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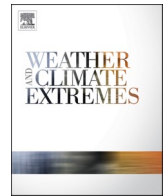
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A bivariate approach to estimating the probability of very extreme precipitation events

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ABSTRACT

We describe in this paper a semi-parametric bivariate extreme value approach for studying rare extreme precipitation events considered as events that result from a combination of extreme precipitable water (PW) in the atmospheric column above the location where the event occurred and extreme precipitation efficiency, described as the ratio between precipitation and PW. An application of this framework to historical 6-h precipitation accumulations simulated by the Canadian Regional Climate Model CanRCM4 shows that uncertainties and biases of very long-period return level estimates can be substantially reduced relative to the standard univariate approach that fits Generalized Extreme Value distributions to samples of annual maxima of extreme precipitation even when using modest amounts of data.

1. Introduction

An ongoing challenge in the engineering design of water management infrastructure is the determination of design values for extreme precipitation (P). Such design values often require return level estimates for intervals that well exceed the length of the available precipitation record, and thus require extrapolation outside the observed data. An approach that is frequently used is to appeal to statistical extreme value theory, using either the block maximum (BM) or peaks-over-threshold (POT) approaches (Coles, 2001). The BM approach is appealing because it enables one to consider the possibility of using a year as a natural block length, thus avoiding the impact of the annual cycle on the annual variation of the likelihood and magnitude of extremes. On the other hand, careful application of the POT approach may, in some circumstances, provide larger samples of extremes than can be obtained with the BM approach, thereby using more of the available observational data. The former results in the use of the generalized extreme value distribution (GEV) to describe the probability distribution of the intensity of maxima of block (Fisher and Tippett 1928; Gnedenko 1943; Leadbetter et al., 1983), while the latter results in the use of the generalized Pareto distribution to model excesses over a threshold (Davison and Smith 1990). Because the application of the POT method generally requires more decisions from the users (declustering of

extremes, specification of a sufficiently high threshold, modelling of the annual cycle, and so on) the BM approach tends to be more frequently used in hydrology and climatology. Furthermore, in many cases only annual maxima are available for the analysis.

A limitation, however, is that the supporting theory only predicts the limiting distributions that are obtained as block lengths or exceedance thresholds are increased without bound when a limiting distribution exists. Thus, in both cases, two implicit assumptions are (i) that a limiting distribution does exist, and (ii) that the sampling of extremes from the records that are being analyzed is performed at a high enough point in the upper tail to ensure that the sampled extremes lie well within the domain of convergence to the limiting extreme value distribution (De Haan and Ferreira 2007; Gumbel 1958). These assumptions are required to confidently extrapolate the fitted distribution, and to be assured that tail stability – the lack of “surprises” in the upper tail that deviate from the predicted max-stable behaviour – holds.

It is, however, well recognized that the upper tail of the distribution of extreme precipitation and stream flow may have complex behaviour that is associated with, for example, unusual storm activity (e.g., Barlow 2011; Barth et al., 2019; England Jr et al., 2019; Ralph and Dettinger 2012; Rossi et al., 1984). Climate model output is not free of such surprises, as has been shown by Ben Alaya et al. (2020a) using a large ensemble of regional climate simulations for North America. Indeed,

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they showed that convergence to an eventual stable GEV distribution occurs slowly over much of North America in the climate for the 1951–2000 period simulated by the CanRCM4 (Scinocca et al., 2016) regional climate model. They also showed that estimates of very long-period return level estimates that are obtained by fitting GEV distributions to large samples of model simulated annual maxima of daily and sub-daily precipitation have substantial biases. That is, assuming that annual maxima are max-stable leads to large biases in very long period return level of extreme precipitation in the climate simulated by CanRCM4 in many locations. In the climate modelling world, these biases can be reduced by using multi-year blocks to ensure that the processes generating the largest events influence every block maximum, but this is clearly not possible when using observational records of limited length (often less than 50-years). Many of the approaches that have been developed to deal with unusual events that are used operationally, such as storm maximization (WMO 2009), require detailed knowledge of the environment and circumstances in which individual events occur and are therefore difficult to apply in an automated manner across a large domain when analyzing model-based data products, such as those from a historical reanalysis (e.g., ERA5, Hersbach (2016)) or a large climate simulation experiment. This therefore motivates the approach that is considered in this paper.

We begin by considering the possibility that the amount of precipitable water (PW) in the atmospheric column at the time and location of an extreme precipitation event may contain useful information about the form of the upper tail of the distribution of the amount of precipitation P that is produced. While P is apt to be small when PW is near zero, P can be larger than PW if there is continual replenishment of the moisture in the atmospheric column at the location where precipitation is occurring as the precipitation event proceeds, suggesting an important role for the circulation that converges moisture into the atmospheric column. In order to obtain extra information about P from PW, it is therefore useful to model P and PW jointly. To do this, we will first perform a simple transformation of variables by which the joint distribution of (P, PW) is transformed to the joint distribution of (PE, PW) , where $PE = P/PW$. This is a one-to-one and onto mapping of the positive quadrant of the two-dimensional vector space \mathbb{R}^2 onto itself, and thus no information about P or its relation to PW is lost (or gained). We then model the joint distribution of the extremes of PE and PW using the approach of Heffernan and Tawn (2004)– hereafter HT (2004). Once the joint model has been established, we implicitly perform an inverse transformation of extreme (PE, PW) to a pair of variables (P, Z) , and integrate over Z via Monte Carlo integration to obtain the marginal distribution for extreme P . Note that the use of a Monte Carlo integration approach makes it unnecessary to explicitly specify Z . The result is a semi-parametric representation of the distribution of extreme P , where the shape of the upper tail is affected by the shapes of the upper tails of the PE and PW distributions and the dependence between these variables.

This has benefits because PE and PW, individually, tend to have different tail behaviours (we will show that PE is often heavy tailed, whereas PW tends to be light tailed). Furthermore, the semi-parametric HT (2004) approach also allows relatively more flexible modelling of the tail dependence between PE and PW than parametric copula based models such as has been used by (Ben Alaya et al. 2018b, 2019, 2020b) in the estimation of probable maximum precipitation from PE and PW. From a physical perspective, jointly modelling PE and PW is beneficial because PE reflects the effects of circulation variability via the impact of circulation on moisture convergence into the atmospheric column that is producing precipitation, whereas PW can be expected to be related to the temperature of the atmospheric column since the moisture holding capacity of a warmer atmospheric column is higher (e.g., Allen and Ingram 2002; Santer et al., 2007). These two variables are clearly related in the sense that a given circulation can converge more moisture into the atmospheric column when the atmosphere is warmer and wetter, and thus we might also expect to gain useful information by considering

their upper tail dependence.

Consideration of the idea of using the joint multiple component variables to obtain the univariate distribution of a target variable dates back, in hydrology, to at least Klemes (1993) who also decomposed precipitation into a product of precipitable water with precipitation efficiency where the latter was further decomposed into the sum of synoptic efficiency and convective efficiency. The Klemes (1993) approach was not however, well supported by statistical theory. From the statistical literature Coles and Tawn (1994) suggested applying a multivariate extreme value approach when a number of processes simultaneously act on a structure, presenting numerous examples. In hydrology the idea has been also applied in other contexts, although not specifically to precipitation extremes, by De Michele et al. (2005), Michailidi and Bacchi (2017), Serinaldi (2015) and Volpi and Fiori (2014) among others.

The relevance of this study for a special issue that is focused on compound events is that their often involves the modelling of the joint extreme behaviour of two or more variables. The flexible semi-parametric multivariate extremes model of HT (2004) that is used in this study is well suited for such compound events problems, as is discussed further in Section 4. Indeed, the approach used in this study in effect, applies a compound events approach to a pair of variables, PE and PW, as an intermediate step towards obtaining a distribution for the variable of interest, which is a function of PE and PW. This is analogous to many other problems involving compound events, where an extreme impact can be considered as a function of the simultaneous exposure to more than one type of extreme. The paper is organized as follows: the datasets and the proposed method are introduced respectively in Section 2 and Section 3. Results and discussions are presented in Section 4 and finally, conclusions are given in section 4.

2. Data

Physically-based numerical atmospheric models developed over the past three decades have come to play a predominant role in climate research. Numerical models are useful since they are able to simulate three-dimensional data representative of long periods. Output from the CanRCM4 regional climate model is used in this study over the period 1951–2000. The model has 0.44° spatial horizontal resolution (155×130 grid points) over a North American domain. A detailed description of CanRCM4 is provided in Scinocca et al. (2016), von Salzen et al. (2013) and Diaconescu et al. (2015). CanRCM4 is a participant in the Coordinated Regional climate Downscaling EXperiment (CORDEX) framework (Giorgi et al., 2009) and is developed by the Canadian Center for Climate Modelling and Analysis (CCCma) to make quantitative projections of future long-term climate change.

The CanRCM4 simulation used in this study is driven by a global climate simulation produced with the second generation of Canadian earth system model (CanESM2) under the historical “all” forcing prescription of the Coupled Model Intercomparison Project Phase 5 (CMIP5; Taylor et al., 2012) including solar and volcanic forcing, greenhouse gases, aerosols, ozone and land use. From the numerous variables available in the CCCma archives, we used precipitation accumulation and PW (vertically integrated water vapor through the atmospheric column), both at a 6-hourly temporal resolution, for the period 1951–2000. PW is the depth of water that would be produced at a given location if all the water in the atmospheric column above that location were precipitated as rain or snow. Precipitation efficiency (PE) was obtained as the ratio of precipitation to PW.

For evaluation and comparison purposes with the traditional univariate approach, we also use 6-h precipitation accumulations for the same period 1951–2000 from an ensemble of 35 CanRCM4 simulations, each of which was driven by a different member of a large CanESM2 initial conditions ensemble of global simulations. The differences amongst different ensemble members are therefore due to the different, independent, realizations of internal variability that are present in the

different ensemble members. At a given location, the 35 simulations from the CanRCM4/CanESM2 combination can be considered as 35 realizations of a climate that is similar to that of the real world (Deser et al., 2012). Hence, at a given location, the ensemble provides 35 times as much data as an observational record for the period 1951–2000. Note that PW and PE are used from a single run, while precipitation from the 35 runs is used for evaluation and comparison purposes.

3. Methodology

The bivariate event framework being proposed here and the univariate approach that is used for comparison are calibrated using only one single CanRCM4 simulation of the period 1951–2000. We evaluate the ability of these two approaches to estimate very extreme quantiles as represented by the 1000-year return level (RL), using empirical estimates of the 1000-year RL based on the sample from the 35 CanRCM4 simulations as reference. The univariate approach involves fitting a GEV distribution to annual maxima of precipitation via the maximum likelihood method, which is used in this paper. More details about the GEV distribution and univariate extreme value theory can be found, for example, in Coles (2001); see also Balkema and De Haan, 1974; Pickands, 1975). A brief description is also presented in the supporting information.

3.1. The bivariate event framework

As with the univariate approach, we also use an asymptotic extreme value framework for studying rare extreme values of a variable of interest W that is driven by more than one component. The variable W is regarded as being the result of a bivariate process (for simplicity) given by

$$W = f(X_1, X_2) \tag{1}$$

where $X = (X_1, X_2)$ are its constituent process variables and f links realizations of X to those of W . This function may follow from physical theory (when applied to a physical process), it could represent the impact of a compound event that is characterized by f , or it could be defined by stakeholders and engineers when applied to structural design problems or natural hazards (see Coles and Tawn (1994) for some examples of f for structural design problems). If the analytical form of f is not known it could also be inferred from empirical evidence.

The modelling task we undertake has two distinct parts. The first involves the construction of a bivariate extreme value model of $X = (X_1, X_2)$, while the second part involves a derivation of the univariate distribution of the variable of interest W from the distribution of X . The latter step is performed via Monte-Carlo integration by repeatedly simulating X values from the fitted bivariate distribution, using the function f to obtain a value of W for each realization of X , and tabulating the resulting sample of W values into an empirical estimate of its distribution. Within the multivariate approach the upper tail behaviour of each constituent variable is approximated by a univariate extreme value model. The upper tail behaviour of w is determined by these characteristics and by the dependence structure between these variables.

We illustrate this framework by an application to historical 6-h precipitation accumulations simulated with the CanRCM4 regional climate model. In the current application, the variable of interest $W = X_1 \times X_2$ is 6-hourly precipitation accumulation, which is considered to have inherent bivariate character from two constituent processes: precipitable water X_1 , and precipitation efficiency X_2 . This in this application the functional f is simply the product function. The bivariate extreme value model we apply is presented in the following subsection.

3.2. The bivariate extreme value model

A key consideration in the construction of a bivariate extreme value

model is whether the two variables that are involved are asymptotically independent or asymptotically dependent (Coles et al., 1999; Poon et al., 2003). For asymptotically independent random variables, as one variable tends to its upper limit, the chance of the other variable also being close to its upper limit goes to zero, and hence extreme values of the individual variables almost always arise at separate times. On the other hand, for asymptotically dependent random variables, as one variable tends to its upper limit, the chance of the other variable being close to its upper limit goes to a non-zero limit, and thus extreme values the individual variables can occur simultaneously. These two different types of the asymptotic dependence structure may result in different upper tail characteristics of the resultant variable w . The application of multivariate extreme value models therefore requires careful consideration of whether the component variables are asymptotically independent or dependent.

The multivariate extreme value approach commonly used in the hydrometeorological literature is based on limiting arguments in which all component variables become large at the same rate, known as multivariate regular variation. These approaches often involve the use of an extreme value copula in which it is assumed that the variables are asymptotically dependent. In contrast to those models, Heffernan and Tawn (2004) proposed a semiparametric method based on the modelling of the conditional behaviour of the elements of X . This method provides a more general framework that incorporates both asymptotically dependent and independent cases. The method is most easily described in the case of two variables, but can be extended to higher dimensions. We will use this approach to model the joint distribution of precipitable water (X_1) and precipitation efficiency (X_2).

3.2.1. The conditional approach to multivariate extreme values

Here we describe the conditional approach to multivariate extremes in the case of two variables (X_1, X_2). Fitting the multivariate distribution using the conditional approach to multivariate extreme values involves three steps: (i) fitting the marginal distributions, (ii) marginal transformation to the Gumbel or Laplace scale, and (iii) finally fitting the extremal dependence model.

3.2.1.1. Marginal distribution. Each of the two marginal distributions used in the Heffernan and Tawn (2004) approach is composed of a semi-parametric model consisting of the empirical distribution function for observations below a high threshold u_{X_i} and the generalized Pareto distribution (GPD) for observations above that threshold (see supporting information for more details about the GPD). More precisely, let μ_{X_i} for $i = 1, 2$ be a high threshold for which the distribution of exceedances ($X_i - \mu_{X_i}$) is well-described by the GPD. The marginal distribution F_{X_i} for $i = 1, 2$ is then estimated as:

$$\hat{F}_{X_i}(x) = \begin{cases} \tilde{F}_{X_i}(x) & \text{for } x \leq \mu_{X_i} \\ 1 - \left[1 - \tilde{F}_{X_i}(\mu_{X_i}) \right] \left[1 + \hat{\xi}_i(x - \mu_{X_i}) / \hat{\phi}_i \right]_+^{-1/\hat{\xi}_i} & \text{for } x > \mu_{X_i} \end{cases} \tag{2}$$

where $x = (x_1, x_2)$ is a realization of $X = (X_1, X_2)$, $\tilde{F}_{X_i}(x)$ is the empirical distribution of the variable X_i and $[\cdot]_+ = \max(0, \cdot)$. Here $\hat{\phi}_i > 0$ and $\hat{\xi}_i$ are the estimated GPD scale and shape parameters, respectively. To avoid the declustering step related to the application of the POT method, we make use of the asymptotic duality of the GEV and GPD distributions (see supporting information) to estimate the threshold and the parameters of the GPD distribution using the annual maxima of X_i . Hence we fit the GEV distribution to the annual maxima of X_i using the method of maximum likelihood and then, for simplicity, take the corresponding threshold for the GPD, μ_{X_i} , to be the estimated location parameter of the GEV distribution. This choice allows us to use the estimated shape and scale parameters of the fitted GEV distribution as estimates of the shape and scale parameters of the GPD distribution. This enables automated

fitting of the GPD and avoids declustering steps, which are necessary simplifications when repeatedly applying the method at a large number of locations as in the case of the gridded output of a climate model over a large region.

3.2.1.2. Marginal transformations. The next step is to transform the original variable $X = (X_1, X_2)$ to the variable $Y = (Y_1, Y_2)$ having standard Laplace margins F_{Y_i} , for $i = 1, 2$, as follows:

$$Y_i = F_Y^{-1} \left[\widehat{F}_{X_i}(X_i) \right] = \begin{cases} \log \left[2 \widehat{F}_{X_i}(X_i) \right] & \widehat{F}_{X_i}(X_i) \leq 0.5 \\ -\log \left[2 \left(1 - \widehat{F}_{X_i}(X_i) \right) \right] & \widehat{F}_{X_i}(X_i) > 0.5 \end{cases} \quad (3)$$

We use Laplace rather than Gumbel margins as originally proposed by [Heffernan and Tawn \(2004\)](#) in order to allow both positive and negative dependencies, as suggested in [Keef et al. \(2013\)](#) and previously used by [Hilal et al. \(2011\)](#). The dependence structure of the transformed variables (Y_1, Y_2) is the same as that of the original variables (X_1, X_2) ; they have the same copula function due to the invariance of copulas under strictly increasing transformations of the margins. The focus then will be on characterizing and modelling the extremal dependence structure of the transformed random variables (Y_1, Y_2) .

3.2.1.3. Extremal dependence model. The fitting of the [Heffernan and Tawn \(2004\)](#) model is completed by specifying a parametric form for the conditional distribution of one variable given a large value of the other variable $Y_j | Y_i = y$ as $y \rightarrow \infty$ for $i, j \in \{1, 2\}$ with $i \neq j$

$$Y_j = \alpha_{ji} y + y^{\beta_{ji}} Z_{ji} \text{ .for } Y_i = y > v_{Y_i} \quad (4)$$

for an appropriate threshold v_{Y_i} , where $\alpha_{ji} \in [-1, 1]$ and $\beta_{ji} \in (-\infty, 1]$ are parameters of the model, and where Z_{ji} is a residual random variable independent of Y_i that converges in distribution to a non-degenerate limiting distribution G_{ji} as $y \rightarrow \infty$. Using a sample (y_{ik}, y_{jk}) for $k = 1, \dots, n_i$ of Y_i and Y_j , and by assuming Z_{ji} to be normally distributed (only for fitting purposes) with mean parameter m_{ji} and a standard deviation σ_{ji} , the parameters of the model are estimated by maximizing the following log likelihood function:

$$LF(\alpha_{ji}, \beta_{ji}, m_{ji}, \sigma_{ji}) = \sum_{k=1}^{n_i} \left\{ \log [\sigma_{ji} y_{ik}^{\beta_{ji}}] + \frac{1}{2} \left[\frac{y_{jk} - \alpha_{ji} y_{ik} - m_{ji} y_{ik}^{\beta_{ji}}}{\sigma_{ji} y_{ik}^{\beta_{ji}}} \right]^2 \right\} \quad (5)$$

The residuals $\widehat{z}_{jk} = \frac{y_{jk} - \widehat{\alpha}_{ji} y_{ik}}{\widehat{y}_{ik}^{\widehat{\beta}_{ji}}}$ for $k = 1, \dots, n_i$ are then used to obtain the empirical cumulative distribution \widehat{G}_{ji} as an estimate of G_{ji} where $\widehat{\alpha}_{ji}$ and $\widehat{\beta}_{ji}$ are the estimated values of α_{ji} and β_{ji} respectively.

It is worth noting that the thresholds v_{Y_i} should not be confused with the thresholds μ_{X_i} ; the former are used for the extremal dependence model while the latter are used for modelling the upper tails of the marginal distributions. The threshold v_{Y_i} is usually selected by inspecting a number of model fit diagnostics to identify the smallest value of threshold v_{Y_i} for which acceptable model fit diagnostics are observed, thus allowing the largest possible sample for model fitting. This is not feasible when such models need to be fitted at a large number of locations (20150 grid boxes in our case). For this reason, and because we are interested in extreme precipitation, we therefore obtain the thresholds v_{Y_i} for $i = 1, 2$ simultaneously using the 90th percentile of annual maximum precipitation series. Specifically, we set v_{Y_1} and v_{Y_2} such that $v_{Y_1} = v_{Y_2}$ and the product $F_{X_1}^{-1}[F_L(v_{Y_1})] \times F_{X_2}^{-1}[F_L(v_{Y_2})]$ is equal to the 90th percentile of the precipitation annual maxima series, where F_L is the marginal standard Laplacian distribution of Y_i for $i = 1, 2$. We use the 90th percentile of annual maximum of precipitation to ensure that the two selected thresholds are high enough to be consistent with extreme precipitation, but also low enough for efficient estimation of the

extremal dependence parameters.

3.3. Simulation of precipitation annual maxima

Simulated precipitation values can be generated from the fitted bivariate model by simulating independent realizations of (X_1, X_2) and calculating products $W = X_1 \times X_2$. In this step we simulate $N = 1000000$ annual maxima of precipitation by randomly resampling from the available annual maxima series $0.9 \times N$ values that are lower than its 0.9 empirical quantile and $0.1 \times N$ values higher than the 0.9 empirical quantile simulated from the joint distribution. To this end we repeatedly simulate random (X_1, X_2) pairs from the bivariate distribution until $0.1 \times N$ values of $X_1 \times X_2$ exceeding the 0.9 empirical quantile of precipitation annual maxima are obtained. Finally, the N simulated annual maxima values were randomly permuted. The 0.9 quantile is chosen here to be consistent with the fixed thresholds used in the extremal dependence model. The 1000000 annual maxima can then be used to empirically estimate high return levels (such as the 1000-year RL). The procedure for simulating (X_1, X_2) from the bivariate distribution follows that of [Hilal et al. \(2011\)](#).

3.4. Uncertainty quantification for 1000-year return levels

To apply the [Heffernan and Tawn \(2004\)](#) model it is necessary to assume that the available sample of (X_1, X_2) observations represent an independent and identically distributed sample of 2-dimensional random vectors. Nevertheless, persistence in meteorological conditions is likely to induce temporal dependence, so that 6-h or daily precipitation events are likely to occur in groups. Declustering is commonly used to obtain a set of observations that are approximately independent. For marginal modelling we used the duality between the GEV and the GPD distributions to avoid the univariate declustering of exceedances when applying the POT method. This does not, however, deal with the impact of clustering on the modelling of extremal dependence. Treating data as though they were independent when they are dependent should mainly affect resampling uncertainties in parameter estimates ([Beirlant et al., 2006](#)). Fortunately, bootstrapping approaches that account for temporal dependence structures can be used to reliably evaluate resampling uncertainties while avoiding declustering ([Davison and Hinkley 1997](#); [Davison et al., 1986](#)).

Furthermore, data pooling can be used as a means of incorporating additional information from nearby locations to improve the estimation of both marginal and extremal dependence models. For a given location we exploit the gridded nature of climate model output to incorporate additional information by pooling data in 3×3 moving windows (pooling groups) of grid boxes. Since the data within a pooling group will generally exhibit spatial correlation, a procedure that accounts for its effects is required to reliably evaluate the related uncertainties. Again, an appropriate bootstrapping approach can be used for this purpose ([Ben Alaya et al., 2018a](#); [Burn 2003](#); [GREHYS 1996](#)).

We use a vector bootstrapping approach ([Burn 2003](#); [GREHYS 1996](#)) to account for spatial and temporal dependence in the pooling group for both the univariate and bivariate approaches. In vector bootstrapping, resampling is done on years. A single vector bootstrap sample is obtained by first drawing a sample of $m = 50$ years with replacement from the available 50-year period, 1951–2000. Then for each selected year all data values in the 3×3 pooling region for that year are included in the bootstrap sample. For example, for daily precipitation each bootstrap sample is of size $3 \times 3(\text{pooling}) \times 365(\text{day}) \times 50(\text{year}) = 164250$. Having drawn a bootstrap sample, we apply both the bivariate approach and the univariate approach described above to obtain a single estimate of the 1000-year precipitation RL. Finally, repeating this procedure several times (1000 times in the current study) allows us to simulate the expected variation of 1000-year RL estimates that would occur in repeated analyses of independent 50-year periods under the same hydro-climatic conditions. This approach ensures that both the spatial correlation

structure and intra-annual temporal dependence structures are preserved in the bootstrap sample. This resampling approach also accounts for sampling variability in threshold level and model parameter estimates.

Note that in the proposed approach we assume that the properties of the PW, PE and precipitation extremes do not vary substantially from one year to another; that is, stationarity is assumed. Nevertheless, it is recognized that climate change will alter climatic extremes, and that the stationarity assumption will be increasingly difficult to justify as the climate continues to warm. The application of the method in this paper considers a period, 1951–2000 in which there is still only relatively weak evidence of non-stationarity in the behaviour of precipitation (e.g., Li et al., 2019; Sun et al., 2020). Also, it should be noted that the proposed approach does not account for inter-annual serial correlation.

4. Results and discussions

The univariate extreme value approach using the GEV fitted to annual maxima and the proposed bivariate approach are applied to 6-h and 24-h precipitation accumulations over North America at each CanRCM4 grid box using a single 50-year CanRCM4 simulation of the period 1951–2000. In case of daily (24-h) precipitation events, daily PW values were obtained by averaging the 6-hourly PW values for each day and daily PE values were obtained as the ratio of 24-h precipitation to these 24-h PW averages. Maps of the estimated parameters for both the univariate and the bivariate approaches are shown in the supplementary figures in the supporting information (see Figures S1 to S5). Fig. 2 shows estimated 6-h and 24-h precipitation RLs for a given return period from the single CanRCM4 run using the two approaches for two grid boxes GB1 and GB2 that are located at the positions shown in Fig. 1. These locations were chosen because they exhibit two markedly different aspects of the fluctuation in the upper tail beyond annual maxima for 24 h durations, as will be seen later.

Empirical estimates using 1750 annual maxima of 6-h and 24-h precipitation from the ensemble of 35 CanRCM4 simulations are used for evaluation purposes. 80% confidence intervals obtained with the resampling approach described in Section 3.2.3 are also given in each plot for both the univariate and the bivariate approaches. As can be seen, the univariate approach leads to overestimation of high RLs at GB1 and underestimation at GB2 for both 6-h and 24-h durations, while the bivariate approach provides a more satisfactory representation of the empirical estimates for both locations and durations. Plots of estimated

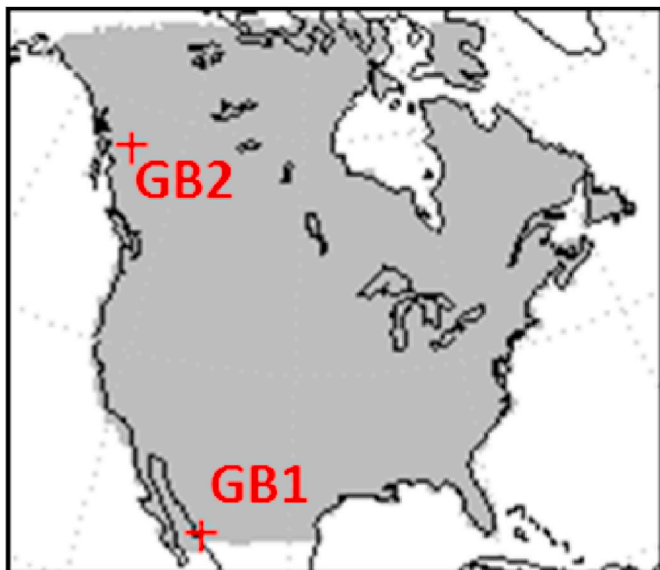


Fig. 1. Geographical positions of GB1 and GB2 in North America. 33.

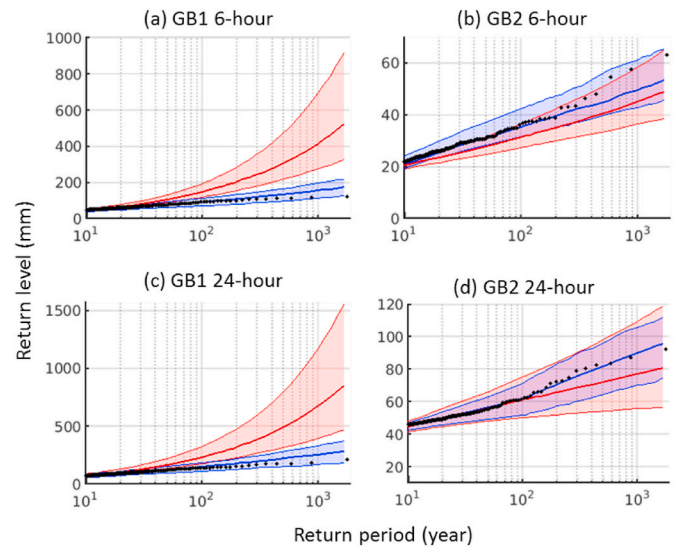


Fig. 2. Estimated 6-h and 24-h precipitation return levels using the univariate approach (red curve) and the bivariate approach (blue curve) for GB1 and GB2. Black dots show empirical estimates obtained using the 1750 annual maxima. The 80% confidence intervals obtained by calculating the 10th and the 90th percentiles obtained by the vector bootstrapping resampling approach are shaded. 34. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

6-h precipitation RLs as a function of return period (similar to plots in Fig. 2a) at 9 additional grid boxes P1, P2 ..., P9 located at the positions shown in the supplementary Figure S6 are shown in the supporting information (see Figure S7). As we can see, the bivariate approach seems to outperform the univariate approach in representing the empirical estimates of 6-h precipitation RLs at most of these locations.

As shown in Ben Alaya et al. (2020a), the reliability of extrapolation to very long return periods using univariate extreme value analysis depends strongly on whether the sample block maxima exhibit the max-stability property that characterizes the GEV distribution. To assess max-stability, the 1750 annual maxima at locations GB1 and GB2 are used to explore how the GEV shape parameter estimate varies when the GEV distribution is fitted to maxima of increasing block lengths. Fig. 3 shows the estimated shape parameters against block length using the univariate and the bivariate approaches at GB1 and GB2 for both 6-h and 24-h durations. Estimated shape parameters using the 1750 annual maxima grouped into blocks of different lengths are also shown in Fig. 3 for evaluation purposes. As we can see, for 6-h duration, the shape parameter decreases as the length of the block increases for both GB1 and GB2. For 24-h duration the shape decreases with increasing block length at GB1, but increases at GB2.

The univariate GEV distribution, which is inherently max-stable, is often not able to well-estimate very long-period RLs in the climate simulated by CanRCM4 because annual maxima appear not to be max-stable at many locations (Ben Alaya et al., 2020a). The additional parametrisation through the bivariate approach may help to accommodate more flexible tail shapes when the max-stability property is not supported by the data. As we can see (Fig. 3), the bivariate approach helps to better depict the fluctuation of the upper tail of the parent distribution beyond annual maxima for both 6-h and 24-h durations at GB1 and GB2, as is also the case at most of the 9 additional locations for 6-h precipitation shown in the supporting information (Figure S8).

In order to better understanding how the bivariate approach leads to a more flexible representation of extreme precipitation, we take a closer look at the fitted extremal conditional models. Fig. 4 shows the fitted conditional models of PE given that PW is extreme at GB1 and GB2 for 6-h precipitation. PE increases with PW at both locations suggesting a positive dependence structure between PE and PW when PW is extreme.

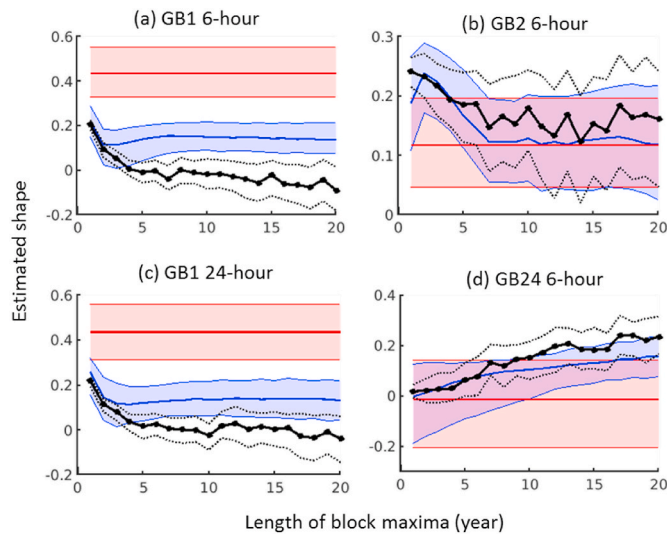


Fig. 3. Estimated 6-h and 24-h precipitation shape parameters against block length using the univariate approach (red line) and the bivariate approach (blue curve) for locations GB1 and GB2 based on 50 years of data. Straight horizontal lines indicate the shape parameter estimated via the univariate approach using the 50 annual precipitation maxima. For the compound approach the pattern of the shape parameter is obtained based on a sample of 10000 annual maxima values generated from the fitted bivariate model using 50 years of data. The GEV distribution is fitted via maximum likelihood to the 10000 values and using longer block lengths ranging from 2 (5000 values) to 20 years (500 values). The shape parameter estimates are retained for each block length. Uncertainties are indicated by 80% confidence intervals (10th and 90th percentiles) obtained via vector bootstrapping as described in the text. The black curve shows the variation of the estimated shape parameters with block length that is obtained via the univariate approach when using all 1750 annual maxima from the 35-member CanRCM4 ensemble. The 80% confidence intervals for the latter are indicated by the 10th and the 90th percentiles of shape parameter estimates from 1000 bootstrap samples drawn from the 1750 annual maxima. 35. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

These relationships are linear on the Laplacian scale (Fig. 4a and b), but deviate substantially from linearity after transformation back to their original scales, reflecting a strengthening influence of PW on the upper tails of PE distributions for larger values of PW (Fig. 4c and d). As a consequence, extreme precipitation increases nonlinearly with PW given that PW is extreme at both GB1 and GB2 for 6-h precipitation. From Fig. 5 we can see that PW increases with PE given that PE is extreme at GB1 (see Fig. 5a and c), but decreases at GB2 (see Fig. 5b and d). We note that, based on GEV distributions fitted to annual maxima (based on 50 years data), PE seems to be bounded at GB1 (negative shape parameter) but heavy tailed (positive shape parameter) at GB2. Given that PE is not bounded in the right-hand upper tail at GB2, the quantiles of precipitation should increase substantially when PE attains large values. Nevertheless, the fact that PW decreases sharply with PE given that PE is extreme (Fig. 5d) offsets the increase of precipitation with increasing extreme PE. As a result, extreme precipitation also begins to decrease when PE increases (see Fig. 5f). The contrast between the two locations can perhaps be understood in terms of their climates and factors that influence extreme precipitation. Annual extreme precipitation events at GB2 occur most frequently in fall, and thus are likely to be associated with atmospheric rivers, which are well simulated in the global model that drives CanRCM4 (Tan et al., 2020). Atmospheric rivers are characterized by large moisture advection and thus high PE, but tend to occur during fall and winter at this location, when atmospheric temperatures are lower, presumably reducing PW. In contrast, annual precipitation extremes at GB1 occur either during the cool season when atmospheric rivers occur, or in mid-to late summer when

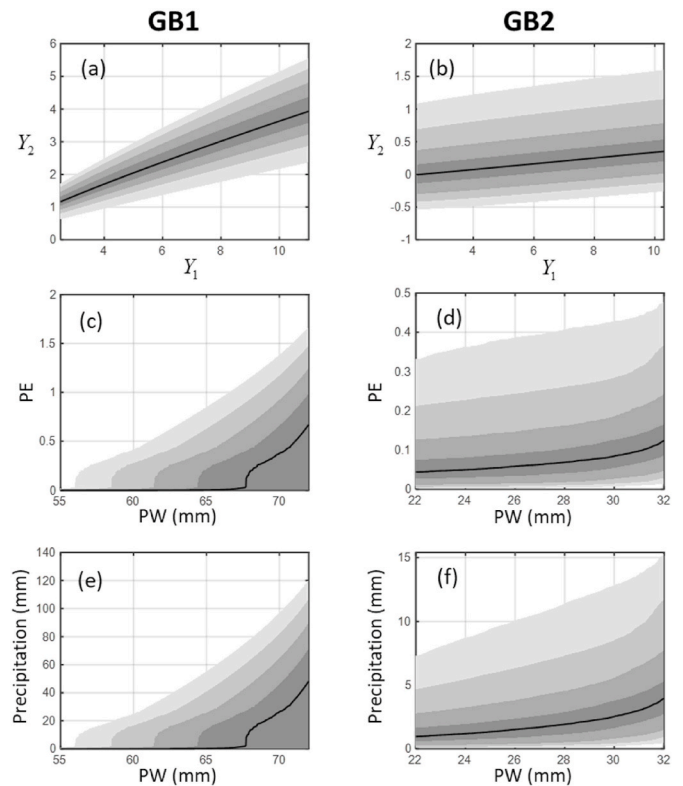


Fig. 4. The fitted conditional models of PE given that PW is extreme at GB1 and GB2 for 6-hourly precipitation on both Laplacian ((a) and (b)) and original ((c) and (d)) scales. The resultant conditional models of extreme precipitation given that PW is extreme are shown in (e) and (f) at GB1 and GB2 respectively. Shading shows conditional percentiles in 10% increments while the black line shows conditional median. 36.

convective activity associated with the North American monsoon dominates, leading to events with higher PW when PE is extreme.

Both examples illustrate that by accounting for the dependence between the two precipitation components, we can more flexibly represent the far upper tail of the extreme precipitation distribution, as seen in Fig. 3. Nevertheless, we should worry about max-stability of the upper tail for each marginal component since this is a presumed property of the upper tail for each component. We note, however, that while it is necessary to be cautious when fitting marginal distributions with stable upper tails to data whose stability properties are unknown, it is nevertheless often reasonable to use univariate asymptotic theory for light extrapolation (such as from annual maxima to 100-year RL in the case of extreme precipitation simulated by CanRCM4; Ben Alaya et al. (2020a)). Confidence in extrapolation of precipitation extremes to very long RLs using our conditional extremes model will be affected by the strength of dependence between the constituent extremes model. When dependence is not very strong, extreme precipitation events would tend to occur for combinations of PE and PW that individually are substantially less rare than the rarity of the P events that are produced. Greater caution is required, however, when PE and PW are strongly asymptotically dependent. For example, in the limit case of comonotonicity (complete asymptotic dependence) the values of the component variables will be about as extreme as P itself, heightening concern about tail stability of the constituent variables when estimating very long period RLs. Obviously in this case no information can be gained about precipitation extreme from the bivariate comonotonic components, and thus no improvement is expected by using the bivariate approach.

Fig. 6 shows maps of relative biases (RB) and relative root mean square errors (RRMSE) of the 1000-year RL estimates for 6-h precipitation accumulation over North America for both the bivariate and the

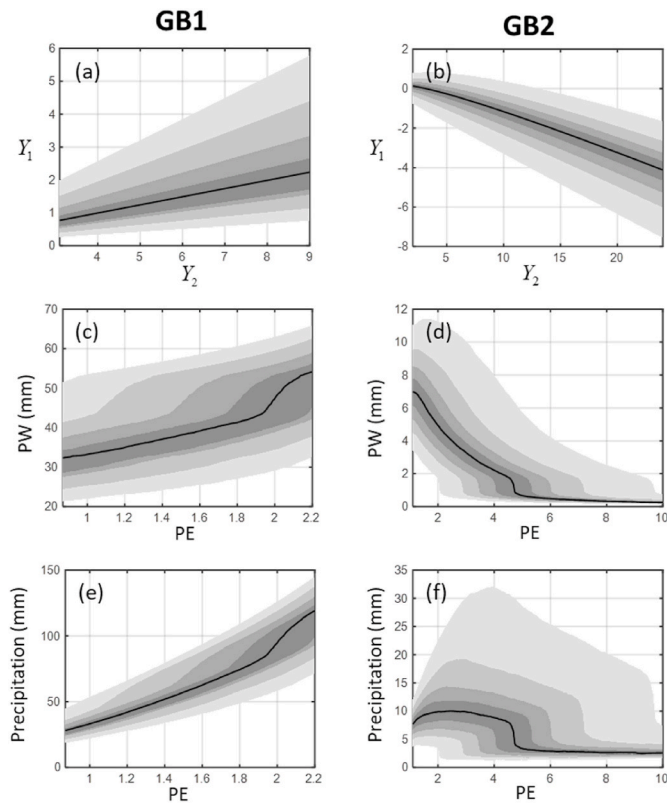


Fig. 5. As Figure 4, except for models of PW conditional on PE being extreme. 37.

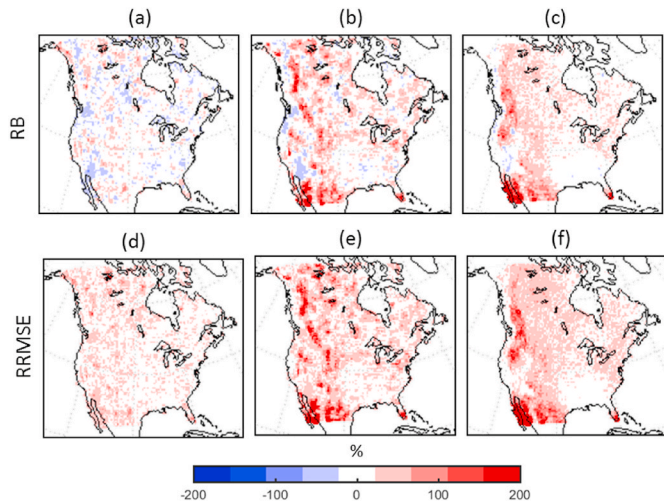


Fig. 6. Maps of the relative bias (RB) and the relative root mean square errors (RRMSE) of 1000-year RL estimates of 6-h precipitations, over North America, using the bivariate approach in (a) and (d), the univariate approach in (b) and (e), and the univariate approach using the 1750 annual maxima in (c) and (f). 38.

univariate approaches, respectively. RB and RRMSE for 24-h durations are shown in Fig. 7. Empirical 1000-year RL estimates obtained using the large ensemble (the 1750 annual maxima) are used as reference when calculating relative biases. In addition, the RB and RRMSE of the univariate approach using the full large ensemble (meaning a GEV distribution fitted to the 1750 annual maxima) are shown in Figs. 6c and 7c for 6-h and 24-h durations respectively. As we can see, the proposed bivariate approach outperforms the univariate approach, and avoids

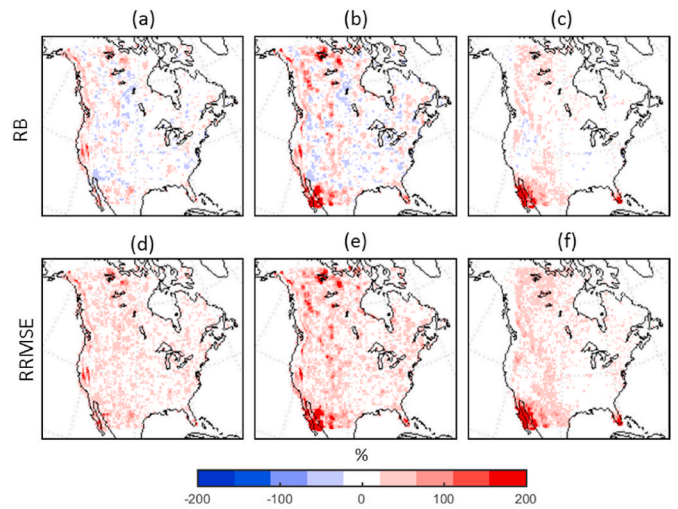


Fig. 7. Similar to Figure 6 but for 24-h precipitation accumulations 39.

substantial overestimation in some locations over North America for both 6-h and 24-h durations. Biases using the univariate approach applied to 50 years of data are mainly affected by the reliability of the GEV parameter estimates and the suitability of the GEV distribution, which reflects the validity of the max-stability assumption. As we can see

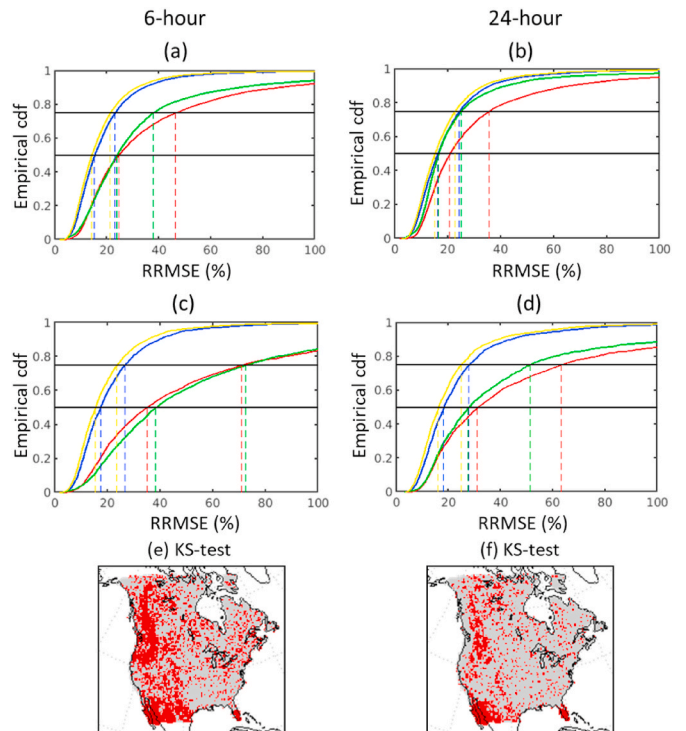


Fig. 8. Empirical CDF of the RRMSE for 6-h and 24-h 1000-year RL precipitation for the bivariate approach using 50 years of data (blue) and using 250 years of data from 5 CanRCM4 runs (yellow), the univariate approach using 50 years of data (red) and 1750 annual maxima (green). Empirical CDFs of RRMSE using all grid boxes over North America are shown for 6-h and 24-h precipitation in (a) and (b) respectively. (c) and (d) show corresponding plots using grid boxes where the null hypothesis that the sample of 1750 annual precipitation maxima is GEV distributed is rejected at 5% significance level by the Kolmogorov-Smirnov test. Red dots in (e) and (f) show grid boxes where rejection occurs for 6-h and 24-h precipitation respectively. 40. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

from Fig. 8a, the bivariate approach using just 50 years of data appears to substantially outperform the univariate approach for 6-h precipitation extremes even when the univariate approach is applied using a sample of 1750 annual maxima. For 24-h precipitation extremes, performance of the bivariate approach based on 50 years of data is also better than the univariate approach when based on 50 years of data, and comparable to that of the univariate approach based on 1750 years of data. The application of the bivariate approach using an ensemble of 5 CanRCM4 runs (250 years of data) improves RL estimates, but the improvement is small compared to that when using bivariate rather than univariate approach. As it can be seen from Fig. 8c and d, for both 6-h and 24-h precipitations, the improvement using the bivariate approach is more striking for grid boxes where a Kolmogorov-Smirnov goodness-of-fit indicates that the GEV distribution does not describe the distribution of the full sample of 1750 annual maxima well. For those grid boxes, the univariate approach exhibits relatively high RRMSE values for both durations; rejection of the GEV distribution occurs at 35% of grid boxes (see Fig. 8e) for 6-h precipitation and at a lower frequency of approximately 20% for 24-h precipitation occurs (see Fig. 8f). The higher rejection rate for 6-h precipitation reflects the higher complexity of sub-daily extreme precipitation compared to daily (Ben Alaya et al., 2020a; Zhang et al., 2017).

The Heffernan and Tawn (2004) semi-parametric extreme value model was able to effectively reproduce the fluctuation seen in the upper tail of the univariate precipitation distribution by modelling the joint behaviour of the extremes of precipitation efficiency and precipitable water as an intermediate step. The method is general and can be used for modelling and understanding environmental risks caused by the combination of multiple drivers. Many hazardous events such as tropical cyclones, floods, fires and droughts are due to unusual combinations of climate and weather variables such as rainfall, temperature and circulation interacting across a wide range of spatial and temporal scales. Combinations of such drivers that lead to societal or environmental impacts are often referred to as compound events (Leonard et al., 2014; Zscheischler et al. 2018, 2020). Understanding and modelling compound events is a rapidly developing area. From a statistical perspective, copula functions have been widely used to describe the dependence structure of climate variables and compound events (AghaKouchak 2014; Ben Alaya et al. 2014, 2016; Bevacqua et al., 2017; Chebana and Ouarda 2007; Hao et al., 2018; Manning et al., 2018; Mofstakhari et al., 2017; Zscheischler and Seneviratne 2017). Extreme value copula functions, such as the Gumbel copula, are now commonly used for modelling the dependence between rare compound events (Salvadori and De Michele 2010; Wahl et al., 2015). Those functions, however, are only suitable for asymptotically dependent divers. This fact is often ignored and only occasionally discussed in the compound events literature. Given its flexibility to incorporate both asymptotic dependence and asymptotic independence, it is expected that the Heffernan and Tawn (2004) approach will play a valuable role to increase our understanding of compound events.

The proposed bivariate approach is applied to data derived from CanRCM4, which provides a useful environment for the comparison and evaluation of extreme value analysis approaches thanks to the availability of a large ensemble of simulation. The final objective, however, is real-world application using observational data records. This requires PW data records to be available alongside with precipitation data. One option in case of missing PW may be to cautiously use PW from reanalysis products, noting that all reanalysis variables are not equally reliable. Kalnay et al. (1996) classify PW as a “type B” variable that is directly affected by observational data, but is also strongly influenced by the forecast model used to perform the reanalysis. More recent reanalyses, which benefit from improved models and data assimilation schemes, and the availability of improved remote sensing of atmospheric water vapor content, likely produce substantially higher quality analyzed PW values, particularly for more recent periods. Note that in contrast with PW, Kalnay et al. (1996) classified precipitation from the

NCEP/NCAR reanalysis, as a “type C” variable that is strongly model dependent. Thus confidence in PW from reanalysis is generally higher than in precipitation.

5. Conclusions

The reliable estimation of extreme quantiles deep in the upper tail from a GEV distribution that has been fitted to a sample of block maxima relies crucially on the max-stability property of GEV distributed random variables. The underlying process generating precipitation extremes, may however, be very complex, implying that max-stability might only be attained (if a stable law exists) once blocks are large enough to consistently sample extremes from the physical process responsible for the largest events (Ben Alaya et al., 2020a). Large multi-year blocks are, however, infeasible with short historical records. Hence this study has proposed an alternative bivariate approach that uses additional observable information available in the historical record concerning the constituent variables that produce precipitation extremes. By decomposing precipitation as the product of precipitable water and precipitation efficiency the bivariate model is able to capture fluctuations in extreme precipitation that result from physical relationships between the component variables. Bias in estimating 1000-year RLs is, consequently, considerably reduced, even when using a modestly short 50-year sample.

The proposed framework not only makes possible the use of additional information from our knowledge about the physical process that produces extremes, but also takes full advantage of a multivariate extreme value theory to carefully extract information from available data records of the different components involved, and thus allows a better scientific and empirical scrutiny.

CRedit authorship contribution statement

Mohamed Ali Ben Alaya: Conceptualization, Methodology, Formal analysis, Writing - original draft, Writing - review & editing. **Francis W. Zwiers:** Methodology, Formal analysis, Writing - review & editing. **Xuebin Zhang:** Formal analysis, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.wace.2020.100290>.

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