A MANIPULATIVE MATERIALS APPROACH
TO TEACHING FRACTIONS
AT THE GRADE 4 LEVEL:
A COMPARATIVE STUDY

by

EDWARD WILLIAM RICHMOND

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Supervisor: Dr. J. H. Vance

ABSTRACT

The purpose of the study was to investigate the relative effectiveness of a concrete materials centered approach for teaching introductory topics in fractions to Grade 4 pupils. The set of materials consisted of a kit of colored paper strips of varying lengths constructed by each pupil.

The students in the two Grade 4 classes in a Greater Victoria elementary school were assigned on a stratified random basis to two groups for the study. Both groups were taught by the same teacher using large group instruction for the treatments which lasted for fifteen consecutive instructional periods.

Subjects in the Experimental Group each constructed a fraction kit and made use of the strips for approximately half of the instructional time. Subjects in the Control Group studied fractions as outlined in the prescribed textbook and did not use concrete materials.

Instruments were developed to compare the two groups on initial achievement, retention, and transfer and to ascertain the reaction of students to the instructional treatments. Both pencil and paper tests and performance type tests requiring the manipulation of physical materials were developed for the study.

No significant differences were found between the group mean scores on any of the four tests either immediately following

the treatments or two months after the treatments. Neither treatment was found to be more suitable than the other for students of any of the three ability levels. Each group responded favorably to its treatment. Within the Experimental Group students of average ability tended to respond more favorably to the use of the fraction kit than either the high or low ability students.

Examiners: [redacted]

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CHAPTER 1

INTRODUCTION

BACKGROUND AND NEED FOR THE STUDY

The study of fractions has long been a source of confusion and difficulty for both students and teachers. Price (1969) writes:

> Fractions probably give trouble to more teachers, and thus more pupils, than any other single topic in elementary school mathematics (p.111).

Because of the difficulty caused by fractions, researchers for many years have been investigating this topic in an attempt to determine the most effective methods for teaching it. It is noteworthy that a lack of mastery of the early work, particularly skills involving equivalent fractions, seem to account for a large portion of pupil errors in computation involving fractions (Brueckner, 1928; Hinkelman, 1956; Anderson, 1966).

With the coming of "modern Mathematics" to North America in the early 1960's, a renewed interest developed in the role of concrete materials in mathematics instruction. The theories and work of educators and psychologists such as Piaget, Bruner, and Dienes also lent support for the use of such materials with elementary school students. With this new emphasis on manipulation of physical objects, approaches such as the Cuisenaire method (Cuisenaire and Gattegno, 1955) and the mathematics laboratory have become popular.

Kieren (1971) defines two broad questions concerning concrete materials which research has left unanswered:

> In what particular ways does an action-image-symbol sequence, or components thereof, affect mathematics learning? For whom, for which topics, and with what materials are manipulative and play-like activities valuable? (p. 232)

A wide variety of concrete materials have been recommended by researchers and teachers for the teaching of fractions. One inexpensive and easy to construct aid is a fraction kit consisting of a variety of colored paper strips which represent the more common unit fractions.

THE PROBLEM

The purpose of the study was to determine the relative effectiveness of using pupil constructed fraction kits for teaching introductory topics in fractions at the Grade 4 level. Answers to the following questions were sought:

(1) Would the use of this manipulative materials approach be more effective in introducing fractions than a textbook centred approach?

(2) Would the effectiveness of either of these two methods vary with the ability level of the students?

(3) Are there any differences in transfer benefits between the concrete and the textbook approaches?

(4) Would performance type concrete materials tests detect a different level of learning effectiveness?

(5) What would be the reaction of students to learning fractions by this manipulative approach?

THE EXPERIMENTAL SETTING

Two groups were formed from the two Grade 4 classes in a single elementary school.

The Experimental Group

The Experimental Group was introduced to the study of fractions through the use of a fraction kit (Appendix A) which each student first constructed. This kit consisted of a number of colored paper strips cut to varying lengths to correspond to common unit fractions. The class was instructed in one large group. About half of the fifteen instructional periods taken for the treatment were used for constructing and manipulating the fraction strips.

The Control Group

The Control Group studied fractions using the prescribed textbook, Seeing Through Arithmetic (Gage, 1960). Activities for this treatment included blackboard and textbook illustrations and pencil and paper sketches, but no manipulation of physical materials (Appendix B). The same teacher instructed both the Experimental and Control Groups.

OUTLINE OF THE REPORT

The present chapter has introduced the problem. Chapter 11 reviews the literature related to the study. Chapter 111 includes a description of the development of the experimental programme and the design of the study. The results of the investigation are reported in Chapter 1V, and Chapter V contains a summary of the study and the conclusions and implications drawn from it.

CHAPTER 11

REVIEW OF THE LITERATURE

In this chapter literature relating to the use of concrete materials in mathematics instruction is reviewed. The first part of the chapter examines the theoretical basis for manipulative materials. This is followed by a survey of statistical research which explores the effectiveness of using concrete materials in teaching mathematics. Finally, action research and statistical research investigating the use of physical materials in the teaching of fractions is discussed.

THEORETICAL BASIS

The idea of using concrete materials in mathematics instruction is not new. Pestalozzi in 1801 and Colburn in 1821 initiated mathematics instruction utilizing physical materials. Recent mathematics curriculum revisions in North America and current practice demonstrate a renewed emphasis on the role of concrete manipulation in mathematics learning. Support for this trend can be found in current theories in developmental psychology. The work of Piaget, Bruner, and Dienes has indicated that concrete materials can play a strategic role in the instructional sequence in mathematics.

Piaget has suggested that as a child progresses from infancy to maturity he passes through four distinct stages of mental growth. The child between the ages of seven and eleven is usually in the stage of "concrete operations". These mental operations are called "concrete" because their starting point is always in some real system of objects and relations perceived by the child. One implication of Piaget's theory to mathematics teaching, according to Adler (1966), is that physical action is one basis of learning. To develop the concept of number and space, a child must not only look at things but he must experience them through physical manipulation.

Duquette (1972) drew specific implications for the teaching of fractions from Piaget's learning model. He concluded that the teaching of equivalent fractions to children below Grade 5 is effective only when concrete materials are at hand and that the teaching of addition of fractions should be done through manipulation of physical materials. Duquette suggested, therefore, that the teaching of fractions begin at the concrete level and remain there until the child is capable of meaningfully dealing with symbols alone.

Bruner (1966) outlined an optional sequence for learning mathematical ideas proceeding from an "enactive representation" (physical manipulation of objects), to an "iconic representation" (mental manipulation of images), and finally to a "symbolic representation". He suggested that to effectively teach an idea or concept one should first provide an opportunity for concrete manipulation leading to the association or concept. He emphasized the importance of teaching so that students grasp the structure of the subject. This, he claimed, will enable them to generalize to new situations.

Dienes (1960) described three stages in conceptual development. The initial stage involves an exploration period in which the "concept realization" begins. The second stage adds more awareness and direction in thinking. At this stage a large number of different experiences are required to prepare the foundation for developing the concept. These two stages prepare the way for "insight" which completes the learning cycle. Dienes emphasized that the child's ideas grow out of his personal experiences. Kieren (1971) described Dienes' theory as a two-phase sequence for learning mathematical ideas. The first is the "constructive phase" in which ideas are developed upon physical experience. The second is the "analytic phase" in which the idea or concept is analyzed from a logical point of view.

In summary, the theories of Piaget, Bruner, and Dienes support a progression from concrete materials to semi-concrete representations, and finally to symbolic representation in the learning of mathematical ideas at the elementary school level.

RESEARCH STUDIES

Use of Concrete Materials in Mathematics Learning

The teaching methods and materials of Cuisenaire (1955) were a significant development in the teaching of elementary school mathematics. Studies examining the effectiveness of these materials (colored rods) usually evaluate them in terms of general arithmetic achievement which includes work with fractions. Canadian experience with the Cuisenaire method was investigaged by the Canadian Council for Research in Education (1964). The studies now reviewed are cited in their publication.

In a Vancouver experiment a comparison was made between the arithmetic achievement of Cuisenaire-instructed pupils and the achievement of pupils instructed according to the prescribed British Columbia mathematics curriculum. At the end of the first grade the Cuisenaire group scored slightly higher on the Detroit Test of Arithmetic and significantly higher on the Cuisenaire test. The Cuisenaire group was also able to cover more arithmetic topics (including fractions) than the Control Group. In a Burnaby study two groups of Grade 1 pupils were subjected to the same instructional treatments as the groups in the Vancouver study. On entering Grade 2 the pupils were tested to determine retention of mathematical ideas learned in Grade 1. It was found that the Cuisenaire group had a significantly higher mean score than did the Control Group.

An experiment in Brandon, Manitoba compared three groups of pupils in knowledge of addition and subtraction of whole numbers, number relations, and fractions. One group used Cuisenaire materials in the first two grades, a second group used Cuisenaire materials in Grade 1 and the prescribed course in Grade 2, and a third group used the prescribed course in Grades 1 and 2. The achievement was found to vary directly with the amount of Cuisenaire instruction received. The results of experiments conducted in other parts of

Manitoba and in Saskatchewan support these findings, but other studies produced conflicting results.

A study in North York, Ontario compared Cuisenaire and Number Patterns programmes with the prescribed curriculum at the first grade level. On general arithmetic achievement the highest mean score was achieved by pupils on the prescribed curriculum. The pupils using Number Patterns achieved the second highest mean score, and the Cuisenaire group achieved the lowest mean score.

The authors of the survey of Canadian experience with Cuisenaire concluded that the Cuisenaire method effectively develops understanding in arithmetic, but that the effectiveness of these materials as aids in arithmetic computation reaches a peak in about Grade 3. They also observed that symbol manipulation outstrips comprehension of these manipulations.

Haevin (1965) conducted a study involving 2800 children in which the frequency of use of instructional materials was the independent variable. At the end of the year-long study the classes in which the concrete materials were used frequently, achieved significantly higher mean scores than did the classes that made infrequent use of concrete materials.

Weber (1970), using 24 classes of Grade 1 pupils, compared the effectiveness of reinforcing mathematical concepts through pencil and paper activities ("Gagné based") and concrete manipulative materials ("Piaget based"). It was observed that children using concrete materials scored significantly higher on an oral test of understanding, both in the number of correct responses and in levels of understanding.

Fennema (1970) compared a meaningful concrete approach with a meaningful symbolic approach in teaching multiplication to Grade 2 pupils. On a symbolic transfer test the group receiving the meaningful symbolic treatment scored significantly higher than the group receiving the meaningful concrete treatment. Trends in the same direction were detected in overall learning of the principle and in direct recall. However, these differences were not significant.

Trueblood (1968) investigated two approaches to the use of concrete materials in the teaching of exponents and non-decimal bases to children aged nine to eleven. In one approach pupils manipulated the concrete devices. In the second approach the pupils instructed the teacher how to manipulate these concrete devices, and the pupils observed this manipulation. On the posttest the teacher demonstration method resulted in a higher mean score which was significant at the $P < .1$ level. No significant differences were detected on the retention test.

Shuster and Pigge (1965) studied the relative effectiveness of three approaches to teaching fractions to pupils in the fifth grade. They varied the amount of developmental activities (discussion, audio-visual presentations, and manipulative materials) and drill activities (oral questions, flash cards, drill books and textbooks). The amount of developmental activities given to the different groups was 75 percent, 50 percent and 25 percent. The remaining time was spent in drill activities. On the retention tests the groups receiving 75 percent and 50 percent developmental activities did significantly better in the areas of understanding of fractions and of addition and subtraction of fractions than the group receiving only 25 percent developmental activities. These findings were consistent with the results obtained earlier by Shipp and Deer (1960) in a similar study involving pupils in Grades 4, 5, and 6.

On the other hand, Surinam and Weaver (1971), in a review of the research conducted in 1970, reported little support for the superiority of the manipulative approach. They stated that the "use of concrete materials may not always be as essential to the development of meaning as has been hypothesized ..."(p. 3).

Kieren (1971) in reviewing the research on manipulative activity in mathematics learning cited studies by Fennema (1970), Trueblood (1967), and Vance (1969) which reported that meaningful instruction at the symbolic or visual-image level was as effective as manipulation of physical materials in terms of achievement. Thus the evidence for research on the effectiveness of using concrete manipulative materials is not conclusive.

Use of Concrete Materials in Teaching Fractions

Surinam and Weaver (1969) in summarizing the literature prior to 1969 concluded that the use of manipulative materials was helpful in the teaching of fractions. Similarly, Anderson (1969), after reviewing the research on fractions, suggested that the teaching of fractions should make use of concrete experiences with real objects.

The literature on the use of concrete materials in the teaching of fractions can be subdivided into action research and statistical research. Both action research and statistical research have contributed to the total body of knowledge on teaching and learning topics relating to fractions. Both kinds of research have a significant influence on practices in the mathematics classroom.

Action research. Howell (1972) observed low ability students being taught fractions using concrete objects or some visual representation to develop understanding of each concept. She concluded that the level of motivation, understanding, and enjoyment during the study of fractions was positively influenced by the use of these materials.

Braunfield and Wolfe (1966) reported on the work done by low achievers in a programme sponsored by the Illinois Project. Ideas of fractions and operations with fractions were tied to a concrete model, namely sticks and their lengths. It was observed that by using this approach fractions and operations with fractions could be taught to "almost non-learners" of arithmetic.

Gunderson (1958) systematically observed seven-year-old children working with fractions in the classroom. Fractional discs were made available to the pupils. After becoming familiar with the discs, pupils made up their own problems involving fractions. These children were able to extend their learning of fraction concepts to include equivalent fractions and addition of fractions, work which is normally left to Grade 4 or 5. No algorithms were used at this time.

A great variety of concrete devices have been recommended by intermediate grade teachers for the teaching of fractions. Glenn (1957) used a kit of paper fractional strips to introduce fractions. Hall (1950) used student-constructed flannel boards and fraction kits to permit each child to discover relationships between fractions. Cunningham and Raskin (1968) utilized pegboards, and Hyde and Nelson (1967) incorporated egg cartons as aids in teaching fractions.

Statistical research. An empirical study comparing two methods of teaching addition of fractions was conducted by Pickering (1969). The control treatment was a large group, textbook centred approach. The Experimental Group received individualized instruction with a heavy emphasis on the use of manipulative materials. Significantly greater gain scores and retention scores in addition of fractions were achieved by the pupils using the manipulative individual approach.

Toney (1968) conducted a study comparing the effectiveness of pupil manipulation of materials and teacher demonstration in developing understanding of fractions in Grade 4 pupils. Although no significant differences were detected, trends were discernible. Both boys and girls who manipulated the materials made greater gains on a test of "proficiency in mathematical principles" than did those observing the demonstrations by the teacher. The boys who manipulated the materials made greater gains on a general achievement test than did the boys in the teacher demonstration group.

Bohan (1971) studied the effectiveness of three learning sequences for equivalent fractions. The first approach was a composite of approaches used in modern textbooks, and utilized diagrams and sets of objects to introduce equivalent fractions. Equivalent fractions were generated symbolically by multiplying the numerator and denominator of some fraction by a sequence of natural numbers. Addition and finally multiplication of fractions were then developed. The second treatment was similar to the first, but paperfolding activities were used to generate sets of equivalent fractions. The third treatment followed the sequence: multiplication of fractions, equivalent fractions, and finally addition of fractions with no paperfolding activities. Sets of equivalent fractions were generated for given fractions by applying the property of multiplication by one. Both Treatment One and Treatment Two produced significantly higher mean scores on an equivalent fractions posttest than did Treatment Three. Pupils in Treatment Two which included paperfolding achieved significantly higher mean scores on a retention test of equivalent fractions and also on an attitude test than did pupils in either of the other treatments. However, there were no significant differences among treatment groups on tests of addition of fractions or converting fractions to lowest terms.

Green (1970) compared the use of diagrams and the use of pupil-manipulated cardboard strips in the teaching of multiplication of fractions to Grade 5 pupils. Each approach was used to teach multiplication of fractions--by finding the area of a rectangular region to one group of pupils, and by finding a fractional part of a rectangular region or set of objects to a second group. The area approach produced significantly higher mean scores than did the method of finding a part of an area or set. No significant differences were detected between group mean scores of pupils using the cardboard strips and those using diagrams.

In a study involving Grade 5 pupils, Bisio (1971), compared three methods of teaching addition and subtraction of fractions. The first treatment did not include any manipulation of physical materials by either pupils or teacher. In the second treatment the teacher demonstrated the manipulation of the concrete materials. In the third treatment both teachers and pupils manipulated the materials. No significant differences were detected among treatment groups. However, for high socio-economic students, teacher demonstration produced a significantly higher mean score than did the first treatment in which there was no manipulation. Pupil manipulation was not found to be significantly better than either of the other treatments.

SUMMARY OF THE LITERATURE

According to Piaget, Bruner, and Dienes, the child should become actively involved in the learning process, and physical materials can play an important role in providing experiences leading to the development of mathematical ideas.

Research findings on the use of concrete materials in mathematics instruction is not conclusive. Some studies have shown that physical aids do promote increased learning, but other investigations have found that meaningful symbolic instruction and teacher demonstration are at least as effective in the teaching of many topics.

A great variety of manipulative materials have been described and advocated as valuable aids in teaching fraction concepts to elementary pupils. The results of several empirical studies have revealed that concrete centred instruction produced equivalent and often superior learning of fraction ideas to approaches in which no physical aids were used.

This study was based on the theory and research cited in this chapter. The purpose of the investigation was to develop an appropriate set of concrete materials to be used by children studying fractions in Grade 4, and to examine the effects of using these materials on achievement, retention, and transfer of learning. Pupil reaction to these materials was also of interest.

CHAPTER 111

EXPERIMENTAL DESIGN AND RESEARCH PROCEDURES

The literature on the teaching of fractions and the theories of Piaget, Bruner, and Dienes suggested that pupils in the fourth grade (ages eight and nine) would benefit by using appropriate concrete materials while learning basic fraction concepts. The purpose of this study was to investigate the suitability and effectiveness of a particular learning aid, a pupil-constructed fraction kit consisting of a number of colored paper strips of varying lengths.

THE TREATMENTS

The Experimental Group

Students in the Experimental Group began their study of fractions by constructing a fraction kit (Appendix A). The kit consisted of a variety of colored paper strips which were cut so as to form halves from the red strip, quarters from the orange strip, eighths from the yellow strip, thirds from the brown strip, sixths from the black strip and twelfths from the blue strip. Additional strips of appropriate color were cut to represent the fractions two-thirds and three-quarters. A white cardboard strip was retained uncut to later represent the unit.

Relationships between the different colors were developed by repeatedly naming any particular colored strip as the unit and defining the others in terms of that unit. Eventually the longest colored strip (white) was designated as the unit, and each of the other colored strips was given an appropriate fractional name in relation to it. Practice was provided in determining a variety of different names to describe each fraction strip (e.g. $\frac{1}{2} = \frac{1}{4} + \frac{1}{4}$, $\frac{2}{3} = \frac{4}{6}$, etc.) and in comparing fractions to determine order relations. Extra colored paper strips were provided when the above procedures were extended to improper fractions.

During the experimental treatment the concrete materials based instruction replaced the existing programme, with the authorized textbook serving only as a source of exercises which were duplicated for the students. Instruction was in one large group. The discovery of relationships between fractions was achieved through precise guidance from the teacher. The fraction kits were always available during sessions of practice exercise. The instruction extended over a period of fifteen lessons taught on consecutive school days, and seven additional days were used for testing. Approximately fifty percent of the instructional time was spent constructing or working with the fraction kits.

The Control Group

To determine the effects of using the fraction kit a Control Group was established. Students in this group followed the procedure for learning about fractions outlined in the authorized textbooks, Seeing Through Arithmetic (Gage, 1960). Material from both the Grade 4 and Grade 5 textbooks was covered. Activities for this group included examining blackboard and textbook illustrations, making pencil and paper sketches, and performing symbolic manipulations (Appendix B).

The basic difference between the two treatments was the inclusion or exclusion of manipulation of physical materials. In the experimental treatment fifty percent of the time was spent working with the fraction strips. In the control treatment no concrete aids were used, and students worked with pencil and paper from the textbook and chalkboard.

DESIGN OF THE STUDY

The Subjects

The study was conducted at Campus View Elementary School in Victoria, British Columbia. Students enrolled in the school are generally from upper-middle socio-economic homes. The two Grade 4 classes were selected for the study. Because the students had not been randomly assigned to classes the previous September, the following procedure was adopted. A Lorge-Thorndike Non-verbal I.Q. test was administered to all Grade 4 students, and the resulting scores were used to classify the students as being of high ability (above 113), average ability (103-113), or low ability (below 103). Students were then divided into two groups by stratified random assignment. The treatment each group was to receive was then determined by tossing a coin. Table 1 gives the final number of students in each experimental cell.

TABLE 1

CELL FREQUENCIES FOR TREATMENT GROUP
AND ABILITY LEVEL CLASSIFICATIONS

Achievement Level	Treatment EXP	Treatment CON	Total
High	11	11	22
Average	10	10	20
Low	11	10	21
TOTAL	32	31	63

The Teacher and Inservice

The same teacher instructed both the Experimental and Control Groups. He was teaching Grade 5 at Campus View School at the time the study was conducted. His class had been used for the experimental treatment in the pilot study conducted the previous year. He held a B. Sc. degree and had been successful in teaching at the elementary and secondary level.

The teacher had previously taught the authorized programme, and therefore had little difficulty in progressing through the sequence of lessons with the Control Group. Also, he was familiar with a teaching procedure using fraction kits which closely parallelled the experimental treatment. A detailed set of lesson plans for the experimental procedure was developed by the investigator to insure that the desired emphasis and sequence were realized (Appendix A). The lesson outline for the Control Group consisted of references to specific pages in the teacher's guide and pupil textbook (Appendix B). The two sets of procedures were reviewed with the teacher by the investigator.

RESEARCH QUESTIONS AND MEASURING INSTRUMENTS

All instruments (Appendix C) used to measure mathematics achievement and investigate student attitude toward the treatments were constructed by the investigator and were examined by members of the Faculty of Education at the University of Victoria. Indeed, a significant part of the study was the construction of instruments to assess student understanding of basic fraction concepts. A good pencil and paper test of these topics was not found, although appropriate items were usually located in tests of general mathematics achievement. In addition, it was felt desirable to construct performance tests which would require the manipulation of concrete materials. No such tests in fractions were available.

Initial Achievement and Transfer of Learning

How well were pupils in the two treatment groups able to learn material studied during the fifteen instructional periods? How would the groups compare in ability to demonstrate their learning using concrete materials? Would there be a difference in the ability of pupils in the two groups to transfer what had been learned to situations involving addition, subtraction, multiplication, and division of fractions at the concrete and the symbolic levels?

Symbolic Achievement Test. This test consisted of thirty-eight multiple choice items based on the subject matter covered in the study. The items were selected from a pilot test of sixty-one items which had been administered to four Grade 5 classes. These Grade 5 classes had just completed studying the materials included in this study and were ready to begin addition of fractions. A Kuder-Richardson Formula 20 reliability coefficient of .81 was computed for the thirty-eight item test administered following the treatments.

Concrete Achievement Test. This test consisted of nine performance items in which the pupils were required to demonstrate their answers using two different types of concrete materials. The first type consisted of a set of blocks similar to Dienes' Base 5 Blocks. The second type consisted of two open boxes of different dimensions and twenty sugar cubes. Items required pupils to demonstrate knowledge of fractions as part of a whole and comparison of fractions. Observers recorded and evaluated pupil responses to each item. Both sets of materials were substantially different from the fraction kit used by the Experimental Group during the treatment.

Symbolic Transfer Test. This test consisted of eight items, two each involving addition, subtraction, multiplication, and division of fractions. Pupils were required to determine and record answers for each item using only pencil and paper.

Concrete Transfer Test. This test consisted of eight performance items which closely parallelled the items in the symbolic transfer test. Pupils were provided with a cardboard two-dimensional fraction kit, to determine and demonstrate answers to each item. This kit consisted of a cardboard unit square; rectangular-shaped halves two of which had been cut to form square quarters; triangular eighths which were made by cutting square quarters along the diagonal; rectangular thirds; rectangular sixths which had been made by cutting thirds in half. Observers recorded and evaluated the pupil responses to each item.

Delayed Achievement and Transfer of Learning

How well did pupils in the Experimental and Control Groups retain and transfer, after a two-month period, concepts learned during the fifteen instructional periods?

Retention Tests. The symbolic and concrete achievement tests and symbolic and concrete transfer tests were readministered as retention tests.

Student Reaction to the Experimental and Control Treatments

An important aspect of the study was to determine student reaction to the use of the fraction kit. How favorably accepted was this treatment compared with the prescribed textbook approach which had been used prior to the study and in the control treatment?

Student Questionnaire. The instrument consisted of nine statements to which the pupil was required to indicate agreement, disagreement or uncertainty. Two parallel forms of the questionnaire were developed, one for the Experimental Group and one for the Control Group.

THE TESTING PROGRAMME

All tests and the questionnaire were administered to both groups by the investigator. To ensure that the students' reading ability did not affect test achievement, each item on every measuring instrument was read to the students by the investigator. The symbolic tests and the questionnaire were all administered in the students' regular classroom setting. All performance tests were administered in a large empty school library. Students, in groups of about fifteen were taken to the library and were assigned to a large working area of carpeted floor. Each area was provided with the concrete materials necessary to perform the required tasks. Five observers recorded student activity and responses.

The tests and the questionnaire were administered during the four days following the completion of instruction. The retention tests were administered during the ninth week following instruction.

THE NULL HYPOTHESES

The research questions previously discussed gave rise to the following null-hypotheses.

Initial Achievement and Transfer of Learning

Hypothesis 1. There is no significant difference between group mean scores obtained by the Experimental and Control Groups on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer test

Hypothesis 2. There is no significant interaction between treatment and ability level on scores on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer test

Delayed Achievement and Transfer of Learning

Hypothesis 3. Two months following instruction there is no significant difference between group mean scores of the Experimental and Control Groups on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer test

Hypothesis 4. Two months following instruction there is no significant interaction between treatment and ability level on scores on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer test

Student Reaction to the Experimental and Control Treatments

Hypothesis 5. There is no significant relationship between responses to any item on the student questionnaire and treatment group.

Hypothesis 6. For subjects in the Experimental Group there is no significant relationship between the responses to any item on the student questionnaire and:

(a) ability level

(b) sex

Hypothesis 7. For subjects in the Control Group there is no significant relationship between the responses to any item on the student questionnaire and:

(a) ability level

(b) sex

STATISTICAL PROCEDURES

Sample

The sample consisted of all Grade 4 students at Campus View School who had:

(1) been present for all fifteen instructional periods.

(2) written all tests and completed the questionnaire.

Procedure for Testing Hypotheses

The probabilities for all statistical tests are recorded in Chapter lV. The level of significance established for this study was $p < .05$. Hypotheses 1 and 3 were tested using t-tests (Popam, 1967, p. 129). Hypotheses 2 and 4 were tested using a two-way analysis of variance (Popam, 1967, p. 193). Hypotheses 5, 6, and 7 were tested using the Chi square test for independence (Popam, 1967, p. 277, 281).

CHAPTER 1V

RESULTS OF THE STUDY

The purpose of the study was to compare the effectiveness of a concrete materials approach and a textbook centred approach in teaching fractions to Grade 4 pupils. To make this comparison student scores on several measures were analyzed statistically. The findings are now reported under headings corresponding to the research questions discussed in Chapter 111. In presenting the findings related to each question, the null hypothesis is restated, the testing procedure is described, and the results of the analysis and the conclusions are given.

Initial Achievement and Transfer of Learning

Hypothesis 1 Restated. There is no significant difference between group mean scores of the Experimental and Control Groups on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer test

Testing Procedure

The four parts of Hypothesis 1 were tested by means of t-tests.

Results

Table 11 summarizes the comparison of initial group mean scores on the four tests. As indicated in the table, there was no significant difference between group mean scores of the Experimental and Control Groups on any of the four measures.

TABLE 11

T-TEST ANALYSIS OF INITIAL ACHIEVEMENT AND TRANSFER SCORES

	EXP		CON		T	P
	Mean	S.D.	Mean	S.D.		
ACHIEVEMENT						
Symbolic	11.88	5.94	14.16	6.98	-1.38	0.17
Concrete	2.06	1.66	2.58	1.31	-1.35	0.18
TRANSFER						
Symbolic	2.03	1.85	1.65	1.98	0.79	0.43
Concrete	5.47	2.03	5.10	2.22	0.68	0.50

Conclusions

Null Hypothesis 1 was not rejected. No significant differences in achievement were found to exist between the experimental and control students in initial achievement or ability to transfer.

Hypothesis 2 Restated. There is no significant interaction between treatment and ability level on scores on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer

Test Procedure

The four parts of Hypothesis 2 were tested by means of a two-way analysis of variance.

Results

Tables 111, V, V11 and 1X list cell means for each test. Tables 1V, V1, V111 and X summarize the analysis of variance performed on the scores obtained from these measures. As indicated in these tables, there are no significant treatment by ability level interactions.

TABLE III

INITIAL SYMBOLIC ACHIEVEMENT TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	15.45	17.33
Average	11.90	16.00
Low	8.27	8.70
GROUP MEANS	11.88	14.16

TABLE IV

ANALYSIS OF VARIANCE OF INITIAL SYMBOLIC ACHIEVEMENT TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	349.62	10.47	0.00
Treatment	1	66.97	2.01	0.16
Interaction	2	16.93	0.51	0.60
Within	57	33.38		

TABLE V

CONCRETE ACHIEVEMENT TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	2.82	2.83
Average	2.00	2.89
Low	1.36	2.00
GROUP MEANS	2.06	2.58

TABLE VI

ANALYSIS OF VARIANCE OF CONCRETE ACHIEVEMENT TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	7.26	3.32	0.04
Treatment	1	3.71	1.69	0.20
Interaction	2	1.08	0.49	0.61
Within	57	2.19		

TABLE VII

SYMBOLIC TRANSFER TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	2.45	2.83
Average	2.50	1.33
Low	1.18	0.50
GROUP MEANS	2.03	1.65

TABLE VIII

ANALYSIS OF VARIANCE OF SYMBOLIC TRANSFER TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	18.19	5.54	0.01
Treatment	1	3.05	0.93	0.34
Interaction	2	3.33	1.01	0.37
Within	57	3.28		

TABLE IX

CONCRETE TRANSFER TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	6.27	5.58
Average	5.50	4.78
Low	4.64	4.80
GROUP MEANS	5.47	5.10

TABLE X

ANALYSIS OF VARIANCE OF CONCRETE TRANSFER TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	8.37	1.80	0.17
Treatment	1	2.71	0.58	0.45
Interaction	2	1.32	0.28	0.75
Within	57	4.65		

Conclusions

Null Hypothesis 2 was not rejected. There was no significant differential treatment effects across three ability levels.

Delayed Achievement and Transfer of Learning

Hypothesis 3 Restated. Two months following the instruction there is no significant difference between group mean scores of the Experimental and Control Groups on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer test

Test Procedure

The four parts of Hypothesis 3 were tested by means of t-tests.

Results

Table XI summarizes the comparison of retention group mean scores on the four measuring instruments. As indicated in the table there is no significant difference between group mean scores of the Experimental and Control Groups on any of the four measures.

TABLE XI

T-TEST ANALYSIS OF DELAYED ACHIEVEMENT AND TRANSFER SCORES

	EXP Mean	EXP S.D.	CON Mean	CON S.D.	T	P
ACHIEVEMENT						
Symbolic	12.16	5.14	14.39	6.17	-1.54	0.13
Concrete	3.13	1.92	3.94	2.40	-1.46	0.15
TRANSFER						
Symbolic	1.69	1.88	2.39	2.34	-1.29	0.20
Concrete	5.97	1.79	5.48	1.92	1.02	0.31

Conclusions

Null Hypothesis 3 was not rejected. No significant differences in achievement were found to exist between the experimental and control students on retention of achievement and ability to transfer.

Hypothesis 4 Restated. Two months following the instruction there is no significant interaction between treatment and ability level on scores on the:

(a) symbolic achievement test

(b) concrete achievement test

(c) symbolic transfer test

(d) concrete transfer test

Test Procedure

The four parts of Hypothesis 4 were tested by means of a two-way analysis of variance.

Results

Tables Xll. XlV, XVl and XVlll list cell means for each measure. Tables Xlll, XV, XVll and XlX summarize the analysis of variance performed on the scores obtained from these measures. As indicated in these tables, there is no significant treatment by ability level interactions.

TABLE XII

DELAYED SYMBOLIC ACHIEVEMENT TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	14.91	18.25
Average	12.50	14.33
Low	9.09	9.80
GROUP MEANS	12.16	14.39

TABLE XIII

ANALYSIS OF VARIANCE OF DELAYED SYMBOLIC ACHIEVEMENT TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	279.11	10.97	0.00
Treatment	1	63.43	2.49	0.12
Interaction	2	9.59	0.38	0.69
Within	57	25.44		

TABLE XIV

DELAYED CONCRETE ACHIEVEMENT TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	3.45	4.25
Average	3.30	3.67
Low	2.64	3.80
GROUP MEANS	3.13	3.94

TABLE XV

ANALYSIS OF VARIANCE OF DELAYED CONCRETE ACHIEVEMENT TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	2.28	0.45	0.64
Treatment	1	9.78	1.93	0.17
Interaction	2	.79	0.16	0.85
Within	57	5.07		

TABLE XVI

DELAYED SYMBOLIC TRANSFER TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	2.82	3.75
Average	1.90	2.00
Low	0.36	1.10
GROUP MEANS	1.69	2.39

TABLE XVII

ANALYSIS OF VARIANCE OF DELAYED SYMBOLIC TRANSFER TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	36.04	9.87	0.00
Treatment	1	5.96	1.63	0.21
Interaction	2	.95	0.26	0.77
Within	57	3.65		

TABLE XVIII

DELAYED CONCRETE TRANSFER TEST CELL MEANS

GROUP	EXPERIMENTAL	CONTROL
High	6.82	6.17
Average	6.10	5.33
Low	5.00	4.80
GROUP MEANS	5.97	5.48

TABLE XIX

ANALYSIS OF VARIANCE OF DELAYED CONCRETE TRANSFER TEST SCORES

SOURCE	df	MS	F	P
Ability Level	2	13.99	4.25	0.02
Treatment	1	4.51	1.37	0.25
Interaction	2	.46	0.14	0.87
Within	57	3.29		

Conclusions

Null hypothesis 4 was not rejected. There was no significant differential treatment effects across three ability levels.

Student Reaction to the Experimental and Control Treatments

Hypothesis 5 Restated. There is no significant relationship between the responses to any item on the student questionnaire and treatment group.

Test Procedure

Chi square tests were performed on the percentages of "Agree", "Not Sure" and "Disagree" responses made by experimental and control students to each item on the student questionnaire.

Results

Table XX gives the percentages of "Agree" (A), "Not Sure" (NS) and "Disagree" (D) responses for the two groups on each of the nine items, the value of Chi square (χ^2) testing the relationship between treatment and response, and the probability of the observed Chi square.

TABLE XX

PERCENTAGES AND SIGNIFICANCE OF RESPONSES OF EXP AND CON STUDENTS TO QUESTIONNAIRE ITEMS

ITEM		Percentages of Responses A	NS	D
1. I like arithmetic.	EXP	47	37	16
	CON	59	25	16
	$\chi^2 = 1.27$; P = .53			
2. I enjoyed the work just completed on fractions.	EXP	44	44	12
	CON	56	41	3
	$\chi^2 = 2.34$; P = .31			
3. I now understand fractions much better than I did before I completed this work on fractions.	EXP	66	25	9
	CON	73	18	9
	$\chi^2 = .38$; P = .72			
4. The work on fractions was boring most of the time.	EXP	21	19	60
	CON	19	19	62
	$\chi^2 = .1$; P = .95			
5. I think the work on fractions has confused me.	EXP	16	40	44
	CON	10	12	78
	$\chi^2 = 8.37$; P = .02			
6. The way fractions were taught made it easy for me to learn.	EXP	82	9	9
	CON	66	18	16
	$\chi^2 = 2.03$; P = .37			
7. I think I learned material about fractions that I will use in Grade 5.	EXP	53	41	6
	CON	72	25	3
	$\chi^2 = 2.42$; P = .30			

TABLE XX

PERCENTAGES AND SIGNIFICANCE OF RESPONSES OF EXP AND CON STUDENTS TO QUESTIONNAIRE ITEMS

ITEM		Percentages of Responses A	NS	D
8. I would rather have studied fractions using {fraction strips / the textbook} than using the {textbook. / fraction kit.}	EXP	22	22	56
	CON	21	41	38
	$\chi^2 = 3.0$;		P =	.23
9. I felt I was wasting time using the {textbook. / fraction kit.}	EXP	19	22	59
	CON	22	22	56
	$\chi^2 = .1$;		P =	.95

Conclusions

Hypothesis 5 was not rejected for any item except item 5. For this item a significant relationship between treatment and student opinion was found to exist. Inspection of the percentages of students in the two groups who agreed and disagreed with this statement suggest that the control students found their treatment less confusing than did the Experimental Group.

Hypothesis 6 Restated. For subjects in the Experimental Group there is no significant relationship between the responses to any item on the student questionnaire and:

(a) ability level

(b) sex

Test Procedure

The above hypothesis was tested using the Chi square test for independence.

Results

The data for testing the above hypothesis, the Chi square values, and the probabilities of the observed Chi square values for all items are contained in Appendix D. This data for selected items is contained in Table XXl.

TABLE XXI

PERCENTAGES AND CHI SQUARE TESTS OF SIGNIFICANCE OF RESPONSES TO SELECTED STUDENT QUESTIONNAIRE ITEMS BY EXPERIMENTAL STUDENTS CLASSIFIED BY ABILITY LEVEL

ITEM		Percentages of Responses A	NS	D
4. The work on fractions was boring most of the time.	H	18	0	82
	A	0	30	70
	L	46	27	27
	$\chi^2 = 11.25$;		$P = .03$	
9. I would rather have studied fractions using the textbook than using the fraction kit.	H	27	27	46
	A	0	0	100
	L	27	36	37
	$\chi^2 = 10.24$;		$P = .04$	

Conclusions

Hypothesis 6(a) was rejected for items 4 and 9. For these items a significant relationship between ability level and student opinion was found to exist. Inspection of the percentages of students in the three ability groups who agreed and disagreed with statement 4 suggests that the high and average ability students in the Experimental Group responded more favorably to the work on fractions than did the low ability students. Inspection of the percentages of students who agreed and disagreed with statement 9 suggests that average ability students in the Experimental Group responded more favorably to their experiences manipulating the fraction strips than did either the high or low ability students.

Hypothesis 6(b) was not rejected for any item on the questionnaire. Within the Experimental Group no significant relationship was detected between the response to any item on the student questionnaire and sex.

Hypothesis 7 Restated. For subjects in the Control Group there is no significant relationship between responses to any item on the student questionnaire and:

(a) ability level

(b) sex

Test Procedure

The above hypothesis was tested using the Chi square test for independence.

Results

The data for testing the above hypothesis, the Chi square values, and the probabilities of the observed Chi square values for all items are contained in Appendix D.

Conclusions

Hypotheses 7(a) and 7(b) were not rejected for any item on the student questionnaire. For subjects in the Control Group there was no significant relationship between responses to any item on the student questionnaire and either ability level, or sex.

CHAPTER SUMMARY

In this chapter statistical comparisons of the two groups with respect to initial achievement and transfer and retention of learning as measured by both symbolic and concrete tests were reported. Student reaction to the two instructional treatments as obtained through a questionnaire was also analyzed. Analysis of the data indicated the following conclusions and trends:

(1) There was no significant difference between the two groups on any of the initial or delayed fraction tests.

(2) There was no interaction between ability level and treatment on any of the initial or delayed fraction tests.

(3) There was a significant relationship between the responses to one item on the student questionnaire and treatment group. An examination of data suggests that the control students found their treatment less confusing than did students in the Experimental Group.

(4) Within the Experimental Group average ability students tended to respond more favorably to manipulating the fraction strips than did either the high ability group or the low ability group. The average ability group and the high ability group reacted more favorably to the unit on fractions than did the low ability group.

(5) Within the Control Group there was no significant relationship between ability group and response to any item relating to treatment.

(6) There was no significant relationship between sex and response to items within either treatment group.

CHAPTER V

SUMMARY AND CONCLUSIONS

PURPOSE AND DESIGN OF THE STUDY

The purpose of the study was to investigate the effects of using a concrete materials centred instructional approach to introduce topics in fractions at the Grade 4 level. The method, which involved the construction and use of a fraction kit for approximately half of the fifteen instructional periods devoted to this topic, was adapted for the study by the investigator. To assess the relative effectiveness of using these materials a Control Group was established in which the students studied fractions using the regular mathematics textbook, Seeing Through Arithmetic (Gage, 1960). No manipulation of concrete materials was involved in this treatment.

The subjects for the study were the two classes of Grade 4 students at Campus View School, Victoria, British Columbia. Students were designated as being of high, average, or low ability according to their non-verbal I.Q. scores and were assigned to treatment group by stratified random assignment. The same teacher taught both classes.

Instruments were developed to compare the two groups on initial learning, transfer of learning, and retention of learning and to assess the feelings of the students toward the treatments. In addition to the traditional pencil and paper tests, concrete materials performance tests were developed to measure the initial learning, transfer of learning, and retention of learning. The experiment was designed to obtain answers to the following questions:

(1) What was the relative effectiveness of the experimental and control treatments in promoting:

a. initial achievement?
b. transfer of learning?
c. delayed achievement (retention)?
d. delayed transfer of learning?

(2) Was there significant interaction between treatment and ability level on:

a. initial achievement?
b. transfer of learning?
c. delayed achievement?
d. delayed transfer of learning?

(3) How did the experimental and control students react to their respective instructional treatments?

Hypotheses comparing the two groups on the four measures of learning were tested using t-tests. Hypotheses investigating interaction between treatment group and ability level were tested using two-way analysis of variance. Hypotheses investigating student reactions were tested using the Chi square test for independence.

SUMMARY OF THE RESULTS

Analysis of the data obtained during the investigation revealed the following results:

(1) There was no significant difference between the two groups in their:

a. initial achievement
b. transfer of learning
c. delayed achievement
d. delayed transfer of learning

as measured by both symbolic and concrete tests.

(2) There was no significant interaction between treatment and ability level in:

a. initial achievement
b. transfer of learning
c. delayed achievement
d. delayed transfer of learning

as measured by both symbolic and concrete tests.

(3) There was no significant relationship between responses to any item on the student questionnaire and treatment group except for one item. This item indicated that students in the Control Group found that treatment less confusing than did students in the experimental treatment. Within the Experimental Group one questionnaire item suggested that the average ability students responded more favorably to manipulating the fraction strips than did either the high ability or low ability students and one item suggested that both the high and average ability students responded more favorably to the manipulative activity than did the low ability students. Within the Control Group no significant relationship between ability group and response to a questionnaire statement was found. No significant relationship between sex and response to a questionnaire statement was detected within either treatment.

CONCLUSIONS AND DISCUSSION

The four tests -- symbolic achievement, concrete achievement, symbolic transfer, and concrete transfer -- were administered to the subjects twice, once immediately following the treatment and again two months later. No significant differences were revealed between the two groups by any of the eight statistical comparisons(Tables 2 and 11). Thus neither of the two treatments can be considered superior on these measures. However, several observations and trends are worth noting.

On all four measures of achievement the mean scores of the Control Group were higher than the mean scores of the Experimental Group with corresponding P values ranging between .1 and .2 . No such consistent trend was detected for the transfer tests.

While it is perhaps not surprising that the Control Group did at least as well as the Experimental Group on the symbolic tests, it might have been expected that the concrete tests would favor the Experimental Group who had manipulated fraction strips during the treatment. Apparently this experience did not provide any particular advantage in relating fractions to new physical settings. An examination of the item analyses of the concrete achievement and concrete and symbolic transfer tests (Appendix C) reveals some interesting data.

The concrete achievement test yielded vastly different scores when administered as an initial test and as a retention test. On the initial test, the first group of three items on which students had to make fractional comparisons between objects in a Base five-Multibase Blocks kit were answered correctly by approximately one-half of the students. The second group of three items requiring students to place sugar cubes in a given open box until it was filled to a specific fraction of its capacity, was successfully completed by less than one-third of the students. The final group of three questions required the students to place a designated number of sugar cubes in a specific open box and determine what fraction of the box was filled. A total of only three correct responses were made by the sixty-three students. On the retention test, a marked increase in the number of correct responses to eight of the nine items was recorded. It would appear that the relative unfamiliarity by the subjects with performance type tests resulted in a practice effect which accounted for the marked improvement in performance on the test when used to measure retention.

The concrete transfer test required students to demonstrate with concrete materials answers to questions involving addition, subtraction, multiplication and division of fractions. On both administrations of the test group mean scores were high, with a mean frequency of approximately 5.5 out of a possible score of 8. The symbolic transfer test which was parallel to the concrete transfer test resulted in very low group mean scores, a mean of approximately 1.9 correct responses on eight items. This suggests that while students could manipulate physical materials to demonstrate answers to questions involving computation with fractions, even though these skills had not yet been formally taught, they were unprepared to solve parallel items at the symbolic level.

Some writers have suggested that a concrete materials approach would be more suitable for low ability students and a symbolic approach more appropriate for high ability students. There is no evidence in this study that one treatment was particularly better suited for a given ability level than another.

Analysis of the student questionnaire responses indicated that both the experimental and control treatments had been fairly well received by the students. Only three percent of the Control Group and twelve percent of the Experimental Group indicated that they did not enjoy their work with fractions. Item #5, "I think the work on fractions confused me." received agreement from nine percent of the Control Group and sixteen percent of the Experimental Group. This difference was statistically significant. However, Item #6, "The way fractions were taught made it easy for me to learn." received eighty-two percent agreement from the Experimental Group and sixty-six percent agreement from the Control Group. Of the Experimental Group sixty-five percent indicated they understood fractions much better and only twenty-one percent indicated they would rather have worked with the textbook than with the fraction kit.

Within the Experimental Group students of average ability tended to respond more favorably to the construction and manipulation of fraction strips than did the low ability students and to a certain extent the high ability students. The reaction by the low ability students is consistent with a statement in a report on mathematics teaching in Israel by Weiss(1973). He suggests that:

> Although the value of manipulative and other physical materials in the teaching of mathematics to low achievers is widely accepted in the United States, one encounters a great deal of scepticism among Israelis about the value of such materials. Dr. Maschler, for example, believes that while such materials are of occasional help, they more often hinder the students grasping an abstraction. (p.311)

This trend could also have been partially due to the fact that large group instruction tends to be best suited to the average student. With the overt manipulation of materials and the recording of answers, low ability students could have been continually under pressure to "keep up", since their progress in working with the strips would be immediately evident to the teacher. It is possible that the less enthusiastic responses of the high ability group were due to the fact that these students did not need the amount of concrete manipulation provided in the experimental treatment.

LIMITATIONS OF THE STUDY

Persons drawing conclusions from this study or projecting its results to the general population should do so with the following limitations in mind:

(1) The subjects in this study were limited to sixty-three Grade 4 students in one school.

(2) The study was limited to introductory work with common fractions and mixed numbers. Computation with fractions was not studied.

(3) The experimental treatment was limited to specific sequences and exercises developed by the investigator.

(4) The symbolic and concrete achievement tests used recorded low group mean scores and standard deviations.

RECOMMENDATIONS

On the basis of this study the investigator recommends the experimental method as an alternative to a textbook approach to introducing fractions. The materials required are inexpensive and easy to construct. Little inservice training is required for the teacher. Although the method did not produce superior learning in the study, the students did about as well as those in the Control Group and their reaction was generally positive. Furthermore, such a method provides a break from a textbook centred teaching approach.

The experimental treatment may be more suitable for grouped rather than whole class instruction. This treatment appeared most suitable for the average ability group. A slower pace is recommended for the low ability students. It is questionable that the high ability group needs as much manipulative experience as was provided.

SUGGESTIONS FOR FURTHER RESEARCH

To further investigate the problem defined in this study the experiment should be replicated in other schools, perhaps in different socio-economic areas and using a larger number of subjects. The materials might also be tried at the Grade 3 level or the Grade 5 level. In the latter case the treatment could be expanded to include addition and subtraction of fractions. Further refinement of the testing instruments is suggested, particularly the performance tests requiring manipulation of concrete materials.

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APPENDIX A

THE EXPERIMENTAL TREATMENT

LESSON 1

A. Distribute the following 12 inch strips to each student:

- cardboard strip - 1 white
- construction paper strips - 1 red, 1 blue, 2 orange, 2 brown, 1 yellow, 1 black

B. To construct the fraction kit, students will cut:

- the red strip into 2 equal parts
- the first orange strip into 4 equal parts
- the yellow strip into 8 equal parts
- the first brown strip into 3 equal parts
- the black strip into 6 equal parts
- the blue strip into 12 equal parts
- the second orange strip to form $\frac{1}{4}$ and $\frac{3}{4}$ pieces
- the second brown strip to form $\frac{1}{3}$ and $\frac{2}{3}$ pieces

The white cardboard strip will be retained uncut.

C. The kits will be displayed on the pupils' desks keeping similar colors together to form the original strips. The number of parts into which each color was divided is thoroughly reviewed.

LESSON 2

A. A foot ruler is identified as the unit length "one whole one". Pupils are required to identify the yardstick in terms of the unit. This procedure is repeated with several different pairs of objects in which the length of the second is a multiple of the length of the first.

B. The above procedure is repeated using colored strips such as:

Basic Unit	Comparison	Measure
red strip	yardstick	6 units
short orange strip	yardstick	12 units
black strip	yardstick	18 units

C. The process is now reversed and the yardstick is identified as the unit and the foot ruler is defined in terms of the unit. Several combinations are examined such as:

Basic Unit	Comparison	Measure
yardstick	red strip	$\frac{1}{6}$ unit
yardstick	orange strip	$\frac{1}{12}$ unit

D. Procedure 2A is repeated with the strip to be identified always being a multiple of the unit. All answers are <u>shown</u> using the kit.

Basic Unit	Comparison	Measure
black strip	brown	2
	red	3
	white	6

LESSON 3

A. Review procedure Lesson #2C with the unit always being a multiple of the unknown strip.

i.e. "If the red strip is one whole one, what is the yellow?" Ans. $\frac{1}{4}$

"How many $\frac{1}{4}$'s does it take to make up one whole one?"

B. After reviewing how many $\frac{1}{4}$'s, $\frac{1}{6}$'s, etc. there are in one whole one, pupils should be required to extrapolate beyond the fractions represented in the kit and answer such questions as "How many $\frac{1}{5}$'s, $\frac{1}{13}$'s etc. are there in one whole one?"

C. A complete review of 3B using the fraction strips is made before proceeding.

D. Random colored strips are designated as the unit and pupils are required to show a particular fractional part. i.e. If red is one whole one (a) show $\frac{1}{4}$ (b) what is the black?

E. Using the white cardboard strip as the unit pupils are required to find a specific number (i.e. 4) of the strips of the same color and length that end-to-end match the length of the unit and they are required to answer the questions:

(i) What color is it? (orange)
(ii) One strip (orange) is what part of one whole one? ($\frac{1}{4}$)
(iii) How many $\frac{1}{4}$'s make up a whole? (4)

Repeat with 6 strips etc.

LESSON 4

A. Introduce the idea of different names for the same number by asking pupils to select the combinations that represent the number 5.

0×5 $7 - 3$ $3 - 4$ $14 - 10$ $15 \div 3$ $1 + 2 + 2$

- Allow students to generate a variety of names for the number 5.
- Require students to generate names involving fractions to identify the white cardboard strip i.e. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
- Repeat for other strips.

B. Thoroughly review relationship between strips by selecting random strips as the unit and identifying others in terms of the unit. The strips to be compared to the unit should include both multiples and fractional parts of that unit.

C. The white cardboard strip is now identified as the permanent unit and all colored strips are examined and given their correct fractional names in relation to this unit. The association between color and fractional name is strongly reinforced.

D. Other names for various strips including the $\frac{2}{3}$ and $\frac{3}{4}$ strips are examined. Pupils both show their answers and write them symbolically. The meaning of numerator and denominator should be stressed.

LESSON 5

A. This is a review lesson in which the fraction kits are used. Lessons 3 and 4 are reviewed with emphasis on Lesson 4 C and D.

LESSON 6

A. Introduce improper fractions by placing the white unit strip and the $\frac{1}{3}$ strip end to end. Have pupils:

(i) identify the total length in units. ($1\frac{1}{3}$)

(ii) determine how many $\frac{1}{3}$'s there are in $1\frac{1}{3}$ by using the kit. (4)

Repeat the various examples in which the answers are demonstrated and written symbolically. The types of questions are:

$1\frac{2}{8} = \frac{?}{8}$ $\qquad\qquad$ $\frac{11}{8} = 1\frac{?}{8}$

B. Give several questions of the above type which pupils will solve symbolically or if necessary using the strips.

LESSON 7

A. Review the changing of mixed fractions to mixed numbers and mixed numbers to improper fractions.

B. Review the names associated with each color of strip.

C. Distribute a second full length cardboard strip, orange strip, and a blue strip. Have the orange strip divided into $\frac{1}{4}$'s and the blue strips into $\frac{1}{12}$'s.

D. Working in pairs, students will place two white strips end to end and determine how many $\frac{1}{4}$'s, $\frac{1}{8}$'s etc. are required to make up two wholes. Answers are demonstrated in strips and written symbolically.

LESSON 8

A. Review Lesson 7 D.

B. Review Lesson 6 A and B using mixed numbers less than 3. Give sufficient time and guidance for pupils to discover the rules for symbolically converting improper fractions to mixed numbers and mixed numbers to improper fractions.

C. Provide practice time for pupils to apply these rules to a variety of examples which can be demonstrated with the fraction strips.

LESSON 9

A. Each pupil is issued with an additional red, yellow, brown and black strip. Each is cut into the appropriate fractional units.

B. Review Lesson 8 using fraction strips. Reinforce the two rules.

C. Working in pairs pupils should work with larger improper fractions such as:

$4\frac{3}{4} = \frac{?}{4}$ $\qquad$ $2\frac{1}{2} =$

$\frac{10}{3} =$ $\qquad$ $2\frac{8}{12} =$

D. An alternative method is investigated using fraction strips.

$\frac{4}{3} = \frac{3}{3} + \frac{1}{3}$ or $1\frac{1}{3}$

$\frac{37}{12} = \frac{12}{12} + \frac{12}{12} + \frac{12}{12} + \frac{1}{12}$

LESSON 10

A. Pupils will use the $\frac{1}{12}$ pieces as individual squares to make up various groups. The procedure in S.T.A. 5 pupil text, pages 129-130 will be used with these squares to introduce the fractions as part of a group. Several examples should be done.

- Comparisons of fractions such as $\frac{1}{8}$ $\frac{1}{6}$; $\frac{1}{3}$ $\frac{1}{4}$; etc will be made using groups of squares.

B. Simplifying fractions is introduced with these squares: i.e. Each child arranges 24 squares (X's) on a sheet of newsprint.

(x x x x)
(x x x x)
(x x x x)
(x x x x)
x x x x
x x x x

Step 1 - The students circle groups of 4 squares until 16 are circled. They are required to name the fraction of the squares circled.

$\frac{4}{6}$ and/or $\frac{16}{24}$

Step 2 - The students arrange the squares in the above configuration and circle groups of 8 squares until 16 are again circled and the fraction squares circled as identified.

$\frac{2}{3}$

Because 16 squares were circled each time,

$$\frac{16}{24} = \frac{4}{6} = \frac{2}{3}$$

Several such examples are done.

C. Equivalent fractions are continued allowing sufficient time for pupils to discover that when both the numerator and denominator of a fraction are divided or multiplied by the same number the result is an equivalent fraction.

LESSON 10

D. The following generalizations are emphasized at the symbolic level:

$$\frac{8 \div 4}{12 \div 4} = \frac{2}{3} \qquad \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

LESSON 11

A. Review equivalent fractions with emphasis on simplest form allowing pupils to work with the fraction kit if necessary.

B. Pupils should be given practice in generating sets of fractions equal to a specific fraction.

C. Using the kit pupils will determine the missing term in sets of equivalent fractions.

i.e. $\frac{1}{2} = \frac{?}{4}$

* The first half of page 1 of the duplicated practice sheets should be completed.

LESSON 12

A. Simplification of fractions and equivalent fractions are reviewed.

B. Complete page 1.

* S.T.A. 5 p. 146

LESSON 13

A. Review simplification of fractions and complete the first

* half of sheet #2. Fraction kits are available to students.

B. Review converting improper fractions to mixed numbers.

C. Commence sheet #2 second half with fraction kits available.

LESSON 14

A. * Sheet #2 and #3 are completed and sheet #4 is started.

Fractions kits should be used if needed.

LESSON 15

A. Complete sheet #4.

B. Pairs of fractions strips will be compared to determine the

$>$, =, or $<$ relationships by directly comparisons of length.

C. Pairs of fractions are compared by changing them to a common

denominator using the fraction strips.

D. Examples A-H from p.162 of the S.T.A. 5 pupil text are

printed on the chalkboard and solved.

* Sheet #2 - S.T.A. 5 p. 146, 153
Sheet #3 - S.T.A. 5 p. 153, 155
Sheet #4 - S.T.A. 5 p. 155

APPENDIX B

THE CONTROL TREATMENT

CONTROL GROUP TREATMENT

LESSON 1 (Seeing Through Arithmetic 4 - pp. 208-211)

A. Meaning of fractions - part of a whole

- part of a group

LESSON 2 (S.T.A. 4 pp. 212-214)

A. Review meaning of fractions

B. Equivalent fractions

LESSON 3 (S.T.A. 4 p. 215)

A. Review equivalent fractions

B. Comparison of unit fractions

LESSON 4 (S.T.A. 4 pp. 216-219)

A. Meaning of improper fractions and mixed numbers

B. Reading and writing of improper fractions and mixed numbers

C. Relationship between mixed numbers and improper fractions

LESSON 5 (S.T.A. 4 pp. 220-221)

A. Review meaning of mixed numbers

B. Fractions in ruler measurement

LESSON 6 (S.T.A. 4 p. 222)

A. Review meaning of fractions

B. Fraction meaning - equivalent parts

LESSON 7 (S.T.A. 4 p. 247)

A. General review using "Looking back" exercise

LESSON 8 (S.T.A.5 pp. 143-145)

A. Determining fractions equivalent to a given fraction by multiplying or dividing the numerator and denominator by the same number

LESSON 9 (S.T.A.5 pp. 145-6 top section)

A. Review lesson 8

B. Emphasize procedure for finding the missing term in a pair of equivalent fractions

LESSON 10 (S.T.A. 5 p. 146)

A. Review least common divisor

B. Reducing fractions to simplest form

LESSON 11 (S.T.A. 5 pp. 148-150)

A. Review lessons 8, 9 and 10

B. Review the meaning of improper fractions and mixed numbers

<u>LESSON 12</u> (S.T.A. 5 pp. 151-153)

A. Fractional names for 1

B. Converting improper fractions to mixed numbers and mixed numbers to improper fractions

<u>LESSON 13</u> (S.T.A. 5 p. 155)

A. Review equivalent fractions and simplest form

B. Checking up test 1 and 2

<u>LESSON 14</u> (S.T.A. 5 p. 155)

A. Checking up test 3

B. General review

<u>LESSON 15</u> (S.T.A. 5 pp. 158-159)

A. Comparing fractions on the number line

APPENDIX C

TESTING INSTRUMENTS

AND ITEM ANALYSIS

Symbolic Achievement Test

Concrete Achievement Test

Concrete Transfer Test

Symbolic Transfer Test

Experimental Group Student Questionnaire

Control Group Student Questionnaire

SYMBOLIC ACHIEVEMENT TEST

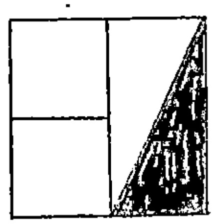
Instructions: For each of the following questions draw a circle around the best answer.

1. What fraction of the squares is shaded?

A. 6/7 C. 7/13

B. 6/13 D. 7/6

2. What fraction of the square is shaded?



A. 1/2 C. 1/4

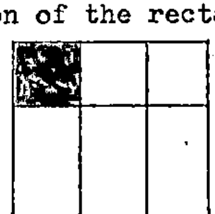
B. 1/3 D. 4/1

3. What part of the rectangle is shaded?

A. 5/3 C. 3/5

B. 3/4 D. 3/8

4. What fraction of the rectangle is shaded?



A. 1/5 C. 6/1

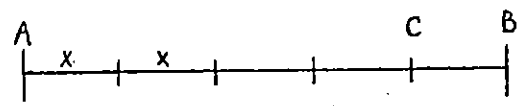
B. 1/6 D. None of these

5. In the mixed number $2\frac{4}{15}$ what is the numerator?

A. 2 C. 15

B. 4 D. 60

Use this line segment for questions 6 and 7.



6. What fraction of the line segment has X 's above it?

A. 2/5 C. 2/6

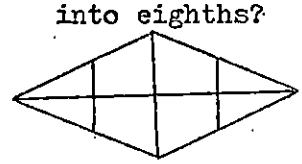
B. 2/3 D. 3/2

7. The point C is what fraction of the distance from A to B?

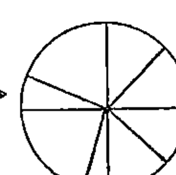
A. 4/5 C. 1/6

B. 5/6 D. 5/1

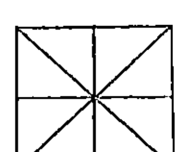
8. Which figure is divided into eighths?



A



C

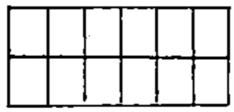


B

D. All of these

SYMBOLIC ACHIEVEMENT TEST - Continued

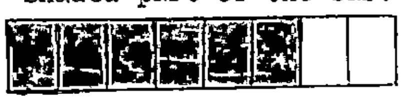
9. How many small rectangles are there in one-third of the large rectangle?



A. 4 C. 2

B. 3 D. 6

10. What two fractions name the shaded part of the bar?



A. 2/8 and 1/4 C. 6/8 abd 3/4

B. 6/8 and 1/2 D. 3/4 and 2/8

11. Which pair of fractions name the same amount?

A. 1/3 and 1/2 C. 1/2 and 3/6

B. 1/2 and 3/4 D. 3/4 and 3/6

12. Which of the following is equal to 2/5?

A. 3/6 C. 5/10

B. 5/7 D. 6/15

13. Which of the following is equal to 6/24?

A. 2/8 C. 6/12

B. 4/8 D. 12/18

14. Which set of fractions is made up of equivalent fractions?

A. 1/2, 1/3, 1/4, 1/5, 1/6

B. 1/8, 2/8, 3/8, 4/8, 5/8

C. 2/3, 4/6, 6/9, 8/12,10/15

D. 1/10,3/20,5/30,7/40, 9/50

15. In the equation 3/7 = 12/ , what number can replace ?

A. 21 C. 32

B. 28 D. 35

16. In the equation $\frac{\quad}{7} = 1$, what number can replace box?

A. 1 C. 8

B. 7 D. 14

17. Which of the following is expressed in lowest terms (simplest form)?

A. 7/14 C. 3/24

B. 18/28 D. 7/22

SYMBOLIC ACHIEVEMENT TEST - Continued

18. Which of the following shows another name for 3/4?

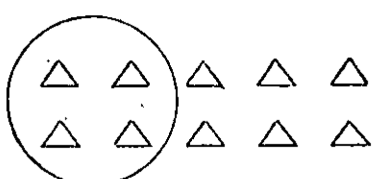
A. $3/4 = \frac{3 - 2}{4 - 2}$

B. $3/4 = \frac{2 \times 3}{2 \times 4}$

C. $3/4 = \frac{3 \times 3}{4 \times 4}$

D. $3/4 = \frac{3 + 1}{4 + 1}$

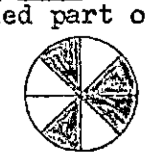
19. What fraction of the set of triangles is circled?



A. 2/4 C. 4/6

B. 2/5 D. 4/8

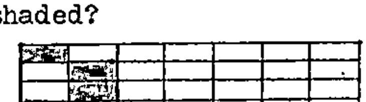
20. Choose the fraction in simplest form that represents the shaded part of the circle.



A. 1/2 C. 3/8

B. 4/8 D. 2/4

21. What fraction of the bar is shaded?



A. 1/7 C. 1/12

B. 4/24 D. 1/8

22. Which of the following shows how to change 9/12 to simplest form?

A. $\frac{9 \div 3}{12 \div 4}$

B. $\frac{9 \div 3}{12 \div 3}$

C. $\frac{3 \times 9}{3 \times 12}$

D. $\frac{3 \times 9}{4 \times 12}$

23. When reduced to simplest form, 16/24 would be written as what fraction?

A. 8/12 C. 2/3

B. 4/6 D. 3/4

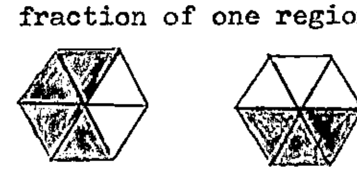
24. What are the two missing terms in the following set of fractions?

(2/5, /10, 6/15, /20, 10/25)

A. 2 and 6 C. 6 and 8

B. 4 and 6 D. 4 and 8

25. The total shaded area is what fraction of one region?



A. 4/6 C. 7/6

B. 3/2 D. 1/2

SYMBOLIC ACHIEVEMENT TEST - Continued

26. What mixed number describes the shaded part of the regions in question 25?

A. $1\frac{1}{6}$ C. $1\frac{1}{2}$

B. $1\frac{2}{3}$ D. $1\frac{7}{6}$

27. Which of the following is another name for 11/4?

A. $1\frac{1}{4}$ C. $\frac{4}{11}$

B. $2\frac{3}{4}$ D. $2\frac{1}{4}$

28. In the equation $\quad /3 = 4\frac{2}{3}$ the is equal to what number?

A. 14 C. 11

B. 12 D. 6

29. Which one of the following sets of fractions is arranged in order from largest to smallest?

A. 1/3 1/4 1/7 1/9

B. 1/9 1/7 1/4 1/3

C. 1/7 1/9 1/3 1/4

D. 1/4 1/7 1/9 1/3

30. Which is longest:

A. 6 inches C. $6\frac{2}{3}$ inches

B. $6\frac{1}{2}$ inches D. $6\frac{3}{5}$ inches

31. In which set of fractions is 1/5 larger than all the other fractions?

A. 1/8 1/5 1/3

B. 1/6 1/5 1/8

C. 1/4 1/5 1/6

D. 1/2 1/3 1/5

32. What fraction is less than 4/2 ?

A. 15/8 C. 10/4

B. 7/3 D. 21/10

33. Which of the following is less than 3 1/2 ?

A. 16/6 C. 7/2

B. 11/3 D. 30/6

34. What fraction is the largest?

A. 4/7 C. 4/5

B. 4/9 D. 4/16

35. How many of the fractions in the set are less than 1/2 ?

(4/10, 1/3, 5/8, 3/16, 4/6, 3/8, 5/12)

A. 2 C. 4

B. 3 D. 5

SYMBOLIC ACHIEVEMENT TEST - Continued

36. Which of the following is largest?

A. 22 2/2 C. 2 22/2

B. 22/22 D. 222/2

37. What number can be used to replace □ ? Remember, both □ 's must be replaced by the same number.

2/3 = /□

2/5 = /□

A. 20 C. 10

B. 15 D. 5

38. Which set of fractions is in order from smallest to largest?

A. 1/2 3/5 4/10

B. 4/10 1/2 3/5

C. 3/5 4/10 1/2

D. 3/5 1/2 4/10

ITEM ANALYSIS OF SYMBOLIC ACHIEVEMENT TEST

	INITIAL ACHIEVEMENT		RETENTION ACHIEVEMENT	
ITEM	Biserial Correlation	Item Difficulty	Biserial Correlation	Item Difficulty
1	.539	.647	.332	.653
2.	.500	.647	.485	.736
3	.599	.279	.479	.292
4	.520	.368	.409	.417
5	.416	.471	.268	.389
6	.468	.735	.485	.653
7	.373	.397	.429	.458
8	.162	.221	.109	.389
9	.470	.353	.691	.361
10	.324	.412	.527	.500
11	.666	.574	.576	.486
12	.589	.206	.646	.181
13	.702	.132	.928	.111
14	.417	.397	.387	.292
15	.563	.206	.555	.194
16	.604	.574	.601	.639
17	.156	.147	.259	.097
18	.245	.324	.337	.264
19	.422	.118	.599	.181
20	.574	.353	.815	.208
21	.753	.103	.926	.069
22	.412	.338	.562	.153
23	.691	.265	.794	.181
24	.305	.265	.440	.194
25	.334	.485	.409	.417
26	.400	.103	.245	.486
27	.423	.250	.701	.183
28	.720	.368	.290	.352
29	.621	.559	.425	.535
30	.787	.279	.696	.211
31	.593	.471	.356	.451
32	.871	.206	.825	.169
33	.694	.118	.468	.155
34	.532	.647	.592	.761
35	.244	.294	.391	.366
36	.133	.485	.240	.310
37	.473	.176	.297	.129
38	.255	.250	.366	.329

	Initial	Retention
KR-20 Reliability	.806	.787
Test-Retest Reliability (after two months)		.848
Number of subjects	63	63

CONCRETE ACHIEVEMENT TEST

Examiner's Guide

1. As each piece of concrete material is referred to, the examiner must hold up that piece to ensure that children are working with the correct materials.

2. All answers will be recorded on the separate answer sheet.

3. Read all questions orally to the group as many times as is necessary.

I Materials: Multibase Blocks - base five

Question 1. "A single unit block is what fraction of a strip?"

Question 2. "A single unit block is what fraction of a flat?"

Question 3. "A flat is what fraction of a large block?"

II Materials: Sugar cubes and open box (dimensions 2 x 3 x 4 cubes)

Question 4. "Fill this box $\frac{1}{4}$ full of sugar cubes. How many sugar cubes did you use?"

Question 5. "Fill this box $\frac{1}{12}$ full of sugar cubes. How many sugar cubes did you use?"

Question 6. "Fill this box $\frac{1}{24}$ full of sugar cubes. How many sugar cubes did you use?"

III Materials: Sugar cubes and open box (dimensions 2 x 2 x 4 cubes)

Question 7. "Place four sugar cubes in this tall box. What fraction of the box is filled?"

Question 8. "Place one sugar cube in this box. What fraction of the box is filled?"

Question 9. "Place seven sugar cubes in this box. What fraction of the box is filled?"

ITEM ANALYSIS OF CONCRETE ACHIEVEMENT TEST

CONCRETE ACHIEVEMENT TEST

	INITIAL			RETENTION		
Item Number	EXP (Correct)	CON (Correct)	TOTAL (Correct)	EXP (Correct)	CON (Correct)	TOTAL (Correct)
1	16	26	42	27	28	55
2	18	24	42	23	28	51
3	4	4	8	10	8	18
4	10	8	18	12	12	24
5.	6	6	12	8	10	18
6	9	12	21	11	9	20
7	0	0	0	15	19	34
8	1	1	2	5	11	16
9	1	0	1	1	5	6
TOTAL	65	81	146	112	130	242

Test-Retest (after 2 months) Reliability .600

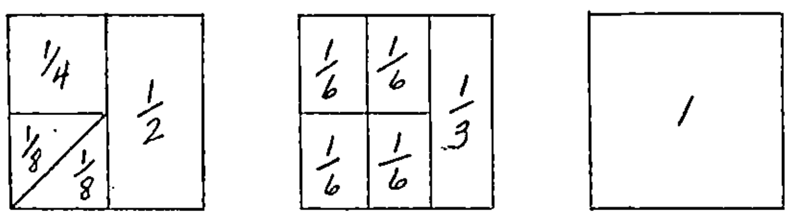
Number of Subjects 63

CONCRETE TRANSFER TEST

Examiner's Guide

Materials: The fraction kits were constructed from light cardboard as shown below. Each kit contained the following marked pieces:

1 whole
2 halves
4 fourths
12 eighths
3 thirds
6 sixths



1. As each item in the concrete materials kit is referred to, the examiner must hold up that piece to ensure that children are working with the correct materials.

2. On the record sheet for each pupil, observers will record a (✓) if the pupil has correctly demonstrated the answer and an (X) if he has not.

3. The pupil will be required to record his answer to each question on the separate answer sheet.

4. All answers must be shown using only one kind of fractional part. i.e. $\frac{1}{2} + \frac{1}{4}$ must be shown as three $\frac{1}{4}$ pieces or six $\frac{1}{8}$ pieces.

5. Read all questions to the class as many times as is necessary.

1. Question 1. $\frac{1}{2} + \frac{1}{4} =$ ________

Question 2. $\frac{3}{4} + \frac{3}{8} =$ ________

Question 3. $\frac{3}{4} - \frac{1}{2} =$ ________

CONCRETE TRANSFER TEST - Continued

Question 4. $1\frac{1}{3} - \frac{1}{6}$ = ________

Question 5. $\frac{1}{2}$ of $\frac{1}{3}$ = ________

Question 6. 2 groups of $\frac{2}{6}$ = ________

Question 7. How many $\frac{1}{8}$'s are there in $\frac{3}{4}$?

Question 8. Divide $\frac{3}{4}$ into 2 equal parts. How much is one of these equal parts?

SYMBOLIC TRANSFER TEST

Name: ______________

A. $\frac{1}{3} + \frac{1}{6} =$ ________

B. $\frac{2}{3} + \frac{2}{6} =$ ________

C. $\frac{4}{6} - \frac{1}{2} =$ ________

D. $1\frac{1}{4} - \frac{1}{8} =$ ________

E. $\frac{1}{2}$ of $\frac{1}{4}$ = ________

F. 2 groups of $\frac{2}{3}$ = ________

G. How many $\frac{1}{8}$'s are there in $\frac{2}{4}$? ________

H. Divide $\frac{1}{4}$ into 2 equal parts. One equal part = ______

ITEM ANALYSIS

CONCRETE (C) AND (PARALLEL) SYMBOLIC (S) TRANSFER TESTS

Item Number	S/C	INITIAL EXP	INITIAL CON	INITIAL Total	RETENTION EXP	RETENTION CON	RETENTION Total
Addition							
1	S	5	4	9	4	9	13
	C	21	18	39	20	23	43
2	S	4	5	9	3	7	10
	C	18	17	35	23	19	42
Subtraction							
3	S	4	1	5	1	5	6
	C	24	24	48	26	25	51
4	S	5	3	8	5	6	11
	C	24	22	46	27	25	52
Multiplication							
5	S	8	10	18	10	12	22
	C	33	26	59	33	27	60
6	S	10	8	18	10	9	19
	C	24	22	46	25	18	43
Division							
7	S	16	13	29	10	13	23
	C	10	8	18	29	28	57
8	S	8	10	18	11	12	23
	C	31	27	58	14	11	25
TOTAL	S	60	54	114	54	73	127
	C	185	164	349	197	176	373

Test-Retest	S	.695	Number of Subjects	63
Reliability	C	.580		

STUDENT QUESTIONNAIRE - EXPERIMENTAL GROUP

A. The following statements tell how a student might feel about arithmetic and working with the fraction strips. For each statement circle whether you agree (A), are not sure (NS), or disagree (D).

Name: ________________

A - Agree
NS - Not Sure
D - Disagree

1.	I like arithmetic.	A	NS	D
2.	I enjoyed the work just completed on fractions.	A	NS	D
3.	I now understand fractions much better than I did before I completed this work on fractions.	A	NS	D
4.	This work on fractions was boring most of the time.	A	NS	D
5.	I find that the work with fractions has confused me.	A	NS	D
6.	The way fractions were taught made it easy for me to learn.	A	NS	D
7.	I think I learned material about fractions that I will use in Grade 5.	A	NS	D
8.	I would rather work with the textbook than using the fraction kit.	A	NS	D
9.	I felt I was wasting time using the fraction kit.	A	NS	D

STUDENT QUESTIONNAIRE - CONTROL GROUP

A. The following statements tell how a student might feel about arithmetic and studying fractions using the two textbooks. For each statement circle whether you agree (A), are not sure (NS), or disagree (D).

Name: ____________________

A - Agree
NS - Not Sure
D - Disagree

1.	I like arithmetic.	A	NS	D
2.	I enjoyed the work just completed on fractions.	A	NS	D
3.	I now understand fractions much better than I did before I completed this work on fractions.	A	NS	D
4.	The work on fractions was boring most of the time.	A	NS	D
5.	I find that the work with fractions has confused me.	A	NS	D
6.	The way fractions were taught made it easy for me to learn.	A	NS	D
7.	I think I learned material about fractions that I will use in Grade 5.	A	NS	D
8.	I would rather have studied fractions using fraction strips than using the textbook.	A	NS	D
9.	I felt I was wasting time using the textbook.	A	NS	D

ITEM ANALYSIS

STUDENT QUESTIONNAIRE - GROUP TOTAL RESPONSES

ITEM		AGREE (A)	NOT SURE (NS)	DISAGREE (D)
1	E	15	12	5
	C	18	8	5
2	E	14	14	4
	C	17	13	1
3	E	21	8	3
	C	22	6	3
4	E	7	6	19
	C	6	6	19
5	E	5	13	14
	C	3	4	24
6	E	26	3	3
	C	20	6	5
7	E	17	13	2
	C	22	8	1
8	E	7	7	18
	C	7	13	11
9	E	6	7	19
	C	7	7	17

Number of Subjects	E	32
	C	31

APPENDIX D

PERCENTAGES AND CHI SQUARE TESTS OF SIGNIFICANCE OF RESPONSES TO EXP AND CON QUESTIONNAIRES OF STUDENTS CLASSIFIED BY ABILITY LEVEL AND SEX

EXPERIMENTAL GROUP STUDENT QUESTIONNAIRE

Item		ABILITY LEVEL A	NS	D		SEX A	NS	D
1	H	55	27	18	M	50	31	19
	A	50	30	20	F	44	44	12
	L	36	55	9				
χ^2			2.2				.6	
P			.70				.75	
2	H	55	27	18	M	44	50	6
	A	60	40	0	F	44	37	19
	L	18	64	18				
χ^2			6.07				1.29	
P			.20				.50	
3	H	64	27	9	M	75	19	6
	A	80	10	10	F	56	31	13
	L	55	36	9				
χ^2			2.02				1.26	
P			.74				.50	
4	H	18	0	82	M	31	13	56
	A	0	30	70	F	13	25	62
	L	46	27	27				
χ^2			11.25				2.01	
P			.03				.38	
5	H	9	27	64	M	19	44	37
	A	10	40	50	F	13	37	50
	L	27	55	18				
χ^2			5.19				.56	
P			.28				.75	
6	H	91	9	0	M	88	6	6
	A	100	0	0	F	75	12	13
	L	55	27	18				
χ^2			8.62				.82	
P			.92				.66	

EXPERIMENTAL GROUP STUDENT QUESTIONNAIRE

Item		ABILITY LEVEL A	NS	D		SEX A	NS	D
7	H	55	45	0	M	56	38	6
	A	40	50	10	F	50	44	6
	L	64	27	9				
χ^2			2.37				.14	
P			.67				.95	
8	H	18	18	64	M	13	25	62
	A	20	20	60	F	31	19	50
	L	27	27	46				
χ^2			.82				1.55	
P			.94				.93	
9	H	27	27	46	M	31	13	56
	A	0	0	100	F	6	31	63
	L	27	36	37				
χ^2			10.24				4.01	
P			.04				.13	

CONTROL GROUP STUDENT QUESTIONNAIRE

Item		ABILITY LEVEL A	NS	D		SEX A	NS	D
1	H	64	18	18	M	64	12	24
	A	70	20	10	F	50	43	7
	L	40	40	20				
χ^2			2.38				4.44	
P			.68				.13	
2	H	64	27	9	M	65	35	0
	A	60	40	0	F	43	50	7
	L	40	60	0				
χ^2			3.77				2.28	
P			.45				.32	
3	H	100	0	0	M	65	24	6
	A	70	20	10	F	79	14	7
	L	40	40	20				
χ^2			9.16				.54	
P			.06				.77	
4	H	36	0	64	M	24	17	59
	A	10	20	70	F	14	22	64
	L	10	40	50				
χ^2			6.85				.43	
P			.15				.80	
5	H	0	18	72	M	5	24	71
	A	0	10	90	F	14	0	86
	L	30	10	60				
χ^2			7.29				4.08	
P			.14				.14	
6	H	64	27	9	M	59	23	18
	A	70	20	10	F	72	14	14
	L	60	10	30				
χ^2			2.65				.58	
P			.62				.75	

CONTROL GROUP STUDENT QUESTIONNAIRE

Item		ABILITY LEVEL A	NS	D		SEX A	NS	D
7	H	72	18	0	M	65	29	6
	A	70	30	0	F	79	21	0
	L	60	30	10				
χ^2			2.84				1.22	
P			.59				.56	
8	H	18	36	46	M	29	42	29
	A	30	40	30	F	14	43	43
	L	20	50	30				
χ^2			1.09				1.17	
P			.90				.56	
9	H	18	27	55	M	29	29	42
	A	20	10	70	F	14	14	72
	L	30	30	40				
χ^2			2.24				2.84	
P			.70				.25	

APPENDIX E

RAW SCORES

APPENDIX E

RAW SCORES FOR ALL SUBJECTS

ID Numbers - The first three digits of the four digit identification number for each student indicate his treatment group, ability level, and sex as follows:

ID	GROUP	ABILITY LEVEL	SEX
111	EXP	H	M
112	EXP	H	F
121	EXP	A	M
122	EXP	A	F
131	EXP	L	M
132	EXP	L	F
211	CON	H	M
212	CON	H	F
221	CON	A	M
222	CON	A	F
231	CON	L	M
232	CON	L	F

The last of the four digits indicates the student's individual number.

Column Heading	Measure
1	Non-Verbal I. Q.
2	Initial Symbolic Achievement Test
3	Initial Concrete Achievement Test
4	Initial Symbolic Transfer Test
5	Initial Concrete Transfer Test
6	Delayed Symbolic Achievement Test
7	Delayed Concrete Achievement Test
8	Delayed Symbolic Transfer Test
9	Delayed Concrete Transfer Test

ID	1	2	3	4	5	6	7	8	9
1121	139	25	4	4	8	26	5	7	3
1122	136	24	5	2	8	16	7	7	4
1112	131	15	5	6	8	14	7	8	5
1123	129	19	5	4	5	19	3	8	5
1124	124	12	3	1	8	13	2	8	2
1125	120	08	3	0	3	13	7	3	0
1113	120	14	2	1	6	13	1	7	2
1126	119	09	1	0	5	09	0	7	0
1114	118	16	1	3	3	11	1	6	6
1115	117	20	1	2	8	16	1	8	3
1116	117	08	1	4	7	14	4	6	1
1211	114	09	1	3	7	09	4	8	0
1212	113	27	6	6	8	28	7	8	6
1222	112	18	0	2	7	15	3	3	3
1223	110	06	3	3	5	11	3	7	3
1224	110	11	2	1	5	14	4	5	2
1213	109	04	1	0	6	10	4	5	0
1225	109	10	2	2	1	05	3	7	0
1214	107	11	1	2	6	14	2	6	2
1215	106	16	4	6	5	13	2	7	3
1226	104	07	0	0	5	06	1	5	0
1321	104	07	1	0	4	09	3	4	0
1311	104	09	0	2	4	06	2	7	1
1322	103	11	3	1	2	14	3	6	1
1324	099	07	0	0	6	06	5	5	0
1312	098	12	2	3	7	12	2	6	0
1313	096	10	2	4	7	12	2	6	1
1314	095	09	0	2	7	10	3	7	1
1325	092	09	0	1	6	07	0	4	0
1315	091	03	3	0	4	06	2	6	0
1326	085	06	2	0	1	07	3	0	0
1317	074	08	2	0	3	11	4	4	0
2111	143	36	6	7	8	36	9	8	8
2121	129	22	4	3	7	23	6	7	6
2112	128	17	3	4	7	21	9	7	6
2113	121	11	2	0	2	14	2	5	0
2122	118	11	2	1	4	16	3	3	0
2123	118	17	4	7	4	21	4	5	6
2114	117	22	2	2	4	15	5	5	6
2124	116	14	2	1	7	16	3	8	2
2115	115	13	3	4	7	13	2	7	5
2116	115	08	2	0	3	12	3	4	0
2125	115	14	2	0	6	10	3	7	3
2126	115	23	2	5	8	22	2	8	3

ID	1	2	3	4	5	6	7	8	9
2221	114	28	5	2	7	22	8	8	3
2211	113	07	2	0	0	13	1	2	1
2212	112	12	4	3	5	18	6	6	3
2213	111	18	4	0	7	15	8	4	0
2214	111	20	4	2	7	18	5	7	4
2223	110	16	3	3	7	12	2	5	5
2215	109	15	2	1	0	11	3	5	0
2224	107	12	2	1	4	07	0	4	0
2226	104	16	0	0	6	13	0	7	2
2321	102	09	0	0	3	10	5	7	0
2322	101	10	3	1	8	13	5	5	2
2311	098	08	1	0	5	11	5	5	2
2312	096	06	2	0	6	09	4	5	0
2313	094	01	4	1	6	08	4	7	1
2323	093	16	3	1	5	15	5	8	0
2314	091	11	2	0	5	09	4	4	3
2315	089	10	1	2	6	09	0	4	0
2316	087	11	2	0	2	11	4	1	3
2325	077	05	2	0	2	03	2	2	0

APPENDIX F

CORRELATION MATRIX FOR ALL TESTS

TEST	1	2	3	4	5	6	7	8	9
1	1.000	.624	.444	.490	.366	.661	.344	.567	.486
2		1.000	.563	.593	.447	.848	.439	.721	.443
3			1.000	.528	.251	.652	.600	.557	.309
4				1.000	.445	.641	.267	.695	.492
5					1.000	.448	.272	.393	.580
6						1.000	.558	.693	.407
7							1.000	.393	.173
8								1.000	.378
9									1.000

V I T A

Surname: Richmond Given Names: Edward William

Place of Birth: Whitefox, Sask. Date of Birth: May 12, 1939

Educational Institutions Attended, with Dates of Entering and Leaving:

Victoria College (University of Victoria)	1957	to	1960
University of British Columbia	1961	to	1962
University of Victoria	1962	to	1964
University of British Columbia	1970	to	1970

Degrees, Diplomas, Etc., Awarded, with Dates and Names of Institutions:

B. A.	1965	University of Victoria

Honours and Awards:

Publications:

"Let's Make Values A Basis For Curriculum Development", The B. C. Teacher, 1973, 52, 190-192

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