ALTERNATIVE MODELS OF EXPECTATIONS FORMATION: A REVIEW AND EMPIRICAL TESTS

by

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ABSTRACT

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Individuals formulate expectations of key economic variables and behave according to these expectations. These expectations play major roles in the effectiveness of discretionary monetary and fiscal policy. In particular, monetary and fiscal policy effectiveness depends on whether inflation rate and interest rate expectational errors are randomly distributed around their realized values, or whether the expectational errors are systematically and persistently biased. An individual's expectations of the inflation rate or interest rate are unobservable. To evaluate policy effectiveness, some assumptions have to be made concerning the nature of expectations formation. This thesis reviews the major hypotheses of expectations formation and generates empirical proxies for the inflation rate and interest rate using alternative weak-form models of expectations formation; i.e., models using information found in past values of the economic variable. The proxies generated by these models are subjected to a number of tests to determine the nature of the expectational errors and the implications for policy effectiveness. The results of the tests indicate that weak-form unconstrained or stochastic models of expectations for-

mation, as represented by the Box and Jenkins ARIMA methodology, generate empirical proxies that are considerably more accurate and random in nature than the empirical proxies generated by constrained models or by models that are based on market determined nominal interest rates. By assuming that individuals weigh the costs of using complex expectations models with the benefits derived from generating accurate forecasts from these models, this thesis concludes that economic agents will use ARIMA models in formulating actual expectations. Because expectational errors using ARIMA models are minimal and random in nature, this thesis concludes that the economic impact of government stabilization policy will be limited.

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DEDICATION

To MLM

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Chapter I

INTRODUCTION

An individual's current economic behavior is influenced by anticipated future values of economic variables. As expectations are believed to guide and influence current decisions (employment, investment, etc....) and behavior, an understanding of how expectations are formulated and revised is an obvious area of concern for economic analysis intent on specifying policy actions to improve macroeconomic performance. Two significant examples of the implications of expectations formation on policy effectiveness are the nature of the inflation/unemployment trade-off and the determination of the real interest rate.

1.1 THE INFLATION/UNEMPLOYMENT TRADE-OFF

An important example of the implications of expectations formation on policy effectiveness is the nature of the inflation/unemployment trade-off. It is noted that prior to the late 1960's economic experience had been consistent with the existence of a stable trade-off between inflation and unemployment[1]. As well, it had been commonly believed that government economic policy could achieve low levels of unemployment at the expense of high rates of inflation because everyone anticipated that nominal prices would remain stable regardless of what actually happened to the price level. An explanation of how government economic policy can achieve a low level of unemployment at the expense of a high inflation rate clarifies the school of thought that existed prior to the late 1960's. Suppose an increase in government expenditures leads to an increase in aggregate nominal demand. The increase in aggregate demand is perceived by employers to mean an eventual increase in the employer's product selling price. As a result, the employer, whose labour demand decisions are based on the current real wage[2], perceives a decrease in the real wage and reacts by increasing the size of his labour force. The increase in aggregate nominal demand eventually results in a general price increase. Because everyone anticipates that general prices will remain stable, labour does not demand an increase in nominal wages to restore the previous real wage. The end result of the increase in government expenditures is a permanently lower unemployment rate at the expense of a high inflation rate. The permanent impact on the unemployment rate is a result of the fact that labour is assumed to bargain for nominal wages not real wages.

By late 1968 an alternative analysis of the inflation/unemployment trade-off emerged. This analysis introduces the role inflation expectations and labour market outcomes play in the inflation/unemployment trade-off. The new theory, labelled the Natural Rate Hypothesis[3], contradicts the stable trade-off between inflation and unemployment. The Natural Rate Hypothesis states that the traditional Phillips curve relating inflation to unemployment is incomplete because it fails to incorporate the fact that anticipations about inflation change, as economic agents experience price level changes. The Natural Rate Hypothesis introduces the notion that the trade-off between inflation and unemployment shifts when inflation expectations are revised[4]. This hypothesis maintains that a transitory trade-off exists not between unemployment and actual inflation but rather between unemployment and the difference between actual and expected inflation. According to the Natural Rate Hypothesis, unemployment less than the natural rate of unemployment is a consequence of errors in inflation expectations formation whereby workers are "fooled"; i.e., wage rate demands and labour markets settlements are associated with expected inflation below actual inflation.

The transitory nature of the effect of government economic policy on the unemployment rate, implied by the natural rate hypothesis, is easier to understand by referring to the example described earlier. As mentioned above, an employer, perceiving an increase in the price for his product, will increase his demand for labour to produce more. The employer may even increase nominal wages to attract more labour but by an amount less than the increase in price he per-labelled the Natural Rate Hypothesis[3], contradicts the stable trade-off between inflation and unemployment. The Natural Rate Hypothesis states that the traditional Phillips curve relating inflation to unemployment is incomplete because it fails to incorporate the fact that anticipations about inflation change, as economic agents experience price level changes. The Natural Rate Hypothesis introduces the notion that the trade-off between inflation and unemployment shifts when inflation expectations are revised[4]. This hypothesis maintains that a transitory trade-off exists not between unemployment and actual inflation but rather between unemployment and the difference between actual and expected inflation. According to the Natural Rate Hypothesis, unemployment less than the natural rate of unemployment is a consequence of errors in inflation expectations formation whereby workers are "fooled"; i.e., wage rate demands and labour markets settlements are associated with expected inflation below actual inflation.

The transitory nature of the effect of government economic policy on the unemployment rate, implied by the natural rate hypothesis, is easier to understand by referring to the example described earlier. As mentioned above, an employer, perceiving an increase in the price for his product, will increase his demand for labour to produce more. The employer may even increase nominal wages to attract more labour but by an amount less than the increase in price he perceives. As the general price level increases, labour will begin to anticipate higher prices and to demand that these anticipations be reflected in higher nominal wages. The real wage may remain below its previous value for a short period of time either because labour is prevented from realizing higher nominal wages (due to fixed labour market contracts), or because inflation rate anticipations are not accurate. With either case, the unemployment rate will remain below its original value. Eventually, however, labour will be able to fully incorporate anticipations into labour market settlements and the real wage will return to its previous value bringing with it a return of the unemployment rate to its natural level. The transitory impact of the increase in government expenditures is a result of the fact that labour is assumed to bargain for real wages not nominal wages as implied by the original Phillips curve analysis.

The implications of the natural rate analysis upset the conventional wisdom of the late 1960's. In brief, the Natural Rate Hypothesis implies that government economic policy can have a systematic and persistent effect on the unemployment rate only if policy actions can generate systematic and persistent expectational errors.

1.2 DETERMINATION OF THE REAL INTEREST RATE

The Natural Rate Hypothesis highlights the effects of expectations formation on the determination of other economic variables besides the unemployment rate. For example, the Natural Rate Hypothesis outlines the role expectations play in the determination of the real interest rate.

Milton Friedman (1968) uses the theoretical underpinnings of the Natural Rate Hypothesis and the Irving Fisher hypothesis, that nominal interest rates are composed of a real interest rate component and an inflation expectations component[5], to show that the monetary authorities can push real interest rates lower than natural rates only by generating inflation expectational errors. Once again, an example helps to make this point easier to understand. Suppose the government acts to increase the money supply growth rate. Individuals respond to an increase in the total money supply by purchasing interest-bearing financial assets. This results in a decrease in the market real interest rate, as increasing security prices drive nominal yields downward while expected inflation remains constant. The lower real interest rate generates an increase in investment which leads eventually to an increase in incomes and the general price level. These effects act to return the nominal interest rate to its previous value. In addition, because of the increase in the money supply growth rate, lenders and borrowers begin to expect higher prices and act to incorporate expected inflation into nominal interest rate values. The end result of the increase in the money supply growth rate is a higher nominal interest rate and a return of the real interest rate to its natural value. The extent to which the monetary authorities can influence real interest rates depends on the extent to which economic agents incorporate accurate inflation rate expectations into nominal interest rate values.

The real interest rate implications of the Natural Rate analysis upset the conventional wisdom. In brief, the Natural Rate Hypothesis implies that monetary authorities can not peg real interest rates, and that nominal interest rates can not be used as indicators of the direction of monetary policy[6].

In general, the investigation of the relationship between inflation expectations and the determination of the real wage rate and real interest rate highlights the major role expectations formation plays in determining the effectiveness of government economic policy. In practice, however, expectations held by economic agents are not observable values. Thus, evaluation of the impact of economic policy can only be made in conjunction with hypotheses formulated on the nature of expectations formation.

The objective of this thesis is to use alternative models of expectations formation to generate empirical proxies for the inflation rate and the interest rate, and to subject the results obtained to a number of tests designed to determine the nature of the expectational errors associated with each model. The results of the tests are then used to draw conclusions about the properties associated with different models of expectations formation; in particular, the economic policy implications of expectations formation.

'Chapter Notes'

[1] A.W. Philips (1958) is noted to be the first person to observe a stable negative relationship between the rate of wage change and the rate of unemployment. This negative relationship was subsequently referred to as the Phillips curve.

Robert Lipsey (1960) provides the theoretical underpinnings of the stable Phillips curve. His specification of the stable Phillips curve can be represented as

$$W = v + g(U)$$

where: W = wage change rate;

v = labour productivity growth rate;

g(U) = unemployment rate

= proxy for excess labour market demand.

The negative relationship between the wage change rate and the unemployment rate is easily extended to a relationship between the inflation rate and the unemployment rate by noting that the inflation rate can be expressed as the wage change rate minus the rate of growth of labour productivity.

In their empirical work, Bodkin, Bond, Reuber and Robinson (1966) investigate the Canadian inflation/unemployment trade-off.

[2] It is generally accepted that an employer equates the revenue received from selling an additional unit of output (marginal revenue) with the cost of producing that

additional unit (marginal cost) in making labour demand decisions. This relationship can be expressed as

$$P = MP_l(W)$$

where: P = price received from additional unit sold;

MP_l = marginal product of labour unit;

W = current wage rate per labour unit.

If the price an employer receives for his product increases while the current wage remains constant; i.e., $P > MP_l(W)$, then the profit maximizing employer will hire an additional unit of labour because the revenue received for the extra unit sold will be greater than the cost of producing it. Labour units will be continually added to the employer's workforce, in this manner, until the marginal revenue received from an extra unit sold equals the marginal cost of producing that unit. Another way to express this same point is that if the real wage, W/P, facing the employer decreases, with a perceived or actual increase in price, the employer will hire additional units of labour.

[3] The Natural Rate Hypothesis is generally attributed to the work of Milton Friedman (1968) and Edmund Phelps (1970). The natural rate of unemployment is defined by Friedman as the value of unemployment at which real wages rise at normal or secular rates. Normal increases are dictated by factors such as capital formation and technology. Rates of unemployment above or below the

natural value are associated with downward or upward pressures on the real wage rate respectively. According to Friedman, the natural rate is not a numerical constant. Its value is dictated by real factors such as the effectiveness of the labour market.

[4] The expectations augmented Phillips curve can be expressed as

$$W = v + g(U) + P^e$$

where: P^e = expected inflation rate(shift parameter).

[5] According to I. Fisher (1907), the relationship between nominal and real rates of interest is given by

$$i = r + (1+r)\ P^e$$

where: i = nominal rate of interest;

r = real rate of interest;

P^e = expected inflation rate.

[6] The Natural Rate Hypothesis reveals that a change in monetary policy has a short-run impact that is opposite in direction to the long-run impact. It shows that the long-run consequence of an easy monetary policy is a higher nominal interest rate and the long-run consequence of a tight monetary policy ia a lower nominal interest rate. The Natural Rate Hypothesis refutes the conventional wisdom of using nominal interest rates as indicators of monetary policy.

Chapter II

A REVIEW OF THE ECONOMIC LITERATURE

2.1 GENERAL PRINCIPLES OF GENERATING EXPECTED VALUES

When discussing the process of expectations formation, it is important to differentiate between the meaning of expected value in the conditional mathematical sense and expected value in the subjective or predictive sense. The difference between the two interpretations of expected value is best explained with reference to probability theory.

Suppose an economic variable can be thought of as a random variable, X, that takes on different values; i.e., $X_1,...X_n$, and that this random variable has a probability distribution describing its stochastic behavior. The probability distribution assigns a probability of occurrence to each value of the random variable. It is important to note that this probability distribution also describes the process that generates the values of the random variable. When the probability distribution is conditional on, or caused by, a certain information set affecting its stochastic behavior, it is referred to as a conditional probability distribution. A mathematical expectation is nothing more than a probability weighted average. It is calculated by taking the sum of the products of each value of the random variable and its probability of occurrence. A conditional mathematical expectation is calculated by using a conditional probability distribution in the weighted average calculation. An example of a conditional mathematical expectation makes this easier to understand[1]. Suppose 300 cards are to be drawn from a bag containing 3000 cards; 1000 of which are numbered 4, 1000 numbered 5 and 1000 numbered 6. The probability of occurrence of each type of card is 1/3 and the mathematical expectation is 1/3(4) + 1/3(5) + 1/3(6) = 5. Now suppose you are told that, of the 300 cards drawn, 200 are numbered 4. The probability distribution has changed and is 'conditional' on this extra information. The probability of drawing a 4 is now given at 200/300 and the probability of drawing a 5 or 6 becomes 50/300. Given this additional information, the conditional mathematical expectation becomes 200/300(4) + 50/300(5) + 50/300(6) = 4.5.

A subjective expectation is quite different from a mathematical expectation. It is a personal interpretation about the likely outcome of an event rather than a specific calculation based on a conditional probability distribution. In the example given above, a subjective expectation for the average of the 300 cards drawn might be 4.8.

It is unrealistic to think that economic agents calculate conditional mathematical expectations when they formulate anticipations of future values of an economic variable.

Rather, it is more reasonable to think that expectations are subjective in nature. This thesis is concerned with subjective expectations formation and alternative methods by which economic agents might calculate proxies for non-observable economic values.

2.2 STRONG-FORM VERSUS WEAK-FORM EXPECTATIONS FORMATION MODELS

It is useful to distinguish between two classes of subjective expectations formation models - strong-form models and weak-form models. These two classes of models can be conveniently explained in terms of efficient market theory[2]. The concept of efficient markets maintains that a market is efficient if the price determined in that market fully reflects a certain subset of information. According to this theory, strong-form efficient markets are characterized by prices that reflect all the information that actually determines the prices; whereas weak-form efficient markets are characterized by prices that reflect only a subset of the information that actually determines the prices. Applying the efficient market terminology to expectations formations, strong-form expectations models are defined as models that use all the information determining the economic variable and weak-form expectations models are defined as models that use a subset of the information that actually determines the economic variable.

Almost all of the models used in the literature to generate expected values of an unobservable economic variable have restricted the information set to past and present values of the economic variable of interest. Thus, these models can be classified as weak-form models because expectations are formed using a subset of the information base that actually determines the economic variable[3]. The various weak-form approaches to expectations formation differ in their assumptions about how the information contained in past values of the economic variable is used to generate expectations. For example, some models are simple extrapolations of past values of the economic variable. They use information on past movements of the economic variable to predict future movements. Moreover, they do not assume that the evolution of the economic variable over time is a result of a stochastic process. Because future movements in the economic variable are constrained to follow a predefined pattern, these models can be considered weak-form deterministic or constrained models. This approach is attractive because it is easy to use and cheap to administer. Although rather simplistic, these weak-form constrained models can generate accurate predictions of the economic variable under certain circumstances; namely, when past trends can be accurately identified and when there is no reason to believe that past trends will not repeat themselves in the future.

Other commonly-used weak-form models presume that the economic variable is generated by a stochastic process. In this approach, the expectations model attempts to characterize or describe the stochastic or random process of the economic variable by looking at the randomness embodied in the historical values of the economic variable. By determining the significant correlations between current and past values of the variable, inferences can be made about the probabilities associated with different future values of the variable. This type of model can be considered a weak-form unconstrained model, and provides an appropriate forecast model whenever the stochastic process underlying the economic variable is constant.

2.3 CHRONOLOGICAL DEVELOPMENT OF EXPECTATIONS FORMATION HYPOTHESES

2.3.1 Autoregressive Expectations Formation

Irving Fisher (1907) is acknowledged to have been the first person to distinguish between the nominal rate of interest and the real rate of interest. He hypothesized that lenders and borrowers are willing to incorporate the rate of inflation, expected to apply over the duration of the loan, into the prevailing cost of borrowing money. According to Fisher, the relationship between nominal and real rates of interest is given by

$$i = r + (1+r)\, P^e \qquad (2.1)$$

where: i = nominal rate of interest;

r = real rate of interest;

P^e = expected inflation rate.

This relationship, postulated by Fisher, between nominal and real interest rates describes a long term relationship, and as such, is based on the assumption of perfect inflation foresight and complete adjustment of nominal interest rates to changes in expected inflation. Fisher's empirical research indicated that changes in the nominal interest rate lagged behind changes in the current inflation rate. He attributed this inertia to the cumulative effect of inflation on business profits and the fact that economic agents formulate inflation expectations on the basis of historical values of inflation. Fisher felt that economic agents suffer from money illusion under which they do not consciously process current information in formulating inflation expectations.

Although Fisher was not primarily concerned with devising a formal model of expectations formation, he was the first to introduce such a model, albeit in an indirect manner. According to Fisher, inflation expectations are formulated on the basis of past price change information. His model of inflation expectations formation can be represented by a general functional form of the nature

$$P^e_t = f(P_t, P_{t-1}, P_{t-2}, \ldots, P_{t-n}) \qquad (2.2)$$

where: P^e_t = this period's expectation of next period's inflation rate;

P_t = current period's actual inflation rate;

P_{t-n} = t-n period's actual inflation rate.

As indicated by equation (2.2), Fisher's model of inflation expectations formation lacks specific functional form. He felt expectations are generated on the basis of past inflation rate information but he did not elaborate on how that information is used.

2.3.2 Adaptive Expectations Formation

Phillip Cagan (1956) is acknowledged to have been the first economist to give an explicit functional form to expectations formation. In his research on the demand for real cash balances in a hyperinflation, he hypothesized that the demand for money is inversely related to the expected rate of change in prices and that "the expected rate of change in prices is revised per period of time in proportion to the difference between the actual rate of change in prices and the rate of change that was expected."[4] This error learning or adaptive expectations model of expectations formation can be expressed as

$$P^e_t - P^e_{t-1} = B(P_t - P^e_{t-1}) \qquad 0<B<1 \qquad (2.3)$$

where: P^e_t = this period's expectation of next period's inflation rate;

P^e_{t-1} = last period's expectation of this period's inflation rate;

P_t = current period's actual inflation rate;

B = coefficient of expectations.

According to equation (2.3), whenever expected inflation differs from actual inflation, expected inflation is adjusted by a portion of the forecast error, $P_t - P^e_{t-1}$, to derive a new expected inflation rate. The Cagan model of expectations formation is adaptive in nature because expectations are adjusted according to the most recently available data; i.e., expectations depend on the error made in forecasting last period's inflation rate.

Equation (2.3) can be rewritten to express the expected inflation rate as a geometrically declining weighted average of current and past values of the inflation rate. By successive substitution for the lagged term in (2.3) we obtain

$$P^e_t = \sum_{i=0}^{\infty} B(1-B)^i P_{t-i}. \qquad (2.4)$$

Equation (2.4) shows that the Cagan model of expectations formation is based on current and past values of the inflation rate where economic agents use a geometrically declining weighted average of current and past inflation rates to formulate the expected value of next period's inflation rate. The Cagan model differs from the Fisher model in that it details exactly how past inflation rate information is used to generate future expectations.

A distinguishing characteristic of the Cagan type expectations formation model is that revisions of expectations involve a systematic prediction error. For example, an unexpected increase in inflation results in subsequent expectations that consistently underpredict the actual inflation rate. These underpredictions continue until the expected inflation rate converges to the actual inflation rate. The adjustment process is partial, distributed over time, and depends on the value of B - the coefficient of expectations.

Because expected values are based exclusively on past values of the inflation rate, the Cagan model is a weak-form constrained or deterministic expectations model. Moreover, revision of the expected value is constrained to follow this partial adjustment process described by a geometrically declining weighted average of past values of the inflation rate.

2.3.2.1 Adaptive Expectations and the Phillips Curve

The introduction to this thesis touched on the important role inflation expectations play in the determination of the effectiveness of stabilization policy. It was mentioned in the introduction that the conventional wisdom concerning the effectiveness of stabilization policy changed with the introduction of the Natural Rate Hypothesis and the incorporation of inflation expectations into the Phillips curve analysis of the trade-off between inflation and unemployment.

The Natural Rate Hypothesis argues that the difference between the actual and natural level of unemployment is due to the difference between actual and expected inflation and not, as implied by Phillips' original study, simply on the current rate of inflation. The Natural Rate Hypothesis introduced the significant role expectations formation plays in determining the efficacy of stabilization policy.

The Cagan model of adaptive expectations formation can be applied to the Phillips curve analysis of the trade-off between inflation and unemployment. According to the Cagan model of expectations formation, expectations converge to the actual inflation rate in a gradual and systematic manner. This implies that discretionary economic policy can exact a transitory short-run impact on unemployment and output, but that in the long run its only lasting effect is a change in the rate of inflation. In the short-run, economic policy that generates higher than expected inflation will have a positive effect on unemployment levels as labour market participants gradually and partially incorporate expectational errors into wage demands. The ability of labour market participants to incorporate expected price changes into wage demands will be influenced by the length of wage contracts, as long as wage contracts delay complete adjustment of wages to the change in inflation. Longer wage contracts will increase the effectiveness of economic policy. In the long run, however, expectations will catch up to actual inflation and new labour contracts will incorporate expectations completely. The end result is a return to the natural level of unemployment and a diffusion of the positive effect of the economic policy.

Application of the Cagan model of expectations formation to the Natural Rate Phillips curve analyis yields radically different policy prescriptions and implications than that obtained from the original Phillips curve analysis. If expectations are formed in a Cagan like manner, systematic government policy undertaken to influence unemployment levels will be effective because expectational errors are systematic and persistent. But given that expectational errors approach zero in the long run then policy efficacy approaches zero in the long run. In effect, a trade-off exists but it is transitory. The duration of the trade-off depends on the length of labour contracts and on the value of B - the coefficient of expectations. This limits the scope of discretionary policy.

2.3.3 Rational Expectations Formation

The distinguishing characteristic of adaptive expectations formation is that expectations are formulated in an adaptive manner; that is, future expectations of economic variables are based on the deviation of actual values from previously expected values. This expectations formation process assumes economic agents do not learn from past errors in genof an economic variable. This rather unreasonable assumption is the focus of the Rational Expectations Formation school of thought.

In contrast to the adaptive expectations model advanced by Cagan, John Muth (1961) argues that a theory of expectations formation should assume rational, maximizing behavior. Muth argues that it is unreasonable to think economic agents are fooled consistently. He contends that economic agents learn from their expectational errors and anticipate movements in economic variables that affect them. Muth's Rational Expectations Hypothesis (REH) states that economic agents do not generate expectations that involve a systematic prediction error as implied by the adaptive process of expectations formation. Rather, Muth asserts that economic agents know the true specification of the economic model that actually generates the economic variable of interest. Thus, if economic agents use the true structural model in formulating expectations, their subjective expectations will be "essentially the same as the predictions of the relevant theory"[5] and not simply a weighted average of current and past experience.

Muth's rational expectations formation hypothesis can be explained in terms of the general principles outlined in section one. By simulating their version of the true structural model generating the economic variable, economic agents are able to subjectively characterize the stochastic behavior of the system in a manner that equals the actual stochastic behavior. B. Friedman explains this point in the following manner.

> Peoples' subjective probability distributions describing future outcomes are identical to the corresponding objective probability distribution conditional on the true model of the economy[6].

In notational form, Muth's rational expectations formation hypothesis can be expressed as

$$_{t-1}X^e_t = E(X_t | I_{t-1}) \tag{2.5}$$

where: $_{t-1}X^e_t$ = subjective expectation of X, for period t, generated in period t-1;

$E(X_t | I_{t-1})$ = conditional mathematical expectation of X, for period t, based on information set I, in period t-1.

Because the information set used in the model, I_{t-1}, contains all the information that actually determines the economic variable, Muth's REH is an example of strong-form expectations formation.

In summary, there are two main differences between Muth's rational expectations formation model and Cagan's error learning expectations formation model. Firstly, according to the REH, forecast errors are random and not persistently negative or positive as postulated by Cagan's error learning model. Expressed in a different way, economic agents' subjective expectations equal actual values on average. Secondly, expectations anticipate future changes in the economic variable by observing present changes in the independent variables affecting the economic variable under study. This is different from the error learning model which contends that expectations are altered only after changes in the economic variable have been observed.

2.3.3.1 Rational Expectations and the Phillips Curve

Muth's model of rational expectations formation can be applied to the Natural Rate Phillips curve analysis. The economic policy implications of rational expectations formation contrasts sharply with those obtained under the assumptions of the adaptive expectations formation model.

The Natural Rate Phillips curve analysis maintains that the difference between the actual and natural level of unemployment is caused by the difference between actual and expected inflation. Application of adaptive expectations formation to the Natural-Rate analysis of the Phillips curve implies a systematic trade-off between inflation and unemployment in the short-run because of persistently biased expectational errors. In contrast, the REH implies that expectational errors will be random in nature and that economic agents will, on average, correctly anticipate the effect of correctly perceived government economic policy actions. When rational expectations are incorporated into wage contracts, real wages are not persistently lower than the market clearing value; consequently, there is no significant short-run or long-run impact of economic policy on the unemployment rate. In its extreme form, the REH contends that an inflation/unemployment trade-off exists in neither the short-run nor long-run because inflation expectations are incorporated immediately into wage contracts.

Less extreme applications of the REH have been applied to the Natural Rate Phillips curve analysis. The major difference between these less extreme versions of the REH and the extreme version discussed above centers around the assumptions each makes about the accessibility and processibility of information required to generate rational expectations and the ability of economic agents to incorporate rational expectations immediately into wage settlements. Sargent and Wallace (1975,1976) argue that if government economic policy is systematic in its application, economic agents will eventually determine the nature of the systematic influence and use that information to generate rational expectations. The result is that economic policy will affect nominal variables only. Other versions of the less extreme REH argue that economic policy will be effective if either economic policy is unsystematic in nature or if economic agents are unable to incorporate rational expectations into wage settlements continuously. If economic policy is unsystematic, economic agents will be unable to correctly anticipate changes in the inflation rate and economic policy might have a positive impact on macroeconomic variables. However, in this situation, government stabilization policy can be a source of instability if economic agents' anticipations counteract government economic policy. Also, if economic agents are unable to incorporate rationally held expectations into wage settlements because of explicit or implicit contracts, economic policy will have a positive impact. More will be said on the non-continuous auction market characteristics of the labour market in section 2.4.2.

2.3.3.2 Rational Expectations and The Real Interest Rate

Muths's model of rational expectations formation can also be applied to the question of whether monetary policy can control the real rate of interest. This topic was discussed briefly in the introduction to this thesis.

The commonly held view is that expansionary monetary policy leads to a decrease in nominal and real interest rates, in the short-run at least, via the liquidity effect. This point can be explained by rewriting equation (2.1) as

$$r = (i - P^e)/(1 + P^e) \qquad (2.6)$$

where: i = nominal rate of interest;

r = real rate of interest;

P^e = expected inflation rate;

and noting that a decrease in the nominal interest rate (i) leads to a decrease in the real interest rate (r) because expected inflation (P^e) has not changed in the short-run. As purchases of consumer and producer durable goods depend on the real interest rate rather than simply the nominal interest rate, a decrease in the real rate has a major effect on investment and consumer spending and hence on aggregate macro variables such as the unemployment rate.

Following an expansionary monetary policy, it is usually thought that the nominal rate eventually increases due to price and income effects as well as a price expectations effect[7]. From equation (2.6), a rise in the nominal interest rate, that incorporates the increase in inflation expectations and offsets the initial decrease in the nominal interest rate, forces the real interest rate back to its equilibrium level. Under the adaptive expectations hypothesis, expected inflation is gradually revised upwards and thus there is a transitional impact on the real interest rate. In contrast, the REH implies that economic agents correctly anticipate these longer run effects and act to incorporate these effects into current nominal interest rate values at once. Thus, as the nominal interest rate adjusts to the revised value of expected inflation, the result is a constant real interest rate. Accordingly, the expansionary monetary policy fails to exert a short-run impact. This application of the REH to interest rate determination assumes that nominal interest rates will be allowed to adjust immediately to rationally formed expectations, as is likely in a continuous auction market. More will be said about the continuous auction market characteristics of actively traded financial assets in section 2.4.1.

2.3.4 Economically Rational Expectations Formation

It is noted that during the 1960's there was very little attention given to testing the REH, with the empirical work requiring expected values of an economic variable employing rather simple methods of generating expected value proxies. Most of the models disregard the logic of Muth's REH in that they do not attempt to define the true structural model generating the economic variable. For example, Perry (1970) uses the lagged one-period rate of change in prices and, Eckstein and Brenner (1972) apply a moving four-quarter average of actual inflation rates plus the positive excess of the accumulated price inflation rates over 5 percent for two years as empirical proxies for the inflation expectations variable. A more sophisticated approach is adopted by Gordon (1971) who utilizes the Fisher relationship between expected price changes and nominal interest rates to generate empirical approximations of the expected inflation rate. In this approach, a polynomial distributed lag of past inflation rates is substituted for the price expectations variable in the Fisher equation (see eq. 2.1) and the lagged coefficients from the best fitting polynomial are used to generate an independent measure of expected inflation.

Somewhat different approaches are followed by Frankel (1975) and Diller (1969). They hypothesize that economic agents adjust or regress short-run expectations toward some long-run or normal level. Frankel applies this 'return to normality' model to generate expected inflation rates and Diller adheres to the same concept when generating expected values for interest rates. All of the models used in the late 1960's and early 1970's are similar in that they generate expectations based on the information contained in past and present values of the economic variable. Consequently, they can all be categorized as weak-form constrained or deterministic models of expectations formation in that they all extrapolate past values of the economic variable to generate expected value proxies.

Feige and Pearce (1976) offer an explanation for the apparent lack of empirical attention given to the expectations formation model formulated by J. Muth. These authors argue that expectations formation, according to Muth's REH, assumes that the costs of acquiring and processing the information required to actually use the true structural model are zero. They contend that this is an unrealistic assumption and argue that an 'economically rational' agent considers the costs and benefits of using additional information when generating expectations. From this viewpoint, an economically rational agent, in equating the costs of additional information with the benefits of a smaller standard error of forecast, might trade-off the expensive rational expectations model for a less accurate but less expensive expectations model.

The Feige and Pearce hypothesis of economically rational expectations formation is significant because it gives credibility to the many simple methods used in the early 1970's. According to this hypothesis, economic agents are motivated/encouraged to use fairly simple methods of generating expectations when they find that the costs associated with collecting and processing the extra information required to form rational expectations, in the Muth sense, outweigh the benefits. Moreover, the Feige and Pearce hypothesis supports the use of expectations models that characterize or describe the stochastic process underlying the economic variable by using only past and present values of the economic variable. As mentioned in section 2.2, this type of model is classified as a weak-form unconstrained expectations formation model. The reason Feige and Pearce support the use of a weak-form unconstrained expectations model can be explained in the following manner. An expectations model that incorporates all the predictive information contained in past and present values of the variable, might generate rational or near rational expectations if the model is able to correctly identify the stochastic process underlying the economic variable. Accordingly, it would be economically rational for the economic agent to use such an expectations model because the costs of using a more expansive and rational model would outweigh the benefits derived from using that model, if both approaches generated rational expectations.

An example of a weak-form unconstrained model approach to expectations formation is that devised by Box and Jenkins (1970). The Box/Jenkins methodology is an example of weak-form expectations formation in that it assumes the time series under consideration is a result of a stochastic process and it employs present and past values of the economic variable to identify this process. In their empirical work, Feige and Pearce (1976) argue that if the economic variable under consideration is generated in a systematic manner, the Box/Jenkins methodology might generate expectations close to expectations generated by the true structural model[8].

2.3.4.1 Economically Rational Expectations and the Phillips Curve

The concept of economically rational expectations formation (EREH) can be applied to the Natural Rate Phillips curve analysis to determine the economic policy implications of economically rational expectations formation.

The EREH states that agents trade-off forecast accuracy for a less costly expectations model. The effectiveness of economic policy, in this context, depends on the extent to which economic agents trade-off smaller expectational errors for a less costly expectations model. This, in turn, maybe significantly influenced by the decision of economic agents to use either constrained or unconstrained expectations formation models.

Constrained models of expectations formation may follow an adaptive approach where expectational errors are systematically biased, or they may follow simple extrapolative rules where expectational errors are indeterminate in the sense that they can be either random or biased in nature. The economic policy implications of adaptive expectations formation have been discussed earlier. If expectational errors are indeterminate in nature, the economic policy implications are indeterminate as well. That is, because expectational errors maybe either persistently biased or random in nature, there is no way of determining the effectiveness of systematic economic policy.

The economic policy implications of the unconstrained model of expectations formation contrasts sharply with the constrained models. Because the unconstrained model identifies the stochastic nature of the economic variable, it is able to anticipate movements in the economic variable. Therefore, expectational errors will be zero, on average, in the short-run and long-run, and stabilization policy will have random impacts only on macroeconomic variables. In this extreme version of the unconstrained model approach, the stabilization policy implications are rational in nature. It should be noted that in its extreme form the expectational errors of the unconstrained model of expectations formation exhibit rational characteristics only if the unconstrained model is able to correctly identify the systematic nature of the economic variable. In less extreme versions of the unconstrained model approach, the economic policy implications are subject to the same qualifications made earlier when discussing the less extreme versions of the rational expectations formation model. In recalling those models, labour contracts and unsystematic policy mitigate the rational expectation implications of stabilization policy, and therefore, economic policy can have a non-neutral impact on macroeconomic variables.

2.4 ESTIMATING EXPECTED VALUES USING SPECIFIC MARKET OUTCOMES

It is mentioned in Chapter 1 that because expectations of economic variables are unobservable it is impossible to actually determine the efficacy of stabilization policy. Furthermore, it is noted in that chapter that the efficacy of stabilization policy can only be assessed and tested in conjuction with hypotheses concerning actual expectations formation and empirical proxies of these expectations. Generally speaking, there are two approaches to generating empirical proxies of unobservable expectations formation. The first approach is to use actual market determined outcomes that incorporate actual market expectations. This approach generates independent empirical proxies by using alternative methods of extracting expectations from the market determined outcomes. Each alternative method makes different assumptions about how expectations are formed. The second approach is to use independent statistical techniques to generate expected value proxies. These may be constrained (simple extrapolative methods) or unconstrained (Box/Jenkins) techniques. In this thesis both general approaches are used. The independent statistical technique approach will be discussed further in the next chapter when the Box/ Jenkins model of expectations formation is discussed. In the remainder of this chapter, the market determined outcome approach is discussed in more detail.

Actual inflation rate expectations are implicit in both market determined nominal interest rates and market determined wage rates. This thesis is concerned with whether the determination of interest rates in financial markets is more conducive to rational expectations formation, in the Muth sense, than the determination of wage rates in labour markets, and also, whether there is a difference in the ability of economic agents in each market to act on changes in expectations. These two market characteristics are major factors in allowing the researcher to draw conclusions about actual expectations formation from market determined outcomes. If one market is more conducive to rational expectations formation than another, the researcher is better off using information produced in this market when generating expectational proxies of economic variables.

In order for the market consequences of rational expectations to be revealed, specific markets must exhibit two important characteristics. Firstly, each economic agent in the market must have access to all relevant information pertinent to the true structural model. Secondly, each economic agent must be able to act immediately on any new information that affects the true structural model. This in turn assumes that each economic agent understands the economic structure affecting the variable and that each agent is able to correctly perceive changes in the structural model. We now turn to an analysis of financial and labour markets to see if either of these markets exhibits these characteristics.

2.4.1 Financial Markets

Poole (1976) argues that the market for actively traded financial assets is the most plausible arena in which to find expectations which are rational in the Muth sense. The major explanation for this contention is that the market for actively traded financial assets is an auction market that is well organized with a large number of traders and for which there exists continuous price determining transactions. As an auction market, it is able to reflect new and relevant information quickly. There are two reasons supporting this conjecture. Firstly, the existence of speculators in an auction market ensures that prices are forced to levels consistent with all available information. Speculators are able to earn excess profits with new information[9] and the existence of excess profits results in greater competition to exploit the new information. Secondly, the nature of an auction market is conducive to exploiting new information quickly because a convergence between asking and selling price is always achieved and contracts are signed frequently. As a result, financial market prices always tend to reflect anticipated conditions quite closely because interest rates and security prices are able to incorporate all available and pertinent information. It is expected, therefore, that financial markets will be useful markets from which to deduce accurate characteristics of the expectations formation process.

2.4.2 Labor Markets

Unlike the financial market, the labour market does not exhibit the characteristics required for an efficient market. The primary reason for this situation is that labour markets are not continuous auction markets, and therefore, it is difficult for prevailing wage settlements to reflect all available information. Labour markets are characteristically non-continuous auction markets because of two-conditions. Firstly, there are no labour market contract traders that persistently force prices to levels consistent with all available information. As new information is costly to gather and process, relevant information may not be reflected in current wage settlements. Secondly, because labour markets are not continuous auction markets where contracts are signed frequently, there is no automatic convergence between the asking and selling wage rate. Thus, new information is reflected in current wage settlements very slowly. Consequently, labour market wages may not reflect anticipated conditions accurately and hence labour markets will not be useful markets from which to deduce accurate characteristics of the expectations formation process.

'Chapter Notes'

[1] This example is taken from Parkin, 1982. P. 382.

[2] The concept of an efficient market is explained clearly by E. Fama in his paper "Efficient Capital Markets: A Review of the Theoretical and Empirical Work." Journal of Finance. Pp. 383-417. Vol.25. 1970.

[3] However, Pesando (1975) makes the point that autoregressive models will be strong-form models when the true economic model, determining the relevant economic variable, can be reduced to an autoregressive structure. In this situation, economic agents, using only current and past values of the relevant economic variable, would be using the strong-form information base or the true structural model.

[4] Cagan, 1956. P. 37.

[5] Muth, 1961. P. 316.

[6] B. Friedman, 1979. P. 23.

[7] Milton Friedman (1979. P. 6.) is acknowledged to have been the first writer to explain how a change in the money supply growth rate will affect price expectations and subsequently nominal interest rate values.

[8] David Laidler (1982. P. 85.) explains this point in a different way. Laidler argues that economic agents, intending to generate rational expectations, need not consider determining the systematic money supply behavior or the systematic behavior of any other factors which affect price level changes, if the past history of the price level reflects the consequences of that systematic

behavior.

[9] It is assumed that speculation in financial markets results in stable markets which allows for the existence of profits.

Chapter III

ALTERNATIVE MODELS OF INFLATION RATE EXPECTATIONS FORMATION

In this chapter, three different models for obtaining expected values for the inflation rate are described and used to generate proxy values for expected inflation. Each model makes different assumptions about how expectations are formed.

Model one uses the Fisher equation (see equation 2.1) to generate proxy values for expected inflation. Three different applications of the Fisher equation are employed to generate three independent measures of expected inflation. All three proxies of expected inflation use market determined nominal interest rates to generate expected inflation rates[1]. The three applications differ in their assumptions concerning the manner in which the expected inflation rate variable in the Fisher equation is calculated. Application one of model one subtracts an estimate of the real interest rate from nominal interest rate values to generate proxies for expected inflation. Applications two and three assume that the expected inflation rate variable in the Fisher equation can be represented by a geometrically distributed lag of past inflation rates and a polynomial distributed lag of past inflation rates respectively. Proxies for expected inflation are calculated, using applications two and three, by estimating the lagged coefficients for each model over a sample period and then employing these coefficients to generate independent measures of expected inflation.

Model two of the inflation rate expectations models is similar to model one in that it uses nominal interest rates as the basis for generating expected inflation rate proxies. However, unlike applications 1 and 2 of model 1, model 2 assumes that the real interest rate is variable in the short-run and converges to a long-run equilibrium value. A proxy for long-run expected inflation is calculated in a two-step process. The first step is to extrapolate two different maturity values to determine the long-run equilibrium nominal interest rate value. The second step is to subtract, from the long-run nominal interest rate value, an estimate of the long-run real interest rate thus producing a proxy for the expected inflation rate.

Model three differs from models one and two in that financial market determined interest rates are not the basis for generating proxy values for expected inflation. Model three is based on the statistical time series methodology of Box/Jenkins which identifies and estimates the stochastic nature of the inflation rate variable. It employs this estimate to generate empirical proxies for the expected inflation rate.

Expected values for the monthly inflation rate are generated for the test period January 1960 to December 1979 - a time period when the inflation rate shows considerable variation. This is a good period to test the accuracy of alternative models of inflation rate expectations formation because it is expected that inflation rate expectations changed considerably during this period[2].

The monthly inflation rate in this thesis is the percentage change in the Canadian Consumer Price Index (CPI) from each month in a given year to the same month in the following year[3]. Appendix A lists the monthly values for the inflation rate calculated in this manner for the test period 1960/1 to 1979/12 and the Statistics Canada Cansim (Canadian Socio-Economic Information Management) data base source. Also included in Appendix A is a graph of the monthly values for the test period.

The nominal interest rate values used in the inflation rate expectations models are Government of Canada 3-month treasury bill rates and 10-year average bond yields. Appendix B lists the monthly yield values for these Government of Canada securities and the Statistics Canada Cansim data base source. Also included in Appendix B is a graph of the 3-month nominal interest rate monthly values for the test period under study.

3.1 MODEL 1: FISHER EQUATION APPLICATIONS

3.1.1 Application 1 - The Fama Model

As indicated by equation (2.1), Fisher (1907) states that the nominal interest rate is equal to the expected real interest rate plus the market's assessment of the expected rate of inflation. This equation implies that the nominal interest rate is a good predictor of inflation if three conditions apply; namely, if expected inflation is perfectly anticipated, real interest rates are constant and nominal interest rates are adjusted instantaneously. Fama (1975), who tests this equation, hypothesizes that each month market participants set the nominal interest rate as though they perceive the expected real return to be constant[4]. Therefore, all the variation in the nominal interest rate is a direct reflection of the variation in the market's assessment of the expected value of inflation. Consequently, the market determined nominal interest rate is a good predictor of inflation.

In order to test his hypothesis and to determine the value of the real interest rate, Fama regresses inflation rates, observable at the end of the current period, on nominal interest rates applicable for the current period. His relevant estimating equation is

$$P_t = a_0 + a_1(i_t) + v_t \qquad (3.1)$$

where: P_t = the inflation rate observable at the end of time period t;

i_t = the nominal interest rate prevailing during the tth time period;

a_0 = a constant expected real interest rate;

v_t = a non-serially correlated error term

and his hypothesis of a constant real interest rate is given by the null hypothesis[5]

$$H_0: a_0 = E(r), \ a_1 = 1$$

where: $E(r)$ = a constant expected real interest rate.

Fama theorizes that if the coefficient estimates derived from running the relevant estimating equation over a sample period are consistent with the null hypothesis, and if there is no autocorrelation in the disturbance term, then the nominal interest rate is a good predictor of inflation. Fama postulates that an autocorrelation problem in the sample results indicates that market participants are not efficiently processing information from past values of the inflation rate to improve the accuracy of their inflation expectations. He feels that market participants should be able to learn from past expectational errors and be able to incorporate this knowledge into subsequent expectations. In terms of its effect on the regression results, an autocorrelation problem hides the fact that the true standard errors are larger than the standard error estimates and thus, parameter estimates are less precise than they are reported.

Fama's procedure is one way to test if nominal interest rates are good predictors of inflation. It tests whether nominal interest rates efficiently incorporate all the relevant information determining the inflation rate and whether finanicial markets quickly record changes in inflation expectations. This test procedure also indirectly evaluates the hypothesis that economic agents efficiently use the information in past inflation rates.

Table 3.1 shows the results of applying Ordinary Least Squares (OLS) to equation (3.1) for the test period under study. Student-t tests are performed on a_0 and a_1 according to the null hypothesis. The t-observed values are reported in parenthesis under the estimated coefficients. The significance level of the tests is 95% so that the critical t-value is 1.96. The expected inflation rate proxy is derived by subtracting the estimate of the real interest rate found in equation (3.1) from the monthly nominal interest rate. The expected values of inflation calculated for the test period using this procedure are given in Appendix A.

TABLE 3.1

OLS Results for Equation (3.1)

$$P_t = a_0 + a_1(i_t) + v_t \tag{3.1}$$

a_0	a_1	R^2	D_W.
-1.21 (-3.5)	1.07 (1.34)	.62	.07

The results in Table 3.1 offer some support for acceptance of Fama's null hypothesis. The observed t-value for the first part of the null hypothesis ($a_0 = 0$) is -3.5 which is greater than -1.96. This means that the null hypothesis is rejected in favour of the alternate hypothesis that a_0 equals a positive number[6]. Also, the observed t-value for the second part of the null hypothesis ($a_1 = 1$) is 1.34 which is less than 1.96. This means that the second part of the null hypothesis cannot be rejected at the 95% level.

In general, the results in Table 3.1 are contradictory. On the one-hand, the fact that the second part of the null hypothesis ($a_1 = 1$) cannot be rejected indicates that financial markets efficiently incorporate the factors influencing the actual inflation rate into nominal interest rate values. On the other-hand, however, there is a significant autocorrelation problem as indicated by the low Durbin Watson (DW) statistic[7]. As mentioned earlier, a significant autocorrelation problem signifies that agents do not use the information contained in past inflation rates efficiently when predicting future inflation rates. The contradictory nature of the results might be explained by the possibility that the significance of the test results are distorted by the autocorrelation problem. The significance of the autocorrelation problem revealed in equation (3.1) will be discussed further, in Chapter 5, when the results of the Fama model are tested for rationality characteristics.

3.1.2 Application 2 - Adaptive Expectations Model

As noted in the above section, the Fisher equation (see equation 2.1) can be used to generate empirical proxies for the expected inflation rate by making particular assumptions about the formulation of the expected inflation variable in that equation. Fama's model assumes that the expected inflation variable can be represented by perfect inflation foresight. Another assumption concerning the expected inflation rate variable is that it can be represented by a geometrically declining weighted average of current and past inflation rate values. This is the familiar adaptive approach to expectations formation discussed in section 2.3.2.

If we assume that expectations of inflation are generated in an adaptive manner and subsequently incorporated into nominal interest rate values according to the Fisher equation, nominal interest rate movements should reflect a slow adjustment process mirroring changes in inflation rate expectations formation. The fact that nominal interest rates are likely to reflect actual expectations quickly, (reference section 2.4), allows the researcher the opportunity to indirectly test this hypothesis of expectations formation. In other words, by determining how well a geometrically declining weighted average of past inflation rates explains nominal interest rate movements, the research analyst is able to draw general conclusions about how expectations are formed.

Using this application of the Fisher equation, an independent measure of expected inflation is generated over a sample test period in a two step process. The first step is to regress nominal interest rate values, for the sample period, on a geometrically declining weighted average of current and past rates of inflation. This determines the weighting structure that best explains interest rate movements. The second step is to use the estimated coefficients from the regression to generate an independent measure of expected inflation. Because the number of lagged variables in this approach is theoretically infinite, the number of coefficients used in the independent measure of expected inflation is cut off at a point where the marginal coefficient is not noticeably different from zero. Following this procedure, equation (2.1) is re-expressed as

$$i_t = a_0 + a_1 (P^e{}_t) + u_t \qquad (3.2)$$

where $P^e{}_t = \sum^{\infty}{}_{i=0} w_i (P_{t-i})$ (3.3)

$a_0 = r;$

$a_1 = (1+r);$

and the weights associated with current and past values of the inflation rates, w_i, are a function of the degree of geometric decline. Equation (3.2) is estimated over the test period by a computer programme that automatically determines the weights of the lagged coefficients[8]. The calculated set of weights is given in Appendix D. These weights are used to generate the independent measure of expected inflation listed in Appendix A.

Table 3.2 gives the OLS results for equation (3.2) corrected for autocorrelation using the Cochrane-Orcutt procedure. The t-ratios for the significance of the estimated coefficients are given in parentheses.

TABLE 3.2

OLS Results for Equations (3.2) and (3.3)

a_0	a_1	R^2	D.W.	RHO	b
2.19	0.88	0.96	2.03	0.31	0.95
(1.3)	(2.88)				

The Table 3.2 results are quite interesting. The Durbin Watson statistic indicates that there is no autocorrelation problem for the equation at the 95% level of significance. The high R^2 statistic indicates that movements in the nominal interest rate over the test period can be well explained by a geometrically declining weighted average of current and past inflation rate values where the declining pattern is very gradual; i.e., the calcualted value of b is 0.95. (See footnote 8 for explanation of b.) These results, combined with the contention that financial market outcomes likely reflect actual expectations formation, appear to support the theory that expectations are formed in an adaptive manner.

3.1.3 Application 3 - Polynomial Distributed Lag Model

Another assumption that can be made concerning the expected inflation rate variable in the Fisher equation is that it can be represented by a polynomial distributed lag structure. This implies that economic agents formulate inflation expectations based on a polynomial weighted distributed lag of current and past inflation rates. This assumption concerning expectations formation is less restrictive than assuming economic agents formulate expectations in an adaptive manner because it allows the estimated coefficients, on current and past inflation rate values, to follow a weighting scheme that is not predefined in nature. By allowing the user to select the degree of the polynominal, the length of the distributed lag and whether there should be any end point restrictions on the lag, the polynomial distributed lag approach to expectations formation enables the user to determine if there is a weighting scheme different from that suggested by Cagan's adaptive expectations model that explains nominal interest rate movements the best. Accordingly, the estimated coefficients in the polynomial model are free to take on values that best explain nominal interest rate movements thus reflecting accurate inflation expectations formation.

Yohe and Karnosky (1969) and Pyle (1972) employ a polynomial lag to represent the expected inflation variable in an extended Fisher equation. The Fisher equation is extended in order to include variables accounting for changes in the real interest rate. The extended Fisher equation and polynomial distributed lag model are represented by

$$i_t = a_0 + a_1 P^e{}_t + a_2(MCHG) + a_3(ICHG) + u_t \quad (3.4)$$

where: $P^e{}_t = \sum^n{}_{i=0} w_i(P_{t-i})$ (3.5)

MCHG = money supply growth rate (M1);

ICHG = Canadian industrial production index growth rate;

u_t = error term.

The expected inflation time series, for the test period, is derived using a process similar to the one employed by application two (2) of the Fisher equation (adaptive expectations model). That is, the best fitting polynomial lag structure of past inflation rates is substituted for the expected inflation variable in equation (3.4) to generate estimated coefficients for current and past inflation rate values. These estimated coefficients are then employed to produce an independent measure of expected inflation[9].

Table 3.3 gives the OLS results for equation (3.4) using a first degree polynomial with a 24 month lag and a tail restriction. This polynomial distributed lag structure is found to be the best among a number of alternative models tried[10].

The results in Table 3.3 show correct signs for all the variables and the R^2 of 0.79 indicates that this model ex-

TABLE 3.3

OLS Results for Equations (3.4) and (3.5)

a_0	a_1	a_2	a_3	R^2	D.W.
2.96	0.79	-0.18	0.12	0.79	0.19
(12)	(27)	(-10)	(5.8)		

plains a significant portion of the changes in the nominal interest rates over the test period. However, the Durbin Watson statistic reveals a significant autocorrelation problem. The Cochrane-Orcutt autocorrelation adjustment procedure is employed to correct for serial correlation. However, the results generated from the application of the procedure are unrealistic and abandoned.

One particularly interesting characteristic of application three's best fitting polynomial model is that despite being unrestricted in its choice of distributed lag weighting structure, the estimated coefficients follow a weighting structure that is similiar to the estimated coefficient weighting structure of the more restrictive geometrically-distributed lag model (application 2). That is, a 1st degree polynomial lagged 24 months with an end-point restriction is similar to a geometrically declining weighting structure. Moreover, it is interesting to note that the more restrictive geometrically declining weighting structure seems to explain more of the movements in the nominal interest rate than the polynomial distributed lag weighting structure; i.e., the R^2 for the adaptive expectations model is 0.96 and for the polynomial lag model is 0.79. These results appear to indicate that economic agents follow the more orthodox adaptive approach to expectations formation.

3.2 MODEL 2: EXTRACTING EXPECTED INFLATION FROM THE TERM STRUCTURE

A method of extracting a time series for expected inflation and instantaneous short-term interest rates from the interest rate term structure has been advanced by Frankel (1982)[11]. The Frankel model is another method by which market determined nominal interest values are used to generate proxies for expected inflation. Unlike the Fisher equation applications discussed above, Frankel does not assume that the real interest is constant or that nominal interest rates immediately reflect changes in expected inflation.

Frankel's term structure model is based on two assumptions concerning the relationship between the expected inflation rate and the nominal interest rate. Firstly, he contends that the expected inflation rate is gradually incorporated into the nominal interest rate with the passage of time, as lenders and borrowers react to actual inflation. In effect, financial markets are not immediately responsive to expected inflation rate values as was assumed by Fama. Under this assumption, long-term interest rates will reflect expected inflation more fully than short-term interest rates. Secondly, Frankel contends, that in the long run, the real rate of return converges to a long-run equilibrium value; consequently, the nominal interest rate converges to a long-run equilibrium value which reflects only the expected inflation rate. According to Frankel, this assumption implies that the rate of return on a given term of maturity[12] can be regarded as a weighted average of the instantaneous short-term rate, that is sensitive to monetary policy, and the infinitely long-term rate that reflects only the expected inflation rate.

Frankel's postulated relationship between the short-term and long-term interest rate is given as

$$i^r = (1-w_r)(P^e{}_t + a) + (w_r)(i_t) \qquad (3.6)$$

where: i^r = interest rate on bond with term to maturity of r months/years;

$P^e{}_t + a$ = the long-term interest rate;

i_t = instantaneous short-term interest rate;

w_r = weight attached to instantaneous short term interest rate for bond with term to maturity of r months/years.

Frankel asserts that the rate at which the yield on a given term of maturity converges to its long run equilibrium value is given by C in the equation

$$di^r/dt = -C(i^r - P^e{}_t - a) \qquad (3.7)$$

where: C = the rate at which the yield on a given maturity converges to its long run equilibrium the real rate of return converges to a long-run equilibrium value; consequently, the nominal interest rate converges to a long-run equilibrium value which reflects only the expected inflation rate. According to Frankel, this assumption implies that the rate of return on a given term of maturity[12] can be regarded as a weighted average of the instantaneous short-term rate, that is sensitive to monetary policy, and the infinitely long-term rate that reflects only the expected inflation rate.

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$P^e{}_t + a$ = the long-term interest rate;

i_t = instantaneous short-term interest rate;

w_r = weight attached to instantaneous short term interest rate for bond with term to maturity of r months/years.

Frankel asserts that the rate at which the yield on a given term of maturity converges to its long run equilibrium value is given by C in the equation

$$di^r/dt = -C(i^r - P^e{}_t - a) \qquad (3.7)$$

where: C = the rate at which the yield on a given maturity converges to its long run equilibrium value;

p^e_t = the expected rate of inflation at the current period;

a = the long run real interest rate.

Frankel argues that given these two assumptions an equation similar to (3.6) can be constructed for each of any two different maturities at any time. Once the weights are calculated for each equation, the associated two-equation system can be solved for the long-run, steady-state expected rate of inflation, p^e_t, and the instantaneous expected short-term interest rate, i_t. These two reduced form equations are

$$p^e_t=(w_{r1}(i^{r2}-k_{r2})-w_{r2}(i^{r1}-k_{r1})/w_{r1-r2}) - a \qquad (3.8)$$

$$i_t=(w_{r1}-1)(i^{r2}-k_{r2})-(w_{r2}-1)(i^{r1}-k_{r1})/w_{r1}-w_{r2} \qquad (3.9)$$

where: k = a liquidity premium;

a = the real interest rate.

To make the method operational, a technique for determining the weights for each reduced form equation is required. Frankel determines the weights (W_{r1},W_{r2}) in the following manner. Firstly, he hypothesizes that if equation (3.6) holds, the weight attached to the instantaneous short-term rate of return, W_r, is a function of the rate at which the yield on a given maturity converges to its long run equilibrium value, C. This he represents as

$$w_r = (1-\exp(-C_r))/C_r. \quad (3.10)$$

He then contends that equation (3.7) suggests that the rate at which the real interest rate converges to its long-run equilibrium value, B, is also a function of C and can be represented as

$$B = \exp(-C/n) \quad (3.11)$$

where: n = number of observations per year for the given term of maturity[13].

His final step is to generate an empirical estimate for the value of B. To do this he regresses the interest rate spread between two different r-maturity bonds against its lagged value. This equation he represents as

$$(i_t^{r2}-i_t^{r1}) = A + B(i_{t-1}^{r2}-i_{t-1}^{r1}) + u_t. \quad (3.12)$$

Once the value of B is determined, values for C and W_r can easily be determined by using equations (3.11) and (3.10) respectively. Once the values of C and W_r are calculated, the two-equation system (3.8,3.9) can be solved for the long-run expected rate of inflation, $p^e{}_t$, and the instantaneous expected short-term interest rate, i_t.

The two maturities used in the Frankel model are 3-month Government of Canada treasury bill yields and 10-year Government of Canada average bond yields. Table 3.4 gives the estimate of B obtained from running regression equation

(3.12) over the sample period. Table 3.4 also gives the implied values for C, W_{r1} and W_{r2}. The estimated value of B, 0.98, suggests that the nominal interest rate value converges quickly to a long-run equilibrium value. This result is further supported by the estimated value for w_{r1} of 0.97 which proposes that at any time, the nominal interest rate is influenced predominantly by its long-run equilibrium value. The time series for expected inflation and expected short-term interest rates are generated using equations (3.8) and (3.9) and the estimated values for w_{r1} and w_{r2}.

TABLE 3.4

Estimation of B - The Speed of Adjustment

$$(i_t^{r2}-i_t^{r1}) = A + B(i_{t-1}^{r2}-i_{t-1}^{r1}) + u_t \qquad (3.12)$$

Terms of Maturity		Regression Results				Implied Parameters		
r_1	r_2	A	B	R^2	D.W.	C	w_{r1}	w_{r2}
3mths	10yr	0.01	0.98	0.93	1.62	0.21	0.97	0.42

Following Frankel's procedure, the time series for the long-run expected inflation rate is adjusted for the real rate of interest and liquidity premium (see equation 3.8), by subtracting, from each month's estimated expected value of inflation, the average of the difference between the actual inflation rate and the estimated expected value of inflation. The average value of this difference, which represents an estimate of the real interest rate plus a liquidity premium, is calculated to be 3.15. Appendix A lists the expected values of inflation over the sample period using this procedure. The expected values for the 3-month nominal interest rate are listed in Appendix B.

3.3 MODEL 3: ARIMA TIME SERIES MODEL

As mentioned earlier in section 2.4, an alternative approach to using financial market outcomes for generating empirical proxies of the expected inflation rate is to use independent statistical techniques. One such technique is the time series methodology of Box/Jenkins (1970). This methodology is an example of the weak-form unconstrained approach to expectations formation in that it allows the user to identify and estimate the stochastic process of the time series using only current and past values of the economic variable. This is accomplished by analysing the pattern of randomness embodied in current and past values of the time series. In this section the Box/Jenkins time series methodology is used to generate a time series for expected inflation[14].

The Box/Jenkins methodology assumes there is a basic underlying pattern in a time series which is represented by historical data. By identifying the pattern, future forecasts of the variable are possible. The Box/Jenkins methodology follows a three step procedure. The first step is to identify a preliminary model that fits the pattern. The second step is to estimate the model parameters from historical data and generate estimated values for the variable over a sample period. The final step is to apply various tests to the estimated values to ascertain if the model selected fits the pattern and minimizes the error of the pattern[15]. If the model does not fit the data, a new model is considered and the process begins again. Once the best model is selected, the estimated parameter coefficients are used to generate forecasts of the time series.

This approach to generating expected values of an economic variable postulates three general classes of models that can describe any time series pattern: (a) Autoregressive (AR), (b) Moving Average (MA), and (c) mixed AutoRegressive-Moving Average (ARMA). The autoregressive model is applicable when future values of a variable are influenced by those of the past in a specific fashion. An autoregressive model can be represented as

$$X_t = a_1 X_t + a_2 X_{t-1} + \ldots a_n X_{t-n} + u_t \qquad (3.13)$$

where X_t is a future value of the variable; X_{t-1}, X_{t-2},.., X_{t-n} are values of the variable currently and at previous time periods; and u_t is an error term that cannot be explained by the model.

The Moving Average model implies that future values of an economic variable depend on previous values of the forecast error term and not on the variable itself. This model is represented as

$$X_t = u_t - b_1 u_{t-1} - b_2 u_{t-2} - \ldots b_n u_{t-n} \qquad (3.14)$$

where $u_t, u_{t-1}, \ldots, u_{t-n}$ are current and past forecast error terms.

The last class of model is a mixed AutoRegressive-Moving Average model which incorporates both AutoRegressive and Moving Average characteristics. This model implies that future values of a variable depend on both past values of the variable and past forecast errors.

To choose the appropriate type of model, the autocorrelations and partial autocorrelations of the historical data are analysed. This allows the researcher to identify the significant correlations between current and past values of the variable plus current and past values of the error term. In this manner, the researcher is able to identify the order of the model. An example makes this easier to understand. Suppose the sample autocorrelations of a monthly time series reveal that the variable is significantly related to the error term in the previous two periods. The non-seasonal order of the model, q, is two (2) where q equals the number of non-seasonal MA parameters. If, in addition, the autocorrelations for the monthly series reveal one (1) significant seasonal component; i.e., the current value of the variable is significantly correlated with the error term 12 periods before, the seasonal order of the model, Q, is one (1) where Q equals the number of seasonal MA components. In a similar manner, the partial autocorrelations of the historical data are used to identify the non-seasonal (p) and seasonal (P) autoregressive components of the model.

An important part of the Box/Jenkins approach to time series forecasting is rendering the time series stationary prior to choosing a model that fits the data. A time series is stationary when the underlying stochastic process is invariate with respect to time. If a time series is nonstationary, it is inappropriate to represent it with a fixed coefficient model. Therefore, by rendering a time series stationary, the researcher can argue that the stochastic process is stable and thus can be represented by a fixed coefficient model[16]. The autocorrelations of the time series are used to identify whether the series is stationary or non-stationary. A time series showing random deviations around its mean is considered stationary. If a series is non-stationary, non-seasonal and seasonal differencing are used to render it stationary. The degree of differencing (number of times of differencing) required to render a series stationary is denoted as d, for non-seasonal differencing, and D for seasonal differencing. Once the time series has been rendered stationary, and the Autoregressive and Moving Average elements identified, the model is expressed in the notational form

$$\text{ARIMA}\ (p,d,q)(P,D,Q)_S$$

where: s = seasonal time frame.

Following the procedure described above, the Box/Jenkins time series methodology is applied to the Canadian CPI over the sample time period. It is found that a model of the form ARIMA $(0\ 2\ 1)(2\ 0\ 2)_{12}$ is the best fitting model. It is best fitting in the sense that if the CPI time series is differenced twice and if one non-seasonal Moving Average component as well as two seasonal Moving Average and AutoRegressive components are incorporated into the model, the model parameter estimates are significant and the empirical proxies of the inflation rate generated by the model exhibit randomly distributed residuals.

There are two general approaches for applying the estimated ARIMA model to produce expected values for the Canadian CPI. The first method is based on the assumption that estimated parameters of the model remain fixed over time. The fixed coefficients are used in each period to generate an empirical proxy for the following period. The second method assumes that the model parameters fluctuate over time reflecting changes in the stochastic nature of the variable. This method requires re-estimating the model parameters every period as new information about the stochastic nature of the variable becomes available. Although this approach involves a considerable amount of data processing, it represents a more realistic approach to expectations formation since it means that expectations are based only on the information available to the economic agent at the time a forecast is made. In contrast, the first application of the ARIMA model infers that economic agents have access to future information when they formulate expectations[17]. A rational forecast, in the Muth sense, requires that all available information concerning the true structural model be used in generating forecasted values of the economic variable. This necessitates the second application of the ARIMA model be employed in generating expected value proxies.

Following Ridell and Smith (1976), the second application of the ARIMA model is used to generate empirical proxies for the Canadian CPI. The model parameters are re-estimated monthly using a 10-year moving average estimation period starting from January/1950 - December/1959 to December/1969 - November/1979. Table 3.5 shows some of the re-estimations over the sample test period. One-month ahead and two-month ahead expected values for the Canadian CPI are produced each month over the sample period using the re-estimated model parameters. These one-month and two-month ahead expected values are compared to actual CPI values one year earlier to produce expected inflation rate proxies. The one-month and two-month ahead expected inflation rate proxies derived in this manner are listed in Appendix A.

TABLE 3.5

ARIMA Model Parameter Estimation Results for the CPI

Sample	NSAR1	NSAR2	NMA1	NSMA1	NSMA2	Var
1952(1)-1961(12)	.01	.99	.90	.03	.82	.04
1954(1)-1963(12)	.01	.99	.94	.02	.82	.04
1956(1)-1965(12)	.30	.65	.96	.69	.13	.04
1958(1)-1967(12)	.02	.93	.97	.36	.45	.04
1960(1)-1969(12)	.01	.98	.98	.03	.82	.20
1962(1)-1971(12)	.01	.99	.95	.01	.79	.05
1964(1)-1973(12)	.01	.98	.85	.09	.78	.05
1966(1)-1975(12)	.03	.96	.85	.03	.77	.10
1968(1)-1977(12)	.03	.95	.82	.04	.78	.13

NSAR = number of seasonal AR parameters
NMA = number of MA parameters
NSMA = number of seasonal MA parameters
Var = in-sample variance

The results in table 3.5 show that the parameter estimates change over the sample period. These results support the argument that the model parameters should be re-estimated monthly to reflect model parameter changes. In most cases, however, the parameter changes are minimal except for the 1956-1965 and 1958-1967 periods when the parameter estimates changed significantly. It is interesting to note that the model exhibited a slowly increasing variance during the 1970's coinciding with the volatile inflation environment during this period.

'Chapter Notes'

[1] Because financial markets are conducive to rational expectation formation (reference section 2.4.1), nominal interest rate values, which incorporate inflation expectations according to the Fisher equation, present a valuable source of information from which to generate proxies for inflation rate expectations.

[2] The entire 1960/1 to 1979/12 test period does not exhibit the same degree of inflation rate variability. The period 1960/1 to 1970/12 reflects a more stable inflation rate pattern than the 1970/1 to 1979/12 period. This implies that expectations are revised more quickly in the second half of the test period than the first half. (See Appendix A for a graph of the monthly inflation rate over the test period.)

[3] The month-over-month inflation rate is chosen over the month-to-month inflation rate for the following reasons. Firstly, the month-over-month inflation rate is the so called 'official' inflation rate that is announced each month by Statistics Canada. As the 'official' inflation rate, the announcement effect will be large; that is, economic agents will most likely respond to this estimate and not to the month-to-month inflation rate calculation when determining nominal interest values in financial markets. Secondly, the month-to-month calculation shows more variation than the month-over-month estimate and therefore, it seems improbable that

economic agents will use this figure. Thirdly, the empirical proxies of the expected inflation rate generated by the alternative models considered in this thesis were compared to both versions of the actual inflation rate. Because the expected value proxies were closer to the month-over-month inflation rate calculation, this inflation rate calculation was considered a more appropriate value to employ.

[4] Implicit in Fama's hypothesis of a constant expected real interest rate is the assumption that financial markets are efficient and able to immediately incorporate changes in inflation expectations into current nominal interest rate values.

[5] In the Fama test, there are really two null hypotheses to test. The first hypothesis, that the real interest rate is a positive constant, is tested by speculating that the constant in the estimating equation is significantly different from zero (i.e., $a_0 = E(r)$). The second hypothesis is that the prevailing period's interest rate fully incorporates the following period's expected inflation rate. In other words, nominal interest rates move concurrently with the expected inflation rate. Fama tests this assumption by speculating that the interest rate variable coefficient, in equation (3.1), equals 1 (i.e., $a_1 = 1$).

[6] Equation (3.1) is a rearrangement of the Fisher equation. (Reference equation 2.1.) A negative value for

a_0 in equation (3.1) implies a positive real interest rate in the Fisher equation.

[7] The Cochrane-Orcutt adjustment procedure is applied to correct for the autocorrelation problem. The results from this procedure are not used because they are unrealistic.

[8] The computer software for the OLS regression is I.P. Sharp's 'EASY' regression package. The 'EASY' regression programme calculates the weights of the lagged coefficients in the following manner. Firstly, it assumes that the geometrically declining weights can be represented as

$$w_i = kb^i;$$

where: $w_1 = k;$

$w_2 = kb = bw_1 < w_1;$

$w_3 = kb^2 = bw_2 < w_2;$

and k = a positive constant which must be determined;

b = reflection of the behavior of the lag.

A high value of b reflects a very slow decline in the lag structure; in other words, the impact on the expected inflation rate, of the inflation rate 12 periods before, is little different to the impact of last period's inflation rate.

Secondly, by applying a Koyk transformation to equation (3.2), the equation

$$i_t = a_0(1-w) + k(P_t) + b(i_{t-1}) + u_t$$

is derived which can be used to generate estimates for k and b. Once these values are estimated, the complete set of geometrically declining weights can be calculated and used to generate the independent measure of inflation.

It should be noted that the geometrically declining factor employed in the calculation is (1-b) so that the long-run impact of a permanent change in the inflation rate on the nominal interest rate is k/(1-b).

[9] Two (2) criteria are used to determine the best fitting lag structure: 1) the estimated coefficients must have significant t-values, and 2) the regression's R^2 must be the largest among alternative lag structures. These are the criteria recommended by Pyle (1972).

Assuming that nominal interest rates reflect inflation rate expectations accurately, the value of the R^2 becomes an important statistic because it indicates whether the inflation expectations model best explains nominal interest rate movements.

[10] The estimated coefficients for a first degree polynomial with a 24 month lag of current and past inflation rates is given in Appendix D. These coefficients are used to generate the independent measure of expected inflation given in Appendix A. A number of other polynomial distributed lags are tried in equations (3.4) and (3.5); however, the results generated from these alternative models are not as good as the first-degree,

24 month lag model. Appendix D reports the results of some of the other models tried.

[11] Frankel's method for generating both an expected inflation rate time series and an expected nominal interest rate time series is discussed in this section. Model 1 - Application 2 of the interest rate models discussed in the next chapter is based on the Frankel interest rate expectations model discussed in this section.

[12] The term to maturity of a bond refers to the length of time the bond principle is loaned.

[13] For example, a one-year government bond has 1 observation per year whereas a one-month government bond has 12 observations per year.

[14] The Box/Jenkins time series methodology is used by Feige and Pearce (1970) and Riddell and Smith (1978) to generate empirical proxies for the expected inflation rate.

[15] The most common test is to determine if the model errors are randomly distributed.

[16] A stationary time series, with fixed parameter coefficients, is similar to a regression equation, with fixed coefficients, in that both assume the parameter estimates are stable over time.

[17] The first approach generates expectations based on parameter estimates calculated using information from the entire sample period. This means that expectations generated in the first part of the sample period use

information about the randomness of the variable that was not available until the last part of the sample period.

Chapter IV

ALTERNATIVE MODELS OF INTEREST RATE EXPECTATIONS FORMATION

In this chapter two different models for obtaining expected values proxies for the nominal interest rate are described. Model 1 utilizes the information contained in the interest rate term structure to generate empirical proxies for the expected nominal interest rate. Two different applications of the interest rate term structure model are employed to produce two independent measures of the expected nominal interest rate. The distinguishing characteristic of both applications of model 1 is that they both use financial market determined nominal interest rates to generate expected value proxies. Model 2 differs from model 1 in that market determined nominal interest rates are not the basis for generating empirical proxies. Model 2 applies the Box/Jenkins methodology to generate expected value proxies.

The nominal interest rates used in the interest rate expectations models are Government of Canada 3-month and 6-month treasury bill rates. Appendix B lists the monthly values for these two time series over the the test period as well as their data base source. In addition, a graph of the 3-month nominal interest rate is given in Appendix B.

Empirical proxies for the expected nominal interest rate are generated for the period January, 1960 to December, 1979. Since, the nominal interest rate fluctuates considerably during this period, it is a good sample period to test the accuracy of alternative models of interest rate expectations formation.

4.1 MODEL 1: INTEREST RATE TERM STRUCTURE APPLICATIONS

4.1.1 Application 1 - The Pure Expectations Hypothesis

The term structure of interest rates refers to the relationship among bond yields that differ in their length of time to maturity. The Pure Expectations Hypothesis Theory of the term structure of interest rates defines a particular relationship among bond yields. This theory supposes that there are no transaction costs involved in moving between bonds differing in lengths of time to maturity, and that investors will be indifferent toward holding a two-year bond for two years or purchasing two consecutive one-year bonds as long as the total sum accured to both investments is the same[1]. If the sum is different, the investor will purchase that maturity which generates the higher yield, and in so doing the yield on the other maturity will rise, equating the two yields in the long-run. This process is commonly referred to as financial market arbitrage and is the distinguishing characteristic of the Pure Expectations Hypothesis Theory of the interest rate term structure.

Assuming arbitrage holds, the Pure Expectations Theory suggests that a long-term maturity yield is a geometric average of the current and expected short-term rates. This can be expressed as

$$(1 + R_n)^n = (1+r_1)(1+r^e_2)\ldots(1+r^e_n) \tag{4.1}$$

where: R_n = market observable rate for n-month long-term bond;

r^e_n = market expected value for a short-term bond in the nth month.

At any time the long-term bond rate, as defined by equation (4.1), contains an implicit set of forward interest rates. The Pure Expectations Hypothesis Theory infers that these forward rates are unbiased estimates of expected future rates; for the reason that, if the forward rates differ from expected rates, profit opportunities will exist and arbitrage activity will result in investors shifting investments to equate the yields on the different maturities.

Equation (4.1) can be used to derive expected monthly interest rate values by using two consecutive, annualized, market-observable interest rate values. An example makes this easier to understand. By employing two maturities; i.e., the annualized monthly yields on 3-month and 6-month Government of Canada securities, equation (4.1) can be expressed as

$$(1+ {}_tR_6)^2 = (1+ {}_tr_3)(1+ {}_{t+1}r^e_3) \tag{4.2}$$

where: ${}_{t}R_{6}$ = monthly rate for 6-month Government of Canada securities observable in time period t;

${}_{t}r_{3}$ = monthly rate for 3-month Government of Canada securities observable in time period t;

${}_{t+1}r^{e}_{3}$ = monthly rate for 3-month Government of Canada securities expected in time period t+1.

By rearranging equation (4.2), the implied 3-month yield that is expected for time period t+1. can be expressed in terms of the currently observable values for r_3 and R_6. This is expressed as

$$(1+ {}_{t+1}r^{e}_{3}) = (1+ {}_{t}R_{6})^{2} / (1+ {}_{t}r_{3}) \qquad (4.3)$$

A time series for expected one-month ahead nominal interest rate values, for the test period under consideration, is generated by equation (4.3). This time series is listed in Appendix B.

4.1.2 Application 2 - The Frankel Method

A distinguishing characteristic of the Pure Expectations Theory application of the interest rate term structure is that it requires two consecutive market observable interest rate values to derive an implied/expected value. The Frankel method, which extracts proxies for the expected interest rate from the interest rate term structure, does not have this restriction. (Reference section 3.2 for a review of the Frankel method.) Frankel advocates that because expected inflation is gradually incorporated into the nominal interest rate, the real interest rate gradually converges to an equilibrium level. As a result, at any time, the yield on a particular maturity is a function of the time elapsed for the nominal and real interest rate to reach its long-run equilibrium value. Using this assumption and a number of complex manipulations, Frankel is able to take any two maturity values and forecast both a long-term and short-term nominal interest rate value. His formula for deriving the expected short-term nominal interest rate is

$$i_t=(w_{r1}-1)(i^{r2}-k_{r2})-(w_{r2}-1)(i^{r1}-k_{r1})/w_{r1}-w_{r2} \quad (4.4)$$

where: k = a liquidity premium;

i_t = expected short-term interest rate;

i^r = interest rate on bond with term to maturity of r months/years;

w_r = weight attached to instantaneous short term interest rate for bond with term to maturity of r months/years.

A time series for expected, one-month ahead nominal interest rate values, for the test period, is generated using equation (4.4). This time series is listed in Appendix B.

4.2 MODEL 2: ARIMA TIME SERIES MODEL

An alternative approach to generating empirical proxies for the nominal interest rate that does not rely on the interest rate term structure is to utilize the time series methodology of Box/Jenkins. As noted earlier, the time series methodology of Box/Jenkins is a means of applying a weak-form unconstrained model of expectations formation. The procedure for indentifying and estimating an ARIMA model for short-term nominal interest rates and applying that model specification to generate one-month and two-month ahead forecasts is the same procedure as the one followed in section 3.3 for identifying a model and generating expected values for the inflation rate.

By applying the Box/Jenkins methodology to Government of Canada 3-month treasury bill rates over the test period, it is found that a model of the form ARIMA (1 1 0) is the best fitting model. Following the rationale for re-estimation of the model parameters on a monthly basis outlined in section 3.3, the interest rate ARIMA model is re-estimated monthly from January/1950 - December/1959 to December/1969 - November/1979, and the re-estimated parameters are used to generate one-month ahead and two-month ahead expected values for the 3-month treasury bill rate. Table 4.1 shows some of the re-estimations of the model parameters over the sample period. Appendix B lists the one-month ahead and two-month ahead expected nominal interest rate values for the test period under study using this ARIMA model.

TABLE 4.1

ARIMA Model Parameter Estimation Results for 3-month Treasury Bill Rates

Sample	NAR(1)	Variance
1952(1)-1961(12)	.11	.12
1954(1)-1963(12)	.13	.16
1956(1)-1965(12)	.12	.16
1958(1)-1967(12)	.14	.17
1960(1)-1969(12)	.18	.14
1962(1)-1971(12)	.35	.09
1964(1)-1973(12)	.52	.05
1966(1)-1975(12)	.56	.08
1968(1)-1977(12)	.55	.08

NAR(1) - number of significant autoregressive terms equals one.
VAR = in-sample variance

Table 4.1 reveals a gradual increase in value of the autoregressive parameter estimate beginning in the late 1960's and early 1970's. This represents a change in the underlying stochastic nature of the nominal interest rate variable. It is interesting to compare these results with the ARIMA inflation model results reported in Table 3.5 to determine if the Fisherian relationship between nominal interest rates and expected inflation (Reference equation 2.1) is true. That is, we would expect that changes in inflation rate expectations would be reflected in nominal interest rate changes. This postulated relationship is reflected in the test results reported in Tables 3.5 and 4.1. In Table 3.5, an increase in the variability of inflation rate forecasts starting in the early 1970's is indicated by an increase in the estimated model variance. The increase in expectations variability appears to be directly reflected in nominal interest rate movements. There is a change in the underlying stochastic process of the nominal interest rate beginning in the early 1970's; i.e., the 1st order autoregressive parameter estimates in Table 4.1 change abruptly in the early 1970's.

'Chapter Notes'

[1] The Pure Expectations Theory postulates that securities, with different terms to maturity, are perfect substitutes for each other. According to this theory, investors do not attach different risk premiums to maturities with different terms.

Chapter V

RATIONALITY TESTS FOR EXPECTATIONAL ERRORS

The objective of this Chapter is to subject the expected value proxies generated by the interest rate and inflation rate expectations models described in chapters 3 and 4 to a number of tests designed to determine their rationality, in the Muth sense. This thesis follows other studies in defining rationality in expectations formation as expectations which are unbiased, efficient, consistent and have forecast residuals that are uncorrelated with other costless information[1]. By assessing the rationality of the expectations generated in Chapters 3 and 4, conclusions can be made concerning the economic policy implications of various models of expectations formation. Also, the rationality test results can be used to determine the extent to which interest rate expectations formation differ from inflation rate expectations formation. These conclusions are discussed in chapter 6.

1. TESTS OF UNBIASEDNESS

An expectations model is rational, in the Muth sense, if the expected values generated by the model equal the mathematical expectations of the economic variable. An expectations model that has this characteristic is classified as unbiased. There are two alternative ways to test for unbiasedness. One way to test for this property is to perform the regression

$$X_t = a_1 + a_2\,({}_{t-1}X^e{}_t) + e_t \qquad (5.1)$$

where: X_t = observed value of the economic variable at time t;

${}_{t-1}X^e{}_t$ = expected value of the economic variable for time t generated at time t-1;

e_t = error term.

Unbiasedness, in expectations, is given by the null hypothesis

$$H_0\colon (a_1, a_2) = (0, 1);$$

where it is hypothesized that the model generates expectations that equal realized values ($a_2 = 1$) subject to random errors. Thus the null hypothesis implies that

$$X_t - {}_{t-1}X^e{}_t = e_t$$

where: e_t = random error term.

The test for unbiasedness requires that the null hypothesis is not rejected[2] and that the error term in equation (5.1) is not serially correlated[3].

A second way to test for unbiasedness is to calculate the mean square error (MSE) of the expectations and the Thiel[4] decomposition of each model's forecast errors. The MSE is calculated by taking the arithmetic average of the sum of each period's squared forecast error. The MSE calculation gives an indication of the accuracy or inaccuracy of each model. A MSE value of zero means the model is perfectly accurate. By comparing the actual inflation average over the sample period to the model inflation average, the nature of the inaccuracy (bias) is determined. In addition, because the MSE statistic implicitly incorporates a standardizing procedure; i.e., the actual inflation average is the same for each expectations model, it is an appropriate measure of the relative degree of forecast accuracy among the alternative models considered.

The Thiel decomposition procedure decomposes the forecast error into: (1) a bias component (U^m), which indicates any systematic error in the expectations; (2) a regression component (U^r), which indicates if the model replicates the degree of variability in the economic variable; and (3) a disturbance component (U^d), which indicates the unsystematic or random error component of the expectations. According to the Thiel decomposition tests, unbiased forecasts have values of U^m and U^r close to zero and U^d close to one.

5.1.1 Unbiasedness Test Results For Inflation Rate Models

In this section the two tests for unbiasedness are applied to the empirical proxies of expected inflation generated by the three models of inflation expectations formation discussed in chapter 3. The tests for unbiasedness will determine if any of the inflation expectations models are rational in the sense that expectations equal realized values subject to a random error.

Table 5.1 presents the test results using the first method of unbiasedness testing which is the method based on equation (5.1). The t-ratios are reported in parentheses. With the critical value of F, F_C .05 = 3.0, and the observed values of F, $F_O(.95,2,238)$ as shown in Table 5.1, the results indicate that the null hypothesis of an unbiased forecast is rejected at the 95% significance level for all the one-month ahead forecasts as well as for model 3's two-month ahead forecast. Also, as the critical lower bound for the D.W. test is 1.65, the D.W. statistic for each model indicates positive serial correlation in the disturbance term. This result further supports the rejection of the null hypothesis which requires that the error term be randomly distributed.

In summary, the Table 5.1 results indicate that, according to the first method of unbiasedness testing, none of the expectations formation models can be classified as unbiased.

Table 5.2 presents the test results using the second method of unbiasedness testing which is the method based on the MSE and the Thiel decomposition of the MSE. The decomposition of the forecast error indicates that the bias component is very significant for applications 2 and 3 of model 1 but much less significant for models 2 and 3 and application 1 of model 1. Over the test period, the mean rate of inflation is 4.96 percent. Comparing the actual inflation mean to the forecasted means given in Table 5.2 reveals that the nature of the bias in the forecast error for models 1 and 2 is downward, while the bias for model 3 is upward. In terms of the disturbance term results, models 2, 3 and application 1 of model 1 have a disturbance component (U^d) that is significantly larger than that of applications 2 and 3 of model 1. In terms of overall forecast accuracy, denoted by the MSE for each model, the one-month ahead MSE value of 0.26 for model 3 indicates that that model is the most accurate expectations model.

In summary, the results in Table 5.2 point out that, according to the second method of unbiasedness testing, there are no models that reveal completely unbiased results. However, application 1 of model 1 and models 2 and 3 exhibit unbiasedness test results superior to applications 2 and 3 of model 1.

TABLE 5.1

Unbiasedness Test One: Inflation Rate Models

$$X_t = a_1 + a_2 \left({}_{t-1}X_t^{e}\right) + e_t \qquad (5.1)$$

	Model 1			Model 2	Model 3	
Results	Appl 1	Appl 2	Appl 3		1mth	2mth
a_1	0.07	0.40	0.28	-2.49	0.05	0.22
	(0.24)	(2.38)	(2.32)	(-5.4)	(0.94)	(2.18)
a_2	1.09	1.28	1.27	1.51	0.96	0.91
	(19)	(32)	(46)	(17)	(104)	(51)
F(.95;2,238)	6.81	141	225	16	17	31
D.W.	0.07	0.08	0.12	0.07	1.55	1.05

where: F_c .05 = 3.0;

$D.W._c(.05;d_l)$ = 1.65.

Model 1 - Appl 1 = Fama Model

- Appl 2 = Adaptive Expectations Model

- Appl 3 = Polynomial Lag Model

Model 2 = Frankel Model Of Term Structure

Model 3 = ARIMA Model

TABLE 5.2

Unbiasedness Test Two: Inflation Rate Models

Results	Model 1 Appl 1	Appl 2	Appl 3	Model 2	Model 3 1mth	2mth
MSE	4.55	4.32	3.08	5.57	0.26	0.93
(U^m)	0.05	0.45	0.52	0.01	0.06	0.09
(U^r)	0.01	0.10	0.14	0.12	0.06	0.12
(U^d)	0.95	0.46	0.34	0.88	0.90	0.79
Mean	4.50	3.57	3.70	4.93	5.09	5.24

where: Model 1 - Appl 1 = Fama Model
- Appl 2 = Adaptive Expectations Model
- Appl 3 = Polynomial Lag Model

Model 2 = Frankel Model Of Term Structure

Model 3 = ARIMA Model

5.1.2 <u>Unbiasedness Test Results For Interest Rate Models</u>

In this section the two tests for unbiasedness are applied to the empirical proxies for the expected nominal interest rate generated by the three interest rate expectations models discussed in chapter 4.

Table 5.3 presents the test results using the first method of unbiasedness testing. The t-ratios are reported in parentheses. As the critical value of F, F_c .05 = 3.0, and the observed values of F, $F_o(.95,2,238)$ are as given in Table 5.3, the results indicate that the null hypothesis of unbiasedness is rejected at the 95% level for the one-month ahead forecasts of applications 1 and 2 of model 1 but is not rejected for the one and two-month ahead forecasts of model 2. As the critical value for the D.W. statistic is 1.65, the observed D.W. statistic specifies no serial correlation problem for application 1 of model 1 or the one-month ahead forecast of model 2 but does show a positive serial correlation problem for application 2 of model 1 and the two-month ahead forecast of model 2.

These results reveal that, according to the first method of unbiasedness testing, model 2 is an unbiased expectations formations model at least in its one-month ahead forecasts. All the other other models are biased.

Table 5.4 presents the test results using the second method of unbiasedness testing which is the test based on the Thiel decomposition of the forecast error. The results show that the bias component (U^m) is non-existent for model 2, virtually non-existent for application 2 of model 1 but relatively large for application 1 of model 1. In the test period the mean value of the interest rate is 5.74 percent. A comparison of the actual interest rate mean to the means of the expectations generated by the 2 models reveals that application 1 of model 1 is biased upward, whereas application 2 is biased downward. Model 2 reveals no bias in its mean value. Model 2 (ARIMA model) reports the lowest MSE value of the three models.

In summary, according to the second test for unbiasedness, the ARIMA model's 1-month and 2-month ahead expectations are unbiased, whereas both applications of model 1 yield biased expectations with application 2 (Frankel model) exhibiting less bias in its forecasts than application 1 (Pure Expectations Theory).

TABLE 5.3

Unbiasedness Test One: Interest Rate Models

$$X_t = a_1 + a_2\,({}_{t-1}X_t{}^e) + e_t \qquad (5.1)$$

Results	Model 1 Appl 1	Appl 2	Model 2 1mth	2mth
a_1	-0.52	0.14	0.05	0.11
	(-7.54)	(2.14)	(0.80)	(1.10)
a_2	1.04	0.99	0.99	0.98
	(96)	(96)	(101)	(61)
F(.95;2,238)	67	8.10	0.34	0.63
D.W.	1.73	1.39	2.02	1.07

where: F_c .05 = 3.0;

D.W.$_c$(.05;d_l) = 1.65.

Model 1 - Appl 1 = Pure Expectations Model

- Appl 2 = Frankel Model

Model 2 = ARIMA Model

TABLE 5.4

Unbiasedness Test Two: Interest Rate Models

	Model 1		Model 2	
Results	Appl 1	Appl 2	1mth	2mth
MSE	0.23	0.15	0.13	0.35
(U^m)	0.32	0.06	0.00	0.00
(U^r)	0.05	0.002	0.002	0.005
(U^d)	0.62	0.94	0.98	0.99
Mean	6.01	5.64	5.74	5.73

where: Model 1 - Appl 1 = Pure Expectations Model

- Appl 2 = Frankel Model

Model 2 = ARIMA Model

5.2 TESTS OF EFFICIENCY

An expectations model is defined to be rational if it efficiently incorporates all relevant information. Pesando (1975) devises a way to test for 'weak-form' efficiency; that is, for the property that expected value proxies efficiently utilize all the information contained directly or indirectly in realized values of the economic variable being forecast. According to this criterion, the weak-form efficiency test requires that expectations and realized values both follow the same stochastic process. Pesando argues that this criterion can be tested by determining whether empirical expectations and their corresponding realizations share a common autoregressive pattern; i.e., do expectations and realizations evolve through time in a similar fashion.

Pesando tests for efficiency, in this context, by first regressing each period's realized value on past values of the economic variable and then regressing each period's expected value on the same past values. In this manner, he determines if they both follow identical patterns. The number of past values chosen in the initial regression is the number that produces the lowest standard error of estimate. In notational form, Pesando's 'weak-form' efficiency test is represented by the two equations

$$X_t = a_1X_{t-1} + a_2X_{t-2} \ldots a_nX_{t-n} + u_t; \qquad (5.2)$$

and $${}_{t-1}X^e_t = b_1X_{t-1} + b_2X_{t-2} \ldots b_nX_{t-n} + v_t. \qquad (5.3)$$

where: X_t = observed value of the economic variable at time t;

${}_{t-1}X^e_t$ = expected value of the economic variable for time t generated at time t-1;

X_{t-n} = actual value of X in t-n;

u_t/v_t = error terms.

Pesando argues that efficiency is indicated by the null hypothesis

H_0: $a_n=b_n$ for all n.

The appropriateness of the Pesando efficiency test depends on the distribution of the two error terms, u_t and v_t, in equations (5.2) and (5.3). In particular, Mullineaux (1978) argues that the Pesando test yields incorrect results if the error terms are not identically distributed. In place of equations (5.2) and (5.3), Mullineaux proposes a test for non-identical error distributions using the Bartlett statistic, and a test for efficiency using a regression which does not require the homogeneity assumption. Mullineaux estimates the regression

$$X_t - {}_{t-1}X^e_t = a_o + (a_1-b_1)\ X_{t-1} + (a_2-b_2)\ X_{t-2} \ldots (a_n-b_n)X_{t-n} + (u_t-v_t) \qquad (5.4)$$

and indicates efficiency by the null hypothesis

H_0: $(a_n - b_n) = 0$ for all n.

5.2.1 Efficiency Test Results For Inflation Rate Models

In this section both the Pesando and Mullineaux tests for weak-form efficiency are applied to the empirical proxies of expected inflation generated by the inflation rate expectations models. Following Pesando, the number of lagged dependent variables used in the test is determined by finding the regression that has the smallest standard error of the estimate. It is found that lagging the inflation rate 10 times produces the smallest standard error.

The Bartlett test determines if the error distributions of Pesando's two estimating equations are identical. If the Bartlett statistic indicates non-equal error distributions, the Mullineaux test is preferred to the Pesando test. The Bartlett testing procedure is similar to other statistic testing procedures with the only difference being that it uses the Chi-Squared test statistic (χ^2). An observed χ^2 value, χ^2_o, greater than the critical χ^2 value, $\chi^2_c.05$, denotes the error distributions are not identical. Such a situation warrants the use of the Mullineaux efficiency test. Table 5.5 shows the results of applying the Bartlett test to the Pesando efficiency test results. As the critical value of χ^2, $\chi^2_c.05 = 3.84$, is lower than the observed χ^2 values for all 3 models, the null hypothesis of equal error distributions is rejected. This result supports the use of Mullineaux's 'weak-form' efficiency test.

Using the Mullineaux test results, the critical value of F, $F_C(.95;10,219) = 1.83$, is lower than the observed F-values for all the one-month and two-month ahead inflation expectations models except model 3's one-month ahead forecasts. Accordingly, the ARIMA model (model 3) is the only one-month ahead model of inflation expectations formation that efficiently utilizes all the information contained directly or indirectly in realized values of the inflation rate.

TABLE 5.5

Efficiency Test Results: Inflation Rate Models

	Model 1			Model 2	Model 3	
Results	Appl 1	Appl 2	Appl 3		1mth	2mth
Pesando[1]	20.69	223.00	474.00	12.40	46.00	22.00
Mullineaux[2]	30.82	107.00	216.00	80.84	.99	4.10
Bartlett[3]	39.00	450.00	66.00	490.00	11.00	146.00

where: 1. $F_c(.95;10,460) = 1.83$

2. $F_c(.95;10,219) = 1.83$

3. $\chi^2_c.05 = 3.84$

Model 1 - Appl 1 = Fama Model

- Appl 2 = Adaptive Expectations Model

- Appl 3 = Polynomial Lag Model

Model 2 = Frankel Model Of Term Structure

Model 3 = ARIMA Model

5.2.2 Efficiency Test Results For Interest Rate Models

In this section the Pesando and Mullineaux F-tests for weak-form efficiency are applied to the empirical proxies generated by the interest rate expectations models. It is found that 12 lagged values of the nominal interest rate produces the smallest standard error of the regression in the initial Pesando estimating equation.

Table 5.6 gives the results of applying the Bartlett test to the Pesando efficiency test results. As the critical value of χ^2, $\chi^2{}_c.05 = 3.84$, is lower than the observed χ^2 values for model 2 and application 2 of model 1, the null hypothesis of equal error distributions is rejected and the Mullineaux 'weak-form' efficiency test is the appropriate test. Because the null hypothesis of equal error distributions is not rejected for application 1 of model 1, the Pesando efficiency test is the appropriate test.

Using the Mullineaux test results, the critical value of F, $F_c(.95;10,219) = 1.75$, is lower than the observed F-values for application 2 of model 1 and the 2-month ahead value of model 2 but it is higher than the one-month value for model 2. Therefore, the null hypothesis of efficiency is not rejected, at the 95% confidence level, for the one-month ahead forecasts of model 2 but is rejected for application 2 of model 1 and the two-month ahead forecasts of model 2. The Pesando null hypothesis of efficiency is rejected for application 1 of model 1 as the observed F-value is greater than the critical F-value.

In summary, the test results reveal that the one-month ahead ARIMA model (model 2) is the only model of interest rate expectations formation that efficiently utilizes all the information contained directly or indirectly in realized values of the interest rate.

TABLE 5.6

Efficiency Test Results: Interest Rate Models

	Model 1		Model 2	
Results	Appl 1	Appl 2	1mth	2mth
Pesando[1]	64.00	756.00	124.00	155.00
Mullineaux[2]	2.89	3.59	0.53	33.00
Bartlett[3]	2.79	493.00	422.00	195.00

where: 1. $F_C(.95;12,456) = 1.75$

2. $F_C(.95;12,215) = 1.75$

3. $\chi^2_C .05 = 3.84$

Model 1 - Appl 1 = Pure Expectations Model

- Appl 2 = Frankel Model

Model 2 = ARIMA Model

5.3 TESTS OF CONSISTENCY

Pesando argues that an expectations formation model is consistent if expectations for time period t, generated at different times in the past; i.e., 1 period or 2 periods prior to time period t, are the same, subject to a random error. According to Pesando, a consistent expectations model is able to identify the stochastic nature of the economic variable and use this information to forecast identical future values of the variable regardless of when the forecasting is done. Pesando states that this criterion can be tested by determining whether the two different time period expectations share a common autoregressive pattern. He tests for consistency, in this context, using the two estimating equations

$$ {}_{t-1}X^e_t = b_1X_{t-1} + b_2X_{t-2} \ldots b_nX_{t-n} + v_t \qquad (5.5)$$

$$ {}_{t-2}X^e_t = c_1\,{}_{t-2}X^e_{t-1} + c_2X_{t-2} \ldots c_nX_{t-n} + w_t \qquad (5.6)$$

where: ${}_{t-1}X^e_t$ = expected value of X, for period t, generated in period t-1;

${}_{t-2}X^e_t$ = expected value of X, for period t, generated in period t-2;

${}_{t-2}X^e_{t-1}$ = expected value of X, for period t-1, generated in period t-2;

X_{t-n} = realized value of X in period n;

v_t/w_t = error terms.

According to Pesando, consistency is demonstrated by the null hypothesis

$$H_0: b_n = c_n \text{ for all } n$$

which shows that different time period expectations follow the same pattern or evolve in the same manner.

Pesando's test for consistency, like his test for efficiency, assumes that the error terms in the two estimating equations are identically distributed. Mullineaux proposes a test for conistency that does not require this restrictive assumption. He estimates the two equations

$$\begin{aligned}({}_{t-1}X^e{}_t - {}_{t-2}X^e{}_t) = {} & b_1X_{t-1} - c_{1\,t-2}X^e{}_{t-1} \\ & + (b_2 - c_2)X_{t-2} \dots \\ & (b_n - c_n)Xt{-}n \\ & + (v_t - w_t) \qquad (5.7)\end{aligned}$$

$$({}_{t-1}X^e{}_t - {}_{t-2}X^e{}_t) = b_1(X_{t-1} - {}_{t-2}X^e{}_{t-1}) \qquad (5.8)$$

and indicates consistency by the null hypothesis

$$H_0: b_n = c_n \text{ for all } n.$$

The possibility of different error terms in the estimating equations is not a factor in the Mullineaux test for consistency, since he subtracts one of Pesando's estimating equations from the other (eq. 5.7 = eq. 5.5 - eq. 5.6) to yield an equation with one error term only. He tests to determine if the estimating equations are identical by using the F-test.

5.3.1 Consistency Test Results: Inflation/Interest Rate Models

In this section, the Pesando and Mullineaux F-tests for consistency are applied to the expected value proxies generated by both the two-month ahead inflation rate ARIMA model and the two-month ahead interest rate ARIMA model. These models are chosen because they are the only ones that have the capability to generate 2-month ahead as well as 1-month ahead expected value proxies. The number of lagged inflation rate values employed in the consistency test is 10 and the number of lagged interest rate values used is 12.

Table 5.7 shows the results of applying the Bartlett test to the Pesando consistency test results. As the observed χ^2 values are greater than the critical χ^2 value of 3.84, the Mullineaux test is the preferable test. The results in Table 5.7 show that the observed inflation rate and interest rate F-values (2.96 and 6.77 respectively) in the Mullineaux test for consistency are both greater than the appropriate critical F-values (1.88 and 1.75 respectively). Therefore, the null hypothesis of consistency is rejected for both the inflation rate and nominal interest rate ARIMA expectations models.

In summary, the consistency test results for the sample period under study indicate that the ARIMA models of expectations formation are unable to generate consistent expectations using the information in past values of the relevant economic variable.

TABLE 5.7

Consistency Test Results: Inflation/Interest Rate Models

Results	Model 3	Model2
Pesando	64.34	2602
Mullineaux	2.96[1]	6.77[2]
Bartlett[3]	16.6	47.3

where: 1. $F_C(.95;9,230) = 1.88$

2. $F_C(.95;11,228) = 1.75$

3. $\chi^2_C .05 = 3.84$

Model 3 - Inflation rate ARIMA model

Model 2 - Interest rate ARIMA Model

5.4 TESTS OF ERROR UNPREDICTABILITY

The test for efficiency outlined in section 5.2 is a 'weak-form' efficiency test in that it determines if expectations efficiently incorporate all the predictive information available in past values of the economic variable. Rational expectations, in the Muth sense, are efficient in a broader sense. They incorporate all information that is relevant to the determination of the economic variable. For expectations to be rational, in the Muth sense, there should be no correlation between a forecast error and any relevant information set available at the time the forecast is made. In other words, the error term should be unpredictable.

A test for efficiency in this broader sense is achieved by regressing forecast errors on any relevant information set. A relevant information set is defined as any variable found in the true economic model. The relevant information set may be the current period value of a variable or it may be lagged values of the variable. Following B. Friedman (1979), a test for error unpredictability is given by the regression equation

$$({}_{t-1}X^e_t - X_t) = a_0 + \textstyle\sum^n_{t=1} a_i\ (I_{t-i}) \qquad (5.9)$$

where: ${}_{t-1}X^e_t$ = expected value of the economic variable for time t, generated in time t-1;

X_t = realized value of the economic variable in time t;

I_{t-i} = a relevant information set available at time t-i.

and error unpredictability is indicated by the null hypothesis

H_0: a_i=0. for all i.

The error unpredictability test, as represented by equation (5.9), determines if there is a significant correlation between lagged values of the relevant variable and the forecast error. In applying the F-test to equation (5.9), if the observed F-value is greater than the critical F-value, the null hypothesis of no correlation is rejected in favour of the alternate hypothesis of a significant correlation. This implies that the expectation could have been more accurate if this additional information had been considered[5].

There are many economic variables that can be considered relevant for the error unpredictability test. Following Friedman (1979), the month-over-month inflation rate, the seasonally adjusted unemployment rate, the money supply growth rate (M1), and the industrial production index growth rate are the relevant information sets chosen for the test. Appendix C lists the values of these time series over the test period. The inflation rate, the unemployment rate and the industrial production index growth rate are all lagged 12 periods in the error unpredictability tests. The money supply growth rate is lagged 6 periods to relect the assumption that money supply changes have an immediate impact on macroeconomic variables relative to the other variables considered.

5.4.1 Unpredictability Test Results For Inflation Rate Models

In this section, the error unpredictability test is applied to the forecast errors of the three inflation rate expectations formation models. The relevant information sets employed in the unpredictability tests are the unemployment rate, the industrial production index growth rate, and the money supply growth rate (M1). Table 5.8 presents the F-test results for the inflation rate model error unpredictability tests.

The results in Table 5.8 are mixed. Applications 2 and 3 of model 1 do not efficiently utilize the information contained in any of the three time series as indicated by the fact that the observed F-values are greater than the critical F-values in all three cases. Application 1 of model 1 and model 2 fare somewhat better. Application 1 of model 1 efficiently incorporates unemployment information, however it does not to so for the money supply and industrial production index growth rates. Model 2's expectations consolidate the information found in the industrial production index growth rate but does not incorporate the information found in the unemployment rate and money supply growth rate variables. In contrast, as the observed F-values are less than the critical F-value for all the model 3 tests, we conclude that model 3 efficiently embodies the information contained in all three economic variables.

In summary, of the 3 models tested, the only expectations model that utilizes the information in all three economic variables is the the ARIMA model (Model 3).

TABLE 5.8

Error Unpredictability Test Results: Inflation Rate Models

$$({}_{t-1}P^{e}_{t} - P_t) = a_0 + \sum^{n}_{i=1} a_i \, (I_{t-i}) \qquad (5.9)$$

	Model 1			Model 2	Model 3	
	Appl 1	Appl 2	Appl 3		1mth	2mth
U^1	0.57	1.86	2.75	2.39	0.41	0.71
I^1	2.61	3.72	3.13	1.58	0.68	0.87
M^2	30	16	17	10	0.31	0.59

where: 1. $F_c(.95;12,215) = 1.75$

2. $F_c(.95;6,227) = 2.10$

U = Unemployement Rate

I = Growth Rate of Industrial Production Index

M = Growth Rate of Money Supply (M1)

Model 1 - Appl 1 = Fama Model

- Appl 2 = Adaptive Expectations Model

- Appl 3 = Polynomial Lag Model

Model 2 = Frankel Model Of Term Structure

Model 3 = ARIMA Model

5.4.2 Unpredictability Tests Results For Interest Rate Models

In this section, error unpredictability tests are applied to the forecast errors of the two interest rate expectations formation models. The relevant information sets employed in the unpredictability tests are the unemployment rate, the industrial production index growth rate index, the M1 money supply growth rate and the month-over-month inflation rate. Table 5.9 presents the F-test results for the interest rate model unpredictability tests.

Once again, the results in Table 5.9 are mixed. Application 1 of Model 1 integrates the information found in all the economic variables except for the inflation rate, where the observed F-value of 2.34 is greater than the critical F-value of 1.75, thus indicating a rejection of the null hypothesis of error unpredictability. Application 2 of Model 1 incorporates the information found in all the variables except for the money supply growth rate variable and the unemployment rate variable. As the observed F-value is less than the critical F-value for all the error unpredictability tests for Model two's one-month ahead forecasts, the null hypothesis of error unpredictability cannot be rejected. The two-month ahead forecasts of Model 2 embody the information contained in all the relevant variables except for the money supply growth rate variable.

In summary, the Table 5.9 results indicate that the one-month ahead ARIMA ahead forecast model is the only interest rate expectations model that completely incorporates the information found in the relevant economic variables chosen.

TABLE 5.9

Error Unpredictability Test Results: Interest Rate Models

$$({}_{t-1}r^{e}_{t} - r_{t}) = a_0 + \sum^{n}_{i=1} a_i \, (I_{t-i}) \qquad (5.9)$$

Results	Model 1 Appl 1	Model 1 Appl 2	Model 2 1mth	Model 2 2mth
U^1	0.71	1.86	1.13	1.71
I^1	0.81	0.65	0.52	0.71
M^2	1.04	5.09	1.72	3.43
P^1	2.34	1.47	1.45	1.21

where: 1. $F_c(.95;12,215) = 1.75$

2. $F_c(.95;6,227) = 2.10$

U = Unemployement Rate

I = Growth Rate of Industrial Production Index

M = Growth Rate of Money Supply (M1)

P = Month-over-Month Inflation Rate

Model 1 - Appl 1 = Pure Hypothesis Model

- Appl 2 = Frankel Model

Model 2 = ARIMA Model

'Chapter Notes'

[1] B. Friedman (1970) was one of the first writers to use these four tests for rationality.

[2] The usual hypothesis testing procedure is followed. That is, an observable test statistic is calculated from the sample data and compared to a critical test statistic value. The critical test statistic value is a function of the type of test statistic used and the level of significance chosen for the test. For the first unbiasedness test, as well as for the other rationality tests considered in this thesis, the F-statistic is the relevant test statistic. The F-test procedure is completed by comparing the observable F-statistic, $F_o(.95;r1,r2)$, to the critical F- statistic, $F_c.05$, and rejecting the null hypothesis (accepting the alternate hypothesis) if the observed F-value is greater than the critical F-value. Acceptance of the alternate hypothesis means that the sample data does not support the hypothesis made concerning the population parameter. The r1 and r2 variables in the observed F-statistic represent the degrees of freedom for the numerator and denominator respectively in the F-statistic.

All the tests are conducted using a 95% confidence level. This in noted as .05 in the F_c statistic and .95 in the F_o statistic. A 95% confidence level implies that if a large number of samples are taken to test the null hypothesis, 95% of them will reject the null hypothesis when the null hypothesis is in fact false. The level of significance is a means of attaching a measure of accuracy to the test results.

[3] If the error term is serially correlated it means that the expectation fails to incorporate information available in past forecast errors. The Durbin Watson test is one way to check for an autocorrelation problem in the test results. If the observed Durbin Watson statistic, D.W., is less than the D.W. critical lower bound, $D.W._c(.05;d_l)$, the null hypothesis of non-serial correlated residuals is rejected.

[4] This Thiel decomposition procedure is the one suggested by Granger and Newbold (1977).

[5] Appendix E gives an example of the error unpredictability test where the relevant information set used is the M1 money supply growth rate and the forecast errors are those associated with the ARIMA one-month ahead inflation rate model and the ARIMA one-month ahead interest rate model.

Chapter VI

ANALYSIS OF TEST RESULTS AND CONCLUSIONS

This chapter summarizes the rationality test results for each inflation rate and interest rate expectations model. Inferences are drawn about the characteristics of each expectations formation model based on these test results; in particular, the economic policy implications of each model. In the conclusion to this thesis, the test results are employed in hypothesizing how expectations are actually formulated. Statements about the efficacy of government stabilization policy are made in light of the hypothesis fashioned about actual expectations formation.

6.1 ANALYSIS OF TEST RESULTS

6.1.1 Inflation Rate Expectations Models

Table 6.1 summarizes the results of applying the rationality tests to the expected value proxies generated by the inflation models discussed in chapter 3. For each model, an asterisk (*) indicates acceptance of the null hypothesis of rationality for the test under consideration and no asterisk indicates rejection of the null hypothesis. All tests are conducted at the 95% confidence level.

6.1.1.1 Model 1: Application 1 (Fama Model)

The Fama model tests the hypothesis that nominal interest rates are good predictors of inflation. Because financial markets are characteristically efficient markets, the hypothesis that nominal interest rates incorporate rational inflation rate expectations, according to the Fisherian equation, is plausible. The Fama model tests the hypothesis that real interest rates are constant and that changes in short-run nominal interest rates reflect changes in expected inflation rates only.

The Table 6.1 results suggest that application 1 of Model 1 produces expectations that are biased, inefficient and fail to incorporate information pertinent to the determination of the actual inflation rate. The biased nature of the model is a function of its failure to efficiently incorporate relevant information. Although application 1 fails both versions of the unbiasedness test, it reports a relatively small systematic bias component (U^m) especially when compared to applications 2 and 3 of Model 1. A small systematic bias component means that the expectational errors are not persistent or consistent in their biasedness, in other words, they exhibit some degree of randomness.

The rationality test results for application 1 are not surprising. It is generally recognized that real interest rates are not constant in the very short-run due to liquidity effects of changes in economic policy and the likelihood that changes in expected inflation are slowly incorporated into nominal interest rate values. Therefore, using a constant real interest rate will fail to identify rational expectations even if financial markets do identify future price movements correctly.

In summary, the rationality test results for model 1: application 1 support the conclusion that if economic agents assume constancy of the real interest rate and use nominal interest rates as predictors of inflation, they will make systematically biased expectational errors; however, the biasedness will not be persistent or consistent. If people behave in this manner, economic policy will be effective in influencing economic aggregate variables such as the unemployment rate but the duration and consistency of the effect will be indeterminant.

6.1.1.2 Model 1: Applications 2 and 3

Applications 2 and 3 of model 1 use the postulated Fisherian relationship to generate expectations of the inflation rate by substituting current and past values of the inflation rate for the expected price variable in the Fisher equation. Application 2 substitutes a geometrically declining distributed lag of current and past values of the inflation rate, for the expected inflation variable, whereas application 3 substitutes a polynomial distributed lag of current and past inflation rates for the expected inflation variable. These two applications of the Fisher equation are examples of weak-form constrained expectations formation models in that expectations are based on past values of the inflation rate in a constrained or deterministic manner.

As the Table 6.1 results indicate, both models produce expectations that are biased, inefficient and fail to incorporate any of the information assumed pertinent to the determination of the actual inflation rate. The degree of bias associated with both models, as indicated by the second test for unbiasedness (U^m), is markedly larger than the degree of bias associated with models 2 and 3 as well as application 1 of model 1. The fact that both models exhibit significant forecast bias and fail to efficiently incorporate information available in past inflation rates may be caused by their deterministic approach to expectations formation. That is, both models do not consider the stochastic properties of the inflation rate variable. The lower MSE result for application 3, relative to application 2, indicates the polynomial distributed lag approach to expectations formation yields expectations that are closer to realized values.

The rationality test results in Table 6.1 imply that if economic agents use an adaptive approach, or a polynomial distributed lag approach to expectations formation, they will generate systematically biased and persistent expectational errors. If people behave according to either of these two approaches, economic policy will be effective in influencing economic aggregate variables in a systematic and persistent fashion.

6.1.1.3 Model 2 (Frankel Model)

Model 2 is another test of the hypothesis that nominal interest rates are good predictors of inflation. It differs from applications 1 and 2 of model 1 in that it assumes real interest rates are variable in the short-run but constant in the long run when expected inflation is fully incorporated into nominal interest rate values. The rationality test results for model 2 are similar to the test results for application 1 of model 1 in that the expectations are biased, inefficient and fail to incorporate pertinent information. The fact that the model 2 expectations are biased is likely a result of the model not efficiently utilizing relevant informtion. This is evidenced by the failure of the model to pass the efficiency and error unpredictability tests.

The model 2 test results are not entirely surprising given the nature of the expectations formation model. The large MSE of model 2 is probably a result of the fact that the expected inflation rate generated by model 2 is a long-run, steady-state expected inflation rate. This expected value proxy is somewhat inconsistent with the short-run (one-month ahead) actual value to which the expected value is compared when calculating the expectational errors. The very small systematic bias component in the forecast error (U^m) suggests that this model is able to identify the underlying movements in the inflation rate to a certain degree, which is not totally unexpected given the fact that real interest rates do vary and this model is able to identify this variation.

The rationality test results for model 2 have the same policy implications as the test results for application 1 of model 1. The test results for model 2 imply that if economic agents assume the real interest rate is variable and use nominal interest rates as predictors of inflation they will make large prediction errors. The errors will biased but not persistently or consistently so as indicated by the small value of the bias component in the forecast errors. If people behave in this manner, economic policy will be effective although its effectiveness may be inconsistent.

6.1.1.4 Model 3 (ARIMA model)

Model 3 of the inflation rate expectations models is similar to model 1 in that it is a weak-form approach to expectations formation. It assumes that economic agents use past values of the inflation rate to approximate future values. Unlike the model 1 applications, this approach identifies the stochastic properties of the inflation rate variable and uses this information to generate expected value proxies. As an unconstrained model of expectations formation, this approach has two distinguishing characteristics. Firstly, if the stochastic process identified by the model is stable, the ARIMA model approach is theoretically capable of generating expectations that are rational in the Muth sense. Secondly, if economic agents are faced with equating the costs of expectations formation with the benefits of forecast accuracy, the weak-form unconstrained approach to expectations formation is a viable alternative to more rational models since it is able to identify the stochastic nature of the economic variable at a cost lower than that involved with using the true structural model.

The test results shown in Table 6.1 reveal that the unconstrained model approach to expectations formation exhibits a number of rational characteristics implying that this type of model is able to identify the stochastic nature of the inflation rate variable to a large extent. The test results of model 3 have more rational characteristics than the constrained models' test results (applications 2 and 3 of model 1) or the test results of the models based directly on nominal interest rates (application 1 of model 1 and model 2). This is evident by the fact that model 3's one-month expected inflation rate proxies efficiently utilize the predictive information in past values of the inflation rate (test 2) and in all other pertinent economic variables considered (test 3). This is not evident for the other one-month ahead models considered. The significantly smaller

MSE for model 3 is a logical consequence of efficiently incorporating information affecting the inflation rate. Although the bias component for model 3 (U^m) is larger than model 2 or application 1 of model 1, it is close to zero indicating that the systematic bias component of the model is very small.

The fact that both the 1-month ahead and 2-month ahead expectations are biased demonstrates that the unconstrained approach to expectations formation is not completly rational in the Muth sense. That is, it is unable to identify the stochastic nature of the inflation rate completely which means it neglects to account for some influences on the inflation rate variable.

The test results for model 3 support the contention that an unconstrained model approach to expectations formation is economically rational. It would be unlikely that the benefits of using a more rational based model; i.e., a more complex and completely specified model, would outweigh the cost of collecting and processing the information required to use the model.

In summary, if economic agents use the unconstrained model approach to expectations formation, as represented by the time series methodology of Box/Jenkins, they will generate expectational errors that are small and not significantly biased, in other words, the expectational errors will be small and very near random in nature. If expectations are formulated in this manner, the economic policy implications are significant. That is, government economic policy undertaken to affect macroeconomic variables will be minimal and random in nature because economic agents will diffuse, for the most part, the impact of the policy.

TABLE 6.1

Summary of Rationality Test Results: Inflation Rate Models

	TEST 1			TEST 2	TEST 3	TEST 4		
	1.	2.						
		U^m	MSE			M	U	I
Model 1 Appl		.05	4.5		N/A		*	
Model 1 App2		.45	4.3		N/A			
Model 1 App3		.52	3.1		N/A			
Model 2		.01	5.5		N/A			*
Model 3-1mth		.06	0.3	*	N/A	*	*	*
Model 3-2mth		.09	0.9			*	*	*

where: Model 1 Appl 1 = Fama Model

Model 1 Appl 2 = Adaptive Expectations Model

Model 1 Appl 3 = Polynomial Lag Model

Model 2 = Frankel Model Of Term Structure

Model 3 = ARIMA Model

Test 1 = Test of Unbiasedness (2 parts)

U^m - systematic bias component of error

Test 2 = Test of Efficiency

Test 3 = Test of Consistency

Test 4 = Test of Error Unpredictability

M - Money supply growth rate

U - Unemployment rate

I - Industrial Production Growth Rate

N/A - Not Applicable for this model

6.1.2 Interest Rate Expectations Models

Table 6.2 summarizes the results of applying the rationality tests to the expected values generated by the interest rate models discussed in Chapter 4. For each model, an asterisk (*) indicates acceptance of the null hypothesis of rationality for the test under consideration and no asterisk indicates rejection of the null hypothesis.

6.1.2.1 Model 1: Application 1 (Pure Expectations Model)

Model 1 - application 1 generates expected value proxies for the nominal interest rate based on the assumption that the Pure Expectations Hypothesis Theory of the interest rate term structure is relevant, and that expected values derived using the term structure formulae are unbiased estimates of actual interest rate expectations. As the results in Table 6.2 pinpoint, the Pure Expectations Hypothesis model generates expectations that are biased and inefficient, yet they are able to synthesize information available in other relevant economic variables. The biased nature of the model, as indicated by the value of U^m, is significant relative to the other expectations models and is a result of the model not efficiently incorporating relevant information.

The rationality test results for application 1 of model 1 infer that if economic agents generate interest rate expectations according to the Pure Expectations Hypothesis, expectations will be systematically and persistently biased and therefore economic policy will be able to systematically and persistently influence the real interest rate value, and subsequently the economic activity (investment and consumer expenditures) that is a function of the real interest rate.

The poor rationality test results for application 1 of model 1 are not unexpected. This model supposes that the Pure Expectations Hypothesis theory of the term structure holds true. However, this theory fails to consider liquidity premiums and segmented market influences on expected interest rates[1]. Consequently, the expectations generated by application 1 may not reflect actual expectations formation as initially hypothesized. This may bias the rationality tests in favour of rejecting the null hypotheses of rationality. An example makes this easier to understand. Suppose that investors decide, after considering the added risk in holding less liquid long-term securities, to hold liquid short-term bonds. This situation will result in long-term bonds exhibiting higher yields than short-term bonds. But according to the Pure Expectations Hypothesis, the yields on different terms to maturity should be equal. Therefore, estimates of expectations based on the Pure Expectations theory will be persistently biased because the estimates do not reflect the market's ingestion of liquidity premiums into nominal interest rate values. Accordingly, the rationality test results will be biased in favour of rejecting the null hypotheses of rational expectations.

6.1.2.2 Model 1: Application 2 (Frankel Model)

Application 2 of model 1 uses the Frankel technique for generating short-run expected nominal interest rate proxies. Frankel's model is based on the hypothesis that maturity rates, observable in financial markets, can be used to approximate short-term interest rates which are influenced primarily by economic policy rather than expected inflation. The Frankel model is more flexible than the Pure Expectations Model because it does not make any restrictive assumptions about maturity substitutability and liquidity premiums. For this reason, it is more reflective of actual interest rate expectations formation.

The Frankel model results in Table 6.2 appear to reflect the above analysis. Although the model is biased, inefficient and does not incorporate all the information available in other relevant economic variables, it does exhibit a lower MSE than application 1 of model 1 and a significantly lower bias component (U^m) of the forecast error. These results support the argument that the Frankel model is able to identify, and incorporate into expected value proxies, many of the factors influencing nominal interest rate movements. However, the fact that the model is inefficient and biased means that it does not consider all the factors affecting the nominal interest rate.

To summarize the Table 6.2 results for model 1 application 2, if economic agents generate interest rate expectations based on the Frankel model they will generate systematically biased expectational errors; however, the errors will not be consistently biased. Consequently, economic policy will be effective in influencing the real interest rate in a systematic manner but the effect will be inconsistent.

6.1.2.3 Model 2 (ARIMA Model)

Model 2 uses the time series methodology of Box/Jenkins to estimate the stochastic properties of the nominal interest rate. As mentioned earlier, the distinguishing characteristic of the Box/Jenkins unconstrained model approach to expectations formation is that it assumes the economic variable under study is a result of a stochastic process.

The Table 6.2 test results support the contention that the unconstrained approach to expectations formation is able to accurately identify the stochastic nature of the interest rate variable, and in so doing, generate rational expectations. The one-month ahead expectations of model 2 pass the rationality tests for unbiasedness, efficiency and error unpredictability and the two-month ahead expectations pass the rationality test for unbiasedness. In addition, the bias component of the forecast errors (U^m) for both the one-month and two-month ahead expected value proxies are non-existent implying that the expectational errors associated with these models are random in nature.

The test results for model 3 infer that if expectations are formulated according to the Box/Jenkins unconstrained approach and economic agents behave according to these expectations, government economic policy will have only random effects on the real interest rate because expectations will equal realized values subject to random errors. The fact that the test results show that the two-month ahead expectations of model 3 are inefficient, inconsistent, and less accurate (higher MSE) than the one-month ahead expectations of model 3, demonstrates that expectations formed in this manner are unable to identify the stochastic nature of the interest rate variable completely.

TABLE 6.2

Summary of Rationality Test Results: Interest Rate Models

	TEST 1			TEST 2	TEST 3	TEST 4			
	1.	2.							
		U^m	MSE			M	U	I	P
Model 1 App1		.32	.23		N/A	*	*	*	
Model 1 App2		.06	.15		N/A		*	*	*
Model 2-1mth	*	.00	.13	*	N/A	*	*	*	*
Model 2-2mth	*	.00	.35			*		*	*

where: Model 1 App1 = Pure Hypothesis Model

Model 1 App2 = Frankel Model

Model 2 = ARIMA Model

Test 1 = Test of Unbiasedness (2 parts)

U^m - systematic bias component of error

MSE - Mean Square Error

Test 2 = Test of Efficiency

Test 3 = Test of Consistency

Test 4 = Test of Error Unpredictability

M - Money Supply Growth Rate

U - Unemployment Rate

I - Industrial Production Growth Rate

P - Month-Over-Month Inflation Rate

N/A - Not Applicable for this model

6.1.3 Comparing Inflation Rate and Interest Rate Rationality test Results

In this section, the rationality test results for the interest rate expectations models and the inflation rate expectations models are analysed in context of the general principles of expectations formation outlined in Chapter 2.

The rationality test results for both the interest rate and inflation rate expectations models reveal that the 'weak-form' unconstrained or stochastic approach to expectations formation (ARIMA Models) exhibits more rational characteristics than any of the other 'weak-form' expectations models. The unconstrained models are the only models to pass some of the rationality tests for unbiasedness and efficiency and they exhibit a higher degree of successes in the rationality unpredictability tests. This finding suggests that unconstrained models are superior to constrained models or models based on nominal interest rate values in identifying the stochastic nature of the economic variable and predicting future movements of that variable.

Although the inflation rate models based exclusively on nominal interest rates (application 1 of model 1 and model 2) did not exhibit as many rational characteristics as the ARIMA models, they did display expectations with very little systematic bias especially when compared to the constrained models of expectations formation (applications 2 and 3 of model 1). This result implies that these models are able to identify, to a certain degree, the underlying stochastic nature of the inflation rate variable. This conclusion supports the hypothesis made in chapter 2 that financial markets efficiently incorporate the factors influencing inflation rate movements.

The rationality test results in this thesis also reveal that the interest rate expectations models exhibit more rational characteristics than the inflation rate expectations models. This is evident in their lower MSE statistics and their higher degree of successes in the error unpredictability tests. To the extent that economic agents use the inflation rate and interest rate expectation models outlined in this thesis, this conclusion infers that economic policy will have a greater impact on the macroeconomic variables that depend on inflation rate expectational errors (unemployment rate), than on the macroeconomic variables that depend on the real interest rate (housing starts). This conclusion also, indirectly, supports the contention made in Chapter 2 that financial markets are characteristically more efficient and generally more condusive to rational expectations formation than labour markets.

6.2 CONCLUSIONS

As noted in the introduction to this thesis the nature of inflation rate and interest rate expectational errors play major roles in the efficacy of discretionary economic policy. In particular, if actual inflation rate and nominal interest rate expectational errors are systematically and persistently biased, economic policy will have a systematic and persistent impact on the unemployment rate and the real interest rate. Consequently, a strong argument can be made for the use of stabilization policy to affect key macroeconomic variables. On the other hand, if actual inflation rate and nominal interest rate expectational errors are not systematically and persistently biased; i.e., random, economic policy will have only random effects on the unemployment rate and real interest rate; accordingly, the case for discretionary use of stabilization policy becomes much more uncertain. However, because actual market expectations are unobservable, it is impossible to determine if economic policy will have systematic or random effects.

This thesis uses alternative models of expectations formation to generate empirical proxies for the expected inflation rate and expected nominal interest rate over a 20 year test period from January, 1960 to December, 1979. It then subjects these empirical proxies to a number of tests to infer the nature of the expectational error associated with each model. By determining the nature of the expectational error associated with each model, it is possible to make inferences about the efficacy of economic policy when expectations are formulated according to that particular model.

The rationality test results reveal some major differences in the nature of the expectatonal errors associated with alternative models of expectations formation, and therefore, the test results imply that the efficacy of stabilization policy will be significantly affected by the manner in which expectations are formed. In particular, among the various weak-form models of expectations formation analysed in this thesis, the unconstrained model approach, as represented by the Box/Jenkins methodology, generates expected value proxies that are considerably more accurate and exhibit more rational characteristics; i.e., expectational errors that are random in nature, than the proxies generated by other weak-form models. Based on these test results, this thesis concludes that economic policy will have a significantly smaller impact on macroeconomic variables if economic agents use an unconstrained expectations model rather than a model that relies exclusively on nominal interest rate values or one that is constrained or extrapolative in nature.

The rationality test results for the alternative expectations models analysed in this thesis do not, however, provide any evidence on how people actually formulate expectations. If the actual expectations formation process used by economic agents can be identified as one of the alternative models considered in this paper, and if we assume that the test results reported in this thesis are accurate reflections of the rationality characteristics of each model, the conclusions drawn earlier in this chapter about the stabilization policy ramifications of each expectations model are of particular interest to the economic analyst intent on determing the appropriate policy actions to improve macroeconomic performance.

Although the task of identifying the expectations model actually used by economic agents is difficult because expectations formation is unobservable, it is possible to use the test results reported in this thesis to deduce how people actually formulate expectations and in so doing, make some conclusions on the efficacy of stabilization policy. Toward this end we can ask the following question about the alternative expectations formation models considered in this paper. Is the assumption that people act in a economically rational manner by weighing the cost and benefits associated with alternative expectations models more plausible than to assume that economic agents are content to use simplistic approaches to expectations formation such as those based on market determined nominal interest rates or those that are simple extrapolations of past data? It seems logical to believe that economic agents will choose the expectations model that exhibits the most economically rational characteristics. If we make this assumption, the test results reported in this thesis lead us to believe that the ARIMA unconstrained approach to expectations formation is the model economic agents prefer to use in generating actual inflation rate and interest rate expectations. This conclusion can be supported in the following manner. The test results show that the ARIMA inflation rate and interest rate expectations models generate empirical proxies that have many rational characteristics. These particular models exhibit more rational characteristics than any of the other models yet they all use the same information base. In this sense the ARIMA model approach is the most economically rational model of the alternative models considered because it is able to generate the most accurate empirical proxies at the same cost. In addition, it seems unlikely that a more complex expectations model incorporating more information than just past values of the economic variable will yield more economically rational expectations. That is, it is improbable that the benefits associated with more rational expectations will be greater than the costs of the information required to generate these expectations.

Following the above argument, we conclude that if economic agents act in an economically rational manner they will choose to employ the ARIMA, unconstrained model approach to expectations formation. This conclusion does not not preclude the possibility that economic agents employ other expectations models at the present time. Rather, it implies that economic agents will revert to an ARIMA unconstrained model approach to expectations formation as they become more sophisticated in their forecasting methods and as they weigh the costs and benefits of using models other than ARIMA unconstrained models[2].

Based on the rationality test results reported in this thesis, if economic agents use ARIMA forecasting models to generate expected values for key economic variables such as the inflation rate and nominal interest rate, expected values will be very close if not equal to realized values and thus would potentially fustrate the effectiveness of government stabilization policy, as contended by Sargent and Wallace (1975). However, this conclusion requires some qualification. Firstly, although expected values may equal realized values, participants in labour markets will be prevented from acting on these expectations because of the existence of long-run bargaining contracts. The resulting discrepancies between actual and expected values consequently allow stabilization policy to be effective. Secondly, even though cost-of-living-allowance (COLA) clauses are incorporated into labour market settlements to mitigate the above mentioned problem, the facts remain that COLA clauses are not perfect and they are periodic in nature, once again allowing discrepancies between actual and expected values to exist. Thirdly, it is important to note that the above conclusion is based on empirical results completed for a period which saw realized values change abruptly in the second half of the period. Consequently, employing a model to represent the whole period precludes incorporating any stochastic structural changes into the ARIMA process and thus may bias the results generated by the model. For this reason, the ARIMA models used in the empirical work cannot be considered completely reliable and thus the results and conclusions drawn from the empirical work need to be evaluated in light of these qualifications.

'Chapter Notes'

[1] A liquidity premium influence refers to the fact that long-term bonds are less liquid than short-term bonds and therefore long-term bonds show a higher yield than short term bonds in order to attract investment money. A segmented market influence may have two interpretations. It may refer either to the fact investors prefer one type of maturity more than another at a particular time or that the relative supply of short-term debt is greater/smaller than long-term debt. Because segmented market influences exist, the substitutibility of different maturities is reduced. This is not accounted for in the Pure Expectations Theory approach.

[2] It is not unrealistic to believe that economic agents use ARIMA modelling techniques. These techniques are relatively easy forecasting models to learn and inexpensive to apply. In addition, most of the people involved with generating expectations affecting key economic variables are trained in sophisticated forecasting methods; i.e., trade unions hire economists to forecast future movements in the inflation rate.

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Appendix A

Actual and Expected Value Proxies

Month-Over-Month Inflation Rate

1960/1 To 1979/12

MONTH-OVER-MONTH ACTUAL INFLATION RATE

1960	1.09	1.23	1.10	1.65	1.37	2.21	1.37	1.23	.95	1.08	1.08	1.35
1961	1.35	1.36	1.77	1.22	1.22	.94	1.08	.94	.54	-.13	.13	.13
1962	.40	.67	.53	.93	.94	1.20	1.60	1.74	1.47	1.73	1.59	1.59
1963	1.73	1.73	1.73	1.59	1.72	1.85	1.84	1.97	1.84	1.57	1.57	1.83
1964	1.70	1.83	1.96	1.95	1.95	1.82	2.07	1.54	1.68	1.55	1.54	1.93
1965	2.05	2.05	2.05	2.04	2.30	2.80	2.53	2.53	2.54	2.67	3.04	2.9
1966	3.02	3.64	3.64	4.01	3.87	3.35	3.33	3.96	4.34	4.33	3.81	3.55
1967	3.42	2.79	2.91	3.25	3.25	3.60	4.18	4.16	3.80	3.56	3.79	4.14
1968	4.49	4.48	4.59	4.20	4.07	3.82	3.56	3.31	3.89	4.24	4.34	4.09
1969	3.84	3.72	3.94	4.47	4.70	5.24	4.98	5.08	4.52	4.51	4.49	4.58
1970	4.57	5.01	4.55	4.07	3.85	3.18	3.16	2.84	2.85	2.73	2.30	1.46
1971	1.66	1.66	1.86	1.95	2.37	2.36	2.76	3.48	3.48	3.48	3.89	5.04
1972	.91	4.89	4.67	4.54	4.22	4.11	4.58	4.64	5.25	5.24	5.12	5.09
1973	5.66	5.83	6.02	6.56	7.23	8.09	7.71	8.31	8.56	8.74	9.28	9.12
1974	9.05	9.55	10.35	9.96	10.97	11.41	11.22	10.81	10.83	11.58	11.92	12.46
1975	12.11	11.82	11.29	11.04	10.13	10.32	11.04	11.09	10.63	10.61	10.42	9.48
1976	9.59	9.15	9.02	8.90	8.90	7.83	6.80	6.23	6.50	6.23	5.62	5.82
1977	6.13	6.73	7.39	7.56	7.57	7.80	8.37	8.33	8.43	8.77	9.13	9.50
1978	8.96	8.69	8.79	8.42	9.05	9.23	9.83	9.42	8.63	8.67	8.85	8.43
1979	8.88	9.18	9.25	9.75	9.27	8.85	8.10	8.44	9.58	9.26	9.40	9.76

SOURCE: Statistics Canada Cansim System (D484000)

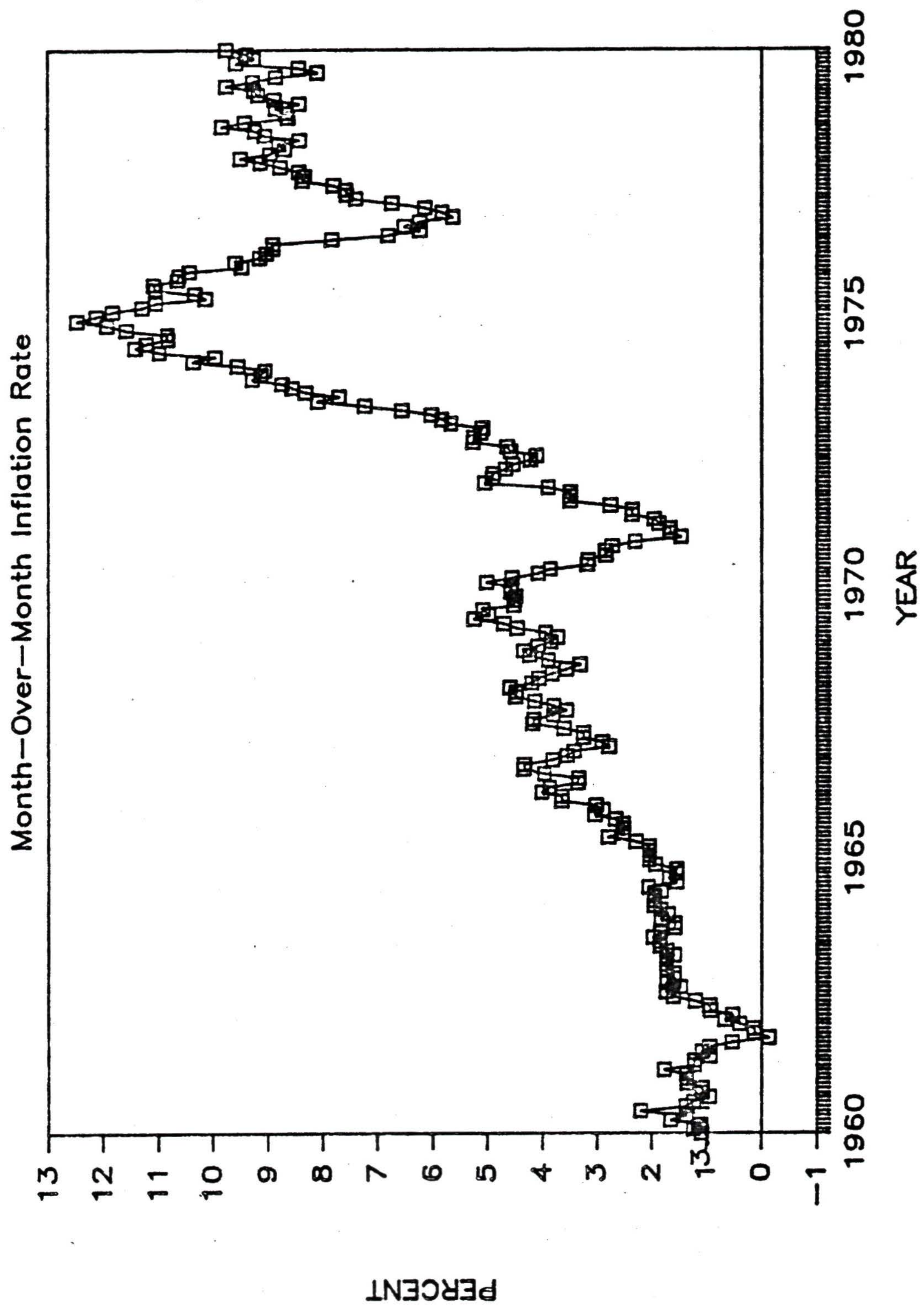

Figure A.1: Monthly Inflation Rate -1960/1 To 1979/12

ESTIMATES OF THE EXPECTED INFLATION RATE

MODEL 1 - APPLICATION 1 (Fama Model)

1960	3.91	3.39	3.40	1.80	2.05	1.80	1.86	1.71	.8	.49	1.82	2.74
1961	2.04	1.83	1.90	2.00	2.07	1.93	1.36	1.34	1.05	1.38	1.29	1.29
1962	1.78	1.86	2.00	1.91	1.86	2.31	4.24	4.26	3.74	3.78	2.95	2.50
1963	2.70	2.44	2.47	2.41	2.45	1.98	2.03	2.22	2.50	2.35	2.38	2.42
1964	2.57	2.56	2.67	2.67	2.49	2.37	2.38	2.46	2.59	2.52	2.49	2.66
1965	2.61	2.53	2.53	2.41	2.56	2.69	2.72	2.84	2.87	2.92	2.94	2.95
1966	3.33	3.43	3.48	3.85	3.87	3.90	3.79	3.81	3.83	3.80	3.98	3.94
1967	3.75	3.47	3.37	2.92	2.79	3.03	3.07	3.11	3.13	3.55	3.74	4.25
1968	4.74	5.08	5.59	5.77	5.78	5.74	5.35	4.82	4.27	4.45	4.36	4.45
1969	5.03	5.17	5.22	5.37	5.59	5.53	5.92	6.41	6.48	6.56	6.39	6.55
1970	6.60	6.57	6.39	5.79	5.57	5.13	4.73	4.49	4.30	4.18	3.80	3.19
1971	3.23	3.47	2.85	1.95	1.79	1.82	2.16	2.47	2.58	2.85	2.26	2.03
1972	2.00	2.15	2.24	2.36	2.43	2.52	2.29	2.25	2.29	2.41	2.36	2.47
1973	2.44	2.69	2.78	3.25	3.69	3.97	4.27	4.53	4.97	5.29	5.32	5.22
1974	5.14	5.01	4.86	5.30	6.43	7.42	7.54	7.89	7.90	7.73	7.10	6.28
1975	5.91	5.19	5.05	5.12	5.64	5.66	5.78	6.23	6.66	7.20	6.95	7.31
1976	7.43	7.38	7.58	7.86	7.78	7.73	7.77	7.86	7.92	7.90	7.80	7.38
1977	6.93	6.83	6.44	6.33	6.37	5.84	5.86	5.93	5.93	5.89	6.03	6.05
1978	5.96	5.92	6.09	6.52	6.98	6.99	7.05	7.45	7.59	7.96	8.64	9.15
1979	9.25	9.64	9.61	9.71	9.61	9.63	9.57	10.03	10.24	10.43	12.40	12.41

ESTIMATES OF THE EXPECTED INFLATION RATE

MODEL 1 - APPLICATION 2

(Adaptive Expectations Model)

1960	1.48	1.45	1.43	1.41	1.41	1.40	1.42	1.41	1.38	1.35	1.32	1.29
1961	1.28	1.26	1.25	1.25	1.23	1.20	1.17	1.15	1.12	1.08	1.01	.96
1962	.91	.87	.85	.82	.81	.80	.79	.81	.84	.85	.87	.89
1963	.91	.93	.94	.96	.98	1.01	1.03	1.06	1.09	1.11	1.12	1.12
1964	1.14	1.15	1.17	1.19	1.21	1.23	1.24	1.26	1.26	1.26	1.27	1.27
1965	1.28	1.30	1.32	1.34	1.35	1.38	1.43	1.46	1.49	1.52	1.56	1.61
1966	1.66	1.70	1.77	1.84	1.91	1.98	2.02	2.06	2.12	2.19	2.27	2.31
1967	2.34	2.37	2.36	2.37	2.38	2.40	2.43	2.48	2.53	2.56	2.58	2.61
1968	2.65	2.70	2.76	2.81	2.84	2.87	2.89	2.89	2.88	2.90	2.93	2.97
1969	2.99	3.00	3.01	3.02	3.06	3.10	3.16	3.21	3.26	3.28	3.30	3.32
1970	3.35	3.37	3.40	3.42	3.41	3.39	3.35	3.31	3.26	3.21	3.15	3.08
1971	2.98	2.90	2.82	2.75	2.69	2.65	2.61	2.59	2.59	2.60	2.60	2.63
1972	2.70	2.76	2.82	2.86	2.89	2.92	2.93	2.97	3.01	3.07	3.13	3.18
1973	3.22	3.29	2.36	3.44	3.53	3.65	3.79	3.92	4.06	4.20	4.35	4.51
1974	4.66	4.80	4.95	5.12	5.28	5.47	5.67	5.86	6.02	6.17	6.35	6.53
1975	6.73	6.91	7.07	7.20	7.31	7.38	7.45	7.54	7.64	7.70	7.77	7.82
1976	7.83	7.84	7.83	7.81	7.80	7.78	7.72	7.62	7.5	7.40	7.29	7.16
1977	7.04	6.94	6.87	6.83	6.80	6.77	6.74	6.74	6.74	6.74	6.75	6.78
1978	6.82	6.83	6.83	6.84	6.83	6.84	6.86	6.90	6.92	6.91	6.90	6.90
1979	6.87	6.87	6.88	6.89	6.93	6.94	6.94	6.91	6.89	6.92	6.94	6.96

ESTIMATES OF THE EXPECTED INFLATION RATE

MODEL 1 - APPLICATION 3 (Polynomial Lag Model)

1960	1.13	1.09	1.05	1.05	1.03	1.07	1.06	1.04	1.01	1.00	.98	.98
1961	.99	1.00	1.03	1.04	1.04	1.02	1.01	.99	.94	.86	.79	.73
1962	.69	.66	.63	.63	.62	.64	.67	.72	.75	.80	.83	.87
1963	.91	.95	.99	1.02	1.06	1.10	1.14	1.19	1.23	1.25	1.27	1.30
1964	1.31	1.33	1.36	1.38	1.40	1.41	1.43	1.41	1.41	1.40	1.38	1.40
1965	1.41	1.43	1.44	1.46	1.49	1.55	1.58	1.62	1.66	1.70	1.76	1.81
1966	1.87	1.96	2.04	2.15	2.23	2.29	2.34	2.42	2.52	2.62	2.67	2.71
1967	2.73	2.71	2.70	2.70	2.70	2.72	2.77	2.82	2.84	2.85	2.86	2.90
1968	2.95	3.00	3.05	3.07	3.09	3.09	3.07	3.04	3.05	3.07	3.11	3.12
1969	3.12	3.12	3.12	3.16	3.20	3.28	3.33	3.39	3.41	3.43	3.45	3.46
1970	3.48	3.52	3.54	3.52	3.49	3.43	3.36	3.27	3.19	3.10	2.99	2.83
1971	2.70	2.56	2.45	2.34	2.27	2.20	2.16	2.17	2.18	2.19	2.24	2.35
1972	2.46	2.56	2.65	2.73	2.80	2.85	2.94	3.02	3.14	3.25	3.35	3.44
1973	3.56	3.68	3.80	3.94	4.11	4.32	4.49	4.69	4.89	5.08	5.30	5.49
1974	5.67	5.87	6.10	6.30	6.55	6.80	7.03	7.22	7.39	7.59	7.80	8.02
1975	8.21	8.36	8.46	8.54	8.54	8.55	8.60	8.64	8.65	8.65	8.63	8.56
1976	8.49	8.39	8.28	8.17	8.06	7.88	7.66	7.41	7.19	6.96	6.71	6.48
1977	6.29	6.14	6.06	5.99	5.93	5.89	5.90	5.91	5.93	5.97	6.04	6.14
1978	6.21	6.26	6.33	6.37	6.45	6.54	6.66	6.76	6.80	6.84	6.88	6.89
1979	6.93	6.97	7.01	7.07	7.10	7.10	7.06	7.03	7.07	7.09	7.11	7.16

ESTIMATES OF THE EXPECTED INFLATION RATE

MODEL 2 (Frankel Model)

1960	2.55	3.15	2.89	3.93	3.72	3.74	3.37	3.63	3.82	4.04	3.49	3.22
1961	3.75	3.78	3.45	3.55	3.56	3.58	3.63	3.67	3.92	3.72	3.65	3.6
1962	3.27	3.28	3.12	3.05	2.96	2.97	2.03	2.27	2.6	2.53	2.69	2.98
1963	2.87	2.98	3.06	3.07	2.88	3.14	3.13	3.27	3.34	3.15	3.09	3.17
1964	3.05	3.10	3.01	3.15	3.28	3.31	3.29	3.26	3.18	3.2	3.13	2.91
1965	2.81	2.75	2.87	3.02	2.89	2.91	2.96	3.08	3.18	3.09	3.16	3.2
1966	2.91	2.86	3.17	2.83	2.85	2.85	3.02	3.14	3.48	3.17	2.96	3.35
1967	3.23	3.16	3.31	3.37	3.61	3.71	3.94	3.93	4.11	4.14	4.3	3.99
1968	3.84	3.58	3.51	3.71	3.19	3.84	3.52	3.69	4.01	4.17	4.65	4.79
1969	4.91	4.61	4.64	4.56	4.52	4.9	4.63	4.29	4.26	4.69	4.84	5.3
1970	5.58	5.57	5.39	5.50	5.86	6.53	6.59	6.46	6.77	6.65	7.05	6.74
1971	5.80	5.05	5.85	6.38	6.88	7.58	7.17	7.27	6.59	6.06	6.05	5.97
1972	5.99	6.17	6.41	6.91	6.91	6.97	7.34	7.44	7.32	7.27	6.95	6.55
1973	6.64	6.52	6.54	6.34	6.16	6.53	6.33	6.11	5.93	5.51	5.27	5.42
1974	5.59	5.78	5.88	6.33	6.56	5.98	6.86	6.89	7.25	7.08	6.74	6.78
1975	6.89	6.61	6.49	6.97	7.58	6.98	7.18	7.65	7.41	7.58	7.08	7.25
1976	6.99	6.68	6.49	6.49	6.46	6.46	6.48	6.45	6.17	6.05	6	5.85
1977	5.57	5.74	6.22	6.67	6.68	6.94	6.84	6.75	6.52	6.62	6.67	6.73
1978	6.85	7.40	7.42	7.13	6.86	6.87	6.83	6.41	6.29	5.98	6.05	5.76
1979	5.93	5.88	6.17	5.98	5.62	5.64	5.77	5.61	6	6.26	6.12	5.73

ESTIMATES OF THE EXPECTED INFLATION RATE

MODEL 3 -1 MONTH (ARIMA MODEL)

1960	1.36	1.17	.91	1.10	1.36	2.65	1.38	1.21	.41	.28	1.55	1.19
1961	1.58	1.09	1.30	1.35	1.18	.37	1.28	.70	.80	-.50	-.49	.09
1962	.75	.97	.53	1.65	1.19	1.10	1.57	2.47	2.15	.77	1.73	1.72
1963	1.70	1.95	1.86	1.62	1.67	1.08	1.79	2.09	2.66	1.56	1.67	1.81
1964	2.01	1.83	1.84	2.44	1.58	1.94	1.14	1.67	1.85	1.86	1.84	1.51
1965	1.73	2.02	1.64	2.01	2.78	2.24	2.89	3.04	2.69	2.60	2.79	2.68
1966	3.27	3.22	3.95	3.80	3.92	3.53	3.18	3.63	4.33	4.43	4.10	3.81
1967	3.57	3.29	2.84	2.78	3.55	3.34	3.63	3.90	4.31	3.94	3.87	4.02
1968	4.01	4.99	4.17	4.10	4.52	4.41	3.97	3.82	3.84	4.47	4.47	4.45
1969	4.56	4.46	4.25	4.54	5.21	5.44	5.84	5.04	4.71	4.31	4.49	4.59
1970	6.71	6.81	7.07	5.88	6.13	3.64	3.28	3.19	2.76	2.53	2.60	2.14
1971	1.00	1.33	2.04	2.03	2.40	2.99	2.37	3.29	3.93	3.26	3.98	5.04
1972	5.83	5.08	5.00	4.76	4.79	4.06	3.59	5.46	5.21	5.6	5.21	4.84
1973	5.62	5.96	6.42	6.38	7.32	8.02	7.94	7.87	8.71	9.28	9.27	9.70
1974	8.93	9.39	10.10	10.50	9.86	11.32	12.16	10.98	11.33	11.41	11.80	12.63
1975	13	11.99	11.86	11.87	10.09	9.85	10.87	11.59	11.31	10.15	10.60	10.47
1976	9.66	9.73	9.22	9.28	8.79	8.19	7.13	6.39	6.43	6.11	6.03	6.02
1977	5.69	6.53	6.99	8.14	7.52	8.17	8.38	8.98	6.28	8.49	9.41	9.45
1978	9.48	8.64	8.35	9.43	8.31	9.39	9.28	10.73	9.62	8.08	9.19	9.03
1979	8.85	9.12	8.90	10.26	9.27	9.55	8.36	9.09	9.21	9.64	9.51	10.34

ESTIMATES OF THE EXPECTED INFLATION RATE

MODEL 3 -2 MONTH (ARIMA MODEL)

1960 1.91 1.59 .97 .75 .58 2.62 2.09 1.18 .52 -.38 .01 2.13

1961 1.35 1.62 .84 .65 1.51 .55 .38 1.19 .23 .13 -1.2 -.78

1962 .50 1.35 1.23 .95 2.46 1.79 .84 2.26 3.33 2.68 -.60 1.93

1963 1.81 1.82 2.31 1.69 1.93 1.33 .61 2.01 2.92 2.64 1.31 1.82

1964 1.98 2.32 2.05 1.99 2.74 1.20 1.71 .67 1.75 1.97 1.67 1.96

1965 1.73 1.78 2.09 1.33 2.08 3.37 1.89 3.80 3.45 2.75 2.50 2.54

1966 2.64 3.67 3.44 4.21 3.75 3.67 3.47 3.21 3.96 4.62 4.18 4.07

1967 3.82 3.26 3.53 2.58 3.08 3.73 3.36 3.33 4.02 4.46 4.22 4.10

1968 4.27 4.26 5.16 3.65 3.90 4.68 5.51 4.41 4.41 4.38 4.75 4.59

1969 4.56 4.93 5.00 4.83 5.29 5.96 6.09 6.58 4.73 4.52 4.28 4.58

1970 4.68 9.10 8.50 8.94 7.79 7.89 4.03 3.21 3.68 2.38 2.16 2.48

1971 2.27 .23 1.53 2.25 2.33 3.07 3.26 2.58 3.67 4.44 3.38 4.91

1972 5.20 6.78 5.22 5.03 4.75 5.50 3.47 2.88 7.28 5.68 5.74 4.80

1973 5.16 6.06 6.68 6.85 6.86 8.33 7.73 8.15 7.97 9.76 10.07 9.5

1974 9.75 9.13 9.89 10.16 10.68 9.72 12.17 12.19 11.27 12.32
11.57 12.41

1975 13.26 13.22 11.87 12.56 11.13 9.73 10.31 11.14 12.04 11.26
9.81 10.7

1976 11.12 9.59 10.06 9.45 9.23 8 7.66 6.75 6.61 6 5.84 6.61

1977 5.94 5.83 6.83 7.50 8.46 8.06 8.86 8.82 9.19 8.26 9.01 9.89

1978 9.35 9.44 8.17 8.61 9.94 8.37 9.55 9.83 11.36 9.59 7.81
9.68

1979 9.29 8.98 8.88 9.58 10.19 9.39 9.32 9.19 10.16 9 9.88 10.25

Appendix B

Actual And Expected Value Proxies

3-Month Government of Canada Treasury Bill Rate

1960/1 To 1979/12

3-MONTH GOVERNMENT OF CANADA TREASURY BILLS

1960 4.6 4.61 3.01 3.26 3.01 3.07 2.92 2.01 1.7 3.03 3.95 3.25

1961 3.04 3.11 3.21 3.28 3.14 2.57 2.55 2.26 2.59 2.5 2.5 2.99

1962 3.07 3.21 3.12 3.07 3.52 5.45 5.47 4.95 4.99 4.16 3.71 3.91

1963 3.65 3.68 3.62 3.66 3.19 3.24 3.43 3.71 3.56 3.59 3.63 3.78

1964 3.77 3.88 3.88 3.7 3.58 3.59 3.67 3.8 3.73 3.7 3.87 3.82

1965 3.74 3.74 3.62 3.77 3.90 3.93 4.05 4.08 4.13 4.15 4.16 4.54

1966 4.63 4.69 5.06 5.08 5.11 5 5.02 5.04 5.01 5.19 5.15 4.96

1967 4.68 4.58 4.13 4 4.24 4.28 4.32 4.34 4.76 4.95 5.46 5.95

1968 6.29 6.8 6.98 6.99 6.95 6.56 6.03 5.48 5.66 5.57 5.66 6.24

1969 6.38 6.43 6.58 6.8 6.74 7.13 7.62 7.69 7.77 7.6 7.76 7.81

1970 7.78 7.6 7 6.78 6.34 5.94 5.7 5.51 5.39 5.01 4.4 4.44

1971 4.68 4.06 3.16 3 3.03 3.37 3.68 3.79 4.06 3.47 3.24 3.21

1972 3.36 3.45 3.57 3.64 3.73 3.5 3.46 3.5 3.62 3.57 3.68 3.65

1973 3.9 3.99 4.46 4.9 5.18 5.48 5.74 6.18 6.5 6.53 6.43 6.35

1974 6.22 6.07 6.51 7.64 8.63 8.75 9.10 9.11 8.94 8.31 7.49 7.12

1975 6.4 6.26 6.33 6.85 6.87 6.99 7.44 7.87 8.41 8.16 8.52 8.64

1976 8.59 8.79 9.07 8.99 8.94 8.98 9.07 9.13 9.11 9.01 8.59 8.14

1977 8.04 7.65 7.54 7.58 7.05 7.07 7.14 7.14 7.1 7.24 7.26 7.17

1978 7.13 7.3 7.73 8.19 8.2 8.26 8.66 8.8 9.17 9.85 10.36 10.46

1979 10.85 10.82 10.92 10.82 10.84 10.78 11.24 11.45 11.64 13.61
13.62 13.66

SOURCE: Statistics Canada Cansim System (B14007)

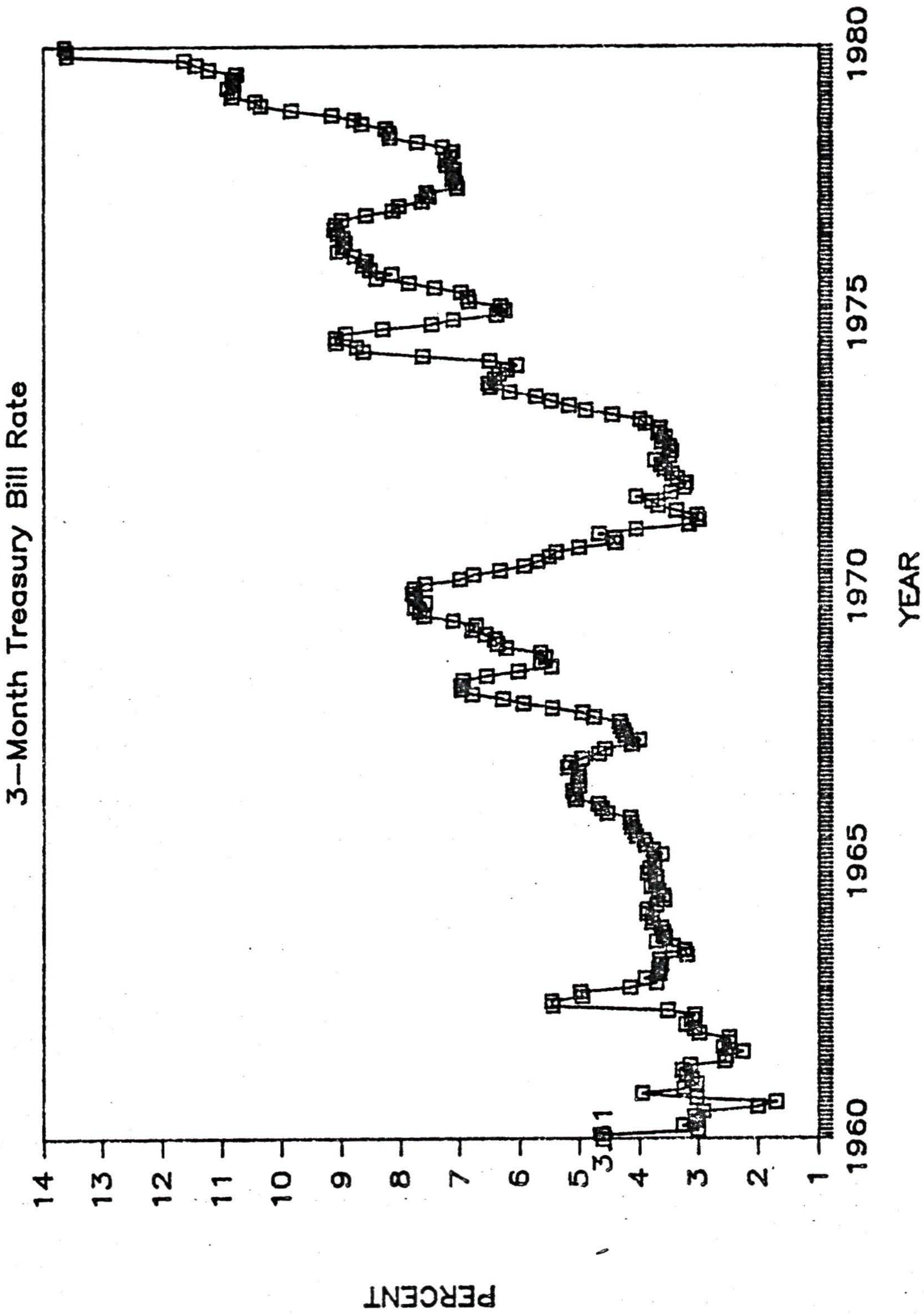

Figure B.1: 3-Month Treasury Bill Rate-1960/1 To 1979/12

ESTIMATES OF THE EXPECTED INTEREST RATE

MODEL 1 - APPLICATION 1

(Pure Expectations Theory)

1960	5.82	5.22	5.11	3.45	3.60	3.71	3.39	3.38	2.47	2.40	3.65	4.19
1961	3.83	3.68	3.63	3.53	3.52	3.46	2.91	3.03	2.84	3.19	2.94	2.96
1962	3.29	3.45	3.61	3.46	3.31	3.90	6.01	5.83	5.31	5.41	4.44	3.95
1963	4.11	3.89	3.90	3.86	3.94	3.41	3.44	3.77	4.19	3.86	3.95	3.89
1964	4.20	4.11	4.16	4.20	4.00	3.90	3.91	3.99	4.10	4.05	4.02	4.25
1965	4.10	3.92	3.98	3.84	3.97	4.12	4.15	4.41	4.52	4.57	4.65	4.74
1966	5.00	5.05	5.11	5.54	5.38	5.45	5.20	5.48	5.50	5.29	5.39	5.29
1967	5.10	4.66	4.60	4.09	4.02	4.62	4.76	4.88	4.84	5.26	5.49	5.80
1968	6.31	6.45	6.82	6.98	6.71	7.07	6.46	5.77	5.38	5.84	5.75	5.80
1969	6.70	6.74	7.15	7.02	6.76	7.10	7.39	7.94	7.87	7.87	7.80	7.94
1970	7.95	7.74	7.52	6.52	6.86	6.36	6.02	5.92	5.81	5.55	5.27	4.50
1971	4.60	5.10	4.10	3.34	3.26	3.55	3.73	4.24	4.13	4.26	3.69	3.96
1972	3.41	3.70	4.03	4.15	4.22	4.47	4.12	4.14	4.26	4.28	4.05	4.20
1973	4.09	4.48	4.61	5.12	5.84	6.28	6.38	6.56	7.14	7.02	6.85	6.71
1974	6.67	6.56	6.05	6.59	8.28	9.23	9.05	9.46	9.31	8.86	8.05	6.77
1975	6.82	6.74	6.20	6.55	7.77	7.43	7.45	7.92	8.57	8.89	8.34	8.94
1976	9.02	8.49	9.01	9.29	9.05	8.78	8.90	9.01	8.95	8.79	8.81	8.41
1977	7.72	7.52	7.39	7.54	7.60	7.15	7.21	7.36	7.44	7.32	7.54	7.56
1978	7.55	7.59	7.86	8.41	8.85	8.88	9.04	9.06	9.14	9.51	10.35	10.68
1979	10.96	11.11	11.00	11.00	10.72	10.82	10.78	11.24	12.19	12.16	14.25	13.36

ESTIMATES OF THE EXPECTED INTEREST RATE

MODEL 1 - APPLICATION 2 (Frankel Model)

1960	5.10	4.56	4.57	2.90	3.17	2.91	2.98	2.82	1.88	1.56	2.94	3.89
1961	3.16	2.94	3.02	3.12	3.19	3.05	2.46	2.44	2.14	2.48	2.39	2.39
1962	2.90	2.98	3.13	3.04	2.99	3.45	5.46	5.47	4.93	4.97	4.12	3.65
1963	3.86	3.59	3.61	3.55	3.60	3.11	3.16	3.35	3.64	3.49	3.52	3.56
1964	3.72	3.71	3.82	3.82	3.63	3.51	3.52	3.60	3.73	3.66	3.63	3.81
1965	3.76	3.68	3.68	3.55	3.71	3.84	3.87	3.99	4.02	4.08	4.09	4.10
1966	4.50	4.59	4.65	5.04	5.06	5.09	4.97	4.99	5.00	4.98	5.17	5.12
1967	4.92	4.64	4.53	4.07	3.93	4.17	4.21	4.25	4.26	4.69	4.89	5.42
1968	5.92	6.28	6.80	6.98	7.01	6.95	6.56	6.01	5.44	5.62	5.51	5.60
1969	6.19	6.34	6.39	6.55	6.78	6.71	7.11	7.62	7.70	7.77	7.59	7.74
1970	7.79	7.76	7.58	6.96	6.72	6.25	5.84	5.60	5.40	5.28	4.88	4.26
1971	4.32	4.59	3.93	2.99	2.82	2.83	3.19	3.51	3.64	3.93	3.32	3.09
1972	3.06	3.21	3.29	3.40	3.47	3.56	3.32	3.28	3.32	3.44	3.40	3.52
1973	3.49	3.75	3.84	4.33	4.79	5.06	5.38	5.65	6.10	6.44	6.48	6.37
1974	6.29	6.15	5.99	6.43	7.59	8.62	8.72	9.08	9.08	8.91	8.27	7.43
1975	7.04	6.31	6.17	6.23	6.75	6.79	6.90	7.35	7.80	8.35	8.11	8.47
1976	8.60	8.56	8.77	9.06	8.97	8.92	8.96	9.06	9.12	9.11	9.01	8.58
1977	8.12	8.02	7.61	7.48	7.52	6.97	6.99	7.07	7.07	7.03	7.17	7.19
1978	7.10	7.04	7.22	7.66	8.14	8.15	8.22	8.64	8.78	9.17	9.87	10.40
1979	10.50	10.90	10.86	10.97	10.87	10.89	10.83	11.30	11.51	11.70	13.72	13.74

ESTIMATES OF THE EXPECTED INTEREST RATE

MODEL 2 - 1 MONTH (ARIMA Model)

1960	5.21	4.53	4.64	2.71	3.31	3.01	3.09	2.93	1.92	1.67	3.16	4.13
1961	3.19	3.03	3.14	3.24	3.31	3.14	2.51	2.56	2.24	2.64	2.50	2.51
1962	3.06	3.10	3.24	3.13	3.08	3.59	5.80	5.50	4.91	5.02	4.08	3.66
1963	3.96	3.63	3.70	3.63	3.68	3.14	3.26	3.47	3.76	3.55	3.61	3.65
1964	3.81	3.78	3.91	3.90	3.69	3.58	3.61	3.70	3.84	3.74	3.71	3.91
1965	3.84	3.75	3.76	3.62	3.81	3.93	3.95	4.08	4.10	4.15	4.16	4.17
1966	4.60	4.66	4.71	5.12	5.10	5.13	5.00	5.04	5.06	5.02	5.22	5.16
1967	4.95	4.65	4.57	4.08	3.99	4.27	4.29	4.33	4.35	4.82	4.98	5.54
1968	6.04	6.36	6.90	7.04	7.03	6.98	6.54	5.98	5.42	5.71	5.58	5.69
1969	6.36	6.42	6.45	6.62	6.85	6.74	7.20	7.72	7.72	7.80	7.59	7.81
1970	7.84	7.80	7.59	6.87	6.75	6.24	5.85	5.66	5.47	5.37	4.89	4.20
1971	4.46	4.77	3.87	2.84	2.94	3.04	3.49	3.80	3.84	4.17	3.26	3.16
1972	3.20	3.41	3.48	3.62	3.67	3.76	3.39	3.44	3.51	3.67	3.55	3.73
1973	3.64	4.01	4.03	4.69	5.12	5.33	5.64	5.88	6.42	6.68	6.56	6.39
1974	6.32	6.16	6.00	6.75	8.29	9.24	8.84	9.31	9.14	8.87	7.97	7.02
1975	6.92	5.98	6.19	6.38	7.17	6.89	7.07	7.71	8.13	8.74	8.03	8.74
1976	8.72	8.58	8.92	9.24	8.96	8.93	9.02	9.14	9.18	9.11	8.97	8.37
1977	7.90	8.00	7.44	7.49	7.61	6.76	7.09	7.19	7.15	7.09	7.32	7.28
1978	7.12	7.11	7.39	7.97	8.45	8.21	8.30	8.89	8.89	9.39	10.25	10.67
1979	10.53	11.09	10.82	10.99	10.78	10.87	10.76	11.51	11.58	11/76	14.78	13.65

ESTIMATES OF THE EXPECTED INTEREST RATE

MODEL 2 - 2 MONTH (ARIMA Model)

1960	4.88	5.26	4.54	4.67	2.67	3.33	3.03	3.12	2.94	1.93	1.67	3.19
1961	4.18	3.21	3.05	3.16	3.26	3.33	3.16	2.52	2.58	2.24	2.66	2.51
1962	2.53	3.08	3.11	3.26	3.14	3.10	3.61	5.88	5.54	4.93	5.06	4.09
1963	3.67	3.98	3.64	3.72	3.64	3.70	3.14	3.27	3.48	3.78	3.56	3.62
1964	3.66	3.83	3.80	3.93	3.92	3.71	3.60	3.63	3.72	3.86	3.76	3.74
1965	3.94	3.86	3.77	3.78	3.64	3.83	3.96	3.97	4.10	4.12	4.17	4.18
1966	4.19	4.62	4.67	4.73	5.15	5.12	5.15	5.02	5.06	5.07	5.03	5.24
1967	5.17	4.95	4.66	4.58	4.07	3.99	4.28	4.29	4.33	4.35	4.84	5.00
1968	5.57	6.07	6.40	6.94	7.09	7.08	7.03	6.57	6.00	5.43	5.73	5.60
1969	5.71	6.40	6.44	6.47	6.64	6.87	6.75	7.23	7.75	7.74	7.83	7.61
1970	7.83	7.87	7.82	7.61	6.86	6.76	6.24	5.84	5.67	5.48	5.38	4.85
1971	4.14	4.48	4.81	3.81	2.73	2.92	3.05	3.54	3.85	3.86	4.21	3.20
1972	3.31	3.20	3.43	3.50	3.64	3.68	3.76	3.34	3.42	3.51	3.69	3.54
1973	3.75	3.63	4.07	4.05	4.80	5.24	5.41	5.73	5.96	6.55	6.78	6.58
1974	6.38	6.31	6.14	5.97	6.88	8.66	9.62	8.90	9.45	9.17	8.85	7.80
1975	6.75	6.81	5.73	6.15	6.42	7.36	6.92	7.13	7.88	8.30	8.96	7.97
1976	8.87	8.78	8.58	9.00	9.36	8.96	8.93	9.05	9.19	9.22	9.13	8.96
1977	8.25	7.77	7.98	7.33	7.47	7.65	6.60	7.11	7.23	7.16	7.09	7.38
1978	7.29	7.10	7.10	7.44	8.10	8.60	8.22	8.33	9.03	8.95	9.52	10.50
1979	10.85	10.59	11.24	10.83	11.05	10.77	10.89	10.76	11.67	11.67	11.84	15.49

Appendix C

6-Month Government of Canada Treasury Bill Rates

10-Year Government of Canada Treasury Bill Rates

Seasonally Adjusted Unemployment Rates

Canadian Industrial Index Growth Rates

Non-Seasonally Adjusted Money Suppy Growth Rates (M1)

6-MONTH GOVERNMENT OF CANADA TREASURY BILL RATES

1960/1 To 1979/12

1960	4.91	4.86	3.23	3.43	3.36	3.23	3.15	2.24	2.05	3.34	4.07	3.54
1961	3.36	3.37	3.37	3.4	3.3	2.74	2.79	2.55	2.89	2.72	2.73	3.14
1962	3.26	3.41	3.29	3.19	3.71	5.73	5.65	5.13	5.2	4.3	3.83	4.01
1963	3.77	3.79	3.74	3.8	3.3	3.34	3.6	3.95	3.71	3.77	3.76	3.99
1964	3.94	4.02	4.04	3.85	3.74	3.75	3.83	3.95	3.89	3.86	4.06	3.96
1965	3.83	3.86	3.73	3.87	4.01	4.04	4.23	4.3	4.35	4.4	4.45	4.77
1966	4.84	4.9	5.3	5.23	5.28	5.1	5.25	5.27	5.15	5.29	5.22	5.03
1967	4.67	4.59	4.11	4.01	4.43	4.52	4.6	4.59	5.01	5.22	5.63	6.13
1968	6.37	6.81	6.98	6.85	7.01	6.51	5.90	5.43	5.75	5.66	5.73	6.47
1969	6.56	6.79	6.80	6.78	6.92	7.26	7.78	7.78	7.82	7.70	7.85	7.88
1970	7.76	7.56	6.76	6.82	6.35	5.98	5.81	5.66	5.47	5.14	4.45	4.52
1971	4.89	4.08	3.25	3.13	3.29	3.55	3.96	3.96	4.16	3.58	3.60	3.31
1972	3.53	3.74	3.86	3.93	4.10	3.81	3.80	3.88	3.95	3.81	3.94	3.87
1973	4.19	4.3	4.79	5.37	5.73	5.93	6.15	6.66	6.76	6.69	6.57	6.51
1974	6.39	6.06	6.55	7.96	8.93	8.90	9.28	9.21	8.90	8.18	7.13	6.97
1975	6.57	6.23	6.44	7.31	7.15	7.22	7.68	8.22	8.65	8.25	8.73	8.83
1976	8.54	8.90	9.18	9.02	8.86	8.94	9.04	9.04	8.95	8.91	8.50	7.93
1977	7.78	7.52	7.54	7.59	7.10	7.14	7.25	7.29	7.21	7.39	7.41	7.36
1978	7.36	7.58	8.07	8.52	8.54	8.65	8.86	8.97	9.34	10.10	10.61	10.71
1979	10.98	10.91	10.96	10.77	10.83	10.78	11.24	11.82	11.90	13.93	13.49	13.60

SOURCE: Statistics Canada Cansim System (B14008)

10-YEAR GOVERNMENT OF CANADA AVERAGE BOND YIELD RATES 1966/1 To 1979/12

Note: The Cansim data-base printout (B14013) listing for the 10-year average bond yield rates for the period 1960/1 to 1979/12 could not be located. The following is the same data but taken from the Bank of Canada Statistical Summaries. Unfortunately, the data for the period 1960/1 to 1965/12 was not available from this source.

1966	5.41	5.61	5.58	5.60	5.61	5.66	5.74	5.94	5.75	5.71	5.91	5.76
1967	5.60	5.64	5.48	5.56	5.72	5.87	5.88	5.99	6.19	6.36	6.41	6.54
1968	6.54	6.72	6.91	6.62	6.97	6.62	6.49	6.43	6.60	6.83	6.95	7.27
1969	7.16	7.20	7.22	7.29	7.48	7.50	7.52	7.53	7.81	7.82	8.15	8.33
1970	8.31	8.13	7.93	8.04	8.23	8.09	7.91	8.00	7.88	7.94	7.50	6.99
1971	6.67	6.85	6.76	6.97	7.38	7.30	7.49	7.15	6.97	6.71	6.56	6.56
1972	6.73	6.90	7.24	7.27	7.34	7.45	7.49	7.44	7.46	7.26	7.08	7.12
1973	7.16	7.21	7.30	7.39	7.72	7.74	7.73	7.82	7.72	7.60	7.64	7.70
1974	7.75	7.74	8.19	8.81	8.91	9.46	9.63	9.84	9.67	9.20	8.87	8.77
1975	8.30	8.17	8.47	9.04	8.71	8.88	9.34	9.39	9.72	9.33	9.58	9.49
1976	9.29	9.27	9.39	9.34	9.32	9.35	9.37	9.24	9.16	9.09	8.82	8.47
1977	8.52	8.62	8.83	8.85	8.77	8.72	8.70	8.57	8.61	8.70	8.74	8.77
1978	9.06	9.15	9.17	9.22	9.23	9.23	9.17	9.16	9.15	9.48	9.54	9.68
1979	9.82	9.97	9.91	9.66	9.68	9.73	9.84	10.15	10.38	11.16	10.94	11.32

SEASONALLY ADJUSTED UNEMPLOYMENT RATE

1966/1 To 1979/12

Note: The Cansim data-base listing (D767611) for the seasonally adjusted unenployment rate was only available for the period 1966/1 to 1979/12. As a result, the error unpredictability tests performed using this economic variable were based only on this time period.

1966	3.40	3.40	3.30	3.20	3.20	2.90	3.20	3.50	3.60	3.30	3.50	3.40
1967	3.60	3.70	3.80	3.90	3.70	3.60	3.70	3.60	3.70	4.10	4.00	4.40
1968	4.30	4.40	4.60	4.50	4.40	4.70	4.70	4.60	4.30	4.40	4.50	4.50
1969	4.30	4.30	4.20	4.30	4.40	4.40	4.30	4.50	4.30	4.70	4.60	4.50
1970	4.50	4.90	5.20	5.40	5.80	6.10	6.40	6.10	6.00	6.00	6.10	6.20
1971	6.20	6.30	6.20	6.60	6.10	6.10	6.10	6.00	6.20	6.30	6.20	5.90
1972	6.00	5.80	6.10	5.80	6.10	6.30	6.30	6.40	6.40	6.60	6.40	6.40
1973	6.10	5.90	5.60	5.50	5.30	5.40	5.30	5.30	5.60	5.70	5.50	5.40
1974	5.30	5.20	5.20	5.20	5.40	5.00	5.10	5.30	5.40	5.40	5.50	6.00
1975	6.90	6.60	6.60	6.50	7.00	7.00	6.90	7.20	6.90	7.20	7.20	7.00
1976	7.00	7.00	6.60	6.90	7.00	7.00	7.40	7.10	7.10	7.50	7.30	7.50
1977	7.70	8.00	8.00	8.00	7.80	7.90	8.30	8.20	8.20	8.30	8.40	8.50
1978	8.30	8.40	8.50	8.40	8.50	8.40	8.40	8.40	8.50	8.10	8.10	8.10
1979	8.00	7.80	7.80	7.90	7.50	7.30	7.20	7.10	7.00	7.30	7.20	7.10

SOURCE: Statistics Canada Cansim System (D767611)

CANADIAN INDUSTRIAL INDEX GROWTH RATES

1960/1 To 1979/12

1960 9.21 6.20 7.07 1.81 3.00 2.14 .11 .96 -.10 -1.74 0.00 -1.04

1961 -2.24 -1.33 -3.25 2.72 1.98 4.18 7.55 6.74 7.20 7.48 8.58 9.56

1962 8.73 9.66 12.92 9.07 11.53 10.44 10.28 9.97 9.06 7.35 7.03 6.52

1963 6.60 6.53 6.23 6.17 5.86 5.73 2.78 5.12 6.61 7.21 9.09 10.35

1964 11.21 12.00 8.93 12.06 9.85 9.89 11.77 10.76 8.71 9.08 9.08 7.83

1965 8.31 5.95 9.97 6.36 7.32 7.51 8.35 9.10 8.86 10.32 9.15 10.97

1966 9.38 11.24 8.77 9.90 8.87 8.22 5.90 5.09 5.94 5.31 4.02 2.79

1967 2.38 1.21 .40 2.49 1.28 1.55 2.31 3.03 1.80 .20 2.93 3.98

1968 2.86 2.59 4.42 3.28 5.59 6.23 5.85 4.18 6.43 8.34 7.12 5.87

1969 7.37 9.08 9.68 6.73 5.29 4.61 5.15 3.01 2.40 1.71 2.66 3.73

1970 3.13 3.15 -.88 1.73 1.38 1.73 1.25 2.80 1.44 1.26 .18 -1.39

1971 .06 -.58 1.71 1.52 3.01 2.93 3.54 5.74 6.65 7.59 6.70 7.77

1972 8.55 6.20 7.00 9.56 7.09 7.54 7.32 4.59 6.51 8.03 10.05 9.46

1973 9.70 14.41 14.34 10.41 11.73 11.03 13.36 10.18 8.76 8.24 8.60 7.85

1974 8.93 6.13 5.39 4.50 5.67 3.87 1.40 4.24 1.93 .66 -2.20 -2.21

1975 -7.15 -6.98 -8.63 -6.10 -9.29 -7.13 -5.53 -6.26 -4.86 -6.00 -2.42 -.50

1976 2.68 3.10 6.03 6.49 10.07 7.33 5.68 7.63 7.44 6.39 6.33 4.62

1977 7.00 4.43 3.30 .89 .72 2.93 2.61 1.37 .72 3.70 1.53 1.45

1978 -.55 1.60 .96 4.03 1.43 2.45 1.59 2.46 5.60 5.08 6.26 8.23

1979	8.56	8.20	8.14	5.34	9.03	6.09	8.67	8.15	5.60	5.06	3.13	.07

SOURCE: Statistics Canada Cansim System (D144312)

NON-SEASONALLY ADJUSTED MONEY SUPPLY GROWTH RATES (M1)

1960/1 To 1979/12

1960	-2.61	-2.29	-1.56	.31	.65	1.96	.23	1.08	3.21	4.83	4.43	5.08
1961	5.41	6.25	6.47	3.89	4.14	3.28	5.37	6.69	4.37	4.01	6.14	6.40
1962	4.41	3.38	2.82	4.81	4.75	4.29	2.11	-0.18	2.64	4.15	2.72	3.86
1963	4.74	4.31	4.74	6.75	7.92	7.15	8.54	7.62	6.21	4.71	4.63	3.60
1964	5.54	5.47	6.92	5.24	3.67	5.07	4.27	5.44	4.20	4.63	4.41	6.31
1965	4.41	5.37	4.59	5.41	5.65	6.46	7.45	8.59	7.36	6.46	7.84	6.53
1966	6.97	7.65	7.44	6.62	6.12	6.01	5.77	5.41	6.85	7.64	6.86	8.25
1967	8.08	10.02	10.40	10.54	10.87	9.95	10.22	9.64	11.07	10.45	8.07	8.25
1968	7.92	4.10	1.69	1.54	1.79	2.02	4.34	7.20	4.48	4.51	6.05	6.28
1969	8.00	7.36	9.73	10.89	10.97	10.37	8.08	3.51	4.42	5.07	2.86	3.60
1970	.28	2.49	2.01	1.15	-.27	.71	2.46	3.56	3.44	2.07	3.90	5.46
1971	6.61	7.60	9.59	9.02	12.28	14.99	11.94	13.89	16.63	15.81	17.04	17.57
1972	16.29	16.09	14.76	14.87	13.45	11.98	14.26	14.00	13.06	14.15	15.22	13.73
1973	15.76	15.58	14.07	15.59	16.85	15.64	15.96	15.20	14.10	13.85	10.92	11.06
1974	11.11	11.03	11.66	13.35	13.90	10.85	8.86	7.34	6.48	5.78	6.13	5.73
1975	7.62	10.25	12.00	8.35	6.64	11.07	13.41	16.05	16.36	18.10	23.64	23.11
1976	15.87	13.51	9.71	9.36	9.40	10.11	8.22	6.98	7.40	5.26	.34	1.84
1977	4.92	5.58	7.67	7.67	7.83	9.30	8.66	9.16	8.68	9.34	10.20	11.63

1978	10.60	10.95	8.89	8.70	9.92	8.32	9.86	10.26	10.59	11.83	12.26	8.44
1979	7.76	7.35	6.08	6.83	8.57	8.83	8.08	8.56	7.41	6.28	3.67	3.54

SOURCE: Statistics Canada Cansim System (B2013)

Appendix D

Estimated Lag Coefficients

Adaptive Expectations Model (Eq. 3.2)

Polynomial Lag Models (Eq. 3.4)

ESTIMATED LAGGED COEFFICIENTS

ADAPTIVE EXPECTATIONS MODEL (EQUATION 3.2)

$$i_t = a_0 + a_1 (P^e{}_t) + u_t \tag{3.2}$$

$R^2 = .96$

D.W. = 2.03

SER = 0.35

$a_o = 2.19(1.3)$

$a_1 = .88(2.88)$

lag 0 .042

lag 1 .040	lag 13 .022	lag 25 .012	lag 37 .007
lag 2 .038	lag 14 .021	lag 26 .012	lag 38 .007
lag 3 .036	lag 15 .020	lag 27 .011	lag 39 .006
lag 4 .034	lag 16 .019	lag 28 .011	lag 40 .006
lag 5 .033	lag 17 .018	lag 29 .010	lag 41 .006
lag 6 .031	lag 18 .017	lag 30 .010	lag 42 .005
lag 7 .030	lag 19 .017	lag 31 .009	lag 43 .005
lag 8 .028	lag 20 .016	lag 32 .009	lag 44 .005
lag 9 .027	lag 21 .015	lag 33 .008	lag 45 .005
lag 10 .026	lag 22 .014	lag 34 .008	lag 46 .004
lag 11 .024	lag 23 .014	lag 35 .008	lag 47 .004
lag 12 .023	lag 24 .013	lag 36 .007	lag 48 .004

ESTIMATED LAGGED COEFFICIENTS FOR 1ST DEGREE POLYNOMIAL

WITH 24 MONTH LAG AND TAIL RESTRICTION

EQUATION (3.4)

$$i_t = a_0+a_1P^e{}_t+a_2(MCHG)+a_3(ICHG)+u_t \qquad (3.4)$$

R^2	.78
D.W.	.19
SER	1.08
a_0	2.96(12.35)
a_1	.79(26.89)
a_2	-.18(-10.06)
a_3	.12(5.85)

NOTE: The t-ratio is 26 for all 24 of the lagged coefficients.

LAG	Coefficient	LAG	Coefficient
lag 0	.061		
lag 1	.058	lag 13	.029
lag 2	.056	lag 14	.027
lag 3	.053	lag 15	.024
lag 4	.051	lag 16	.022
lag 5	.048	lag 17	.019
lag 6	.046	lag 18	.017
lag 7	.044	lag 29	.015
lag 8	.041	lag 20	.012
lag 9	.039	lag 21	.010
lag 10	.036	lag 22	.007
lag 11	.034	lag 23	.005
lag 12	.031	lag 24	.002

ESTIMATED LAGGED COEFFICIENTS FOR ALTERNATIVE POLYNOMIAL LAG MODELS EQUATION (3.4)

It is found that without a tail restriction the lag structure for all the polynomial models is unrealistic. Therefore, the models considered all have a tail restriction where the lag structure declines to zero. In general, there are a number of models that generate results which are very close in value. The selection criteria for choosing the best fitting model includes: 1) the R^2 value, 2) the t-values of the lagged coefficients, 3) the size of the Durbin Watson statistic, 4) the standard error of the regression, and 5) the significance of the coefficient values for the equation.

Among the polynomial models tried, 4 models performed better than the others. The results of these models are presented below (t-ratios in parentheses).

$$i_t = a_0+a_1P^e{}_t+a_2(MCHG)+a_3(ICHG)+u_t \qquad (3.4)$$

	Degree	Lag	Degree	Lag	Degree	Lag	Degree	Lag
	3	36	1	18	3	24	2	12
R^2	.79		.78		.79		.78	
D.W.	.17		.18		.19		.17	
SER	1.09		1.10		1.06		1.13	
a_0	2.95(11.7)		2.93(12.2)		2.97(12.5)		2.95(12.7)	
a_1	.80(25.3)		.79(26.8)		.80(27)		.77(26.9)	
a_2	-.18(-9.42)		-.18(-10.1)		-.18(-10)		-.18(-9.1)	
a_3	.12(5.67)		.11(5.6)		.10(5.5)		.11(5.6)	
lag 0	.067(4.5)		.079(26)		.115(4.2)		.040(1.3)	
lag 1	.062(5.3)		.075		.092(5.0)		.053(2.5)	
lag 2	.058(6.5)		.070		.072(6.4)		.063(4.9)	
lag 3	.053(8.2)		.066		.056(8.5)		.071(11.1)	
lag 4	.049(10.5)		.062		.042(7.7)		.076(26.9)	
lag 5	.046(12.8)		.058		.032(4.4)		.079(14.9)	
lag 6	.042(13.1)		.054		.024(2.6)		.078(9.4)	
lag 7	.039(10.9)		.050		.018(1.7)		.075(7.2)	
lag 8	.036(8.6)		.046		.015(1.3)		.069(1.0)	
lag 9	.033(6.9)		.041		.013(1.1)		.061(5.3)	
lag 10	.030(5.8)		.037		.012(1.1)		.050(4.8)	
lag 11	.028(5.0)		.033		.013(1.3)		.036(4.5)	
lag 12	.025(4.4)		.029		.015(1.8)		.019(4.2)	
lag 13	.023(4.1)		.025		.017(2.6)			

lag 14	.021(3.8)	.021	.020(3.9)
lag 15	.019(3.6)	.017	.023(5.6)
lag 16	.018(3.5)	.012	.026(6.5)
lag 17	.016(3.5)	.008	.028(5.9)
lag 18	.015(3.6)	.004	.030(5.1)
lag 19	.014(3.8)		.030(4.5)
lag 20	.012(4.0)		.030(4.0)
lag 21	.011(4.3)		.028(3.7)
lag 22	.010(4.4)		.024(3.5)
lag 23	.010(4.2)		.019(3.3)
lag 24	.009(3.6)		.011(3.2)
lag 25	.008(2.9)		
lag 26	.007(2.4)		
lag 27	.007(1.9)		
lag 28	.006(1.6)		
lag 29	.005(1.3)		
lag 30	.005(1.1)		
lag 31	.004(1.0)		
lag 32	.004(.9)		
lag 33	.003(.8)		
lag 34	.002(.7)		
lag 35	.002(.6)		
lag 36	.001(.6)		

Appendix E

Error Unpredictability Test Results

ARIMA Inflation Rate and Interest Rate Models

This appendix provides an example of the error unpredictability test. The information set used in this example is the money supply growth rate (M1) lagged 6 periods and the forecast errors are those generated by the one-month ahead ARIMA inflation rate model (model 2) and the one-month ahead ARIMA nominal interest rate model (model 3). The general form of the error unpredictability test, using the money supply growth rate lagged 6 periods, is

$$({}_{t-1}X^e{}_t - X_t) = a_0 + \sum^6{}_{i=1} a_i \, (I_{t-i}) \qquad (5.9)$$

where: ${}_{t-1}X_t{}^e$ = expected value of inflation rate/ interest rate for time t, generated in time t-1;

X_t = realized value of inflation rate/ interest rate in time t;

I_{t-i} = money supply growth rate in time period t-i.

The F-test is used to determine if the estimated coefficients in equation (5.9) are significantly different from zero. If $F_{obs} < F_c$, the null hypothesis of error unpredictability is accepted.

TEST RESULTS (t-ratios in brackets)

	ARIMA Inflation Rate Model	ARIMA Interest Rate Model
R^2	.008	.02
SER	.500	.34
D.W.	1.57	2.03
a_o	-.16(-2.14)	-.10(-.21)
1lagM1	.01(.60)	.02(1.38)
2lagM1	-.02(-.79)	.007(.33)
3lagM1	.02(.51)	-.02(-.94)
4lagM1	.006(.20)	.007(.34)
5lagM1	.01(.34)	.006(.30)
6lagM1	-.018(-.89)	-.006(-.41)

$F_o(.95;6,227) = .31$	$F_o(.95;6,227) = 1.72$
$F_c(.95;6,227) = 2.10$	$F_c(.95;6,227) = 2.10$

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- Research completed for Economics Department on advantages/disadvantages of competition in the Canadian telecommunications industry.

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AEL Microtel, Burnaby, B.C.

<u>Market Research Assistant</u>

- Model Microtel sales using econometric and time series models.

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