MATERIALS FOR

"EDUCATION FINANCE AND OPTIMAL INVESTMENT

WITH GESTATION LAGS"*

by

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These materials are preliminary, and circulated for discussion only. Please do not quote.

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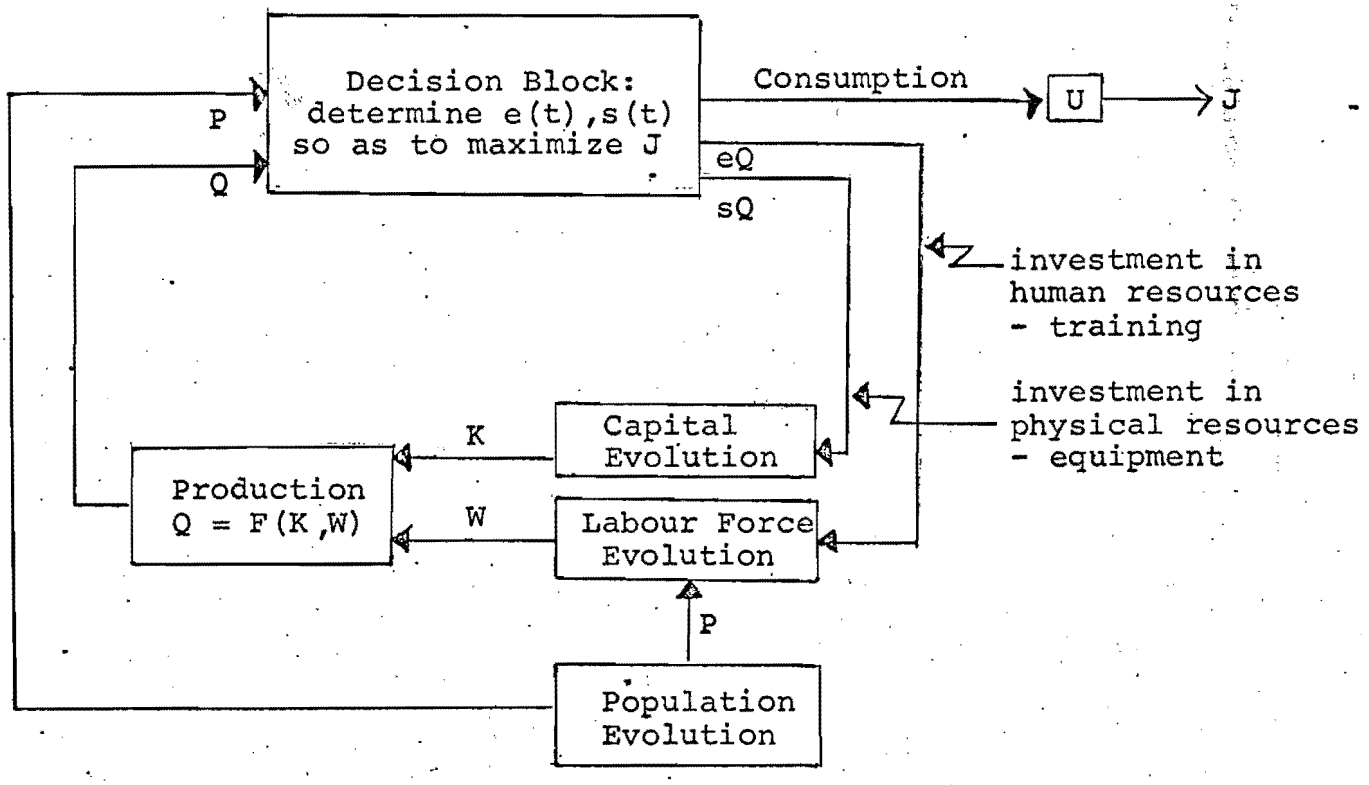
1. The Macroeconomic Approach [3,4,5,7,8,11]



Model I

$$\max J = \int_0^{t_f} U(c(t))\exp(-\gamma t)dt$$

$$\text{s.t. } c(t) = (1-s(t)-e(t))Q/P(t) = (1-s(t)-e(t))wf(k(t)/w(t))$$

$$\dot{k} = -\sigma k + swf(k/w) \qquad k(0) = k_o, k(t_f) \geq k_f$$

$$\dot{w} = ewf/d - aw \qquad w(0) = w_o, w(t_f) \geq w_f$$

$$0 \leq s(t) \leq 1$$

$$0 \leq e(t) \leq 1$$

$$0 \leq c(t) = (1-s-e)f(k)$$

$$0 \leq w \leq 1,\ 0 \leq k$$

Model II (more detail on population dynamics and skill structure)

Define the productivity-weighted measure of the effective labour force by:

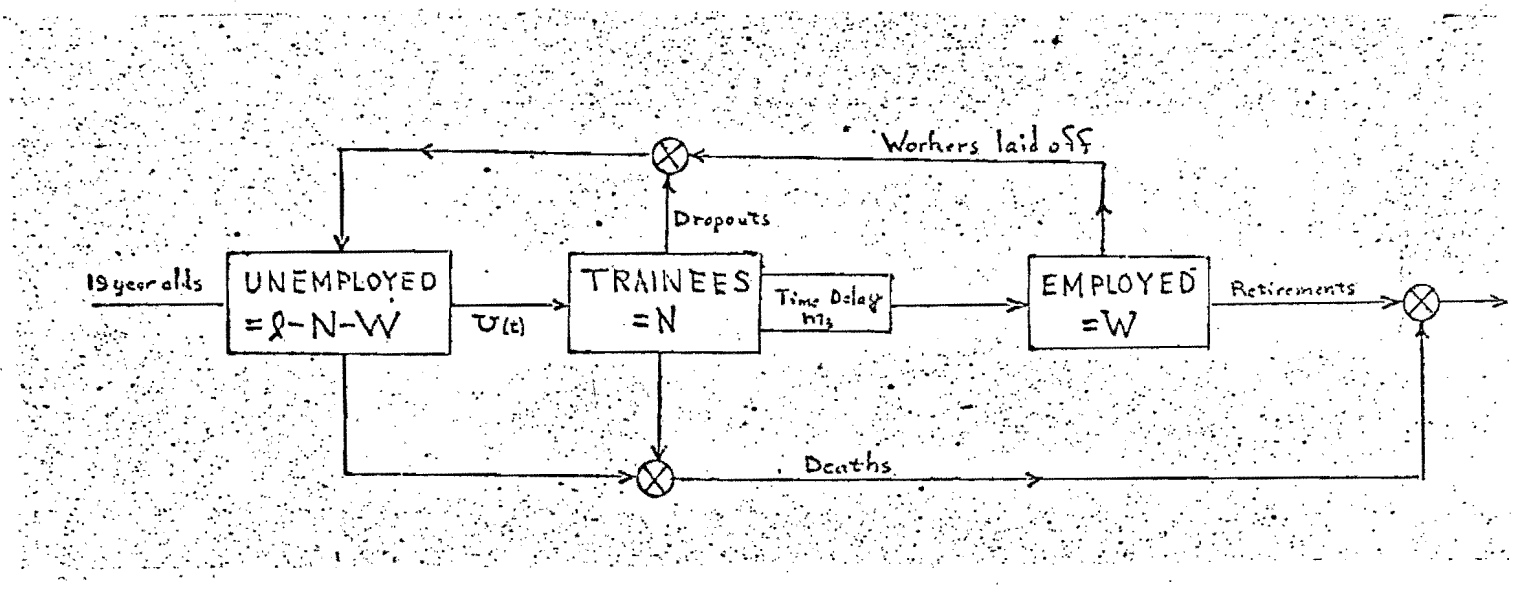
$$W^*(t) = \int_{t-m}^{t} w(t,\tau)A(t,\tau)d\tau$$

where $w(t,\tau)$ represents the fraction of the labour force entering employment at time τ surviving to time t, and $A(t,\tau)$ represents the productivity factor attaching to them.

criterion	$J = \int_{0}^{t_f} U(c(t))\exp[-\gamma t]\,dt$
dynamics	$\dot{w}^*(t) = -aw^*(t) + b_1 u(t)A(t) - b_1 b_2 u(t-m)A(t-m)$
	$\dot{k}(t) = -\sigma k(t) + s(t)w^*(t)f(k(t)/w^*(t))$
control variables	$A(t)$, $s(t)$, $u(t)$
state variables	$k(t)$, $w^*(t)$
constraints	$1 \geq s \geq 0$, $1 \geq u \geq 0$, $c = [(1-s)w^*f(k/w^*) - b_1 ud(A)]\,\theta \geq 0$

Source: Budelis [3]

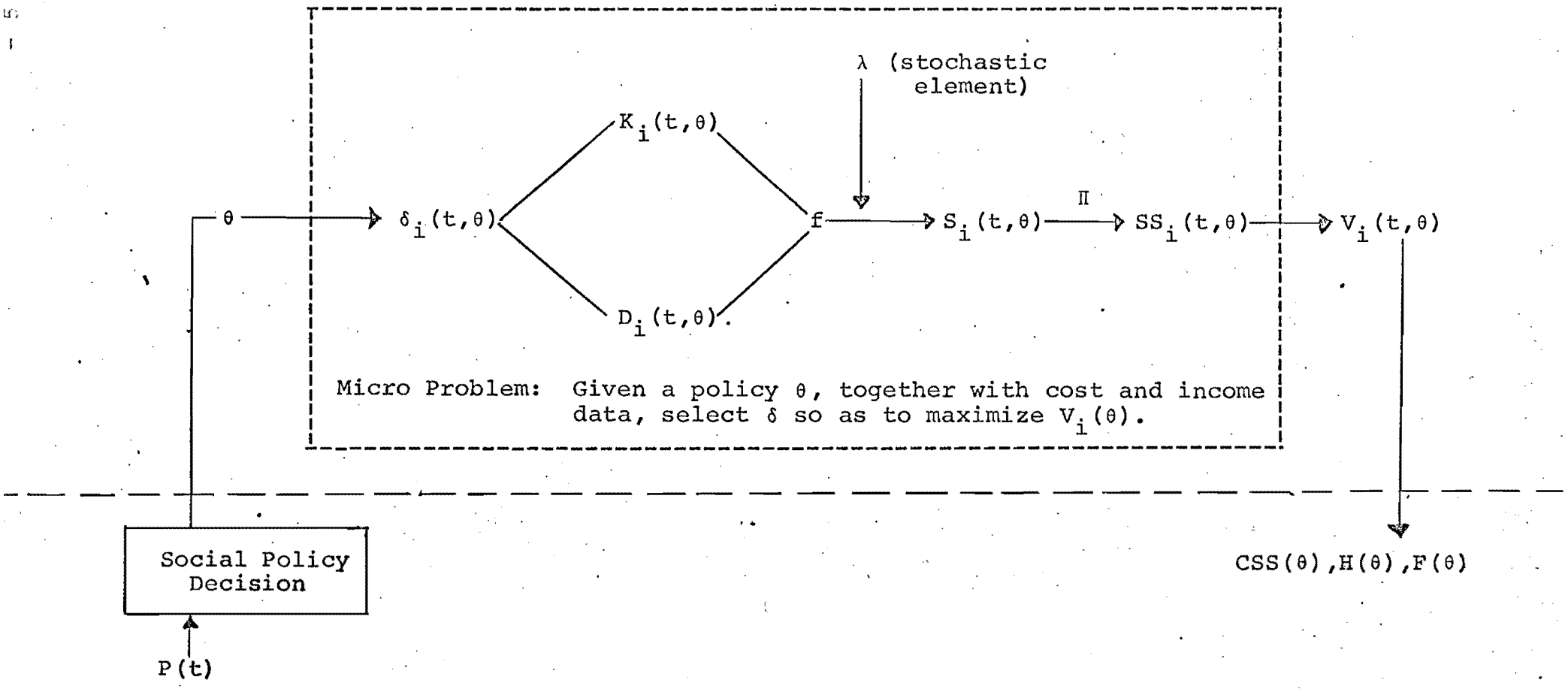
Model III (expand labour force block to take account of gestation lags)



criterion	$J_1 = \int_0^{t_f} \frac{c(t)}{\theta} \exp[-\gamma t]dt$
dynamics	$\dot{w}(t) = -aw(t) + \exp[-bm_3]\, u(t-m_3)$
	$\dot{n}(t) = -bn(t) - \exp[-bm_3]\, u(t-m_3) + u(t)$
	$\dot{k}(t) = -\sigma k(t) + s(t)w(t)f(k/w)$
control variables	$s(t), u(t)$
state variables	$k(t), n(t), w(t)$
constraints	$1 \geq s \geq 0,\ \bar{u} \geq u,\ c = (1-s)wf - dn \geq 0,\ 1 \geq w+n$

Source: Budelis [3]

Simulation Model (Schematic Diagram)



Macro Problem: Given population data (cohort sizes) and a statistical approximation to the individual decision rules, select the policy θ yielding the "optimal" configuration of cohort monetary flows, CSS, fund flows, F, and distributional results, H.

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