ON THE APPLICATION OF OPTIMUM INTERPOLATION TO THE ANALYSIS OF PRECIPITATION IN COMPLEX TERRAIN

by

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Abstract

Optimum interpolation is a procedure that allows the combination of observations with preliminary trial fields of the same quantities in order to produce an updated field in which the error variance is minimized. In this thesis, we describe an operational method to analyze observed precipitation amounts, based on optimum interpolation. Since the area that we deal with is topographically complex, this factor has been included in the operational method. The trial fields are provided by a 3-dimensional numerical weather prediction model. Our thesis presents an estimation of the covariances of observational and trial field errors. Two assumptions are made: 1. Trial field errors and observational errors are independent of each other, 2. Observational errors and the deviations of the trial field values from the observations are uncorrelated. The first assumption is customarily made in any application of optimum interpolation. The second assumption is specific to this thesis. These two statements together imply that observational errors are uncorrelated. A technique is derived to determine which observations influence a given grid point and their respective weights. The selection of influencing observations is done by calculating the spatial dependence of τ, the trial field error covariance. A cut-off point is determined on the smoothed curve where the τ value is a small fraction of the τ value at the origin. The procedure is applied to the heavy rainstorm of 11-13 July 1983 in the upper Columbia River watershed. Certain practical problems do arise in the implementation. The overlap of model day and climate day tends to introduce systematic errors within the observations. This result conflicts with our assumption that observational errors are uncorrelated. Additionally, the observing system is not designed to make allowance for topographical detail. Errors are thus introduced in the observations from a variety of sources. These are discussed in detail in the thesis.

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Dedication

To my parents who provided me with the motivation, moral support, love and who were ever so patient with me.

Chapter 1

Introduction

1.1 The General Problem of Numerical Weather Prediction

Classically, the problem of numerical weather prediction has always been viewed as a mathematical initial value problem. The problem includes a determination of the initial values of many time-dependent variables which in turn define the state of the numerical atmospheric model. Unfortunately, the problem of specifying initial conditions for numerical weather prediction has not been satisfactorily solved.

1. Pressure, temperature and wind observations which are usually taken simultaneously at fixed times, tend to be irregularly distributed over the planet. This tends to leave large geographical regions for which no data are available.

2. These observations tend to be point observations. Small scale features may be inadequately represented due to their spatial variability.

3. The measured data may have significant random instrumental errors.

4. Remote polar orbiting space observations tend to be non-synoptic. Thus at any particular time, any information obtained from these observations is incomplete.

5. Observations from geostationary satellites are synoptic. However, they are unable to provide detailed vertical resolution of the wind field.

6. Remote space measurements, in most of the cases involve assumptions made on some physical properties of the field. This provides another source of error in the data.

We see that single-time data sets may be inaccurate and by themselves provide insufficient information as to be able to determine an adequate account of the state of the atmosphere.

Thus any analysis procedure must, in addition to the single-time data sets, use any information predicted by data obtained at previous times in order to obtain a reasonable analysis. The procedure must be carried out at a chosen time $t = 0$ by merging the new data with a preliminary estimate of the meteorological fields which in turn was determined by previous observations obtained at times $t < 0$.

The problem of specifying complete initial conditions at a given time from observations distributed in space and time is known as four-dimensional data assimilation. No method has been devised which takes full advantage of available observations. There remain many unsolved questions all of which are relative to the dimensional aspects of the problem at hand. Talagrand [19] poses two of these questions. The first question is mostly theoretical in nature. Under what conditions can the time-space distribution of observations which define a unique solution to the model equations, as do complete observations at a given instant of time, replace the initial conditions at a time ? Linked to the above question, but closer to the practical problem at hand is the following. Given observations distributed over space and time and the knowledge that these observations define a unique solution to some basic equation, through which numerical procedures can the corresponding complete initial conditions at a given time be determined?

Most proposed procedures are variations of the approach of correcting the most recent forecast with the most recent observations. In these methods, we integrate the numerical model over the time period of available observations. Whenever model time reaches a instant in time when observations are available, the state predicted by the model is corrected with the observations. The underlying heuristic principle is that, provided the successive corrections are applied frequently enough, the model flow will be induced towards the particular solution out of which the observations have taken place. Remarkably, the simple repetition of updating the model with new data suffices for a reconstruction of observed flow. At locations where only some of the model variables have available observations, we hope that the internal dynamics of the model would induce reasonable and consistent values of the missing variables.

For the remainder of this literature review, we will cover one facet of four-dimensional data assimilation, namely the specification of meteorological variables at the beginning of a forecast period. Alternatively, as Gustafsson [10] says "It is the procedure of transfer of information from irregularly distributed observations into a form convenient for insertion into numerical forecast models.". The term objective analysis is used to distinguish this procedure.

1.2 Review of Relevant Objective Analysis Techniques

1.2.1 Introduction

The very first numerical schemes proposed to do such an analysis were introduced in the 1940's. These schemes simply performed a two dimensional interpolation of the observed data onto a regular network of grid points. A short range numerical forecast or a "first guess" field was used as a preliminary field in order to obtain a better quality forecast. Since, at the time, the numerical forecast models were relatively simple, the two dimensional interpolation procedures provided an adequate initial analyses for the models. Subsequently, the forecast models have been improved upon and refined. With the refinement of the models, the sensitivity of the models to the quality of the initial analyses has also increased. Furthermore, with the increase in the complexity of the observational infrastructure, we now have various data sources with variable quality of data.

Over the last 40 to 50 years of work that has been done in numerical analysis, most schemes belong to one of the following four classes of numerical analysis procedures.

1. Polynomial Interpolation Methods.

2. Successive Correction Methods

3. Variational Numerical Analysis Methods.

4. Optimum Interpolation Methods.

We will now provide a brief discussion of the different numerical analysis techniques. Much emphasis has been put into the discussion of optimum interpolation which forms the crux of the thesis. Most of the following has been summarized from [10].

1.2.2 Polynomial Interpolation

The underlying idea of polynomial interpolation is fitting a polynomial function to the observed data in the vicinity of the grid point being analyzed. We illustrate the theory by considering the two dimensional analysis of a field variable z. We approximate the variation of the field variable, in the vicinity of the grid point being analyzed, by

$$\hat{z}(x,y) = \sum_{ij} a_{ij} x^i y^j \qquad i,j \geq 0 \;\; i+j \leq n_p \tag{1.1}$$

where x, y are the coordinates of the grid point and n_p is the degree of the fitting polynomial. We are interested in determining the interpolation coefficients a_{ij} such that the interpolation error E is minimized. We use the least square procedure to minimize the error. On taking the partial derivative of E with respect to a_{ij}, $i,j \geq 0$, $i+j \leq n_p$, we arrive at a system of linear equations for the determination of the interpolation coefficients a_{ij}, $i,j \geq 0$, $i+j \leq n_p$. Clearly the more the number of observations the better the fit, as any small amplitude observational errors can be filtered out.

Practical schemes based on the polynomial interpolation method often use numerical forecast values in the minimization procedure. One of the major drawbacks of this method, however, is the arbitrary manner of selection of the interpolation function. This selection does not use any relevant past information that may be available to us and thus the quality of the procedure diminishes. A problem also arises in sparse data areas.

1.2.3 Successive Correction Methods

The underlying idea in successive correction methods is to iteratively correct an initial preliminary field f_{gi}^{P} for all grid points to be analyzed, whereby the initial preliminary field is determined beforehand. In each iteration the corrections are computed by interpolation of the deviations from the preliminary field of the observed values i.e. as in optimum interpolation (section 1.2.5). The new analyzed field which we denote by f^N is

$$f_{gt}^N = f_{gt}^P + \sum_{i=1}^{n_o} \lambda_i (f_{it}^O - f_{it}^P) \tag{1.2}$$

where f_{it}^O is the observed grid value, f_{it}^P is the preliminary field value and n_o is the number of influencing observations.

Unlike optimum interpolation procedure, where the weights are calculated by solving a system of linear equations, this method explicitly calculates the interpolation coefficients according to some determined formula. For example, the Swedish Weather Service uses the formula given below to determine the interpolation coefficients λ_i.

$$\lambda_i = \frac{\psi(r_{ig}) \cdot h(\rho_i)}{\lambda_p + \sum_{k=1}^{n_o} \psi(r_{kg}) \cdot h(\rho_k)}$$

where ψ is a function dependent on the distance r and r_{ig} is the distance between the observational point i and the grid point g. The station density function, $h(\rho_i)$, corrects for the irregular and uneven distribution of data around the grid points. The number of influencing observations, within a certain distance from observation i is denoted by ρ_i. Normalization of the weights is done by the sum of all weights. An additional term λ_p is added to the normalizing factors for areas of the preliminary fields where there are sparse data.

The new corrected analyzed field then serves as a preliminary field for the next analysis and the procedure is repeated over this field. Thus as the name suggests, we are applying successive corrections to the field at each iteration.

Although the successive correction method is rather empirical in nature as compared to the optimum interpolation procedure, its main advantage lies in the relatively low number of computations that are required.

1.2.4 Variational Techniques

Variational methods involve forcing the complete or partial fulfillment of some given relationship among the different meteorological variables. Effectively, we are minimizing the errors of the analyzed fields by mutually adjusting the analyzed fields. The idea was initially presented by Sasaki in 1958 and proposed using the calculus of variations to optimize the adjustment procedure.

We assume that we have an analyzed field of the basic meteorological variables, the geopotential Z, the temperature T, the wind components U, V and the humidity Q. Since these are the meteorological variables, one can assume that there exists some valid relationship between these variables :

$$F_i(Z, U, V, T, Q) = O, \qquad i = 1, \cdots, n_f \tag{1.3}$$

where n_f is the number of functional relationships. We add small corrections (dZ, dU, dV, dT, dQ) to the original analyses (Z^N, U^N, V^N, T^N, Q^N) as the small adjustments made to achieve the partial or complete fulfillment of the relationship. Clearly, our objective is to minimize the magnitude of these corrections. We do this by minimizing the analysis volume integral of the square of these corrections :

$$\min I = \int_x \int_y \int_p \{\alpha_Z (dZ)^2 + \alpha_U (dU)^2 + \alpha_V (dV)^2 + \alpha_T (dT)^2 + \alpha_Q (dQ)^2\} dx dy dp \tag{1.4}$$

constrained to the condition

$$F_i(Z^N + dZ, U^N + dU, V^N + dV, T^N + dT, Q^N + dQ) \approx O, \;\; i = 1 \ldots n \tag{1.5}$$

Such a minimization problem can be solved using the calculus of variations.

Usually, it is necessary to use the variational analysis techniques along with some space interpolation procedure that provides the analyzed fields. The quality of the procedure increases if the coefficients α_Z, $\alpha_U, \alpha_V, \alpha_T, \alpha_Q$ in the integral are some functions of the quality of the corresponding analyses, e.g. by using the mean square interpolation errors obtained by a statistical method such as optimum interpolation.

1.2.5 Optimum Interpolation Methods

Optimum interpolation procedures were initially introduced to the field of meteorology by A. Eliassen [7] in 1954 and L. S. Gandin [9] in 1963. Interpolation in the conventional mathematical usage means that measured values are exactly reproduced. This however is not the case with optimum interpolation. Here, interpolation takes on a different meaning. Let a function $f(r)$ be defined at n points whose coordinates are $(x_1, y_1, \ldots, t_1), (x_2, y_2, \ldots, t_2), \cdots, (x_n, y_n, \ldots, t_n)$. Let us find the minimum x_m, the maximum x_M of all values of x, the minimum y_m, the maximum y_M of all values of y and so on. Interpolation in such a coordinate system, would be the determination, according to the known values $f(r_1)$, $f(r_2), \ldots, f(r_n)$ of the value of $f(r)$ at any point in the multidimensional prism $x_m \leq x \leq x_M, y_m \leq y \leq y_M, \ldots, t_m \leq t \leq t_M$ (see [9]).

Optimum interpolation procedures use past history about the nature and behavior of the atmosphere to determine the interpolation weights. The prerequisite imposed on us when using the above method is that the data be spatially correlated. This implies that observations that are spatially close to each other have similar tendencies and as the observations get farther apart, this correlation decreases. The method involves determining the weights to be assigned to the data, exploiting this correlation. The method, in other words, can be said to produce the optimal linear unbiased estimator of a variable at a particular location. Since the theory incorporates statistical information on the structure of variables through covariance and correlation factors, we are assured of optimality in the least squares sense. Basic statistical concepts are outlined in appendix 1 at the end of this thesis.

Let f be some meteorological field. We consider a grid point g with n observations f_{it}^{O} around it that will be used to calculate the analyzed value f_{gt}^{N} at that particular point g at a certain time t. We determine f_{gt}^{N} as the linear combination of a initial field value f_{gt}^{P} and the deviations of the observed values from preliminary values $(f_{it}^{O} - f_{it}^{P})$, $i = 1, \cdots, n$.

$$f_{gt}^{N} = f_{gt}^{P} + \sum_{i=1}^{n} \lambda_i (f_{it}^{O} - f_{it}^{P}) \tag{1.6}$$

The interpolation weights are the λ_i's. By intuition, we expect that the λ_i's should be positive and that they decrease monotonically with increasing distance from the grid point g. However, we do not impose any conditions on the λ_i's. We determine these by the imposition of the condition that the mean square error of interpolation is a minimum. Thus, we would like to minimize the equality

$$E = \overline{(f_{gt} - f_{gt}^{N})^2} \tag{1.7}$$

where f_{gt} is the true value of the variable in question and $\overline{()}$ is the estimated ensemble average (see Appendix 1).

Substituting equation (1.6) in equation (1.7) and on evaluating the square, we obtain the following.

$$\begin{aligned} E = \overline{(\delta f_{gt})^2} \quad &+ \quad \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j (\overline{\delta f_{it} \cdot \delta f_{jt}} + \overline{\triangle f_{it} \cdot \delta f_{jt}} + \overline{\delta f_{it} \cdot \triangle f_{jt}} + \overline{\triangle f_{it} \cdot \triangle f_{jt}}) \\ &- \quad 2 \sum_{i=1}^{n} \lambda_i (\overline{\delta f_{gt} \cdot \delta f_{it}} + \overline{\delta f_{gt} \cdot \triangle f_{it}}) \end{aligned} \tag{1.8}$$

where $\delta f_{it} = (f_{it}^{P} - f_{it})$ are the deviations from the first guess field of the true field values and $\triangle f_{it} = (f_{it}^{O} - f_{it})$ are the observational deviations.

In the above expression for the error E, the terms of the form $\overline{\triangle f_{it} \cdot \delta f_{jt}}$ are the cross-covariances between the preliminary and observational field errors. We can make the reasonable assumption that the observational errors and preliminary field errors are independent of each other. Thus any terms of the form $\overline{\triangle f_{it} \cdot \delta f_{jt}}$ may be assumed to be 0. The expression for E then reduces to

$$E = \sigma_g^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda j(\tau_{ij} + \eta_{ij}) - 2\sum_{i=1}^{n} \lambda_i \tau_{gi} \tag{1.9}$$

where σ_g^2 is the variance of the deviation of f_{gt}^P from the true value, τ_{ij} denotes the covariance for the deviations of the true values from any preliminary field value f_{it}^P and η_{ij} denotes the covariance of observational errors, namely

$$\begin{aligned} \tau_{ij} &= \overline{(f_{it}^P - f_{it})(f_{jt}^P - f_{jt})} = \overline{\delta f_{it} \cdot \delta f_{jt}} \\ \eta_{ij} &= \overline{(f_{it}^O - f_{it})(f_{jt}^O - f_{jt})} = \overline{\Delta f_{it} \cdot \Delta f_{jt}} \end{aligned}$$

The minimum of E, with respect to the interpolation weights, will necessarily occur when the derivative of E with respect to the weights is zero. Then

$$\frac{\partial E}{\partial \lambda_k} = 2(\sum_{i=1}^{n} \lambda_i(\tau_{ik} + \eta_{ik}) - \tau_{kg}) = 0 \tag{1.10}$$

for $k = 1, \ldots, n$, or

$$\sum_{i=1}^{n} \lambda_i(\tau_{ik} + \eta_{ik}) = \tau_{kg} \tag{1.11}$$

for $k = 1, \ldots, n$.

The above is a system of n linear equations in the n unknowns λ_i, given that we know the covariances τ_{ik}, τ_{kg} of the preliminary field errors and the covariance η_{ik} of the observational errors. A unique solution can then be obtained giving the values of the n interpolation weights. The analyzed field f_{it}^N can be calculated using equation (1.6).

By multiplying equation (1.11) by λ_k and summing over $k = 1, \ldots, n$ we obtain

$$\sum_{k=1}^{n}\sum_{i=1}^{n} \lambda_i \lambda_k(\tau_{ik} + \eta_{ik}) - \sum_{k=1}^{n} \lambda_k \tau_{kg} = 0 \tag{1.12}$$

By subtracting equation (1.12) from equation (1.9), we obtain the minimized interpolation error E^{MIN},

$$E^{MIN} = \sigma_g^2 - \sum_{i=1}^{n} \lambda_i \tau_{ig} \qquad (1.13)$$

When designing a practical scheme for such an interpolation analysis, the following tasks and decisions need to be involved.

- Decision of which meteorological variable to analyze.
- Decision of which measured quantities shall enter into the interpolation.
- Decision of the manner of modularization of the procedure to reduce needed computer resources and optimize the running time.
- Determination of a preliminary field or first guess field f_{gt}^P over the area of interest including the grid points and observation points.
- Estimation of the covariances τ_{ij} of the preliminary field errors where

$$\tau_{ij} = \overline{(f_{it}^P - f_{it})(f_{jt}^P - f_{jt})}$$

- Estimation of the covariances η_{ij} of the observational field errors where

$$\eta_{ij} = \overline{(f_{it}^O - f_{it})(f_{jt}^O - f_{jt})}$$

- Selecting which of the neighboring observations f_{it}^O will affect the grid point value f_{gt}^N.
- Solving the system of linear equations to determine the interpolation weights λ_i $(i = 1, \ldots, n)$ using the decided upon numerical procedure.

We discuss some of these issues in the following sections.

Estimation of the Covariances and Correlations

In order to solve system (1.11), we need to be able to estimate the covariances for both the observational and preliminary field errors. At a given time and location, these errors tend to be a function of several factors such as

1. The inaccuracy and uncertainty of the initial data.
2. The ease of prediction of the actual state of atmosphere.
3. Physical and numerical shortcomings of the mathematical forecast model.

Practically, the estimation of these covariances is done by constructing models based on certain assumptions that simplify the theory and on past historical forecast errors.

Modeling Error Correlations

Most practical models accept certain assumptions based on empirical observations. The major assumptions are the fulfillment of isotropy and homogeneity by the correlation fields of the empirical observations. We define isotropy as a characteristic of the field when the covariance is independent of a rotation in the field around the middle point on the line between the two positions. Equivalently, when the variable under analysis exhibits the same properties in different directions, the field is said to be isotropic. Homogeneity in reference to the the covariance describes the independence of the covariance with respect to a translation of the two positions.

There are several considerations that must dictate the choice of the functional form of the the covariance and or correlation. Some of these as suggested by Julian and Thiebaux [12], who based most of their work on geopotential fields, are

1. The "goodness of fit" of the possible functions to the sample correlations derived from observed data.

2. The satisfaction of constraints placed upon the selected functions by the numerical models.

3. The existence of derivatives required for the geostrophic derivation of wind field correlations.

4. The selected correlation function should be positive definite [9].

Since the cosine transform of correlation functions, i.e. the wavenumber spectrum contains equivalent statistical information, we can measure the goodness of fit. This can be done by comparing the characteristics of the spectra of the fitted function to the properties demonstrated by the actual spectra of the meteorological variable. A good match would indicate a good choice of function. Thiebaux and Pedder [18] discuss these considerations in detail.

One of the often used correlation functions for geopotential is the Gaussian function which is a distance dependent function.

$$\beta(r) = e^{-\frac{r^2}{2b^2}}$$

where r is the distance and b^2 is a coefficient defining the scale of the parameter under analysis. However, the Gaussian error function is not an appropriate choice when dealing with geopotential as the correlation function produced does not possess an energy spectrum suitable for calculation of vorticity advection by finite differences (see [12]). Moreover, as pointed out by Julian and Thiebaux [12], the geostrophic kinetic energy spectrum of the geopotential field does not correspond to that of the atmosphere.

The covariances for the wind fields are more complex. In addition, the wind field is anisotropic. The correlations are commonly obtained by applying the geostrophic wind equation to the isotropic correlation function for the geopotential e.g.,

$$\tau(u_i, v_j) \approx \tau(-\frac{g}{f}(\frac{\partial z}{\partial y})_i, \frac{g}{f}(\frac{\partial z}{\partial x})_j) \tag{1.14}$$

The correlation between the wind and geopotential is obtained in the same way. Equation (1.14) assumes non-divergent winds and fulfillment of isotropy. For a consistent analysis of the divergent part of the wind, the model should be improved using the velocity potential.

Julian and Thiebaux [12] experimented with fitting analytic functions to the observed correlations of geopotential and temperature fields. They noted that in modeling the correlations, the procedure does not show increased sensitivity to the actual statistical information in the variable. They then raise a very basic concern,

"The inability of the scheme to reproduce the proper spectral distribution of the variable being interpolated suggests that the practical limitations imposed on how statistical information of the population is in fact used, limits the sense of optimality of the scheme. This raises the fundamental question of just what statistical information is important for incorporation in an objective analysis scheme."

Clearly, this is a limitation of the optimum interpolation method. This weakness in the technique should not, however, imply that optimum interpolation need be abandoned. Thiebaux experimented fitting both isotropic and anisotropic functions to the correlations [16, 17].

Modeling Standard Deviations for Preliminary Field

Historical forecast errors are utilized to estimate the standard deviations of the forecast errors. One of the methods proposed by Bengtsson and Gustafsson [2] is the one used in the majority of the operational schemes for the geopotential forecast errors. The variance of the error is calculated from the estimated mean interpolation error every time the analysis is performed on the data.

$$\sigma_g^A(t)^2 = \sigma_g^P(t)^2 - \sum_{i=1}^{n} \lambda_i \tau_{ig}^P(t) \tag{1.15}$$

The forecast variance at a later time $t + \Delta t$ is then given by a linear increase

$$\sigma_g^P(t + \Delta t)^2 = \sigma_g^A(t)^2 + \frac{\Delta t}{T}\sigma_g^2 \tag{1.16}$$

where σ_g^2 is the climatology error variance and T is the time to reach this error level in the forecast model.

Selection of Influencing Information

Initially, when the single variable two-dimensional schemes were introduced, it was sufficient to take about 6 to 8 observations surrounding the grid point. Usually, the closest 6 to 8 observations were a suitable choice. With the increase in sensitivity of the analysis to the data due to more complex schemes, the selection of influencing observations has become more difficult. The naturally obvious choice of taking a number of closest observations is no longer a viable one. We may have some observations that have a small correlation with the analyzed grid point. However, they may be of importance due to the large cross-correlation with other observed quantities.

Gustafsson [10] mentions the two types of selection algorithms that are in use currently.

1. A certain analysis volume is defined. All observations within this volume are selected. The covariance matrix is inverted and the inverted matrix is used to calculate the analyzed values of all the variables in the central part of the selected volume.

2. For each grid point, analysis level and, possibly, each analyzed variable a local selection of influencing observations is made. Empirical algorithms are then employed to choose cross-correlated information.

Computationally, it is possible to only select about 8 to 12 influencing observations. This is because of the cubic increase in computing power time as a function of this number.

Kriging

Kriging is a modified optimum interpolation method originally developed by Matheron and his coworkers [14] to analyze distribution of ore bodies. In the last decade, Kriging has been frequently used in the spatial analysis of the acidity (pH) of precipitation.

Kriging is similar to the optimum interpolation that we introduced previously in the sense that the statistical theory behind the two is similar. Kriging, however, does not incorporate past knowledge in the form of preliminary trial fields. Finkelstein [8] provides the detailed mathematics of Kriging. Essentially, the estimate, $\hat{f}_0$, of a quantity whose true value is f_0 is assumed to be of the form

$$\hat{f}_0 = \sum_{j=1}^{N} \lambda_j f_j \tag{1.17}$$

where the weights λ_j are assumed to be independent of the f_j's, the observations. In addition, each observation as well as $\hat{f}_0$ is assumed to be an unbiased estimator of f_0. This requires that

$$\sum_{j=1}^{N} \lambda_j = 1 \tag{1.18}$$

where N is the number of observations influencing the value f_0.

The procedure to determine λ_j's is then one where the variance of the error of the analysis $V = \mathcal{E}[(\hat{f}_0 - f_0)^2]$ is to be minimized, where $\mathcal{E}$ is the expectation operator. Given the above information, we arrive at the set of $(N+1)$ equations in $(N+1)$ unknowns.

$$\begin{aligned} \sum_{j=1}^{N} \lambda_j \gamma_{ij} + \Lambda &= \gamma_{i0} \qquad i = 1, \cdots, N \\ \sum_{j=1}^{N} \lambda_j - 1 &= 0 \end{aligned} \tag{1.19}$$

where Λ is the Lagrange multiplier and γ_{ij} is the the semivariogram defined as

$$\gamma_{ij} = \frac{1}{2}\mathcal{E}[(f_i - f_j)^2] \tag{1.20}$$

We assume that the semivariogram is a function of distance only, i.e, the values of the above are plotted as a function of separation distance $h = |x_i - x_j|$ and a smooth curve fitted to the data.

$$\gamma_{ij} = \gamma(|x_i - x_j|) = \gamma(h) \tag{1.21}$$

The Kriging error E is given by the expression below

$$E = \sum_{i}^{N} \lambda_i \gamma_{i0} + \Lambda \tag{1.22}$$

Clearly, the usefulness of the estimate for γ_{ij} relies on

(A). The number of points $N(h)$ at the distance of interest h,

(B). The validity of our assumptions on γ_{ij}.

(A) implies that part of the success of Kriging is dependent on the spatial density of the observations. (B) implies that the model for γ_{ij} plays a crucial role in determining the error. Some of the common variogram models observed in practice [21] are

1. Linear:

$$\gamma(h) = a \cdot h \tag{1.23}$$

2. Linear with constant:

$$\begin{aligned} \gamma(h) &= a \cdot h + b, && \text{for } h > 0, \\ &= 0, && \text{for } h = 0. \end{aligned}$$

3. Power function with constant:

$$\begin{aligned}\gamma(h) &= a \cdot h^k + b, && \text{for } h > 0,\ (1 < k < 2),\\ &= 0, && \text{for } h = 0.\end{aligned}$$

4. Spherical:

$$\begin{aligned}\gamma(h) &= C \cdot [\frac{3}{2}\frac{h}{a} - \frac{1}{2}(\frac{h}{a})^3], && \text{for } 0 \leq h < a,\\ &= C, && \text{for } h \geq a.\end{aligned}$$

1.3 Concluding Remarks

We are aware of several methods that have been developed to prepare initial data for forecast models. We looked in detail at the statistical interpolation techniques. These form an integral part of four-dimensional data assimilation and are utilized by most operational schemes. However, the methods are far from perfect and can be improved upon. With this point in mind, we present the main objective of this thesis in the next chapter.

Chapter 2

Objective of the Thesis

The main aim of this thesis is the application of optimum interpolation to the analysis of precipitation amounts over complex terrain. To the author's knowledge, optimum interpolation has not been used before in this specific context.

The optimum interpolation method for the objective analysis of meteorological fields produces the best solution in the sense that the total interpolation error is, on average, minimum. The method allows for the extraction of as much useful information as the observations provide. Nevertheless, it is a complex scheme which has only been put into use much later after it was initially proposed by Eliassen [7]. The reason for this delay is twofold

- The scheme requires a knowledge of covariances. These are often not known and thus an estimate is required. Establishing such an estimate is often fraught with difficulty as a host of local factors are involved.

- Most schemes for objective analysis assign weights to observations primarily on the basis of the position of each observation relative to the point of interpolation. This dependence or basis is determined from an arbitrarily chosen weighting model. This is no longer the case with optimum interpolation. Essentially, there is now a need to determine a region of influence around the point of interpolation. These observations would be used to resolve the value at the gridpoint in question. It would seem that the greater the number of observation points included in the interpolation, the greater the accuracy of the interpolation results. Unfortunately, due to the uncertainty in the available data, at some stage of inclusion, we would reach some number n, where a further inclusion of data will not lead to a decrease but to an increase in the actual interpolation error. There is now a need to determine *a priori* which observations are greatly correlated with the value at the point of interpolation. Clearly, a host of local factors greatly complicate such a determination.

Over complex terrain, where topography influences precipitation, the method must allow for the inclusion of this factor into the analysis.

Given rainfall data, irregularly distributed in time and space and a first guess field we will apply the method of optimum interpolation to the objective analysis of precipitation amounts over a complex terrain. The first guess field is usually obtained from climatological data or from model simulations or both. Our aim is to construct a covariance/correlation model that will improve the overall performance of the implementation by taking the topographical factor into consideration . We measure the performance of the implementation by observing the magnitude of the interpolation error. In this thesis, we propose the following solution that addresses the questions raised above.

- Estimation of covariances using observed quantities
- Using the spatial structure of the covariances, we attempt to define a region of influence. This will provide a mechanism that would allow us to selectively choose the number of influencing observations that neither leads to a loss of information from the non-selected data nor to inaccuracy from the inclusion of erroneous or non-significant data.

Chapter 3

A Practical Application of Optimum Interpolation

3.1 Introduction

The major problem that arises in the implementation of optimum analysis techniques is the representation of covariances and correlations of the observational and preliminary data. The linear system (1.11) requires a knowledge of covariances of both the observational and preliminary field errors. These in turn require a knowledge of hypothetical true values which are unavailable. Thus we need to transform the linear system (1.11) into a form that requires measured and not hypothetical quantities. We propose one such solution.

3.2 A Proposed Solution

The preliminary field error covariance, τ_{ij}, is defined as

$$\overline{(f_{it}^{P} - f_{it}) \cdot (f_{jt}^{P} - f_{jt})} \tag{3.1}$$

In the derivation of the linear system in equation (1.11), it was assumed that the trial field errors and the observational errors are independent of each other, i.e.

$$\overline{(f^P_{it} - f_{it}) \cdot (f_{jt} - f^O_{jt})} = 0 \tag{3.2}$$

On subtracting equation(3.2) from equation(3.1), we obtain

$$\tau_{ij} = \overline{(f^P_{it} - f_{it}) \cdot (f^O_{jt} - f^P_{jt})} \tag{3.3}$$

We now make an additional assumption that the observational errors are not correlated with the deviations of the trial field values from the observations, i.e.

$$\overline{(f_{it} - f^O_{it}) \cdot (f^O_{jt} - f^P_{jt})} = 0 \tag{3.4}$$

Then, subtracting equation(3.4) from equation(3.3), we obtain

$$\tau_{ij} = \overline{(f^O_{it} - f^P_{it}) \cdot (f^O_{jt} - f^P_{jt})} \tag{3.5}$$

Applying a similar analysis on η_{ij}, where η_{ij} is defined as

$$\eta_{ij} = \overline{(f_{it} - f^O_{it}) \cdot (f_{jt} - f^O_{jt})} \tag{3.6}$$

subtracting equation(3.4) from equation(3.6), we obtain

$$\begin{aligned} \eta_{ij} &= \overline{(f_{it} - f^O_{it}) \cdot (f^O_{jt} - f^P_{jt})} \\ &= 0 \end{aligned}$$

from assumption (3.4).

In the limiting case, when $i = j$, $\eta_{ii} = 0$. This means that there is zero observation error. This appears to contradict our discussion in chapter 4 on observational errors. With these modified definitions for τ and η, equation(1.11) reduces to

$$\sum_{i=1}^{n} \lambda_i \tau_{ik} = \tau_{kg} \tag{3.7}$$

for $k = 1, \ldots, n$ and gridpoint g.

Assumptions 3.2 and 3.4 together imply that observational errors are uncorrelated with each other. A justification of when such an assumption is valid is given in chapter 4.

3.3 Analysis

The main premise of optimum interpolation is that empirical data be used to determine the spatial structure of the covariances. The r_{ij}'s were computed for four different observation sets, shown in figures 3.1 - 3.4, over the Columbia river basin in southeastern British Columbia. Since by the assumption (3.4) η equals 0, we shall ignore η altogether. These four data sets are the precipitation amounts for the three days 83/07/11, 83/07/12, 83/07/13, and the 3-day total. Initial trial fields of the precipitation were obtained from a 3-dimensional initial value model which incorporates topographical effects. This initial value model is described in appendix B. These trial fields over an 18 × 18 grid covering the region are shown in figures 3.5 - 3.8. Initial value models need accurate initial conditions to be successful. Both surface and upper air data are sparse in southeastern British Columbia. Therefore, discrepancies between model predicted and observed precipitation amounts are to be expected. The trial fields were interpolated using bi-cubic splines to the observation sites. Table 3.1 shows the correlations obtained between the trial and observed precipitation. Observational errors also contribute to a low correlation and these will be discussed further on in chapter 4.

Date	Correlation
83/07/11	0.623
83/07/12	0.139
83/07/13	0.587
3-day total	0.630

Table 3.1: Correlation coefficients between trial and observed precipitation amounts

The r_{ij}'s, for each precipitation interval, were calculated using equation (3.5) for observation points i and j. The great circle distance between points i and j determined the bin in which τ_{ij} would be included. Each bin was an interval of 50 km. Thus, for any two points that lay within 0 to 50 km of each other, the τ would go in the first bin. Points that lay within 50 to 100 km of each other, were included in the second bin, etc. For each bin, an approximation to the ensemble average was obtained by averaging the contents of that bin. The reason why the ensemble average was calculated in such a manner was due to the lack of archived data. A more correct estimate could be determined if we had access to some operational weather prediction data archives. Since $\tau_{ij} = \tau_{ji}$, each τ_{ij} was included in the appropriate bin exactly once. When observation points i and j were coincidental, the τ_{ij} was included in the zero bin. The ensemble averages were then plotted against the midpoints of the different bins. Figures 3.9 - 3.12 illustrate the plots obtained for the spatial structure of τ.

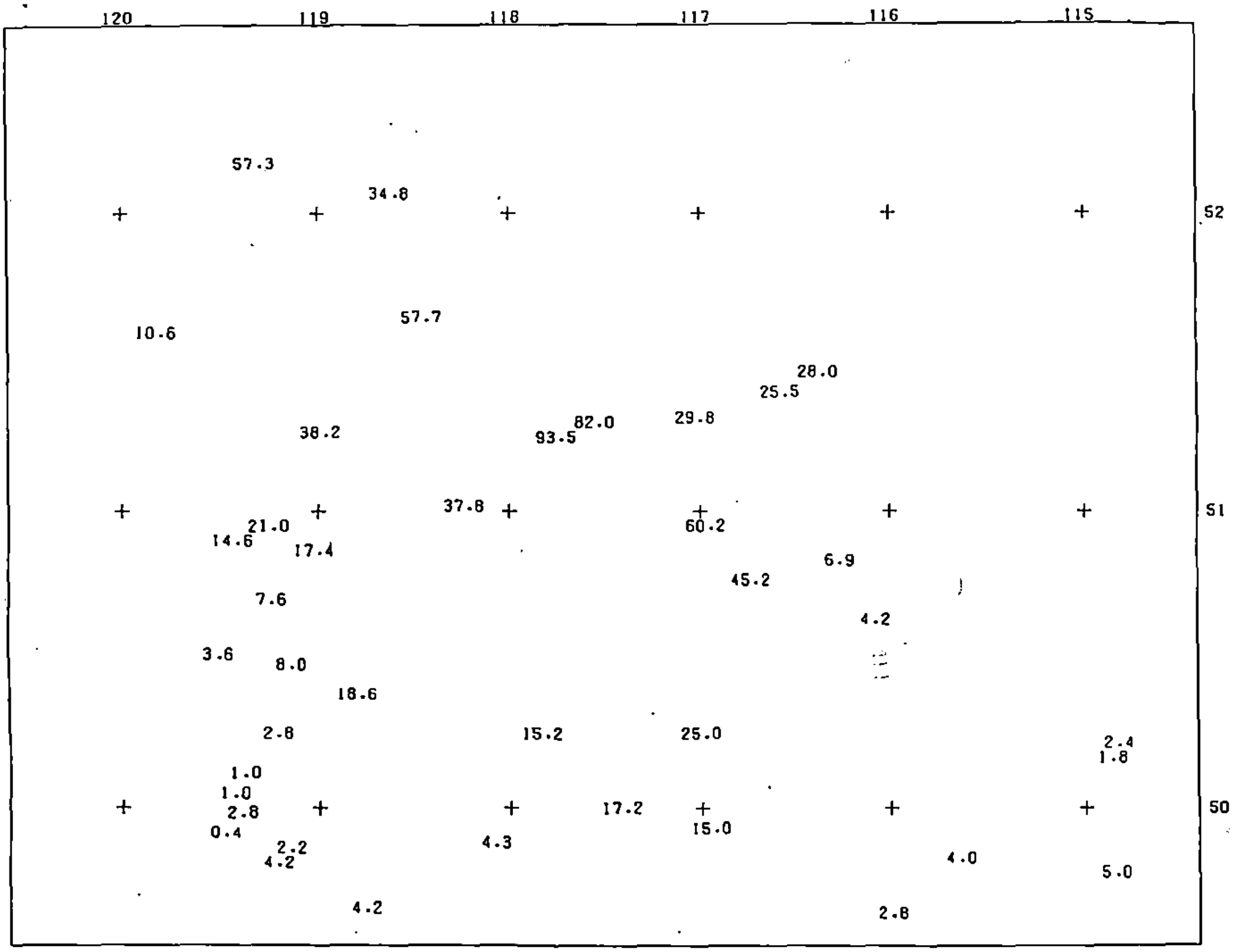

Figure 3.1: Observed precipitation amounts (mm) for 83/07/11

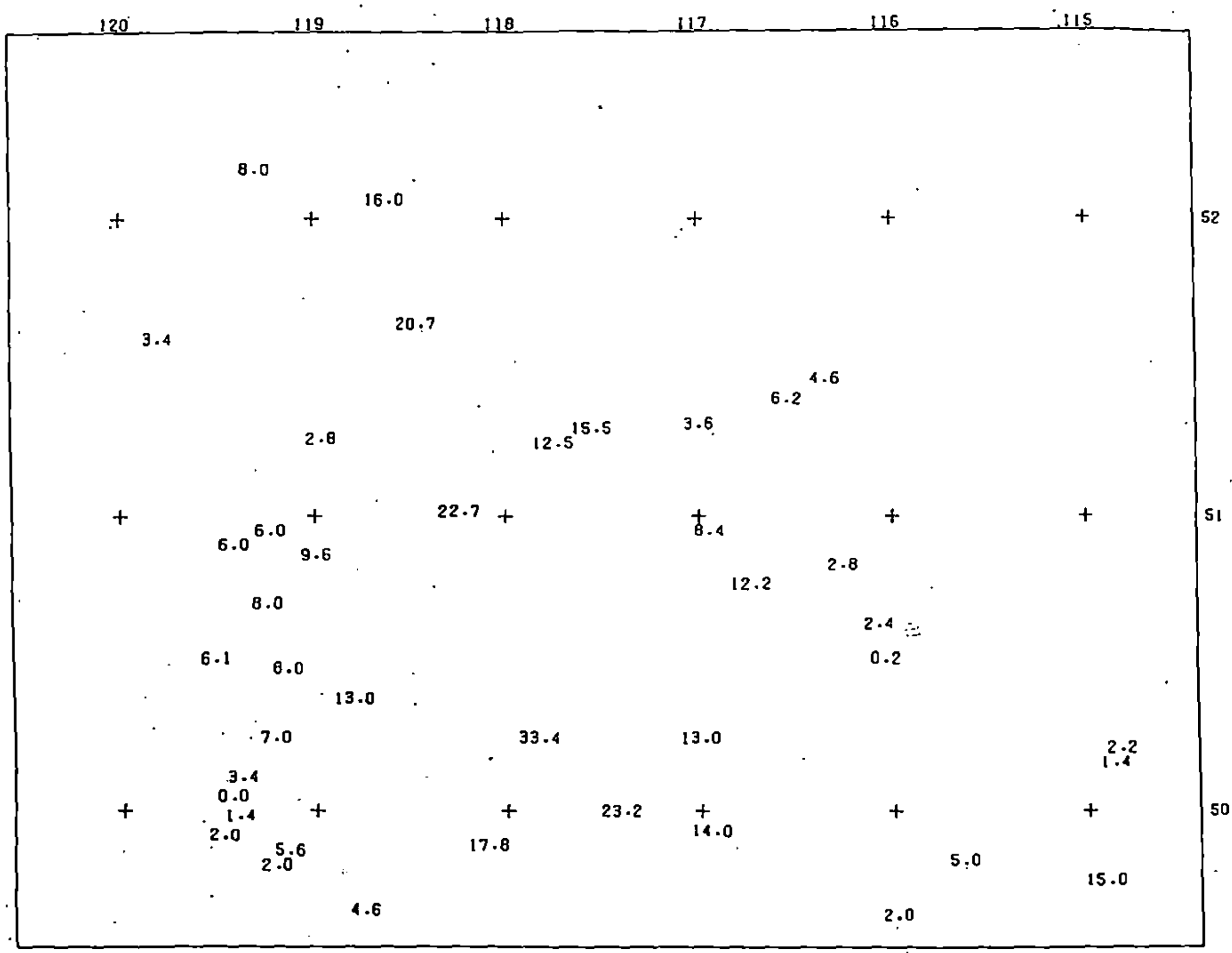

Figure 3.2: Observed precipitation amounts (mm) for 83/07/12

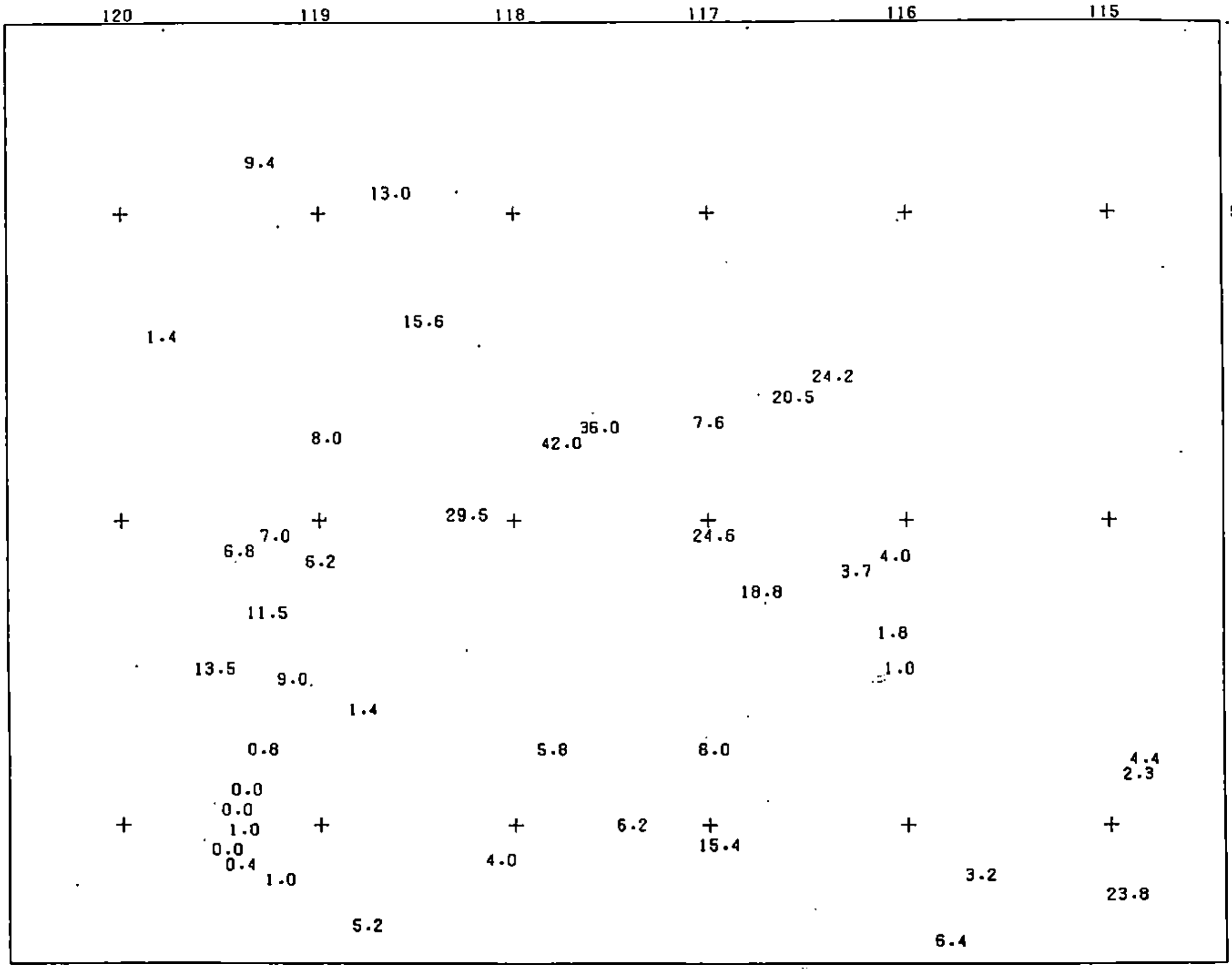

Figure 3.3: Observed precipitation amounts (mm) for 83/07/13

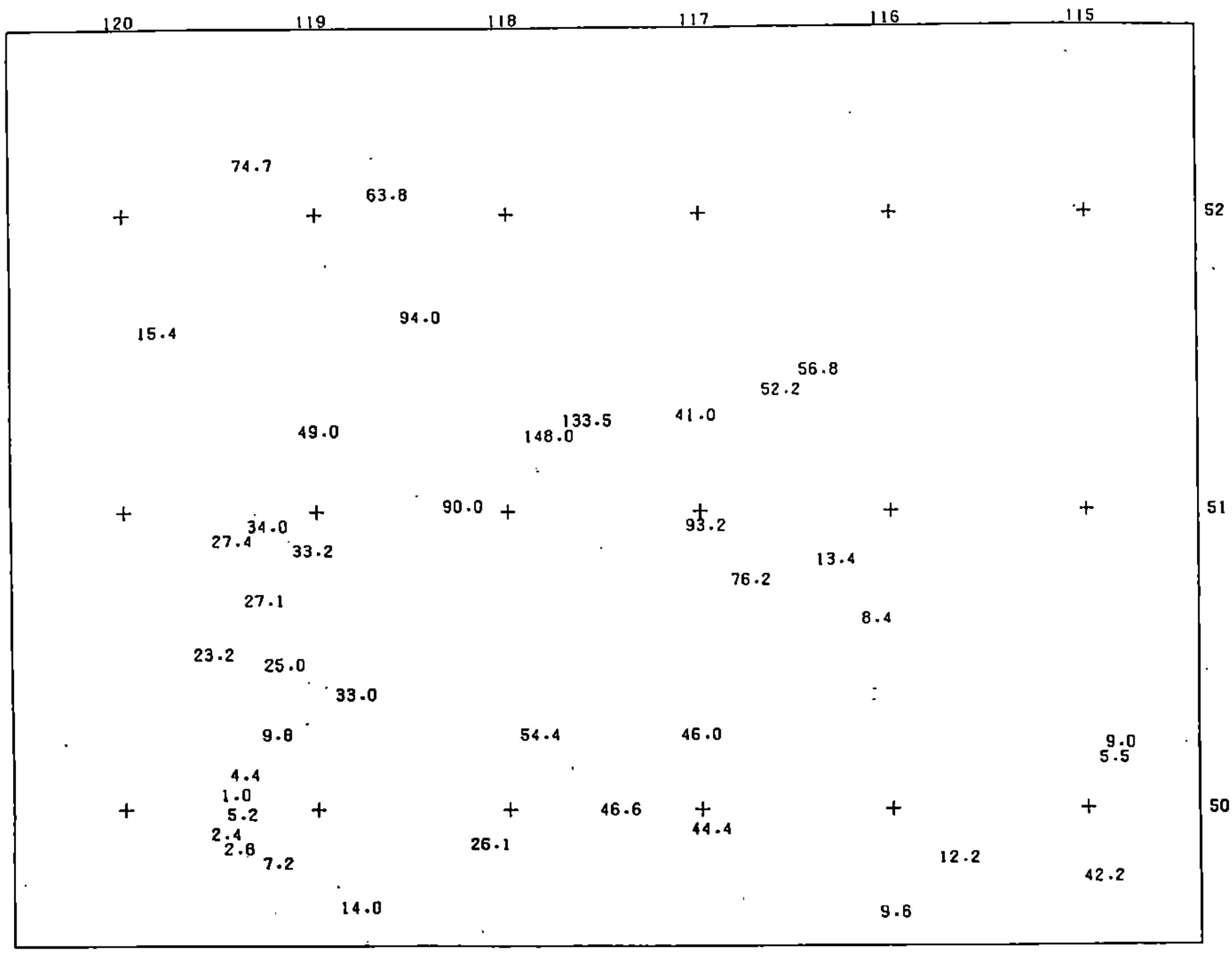

Figure 3.4: 3-day total observed precipitation amounts (mm)

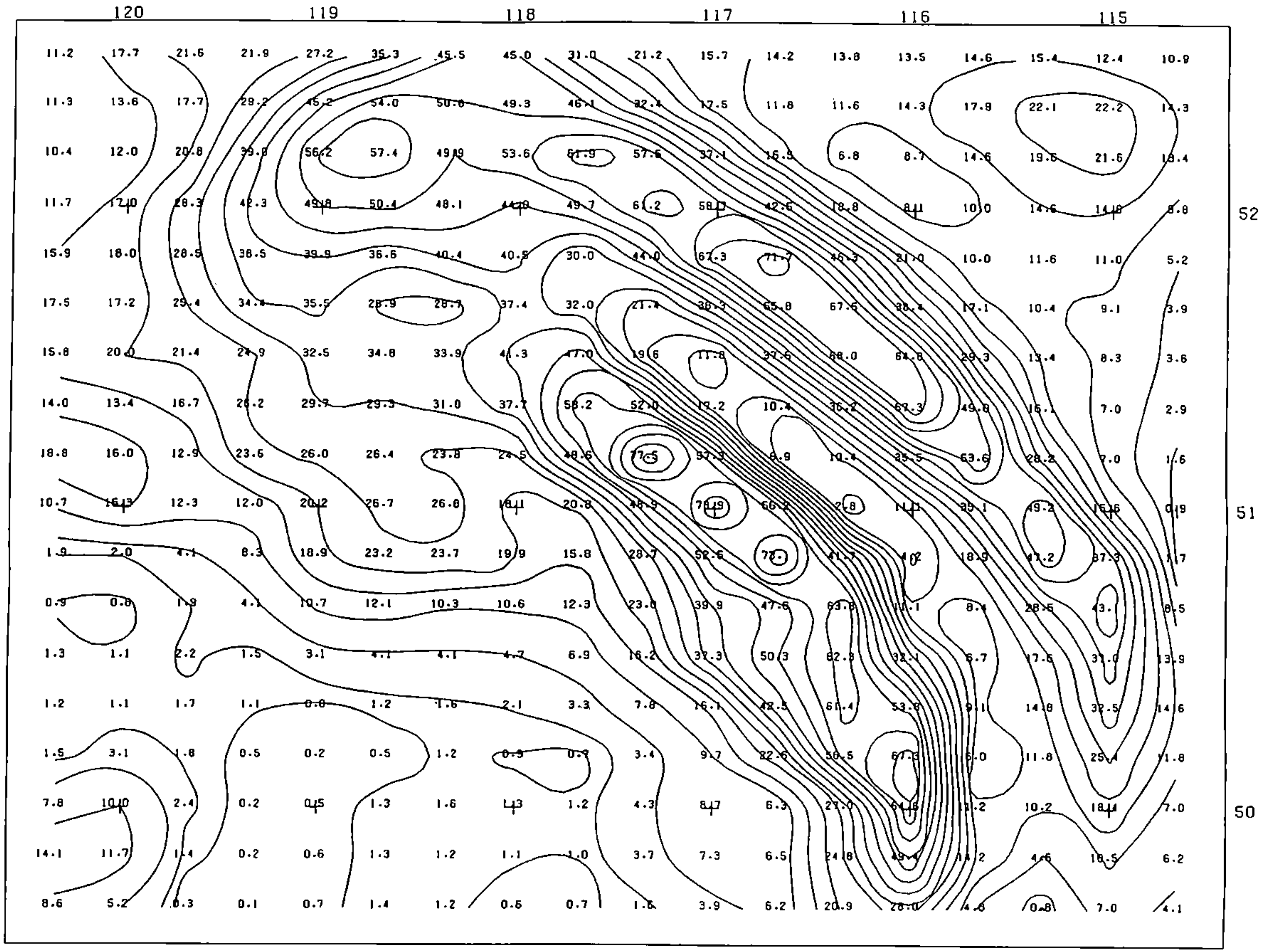

Figure 3.5: Trial field of precipitation amounts (mm) for 83/07/11

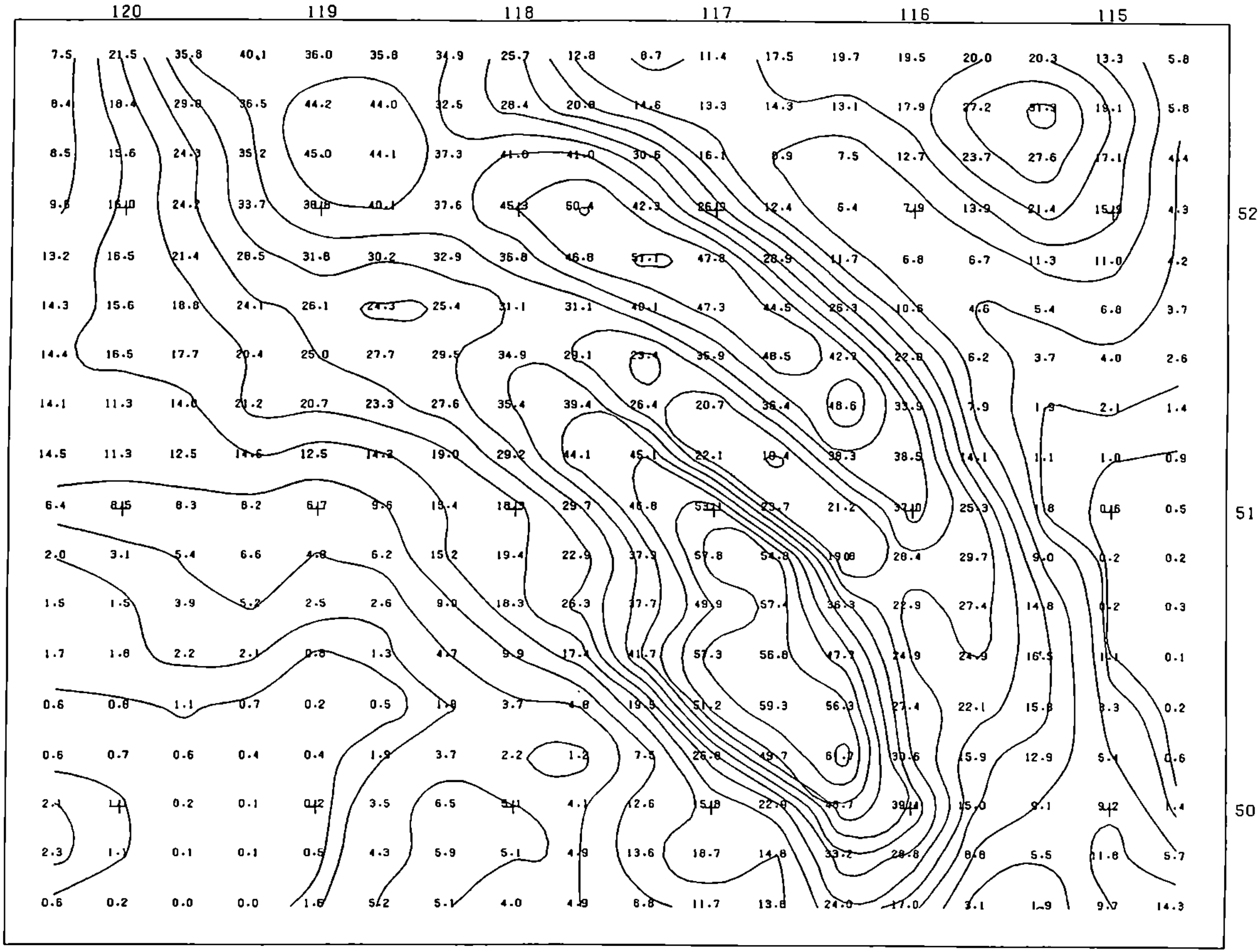

Figure 3.6: Trial field of precipitation amounts (mm) for 83/07/12

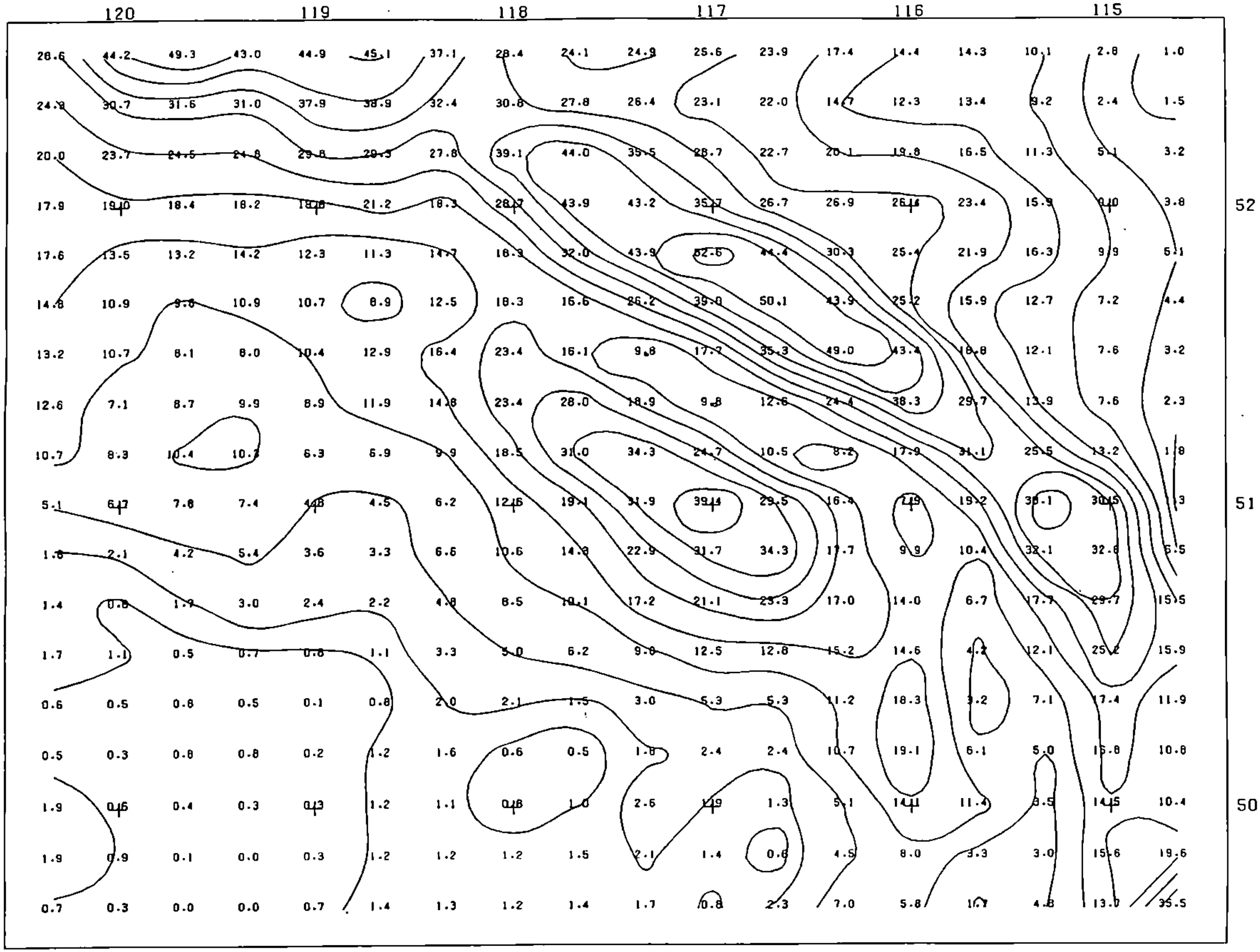

Figure 3.7: Trial field of precipitation amounts (mm) for 83/07/13

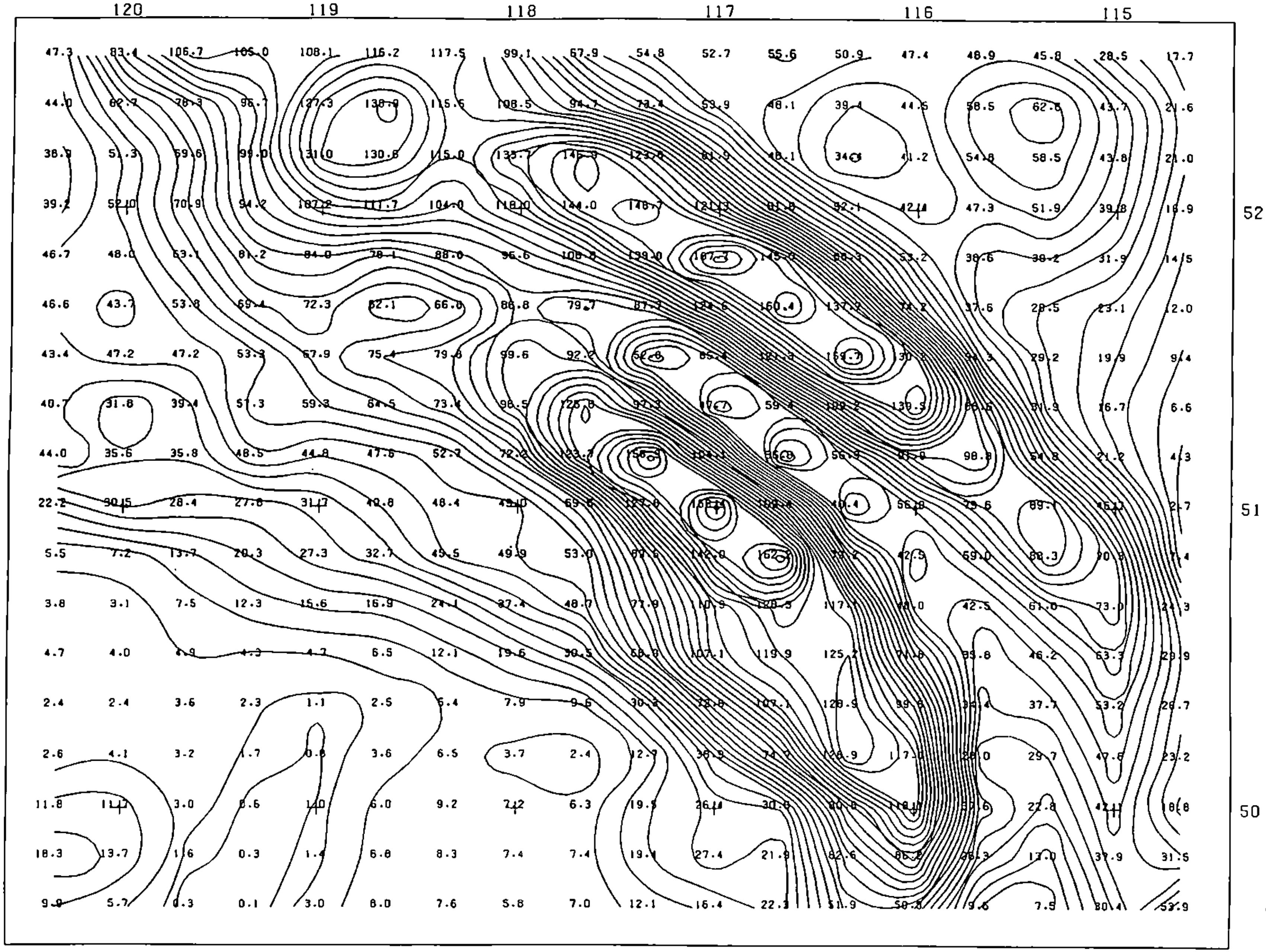

Figure 3.8: Trial field of precipitation amounts (mm) for the 3-day total

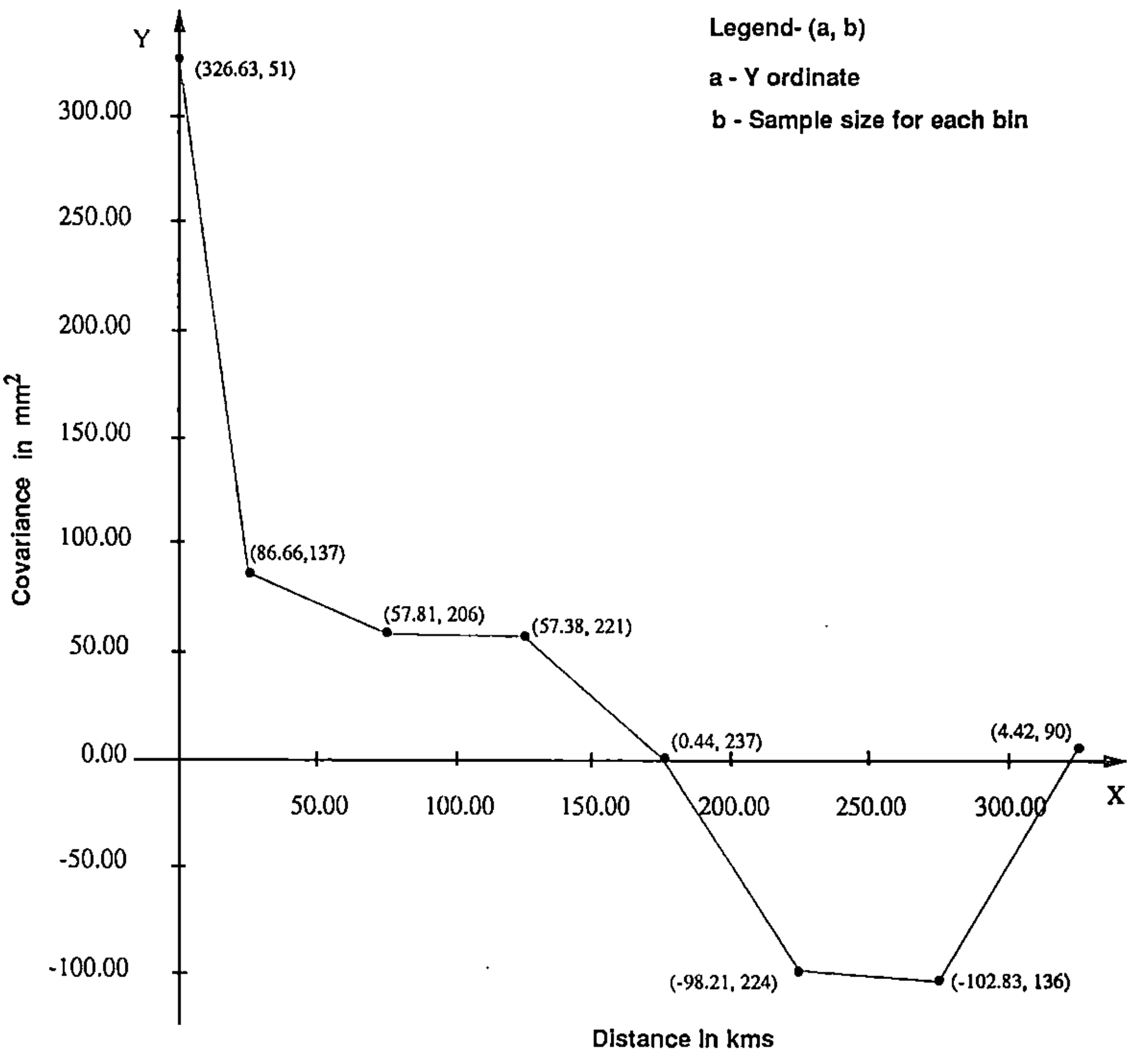

Figure 3.9: Spatial structure of covariances obtained from observations for 83/07/11

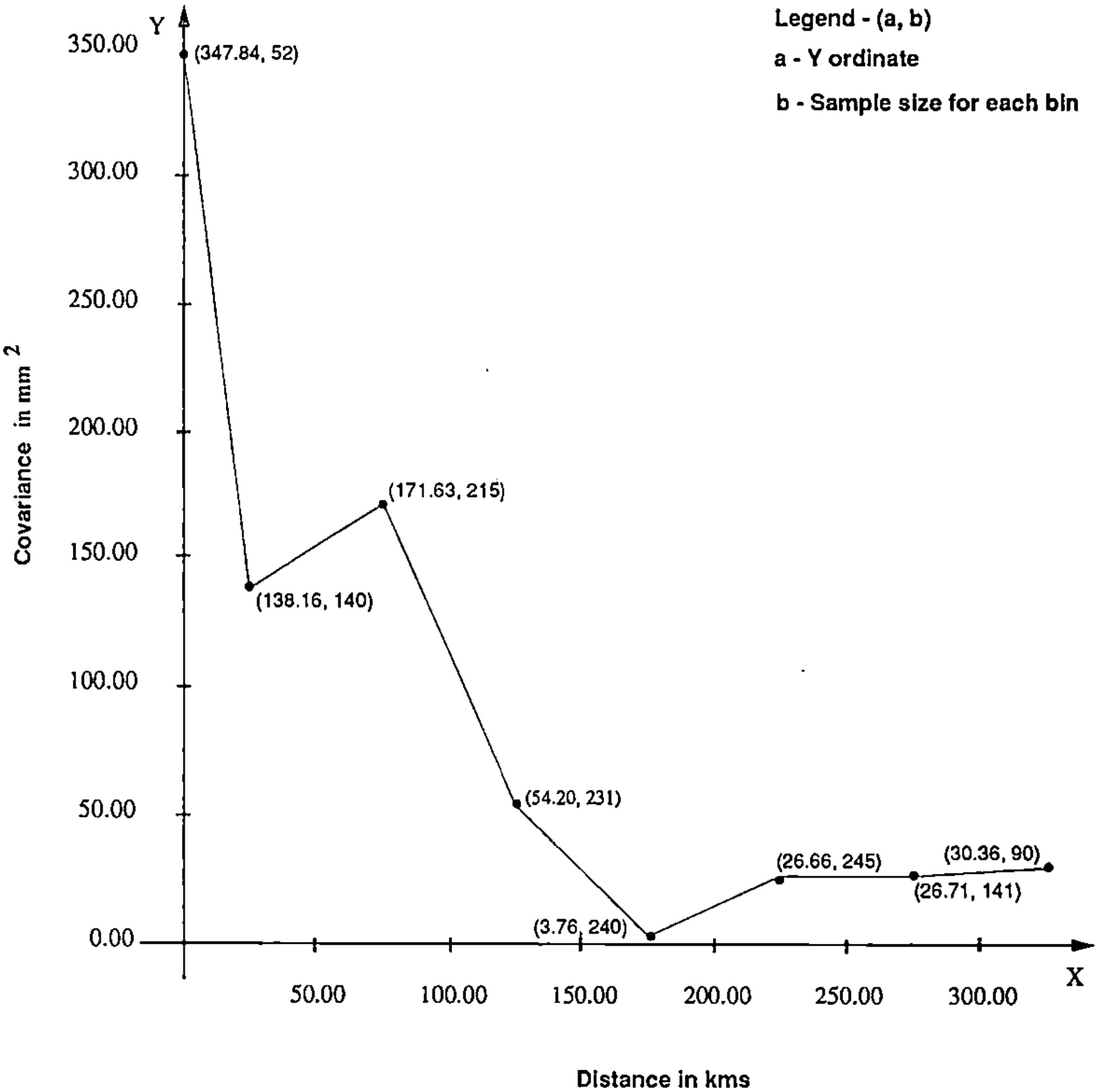

Figure 3.10: Spatial structure of covariances obtained from observations for 83/07/12

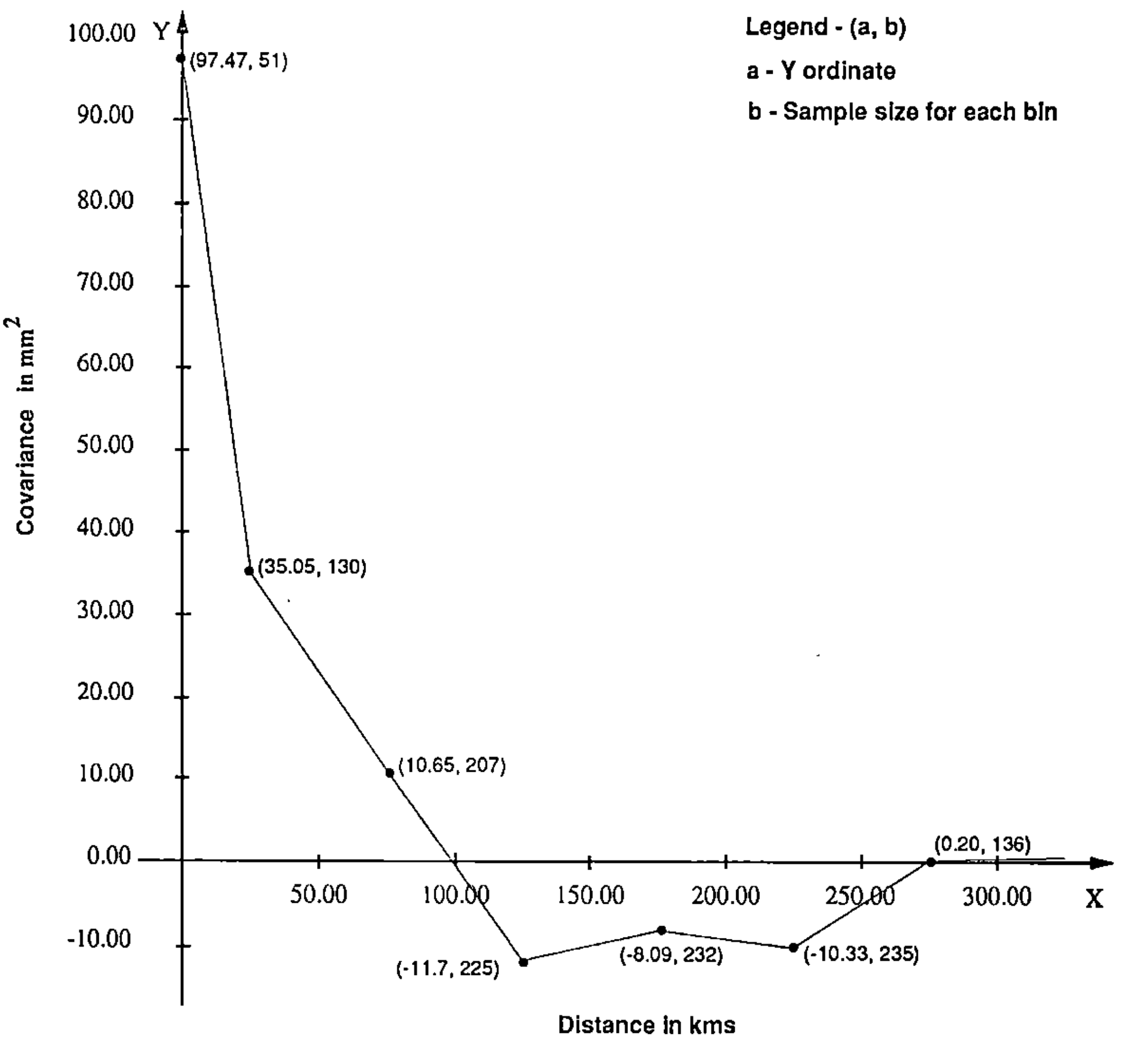

Figure 3.11: Spatial structure of covariances obtained from observations for 83/07/13

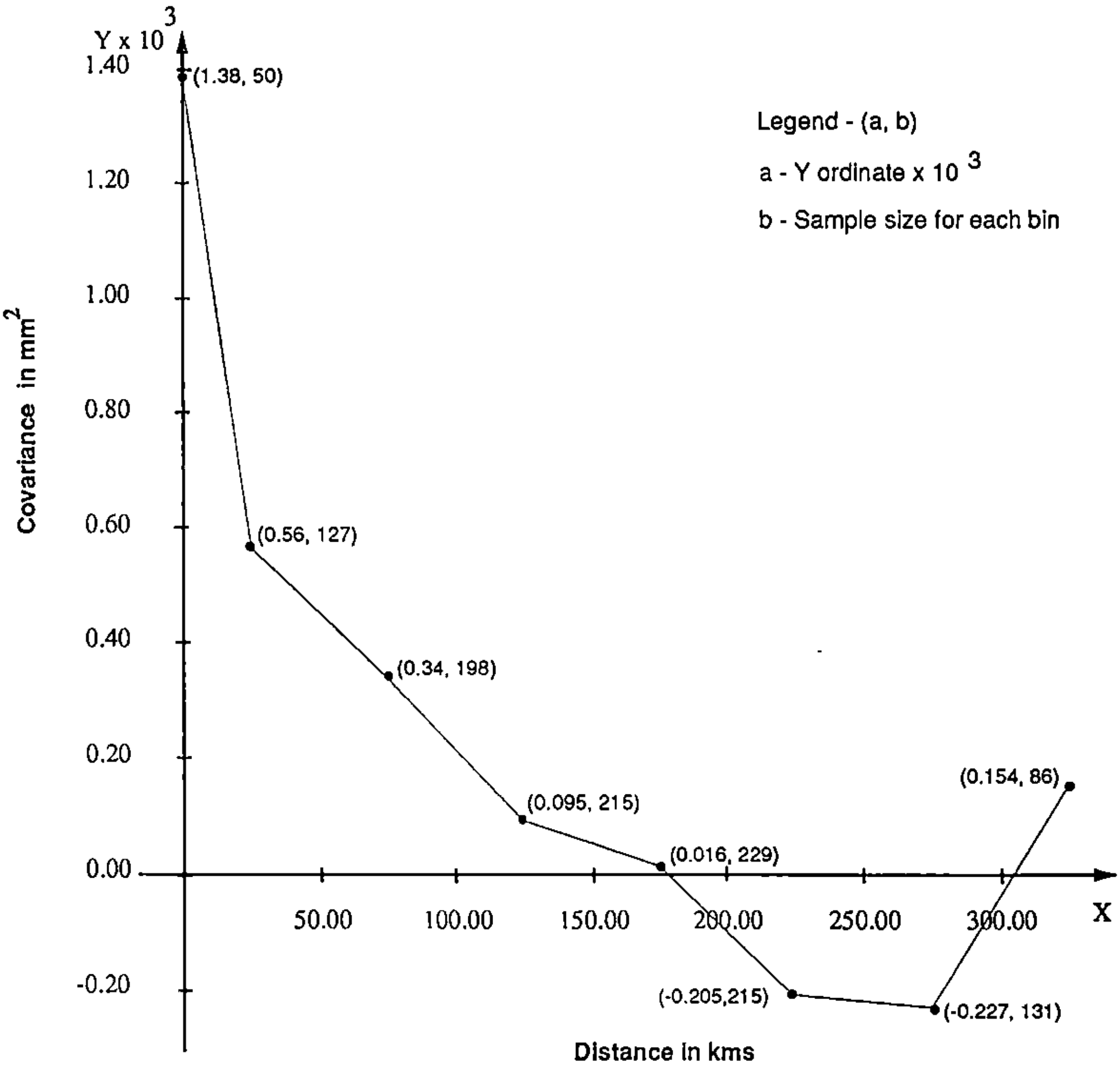

Figure 3.12: Spatial structure of covariances obtained from observations for the 3-day total

Clearly, interpolating between any two points using this raw data curve would run into some predictable obstacles. The fact that the raw curves do not decrease monotonically gives rise to the singularity of the matrix of τ values obtained. We decided to approximate figures 3.9 - 3.12 with a cubic polynomial. The approximation was done using Chebychev polynomials [3]. The approximation was a least squares fit of a linear combinations of Chebychev polynomials. A Chebychev polynomial is defined as

$$T_n(x) = \cos[n \arccos x], \quad x \in [-1, 1],\ n = 0, 1, 2, \cdots$$

Thus the polynomial fitted was of the form

$$\sum_{i=0}^{3} c_i T_i(x)$$

where the coefficients c_i were determined using least squares. Figures 3.13 - 3.16 show the original plots and the fitted cubic polynomials for all of the data sets. The graphs obtained are much smoother and intuitively should provide a more "non-singular" matrix of τ values. All the polynomials, with the exception of the one from the observations from July 12 exhibited a root in the interval of 100 - 200 km. In all cases, the value of the covariance at 100 km is a small fraction of the maximum covariance value. It was decided to define the radius of influence as 100 km. Thus any observation point that lay within 100 km of the grid point was taken as an influencing observation, and all observations beyond this distance were ignored. The τ values were determined using the fitted curve. The matrix of τ values so obtained was positive definite and well conditioned. The resulting system of linear equations was solved using Cholesky factorization [3].

3.4 Estimating the Interpolation Error

In chapter 1, we arrived at the expression for the minimized interpolation error variance, E^{MIN} (1.13),

$$E^{MIN} = \sigma_g^2 - \sum_{i=1}^{n} \lambda_i \tau_{ig} \tag{3.8}$$

where $\sigma_g^2 = \overline{(\delta f_{gt})^2} = \overline{(f_{gt} - f_{gt}^P)^2}$ is the variance of the preliminary field gridpoint value from the hypothetical true value. Since the hypothetical true gridpoint value is not known, σ_g^2 has to be estimated. Neglecting interpolation error in f_{gt}^P in order to simplify the estimation and dropping the subscript t, we have

$$(f_g^O - f_g^P) = (f_g^O - f_g) + (f_g - f_g^P) \tag{3.9}$$

If we assume that all gridpoints coincide with the observation points, we can drop the subscript g. On squaring the whole expression, taking the ensemble average and rearranging, we obtain

$$\begin{aligned} \overline{(f - f^P)^2} &= \overline{(f^O - f^P)^2} - \overline{(f^O - f)^2} \\ \sigma_g^2 &= \delta^2 - \epsilon^2 \end{aligned} \tag{3.10}$$

The cross products are eliminated due to equation (3.2). We can determine δ^2 from available data. Now

$$\epsilon^2 = \overline{(f^O - f)^2} \tag{3.11}$$

If we assume that $(f^O - f)$ is proportional to (f^O) then

$$\epsilon^2 = r^2\overline{(f^O)^2} \tag{3.12}$$

where r is the constant of proportionality that reflects the reliability of the observations. Since $\overline{(f^O)^2}$ can be ascertained from given data, we need an indication of the accuracy of the observations in order to be able to evaluate ϵ^2. We decided to use a value of 0.5 for r. This effectively says that the standard error of the observations is assumed to be 50% of the root mean square observed precipitation. One way to determine a value for r would be to have a high resolution network of accurate precipitation measurements. In the absence of such a network, a value of 0.5 assigned to r does not seem unreasonable. Clearly, its exact value is open to question.

We compute σ_g^2 by substituting equation (3.11) in equation (3.10). We are now in a position to evaluate the minimum error variance E^{MIN} according to equation (3.8).

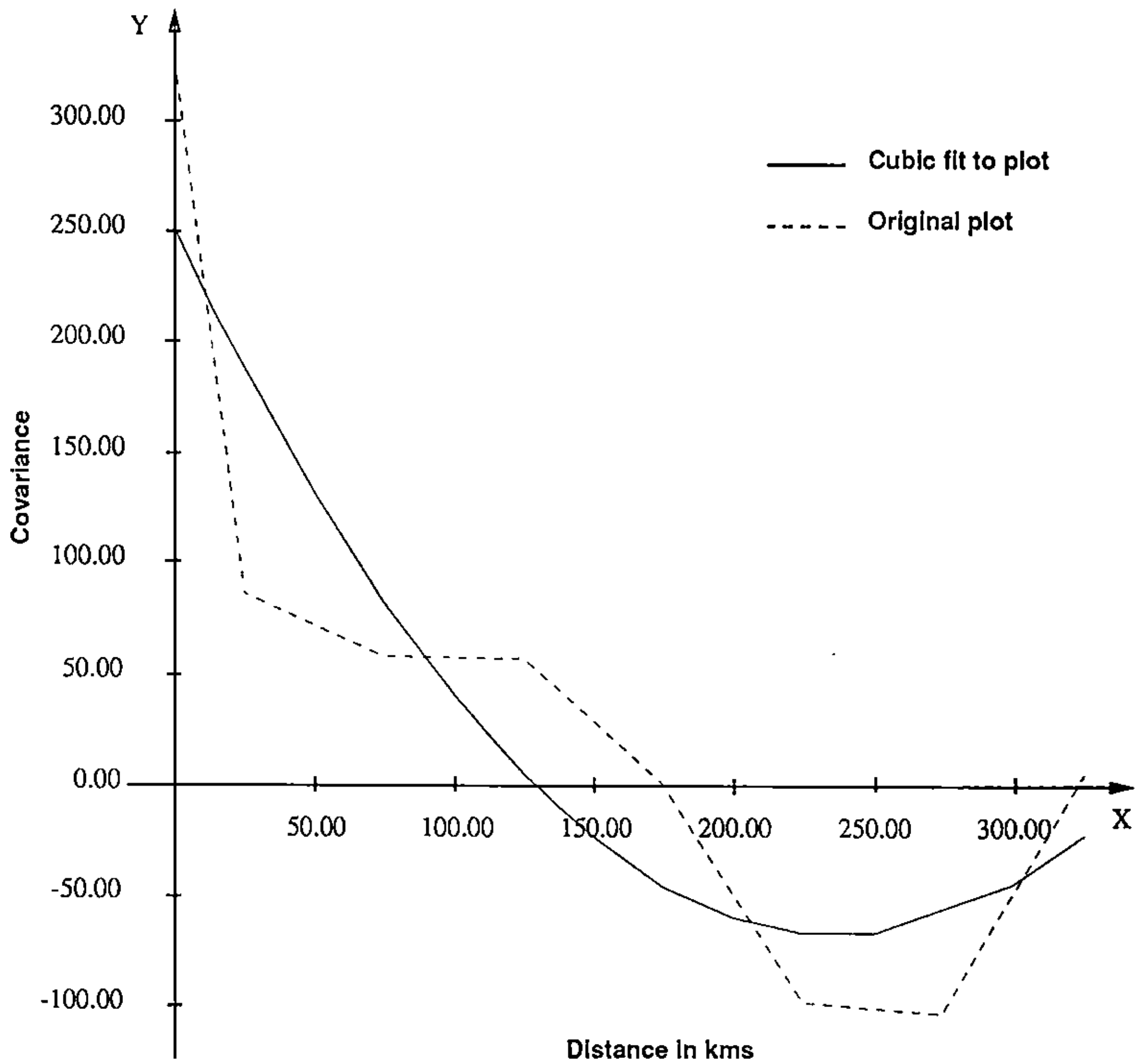

Figure 3.13: Chebychev approximation to the spatial structure of covariances for 83/07/11

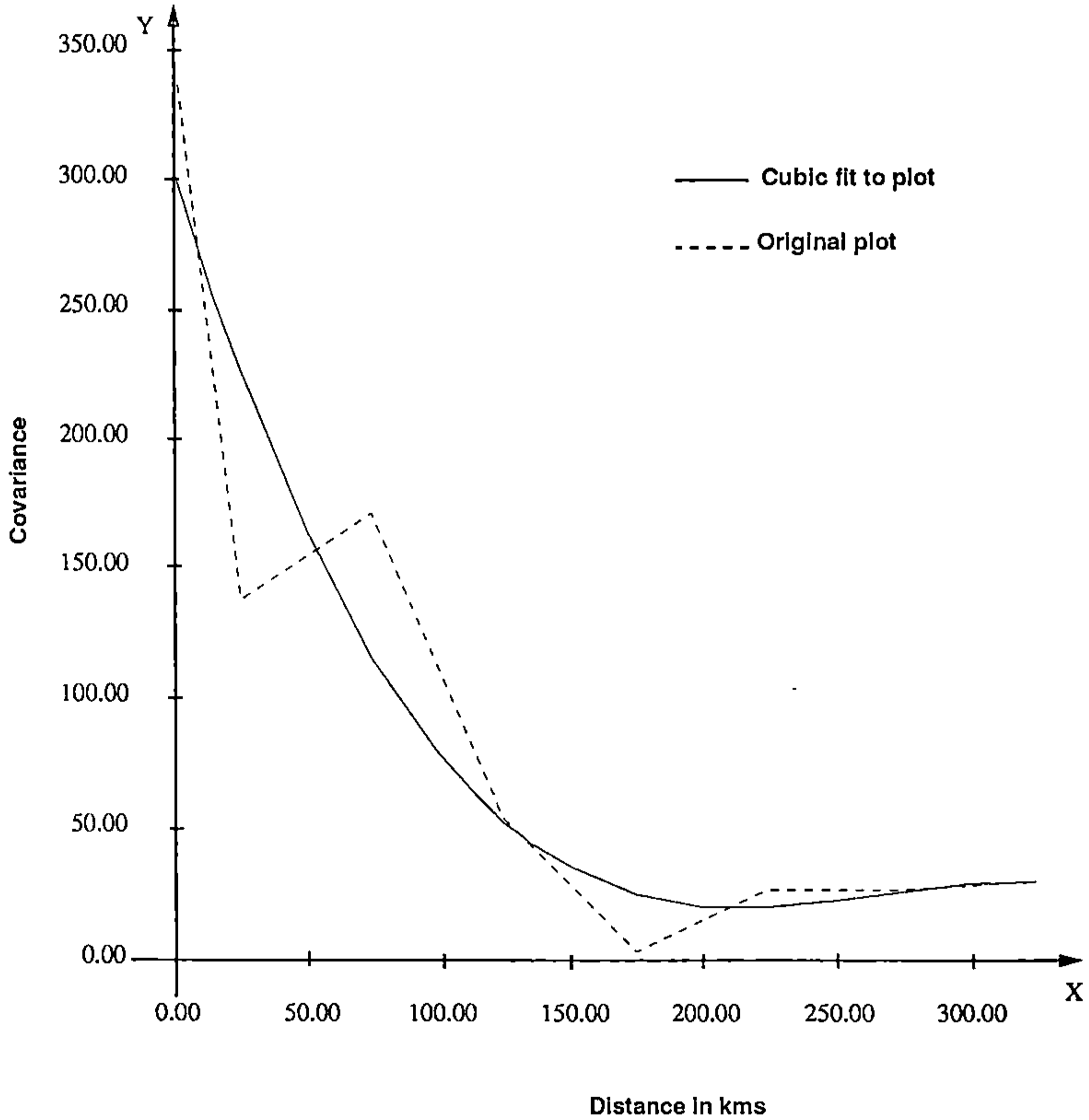

Figure 3.14: Chebychev approximation to the spatial structure of covariances for 83/ 07/12

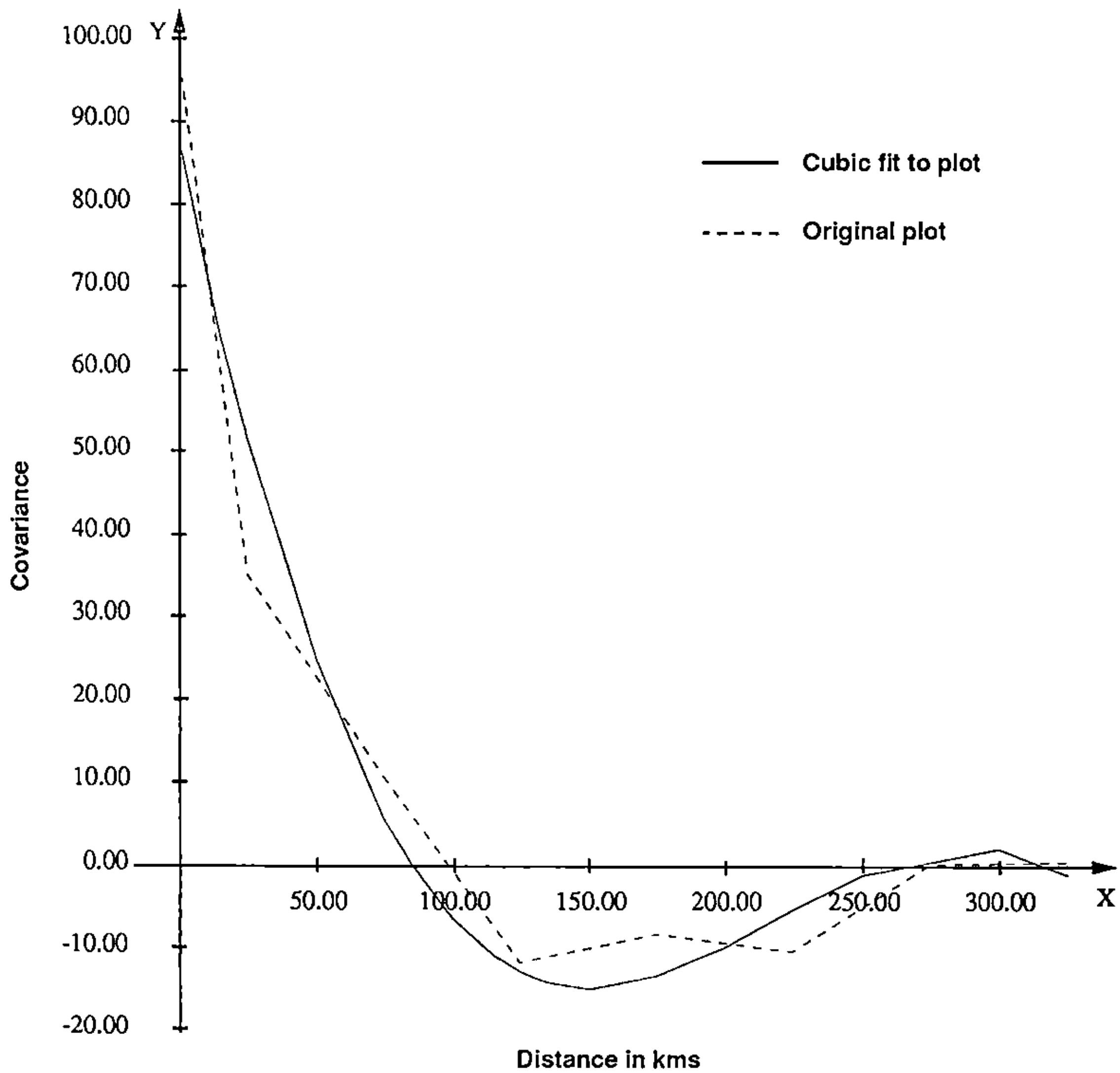

Figure 3.15: Chebychev approximation to the spatial structure of covariances for 83/07/13

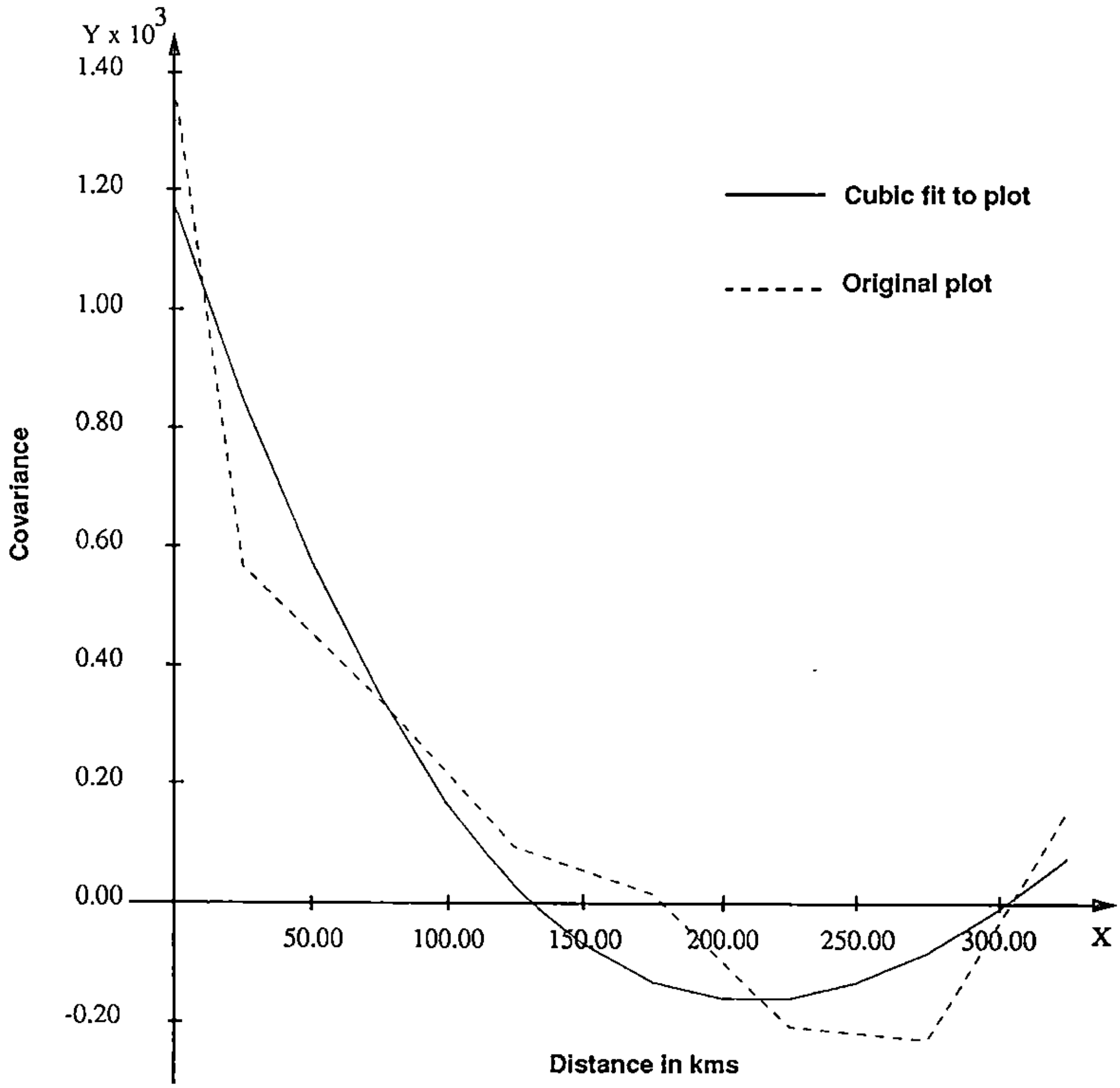

Figure 3.16: Chebychev approximation to the spatial structure of covariances from the three day total

Chapter 4

Discussion of Results

4.1 Introduction

In this chapter, we shall present the results obtained from our implementation and discuss the significance of these results. We also discuss how our model is constrained by assumptions made in our proposed solution and how justified are these assumptions. In addition, we will mention possible sources of errors that may affect the accuracy of the solution.

4.2 Analysis of Results

For each gridpoint, the linear equation (3.7) was solved. The matrix $\{\tau_{ik}\}$ was obtained from the Chebychev curve fitted to the spatial profile of τ. The right hand side vector $\{\tau_{kg}\}$ was also obtained from the fitted Chebychev curve. This system was then solved using Cholesky's method of factorization. We shall present detailed results for a few gridpoints in tables 4.1 - 4.4. The analyzed fields are given in figures 4.1 - 4.4.

Distance of observation from grid point	Weights obtained
99.26 (km)	-0.0122
93.39	0.0002
89.43	-0.0053
79.56	-0.0122
77.94	0.0006
70.59	-0.0264
66.16	-0.001
64.58	-0.0043
54.85	-0.0112
54.92	0.0024
50.52	-0.0048
43.22	-0.0462
42.37	0.0129
35.22	0.0390
22.50	0.1249
20.64	0.3595
18.07	0.5245

Trial field value	Analyzed field value	$\frac{(\sigma_g^2 - E_{MIN})}{\sigma_g^2}$ %
0.70 (mm)	3.746 (mm)	15.11 %

Table 4.1: Results for gridpoint (18,5) and observations from 83/07/11

Distance of observation from grid point	Weights obtained
98.79 (km)	0.1491
78.55	0.1049
63.96	0.3573

Trial field value	Analyzed field value	$\frac{(\sigma_g^2 - E_{MIN})}{\sigma_g^2}$ %
41.00 (mm)	26.897 (mm)	2.114 %

Table 4.2: Results for gridpoint (3,9) and observations from 83/07/12

Distance of observation from grid point	Weights obtained
97.83 (km)	-0.0289
81.71	-0.1602
69.23	-0.0603
55.69	0.1645
52.00	0.0625
45.12	0.1587
40.82	0.2909

Trial field value	Analyzed field value	$\frac{(\sigma_g^2 - E_{MIN})}{\sigma_g^2}$ %
39.00 (mm)	38.098 (mm)	2.533 %

Table 4.3: Results for gridpoint (6,11) and observations from 83/07/13

Distance of observation from grid point	Weights obtained
93.34 (km)	-0.0295
92.49	0.0138
87.84	0.0438
81.12	-0.0867
79.59	0.0261
66.89	-0.0179
53.92	0.2305
52.35	0.1127
29.89	0.2283
26.26	0.4153

Trial field value	Analyzed field value	$\frac{(\sigma_g^2 - E_{MIN})}{\sigma_g^2}$ %
92.20 (mm)	110.942 (mm)	0.563 %

Table 4.4: Results for gridpoint (7,9) and observations from the 3-day total

From tables 4.1 - 4.4, it is seen that the weights generally decrease with increasing distance of the observation from the gridpoint.We notice that the decrease in the error variance, the third column of the last row, is quite small due to data sparsity and observational errors. This raises the question of whether optimum interpolation is a good objective analysis technique. Theoretically, it is but practically, many factors influence the success of the technique. We shall discuss sources of observational errors in the next section.

Figures 4.1 - 4.4 may be compared to the observed values in figures 3.1 - 3.4 and to the trial fields in figures 3.5 - 3.8. The general patterns of the trial fields are reflected in the final analyses. The maximum amounts appear to be accurately analysed.

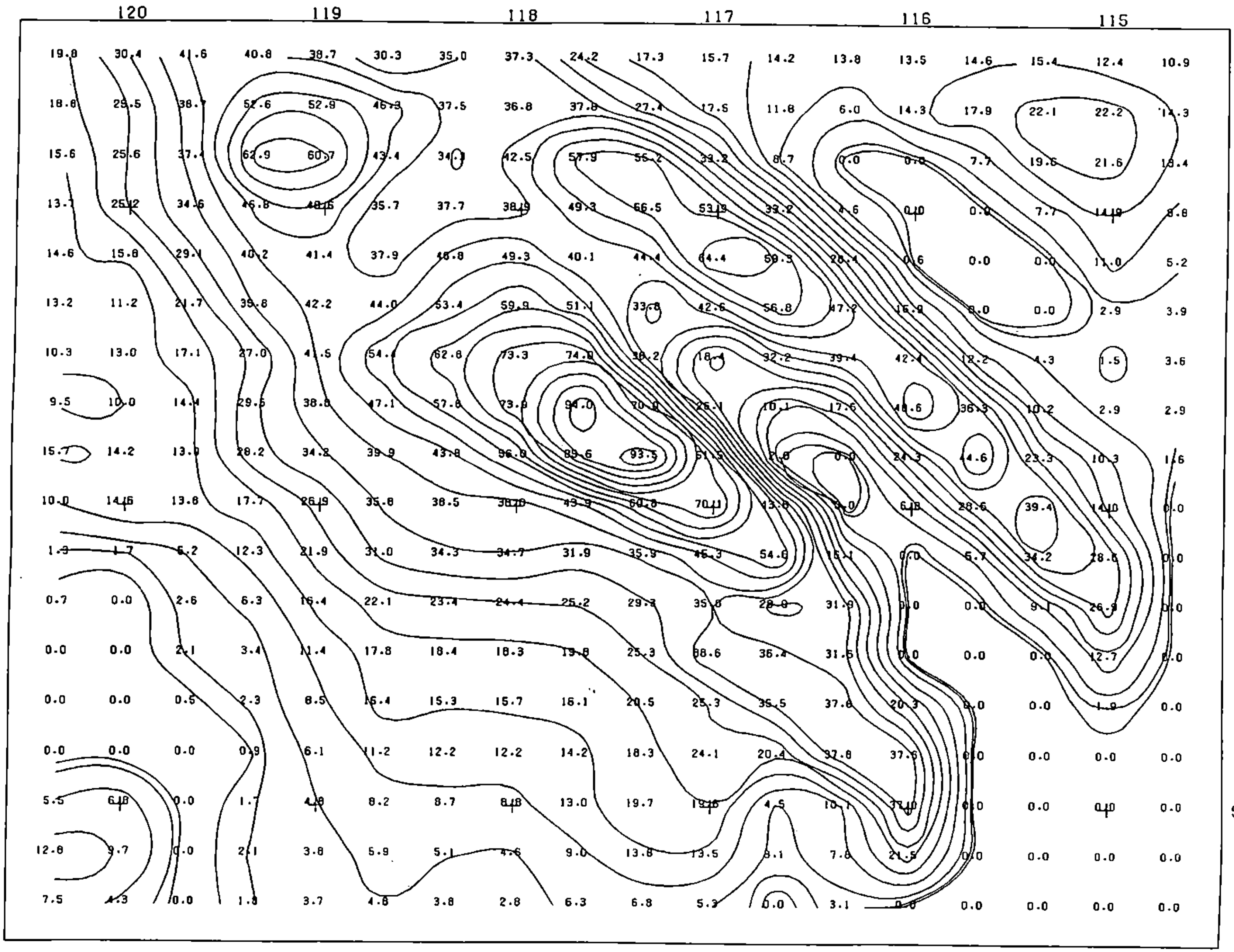

Figure 4.1: Analyzed field of precipitation amounts (mm) for 83/07/11

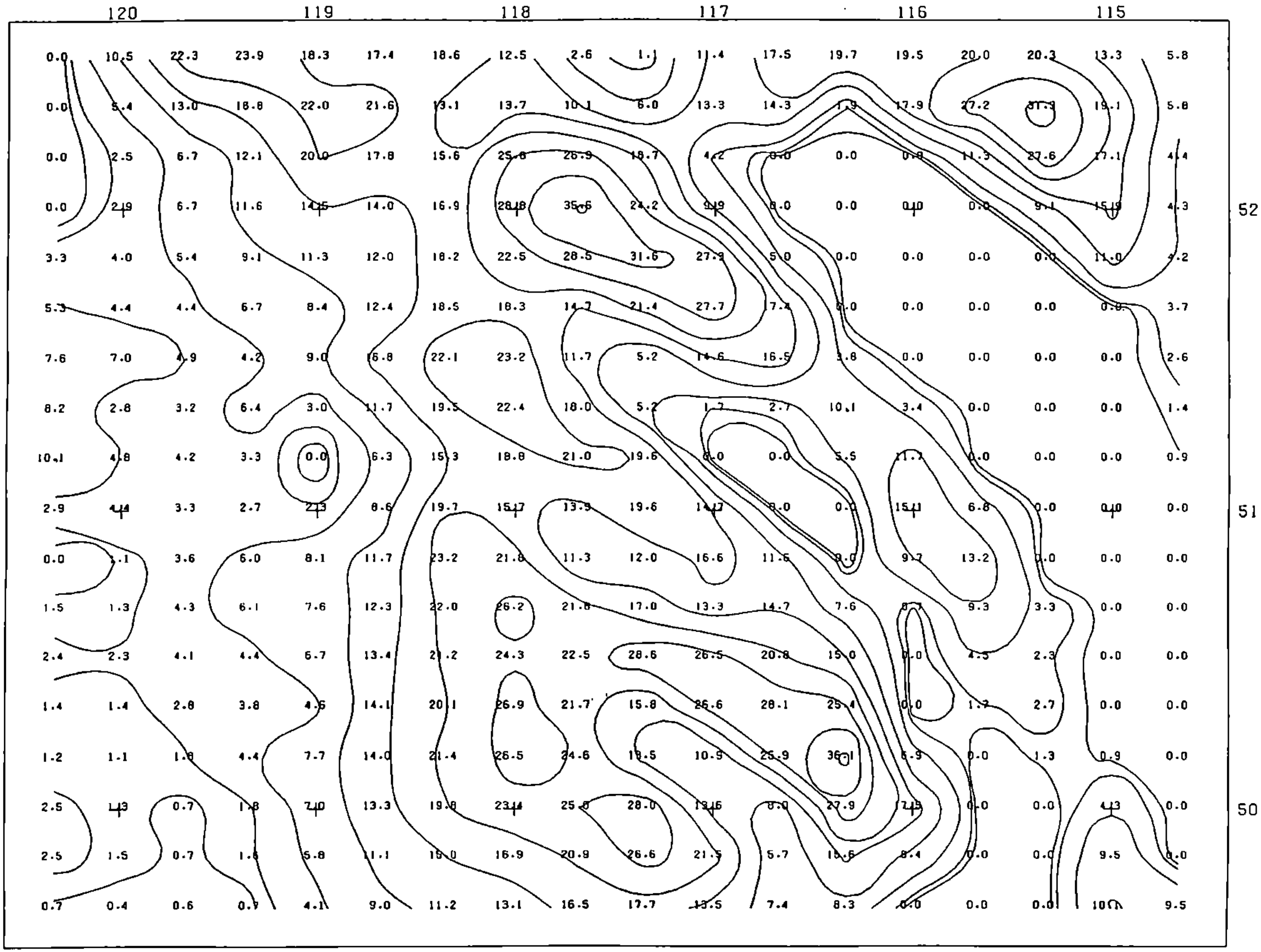

Figure 4.2: Analyzed field of precipitation amounts (mm) for 83/07/12

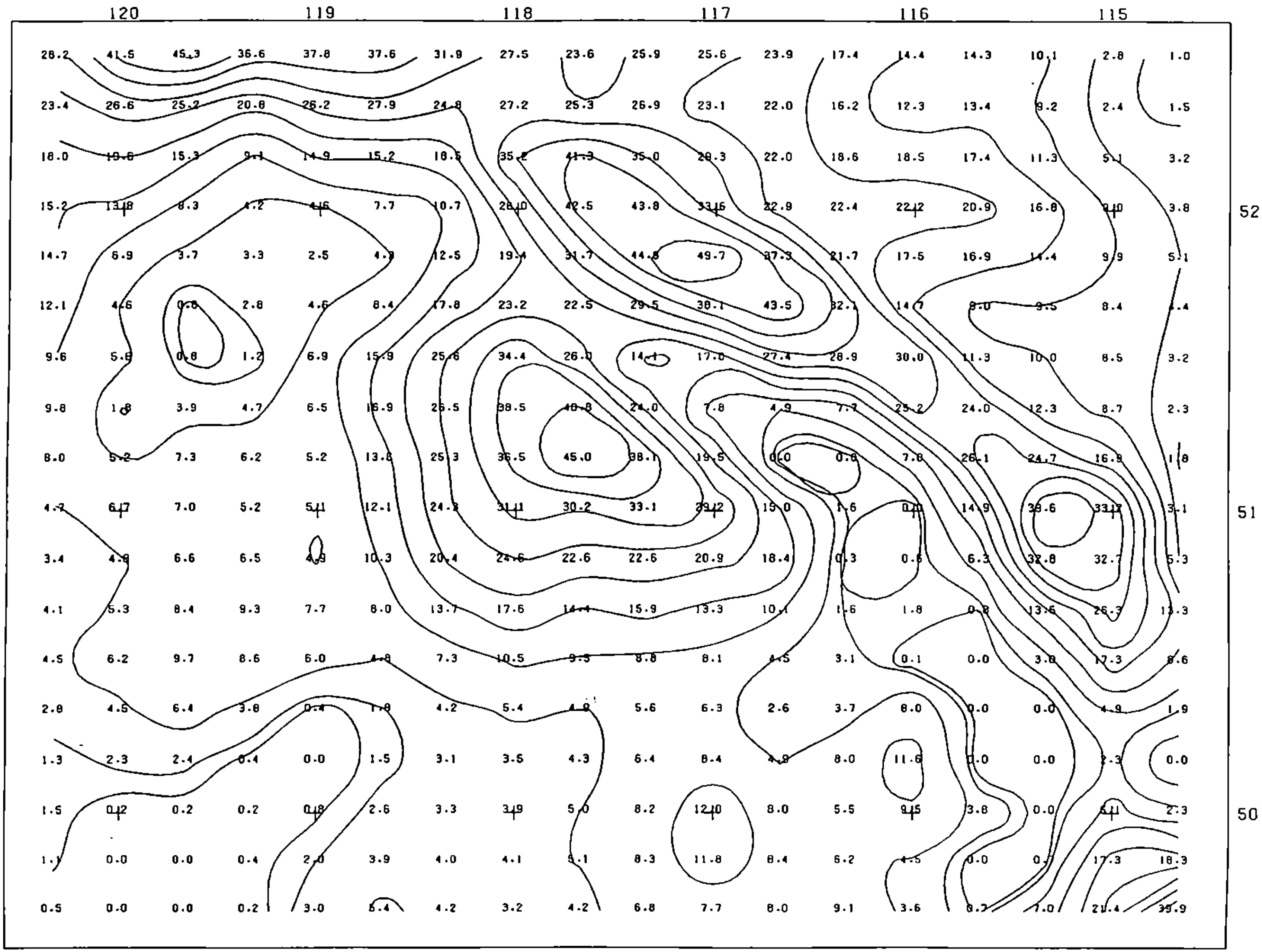

Figure 4.3: Analyzed field of precipitation amounts (mm) for 83/07/13

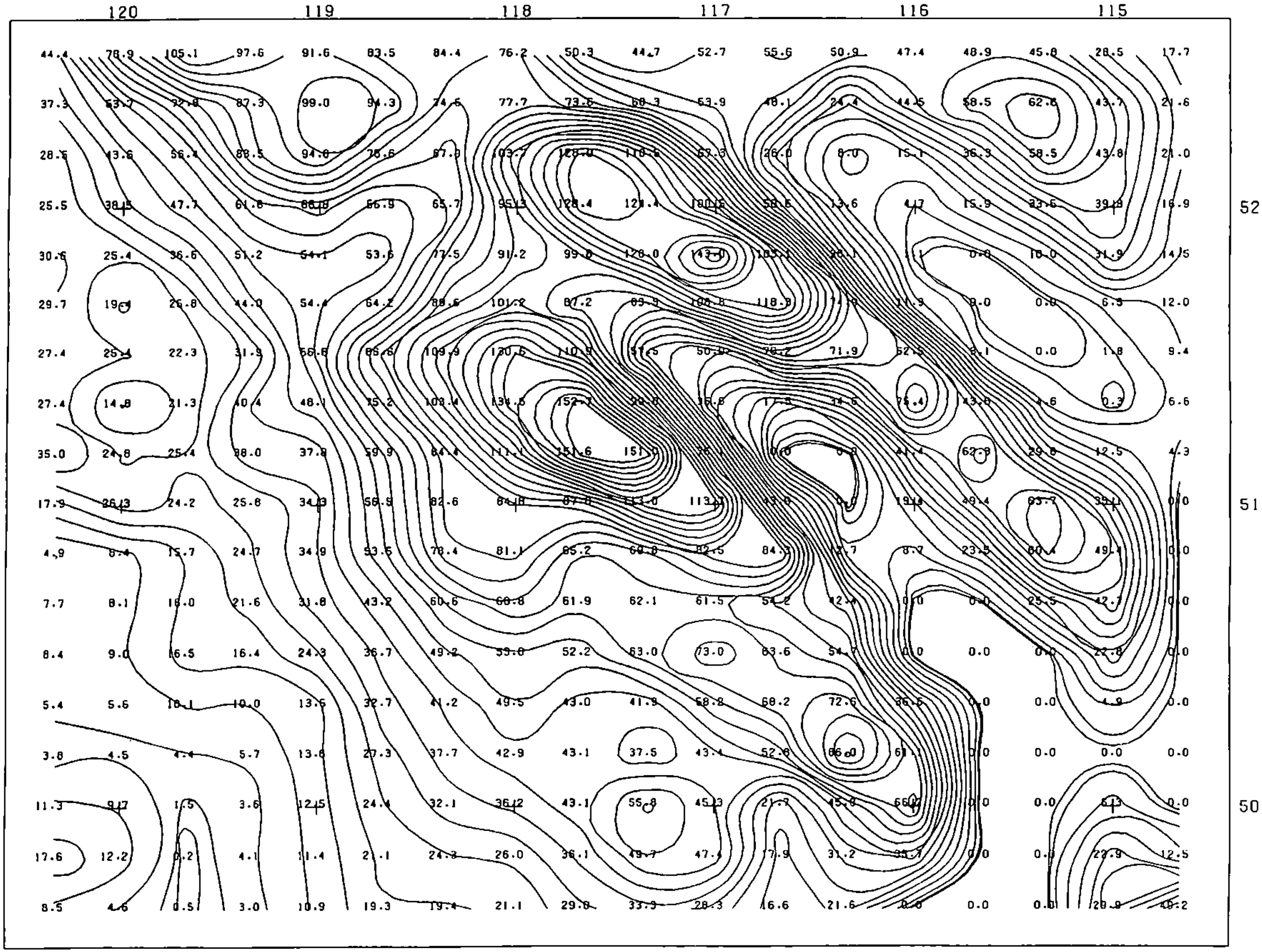

Figure 4.4: Analyzed field of precipitation amounts (mm) for the 3-day total

4.3 Sources of Errors

4.3.1 Measurement Errors

One of the major sources of errors that arise in our application of optimum interpolation is observational errors. The meteorological variable under study, precipitation, is usually measured using rain gauges. However due to a combination of factors, the observed precipitation cannot be said to be the same as the actual precipitation. This observational error can be thought of as made of three components, all of which arise from different circumstances. It should be remembered that these error components are often indistinguishable and thus, we are unable to study them individually. These three components are

1. Local errors.

2. Subgrid scale errors (Spatial errors).

3. Temporal errors.

Of the three components, the best known component has been the local error. Local errors arise due to the local aerodynamics above the mouth of the rain gauge. Ideally, the gauge would measure the amount falling on a unit area of level ground. Inclusion of a rain gauge in an area tends to affect the wind pattern over that area. In the complete absence of winds, observed precipitation approximates actual precipitation, except for some loss due to splash out . However, such a situation practically almost never arises and thus local aerodynamics tend to add noise to the measurements. Certain measures are taken so as to either try and make the wind above the gauge as horizontal as possible or to minimize the effect. Systematic errors can arise if the gauge is not level [13]. Often, physical devices such as pits, fences and shields are used to shield the gauge from the wind. In addition, large obstructions near the vicinity of the gauge also tend to change the flow of the wind near the gauge. A rule often followed is that the gauge should not be closer to the obstruction than the height of the obstruction (see [15]). However, in actual situations, such a rule may be difficult to apply. Precipitation measurements are frequently underestimated. Louie and Goodison [13] give average underestimates of 0.4 to 2.5 % for three locations in Canada.

Subgrid scale errors or spatial errors arise due to the displacement of observation sites by small amounts. Since the grid distance used is approximately 20 kms, a slight displacement of the observation sites, say by 10 m, can be considered as negligible. Any difference in observed amounts over such small distances is regarded as a local error. Nevertheless, over complex terrain, due to the nature of the topography, a displacement in the observation site, which is small compared to the grid size, often produces large discrepancies in the observations of the two different measuring sites. This spatial characteristic of the observations is often ignored when collecting data and is thus a source of noise in the data.

Temporal errors are not exactly observational errors in the conventional sense of the term. They arise mostly due to the difference in the model defined day and the actual observation day. Most models operate under GMT. Thus a model day starts at 12.00 GMT (4 a.m. PST) and ends at 12.00 GMT the next day, i.e. a duration of 24 hours. However, observations are not taken in a model day but over the climate day which usually ends on or about 8 a.m. PST. Thus the climate day usually starts and ends about 4 hours later than the model day. This can lead to systematic errors within the observations. These errors are not per se in the observations themselves but are the "difference" between observed and model predicted values. Thus the assumption that the observational errors are uncorrelated (see section 3.2) is strictly not true. It may be remarked that in the Maritime Provinces, where 12.00 GMT is 8 a.m. local standard time, the assumption of uncorrelated errors is more likely to be correct.

Because the terrain is so complex and the grid size is large, probably the greatest contribution to measurement error in this application comes from subgrid scale variations [5]. The only way to assess this effect is to have a high resolution network of accurate precipitation measurements. To simulate such a network, precipitation was abstracted for a 4×4 grid superimposed on a $15' \times 15'$ area from an analysis of observed 5-day precipitation in Colorado ([20], figure 2.22, p 45). The ratio of the standard deviation to the mean precipitation was around 0.3. The value of 0.5 assigned to r in equation (3.12) attempts to account for all three types of errors discussed above. We refer the reader to section (3.4).

4.3.2 Sensitivity of The Solution to Perturbations in Data

To determine how sensitive the solution, i.e. the weights obtained, was to the initial data, we decided to perturb the original precipitation data slightly. This perturbation was done by adding "small" random numbers to the observations. The random numbers were generated from an uniform distribution over the interval (-1.5,1.5). Clearly, this affected the spatial structure and as a result, the cubic polynomial fitted to this structure. Thus, both the matrix on the left hand side and the vector on the right hand side of equation (3.7) are affected by this small perturbation in the observations. The system was solved as in the non-perturbed case. From a comparison of the weights obtained from the two cases (tables 4.5 - 4.8), one may conclude that the weights are generally insensitive to small perturbations in the data. It may be prudent to note at this time that the perturbations added to the observations were all small. In section (4.3), we talked about the many sources of errors that arise in the observations. The order of magnitude of these errors is much greater than the perturbations we applied.

Weights from original system	Weights from perturbed system
-0.0122	-0.0121
0.0002	0.0002
-0.0053	-0.0053
0.0006	0.0006
-0.0122	-0.0121
-0.0010	-0.0010
-0.0043	-0.0043
-0.0112	-0.0112
0.0024	0.0025
-0.0048	-0.0049
-0.0462	-0.0463
0.0129	0.0130
-0.0264	-0.0262
0.0390	0.0391
0.1249	0.1249
0.3595	0.3596
0.5245	0.5247

Table 4.5: Comparison of weights obtained for gridpoint (18,5) for 83/07/11

Weights from original system	Weights from perturbed system
0.3573	0.3677
0.1049	0.1022
0.1491	0.1524

Table 4.6: Comparison of weights obtained for gridpoint (3,9) for 83/07/12

Weights from original system	Weights from perturbed system
-0.0289	-0.0114
0.0625	0.0771
0.1587	0.1642
0.2909	0.2986
0.1645	0.1720
-0.0603	-0.0535
-0.1602	-0.1551

Table 4.7: Comparison of weights obtained for gridpoint (6,11) for 83/07/13

Weights from original system	Weights from perturbed system
0.0438	0.0450
0.2305	0.2313
0.0138	0.0148
0.0261	0.0265
0.1127	0.1134
0.4153	0.4161
-0.0295	-0.0298
0.2283	0.2286
-0.0179	-0.0179
-0.0866	-0.0871

Table 4.8: Comparison of weights obtained for gridpoint (7,9) for the 3-day total

Chapter 5

Conclusions And Future Work

In this thesis, we have applied optimum interpolation to the analysis of precipitation in complex terrain. Topography is an influential factor and we have included it in the trial fields employed in our application. To the author's knowledge, the use of optimum interpolation in this specific context is unique.

Theoretically, optimum interpolation offers the most favorable solution to the problem of obtaining as much useful information as possible from observations of meteorological elements. In reality it is complex to apply and requires much more effort, both computer and human, than many other objective analysis methods. The analysis requires knowledge of covariances which in turn require true values of the meteorological elements under analysis. These are practically unobtainable. Establishing accurate covariance estimations which are so fundamental for computing weights is thus a major problem. In data dense regions (as in Western Europe), this problem is often addressed by taking the analyzed value as the true value. For data sparse regions, approximations and assumptions are necessary in order to implement such a scheme.

In our work, we make use of two assumptions in order to make optimum interpolation tractable. The first assumption, routinely made in any application of optimum interpolation,is that trial field errors and observational errors are independent of each other. The second assumption, which is specific to this thesis, states that there is no correlation between the observational errors and the deviations of trial field values from the observations. These two assumptions together imply that observational errors are uncorrelated with each other. This allows us to establish a formula for covariances that utilizes available measurements. The assumption of uncorrelated observational errors is strictly not valid when observations have systematic errors. Future work may want to look at ways to eliminate systematic errors from observations or to develop a methodology that accounts for the presence of the same.

In the estimation of the error, we assigned a value to r, our constant of proportionality. Essentially, r is a measure of the representativity of our observed measurements of the actual precipitation. Though our value of 0.50 seems like a reasonable value, nonetheless, it is an assigned value. However, it only affects the estimate of the error of the final analysis i.e. equation (1.13). It has no influence on the station weights or the final analyzed precipitation. Future work may want to look at ways of determining a more accurate value of this parameter.

The method was applied to the three historical cases of 83/07/11, 83/07/12, 83/07/13 and the three-day total. In our procedure, the ensemble average is meant to be computed as the average over many cases spread over time. In practice, since we have only limited data, we have computed our ensemble average as a spatial rather than temporal average. This should not affect the theory behind the analysis.

In the application of equation (3.7), the radius of influence was chosen so that all terms on the right hand side (i.e., all the τ_{kg}'s) are non-negative. That is, all the observations used are positively correlated with the precipitation at point g. However, some of the λ_i's turn out to be negative, which seems inconsistent to the positive covariances. This apparent paradox has not yet been adequately resolved as of this writing.

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APPENDIX 1 - Basic Statistical Concepts

In this appendix, we introduce some statistical terminology that provides some background information to the statistical interpolation method.

We denote the meteorological variable under consideration by f. The true value of the variable in point i at time t is then denoted by f_{it}. The observed value f_{it}^O can then be viewed as a small perturbation of the true value, the perturbation $\triangle f_{it}$ including any observational error. Then

$$f_{it}^O = f_{it} + \triangle f_{it}$$

The ensemble mean value is often approximated as the mean value over a large number M of repeated experiments, namely

$$\overline{f_i} = \frac{1}{M} \sum_{m=1}^{m=M} f_{it_m}$$

where t_m denotes the distinct times at which the function is evaluated.
The departures of the true values from the estimated ensemble mean are denoted by

$$\delta f_{it} = f_{it} - \bar{f}_i$$

The mean square of the deviations, the variance, measures the dispersion of the meteorological variable around its mean value. It is defined as

$$\sigma_i^2 = \overline{(\delta f_{it})^2} = \overline{(f_{it} - \bar{f}_i)^2}$$

To characterize the spatial variation of different meteorological variables we use the covariance. The covariance σ_{ij} of a variable f between points i and j is given by

$$\sigma_{ij} = \overline{\delta f_{it} \cdot \delta f_{jt}}$$

The generalized covariance σ_{ij} denotes the expected value of the product of the departures from the expected value of the function. Clearly, when both the points coincide, the covariance reduces to the variance.

When working with different meteorological variables, we generalize the covariance to the cross-covariance $\sigma_{f_i g_j}$ where

$$\sigma_{f_i g_j} = \overline{\delta f_{it} \delta g_{jt}} = \overline{(f_{it} - \bar{f}_i)(g_{jt} - \bar{g}_j)}$$

To obtain a relative measure of the spatial variation, we use the auto-correlation τ_{ij}

$$\tau_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_i \sigma_j}}$$

We can generalize the auto-correlation to the cross-correlation where

$$\tau_{f_i g_j} = \frac{\overline{(f_{it} - \overline{f_i})(g_{jt} - \overline{g_i})}}{\sqrt{\overline{(f_{it} - \overline{f_i})^2}\,\overline{(g_{jt} - \overline{g_i})^2}}}$$

APPENDIX 2 - Description of the 3-dimensional Initial Value Model

Introduction

The initial trial fields, which were used when conducting the interpolation, were obtained from a 3-dimensional initial value model. The fields were obtained from Dr. Danard, who developed this 3-dimensional model. We will give a brief description of this model, and how it generates the trial fields that are of interest to us. The following has been summarized from [3].

General Description of The Model

The model uses the σ coordinate system, in which the vertical coordinate is defined by

$$\sigma = \frac{p}{p_s}$$

The dependent variables are $u, v, T, r, p_s, \phi', \dot{\sigma}$ and ω [1]. The predicted values of these eight dependent variables are determined by integrating forward in time from an initial state, the following eight equations.

$$\begin{aligned}\frac{\partial}{\partial t}(p_s T) &= -m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s u T}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s v T}{m_x}\right)\right] - p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} T) \\ &\quad + \frac{R T_v \omega}{c_p \sigma} + \frac{p_s H_l}{c_p} + D_{hT}\end{aligned} \tag{.1}$$

$$\frac{\partial p_s}{\partial t} = -m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s u}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s v}{m_x}\right)\right] - p_s \frac{\partial \dot{\sigma}}{\partial \sigma} \tag{.2}$$

$$\begin{aligned}\frac{\partial}{\partial t}(p_s r) &= -m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s u r}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s v r}{m_x}\right)\right] - p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} r) \\ &\quad - p_s C + D_{hr}\end{aligned} \tag{.3}$$

[1] Symbols defined in the list of symbols at the end of this appendix.

$$\frac{\partial \phi'}{\partial ln\sigma} = -RT'_v \tag{.4}$$

$$\begin{aligned}
\frac{\partial}{\partial t}(p_s u) &= -m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s u^2}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s uv}{m_x}\right)\right] - p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} u) \\
&+ \left[f + \left(u\frac{m_y}{m_x}\frac{\partial m_x}{\partial Y} - v\frac{m_x}{m_y}\frac{\partial m_y}{\partial X}\right)\right] p_s v \\
&-m_x p_s \left(\frac{\partial \phi'}{\partial X} + RT'_v \frac{\partial ln p_s}{\partial X}\right) \\
&+W_x + B_x + G_x + F_{hx} + F_{vx}
\end{aligned} \tag{.5}$$

$$\begin{aligned}
\frac{\partial}{\partial t}(p_s v) &= -m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s uv}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s v^2}{m_x}\right)\right] - p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} v) \\
&- \left[f + \left(u\frac{m_y}{m_x}\frac{\partial m_x}{\partial Y} - v\frac{m_x}{m_y}\frac{\partial m_y}{\partial X}\right)\right] p_s u \\
&-m_y p_s \left(\frac{\partial \phi'}{\partial Y} + RT'_u \frac{\partial ln p_s}{\partial Y}\right) \\
&+W_y + B_y + G_y + F_{hy} + F_{vy}
\end{aligned} \tag{.6}$$

(.7)

$$\begin{aligned}
p_s \dot{\sigma} &= -(1-\sigma)\int_0^{\sigma} m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s u}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s v}{m_x}\right)\right] d\sigma \\
&+\sigma \int_{\sigma}^{1} m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s u}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s v}{m_x}\right)\right] d\sigma
\end{aligned} \tag{.8}$$

$$\begin{aligned}
\omega &= \sigma \left(u m_x \frac{\partial p_s}{\partial X} + v m_y \frac{\partial p_s}{\partial Y}\right) \\
&- \int_0^{\sigma} m_x m_y \left[\frac{\partial}{\partial X}\left(\frac{p_s u}{m_y}\right) + \frac{\partial}{\partial Y}\left(\frac{p_s v}{m_x}\right)\right] d\sigma
\end{aligned} \tag{.9}$$

Given initial values for the eight dependent variables, these eight equations are numerically integrated forward in time to obtain values at a later time. The model has seven different levels and these eight equations are applied to each and every level.

Calculating Precipitation

The model considers precipitation as made of two components, the resolvable scale precipitation P_l and the sub-grid scale precipitation P_c. Resolvable scale precipitation, during a time-step $\triangle t$, is calculated using the equation

$$P_l = \left[-\frac{p_s}{g}\int \alpha\omega \left(\frac{dr}{dp}\right)_s d\sigma\right]\triangle t \qquad (.10)$$

Both components are assumed to occur over a fraction α of the grid area defined as

$$\begin{aligned}\alpha &= \frac{h - h_0(\sigma)}{1.0 - h_0(\sigma)}, \quad & h > h_0(\sigma) \\ &= 0, & h \leq h_0(\sigma)\end{aligned} \qquad (.11)$$

where $h = r/r_s$ defines the relative humidity and h_0 is a threshold value for relative humidity for the occurrence of precipitation. In (.10), $(dr/dp)_s$ is the substantial rate of change of mixing ratio with pressure in the saturated state along a moist adiabat. The integration is done over all levels for which $\alpha > 0$ and for all upward motion, i.e. , $\omega < 0$.

Convective adjustment is used to calculate sub-grid scale precipitation. A fraction α of the grid area is said to be saturated in which the equivalent potential temperature is Θ_e where

$$\Theta_e = \left(T + \frac{Lr_s}{C_p}\right)\left(\frac{p_0}{p}\right)^{\frac{R}{C_P}} \qquad (.12)$$

The following criteria must be satisfied at all levels involved in the adjustment in order for convective overturning to take place at adjacent levels.

$$\frac{\partial \Theta_e}{\partial p} > 0 \quad (.13)$$

$$\alpha > 0 \quad (.14)$$

$$\omega < 0 \quad (.15)$$

After overturning, the final state is assumed to satisfy

$$\frac{\partial \Theta_e}{\partial p} = 0 \quad (.16)$$

$$(.17)$$

If (.13) initially holds over N adjacent levels, equation (.16) gives $(N-1)$ equations in $2N$ unknowns, the final temperature T_f and saturation mixing ratio at each of the N levels. $(N+1)$ more equations are required. N of these equations are obtained from the Clausius-Clapeyron equation applied to the N levels.

$$\delta r = \frac{0.622 r_{si} L}{R T_i^2} \frac{p}{(p - e_{si})} \delta T \quad (.18)$$

In (.18), $\delta r = r_{sf} - r_{si}$ and $\delta T = T_f - T_i$ are the changes from the initial to final states. The final equation is obtained from the relation below which holds provided motion arising from convection is dissipated as heat.

$$-\frac{C_p}{g} \int \delta T dp = \frac{L}{g} \int \delta r dp \quad (.19)$$

The integration is spread over all levels participating in the adjustment. This system of $2N$ equations and variables is then solved simultaneously. The subgrid scale precipitation resulting from the overturning is then given by

$$P_c = -\bar{\alpha} \int \delta r \frac{dp}{g} \qquad (.20)$$

where $\bar{\alpha}$ is the vertical average of α of the levels involved.

List of Symbols

σ	Vertical coordinate ($= p/p_s$)
u, v	Horizontal velocity components
$\dot{\sigma}$	Substantial derivative of σ (vertical velocity)
p_s	Surface pressure
m_X, m_Y	Map scale factors in X and Y directions
f	Coriolis parameter
ϕ'	Departure of geopotential from value in a reference atmosphere
T_v'	Departure of virtual temperature from value in a reference atmosphere
W_x, W_y	Components of term to nudge model vorticity tendencies towards observed large scale tendencies
B_x, B_y	Components of term to smooth divergence tendencies
G_x, G_y	Components of term to smooth divergence
F_{hx}, F_{hy}	Components of term representing horizontal mixing of momentum
F_{vx}, F_{vy}	Components of term representing vertical mixing of momentum
T, T_v	Temperature, virtual temperature
R	Gas constant for 1 kg of dry air
ω	Substantial derivative of pressure
C_p	Specific heat of dry air at constant pressure
H_L	Latent heat of vaporization
D_{hT}	Horizontal eddy diffusion of temperature
D_{hr}	Horizontal eddy diffusion of moisture
C	Rate of condensation of water vapor

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