SOME q-GENERATING FUNCTIONS ASSOCIATED WITH BASIC MULTIPLE HYPERGEOMETRIC SERIES

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The authors derive the basic (or q–) extensions of certain generalized hypergeometric generating functions of H.M. Srivastava. Some remarkable consequences of these q–generating functions are also considered.

1. INTRODUCTION AND DEFINITIONS

Motivated by some interesting generating functions for the classical Jacobi polynomials, Srivastava [11] established the following hypergeometric generating function:

$$(1.1)\qquad \sum_{n=0}^{\infty} \frac{(\lambda)_n(\mu)_n}{(\rho)_n}\ {}_{A+1}F_B\left[\begin{matrix}-n,\ (a);\\ (b);\end{matrix}\ x\right]\frac{t^n}{n!}$$

$$= (1-t)^{-\lambda}\ F^{1:1;A+1}_{1:0;B}\left[\begin{matrix}\lambda:\ \rho-\mu;\ \mu,\ (a);\\ \rho:\ \text{—};\ (b);\end{matrix}\ \frac{t}{t-1},\ \frac{xt}{t-1}\right]\quad (|t| < 1),$$

where $(\lambda)_n = \Gamma(\lambda+n)/\Gamma(\lambda)$, (a) abbreviates the array of A parameters $a_1,\cdots,a_A$, and $F^{A:B;B'}_{C:D;D'}$ is a generalized Kampé de Fériet function (see, *e.g.*, [7]; see also Srivastava and Karlsson [14, p. 27]). For $\rho = \mu$, (1.1) would reduce immediately to Chaundy's result [2, p. 62, Equation (25)]:

$$(1.2)\qquad \sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!}\ {}_{A+1}F_B\left[\begin{matrix}-n,\ (a);\\ (b);\end{matrix}\ x\right]t^n$$

$$= (1-t)^{-\lambda}\ {}_{A+1}F_B\left[\begin{matrix}\lambda,\ (a);\\ (b);\end{matrix}\ \frac{xt}{t-1}\right]\quad (|t| < 1),$$

which indeed contains, as special cases, several familiar generating functions for Jacobi and Laguerre polynomials.

Srivastava [12] gave a further generalization of (1.1) in the following form:

$$(1.3)\qquad \sum_{n=0}^{\infty} \frac{(\lambda)_n(\mu)_n}{(\rho)_n} F^{C:A+1;A'+2}_{D:\ B;B'+1}\left[\begin{matrix}(c):\ -n,\ (a);\ \lambda+n,\ \mu+n,\ (a');\\ (d):\qquad (b);\qquad \rho+n,\ (b');\end{matrix}\ x,\ y\right]\frac{t^n}{n!}$$

$$= (1-t)^{-\lambda}\ F^{(3)}\left[\begin{matrix}\lambda::\ -;\ \mu,\ (c);\ -:\ \rho-\mu;\ (a);\ (a');\\ \rho::\ -;\qquad (d);\ -:\ -;\ (b);\ (b');\end{matrix}\ \frac{t}{t-1},\ \frac{xt}{t-1},\ \frac{y}{1-t}\right] (|t| < 1),$$

where $F^{(3)}[x,y,z]$ denotes a generalized triple hypergeometric series (introduced by Srivastava [10, p. 428]). The main object of this paper is to establish the basic (or q-) extensions of Srivastava's results involving hypergeometric generating functions. We also consider several remarkable consequences of these hypergeometric q-generating functions.

For real or complex q ($|q| < 1$), put

$$(1.4)\qquad (\lambda;q)_\infty = \prod_{j=0}^{\infty} (1-\lambda q^j)$$

and let $(\lambda;q)_\mu$ be defined by

$$(1.5)\qquad (\lambda;q)_\mu = \frac{(\lambda;q)_\infty}{(\lambda q^\mu;q)_\infty}$$

for arbitrary parameters λ and μ, so that

$$(1.6)\qquad (\lambda;q)_n = \begin{cases} 1, \text{ if } n = 0, \\ (1-\lambda)(1-\lambda q)\cdots(1-\lambda q^{n-1}),\ \forall n \in \mathbb{N} = \{1,2,3,\ldots\}. \end{cases}$$

Then a generalized basic (or q-) hypergeometric function is defined by (*cf.*, *e.g.*, Slater [9, Chapter 3] and Exton [4]; see also Srivastava and Karlsson [14, p. 347])

$$(1.7) \qquad {}_A\Phi_B\left[\begin{matrix}(a); & q\,;\,z\\ (b); & i\end{matrix}\right] = \sum_{n=0}^{\infty} q^{in(n-1)/2}\,\frac{\prod_{j=1}^{A}(a_j;q)_n}{\prod_{j=1}^{B}(b_j;q)_n}\,\frac{z^n}{(q;q)_n},$$

where, for convergence,

$$|q| < 1 \quad \text{and} \quad |z| < \infty \quad \text{when} \quad i \in \mathbb{N},$$

or

$$\max\{|q|,\ |z|\} < 1 \quad \text{when} \quad i = 0,$$

provided that no zeros appear in the denominator.

For $i,j,k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, a generalized basic (or q–) hypergeometric function of two variables is defined by (*cf.* Srivastava and Karlsson [14, p. 349])

$$(1.8) \qquad \Phi^{A:B;B'}_{C:D;D'}\left[\begin{matrix}(a):\ (b);\ (b');\ q\,;x,y\\ (c):\ (d);\ (d');\ i,j,k\end{matrix}\right]$$

$$= \sum_{m,n=0}^{\infty} q^{kmn+\{im(m-1)+jn(n-1)\}/2}$$

$$\cdot\ \frac{\prod_{\tau=1}^{A}(a_\tau;q)_{m+n}\ \prod_{\tau=1}^{B}(b_\tau;q)_m\ \prod_{\tau=1}^{B'}(b'_\tau;q)_n}{\prod_{\tau=1}^{C}(c_\tau;q)_{m+n}\ \prod_{\tau=1}^{D}(d_\tau;q)_m\ \prod_{\tau=1}^{D'}(d'_\tau;q)_n}\ \frac{x^m}{(q;q)_m}\ \frac{y^n}{(q;q)_n},$$

provided that the series converges or terminates.

Finally, a basic (or q–) extension of the triple hypergeometric series $F^{(3)}[x,y,z]$ of Srivastava [10, p. 428] is defined by (*cf.* Denis [3])

(1.9) $$\Phi^{A::B;B';B'':C;C';C''}_{E::G;G';G'':H;H';H''}\left[\begin{matrix}(a)::(b);(b');(b''):(c);(c');(c''); & q;x,y,z \\ (e)::(g);(g');(g''):(h);(h');(h''); & i,j,j,u,v,w\end{matrix}\right]$$

$$= \sum_{m,n,p=0}^{\infty} q^{umn+vnp+wpm+\{im(m-1)+jn(n-1)+kp(p-1)\}/2}$$

$$\cdot \frac{\prod_{\tau=1}^{A}(a_\tau;q)_{m+n+p} \prod_{\tau=1}^{B}(b_\tau;q)_{m+n} \prod_{\tau=1}^{B'}(b'_\tau;q)_{n+p} \prod_{\tau=1}^{B''}(b''_\tau;q)_{p+m}}{\prod_{\tau=1}^{E}(e_\tau;q)_{m+n+p} \prod_{\tau=1}^{G}(g_\tau;q)_{m+n} \prod_{\tau=1}^{G'}(g'_\tau;q)_{n+p} \prod_{\tau=1}^{G''}(g''_\tau;q)_{p+m}}$$

$$\cdot \frac{\prod_{\tau=1}^{C}(c_\tau;q)_m \prod_{\tau=1}^{C'}(c'_\tau;q)_n \prod_{\tau=1}^{C''}(c''_\tau;q)_p}{\prod_{\tau=1}^{H}(h_\tau;q)_m \prod_{\tau=1}^{H'}(h'_\tau;q)_n \prod_{\tau=1}^{H''}(h''_\tau;q)_p} \frac{x^m}{(q;q)_m} \frac{y^n}{(q;q)_n} \frac{z^p}{(q;q)_p},$$

provided that the series converges or terminates.

In the special case when $i = j = k = 0$, the first member of (1.8) will be written simply as

$$\Phi^{A:B;B'}_{C:D;D'}\left[\begin{matrix}(a):(b);(b'); \\ (c):(d);(d');\end{matrix} \; q;x,y\right],$$

and a similar notational simplification will also be made for writing the first member of (1.7) when $i = 0$.

It should be remarked in passing that, in the definition (1.8), the double series converges absolutely for all bounded values of the complex arguments x and y when $i,j,k \in \mathbb{N}$ and $|q| < 1$, and also when $i = j = k = 0$, provided further that

$$\max\{|q|,|x|,|y|\} < 1.$$

The conditions of convergence of the triple series in (1.9) can be stated in an analogous manner.

The following results will be required in our analysis (see, *e.g.*, Slater [9, pp. 247–248]).

I. The q–binomial theorem:

$$(1.10)\qquad {}_1\Phi_0\left[\begin{matrix}\lambda; \\ -;\end{matrix}\; q\,;z\right] = \frac{(\lambda z\,;q)_\infty}{(z\,;q)_\infty} \quad (\max\{|q|,|z|\} < 1).$$

II. The q–Gauss theorem:

$$(1.11)\qquad {}_2\Phi_1\left[\begin{matrix}\alpha,\ \beta; \\ \gamma;\end{matrix}\; q;\ \frac{\gamma}{\alpha\beta}\right] = \frac{(\gamma/\alpha;q)_\infty(\gamma/\beta;q)_\infty}{(\gamma\,;q)_\infty(\gamma/\alpha\beta;q)_\infty},$$

which, for $\beta = q^{-n}$ $(n \in \mathbb{N}_0)$, yields the terminating version:

$$(1.12)\qquad {}_2\Phi_1\left[\begin{matrix}\alpha,\ q^{-n}; \\ \gamma;\end{matrix}\; q;\ \frac{\gamma}{\alpha}\,q^n\right] = \frac{(\gamma/\alpha;q)_n}{(\gamma;q)_n} \qquad (n \in \mathbb{N}_0).$$

It should be mentioned here that, upon reversal of the order of its terms, this last result (1.12) would lead us to

III. The q–Chu–Vandermonde theorem:

$$(1.13)\qquad {}_2\Phi_1\left[\begin{matrix}\alpha,\ q^{-n}; \\ \gamma;\end{matrix}\; q\,;q\right] = \frac{(\gamma/\alpha;q)_n}{(\gamma;q)_n}\,\alpha^n \qquad (n \in \mathbb{N}_0),$$

which incidentally is a widely useful result.

2. HYPERGEOMETRIC q-GENERATING FUNCTIONS

Such hypergeometric generating functions as (1.1), (1.2), and (1.3), and their various generalizations and q-extensions, were considered by Srivastava [13]. (See also a recent book on the subject of generating functions by Srivastava and Manocha [15].) In this section we give a direct proof of the following q-extension of Srivastava's generating function (1.3):

$$(2.1)\quad \sum_{n=0}^{\infty} \frac{(\lambda;q)_n(\mu;q)_n}{(\rho;q)_n}\, \Phi^{C:A+1;A'+2}_{D:\quad B;B'+1}\left[\begin{matrix}(c):\ q^{-n},(a);\ \lambda q^n,\mu q^n,(a'); \\ (d):\quad (b);\quad \rho q^n,(b');\end{matrix}\ q;xq^n,y\right] \frac{t^n}{(q;q)_n}$$

$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty}\, \Phi^{1::0;C+1;0:1;A;A'}_{2::0;\quad D;0:0;B;B'}\left[\begin{matrix}\lambda::\text{—};\ \mu,\ (c);\ \text{—}: \\ \rho,\ \lambda t::\text{—};\quad (d);\ \text{—}:\end{matrix}\right.$$

$$\left.\begin{matrix}\rho/\mu;\ (a);\ (a');\ q;\ -\mu t,\ -xt,\ y \\ \text{—};\ (b);\ (b');\ 1,1,0,1,0,1\end{matrix}\right] \quad (\max\{|q|,\ |t|\} < 1).$$

Proof. Denote, for convenience, the right-hand side of the q-generating function (2.1) by $\Omega(x,y;t)$. Then, by appealing to the definition (1.9), we have

$$\Omega(x,y;t) = \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \sum_{\ell,m,p=0}^{\infty} \frac{(\lambda;q)_{\ell+m+p}(\mu;q)_{m+p}(\rho/\mu;q)_\ell}{(\rho;q)_{\ell+m+p}(\lambda t;q)_{\ell+m+p}}$$

$$\cdot\ \Delta(m,p)q^{\ell(m+p)+\{\ell(\ell-1)+m(m-1)\}/2} \frac{(-\mu t)^\ell}{(q;q)_\ell} \frac{(-xt)^m}{(q;q)_m} \frac{y^p}{(q;q)_p}$$

or, equivalently,

$$(2.2)\qquad \Omega(x,y;t) = \sum_{\ell,m,p=0}^{\infty} \frac{(\mu;q)_{m+p}(\rho/\mu;q)_{\ell}}{(\rho;q)_{\ell+m+p}} \Delta(m,p) q^{\ell(m+p)+\{\ell(\ell-1)+m(m-1)\}/2}$$

$$\cdot \frac{(-\mu)^{\ell}}{(q;q)_{\ell}} \frac{(-x)^m}{(q;q)_m} \frac{y^p}{(q;q)_p} \sum_{n=0}^{\infty} \frac{(\lambda;q)_{\ell+m+n+p}}{(q;q)_n}$$

$t^{\ell+m+n}$

$$(\max\{|q|,\ |t|\} < 1),$$

where we have also used the q-binomial theorem (1.10), and the definitions (1.5) and (1.7), $\Delta(m,p)$ being given by

$$(2.3)\qquad \Delta(m,p) = \frac{\prod_{\tau=1}^{C}(c_\tau;q)_{m+p} \prod_{\tau=1}^{A}(a_\tau;q)_m \prod_{\tau=1}^{A'}(a'_\tau;q)_p}{\prod_{\tau=1}^{D}(d_\tau;q)_{m+p} \prod_{\tau=1}^{B}(b_\tau;q)_m \prod_{\tau=1}^{B'}(b'_\tau;q)_p}.$$

Upon writing $n-\ell-m$ for n in (2.2), and noting the elementary identity:

$$(2.4)\qquad (q;q)_{n-k} = (-1)^k q^{k(k-2n-1)/2} \frac{(q;q)_n}{(q^{-n};q)_k} \qquad (0 \leqq k \leqq n),$$

we find from (2.2) that

$$(2.5)\qquad \Omega(x,y;t) = \sum_{n,p=0}^{\infty} \sum_{\ell,\,m=0}^{\ell+m\leqq n} \frac{(\lambda;q)_{n+p}(\mu;q)_{m+p}(\rho/\mu;q)_{\ell}(q^{-n};q)_{\ell+m}}{(\rho;q)_{\ell+m+p}}$$

$$\cdot\ \Delta(m,p) q^{mn+\ell(n+p)} \frac{\mu^{\ell}}{(q;q)_{\ell}} \frac{x^m}{(q;q)_m} \frac{t^n}{(q;q)_n} \frac{y^p}{(q;q)_p},$$

which readily yields

$$(2.6)\qquad \Omega(x,y;t) = \sum_{n,p=0}^{\infty} \sum_{m=0}^{n} \frac{(\lambda;q)_{n+p}(\mu;q)_{m+p}(q^{-n};q)_m}{(\rho;q)_{m+p}} \Delta(m,p)$$

$$\cdot\ {}_2\Phi_1\left[\begin{matrix}\rho/\mu,\ q^{-n+m}; \\ \rho q^{m+p};\end{matrix}\ q;\ \mu q^{n+p}\right] \frac{(xq^n)^m}{(q;q)_m} \frac{t^n}{(q;q)_n} \frac{y^p}{(q;q)_p}.$$

Finally, we sum the hypergeometric ${}_2\Phi_1$ series in (2.6) by means of the known result (1.12) with, of course,

$$\alpha = \frac{\rho}{\mu},\quad \gamma = \rho q^{m+p},$$

and n replaced by $n-m$, and we thus obtain the left–hand side of (2.1). The derivation of the q–generating function (2.1) is evidently completed.

3. APPLICATIONS

For $c_j = 0$ $(j = 1,\dots,C)$ and $d_j = 0$ $(j = 1,\dots,D)$, (2.1) reduces immediately to the bilinear generating relation:

$$(3.1)\quad \sum_{n=0}^{\infty} \frac{(\lambda;q)_n(\mu;q)_n}{(\rho;q)_n} {}_{A+1}\Phi_B\left[\begin{matrix}q^{-n},(a); \\ (b);\end{matrix}\ q;xq^n\right] {}_{A'+2}\Phi_{B'+1}\left[\begin{matrix}\lambda q^n,\mu q^n,(a'); \\ \rho q^n,(b');\end{matrix}\ q;y\right]\frac{t^n}{(q;q)_n}$$

$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \Phi^{1::0;1;0:1;A;A'}_{2::0;0;0:0;B;B'}\left[\begin{matrix}\lambda::\ —;\ \mu;\ —: \\ \rho,\ \lambda t::\ —;\ -;\ —:\end{matrix}\right.$$

$$\left.\begin{matrix}\rho/\mu;\ (a);\ (a');\ \ q;\ -\mu t,\ -xt,\ y \\ —;\ (b);\ (b');\ 1,\ 1,\ 0,\ 1,\ 0,\ 1\end{matrix}\right] (\max\{|q|,|t|\} < 1).$$

Upon setting $\rho = \mu$ in (3.1), if we simplify the right–hand side by applying the definition (1.9), we obtain the formula:

$$(3.2) \qquad \sum_{n=0}^{\infty} \frac{(\lambda;q)_n}{(q;q)_n} \, {}_{A+1}\Phi_B\left[\begin{matrix} q^{-n}, (a); \\ (b); \end{matrix} \; q;\; xq^n\right] {}_{A'+1}\Phi_{B'}\left[\begin{matrix} \lambda q^n, (a'); \\ (b'); \end{matrix} \; q;\; y\right] t^n$$

$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \, \Phi^{1:A;A'}_{1:B;B'}\left[\begin{matrix} \lambda: (a); (a'); \; q; \text{-}xt, \; y \\ \lambda t: (b); (b'); \; 1, 0, 0 \end{matrix}\right] (\max\{|q|,|t|\} < 1),$$

which provides a q-extension of a known bilinear generating function (*cf.*, *e.g.*, [15, p. 229, Equation 4.1(35)]). More generally, a q-extension of another known bilinear generating function [15, p. 231, Equation 4.1(42)] is provided similarly by the following special case of (2.1) when $\rho = \mu$:

$$(3.3) \qquad \sum_{n=0}^{\infty} \frac{(\lambda;q)_n}{(q;q)_n} \, \Phi^{C:A+1;A'+1}_{D:\;\;B;\;\;B'}\left[\begin{matrix} (c): \; q^{-n}, (a); \; \lambda q^n, (a'); \\ (d): \;\; (b); \;\; (b'); \end{matrix} \; q;\; xq^n, \; y\right] t^n$$

$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \, \Phi^{C+1:A;A'}_{D+1:B;B'}\left[\begin{matrix} \lambda, (c): (a); (a'); \; q; \text{-}xt, \; y \\ \lambda t, (d): (b); (b'); \; 1, 0, 0 \end{matrix}\right]$$

$$(\max\{|q|,|t|\} < 1).$$

A further special case of (3.1) when $y = 0$ yields

$$(3.4) \qquad \sum_{n=0}^{\infty} \frac{(\lambda;q)_n(\mu;q)_n}{(\rho;q)_n} \, {}_{A+1}\Phi_B\left[\begin{matrix} q^{-n}, (a); \\ (b); \end{matrix} \; q;\; xq^n\right] \frac{t^n}{(q;q)_n}$$

$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \, \Phi^{1:1;A+1}_{2:0;\;\;B}\left[\begin{matrix} \lambda: \rho/\mu; \; \mu, (a); \; q; \text{-}\mu t, -xt \\ \rho, \lambda t: \text{---}; \;\; (b); \; 1, 1, 1 \end{matrix}\right]$$

$$(\max\{|q|,|t|\} < 1),$$

which is a q-extension of the hypergeometric generating function (1.1).

For $\rho = \mu$, (3.4) assumes the form:

$$(3.5)\qquad \sum_{n=0}^{\infty} \frac{(\lambda;q)_n}{(q;q)_n}\ {}_{A+1}\Phi_B\left[\begin{matrix} q^{-n},\ (a); \\ (b); \end{matrix}\ q;\ xq^n\right] t^n$$

$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty}\ {}_{A+1}\Phi_{B+1}\left[\begin{matrix} \lambda,\ (a);\ q;\ -xt \\ \lambda t,\ (b);\ 1 \end{matrix}\right] (\max\{|q|,|t|\} < 1),$$

which follows also from (3.2) and (3.3) when $y = 0$.

Formula (3.5) provides a q-extension of Chaundy's result (1.2).

Next, setting

$$a_j = 0 \quad (j = 1,\dots,A) \quad \text{and} \quad b_j = 0 \quad (j = 1,\dots,B)$$

in (3.4), and applying the q-binomial theorem (1.10) with $\lambda = q^{-n}$ and $z = xq^n$, we get

$$(3.6)\qquad {}_3\Phi_1\left[\begin{matrix} \lambda,\ \mu,\ x; \\ \rho; \end{matrix}\ q;\ t\right]$$

$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty}\ \Phi^{1:1;1}_{2:0;0}\left[\begin{matrix} \lambda:\ \rho/\mu;\ \mu;\ q;\ -\mu t,\ -xt \\ \rho,\ \lambda t:\ \text{—};\ \text{–};\ 1,\ 1,\ 1 \end{matrix}\right]$$

$$(\max\{|q|,|t|\} < 1),$$

which, for $x = 0$, yields Jackson's transformation (*cf.* [6, p. 145, Equation (4)] and [14, p. 348, Equation 9.4 (279)]):

$$(3.7)\qquad {}_2\Phi_1\left[\begin{matrix} \lambda,\ \mu; \\ \rho; \end{matrix}\ q;\ t\right] = \frac{(\lambda t;q)_\infty}{(t;q)_\infty}\ {}_2\Phi_2\left[\begin{matrix} \lambda,\ \rho/\mu;\ q;\ -\mu t \\ \rho,\ \lambda t;\ 1 \end{matrix}\right]$$

$$(\max\{|q|,|t|\} < 1).$$

Many of the hypergeometric q-generating functions, considered in this paper, can be applied also to the various families of q-orthogonal polynomials including, for example, the little q–Jacobi polynomials defined by (*cf.* [5, p. 29] and [1, p. 48, Equation (3.34)])

$$p_n^{(\alpha,\beta)}(x;q) = \frac{(\alpha q;q)_n}{(q;q)_n}\, {}_2\Phi_1\left[\begin{matrix} q^{-n}, & \alpha\beta q^{n+1}; & \\ & \alpha q; & \end{matrix}\; q;\; qx\right] \tag{3.8}$$

and the q–Laguerre polynomials defined by (*cf.* [5, p. 29], [4, p. 188], and [8, p. 21, Equation (2.3)]

$$L_n^{(\alpha)}(x;q) = \frac{(\alpha q;q)_n}{(q;q)_n}\, {}_1\Phi_1\left[\begin{matrix} q^{-n}; & q; & xq^n \\ \alpha q; & 1 & \end{matrix}\right] \tag{3.9}$$

$$= \lim_{\beta\to 0}\left\{p_n^{(\alpha,\beta)}\left[-\frac{x}{\alpha\beta q^2};\; q\right]\right\}.$$

The details may be omitted.

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