathcal{C}_N\} \subseteq \omega(\lambda_p^{j-1} + \cdots + \lambda_p + 1)\mathcal{B}(e),\end{aligned}\tag{5.33}$$

where the production $\mathcal{A}_{m_{c_i^j}^{c_i^j}} \tilde{A}_{m^j}^{\nu^j}$ represents the j th iteration of system matrices evolution in (5.29) through Algorithm 5.2 under the c_i faulty combination, $c_i \in \mathcal{C}_N$. Considering a Hausdorff metric space formed by a series of compact set in \mathbb{R}^{n_x} [149], define the Hausdorff distance as $\mathcal{D}(\mathcal{G}, \mathcal{H}) := \max\{\sup_{g \in \mathcal{G}} D(g, \mathcal{H}), \sup_{h \in \mathcal{H}} D(h, \mathcal{G})\}$. Since $\mathcal{Z}_j \subseteq \omega(\lambda_p^{j-1} + \cdots + \lambda_p + 1)\mathcal{B}(e)$, it indicates that the Hausdorff distance between \mathcal{Z}_{j+1} and \mathcal{Z}_j is bounded by $\omega \lambda_p^j \mathcal{B}(e)$. As $j \rightarrow \infty$, $\lambda_p^j \rightarrow 0$, therefore, \mathcal{Z}_∞ exists. Moreover, a series of sets \mathcal{Z}_j , $j \in \mathbb{Z}^+$, obtained from Algorithm 5.2 is Cauchy and converges to \mathcal{Z}_∞ .

(iv) The proof is similar to (iii) and omitted here.

(v) The proofs of (iii) and (iv) guarantee the existence of \mathcal{L}_∞ and \mathcal{Z}_∞ . Similar to the proof in (i), $\mathcal{L}_\infty = \text{Co}\{\mathcal{Z}_\infty\}$ holds. ■

5.5.3 Minimum \mathcal{L}_∞

The minimal characteristics of the FD-MDT set \mathcal{L}_∞ is proved in Lemma 5.2.

Lemma 5.2. *Suppose Assumptions 5.1 and 5.2 are satisfied and an FD-MDT set \mathcal{L}_∞ is obtained by using Algorithm 5.2. Then,*

- (i) \mathcal{L}_∞ is the minimal FD-MDT invariant set;
- (ii) For every admissible switching sequence $\xi_{\mathcal{T}[\Delta]}(\cdot)$ under the allowable disturbance sequence $\xi_{\mathcal{T}[\Delta]}^w(\cdot)$, the state trajectory starting from any x_0 will converge to \mathcal{L}_∞ .

Proof. (i) The set \mathcal{L}_∞ is acquired once $\mathcal{L}_{j'+1} = \mathcal{L}_{j'}$ is satisfied for a finite $j' \in \mathbb{Z}^+$. Thus, we have the inclusion

$$\text{Co}\{\mathcal{R}^{c_i}(\tilde{\mathcal{R}}_m^\nu(\mathcal{L}_\infty)), \forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}, \forall c_i \in \mathcal{C}_N\} \subseteq \mathcal{L}_\infty,$$

since $\mathcal{L}_\infty = \mathcal{L}_{j'}$. Then, the invariance of the FD-MDT set \mathcal{L}_∞ is ensured based on the necessary and sufficient condition (i) in Theorem 5.2. From (5.28), since the initial \mathcal{L}_0 is chosen as $\{\mathbf{0}\}$, the computed \mathcal{L}_∞ is the minimal FD-MDT invariant set.

(ii) As $k \rightarrow \infty$, the production $\mathcal{A}_{m_j}^{c_j} \tilde{A}_{m_j}^{\nu_j} \cdots \mathcal{A}_{m_1}^{c_1} \tilde{A}_{m_1}^{\nu_1} x_0$ will converge to the origin. Therefore, the state trajectories starting from any x_0 with admissible switching sequence $\xi_{\mathcal{T}[\Delta]}(k)$ under the allowable disturbance sequence $\xi_{\mathcal{T}[\Delta]}^w(k)$ converge to the Minkowski sum of the rest of the terms except the first term of (5.33), i.e., \mathcal{L}_∞ . ■

5.6 CAFD-MDT Set Design and Stability Results

The design algorithm of CAFD-MDT sets and a sufficient and necessary stability condition of switched systems (5.13) with multiple faults as well as the critical properties of a CAFD-MDT set are studied in this section. The primary reason to develop such a set is to include all feasible initial states of the perturbed system (5.13) along all admissible switching sequences under multiple faults such that the state trajectories are driven innerly into this set without violating the state constraint. An essential Assumption 5.3 is presented as follows:

Assumption 5.3. *It is assumed that \mathcal{W} is small enough such that $\mathcal{Z}_\infty \subset \mathcal{X}$.*

Algorithm 5.3 Computation of a contractive CAFD-MDT set \mathcal{Q}_∞^β

- 1: **Input:** The state constraint \mathcal{X} ; system matrices sets \mathcal{A} , $\tilde{\mathcal{A}}$, and $\hat{\mathcal{A}}$; the upper bound $\tilde{\Delta}$; a total number N ; a contraction factor β ; MDT sets \mathcal{T}^F and \mathcal{T}^S ; the disturbance constraint set \mathcal{W} .
 - 2: Initialize $y = 0$ and $p = 1$ and calculate the initial set $\mathcal{Q}_0^\beta = \mathcal{X} \cap (\cap_{m \in \mathcal{M}} (\cap_{\nu \in \mathcal{T}_m^F} \tilde{\mathcal{P}}_m^\nu(\mathcal{X})))$.
 - 3: **while** $p \leq N$ **do**
 - 4: Compute $\Xi = \cap_{m \in \mathcal{M}} (\cap_{\nu \in \mathcal{T}_m^S} \tilde{\mathcal{P}}_m^\nu(\mathcal{P}^{c_p}(\beta \mathcal{Q}_y^\beta)))$;
 - 5: Calculate $\mathcal{Q}_{y+1}^\beta = \mathcal{Q}_y^\beta \cap \Xi$;
 - 6: Set $p = p + 1$;
 - 7: **end while**
 - 8: **If** $\mathcal{Q}_{y+1}^\beta \equiv \mathcal{Q}_y^\beta$, set $\mathcal{Q}_\infty^\beta = \mathcal{Q}_y^\beta$ and **stop**; else set $y = y + 1$ and goto **step 2**.
-

Remark 5.9. *Assumption 5.3 guarantees the existence of a CAFD-MDT invariant set. It is a reasonable assumption which ensures the set of the accumulated effect of uncertainties will not exceed the state constraint.*

5.6.1 Design of CAFD-MDT Invariant Sets

According to (5.21) and (5.24), let $\mathcal{Q}_y^\beta \subset \mathbb{R}^{n_x}$, $y \in \mathbb{Z}^+$, denote y th stage contractive backward set as

$$\begin{aligned} \mathcal{Q}_y^\beta &:= \mathcal{P}(\beta \mathcal{Q}_{y-1}^\beta, \mathcal{W}) \\ &= \cap_{m \in \mathcal{M}} (\cap_{\nu \in \mathcal{T}_m^S} (\cap_{i=1}^N \tilde{\mathcal{P}}_m^\nu(\mathcal{P}^{c_i}(\beta \mathcal{Q}_{y-1}^\beta))))), \end{aligned} \quad (5.34)$$

with $\mathcal{Q}_0^\beta = \mathcal{X} \cap (\cap_{m \in \mathcal{M}} (\cap_{\nu \in \mathcal{T}_m^F} \tilde{\mathcal{P}}_m^\nu(\mathcal{X})))$ where $\beta \in (0, 1)$ is a contraction factor.

In the following, the algorithm to calculate the contractive CAFD-MDT set \mathcal{Q}_∞^β is presented. Step 2 of Algorithm 5.3 is designed to acquire all the system states evolved along the truncated switching sequence. The state trajectories with the admissible switching sequence under each faulty combination are collected by using the backward set operators (5.21) and (5.24) from step 3 to step 7. Algorithm 5.3 is terminated until $\mathcal{Q}_{y+1}^\beta = \mathcal{Q}_y^\beta$.

5.6.2 A Non-conservative Stability Criterion

To begin with, the concepts of asymptotic stability and UAS with respect to the switched closed-loop system (5.13) with multiple faults and $\xi_{\mathcal{T}[\tilde{\Delta}]}^w(\cdot)$ are presented as

follows:

Definition 5.7. [2, 57] *The switched system (5.13) under multiple faults is said to be asymptotically stable, if for $\|x_0\| \leq \hat{\delta}$ with a existing positive constant $\hat{\delta}$, x_t satisfies $D(x_t, \mathcal{Z}_\infty) \rightarrow 0$ as $t \rightarrow \infty$.*

Definition 5.8. [2, 57] *The switched system (5.13) under multiple faults is said to be uniformly asymptotically stable along all the admissible switching sequences $\xi_{\mathcal{T}[\bar{\Delta}]}(t)$ and the allowable disturbance sequence $\xi_{\mathcal{T}[\bar{\Delta}]}^w(t)$, if for $\|x_0\| \leq \hat{\delta}$ with a existing positive constant $\hat{\delta}$, x_t satisfies $D(x_t, \mathcal{Z}_\infty) \rightarrow 0$ as $t \rightarrow \infty$.*

Subsequently, a necessary and sufficient stability condition for the switched closed-loop system (5.13) with multiple faults is proposed.

Theorem 5.4. *Suppose Assumptions 5.1 to 5.3 are satisfied. The following two statements are equivalent.*

(i) *The CAFD-MDT set $\mathcal{Q}_\infty^\beta := \lim_{y \rightarrow \infty} \mathcal{Q}_y^\beta$ generated by Algorithm 5.3 with an MDT set Φ^w exists.*

(ii) *The switched closed-loop system (5.13) with multiple faults along the admissible switching sequence $\xi_{\Phi^w[\bar{\Delta}]}(k)$ and the allowable disturbance sequence $\xi_{\Phi^w[\bar{\Delta}]}^w(k)$ is uniformly asymptotically stable in the sense that $D(x_k, \mathcal{Z}_\infty) \rightarrow 0$ as $k \rightarrow \infty$.*

Proof. (i) \Rightarrow (ii): The sufficiency proof is presented in this part. The target is to show that the existence of such a CAFD-MDT set \mathcal{Q}_∞^β implies the state trajectories along the admissible switching sequence with $x_0 \in \mathcal{Q}_\infty^\beta$ converging to \mathcal{Z}_∞ . Consider the iteration from $(y-1)$ th stage to y th stage by using Algorithm 5.3. For $x_0 \in \mathcal{Q}_{y-1}^\beta$, we have

$$\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^{\Omega_{1,y}} x_0 \in (\beta \mathcal{Q}_{y-1}^\beta \ominus \mathcal{D}_{[m, \Omega_{1,y}]}^{c_i}), y \in \mathbb{Z}^+ \quad (5.35)$$

from (5.29), $m \in \mathcal{M}$, $c_i \in \mathcal{C}_N$. In order to ensure the state trajectory converging to \mathcal{Z}_∞ , it is desirable that $\rho(\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^{\Omega_{1,s}}) < 1$ holds for all $m \in \mathcal{M}$ and for all $c_i \in \mathcal{C}_N$. As mentioned in Assumption 5.2, the faulty portion is upper bounded. Thus, by designing an MDT set Φ^w with appropriate lengths of each element, $\rho(\mathcal{A}_{m_{c_i}}^{c_i} \tilde{A}_m^{\Omega_{1,s}}) < 1$ can be achieved for all admissible switching sequences exerted by the allowable disturbance sequence. Therefore, the inequality $\beta^{\tilde{y}} \varrho_k^w < 1$ is satisfied for all $m \in \mathcal{M}$ and for all $c_i \in \mathcal{C}_N$ where ϱ_k^w is the production with respect to the maximal Euclidean norm of the eigenvalues regarding every system matrix of $\mathcal{A}_{\sigma(k_{2s})_{c_i}^{\tilde{y}}}^{c_i^{\tilde{y}}} \tilde{A}_{\sigma(k_{2s})}^{\Omega_{1,s}} \cdots \mathcal{A}_{\sigma(k_0)_{c_i}^y}^{c_i^y} \tilde{A}_{\sigma(k_0)}^{\Omega_{1,0}}$ where $\tilde{y} := \lfloor \frac{\Omega_{1,0}}{\phi_{\sigma(k_0)}^w} \rfloor + \lfloor \frac{\Omega_{1,1}}{\phi_{\sigma(k_2)}^w} \rfloor + \cdots + \lfloor \frac{\Omega_{1,s}}{\phi_{\sigma(k_{2s})}^w} \rfloor$, $s \in \mathbb{N}$ and $c_i^y \in \mathcal{C}_N$ is associated with

y th stage. Since $\beta \in (0, 1)$ and $\tilde{y} \rightarrow \infty$ as $k \rightarrow \infty$, the trajectories of system states will finally converge to \mathcal{Z}_∞ if $x_0 \in \mathcal{Q}_\infty^\beta$, i.e., $D(x_k, \mathcal{Z}_\infty) \rightarrow 0$ as $k \rightarrow \infty$. Thus, the closed-loop UAS of (5.13) is claimed. The proof of sufficiency is completed.

(ii) \Rightarrow (i): The argument of this part is to demonstrate that the contractive CAFD-MDT invariant set \mathcal{Q}_∞^β can be obtained through finite-time iterations by employing Algorithm 5.3. Define $\|\cdot\|_{\mathcal{Q}_\infty^\beta}$ as the induced norm of the CAFD-MDT set \mathcal{Q}_∞^β . Assuming there exist a contraction factor $\bar{\beta} \in (0, 1)$ such that

$$\|\mathcal{A}_{m_y^{c_y}}^{\nu_y} \tilde{A}_{m_y}^{\nu_y} \cdots \mathcal{A}_{m_1^{c_1}}^{\nu_1} \tilde{A}_{m_1}^{\nu_1}\|_{\mathcal{Q}_\infty^\beta} < \bar{\beta}^y \varrho_k^w < 1 \quad (5.36)$$

for all $\nu_y \in \mathcal{T}_m^S$, $m^y \in \mathcal{M}$ and for all $c_i^y \in \mathcal{C}_N$, $y \in \mathbb{Z}^+$. Rewrite the set $\mathcal{Q}_0^{\bar{\beta}}$ as $\mathcal{Q}_0^{\bar{\beta}} := \{x | \tilde{H}_M x \leq \mathbf{1}, \forall M \in \mathbb{Z}^+\}$. Consider the iteration of \mathcal{Q}_y^β to \mathcal{Q}_{y+1}^β from step 3 to step 7 of Algorithm 5.3 for any $\tilde{\beta} > \bar{\beta}$. An additional inequality is supposed to be applied to $\mathcal{Q}_y^{\tilde{\beta}}$ in the form of

$$\begin{aligned} & \tilde{H}_M (\tilde{A}_{m_1}^{\nu_1} \mathcal{A}_{m_1^{c_1}}^{\nu_1} \cdots \tilde{A}_{m_{y+1}}^{\nu_{y+1}} \mathcal{A}_{m_{y+1}^{c_{y+1}}}^{\nu_{y+1}}) x \leq [\tilde{\beta}^{y+1} \mathbf{1} - \\ & \tilde{\beta}^y \bar{E}_{[m^1, \nu_1, w]}^{c_1}(\tilde{H}_M) - \tilde{\beta}^{y-1} \bar{E}_{[m^2, \nu_2, w]}^{c_2}(\tilde{H}_M \tilde{A}_{m_1}^{\nu_1} \mathcal{A}_{m_1^{c_1}}^{\nu_1}) - \\ & \cdots - \bar{E}_{[m^{y+1}, \nu_{y+1}, w]}^{c_{y+1}}(\tilde{H}_M \tilde{A}_{m_1}^{\nu_1} \mathcal{A}_{m_1^{c_1}}^{\nu_1} \cdots \tilde{A}_{m_y}^{\nu_y} \mathcal{A}_{m_y^{c_y}}^{\nu_y})] \end{aligned} \quad (5.37)$$

for all $\nu_y \in \mathcal{T}_m^S$, $m^y \in \mathcal{M}$, and for all $c_i^y \in \mathcal{C}_N$, $y \in \mathbb{Z}^+$, where $\mathcal{A}_{m_{c_i}}^{\nu_{c_i}} := A_{m_{c_i}}^{s_1^{c_i}} \hat{A}_{m_{c_i}, n}^{s_1^{c_i}} \cdots A_{m_{c_i}}^{s_r^{c_i}} \hat{A}_{m_{c_i}, n}^{s_r^{c_i}}$ and

$$\begin{aligned} \bar{E}_{[m^y, \nu_y, w]}^{c_y}(\tilde{H}_M) & := \max_{w \in \mathcal{W}} \tilde{H}_M \tilde{A}_{m^y}^{\nu_y-1} \mathcal{A}_{m^y^{c_y}}^{\nu_y} w + \cdots + \\ & \max_{w \in \mathcal{W}} \tilde{H}_M \tilde{A}_{m^y}^{\nu_y-1} A_{m_{c_i}} w + \max_{w \in \mathcal{W}} \tilde{H}_M \tilde{A}_{m^y}^{\nu_y-1} w + \\ & \cdots + \max_{w \in \mathcal{W}} \tilde{H}_M \tilde{A}_{m^y} w + \max_{w \in \mathcal{W}} \tilde{H}_M w. \end{aligned} \quad (5.38)$$

The right-hand side of (5.37) is greater than 0 because each $\mathcal{Q}_y^{\tilde{\beta}}$ is non-empty. The following proof will show that the additional inequality (5.37) is redundant when $\mathcal{Q}_{y+1}^{\tilde{\beta}}$ is evolved from $\mathcal{Q}_y^{\tilde{\beta}}$. Inspired by [138], for each row of (5.38), we assume that $\bar{E}_{[m^j, \nu_j, w]}^{c_j}(\tilde{H}_M) = \kappa$. Then, for all $\nu_v \in \mathcal{T}_m^S$, $m^v \in \mathcal{M}$, and for all $c_i^v \in \mathcal{C}_N$, $v \in [1, y]$, $y \in \mathbb{Z}^+$, we have $\bar{\beta} \bar{E}_{[m^v, \nu_v, w]}^{c_v}(\tilde{H}_M \tilde{A}_{m^v}^{\nu_v} \mathcal{A}_{m^v^{c_v}}^{\nu_v}) \leq \bar{\beta} g_v \varrho_v \kappa$ where $g_v > 0$ and $\varrho_v \in (0, 1)$.

Thus, let

$$\bar{\kappa} := \max_{\nu_y \in \mathcal{T}_m^S, m \in \mathcal{M}, c_i \in \mathcal{C}_N} \bar{E}_{[m^y, \nu_y, w]}^{c_i^y}(\tilde{H}_M).$$

Therefore, a $g > 0$ and a $\varrho \in (0, 1)$ can be found such that

$$\max_{\nu_v \in \mathcal{T}_m^S, m^v \in \mathcal{M}, c_i^v \in \mathcal{C}_N} \bar{\beta} \bar{E}_{[m^v, \nu_v, w]}^{c_i^v}(\tilde{H}_M \tilde{A}_{m^v}^{\nu_v} \tilde{\mathcal{A}}_{m_{c_i^v}^v}^{c_i^v}) \leq \bar{\beta} g \varrho \bar{\kappa}.$$

Subsequently, each row of the item in the bracket of (5.37) is bounded by

$$\begin{aligned} & \bar{\beta}^{y+1} - (\bar{\beta}^y \bar{\kappa} + \bar{\beta}^{y-1} g \varrho \bar{\kappa} + \cdots + g \varrho^y \bar{\kappa}) \\ &= \bar{\beta}^{y+1} - \bar{\beta}^y \bar{\kappa} - \bar{\beta}^{y-1} g \varrho \bar{\kappa} \frac{1 - (\varrho/\bar{\beta})^y}{1 - \varrho/\bar{\beta}} \\ &= \bar{\beta}^{y+1} (1 - \bar{\kappa}/\bar{\beta} + g \varrho \bar{\kappa} \frac{1 - (\varrho/\bar{\beta})^y}{\bar{\beta}^2(\varrho/\bar{\beta} - 1)}). \end{aligned} \quad (5.39)$$

For the left-hand side of (5.37), from (5.12), it follows that

$$\tilde{H}_M(\tilde{A}_{m^1}^{\nu_1} \tilde{\mathcal{A}}_{m_{c_1}^1}^{c_1^1} \cdots \tilde{A}_{m^{y+1}}^{\nu_{y+1}} \tilde{\mathcal{A}}_{m_{c_i^{y+1}}^{y+1}}^{c_i^{y+1}})x \leq \bar{\beta}^{y+1} \varrho_k^w \eta' \|\tilde{H}_M\| \|x\| \leq \tilde{\beta}^{y+1}, \quad (5.40)$$

where $\eta' > 0$ is a positive real number. Since $\bar{\beta} < \tilde{\beta}$, the last inequality of (5.40) holds. Hence, similar to Theorem 5.1, inequality (5.40) is also eligible for a larger $\tilde{\beta}$.

Since the magnitude of ϱ and $\bar{\beta}$ are adjustable, from (5.39) and (5.40), a sufficiently large \hat{y} must exist such that

$$\max\{\varrho_k^w \eta' \|\tilde{H}_M\| \|x\| \|x\| \in \mathcal{Q}_0^\beta, M \in \mathbb{Z}^+\} < 1 - \bar{\kappa}/\bar{\beta} + g \varrho \bar{\kappa} \frac{1 - (\varrho/\bar{\beta})^{\hat{y}}}{\bar{\beta}^2(\varrho/\bar{\beta} - 1)} \quad (5.41)$$

suffices for all $y \geq \hat{y}$. Hence, the contractive CAFD-MDT invariant set $\mathcal{Q}_\infty^\beta \equiv \mathcal{Q}_{\hat{y}}^\beta$ can be obtained with finite-time iterations by using Algorithm 5.3. The proof of necessity is accomplished. \blacksquare

Similar to Corollaries 5.1 and 5.2, the convergence performance of system states along two categories of particular switching sequences which violate the requirement of admissible switching sequences in Definition 5.2 is studied next.

Corollary 5.3. *Suppose that the synchronous portion $\Omega_{1,s}$ does not fulfill the MDT constraint and the length of faulty portions $\Omega_{2,s} \equiv 0, \forall s \in \mathbb{N}$. The switched closed-loop system (5.13) is asymptotically stable in the sense that $D(x_k, \tilde{\mathcal{Z}}_\infty) \rightarrow 0$ as $k \rightarrow \infty$, if the mode transitions are well scheduled, i.e., $\varrho_k^w < 1$, where $\tilde{\mathcal{Z}}_\infty$ is the FD-MDT set*

designed for the prescribed switching sequence.

Corollary 5.4. *Suppose the length of synchronous portions $\Omega_{1,s} \equiv 0$ for all $s \in \mathbb{N}$. The asynchronously switched system (5.13) under controller failures and the arbitrary switching fault is asymptotically stable in the sense that $D(x_k, \tilde{Z}_\infty) \rightarrow 0$ as $k \rightarrow \infty$, if the switching sequence is elaborately designed, i.e., $q_k^w < 1$.*

Remark 5.10. *By adopting Algorithm 5.3 with the above predefined switching sequence, the constraint satisfaction of the closed-loop system can also be guaranteed if $x_0 \in \mathcal{Q}_\infty^\beta$ is satisfied.*

5.6.3 Maximum \mathcal{Q}_∞^β

The maximal characteristic of the set \mathcal{Q}_∞^β with respect to a contraction factor β is rigorously proved in Lemma 5.3.

Lemma 5.3. *Suppose Assumptions 5.1 to 5.3 are satisfied. An non-empty $\mathcal{Q}_y^\beta, \forall y \in \mathbb{N}$, is obtained by using Algorithm 5.3. Then, the inclusion $\mathcal{Q}_y^\beta \in \mathcal{X}$ and $\mathcal{Q}_{y+1}^\beta \subseteq \beta \mathcal{Q}_y^\beta$ hold, $\forall y \in \mathbb{N}$. In addition, \mathcal{Q}_∞^β is the maximal CAFD-MDT invariant set with respect to a contraction factor $\beta \in (0, 1)$.*

Proof. From step 2 and step 5 in Algorithm 5.3, it directly follows that $\mathcal{Q}_y^\beta \in \mathcal{X}$ and $\mathcal{Q}_{y+1}^\beta \subseteq \beta \mathcal{Q}_y^\beta$.

The contractive CAFD-MDT invariant property of \mathcal{Q}_∞^β is firstly proved. Since \mathcal{Q}_∞^β is generated once the condition $\mathcal{Q}_{y^*+1}^\beta = \mathcal{Q}_{y^*}^\beta$ is satisfied, $y^* \in \mathbb{N}$. Therefore,

$$\mathcal{Q}_\infty^\beta \subseteq \mathcal{P}(\beta \mathcal{Q}_\infty^\beta, \mathcal{W})$$

is satisfied, $\forall \nu \in \mathcal{T}_m^S, m \in \mathcal{M}, \forall c_i \in \mathcal{C}_N$. Hence, \mathcal{Q}_∞^β is FD-MDT invariant because of the necessary and sufficient condition (ii) proposed in Theorem 5.2. Since the inclusions $\mathcal{Q}_\infty^\beta \subseteq \mathcal{X}$ and $\mathcal{Q}_{y+1}^\beta \subseteq \beta \mathcal{Q}_y^\beta$ follow from step 1 and step 4 of Algorithm 5.3, the computed \mathcal{Q}_∞^β is also a contractive CAFD-MDT invariant set for all admissible switching sequence $\xi_{\mathcal{T}[\bar{\Delta}]}(\cdot)$ under the disturbance sequence $\xi_{\mathcal{T}[\bar{\Delta}]}^w(\cdot)$.

The proof that \mathcal{Q}_∞^β is the maximal CAFD-MDT set with respect to a contraction factor $\beta \in (0, 1)$ is similar to Lemma 5.1 and omitted here. \blacksquare

5.7 Simulation Results

The simulation regarding each algorithm is conducted via MATLAB MPT3 toolbox [143]. The following switched linear system with two modes is considered

$$A_1 = \begin{bmatrix} 0.94 & 0.56 \\ 0.3 & -0.46 \end{bmatrix}, A_2 = \begin{bmatrix} 0.75 & 0.23 \\ 0.75 & 0.53 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.69 & 2.65 \end{bmatrix}^T, B_2 = \begin{bmatrix} 1 & -1 \end{bmatrix}^T.$$

The state constraint is $\mathcal{X} := \{x_k \in \mathbb{R}^2 \mid \|x_k\|_\infty \leq 3\}$ and the disturbance constraint is $\mathcal{W} := \{w_k \in \mathbb{R}^2 \mid \|w_k\|_\infty \leq 0.008\}$. The stabilizing feedback gains are $K_1 = \begin{bmatrix} -0.5 & 0.3 \end{bmatrix}$ and $K_2 = \begin{bmatrix} -0.2 & -0.3 \end{bmatrix}$ and the corresponding eigenvalues of \tilde{A}_1 and \tilde{A}_2 are $0.81 \pm 0.3691i$ and $0.69 \pm 0.2166i$, respectively. When the controller failure occurs during the faulty portion, there is no control action which results in the switched systems (5.2) and (5.14) with unstable system matrices back into use. The eigenvalues of A_1 and A_2 are 1.0512, -0.5712 and 1.0697, 0.2103, respectively. Note that there is no specific assumption requiring that the asynchronous system matrices in (5.3) and (5.15) are supposed to be stable. Therefore, two unstable asynchronous system matrices $\hat{A}_{1,2}$ and $\hat{A}_{2,1}$ which the eigenvalues are respectively 0.9998, -1.1768 and 1.054, -0.574 are employed to validate the proposed control strategy. The contraction factors are respectively chosen as $\lambda = 0.999$ and $\beta = 0.98$.

Table 5.1 and Table 5.2 show the obtained minimum MDT requirement for switched systems (5.1) and (5.13) under 3 distinct durations of faulty portions, respectively. It can be observed that, as the length of faulty portions increasing, a much longer MDT restriction is required. The reason behind this is that the incremental length of the faulty portions will exacerbate the adverse effects on the convergence of system states. Note that the minimum MDT constraint regarding the perturbed system (5.13) is larger than or at least equal to the MDT requirement with respect to the nominal switched system (5.1). This is because the disturbed system (5.13) employing the minimum MDT constraint regarding the nominal system in Table 5.1 may lead to the minimum FD-MDT set exceeding the system constraint, i.e., $\mathcal{Z}_\infty \not\subset \mathcal{X}$. Thus, no solutions of the feasible initial states can be found for the switched system (5.13) under additive uncertainties.

Figure 5.5 illustrates the computed CAF-MDT contractive set in the presence of multiple faults $\bar{\Delta} = 2$ under 3 distinct combinations of admissible MDTs via Algo-

Table 5.1: The minimum MDT requirement under different lengths of faulty portions regarding the nominal system (5.1)

The Length of Faulty Portions	Minimum Values of MDTs
$\bar{\Delta} = 1$	$\tau_1 = 3, \tau_2 = 2$
$\bar{\Delta} = 2$	$\tau_1 = 4, \tau_2 = 3$
$\bar{\Delta} = 3$	$\tau_1 = 6, \tau_2 = 4$

Table 5.2: The minimum MDT requirement under different lengths of faulty portions regarding the perturbed system (5.13)

The Length of Faulty Portions	Minimum Values of MDTs
$\bar{\Delta} = 1$	$\tau_1 = 4, \tau_2 = 2$
$\bar{\Delta} = 2$	$\tau_1 = 5, \tau_2 = 3$
$\bar{\Delta} = 3$	$\tau_1 = 6, \tau_2 = 4$

algorithm 5.1. It is worth mentioning that the inclusion $\Theta_\infty^\lambda(\tau_1 = 4, \tau_2 = 3) \subset \Theta_\infty^\lambda(\tau_1 = 5, \tau_2 = 3) \subset \Theta_\infty^\lambda(\tau_1 = 5, \tau_2 = 4)$ holds where the applied MDT constraint is described in the bracket. The existence of such an inclusion is because extending the duration of the synchronous portion will enhance the performance of state convergence. Figure 5.6 demonstrates the calculated CAF-MDT set under 3 distinct lengths of faulty portions from $\bar{\Delta} = 1$ to $\bar{\Delta} = 3$. The identical MDT constraint $\tau_1 = 6, \tau_2 = 4$ is applied in the 3 sets computation since it can guarantee the existence of a CAF-MDT set when $\bar{\Delta} = 3$. It can be seen that, as the length of faulty portions increased, a tighter CAF-MDT set is obtained. In other words, it means that the region for choosing the initial state x_0 of closed-loop system (5.1) which ensures that the evolved states to be constraint admissible is getting smaller. Four state trajectories in Figure 5.7 which are respectively denoted by traj1 to traj4 are plotted to demonstrate the convergence of system states under multiple faults $\bar{\Delta} = 2$. The initial states of traj1 to traj4 are chosen as the four vertices of the CAF-MDT set $x_{10} = [-0.668; 3]$, $x_{20} = [-1.9463; 3]$, $x_{30} = [1.9463; -3]$, and $x_{40} = [0.668; -3]$, respectively. The corresponding admissible switching sequences are generated arbitrarily satisfying restrictions in Definition 5.2.

Similarly, Figure 5.8 displays the acquired FD-MDT sets and CAFD-MDT sets under 3 sets of admissible MDTs. The inclusion $\mathcal{Q}_\infty^\beta(\tau_1 = 5, \tau_2 = 3) \subset \mathcal{Q}_\infty^\beta(\tau_1 = 5, \tau_2 = 4) \subset \mathcal{Q}_\infty^\beta(\tau_1 = 6, \tau_2 = 4)$ still holds. However, for the FD-MDT set, the opposite inclusion $\mathcal{L}_\infty(\tau_1 = 6, \tau_2 = 4) \subset \mathcal{L}_\infty(\tau_1 = 5, \tau_2 = 4) \subset \mathcal{L}_\infty(\tau_1 = 5, \tau_2 = 3)$ stands since a relatively long synchronous portion leads to a smaller convex hull in

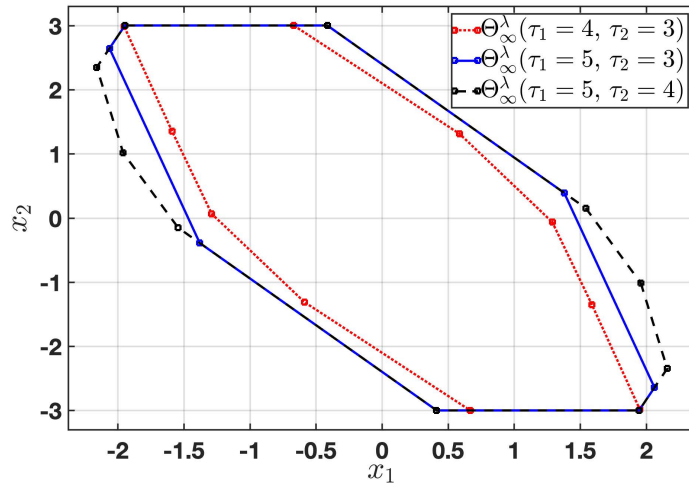


Figure 5.5: Illustration of the CAF-MDT set in the presence of multiple faults $\bar{\Delta} = 2$ under 3 distinct combinations of admissible MDTs.

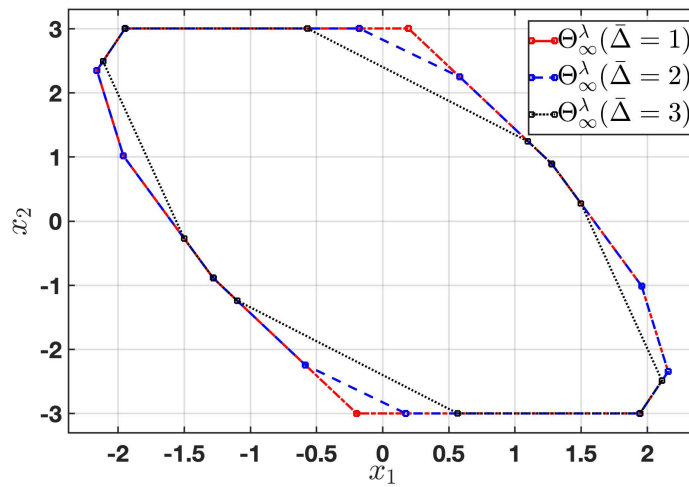


Figure 5.6: Illustration of the CAF-MDT set under different lengths of faulty portions (MDT constraint: $\tau_1 = 6, \tau_2 = 4$).

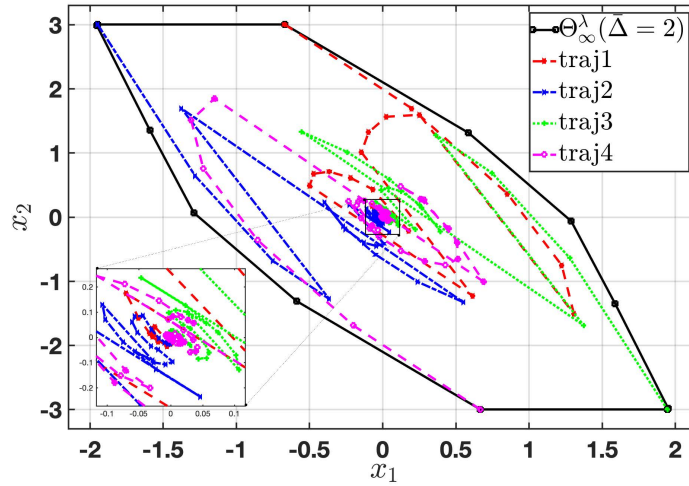


Figure 5.7: Illustration of the CAF-MDT set in the presence of multiple faults $\bar{\Delta} = 2$ and state trajectories.

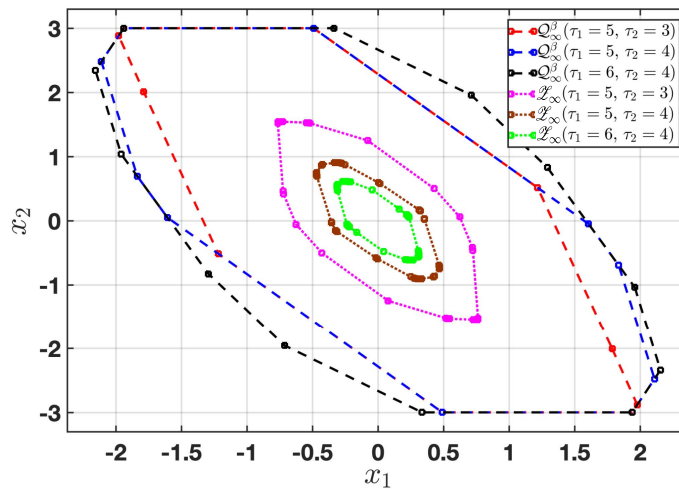


Figure 5.8: Illustration of the FD-MDT and CAFD-MDT sets in the presence of multiple faults $\bar{\Delta} = 2$ under 3 distinct combinations of admissible MDTs.

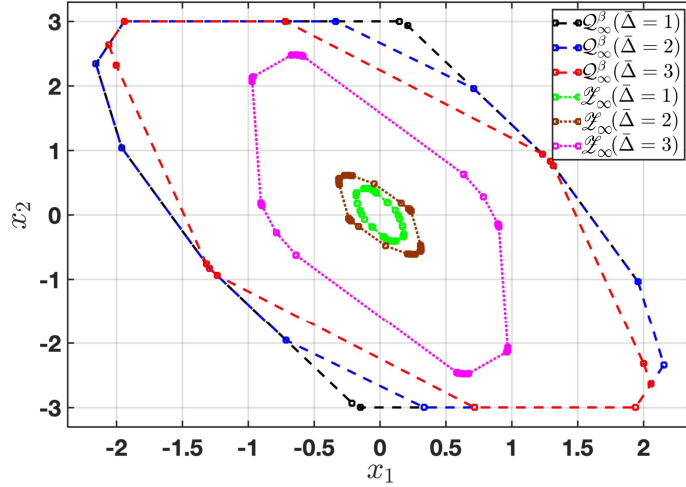


Figure 5.9: Illustration of the FD-MDT and CAFD-MDT sets under different lengths of faulty portions (MDT constraint: $\tau_1 = 6, \tau_2 = 4$).

the calculation process in Algorithm 5.2. In Figure 5.9, under 3 faulty portions from $\bar{\Delta} = 1$ to $\bar{\Delta} = 3$, the computed FD-MDT and CAFD-MDT sets are demonstrated. The similar inclusions $\mathcal{Z}_\infty(\bar{\Delta} = 1) \subset \mathcal{Z}_\infty(\bar{\Delta} = 2) \subset \mathcal{Z}_\infty(\bar{\Delta} = 3)$ and $\mathcal{Q}_\infty^\beta(\bar{\Delta} = 3) \subset \mathcal{Q}_\infty^\beta(\bar{\Delta} = 2) \subset \mathcal{Q}_\infty^\beta(\bar{\Delta} = 1)$ hold. Comparing with Figure 5.6 and Figure 5.9, it can be observed that the region of the CAFD-MDT set is smaller than the CAF-MDT set in the presence of the identical MDT constraint and the same duration of multiple faults. The reason behind is that the existence of additive disturbances results in the shrinking region for choosing the feasible initial states. Figure 5.10 shows the state responses for closed-loop system (5.13) under multiple faults $\bar{\Delta} = 2$. The initial states of *traja* and *trajb* are chosen as $x_{a0} = [-0.4908; 3]$ and $x_{b0} = [1.9385; -3]$ which are two vertices of $\mathcal{Q}_\infty^\beta(\tau_1 = 5, \tau_2 = 3)$ in Figure 5.8. The activated modes and controllers of *traja* and *trajb* are respectively shown on the top and bottom of this figure. The convergence of system states verifies the effectiveness of the proposed control scheme in Theorem 5.4. Another interesting finding is that the maximum tolerance of $\|w\|_\infty$ regarding the switched system (5.13) with multiple faults can be explored by testing whether the solution of a \mathcal{Q}_∞^β exists or not.

5.8 Conclusion

In this chapter, the non-conservative stability criteria for a class of constrained switched linear systems under multiple faults have been proposed from a set-theory

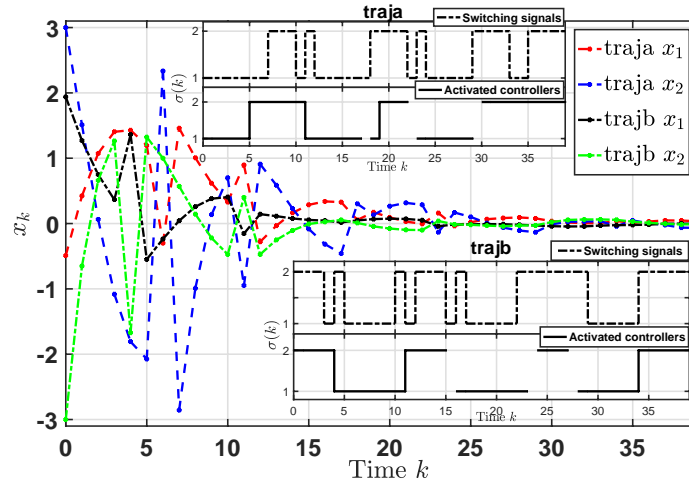


Figure 5.10: State trajectories in the presence of multiple faults $\bar{\Delta} = 2$ and additive uncertainties as well as the corresponding activated modes and controllers.

perspective. For nominal switched systems, an algorithm to calculate the maximal CAF-MDT contractive set has been established. In addition, a necessary and sufficient UAS condition for the switched closed-loop system under the multiple faults is provided with a rigorous proof. In the presence of additive uncertainties, computational algorithms to construct the minimal FD-MDT set and maximal contractive CAFD-MDT set have been designed. In the sense of the FD-MDT set serving as the equilibrium, the switched closed-loop system under multiple faults is guaranteed to be uniformly asymptotically stable as long as the designed MDT constraint is satisfied. Furthermore, the satisfaction of the state constraint is ensured. Numerical examples verify the validity of the theoretical results.

Chapter 6

Robust Model Predictive Control for Asynchronously Switched Linear Systems with Intermittent Controller Failures

Chapter 5 presents the non-conservative stability strategy for a class of constrained switched systems in the joint effects of multiple faults and external disturbances. In this chapter, we aim to utilize the superior stability results in handling the RMPC problem for switching dynamics.

6.1 Introduction

In engineering practice, switching dynamics are considered a powerful modeling tool. However, such systems are vulnerable to the variations from the environment, such as external disturbances, as well as unreliable networks, such as desired controllers delayed to enabled subsystems or even controllers disconnecting with activated modes. These challenges may result in the malfunction of switched systems and eventually pose risks to safe and reliable operations. Consequently, to confront these issues, this chapter aims to address the RMPC problem for asynchronously switched systems with controller failures. Figure 6.1 shows a disturbed asynchronously switched system with controller failures in a typical airplane control system by utilizing mode-dependent MPC controllers. The primary objective of this illustration is to provide an intuitive

understanding of the theoretical problem that the paper aims to tackle.

Representative works summarized in Table 1.3 indicate that very few efforts have been conducted on the challenging yet significant asynchronously switched MPC problem, particularly when considering the joint effects of external disturbances and controller failures [114–126, 128–130]. In [117], the MPC problem for switched systems with switching detection delays is explored. However, the use of ADT conditions may cause the shrink of feasible regions for all modes. Inspired by RMPC design for switched linear systems presented in [126], a less conservative methodology subject to MDT constraints is proposed to address this problem via switching logic scheduling so as to reduce the sacrifice of system performance to some extent. The **main contributions** of this chapter are threefold:

- This chapter addresses the significant yet challenging RMPC problem for a class of asynchronously switched linear systems with controller failures via the tube-based MPC approach. Constraint satisfaction is guaranteed by properly tightening original system constraints.
- Regarding the switching dynamics with asynchronous switching and controller failures, the persistent feasibility of the switched MPC design is ensured by forcing the evolved reachable sets to be included in a target feasible region. Thereby, the minimum MDT is determined offline. Moreover, the attractivity of the closed-loop solutions is achieved with the online stage MDT computation.
- In order to guarantee the uniform asymptotic stability (UAS) of the closed-loop system, a common terminal set is designed for all switching subsystems. Meanwhile, a non-conservative stability condition is proposed for switched systems with the state feedback control law. Furthermore, two UAS criteria are developed by driving the state trajectory into the common terminal set and the determination of MDT is achieved offline.

The remainder of this chapter is organized as follows. Section 6.2 introduces useful definitions and preliminaries. Section 6.3 provides the preliminary results of constructing disturbance invariant sets, which are crucial for the employment of tube-based MPC techniques. Then, Section 6.4 formulates the problem of RMPC for switched linear systems. Persistent feasibility results and stability analysis are presented in Section 6.5 and Section 6.6, respectively. In Section 6.7, the numerical

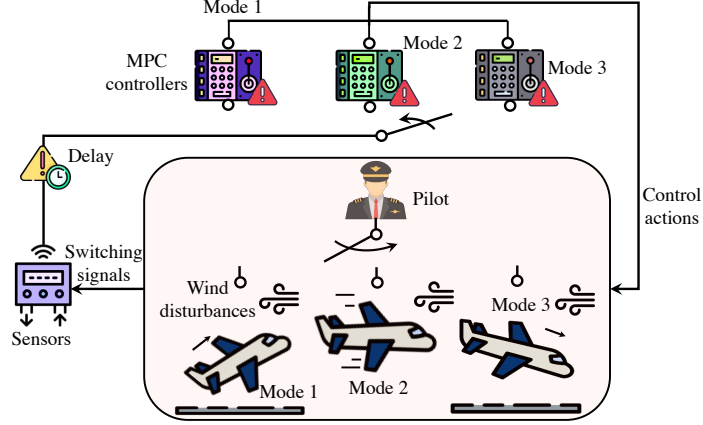


Figure 6.1: A practical example of an asynchronously switched system under the joint effects of external disturbances and controller failures with three modes (Mode 1: taking off mode; Mode 2: cruise mode; Mode 3: landing mode).

simulation results are demonstrated. Finally, Section 6.8 concludes this paper and highlights promising research directions.

6.2 Preliminaries

Consider a constrained discrete-time switched linear system in the presence of additive disturbances with unstable mode matrices $A_m \in \mathbb{R}^{n_x \times n_x}$, $m \in \mathcal{M}$, where $\mathcal{M} := \{1, 2, \dots, m, \dots, M\}$ contains all admissible switching modes and $M \in \mathbb{Z}^+$ represents the total numbers of subsystems. The disturbed switched system is given as

$$x_{k+1} = A_{\sigma(k)}x_k + B_{\sigma(k)}u_k + w_k, \quad \sigma(k) \in \mathcal{M}, \quad (6.1)$$

where the time instant $k \in \mathbb{N}$; $x_k \in \mathbb{R}^{n_x}$ and $u_k \in \mathbb{R}^{n_u}$ are state and control input vectors, respectively. $w_k \in \mathbb{R}^{n_x}$ is the unknown and time-varying disturbance vector. The switching signal $\sigma(\cdot) : \mathbb{N} \rightarrow \mathcal{M}$ is a piecewise constant function of time k and it indicates the currently activated mode in (6.1). In addition, the switching signal $\sigma(\cdot)$ is considered to be unknown *a priori*, but is known instantly. The system constraints are denoted as follows:

$$x_k \in \mathbb{X}_{\sigma(k)} \subseteq \mathbb{R}^{n_x}, u_k \in \mathbb{U}_{\sigma(k)} \subseteq \mathbb{R}^{n_u}, \quad (6.2a)$$

$$w_k \in \mathbb{W} \subseteq \mathbb{R}^{n_x}, \quad (6.2b)$$

where the state constraint \mathbb{X}_m and control input constraint \mathbb{U}_m , $m \in \mathcal{M}$, are mode-dependent while a unified disturbance constraint \mathbb{W} is imposed on all subsystems in (6.1). It is assumed that the constraint sets are all compact, convex polytopes and the origin is contained in each of their interiors. In addition, we also assume that the subsystem pairs (A_m, B_m) are stabilizable, $\forall m \in \mathcal{M}$, and all the state information can be measured at any time $k \in \mathbb{N}$.

Before introducing advanced control techniques, namely mode-dependent dwell time (MDT) and stage MDT, we first define the switching instants $k_{s-1} \in \mathcal{S} := \{k_0, k_1, \dots, k_{s-1}, \dots\}$, $s \in \mathbb{Z}_{[1,+\infty)}^+$, with $k_0 = 0$ and $k_{s-1} < k_s$. Thus, the switched system (6.1) stays in subsystem $\sigma(k_{s-1})$ when $k \in [k_{s-1}, k_s)$. The definition of MDT is given below.

Definition 6.1. [137] Consider the switched system in (6.1) with switching instants $k_0, k_1, \dots, k_{s-1}, \dots$. The MDT $\tau_m \in \mathbb{Z}^+$, associated with mode $m \in \mathcal{M}$ is the minimum amount of time that the system (6.1) has to stay in subsystem m . Thus, the MDT constraint $k_s - k_{s-1} \geq \tau_m$ is imposed when $\sigma(k) = m$ for $k \in [k_{s-1}, k_s)$, $s \in \mathbb{Z}^+$.

The set of all admissible MDTs which satisfy the MDT constraint is denoted as $\mathcal{T} := \{\tau_1, \tau_2, \dots, \tau_M\}$. Consider the s th stage mode transition beginning with k_{s-1} and ending with k_s , $s \in \mathbb{Z}^+$. The definition of stage MDT is presented as follows:

Definition 6.2. [126] Consider switched system (6.1) with the switching instants $k_0, k_1, \dots, k_{s-1}, \dots$. The MDT $\tau_m^{(s)} \in \mathbb{Z}^+$, $s \in \mathbb{Z}^+$, associated with mode $m \in \mathcal{M}$ is said to be the s th stage MDT if $k_s - k_{s-1} \geq \tau_m^{(s)}$ when $\sigma(k) = m$ for $k \in [k_{s-1}, k_s)$.

Due to the delay in transmitting switching signals to the mode-dependent controller side, it is impractical to consider the mode-dependent controller can be applied to activated subsystem in real time [44, 66]. What is even worse, the controller may disconnect with the enabled subsystem, i.e., $u_k = \mathbf{0}$. Hence, the adverse effects of asynchronous switching and controller failures are essential to be considered. To have a concrete description of the above two challenging issues, let $T_{[A]} \in \mathbb{N}$ represent the duration of the asynchronous switching and denote $T_{[F]} \in \mathbb{N}$ as the duration of controller failures. Moreover, we define the synchronous portion $\Omega_{1,s-1}$, $s \in \mathbb{Z}^+$, which obeys the MDT constraint and the faulty portion $\Omega_{2,s-1}$ affected by the asynchronous switching and controller failures as

$$\begin{cases} \Omega_{1,s-1} := [\bar{k}'_{s-1}, k'_s), \\ \Omega_{2,s-1} := [k'_s, \bar{k}'_s), s \in \mathbb{Z}^+, \end{cases} \quad (6.3)$$

where $k'_0, k'_1, \dots, k'_{s-1} \dots$ with $k'_0 = k_0$ denote the ideal switching instants at which the aligned mode-dependent controller is supposed to be implemented and $\bar{k}'_0, \bar{k}'_1, \dots, \bar{k}'_{s-1} \dots$ with $\bar{k}'_0 = k_0$ representing the time instants at which the switched system (6.1) reverts to synchronous portions.

Assumption 6.1. *The length of the faulty portion $\Omega_{2,s-1}$, $s \in \mathbb{Z}^+$, satisfies $0 \leq \bar{k}'_s - k'_s \leq \bar{\Delta}$ where $\bar{\Delta} \in \mathbb{Z}^+$ denotes the upper bound duration of $\Omega_{2,s-1}$. Moreover, the time instants k'_s and \bar{k}'_s coincide with the sampling instant k .*

Remark 6.1. *Assumption 6.1 ensures the faulty portion $\Omega_{2,s-1} \in [0, \bar{\Delta}]$, $\forall s \in \mathbb{Z}^+$ and $\Omega_{2,s-1} = 0$ indicates that there is no occurrence of either asynchronous switching or controller failures. In addition, the durations $T_{[A]} \in [0, \bar{\Delta}]$ and $T_{[F]} \in [0, \bar{\Delta}]$ are also fulfilled, $\forall s \in \mathbb{Z}^+$.*

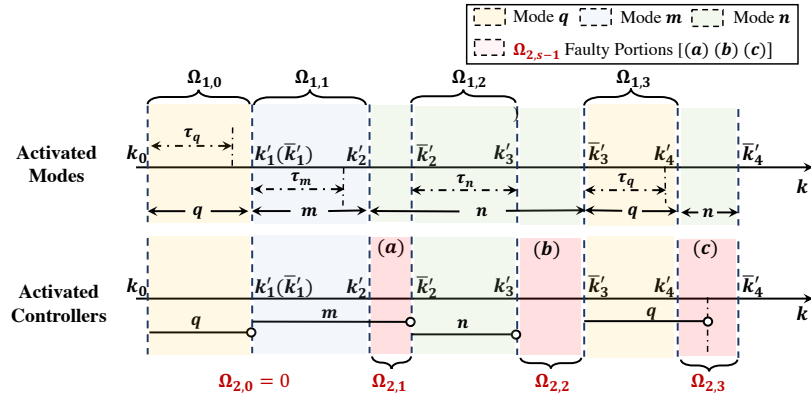


Figure 6.2: Illustration of the activated subsystems and controllers during the faulty portions, $m, n, q \in \mathcal{M}$ ($k'_1 - k'_4$ denoting ideal switching instants and $\bar{k}'_1 - \bar{k}'_4$ representing time instants that (6.1) reverts to synchronous portions).

Figure 6.2 illustrates the enabled subsystems and practically applied controllers during the faulty portions. The asynchronous switching which happens during $\Omega_{2,1}$ is displayed in (a) while the controller failure scenario with $\Omega_{2,2}$ is demonstrated in (b). The combining effect of the asynchronous switching and controller failures with $\Omega_{2,3}$ is shown in (c). It is seen that the controller with respect to subsystem q , $q \in \mathcal{M}$, fails to perform in the second half of $\Omega_{2,3}$. Subsequently, the definition of an admissible switching sequence with faulty portion $\Omega_{2,s-1}$ and additive uncertainties is presented. Define the allowable disturbance sequence as $\zeta_{\mathcal{T}[\bar{\Delta}]}^w(k) := \{w_0, w_1, \dots, w_{k-1}\}$ with each $w_{k-1} \in \mathbb{W}$.

Definition 6.3. A switching sequence in the presence of allowable disturbance sequences $\zeta_{\mathcal{T}[\bar{\Delta}]}^w(\cdot)$ with an MDT set \mathcal{T} , synchronous portions $\Omega_{1,s-1}$, and faulty portions $\Omega_{2,s-1}$, $s \in \mathbb{Z}^+$ is said to be admissible if the MDT constraint $k'_s - \bar{k}'_{s-1} \geq \tau_m$, $\forall m \in \mathcal{M}$, $s \in \mathbb{Z}^+$, is satisfied and $\Omega_{2,s-1} \in [0, \bar{\Delta}]$, $\forall s \in \mathbb{Z}^+$, is bounded.

For notational convenience, let $\zeta_{\mathcal{T}[\bar{\Delta}]}(k) := \{\sigma(0), \sigma(1), \dots, \sigma(k-1)\}$ denote the admissible switching sequence during interval $[0, k-1]$. In order to have an intuitive understanding of $\zeta_{\mathcal{T}[\bar{\Delta}]}(\cdot)$, an elementary example is exploited to demonstrate the admissibility of a switching sequence.

Example 6.1. Suppose $\mathcal{M} = \{1, 2\}$, $\tau_1 = 3$, $\tau_2 = 2$, and $\bar{\Delta} = 2$ then $\zeta_{\mathcal{T}^{[2]}}(7) = \{\bar{2}, \bar{2}, \tilde{1}_2, \tilde{1}_2, \bar{1}, \bar{1}, \bar{1}\}$ and $\zeta_{\mathcal{T}^{[2]}}(7) = \{\bar{1}, \bar{1}, \bar{1}, 2, 2, \bar{2}, \bar{2}\}$ are two admissible switching sequences where $\bar{1}$ and $\bar{2}$ denote that the mode-dependent controller is aligned with corresponding subsystems 1 and 2, respectively. $\tilde{1}_2$ means the subsystem 1 is controlled by the lagged controller with respect to mode 2 and 2 in $\zeta_{\mathcal{T}^{[2]}}(7)$ denotes that no control action is applied when Subsystem 2 is enabled. However, $\zeta_{\mathcal{T}^{[2]}}(8) = \{\bar{2}, \bar{2}, \tilde{1}_2, 1, \tilde{1}_2, 1, \bar{1}, \bar{1}\}$ is not an admissible switching sequence since the MDT constraint is violated and $\bar{\Delta}$ is exceeded.

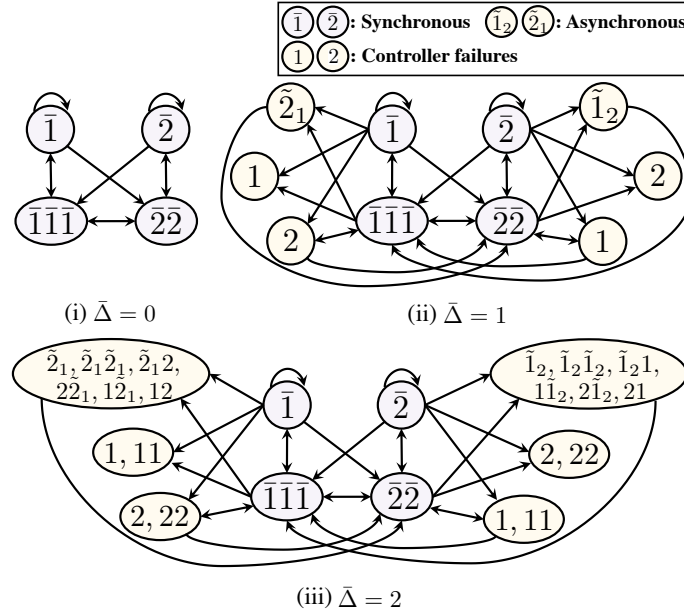


Figure 6.3: Admissible switching sequences with automaton-type representations for (i) $\bar{\Delta} = 0$ (ii) $\bar{\Delta} = 1$ (iii) $\bar{\Delta} = 2$.

Inspired by [147, 148], a directed graph is employed to display all possible combinations in the faulty portion. Figure 6.3 illustrates the admissible switching sequence

with automaton-type expressions. The finite length of the sequence is represented by a node while feasible switching sequence transitions are depicted by paths that point to a node. Define $V \in \mathbb{Z}^+$ as the total number of the faulty combinations for the mode transition from m to n including one fault-free situation, i.e., $\Omega_{2,s-1} = 0$. For notational convenience, label each faulty combination for the mode transition from m to n as $c_{[mn,v]}$, $v \in \mathbb{Z}^+$, and $c_{[mn,v]} \in \mathcal{C}_V^{mn} := \{c_{[mn,1]}, c_{[mn,2]}, \dots, c_{[mn,V]} \mid m, n \in \mathcal{M}, n \neq m\}$ where \mathcal{C}_V^{mn} is the set encompassing all the faulty combinations from subsystem m to n and $\mathcal{C}_V^{mn} \subset \mathcal{C}_V^m =: \{\mathcal{C}_V^{m1}, \mathcal{C}_V^{m2}, \dots, \mathcal{C}_V^{mn}\} \subset \mathcal{C}_V := \{\mathcal{C}_V^1, \mathcal{C}_V^2, \dots, \mathcal{C}_V^M\}$, $m \neq n$. For the sake of clarity, taking (ii) in Figure 6.3 as an example, $V = 4$ is obtained including the fault-free situation representing as $c_{[12,1]}$, $\tilde{2}_1$ as $c_{[12,2]}$, 1 as $c_{[12,3]}$, and 2 as $c_{[12,4]}$ when mode 1 switches to mode 2.

To facilitate the theoretical analysis, an admissible switching sequence with MDT constraint and bounded faulty portions is reconstructed by an arbitrarily *stage-based* concatenation of nodes [54, 137]. Each stage of an admissible switching sequence contains a synchronous portion, i.e., $\{m^j\}$, $j \in \mathcal{T}_m := \{\tau_m, \tau_m + 1, \dots, 2\tau_m - 1\}$, $m \in \mathcal{M}$, where j represents the length of the switched system (6.1) staying in mode m and a faulty portion $c_{[mn,v]} \in \mathcal{C}_V^{mn}$. Employing (iii) in Figure 6.3 as an example, a stage segment of an admissible switching sequence is $\{\bar{1}^j, c_{[12,v]}\}$, $j \in \{3, 4, 5\}$ or $\{\bar{2}^j, c_{[12,v]}\}$, $j \in \{2, 3\}$ with $c_{[12,v]} \in \mathcal{C}_{11}^{12}$. Figure 6.4 illustrates the structure of theoretical analysis in the following sections.

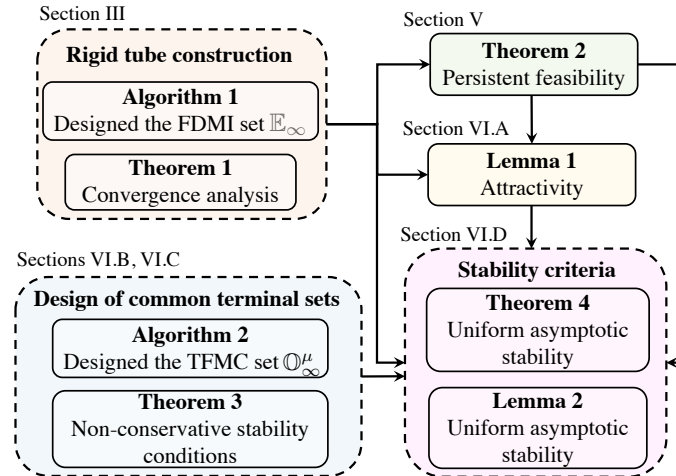


Figure 6.4: The roadmap of theoretical analysis.

6.3 FDMI set design and analysis

Motivated by [79, 97], we intend to employ the tube-based MPC robust control technique to attenuate the adverse effect of additive uncertainties. Thus, this section provides the preliminary results for the facilitation to construct the disturbance invariant set [96] serving as the rigid tube. To begin with, let the nominal switched system be defined as

$$\bar{x}_{k+1} = A_{\sigma(k)}\bar{x}_k + B_{\sigma(k)}\bar{u}_k, \quad \sigma(k) \in \mathcal{M}, \quad (6.4)$$

where $\bar{x}_k \in \mathbb{R}^{n_x}$ is the nominal system states and $\bar{u}_k \in \mathbb{R}^{n_u}$ is the nominal control input vector. Considering the MPC control strategy, the following robust control law is derived

$$u_k = \bar{u}_k + K_{\sigma(k)}(x_k - \bar{x}_k), \quad (6.5)$$

where K_m , $m \in \mathcal{M}$, is the mode-dependent feedback gain such that $\bar{A}_m = A_m + B_m K_m$, $\forall m \in \mathcal{M}$, is Schur stable, i.e., $\rho(\bar{A}_m) < 1$, $\forall m \in \mathcal{M}$, and $\bar{A}_m \in \bar{\mathcal{A}} := \{\bar{A}_1, \bar{A}_2, \dots, \bar{A}_M\}$. Form (6.1)(6.4)(6.5), the error dynamics between the actual state x_k and the nominal state \bar{x}_k is obtained

$$e_{k+1} = \bar{A}_{\sigma(k)}e_k + w_k, \quad (6.6)$$

where $e_k := x_k - \bar{x}_k$. Considering the mode switching from mode m to n with $m, n \in \mathcal{M}$, the error dynamics (6.6) in the presence of asynchronous switching is represented as

$$e_{k+1} = \tilde{A}_{m,n}e_k + w_k, \quad (6.7)$$

where $\tilde{A}_{m,n} = A_n + B_n K_m$; $\tilde{A}_{m,n} \in \tilde{\mathcal{A}}_m := \{\tilde{A}_{m,1}, \tilde{A}_{m,2}, \dots, \tilde{A}_{m,M}\}$ and $\tilde{\mathcal{A}} := \bigcup_{m \in \mathcal{M}} \tilde{\mathcal{A}}_m$ is the union of all asynchronous system matrices. Moreover, the error dynamics (6.7) under controller failures is described by

$$e_{k+1} = A_m e_k + w_k, \quad (6.8)$$

where $A_m \in \mathcal{A} := \{A_1, A_2, \dots, A_M\}$.

Note that the conventional rigid tube construction through $\sum_{i=0}^{\infty} \tilde{A}_m^i \mathbb{W}$ [96] is not competent for switching dynamics, especially for a class of asynchronously switched systems under controller failures, since one can seldom obtain a converged disturbance

invariant set. Hence, a generalized fault-tolerant disturbance MDT invariant (FDMI) set is required and the definition of FDMI set is given as follows.

Definition 6.4. [138] *A set $\Theta \subseteq \mathbb{R}^{n_x}$ is said to be an FDMI set for the system in (6.6) in the presence of asynchronous switching and controller failures with an MDT set $\Phi := \{\phi_1, \phi_2, \dots, \phi_M\}$, if $e_0 \in \Theta$ implies $e_k \in \Theta$ for every admissible switching sequence $\zeta_{\Phi[\bar{\Delta}]}(k)$ and allowable disturbance sequences $\zeta_{\Phi[\bar{\Delta}]}^w(k)$, $k \in \mathbb{Z}^+$.*

6.3.1 Algorithm Design

For a given set $\Theta \subseteq \mathbb{R}^{n_x}$, the ℓ -step reachable set computational operator, $\ell \in \mathbb{N}$, along system (6.6) is characterized as

$$\begin{aligned} \bar{\mathcal{H}}_m^\ell(\Theta) &:= \{\bar{A}_m^\ell e + \dots + \bar{A}_m w + w \mid e \in \Theta, w \in \mathbb{W}, m \in \mathcal{M}\} \\ &= \bar{A}_m^\ell \Theta \oplus \bar{A}_m^{\ell-1} \mathbb{W} \oplus \dots \oplus \bar{A}_m \mathbb{W} \oplus \mathbb{W}, \end{aligned} \quad (6.9)$$

with $\bar{\mathcal{H}}_m^0(\Theta) = \Theta$. The operator for computing ℓ -step reachable set along the asynchronous error system (6.7) is defined as

$$\begin{aligned} \tilde{\mathcal{H}}_{m,n}^\ell(\Theta) &:= \{\tilde{A}_{m,n}^\ell e + \dots + \tilde{A}_{m,n} w + w \mid e \in \Theta, w \in \mathbb{W}, m, n \in \mathcal{M}, m \neq n\} \\ &= \tilde{A}_{m,n}^\ell \Theta \oplus \tilde{A}_{m,n}^{\ell-1} \mathbb{W} \oplus \dots \oplus \tilde{A}_{m,n} \mathbb{W} \oplus \mathbb{W}, \end{aligned} \quad (6.10)$$

with $\tilde{\mathcal{H}}_{m,n}^0(\Theta) = \Theta$. Similarly, the ℓ -step reachable set for the system (6.8) is denoted as

$$\begin{aligned} \mathcal{H}_m^\ell(\Theta) &:= \{A_m^\ell e + \dots + A_m w + w \mid e \in \Theta, w \in \mathbb{W}, m \in \mathcal{M}\} \\ &= A_m^\ell \Theta \oplus A_m^{\ell-1} \mathbb{W} \oplus \dots \oplus A_m \mathbb{W} \oplus \mathbb{W}, \end{aligned} \quad (6.11)$$

with $\mathcal{H}_m^0(\Theta) = \Theta$. Thus, for the given set Θ , the reachable set evolved after $c_{[mn,v]}$ faulty combination, $c_{[mn,v]} \in \mathcal{C}_V^{mn}$ under $\zeta_{\mathcal{T}[\bar{\Delta}]}^w(\cdot)$ is represented as

$$\mathbb{H}^{c_{[mn,v]}}(\Theta, \mathbb{W}) := \tilde{\mathcal{H}}_{m,n}^{\tilde{d}_r^{c_{[mn,v]}}}(\mathcal{H}_{m_{c_{[mn,v]}}}^{d_r^{c_{[mn,v]}}} \dots \tilde{\mathcal{H}}_{m,n}^{\tilde{d}_1^{c_{[mn,v]}}}(\mathcal{H}_{m_{c_{[mn,v]}}}^{d_1^{c_{[mn,v]}}}(\Theta))), \quad (6.12)$$

where $\tilde{d}_r^{c_{[mn,v]}}$ and $d_r^{c_{[mn,v]}}$ denote the duration of the r th asynchronous switching portion and the r th controller faulty portion, respectively. In addition, the summation satisfies $\Omega_{2,s-1} = \sum_{z=1}^r (\tilde{d}_z^{c_{[mn,v]}} + d_z^{c_{[mn,v]}}) \in [0, \bar{\Delta}]$, $\forall s \in \mathbb{Z}^+$. $m_{c_{[mn,v]}} \in \mathcal{M}$ denotes the index of the subsystem of (6.8) depending on the specific faulty combination.

Algorithm 6.1 Computation of an FDMI set \mathbb{E}_∞

- 1: **Input:** \mathbb{W} , $\bar{\mathcal{A}}$, $\tilde{\mathcal{A}}$, \mathcal{A} , \mathbb{C}_V , M , and \mathcal{T} .
 - 2: Set $p = 0$, $m = 1$, $\ell = \tau_m$, $v = 1$, and $\mathbb{E}_0 = \{\mathbf{0}\}$.
 - 3: **for every** $m \leq M$, $m \in \mathbb{Z}^+$,
 - 4: **for every** $\ell \leq 2\tau_m - 1$, $\ell \in \mathbb{Z}^+$,
 - 5: **for every** $v \leq V$, $v \in \mathbb{Z}^+$, and $n \in \mathcal{M}$, $n \neq m$,
 - 6: $\mathbb{E}_{p+1} = \text{Co}\{\mathbb{E}_p, \mathbb{H}^{\mathbb{C}[mn,v]}(\bar{\mathcal{H}}_m^\ell(\mathbb{E}_p))\}$;
 - 7: **end**
 - 8: **end**
 - 9: **end**
 - 10: **If** $\mathbb{E}_{p+1} = \mathbb{E}_p$, then output $\mathbb{E}_\infty = \mathbb{E}_p$ and **stop**; else set $p = p + 1$ and goto **step 2**.
-

Define $\mathcal{T} := \bigcup_{m \in \mathcal{M}} \mathcal{T}_m$ and $|\mathcal{T}|$ the total number of elements contained in \mathcal{T} . Algorithm 6.1 presents the computational procedure to obtain an FDMI set \mathbb{E}_∞ . Note that the convex hull of each iterative set \mathcal{E}_p is exploited below

$$\mathbb{E}_p := \text{Co}\{\mathcal{E}_p\} \quad (6.13)$$

since the convex hull operation of a group of convex sets can preserve the convexity of the result set. From Algorithm 6.1, it is seen that all the possible reachable sets are considered in the one-stage convex hull calculation from step 3 to step 9. \mathbb{E}_∞ is achieved until the condition $\mathbb{E}_{p+1} = \mathbb{E}_p$ is fulfilled.

6.3.2 Convergence Analysis

This subsection aims to provide the theoretical basis for the FDMI set design.

Theorem 6.1. *Suppose Assumption 6.1 holds and \mathbb{E}_p , $p \in \mathbb{Z}^+$, is obtained through Algorithm 6.1. (i) We have $\mathbb{E}_p = \text{Co}\{\mathcal{E}_p\}$ for all $p \in \mathbb{N}$; (ii) For an FDMI set $\mathbb{E}_\infty := \lim_{p \rightarrow +\infty} \mathbb{E}_p$, the inclusion $\mathbb{E}_p \subseteq \mathbb{E}_\infty$ holds, $\forall p \in \mathbb{N}$.*

Proof. (i) We first define the superimposed disturbance set Λ for a p th-stage faulty portion as

$$\Lambda_{[mn,v]}^p := \left(\bigoplus_{g=1}^{g=\ell-1} \mathbb{F}_{[mn,v]}^p \bar{A}_m^g \mathbb{W} \right) \bigoplus \mathbb{F}_{[mn,v]}^p \mathbb{W} \bigoplus \cdots \bigoplus \tilde{A}_{m,n} \mathbb{W} \bigoplus \mathbb{W}, \quad (6.14)$$

where $\ell \in \mathcal{T}_m$, $m \in \mathcal{M}$, and the product of system matrices $\mathbb{F}_{[mn,v]}^p$ with $c_{[mn,v]}$ is

$$\mathbb{F}_{[mn,v]}^p := \prod_{k=\bar{k}'_p - \bar{d}_r^{c[mn,v]} - 1}^{\bar{k}'_p - 1} \tilde{A}_{m,\sigma(k)}^{\bar{d}_r^{c[mn,v]}} \cdots \prod_{k=k'_p}^{k'_p + d_1^{c[mn,v]}} A_{\sigma(k)}^{d_1^{c[mn,v]}}.$$

Thus, from Algorithm 6.1, we have

$$\mathbb{E}_1 = Co\left\{ \bigcup_{\ell=\tau_m}^{\ell=2\tau_m-1} \bigcup_{v=1}^{v=V} \Lambda_{[mn,v]}^1, \forall m, n \in \mathcal{M} \right\} = Co\{\mathcal{E}_1\}. \quad (6.15)$$

Inspired by [138], the subsequent proof is conducted by the induction method. Assuming that $\mathbb{E}_p = Co\{\mathcal{E}_p\}$ stands. Then, for \mathcal{E}_{p+1} , we have

$$\begin{aligned} \mathcal{E}_{p+1} &= \bigcup_{\ell=\tau_m, m \in \mathcal{M}}^{\ell=2\tau_m-1} \bigcup_{v=1}^{v=V} \left(\mathbb{F}_{[mn,v]}^{p+1} \bar{A}_m^\ell \mathcal{E}_p \oplus \Lambda_{[mn,v]}^{p+1} \right) = \\ &= \bigcup_{l=1}^{l=\xi_p} \bigcup_{\ell=\tau_m, m \in \mathcal{M}}^{\ell=2\tau_m-1} \bigcup_{v=1}^{v=V} \left(\mathbb{F}_{[mn,v]}^{p+1} \bar{A}_m^\ell \mathcal{G}_l \oplus \Lambda_{[mn,v]}^{p+1} \right) = \bigcup_{f=1}^{f=\xi_{p+1}} \mathcal{O}_f, \end{aligned} \quad (6.16)$$

where $\xi_p = (V|\mathcal{T}|)^p$ represents the total number of possible switching sequences in p th stage and \mathcal{G}_l denotes the reachable set along the l th switching sequence. Thus, \mathcal{E}_p and \mathcal{E}_{p+1} can be rewritten as $\mathcal{E}_p = \bigcup_{l=1}^{l=\xi_p} \mathcal{G}_l$ and $\mathcal{E}_{p+1} = \bigcup_{f=1}^{f=\xi_{p+1}} \mathcal{O}_f$, respectively.

From (6.16), it follows that $\mathbb{E}_p = Co\{\mathcal{E}_p\} = \{\sum_{l=1}^{l=\xi_p} g_l \mathcal{G}_l \mid \sum_{l=1}^{l=\xi_p} g_l = 1\}$. Then, we have

$$\begin{aligned} \mathbb{E}_{p+1} &= \left\{ \sum_{l=1}^{l=\xi_p} \sum_{a=1}^{a=\xi_1} y_a g_l \left(\mathbb{F}_{[mn,v]}^{p+1} \bar{A}_m^\ell \mathcal{G}_l \oplus \Lambda_{[mn,v]}^{p+1} \right) \mid \sum_{l=1}^{l=\xi_p} \sum_{a=1}^{a=\xi_1} y_a g_l = 1 \right\} \\ &= Co\left\{ \sum_{l=1}^{l=\xi_p} \sum_{a=1}^{a=\xi_1} z_{a,l} \mathcal{O}_f \mid \sum_{l=1}^{l=\xi_p} \sum_{a=1}^{a=\xi_1} z_{a,l} = 1, z_{a,l} = y_a g_l \right\} \\ &= Co\{\mathcal{E}_{p+1}\}. \end{aligned}$$

The last equation holds because of $\sum_{l=1}^{l=\xi_p} \sum_{a=1}^{a=\xi_1} z_{a,l} = \sum_{f=1}^{f=\xi_{p+1}} z_f = 1$. Hence, $\mathbb{E}_p = Co\{\mathcal{E}_p\}$ holds, $\forall p \in \mathbb{N}$.

(ii) The convergence analysis of Algorithm 6.1 is conducted as follows. Define the convex hull of (6.14) as $\mathbb{Q} = Co\{\bigcup_{\ell=\tau_m}^{\ell=2\tau_m-1} \bigcup_{v=1}^{v=V} \Lambda_{[mn,v]}^p, \forall m, n \in \mathcal{M}\}$. Therefore, following [138], it is obtained that $\mathbb{E}_p \subseteq Co\{\bigcup_{\ell=\tau_m}^{\ell=2\tau_m-1} \bigcup_{v=1}^{v=V} \mathbb{F}_{[mn,v]}^p \bar{A}_m^\ell \mathbb{E}_{p-1} \oplus \mathbb{Q}, \forall m, n \in \mathcal{M}\}$.

\mathcal{M} . Let a norm ball with the radius of $l > 0$ be defined by $\mathcal{B}(l) \subset \mathbb{R}^{n_x}$. Since \mathbb{W} is compact, for an appropriate $l_b > 0$ and a proper $\nu \in (0, 1)$, the inclusions $\mathbb{Q} \subseteq \mathcal{B}(l_b)$ and $\mathbb{F}_{[mn,v]}^p \bar{A}_m^g \mathbb{Q} \subseteq \nu \mathcal{B}(l_b)$ hold, $\forall g \in \mathcal{T}_m, c_{[mn,v]} \in \mathcal{C}_V^m, m \in \mathcal{M}$, and $p \in \mathbb{N}$. Thus, we have the following inclusion

$$\begin{aligned} \mathbb{E}_p &\subseteq \text{Co}\{\mathbb{F}_{[mn,v]}^p \bar{A}_m^{\ell_p} \cdots \mathbb{F}_{[mn,v]}^1 \bar{A}_m^{\ell_1} \mathbb{E}_0 \bigoplus \mathbb{F}_{[mn,v]}^p \bar{A}_m^{\ell_p} \cdots \\ &\quad \mathbb{F}_{[mn,v]}^2 \bar{A}_m^{\ell_2} \mathbb{W} \bigoplus \cdots \bigoplus \mathbb{F}_{[mn,v]}^p \bar{A}_m^{\ell_p} \mathbb{W} \bigoplus \mathbb{W}\} \\ &\subseteq (\nu^{p-1} + \cdots + \nu + 1) \mathcal{B}(l_b), \end{aligned} \quad (6.17)$$

where $\ell_p \in \mathcal{T}_m$ denotes the duration of the p th-stage synchronous portion. From (6.17), it follows that Hausdorff distance between \mathbb{E}_{p+1} and \mathbb{E}_p is bounded by $\nu^p \mathcal{B}(l_b)$. Thus, a finite number of sets $\{\mathbb{E}_p \mid p \in \mathbb{N}\}$ is a Cauchy sequence and the inclusion $\mathbb{E}_p \subseteq \mathbb{E}_\infty$ stands with $\mathbb{E}_\infty := \lim_{p \rightarrow +\infty} \mathbb{E}_p$. ■

Remark 6.2. *The theoretical studies of Theorem 6.1 ensure the computation of an FDMI set can be obtained within a finite number of iterations with Algorithm 6.1.*

6.4 RMPC for switched systems

The problem formulation of RMPC for nominal switched systems (6.4) is introduced and then the control objectives for the successive proposed feasibility and stability criteria are presented. The cost function $J_m(\cdot)$ with respect to subsystem m , $m \in \mathcal{M}$, is defined as

$$J_m(x_k, \bar{\mathbf{u}}_k) := \sum_{i=0}^{N_m-1} L_m(\bar{x}_{k+i|k}, \bar{u}_{k+i|k}) + F_m(\bar{x}_{k+N_m|k}), \quad (6.18)$$

where the initial state $\bar{x}_k = x_k = \bar{x}_{k|k}$ is considered and $\bar{\mathbf{u}}_k := \{\bar{u}_{k|k}, \bar{u}_{k+1|k}, \cdots, \bar{u}_{k+N_m-1|k}\}$ is the control sequence to be determined. $\bar{x}_{k+i|k}$ and $\bar{u}_{k+i|k}$ represent the i th step predicted state and control input, respectively. N_m , $m \in \mathcal{M}$, is the mode-dependent prediction horizon. The stage cost function is $L_m : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}_{[0,+\infty)}$ and $L_m := \|\bar{x}_{k+i|k}\|_{Q_m}^2 + \|\bar{u}_{k+i|k}\|_{R_m}^2$ with $L_m(0, 0) = 0$ where $Q_m \succ 0$ and $R_m \succ 0$ are the mode-dependent weighting matrices. The terminal cost function is $F_m : \mathbb{R}^{n_x} \rightarrow \mathbb{R}_{[0,+\infty)}$ and $F_m := \|\bar{x}_{k+N_m|k}\|_{P_m}^2$ with $F_m(0) = 0$ where $P_m \succ 0$ is the mode-dependent terminal weighting matrix.

Based on the constructed FDMI set, the tightened constraint associated with

mode m , $m \in \mathcal{M}$, is expressed as

$$\bar{\mathbb{X}}_m = \mathbb{X}_m \ominus \mathbb{E}_\infty, \quad \bar{\mathbb{U}}_m = \mathbb{U}_m \ominus K_m \mathbb{E}_\infty. \quad (6.19)$$

The constrained optimization problem is formulated as

$$\bar{\mathbf{u}}_k^* = \arg \min_{\bar{\mathbf{u}}_k} J_m(x_k, \bar{\mathbf{u}}_k) \quad (6.20a)$$

$$\text{s.t. } \bar{x}_{k+i+1|k} = A_m \bar{x}_{k+i|k} + B_m \bar{u}_{k+i|k}, i \in \mathbb{N}_{[0, N_m-1]} \quad (6.20b)$$

$$\bar{x}_{k|k} = x_k \quad (6.20c)$$

$$\bar{x}_{k+i|k} \in \bar{\mathbb{X}}_m, i \in \mathbb{N}_{[0, N_m-1]} \quad (6.20d)$$

$$\bar{u}_{k+i|k} \in \bar{\mathbb{U}}_m, i \in \mathbb{N}_{[0, N_m-1]} \quad (6.20e)$$

$$\bar{x}_{k+N_\sigma(k)|k} \in \Xi_m \quad (6.20f)$$

where $\Xi_m \subseteq \bar{\mathbb{X}}_m$, $m \in \mathcal{M}$, is the mode-dependent terminal constraint. By solving the optimization problem at k , the optimal control sequence and optimal state sequence are obtained respectively as $\bar{\mathbf{u}}_k^* := \{\bar{u}_{k|k}^*, \bar{u}_{k+1|k}^*, \dots, \bar{u}_{k+N_m-1|k}^*\}$ and $\bar{\mathbf{x}}_k^* := \{\bar{x}_{k|k}^*, \bar{x}_{k+1|k}^*, \dots, \bar{x}_{k+N_m|k}^*\}$. In addition, only the first element of $\bar{\mathbf{u}}_k^*$ is implemented. Thus, the following switched MPC law is applied

$$u_m^{\text{MPC}}(x_k) := \bar{u}_{k|k}^*. \quad (6.21)$$

Let $\mathcal{X}_m^{N_m}$ denote the feasible set below associated with mode m , $m \in \mathcal{M}$,

$$\begin{aligned} \mathcal{X}_m^{N_m} := & \{x_k \in \bar{\mathbb{X}}_m \mid \bar{x}_{k+i|k} \in \bar{\mathbb{X}}_m, \bar{u}_{k+i|k} \in \bar{\mathbb{U}}_m, \bar{x}_{k|k} = x_k, \\ & \bar{x}_{k+N_m|k} \in \Xi_m, \bar{\mathbf{u}}_k \neq \emptyset, \forall i \in \mathbb{N}_{[0, N_m-1]}\}. \end{aligned} \quad (6.22)$$

Remark 6.3. *Following [117, 126], we adopt the prediction scheme which only takes into consideration the currently enabled subsystem within the prediction horizon in (6.20). We aim to utilize the scheduling of switching signals to mitigate the negative impact of asynchronous switching and controller failures. Compared with the prediction strategy in [120] which takes all the possible future switching sequences into account, the advantage of this control design is that the massive sacrifice of the system performance can be avoided to a great extent with respect to slow switching dynamics.*

We have the following two general assumptions regarding (6.20).

Assumption 6.2. (i) The constraint sets $\bar{\mathbb{X}}_m$ and $\bar{\mathbb{U}}_m$, $m \in \mathcal{M}$, are non-empty polytopes. For some proper matrix $Z_m \in \mathbb{R}^{n_z \times n_x}$, $\bar{\mathbb{X}}_m$ can be denoted by $\bar{\mathbb{X}}_m = \{Z_m \bar{x} \leq \mathbf{1}\}$. Moreover, (\bar{A}_m, Z_m) is observable for at least one $\bar{A}_m \in \bar{\mathcal{A}}$. (ii) The terminal constraint set $\bar{\Xi}_m$, $m \in \mathcal{M}$, satisfies $\bar{A}_m \bar{\Xi}_m \subset \bar{\Xi}_m$ and $\bigcap_{m \in \mathcal{M}} \bar{\Xi}_m \subset \bigcap_{m \in \mathcal{M}} \bar{\mathbb{X}}_m$.

Remark 6.4. Assumption 6.2(i) is made to facilitate the common terminal set calculation. Assumption 6.2(ii) is a standard assumption which is a prerequisite for the subsequent feasibility analysis.

Assumption 6.3. Suppose there exists a state feedback control law $\mathcal{K}_m(\cdot)$ associated with subsystem m , $m \in \mathcal{M}$, such that the terminal constraint $\bar{\Xi}_m$ is control invariant for the closed-loop system $\bar{x}_{k+1} = A_m \bar{x}_k + B_m \mathcal{K}_m(\bar{x}_k)$ with $\mathcal{K}_m(\bar{x}_k) \in \bar{\mathbb{U}}_m$. Moreover, $\forall \bar{x} \in \bar{\Xi}_m$, the following inequality is required

$$F_m(A_m \bar{x} + B_m \mathcal{K}_m(\bar{x})) - F_m(\bar{x}) \preceq -L_m(\bar{x}, \mathcal{K}_m(\bar{x})). \quad (6.23)$$

Remark 6.5. Assumption 6.3 is necessary for analyzing the closed-loop stability under the switched MPC control law (6.21) [79]. Furthermore, inequality (6.23) ensures that the feasible set $\mathcal{X}_m^{N_m}$ is a region of attraction for the subsystem m in the closed-loop.

The control objectives of this chapter are stated in two points:

- To design a minimum MDT restriction such that the optimal control problem (6.20) is feasible under the joint effects of asynchronous switching and controller failures;
- To find the lower bound of MDT in an executable way such that the closed-loop system is stable.

6.5 Persistent Feasibility

In this section, a strategy to retain the persistent feasibility for (6.20) is proposed. The switched MPC problem (6.20) is said to be *persistently feasible* if the solutions of (6.20) are obtained continuously within a mode or at switching instants [126].

Followed by the reachable set defined in (6.9)-(6.11), one-step reachable set along subsystem m , $m \in \mathcal{M}$, of (6.4) with the switched MPC law (6.21) is

$$\bar{S}_m^1(\Theta) := \{\bar{x}_{k+1} \mid \bar{x}_k \in \Theta, \bar{x}_{k+1} = A_m \bar{x}_k + B_m u_m^{\text{MPC}}(\bar{x}_k)\}, \quad (6.24)$$

and the ℓ -step reachable set, $\ell \in \mathbb{Z}^+$, is obtained by

$$\bar{\mathcal{S}}_m^\ell(\Theta) = \bar{\mathcal{S}}_m^1(\bar{\mathcal{S}}_m^{\ell-1}(\Theta)),$$

with $\bar{\mathcal{S}}_m^0(\Theta) = \Theta$. Consider the mode transition from mode m to mode n with $m, n \in \mathcal{M}$. The one-step reachable set of (6.4) under the asynchronous switching is

$$\tilde{\mathcal{S}}_{m,n}^1(\Theta) := \{\bar{x}_{k+1} \mid \bar{x}_k \in \Theta, \bar{x}_{k+1} = A_n \bar{x}_k + B_n u_m^{\text{MPC}}(\bar{x}_k)\}, \quad (6.25)$$

with ℓ -step reachable set $\tilde{\mathcal{S}}_{m,n}^\ell(\Theta) = \tilde{\mathcal{S}}_{m,n}^1(\tilde{\mathcal{S}}_{m,n}^{\ell-1}(\Theta))$ and $\tilde{\mathcal{S}}_{m,n}^0(\Theta) = \Theta$. Thus, under controller failures, the one-step reachable set along subsystem m of (6.4) is

$$\mathcal{S}_m^1(\Theta) := \{\bar{x}_{k+1} \mid \bar{x}_k \in \Theta, \bar{x}_{k+1} = A_m \bar{x}_k\}, \quad (6.26)$$

with ℓ -step reachable set $\mathcal{S}_m^\ell(\Theta) = \mathcal{S}_m^1(\mathcal{S}_m^{\ell-1}(\Theta))$ and $\mathcal{S}_m^0(\Theta) = \Theta$. Based on (6.25) and (6.26), for a certain faulty combination $c_{[mn,v]} \in \mathcal{C}_V^{mn}$, the evolved reachable set is achieved by

$$\mathbb{S}^{c_{[mn,v]}}(\Theta) := \tilde{\mathcal{S}}_{m,n}^{c_{[mn,v]}}(\mathcal{S}_{m_c[mn,v]}^{c_{[mn,v]}} \cdots \tilde{\mathcal{S}}_{m,n}^{c_{[mn,v]}}(\mathcal{S}_{m_c[mn,v]}^{c_{[mn,v]}}(\Theta))). \quad (6.27)$$

The proposed feasibility strategy is presented in Theorem 6.2.

Theorem 6.2. *Suppose that Assumptions 6.1-6.3 hold with $\mathcal{X}_m^{N_m}$, $\forall m \in \mathcal{M}$, being the feasible set of mode m . The RMPC problem (6.20) for nominal switched system (6.4) in the presence of asynchronous switching and controller failures with \mathcal{C}_V^m is persistently feasible, if the stage MDT $\tau_m^{(s)} \equiv \tau_m$, $s \in \mathbb{Z}^+$, where τ_m satisfies*

$$\bigcup_{n \in \mathcal{M}, n \neq m} \bigcup_{v=1}^{v=V} \bigcup_{\alpha=1}^{\alpha=r} \mathcal{S}_{m_c[mn,v]}^{c_{[mn,v]}} \cdots (\tilde{\mathcal{S}}_{m,n}^{c_{[mn,v]}}(\mathcal{S}_{m_c[mn,v]}^{c_{[mn,v]}}(\bar{\mathcal{S}}_m^{\tau_m}(\mathcal{X}_m^{N_m})))) \subseteq \bigcap_{m_c[mn,v] \in \mathcal{M}} \bar{\mathcal{X}}_{m_c[mn,v]} \quad (6.28)$$

for $k \in [k'_s, \bar{k}'_s)$ with $\sum_{\alpha=1}^r d_\alpha^{c_{[mn,v]}} \neq 0$ and $\Omega_{2,s-1} \in [2, \bar{\Delta}]$, $\bar{\Delta} \geq 2$, $s \in \mathbb{Z}^+$, and

$$\bigcup_{n \in \mathcal{M}, n \neq m} \bigcup_{v=1}^{v=V} \bigcup_{\alpha=1}^{\alpha=r} \tilde{\mathcal{S}}_{m,n}^{c_{[mn,v]}}(\mathcal{S}_{m_c[mn,v]}^{c_{[mn,v]}} \cdots \tilde{\mathcal{S}}_{m,n}^{c_{[mn,v]}}(\mathcal{S}_{m_c[mn,v]}^{c_{[mn,v]}}(\bar{\mathcal{S}}_m^{\tau_m}(\mathcal{X}_m^{N_m})))) \subseteq \mathcal{X}_m^{N_m} \quad (6.29)$$

for $k \in [k'_s, \bar{k}'_s)$ with $\sum_{\alpha=1}^r \tilde{d}_\alpha^{c_{[mn,v]}} \neq 0$ and $\Omega_{2,s-1} \in [2, \bar{\Delta}]$, $\bar{\Delta} \geq 2$, $s \in \mathbb{Z}^+$. In addition,

τ_m also satisfies

$$\bigcup_{n \in \mathcal{M}, n \neq m} \bigcup_{v=1}^{v=V} \mathbb{S}^{c_{[mn,v]}} (\bar{\mathcal{S}}_m^{\tau_m}(\mathcal{X}_m^{N_m})) \subseteq \bigcap_{n \in \mathcal{M}, n \neq m} \mathcal{X}_n^{N_n}, \quad (6.30)$$

with $k = \bar{k}'_s$.

Proof. Assumption 6.2 guarantees that the feasible set $\mathcal{X}_m^{N_m}$, $m \in \mathcal{M}$, is a region of attraction as long as the switched system (6.4) stays in subsystem m , i.e., $\bar{\mathcal{S}}_m^\gamma(\mathcal{X}_m^{N_m}) \subseteq \mathcal{X}_m^{N_m}$, $\gamma \in \mathbb{Z}^+$. Then, it follows that $\bar{\mathcal{S}}_m^{\tau_m}(\mathcal{X}_m^{N_m}) \subseteq \mathcal{X}_m^{N_m}$ and

$$\bar{\mathcal{S}}_m^{k'_s - \bar{k}'_{s-1}}(\mathcal{X}_m^{N_m}) \subseteq \bar{\mathcal{S}}_m^{\tau_m}(\mathcal{X}_m^{N_m}), s \in \mathbb{Z}^+,$$

since $k'_s - \bar{k}'_{s-1} \geq \tau_m$, $m \in \mathcal{M}$, is required. Note that, in the presence of controller failures, the state constraint $\bar{\mathbb{X}}_m$ is supposed to be satisfied. Hence, the reachable set is included in $\bigcap_{m, c_{[mn,v]} \in \mathcal{M}} \bar{\mathbb{X}}_{m, c_{[mn,v]}}$, $c_{[mn,v]} \in \mathcal{C}_V^{mn}$ considering all possible mode transitions, i.e., (6.28). Similarly, to retain the persistent feasibility of the switched MPC problem (6.20), the reachable set with the asynchronous switching is forced to be contained in $\mathcal{X}_m^{N_m}$ which is the feasible set regarding the last enabled switching mode, i.e., (6.29). When the faulty portion switches back to synchronous portions, the target region is designed as $\bigcap_{n \in \mathcal{M}, n \neq m} \mathcal{X}_n^{N_n}$ due to $\mathcal{X}_n^{N_n} \oplus \mathcal{E}_\infty \subseteq \mathbb{X}_n$, $\forall n \in \mathcal{M}$. Since $e_0 = \mathbf{0} \in \mathcal{E}_\infty$, the sufficient condition guarantees the persistent feasibility of (6.20) as long as the MDT τ_m , $m \in \mathcal{M}$, is well designed. ■

Remark 6.6. Note that the sufficient condition proposed in Theorem 6.1 does not imply the closed-loop stability by using switched MPC law (6.21) since the closed-loop solution may stay in the feasible set without converging.

6.6 Stability analysis

6.6.1 Stage-based Attractivity Criterion

This subsection provides a more stringent attractivity criterion with guaranteed persistent feasibility of the switched MPC algorithm.

Lemma 6.1. Suppose that Assumptions 6.1-6.3 hold with $\mathcal{X}_m^{N_m}$, $\forall m \in \mathcal{M}$, being the feasible set of mode m . Within the feasible set $\bigcup_{m \in \mathcal{M}} \mathcal{X}_m^{N_m}$, the closed-loop system with

asynchronous switching and controller failures is attractive, if $\tau_m^{(1)}$ satisfies (6.28)-(6.30) and $\tau_m^{(s)}$, $s \in \mathbb{Z}_{[2,+\infty)}^+$, satisfies

$$\bigcup_{n \in \mathcal{M}, n \neq m} \bigcup_{v=1}^{v=V} \bigcup_{\alpha=1}^{\alpha=r} \mathcal{S}_{m_{c[mn,v]}}^{d_{\alpha}^{c[mn,v]}} \cdots (\tilde{\mathcal{S}}_{m,n}^{c[mn,v]} (\mathcal{S}_{m_{c[mn,v]}}^{d_1^{c[mn,v]}})) \left(\bigcup_{\eta=0}^{\tau_m^{(1)} - \tau_m^{(s)}} \bar{\mathcal{S}}_m^{\tau_m^{(s)} + \eta} \left(\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{s-2} (\mathcal{X}_n^{N_n}) \right) \right) \subseteq \bigcap_{m_{c[mn,v]} \in \mathcal{M}} \bar{\mathbb{X}}_{m_{c[mn,v]}} \quad (6.31)$$

for $k \in [k'_s, \bar{k}'_s)$ with $\sum_{\alpha=1}^r d_{\alpha}^{c[mn,v]} \neq 0$ and $\Omega_{2,s-1} \in [2, \bar{\Delta}]$, $\bar{\Delta} \geq 2$, $s \in \mathbb{Z}_{[2,+\infty)}^+$, and

$$\bigcup_{n \in \mathcal{M}, n \neq m} \bigcup_{v=1}^{v=V} \bigcup_{\alpha=1}^{\alpha=r} \tilde{\mathcal{S}}_{m,n}^{c[mn,v]} (\mathcal{S}_{m_{c[mn,v]}}^{d_{\alpha}^{c[mn,v]}} \cdots \tilde{\mathcal{S}}_{m,n}^{c[mn,v]} (\mathcal{S}_{m_{c[mn,v]}}^{d_1^{c[mn,v]}})) \left(\bigcup_{\eta=0}^{\tau_m^{(1)} - \tau_m^{(s)}} \bar{\mathcal{S}}_m^{\tau_m^{(s)} + \eta} \left(\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{s-2} (\mathcal{X}_n^{N_n}) \right) \right) \subseteq \mathcal{X}_m^{N_m} \quad (6.32)$$

for $k \in [k'_s, \bar{k}'_s)$ with $\sum_{\alpha=1}^r \tilde{d}_{\alpha}^{c[mn,v]} \neq 0$ and $\Omega_{2,s-1} \in [2, \bar{\Delta}]$, $\bar{\Delta} \geq 2$, $s \in \mathbb{Z}_{[2,+\infty)}^+$. Moreover, $\tau_m^{(s)}$, $s \in \mathbb{Z}_{[2,+\infty)}^+$, also satisfies

$$\bigcup_{n \in \mathcal{M}, n \neq m} \bigcup_{v=1}^{v=V} \mathcal{S}^{c[mn,v]} \left(\bigcup_{\eta=0}^{\tau_m^{(1)} - \tau_m^{(s)}} \bar{\mathcal{S}}_m^{\tau_m^{(s)} + \eta} \left(\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{s-2} (\mathcal{X}_n^{N_n}) \right) \right) \subseteq \bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{s-1} (\mathcal{X}_n^{N_n}), \quad (6.33)$$

with $k = \bar{k}'_s$.

Proof. To start with, the proof of persistent feasibility for (6.20) is given. The optimization problem (6.20) is solvable in the first stage since $\tau_m^{(1)}$ obeys the feasibility criteria (6.28)-(6.30) proposed in Theorem 6.2. For $s \in \mathbb{Z}_{[2,+\infty)}^+$, during the faulty portion, by letting the reachable set be contained in the target region $\bigcap_{m_{c[mn,v]} \in \mathcal{M}} \bar{\mathbb{X}}_{m_{c[mn,v]}}$ and $\mathcal{X}_m^{N_m}$, respectively. The possible infeasibility issue is tackled by designing stage MDTs $\tau_m^{(s)}$ via (6.31) and (6.32). Since

$$\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{s-1} (\mathcal{X}_n^{N_n}) \subset \bigcap_{n \in \mathcal{M}, n \neq m} \mathcal{X}_n^{N_n} \quad (6.34)$$

holds for all $s \geq 2$, the persistent feasibility of (6.20) is guaranteed as long as $\tau_m^{(s)}$

satisfies (6.33).

The attractivity proof is given. As $s \rightarrow \infty$, the target region $\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{s-1}(\mathcal{X}_n^{N_n}) \rightarrow \{\mathbf{0}\}$ because Assumption 6.2 ensures the convergence of system states as long as nominal switched system (6.4) with MPC law (6.21) stays in certain mode without mode transitions. Thus, the attractivity is achieved since $D(x_k, \mathcal{E}_\infty) \rightarrow 0$ as $s \rightarrow \infty$. ■

Remark 6.7. Lemma 6.1 provides a sufficient condition to ensure $\bigcup_{m \in \mathcal{M}} \mathcal{X}_m^{N_m}$ serving as a region of attraction for the switched closed-loop system under asynchronous switching and controller failures. Note that the convergence rate of the closed-loop solution can be accelerated by forcing the reachable step of $\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{s-1}(\mathcal{X}_n^{N_n})$ incremented by every two steps or more, e.g., replacing $(s-1)$ -step with $(2s-2)$ -step or $(3s-3)$ -step.

6.6.2 Design of Common Terminal Sets

It is worth pointing out that the criteria proposed in Lemma 6.1 might be computationally intractable since $\tau_m^{(s)}$, $s \in \mathbb{Z}^+$, has to be determined for each stage. To reduce the computational burden, this section aims to design a common terminal set which is competent for all $m \in \mathcal{M}$ such that the calculation of stage MDTs can be terminated once the state trajectory steps into this set. We first introduce the subsequent nominal switched closed-loop system (6.4) with control law \mathcal{K}_m within the terminal set Ξ_m

$$\bar{x}_{k+1} = \bar{A}_m \bar{x}_k, m \in \mathcal{M}. \quad (6.35)$$

Thus, the switching dynamics (6.35) with asynchronous switching and controller failures is respectively given as

$$\bar{x}_{k+1} = \tilde{A}_{m,n} \bar{x}_k, \quad (6.36)$$

$$\bar{x}_{k+1} = A_m \bar{x}_k. \quad (6.37)$$

Then, the definition of the designed common terminal set, namely a terminal constraint admissible fault-tolerant MDT contractive (TFMC) set, is presented.

Definition 6.5. [138] A set $\Theta \subseteq \bigcap_{m \in \mathcal{M}} \Xi_m$ is said to be TFMC regarding closed-loop systems (6.35) with asynchronous switching and controller failures and a contractive factor $\mu \in (0, 1)$ as well as an MDT set Φ , if $x_0 \in \Theta$ indicates that $x_t \in \mu\Theta$ for $t \in \mathbb{Z}_{[1,k]}^+$ and every admissible switching sequence $\zeta_{\Phi[\bar{\Delta}]}(t)$.

Algorithm 6.2 Computation of a TFMC set \mathbb{O}_∞^μ

- 1: **Input:** $\Xi_m, m \in \mathcal{M}, \bar{A}, \tilde{A}, \mathcal{A}, \mathbb{C}_V, M, \mu, \mathcal{T}',$ and \mathcal{T} .
- 2: Set $y = 0$ and define \mathbb{O}_0^μ as

$$\mathbb{O}_0^\mu := \left(\bigcap_{m \in \mathcal{M}} \Xi_m \right) \bigcap \left(\bigcap_{m \in \mathcal{M}} \bigcap_{\ell \in \mathcal{T}'_m} \bar{\mathcal{L}}_m^\ell \left(\bigcap_{m \in \mathcal{M}} \Xi_m \right) \right);$$

- 3: Let \mathbb{O}_{y+1}^μ be defined by

$$\mathbb{O}_{y+1}^\mu := \mathbb{O}_y^\mu \bigcap \left(\bigcap_{m \in \mathcal{M}} \bigcap_{\ell \in \mathcal{T}_m} \bigcap_{c_{[mn,v]} \in \mathbb{C}_V} \bar{\mathcal{L}}_m^\ell (\mathbb{L}^{c_{[mn,v]}}(\mu \mathbb{O}_y^\mu)) \right);$$

- 4: **If** $\mathbb{O}_{y+1}^\mu = \mathbb{O}_y^\mu$, then output $\mathbb{O}_\infty^\mu = \mathbb{O}_y$; else set $y = y + 1$ and goto **step 2**.
-

The backward set technique is employed to design a TFMC set. For a given set $\Theta \subseteq \mathbb{R}^{n_x}$, define the ℓ -step backward set operator, $\ell \in \mathbb{N}$, for closed-loop system (6.35) as

$$\bar{\mathcal{L}}_m^\ell(\Theta) := \{\bar{x} \mid \bar{A}_m^\ell \bar{x} \in \Theta, m \in \mathcal{M}\}, \quad (6.38)$$

with $\bar{\mathcal{L}}_m^0(\Theta) = \Theta$. Likewise, regarding closed-loop systems (6.36) and (6.37), the ℓ -step backward set operators are respectively defined as

$$\tilde{\mathcal{L}}_{m,n}^\ell(\Theta) := \{\bar{x} \mid \tilde{A}_{m,n}^\ell \bar{x} \in \Theta, m, n \in \mathcal{M}\}, \quad (6.39)$$

$$\mathcal{L}_m^\ell(\Theta) := \{\bar{x} \mid A_m^\ell \bar{x} \in \Theta, m \in \mathcal{M}\}, \quad (6.40)$$

with $\tilde{\mathcal{L}}_{m,n}^0(\Theta) = \Theta$ and $\mathcal{L}_m^0(\Theta) = \Theta$. Thus, the backward set before the occurrence of $c_{[mn,v]}$ is denoted as

$$\mathbb{L}^{c_{[mn,v]}}(\Theta) := \mathcal{L}_{m_{c_{[mn,v]}}}^{d_1^{c_{[mn,v]}}} \left(\tilde{\mathcal{L}}_{m,n}^{d_1^{c_{[mn,v]}}} \cdots \mathcal{L}_{m_{c_{[mn,v]}}}^{d_r^{c_{[mn,v]}}} \left(\tilde{\mathcal{L}}_{m,n}^{d_r^{c_{[mn,v]}}}(\Theta) \right) \right). \quad (6.41)$$

Let \mathcal{T}'_m be defined by $\mathcal{T}'_m := \{1, 2, \dots, \tau_m - 1\}$ with $\mathcal{T}' := \bigcup_{m \in \mathcal{M}} \mathcal{T}'_m$.

In the following, Algorithm 6.2 to compute a TFMC set is presented. Once $\mathbb{O}_{y+1}^\mu = \mathbb{O}_y^\mu$ is satisfied, a TFMC set is obtained.

6.6.3 Uniform Asymptotic Stability

To begin with, we introduce the concept of uniform asymptotic stability (UAS).

Definition 6.6. [2] *The system (6.35) in the presence of asynchronous switching*

and controller failures is uniformly asymptotically stable with all admissible switching sequences, if there exists a $\delta > 0$ such that for all $x_0 \in \Theta \cap \delta\mathcal{B}$ implies that $\bar{x}_t \rightarrow \mathbf{0}$ as $t \rightarrow \infty$ where $\Theta \subseteq \mathbb{R}^{n_x}$ is a positive invariant set.

Theorem 6.3. *Suppose Assumptions 6.1-6.3 hold. Two equivalent statements are presented as follows: (i) For a given MDT set Φ , the TFMC set \mathbb{O}_∞^μ obtained from Algorithm 6.2 is non-empty. (ii) The closed-loop system (6.35) is uniformly asymptotically stable with asynchronous switching and controller failures.*

Proof. (i) \Rightarrow (ii): Let the product of s stages system matrices be denoted by

$$\begin{aligned} \mathbb{A}_{\zeta_\Phi[\bar{\Delta}]}(k) := & \left(\prod_{k=\bar{k}'_s-d_r^{c[v]}-1}^{\bar{k}'_s-1} \tilde{A}_{\sigma(\bar{k}'_{s-1}),\sigma(k)} \cdots \prod_{k=k'_s}^{k'_s+d_1^{c[v]}} A_{\sigma(k)} \right) \prod_{k=\bar{k}'_{s-1}}^{k'_s-1} \bar{A}_{\sigma(k)} \\ & \cdots \left(\prod_{k=\bar{k}'_1-d_r^{c[v]}-1}^{\bar{k}'_1-1} \tilde{A}_{\sigma(\bar{k}'_0),\sigma(k)} \cdots \prod_{k=k'_1}^{k'_1+d_1^{c[v]}} A_{\sigma(k)} \right) \prod_{k=\bar{k}'_0}^{k'_1-1} \bar{A}_{\sigma(k)}, \end{aligned} \quad (6.42)$$

where $c_{[v]} \in \mathbb{C}_V$ denotes the simplified label each faulty combination if no specific mode transition is considered. $d_r^{c[v]}$ and $\tilde{d}_r^{c[v]}$ are the duration of the r th controller faulty portion and asynchronous portion, respectively. A non-empty TFMC set indicates that

$$\bar{x}_k = \mu^s \mathbb{A}_{\zeta_\Phi[\bar{\Delta}]}(k) x_0 \in \mu^s \beta \mathbb{O}_\infty^\mu, \quad (6.43a)$$

$$\beta := \max\{\beta > 0 \mid \mathbb{A}_{\zeta_\Phi[\bar{\Delta}]}(k) \mathbb{O}_\infty^\mu \in \beta \mathbb{O}_\infty^\mu, k \geq k'_1 - 1\}, \quad (6.43b)$$

where β represents the maximal Euclidean norm of the eigenvalue with respect to (6.42). Since $0 < \mu < 1$ and $\phi_m \in \Phi$ is a decision variable, it follows that $\bar{x}_k \rightarrow \mathbf{0}$ as $k \rightarrow \infty$.

For two positive constants $\delta_1 > 0$ and $\epsilon_1 > 0$, we have

$$\mathcal{B}(\delta_1) \subseteq \mathbb{O}_\infty^\mu \subseteq \bigcap_{m \in \mathcal{M}} \Xi_m \subseteq \mathcal{B}(\epsilon_1). \quad (6.44)$$

Considering the inclusions $\mathcal{B}(\delta) \subset \mathcal{B}(\delta_1)$ and $\mathcal{B}(\epsilon_1) \subset \mathcal{B}(\epsilon)$, it follows that, for $\|x_0\| \leq \delta$, we have $\|\bar{x}_k\| \leq \epsilon$ since $\bar{x}_k \in \mathcal{B}(\epsilon_1) \subset \mathcal{B}(\epsilon)$. For a norm ball $\mathcal{B}(\epsilon) \subset \mathcal{B}(\epsilon_1)$, from (6.43a), we still have $\mathcal{B}(\epsilon_1) \subset \mu^s \beta \mathcal{B}(\epsilon)$. Likewise, for $\delta_1 < \delta$, it follows that

$0 < \delta < \mu^s \beta \delta_1$. Therefore, for any $x_0 \in \mathcal{B}(\delta)$, it holds that

$$\bar{x}_k \in \mathbb{O}_\infty^\mu \subseteq \bigcap_{m \in \mathcal{M}} \Xi_m \subseteq \mathcal{B}(\epsilon). \quad (6.45)$$

Thus, the closed-loop system (6.35) is uniformly asymptotically stable even with asynchronous switching and controller failures.

(ii) \Rightarrow (i): Convergence of Algorithm 6.2 is proved succeedingly. Rewrite \mathbb{O}_0^μ as $\mathbb{O}_0^\mu = \{\bar{x} \mid Z_M \bar{x} \leq \mathbf{1}, \forall M \in \mathbb{Z}^+\}$. Let the norm induced by set \mathbb{O}_∞^μ be represented as $\|\cdot\|_{\mathbb{O}_\infty^\mu}$. Assuming that there exists a contraction factor $\mu_1 \in (0, 1)$ such that $\|\hat{\mathbb{A}}_{\zeta_{\Phi[\bar{\Delta}]}}(k)\|_{\mathbb{O}_\infty^{\mu_1}} < \mu_1^s \beta < 1$. Thus, considering the computational process to obtain $\mathbb{O}_y^{\mu_2}$ over $\mathbb{O}_{y-1}^{\mu_2}$ with any $\mu_2 > \mu_1$, $\mu_2 \in (0, 1)$, an inequality

$$Z_M \hat{\mathbb{A}}_{\zeta_{\Phi[\bar{\Delta}]}}(k) \bar{x} \leq \mu_2^y \beta \mathbf{1} \quad (6.46)$$

has to be held where

$$\begin{aligned} \hat{\mathbb{A}}_{\zeta_{\Phi[\bar{\Delta}]}}(k) &:= \prod_{k=\bar{k}'_0}^{k'_1-1} \bar{A}_{\sigma(k)} \left(\prod_{k=k'_1}^{k'_1+d_1^{c[v]}} A_{\sigma(k)} \cdots \prod_{k=\bar{k}'_1-d_r^{c[v]-1}}^{\bar{k}'_1-1} \tilde{A}_{\sigma(\bar{k}'_0), \sigma(k)} \right) \cdots \\ &\quad \prod_{k=\bar{k}'_{y-1}}^{k'_y-1} \bar{A}_{\sigma(k)} \left(\prod_{k=k'_y}^{k'_y+d_1^{c[v]}} A_{\sigma(k)} \cdots \prod_{k=\bar{k}'_y-d_r^{c[v]-1}}^{\bar{k}'_y-1} \tilde{A}_{\sigma(\bar{k}'_{y-1}), \sigma(k)} \right). \end{aligned}$$

Inspired by [142], we will show (6.46) is no more required in the y th iteration with Algorithm 6.2 based on the following inequalities for each row of $Z_M \hat{\mathbb{A}}_{\zeta_{\Phi[\bar{\Delta}]}}(k) \bar{x}$:

$$\begin{aligned} Z_M \hat{\mathbb{A}}_{\zeta_{\Phi[\bar{\Delta}]}}(k) \bar{x} &\leq \max_{\theta_1 \in \mathcal{B}(\|\hat{\mathbb{A}}_{\zeta_{\Phi[\bar{\Delta}]}}(k) \bar{x}\|)} Z_M \theta_1 \leq \max_{\theta_2 \in \mathcal{B}(\|\hat{\mathbb{A}}_{\zeta_{\Phi[\bar{\Delta}]}}(k) \bar{x}\|_{\mathbb{O}_\infty^{\mu_1}})} \lambda_1 Z_M \theta_2 \\ &\leq \max_{\theta_2 \in \mathcal{B}(\mu_1^y \beta \|\bar{x}\|_{\mathbb{O}_\infty^{\mu_1}})} \lambda_1 Z_M \theta_2 \\ &\leq \max_{\theta_3 \in \mathcal{B}(\|\bar{x}\|)} \mu_1^y \beta \lambda_1 \lambda_2 G_M \theta_3 \leq \mu_1^y \beta \lambda_1 \lambda_2 \|G_M\| \|\bar{x}\| < \mu_2^y, \end{aligned}$$

where λ_1 and λ_2 are two positive constants. Hence, the inequality (6.46) is no longer required even with $\mu_2 > \mu_1$. Thus, the convergence of Algorithm 6.2 is proved. This proof is completed. \blacksquare

Remark 6.8. *Theorem 1.2 provides the theoretical basis for the stability analysis of*

the switched MPC design. Once the state trajectory enters the common terminal set \mathbb{O}_∞^μ , the nominal state \bar{x}_k is driven to the origin as $k \rightarrow \infty$, i.e., $D(x_k, \mathcal{E}_\infty) \rightarrow 0$ as $k \rightarrow \infty$.

6.6.4 UAS Criteria for Switched MPC Design

In light of the above stability result, two UAS criteria are proposed.

Theorem 6.4. *Suppose that Assumptions 6.1-6.3 hold with $\mathcal{X}_m^{N_m}$, $\forall m \in \mathcal{M}$, being the feasible set of mode m . In addition, a TFMC set \mathbb{O}_∞^μ exists for the closed-loop system (6.35)-(6.37) with an MDT set Φ . Regarding the region of attraction $\bigcup_{m \in \mathcal{M}} \mathcal{X}_m^{N_m}$, the switched closed-loop system (6.4) in the presence of asynchronous switching and controller failures is uniformly asymptotically stable if $\tau_m^{(1)}$ satisfies (6.28)-(6.30) and $\tau_m^{(s)}$, $s \in \mathbb{Z}_{[2, \bar{s}]}$, satisfies (6.31)-(6.33). Moreover, for the stage index $s \geq \bar{s} + 1$, \bar{s} satisfies*

$$\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{\bar{s}-1}(\mathcal{X}_n^{N_n}) \subseteq \mathbb{O}_\infty^\mu \quad (6.47)$$

for $\tau_m^{(s)} = \phi_m$.

Proof. (i) Persistent feasibility: The design of $\tau_m^{(1)}$, $m \in \mathcal{M}$, directly follows the criterion proposed in Theorem 6.2. For $s \in [2, \bar{s}]$, according to Lemma 6.1, system states are required to be included in $\bigcap_{n \in \mathcal{M}, n \neq m} \bar{\mathcal{S}}_n^{\bar{s}-1}(\mathcal{X}_n^{N_n})$ which is a subset of $\bigcap_{n \in \mathcal{M}, n \neq m} \mathcal{X}_n^{N_n}$. The feasibility criterion (6.31)-(6.33) is obeyed. In addition, from Theorem 6.3, the inclusion

$$\mathbb{O}_\infty^\mu \subseteq \bigcap_{m \in \mathcal{M}} \Xi_m \subseteq \bigcap_{m \in \mathcal{M}} \mathcal{X}_m^{N_m} \quad (6.48)$$

holds. Thus, for $s \geq \bar{s} + 1$, the persistent feasibility of optimal control problem (6.20) is guaranteed.

(ii) Uniform asymptotic stability: Note that the nominal state is driven into a common terminal set \mathbb{O}_∞^μ after $(\bar{s} + 1)$ stages. Once $\bar{x}_{\bar{k}'_{\bar{s}+1}} \in \mathbb{O}_\infty^\mu$ the feedback control law \mathcal{K}_m , $m \in \mathcal{M}$, is applied to (6.35) and (6.36). Therefore, from the stability criteria proposed in Theorem 6.3, it implies that

$$\bar{x}_k = \mathbb{A}_{\zeta_{\Phi, [\bar{\Delta}]}}(k) \bar{x}_{\bar{k}'_{\bar{s}+1}} \in \mu^s \beta \mathbb{O}_\infty^\mu$$

for all admissible switching sequences. Then, we have $\|\bar{x}_k\| \rightarrow 0$ as $k \rightarrow \infty$. The

stability proof is similar to Theorem 6.3 and omitted here. This completes the proof. \blacksquare

Remark 6.9. *Compared with the stage-based attractivity criterion in Lemma 6.1, the determination of stage MDTs can be terminated in $(\bar{s} + 1)$ stages, $\bar{s} \geq 2$, once (6.47) is satisfied.*

Lemma 6.2. *Suppose that Assumptions 6.1-6.3 hold with $\mathcal{X}_m^{N_m}$, $\forall m \in \mathcal{M}$, being the feasible set of mode m . In addition, a TFMC set \mathbb{O}_∞^μ exists for the closed-loop system (6.35)-(6.37) with an MDT set Φ . Regarding the region of attraction $\bigcup_{m \in \mathcal{M}} \mathcal{X}_m^{N_m}$, the switched closed-loop system (6.4) in the presence of asynchronous switching and controller failures is uniformly asymptotically stable if $\tau_m^{(1)}$, $m \in \mathcal{M}$, satisfies (6.28) and (6.29) and*

$$\bigcup_{n \in \mathcal{M}, n \neq m} \bigcup_{v=1}^{v=V} \mathbb{S}^{c_{[mn,v]}} (\bar{\mathcal{S}}_m^{\tau_m} (\mathcal{X}_m^{N_m})) \subseteq \mathbb{O}_\infty^\mu. \quad (6.49)$$

Furthermore, the MDT $\tau_m^{(s)} = \phi_m$ for all $s \in \mathbb{Z}_{[2,+\infty)}^+$

Remark 6.10. *The criterion proposed in Lemma 6.2 forces all possible trajectories of system states to be included in \mathbb{O}_∞^μ in the first stage. After the first stage, the stage MDT constraint is used to guarantee the existence of the set \mathbb{O}_∞^μ .*

6.7 Simulation Results

To validate the effectiveness of the proposed control scheme, we consider a switched linear system with two subsystems

$$A_1 = \begin{bmatrix} 1.6 & 0.1 \\ 0.3 & -0.49 \end{bmatrix}, A_2 = \begin{bmatrix} 0.6 & 0.5 \\ 0.85 & 0.5 \end{bmatrix}, \quad (6.50a)$$

$$B_1 = \begin{bmatrix} -0.9 & 2.7 \end{bmatrix}^T, B_2 = \begin{bmatrix} 1.5 & -1.4 \end{bmatrix}^T. \quad (6.50b)$$

The state constraints of mode 1 and mode 2 are respectively given as $\mathbb{X}_1 := \{x_k \in \mathbb{R}^2 \mid \|x_k\|_\infty \leq 14\}$ and $\mathbb{X}_2 := \{x_k \in \mathbb{R}^2 \mid \|x_k\|_\infty \leq 12\}$. The input constraints of both mode 1 and mode 2 are $\mathbb{U}_1 = \mathbb{U}_2 := \{u_k \in \mathbb{R}^2 \mid \|u_k\|_\infty \leq 4\}$. The disturbance constraint is $\mathbb{W} := \{w_k \in \mathbb{R}^2 \mid \|w_k\|_\infty \leq 0.008\}$. The weighting matrices of mode 1 are $Q_1 = [10 \ 0; 0 \ 10]$ and $R_1 = 1$. The weighting matrices of mode 2 are $Q_2 = [5 \ 0; 0 \ 5]$ and $R_2 = 1$. Then the linear quadratic regulator (LQR) feedback gains are computed as

$K_1 = [1.0481 \ 0.1375]$ and $K_2 = [-0.8353 \ -0.6301]$. Additionally, the corresponding terminal weighting matrices are $P_1 = [157.2674 \ -5.3006; -5.3006 \ 10.2219]$ and $P_2 = [55.2919 \ 34.5284; 34.5284 \ 28.7092]$, respectively. Moreover, $N_1 = N_2 = 5$ are served as the prediction horizon. The upper bound of the faulty portion is $\bar{\Delta} = 2$. Note that, in (??), the spectral radii of A_1 , A_2 , $\tilde{A}_{1,2}$, and $\tilde{A}_{2,1}$ are all greater than one, which results in the infeasibility and instability issues of the closed-loop system caused by controller failures and asynchronous switching. We use this general example to verify the theoretical findings.

The simulation results are obtained by employing MATLAB YALMIP [150] and MPT3 [143] toolboxes. Table 6.1 demonstrates the computational results of minimum MDT values by employing the proposed algorithms and criteria. The MDT constraints adopted in Algorithm 6.1 and 6.2 ensure the existence of an FDMI set and a TFMC set, respectively. The lower bound of MDTs obtained by utilizing the designed strategy in Theorem 6.2 guarantees the persistent feasibility of the switched MPC algorithm. For the UAS criterion proposed in Theorem 6.4, it is worth mentioning that $\tau_1^{(2)}$ is greater than $\tau_1^{(1)}$. The reason behind is that the state trajectories reaching a tightened target region, i.e., $\tilde{\mathcal{S}}_2^1(\mathcal{X}_2^{N_2}) \subseteq \mathcal{X}_2^{N_2}$, require a longer stage MDT.

Table 6.1: The simulation results of minimum MDT values with respect to distinct proposed algorithms or strategies.

Algorithms/ Criteria	Lower Bounds of MDTs
Algorithm 6.1	$\tau_1 = 5, \tau_2 = 4$
Theorem 6.2	$\tau_1 = 7, \tau_2 = 13$
Algorithm 6.2	$\tau_1 = 5, \tau_2 = 4$
Theorem 6.4	$\tau_1^{(1)} = 7, \tau_2^{(1)} = 13; \tau_1^{(2)} = 32, \tau_2^{(2)} = 10;$ $\tau_1^{(3)} = 7, \tau_2^{(3)} = 4; \tau_1^{(4)} = 5, \tau_2^{(4)} = 4, \forall s \geq 4$
Lemma 6.2	$\tau_1^{(1)} = 8, \tau_2^{(1)} = 13; \tau_1^{(2)} = 5, \tau_2^{(2)} = 4, \forall s \geq 2$

Figure 6.5 displays the feasible regions of subsystem 1 and 2, i.e., $\mathcal{X}_1^{N_1}$ and $\mathcal{X}_2^{N_2}$, as well as the reachable sets $\tilde{\mathcal{S}}_{1,2}^1$ and $\tilde{\mathcal{S}}_{1,2}^2$ during the faulty portion with $\tilde{\mathcal{S}}_{1,2}^1 \subset \mathcal{X}_1^{N_1}$ and $\tilde{\mathcal{S}}_{1,2}^2 \subset \mathcal{X}_2^{N_2}$. It is also seen that the state trajectory starting from an initial feasible state $x_0^{T2} = [2.7; 10.5]$ of mode 1 along an admissible switching sequence with MDT constraint in Theorem 6.2 successfully enters into the feasible region of mode 2 after leaving the $\tilde{\mathcal{S}}_{1,2}^1$ and $\tilde{\mathcal{S}}_{1,2}^2$ sequentially. Thereby, the persistent feasibility of switched MPC algorithm is ensured. The obtained TFMC set is displayed in Figure 6.6 with a contractive factor $\mu = 0.99$. Four closed-loop state trajectories by using

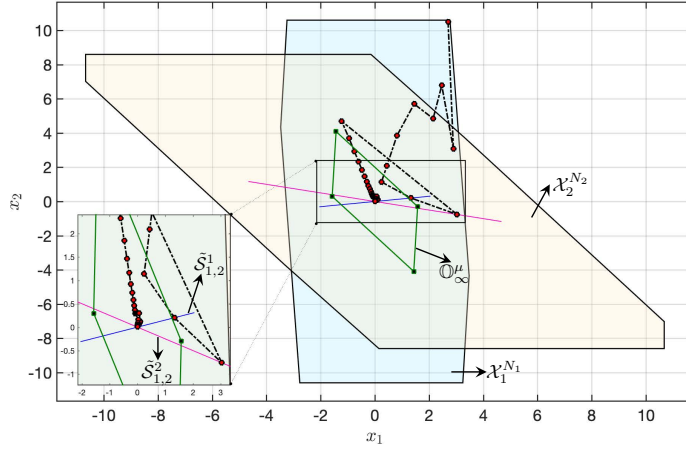


Figure 6.5: The trajectory of system states which satisfies the feasibility requirement in Theorem 6.2 and reachable sets.

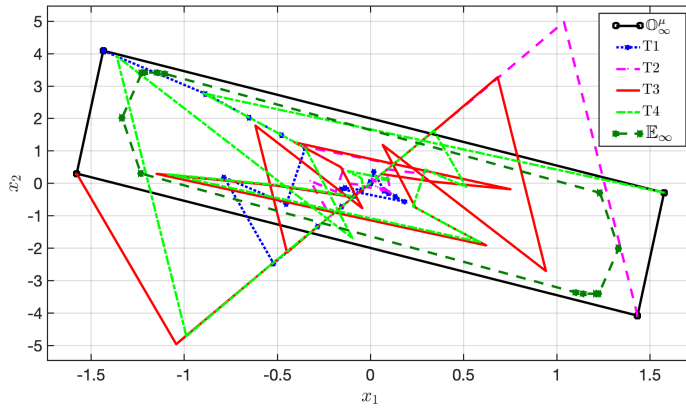


Figure 6.6: Illustration of the FDMI set and TFMC set and state trajectories.

the feedback control law (6.35)-(6.37) are plotted and the initial states are chosen as the four vertices of $\mathbb{O}_{\infty}^{\mu}$, respectively. Note that the system states may leave the constructed common terminal set $\mathbb{O}_{\infty}^{\mu}$. This is because the calculation of $\mathbb{O}_{\infty}^{\mu}$ only collects the system behavior along the admissible switching sequences via Algorithm 6.2. However, the state evolution with all the truncated switching sequences is not considered. The FDMI set \mathbb{E}_{∞} which is used to tighten the original state and input constraints is also shown in Figure 6.6 and the inclusion $\mathbb{E}_{\infty} \subset \mathbb{O}_{\infty}^{\mu}$ holds.

The nominal state responses of the closed-loop system along the admissible switching sequences which satisfy the stage MDT constraints in Theorem 6.4 and Lemma 6.2 are manifested in Figure 6.7 and Figure 6.8, respectively. The initial states are set as $x_0^{T4} = [-2.7; -10]$ and $x_0^{L2} = [-8; 4]$ which are respectively within the initial feasible regions of $\mathcal{X}_1^{N_1}$ and $\mathcal{X}_2^{N_2}$. The corresponding control input trajectories are

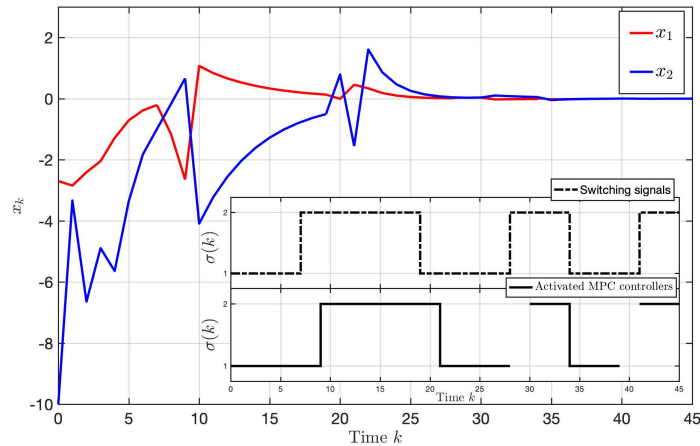


Figure 6.7: Trajectories of system states with the MDT constraint satisfaction in Theorem 6.4.

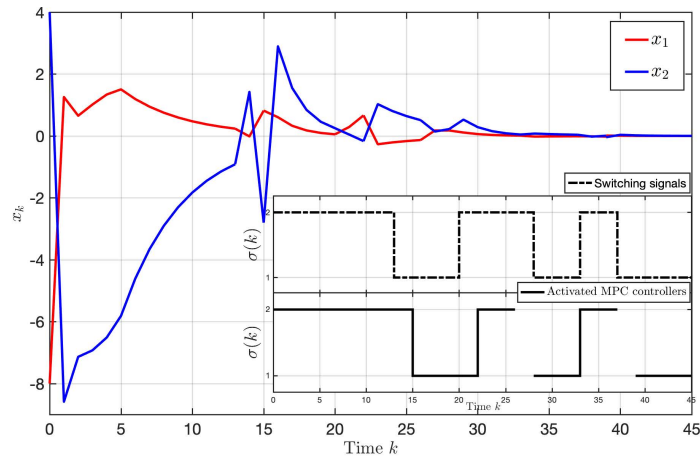


Figure 6.8: Trajectories of system states with the MDT constraint satisfaction in Lemma 6.2.

shown in Figure 6.9. The convergence of closed-loop state trajectories verifies the effectiveness of the proposed UAS criteria to mitigate the joint effects of asynchronous switching and controller failures with respect to switching dynamics.

6.8 Conclusion

In this chapter, the RMPC problem for a class of disturbed asynchronously switched linear systems with controller failures has been investigated. A tube-based switched MPC strategy has been developed by taking the advantage of the appropriately tightened original constraints. To retain the persistent feasibility of the switched MPC

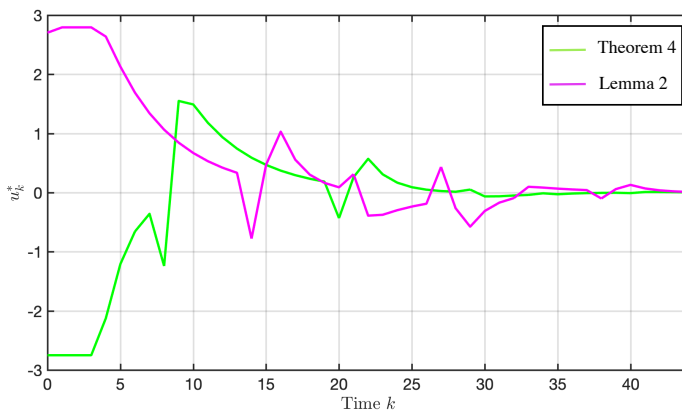


Figure 6.9: Trajectories of control input.

algorithm, we have proposed a criterion to enforce the state trajectory staying in a commonly feasible region for all modes with the MDT constraint. Additionally, we have explored a non-conservative stability property: The existence of a TFMC set is equivalent to the UAS of the switched closed-loop system. This essential property further facilitates the arise of the stability criteria by letting all the possible state along the admissible switching sequences to be contained in the TFMC set. The simulation results have been provided to demonstrate the effectiveness of the theoretical results. Future work involves extending the current theoretical findings for a single switching dynamics to multi-agent switched systems with fault-tolerant control design purposes.

Chapter 7

Conclusions and Future Works

7.1 Conclusions

In this dissertation, three works on switched MPC and one work related to the stabilization of switched systems have been studied. Executable switched MPC strategies are proposed with guaranteed closed-loop stability and persistent feasibility of the switched MPC algorithm. Under the typical faults of switching dynamics, switched MPC design has also been discussed from a theoretical perspective and the stability results have been further extended to constrained switched systems in the presence or absence of external disturbances.

Chapter 3 has studied the MPC problem for a class of constrained switched linear systems with DT constraints. With the known *a priori* switching sequences, a sufficient condition regarding the length of the prediction horizon has been proposed to ensure the closed-loop stability as well as the recursive feasibility of the switched MPC algorithm. By employing the estimated suboptimal parameters, the length of the prediction horizon has been quantitatively determined. Numerical simulations have verified the effectiveness of the theoretical results.

Chapter 4 has investigated the asynchronously switched MPC problem with MDT constraints. For retaining the persistent feasibility, the lower bound of MDT has been determined offline to ensure that every step evolved reachable set can be included in a target feasible area. Moreover, a constraint admissible contractive AMI set has been designed to serve as a common terminal set for all subsystems. Based on the superior properties of the terminal set, two asymptotic stability criteria have been proposed by driving the system states into the constructed terminal set and the

stage MDT has been computed offline. The advantage of theoretical results has been verified via a numerical example.

Chapter 5 has explored the reliable control strategy for constrained switched linear systems with multiple faults. A CAF-MDT set has been established to collect all the admissible initial states subject to MDT constraints. In the light of the computed CAF-MDT set, a non-conservative stability condition has been found and the theoretical findings have been rigorously proven. Then, this condition has been extended to constrained switched systems in the presence of bounded additive disturbances. The simulation studies have demonstrated that the proposed stability condition leads to uniformly asymptotic stability of the closed-loop system.

Chapter 6 has been concerned with the RMPC problem for a class of asynchronously perturbed switched linear systems with controller failures. To collect the disturbance behaviors along all the admissible switching sequences, the algorithm to calculate the FDMI set is designed. Afterward, in order to fulfill the system constraints, a tube-based MPC strategy is used to tighten the original constraints. Based on the modified TFMC set, the proposed persistent feasibility algorithm of asynchronously switched MPC and stability criteria in **Chapter 4** have been developed further to nominal asynchronously switched systems under controller failures with rigorous proof. Simulation studies have been provided to demonstrate the efficacy of the switched MPC scheme.

7.2 Future Works

According to the inspection of the comprehensive literature review in **Chapter 1**, it can be seen that switched MPC as well as switching control problems are still emerging and vibrant research topics. In the following, we list three potential research directions for future studies.

Distributed MPC for constrained switched linear systems with asynchronous communication delays

Asynchronous communication delays in a distributed multi-agent system refers to one agent providing information while other recipients receive the information with a time delay. Compared with the centralized MPC for a multi-agent system in which all control decisions are concentrated on a single node, the distributed formulation can

allow control actions generated across each controlled individual agent such that the scalability is enhanced and the overall reliability of the multi-agent system is improved to some extent. Inspired by [131], we intend to study the distributed MPC for a group of constrained switched linear systems in the presence of asynchronous communication delays. In contrast with distributed MPC design for LTI systems, the information on switching signals will also be sent to the recipients. Hence, considering the severely detrimental effects on the stabilization of a group of switching agents caused by the transmission delays with respect to the switching signals as well as the system states in a distributed MPC network, a feasible and effective solution to tackle this issue is worth investigating.

Event-triggered control of disturbed switched linear systems with multiple faults

In advanced networked control systems, many interacting components may possess identical communication networks to exchange information and share resources. Therefore, the communication cost for periodic signal transmission is supposed to take into consideration due to the limited communication bandwidth. In order to tackle this issue, the aperiodic event-triggered control strategy is employed to circumvent unnecessary information exchange unless some unexpected behaviors happen. Considering the joint adverse effects of external disturbances and multiple faults, the introduction of the event-triggering mechanism can effectively monitor whether the deviation between the real state trajectories and assumed state trajectories exceeds the devised threshold with constraint satisfaction while reducing additional communication effort. Thus, the event-triggered control strategy offers a promising solution for achieving reliable and efficient control synthesis of networked control systems.

MPC for switched linear systems with false-data injection attack

These days, resilient control design for cyber-physical systems against cyber-attacks has received enormous attention in both academia and engineering practice since various cyber-attacks may pose severe threats to the safety and reliability of applications embedded in cyber-physical systems. *False-data injection attacks*, as one of the classes of deception attack, can partially or fully get access to the sensor or actuator channels and acquire model knowledge so as to characterize attack policies for the convenience to achieve stealthy false-data conversion [151]. Since the attack policies may replace

the MPC law originally in the closed-loop, this can result in instability and infeasibility issues of the switched MPC design. Furthermore, the control performance will be affected to a great extent. Hence, an interesting potential research direction is to propose a resilient switched MPC strategy against stealthy false-data injection attacks while preserving the recursive feasibility of the switched MPC algorithm.

Experimental validation

The ultimate goal of my future research is to validate the proposed switched MPC strategies in a real switched system, such as a UAV model with different maneuver action modes (taking-off mode, landing mode, and hovering mode). In the real-time implementation of MPC, whether the optimal control action can be updated within a sampling time is a primary challenge. To this end, a fast-switched MPC algorithm needs to be designed. In addition, an appropriate way to mitigate the transient behavior caused by mode transitions in real switched systems still needs more research endeavor.

Appendix A

Publications

- **Journal papers:**

- J1. L. Zhang, B. Cai, **T. Tan** and Y. Shi, “Stabilization of Non-homogeneous Hidden Semi-Markov Jump Systems with Limited Sojourn-time Information,” *Automatica*, vol. 117, pp. 108963, 2020.
- J2. **T. Tan**, S. Zhuang, and Y. Shi, “Model Predictive Control for Asynchronously Switched Linear Systems with Mode-dependent Dwell Time,” *Automatica*, submitted, February 2022 (Accept provisionally as brief paper).
- J3. **T. Tan**, S. Zhuang, and Y. Shi, “Non-conservative Stability Criteria for Constrained Switched Linear Systems under Multiple Faults,” *IEEE Transactions on Automatic Control*, submitted, July 2022 (Under review).
- J4. **T. Tan**, S. Zhuang, and Y. Shi, “Robust Model Predictive Control for Asynchronously Switched Linear Systems with Intermittent Controller Failures,” *IEEE Transactions on Automatic Control*, submitted, April 2023 (Under review).

- **Conference papers:**

- C1. **T. Tan**, S. Zhuang and Y. Shi, “Switched Model Predictive Control with Scheduled Mode Transitions without Terminal Constraints,” in *Proceedings of 2021 IEEE 4th International Conference on Industrial Cyber-Physical Systems (ICPS 2021)*, Victoria, Canada, May. 10–12, pp. 185–190, 2021.
- C2. **T. Tan**, X. Shang, L. Yang, and Y. Shi, “Model Predictive Control of Asynchronously Switched Systems with Exogenous Disturbances,” in *Proceed-*

ings of the 24th IEEE International Conference on Industrial Technology (ICIT 2023), Orlando, USA, Apr. 04–06, pp. 1-6, 2023.

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