Extended Delivery Time Analysis of Opportunistic Secondary Packet Transmission over Multiple Primary Channels

by

Muhammad Noman Khalid
BS, Syed Babar Ali School of Science and Engineering, LUMS, Pakistan, 2014

A Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of

MASTER OF APPLIED SCIENCE

in the Department of Electrical and Computer Engineering

Extended Delivery Time Analysis of Opportunistic Secondary Packet Transmission over Multiple Primary Channels

by

Muhammad Noman Khalid
BS, Syed Babar Ali School of Science and Engineering, LUMS, Pakistan, 2014

Supervisory Committee

Dr. Hong-Chuan Yang, Supervisor
(Department of Electrical and Computer Engineering)

Dr. Lin Cai, Departmental Member
(Department of Electrical and Computer Engineering)

Supervisory Committee

Dr. Hong-Chuan Yang, Supervisor
(Department of Electrical and Computer Engineering)

Dr. Lin Cai, Departmental Member
(Department of Electrical and Computer Engineering)

ABSTRACT

Cognitive radio (CR) is one of the most prominent technique to deal with the radio spectrum scarcity problem. CR systems can improve radio spectrum utilization by opportunistically accessing the underutilized spectrum resource of the licensed users. In interweave implementation, the secondary user (SU) has to wait and locate spectrum holes before its transmission. Therefore, the extended delivery time (EDT) for the secondary user consist of both wait slots and transmission slots. We study the EDT analysis of fixed size secondary packet transmission over multiple primary channel. In particular, we introduce a birth-death based approach to model the cognitive transmission of the secondary user over multiple primary channels. We use this approach to derive the exact probability density function and probability mass function of EDT of the secondary transmission for both continuous and periodic sensing cases. We also present selected numerical and simulation results to verify our analytical approach and to illustrate the mathematical formulation.

Contents

List of Tables

List of Figures

ACKNOWLEDGEMENTS

I would like to thank:

My parents for providing my education and supporting me in the low moments

Dr. Hong-Chuan Yang and Dr. Mohamed-Slim Alouini for supporting me both financially and academically during my graduate studies. Both of them have been a constant source of knowledge, inspiration and encouragement for me

Dr. Lin Cai for serving on the supervisory committee

Dr. Boualem Khouider for advising me with an alternate strategy to verify my simulation results and also for agreeing to serve on the oral examination committee

DEDICATION

This thesis is dedicated to my parents!

Chapter 1

Introduction

RF spectrum has traditionally been defined as the frequency range from as low as 3 KHz to as high as 300 GHz on electromagnetic (EM) spectrum. RF spectrum is divided into different frequencies categories and bands, as described in Table 1.1. This division is mostly based upon different properties, such as range and attenuation, of RF waves. The difference in these properties also contributes to different usage of these frequency bands.

RF spectrum is a broadcast medium, therefore, all the users that coexist in the same frequency band interfere with the transmission of each other. Due to the exponential growth of wireless systems and services for the past two decades, the availability of wireless spectrum has become extremely scarce. This can be confirmed by looking at NTIAs frequency allocation chart [2], which shows that there is limited bandwidth left that can be used by new wireless services and products. Researchers are looking into novel ways to address this spectrum scarcity problem. Effective resource utilization of currently deployed wireless systems to improve the efficiency and throughput, is one of the main aims of next generation wireless communication systems. Cognitive radio is a promising technique in this regard that exploits temporal/spatial spectrum opportunities over the already licensed frequency bands [3]-[5]. In this thesis we carry out the accurate performance analyses for cognitive radio systems.

Frequency	**Band**	**Description**	**Usage**
3 – 30 KHz	VLF	Very low frequency	Submarine comms, time signals, storm detection
30 – 3000 KHz	LF	Low frequency	Broadcasting (long wave), Navigation beacons
0.3 – 3 MHz	MF	Medium frequency	Broadcasting (medium wave), Maritime comms, analog cordless phones
3 – 30 MHz	HF	High frequency	Broadcasting (short wave), Aeronautical, Amature, Citizen Band
30 – 300 MHz	VHF	Very High frequency	FM broadcasting, Business Radio, Aeronautical
300 – 3000 MHz	UHF	Ultra-high frequency	TV broadcasting, mobile phones, digital cordless phones, Military use
3 – 30 GHz	SHF	Super-high frequency	Point to point links, Satellites, Fixed Wireless Access
30 – 300 GHz	EHF	Extremely high frequency (millimeter waves)	Point to point links, Multimedia wireless systems

Table 1.1: Radio Frequency (RF) Spectrum Band Designation and Usage

1.1 Cognitive Radio Networks

Ever since first introduced by Mitola in 1999 [7], researchers have been quite enthusiastic to the idea of cognitive radio. It was the result of this enthusiasm that FCC had created a spectrum policy task force to provide policy recommendations for cognitive radio innovation, which resulted in policy recommendation in [8]. Cognitive radio improves spectrum utilization by allowing unlicensed users (the secondary user) to make use of under-utilized [9] licensed frequency bands opportunistically without affecting the performance of the licensed user (the primary user). Figure 1.1 shows the representation of a possible cognitive radio network where the licensed users (PU) have a base station (will not accept transmission from the secondary user) to complete their transmission where as the secondary network users make use of primary network's spectrum (without contacting the base station) to complete their transmission.

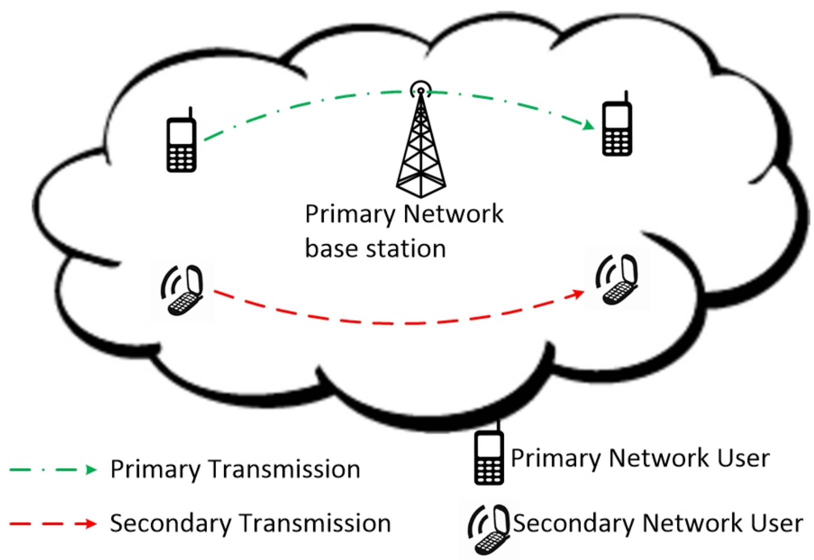

Figure 1.1: Representation of a possible cognitive radio network

Depending upon the transmission strategy used by the secondary user (SU), cognitive radio can be classified into interweave and underlay implementation. In the underlay implementation, both the primary user and the secondary user access the spectrum at the same time. During this transmission, SU makes sure that the interference caused by the SU transmission to the PU transmission does not exceed a predefined limit. At the same time, the SU transmission is affected by the PU network in two different ways: firstly, the interference caused by the primary user transmitter and secondly, the strict interference constraint at the PU receiver. It is also assumed that the SU is aware of the interference caused by its transmitter in the PU receiver(s) [3].

In the interweave implementation, this interference problem is non-existent, mainly because the SU first tries to locate spectrum holes (unoccupied band) in the licensed band of the PU through spectrum sensing. Specifically, the SU can only access the channel and transmit through it when it is not being used by the PU and must stop the SU transmission as soon as the PU reappears. One of the main challenge of this implementation is spectrum sensing by the SU (cognitive user), which is not only challenging but also consumes a lot of power. In order to return the channel to the PU and then to re-access the same or another channel to complete the transmission, spectrum handoff techniques are used. The SU transmission of a given data may involve multiple spectrum handoffs, which will result in transmission delay. The total time needed to transmit a given packet will consist of all the waiting periods before accessing the channel and the time needed for the actual transmission. This is also called the extended delivery time (EDT) [10].

1.2 Performance Analysis of Cognitive Radio

Delay and throughput analysis for secondary systems has been a constant focus for both interweave and underlay cognitive implementation. For interweave transmission, [11] investigates the average service and waiting time for the SU in a single transmission slot for the general primary traffic model. An analytical relation for the probability density function of the service time available to a secondary user during a fixed time slot was derived in [12]. A lower bound on the service time of secondary users for variable packet length that varies according to some general random distribution was derived in [13]. [14] calculates the effect of multiple spectrum handoffs on the average EDT of a secondary packet in a cognitive radio network with multiple channels and users. Probability of successful transmission in cooperative wireless communication with the hard delay scenario was studied in [15]. [10] derived analytical bounds on the throughput and transmission delays of secondary users in cognitive radio network. Also, [22] derived the exact expressions of the probability density function for the EDT of the SU packet of fixed length and further used this expression to calculate average delay time in secondary queue. As far as we know, [22] is the only previous work where exact expressions for the probability density function are derived. However, these expressions are limited to only one PU channel and one

SU channel.

Delay performance of a point-to-multipoint secondary network, for underlay implementation, is analyzed in [16]. The network concurrently shares the spectrum with a point-to-multipoint primary network in the underlay fashion. The probability density function (PDF) of packet transmission time for secondary packets under PU interference and Rayleigh fading channel is derived in [17], which is further used to analyze the timeout probability. An optimum power and rate allocation scheme to maximize the effective capacity for spectrum sharing channels under average interference constraint is proposed in [18].

When the SU transmission is interrupted due to the reappearance of PU, secondary system can adopt either the work preserving strategy or non-work preserving strategy. In the work preserving strategy, the secondary transmission can continue from the point it was interrupted [14], whereas, in non-work preserving strategy, secondary transmission of the interrupted packet must continue from the start; therefore wasting the previous transmission [10]. The work preserving strategy can be achieved with the use of rateless codes, e.g. Fountain code [19] and [20]. This strategy also applies to small and individually coded sub-packet transmission scenarios.

1.3 Contribution

In this thesis, we investigate the delay analysis of secondary transmission in the interweave implementation of cognitive communication over multiple PU channels for work preserving strategy. A direct extension of the conditioning based approach from [21] and [22] is first used to derive the exact statistics for extended delivery time of a fixed-size secondary packet; this approach has some unforeseen complexities that will be discussed in detail. We then introduce a birth-death process based approach to analyze EDT for the SU packet transmission over multiple primary channels; this approach is used to derive the exact probability density function and probability mass function of the EDT of a fixed-size secondary packet. The accuracy of this alternate approach is later verified by numerical simulations. To the best of our knowledge, the use of birth-death process to model a secondary user transmission over multiple primary channels is a completely new concept.

The rest of the thesis is structured as follows:

Chapter 2 presents the system model, problem formulation and the detailed statistical analysis of extended delivery time analysis for multiple primary channels in a cognitive scenario, using the conditioning approach for continuous sensing case. Further, it details the numerical simulation strategy used to verify the analytical results for the conditioning approach. Lastly, this chapter also includes the results from the numerical simulation along with the discussion on the limitation of the conditioning approach.

Chapter 3 comprises of the formulation of a birth-death process as an alternate approach of dealing with the multiple PUs for a cognitive transmission, in both continuous sensing and periodic sensing cases. This process is further used to derive the exact distribution of EDT. Lastly, this chapter also presents the results of comparison between the analytical and numerical simulation results.

Chapter 4 summarizes the thesis and discusses some future research directions.

Chapter 2

Conditioning-based Approach for EDT Analysis

This chapter presents the system model, problem formulation and the detailed statistical analysis of extended delivery time analysis for multiple primary channels in a cognitive scenario. Further, it details the numerical simulation strategy used to verify the analytical results for the conditioning approach. Lastly, this chapter also includes the results from the numerical simulation along with the discussion on the limitations of the conditioning based approach.

2.1 System Model and Problem Formulation

We consider a cognitive transmission scenario where the SU opportunistically accesses the N channels of a primary system, to transmit its packets. The occupancy of these channels by PUs varies independently according to a homogeneous continuous time Markov chain with average busy duration of λ and average free duration of μ. Thus, the duration of busy and free time slots are exponentially distributed. The SU opportunistically accesses these channels in an interweave fashion. To be more precise, the SU can only access a channel when the PU on that channel stops transmission and stop the access of that channel when PU starts transmission. Thus, no interference is caused to PUs transmission.

The SU monitors the activity of PUs through spectrum sensing (we assume perfect sensing here). Specific spectrum sensing techniques have been discussed in detail in [5] and [6]. To ensure no interference with the PU activity, we assume that during transmission, the SU constantly monitors the PU activity of that particular channel. When the PU reappears during the transmission of the SU, the SU user will either switch to another free channel and continue transmission or wait for a free channel before transmission, depending upon the PU activity of remaining channels. Meanwhile, when not transmitting, the SU can monitor the activity of all the PU channels using two different sensing approaches, i.e. continuous and periodic sensing. In the continuous sensing case, the SU continuously senses all the channels and starts its transmission as soon as it finds a free channel, whereas, in periodic sensing case, the SU senses all the channels periodically, with a sensing period of T_s.

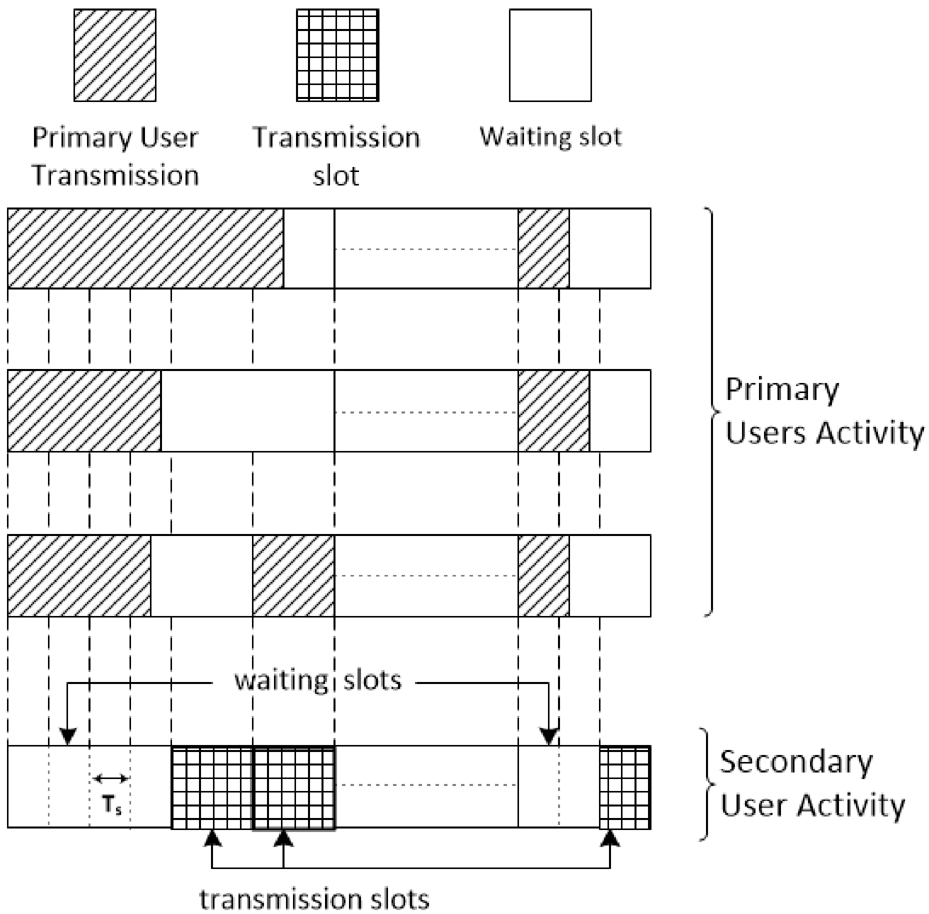

Figure 2.1: Illustration for the activity of PUs and the SU, along with transmission and wait slots for periodic sensing case.

The continuous period of time during which the SU transmits on one particular channel is referred to as a transmission slot. Similarly, continuous period of time during which the SU is not transmitting or all the PUs are in busy time slots (in the periodic sensing case this will also include the time period when one of the PU channels has become free but SU has not yet sensed the channel), is referred to as a waiting slot. Figure 2.1 shows the effect of PU activity on transmission and wait slots, for periodic sensing.

In this work, we have analyzed the packet delivery time of a secondary system, which consists of interleaved sequence of transmission slots and waiting slots. The resulting EDT(Extended Deliver Time) for one packet is given by $T_{ED} = T_{tr} + T_w$, where T_{tr} is the packet transmission time and T_w is total waiting time of SU. In general both T_{tr} and T_w are some random variables, where T_w depends on T_{tr}, the PU behavior and sensing strategies, and T_{tr} depends upon the channel conditions and the SU packet size. In here, we assume T_{tr} is constant. As such, to derive the exact distribution of EDT we need to first derive the exact distribution function of T_w.

2.2 Extended Delivery Time Analysis

In this section, we investigate the EDT of secondary system of a single packet arriving at a random point in time. In particular, we present a conditioning based approach for the EDT analysis with continuous sensing. Without the loss of generality, we assume that the secondary packet arrives at $t = 0$ time instant. The distribution of T_w can be derived by considering following cases:

(i) All the PU channels are busy at $t = 0$, the probability of which is $A = \left(\frac{\lambda}{\lambda+\mu}\right)^N$;

(ii) At least one of the PU channels is free at $t = 0$, whose probability is $B = 1 - A = 1 - \left(\frac{\lambda}{\lambda+\mu}\right)^N$,

where N is the number of PU channels and $\frac{\lambda}{\lambda+\mu}$ is the probability of a PU channel being busy at the instant of packet arrival. We denote the PDF of T_w for case (i) and case (ii) by, $f_{T_w,p_{on}}(t)$ and $f_{T_w,p_{off}}(t)$ respectively. Now, the PDF of total waiting time T_w is given by:

$$f_{T_w}(t) = A \times f_{T_w,p_{on}}(t) + B \times f_{T_w,p_{off}}(t) \tag{2.1}$$

The PDFs $f_{T_w,p_{on}}(t)$ and $f_{T_w,p_{off}}(t)$ are calculated separately.

We classify the transmission slots to Type I and Type II (illustrated in Fig. 2.2), defined as follows:

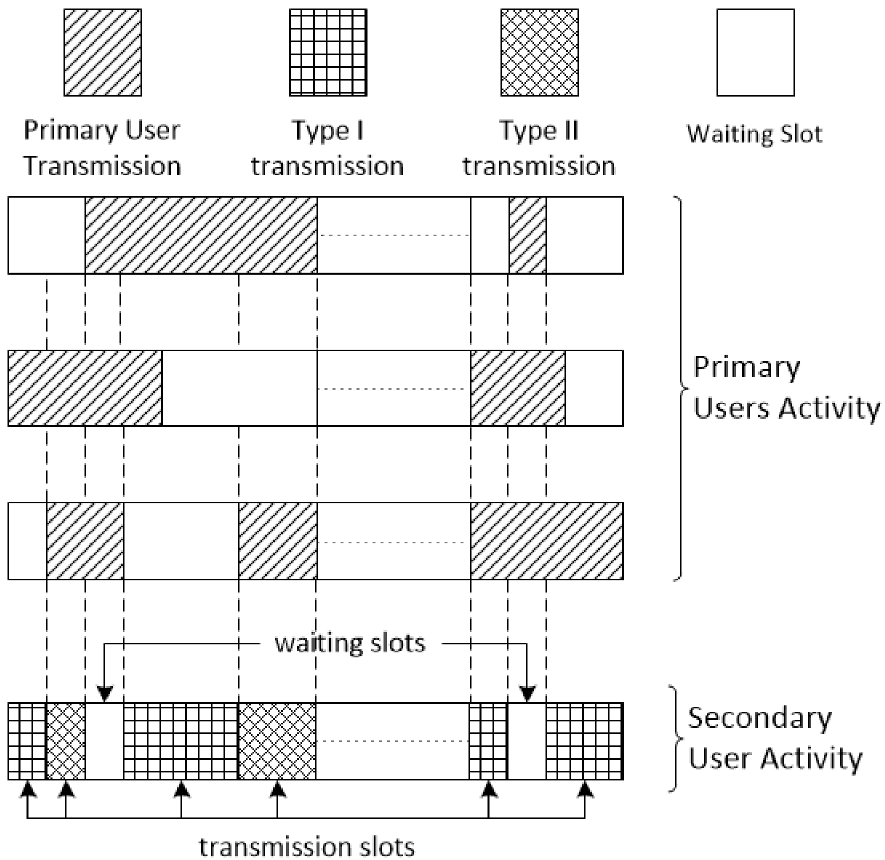

Figure 2.2: Illustration of PUs and the SU activity for continuous sensing case, along with difference in Type I and Type II of transmission slots.

Type I: (transmission slots after waiting slots) the SU is sensing all the channels while waiting and starts transmission as soon as it finds a free channel.

Type II: (transmission slots after other transmission slot) the SU transmission is interrupted as the PU reappears. The SU immediately checks the remaining channels and finds one free channel. The SU switchs to that channel and continues transmission.

Let us first consider case (ii), when at the instant of packet arrival at least one of the PU channels is in free time slot, then depending upon the number of Type I and Type II transmissions, the number of waiting slots can vary anywhere between 0 to the (number of transmission slots)-1, as illustrated in Fig. 2.3. For example, if it takes k transmission slots to send the packet and i number of those slots are Type I and $k-1-i$ are Type II then the SU will experience i waiting slots. (Here first transmission slot is neither Type I nor Type II) Now to derive the PDF of the total waiting time

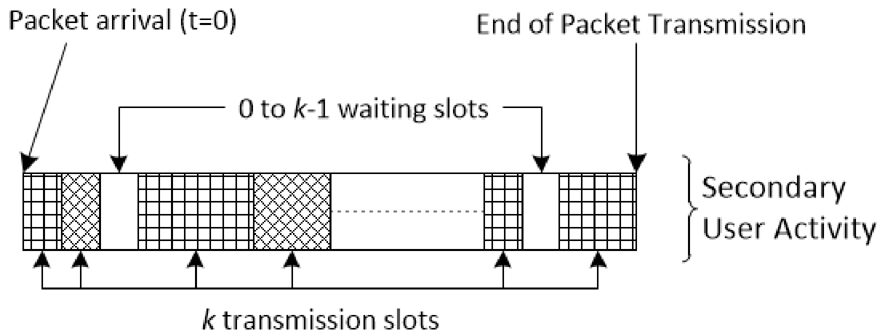

Figure 2.3: Illustration of secondary transmission when atleast one of the PUs is in idle time slot at the instant of packet arrival

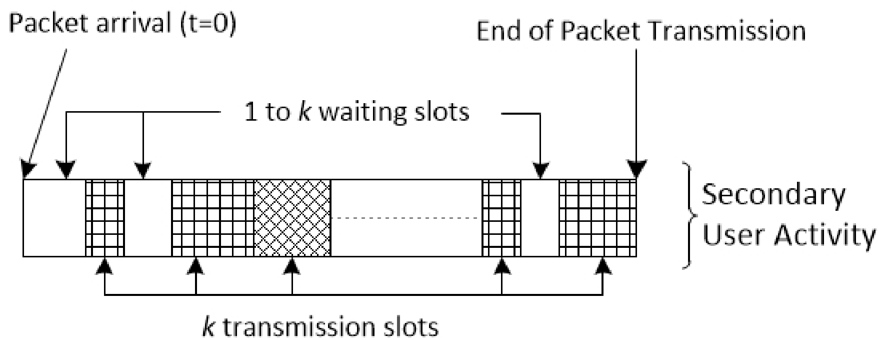

Figure 2.4: Illustration of secondary transmission when all the PUs are in busy time slot at the instant of packet arrival

T_w for the above condition, let's assume that it takes exactly k transmission slots to send the packet. Let P_k denote the probability that SU completes the transmission in k^{th} transmission slots. This can be calculated using the total time for k transmission slots. Since, each of these transmission slot is i.i.d exponential with mean μ, sum of these k transmission slots will be Erlang distribution [23] with parameters k and μ. Then P_k from [21] is given by:

$$P_k = \frac{{T_{tr}}^{k-1} e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \tag{2.2}$$

Then the PDF of total waiting time for the SU, for case (ii) is given by:

$$f_{T_w,p_{off}}(t) = \sum_{k=1}^{\infty} P_k \times \sum_{i=0}^{k-1} \binom{k-1}{i} C^i D^{k-1-i} f_{T_{w,i}}(t) \tag{2.3}$$

where $f_{T_{w,i}}(t)$ represents the PDF of the duration of i waiting slots and C is the probability that at the time of interruption none of the remaining channels is free which is equal to $\left(\frac{\lambda}{\lambda+\mu}\right)^{N-1}$, and D is the probability that at the time of interruption at least one of the remaining channels is free, which is equal to $1-C$. Both C and D from above equation can also be expressed as, $C = Pr\{\text{A transmission slot is Type I}\}$ and $D = Pr\{\text{A transmission slot is Type II}\}$. Here, for each value of i in the summation, T_w is the total time duration of i waiting slots. Also, each wait slot is the minimum of the busy time slots for all the channels, each of which is i.i.d exponential with average λ. Therefore, the wait time for one wait slot is also exponentially distributed with average $\frac{\lambda}{N}$ [23]. So, $f_{T_{w,i}}(t)$ is the PDF of the sum of i i.i.d exponential random variables with rate $\frac{N}{\lambda}$, and is given by the Erlang distribution [23]:

$$f_{T_{w,i}}(t) = \frac{N^i}{\lambda^i}\frac{t^{i-1}}{(i-1)!}e^{\frac{-N}{\lambda}t} \tag{2.4}$$

After Substituting Eqs. (2.2) and (2.4) into (2.3), we get

$$f_{T_w,p_{off}}(t) = \sum_{k=1}^{\infty}\frac{{T_{tr}}^{k-1}e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times \sum_{i=0}^{k-1}\binom{k-1}{i}C^iD^{k-1-i}\frac{N^i}{\lambda^i}\frac{t^{i-1}}{(i-1)!}e^{\frac{-N}{\lambda}t} \tag{2.5}$$

which simplifies to

$$\begin{aligned} f_{T_w,p_{off}}(t) = \sum_{k=1}^{\infty}\frac{{T_{tr}}^{k-1}e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times D^{k-1}\delta(t) + \sum_{k=2}^{\infty}\frac{{T_{tr}}^{k-1}e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \\ \times \sum_{i=1}^{k-1}\binom{k-1}{i}C^iD^{k-1-i}\frac{N^i}{\lambda^i}\frac{t^{i-1}}{(i-1)!}e^{\frac{-N}{\lambda}t} \end{aligned} \tag{2.6}$$

where $\delta(t)$ is the delta function. Also, $\sum_{k=1}^{\infty} P_k \times D^{k-1}\delta(t)$ corresponds to the cases of zero waiting slots and as such no type I transmission slots.

Similarly, we can derive the PDF of total waiting time of SU, for the case (i), illustrated in Fig. 2.4, noting that there is at least one waiting slot and one type I transmission, we can write PDF as:

$$f_{T_w,p_{on}}(t) = \sum_{k=1}^{\infty} P_k \times \sum_{i=0}^{k-1}\binom{k-1}{i}C^iD^{k-1-i}f_{T_{w,i+1}}(t) \tag{2.7}$$

Note that in Eq. (2.7) for k transmission slots, the waiting slots vary anywhere between 1 to k, i.e. we have one extra waiting slot than case (ii). After substituting Eqs. (2.2) and (2.4) into (2.7), we get

$$f_{T_w,p_{on}}(t) = \sum_{k=1}^{\infty} \frac{{T_{tr}}^{k-1} e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times \sum_{i=0}^{k-1} \binom{k-1}{i} C^i D^{k-1-i} \frac{N^{i+1}}{\lambda^{i+1}} \frac{t^i}{i!} e^{\frac{-N}{\lambda}t} \tag{2.8}$$

After substituting Eqs. (2.6) and (2.8) into (2.1), and doing a change of variable of $T_{ED} = T_w + T_{tr}$, the PDF of the EDT T_{ED} for the continuous sensing case is given by:

$$\begin{aligned} f_{T_{ED}(t)} = B\Bigg[\sum_{k=1}^{\infty} \frac{{T_{tr}}^{k-1} e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times D^{k-1}\delta(t-T_{tr}) \Bigg] + u(t-T_{tr}) e^{\frac{-N}{\lambda}(t-T_{tr})} \\ \times \Bigg[A\Bigg(\sum_{k=1}^{\infty} \frac{{T_{tr}}^{k-1} e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times \sum_{i=0}^{k-1} \binom{k-1}{i} C^i D^{k-1-i} \frac{N^{i+1}}{\lambda^{i+1}} \frac{(t-T_{tr})^i}{i!} \Bigg) \\ + B\Bigg(\sum_{k=2}^{\infty} \frac{{T_{tr}}^{k-1} e^{\frac{-T_{tr}}{\mu}}}{\mu^{k-1}(k-1)!} \times \sum_{i=1}^{k-1} \binom{k-1}{i} C^i D^{k-1-i} \frac{N^{i}}{\lambda^{i}} \frac{(t-T_{tr})^{i-1}}{(i-1)!} \Bigg) \Bigg] \end{aligned} \tag{2.9}$$

where $u(.)$ is the step function. Similarly the exact statistic for the periodic sensing case was also derived.

2.3 Monte-Carlo Simulations

Monte-Carlo methods refer to a broad class of computational algorithms, that make use of repeated random number sampling to generate numerical results. Monte-Carlo methods generally follows the same procedure as described below [24]:

1. Define the domain of input parameter
2. Generate random inputs distributed over the domain
3. Perform some deterministic computations on the input
4. Aggregate the final results

2.3.1 Generating Random Variables

One of the essential part of any Monte-Carlo simulation, is generating a large set of random numbers that are distributed over the input domain. Generating random numbers that follow a particular probability distribution can be achieved by using one of the ways listed below [25]:

1. Inverse Transform
2. Composition Algorithm
3. Acceptance Rejection Algorithm

2.3.2 Inverse Transform Method

Inverse transform is the method used to generate random numbers that were used in the numerical verification of our results.

This method makes use of the fact that probability of a random variable is always between 0 and 1. So, if U is a random number between 0 and 1 and F is CDF (Cumulative density function) for some random variable X then:

$$X = F^{-1}(U)$$

will generate a random value for variable X [25].

2.3.3 Simulation Algorithm

Figure 2.5 represents the flow graph of the simulation algorithm used to verify the analytical results for the conditioning approach.

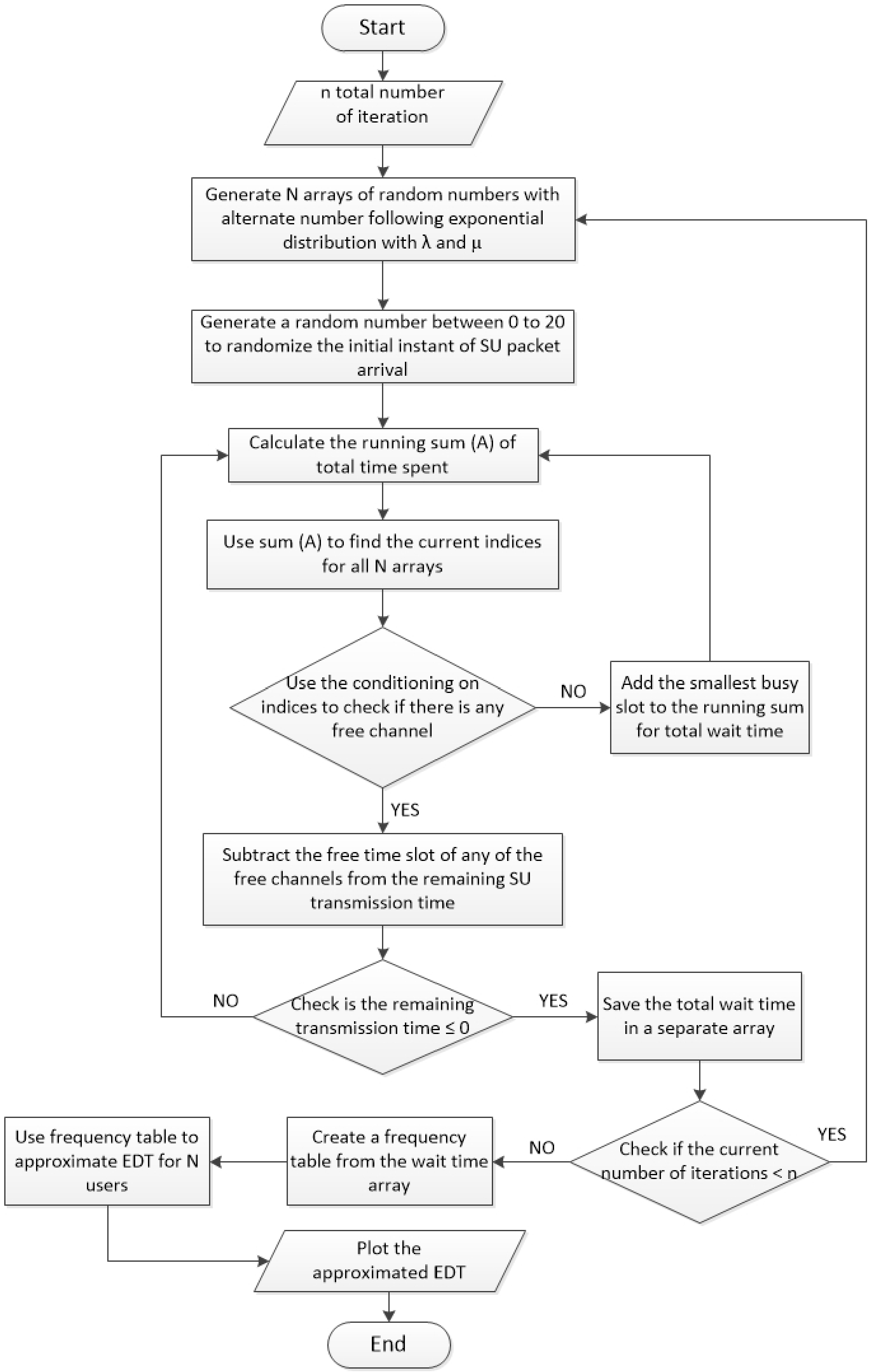

Figure 2.5: Flow graph for the simulation algorithm

2.4 Numerical Results and Discussion

Figure 2.6 plots the analytical result for the PDF of EDT for the continuous sensing scenario, given in Eq. (2.9). The corresponding plots for simulation results and different values of N are also shown. It's clear that the analytical results closely follow the simulation results. Moreover, the general trend of decrease in average EDT, with an increase in the number of primary channels N, also follows as expected.

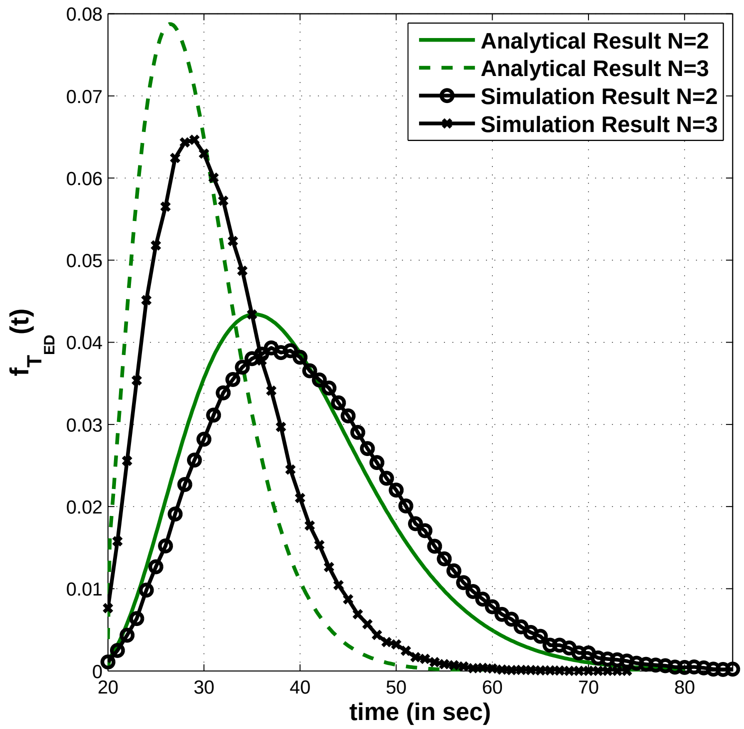

Figure 2.6: Simulation verification of for the analytical PDF of T_{ED} with continuous sensing ($T_{tr} = 20s$, $\lambda = 5$, and $\mu = 2$.)

Although, as seen in Figure 2.6, the graph of the analytical result for the conditioning approach does closely follow the simulation results but there is a clear mismatch.

One of the main reasons for this mismatch is that the memoryless property of exponential distribution doesn't hold true for all the sensing instances. In Eqs. (2.3) and (2.5), C is the probability of not finding a free channel immediately after the transmission of the SU gets interrupted. The calculation assumed that C doesn't depend upon the previous state of PU channels when in fact the SU only transmits after it finds a free time slot on at least one of the PU channels. So, once the SU transmission gets interrupted due to the reappearance of the PU, the probability of finding a free PU channel immediately after the interruption of the SU transmission, from the remaining $N-1$ channels depends upon the previous state (before the start of current transmission) of the PU channels. Thus, our assumption about C does not hold true and EDT analysis needs to take care of this dependence.

One of the solutions for the above mentioned issue, is to keep track of the state of all the PU channels at the most recent sensing instance. That means on each sensing instant, the system knows about the number of free PU channels and number of busy PU channels, from the previous sensing instance. This results in further conditioning on each sensing instance in our analysis. Which means the summation in Eqs.(2.3) and (2.7) turns into a nested summation and EDT analysis becomes far more complicated to accurately calculate the closed form solution. Following example for $N = 2$ will further explain this complexity. For $N = 2$ case, at $t = 0$ (i.e at the instant of SU packet arrival), the SU has following options:

1. All the PU user channels are busy

2. Only one PU channel is free

3. Both the PU user channels are free

Let's consider the case of one PU channel being free. Also, for clarification purpose we label the PU channels as channel#1 and channel#2. We further assume that it is channel#1 which is free at $t = 0$. After packet arrival the SU will immediately start it's transmission on channel#1 and will stop this transmission after τ_1 when the PU at channel#1 restarts its transmission. Here, τ_1 is an exponential random variable with mean μ. Now the SU will check if channel#2 is free then the SU will continue its transmission on channel#2. This probability of channel#2 being free after the first interruption is given by γ_1. Similarly, SU will stop this transmission, when the PU for channel#2 restarts its transmission after τ_2. Again, now the SU will check if the channel#1 is free and will continue its transmission on channel#1 with probability γ_2. The SU will repeat this process to alternate between channel#1 and channel#2 for k transmission slots until the SU transmission gets completed. Now, Eq. (2.3) for this particular case of 0 wait slots gets updated to:

$$f_{T_w,p_{off}}(t)|_{\text{number of wait slots}=0} = \sum_{k=1}^{\infty} P_k \gamma_1 \gamma_2 \cdots \gamma_k \delta(t) \tag{2.10}$$

where γ_i is the probability that a PU channel which was previously busy is free after τ_i time and is given by:

$$\gamma_i = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\frac{1}{\lambda} + \frac{1}{\mu})\tau_i} \tag{2.11}$$

As seen in Eq.(2.10) the PDF of the EDT for the SU packet transmission involves a lot of conditioning and an accurate analysis by taking care of all the conditions is far more complex when N is greater that 2. Therefore, an alternate approach to deal with this complexity is discussed in the following chapter.

Chapter 3

Birth-Death Process based Approach for EDT Analysis

This chapter comprises of the formulation of a birth-death process as an alternate approach of dealing with the multiple PUs for a cognitive transmission, in both continuous sensing and periodic sensing cases. This process is further used to derive the exact distribution of EDT. Lastly, this chapter also presents the results of comparison between the analytical and numerical simulation results.

3.1 Birth-Death Process

We introduce a birth-death process to model the number of free primary channels. The number of free primary channels defines the state space of the systems. An illustration of this birth-death process is shown in Figure 3.1. Here, a birth refers to

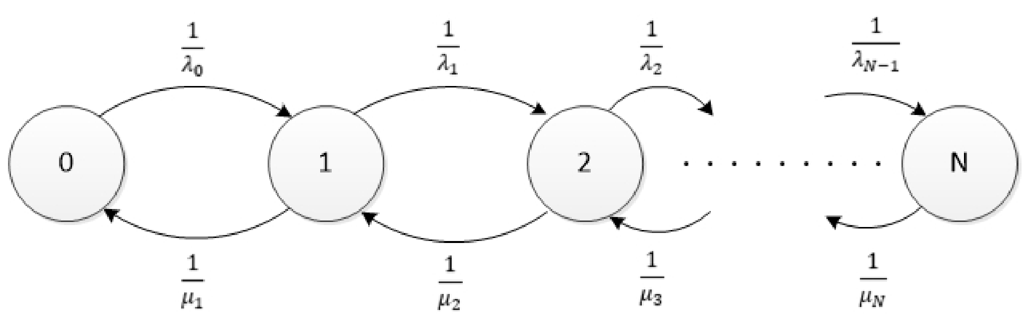

Figure 3.1: Illustration of finite state birth-death process for the number of free primary channels from a total of N primary channels.

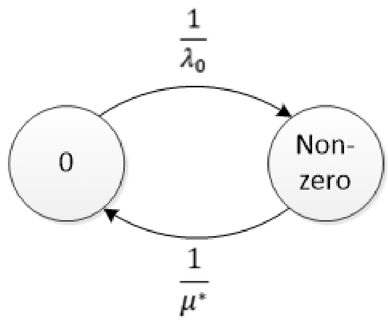

Figure 3.2: Illustration of condensed two state continuous time markov chain

an additional primary channel going from busy state to free state. Similarly, a death corresponds to a primary channel that is in free state, moving to the busy state. So, if the system is in state i, there are i free primary channels and $N - i$ busy primary channels. Since, the death of each free primary channel is exponential with rate $\frac{1}{\mu}$, then the death for state i is minimum of i i.i.d exponential random variables, which is also exponential [23] with rate $\frac{1}{\mu_i} = \frac{i}{\mu}, i = 0, 1, 2, ..., N$. Similarly, the birth is also exponential with rate $\frac{1}{\lambda_i} = \frac{N-i}{\lambda}$.

3.2 Two-State Markov Chain from the Birth-Death Process

Now, applying our assumptions from chapter 2, it becomes clear that the SU will only be in the waiting slot when the system is in state 0. We consolidate the complete multi-state birth death process into a two state continuous time markov chain shown in Fiure. 3.2 where $\frac{1}{\lambda_0} = \frac{N}{\lambda}$ is rate of leaving state 0, and $\frac{1}{\mu^*}$ is the rate of coming back to state 0. Assuming at time $t = 0$ system goes from state 0 to state 1, and T_1 represents the time it will take for the process to go back to state 0. Then

$$\mu^* = E[T_1] \tag{3.1}$$

Now let T_i denote the time, that it takes for the process starting from state i, to enter state $i-1$, for $1 \leq i \leq N$. We will recursively compute $E[T_i]$, $i \leq N$, by starting from $i = N$. Since T_N is exponential with rate $\frac{1}{\mu_N}$, we have

$$E[T_N] = \mu_N = \frac{\mu}{N} \tag{3.2}$$

For $1 \leq i \leq N$, we condition if the first transition is a birth or a death. That is, let

$$I_i = \begin{cases} 1, & \text{if the first transition from } i \text{ is to } i-1 (\text{ is death}) \\ 0, & \text{if the first transition from } i \text{ is to } i+1 (\text{ is birth}) \end{cases}$$

and note that

$$E[T_i \mid \text{death}] = E[T_i \mid I_i = 1] = \frac{1}{\frac{1}{\lambda_i} + \frac{1}{\mu_i}} \tag{3.3}$$

$$E[T_i \mid \text{birth}] = E[T_i \mid I_i = 0] = \frac{1}{\frac{1}{\lambda_i} + \frac{1}{\mu_i}} + E[T_{i+1}] + E[T_i] \tag{3.4}$$

This follows, since independent of whether the first transition is either a birth or a death, the time for this transition is exponential with rate $\frac{1}{\lambda_i}+\frac{1}{\mu_i}$. If this first transition is a death, then the number of free primary channels becomes $i-1$. Therefore, no need for any additional time. If it is a birth, then the number of free primary channels becomes $i+1$ and then the additional time needed to reach $i-1$ is equal to the time that it takes to return to i (this is $E[T_{i+1}]$) plus the time that it takes to reach $i-1$ (this is $E[T_i]$). Now, as the probability that the first transition is a death is $\frac{\frac{1}{\mu_i}}{\frac{1}{\lambda_i}+\frac{1}{\mu_i}}$, using Eqs. (3.3) and (3.4) we get:

$$E[T_i] = \frac{1}{\frac{1}{\lambda_i} + \frac{1}{\mu_i}} + \frac{\frac{1}{\lambda_i}}{\frac{1}{\lambda_i} + \frac{1}{\mu_i}}(E[T_i] + E[T_{i+1}]) \tag{3.5}$$

or, equivalently,

$$E[T_i] = \mu_i + \frac{\mu_i}{\lambda_i} E[T_{i+1}] \tag{3.6}$$

Starting from $E[T_N] = \frac{\mu}{N}$, Eq. (3.6) provides an efficient method to successively compute $E[T_{N-1}]$, $E[T_{N-2}]$, and up to $E[T_1]$. So, starting from $E[T_N]$ and using Eq. (3.6), we see that:

$$E[T_{N-1}] = \mu(N-2)!\Big[\frac{1}{(N-1)!} + \Big(\frac{\mu}{\lambda}\Big)\frac{1}{N!}\Big],$$
$$E[T_{N-2}] = \mu(N-3)!\Big[\frac{1}{(N-2)!} + 2\Big(\frac{\mu}{\lambda}\Big)\frac{1}{(N-1)!} + 2\cdot 1\Big(\frac{\mu}{\lambda}\Big)^2\frac{1}{N!}\Big],$$
$$E[T_{N-3}] = \mu(N-4)!\Big[\frac{1}{(N-3)!} + 3\Big(\frac{\mu}{\lambda}\Big)\frac{1}{(N-2)!} + 3\cdot 2\Big(\frac{\mu}{\lambda}\Big)^2\frac{1}{(N-1)!} + 3\cdot 2\cdot 1\Big(\frac{\mu}{\lambda}\Big)^3\frac{1}{N!}\Big]$$

and, in general,

$$E[T_i] = \mu(i-1)!\Big[\sum_{n=0}^{N-i} \frac{(N-i)!}{n!}\Big(\frac{\mu}{\lambda}\Big)^{N-i-n}\frac{1}{(N-n)!}\Big] \tag{3.7}$$

So, from Eq. (3.7), $E[T_1]$ the average time it takes for the number of free primary channels to go back zero is given by:

$$\mu^* = E[T_1] = \mu\Big[\sum_{n=0}^{N-1} \frac{(N-1)!}{n!}\Big(\frac{\mu}{\lambda}\Big)^{N-1-n}\frac{1}{(N-n)!}\Big] \tag{3.8}$$

3.3 Extended Delivery Time Analysis

We can apply this reduced two-state continuous time markov chain to analyze the EDT for the SU based on the results in [21] and [22].

3.3.1 Continuous Sensing

For continuous sensing case, the PDF of EDT $f_{T_{ED}}(t)$ for one PU channel is given by [22] :

$$\begin{aligned} f_{T_{ED}}(t) = u(t-T_{tr})\frac{1}{\lambda+\mu}e^{\frac{-(t-T_{tr})}{\lambda}}e^{\frac{-T_{tr}}{\mu}} \times \Bigg[I_0\Bigg(2\sqrt{\frac{T_{tr}(t-T_{tr})}{\mu\lambda}}\Bigg) \\ + \sqrt{\frac{T_{tr}\mu}{\lambda(t-T_{tr})}} I_1\Bigg(2\sqrt{\frac{T_{tr}(t-T_{tr})}{\lambda\mu}}\Bigg)\Bigg] + \frac{\mu}{\lambda+\mu}e^{\frac{-T_{tr}}{\mu}}\delta(t-T_{tr}) \end{aligned} \tag{3.9}$$

where $u(\cdot)$ is unit step function, $I_n(\cdot)$ is the modified Bessel function of the first kind of order n, λ is the average busy time for the primary user, μ is the average free time for the primary user and $\delta(t)$ is the delta function.

Now, as seen in Figure 3.2, the average free time for our reduced two-state continuous markov chain is μ^* and the average busy time is $\lambda_0 = \frac{\lambda}{N}$. So, by replacing μ and λ in Eq.(3.9) with μ^* and $\frac{\lambda}{N}$ respectively, the PDF of the EDT T_{ED} for the continuous sensing case over N primary channels is given by:

$$f_{T_{ED}}(t) = u(t-T_{tr})\frac{N}{\lambda+N\mu^*}e^{\frac{-N(t-T_{tr})}{\lambda}}e^{\frac{-T_{tr}}{\mu^*}} \times \left[I_0\left(2\sqrt{\frac{NT_{tr}(t-T_{tr})}{\mu^*\lambda}}\right) + \sqrt{\frac{NT_{tr}\mu^*}{\lambda(t-T_{tr})}}I_1\left(2\sqrt{\frac{NT_{tr}(t-T_{tr})}{\mu^*\lambda}}\right)\right] + \frac{N\mu^*}{\lambda+N\mu^*}e^{\frac{-T_{tr}}{\mu^*}}\delta(t-T_{tr}) \quad (3.10)$$

3.3.2 Periodic Sensing

The PMF of EDT of one PU for periodic sensing case from [22] is given by:

$$\Pr[T_{ED}=nT_s+T_{tr}] = \frac{\lambda}{\lambda+\mu}(1-\beta)\beta^{n-1}e^{\frac{-T_{tr}}{\mu}} \times {}_1F_1\left(1-n;1;\frac{-T_{tr}(1-\beta)}{\mu\beta}\right)u[n] + \frac{\mu}{\lambda+\mu}\left[\left(\frac{T_{tr}(1-\beta)\beta^{n-1}}{\mu}\right)e^{\frac{-T_{tr}}{\mu}} \times {}_1F_1\left(1-n;2;\frac{-T_{tr}(1-\beta)}{\mu\beta}\right)u[n-1]\right] + \frac{\mu}{\lambda+\mu}e^{\frac{-T_{tr}}{\mu}}\delta[n] \quad (3.11)$$

where n is total number of sensing instances, ${}_1F_1(.,.,.)$ is the generalized Hypergeometric function and β is the probability that the primary user is on at the sending instance and from [22] it is given by:

$$\beta = \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu}e^{-(\frac{1}{\lambda}+\frac{1}{\mu})T_s} \quad (3.12)$$

Here T_s is the sensing period.

Now, similar to the continuous sensing case, substituting μ and λ from Eqs. (3.11) and (3.12) by μ^* and $\frac{\lambda}{N}$ respectively, we get:

$$\beta_1 = \frac{\lambda}{\lambda+N\mu^*} + \frac{N\mu^*}{\lambda+N\mu^*}e^{-(\frac{N}{\lambda}+\frac{1}{\mu^*})T_s} \quad (3.13)$$

and the PMF of the EDT T_{ED} for the periodic sensing case over N primary channels is given by:

$$\Pr[T_{ED} = nT_s + T_{tr}] = \frac{\lambda}{\lambda + N\mu^*}(1-\beta_1){\beta_1}^{n-1}e^{\frac{-T_{tr}}{\mu^*}} \times {}_1F_1\left(1-n; 1; \frac{-T_{tr}(1-\beta_1)}{\mu^*\beta_1}\right)u[n] + \frac{N\mu^*}{\lambda + N\mu^*}\left[\left(\frac{T_{tr}(1-\beta_1){\beta_1}^{n-1}}{\mu^*}\right)e^{\frac{-T_{tr}}{\mu^*}} \times {}_1F_1\left(1-n; 2; \frac{-T_{tr}(1-\beta_1)}{\mu^*\beta_1}\right)u[n-1]\right] + \frac{N\mu^*}{\lambda + N\mu^*}e^{\frac{-T_{tr}}{\mu^*}}\delta[n] \quad (3.14)$$

3.4 Numerical Examples

We now verify the analytical results using numerical simulation strategies discussed in chapter 2. This section also compares the work done in this thesis with the previous work on EDT analysis in [21] and [22].

Figure 3.3 plots the analytical result for the PDF of EDT for the continuous sensing scenario, given in Eq. (3.10). The corresponding plots for the simulation results for the different values of N and different simulation strategies, are also shown. The near perfect match between the analytical and simulation results verify our birth-death process based analytical approach. Similar to the anlytical results from chapter 2, the general trend of decrease in average EDT, with an increase in the number of primary channels N, also gets verified.

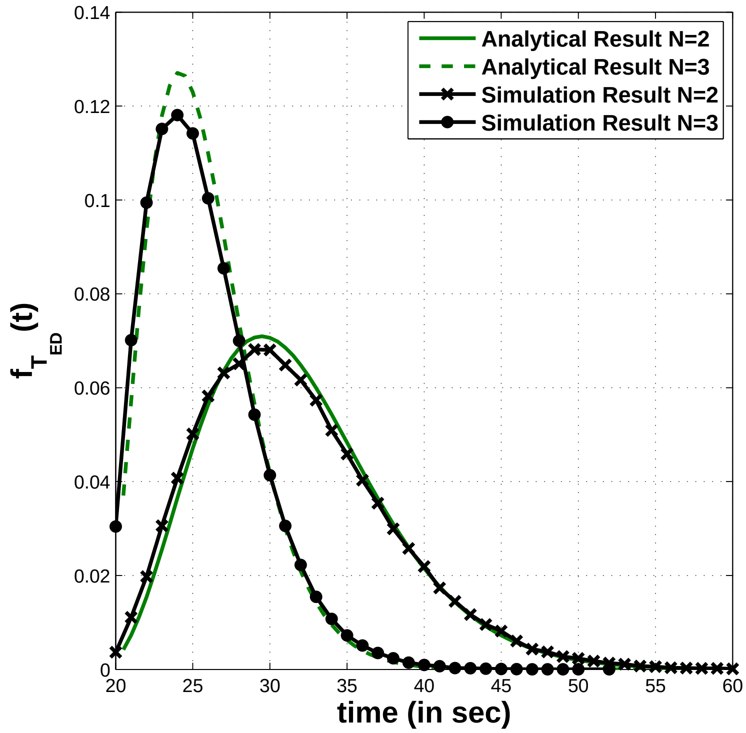

Figure 3.3: Simulation verification of the analytical PDF of T_{ED} with continuous sensing ($T_{tr} = 20s$, $\lambda = 3$ and $\mu = 2$.)

Figure 3.4 plots the PMF of EDT for periodic sensing case for different number of primary channels N, corresponding simulation results are also shown. The plots show that the analytical result reconcile with the simulations results. Also, these plots verify the expected trend of decrease in average EDT for periodic sensing as the number of primary channels N increase.

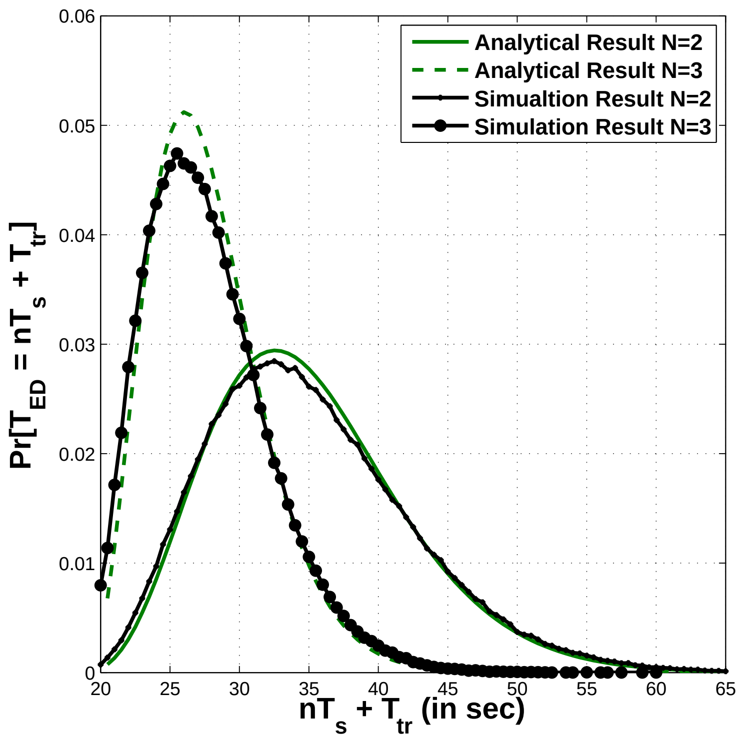

Figure 3.4: Simulation verification of the analytical PMF of T_{ED} with periodic sensing ($T_{tr} = 20s$, $\lambda = 3$, $\mu = 2$ and $T_s = 0.5s$).

Figure 3.5 shows the analytical PMF envelope of the EDT with the periodic sensing case corresponding to the different sensing periods T_s along with the analytical PDF of the continuous sensing case. As T_s approaches 0, the performance of periodic sensing comes closer to that of continuous sensing case, which is the expected outcome as continuous sensing case is essentially periodic sensing case with the sensing period as $T_s = 0$.

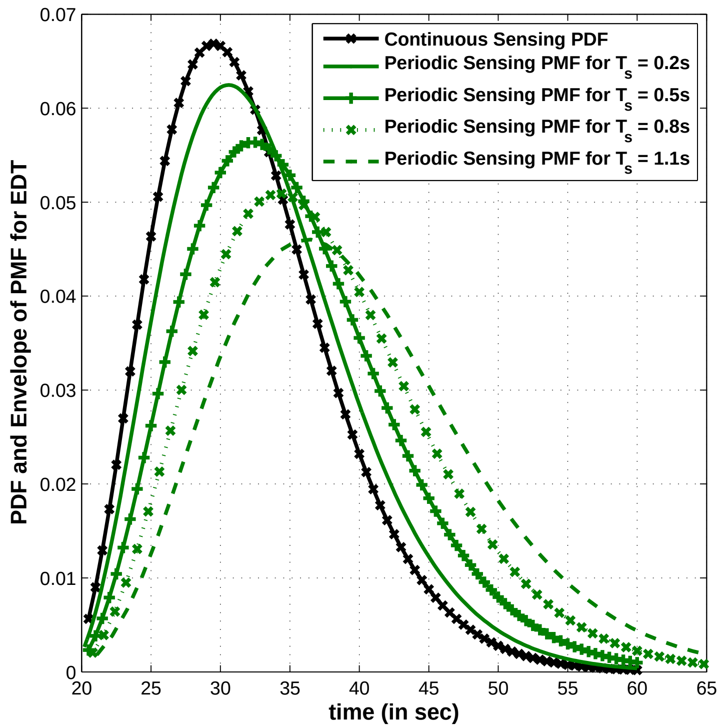

Figure 3.5: Analytical distribution of EDT with the continuous and periodic sensing ($N = 3$, $T_{tr} = 10s$, $\lambda = 5$ and $\mu = 2$).

Figure 3.6 plots the analytical PDF of the total wait time T_w (EDT-transmission time) for the continuous sensing case, for different SU packet sizes. The plots show that as the SU packet size increases the average wait time for the complete transmission of those packets also increases which is the expected output.

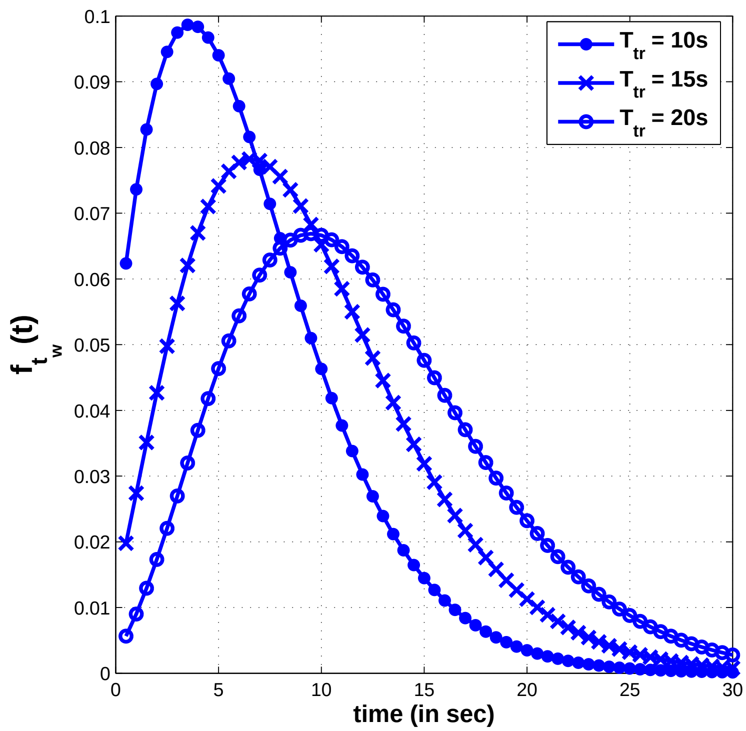

Figure 3.6: Analytical distribution of T_w for the continuous sensing case for different SU packet sizes($N = 3$, $\lambda = 5$ and $\mu = 2$).

Chapter 4

Conclusions

This chapter summarizes the thesis and discusses some future research directions.

4.1 Summary

Radio frequency spectrum scarcity and under-utilization is one of the major problems faced by wireless communication industry today. This thesis explored one the solutions proposed to deal with this problem.

In this work we studied the extended delivery time of a data packet appearing at the secondary user in an interweave cognitive setup with multiple primary user channels. We used two different approaches (i.e. conditioning based approach and birth-death process based approach) to solve this particular problem. Approximate statistical results for the probability distribution of the EDT for a fixed size data packet for the continuous sensing case with the conditioning based approach was derived and it was compared with the monte-carlo simulation results. Analytical results obtained from the conditioning based approach werent completely accurate which resulted in mismatch between the analytical and the simulation outputs. Some of the possible causes for this error were discussed.

Later, we introduced a birth-process based approach to model a cognitive system that has multiple primary channels. The exact statistical results for the probability distribution of EDT for fixed size data packet for both continuous sensing and periodic sensing based upon this birth-death process were obtained. Simulation results were presented to verify the analytical results. Near perfect match between the analytical results based upon the birth-death process based approach and the simulation results verified that a birth-death process can be used to model and multiple primary user cognitive system. The slight mismatch in this case can be attributed to either the assumption that the system remains markovian after the reduction of the birth-death process into a two-state markov chain or the Bessel function approximation in the results from [22]. Both of these possibilities can be further explored in some of the future work.

4.2 Future Work

Current work for cognitive radio systems can be extended into multiple ways. First of all, during the analysis of our results fixed size secondary packets were considered. These can be replaced with variable secondary packet sizes that can follow some general probability distribution. Also, it was assumed that channel sensing is perfect, which isn't true for any real world application. Effect of imperfect channel sensing, on EDT can be studied in detail. Similarly, the primary user activity on the targeted channels was assumed to be Markovian. This assumption can be replaced with non-Markovian distribution, where either one of the on and off times, or both of them don't follow exponential distribution. Further, this analysis considered multiple PU channels only, this can be extended to multiple SUs. Lastly, all of these extensions can not only be applied to the work-preserving strategy discussed in this work, but these extensions can also be applied to non-work preserving strategy.

Bibliography

[1] "Introduction to wireless communications," in *Design and Performance of 3G Networks and Wireless Lans,* pp. 1-12, Springer US, 2006.

[2] National Telecommunication and Information Administration (NTIA), "FCC Frequency Allocation Chart," 2003. Download available at www.ntia.doc.gov/files/ntia/publications/2003-allochrt.pdf

[3] A. Goldsmith, S. Jafar, I. Maric, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. of the IEEE,* pp. 894-914, May 2009.

[4] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.,* vol. 23, pp. 201-220, Feb. 2005.

[5] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks J.,* vol. 50, no. 13, pp. 2127 - 2159, Sep. 2006.

[6] M. Islam, C. Koh, S. W. Oh, X. Qing, Y. Lai, C. Wang, Y.-C. Liang, B. Toh, F. Chin, G. Tan, and W. Toh, "Spectrum survey in Singapore: Occupancy measurements and analyses," in *Proc. 3rd Int. Conf. Cognitive Radio Oriented Wireless Netw. and Commun.,* pp. 1-7, May 2008.

[7] Joseph Mitola, "Congnitive Radio: An Integrated Agent Architecture for Software Defined Radio," *PhD Dissertation,* KTH, Stockhol, Sweden, December 2000.

[8] Federal Communication Commission Spectrum Policy Task Force, "Facilitating Opportunities for Flexible Efficient, and Reliable Spectrum Use Employing Cognitive Radio Technologies," *Technical Report no. 03-322,* December 2003. Download available at web.cs.ucdavis.edu/ liu/289I/Material/FCC-03-322A1.pdf.

[9] "Report of the spectrum efficiency working group," Federal Communications Commission Spectrum Policy Task Force, Tech. Rep., Nov. 2002. Download available at http://transition.fcc.gov/sptf/files/SEWGFinalReport1.pdf

[10] F. Borgonovo, M. Cesana, and L. Fratta, Throughput and delay bounds for cognitive transmissions, in *Advances in Ad Hoc Networking (P. Cuenca, C. Guerrero, R. Puigjaner, and B. Serra, eds.), vol. 265 of IFIP International Federation for Information Processing,* pp. 179-190, Springer US, 2008.

[11] F. Gaaloul, H.-C. Yang, R. Radaydeh, and M.-S. Alouini, "Switch based opportunistic spectrum access for general primary user traffic model," *IEEE Wireless Commun. Lett.,* vol. 1, pp. 424-427, October 2012.

[12] Z. Liang and D. Zhao, "Quality of service performance of a cognitive radio sensor network," in *Proc. IEEE Int. Conf. Commun. (ICC),* 2010, pp. 1-5, May 2010.

[13] M. Luis, R. Oliveria, R. Dinis and L. Bernardo, "Characterization of Opportunistic Service Time in Cognitive Radio Networks," *IEEE trans. on Cognitive communications and Networking,* vol. 2, no.3, pp. 288-300, September 2016.

[14] C.-W. Wang and L.-C. Wang, "Analysis of reactive spectrum handoff in cognitive radio networks," *IEEE J. Sel. Areas Commun.,* vol. 30, pp. 2016-2028, November 2012.

[15] S. Kandeepan, C. Saradhi, M. Filo, and R. Piesiewicz, "Delay analysis of cooperative communication with opportunistic relay access," in *Proc. IEEE 73rd Veh. Technol. Conf. (VTC Spring),* 2011, pp. 1-5, May 2011.

[16] F. Khan, K. Tourki, M.-S. Alouini, and K. Qaraqe, "Delay performance of a broadcast spectrum sharing network in Nakagami-m fading," *IEEE Trans. Veh. Technol.,* vol. 63, pp. 1350-1364, March 2014.

[17] L. Sibomana, H.-J. Zepernick, H. Tran, and C. Kabiri, "Packet transmission time for cognitive radio networks considering interference from primary user," in *9th Int. Wireless Commun. and Mobile Computing Conf. (IWCMC),* 2013, pp. 791-796, July 2013.

[18] L. Musavian and S. Aissa, "Effective capacity of delay-constrained cognitive radio in nakagami fading channels," *IEEE Trans. Wireless Commun.,* vol. 9, pp. 1054-1062, March 2010.

[19] D. J. C. MacKay, "Fountain codes," *IEEE Proc.-Commun.,* vol. 152, pp.1062-1068, Dec 2005.

[20] J. Castura and Y. Mao, "Rateless coding over fading channels," *IEEE Commun. Lett.,* vol. 10, pp. 46-48, Jan 2006.

[21] M. Usman, H.-C. Yang, M.-S. Alouni, "Service Time Anaysis of Secondary Packet Transmission with Opportunistic Channel Access," in *Proc. of IEEE 80th Vehicular Technology Conference (VTC2014-Fall),* pp. 1-5, 2014.

[22] M. Usman, H.-C. Yang, M.-S. Alouni, "Extended Delivery Time Analysis for Cognitive Packet Transmission with Application to Secondary Queuing Analysis," *IEEE Trans. on Wireless Communications,* vol. 14, pp. 5300 - 5312, October 2015.

[23] "The Exponential Distribution and the Poission Process," in *Introduction to Probability Models,* pp. 302-317, Elsevier US, 2010.

[24] D. P. Kroese, T. Brereton, T. Taimre, Z. I. Botev, "Why the Monte Carlo method is important today," *WIREs Comput. Stat.* vol. 6, pp. 386-392, June 2014.

[25] Martin Haugh, "Generating Random Variables and Stochastic Processes," *Monte Carlo Simulation: IEOR E4703,* 2004.