ON OPTIMUM SYSTEM DESIGN FOR WIRELESS COMMUNICATIONS

by

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ABSTRACT

This dissertation addresses the issue of optimum system design to achieve reliable communication in the presence of various types of interference. Multiobjective formulation is used with noncooperative and cooperative approaches owing to the nature of the problems under consideration.

Since intentional jamming is one of the most severe kinds of interference. anti-jam techniques are crucial for communications in a hostile environment. The jam and anti-jam problem is modeled as a two-person zero-sum game in which the communicator and the jammer have antagonistic objectives and are viewed as the two players. The concept of Nash equilibrium is introduced and its characterizations such as existence. uniqueness, stability, robustness, and sensitivity are investigated. This model is then applied to a frequency-hop spread spectrum M-ary frequency-shift-keying system where ratio-threshold diversity is used to combat partial-band noise and multitone jamming. Equilibrium performance in terms of cutoff rate and bit error rate is shown to be superior to that predicted by worst-case analysis.

When mutual interference caused by simultaneous transmissions is the major concern in a heterogeneous packet network, a multiobjective framework is proposed in this dissertation with the objectives and constraints of the individual users taken into consideration. Near-far effect and Rayleigh fading may occasion packet capture and therefore create unfairness in favor of closer users. Thus, multiobjective optimality is introduced, in which criterion of fairness is embedded. Optimum strategies controlling transmission probability and/or power are examined to yield the Pareto optimal solution in a slotted ALOHA network. Then, the same control strategies are studied with the channel utilization being the maximization objective. Optimization results are obtained in various situations. and effectiveness of different strategies is compared.

A multimedia direct-sequence spread spectrum system may support multiple services with different transmission rates and diverse quality-of-service requirements. To facilitate multimedia applications and maximize the system capacity, average power control, error correction coding, and time diversity are incorporated into the system. The capacity of such a system is evaluated in multipath Rayleigh fading channels. Average bit error rate, outage probability, and corresponding information theoretic bounds are discussed. Concatenation of Reed-Solomon codes and convolutional codes is considered for error correction to account for different quality and delay constraints. It is shown through a numerical example that the system capacity can be increased significantly by an appropriate system design.

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List of Abbreviations

ABR	available bit rate
ARQ	automatic repeat request
AWGN	additive white Gaussian noise
BDPSK	binary differential phase shift keying
BER	bit error rate
BFSK	binary frequency shift keying
BPSK	binary phase shift keying
BSC	binary symmetric channel
CBR	constant bit rate
CDMA	code division multiple access
DS	direct sequence
ECC	error correction coding
FDMA	frequency division multiple access
FEC	forward error correction
FFH	fast frequency-hop
FH	frequency-hop
MAC	media access control
MAI	multiple access interference
MDS	maximum distance separable
MFSK	M-ary frequency shift keying
MRC	maximum ratio combining
MSC	M-ary symmetric channel
MT	multitone
PBN	partial band noise
PCS	personal communication services
pdf	probability density function
PDF	probability distribution function

PN	pseudonoise
QOS	quality-of-service
R-T	ratio-threshold
SFH	slow frequency-hop
SNR	signal to noise ratio
SS	spread spectrum
TDMA	time division multiple access
VBR	variable bit rate

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To my parents and Qing.

Chapter 1

Introduction

In wireless communication systems, in addition to background thermal noise, there exist other forms of disturbance to the transmitted signals. The disturbance is referred to as interference and may be characterized as any combination of the following: intentional or unintentional interference from other users; multiple access interference due to spectrum sharing by coordinated and non-coordinated users; and multipath interference or self-jamming by delayed signal. This dissertation is concerned with the issue of optimum system design for wireless communications where suppression of various types of interference is crucial to achieve reliable transmissions. Multiobjective optimization problems arise in these situations where several objectives are to be satisfied [1, 2] and game theoretic approaches are often called for.

Game theory is the mathematical study of conflict and cooperation between intelligent rational decision-makers [3]–[5]. Although it was originated in economics and behavior science, nowadays we can find its spectacular applications in many other branches of social science as well as in engineering areas. Basically, game theory is the study of *equilibrium*, which provides the evaluation of an *optimal* situation where there are a number of individual objectives and independent and/or dependent constraints. In a typical game, there are a number of players with individual strategy sets and payoffs. Depending on whether the players are able to make binding agreements, games are divided into two classes: noncooperative games and cooperative games, and this categorizes the problems under study.

1.1 Noncooperative Approaches

In noncooperative situations, each player acts individually to optimize his objective without concern for the others' objectives. The outcome obtained in this way is the Nash equilibrium [6]. The main application of the noncooperative approach in wireless communications is in anti-jam communications. It is known that intentional jamming may degrade the system performance significantly if no protective measures are taken by the communicator. Thus, various kinds of anti-jam techniques have been studied [7, 8] where an intelligent jammer is often assumed and the worst case performance is considered. In reality, however, the communicator may also adopt intelligent countermeasures in order to obtain a performance better than that predicted by the worst case analysis. Since the communicator and the jammer are in antagonistic positions and can be viewed as two players in a game. Blachman [9] and Dobrushin [10] modeled the jam and anti-jam problem as a two-person zero-sum game. In recent years, there have been many attempts to apply this model to anti-jam communications [11]–[16] where an exact mathematical model is implicitly assumed. In practice, approximation in modeling and perturbation due to noise are inevitable, and characterizations of equilibrium require further investigation, to which Chapter 2 is devoted.

In [12], Chang analyzed the ratio-threshold (R-T) anti-jam technique in a frequency-hop spread spectrum (FH/SS) binary frequency-shift-keying (BFSK) system from the information theoretic point of view. She used game theory approach in the analysis and showed the performance improvement of a ternary output channel generated by the R-T technique over a binary symmetric channel (BSC) resulted from hard decision reception. By using a R-T test, an estimation of the channel condition can be obtained. This estimation can then be used to form a soft decision decoding metric. By applying the game theoretic model studied in Chapter 2, we extend Chang's work to an M-ary FSK scheme in Chapter 3 where the R-T test technique is used in diversity combining with quaternary output. We assume that the jammer knows everything about the system except the pseudonoise (PN) code which controls the carrier frequency-hop sequence. We also assume that the total jamming power of the jammer is finite and fixed. Two types of intelligent jamming are considered, partial-band noise (PBN) and multitone (MT) jamming. PBN jamming concentrates the total jamming power in a fraction of the spread spectrum signal bandwidth and injects jamming power into a receiver in the form of additive Gaussian noise. MT jamming injects the total jamming power into a finite number of tones, which coincide with some of the FSK signal tones used by the communicator. According to the distribution of the jamming tones, MT jamming can be divided into two classes: band multitone jamming and independent multitone jamming. It is shown that the worst case MT jamming usually occurs when there is only one tone in a jammed hopping channel.

1.2 Cooperative Approaches

In cooperative situations where binding agreements are possible, each player tends to choose a cooperative equilibrium strategy, which is applicable to multiple access networks where users are coordinated in a distributed way or are compliant to central control. In fact, the cooperative approach has already been applied to admission control, flow control, routing, and resource sharing in wireline networks [17]–[20]. In wireless networks, the emerging personal communication services (PCS) will support a variety of services, e.g., voice, video, image, and data, with different rates and diverse quality-of-service (QOS) requirements. Therefore, maximizing the spectrum efficiency while meeting the QOS requirements, which is a multiobjective problem in nature, remains a challenge in the design of a PCS system. To address this problem, we present a mathematical framework in Chapter 4 using cooperative approaches.

In multiple access networks, slotted ALOHA is a simple random access technique [21] and is often a component of more complex protocols [22, 23]. In such a network, the overall performance suffers severely from the collision between two or more packets that arrive at a base station simultaneously. Classically, all the packets involved in a collision are assumed to be destroyed, which plagues the system with a low throughput and a high average delay. In a wireless scenario, however, the near-far effect and the channel fading result in significantly different power levels among received packets, and the strongest packet may capture the base station in the presence of a number of colliding packets. This is known as the packet capture [24, 25]. Although the capture effect increases the network throughput considerably [26], it may also create unfairness among remote users in a heterogeneous network where they have different requirements and/or locations. For instance, if the transmission power level is identical for every user in a network, a user closer to the base station may have a higher probability of capturing the base station and therefore receive a better service in terms of higher throughput, lower average delay, and lower packet loss probability. The unfairness is highly undesirable and may even make the whole network drift to an unstable state under a high load. To rectify this problem, certain control strategies have to be employed to ensure that each user has an optimal and fair access to the base station. This is usually done through a network controller at the base station by adjusting each user's transmission power, transmission (or retransmission) probability, error correction capability, or other controllable parameters. Though capture effect in mobile radio channels has been widely studied, few contributions have addressed the optimization problem especially for a heterogeneous network, which is the focus of Chapters 4 and 5 where strategies of controlling transmission probability and power are considered.

Direct sequence (DS) spread spectrum is another technique for multiple access in PCS and has drawn much attention due to its inherent advantages such as statistical multiplexing gain, anti-multipath and anti-jam capability, and privacy [27]–[30]. In Chapter 6, user numbers of different services are used as the optimization objectives, and the effect of error correction in multipath Rayleigh fading is studied in conjunction with average power control and time diversity in the form of RAKE reception.

1.3 Contributions of the Dissertation

The main contributions of this dissertation are as follows:

- Study of the characterizations of equilibrium for a two-person zero-sum game that is used to model an anti-jam communication system.
- Game theoretic study of ratio-threshold diversity for a fast frequency-hopped MFSK system in the presence of partial-band or multitone jamming.
- Construction of a multiobjective framework for cooperative heterogeneous networks with optimality and fairness addressed.
- Optimum strategies by controlling transmission probability and power to maximize the channel utilization in a slotted ALOHA heterogeneous network.
- Evaluation of the system capacity for a multimedia CDMA network in multipath Rayleigh fading channels with multiple services having different rates and quality requirements.

Chapter 2

A Game Theoretic Model for Anti-Jam Communications

2.1 Introduction

In a hostile communication environment, the communicator and the jammer are in antagonistic positions and each tries to use his best strategies to defeat the other's purpose. For instance, the jammer can select different types of jamming, such as partial band jamming, single or multi-tone jamming, follower jamming, repeater jamming, and predictive jamming, and adjust corresponding jamming parameters to achieve the best jamming effect. On the other hand, in order to combat jamming, the communicator may choose from various system considerations, including modulation schemes, channel codes, diversity combining, interleaving, and signal energy. Since jamming may degrade the system performance considerably, it has to be effectively counteracted by the communicator using certain kinds of anti-jam techniques [7]. The jam and anti-jam problem is often formulated as a two-person zero-sum game with the communicator and the jammer being the two players. The payoff of the game may be one of the figures of merit, such as channel capacity, cut-off rate, bit error rate, and throughput.

2.2 Game-Theoretic Model

Assume complete information for both sides and consider the single-period situation in which the communicator and the jammer determine their optimal strategies simultaneously and continue to use them thereafter. Let a k-dimensional vector $a_c \in A_C$ be the communicator's strategy and an l-dimensional vector $a_j \in A_J$ be the jammer's strategy, where A_C and A_J are their respective strategy sets. The Cartesian product $A = A_C \times A_J$ is the strategy set of the game. Assume that A_C, A_J, and A are appropriate metric spaces. A strategy combination $a = (a_c, a_j) \in A$ is known as a *pure strategy*. Denote the communicator's payoff as $u(a)$, which is a function defined from A to the real set $\mathbf{R}$. Then the payoff of the jammer is $-u(a)$ in this zero-sum game. Therefore, the game can be represented by a *strategic form* (A, u). In game theory, the most important concept is *Nash equilibrium*, a vector of strategies that neither player would deviate from given the other's strategy.

Definition 2.1 *For a game* (A, u), *a* Nash equilibrium *is a strategy combination* $a^* = (a_c^*, a_j^*) \in A$ *such that* $u(a^*) \geq u(a_c, a_j^*)$ *for all* $a_c \in A_C$ *and* $u(a^*) \leq u(a_c^*, a_j)$ *for all* $a_j \in A_J$.

$u(a^*)$ is known as the value of the game with a_c^* and a_j^* being the respective optimal strategies for the two players. The value of a game and the optimal strategies are regarded as the solution of the game.

If the jammer knows the communicator's strategy, he can choose a strategy to achieve the best jamming effect, or minimize the communicator's payoff, yielding the worst case jamming. The communicator may then choose a strategy to minimize the worst case jamming effect, resulting in a payoff of u_{min},

$$u_{min} = \max_{a_c \in A_C} \min_{a_j \in A_J} u(a), \tag{2.1}$$

which is the communicator's value or security level. If a_c is chosen in this way, the communicator's payoff can be guaranteed to be no lower than u_{min} against any jammer.

On the other hand, if the communicator knows the jammer's strategy, he may choose a strategy to maximize his payoff, which corresponds to the best anti-jam system. The jammer may then choose an appropriate a_j to maximize his jamming effect for the best anti-jam system, resulting in a payoff of u_{max},

$$u_{max} = \min_{a_j \in A_J} \max_{a_c \in A_C} u(a), \tag{2.2}$$

which is the jammer's value or security level. If a_j is chosen in this way, the communicator's payoff cannot be higher than u_{max}.

It is easy to see that $u_{min} \leq u_{max}$. If there exists $a^o = (a_c^o, a_j^o)$ such that

$$u(a^o) = u_{min} = u_{max}, \tag{2.3}$$

this point is a *saddle-point* equilibrium. In this case, the two sides tend to use the optimum pure strategies a_c^o and a_j^o, respectively. If one side does not use the optimum strategy, the opponent may gain advantage. Thus, neither player would have incentives to deviate from the saddle-point strategy.

In general, u_{min} and u_{max} are unequal; that is, equilibrium cannot be achieved using pure strategies. In this case, both sides may use randomly varying parameters instead of fixed parameters with the payoff function replaced by the expected value. A *mixed strategy* is obtained which is determined by a probability distribution over pure strategies as follows. The communicator selects an appropriate probability distribution function (PDF) of $F(a_c)$ and uses his strategy accordingly, whereas the jammer selects his strategy according to a PDF of $G(a_j)$. The payoff is then the expectation of $u(a)$ over a_c and a_j. Let $\Delta(A_C)$ and $\Delta(A_J)$ denote the sets of PDFs on Borel subsets of A_C and A_J, respectively. A mixed strategy equilibrium results if the following equation holds:

$$\begin{aligned} \bar{u}_o &= \max_{F(a_c) \in \Delta(A_C)} \min_{G(a_j) \in \Delta(A_J)} \int_{A_J} \int_{A_C} u(a_c, a_j)\, dF(a_c)\, dG(a_j) \\ &= \min_{G(a_j) \in \Delta(A_J)} \max_{F(a_c) \in \Delta(A_C)} \int_{A_J} \int_{A_C} u(a_c, a_j)\, dF(a_c)\, dG(a_j). \end{aligned} \tag{2.4}$$

If a_c and a_j have discrete values of a_{c_m} and a_{j_n} with corresponding probabilities of $p(a_{c_m}) \in \Delta^d(A_C)$ and $p(a_{j_n}) \in \Delta^d(A_J)$ where $\Delta^d(D)$ is defined as the set of probability distributions over the discrete set D; that is,

$$\Delta^d(D) = \{p : D \to \mathbf{R} \mid \sum_{l \in D} p(l) = 1, \text{ and } p(l) \geq 0, \forall l \in D\}. \tag{2.5}$$

Then, we have the discrete form of (2.4)

$$\begin{aligned} \bar{u}_o^d &= \max_{p(a_c) \in \Delta^d(A_C)} \min_{p(a_j) \in \Delta^d(A_J)} \sum_m \sum_n p(a_{c_m}) p(a_{j_n}) u(a_{c_m}, a_{j_n}) \\ &= \min_{p(a_j) \in \Delta^d(A_J)} \max_{p(a_c) \in \Delta^d(A_C)} \sum_m \sum_n p(a_{c_m}) p(a_{j_n}) u(a_{c_m}, a_{j_n}). \end{aligned} \tag{2.6}$$

In summary, from the communicator's viewpoint, if there exists a saddle point, the optimum a_c can be obtained by maximizing the worst case $u(a)$. This is optimum in the sense that using any other strategy may result in a worse performance if the jammer employs his optimum strategy. If a saddle point does not exist, mixed strategies can be employed, and equilibrium performance of this system is better than that of a system using pure strategies.

2.3 Existence and Uniqueness of Equilibrium

The finite two-person zero-sum games have been extensively studied, and the key result is the well-known *von Neumann Minimax Theorem* [3], which guarantees the existence of an equilibrium. However, further analysis is needed for the above game theoretic model, which is usually infinite in practice.

2.3.1 Existence of Equilibrium

Existence of equilibrium is usually addressed by applying mathematical fixed point theorems. In the following, we present existence theorems for the game (A, u) under study.

First, we assume that the payoff function $u(a)$ is continuous on A and only pure strategies are used. The first existence theorem is given as follows.

Theorem 2.1 *If for any $a_j \in A_J$, $\max_{a_c \in A_C} u(a)$ has a unique solution; for any $a_c \in A_C$, $\min_{a_j \in A_J} u(a)$ has a unique solution; $u(a)$ is continuous on A; and either A_C or A_J is a compact convex set, then there exist $a_c^* \in A_C$ and $a_j^* \in A_J$ such that $a^* = (a_c^*, a_j^*)$ is an equilibrium, i.e.,*

$$\min_{a_j \in A_J} \max_{a_c \in A_C} u(a) = \max_{a_c \in A_C} \min_{a_j \in A_J} u(a) = u(a^*). \tag{2.7}$$

Proof. Suppose first that A_J is a compact convex set. Since $\max_{a_c \in A_C} u(a)$ has a unique solution for any $a_j \in A_J$, denote this solution by $f(a_j) : A_J \rightarrow A_C$,

$$f(a_j) = \arg \max_{a_c \in A_C} u(a),$$

where $f(a_j)$ is the best response of the communicator if the jammer takes the strategy a_j.

Similarly, let $g(a_c)$ be the best response of the jammer if the communicator's strategy is a_c, i.e., $g(a_c) : A_C \rightarrow A_J$,

$$g(a_c) = \arg \min_{a_j \in A_J} u(a).$$

By assumption, f is a function from A_J to A_C, and g is a function from A_C to A_J.

Since $u(a)$ is continuous on A, by *Maximum Theorem* [31] we have that f and g are continuous. Now, define h: $A_J \rightarrow A_J$ as

$$h(a_j) = g(f(a_j)).$$

If the jammer takes strategy a_j and the communicator takes his best response $f(a_j)$, then $h(a_j) = g(f(a_j))$ is the best response that the jammer can make to the communicator's $f(a_j)$. Hence, we know that if there exists $\bar{a}_j$ such that $h(\bar{a}_j) = \bar{a}_j$, then the jammer will choose $\bar{a}_j$.

Since g and f are continuous, h is continuous. By the *Brouwer Fixed Point Theorem* [31] and the assumption that A_J is a compact convex set, there exists

$a_j^* \in A_J$ such that $h(a_j^*) = a_j^*$. Let $a_c^* = f(a_j^*)$. Then we have $u(a^*) \geq u(a_c, a_j^*)$ for any $a_c \in A_C$ and $u(a^*) \leq u(a_c^*, a_j)$ for any $a_j \in A_J$. Thus, $a^* = (a_c^*, a_j^*)$ is an equilibrium.

We know that $u(a^*) \geq u(a_c, a_j^*)$, $\forall a_c \in A_C$. Therefore,

$$u(a^*) = \max_{a_c \in A_C} u(a_c, a_j^*),$$

and

$$u(a^*) \geq \min_{a_j \in A_J} \max_{a_c \in A_C} u(a).$$

Similarly, $u(a^*) \leq \max_{a_c \in A_C} \min_{a_j \in A_J} u(a)$. Hence,

$$u(a^*) = \min_{a_j \in A_J} \max_{a_c \in A_C} u(a) = \max_{a_c \in A_C} \min_{a_j \in A_J} u(a).$$

When A_C is a compact convex set instead of A_C, the proof is similar. ■

The result of Theorem 2.1 is remarkably strong and reveals some important properties. First, we do not need to randomize the communicator's and jammer's strategy sets to guarantee the existence of an equilibrium. Second, we do not need both A_C and A_J to be compact convex sets in a finite dimensional space. In contrast, the proof of the existence of the Nash equilibrium in an n-player game requires that each player's set of (mixed) strategies be a compact convex set in a finite dimensional space.

With the conditions of Theorem 2.1 changed slightly, the following existence theorems result.

Theorem 2.2 *If for any $a_j \in A_J$, $\max_{a_c \in A_C} u(a)$ has a unique solution: for any $a_c \in A_C$, $u(a)$ is quasi-convex on a_j; $u(a)$ is continuous on A; and A_J is a compact convex set, then there exists an equilibrium.*

Proof. Define f, g, and h in the same way as in the proof of Theorem 2.1. We know that f is a continuous function from A_J to A_C by *Maximum Theorem*, and that g is a convex-valued upper semi-continuous correspondence from A_C to A_J by the fact that $u(a)$ is quasi-convex on a_j. Because A_J is a compact set and $u(a)$ is continuous on A, $g(a_c)$ is a compact set in A_J for any a_c.

We also know that h is a convex-valued correspondence from A_J to A_J and has a closed graph. Then, by the *Kakutani Fixed Point Theorem* [31], there exists $a_j^* \in A_J$ such that $a_j^* \in h(a_j^*)$. Let $a_c^* = f(a_j^*)$. Hence, we know that (a_c^*, a_j^*) is an equilibrium. ■

Theorem 2.3 *If for any $a_c \in A_C$, $\min_{a_j \in A_J} u(a)$ has a unique solution; for any $a_j \in A_J$, $u(a)$ is quasi-concave on a_c; $u(a)$ is continuous on A; and A_C is a compact convex set, then there exists an equilibrium.*

The proof is analogous to that of Theorem 2.2.

Theorem 2.4 *If for any $a_c \in A_C$, $u(a)$ is quasi-convex on a_j; for any $a_j \in A_J$, $u(a)$ is quasi-concave on a_c; $u(a)$ is continuous on A; and A_C and A_J are compact convex sets, then there exists an equilibrium.*

Proof. Define r: $A \to A$ such that

$$r(\bar{a}) = (\arg\max_{a_c \in A_C} u(a_c, \bar{a}_j), \arg\min_{a_j \in A_J} u(\bar{a}_c, a_j)\,), \tag{2.8}$$

where $\bar{a} = (\bar{a}_c, \bar{a}_j)$, and r is the best response correspondence of the game. By assumptions, r is a convex-valued function having a closed graph. By applying the *Kakutani Fixed Point Theorem*, we know that there exists a fixed point, i.e., an equilibrium exists. ■

The above theorems are concerned with the existence of saddle points for a continuous payoff function where only pure strategies are used. When a saddle point does not exist, we need to apply mixed strategies and reformulate the existence problem as follows. The strategies a_c and a_j are replaced by PDFs $F(a_c)$ and $G(a_j)$, and the strategy sets A_C and A_J are replaced by $\Delta(A_C)$ and $\Delta(A_J)$.

respectively. The payoff function is then the expected value of $u(a)$, as expressed in (2.4) and (2.6) at an equilibrium. If A_C and A_J are nonempty compact sets, then there exists an equilibrium for the mixed strategy (see Theorem 1.3 in [5]).

2.3.2 Uniqueness of Equilibrium

In general, equilibrium is nonunique because there may exist more than one fixed point in the equation $a = r(a)$. An equilibrium may or may not be achieved when multiple equilibria exist in a noncooperative game, since even if each player selects a strategy associated with an equilibrium, the resulting combination may not be an equilibrium. Hence, the issue of uniqueness for equilibrium arises. Paralleling to Theorems 3.4 and 3.5 in [32], we present two uniqueness theorems below for the game (A, u).

Assume that the conditions of Theorem 2.1 are satisfied. Then, equilibria exist and the best response $r(a)$ defined in (2.8) is a single-valued function. The first uniqueness theorem requires that $r(a)$ be a contraction, which means that there exists a positive scalar $F < 1$ such that for any $x, y \in A$,

$$d(r(x), r(y)) \leq Fd(x, y).$$

where $d(x, y)$ is the distance from x to y on A.

Theorem 2.5 *If the conditions of Theorem 2.1 are satisfied and $r(a)$ is a contraction, then there exists a unique equilibrium.*

Proof. From Theorem 2.1, we know that there exists at least one equilibrium. Suppose that $a_1 \in A$ and $a_2 \in A$ are equilibria, i.e., $a_1 = r(a_1)$ and $a_2 = r(a_2)$. Then, $d(a_1, a_2) = d(r(a_1), r(a_2))$. Since $r(a)$ is a contraction, we have that $d(r(a_1), r(a_2)) \leq Fd(a_1, a_2)$ for $F < 1$. This can only hold for $a_1 = a_2$, which implies that the equilibrium is unique. ■

The second uniqueness theorem does not require $r(a)$ to be a contraction, but needs the differentiability of the payoff function.

Let $\mathring{A}$ be the *interior* of A and $C^n(\Psi)$ be the set of all the real functions continuously differentiable to the nth order on an open set Ψ. Denote $u'(a)$ as

$$u'(a) = \begin{bmatrix} \partial u/\partial a_c \\ -\partial u/\partial a_j \end{bmatrix}^T,$$

which is a function from $\mathring{A}$ to $\mathbf{R}^{k+l}$, and its Jacobian as $J(u)$.

Theorem 2.6 *Assume that the conditions of Theorem 2.1 are satisfied and* $u \in C^2(\mathring{A})$. *If* $J(u)$ *is negative quasidefinite for all* $a \in \mathring{A}$, *and* $r(a) \in \mathring{A}$ *for any* $a \in A$, *then there exists a unique equilibrium.*

Proof. By assumption, we know that equilibria exist. Since $r(a) \in \mathring{A}$, there is no equilibrium on the boundary of A. Thus, all equilibria satisfy the first-order condition:

$$\partial u/\partial a_c = \partial u/\partial a_j = 0.$$

An equilibrium a^* corresponds to $u'(a^*) = 0$. Since $u'(a)$ is a function from $\mathring{A}$ to $\mathbf{R}^{(k+l)}$ and its Jacobian is negative quasidefinite, by the *Gale-Nikaido Univalence Theorem* [32], $u'(a)$ is univalent and achieves the value $u'(a) = 0$ only once. This establishes the uniqueness of the equilibrium. ■

By extending the differentiability and quasidefinitness conditions from $\mathring{A}$ to A and applying the *Rosen Uniqueness Theorem* [33], we can obtain a stronger global uniqueness result and eliminate the restriction in Theorem 2.6 that all equilibria must be in $\mathring{A}$.

2.4 Characterizations of Equilibrium

The above model is assumed to give an exact description of the system's behavior. However, approximation may occur in constructing the model, and small perturbation caused by system error or noise is inevitable in practice. For a game of (A, u), deviation or perturbation may exist in the optimal strategies and payoff function. We thus seek to evaluate the effectiveness of changing parameters at an equilibrium for the following two cases: stability and robustness of equilibrium with respect to deviations from optimal strategies, and sensitivity of equilibrium with respect to parameter perturbation.

2.4.1 Stability and Robustness of Equilibrium

When either of the two players deviates his strategy slightly from an equilibrium $a^* = (a_c^*, a_j^*)$ intentionally or unintentionally, the opponent might adjust his strategy by choosing the best response to the deviated strategy. Continuing to iterate the process results in a dynamic process. Assuming simultaneous moves with f, g, and r defined in the same way as before, we have the process as:

$$a^t = (a_c^t, a_j^t) = (f(a_j^{t-1}), g(a_c^{t-1})) = r(a^{t-1}) \tag{2.9}$$

with $a^0 = (a_c^0, a_j^0) \in A$ being the initial point. If the process converges to a^* for any initial point, the equilibrium is said to be *stable*. If the convergence is valid only under small initial deviations, the equilibrium is said to be *locally stable*. Otherwise, it is *unstable*.

From the results of dynamic analysis, we have the following stability theorem [5].

Theorem 2.7 *A fixed point a^* of r is* locally stable *in the process of* (2.9) *if all the eigenvalues of $\left[\frac{dr(a^*)}{da}\right]$ have real parts whose absolute values are less than 1.*

To obtain a corollary for one-dimensional case, the following lemma is required.

Lemma 2.1 *If X and Y are open, $\psi(x,y) : X \times Y \to \mathbf{R}$ satisfies $\psi \in C^2(X \times Y)$, and $\min_{x\in X}$ (or $\max_{x\in X}$) $\psi(x,y)$ has a unique solution denoted by $v(y)$, i.e., $v(y) = \arg\min_{x\in X}$ (or $\arg\max_{x\in X}$) $\psi(x,y)$, then*

$$\frac{dv}{dy} = -\left[\frac{\partial^2\psi}{\partial x^2}\right]^{-1}\left[\frac{\partial^2\psi}{\partial x\partial y}\right] \tag{2.10}$$

provided that $\left[\frac{\partial^2\psi}{\partial x^2}\right]^{-1}$ exists.

Proof. Since X and Y are open and $\psi \in C^2(X \times Y)$, then if for any $y \in Y$, $x^* = v(y)$ is the solution of $\min_{x\in X}$ (or $\max_{x\in X}$) $\psi(x,y)$, by the first-order condition,

$$\left.\frac{\partial\psi}{\partial x}\right|_{(x^*,y)} = 0, \quad \text{for any } y \in Y. \tag{2.11}$$

which is

$$\frac{\partial\psi}{\partial x}(v(y),y) = 0, \quad \text{for any } y \in Y. \tag{2.12}$$

Differentiating (2.12) with respect to y yields

$$\frac{\partial^2\psi}{\partial x^2}\frac{dv}{dy} + \frac{\partial^2\psi}{\partial x\partial y} = 0. \tag{2.13}$$

Hence, if $\left[\frac{\partial^2\psi}{\partial x^2}\right]^{-1}$ exists,

$$\frac{dv}{dy} = -\left[\frac{\partial^2\psi}{\partial x^2}\right]^{-1}\left[\frac{\partial^2\psi}{\partial x\partial y}\right]. \tag{2.14}$$

■

From Theorem 2.7 and Lemma 2.1, we can obtain the following corollary.

Corollary 2.1 *When both a_c and a_j are one-dimensional, a sufficient condition to imply that the eigenvalue condition of Theorem 2.7 is satisfied is the following* *condition on the slopes of the best response functions,*

$$\left|\frac{df}{da_j}\right| \left|\frac{dg}{da_c}\right| < 1 \tag{2.15}$$

or

$$\left(\frac{\partial^2 u}{\partial a_c \partial a_j}\right)^2 < \left|\frac{\partial^2 u}{\partial a_c^2}\right| \left|\frac{\partial^2 u}{\partial a_j^2}\right| \tag{2.16}$$

in an open neighborhood of the equilibrium.

Next, we address briefly the issue of robustness of equilibrium with respect to the payoff function perturbation. It is our concern whether an equilibrium of the original game (A, u) is an approximate equilibrium of the game $(A, \tilde{u})$ with a perturbed payoff $\tilde{u}$. An equilibrium of a game is *robust* if there exists a nearby equilibrium for any nearby game.

Definition 2.2 *An equilibrium of the game (A, u), a^*, is* robust *if for any $\varepsilon > 0$ there exists $\eta > 0$ such that for any $\tilde{u}$ satisfying $|u - \tilde{u}| < \eta$, there exists an equilibrium $\tilde{a}$ of the game $(A, \tilde{u})$ such that $d(a^*, \tilde{a}) < \varepsilon$. A game is robust if all its equilibria are robust.*

For simultaneous-move finite games, we have the following robust theorem [5].

Theorem 2.8 *Almost all finite strategic form games are robust.*

For the game (A, u), "almost all" means that the set of finite games with two players' strategy sets fixed is open and dense in the Euclidean space of dimension $2A_C^\# A_J^\#$ where the communicator and the jammer have $A_C^\#$ strategies and $A_J^\#$ strategies, respectively. Those games that are not robust are said to be *exceptional.* Thus, we can roughly say that finite games are robust in general.

2.4.2 Sensitivity Analysis of Equilibrium

The measure of the sensitivity of an equilibrium to a parameter is the sensitivity function with a form of derivatives of the optimal strategies with respect to that parameter. In the proof of the sensitivity theorem, the following lemma is required in addition to Lemma 2.1.

Lemma 2.2 *If $\phi : X \times Y \to X$ satisfies $\phi \in C^1(X \times Y)$, and X is a compact convex set in $\mathbf{R}^k$, then by the* Brouwer Fixed Point Theorem, *there exists x^* such that*

$$\phi(x^*, y) = x^*. \tag{2.17}$$

Furthermore, if x^* is in the interior of X, and Y is open, then

$$\left.\frac{\partial x^*}{\partial y}\right|_{(x^*,y)} = -\left[\frac{\partial \phi}{\partial x} - \mathbf{I}\right]^{-1} \cdot \left.\frac{\partial \phi}{\partial y}\right|_{(x^*,y)} \tag{2.18}$$

provided that $\left[\frac{\partial \phi}{\partial x} - \mathbf{I}\right]^{-1}$ exists where $\mathbf{I}$ is the identity matrix.

Proof. We know that

$$\phi(x^*(y), y) = x^*(y), \quad \text{for any } y \in Y. \tag{2.19}$$

Differentiating (2.19) with respect to y, we have

$$\frac{\partial \phi}{\partial x} \cdot \frac{\partial x^*}{\partial y} + \frac{\partial \phi}{\partial y} = \frac{\partial x^*}{\partial y}. \tag{2.20}$$

Thus,

$$\frac{\partial x^*}{\partial y} = -\left[\frac{\partial \phi}{\partial x} - \mathbf{I}\right]^{-1} \cdot \frac{\partial \phi}{\partial y} \tag{2.21}$$

provided that $\left[\frac{\partial \phi}{\partial x} - \mathbf{I}\right]^{-1}$ exists. ■

Assume the conditions of Theorem 2.1 are satisfied where A_J is a compact convex set and f, g, and h are defined in the same way as in the proof of Theorem 2.1.

We study the sensitivity of an equilibrium with respect to the perturbation of a parameter $s \in (a, b)$. Denote the payoff as $u(a, s)$. We have the following sensitivity theorem.

Theorem 2.9 *If for any $s \in (a,b)$, $u(a,s) \in C^2(A \times (a,b))$ satisfies all the conditions of Theorem 2.1 with an equilibrium $a^* = (a_c^*, a_j^*) \in \mathring{A}$, then*

$$\left.\frac{\partial a_j^*}{\partial s}\right|_{(a^*,s)} = -\left[\frac{\partial h}{\partial a_j} - \mathbf{I}\right]^{-1} \cdot \left.\frac{\partial h}{\partial s}\right|_{(a^*,s)}, \tag{2.22}$$

and

$$\left.\frac{\partial a_c^*}{\partial s}\right|_{(a^*,s)} = \left.\left[\frac{\partial f}{\partial a_j} \cdot \frac{\partial a_j^*}{\partial s} + \frac{\partial f}{\partial s}\right]\right|_{(a^*,s)}, \tag{2.23}$$

where

$$\begin{aligned} \frac{\partial h}{\partial a_j} &= \frac{\partial g}{\partial a_c} \cdot \frac{\partial f}{\partial a_j}, \\ \frac{\partial h}{\partial s} &= \frac{\partial g}{\partial a_c} \cdot \frac{\partial f}{\partial s} + \frac{\partial g}{\partial s}, \end{aligned}$$

with

$$\begin{aligned} \left[\frac{\partial g}{\partial a_c}, \frac{\partial g}{\partial s}\right] &= -\left[\frac{\partial^2 u}{\partial (a_j, s)^2}\right]^{-1} \left[\frac{\partial^2 u}{\partial (a_j, s) \partial a_c}\right], \\ \left[\frac{\partial f}{\partial a_j}, \frac{\partial f}{\partial s}\right] &= -\left[\frac{\partial^2 u}{\partial (a_c, s)^2}\right]^{-1} \left[\frac{\partial^2 u}{\partial (a_c, s) \partial a_j}\right]. \end{aligned}$$

Proof. The proof follows from Lemmas 2.1 and 2.2. ■

The equilibrium that is stable, robust, and insensitive with respect to perturbation is of more interest for a practical problem, since otherwise equilibrium performance is meaningless due to inevitable approximation and perturbation. Computation algorithms for equilibria can be found in [34, 35].

2.5 Conclusions

In this chapter, a single-period complete-information game theoretic model has been presented for the communication-jamming problem. The concept of equilibrium has been emphasized, and related fundamental issues, such as existence, uniqueness, stability, robustness, and sensitivity, have been discussed with corresponding theorems presented. The next chapter considers an application of the game theoretic model to the design of a practical anti-jam communication system.

Chapter 3

Game Theoretic Study for an Anti-Jam FFH/FSK System Using Ratio-Threshold Diversity

3.1 Introduction

In this chapter, we apply the two-person zero-sum model discussed in the previous chapter to a fast frequency-hopped (FFH) M-ary frequency shift keying (MFSK) system under partial-band noise (PBN) or band multi-tone (MT) jamming. The structure of the receiver is shown in Figure 3.1. Ratio-threshold (R-T) diversity combining technique [12, 36] with or without error correction coding (ECC) is used to combat jamming. The communicator chooses the ratio-threshold and the diversity order as his strategies. The jammer selects the fraction of jammed bandwidth in the total bandwidth in PBN jamming or the power ratio of signal to one jamming tone in MT jamming as his strategies. Bit error rate (BER) is used as the payoff function when only diversity combining is employed, and cutoff rate is considered when ECC is also used.

3.2 Ratio-Threshold Diversity Combining

When a R-T diversity combiner is used in a FFH/FSK system, we denote the diversity order as m and the ratio threshold as θ. Assume that the output of the non-coherent matched filter for the ith symbol at the lth hop is X_{il}, where

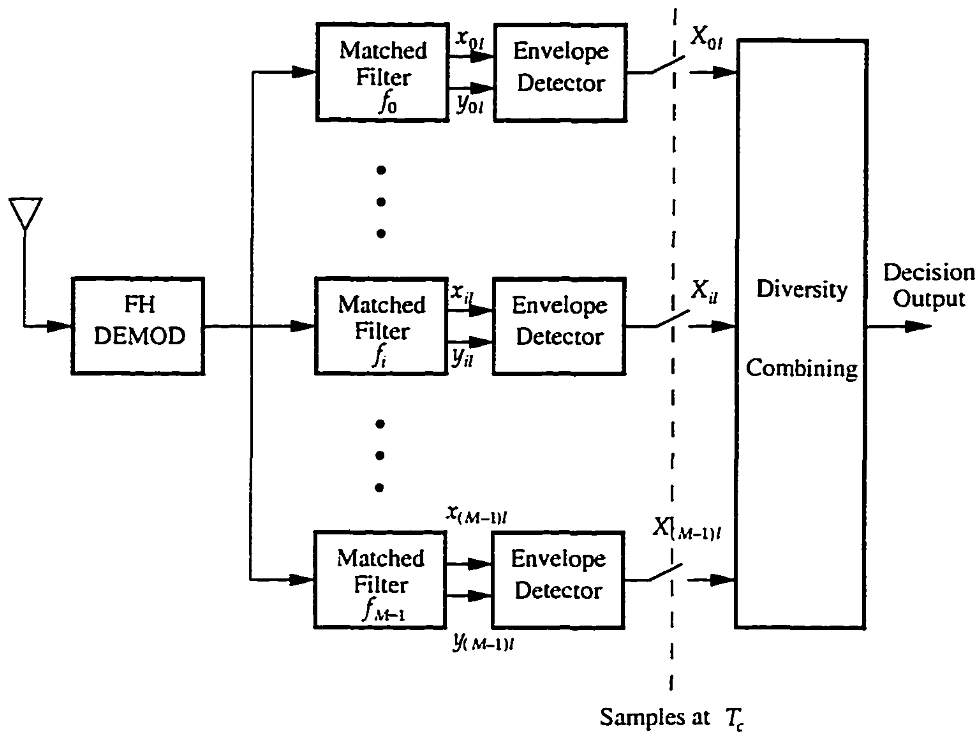

Figure 3.1. *The FFH noncoherent MFSK spread spectrum receiver.*

$i = 0, 1, \ldots, M-1$, and $l = 1, 2, \ldots, m$. The hop decision is made based on which non-coherent matched filter output is the largest. Suppose that, for the lth hop,

$$X_{jl} = \max_{0 \leq i \leq M-1} X_{il}, \tag{3.1}$$

where $l = 1, 2, \ldots, m$, and the quality bit q is set according to [36]

$$q(l) = \begin{cases} 0 \quad (\text{good}) & \text{if } \frac{X_{jl}}{X_{il}} \geq \theta \text{ for all } i \neq j, \\ 1 \quad (\text{poor}) & \text{otherwise.} \end{cases} \tag{3.2}$$

The outputs of all hops are accumulated with good quality bits and poor quality bits, respectively. If there is at least one hop decision with a good quality bit and there is a majority decision without a tie, then the output of the combiner is the majority decision with a good output quality bit attached. If there is a tie between 0 and 1 with good quality bits, or if there is no hop decision with a good quality bit, a decision is made based on hop decisions with poor quality bits (if there is a tie, flip a coin), and this output of the combiner is attached with a poor output quality bit. When the $\log_2 M$ binary information bits associated with each transmitted M-ary symbol are ideally interleaved, the result is a channel with binary inputs and quaternary outputs for each hop.

The transition probabilities of the channel are known to be [36]

$$\begin{aligned} P_C &= F_C(\theta) + \left(\frac{M}{2} - 1\right) F_E(\theta), \\ P_{CX} &= F_C(1) - F_C(\theta) + \left(\frac{M}{2} - 1\right) [F_E(1) - F_E(\theta)], \\ P_{EX} &= \frac{M}{2}[F_E(1) - F_E(\theta)], \\ P_E &= \frac{M}{2} F_E(\theta), \end{aligned}$$

where P_C and P_{CX} are the probabilities of correct reception with good and poor quality, P_E and P_{EX} are the probabilities of error reception with good and poor quality, and

$$\begin{aligned} F_C(\theta) &= \Pr\{X_{jl} \geq \theta X_{il} \text{ for all } i \neq j \mid j \text{ sent}\}, \\ F_E(\theta) &= \Pr\{X_{nl} \geq \theta X_{il} \text{ for a specific } n \neq j \text{ and all } i \neq n \mid j \text{ sent}\}, \end{aligned}$$

which can be derived from the distributions of M matched filter outputs X_{0l}, $X_{1l}, \ldots, X_{(M-1)l}$ using the average computation model for PBN jamming [37] and band MT jamming [38], respectively.

A. PBN Jamming

A PBN jammer is supposed to jam a fraction ρ of the transmission band with noise power density N_J/ρ. We have [37]

$$F_C(\theta) = \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \frac{\theta^2}{\theta^2 + k} \left[\rho \exp\left(-\frac{k}{\theta^2 + k} \frac{\log_2 M E_b}{m(N_0 + N_J/\rho)}\right)\right.$$

$$
\begin{aligned}
&+(1-\rho)\exp\left(-\frac{k}{\theta^2+k}\frac{\log_2 ME_b}{mN_0}\right)\Bigg]. \\
F_E(\theta) &= \sum_{k=0}^{M-2}(-1)^k\binom{M-2}{k}\frac{\theta^2}{\theta^2+k}\frac{1}{\theta^2+k+1}\left[\rho\exp\left(-\frac{\theta^2+k}{\theta^2+k+1}\frac{\log_2 ME_b}{m(N_0+N_J/\rho)}\right)\right. \\
&+(1-\rho)\exp\left(-\frac{\theta^2+k}{\theta^2+k+1}\frac{\log_2 ME_b}{mN_0}\right)\Bigg],
\end{aligned}
$$

where E_b is the energy per information bit and N_0 is the power spectral density of the thermal noise modeled as additive white Gaussian noise (AWGN).

B. MT Jamming

When band MT jamming is assumed where the jammer has a single jamming tone per jammed band with equal power, $F_C(\theta)$ and $F_E(\theta)$ can be computed as [38]

$$
\begin{aligned}
F_C(\theta) &= (1-\mu)F_{C0}(\theta)+\mu F_{C1}(\theta), \\
F_E(\theta) &= (1-\mu)F_{E0}(\theta)+\mu F_{E1}(\theta),
\end{aligned}
$$

where μ is the probability of one M-ary band being jammed

$$
\mu = \frac{\alpha m M}{\log_2 ME_b/N_J}
$$

with α being the ratio of signal power to the power of one jamming tone. The expressions for $F_{C0}(\theta)$, $F_{C1}(\theta)$, $F_{E0}(\theta)$, and $F_{E1}(\theta)$ can be found in [38].

Let o be the output of the diversity combiner before tie-breaking. There are five possible values for o,

$$
o = \begin{cases}
00 & \text{decision is 0, good quality,} \\
01 & \text{decision is 0, poor quality,} \\
x & \text{tie,} \\
11 & \text{decision is 1, poor quality,} \\
10 & \text{decision is 1, good quality.}
\end{cases}
$$

The conditional distribution of o given that "0" is sent is

$$
\begin{aligned}
\Pr\{o = 00 \,|\, 0 \text{ sent}\} &= \sum_{\Omega, k_1 > k_2} C_m^{k_1,k_2,k_3,k_4} P_C^{k_1} P_E^{k_2} P_{CX}^{k_3} P_{EX}^{k_4}, \\
\Pr\{o = 01 \,|\, 0 \text{ sent}\} &= \sum_{\Omega, k_1 = k_2, k_3 > k_4} C_m^{k_1,k_2,k_3,k_4} P_C^{k_1} P_E^{k_2} P_{CX}^{k_3} P_{EX}^{k_4}, \\
\Pr\{o = 11 \,|\, 0 \text{ sent}\} &= \sum_{\Omega, k_1 = k_2, k_3 < k_4} C_m^{k_1,k_2,k_3,k_4} P_C^{k_1} P_E^{k_2} P_{CX}^{k_3} P_{EX}^{k_4}, \\
\Pr\{o = 10 \,|\, 0 \text{ sent}\} &= \sum_{\Omega, k_1 < k_2} C_m^{k_1,k_2,k_3,k_4} P_C^{k_1} P_E^{k_2} P_{CX}^{k_3} P_{EX}^{k_4}. \\
\Pr\{o = x \;|\, 0 \text{ sent}\} &= \sum_{\Omega, k_1 = k_2, k_3 = k_4} C_m^{k_1,k_2,k_3,k_4} P_C^{k_1} P_E^{k_2} P_{CX}^{k_3} P_{EX}^{k_4}.
\end{aligned}
$$

where

$$\Omega = \{k_1, k_2, k_3, k_4 \,|\, 0 \le k_1, k_2, k_3, k_4 \le m, k_1 + k_2 + k_3 + k_4 = m\}$$

and $C_m^{k_1,k_2,k_3,k_4}$ is the multinomial coefficient.

When ECC is not used, the outputs of diversity combiner are also the outputs of the receiver. Suppose the two binary symbols before interleaving are equiprobable. The BER at the diversity combiner output is

$$P_b = \Pr\{o = 11 \,|\, 0 \text{ sent}\} + \Pr\{o = 10 \,|\, 0 \text{ sent}\} + \frac{1}{2} \Pr\{o = x \,|\, 0 \text{ sent}\}. \qquad (3.3)$$

When ECC is used and the cutoff rate of the coding channel is considered, the entire diversity combining channel can be viewed as an equivalent binary inputs quaternary outputs channel with the corresponding transition probabilities being

$$
\begin{aligned}
\overline{P}_C &= \Pr\{o = 00 \,|\, 0 \text{ sent}\}, \\
\overline{P}_{CX} &= \Pr\{o = 01 \,|\, 0 \text{ sent}\} + \frac{1}{2} \Pr\{o = x \,|\, 0 \text{ sent}\}, \\
\overline{P}_{EX} &= \Pr\{o = 11 \,|\, 0 \text{ sent}\} + \frac{1}{2} \Pr\{o = x \,|\, 0 \text{ sent}\}. \\
\overline{P}_E &= \Pr\{o = 10 \,|\, 0 \text{ sent}\}.
\end{aligned}
$$

The normalized cutoff rate R_0 of this channel is related to the transition probabilities by

$$R_0 = 1 - \log_2\left(1 + 2\sqrt{\overline{P}_E\overline{P}_C} + 2\sqrt{\overline{P}_{EX}\overline{P}_{CX}}\right) \text{ (bits/binary symbol).} \qquad (3.4)$$

In the game theoretic model for the system under study, the communicator can adjust θ and m to minimize the BER or maximize the cutoff rate, while the jammer can adjust ρ or α under different types of jamming to maximize the BER or minimize the cutoff rate. Then, $a_c = (\theta, m)$ is the communicator's strategy, and a_j, which is ρ under PBN jamming or α under MT jamming, is the jammer's strategy. The payoff function u is P_b as in (3.3) or R_0 as in (3.4).

Then, we have the strategy set for the communicator

$$\begin{aligned} A_C &= \{(\theta, p) \,|\, 1 \leq \theta < \infty;\ p = (p_1, \ldots, p_i, \ldots),\ p_i \geq 0,\ \sum_{i=1}^{\infty} p_i = 1\} \\ &= \{\theta \,|\, 1 \leq \theta < \infty\} \times \{p \,|\, p = (p_1, \ldots, p_i, \ldots),\ p_i \geq 0,\ \sum_{i=1}^{\infty} p_i = 1\} \\ &= A_C(\theta) \times A_C(p), \end{aligned}$$

where $p_i = \Pr\{m = i\}$. The strategy set for the jammer is

$$A_J(\rho) = \{\rho \,|\, 0 \leq \rho \leq 1\} \qquad (3.5)$$

under PBN jamming, or

$$A_J(\alpha) = \{\alpha \,|\, \alpha_{min} \leq \alpha \leq \alpha_{max}\} \qquad (3.6)$$

under MT jamming where α_{min} is the lower bound small enough to include the lowest point of the worst α, and α_{max} is such an α that causes $\mu = 1$. It is obvious that A_J is a compact convex set.

3.3 Numerical Results

In this section, we apply the game-theoretic model to the anti-jam problem and study the system performance through numerical method where E_b/N_0 is fixed at 13 dB which corresponds to a BER of 2.3×10^{-5} for a system with $m = 1$, $M = 2$, and no jamming.

3.3.1 BER Performance of Uncoded Systems

When R-T diversity is used without ECC, we evaluate the BER as given in (3.3).

A. PBN Jamming

We assume first that the communicator adjust θ at a fixed m. In this case, we can verify that the conditions of Theorem 2.1 are satisfied; thus, the saddle point performance can be achieved. In Figure 3.2, BER versus θ with various ρ at $(M, m, E_b/N_J) = (2, 3, 16 \text{ dB})$ is shown. The value of θ at which the minimum BER is achieved can then be selected by the communicator to obtain the best anti-jam result at different ρ. Similarly, the value of ρ at which the maximum BER is achieved can be adopted by the jammer to obtain the worst case result at different θ. Figure 3.3 shows the worst case BER versus E_b/N_J with various θ at $M = 2$ and $m = 3$. It is observed that the optimum θ is almost the same at different E_b/N_J; that is, there is almost no need to adjust θ when an optimum value of θ is found at a fixed m. The communicator's value can be obtained by selecting the minimum point at each E_b/N_J which corresponds to the saddle point performance. In Figure 3.4, the saddle point performances are shown for various m with $M = 2, 4, 8$, and 16, from which it can be concluded that if the diversity order is greater than a value of $\overline{m}$ (e.g., $\overline{m} = 3$ at $M = 2$ and 4, and $\overline{m} = 5$ at $M = 8$ and 16), then noncoherent combining loss becomes dominant and the system equilibrium performance is uniformly worse than that with a smaller m.

It is obvious that the communicator can gain more if it changes m as well as θ. In this case, a saddle point equilibrium does not exist. The communicator's value can be obtained by choosing m and θ associated with the smallest BER. This corresponds to the lower envelopes of the BER curves in Figure 3.4 at different M. It is evident that the communicator tends to use the largest M to achieve the uniformly best BER performance. A mixed strategy can then be used to obtain the (mixed) equilibrium performance. For example, suppose $m \in \{1, 3\}$, $M = 2$, and a saddle point equilibrium is achieved at (θ_1, ρ_1) for $m = 1$ or (θ_2, ρ_2) for $m = 3$. The communicator can select $m = 1$ and a corresponding θ_1 with a probability of p_c and select $m = 3$ and a corresponding θ_2 with a probability of $1 - p_c$. On the other hand, the jammer can choose ρ_1 with a probability of p_j and choose ρ_2 with a probability of $1 - p_j$. Then we get a game matrix which can be solved to obtain a mixed strategy equilibrium. For instance, when $E_b/N_J = 10$ dB and $M = 2$, the game matrix is shown in Table 3.1, where the value is the BER when corresponding parameters in that row or column are adopted.

Table 3.1. *BER with $E_b/N_J = 10$ dB and $M = 2$*

	$\rho_1 = 0.26$	$\rho_2 = 1.0$
$m = 1, \theta_1 = 1.0$	0.0412	0.0231
$m = 3, \theta_3 = 1.7$	0.0179	0.0590

Using the Shapely-Snow procedure for games [39], we can find that the solution for this game matrix is 0.0341 with $p_c = 0.6943$ and $p_j = 0.6064$. In general, we can obtain a mixed equilibrium by solving a game with an $\overline{m} \times n$ game matrix Υ formed by making ρ into n discrete points as

$$\Upsilon = [\epsilon_{ij}]_{\overline{m} \times n} \tag{3.7}$$

where $\epsilon_{ij} = P_b((\theta_i, m_i), \rho_j)$, $1 \leq i \leq \overline{m}$ and $1 \leq j \leq n$.

B. MT Jamming

Similarly, we can apply the above analysis to MT jamming. The worst case α is found through searching in the interval $[\alpha_{min}, \alpha_{max}]$ numerically. When the communicator's strategy set is $\mathcal{A}_C(\theta)$ with a fixed m, the conditions of Theorem 2.1 are also satisfied and the saddle point performance can be achieved. The corresponding results are shown in Figure 3.5 at various m with $M = 2, 4, 8$, and 16. It is observed again that there is no need to have a diversity order greater than a value of $\overline{m}$ (e.g., $\overline{m} = 4$ at $M = 2$ and 4, and $\overline{m} = 3$ at $M = 8$ and 16). When the communicator further changes m together with θ as his strategies with the strategy set enlarged to $\mathcal{A}_C(\theta) \times \mathcal{A}_C(p)$, a saddle point does not exist and mixed strategies may be used. The performances at the communicator's value correspond to the lower envelopes in Figure 3.5 for different M.

By comparing corresponding lower envelopes of the curves with the same M in Figure 3.4 and Figure 3.5, it is clear that the jammer will almost always use MT jamming (except for a small region of E_b/N_J when $M = 2$) to achieve a better jamming effect, while $M = 8$ is most desirable for the communicator.

3.3.2 Cutoff Rate Performance of Coded Systems

Next, we discuss the equilibrium performance in the presence of ECC. The payoff function is now the cutoff rate as given in (3.4). The results are interpreted through the minimum value of E_b/N_J that is needed for practical and reliable communications. It is determined from the cutoff rate in the following manner. Suppose that the cutoff rate is a function of the symbol signal-to-jamming noise ratio: $R = R_0(E_s/N_J)$. If we use codes of the rate R, then E_b/N_J is given by

$$E_b/N_J = \frac{E_s/N_J}{R \log_2 M}.$$

If $R < R_0(E_s/N_J)$, then we have

$$E_b/N_J > \frac{R_0^{-1}(R)}{R \log_2 M}. \tag{3.8}$$

In the following figures, we plot the lower bound in (3.8) versus R.

A. PBN Jamming

In this case, the result with ECC is analogous to that without ECC. The conditions of Theorem 2.1 are satisfied when the communicator changes θ as his strategy at a fixed m. The saddle point cutoff rate performances at various m with $M = 2, 4, 8$, and 16 are shown in Figure 3.6. The curve of $m = 1$ is the lowest one when code rate R is less than a value (e.g., 0.74 for $M = 2$, 0.81 for $M = 4$ and $M = 8$, and 0.79 for $M = 16$), and its minimum lies below the minima of all the other curves in each figure, which means that no diversity is necessary if a powerful code is used. However, in the higher code rate region, diversity may be concatenated with the ECC to achieve a better performance. It also holds that m need not be greater than $\overline{m}$ (e.g., $\overline{m} = 2$ for $M = 2$, and $\overline{m} = 3$ for $M = 4, 8$ and 16).

The lower envelopes of the curves in Figure 3.6 correspond to the communicator's value. We can reach the same conclusion that the communicator tends to use a larger M.

B. MT Jamming

The saddle point cutoff rate performance at a fixed m with $M = 2$ is shown in the upper left figure in Figure 3.7 when the communicator varies θ as his strategy. In the low code rate region, irregular curves are observed indicating that the cutoff rate does not necessarily decrease with the increase of the jamming power due to the random cancellation of jamming and thermal noise. This means that jamming may even help the communication when it is strong. A similar cancellation phenomenon was also previously observed for a slow frequency-hopped DPSK system [40]. In Figure 3.7, we also present the results for $M = 4, 8$, and 16 with only the regular regions shown, which are of more interest. Here, $\overline{m}$ is 2 at $M = 2$, and 4 at $M = 4$, 8, and 16. The lower envelopes of the curves in Figure 3.7 again correspond to the performances at the communicator's value. It is clear that the performance at $M = 4$ is the best for the communicator.

Again, by comparing the corresponding lower envelopes of the curves with the same M in Figure 3.6 and Figure 3.7, we conclude that in the presence of ECC, the jammer tends to select PBN jamming when $M = 2$, or MT jamming when M is greater than 2, which is slightly different from the conclusion when ECC is not used.

3.4 Conclusions

In this chapter, the game-theoretic model has been applied to a FFH/FSK system where ratio-threshold diversity is used to combat PBN jamming or MT jamming. As for the performance measure, either the cutoff rate or the BER is employed depending on whether ECC is used. System performance in various situations has been evaluated through numerical analysis. It has been shown that the jammer tends to use MT jamming and choose a proper α, and then the communicator has to choose a corresponding (θ, m) at $M = 8$ or $M = 4$ in the absence or presence of coding to achieve a better result.

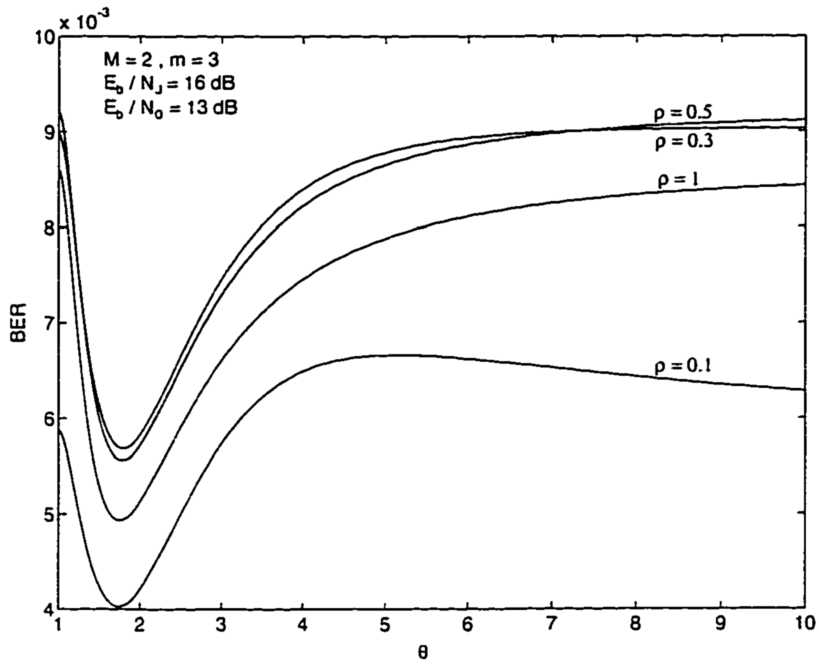

Figure 3.2. *BER versus θ at various ρ with $(M, m, E_b/N_J) = (2, 3, 16\ dB)$ under PBN jamming.*

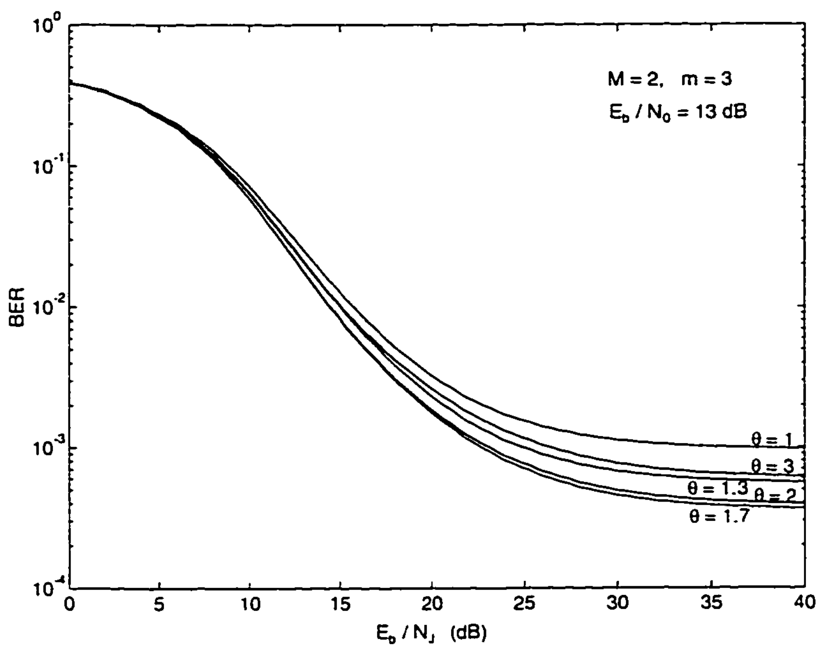

Figure 3.3. *The worst case BER versus E_b/N_J at various θ with $M = 2$ and $m = 3$ under PBN jamming.*

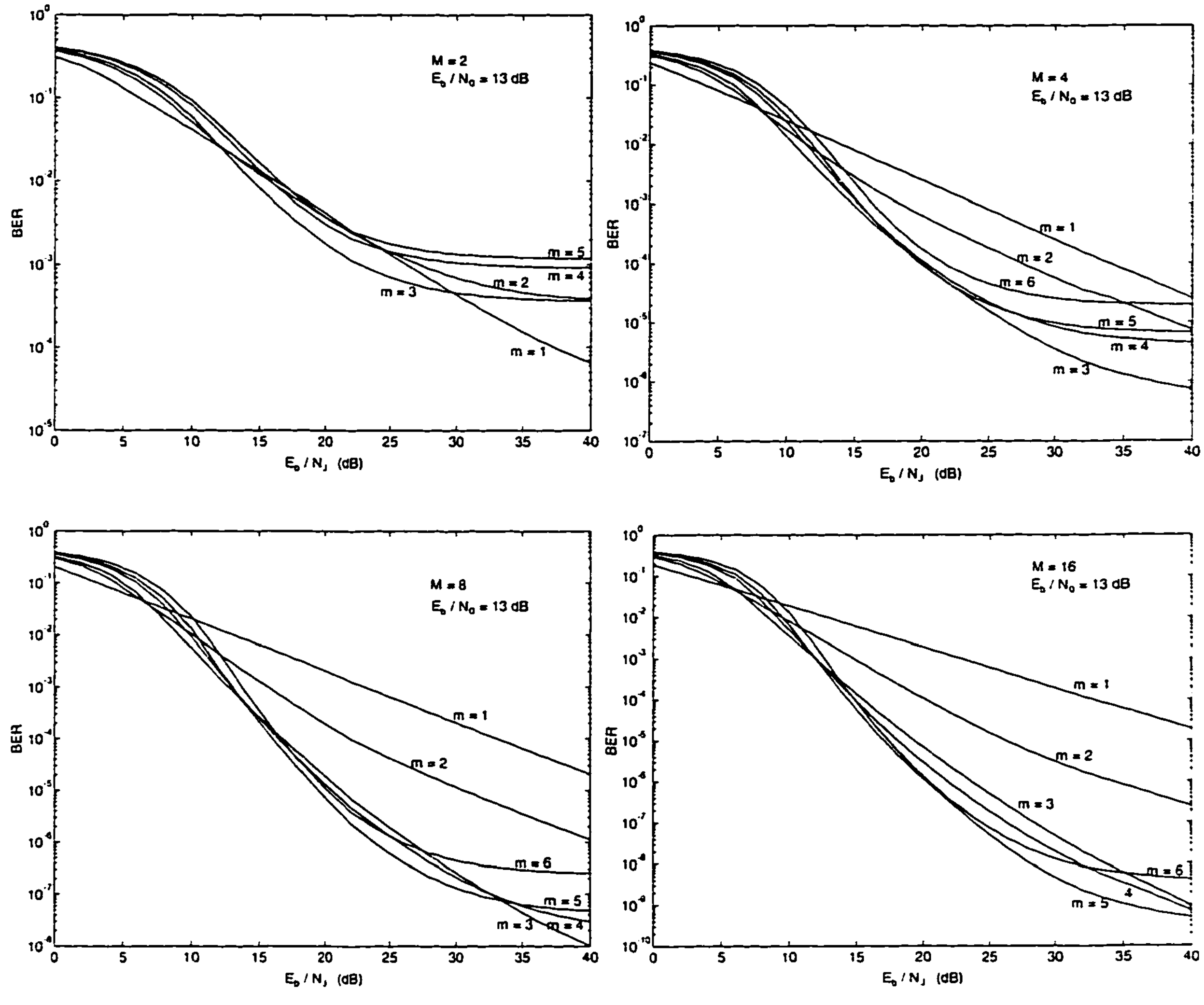

Figure 3.4. *Saddle point BER performances at various m with $M = 2, 4, 8,$ and 16 under PBN jamming.*

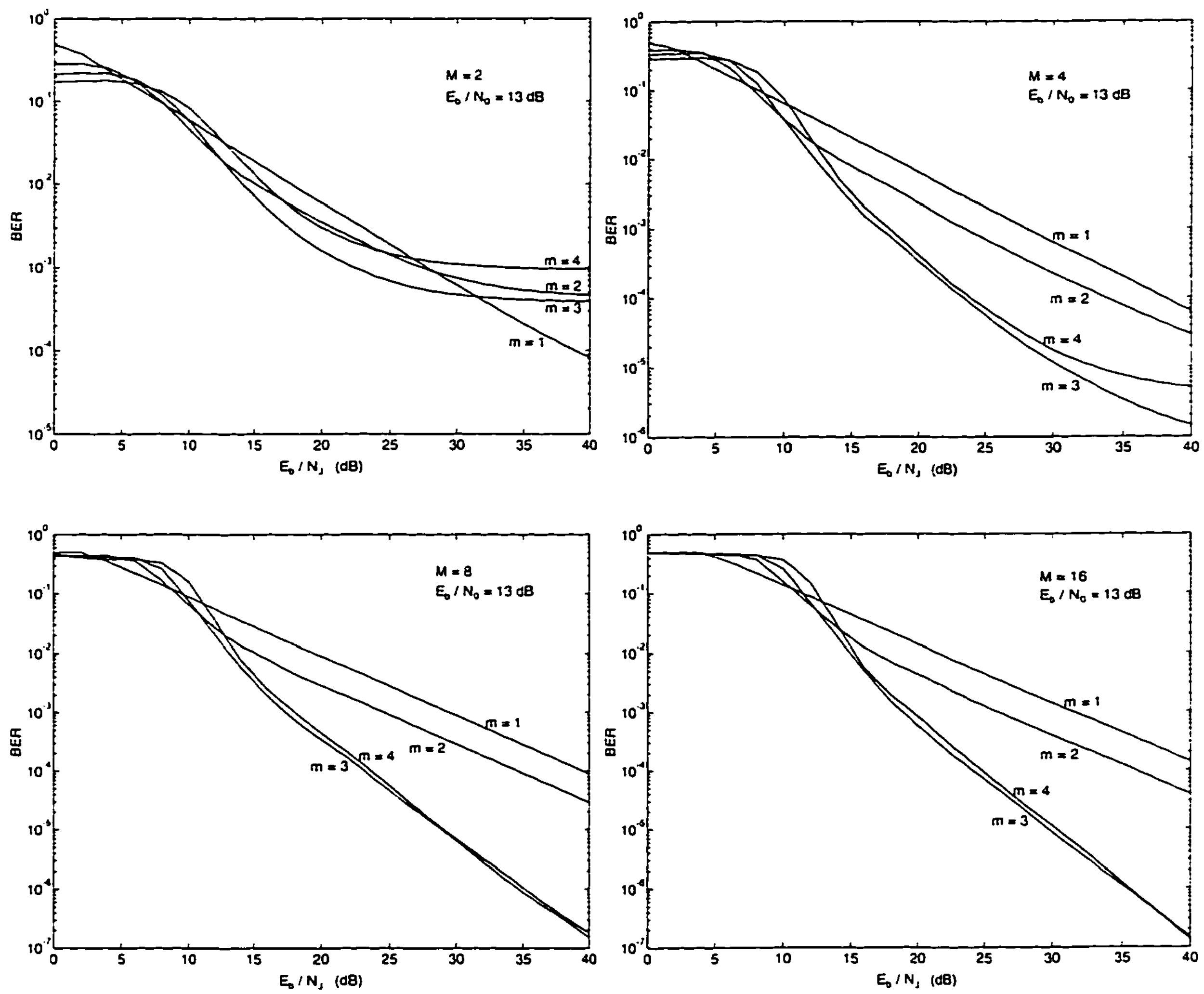

Figure 3.5. *Saddle point BER performances at various m with $M = 2, 4, 8,$ and 16 under MT jamming.*

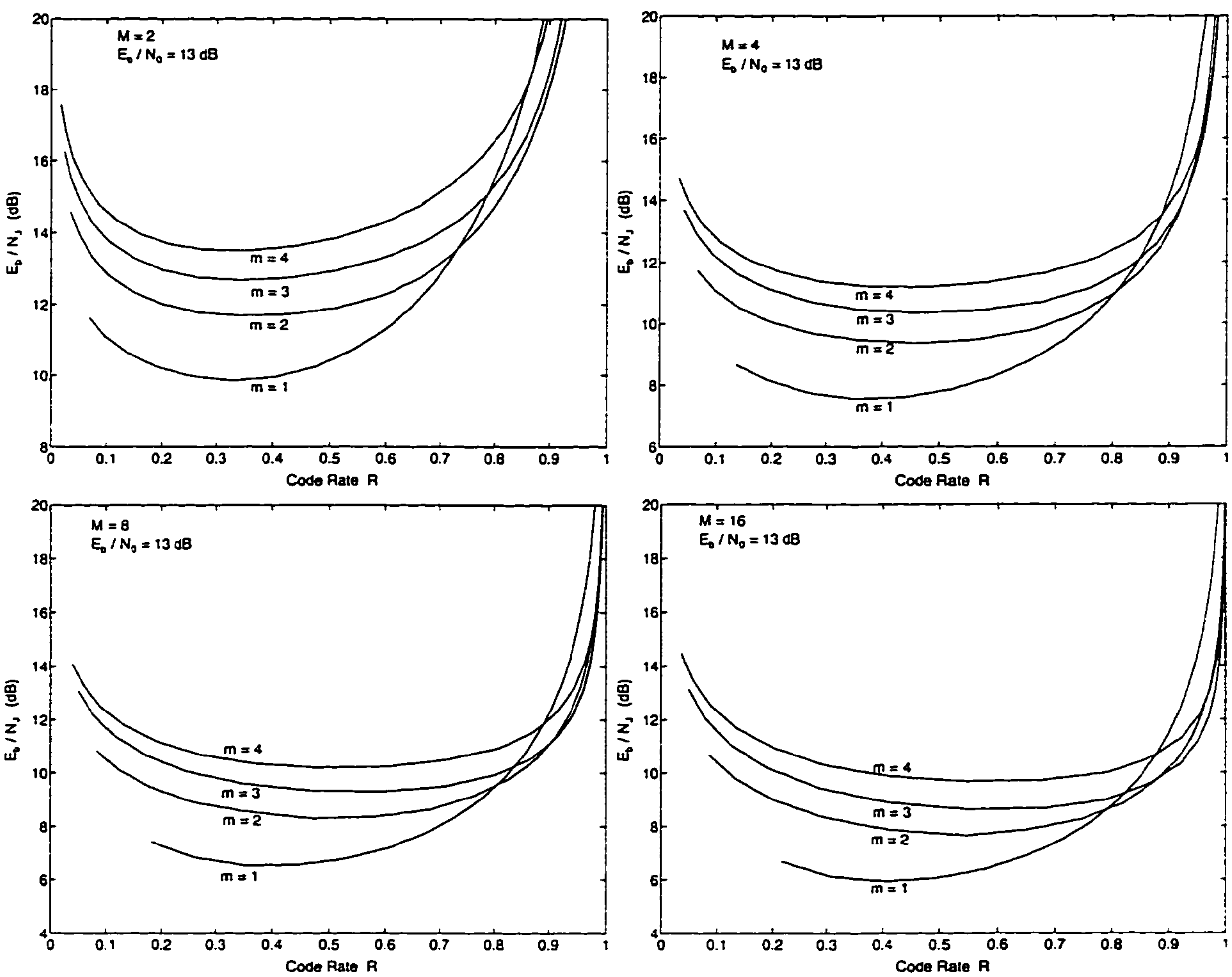

Figure 3.6. *Minimum E_b/N_J needed to achieve the saddle point cutoff rate at various m with $M = 2, 4, 8,$ and 16 under PBN jamming.*

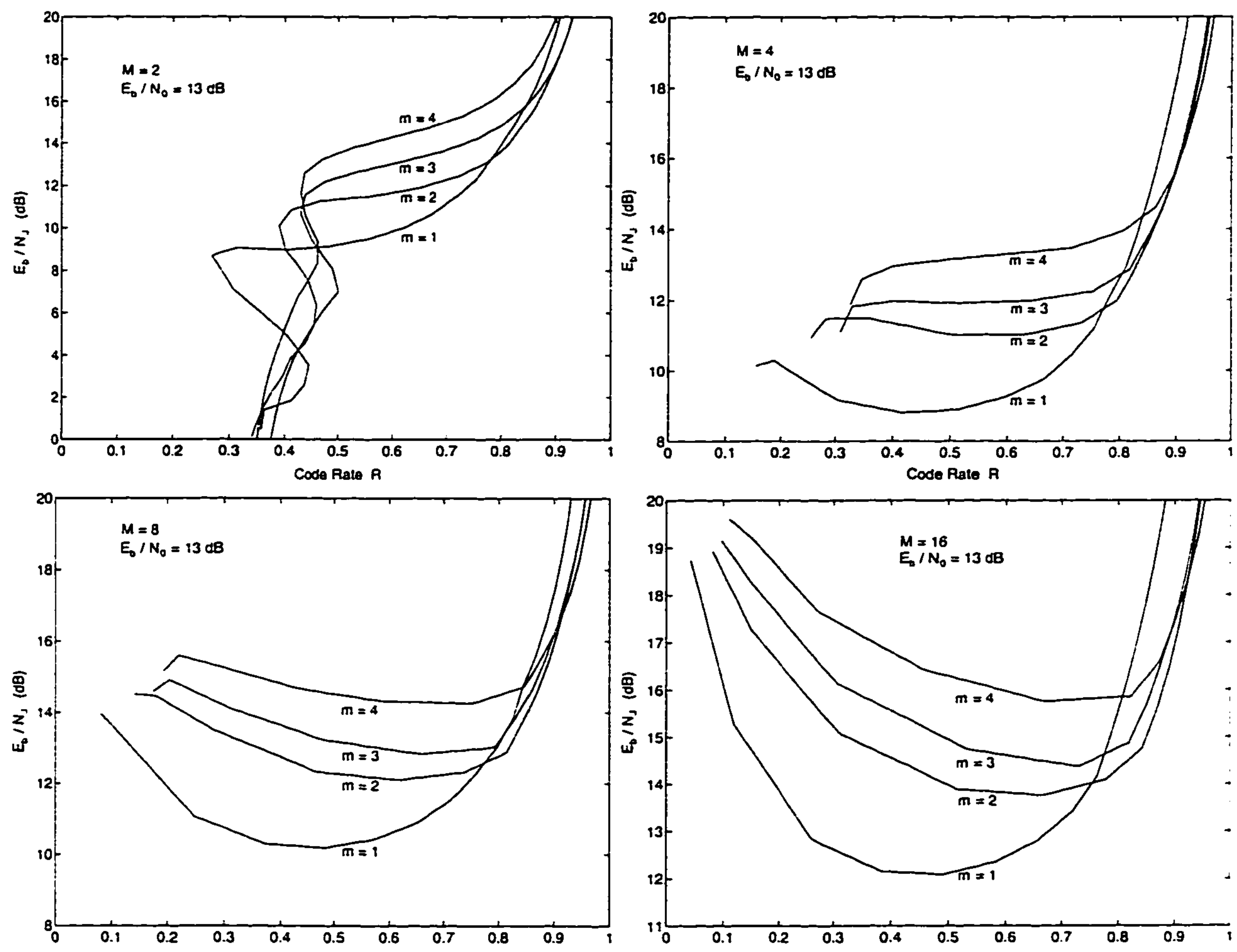

Figure 3.7. *Minimum E_b/N_J needed to achieve the saddle point cutoff rate at various m with $M = 2, 4, 8$, and 16 under MT jamming.*

Chapter 4

A Multiobjective Analytic Framework for Wireless Heterogeneous Networks

4.1 Introduction

In a random access network, users compete with one another for the limited resource of a communication channel. Normally, a single global objective such as network throughput is examined to consider the optimality of the entire network. This approach, however, is only applicable to a network where every user has a common performance objective. When individual users have distinct objectives, the use of a single objective may occasion ignorance of individual objectives, thus sacrificing the performance of some users for the good of the entire network. Therefore, to take individual objectives into consideration, a multiobjective formulation is needed.

In noncooperative situations as studied in the previous chapters, each user acts individually to optimize his objective without concern for others' objectives, resulting in the Nash equilibrium. However, this is appropriate only in a totally individualistic environment. In a cooperative situation where binding agreements are possible, each user tends to act at a cooperative equilibrium which is Pareto optimal in the sense that no user can improve his objective without degrading any other user's objective. The capability of cooperation among users in a network leads naturally to the Pareto optimal approach in terms of performance improvement.

4.2 Multiobjective Optimality and Fairness

Here, we extend the notations in Chapter 2 to the K-person nonzero-sum case. Suppose that there are K decision makers, ν_i, $i = 1, \ldots, K$. Denote ν_i's strategy as a_i and the strategy of the network as the combination: $a = (a_1, a_2, \ldots, a_K)$. Let the strategy set (or admissible set) of ν_i be A_i, which is an appropriate metric space. The strategy set (or admissible set) of the network, A, is the Cartesian product: $A_1 \times A_2 \times \cdots \times A_K$. Denote the individual objective function of ν_i as $u_i(a) : A \to O_i \subset \mathbf{R}$, where O_i is the admissible objective space of ν_i. The admissible objective space of the network, O, is the Cartesian product: $O_1 \times O_2 \times \cdots \times O_K$. The global objective function of the network is defined as $g(a) : A \to \mathbf{R}$. If a is in A, the corresponding $u(a) = (u_1(a), u_2(a), \ldots, u_K(a))$ and $g(a)$ are called *admissible*.

Basically, there are two approaches to the network optimization problem: single-objective overall optimization and multiobjective individual optimization. For the overall network optimization, we maximize a global objective function,

$$\max_{a \in A} g(a) . \tag{4.1}$$

This approach is often used to obtain the optimal performance for the entire network where all users have a common objective and the meaning of optimality is straightforward.

To consider the individual performance objectives, we resort to the multiobjective approach [1]. Since individual objectives depend on the strategies of all users, the optimal strategy of each user depends on the strategies of other users. Consequently, a multiobjective optimization problem is formulated as

$$\max_{a \in A} u_i(a) , \quad i = 1, \ldots, K, \tag{4.2}$$

where we assume that each user attempts to maximize his own objective. We need to find an optimal strategy by considering all individual objectives in A.

As individual objectives usually conflict with one another, simultaneous optimization of all objectives is normally impossible. Therefore, the concept of optimality calls for new formulation. In the following, we will introduce noncooperative and cooperative game theoretic approaches.

In a noncooperative network, the rational users tend to act at the Nash equilibrium [3], which is redefined in the following for a K-person nonzero-sum game.

Definition 4.1 (Nash Equilibrium) *A strategy* $\tilde{a} \in A$ *is a* Nash equilibrium *if for every* $a_i \in A_i$,

$$u_i(\tilde{a}) \geq u_i(\tilde{a}_1, \ldots, \tilde{a}_{i-1}, a_i, \tilde{a}_{i+1}, \ldots, \tilde{a}_K), \quad i = 1, \ldots, K. \tag{4.3}$$

Nash equilibrium actually provides a fault tolerant optimality concept, which is applicable to a totally individualistic environment where individual users compete with one another so that each of them reaches his own optimum [44]. However, it is Pareto inefficient under certain conditions [45].

A related concept is the max-min strategy which is used to obtain the security level in a noncooperative K-person game [3]. Let $a^{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_K) \in A^{-i}$, where A^{-i} is the Cartesian product of the individual admissible sets excluding the ith one.

Definition 4.2 (Max-Min Solution) *A strategy* $\tilde{a}_i \in A_i$ *is the* max-min solution *for the i-th objective if*

$$u_i(a_1, \ldots, a_{i-1}, \tilde{a}_i, a_{i+1}, \ldots, a_K) = \max_{a_i \in A_i} \min_{a^{-i} \in A^{-i}} u_i(a) \,. \tag{4.4}$$

For the multiobjective problem (4.2), a similar idea can be applied to yield the best possible compromise solution when all objectives are observed with equal priority (see Appendix A). With the max-min approach, one may assure a kind of fairness by, e.g., maximizing the minimum objective in the network [43].

In a network where cooperation is possible, a cooperative equilibrium can be obtained by the use of the following Pareto optimality [3].

Definition 4.3 (Pareto Optimality) *A strategy* $\tilde{a} \in A$ *is* Pareto optimal *if there exists no* $a \in A$ *such that*

$$u_i(a) \geq u_i(\tilde{a}), \quad i = 1, \ldots, K, \tag{4.5}$$

with strict inequalities satisfied for at least one i.

It is optimal in the sense that no objective can be improved without worsening at least one other objective. In general, the Pareto optimal solution is superior to the Nash equilibrium solution. Unfortunately, the Pareto optimality almost always gives a set of solutions. There are many techniques to find a Pareto optimal solution or a set of such solutions. One basic idea is to transform the multiobjective optimization problem into a single objective one, by using methods of objective weighting, distance functions, trade-off, and min-max formulation etc. (see Appendix A). An important characteristic of the cooperative equilibrium is that, unlike the Nash equilibrium, it is unstable with respect to unilateral cheating. Each user can improve his objective individually, provided others do not. Thus, an arbitrator or regulator is needed to enforce the cooperative strategy.

In a cooperative game, users may have to bargain between each other in order to pursue the individual optimality. One bargaining approach is the Nash arbitration scheme [3]. For a convex multiobjective problem, the Nash arbitration scheme can be applied to obtain a unique Pareto optimal solution when the so-called fairness axioms are satisfied [18]. However, the application of this scheme is limited due to the fact that the convexity of a practical problem is difficult to be verified. Another approach involves a bargaining process. In a realistic network, for instance, each user may have a minimum admissible requirement, $\bar{y}_i$, $i = 1, \ldots, K$, which is referred to as the pre-game agreement point in bargaining theory. This pre-game agreement point may also be taken as a possible known decision $u(a^*)$ such as the value at the Nash equilibrium. In this case, a point is Pareto optimal in the sense that it is admissible, at least as good as $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_K)$, and cannot be bettered by any other admissible point. The set of these Pareto optimal points is known as the bargaining set.

As for the criterion of fairness, if each user has the same QOS requirement and priority, fairness means equal performance, e.g., equal throughput, equal average delay, or equal packet drop probability, which is termed as individual balancing fairness. However, when users have various QOS requirements and/or different priorities, the notion of fairness can be quite complicated because each definition of optimality implies a fairness criterion, such as Nash equilibrium fairness, max-min fairness, and Pareto fairness. From this standpoint, a fairness criterion is achieved when the corresponding optimal strategy is adopted.

4.3 Network Description

Assume that there are N users transmitting packets to a base station with controllable power levels and probabilities using slotted ALOHA protocol. These N users are divided into K groups, the ith of which is denoted as U_i and consists of N_i users having the same characteristics and requirements. The users in U_i maintain the same mean arrival power level $\bar{P}_i$ by adaptive power control. To avoid the complex Markov chain assumption whereby the number of states increases exponentially with the number of users, we do not distinguish between newly-arrived packets and retransmitted packets. New arrivals may be regarded as being backlogged immediately on arrival, and the stability of this protocol is discussed in [43]. Then, all the users in U_i are assumed to have the same transmission probability p_i, which may represent transmission rate, permission probability [22], or duty cycle in various protocols. Denote a representative user in U_i as the player ν_i. The capture probability of a packet from ν_i in the presence of $m_1, \ldots, m_K$ packets from respective user groups $U_1, \ldots, U_K$ in a particular slot is denoted as $C_i(m_1, \ldots, m_K)$, which is discussed in Appendix B.

In a heterogeneous network, one needs to evaluate the individual user's performance as well as overall network performance. The basic performance measure for

ν_i is his individual throughput, S_i, defined as the average number of successfully received transmissions per slot

$$S_i = p_i \cdot p_{s,i} \tag{4.6}$$

with $p_{s,i}$ being the packet success probability

$$\begin{aligned} p_{s,i} &= \sum_{m_1=0}^{N_1} \cdots \sum_{m_i=1}^{N_i} \cdots \sum_{m_K=0}^{N_K} C_i(m_1, \ldots, m_K) \binom{N_1}{m_1} p_1^{m_1} (1-p_1)^{N_1-m_1} \\ &\cdots \binom{N_i-1}{m_i-1} p_i^{m_i-1} (1-p_i)^{N_i-m_i} \cdots \binom{N_K}{m_K} p_K^{m_K} (1-p_K)^{N_K-m_K}. \end{aligned} \tag{4.7}$$

Thus, S_i is the throughput that ν_i can attain by transmission at a certain power level and probability given other users' choices.

The average delay of ν_i, τ_i, is closely related to the retransmission protocol. We assume a p_i-persistent transmission policy which means that a packet of ν_i is transmitted persistently with a probability of p_i after it reaches the head of the transmission buffer. The number of times that a packet must be transmitted until it is successfully received is geometrically distributed with a composite success probability of $p_i \cdot p_{s,i}$. We further assume that a packet is dropped after it has been retransmitted for Λ_i slots. Then, the delay of a packet from ν_i is defined as the waiting time (in slots) until it is successfully transmitted or is dropped after reaching the head of the buffer, i.e.,

$$\tau_i = \sum_{l=1}^{\Lambda_i} l(1 - p_i\, p_{s,i})^{l-1} p_i\, p_{s,i} + (\Lambda_i + 1)(1 - p_i\, p_{s,i})^{\Lambda_i}. \tag{4.8}$$

Similarly, we can obtain the packet drop probability of ν_i,

$$Z_i = (1 - p_i\, p_{s,i})^{\Lambda_i + 1}. \tag{4.9}$$

The value of Λ_i can be set by ν_i. Apparently, the larger the value of Λ_i, the higher the reliability, and the longer the average delay of a packet from ν_i. When Λ_i

approaches infinity, there is no packet drop, and τ_i is the mean of the geometric distribution, $1/p_i p_{s,i}$.

The overall network throughput is the sum of the individual throughputs, that is, $S = \sum_{l=1}^{K} N_l S_l$. The normalized network throughput is known as the channel utilization, which is the single maximization objective considered in the next chapter. The average network delay is $\tau = \frac{1}{N}\sum_{l=1}^{K} N_l \tau_l$.

In practice, we may have to maximize some objectives such as throughput, or to minimize others such as average delay and packet drop probability. To apply the multiobjective framework, we need to convert the objectives that are to be minimized into the forms that allow for maximization. For instance, we may use the reciprocal of the average delay, $1/\tau_i$, or the complementary packet drop probability, $1 - Z_i$, as the objective function when τ_i or Z_i is the original objective. After these transformations, all the objectives discussed above are converted to their maximization forms, which are limited in the range of $[0,1]$.

Next we consider the admissible set. The strategy of the group U_i is to assign transmission probability and mean arrival power for each user, that is, $a_i = (p_i, \bar{P}_i)$. When there are no other constraints except upper and lower bounds for p_i and $\bar{P}_i$, we denote the admissible set of U_i as

$$A_i^o(p_i, \bar{P}_i) = \{(p_i, \bar{P}_i) \mid 0 \le p_i \le 1,\ 0 \le \bar{P}_i \le \bar{P}_{m_i}\}, \tag{4.10}$$

where $\bar{P}_{m_i}$ is the maximum mean arrival power of users in U_i. The corresponding network admissible set is denoted as A^o. In a heterogeneous network, users may have other constraints, for instance, the individual throughput of ν_i may be required to be greater than a minimum value, and the average delay or packet drop probability of ν_j may be upper-bounded. In this case, the admissible set of U_i is the intersection of A_i^o with the corresponding constraint set. Another related question is which qualities should be treated as objective functions and which as constraints. This can be best determined by users according to their individual requirements, and the optimal solution may therefore vary.

4.4 Application Examples

In this section, we illustrate the application of the multiobjective optimization framework to the above wireless network model with the background noise ignored. $\beta = 4$ and $z = 3.9811$ (6 dB) are assumed in the capture model (see Appendix B) for the numerical computation.

When there is no capture effect, all packets involved in a collision are destroyed. To achieve the individual balancing fairness and maximize the network throughput, it is known that all the transmission probabilities equal $1/N$, yielding a network throughput of $(1 - 1/N)^{N-1}$. Furthermore, by letting N approach infinity, we obtain the well-known maximum utilization of a slotted ALOHA system with infinite population, e^{-1} [21].

In the presence of the capture effect, the network throughput is increased [41], favoring the users with higher average arrival power levels. If each user attempts to maximize his individual throughput selfishly, he tends to transmit with the largest possible probability and power, which results in the Nash noncooperative equilibrium and is highly undesirable in practice. Thus, a certain kind of cooperation among users is necessary to maintain better operation of the network by controlling the transmission probability or power.

In the following example, we assume that each user has an identical transmission power and consider the optimal strategy of transmission probability.

Example 4.1 Assume that there are three users, ν_1, ν_2, and ν_3, with normalized distances $r_1 = 0.4$, $r_2 = 0.5$, and $r_3 = 1$, respectively. Each user tries to maximize his individual throughput.

When only the near-far effect is considered, according to (B.6), we know that $C_1(1,0,1) = C_2(0,1,1) = 1$ and other capture probabilities are zero. Note that the packets from ν_1 and ν_2 are destructive to each other, and packets from ν_3 can capture the base station only when ν_1 and ν_2 are not transmitting. Clearly, unfairness is created due to users' different distances to the base station. In this case, since the packets from ν_3 have no influence on those from ν_1 and ν_2, the optimal strategy of ν_3 is to have p_3 equal one. Thus, the strategy of the network depends only on the actions of ν_1 and ν_2, and the admissible set is

$$A^o(p_1, p_2, p_3) = \{(p_1, p_2, p_3) \mid 0 \leq p_1 \leq 1,\ 0 \leq p_2 \leq 1,\ p_3 = 1\}. \tag{4.11}$$

Any point in this set defines an admissible solution which corresponds to a point in the space of objective functions. By employing the individual balancing fairness, we obtain the optimum solution at $p_1 = p_2 = 0.5$ ($p_3 = 1$) with $S_1 = S_2 = S_3 = 0.25$. The overall throughput is 0.75, which represents a significant increase over the result without capture (0.444), benefiting from the capture effect. Moreover, fairness is guaranteed at the same time.

In Rayleigh fading channels, applying (B.5) yields the capture probabilities in Table 4.1. By employing the individual balancing fairness again, we obtain the optimum solution at $p_1 = 0.437$, $p_2 = 0.544$, and $p_3 = 1$ with equal individual throughput: $S_1 = S_2 = S_3 = 0.263$. The overall throughput is 0.789, which is greater than the result with only the near-far effect (0.75). This is consistent with the known conclusion that in addition to the near-far capture effect, fading can further improve the overall throughput [41].

Table 4.1. *Capture Probabilities in Rayleigh Fading Channels*

$C_1(1,1,0)$	$C_1(1,0,1)$	$C_2(1,1,0)$	$C_2(0,1,1)$	$C_3(1,0,1)$	$C_3(0,1,1)$	$C_1(1,1,1)$	$C_2(1,1,1)$	$C_3(1,1,1)$
0.380	0.908	0.093	0.801	0.006	0.015	0.345	0.074	0.000

Next, distinct objectives and constraints are assumed for two users in order to illustrate the multiobjective solutions graphically.

Example 4.2 Suppose two of the three users, ν_2 and ν_3, are now present in Rayleigh fading channels with $C_2(0,1,1)$ and $C_3(0,1,1)$ the same as in Table 4.1. ν_2, resembling a data user, tries to maximize his throughput S_2 with a minimum requirement $S_2 \geq 0.2$ while ν_3, resembling a packet voice user, attempts to minimize his average delay τ_3 (or maximize $1/\tau_3$) and requires that $S_3 \geq 0.2$ and

$Z_3 \leq 0.01$ with Λ_3 set to 10. The admissible set in this case is

$$\mathcal{A}(p_2, p_3) = \mathcal{A}^o(p_2, p_3) \cap \{(p_2, p_3) \mid S_2 \geq 0.2\} \cap \{(p_2, p_3) \mid S_3 \geq 0.2 \text{ and } R_3 \leq 0.01\}. \tag{4.12}$$

In Figure 4.5, the shaded area shows the admissible set for ν_2 and ν_3. Correspondingly, S_2 is plotted versus $1/\tau_3$ in Figure 4.5, where the shaded area is the set of admissible values in the objective space. Curve BC is the Pareto optimal set in which p_3 always equals one. Point T is the pre-game agreement point which corresponds to the minimum requirement of the two users. Clearly, ν_2 prefers point C and ν_3 prefers point B. If ν_3 has a higher priority, point B may be selected as the operation point at which ν_2's minimum requirement is still met. If there is no benefit for ν_3 to exceed the minimum requirement, or if ν_2 has a higher priority, then point C may be the choice, at which the throughput of ν_2 (0.52) is much higher than his minimum requirement. These two points are actually solutions of the trade-off method. Other methods can also be used to select the final solution from the Pareto optimal set. For instance, point F corresponds to the solution when the product of the two individual objectives is used as the single network objective; point E corresponds to the min-max solution. Thus, different methods may result in different solutions with different criteria of fairness embedded in.

Three or two users are assumed in the above examples with an identical transmission power. When a larger number of users are present, we can extend the same approach with increased complexity, whereby numerical methods can be applied to obtain multiobjective solutions [2]. Another example is given below where there exist two groups of users.

Example 4.3 We assume that there are two groups of users with $N_1 = 5$ and $N_2 = 10$. According to (5.29), we have

$$p_{s,1} = \left(1 - \frac{zp_1}{1+z}\right)^4 \left(1 - \frac{zp_2\bar{P}_2}{\bar{P}_1 + z\bar{P}_2}\right)^{10}, \tag{4.13}$$

and

$$p_{s,2} = \left(1 - \frac{zp_2}{1+z}\right)^9 \left(1 - \frac{zp_1\bar{P}_1}{\bar{P}_2 + z\bar{P}_1}\right)^5. \tag{4.14}$$

The individual objective of ν_1 (or ν_2) is $S_1 = p_1 p_{s,1}$ (or $S_2 = p_2 p_{s,2}$). The total throughput of the network is $S = N_1 S_1 + N_2 S_2$, which is the global optimization objective of the network. As for the constraint, we assume that ν_1 (or ν_2) requires his individual throughput to be greater than 0.005 (or 0.001). When p_1 and p_2 are not equal to 0, we also assume that $\bar{P}_1$ and $\bar{P}_2$ belong to $[P_{min}, P_{max}]$, where P_{min} is actually limited by the neglected background noise and P_{max} is the maximum mean arrival power. Denote P_{max}/P_{min} as c (≥ 1).

In Figure 4.3, S_1 is plotted versus S_2 with $c = 20$. The shaded area is the admissible set with curve BC being the Pareto optimal set. Points T and Q correspond to the minimum requirement and the ideal solution (see Appendix A), respectively. Point D is the solution of maximizing the network throughput S; point E is the min-max solution; point F is the distance function solution; point H is the individual balancing solution. Points B and C are solutions of the trade-off method with S_1 and S_2 being the respective main objectives. The noncooperative Nash equilibrium corresponds to point J which can be used as the starting point in a bargaining process. The bargaining set is the section of curve BC between the two dash-dotted lines. Pareto optimal sets with different c are shown in Figure 4.4. The result of $c = 1$ corresponds to the case where all users have the same average arrival power and are actually in one group. Point V corresponds to the result without capture. It is clear that a larger c is desirable.

4.5 Conclusions

In this chapter, following a game theoretic approach, we have established a multi-objective optimization framework to analyze the performance of a wireless heterogeneous network in the presence of packet capture. Unfairness is a primary concern in such a network, and cooperation is necessary to achieve both the optimality and the fairness under the arbitration of a network controller. A finite-user network model has been constructed in the presence of Rayleigh fading with individual performances taken into account, and illustrative examples have been presented to demonstrate the application of the framework to the design of optimal strategies in terms of transmission probability and power. Various requirements and criteria can be satisfied by using different cooperative strategies in the network, and the network controller can use the gained information to maintain an optimal and fair operation of the network.

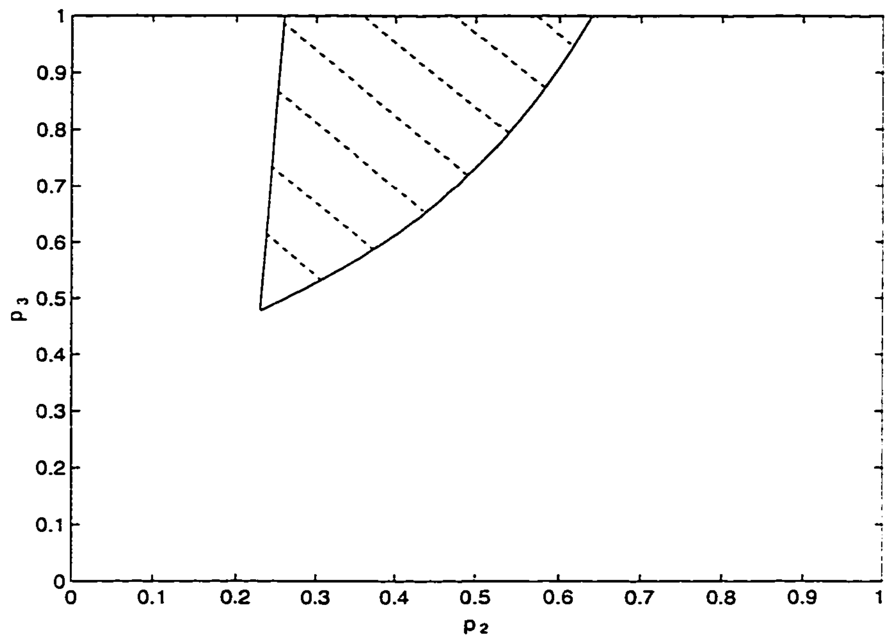

Figure 4.1. *Admissible set for transmission probability of ν_2, p_2, and transmission probability of ν_3, p_3.*

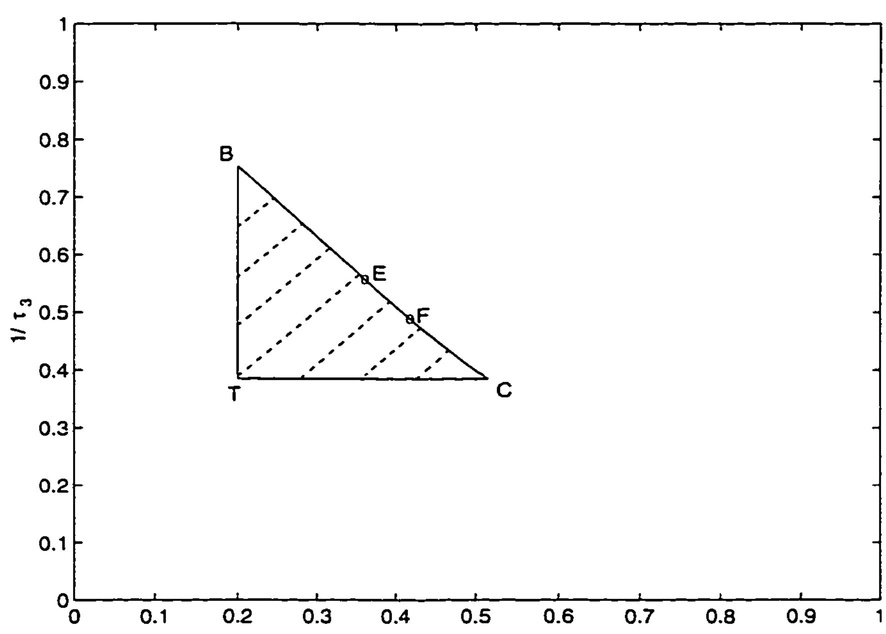

Figure 4.2. *Admissible objective space for throughput of ν_2, S_2, and the reciprocal of average delay of ν_3, $1/\tau_3$.*

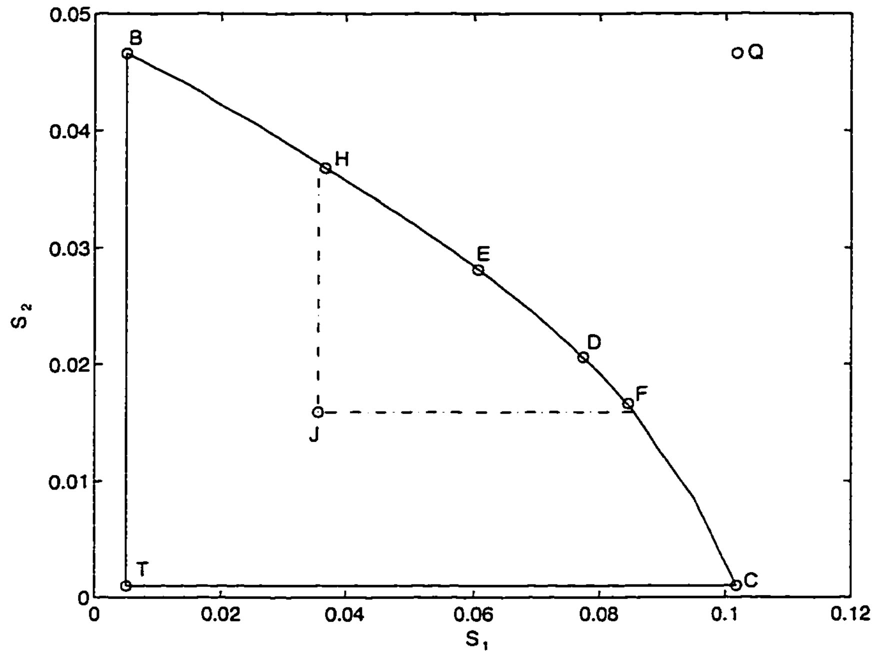

Figure 4.3. *Admissible objective space for* S_1 *and* S_2 *with* $c = 20$.

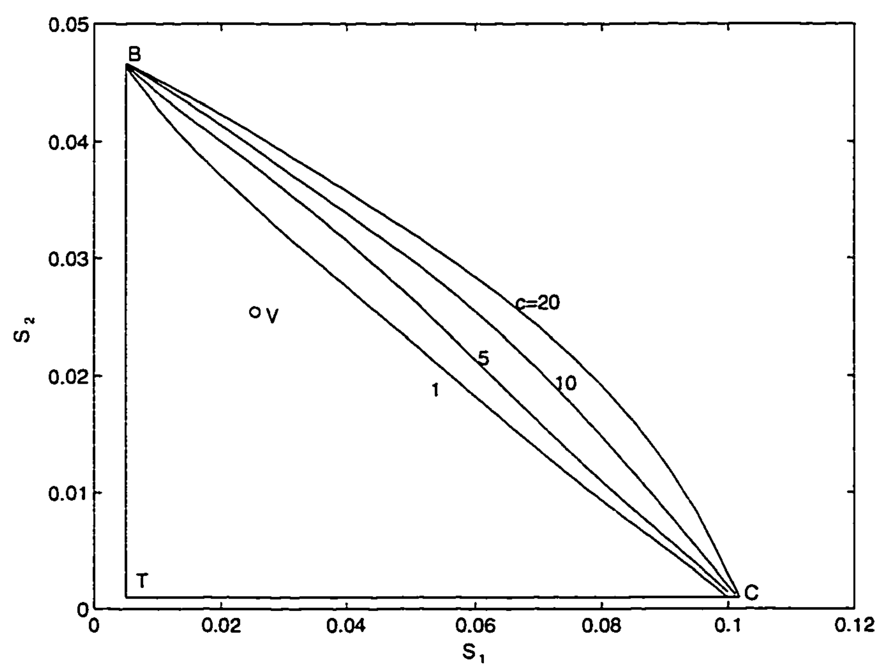

Figure 4.4. *Admissible objective spaces for* S_1 *and* S_2 *with various* c.

Chapter 5

Maximization of Channel Utilization in Wireless Heterogeneous Networks

5.1 Introduction

In this chapter, we study the issue of maximizing the channel utilization in a heterogeneous network. To meet individual requirements fairly, users have to adopt a cooperative strategy, which is possible in a centrally coordinated network. One strategy is to employ power control whereby remote users are assigned appropriate transmission power levels based on their arrival power at the base station. For instance, identical mean arrival power is desirable in single-service packet CDMA (or spread ALOHA) networks to combat the near-far effect [27] [46], while distinct arrival power is assumed in [47] and [48] where users of higher priority or quality are assigned higher arrival power and thus experience a better service (or preferred access). Another strategy is to control transmission probability, which has been addressed in [42] for a homogeneous network.

5.2 Problem Formulation

We adopt the same network model as in the previous chapter. It is known that in a random access network, objectives of individual users are mutually conflicting.

and cooperation among users is necessary in order to achieve both optimality and fairness. In homogeneous cases, maximization of the channel utilization is equivalent to maximization of the individual throughput, and fairness implies equal individual throughput among all users. However, in heterogeneous cases where distinct objectives are involved, a multiobjective optimization problem has to be formulated with users' individual objectives being multiple objectives constrained by individual requirements. When the channel utilization is to be maximized over the set of individual transmission power and probability, the multiobjective problem is transformed to a single-objective problem where the single objective is the sum of individual objectives. Non-negativity and upper limitation for individual transmission power and probability should be included for a complete presentation of the problem; however, we will not repeat these constraints whenever this is obvious. Clearly, the maximization problem under study is sophisticated due to the presence of heterogeneity and nonlinearity, and the solution may vary when there are different constraints. For convenience, we categorize the problem into the following three categories:

1. Maximizing the network throughput without constraints on individual throughput,

$$\max \ S = \sum_{l=1}^{K} N_l S_l. \tag{5.1}$$

2. Maximizing the network throughput as in (5.1) where each user requires an identical throughput,

$$S_1 = S_2 = \cdots = S_K. \tag{5.2}$$

3. Maximizing the network throughput as in (5.1) where users in some groups have specific requirements and the remaining users require an identical throughput,

$$\begin{aligned} &S_i = c_i, \ 1 \leq i \leq j, \\ &S_{j+1} = S_{j+2} = \cdots = S_K. \end{aligned} \tag{5.3}$$

5.3 Strategies to Maximize Channel Utilization

In this section, we discuss solutions to the maximization problem. For all numerical evaluation, we assume a narrow-band system with $\beta = 4$ and $z = 3.981$ (6 dB) in the capture model. We use the superscripts $\star$, $*$, and $\circ$, respectively, to denote the solutions to the three categories of problems described above.

5.3.1 Strategy of Transmission Power

First, we consider a simple case where all the N users require the same QOS: namely, they are in one group with an identical transmission probability p. If each user also transmits with an identical power level, nearby users may experience a higher capture probability and receive a better service unfairly. Therefore, power control is necessary to ensure an equal individual throughput among all users by maintaining equal mean arrival power $\bar{P}$.

A. Near-Far Effect Only

Equal arrival power, which should be greater than zN_A in the presence of background noise, leads to the scenario of standard slotted ALOHA whereby all the packets involved in a collision are destroyed. The arrival packet of one user is successful only when all the other users are not transmitting, yielding an individual throughput of $p(1-p)^{N-1}$. The advantage of capture is undesirably lost and the maximum network throughput is obtained at $p = 1/N$ with

$$S^{\star} = \left(1 - \frac{1}{N}\right)^{N-1}. \tag{5.4}$$

B. Presence of Rayleigh Fading

According to (B.5), the capture probability of a packet among j simultaneously arriving packets is $e^{-z\bar{\gamma}^{-1}}(1+z)^{-j+1}$ where the mean signal-to-noise ratio (SNR).

$\bar{\gamma}$, is identical for all users. Then, each user has the same success probability

$$p_s = e^{-z\bar{\gamma}^{-1}} \sum_{l=1}^{N} \binom{N-1}{l-1} p^{l-1}(1-p)^{N-l}(1+z)^{-l+1} = e^{-z\bar{\gamma}^{-1}} \left(1 - \frac{pz}{1+z}\right)^{N-1}. \quad (5.5)$$

Obviously, the success probability decreases with z and increases with $\bar{\gamma}$. The network throughput is maximized with respect to p at

$$p^* = \frac{1}{N}(1 + \frac{1}{z}) \quad (5.6)$$

with the maximum network throughput being

$$S^* = e^{-z\bar{\gamma}^{-1}} \left(1 + \frac{1}{z}\right) \left(1 - \frac{1}{N}\right)^{N-1}. \quad (5.7)$$

S^* is plotted versus $\bar{\gamma}$ for $N = 10$ and $z = 3.9811$ in Figure 5.1. When $\bar{\gamma} > 12.5$ dB, it is larger than the throughput with only the near-far effect, owing to the capture effect in the presence of Rayleigh fading. However, as $\bar{\gamma}$ decreases below 12.5 dB (but still > 6 dB), this advantage disappears and Rayleigh fading decreases the network throughput instead. Therefore, the maximum utilization is actually determined by the most distant user who imposes a limit on $\bar{P}$ and thus on $\bar{\gamma}$.

5.3.2 Strategy of Transmission Probability

Next, we consider another special case where there is only one user in each group (i.e., $N = K$) and all the N users have an identical transmission power P_T: that is, power control is not used. The action of the only user in U_i, ν_i, is to control his transmission probability p_i with the packet success probability as expressed in (4.7) where $N_j = 1$ and $m_j = 0$ or 1, $1 \leq j \leq N$. Assuming that ν_i's distance to the base station is r_i, the capture probability can be simplified to

$$C_i(m_1, \ldots, m_K) = \begin{cases} 1, & \text{if } z\left(r_i^{\beta} \sum_{l=1, l\neq i}^{K} m_l r_l^{-\beta} + \bar{\gamma}_i^{-1}\right) < 1 \\ 0, & \text{otherwise} \end{cases} \quad (5.8)$$

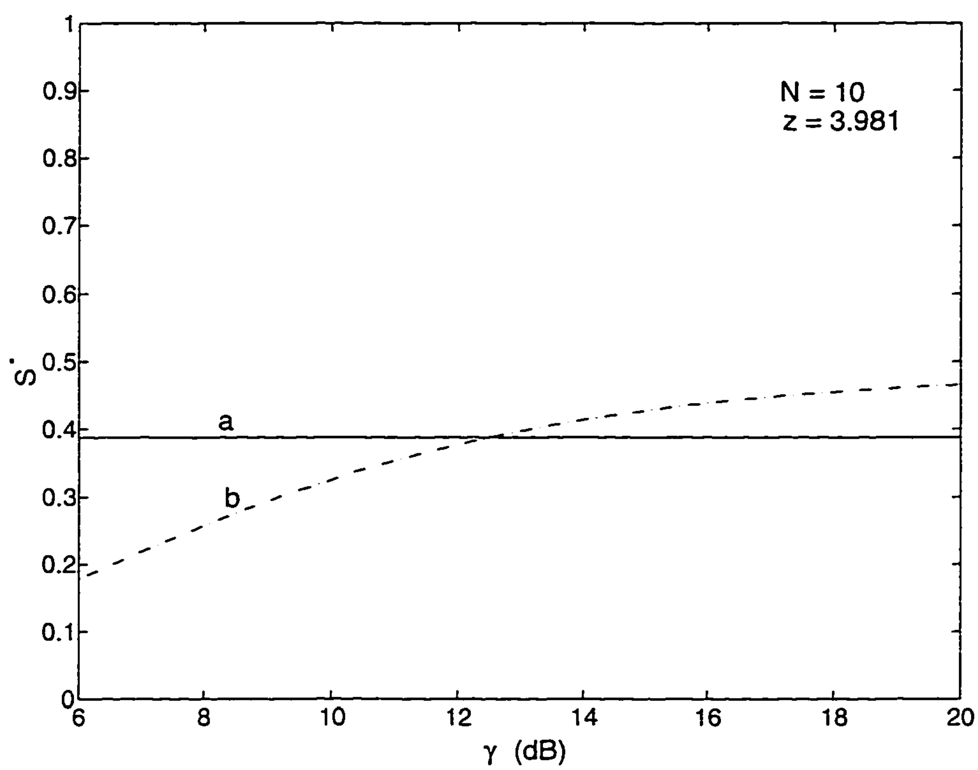

Figure 5.1. *Maximum channel utilization when all users are in one group: a) with near-far effect only; b) in the presence of Rayleigh fading.*

in a near-far environment. When Rayleigh fading is also present, the following occurs

$$C_i(m_1, \ldots, m_K) = e^{-z\bar{\gamma}_i^{-1}} \prod_{l=1, l\neq i}^{K} (1 + zr_l^{-\beta} r_i^{\beta})^{-m_l}. \tag{5.9}$$

Generally, we can not obtain the closed-form solution for the maximization problem in this case, since the individual throughput is a nonlinear function of p_i and r_i, $i = 1, \ldots, N$. Numerical approach is used for an example of three users in the following, while more involved numerical computation can be applied similarly to the case of more users. Another approach is presented in [49] where the distribution of cumulative interference power is studied in a similar situation for a Rayleigh fading channel with infinite population.

Example 5.1 Similarly as in Example 4.1, assume that there are three users, ν_1, ν_2, and ν_3, with normalized distances, $r_1 = 0.4$, $r_2 = 0.5$, and $r_3 = 1$, respectively.

A. Near-Far Effect Only

According to (5.8), we know that $C_1(1,0,1) = C_2(0,1,1) = 1$ at a moderate mean SNR ($\bar{\gamma}_2 > 7.2$ dB) and other capture probabilities are zero. It is clear that without the constraints on individual throughput, the maximum network throughput is one, but the solution is not unique. A trivial solution is at $p_1^* = 1$, $p_2^* = 0$, and an arbitrary p_3^* with $S_1^* = 1$, $S_2^* = 0$, and $S_3^* = 0$. Apparently, neither ν_2 nor ν_3 has access to the base station, causing the extreme unfairness. If they all transmit with an identical probability, maximization of network throughput results: $p_1^s = p_2^s = p_3^s = 0.451$, $S_1^s = S_2^s = 0.248$, $S_3^s = 0.136$, and $S^s = 0.632$. Although the network throughput is increased in comparison with the throughput without capture (0.444) owing to the benefit of capture effect, unfairness favoring the closer users is created due to users' different distances to the base station.

Now, we consider the optimal transmission probabilities under the fairness criterion of equal individual throughput. Obviously, the optimal strategy of ν_3 is to have a unit transmission probability ($p_3^* = 1$), and one only needs to determine the transmission probabilities of ν_1 and ν_2. It is easy to obtain the maximization solution at $p_1^* = 0.5$ and $p_2^* = 0.5$ with $S_1^* = S_2^* = S_3^* = 0.25$. Although the maximum network throughput $S^* = 0.75$ is less than one, it represents a significant increase over the throughput without capture; moreover, fairness is obtained at the same time.

B. Presence of Rayleigh Fading

The capture probabilities can be calculated at different $\bar{\gamma}_3$ by applying (5.9), as listed in Table 5.1 ($C_3(1,1,1)$ equals zero in all the three cases). Constrained by equal individual throughput, the maximization solution is obtained as listed in Table 5.2.

From the above results, we can conclude that in most of the range of the mean SNR of interest the maximum channel utilization in Rayleigh fading channels is

Table 5.1. *Capture Probabilities in Rayleigh Fading Channels*

$\bar{\gamma}_3$(dB)	$C_1(1,1,0)$	$C_1(1,0,1)$	$C_2(1,1,0)$	$C_2(0,1,1)$	$C_3(1,0,1)$	$C_3(0,1,1)$	$C_1(1,1,1)$	$C_2(1,1,1)$
20	0.380	0.907	0.093	0.799	0.006	0.015	0.344	0.074
10	0.376	0.898	0.091	0.781	0.004	0.010	0.341	0.073
5	0.368	0.879	0.086	0.740	0.002	0.004	0.334	0.069

Table 5.2. *Maximization Solution with Equal Individual Throughput*

$\bar{\gamma}_3$(dB)	p_1^*	p_2^*	p_3^*	$S_1^*(=S_2^*=S_3^*)$	S^*
20	0.437	0.544	1.000	0.262	0.786
10	0.436	0.548	1.000	0.259	0.777
5	0.427	0.543	0.950	0.250	0.750

greater than that in the presence of only the near-far effect. Compared with the throughput of (5.7) at $N = 3$, it can be seen that for a narrow-band system, transmission probability control is more effective than power control when equal individual throughput is required among users.

In the case that some users have specific throughput requirements, say, ν_1 requires his individual throughput to be 0.3, we may then seek the optimum strategy to share the remaining throughput equally between ν_2 and ν_3. By maximizing S constrained by $S_1 = 0.3$ and $S_2 = S_3$, we get the solution as given in Table 5.3. As compared with the results in Table 5.2, it is clear that the specific requirement

Table 5.3. *Maximization Solution with $S_1 = 0.3$ and $S_2 = S_3$*

$\bar{\gamma}_3$(dB)	p_1°	p_2°	p_3°	S_1°	$S_2^\circ(=S_3^\circ)$	S°
20	0.496	0.537	0.991	0.300	0.236	0.772
10	0.494	0.529	0.953	0.300	0.231	0.762
5	0.491	0.511	0.874	0.300	0.219	0.738

of ν_1 is satisfied at the expense of a decreased maximum channel utilization. Also, since a larger SNR corresponds to a larger maximum utilization, an increase in transmission power P_T is justified.

5.3.3 Joint Strategy

A better channel utilization is anticipated when a joint control strategy is used when both p_i and $\bar{P}_i$ can be adjusted. Without loss of generality, we assume that the mean arrival power decreases from the first group to the Kth group

$$\bar{P}_1 > \bar{P}_2 > \cdots > \bar{P}_K. \tag{5.10}$$

A. Near-Far Effect Only

In this case, the capture probability of ν_i is

$$C_i(m_1, \ldots, m_K) = \begin{cases} 1, & \text{if } m_j = 0 \ (0 < j < i);\ m_i = 1;\ z\left(\sum_{l=i+1}^{K} m_l \bar{P}_l + N_A\right) < \bar{P}_i \\ 0, & \text{otherwise.} \end{cases} \tag{5.11}$$

By properly assigning transmission power levels, we can realize the so-called group preemptive strategy, which is appealing in terms of increasing the capture probability and, therefore, individual throughput. Here, we define the group preemptive strategy as follows. When exactly one packet from a user in the group with a high arrival power arrives at the same time as one or more packets from users in the groups with lower arrival power levels, it always preempts other packet(s) and succeeds; when two or more packets arrive from the same high arrival power group, they compete like standard slotted ALOHA, irrespective of other packets with lower arrival power levels.

As a sufficient condition to achieve the group preemptive strategy, we further desire the following recursive relation,

$$P_i > z\left(\sum_{l=i+1}^{K} N_l P_l + N_A\right),\ 1 \le i < K. \tag{5.12}$$

The transmission power levels of the users in the Kth group are assigned such that one single packet from a user in U_K can capture the base receiver when it arrives alone. Other power levels can be assigned recursively by applying (5.12), which results in

$$C_i(m_1,\ldots,m_K)=\begin{cases}1, & \text{if } m_j=0\ (0<j<i);\ m_i=1\\ 0, & \text{otherwise.}\end{cases} \tag{5.13}$$

In this case, the success probability is

$$p_{s,i}=\begin{cases}(1-p_1)^{N_1-1}, & i=1\\ (1-p_i)^{N_i-1}\prod_{l=1}^{i-1}(1-p_l)^{N_l}, & i=2,\ldots,K.\end{cases} \tag{5.14}$$

which decreases monotonically from U_1 to U_K. The corresponding network throughput is

$$S=N_1p_1(1-p_1)^{N_1-1}+\sum_{l=2}^{K}N_lp_l(1-p_l)^{N_l-1}\prod_{j=1}^{l-1}(1-p_j)^{N_j}. \tag{5.15}$$

For the maximum of (5.15) without individual throughput constraints, it is shown in Appendix C that the solution is

$$p_i^*=\begin{cases}\frac{1}{N_K}, & i=K\\ \frac{1-(1-p_{i+1}^*)^{N_{i+1}-1}}{N_i-(1-p_{i+1}^*)^{N_{i+1}-1}}, & i=K-1,\ldots,1\end{cases} \tag{5.16}$$

with the maximum network throughput being

$$S^*=(1-p_1^*)^{N_1-1}. \tag{5.17}$$

It is noted that the above maximization is achieved without considering fairness among different groups. For instance, when there is one user in each group, the solution is $S^*=1$ with all transmission probabilities equal to one, which implies that the whole channel is used only by the user in the first group.

In the case of infinite population in each group, by assuming $N_j\to\infty$ for $1\le j\le K$ in (5.16), we get the maximum channel utilization (see Appendix C)

$$\overline{S}^*(K)=\begin{cases}e^{-1}, & K=1;\\ \exp(-1+\overline{S}^*(K-1)), & K\ge 2.\end{cases} \tag{5.18}$$

It is easy to see that $\overline{S}^*(\infty) = 1$. Note that (5.18) is a generalization of a result in [47]. However, as mentioned earlier, unfairness also exists with preferred access achieved for higher arrival power groups. For instance, at $K = 2$, the sum of the throughput of the first and the second groups is 0.336 and 0.196, respectively.

To achieve the fairness of equal individual throughput, it is shown in Appendix C that the solution for the constrained maximization problem is

$$p_i^* = \begin{cases} \frac{1}{N_K}, & i = K \\ \frac{p_{i+1}^*(1-p_{i+1}^*)^{N_{i+1}-1}}{1+p_{i+1}^*(1-p_{i+1}^*)^{N_{i+1}-1}}, & i = K-1,\dots,1. \end{cases} \tag{5.19}$$

The maximum network throughput is then

$$S^* = Np_1^*(1-p_1^*)^{N_1-1}, \tag{5.20}$$

which is equally shared among all users with each individual throughput being $p_1^*(1-p_1^*)^{N_1-1}$.

In the case of infinite population in each group, assuming that $N_1 = N_2 = \cdots = N_K \to \infty$, we have the following maximum channel utilization (see Appendix C)

$$\overline{S}^*(K) = \begin{cases} e^{-1}, & K = 1; \\ \frac{K}{K-1}\overline{S}^*(K-1)\exp(-\frac{\overline{S}^*(K-1)}{K-1}), & K \geq 2, \end{cases} \tag{5.21}$$

which is equally shared among the K groups. It is also clear that $\overline{S}^*(\infty) = 1$. The results of (5.18) and (5.21) are shown in Figure 5.2, from which we observe that fairness is achieved with a slightly reduced channel utilization at a given K.

Considering the special case that there is only one user in each group, according to (5.19), the optimum transmission probability of U_i is

$$p_i^* = \frac{1}{N+1-i}, \tag{5.22}$$

with the maximum channel utilization being one. Then, each user enjoys equal throughput of $1/N$. Perfect channel utilization is achieved for an arbitrary number

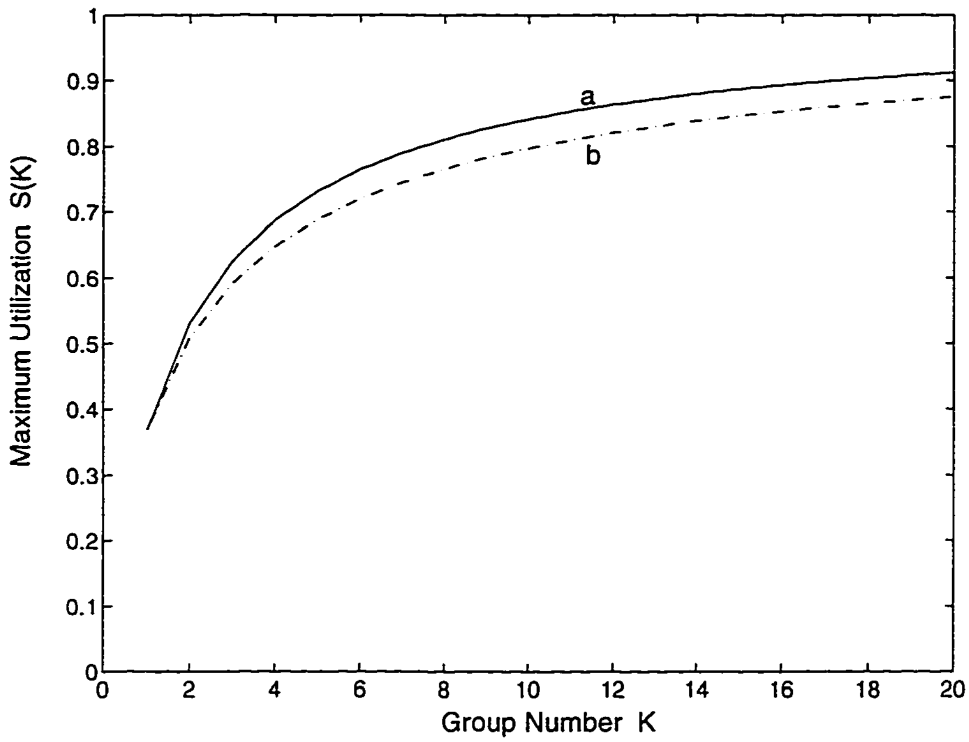

Figure 5.2. *Maximum channel utilization with infinite population using group preemptive strategy in a near-far environment: a) without individual throughput constraints; b) with equal individual throughput among users.*

of users owing to the fact that there is no mutually destructive collision among packets from the same group.

If the high arrival power groups have specific throughput requirements, the optimum solution can be readily obtained to share the remaining throughput equally among the other groups under the preemptive strategy. For instance, if one user in U_1 requires the individual throughput to be c_1, we can choose p_1° to be the solution of

$$p_1^\circ(1 - p_1^\circ)^{N_1 - 1} = c_1, \tag{5.23}$$

and assign transmission probabilities of other groups according to (5.19). The maximum network throughput is then

$$S^\circ = N_1 c_1 + (N - N_1) p_2^\circ (1 - p_2^\circ)^{N_2 - 1} (1 - p_1^\circ)^{N_1}. \tag{5.24}$$

with $S^\circ - N_1 c_1$ equally shared among the $N - N_1$ users of other groups. When there is only one user in each group, assuming that the first j users have specific individual throughput requirements

$$S_i = c_i, \; 1 \leq i \leq j, \tag{5.25}$$

the maximization solution occurs at

$$p_i^\circ = \begin{cases} c_1, & i = 1 \\ \frac{c_i}{1-\sum_{l=1}^{i-1} c_l}, & i = 2, \ldots, j \\ \frac{1}{N+1-i}, & i = j+1, \ldots, N. \end{cases} \tag{5.26}$$

with the individual throughput being

$$S_i^\circ = \begin{cases} c_i, & i = 1, \ldots, j \\ \frac{1-\sum_{l=1}^{j} c_l}{N-j}, & i = j+1, \ldots, N. \end{cases} \tag{5.27}$$

We then have the maximum network throughput $S^\circ = 1$ and perfect channel utilization is achieved again in this case.

Remark: Since the probability of simultaneous transmission of a large number of users in lower arrival power groups may be quite small, inequality (5.12) is actually too strong and may be relaxed to some extent to yield a near preemptive result. For instance, we may relax (5.12) to

$$\bar{P}_i > z(\omega \bar{P}_{i+1} + N_A), \tag{5.28}$$

which assumes, approximately, that a single arrival packet from ν_i is captured if there is no transmission from the same or higher arrival power groups, and no more than ω users in the $(i+1)$th group are transmitting. With the success probability modified correspondingly, the maximum solution can be obtained similarly. This practical approach can also be applied to the case where a finite dynamic range is assumed for transmission power.

B. Presence of Rayleigh Fading

Next, we address briefly the effect of Rayleigh fading on a joint strategy. By applying (4.7) and (5.5), we get the success probability for ν_i

$$p_{s,i} = e^{-z\bar{\gamma}_i^{-1}} \left(1 - \frac{zp_i}{1+z}\right)^{N_i - 1} \prod_{l=1, l\neq i}^{K} \left(1 - \frac{zp_l \bar{P}_l}{\bar{P}_i + z\bar{P}_l}\right)^{N_l} . \qquad (5.29)$$

However, maximization of the network throughput is in general intractable for a practical problem, in which case numerical methods may be resorted to.

When the group preemptive strategy is used where transmission power levels satisfy (5.12), near preemptive results can be obtained with a reduced maximum channel utilization, as illustrated in the following example.

Example 5.2 In the previous three-user example, when only the near-far effect is present, we can assign transmission power levels according to (5.12) to achieve the preemptive group strategy. For instance, $P_{T1} = (0.508\vartheta + 0.026)P_c$, $P_{T2} = (0.249\vartheta + 0.063)P_c$, and $P_{T3} = \vartheta P_c$, where P_c is a power level yielding a mean SNR of 3.9811 and ϑ is a constant greater than one. Then, each user attains equal individual throughput of 1/3 by employing respective transmission probabilities $p_1^* = 1/3$, $p_2^* = 1/2$, and $p_3^* = 1$ according to (5.22).

In Rayleigh fading channels, by assuming $\bar{\gamma}_3 = 10$ dB with $\vartheta = 2.512$, we can obtain the equal individual throughput solution at $p_1^* = 0.350$, $p_2^* = 0.535$, and $p_3^* = 0.728$ with $S^* = 0.690$ which is smaller than that achieved by controlling only the transmission probability. However, we can further adjust transmission power and probability properly to get a better result. In Figure 5.3, the maximum channel utilization versus P_{T1} is shown with different P_{T2} at $\bar{\gamma}_3 = 10$ dB using optimum transmission probabilities under the equal individual throughput requirement. P_{T1} and P_{T2} are normalized with respect to P_{T3}. It may be seen that a better channel utilization can be achieved with appropriately assigned transmission power and probability.

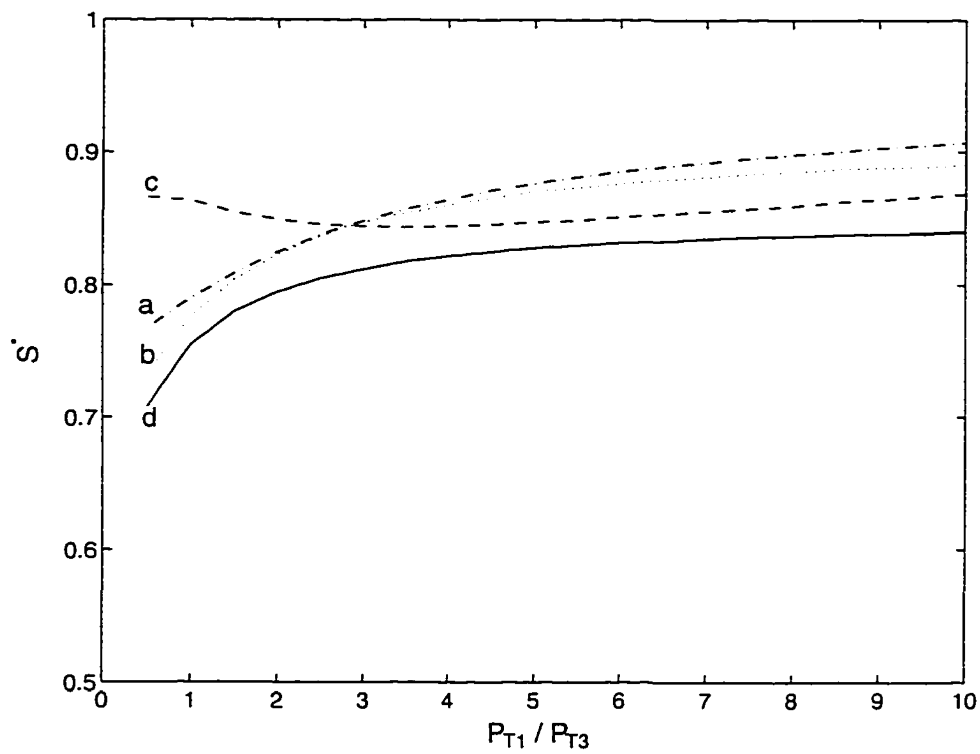

Figure 5.3. *Maximum channel utilization versus P_{T1}/P_{T3} in Rayleigh fading channels under equal individual throughput requirement: a) $P_{T2}/P_{T3} = 2$; b) $P_{T2}/P_{T3} = 1$; c) $P_{T2}/P_{T3} = 10$; d) $P_{T2}/P_{T3} = 0.5$.*

5.4 Conclusions

In this chapter, we have studied the issue of maximizing the channel utilization in a wireless heterogeneous network where slotted ALOHA is used as the random access protocol in the presence of background noise and Rayleigh fading. Unfairness created by the capture effect has been taken into consideration in the maximization problem. Different strategies for controlling transmission power and/or probability are compared in terms of their effectiveness to achieve the maximum channel utilization. It has been shown that for a narrow-band system under equal individual throughput requirement, transmission probability control is more effective than power control, and that perfect or near perfect channel utilization can be achieved through the use of a joint strategy without or with Rayleigh fading.

Chapter 6

Capacity of a Multimedia DS/CDMA System in Multipath Fading Channels

6.1 Introduction

Sharing spectral resource efficiently while meeting different requirements. which is a multiobjective problem in nature. is critically important to the forthcoming multimedia PCS. Various multiple access protocols have been proposed for this purpose including ALOHA, TDMA, and CDMA [50]. In particular, direct sequence (DS) CDMA has drawn much attention due to its inherent advantages such as statistical multiplexing gain, anti-multipath and anti-jam capability, and privacy [27]–[30].

A typical wireless link tends to suffer seriously from multiuser interference and channel fading as well as background noise, yielding a much higher channel error rate than a wireline link. Thus, effective ECC is needed to ensure the required error performance by providing different error protection capabilities to different services in an adaptive way [51] [52]. Forward error control (FEC), automatic repeat request (ARQ), or hybrid ARQ may be employed for different applications, e.g., delay stringent or nonstringent ones. Diversity also plays a crucial role in minimizing transmission power and protecting the link during the short intervals when the channel is severely faded. As the capacity of a CDMA system is mainly interference limited, mitigation of channel fading through the use of ECC and diversity can make the system operate at a lower power level, which in turn leads to a higher capacity sustaining more users. In a single service CDMA system, it is necessary to control transmission power in order to combat the near-far problem [27]; in a multimedia system, power control can be readily adapted to meeting diverse requirements as studied in [53] and [54]. In this chapter, we analyze the maximum admissible objective space for users numbers, or system capacity, in multipath Rayleigh fading channels for multimedia CDMA systems incorporating average power control, ECC, and time diversity in the form of RAKE reception.

6.2 System and Channel Model

Consider the reverse link of a multimedia cellular system where users of different services transmit continuous or bursty information to a base station. CDMA is used by remote users to share a common spread spectrum channel with a total bandwidth of W, and each user is assigned a distinct pseudonoise (PN) sequence code. Similarly as in the previous chapters, we assume that there are K types (or groups) of services with N_i users in the ith type, and a representative user of the ith type service is denoted by ν_i. It is well-known that in a CDMA system using quasi-orthogonal codes, the signals of other active users act as interference to the desired signal. Therefore, it is necessary that the power levels of signals arriving at the base station be tightly controlled. We also assume perfect power control so that the mean arrival signal power is the same for all the users in one group. Denote the source bit rate of ν_i as R_{bi} and the coded bit rate (or channel symbol rate) of ν_i as R_{si} taking account of FEC redundancy and possible ARQ retransmission. Then, the (uncoded) processing gain and the coded processing gain for ν_i are $G_{bi} = W/R_{bi}$ and $G_{si} = W/R_{si}$, respectively.

Unlike a single service system where the user number of the service is the only maximization objective, a multimedia system calls for the formulation of a multiobjective problem with N_i being the individual objective of the ith service ($1 \leq i \leq K$). Then, one needs to evaluate the maximum admissible objective space. As we are primarily concerned with the numbers of users near the capacity of the system, the standard Gaussian approximation for the multiple access interference (MAI) is reasonably accurate when the processing gain is large [55] [56] [27]. Therefore, the average MAI to ν_i when all the other users are active is $\varsigma\left(\sum_{l=1,l\neq i}^{K} N_l P_l + (N_i - 1)P_i\right)$, where $\varsigma = \varsigma_1\varsigma_2$ is a multiplicative coefficient with ς_1 accounting for different detection methods and ς_2 accounting for the interference from users associated with other base stations. For instance, ς_1 may equal one for noncoherent detection [27] or 2/3 for coherent detection [56], and ς_2 may be as high as 1.6 [28].

In terms of bit rate, a multimedia system may offer available bit rate (ABR), variable bit rate (VBR), and constant bit rate (CBR) services. For bursty ABR and VBR services, a simplified Markov traffic flow model is often used with two states: active and idle. For CBR services, a duty cycle may be assumed when the source rate is lower than the peak transmission rate. To take the average traffic into consideration, we attach an activity coefficient δ_i ($0 < \delta_i \leq 1$) to ν_i, which is equivalent to the transmission probability in the previous chapters except that it may not be controllable now. Then, denoting I_0 as the spectral density of the sum of MAI and background noise, we have E_b/I_0 for ν_i as follows:

$$\gamma_{bi} = \frac{P_i/R_{bi}}{\varsigma[\sum_{l=1,l\neq i}^{K} \delta_l N_l P_l + \delta_i(N_i - 1)P_i]/W + N_0} = \frac{P_i G_{bi}}{\varsigma(P_a - \delta_i P_i) + W N_0}, \quad (6.1)$$

where $P_a = \sum_{l=1}^{K} \delta_l N_l P_l$ which is the average total power received at the base station. With E_s denoting the signal energy per coded bit, we have E_s/I_0 for ν_i:

$$\gamma_{si} = \frac{P_i G_{si}}{\varsigma(P_a - \delta_i P_i) + W N_0}, \quad (6.2)$$

from which we can see that the introduction of error correction redundancy decreases G_{si} and thus γ_{si}.

Usually, each service has a minimum QOS requirement in the form of, e.g., average BER or outage probability, which may be translated into an equivalent requirement on E_b/I_0 (or E_s/I_0). We assume that ν_i needs at least $\hat{\gamma}_{bi}$ in E_b/I_0 to meet an acceptable QOS; or, equivalently,

$$P_i \geq \frac{\hat{\gamma}_{bi}[\varsigma(P_a - \delta_i P_i) + WN_0]}{G_{bi}}, \quad 1 \leq i \leq K. \tag{6.3}$$

When the equality holds, users of all services sustain the minimum acceptable QOS and achieve the maximum system capacity [54]. When there is a large number of users in the system, we have $P_a \gg \delta_i P_i$ and can reasonably neglect $\delta_i P_i$ in the right-hand side of (6.3). Then, we can conclude that $\hat{\gamma}_{bi} R_{bi}/P_i$, denoted as κ, should be approximately identical for $i = 1, \ldots, K$ when the system capacity is achieved, which can be used as a criterion to control the transmission power. Furthermore, the admissible objective space for the multiobjective problem satisfies

$$\varsigma \sum_{l=1}^{K} \delta_l N_l \hat{\gamma}_{bl} R_{bl} \leq W(1 - N_0 \kappa). \tag{6.4}$$

When the equality holds, we obtain a hyperplane corresponding to the Pareto optimal solutions of the problem. From (6.4), we know that it is obviously effective to increase the system capacity by reducing the required E_b/I_0 for all services, especially for those having a low power budget and a high rate or quality requirement thus imposing a limit on the admissible objective space. Reduction of the required E_b/I_0 can be well accomplished by ECC, as will be discussed later. From the viewpoint of system design, we can allocate the spectrum and the power for different services according to (6.4) based on their requirements on maximum user number (or individual capacity) and system complexity. Dynamic resource allocation can be realized through the use of a medium access control (MAC) protocol [30], where connections are accepted or denied based on the type of the service and the traffic load. Once a connection is accepted, the system provides a statistical QOS averaged across all connections in the system, known as the statistical connection.

Though we have assumed perfect instantaneous power control in the above discussions, shadowing and multipath fading in a practical mobile channel cause the signal power levels to randomly vary. Since the time constant of the shadowing tends to be relatively small, we do not need to take shadowing into account in the analysis, while fading caused by multipath and motion of the users plays a more important role during the transmission. Hereafter, we assume a Rayleigh fading environment with perfect average power control maintained accordingly. The MAI is also approximated as Gaussian noise in view of the fact that faded interferers look more Gaussian than nonfaded ones [57]. Since the channel coherent bandwidth is much smaller than the typical transmission bandwidth of the DS/CDMA signal, the channel is frequency selective with resolvable paths. In this case, L_d paths or L_d copies of the signal are available at the receiver, where L_d is the ratio of the transmission bandwidth to the coherence bandwidth.

It is known that when resolvable paths are available, diversity combining, such as maximum ratio combining (MRC), is effective in demodulating the transmitted signal. In the system under study, an L-path ($L \leq L_d$) MRC RAKE receiver is used, and then γ_b is the E_b/I_0 of the combined signal,

$$\gamma_b = \sum_{l=1}^{L} \gamma_b^{(l)} \tag{6.5}$$

where $\gamma_b^{(l)}$ is the instantaneous E_b/I_0 of the lth path. Let $\bar{\gamma}_b$ denote the average E_b/I_0 and $p(\gamma_b|\bar{\gamma}_b)$ denote the probability density function (pdf) of E_b/I_0 given $\bar{\gamma}_b$. For illustration purpose, we simply assume that signal of each path has the same average E_b/I_0 of $\bar{\gamma}_b^{(c)}$ and thus $\bar{\gamma}_b = L\bar{\gamma}_b^{(c)}$. By further assuming that each multipath signal is independent, we get the following conditional pdf for E_b/I_0 of a combined signal [58]:

$$p(\gamma_b, L|\bar{\gamma}_b) = \frac{L^L}{(L-1)!}\frac{\gamma_b^{L-1}}{\bar{\gamma}_b^L}e^{-L\gamma_b/\bar{\gamma}_b}. \tag{6.6}$$

6.3 Information Theoretical Considerations

To obtain the theoretical upper bound for the system capacity, we evaluate the information theoretical limits for reliable communication based on Shannon capacity and cutoff rate.

Shannon capacity for the bit rate of a single user in an ideal band-limited Gaussian channel can be expressed as [58]

$$C = W\log_2\left(1 + \frac{R_b}{W}\gamma_b\right). \tag{6.7}$$

When the channel capacity is achieved, i.e., $R_b = C$, we have

$$\hat{\gamma}_b = (2^{1/G_b} - 1)G_b, \tag{6.8}$$

with $\hat{\gamma}_b \to \ln 2$ as $G_b \to \infty$. This is the minimum $\hat{\gamma}_b$ that can be achieved in the average Shannon limit sense where each user is operating independently without cooperative interference cancellation [28].

In Rayleigh fading channels, Shannon capacity is also a random variable. Assuming that the transmitter has no knowledge of the channel parameters, we first calculate the channel capacity in an average sense, which indicates the average best in the memoryless fading environment and is always lower than that of a Gaussian noise channel [59]. In the presence of the RAKE reception, the following expression for average Shannon capacity results:

$$\bar{C}(L) = \int_0^\infty W\log_2\left(1 + \frac{\gamma_b}{G_b}\right)p(\gamma_b, L|\bar{\gamma}_b)\,d\gamma_b. \tag{6.9}$$

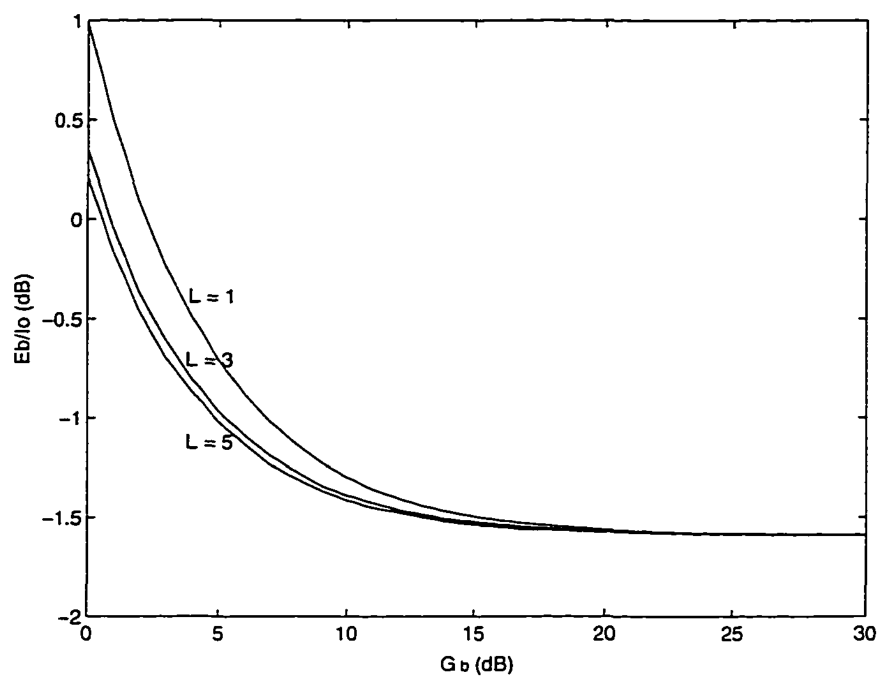

Figure 6.1. *E_b/I_0 required to achieve the average Shannon capacity versus processing gain G_b with various L in Rayleigh fading channels.*

where $p(\gamma_b, L|\bar{\gamma}_b)$ is given in (6.6). Without diversity, i.e., $L = 1$, we can solve (6.9) as [60]

$$\bar{C}(1) = -W \log_2 e \cdot e^{G_b/\bar{\gamma}_b} \cdot \mathrm{Ei}(-G_b/\bar{\gamma}_b), \tag{6.10}$$

where $\mathrm{Ei}(x)$ is the exponential-integral function. It is shown in [59] that $\bar{C}(L)$ approaches the capacity of a Gaussian noise channel when L approaches infinity.

In Figure 6.1, E_b/I_0 that is required to achieve the channel capacity is plotted versus processing gain G_b. From this figure we can see that the required E_b/I_0 decreases drastically as processing gain increases from 0 dB up to about 10–15 dB where E_b/I_0 starts to hit the -1.59 dB floor. Thus, from the viewpoint of average Shannon capacity, a very high processing gain is not necessary and 10–15 dB is sufficient.

It is noted that the average Shannon capacity applies only if the number of transmitted symbols is sufficiently large, which may imply a very large delay. Nevertheless, the maximum admissible objective space in Shannon limit sense constitutes an upper bound for the transmission scheme without stringent delay constraint and is suitable for comparison purposes. Furthermore, capacity under a decoding delay constraint does not exist in the strict Shannon sense [61]. Therefore, we consider the capacity versus outage where the outage probability is defined as the percentage of the time that the instantaneous Shannon capacity is below some acceptable threshold, $\tilde{C}$. That is,

$$P_{out}(\tilde{C}) = \Pr\{C < \tilde{C}\}. \tag{6.11}$$

Assuming $\tilde{\gamma}_b$ is the solution to $C = \tilde{C}$ in (6.8), we get

$$P_{out}(\tilde{C}) = \int_0^{\tilde{\gamma}_b} p(\gamma_b, L|\bar{\gamma}_b)\, d\gamma_b = 1 - e^{-L\tilde{\gamma}_b/\bar{\gamma}_b} \sum_{l=0}^{L-1} \frac{1}{l!} \left(\frac{L\tilde{\gamma}_b}{\bar{\gamma}_b}\right)^l. \tag{6.12}$$

Since $\tilde{\gamma}_b$ is an implicit function of G_b in (6.9) and (6.12), the theoretical minimum of $\bar{\gamma}_b$ at a given G_b can be obtained numerically, which can then be substituted in (6.4) to yield the Shannon limit for the admissible objective space in multipath fading channels.

Next, we consider the computational cutoff rate which represents a practically achievable data rate for reliable transmission. For a binary channel, we know that the cutoff rate can be expressed as [7]

$$R_0(\gamma_s) = 1 - \log_2(1 + D) \tag{6.13}$$

where D is the Bhattacharyya bound which only depends on the coding channel and the choice of decoder metric. When L-path MRC is used for BPSK signals, we have [7]

$$D = \left[\frac{L}{L + \gamma_s}\right]^L. \tag{6.14}$$

Using the code with a rate $R < R_0(\gamma_s)$ yields

$$\gamma_b > R_0^{-1}(R)R^{-1}. \tag{6.15}$$

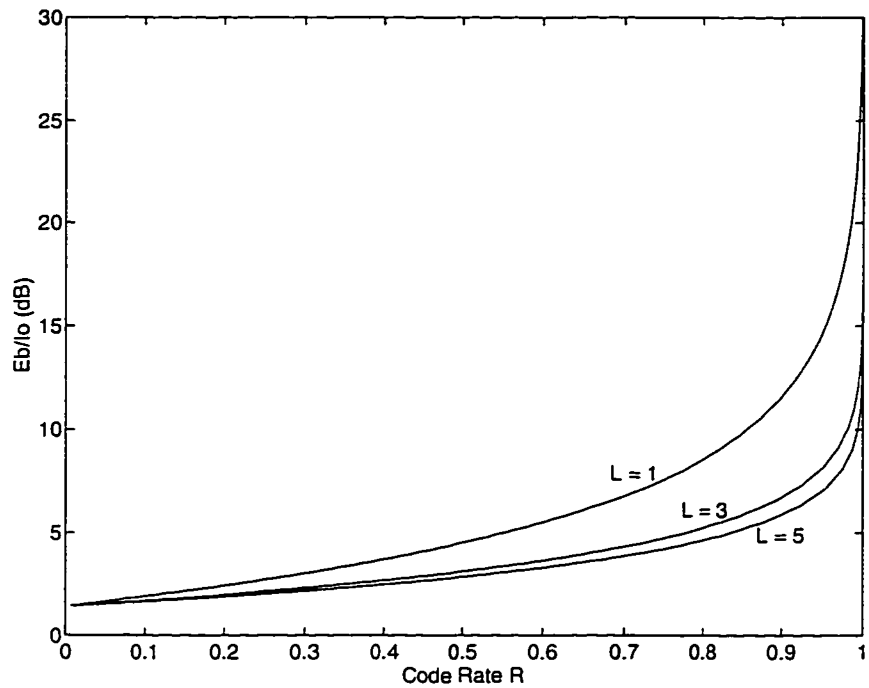

Figure 6.2. *E_b/I_0 required to achieve the cutoff rate versus code rate R with various L in Rayleigh fading channels.*

The minimum required γ_b, which is the lower bound in (6.15), versus the code rate is plotted in Figure 6.2. From it, we can observe that, while the optimum code rate turns out to be zero corresponding to a minimum γ_b of 1.42 dB, an extremely low code rate is practically unnecessary. With minimum diversity, code rates between 0.1 and 0.3 would provide sufficient coding gain from the viewpoint of cutoff rate.

As for hard decision decoding in a memoryless symmetric channel, we know that the corresponding Shannon limit is

$$C = 2W[1 + p\log_2 p + (1-p)\log_2(1-p)]. \tag{6.16}$$

As for the cutoff rate, we have

$$D = \sqrt{4p(1-p)}. \tag{6.17}$$

In (6.16) and (6.17), p is the output BER of a given receiver. Then we can similarly obtain the information theoretical limits in terms of channel capacity and cutoff rate for hard decision decoding.

6.4 Performance Measures

It is noted that different performance measures may be used by different services, e.g., delay stringent and delay nonstringent services. Delay stringent services usually result from voice and video, for which we may be concerned with the outage probability among other performance measures. Delay nonstringent service is typically data, for which the most important error performance measure is the average BER. In the following, we examine the average BER and the outage probability, both of which will be used in a numerical example later. Since the subsequent discussion is applicable to each and every service, we omit the explicit index i where no ambiguity arises.

Denote the instantaneous BER in the presence of L-path MRC receiver as $P_b(\gamma_s, L)$. For BPSK signals, for example, we have

$$P_b(\gamma_s, L) = Q(\sqrt{2\gamma_s}), \tag{6.18}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy$ and γ_s is the E_s/N_0 of the combined signal. If binary DPSK (BDPSK) is used, we have [58]

$$P_b(\gamma_s, L) = \frac{1}{2^{2L-1}} e^{-\gamma_s} \sum_{l=0}^{L-1} b_l \gamma_s^l, \tag{6.19}$$

where b_l is given by

$$b_l = \frac{1}{l!} \sum_{n=0}^{L-1-l} \binom{2L-1}{n}.$$

The average BER is obtained as follows [58]:

$$\bar{P}_b(\bar{\gamma}_s, L) = \int_0^\infty P_b(\gamma_s, L) p(\gamma_s, L|\bar{\gamma}_s) d\gamma_s = \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k. \tag{6.20}$$

where

$$\mu = \sqrt{\frac{\bar{\gamma}_s}{L + \bar{\gamma}_s}}$$

for BPSK, and

$$\mu = \frac{\bar{\gamma}_s}{L + \bar{\gamma}_s}$$

for BDPSK.

The outage probability is now defined as the percentage of the time that the instantaneous BER is above some acceptable threshold, ε. For voice service, the threshold may be as high as 10^{-2} to ensure toll quality speech; for video or data service, the BER requirements are more stringent. Since $P_b(\gamma_s, L)$ is a monotonically decreasing function of γ_s, the outage probability is given by

$$P_{out}(\tilde{\gamma}_s) = \Pr\{\gamma_s < \tilde{\gamma}_s\}, \tag{6.21}$$

where $\tilde{\gamma}_s$ is the solution to $P_b(\gamma_s, L) = \varepsilon$. We can similarly define $\tilde{\gamma}_b$ instead of $\tilde{\gamma}_s$.

For a required value of outage probability $\hat{P}_{out}$, we can obtain a corresponding $\bar{\gamma}_s$ and use it in (6.4) to obtain the admissible objective space. Since P_{out} is a function of the ratio $\lambda = \tilde{\gamma}_s/\bar{\gamma}_s$, reducing $\tilde{\gamma}_s$ at a fixed threshold ε, e.g., with the use of an efficient ECC scheme, can also reduce the required $\bar{\gamma}_s$ by the same amount. In Figure 6.3, we plot the outage probability versus λ. It is clear that for the same outage probability, higher order diversity corresponds to a significantly smaller λ, or a smaller $\bar{\gamma}_s$ at a given $\tilde{\gamma}_s$.

6.5 Error Correction Scheme

ECC is both the ultimate way to approach the theoretical limit on the system capacity and the effective way to increase it and to reduce the power consumption. In a CDMA system, the bandwidth of the transmitted signal is spread over a much larger bandwidth than that of the baseband signal, thus allowing enough coding

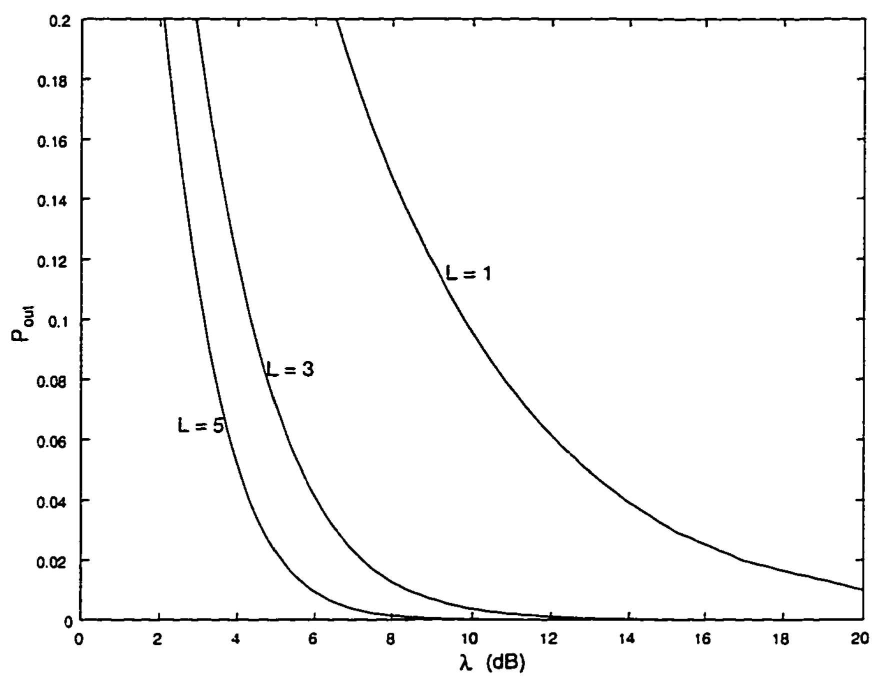

Figure 6.3. *Outage probability versus ratio $\lambda = \bar{\gamma}_s/\tilde{\gamma}_s$ with various L in Rayleigh fading channels.*

redundancy. Since the coding gain is the reduction of E_b/I_0 at a required QOS, the code chosen should provide the highest coding gain at the BER of interest for the service by the use of a low rate code [62] or a moderate low rate code combined with the PN sequence.

FEC may be used for delay stringent services, while hybrid ARQ may be used for delay nonstringent services where an error-free instantaneous feedback channel is assumed. Since the feedback does not increase the capacity of a memoryless channel [63], the theoretical limits obtained previously still provide a comparable upper bound when ARQ is used. It is obvious that both FEC and ARQ increase the transmission rate and thus decrease the coded processing gain or E_s/I_0.

Practically, the encoder and decoder are preferably so designed as to be independent of service type, thus permitting a single encoder/decoder pair to be used for all service types and offering a variable degree of error protection scheme. An adaptive concatenated coding scheme is adopted where convolutional and Reed-

Solomon codes are used as the inner and the outer codes, respectively. A symbol interleaver is placed in between to randomize the burst errors at the output of the Viterbi decoder, and sufficient channel bit interleaving is also assumed to render the fading channel memoryless.

For the Viterbi decoding of a convolutional code, we know that the output BER is bounded as [64]

$$P_b^V = \frac{1}{k} \sum_{j=d_{free}}^{\infty} \delta_j p(j) . \tag{6.22}$$

where k is the number of input bits per encoding interval, d_{free} is the free distance of the convolutional code. δ_j is the number of erroneous bits in the adversary paths at Hamming distance j from the correct path, and $p(j)$ is the probability that a wrong path at distance j is selected. For hard decision decoding, we have [64]

$$p(j) = \begin{cases} \sum_{l=(j+1)/2}^{j} \binom{j}{l} p^l (1-p)^{j-l}, & j \text{ odd}, \\ \frac{1}{2}\binom{j}{j/2} p^{j/2}(1-p)^{j/2} + \sum_{l=j/2+1}^{j} \binom{j}{l} p^l (1-p)^{j-l}, & j \text{ even}. \end{cases} \tag{6.23}$$

In the case of soft decision decoding using L-path RAKE receiver, we have

$$p(j) = \bar{P}_b(j\bar{\gamma}_s, jL) \tag{6.24}$$

as in (6.20).

Reed-Solomon codes are nonbinary maximum distance separable (MDS) codes that are strongly separable and strongly invertible [64]. An important advantage of using the Reed-Solomon code is its simultaneous error correction and detection capability, which is desirable in a hybrid ARQ scheme. Another advantage is that the same hardware can be used in an adaptive way when variable coding rate is required. The basic parameters of a Reed-Solomon (n, k) code are defined as follows: $n = 2^m - 1$ and $d_{min} = n - k + 1$ where $m \geq 2$ is the number of bits per symbol and d_{min} is the minimum distance of the code. The maximum number of correctable symbol errors is $t = \lfloor (d_{min} - 1)/2 \rfloor$. Assuming that P_s is the symbol error rate at the input to the Reed-Solomon decoder and P_b^V is the output BER of the Viterbi decoder, we then have

$$P_s \leq 1 - (1 - P_b^V)^m . \tag{6.25}$$

Let P_{pd}, P_{pu}, and P_{pc} denote the probabilities of a packet containing detectable errors (decoder failure), undetectable errors (decoder error), and no errors, respectively. We have $P_{pd} + P_{pu} + P_{pc} = 1$. When a decoder failure did not occur, the information portion of each coded packet is passed on to the next layer. When a decoding failure occurs, the received information portion is "marked". For video or voice, marked information may lead to blanking and interpolation. When ARQ is applied to high reliability services such as data, marked data indicates a retransmission request. At the output of the bounded distance decoder, we have the probability of no error as

$$P_{pc} = \sum_{j=0}^{t} \binom{n}{j} P_s^j (1 - P_s)^{n-j}. \tag{6.26}$$

The probability of decoder error is [64]

$$P_{pu} = \sum_{l=d_{min}}^{n} B_l P_{l_t}, \tag{6.27}$$

where B_l is the number of code words of weight l given by

$$B_l = \binom{n}{l} (2^m - 1) \sum_{j=0}^{l-d_{min}} (-1)^j \binom{l-1}{j} 2^{m(l-j-d_{min})}$$

and P_{l_t} is given by

$$P_{l_t} = \sum_{v=0}^{t} \sum_{w=0}^{t-v} \binom{n-l}{v} \binom{l}{w} (2^m - 1)^{w-l} \left(1 - \frac{P_s}{2^m - 1}\right)^{w} P_s^{l+v-w} (1 - P_s)^{n-l-v}.$$

The probability of decoder failure is $P_{pd} = 1 - P_{pc} - P_{pu}$.

For a FEC scheme, the output BER of a Reed-Solomon decoder without retransmission is approximately [58]

$$P_b^R = \frac{2^{m-1}}{n(2^m - 1)} \sum_{j=t+1}^{n} j \binom{n}{j} P_s^j (1 - P_s)^{n-j}. \tag{6.28}$$

For a Type-I ARQ scheme, the Reed-Solomon code is used for both error correction and detection, and a truncated protocol is assumed to reduce the transmission delay and simplify the receiver buffer design. Denote the maximum number of retransmission times as Λ. The packet error rate is

$$P_{pe}(\Lambda) = P_{pu} \sum_{j=1}^{\Lambda+1} P_{pd}^{j-1} + P_{pd}^{\Lambda+1} = \frac{P_{pc} P_{pd}^{\Lambda+1} + P_{pu}}{P_{pc} + P_{pu}}. \tag{6.29}$$

The average number of packet transmissions is

$$\Phi(\Lambda) = (1 - P_{pd}) \sum_{j=1}^{\Lambda} j P_{pd}^{j-1} + (\Lambda + 1) P_{pd}^{\Lambda} = \frac{1 - P_{pd}^{\Lambda+1}}{P_{pc} + P_{pu}}. \tag{6.30}$$

For unlimited retransmission $\Lambda \to \infty$, we have $P_{pe} = \frac{P_{pu}}{P_{pc}+P_{pu}}$ and $\Phi = \frac{1}{P_{pc}+P_{pu}}$. Expression of BER for the ARQ scheme can be obtained by exploiting the structure of the Reed-Solomon code:

$$P_b^A(\Lambda) = P_{ub} \sum_{j=1}^{\Lambda+1} P_{pd}^{j-1} + P_b^R P_{pd}^{\Lambda+1}, \tag{6.31}$$

where P_{ub} is the BER of the undetected error packet,

$$P_{ub} = \frac{2^{m-1}}{n(2^m - 1)} \sum_{l=d_{min}}^{n} A_l \bar{P}_{l_t}. \tag{6.32}$$

with $\bar{P}_{l_t}$ being

$$\bar{P}_{l_t} = \sum_{v=0}^{t} \sum_{w=0}^{t-v} (l+v-w) \binom{n-l}{v} \binom{l}{w} (2^m - 1)^{w-l} \left(1 - \frac{P_s}{2^m - 1}\right)^{w} P_s^{l+v-w} (1 - P_s)^{n-l-v}.$$

6.6 Numerical Example

Consider a system accommodating two types of service: voice with $R_{b1} = 8$ kbits/s and data with $R_{b2} = 64$ kbits/s. The voice user has an activity factor of 0.375 and a stringent delay constraint, and requires the outage probability to be less than 0.01 with a BER threshold of 10^{-3}. The data user has a burstiness of 0.2 and a moderate delay constraint, and requires the average BER to be less than 10^{-6}. We are concerned with the reverse link transmission with $W = 5$ MHz and $\varsigma = 1.3$. To take the effect of background noise into consideration, we assume $N_0\kappa = 0.05$. FEC is used for voice users and hybrid ARQ is used for data users, both with a concatenated coding scheme as described in the previous section.

To be compatible with the ATM protocol, we consider a Reed-Solomon code (255, 48) with byte size (8 bit) symbols. Through puncture, this code can be modified to yield a generalized (n', 48) code with byte sized symbols where $255 \leq n' \leq 48$. The puncture operation is performed by deleting $255 - n'$ symbol coordinates from the (255, 48) code, thus resulting in Reed-Solomon codes of rates from 0.1882 to 1 including 1/4, 3/8, 1/2, 5/8, 3/4, and 7/8. By removing one symbol from the code, the minimum distance of the code can be reduced by at most one; thus, the obtained set of punctured code words must also be MDS [64]. As for the convolutional codes, we consider those of rates 2/3, 1/2, 1/3, and 1/4 with a constraint length of 7 [65] which can be obtained through puncture of the same code.

In Tables 6.1–6.4, the required E_b/I_0 is listed for voice and data users in fading channels with $L = 3$ and $L = 5$. In the table, "hard" and "soft" mean hard and soft decision decoding, respectively; the numbers in the first column correspond to the code rates of the convolutional codes while the numbers in the second row correspond to the code rates of the Reed-Solomon codes. The maximum retransmission times for data users is chosen to be three by taking account of the moderate delay requirement. It is observed that different optimum combination rates exist in different case while the outer Reed-Solomon code of rate 7/8 almost always gives the best result. Thus, although a smaller code rate is theoretically desirable, the concatenated scheme presented here shows that a moderate code rate corresponds to the best performance, which may imply that a more involved ECC structure is needed to further approach the theoretical limit in multipath fading channels. However, taking into account the decoding complexity and the negligible theoretical increase in coding gain for very low rate ECC, one can choose a compromise code rate. Higher order diversity yields a smaller required E_b/I_0 for voice users in both BPSK and BDPSK cases and is very effective in reducing the outage probability. However, for data users, it helps slightly for BPSK signals, but increases the required E_b/I_0 for BDPSK signals due to noncoherent combination loss.

Based on the above results, we plot the system capacity region in Figures 6.4 and 6.5 for BPSK and BDPSK signals, respectively, with the information theoretical upper bound shown in Figure 6.4. The increase in capacity due to a properly chosen ECC is drastic, and a hybrid ARQ scheme is desirable for services having moderate delay constraint and low BER requirement. Soft decision of BPSK signals with higher order diversity yields the best result in comparison with other cases. Thus, an adaptive ECC and diversity scheme with appropriately chosen parameters is necessary for optimizing system performance in multimedia applications. On the other hand, the power difference between voice and data users varies in different cases. For instance, in the case of soft decision decoding for BPSK signals, this difference is 2 dB for $L = 3$, which decreases to 0.39 dB for $L = 5$. This eases the design of power control in addition to increasing the system capacity in such a multimedia network.

6.7 Conclusions

In this chapter, we have studied the system capacity for multimedia CDMA systems in terms of the maximum admissible objective space of the user numbers of all services in multipath Rayleigh fading channels, incorporating average power control, RAKE receiver, and error correction coding. To provide theoretical bounds on system capacity, information theoretical limits based on Shannon capacity and cutoff rate have been evaluated. Power control is used to sustain the minimum QOS requirements for all services and achieve the system capacity, while ECC and RAKE reception are employed to increase the admissible objective space and reduce the power consumption. Hybrid ARQ and FEC with hard or soft decision decoding have been considered with a concatenation of Reed-Solomon and convolutional codes. It has been demonstrated through an example that proper design of the system can increase the capacity significantly.

Table 6.1. *Required E_b/I_0 (dB) for voice users using BPSK*

		$L=3$						$L=5$					
		1/4	3/8	1/2	5/8	3/4	7/8	1/4	3/8	1/2	5/8	3/4	7/8
1/4	hard	16.7	15.2	14.2	13.5	13.1	12.9	14.2	12.7	11.7	11.0	10.6	10.4
	soft	14.6	13.1	12.1	11.4	11.0	10.8	12.1	10.6	9.6	8.9	8.5	8.3
1/3	hard	17.0	15.4	14.4	13.6	13.2	13.0	14.5	12.9	11.9	11.1	10.7	10.5
	soft	14.8	13.2	12.2	11.4	11.0	10.8	12.3	10.7	9.7	8.9	8.5	8.3
1/2	hard	18.0	16.4	15.3	14.4	13.9	13.6	15.5	13.9	12.8	11.9	11.4	11.1
	soft	15.7	14.1	13.0	12.2	11.7	11.3	13.2	11.6	10.5	9.7	9.2	8.8
2/3	hard	19.3	17.7	16.6	15.8	15.3	15.0	16.8	15.2	14.1	13.3	12.8	12.5
	soft	16.4	14.8	13.7	12.9	12.4	12.0	13.9	12.3	11.2	10.4	9.9	9.5

Table 6.2. *Required E_b/I_0 (dB) for voice users using BDPSK*

		$L=3$						$L=5$					
		1/4	3/8	1/2	5/8	3/4	7/8	1/4	3/8	1/2	5/8	3/4	7/8
1/4	hard	20.6	19.2	17.9	17.1	16.6	16.2	18.1	16.5	15.4	14.6	14.1	13.7
	soft	18.9	17.4	16.3	15.5	15.0	14.7	16.4	14.9	13.8	13.0	12.5	12.2
1/3	hard	20.4	18.7	17.6	16.8	16.2	15.9	17.9	16.2	15.1	14.3	13.7	13.4
	soft	18.2	16.6	15.6	14.8	14.3	14.1	15.7	14.1	13.1	12.3	11.8	11.6
1/2	hard	20.5	18.8	17.7	16.8	16.3	15.8	18.0	16.3	15.2	14.3	13.8	13.3
	soft	17.9	16.3	15.3	14.5	14.0	13.8	15.4	13.8	12.8	12.0	11.5	11.3
2/3	hard	21.2	19.5	18.4	17.6	17.0	16.6	18.7	17.0	15.9	15.1	14.5	14.1
	soft	18.2	16.6	15.5	14.8	14.3	14.0	15.7	14.1	13.0	12.3	11.8	11.5

Table 6.3. *Required E_b/I_0 (dB) for data users using BPSK*

		L = 3						L = 5					
		1/4	3/8	1/2	5/8	3/4	7/8	1/4	3/8	1/2	5/8	3/4	7/8
1/4	hard	9.0	7.5	6.5	5.8	5.5	5.5	8.7	7.2	6.2	5.5	5.2	5.2
	soft	5.9	4.4	3.5	2.8	2.5	2.6	5.8	4.3	3.4	2.7	2.4	2.5
1/3	hard	9.4	7.9	6.9	6.2	5.8	5.8	9.1	7.5	6.5	5.8	5.4	5.3
	soft	6.1	4.6	3.6	2.9	2.6	2.6	6.0	4.5	3.5	2.8	2.4	2.5
1/2	hard	10.8	9.2	8.2	7.4	7.0	6.9	10.3	8.7	7.6	6.8	6.4	6.2
	soft	7.0	5.4	4.4	3.7	3.3	3.3	6.8	5.2	4.2	3.5	3.1	3.0
2/3	hard	12.9	11.4	10.4	9.7	9.3	9.4	12.1	10.5	9.4	8.7	8.3	8.3
	soft	8.5	6.9	5.9	5.2	4.7	4.7	8.2	6.6	5.6	4.8	4.4	4.2

Table 6.4. *Required E_b/I_0 (dB) for data users using BDPSK*

		L = 3						L = 5					
		1/4	3/8	1/2	5/8	3/4	7/8	1/4	3/8	1/2	5/8	3/4	7/8
1/4	hard	14.7	13.1	12.0	11.2	10.7	10.5	15.2	13.6	12.5	11.7	11.2	11.0
	soft	12.9	11.3	10.3	9.4	8.9	8.6	13.4	11.8	10.7	9.9	9.5	9.2
1/3	hard	14.6	12.9	11.9	11.1	10.6	10.3	15.0	13.4	12.3	11.5	11.0	10.7
	soft	12.6	11.0	9.9	9.1	8.6	8.2	13.0	11.4	10.3	9.5	9.0	8.8
1/2	hard	15.2	13.5	12.4	11.7	11.1	10.8	15.3	13.7	12.6	11.7	11.2	10.9
	soft	12.9	11.3	10.1	9.3	8.8	8.4	12.9	11.3	10.2	9.4	8.9	8.6
2/3	hard	16.7	15.1	14.1	13.3	12.9	12.8	16.4	14.8	13.7	12.9	12.4	12.3
	soft	13.3	11.7	10.6	9.8	9.2	8.9	13.5	11.8	10.7	9.9	9.4	9.1

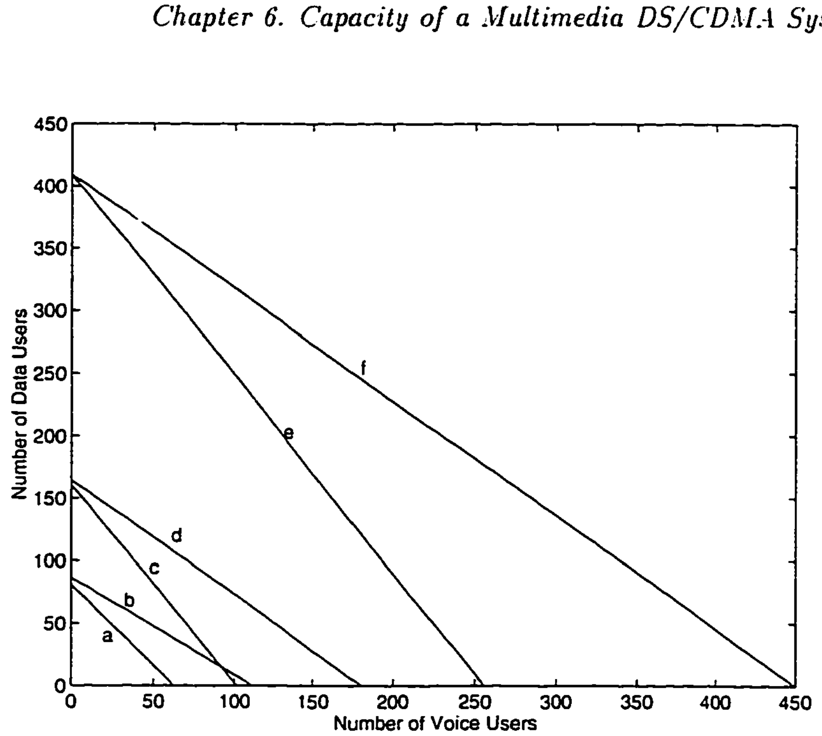

Figure 6.4. *System capacity using BPSK in Rayleigh fading channels: a) $L = 3$ and hard decision; b) $L = 5$ and hard decision; c) $L = 3$ and soft decision; d) $L = 5$ and soft decision; e) Shannon limit with $L = 3$; f) Shannon limit with $L = 5$.*

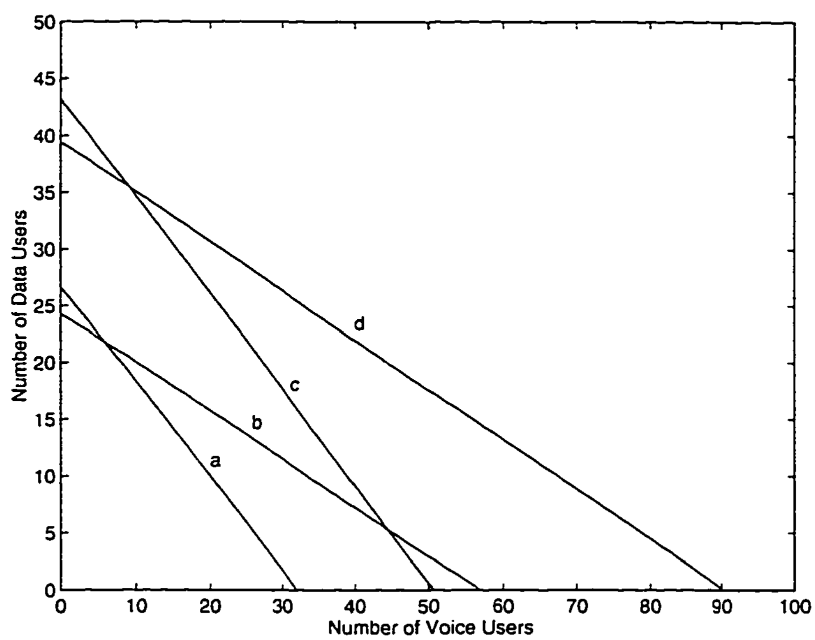

Figure 6.5. *System capacity using BDPSK in Rayleigh fading channels: a) $L = 3$ and hard decision; b) $L = 5$ and hard decision; c) $L = 3$ and soft decision; d) $L = 5$ and soft decision.*

Chapter 7

Conclusions

7.1 Summary of the Dissertation

In this dissertation, several multiobjective applications in wireless communications have been studied based on noncooperative or cooperative approaches. Spread spectrum and ALOHA are used in the system design based on the fact that these two techniques have a common theoretical foundation [24].

In Chapter 2, a game theoretic study has been conducted for an anti-jam communication system. The concept of equilibrium is emphasized, and characterizations of equilibrium, such as existence, uniqueness, stability, robustness, and sensitivity, are examined.

In Chapter 3, the game theoretic model has been applied to a FFH MFSK system where ratio-threshold diversity combining is used to combat the PBN or MT jamming. For the performance measure, either the cutoff rate or the BER is employed depending on whether ECC is adopted. System performance in various situations has been evaluated through numerical analysis. It is shown that the jammer tends to use MT jamming and choose a proper α, and then the communicator has to choose a corresponding (θ, m) at $M = 8$ or $M = 4$ in the absence or presence of coding to achieve a better result.

In Chapter 4, a multiobjective analytic framework has been established for the study of individual performance requirements in a heterogeneous wireless network. Several examples have been given for the design of optimal strategies in a slotted ALOHA network to illustrate the application of this framework. Multiobjective solutions under Pareto optimality are discussed.

In Chapter 5, further to the study in Chapter 4, the issue of maximizing the channel utilization has been addressed for a slotted ALOHA network in the presence of background noise and Rayleigh fading. Optimal strategies controlling transmission probability and/or transmission power are investigated. It is shown that for a narrow-band system under equal individual throughput requirement, transmission probability control is more effective than power control, and that perfect or near perfect channel utilization can be achieved by the use of a joint strategy without or with Rayleigh fading.

In Chapter 6, the system capacity of a multimedia CDMA network in multipath Rayleigh fading channels has been evaluated. To support multimedia applications and maximize the system capacity, average power control, error correction coding, and time diversity are incorporated in the system with an adaptive scheme. An example is presented to show the effectiveness of an appropriate system design.

7.2 Suggestions for Future Research

Though we have described a general game theoretic model in Chapter 2 for anti-jam communications and applied it to a FFH/FSK system, more work is yet to be done to consider practical situations such as incomplete information case and multi-period game. As for the multiobjective framework in a cooperative network, it is important to address characterizations of the optimization solution to provide a solid foundation for its application.

Slotted ALOHA is taken as a representative protocol in Chapters 4 and 5; however, extension of the work to other protocols can be done readily. It is also of interest to study the cases of multi-state assumption and inaccurate parameter assignments. As for system capacity of CDMA systems, more research is needed to further explore the ECC scheme to approach the theoretic limit, e.g., by introducing erasure in decoding Reed-Solomon codes with an optimum erasure threshold based on reliable information from the Viterbi decoder.

Another interesting topic is to establish a general model to include various kinds of interference from both noncooperative and cooperative sources and study the corresponding optimum strategies.

Appendix A

Methods to Find Pareto Optimal Solutions

There are various methods to transform a multiobjective optimization problem into a single-objective optimization problem [1] [2], some of which are listed below.

First, we define the so-called ideal solution which is determined by finding separately admissible maxima for all the objective functions. Let $\hat{y}_i$ denote the maximum value of the ith objective $u_i(a)$, and then the vector $\hat{y} = (\hat{y}_1, \ldots, \hat{y}_K)$ is the ideal solution.

A. Method of Objective Weighting

The global objective is determined by the sum of the single objective functions $u_1, \ldots, u_K$ with the corresponding weighting factors $w_1, \ldots, w_K$:

$$g(a) = \sum_{i=1}^{K} w_i u_i(a), \quad a \in A, \tag{A.1}$$

where

$$0 \leq w_i \leq 1, \quad \sum_{i=1}^{K} w_i = 1. \tag{A.2}$$

B. Method of Distance Function

The single-objective problem is to minimize the distance to the ideal solution:

$$g(a) = \left[\sum_{i=1}^{K} |u_i(a) - \hat{y}_i|^r \right]^{1/r}, \quad a \in A, \quad 1 \leq r < \infty, \tag{A.3}$$

where the variation of r meets various interpretations of the distance, e.g., $r = 2$ corresponds to Euclidean metric.

C. Method of Min-max Formulation

The single-objective problem is to minimize

$$g(a) = \max_{i=1,\ldots,K} t_i(a), \quad a \in A, \tag{A.4}$$

where

$$t_i(a) = \frac{\hat{y}_i - u_i(a)}{\hat{y}_i}, \quad \hat{y}_i > 0, \quad i = 1, \ldots, K. \tag{A.5}$$

This solution corresponds to the variable a with the smallest value of the relative deviations of all objectives to the ideal solution. It is also called min-max optimum as it yields the best possible compromise solution under observance of all objective functions with equal priority. Min-max formulation can also be combined with objective weighting to describe the priority of the individual objective functions.

D. Trade-off Method

A single-objective problem may also be formulated by maximizing only one objective with all others bounded from below:

$$g(a) = u_1(a), \quad a \in A \tag{A.6}$$

with

$$u_i \geq \bar{y}_i, \quad i = 2, \ldots, K. \tag{A.7}$$

Thus, u_1 is called the main objective, and $u_2, \ldots, u_K$ are called secondary objectives with the respective minimum requirements being $\bar{y}_2, \ldots, \bar{y}_K$. This method can be used when some users have lower priorities, or have no objective functions except constraints.

Appendix B

Power Capture Model

A packet is assumed to capture the base station if and only if the arrival power exceeds the sum of instantaneous joint interference power and background noise by at least a threshold factor z. This threshold factor is known as the capture ratio and reflects the capture performance of the employed receiver, modulation, and coding techniques. It is usually greater than one in a narrow-band system and less than one in a spread spectrum system. Assume that a reference user ν_{j_0} is at a distance r_0 to the base station and has a transmission power level P_{Tj_0}. The power of a packet received at the base station from ν_{j_0} is described by the following propagation equation in the presence of Rayleigh fading

$$P_{j_0} = \zeta R_{j_0}^2 r_{j_0}^{-\beta} P_{Tj_0}, \tag{B.1}$$

in which ζ is a scaling constant. $R_{j_0}^2$ is an exponentially distributed random variable with unit mean, and $r_{j_0}^{-\beta}$ accounts for the deterministic power roll-off with β typically ranging from 2 to 4. Then, ν_{j_0}'s mean arrival power $\bar{P}_{j_0}$ is $\zeta r_{j_0}^{-\beta} P_{Tj_0}$, and mean SNR $\bar{\gamma}_{j_0}$ is $\bar{P}_{j_0}/N_A$ where N_A is the received noise power.

When packets from users $\nu_{j_1}, \ldots, \nu_{j_k}$ arrive simultaneously as the one from ν_{j_0}, capture occurs for ν_{j_0} if and only if

$$P_{j_0} > z\left(\sum_{l=1}^{k} P_{j_l} + N_A\right), \tag{B.2}$$

where ν_{j_l} has instant arrival power P_{j_l} and mean arrival power $\bar{P}_{j_l}$. Then, the capture probability of ν_{j_0} can be expressed as

$$C_{j_0} = \Pr\left\{P_{j_0} > z\left(\sum_{l=1}^{k} P_{j_l} + N_A\right)\right\}$$
$$= \Pr\left\{R_{j_0}^2 > z\left(\bar{P}_{j_0}^{-1}\sum_{l=1}^{k} R_{j_l}^2 \bar{P}_{j_l} + \bar{\gamma}_{j_0}^{-1}\right)\right\}, \qquad \text{(B.3)}$$

where $R_{j_l}^2$ has the same distribution as $R_{j_0}^2$. Averaging over $R_{j_l}^2$ for $l = 1, \ldots k$ yields

$$C_{j_0} = \int_0^\infty da_{j_1} e^{-a_{j_1}} \cdots \int_0^\infty da_{j_k} e^{-a_{j_k}} \exp\left[-z\left(\bar{P}_{j_0}^{-1}\sum_{l=1}^{k} a_{j_l}\bar{P}_{j_l} + \bar{\gamma}_{j_0}^{-1}\right)\right]$$
$$= e^{-z\bar{\gamma}_{j_0}^{-1}} \prod_{l=1}^{k}(1 + z\bar{P}_{j_l}\bar{P}_{j_0}^{-1})^{-1}. \qquad \text{(B.4)}$$

By applying the above results, we can obtain the following capture probability in the presence of the near-far effect and Rayleigh fading,

$$C_i(m_1, \ldots, m_K) = e^{-z\bar{\gamma}_i^{-1}}(1+z)\prod_{l=1}^{K}(1 + z\bar{P}_l\bar{P}_i^{-1})^{-m_l}, \qquad \text{(B.5)}$$

where $\bar{\gamma}_i = \bar{P}_i/N_A$. When only the near-far effect is considered, we have

$$C_i(m_1, \ldots, m_K) = \begin{cases} 1, & z\left(\bar{P}_i^{-1}\sum_{l=1}^{K} m_l\bar{P}_l - 1 + \bar{\gamma}_i^{-1}\right) < 1 \\ 0, & \text{otherwise.} \end{cases} \qquad \text{(B.6)}$$

Appendix C

Derivation of Equations (5.16)–(5.21)

To achieve the maximum network throughput without individual throughput constraint, differentiate (5.15) with respect to p_j for $j = 1, \ldots, K$. By letting

$$\frac{\partial S}{\partial p_K} = \left[(1-p_K)^{N_K-1} - (N_K-1)p_K(1-p_K)^{N_K-2}\right] N_K \prod_{j=1}^{K-1}(1-p_j)^{N_j} = 0,$$

we have

$$p_K^* = \frac{1}{N_K}. \tag{C.1}$$

By letting

$$\begin{aligned}\frac{\partial S}{\partial p_{K-1}} &= \Big[N_{K-1}(1-p_{K-1})^{N_{K-1}-1} - N_{K-1}p_{K-1}(N_{K-1}-1)(1-p_{K-1})^{N_{K-1}-2} \\ &\quad - N_K p_K (1-p_K)^{N_K-1} N_{K-1}(1-p_{K-1})^{N_{K-1}-1}\Big] \prod_{j=1}^{K-2}(1-p_j)^{N_j} = 0\end{aligned}$$

and substituting in p_K^*, we find

$$p_{K-1}^* = \frac{1-(1-p_K^*)^{N_K-1}}{N_{K-1}-(1-p_K^*)^{N_K-1}}. \tag{C.2}$$

Continuing this process to p_1 completes (5.16), and substituting (5.16) into (5.15) yields (5.17).

Next, we use induction to prove (5.18). It is clear that $\overline{S}^*(1) = e^{-1}$. For $K = 2$, according to (5.16), we know that $p_2^* = 1/N_2$ and

$$p_1^* = \frac{1-(1-p_2^*)^{N_2-1}}{N_1-(1-p_2^*)^{N_2-1}} = \frac{1-(1-1/N_2)^{N_2-1}}{N_1-(1-1/N_2)^{N_2-1}}. \tag{C.3}$$

By letting $N_2 \to \infty$, we get

$$p_1^* = \frac{1-e^{-1}}{N_1-e^{-1}}, \tag{C.4}$$

and

$$S^*(2) = N_1 p_1^*(1-p_1^*)^{N_1-1} + e^{-1}(1-p_1^*)^{N_1}. \tag{C.5}$$

Further letting $N_1 \to \infty$,

$$\overline{S}^*(2) = (1-e^{-1})e^{-(1-e^{-1})} + e^{-1}e^{-(1-e^{-1})} = e^{-(1-e^{-1})} = e^{-1+\overline{S}^*(1)}. \tag{C.6}$$

Assume that (5.18) is valid for $K = n$. By letting $N_j \to \infty$ from $j = n$ to $j = 2$, we obtain

$$p_1^* = \frac{1-\overline{S}^*(n)}{N_1-\overline{S}^*(n)}, \tag{C.7}$$

and

$$S^*(n+1) = N_1 p_1^*(1-p_1^*)^{N_1-1} + \overline{S}^*(n)(1-p_1^*)^{N_1}. \tag{C.8}$$

By further letting $N_1 \to \infty$,

$$\overline{S}^*(n+1) = (1-\overline{S}^*(n))e^{-1+\overline{S}^*(n)} + \overline{S}^*(n)e^{-1+\overline{S}^*(n)} = e^{-1+\overline{S}^*(n)}. \tag{C.9}$$

Therefore, (5.18) is valid for any K.

When equal individual throughput is required among all users, we equate the individual throughput of one user in U_j to that of one user in U_{j+1} $(1 \le j < K)$

$$p_j^*(1-p_j^*)^{N_j-1}\prod_{l=1}^{j-1}(1-p_l^*)^{N_l} = p_{j+1}^*(1-p_{j+1}^*)^{N_{j+1}-1}\prod_{l=1}^{j}(1-p_l^*)^{N_l}. \tag{C.10}$$

The above equation can be simplified to

$$p_j^* = \frac{p_{j+1}^*(1-p_{j+1}^*)^{N_{j+1}-1}}{1+p_{j+1}^*(1-p_{j+1}^*)^{N_{j+1}-1}}. \tag{C.11}$$

To achieve the maximum utilization, we need $p_K^* = 1/N_K$. Substituting p_K^* in (C.11) from $j = K-1$ to $j = 1$ yields (5.19). Since all the N users have the same individual throughput of $p_1^*(1-p_1^*)^{N_1-1}$, the network throughput can be obtained as in (5.20).

Furthermore, (5.21) can be proved by induction similarly.

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